Size effects in shear force design of concrete beams

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Storlekseffekter vid dimensionering för tvärkraft i betongbalkar

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Abstract

Shear failures in reinforced concrete beams are complex problems that depend on many different mechanisms, such as aggregate interlock and dowel effect. This makes the prediction of shear failures difficult as they are not strictly tied to the material and depend on other factors including the size of the beam. In particular the height of the concrete cross section plays a role in the amount of shear stress the beam can carry. This effect is appropriately named the size effect, and is not well defined for smaller concrete beams, which is why they are studied in this master thesis.

The main objective of this report is to determine if the size effect is present in smaller reinforced concrete beams as well as examining the accuracy of the Eurocode formulas when determining shear failures for smaller concrete beams. The study consists of a theoretical part where the Eurocode is studied along with the mechanisms of shear failure as well as a practical part where concrete beams were cast and tested. In the laboratory beams the conditions, aside from height, were held as constant as possible in order to limit the other factors, thus giving a more accurate representation of the size effect. Both beams with and without shear reinforcement were studied.

The experimental results showed that a size effect is present in the smaller reinforced concrete beams, both shear reinforced and unreinforced, as the stresses at failure decreased in the beams as the height increased. Because the other known factors for shear failure were kept mostly constant, the conclusion is that the size effect contributed to the variation of stress at failure for the different beams. Regarding the Eurocode formulas, differences could be seen between the calculated theoretical shear capacity and the laboratory shear capacity. The theoretical values for shear reinforced concrete beams were closer to the laboratory values than the unreinforced beams, though the reason for this is unknown. The theoretical values were always on the safe side for all the tested beams.

Sammanfattning

Tvärkraftsbrott i armerade betongbalkar är ett komplext problem som beror på olika typer av mekanismer där bland annat friktion mellan sprickytor och dymlingsverkan hos längsgående armeringsjärn spelar stor roll. Ett brott som uppkommer på grund av tvärkraft är inte strikt sammankopplat till materialets egenskaper utan beror på andra faktorer där bland annat effekter som uppkommer på grund av balktvärsnittets storlek påverkar balkens tvärkraftskapacitet. Det har visat sig att höjden på tvärsnittet hos betongbalkar spelar en betydande roll för hur stor tvärkraft som balken kan bära. Den storlekseffekt som finns är inte tydligt definierad för betongbalkar av mindre dimensioner och det är just detta som kommer att studeras i denna rapport.

Huvudsyftet med arbetet är att undersöka om det finns någon tydlig effekt som beror på tvärsnittets storlek vid tvärkraftsdimensionering av små armerade betongbalkar. Studien består av en teorietisk del där de mekanismer som bidrar till att överföra tvärkraft i en armerad betongbalk studeras tillsammans med Eurocodes beräkningsmodeller för att bestämma betongbalkens teoretiska tvärkraftskapacitet. Dessutom ingår en praktisk del där betongbalkar har gjutits och testats till brott för att sedan kunna jämföra verkliga tvärkraftskapaciteter med teoretiskt beräknade. Vid de praktiska testerna hölls alla parametrar som bidrar till tvärkraftskapaciteten konstanta förutom balkhöjden. Detta för att begränsa inflytandet av andra faktorer och bara studera effekter som beror på tvärsnittets storlek. Både balkar utan tvärkraftsarmering och med tvärkraftsarmering har studerats.

De experimentella resultaten visade att en storlekseffekt finns i mindre armerade betongbalkar, både med och utan tvärkraftsarmering, då skjuvspänningarna vid brott i balkarna blev lägre då balkhöjden ökade. Eftersom att de andra kända faktorerna som påverkar tvärkraftkapaciteten hölls konstanta är slutsatsen att storlekseffekten bidrog till de olika brottspänningarna för de olika balktyperna. De beräknade värdena för tvärkraftskapaciteten hos balkarna skilde sig mycket från de värden som uppmättes i laboratoriet. De teoretiska värdena för de tvärkraftsarmerade betongbalkarna var närmare de testade värdena jämfört med värdena för balkar som inte var tvärkraftsarmerade. De teoretiska värdena hamnade alltid på säkra sidan och balkarna klarade i alla av fallen att bära mer last i verkligheten än vad som beräknats.

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List of notations

a	shear span
A_s	longitudinal reinforcement area
A_{sw}	shear reinforcement area
b_w	beam width
c	concrete cover thickness
$C_{Rd,c}$	coefficient dependent on the loading case
d	effective beam height
$d_{reinf.}$	diameter reinforcement bar
E	modulus of elasticity
h	beam height
k	size effect factor
k_1	factor from national annex
f_{cd}	concrete compressive strength
f_{ck}	characteristic concrete compressive strength
f_{cm}	mean concrete compressive strength
f_{yk}	characteristic reinforcement yield strength
f_{ywd}	yield strength shear reinforcement
F_s	force in the reinforcement bars
L	beam length
M	bending moment capacity
n_{bar}	number of reinforcement bars
P_d	vertical load
s	spacing between stirrups
$s_{l,max}$	shear reinforcement ratio
V_{Ed}	shear load effect
V_{Rdc}	shear compression capacity
$V_{Rdc,max}$	maximum shear capacity
V_{Rds}	shear flexural capacity
x	distance to the neutral axis of the beam cross section
z	inner lever arm
α	inclination of stirrups
α_{cw}	inclination of stirrups
β	reduction factor for loads close to support
γ_c	partial factor for concrete
ϵ_{cu}	ultimate compressive strain in concrete
ϵ_s	steel strain
ϵ_{sy}	steel yield strain

θ	strut inclination
ν_1	concrete strength reduction factor
ν_{min}	minimum concrete strength
ρ_l	longitudinal reinforcement ratio
ρ_{min}	minimum shear reinforcement ratio
ρ_w	shear reinforcement ratio
σ_1	normal stress
σ_s	yield stress
ϕ	diameter longitudinal reinforcement

Contents

1 Introduction

1.1 Background

Shear failure in concrete members can be hard to predict and control due to their brittle nature, which is why they're generally avoided at all costs. The majority of shear failures in reinforced concrete (RC) members take place due to bending cracks in the concrete, making the member sensitive to large shear forces that can lead to flexural-shear failure (ASBL, 2008). If failure of a concrete member occurs due to shear force, the consequences can be very unforgiving and due to the unreliability of the calculation models the shear capacity of concrete beams should be determined carefully (SS-EN 1992-1-1:2005, 2005).

The current design models used for predicting shear capacity have been developed during the last century. They have been revised numerous times after a series of laboratory testings and the occurrence of serious structural failures as a result of the shear resistance being insufficient (Yang, 2014). The design models proposed in the Eurocodes are recommended guidelines based on a series of empirical tests, mostly executed during the mid 1900's. Kani (1966) stated the importance of investigating possible size effects in the design of shear capacity of RC members and was one of the first who identified the problem with design rules used in structural analysis. The equations are based on lab test specimens, which are not always representative of the real life specimens. Kani concluded that there is a significant difference in the behavior of actual structural members compared to laboratory test specimen (Kani, 1967).

The effect that size can have on shear capacity in both reinforced and unreinforced concrete beams is a topic that has been discussed for many years. It is commonly accepted that a large concrete beam with the same conditions as a smaller concrete beam will fail at a lower stress due to the so-called size effect. Kani showed that the shear capacity of concrete beams failing in shear-flexure is highly size-dependent and started to investigate the impact of parameters affecting the shear capacity in the 1960's (Kani, 1966). In his work, Kani concluded that the compressive strength of concrete was not of as much importance for the shear capacity as previously thought. Instead other parameters such as the amount of longitudinal reinforcement and the load placement was shown to have a larger impact on the shear capacity (Kani, 1966). Kani also concluded that the shear capacity decreases when the height of the beam increases. This conclusion was made for RC beams without shear reinforcement and Kani showed that the shear capacity of especially slender RC members without shear reinforcement decreases when the height of the beam is increased (Kani, 1966). Other than identifying a clear size effect, Tan and Lu (1999) discovered that there probably exists a critical height where the size effects on the shear resistance of RC beams is no longer obvious (Tan and Lu, 1999).

Studies were also made on RC beams with shear reinforcement by Walraven and Lehwalter (1994), showing that considerable size effects exist for shear reinforced beams as well. According to Walraven and Lehwalter (1994) the size effects in shear reinforced beams depend on the failure mechanism which can be described with strut-and-tie models.

The size effects in RC beams has since been expanded on with many more studies, but none really discussing the size effects impact on smaller sized beams. In regards to this, beams with an effective height of around $200\ mm$ will be investigated.

1.2 Purpose

The main purpose of this thesis is to analyze and evaluate the shear capacity of RC beams with smaller dimensions and the influence size effect has on them. In order to do this, RC beams of varying sizes will be tested experimentally.

In a literature study, the method proposed by Eurocode 2 to calculate shear force capacity is reviewed with regard to the size effect in RC beams. The different parameters affecting the shear capacity in the calculation models proposed by Eurocode will be evaluated and the effect of each parameter studied. The models will then be used to calculate the resistance of each of the RC beams according to Eurocode, in order to compare them to the actual loads at failure for the RC beams tested in the lab. The reason for this is to gain an understanding of how representative the size effect factor used in Eurocode shear capacity calculations pertains to reality in smaller RC beams. It is also in order to gain an understanding of how different parameters such as concrete strength, shear span-to-depth ratio and amount of reinforcement affects the shear resistance of concrete beams.

The laboratory tests will be executed on beams of various cross-sectional sizes and performed on beams with and without shear reinforcement. This is to evaluate the size effects influence on the shear capacity through a comparison of the test values, and examine the nominal stresses at failure for the different beams.

1.3 Limitations

The primary focus of this thesis will be on how the size of the RC beams cross-section affect shear resistance. Therefore, other parameters influencing the shear capacity were held as close to constant as possible throughout the experiments. The only varying parameter was the height of the beams while the width, span-to-depth ratio, amount of reinforcement and concrete strength was kept constant. High performance concrete was excluded from this thesis, due to it having different shear mechanisms from ”regular” concrete.

1.4 Outline

- Chapter 2: General shear force theory is discussed together with different kinds of shear failure modes that can occur when a RC member is subjected to shear loads. Different shear transfer mechanisms are examined and the effect that they have on the shear capacity of RC members is reviewed. The shear behavior in different regions of the beam is discussed together with how modelling of these regions usually is done with strut-and-tie modelling and truss analogy.

- Chapter 3: Review of how shear capacity is calculated according to Eurocode 2 in RC beams with and without shear reinforcement.

- Chapter 4: The different test specimens used in the analysis are presented and the different sizes, materials and other characteristics are discussed. The calculated shear capacities for the beams are presented.

- Chapter 5: The laboratory process including the test set up and measurement procedure is discussed and the results from the practical tests presented.

- Chapter 6: The calculated and tested values are summarized and compared. In addition to this, the nominal shear stresses at failure are examined.
- Chapter 7: Discussion of the results with possible sources of error and suggestions for further research.

2 Basic theory concerning shear behavior in concrete beams

In order to understand the principles surrounding shear behavior in concrete beams, some background knowledge regarding shear force and stress is described in this chapter.

2.1 Shear force

William A. Nash defines shear force as "if a plane is passed through a body, a force acting along this plane is called a shear force or shearing force" (Nash and Potter, 2010). Shear forces occur due to changes in moment forces along a cross-section of a beam and can lead to diagonal tension in the concrete and, in turn, cause cracking and ultimately failure in a beam (Bhatt et al., 2014). The failure modes of a beam are classified by the cracking patterns but are similar in nature. For example a shear crack can be initiated through a flexural crack, known as a flexural shear crack, or through shear tension particularly in the web, both of which create diagonal cracking patterns (Yang, 2014). The cracks are a result of stresses which can be described in a simply supported beam with the help of principal stresses, creating a so-called stress trajectory diagram. A stress trajectory diagram shows the compressive and tensile stresses that the beam cross-section encounters during loading, see figure 1.

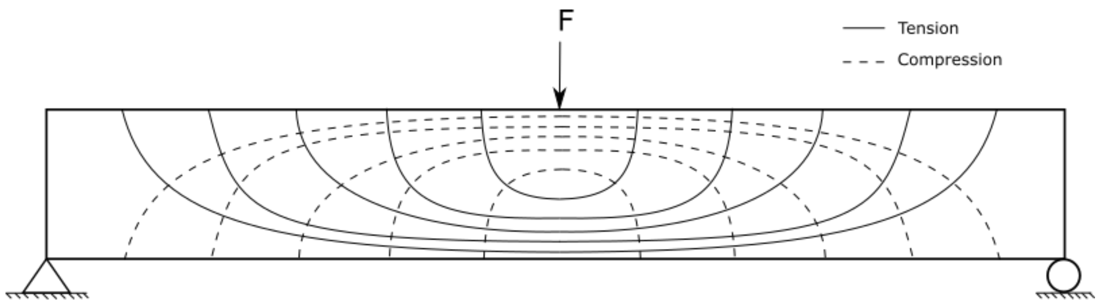


Figure 1: Stress trajectories in a simply supported beam reproduced from Gere and Goodno (2009)

If a small body from the neutral axis of the beam is studied up close, the equilibrium of principal stresses can be shown (Bhatt et al., 2014). Figure 2 shows tensile stresses σ_1 pulling on a small body in one direction with another stress, σ_2, compressing it.

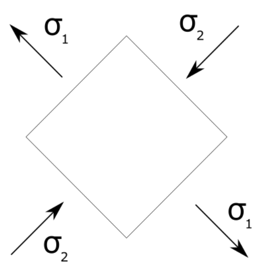


Figure 2: Principal stresses acting on a small body reproduced from Gere and Goodno (2009)

2.2 Flexural shear failure

Flexural shear failure is caused by a combination of flexural and shear stresses (Engström, 2004). The moment forces lead to the concrete cracking, usually on the tensile edge, where the cracks can be further propagated through shear stresses, see figure 3. This leads to diagonal cracking, spanning from the flexural crack to the load placement. This particular type of failure is one of the more common shear failures, as cracking in the tensile edge is very common in concrete members (Yang, 2014).

2.3 Web shear failure

Web shear failure can occur for both compression and tension in RC beams. In compression, the failure of the web can be deduced from the truss model, where the diagonal concrete struts take the compressional stresses. Failure due to compression occurs when the concretes strength is exceeded, causing the concrete to get crushed (Engström, 2004). This can happen, for example, in RC beams that have too much shear reinforcement as the steel will not yield before the crushing of the concrete.

When the web fails in tension, it can be due to a low moment- and high shear force. This causes the shear crack to start near the middle of the web, where the shear stress is highest in most common beams, and propagate diagonally. An example of a beam that can have such failures is an I-beam with a thin web in comparison to its tensile edge (Engström, 2004). Examples of the different types of cracks is shown in figure 3.

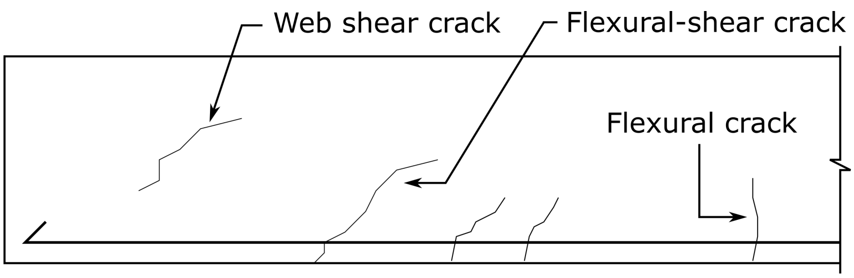


Figure 3: Flexural and web cracks reproduced from Engström (2004)

2.4 Shear transfer mechanisms

In reinforced concrete members without shear reinforcement there are different shear transfer mechanisms that will take part in carrying the shear forces in the concrete. When flexural cracks have developed in a reinforced concrete member there are certain processes that can transfer shear in the concrete, see figure 4. The mechanisms that transfer shear through a RC beam are (Yang, 2014)

- Shear stresses in the uncracked compressive zone
- Aggregate interlock along the cracks
- Dowel action in the bars
- Residual tensile stresses transmitted directly across cracks
- Arch action

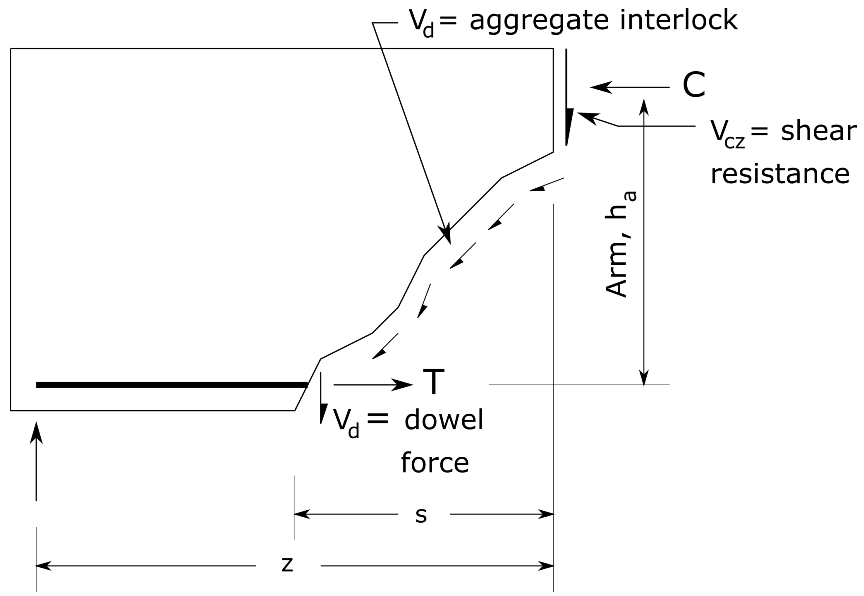


Figure 4: Mechanisms behind shear failure reproduced from Yang (2014)

2.4.1 Shear stresses in the uncracked compressive zone

A certain amount of shear force is taken by the compressive zone of the concrete after flexural cracks have been formed. Since the concrete is uncracked, failure is due to a combination of shear and compressive stresses. This means that the shear force can be represented by the compressive strength of the concrete and the longitudinal steel ratio (Kim and Park, 1996).

2.4.2 Dowel effect

The effect of the longitudinal reinforcement in RC beams has an impact on transferring shear force in the member. When there is a crack in the concrete and the two crack surfaces move in opposite directions, the longitudinal reinforcement bars bend. This creates a resistance between the two separate concrete parts on each side of the open crack, see figure 5 (Engström, 2004).

The steel reinforcement bars are able to transfer shear forces perpendicular to the beam axis when bent. This means that there will be a contribution to the shear resistance in the vertical direction due to the dowel effect (Walraven, 1980).

The size of the dowel effect is proportional to the amount and strength of reinforcement bars as well as their placement in the cross section of the beam. The dowel effect is affected by the distance between the cracks in the concrete along with the thickness of the concrete cover around the reinforcement bars. Due to this, the dowel action is also dependant on the concrete strength in the RC member (Kim and Park, 1996). As well as this, the dowel effect is also influenced by shear reinforcement which increases the effectiveness of the dowel effect (Mårtensson et al., 2005).

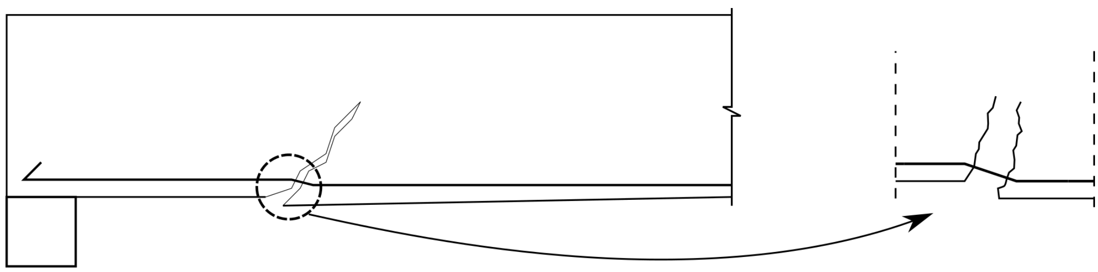


Figure 5: Dowel effect reproduced from Engström (2004)

2.4.3 Aggregate interlock

When concrete members are cracked, the crack surfaces are not flat and even. The cracks are rough and have irregular shapes due to the different sizes of aggregate particles in the concrete. When the cracks are held together by the longitudinal reinforcement, there will be contact between the two crack surfaces. The interaction between the surfaces will create friction and contribute to the shear resistance of the concrete (Engström, 2004). The importance of aggregate interlock was shown by Fenwick and Pauley (1968), who made an investigation of the shear capacity of concrete beams with crack surfaces of varying roughness by comparing beams with smooth and plane cracks to beams with naturally developed cracks. Fenwick and Pauley (1968) concluded that there was a significant impact on the shear resistance of concrete beams from the aggregate interlock mechanism. When the aggregate interlock has developed, the connection between the crack surfaces takes a substantial part in the transmission of shear force through the concrete member, see figure 6 (Bažant and Gambarova, 1980). The mechanism controlling the shear transfer due to aggregate interlock is heavily influenced by the roughness of the two crack surfaces which vary with the size of the aggregates used in the concrete mix (Walraven, 1981).

The interaction between the concrete surfaces, creating the aggregate interlock, is developed due to the strength of the cement being considerably lower than that of the aggregate particles. The parts of the concrete that consist of cement fail and cracks propagate around the aggregate particles. This causes varying crack surfaces due to different aggregate sizes making the surfaces rough and uneven. The two surfaces of the crack are then able to interact and create friction between the aggregate particles on the two surfaces. (Walraven, 1980). In high performance concrete, HPC, the cement can be stronger than the aggregate. This causes the concrete to fail through splitting of the aggregate particles, creating flatter crack surfaces than in normal concrete. The flatter surfaces reduce the amount of friction between rough particles, reducing aggregate interlock (Sagaseta, 2013).

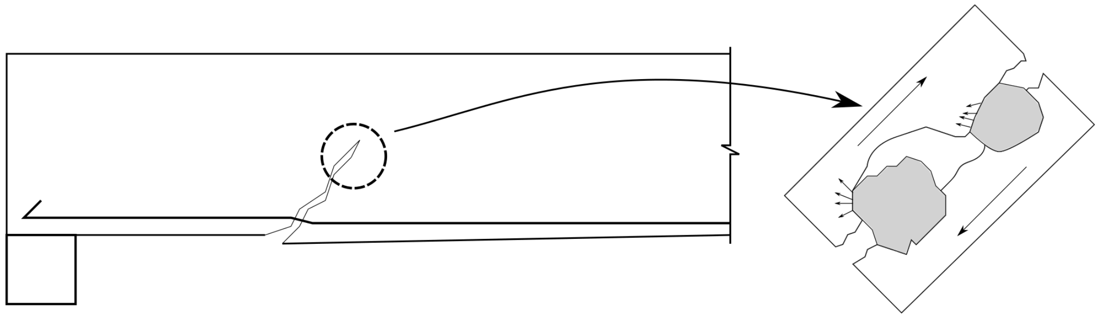


Figure 6: Aggregate Interlock reproduced from Walraven (1980)

Walraven and Lehwalter (1994) examined the impact of aggregate size in RC members. Their studies showed that there were no significant effects on aggregate interlock connected to size of the beam when the aggregate particles were between 8 and 32 mm in diameter.

When cracks develop in concrete members the longitudinal reinforcement will affect the width of the cracks. A RC beam with a large longitudinal reinforcement ratio keeps the cracks in the concrete thin and narrow as the longitudinal reinforcement helps to hold the concrete together. An increase in the reinforcement ratio will then increase the shear capacity due to increased friction and aggregate interlock between two crack surfaces in the concrete beam (Ismail, 2016). Ismail (2016) also showed that the effect of increasing the longitudinal reinforcement ratio is more significant in beams without shear reinforcement. The same principle applies for shear reinforced concrete beams, as the concrete is held together vertically by the reinforcement (Mårtensson et al., 2005).

2.4.4 Residual tensile strength

Concrete has the ability to carry a certain amount of tensile stress after cracking has occurred. This capacity is only relevant at small crack widths, with dimensions around 0.1mm, allowing tensile ties to be formed across the cracks (Ruiz et al., 2015).

2.4.5 Arch action

In RC beams where the shear span is small, or the length of the beam is short, shear force can be resisted by arch action. Arch action can be a result of having concentrated loads close to a support where the load is transferred directly to the support through an inclined strut. When the arch action is developed, strut-and-tie models are often used for designing the member and predicting the shear capacity. The arch action depends on the inclination of the strut which is determined by the inclination of the cracks in the concrete and is mostly developed in beams with short shear spans. As the shear span gets larger, the inclination of the compressive strut flattens out, making it harder for the shear force and arching action to be transferred. When this occurs, vertical ties can be used to connect the struts together, creating a truss system, see figure 7. This is accomplished by having vertical shear reinforcement stirrups in the beam (Engström, 2004).

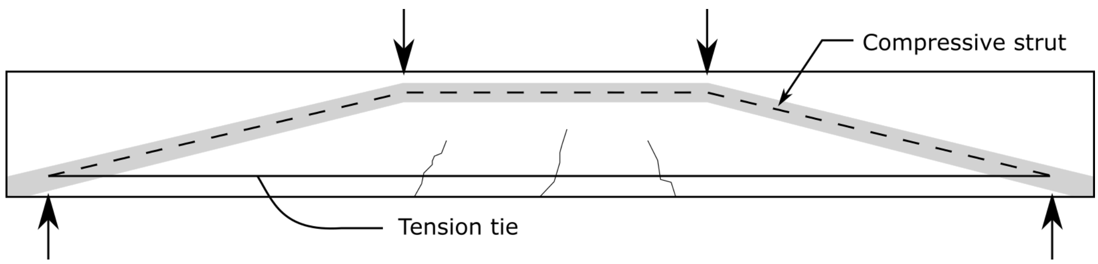


Figure 7: Arch action reproduced from Ramirez et al. (1999)

The arch action in RC beams without shear reinforcement is affected mostly by the length of the shear span and the strength of the strut which is determined by the compressive strength of the concrete and the amount of longitudinal reinforcement in the area of the cross-section of the beam (Kim and Park, 1996).

In his thesis, Ismail (2016) showed that the compressive strength of the concrete in RC beams is connected to the shear capacity, especially in beams with small shear spans. This means that if the shear span-to-depth ratio is decreased due to increased beam height, the concrete strength will have a greater effect on the shear resistance (Ismail, 2016).

2.5 Shear behavior in discontinuity regions

2.5.1 D- and B-regions

The behavior of RC beams subjected to shear forces differ between two region types, meaning that the influence of the different shear transfer mechanisms also differs between the regions. It's therefore important to distinguish between D- and B-regions (Kuo et al., 2010). In a B-region of a beam, the Euler-Bernoulli beam theory hypothesis is applicable where the stresses and strains are assumed to be linearly distributed over the cross section of the beam. The Euler-Bernoulli theory is a simplification of the linear theory of elasticity where plane sections remain plane. D-regions are disturbed or discontinuity regions where the Euler-Bernoulli elasticity simplification of plane sections remaining plane is no longer valid. The stresses and strains in a D-region are nonlinear and often occur close to where a concentrated load is applied, creating a so called static discontinuity in the beam, see figure 8. D-regions also occur if the geometry changes abruptly i.e. a geometric discontinuity. The sectioning of a RC member depends on the geometry of the beam and what kind of load case that is applied. When designing a concrete member with D-regions, a strut-and-tie model can be used to represent the stress fields in the regions (Engström, 2011).

Kuo et al. (2010) investigated the shear behaviour of RC beams by classifying different beam types based on the distribution of D- and B-regions in the beam. One beam class was defined as deep beams, consisting of a single D-region. To analyze the shear behavior in this type of beam, a strut-and-tie model is usually adopted. The most common failure mode of deep beams is the crushing of the compressional struts, also known as shear-compression failure. The second class that Kuo et al. (2010) mentioned was slender beams which are beams of three sections where a B-region is located in-between two D-regions at the ends of the beam. In slender beams two different failure modes can occur, shear-compression failure and shear-tension failure. A slender beam only consisting of D-regions is possible when the shear reinforcement in the beam is insufficient and there is therefore no B-region in the beam. The last beam class is short beams where usually two overlapping D-regions appear and the common failure mode is shear-tension (Kuo et al., 2010).

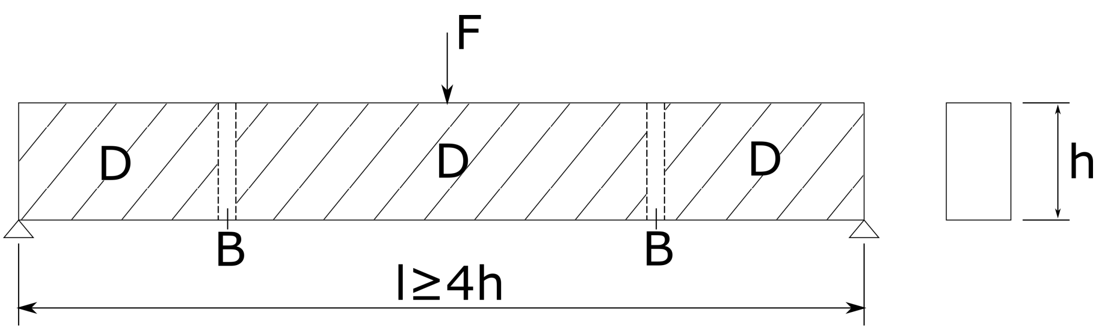


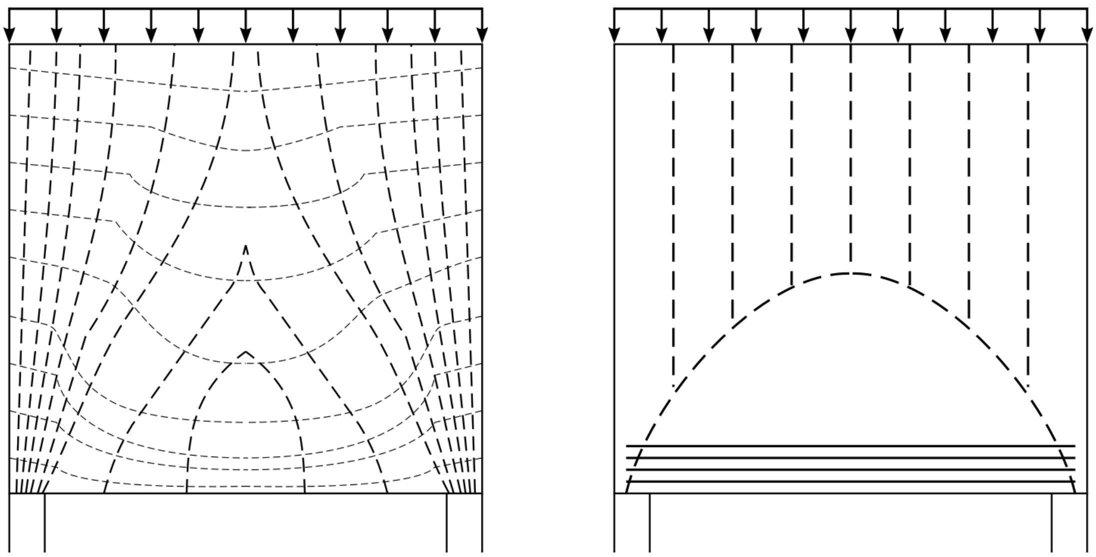
Figure 8: B- and D- regions (Nagarajan and Madhavan Pillai, 2008)

2.5.2 Strut-and-tie method

The strut-and-tie model is based on the theory of plasticity and can be used when designing D-regions in concrete structures. When the RC member is subjected to loads, cracks will develop and the area between two inclined cracks will act as a compressive strut in the concrete. Struts and ties will then represent the internal forces and stresses in the concrete and by connecting the struts and ties in nodes, a truss analogy can be used to describe the concrete member (Engström, 2004).

As mentioned earlier, when a concentrated force is applied close to a support the arch action will develop as the shear force is transferred directly to the support. There is a static discontinuity region, D-region, in this part of the beam. For these regions the strength of the beam is controlled mostly by shear, and strut-and-tie models are used to simplify the shear transfer in RC beams without shear reinforcement (Ismail, 2016).

Figure 9 shows a deep beam with an evenly distributed load where the load makes its way from the top of the beam and downwards towards the two supports. Due to the uneven stress distribution down to the supports the region is referred to as a discontinuous or disturbed region. The stress fields can be simplified, highlighting the compressed and tensile regions of the beam, as shown in 9b. This simplified model is then idealized and transformed into a so called strut-and tie model where the dotted lines represent compression, and the fully drawn lines represent tension, see figure 10.



(a) Stress fields *(b) Simplification of stress fields*

Figure 9: Stress field of a deep beam reproduced from Engström (2011)

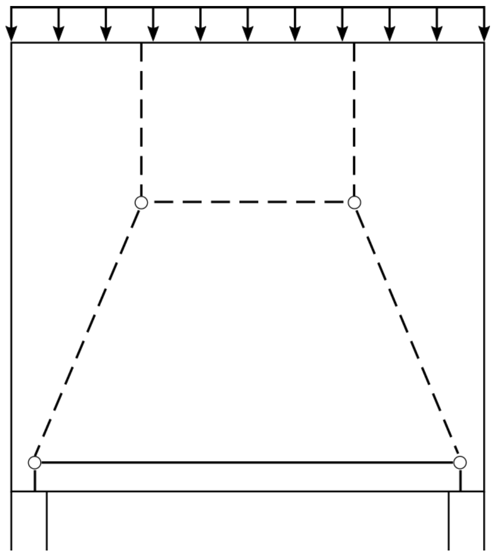


Figure 10: Strut and tie idealization reproduced from Engström (2011)

2.5.3 Truss analogy

When a RC beam is reinforced with shear reinforcement, the vertical stirrups together with inclined struts in a strut-and-tie model can be represented by a truss. This truss analogy can be used when designing shear reinforced beams with the struts and ties being designed to resist the internal shear forces in the RC member. (Engström, 2004)

A truss model is often used when several inclined cracks are developed in the concrete due to combined bending and shear forces. More than one inclined compression strut is then needed to represent the shear transfer in the member. When several struts are used in a strut-and-tie model, tension ties are used to connect the compressive struts to each other, establishing a truss model, see figure 11 (Kim and Kim, 2008).

For deep beams arch action plays an active role in the transfer of forces, with the shear resistance in the D-region often being modelled with strut-and-tie methods. When the RC beam is of a more slender nature, more truss action will be developed compared to deep beams. This is because flatter compressive struts are present and tension ties are needed to connect the struts to each other to establish a model of the shear transfer in the beam. (Ismail, 2016)

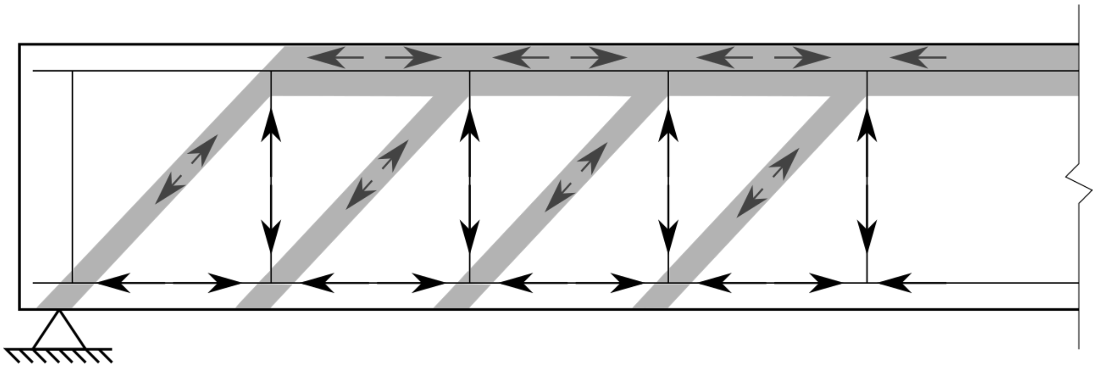


Figure 11: Truss action reproduced from Ismail (2016)

3 Shear capacity according to Eurocode 2

In this chapter the design models used for calculating shear capacity of concrete members is reviewed and analyzed. There are several models and codes used around the globe to design these kinds of structural members, of which the Eurocode (EC) was used in this particular thesis.

3.1 Shear capacity of RC beams without shear reinforcement

3.1.1 Shear flexural failure

In order to derive the Eurocodes equation for calculating shear capacity in a RC beam without shear reinforcement empirical methods were utilized. First of all the most important factors affecting the shear capacity were evaluated and put together into an equation. The equation was then calibrated to fit shear test results from a data collection based on a number of concrete specimen. This was done by adding the factor C. With a regression analysis, the coefficient could be approximated so that the shear capacity curve fit the test results (Yang, 2014). In 1995, König and Fisher evaluated 176 concrete specimen of various sizes and strengths and estimated the coefficient to $C = 0.12$ as the lower boundary of the curve. The approximation of the coefficient has then been analyzed, with different codes using different empirical methods and data collections to fit their own shear capacity curves with different regression factors. Later on the partial safety factor, γ_c, for concrete was introduced in the Eurocode to take different load situations into account when approximating the coefficient C (ASBL, 2008).

Equation (1) is the empirical expression used and is a recommendation for determining the shear-flexural capacity in RC members. The expression takes into account the different factors discussed previously which can contribute to shear capacity. The shear capacity of reinforced concrete beams without shear reinforcement is according to Eurocode 2 determined by

$$V_{Rd,c} = (C \cdot k \sqrt[3]{100 \rho_l f_{ck}})\, b_w d \tag{1}$$

where

$$
\begin{aligned}
k &= 1 + \sqrt{\frac{200}{d}} = \text{size factor} \\
\rho_l &= \text{longitudinal reinforcement ratio} \\
f_{ck} &= \text{concrete strength} \\
C &= \frac{0.18}{\gamma_c} = \text{coefficient dependent on the loading case} \\
d &= \text{effective beam height} \\
b_w &= \text{beam width}
\end{aligned}
$$

The ratio of longitudinal reinforcement is determined as

$$\rho_l = \frac{A_s}{b_w d} \tag{2}$$

where A_s is the cross sectional area of the reinforcement bars in the beam.

Equation (1) takes the longitudinal reinforcement into account. The lower limit of the shear capacity in reinforced concrete beams is given by equation (3) where only the concrete strength contributes to the shear resistance. The lower limit of the shear capacity is

$$V_{Rd,c} = v_{min} \cdot b_w d \tag{3}$$

where the parameter v_{min} depends on concrete strength and the effective height of the beam as shown in equation (4)

$$v_{min} = 0.035\sqrt{k^3 f_{ck}} \tag{4}$$

3.1.2 Web shear failure

Web shear failure in non shear reinforced beams occurs when the concrete struts between cracks in the web of the concrete are crushed. To eliminate the risk of web compressive failure, the load effect V_{Ed} should, according to EC2, fulfill

$$V_{Ed} \leq 0.5\,\nu_1 f_{cd} b_w d \tag{5}$$

where ν_1 is a reduction factor for concrete with shear cracks calculated with

$$\nu_1 = 0.6(1 - \frac{f_{ck}}{250}) \tag{6}$$

3.2 Shear capacity of RC beams with shear reinforcement

If the shear force acting on the concrete beam, V_{Ed} is larger than the calculated capacity, $V_{Rd,c}$, the shear capacity is insufficient with respect to shear failure and shear reinforcement is required.

3.2.1 Shear flexural failure

According to SS-EN 1992-1-1:2005 (2005), shear capacity in shear-reinforced concrete beams can be calculated using

$$V_{Rd,s} = A_{sw} f_{ywd} \frac{0.9d(\cot\theta + \cot\alpha)\sin\alpha}{s} \tag{7}$$

where s is the distance between the reinforcing stirrups limited to $s < 0.75d$ and the parameter α is the angle between the stirrups and the beam axis. It is common to use vertical stirrups, $\alpha = 90°$, as they are generally easier to handle than angled stirrups. The expression for shear capacity of the beam can then be written

$$V_{Rd,s} = A_{sw} f_{ywd} \frac{z\cot\theta}{s} \tag{8}$$

z $= 0.9d =$ inner lever-arm

s = spacing between the vertical stirrups

f_{ywd} = yield strength of the shear reinforcement

A_{sw} = area of shear reinforcement

θ = inclination of concrete strut

The shear reinforcement area, A_{sw}, is the cross sectional area of the two vertical steel bars in the stirrup that span a crack in the concrete.

3.2.2 Web compressive strut failure

For a compressive strut in a shear reinforced beam, the capacity of the concrete is

$$V_{Rdc,max} = b_w z f_{cd} \frac{(\cot\theta + \cot\alpha)\nu_1}{1 + \cot\theta^2} \tag{9}$$

and the capacity of a compressive strut with shear reinforcement at $90°$ is

$$V_{Rdc,max} = b_w z f_{cd} \frac{(\cot\theta)\nu_1}{1 + \cot\theta^2} \tag{10}$$

b_w = beam width
z $= 0.9d =$ inner lever-arm
f_{cd} = concrete compressive strength
θ = inclination of concrete strut
α = inclination of shear reinforcement
ν_1 = reduction factor for concrete with shear cracks

The inclination of a crack is defined by Engström (2004) as the angle between a compressive strut and the beam axis perpendicular to the concentrated shear force acting on the beam. The value of θ is chosen by the designer and is usually in the range so that $1 < \cot\theta < 2.5$ which means that the corresponding inclination, θ, is between 21.8 and 45 degrees. If a lower value of the angle is chosen by the designer, the amount of shear reinforcement needed is reduced but instead the risk for failure in the compressional strut is increased. A smaller value of the crack inclination will also increase the amount of longitudinal reinforcement in the tension zone of the beam (Engström, 2004).

3.3 Concentrated load close to support

When a concrete beam is subjected to a concentrated load close to the support the shear strength is increased due to arch action. The arch action, as discussed in section 2.4.5, enables the load to be directly transferred to the support through a compressive strut which is restrained by a tensile tie (Engström, 2004). Due to this, the shear capacity can be increased by a factor of $\beta = 2d/a$ in the Eurocode, granted the load is placed within $2d$ from the support, under the assumption that the longitudinal reinforcement is sufficiently anchored (SS-EN 1992-1-1:2005, 2005). The effect of arch action can differ between different load cases and conditions. This was, according to ASBL (2008) shown by Regan, who investigated the reduction factor by executing tests to determine appropriate factors to use for different conditions. Regan concluded that there should be different factors for simply supported beams and continuous beams and if the load is concentrated or distributed over the beam (ASBL, 2008).

4 Beam design

4.1 Beams

In order to compare the size effect for smaller dimension beams, four different cross-section sizes were chosen. These different beams were then divided up, one without shear reinforcement and one with shear reinforcement for each cross-section. This brings the total to eight, however, in order to gain more reliable results a control beam of all the original eight beams was also made, making a total of 16 beams. The different beams are summarized in table 1 and the form building and casting procedure can be seen in Appendix B.

Table 1: Beam classification

Beam	Width [mm]	Height [mm]	Length [mm]
A	160	200	1500
B	160	250	1500
C	160	300	1500
D	100	150	800

4.2 Materials

4.2.1 Concrete

The concrete used for the tests had a water-cement ratio (wcr) of 0.55. The exact concrete specifications are shown in table 2. The wcr was chosen for practical reasons; while the cube strength was verified by testing cubes cast in conjunction to the beams with the same concrete. The cube strength of the concrete is shown in table 3 and 4. The cement used was a portland ”bascement” which is a standard portland fly-ash cement mix. The stone aggregates used were of three different sizes.

Table 2: Concrete specifications

Material	Weight $[kg/m^3]$
Cement	380
Water	209
0-2 mm	860
8-12 mm	430
12-16 mm	430

The concrete for beams $C1$, $C2$ and $D1$ was mixed in the laboratory and the cube strength of the concrete is presented in table 3. The plan was to cast all the concrete beams in the laboratory, but due to time restraints a concrete truck had to be bought in to help cast the rest of the beams.

Table 3: Beam C1,C2 and D1 concrete: strength from cube tests

Cube	Strength [MPa]
1	41.33
2	34.67
Mean	38.00

The concrete used for the rest of the beams was delivered to the laboratory by a concrete truck. The cube strength of the delivered concrete is presented in table 4.

Table 4: Imported concrete: strength from cube tests

Cube	Strength [MPa]
1	28.67
2	31.11
3	32.89
Mean	30.89

4.2.2 Reinforcement

The amount of reinforcement for the different beams that were tested are presented below in table 5, where the number of bars and their sizes are given. The reinforcement used for the tests had a yield capacity of $500\ MPa$.

Table 5: Reinforcement

Beam	Width [mm]	Height [mm]	Longitudinal reinforcement	Shear reinforcement
A1	160	200	$2\phi16$	–
A2	160	200	$2\phi16$	$11\phi8\, cc\, 120\, mm$
B1	160	250	$2\phi16$	–
B2	160	250	$2\phi16$	$8\phi8\, cc\, 160\, mm$
C1	160	300	$2\phi16$	–
C2	160	300	$2\phi16$	$6\phi8\, cc\, 190\, mm$
D1	100	150	$2\phi10$	–
D2	100	150	$2\phi10$	$3\phi8\, cc\, 250\, mm$

The amount of longitudinal reinforcement for the different beam types was calculated as

$$\rho_l = \frac{A_{sl}}{b_w d} \tag{11}$$

and the amount of shear reinfrocement was calculated as

$$\rho_w = \frac{A_{sw}}{(s\, b_w\, sin\, \alpha)} \tag{12}$$

where

b_w = beam width
A_{sl} = cross-sectional area of longitudinal reinforcement
d = effective height
A_{sw} = cross-sectional area of shear reinforcement
α = inclination of shear reinforcement
s = distance between shear reinforcement

The percentage of reinforcement was kept relatively similar for all of the beams to avoid the influence of the amount of reinforcement on the shear capacity in the tests. The amount of longitudinal reinforcement and shear reinforcement for the different beams is shown in table 6 below.

Table 6: Amount of reinforcement

Beam	Longitudinal reinforcement [%]	Shear reinforcement [%]
A	1.51	0.52
B	1.16	0.39
C	0.94	0.33
D	1.26	0.40

4.3 Calculated shear capcity

The theoretical shear capacity of the beams were determined according to the equations stated in chapter 3. A calculation example is shown in Appendix A, and the calculated values are presented in table 7.

Table 7: Shear capacity

Beam	Theoretical Shear Capacity [kN]
A1	36.17
A2	62.58
B1	40.33
B2	61.07
C1	44.09
C2	63.33
D1	17.28
D2	22.62

5 Laboratory tests

In this chapter the characteristics of the different beams used in the experiments are discussed.

5.1 Test setup

In order to avoid arch action as much as possible in the beams, concentrated loads should according to Eurocode be placed at a distance of at least $2d$ from the end support, see section 3.3. The load was therefore placed so that the shear span-to-depth ratio was as close to 2.5 as possible for all of the test specimens. To fulfill $a/d = 2.5$ the load was placed at different distances from the left support for the different beam sizes. The beam setup is shown in figure 12.

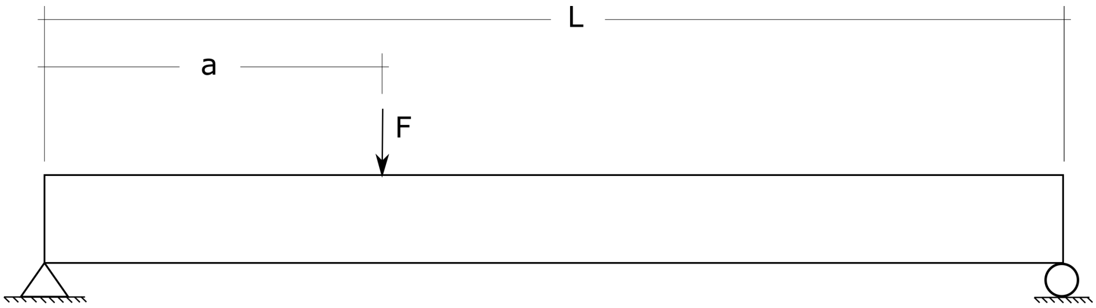


Figure 12: Beam Setup

The ratio a/d for all of the beams is presented in table 8. The length L is the length of the free span between the two supports.

Table 8: Load placements

Beam	a [mm]	d [mm]	a/d	L [mm]
A	415	166	2.50	1300
B	540	216	2.50	1300
C	650	266	2.44	1300
D	315	125	2.52	600

After four weeks of hardening, the concrete beams were tested to failure by being placed on a simple support according to figure 13. The vertical point load was transferred to the beams through a steel plate of 120 x 200 mm for practical reasons. To be as consistent as possible throughout the tests, the machine was set to deform the beams with a constant speed of 0.2 mm/s for all of the beam tests.

Figure 13: Test setup

The deformation of the beams was measured at the lower side, right under the point load according to figure 14.

Figure 14: Measurement device

5.2 Tested shear capacity

To demonstrate the behavior of the tested beams, load-deformation plots for cross-sections $A1$ and $A2$ are presented together with photos of the shear crack propagation. The rest of the beam tests are presented in Appendix C - Lab tests. The first plot, figure 15, shows the results from the two non shear reinforced beams of cross-section size A.

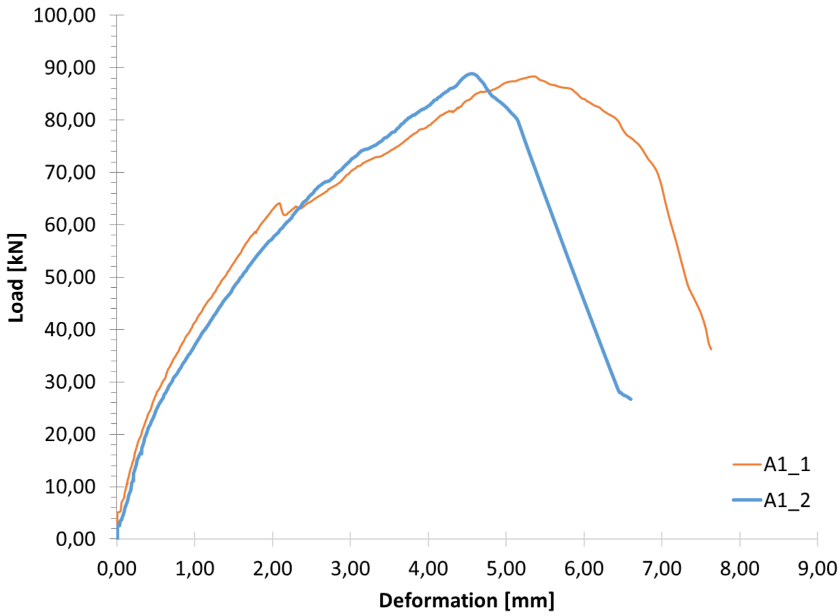


Figure 15: Load-deformation plot A1

(a) Test A1_1 *(b) Test A1_2*

Figure 16: Test for A1 beams

The load-deformation plots for the two shear reinforced beams of cross-section size A are shown in figure 17.

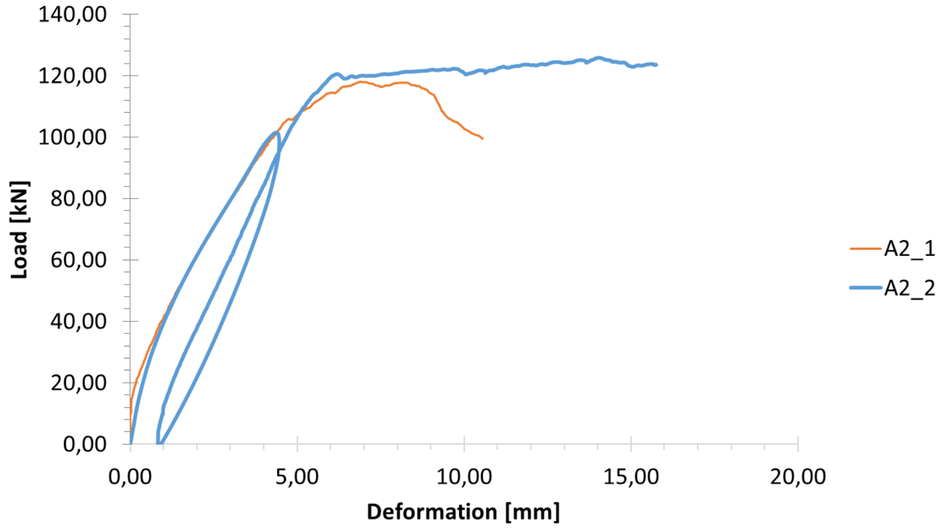


Figure 17: Load-deformation plot A2

(a) Test A2_1

(b) Test A2_2

Figure 18: Test for A2 beams

The shear capacity of the tested beams are presented in table 9.

Table 9: Tested shear capacity

Beam	Tested Shear Capacity [kN]
A1	63.7
A1	64.4
A2	85.4
A2	91.1
B1	64.6
B1	53.1
B2	95.4
B2	100.0
C1	56.7
C1	61.2
C2	113.9
C2	110.5
D1	23.3
D1	27.0
D2	42.3
D2	40.5

6 Results

6.1 Comparison between tested shear capacity and calculated shear capacity

The calculated capacities are compared to the laboratory results in table 10 below. The ratio is determined as

$$ratio = \frac{Tested\,Capacity}{Theoretical\,Capacity}$$

The ratio decreases for the non shear reinforced beams as the effective height increases while the opposite is true for the shear reinforced beams, with the exception of cross section type D.

Table 10: Shear capacity

Beam	Theoretical Capacity [kN]	Tested Capacity [kN]	Ratio [-]
D1	17.28	23.3	1.3
D1	17.28	27.0	1.6
A1	36.17	63.7	1.8
A1	36.17	64.4	1.8
B1	40.33	64.6	1.6
B1	40.33	53.1	1.3
C1	44.09	56.7	1.3
C1	44.09	61.2	1.4
D2	22.62	42.3	1.9
D2	22.62	40.5	1.8
A2	62.58	85.4	1.4
A2	62.58	91.1	1.5
B2	61.07	95.4	1.6
B2	61.07	99.8	1.6
C2	63.33	113.9	1.8
C2	63.33	110.5	1.7

The ratios stated in table 10 are also shown in a graphical format below.

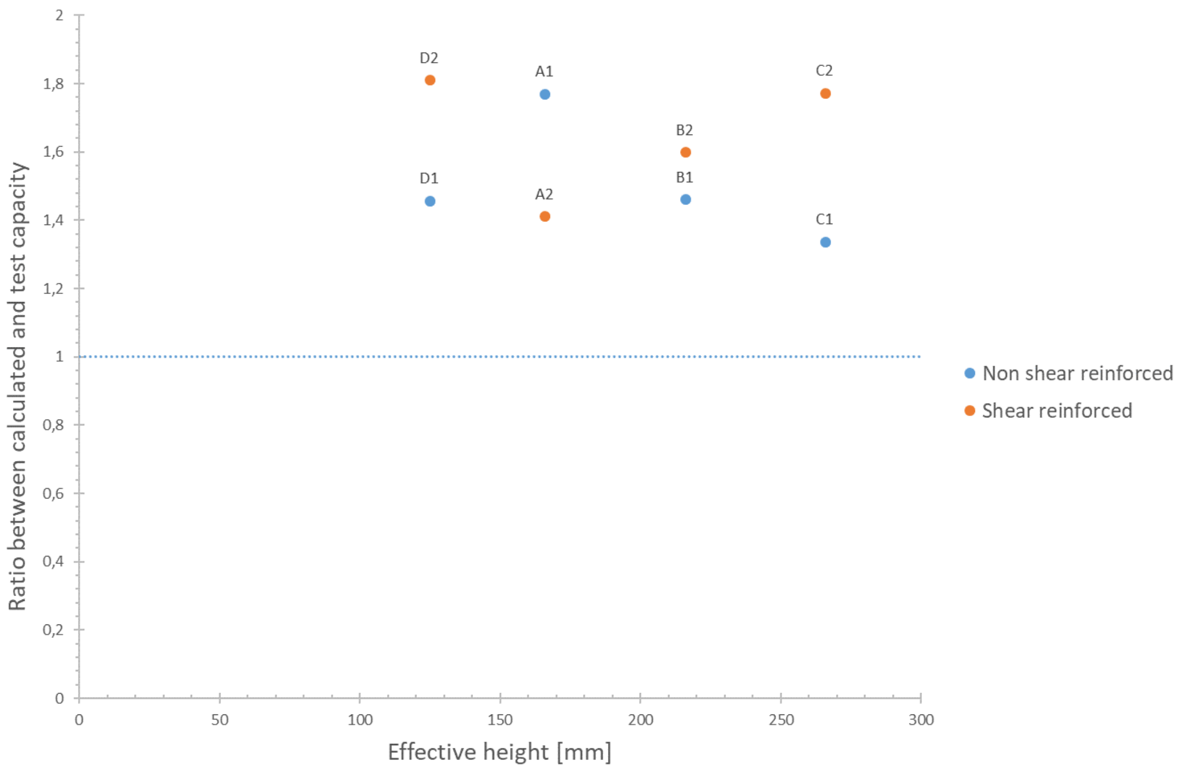


Figure 19: Ratio between tested shear capacity and calculated shear capacity

All of the tested beams carried more load than calculated with the Eurocode models and the ratio between tested and calculated capacity was consequently more than 1 for all of the tests which can be seen in figure 19. The ratio for the shear reinforced beams seemed to become more conservative as the height increased while the opposite was true for the unreinforced beams.

6.2 Nominal shear stress

The shear forces from the lab results were averaged together between the identical beams and transformed into a nominal stress through equation 13.

$$\tau = \frac{V}{b_w d} \tag{13}$$

The results are shown below in table 11.

Table 11: Lab tests: nominal stress at failure

Beam	Averaged nominal stress at failure
D1	2.01
A1	2.41
B1	1.70
C1	1.38
D2	3.31
A2	3.32
B2	2.82
C2	2.64

The results show a decrease in nominal stress with increased effective beam height in both the shear reinforced and non shear reinforced beams, with the exception of cross section type D. This is also shown in a graph format below.

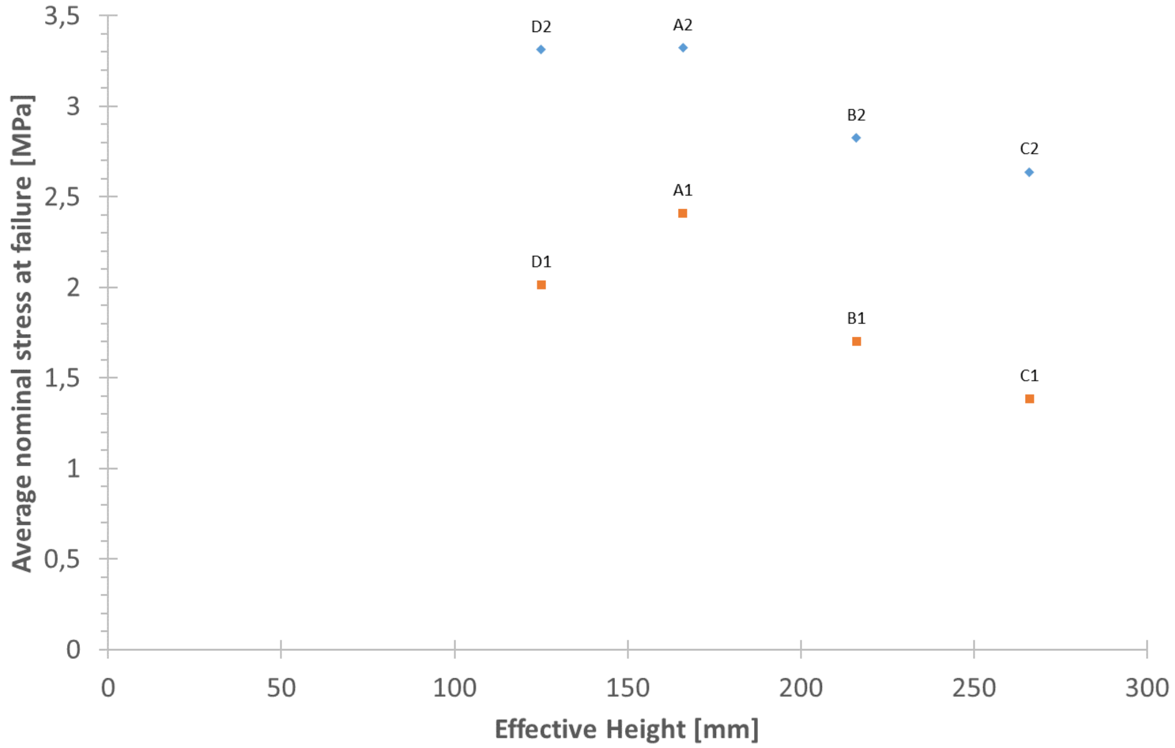


Figure 20: Average nominal stress at failure

7 Conclusions and discussion

7.1 Conclusions

- The average nominal stress decreases with increased beam height for both non shear reinforced and shear reinforced beams, with the exception of the cross section class D, meaning that a size effect is present in the smaller beams.
- The ratio between tested and calculated shear capacity has a general decreasing trend in the non shear reinforced beams when the beam height is increased from size A to C.
- The ratio between tested and calculated shear capacity for the beams with shear reinforcement has a general increasing trend when the beam height is increased from size A to C.
- All of the tested beams carried more load in the laboratory than calculated with the Eurocode models indicating that there is a safety margin present in the shear design formulas for reinforced concrete beams.

7.2 Discussion on Eurocode

The shear capacity formulas utilized in the Eurocode have limitations in the form of limit values. One example is the requirement that the size factor $k = 1 + \sqrt{200/d} \leq 2.0$. This means that the shear force formula for unreinforced concrete beams is valid when the effective height $d \geq 200\,mm$ according to the Eurocode. The reasoning behind this is a little unclear but one can imagine that the lower the effective height becomes the higher the k value becomes. This in-turn has a rather influential impact on the shear capacity the smaller d gets. It seems reasonable to have applied a limit condition so that the shear capacity is not increased unreasonably for lower effective height beams. This requirement impacted cross section type D, and even A, as the effective height was lower than $200\,mm$. Calculations were made disregarding this criteria, and therefore no conclusions with regards to the Eurocode calculations can be made for cross section type D, whilst A was considered a borderline case. The calculations were made despite of this in order to see how well the results fit with the laboratory results. The results showed for cross section A and D that the ratio between tested and calculated value got closer to 1.0 when the calculations were made despite the requirement. It was also not possible to fulfill the restriction of maximum spacing between the stirrups in the shear reinforced beams for cross section type D, so the stirrups were placed without regards to the equation from the Eurocode.

A shortcoming with the formulas for shear capacity of members not reinforced in shear presented in the Eurocode is that the capacity goes to 0 when the amount of tensile reinforcement goes to 0. This is obviously not realistic, as the concrete beam will be able to take a certain amount of shear force even if unreinforced. This does not affect the majority of practical cases, as most beams are reinforced in regards to tension and therefore the capacity will not be 0. However, it shows that the empirical equation has limitations.

Something worth noting about the formulas for shear capacity in the Eurocode is that the concretes contribution is disregarded in the formulas for members that are shear reinforced, see equation (7). The only factors considered in the equation have to do with the shear reinforcements characteristics. This leads to some odd cases where the shear capacity in non shear reinforced beams can be higher than the same beam with shear reinforcement. An example of this is shown in Appendix A - Calculations: Discussion example [1]. In the example, the non shear reinforced beam is calculated to over double the capacity of the shear reinforced beam. This would mean that as soon as the shear reinforcement is added, its capacity would be significantly lower than before the reinforcement was added. In order to gain the same amount of capacity in the shear reinforced beam, a lot of reinforcement would have to be added. In this particular example, a spacing of around $60\,mm$ would be required between the shear reinforcement bars to get the same capacity as the unreinforced beam. This is interesting as the minimum reinforcement of $160\,mm$ was used in this example, but the capacity varied greatly between the non shear reinforced and shear reinforced calculations.

The reasoning behind the total exclusion of the concretes contribution to the shear capacity of shear reinforced concrete beams is unclear, but it could be in order to stay on the ”safe” side when calculating. Due to shear force failures generally being brittle, it is not unreasonable to keep to the safe side by disregarding the concretes contribution. However, this means that caution has to be used when calculating with this method, as it may not be suitable for all RC structures.

7.3 Discussion on laboratory tests

For the beams without shear reinforcement the failure was abrupt and a bit unexpected. When testing these beams, the shear failures appeared with almost no bending cracks, with the shear crack propagating from the load point all the way out to the closest support. This gave a sudden peak in the deformation plots as can be seen in figure 15. For the shear reinforced beams, bending cracks were distributed evenly over the beam span width and the failures were of a more ductile character. The failure mode of the shear reinforced beams was shear flexural failure where the shear failure started from a bending crack at the bottom of the beam (see chapter 2.2). The shear cracks in the shear reinforced beams propagated from the loading point to, most likely, the lower edge of the closest stirrup. The deformation plot for this failure, as seen in figure 17, was different from the non shear reinforced beams and presented a more ductile failure most likely due to the reinforcement yielding.

For the beams of size C, the load was placed in the middle of the beam span as the free span was limited to $1300\,mm$ for practical reasons in the laboratory. Due to this, the span to depth ratio a/d, was a bit lower for these beams than for the rest of the beams. A lower a/d ratio means that the load was applied closer to the support and that the influence of arch action might have been greater. However since the a/d ratio deviated so little from 2.5 it should not have influenced the results in any impacting way.

All of the tested beams carried more load than calculated with the Eurocode models. The beams without shear reinforcement carried between $130\,\%$ and $180\,\%$ of the calculated loads in the laboratory tests and the ratio between tested capacity and theoretical capacity decreased when the beam height was increased. The ratio between tested values and calculated values for the shear reinforced beams varied between 1.4 and 1.9.

[1]It is important to note that the beam discussed in the example is according to the Eurocode not classified as a plate, as the smallest length is not greater than five times the thickness.

The test results show a deviation between the theoretical and test shear capacities of the beams. As it stands, relatively little is still known about shear failures, and so a model uncertainty exists. This could justify a built-in safety factor, in order to increase the safety margins in the Eurocode formulas, especially since shear failures can be brittle. The results for the ratio between tested and theoretical capacity show that the shear reinforced beams become more conservative the larger the effective height. This seems reasonable as the concretes contribution to the shear capacity is ignored in the Eurocode formula, which should lower the theoretical capacity. The opposite is noticed for the non shear reinforced beams, and it would be interesting to see if the value starts to level out for higher effective height beams or if it continues to decrease to a non-conservative value.

In table 11 the nominal stress seems to, aside from $D1$ to $A1$, decrease with increasing beam height. This suggests that there is a size effect present in the smaller beams, as the stress at failure differs between the different sized beams. This trend can also be seen for cross sections $A2$ to $C2$. It is important to note that beam class D is not of the same width as the other beams, and therefore conclusions have to be restricted regarding class D comparisons. The shear reinforced beams seem to follow a similar trend with the nominal stress declining with increased height.

The results from cross section D differ when compared to average nominal stress at failure for the other non shear reinforced beams. As table 11 shows, $D1$ failed at a lower average nominal stress than $A1$. This goes against the trend which can be seen from $A1$ to $C1$, where the average nominal stress decreases with increased effective height. The reasoning for the discrepancy with cross section $D1$ could be due to the differing beam size or the percent longitudinal reinforcement.

7.4 Further research

In order to gain more knowledge on the size effect of smaller concrete beams, more tests with varying cross section sizes could be made. This could add to the beam test pool made in this master thesis and compared for further connections between the different parameters. The height span could even be increased to $150\ mm - 500\ mm$ in order to gain a broader comparison of the size effect.

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Appendix A - Calculations

Shear capacity without shear reinforcement for cross-section A

$$
\begin{aligned}
C_{Rd,c} &= \frac{0.18}{\gamma_C} = \frac{0.18}{1.0} = 0.18 \\
k &= 1 + \sqrt{\frac{200}{d}} = 1 + \sqrt{\frac{200}{166}} = 2.10 \\
\rho_l &= \frac{A_{sl}}{(b_w d)} = \frac{4.02 \cdot 10^{-4}}{(0.16 \cdot 0.166)} = 0.015 \\
k_1 &= 0.15 \\
a &= 2.5\, d = 0.415\, m \\
f_{cm} &= 31\, MPa \\
\sigma_1 &= 0 \\
b_w &= 0.16\, m \\
d &= 0.166\, m \\
V_{Rdc} &= (C_{Rd,c} k \sqrt[3]{100 \rho_l f_{cm}} + k_1 \sigma_1)\, b_w d = \\
&= (0.18 \cdot 2.10 \cdot \sqrt[3]{100 \cdot 0.015 \cdot 31} + 0) \cdot 160 \cdot 166 = \\
&= 36174\, N \rightarrow 36.17\, kN \\
v_{min} &= 0.035 \sqrt{k^3 f_{cck}} = 0.035 \sqrt{2.10^3 \cdot 31} = 0.592 \\
P_d &= \frac{V_{Rd,c} \cdot L}{L - a} = \frac{36.17 \cdot 1.5}{1.5 - 0.415} = 50.01\, kN
\end{aligned}
$$

Requirements according to Eurocode

$$
\begin{aligned}
\rho_l &= 0.015 \leq 0.02 \rightarrow \text{ OK} \\
V_{Rd,c} &\geq V_{min} = (v_{min} + k_1 \sigma_{cp})\, b_w d = 0.592 \cdot 0.16 \cdot 0.166 \cdot 10^3 = \\
&= 18.95\, kN \rightarrow \text{OK} \\
k &= 2.10 \nleq 2.0 \rightarrow \text{ Requirement not fulfilled}
\end{aligned}
$$

The last requirement gives a borderline case, and so the values influence on the capacity is checked both when it is set to 2.0 and 2.1. The impact this value has on the overall shear capacity is minimal, making a difference of around $2\ kN$ between the two cases. However, it has to be noted that according to the Eurocode this requirement is not technically fulfilled for cross-section A. Theoretically it can be seen that this requirement will not be fulfilled as long as the effective height, d, is under $200\ mm$. This puts limitations on the height of beams used when calculating the shear force capacity in smaller beams.

Shear capacity with shear reinforcement for cross-section A

$$
\begin{aligned}
A_{sw} &= 2\,\pi\ \cdot (d/2)^2 = 2\,\pi\ \cdot (0.008/2)^2 = 1.0\ \cdot 10^{-4}\,m^2 \\
s &= 0.12\,m \\
z &= 0.9\,d = 0.9\ \cdot 0.166 = 0.1494\,m \\
f_{ywd} &= 500\,MPa \\
\theta &= 45° \\
\alpha &= 90° \\
V_{Rd,s} &= \frac{A_{sw}}{s} z f_{ywd} (cot\,\theta + cot\,\alpha)\,sin\,\alpha = \\
&= \frac{1.0\ \cdot 10^{-4}}{0.12}\,0.1494\ \cdot 500\ \cdot 10^6\,(cot\,45° + cot\,90°)\,sin\,90° = \\
&= 62581\,N \rightarrow 62.58\,kN
\end{aligned}
$$

$$
\begin{aligned}
\alpha_{cw} &= 1 \\
\nu_1 &= 0.6\,(1 - f_{cm}/250) = 0.526 \\
V_{Rd,max} &= \alpha_{cw}\,b_w\,z\,\nu_1\,f_{cm}\,\frac{cot\,\theta + cot\,\alpha}{1 + cot^2\,\theta} = \\
&= 1\ \cdot 0.16\ \cdot 0.1575\ \cdot 0.526\ \cdot 31\ \cdot 10^6\,\frac{cot\,45° + cot\,90°}{1 + cot^2\,45°} = \\
&= 194741\,N \rightarrow 194.7\,kN
\end{aligned}
$$

Requirements according to Eurocode

$$
\begin{aligned}
s_{l,max} &= 0.75\,d\,(1 + cot\,\alpha) = 0.75\ \cdot 0.166\,(1 + cot\,90°) = 0.125\,m \\
\rho_w &= \frac{A_{sw}}{(s\,b_w\,sin\,\alpha)} = \frac{1.0\ \cdot 10^{-4}}{0.12\ \cdot 0.16\ \cdot sin\,90°} = 5.23\ \cdot 10^{-3} \\
\rho_{min} &= (0.08\sqrt{f_{cm}})/f_{yk} = (0.08\sqrt{31})/500 = 1.1\ \cdot 10^{-3} \\
\rho_w &\geq \rho_{w,min} \text{ is fulfilled}
\end{aligned}
$$

Moment capacity for cross-section A

$$
\begin{aligned}
c &= 26\,mm \\
h &= 200\,mm \\
f_{yk} &= 500\,MPa \\
d_{reinf.} &= 0.016\,m \\
n_{bar} &= 2 \\
A_s &= n_{bar}\,\pi\,(d/2)^2 = 2\,\pi\,\cdot(0.016/2)^2 = 4.02\,\cdot 10^{-4}\,m^2 \\
\sigma_s &= 500\,MPa \\
E &= 200\,GPa \\
x &= \frac{A_s\,\sigma_s}{0.8\,\cdot f_{cm}\,\cdot b} = \frac{4.02\,\cdot 10^{-4}\,\cdot 500}{0.8\,\cdot 31\,\cdot 0.16} = 5.067\,\cdot 10^{-2}\,m \\
F_s &= A_s\sigma_s = 4.02\,\cdot 10^{-4}\,\cdot 500\,\cdot 10^{6} = 201062\,N \rightarrow 201.1\,kN \\
M &= F_s\,(d - 0.4\,x) = 201.1\,\cdot(0.166 - 0.4\,\cdot 5.067\,\cdot 10^{-2}) = 29.30\,kNm \\
P_d &= \frac{M\,\cdot L}{a\,\cdot(L-a)} = \frac{29.30\,\cdot 1.5}{0.415\,\cdot(1.5 - 0.415)} = 97.61\,kN
\end{aligned}
$$

Check of assumptions for cross-section A

$$
\begin{aligned}
\epsilon_{sy} &= \frac{f_{yk}}{E} = \frac{500}{200000} = 2.5\,‰ \\
\epsilon_{cu} &= 3.5\,‰ \\
\epsilon_s &= \epsilon_{cu}\,\frac{d-x}{x} = 3.5\,\frac{0.166 - 5.067\,\cdot 10^{-2}}{0.166} = 7.97\,‰ \\
\epsilon_s &> \epsilon_{sy} \rightarrow \sigma_s = f_{yd}
\end{aligned}
$$

Discussion example

Imagine two simply supported concrete beams of the dimension, $b = 600\,mm$, $h = 250\,mm$, and $L = 1500\,mm$. One beam has only longitudinal reinforcement whilst the other has the same longitudinal reinforcement as well as added shear reinforcement. If the longitudinal reinforcement is for example $8\phi16$ bars and a point load is placed $2.5d$ from the support, the non shear reinforced shear capacity can be calculated

$$\begin{aligned} V_{Rdc} &= (C_{Rd,c} k \sqrt[3]{100 \rho_l f_{cm}} + k_1 \sigma_1)\, b_w d\ = \\ &= (0.18\ \cdot 1.96\ \cdot \sqrt[3]{100\ \cdot 0.012\ \cdot 31} + 0)\ \cdot 600\ \cdot 216 = \\ &= 154534N \rightarrow 155kN \end{aligned}$$

If the same beam is studied but with added $\phi8$ shear reinforcement at $cc\,160\,mm$ the capacity according to Eurocode becomes

$$\begin{aligned} V_{Rd,s} &= \frac{A_{sw}}{s} z f_{ywd} (cot\,\theta + cot\,\alpha)\, sin\,\alpha = \\ &= \frac{1.0\ \cdot 10^{-4}}{0.16} 0.1944\ \cdot 500\ \cdot 10^6\, (cot\,45° + cot\,90°)\, sin\,90° = \\ &= 61073\,N \rightarrow 61.07\,kN \end{aligned}$$

Appendix B - Specimen construction

Figure 21: Building the base of the form

Figure 22: Formwork

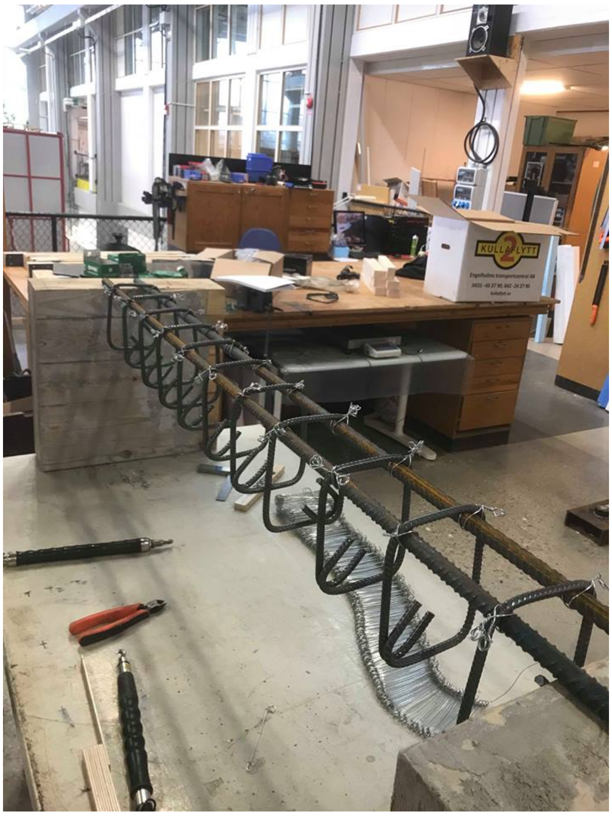


Figure 23: Tying the reinforcement

Figure 24: Finished form

Figure 25: Mini steel form for beam size D

Figure 26: Before casting

Figure 27: Concrete casting

Figure 28: Right after casting

Figure 29: Form removal

Appendix C - Lab tests

Beam A1

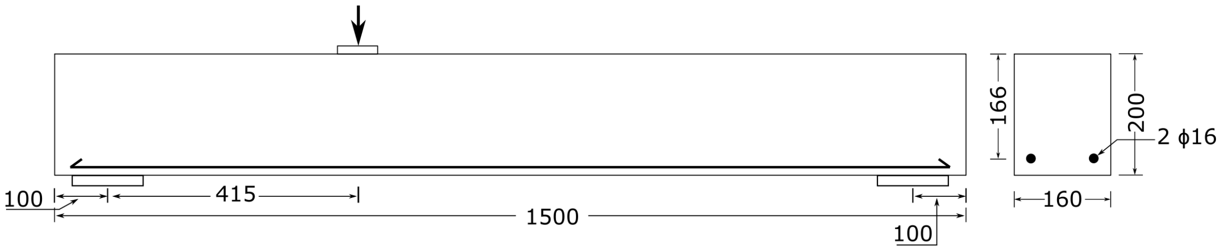


Figure 30: Beam A1

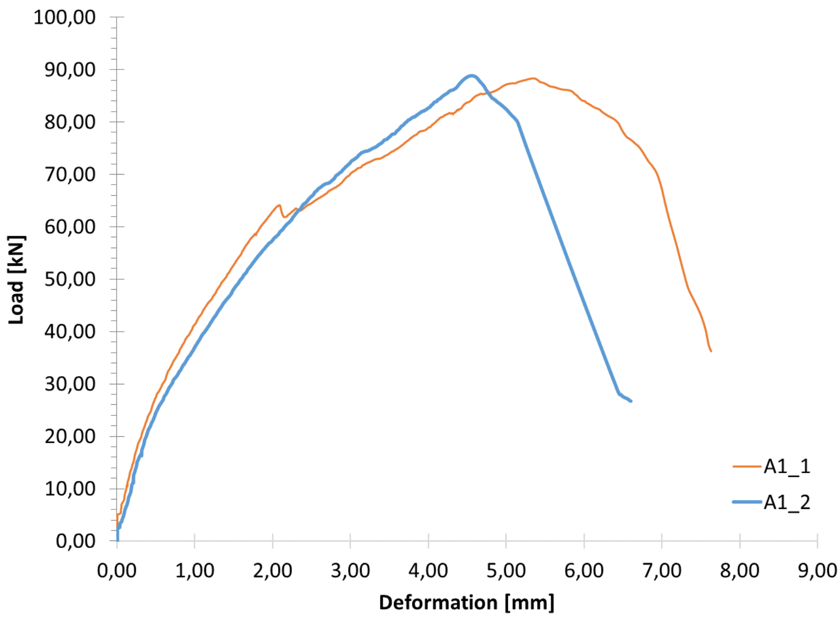


Figure 31: Load-deformation plot A1

(a) Test A1_1

(b) Test A1_2

Figure 32: Crack patterns for the A1 beams

Beam A2

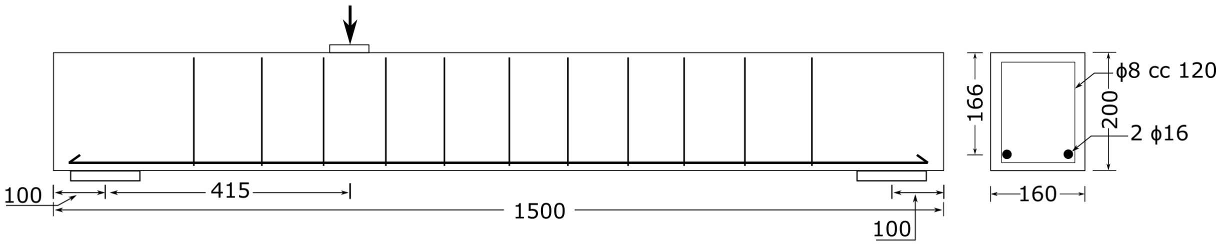


Figure 33: Beam A2

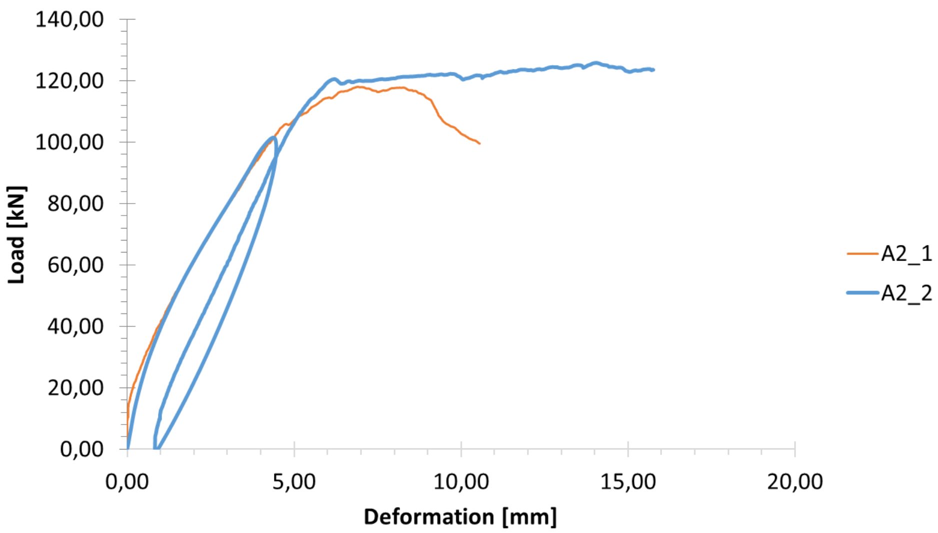


Figure 34: Load-deformation plot A2

(a) Test A2_1

(b) Test A2_2

Figure 35: Crack patterns for the A2 beams

Beam B1

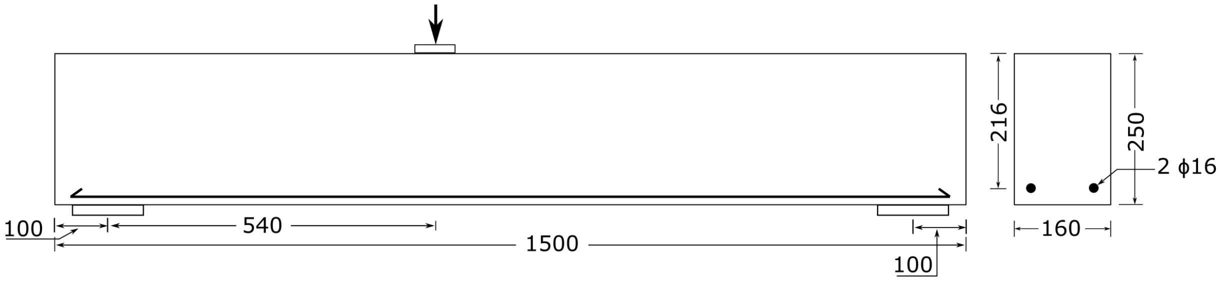


Figure 36: Beam B1

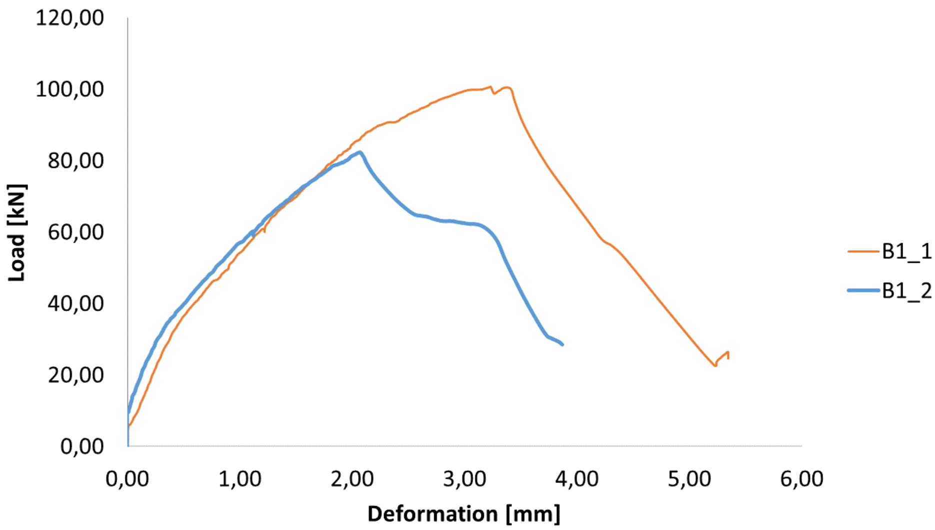
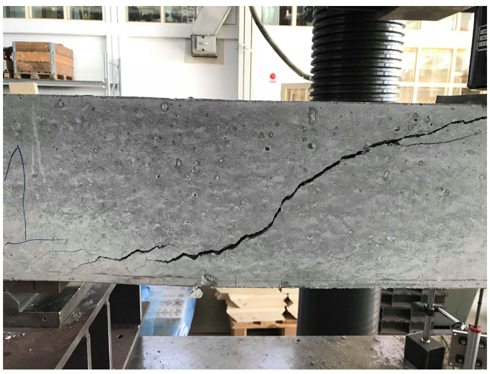
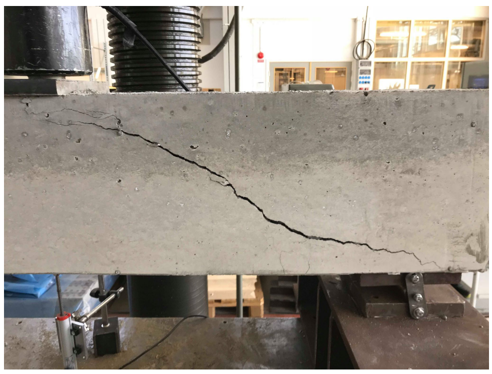


Figure 37: Load-deformation plot B1



(a) Test B1_1



(b) Test B1_2

Figure 38: Crack patterns for the B1 beams

Beam B2

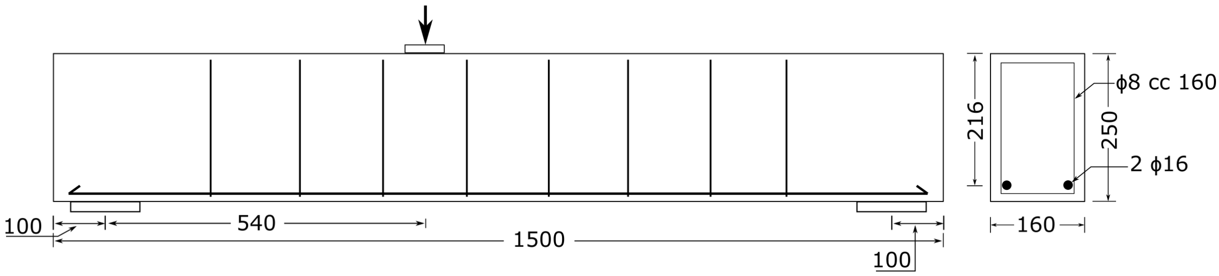


Figure 39: Beam B2

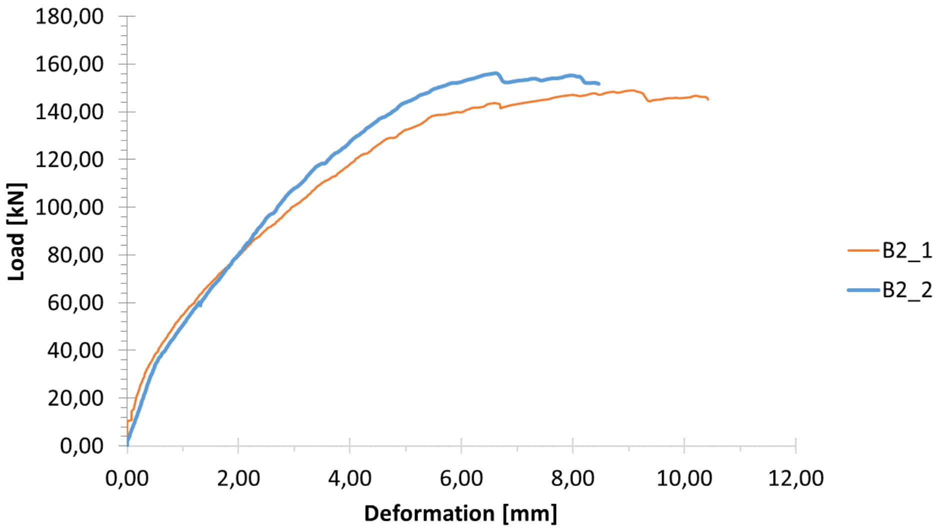
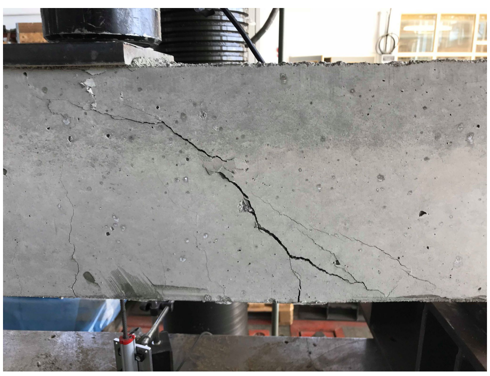


Figure 40: Load-deformation plot B2



(a) Test B2_1

(b) Test B2_2

Figure 41: Crack patterns for the B2 beams

Beam C1

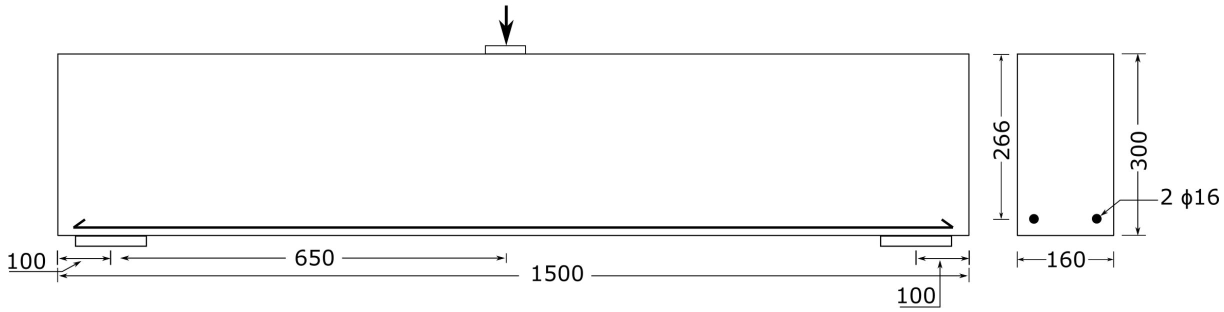


Figure 42: Beam C1

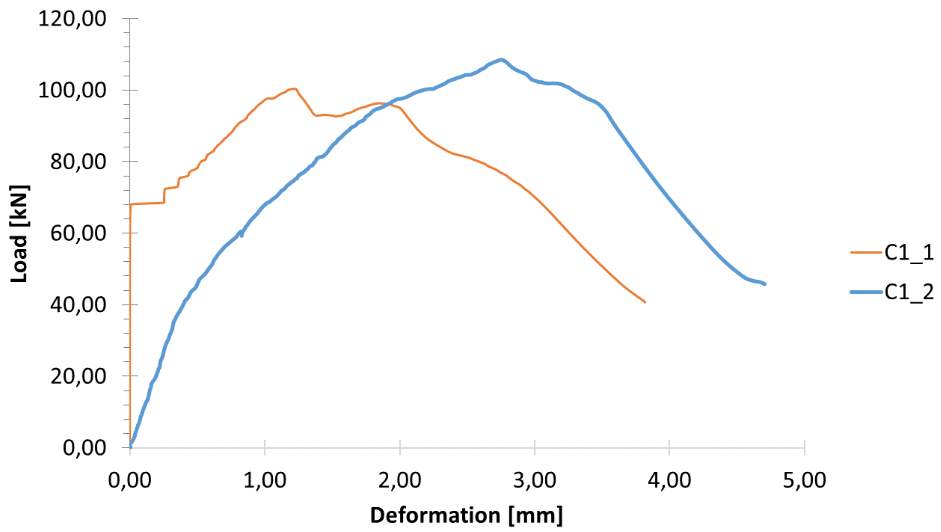


Figure 43: Load-deformation plot C1

(a) Test C1_1 *(b) Test C1_2*

Figure 44: Crack patterns for the C1 beams

Beam C2

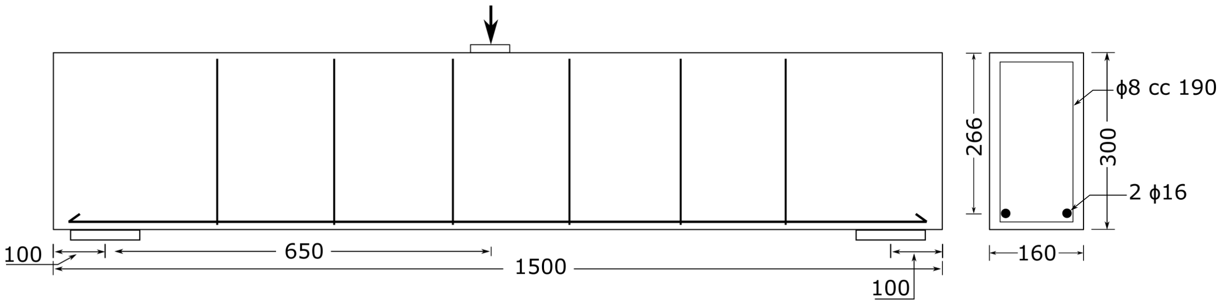


Figure 45: Beam C2

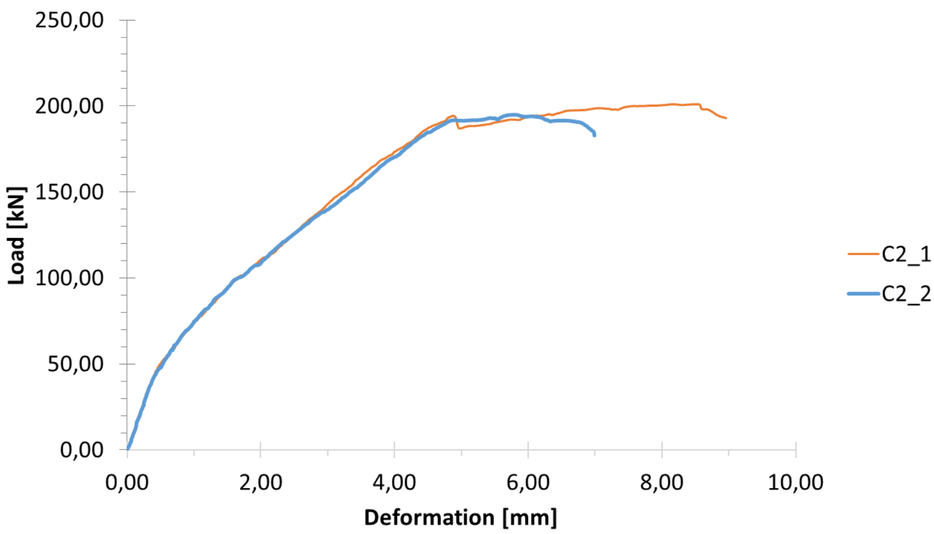
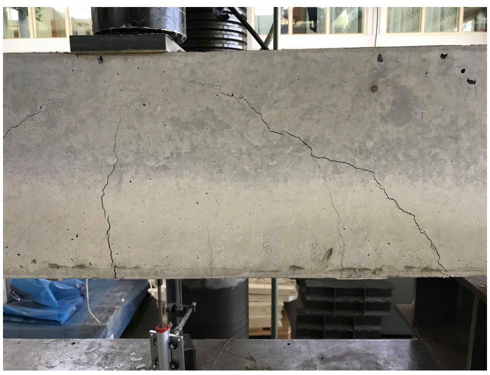
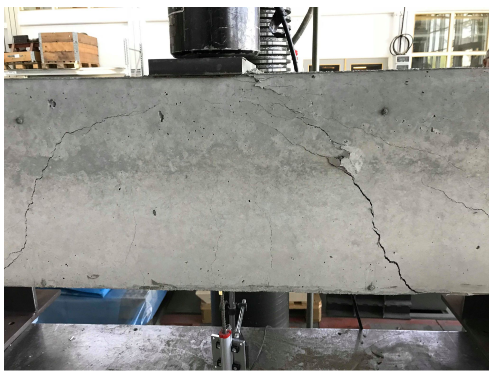


Figure 46: Load-deformation plot C2



(a) Test C2_1



(b) Test C2_2

Figure 47: Crack patterns for the C2 beams

Beam D1

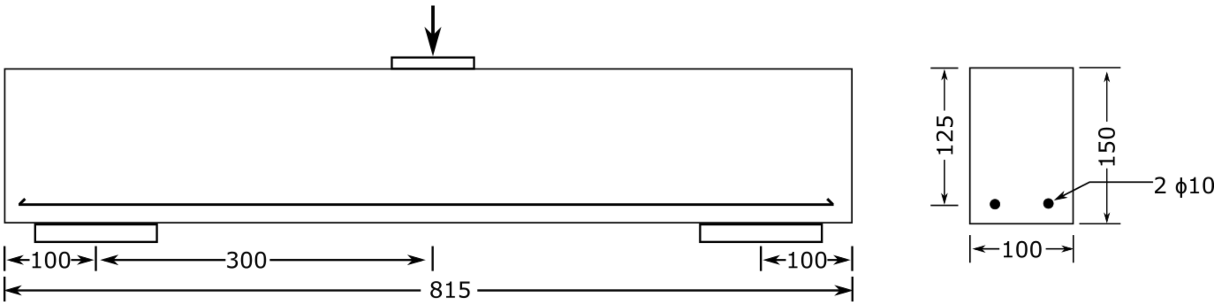


Figure 48: Beam D1

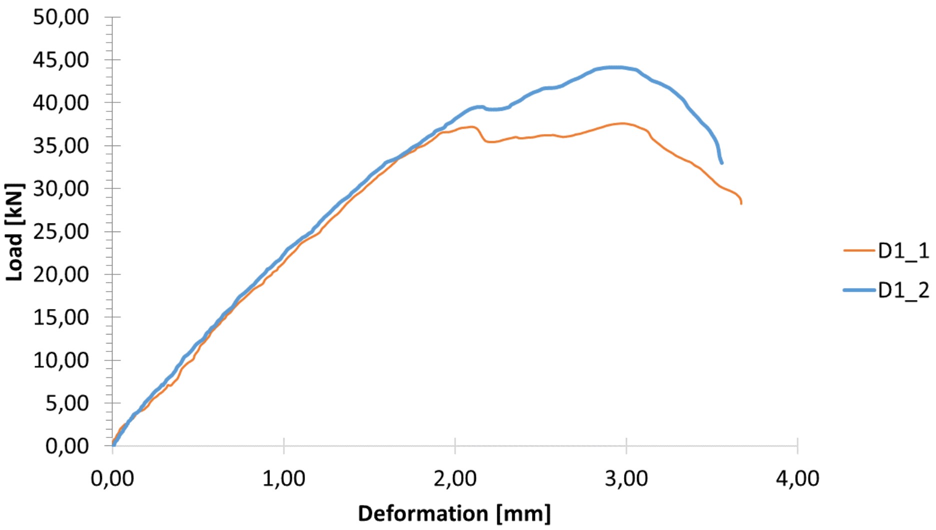


Figure 49: Load-deformation plot D1

(a) *Test D1_1*

(b) *Test D1_2*

Figure 50: Crack patterns for the D1 beams

Beam D2

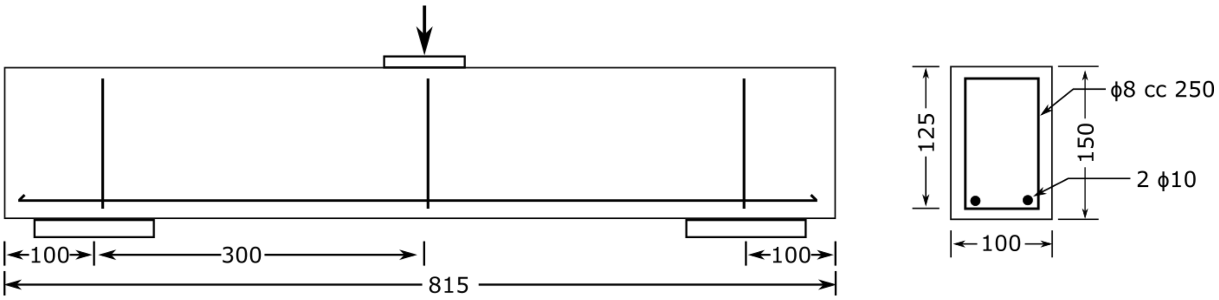


Figure 51: Beam D2

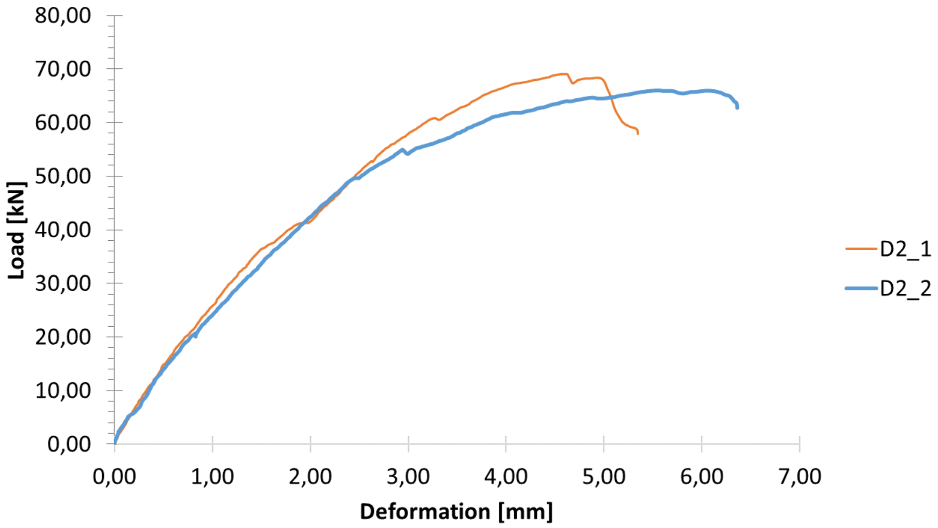


Figure 52: Load-deformation plot D2

(a) Test D2_1

(b) Test D2_2

Figure 53: Crack patterns for the D2 beams