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Quality's relationship to the idiosyncratic volatility puzzle

Master Thesis

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Abstract

This paper examines the well documented relationship between idiosyncratic volatility and mean returns. By using the recently published quality-minus-junk factor this paper attempts to explain both the abnormal performance of portfolios sorted on idiosyncratic volatility as well as the cross-sectional pricing of idiosyncratic volatility. Using data from the U.S. it is shown that the quality factor is able to explain the abnormal performance of the extreme portfolios in the idiosyncratic volatility puzzle, while having no impact on the cross-sectional stock returns. This indicates that the quality-minus-junk factor plays an important role in determining the performance of the portfolios and further research should include it in any model aiming to investigate this puzzle.

Keywords: Idiosyncratic volatility puzzle, quality, portfolio performance, cross-sectional of returns.

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Contents

1 Introduction

The purpose of this study is to investigate the quality factor's (Asness, Franzzini Pedersen 2017) relationship to idiosyncratic volatility. This is done through the framework of the idiosyncratic volatility puzzle (Ang et al, 2009) in which the quality factors' ability to explain the performance of portfolios sorted on idiosyncratic volatility will be investigated. Furthermore the quality factors' effect on the cross-sectional pricing of idiosyncratic volatility will also be investigated to see if it has any effect. A lot of research has investigated the abnormal returns in high volatility stocks and its relationship to other factors (Ang et al 2004; Ang et al 2009; Han, Hu Lesmond 2015; Stambaugh, Yu Yuan 2012; Jiang, Xu Yao 2009). The low returns associated with high idiosyncratic volatility is present throughout all research, with the exception of (Han, Hu Lesmond 2015) when using quote mid-point price based returns. However the relationship between quality and the returns associated with the idiosyncratic volatility has never been explored.

Solving the puzzle behind the pricing of idiosyncratic volatility is not only interesting out of a purely theoretically perspective but also provides valuable practical insights about the returns of stocks. Ang et al (2009) document a vast disparity in the returns of portfolios sorted on past idiosyncratic volatility where low volatility portfolios severely outperform the high volatility portfolio both in terms of excess returns as well as abnormal returns. Solving the puzzle could help investors make more informed decisions as to which assets to hold and how they should be priced.

Theoretically the capital asset pricing model (Sharpe, 1964; Lintner, 1965) posits that an assets return is directly associated with its market risk exposure. This because when portfolios are constructed, given an adequate number of assets, all idiosyncratic volatility should be eliminated and only the market rate of return and each assets covariance with it ought to determine the portfolios return. Campbell (2001) however shows us that idiosyncratic volatility has increased, and the explanatory power of the market model has declined, due to a decreased correlation between individual stocks. Campbell (2001) and Merton (1987) both indicate that idiosyncratic volatility, in a portfolio that is not completely hedged against it, should be positively priced as investors demand a premium for holding an asset with undiversifiable risk

Contrary to this theory Ang et al (2006), while examining volatility and expected returns, find that there is a negative pricing of the idiosyncratic volatility of a stock. By measuring the idiosyncratic volatility relative to the Fama-French 3 factor model (Fama French 1993) each month and developing a trading strategy using the previous periods idiosyncratic volatility, Ang is able to generate robust results showing that stocks with high idiosyncratic volatility have low average returns. In a later paper Ang et al (2009) also establishes through Fama-Macbeth regressions (Fama MacBeth 1973) the presence of a negative pricing of idiosyncratic volatility in the cross-section that is significant and persistent.

Han, Hu, Lesmond (2015) builds upon these results as they investigate the pricing of cross-sectional idiosyncratic volatility through the lens of measurement errors. They develop the same trading strategy as the one employed in Ang(2006). They first employ transaction price based returns and then quote-midpoint price based returns, to control for liquidity bias. They find that once the bid-ask bounce has been accounted for in the idiosyncratic volatility estimates, through quote-midpoint price based returns, the idiosyncratic volatility sorted portfolios no longer generate a significant alpha. The same effect can be observed in the Fama-Macbeth regressions (Fama MacBeth 1973) where the pricing ability of idiosyncratic volatility also disappears in the cross-section. Hou and Loh (2015) evaluate several of the potential explanations for the Idiosyncratic volatility puzzle, including the findings in Han, Hu, Lesmond (2015) and determine that existing explanations do not fully explain the puzzle. Investor lottery preferences and market frictions however do show promise in explaining the puzzle. The quality of a stock however is not one of the explanations explored in solving the idiosyncratic volatility puzzle.

The quality of an asset is interesting because quality as a factor is a unifying measure attempting to aggregate several anomalies like, high profitability outperformance, repurchasing outperformance, low beta, leverage etc (Asness, Franzzini Pedersen 2019). These anomalies cause mispricing of assets. The mispricing caused by these anomalies show up as idiosyncratic volatility in models that do not account for them, such as CAPM and the Fama-French 3-factor model. They also distort the analysis of portfolios by not accurately being able to explain its performance, thereby misconstruing these anomalies as over or underperformance. As such quality as a factor may be able to explain cross-sectional variation and portfolio performance better than the typical models used to investigate the idiosyncratic volatility puzzle. Quality is defined as characteristics of a stock that all else being equal investors should be willing to pay a higher price for (Asness, Frazzini Pedersen 2017). In the following its impact on the idiosyncratic volatility puzzle will be explored by forming quintile portfolios based on idiosyncratic volatility, to asses its power to explain the return of these portfolios and to see if sorting assets in accordance with their idiosyncratic volatility relative to quality impacts its returns. Beyond that Fama-Macbeth cross-sectional regressions (Fama MacBeth 1973) will be applied to determine if quality impacts the pricing ability of idiosyncratic volatility. Most literature on the subject focuses their research on liquidity and market friction (Hou, Loh 2015; Han, Hu Lesmond 2015) with the intention of solving the idiosyncratic volatility puzzle. The approach here will be more to investigate the relationship between quality and idiosyncratic volatility to gage if quality can explain the abnormal return and how it affects the cross-sectional stock returns, if it is shown to contribute towards explaining the idiosyncratic volatility puzzle it may provide a better framework than the current Fama-French 3-factor model.

The paper will be organized as followed; Section 2 will present the data and methodology behind the construction of each factor, the method applied to measure idiosyncratic volatility, the portfolio performance tests and finally the cross-sectional stock return tests. Section 3 will report the results when these methodologies are applied on the data. Section 4 will summarize and discuss the results as well as compare them to previous research. Section 5 will conclude with the key findings of the paper.

2 Data and Methodology

In this section the data will be presented as well as the methodology that accompanies it. First the data itself will be sourced and described. Following that the methodology will be described in 2 subsections. First the different parameters used to measure idiosyncratic will be presented and brief background will be given around idiosyncratic volatility. In the next subsection the test methodology will be described.

2.1 Data

The data gathered for this paper is collected from CRSP and computstat. Return data is collected at a daily level and is used to form the basis for the idiosyncratic volatility measure. The data is filtered in order to give us a smaller and more manageable sample. Following the methodology in Han, Hu, Lesmond (2015) and Jiang, xu, Yao (2009) we filter the data so that only stocks are included that on a monthly level has a price greater than 5 in the same period the portfolio is formed. Furthermore, these are filtered as in Ang et al (2004) so that only months in which there are 17 or more daily returns are kept. This is done to ensure that we have an accurate measure of idiosyncratic volatility. The sample period is 1984-2008 as in Han, Hu, Lesmond (2015). Beyond the daily data, it is also collected at a monthly level for the tests themselves. The Fama-French factors (Fama French 1993) are obtained from Ken French's website. The QMJ (Asness, Franzzini Pedersen 2019) factor is collected from AQR website.

2.2 Measuring Idiosyncratic Volatility

Idiosyncratic volatility is the residual variation within each assets return that is not associated with market return. In the CAPM model this would be the residual after the market return has been adjusted by the assets exposure to market exposure. In the following idiosyncratic volatility will be measured not relative to the CAPM model but relative to the Fama-French 3-factor model, as done in Ang et al (2006) original paper, and the 5-factor model. Both models will be used twice, once in its regular form and once with the addition of Asness, Franzzini and Pedersens quality factor (2019). The reason for using these models instead of the regular CAPM model is that the Fama-French 3-factor model corrects for 2 tendencies in stock returns, namely the size and value effect and therefor is better at evaluating performance Fama French (1993). Quality and the 5-factor model further extends these models to give an even more accurate performance evaluation.

2.2.1 Fama-French 3-factor

The Fama and French (1993) 3-factor model expands upon the regular CAPM model, by adding 2 factors. These factors are the excess market return, the small-minus-big (SMB) and the high-minus-low (HML) factor. Each of them are thought off as representing risk factors not priced in market return. Most literature on the idiosyncratic volatility puzzle uses this model to estimate idiosyncratic volatility (Ang et al (2006)(2009), Han, Hu Lesmond (2014), Hou Loh (2015)). The first factor, the excess market return, is the value weighted return of CRSP firms listed on the New York stock exchange (NYSE), AMEX or NASDAQ, minus the one-month Treasury bill rate. All factors are taken directly from Kenneth French's website.

$$Mkt = Rm - RF \tag{1}$$

SMB refers to the excess return of small, as in small market capitalization, versus big companies, as in big market capitalization. This factor is constructed by first dividing the market into 2 groups small and big. In these 2 groups all assets sorted into to 3 portfolios depending on their book-to-market value. The breakpoints for which portfolio an asset is sorted in to are constructed by taking the bottom 30% (Low), middle 40% (Neutral) and top 30% (High) of the ranked book-to-market ratios of the NYSE. When the portfolios are constructed the big firms are subtracted from the small and divided by 3 to get the average.

$$SMB = \frac{(S/L + S/N + S/H) - (B/L + B/N + B/H)}{3} \tag{2}$$

The next factor HML refers to the excess return exhibited by companies with high book-to-market value versus companies with low book-to-market values, also known as the value factor. We start in an identical manner and divide firms according to their size, we then form portfolios out of these groups based on the book-to-market ratio. As such a high book-to-market portfolio is constructed for big and small firms as well as a low book-to-market portfolio. There are no neutral book-to-market portfolios included in this factor. The high portfolios are then subtracted by the low portfolios and the average return is taken.

$$HML = \frac{(S/H + B/H) - (S/L + B/L)}{2} \tag{3}$$

These factors are then regressed on the excess return of all securities.

$$r_{it} = \alpha + \beta_1 \times Mkt_{it} + \beta_2 \times SMB_t + \beta_3 \times HML_t + \varepsilon_{it} \tag{4}$$

$$\varepsilon_{it} = r_{it} - (\alpha + \beta_1 \times Mkt.r_t + \beta_2 \times SMB_t + \beta_3 \times HML_t) \tag{5}$$

The purpose of this is to estimate the residual for each observation in the month, these are then used to estimate the idiosyncratic volatility by taking the standard deviation of the residuals within the month. As such we have a Idiosyncratic volatility measure for each firm, each month by applying the following formula.

$$\sigma_{it} = \sqrt{250} \times \sqrt{\frac{\sum(\varepsilon - \bar{\varepsilon})^2}{n}} \tag{6}$$

2.2.2 Fama-French 5-factor

Next the Fama-French 5-factor model (Fama French 2014) will be used to estimate idiosyncratic volatility. Most papers concerning idiosyncratic volatility focus on the Fama-French 3-factor model. The 5-factor model takes into account 2 new factors, profitability and investment. These factors take into account additional risk factors, but similar to the Fama-French 3-factor they fail to fully explain returns. It is however interesting to see how idiosyncratic volatility is affected by the presence of these factors and how their addition affects the quality factor impact on idiosyncratic volatility. Both measures are reflected as quality measures in the quality factor but since quality consists of myriad of different measures they are still different from each other.

In the FF5 model SMB is constructed differently from the original FF3 model. The SMB portfolio in this case consists of 9 different portfolios constructed in the following way. The final factor is constructed out average of three different SMB factors.

$$SMB = 1/3 \times (SMB_{B/M} + SMB_{OP} + SMB_{INV}) \tag{7}$$

These are in turn constructed through a similar manner as the HML factor in the FF3 model. The book-to-market factor is constructed by taking the average return of a portfolio consisting of a small minus big position, once again according to market cap, but also sorted upon the book-to-market ratio. In this sort Value (V) is when book-to-market ratio is low, indicating that the value of the company is close to its book value and therefor is not anticipated to grow. Neutral (N) is the middle ground between value and growth. Growth (G) is when the discrepancy between the book and market value is large indicating that the company is anticipated to grow in the future.

$$SMB_{B/M} = \frac{(S/V + S/N + S/G) - (B/v + B/N + B/G)}{3} \tag{8}$$

The same procedure is then repeated for size/operating profitability- and size/investment-portfolios. Where operating profitability is sorted into robust (R), neutral or weak (W), and investment is sorted into conservative (C), neutral or aggressive (A).

$$SMB_{OP} = \frac{(S/R + S/N + S/W) - (B/R + B/N + B/W)}{3} \tag{9}$$

$$SMB_{INV} = \frac{(S/C + S/N + S/A) - (B/C + B/N + B/A)}{3} \tag{10}$$

The first new factor introduced is the robust minus weak (RMW) operating profitability portfolio, where the average return is taken from two portfolios with robust (R) minus two with weak (W) operating profitability portfolios.

$$RMW = \frac{(S/R + B/R) - (S/W + B/W)}{2} \tag{11}$$

The second new factor refers to conservative minus aggressive investment portfolios, where the average return is taken from two portfolios with conservative (C) minus two with agressive investments.

$$RMW = \frac{(S/C + B/C) - (S/A + B/A)}{2} \tag{12}$$

When measuring idiosyncratic volatility using the Fama-French 5-factor model the procedure is identical to the 3-factor model but with the additional factors introduced above.

$$r_{it} = \alpha + \beta_1 \times Mkt_{it} + \beta_2 \times SMB_t + \beta_3 \times HML_t + \beta_4 \times CMA_t + \beta_5 \times RMW_t + \varepsilon_{it} \tag{13}$$

$$\varepsilon_{it} = r_{it} - (\alpha + \beta_1 \times Mkt.r_t + \beta_2 \times SMB_t + \beta_3 \times HML_t + \beta_4 \times CMA_t + \beta_5 \times RMW_t) \tag{14}$$

The errors are then computed as the idiosyncratic volatility in the same manner as in (6).

2.2.3 Quality-minus-Junk

(Asness, Frazzini Pedersens 2017) quality measure provides an aggregate standardized measure of several return anomalies and express them as the quality or junk of an asset, where quality are characteristics that make assets attractive to investors, such as growing, safe, profitable and well managed, assets that do not have these characteristics are on the other hand considered to be junk assets. In the following the characteristics and construction of the Quality Minus Junk (QMJ) will be described.

The factor is constructed in a similar way to the Fama-French factors. Its measure of quality is based on a rewritten version of Gordon's growth model which expresses a stock's price-to-book value as:

$$\text{original: } P = \frac{dividend}{req\text{-}return - growth} \tag{15}$$

$$\text{rearranged: } \frac{P}{B} = \frac{profitability \times payoutratio}{required\text{-}return - growth} \tag{16}$$

To measure theses characteristics several measures are employed for the sake of robustness. Due to a multitude of measures being employed in the creation of the quality characteristic each measurement of a corresponding characteristic is converted into ranks and standardized in order to obtain a z-score. Each individual z-score is obtained through the following formula:

$$z(x) = \frac{r - \mu_r}{\sigma_r} \tag{17}$$

Where x refers to a specific characteristic, r to the rank of an asset and, μ_r and σ_r are the cross sectional means and standard deviation of r. The characteristic score is then the average of the individual z-scores. Starting with the profitability characteristic 6 different measurements are employed. Profitability is defined here as profits per unit of book value, measure through:

1. Gross profits over assets (GPOA)
2. Return on equity (ROE)
3. Return on assets (ROA)
4. Cashflow over assets (CFOA)
5. Gross margin (GMAR)
6. Fraction of earnings composed of cash (ACC)

The purpose of these measures is to get a sense of several profitability characteristics such as margins, profits, earnings, accruals and cash flows. Through the method explained earlier we arrive at the following expression for profitability.

$$Profitability = z(z_{GPOA} + z_{ROE} + z_{ROA} + z_{CFOA} + z_{GMAR} + z_{ACC}) \tag{18}$$

These same measurements are employed once again when growth is measured. By simply taking the prior five-year differences of all these measures we get a sense regarding the growth of an assets. Defined in the following.

$$Growth = z(z_{\Delta GPOA} + z_{\Delta ROE} + z_{\Delta ROA} + z_{\Delta CFOA} + z_{\Delta GMAR} + z_{\Delta ACC}) \tag{19}$$

Moving on we examine the safety of an asset, the intuition behind safety as a quality measure is that investors, all-else-equal, pay a higher price for a safer stock. This characteristic is measured on both return and fundamental based measures of safety. These are.

1. Low Beta (BAB)
2. Low idiosyncratic volatility (IVOL)
3. Low leverage (LEV)
4. Low bankruptcy risk (O) and (Z)
5. Low return on equity volatility (EVOL)

Resulting in the following formula:

$$Safety = z(z_{BAB} + z_{IVOL} + z_{LEV} + z_O + z_Z + z_{EVOL}) \tag{20}$$

The final characteristic that is employed is the payout of an asset. Which is defined as the fraction of profits paid out to shareholders. Payout is measured through 3 measurements:

1. Equity issuance (EISS)
2. Debt issuance (DISS)
3. Total net payout over profits (NPOP)

$$Payout = z(z_{EISS} + z_{DISS} + z_{NPOP}) \tag{21}$$

Each of these scores are then combined into a single quality score for each asset. According to the following formula.

$$Quality = z(Profitability + Growth + Safety + Payout) \tag{22}$$

Once that score has been obtained it is put through the same procedure as the Fama-French factors, by constructing 4 portfolio that goes long in small and big quality and short in small and big junk. Where junk is defined as a low quality score. Resulting in the following factor.

$$QMJ = \frac{(S/Q + B/Q) - (S/J + B/J)}{2} \tag{23}$$

When measuring idiosyncratic volatility with QMJ it is used as a complement to the 3-factor and 5-factor models leading to the following modifications to the the respective models.
For the 3-factor model

$$r_{it} = \alpha + \beta_1 \times Mkt_{it} + \beta_2 \times SMB_t + \beta_3 \times HML_t + \beta_4 \times QMJ_t + \varepsilon_{it} \tag{24}$$

$$\varepsilon_{it} = r_{it} - (\alpha + \beta_1 \times Mkt.r_t + \beta_2 \times SMB_t + \beta_3 \times HML_t + \beta_4 \times QMJ_t) \tag{25}$$

For the 5-factor model

$$r_{it} = \alpha + \beta_1 \times Mkt_{it} + \beta_2 \times SMB_t + \beta_3 \times HML_t + \beta_4 \times CMA_t + \beta_5 \times RMW_t + \beta_6 \times QMJ_t + \varepsilon_{it} \tag{26}$$

$$\varepsilon_{it} = r_{it} - (\alpha + \beta_1 \times Mkt.r_t + \beta_2 \times SMB_t + \beta_3 \times HML_t + \beta_4 \times CMA_t + \beta_5 \times RMW_t + \beta_6 \times QMJ_t) \tag{27}$$

The errors are then computed as the idiosyncratic volatility in the same manner as in (6).

2.3 Portfolio evaluation

In the previous section the ground work has been laid in terms of estimating idiosyncratic volatility. In this section sort test based on the idiosyncratic volatility will be applied to investigate whether or not there is any significant mispricing of the portfolios due to the exclusion of idiosyncratic volatility as a pricing variable.

This is similar to the methodology previous studies have applied. Ang et al (2006)(2009) took this approach both in their original and subsequent paper on idiosyncratic volatility and found significant negative mispricing in the high volatility portfolio. They however only used the Fama-French 3-factor model to evaluate the portfolio while controlling for other factors through double sorting. Each individual double sorted yielded persistent negative 3-factor αs indicating a significant mispricing when sorting on idiosyncratic volatility. Han, Hu, Lesmond (2015) using 2 different datasets, one with transaction price based returns and one with quote-midpoint based returns. Using these 2 datasets Han, Hu and Lesmond are able to show that liquidity bias plays a role in the portfolio return evaluation. The dataset used by Han, Hu and Lesmond does however in their Fama-French 3-factor model show less significant mispricing from the beginning then the data used in this paper, which provides a broader dataset containing more assets.

Using the different measures of idiosyncratic volatility the assets are sorted into different value-weighted quintile portfolios, where each individual assets market capitalization at the beginning of the period is used as weight. This gives us 5 portfolios with idiosyncratic volatility ranging from low to high. Each assets idiosyncratic volatility is measured through one of four models, Fama French 3-factor (FF3), Fama French 3-factor controlling for quality (FF3+QMJ), Fama French 5-factor (FF5) and Fama French 5-factor controlling for quality (FF5+QMJ). Each model measures idiosyncratic volatility on a monthly basis by applying their respective return regression on daily data, for FF3 this would be (5). We then compute the idiosyncratic volatility for that month through equation (6). Using this idiosyncratic volatility measure over the past month we sort each asset at the beginning of the month into quintiles. These portfolios are then held over the next month before the procedure is repeated. We subsequently evaluate each portfolio using the same model as was used in estimating the idiosyncratic volatility, for FF3 this would be (4). Of particular interest in this regression model is the α as its signage and significance level indicates whether there is any mispricing present. A positive (negative) and significantα would indicate that a portfolio overperformed (underperformed) given its risk exposure. A 6'th portfolio is also included which also may be of particular interest, in this portfolio a long position is taken in the high volatility portfolio and a short position in low (H-L). Given a significant α in this portfolio there is a significant mispricing in a net-zero investment portfolio. The other coefficients would indicate the factor loading differences between the portfolios. All standard errors are Newey-West corrected with 9-lags to account for heteroskedasticity and autocorrelation in the error term (Newey West, 1987)

2.4 Cross-sectional return regressions

In the following section Fama-Macbeth regressions (Fama MacBeth, 1973) will be introduced. Fama-Macbeth used this type of regression as a test for the Sharpe-Lintner CAPM. The model tests CAPM by checking if the β exposure of a stock explains the cross-sectional variation in returns. This is done by first estimating β. In a standard CAPM model only the market return is used, but in the following several different models will be estimated to check for variations and consistencies.

There are important similarities here both to Ang et al (2009) and Han, Hu Lesmond (2015). Following their lead we control for book-to-market and size in the cross-section as Daniel and Titman (1997) indicate that the firm characteristics themselves and not the covariance structure of the returns explain the cross-sectional variation. Beyond that we also include the 6-month lagged return to account for the momentum effect (Jegadeesh Titman, 1993) as well as the 1-month lagged return as a control for the reversal effect which Huang et al (2009) demonstrated can cause negative bias in the cross-sectional regressions. Finally we also control for the spread which Han, Hu and Lesmond (2015) demonstrated causes a similar negative bias in the cross-section. Where this paper differs from the previous ones is in the inclusion the quality factor and the 2 new Fama-French factors.

The models applied will be the same ones used in the sort tests, (FF3),(FF3+QMJ),(FF5) and (FF5+QMJ). As such each different model begins with the estimation of each individual assets exposure to each models factors. In the first step using daily data each model is run on a monthly basis to estimate each assets individual factor loading. This creates alot of cross-sectional datasets where for each month there there is a number of observations equal to the number of assets available in that month. These estimates are then used in the following model:

$$r_{it} = \lambda_i \sigma_{it-1} + \sum \gamma_i * \beta it + \sum \delta_i * z_{it} \tag{28}$$

Where σ_{it-1} refers to the previous months idiosyncratic volatility and λ is the risk premium associated with idiosyncratic volatility. β_{it} Refers to each assets individual factor loading estimated in the first step, making γ_i the cross-sectional price of each risk factor. The last part of the equation refers to the various controls used to produce an unbiased estimate. Once each cross-sectional regression is done the mean of all the cross-sectional $\lambda's, \gamma's$ and $\delta's$ are taken to form the coefficients.

In this model each asset is given the same weight, this since each individual asset regardless of size in each cross-section only has one observation. As such there are concerns that smaller-cap assets may distort the findings by having a disordinate amount of weight in the regression. Therefore a special version of the GLS where the diagonal elements equals the inverse of the market cap of an asset and all off diagonal elements are zero, called a weighted-least-squares. All standard errors are Newey-West corrected with 9-lags to account for heteroskedasticity and autocorrelation in the error term (1987)

3 Results

In the following section the results will be presented. In the previous section the methodology was thoroughly discussed and will be applied here to statistically test the performance of the portfolios as well as test the cross-sectional return. First the sort tests will be investigated to check how the portfolios sorted on idiosyncratic volatility perform relative to their respective models, with a particular focus on over- and under-performance as measured through the α of the portfolio. In the second part Fama-Macbeth cross-sectional regressions will be applied to determine each models cross-sectional pricing ability as well as their potential differences with a keen eye on how idiosyncratic volatilities pricing ability is affected by the quality factor.

3.1 Portfolio evaluation

Table 1 shows the descriptive statistics for the 5 portfolios. From the mean returns it can clearly be seen that they are falling monotonously when moving from low to high volatility portfolios. As expected the volatility of the returns increases as well. In general the difference in return between the lowest and highest quintile varies around 1%, with the high volatility portfolios being close to 0% and the low being close to 1%. In the all models a sharp drop off in mean return is present between the 3.rd and 4.th quintile and the 4.th and 5.th. The standard deviation seemingly increases more evenly across the quintiles and do not vary across the different models. From the correlation table in the appendix it is also apparent that all measures of idiosyncratic volatility, regardless of model choice are highly correlated. The smallest correlation being as big as 99.2%, between the FF3 model and the FF5+QMJ, and the largest being 99.8%, between FF3 and FF3+QMJ.

Table 1: Descriptive statistics

In the following table the mean return and standard deviation of 5 portfolios sorted on idiosyncratic volatility are presented, where low refers to the lowest idiosyncratic volatility quintile and (2), (3), (4) and high to subsequently portfolios with a rising idiosyncratic volatility. Each portfolio consists of stocks residing on the NYSE/ AMEX/ NASDAQ exchanges with prices below 5. The sample period is 1984-2008. All t-statistics are Newey-West corrected.

	FF3		FF3+QMJ		FF5		FF5+QMJ	
Quintile	Mean	Std.dev	Mean	Std.dev	Mean	Std.dev	Mean	Std.dev
Low	0.971	3.826	0.964	3.844	0.956	3.896	0.959	3.890
2	0.886	4.818	0.903	4.803	0.823	4.905	0.807	4.925
3	0.857	6.095	0.840	6.117	0.840	6.109	0.790	6.142
4	0.443	7.736	0.510	7.799	0.498	7.750	0.554	7.585
High	−0.014	9.157	−0.073	9.084	0.036	9.071	0.049	9.141

Table 2 shows the results from the regression of portfolio returns on the Fama-French 3-factor model. From the results we can clearly see a monotonically falling FF3-α starting out positive but insignificant in the low-volatility portfolio and then becoming negative in the (2) and (3) finally reaching a significant negative level in the 2 most volatile portfolios. This indicates that the Fama-French 3-factor model is unable to price these portfolios and they underperform relative to their risk exposure. The final portfolio, a net zero portfolio that goes long in high idiosyncratic volatility, and short low volatility is both highly negative and significant compared to other quintile portfolios. These findings are largely to be expected as the model mirrors that of the classic volatility puzzle sort tests. Both Ang et al (2006)(2009) and Han, Hu and Lesmond (2015) document the same relationship using the same model, it should however be noted that Han, Hu and Lesmond noted a far lower degree of mispricing than is present in this sample. Looking at the other variables we can see a monotonous movement in the coefficients, Mkt-RF increases when going from low to high volatility, indicating an increasing exposure to market risk as idiosyncratic volatility increases. HML decreases monotonously across the quintiles, this indicates that in the low volatiltiy portfolio, with a significant positive coefficient, assets are high book-to-market, or growth assets. As volatility increases the portfolio becomes more and more comprised of low book-to-market assets, or value assets. A similar relationship can be observed in the SMB factor where there is a monotonous movement from significant negative in the low portfolio, indicating that the portfolio is comprised of big stocks, towards significant positive in the high portfolio, indicating that the portfolio is comprised of small stocks. In the high minus low portfolio we can clearly see across all characteristics that there is significant relationship, indicating that there is a significant difference in the factor loadings between the high and low portfolios.

Table 3 runs the Fama-French model but this time includes quality minus junk factor both in the estimation of the residuals and in the sort test regression. The relationship observed from table 2 is practically gone. There are negative significant FF3-αs but they are now present in the low volatility portfolios and have lower t-statistics then the results in table 2. This finding runs quite contrary to any of the previous findings concerning sort tests. Ang et al (2006)(2009) is unable to remove the relationship using any of the tests, allthough some such as analyst dispersion does lower the significance. Han, Hu and Lesmond (2015) is able to make the mispricing disappear entirely. These results therefor stand out in that they reverse the signage of the alphas. The high minus low portfolio however shows a very insignificant alpha, indicating that there isn't any abnormal return to be gained from the net zero position portfolio. Interestingly we can see that Mkt-RF is now more evenly distributed with the extreme portfolios having the lowest coefficients and the high minus low portfolio not having any significant difference in market exposure. The HML coefficients has increased in amplituted as well as significance level indicating a larger disparity between the portfolios in HML once quality is added to equation. The reverse effect can however be seen in the SMB portfolio where the amplituted decreases across all coefficients as well as the significance level. The previously documented relationship where the high volatility portfolios consist of smaller and value stocks, and the low volatility portfolios consisting of large growth stocks is still clearly present here. QMJ enters the regression significant across the board, it has a highly positive coefficient in the low portfolio and decreases monotonously as volatility increases. This indicates that the low volatility portfolios consist of quality stocks and the further you move along the quintiles junk stocks tend to take over with the high portfolio having a large negative and significant coefficient. In fact in the high portfolio the amplitude of QMJ is larger than any other variable including Mkt-RF indicating that QMJ is a very important factor in pricing these portfolios.

Table 2: Fama-french 3-factor sort tests

The following table shows the result of a sort test conducted on idiosyncratic volatility. First stocks are sorted in accordance with their idiosyncratic volatility into quintiles. This is done on by first regressing each assets daily return over a month on the Fama-French 3-factor model, and then taking the standard deviation of the returns. Once the portfolios have been constructed each portfolio is regressed relative to the Fama-French 3-factor model. The variable of interest here is particularly the α, which would indicate systematic mispricing of the portfolios. Mkt-RF refers to the market beta, HML to the value factor beta and SMB to the size factor beta, where beta is to be interpreted as the risk exposure relative to each factor. Each portfolio consists of stocks residing on the NYSE/ AMEX/ NASDAQ exchanges with prices below 5. The sample period is 1984-2008. All t-statistics are Newey-West corrected.

	Dependent variable: Excess returns					
	(Low)	(2)	(3)	(4)	(High)	(H-L)
α	0.102	−0.042	−0.052	−0.437***	−0.925***	−1.420***
	1.603	*−0.515*	*−0.533*	*−2.862*	*−3.815*	*−5.042*
Mkt-RF	0.874***	1.065***	1.187***	1.285***	1.346***	0.470***
	38.935	*43.002*	*38.777*	*24.260*	*14.139*	*4.286*
HML	0.179***	0.076*	−0.189***	−0.463***	−0.468***	−0.642***
	2.922	*1.809*	*−2.706*	*−4.319*	*−2.638*	*−2.931*
SMB	−0.225***	−0.010	0.336***	0.713***	1.086***	1.321***
	−9.089	*−0.141*	*4.559*	*9.170*	*10.089*	*10.915*
R^2	0.935	0.929	0.918	0.885	0.803	0.644
Adj R^2	0.934	0.928	0.917	0.884	0.801	0.641

Note: t-statistics in *italics* — *p<0.1; **p<0.05; ***p<0.01

Table 3: Fama-French 3-factor sort test with quality control

The following table shows the result of a sort test conducted on idiosyncratic volatility. First stocks are sorted in accordance with their idiosyncratic volatility into quintiles. This is done on by first regressing each assets daily return over a month on the Fama-French 3-factor model and the quality factor, and then taking the standard deviation of the returns. Once the portfolios have been constructed each portfolio is regressed relative to the Fama-French 3-factor model with the addition of the quality factor. The variable of interest here is particularly the α, which would indicate systematic mispricing of the portfolios. Mkt-RF refers to the market beta, HML to the value factor beta, SMB to the size factor beta and QMJ to the quality beta, where beta is to be interpreted as the risk exposure relative to each factor. Each portfolio consists of stocks residing on the NYSE/ AMEX/ NASDAQ exchanges with prices below 5. The sample period is 1984-2008. All t-statistics are Newey-West corrected.

	Dependent variable: Excess return					
	(Low)	(2)	(3)	(4)	(High)	(H-L)
α	−0.163**	−0.162*	0.144	0.270*	0.166	−0.060
	−2.258	*−1.667*	*1.324*	*1.829*	*0.671*	*−0.224*
Mkt-RF	0.976***	1.113***	1.110***	1.027***	0.925***	−0.054
	42.760	*32.980*	*30.800*	*21.838*	*14.056*	*0.754*
HML	0.220***	0.110***	−0.223***	−0.609***	−0.652***	−0.869***
	4.952	*2.627*	*−3.468*	*−6.703*	*−4.716*	*−6.084*
SMB	−0.129***	0.058	0.253***	0.470***	0.568***	0.706***
	−4.624	*0.863*	*3.515*	*5.641*	*5.408*	*6.552*
QMJ	0.328***	0.172**	−0.280***	−0.809***	−1.447***	−1.781***
	5.256	*2.509*	*−4.154*	*−7.893*	*−8.090*	*−9.861*
R^2	0.959	0.932	0.928	0.916	0.880	0.800
Adj R^2	0.958	0.931	0.927	0.915	0.878	0.797

Note: t-statistics in *italics* — *p<0.1; **p<0.05; ***p<0.01

Table 4 depicts the Fama-French 5-factor model sort test. This test unlike most literature all available literature on the idiosyncratic volatility puzzle focuses on the new and improved 5-factor model. Allthough the model is not without its flaws (Blitz et al 2017), it does significantly improve upon the 3-factor models explanatory power and thus may be of interest in the performance evaluation. Initially it appears as though the systematic mispricing documented in table 2 has evaporated. The previously documented monotonous tendency in the 3-factor models α is gone and now replaced with insignificant coefficients across all quintiles. The idiosyncratic voaltility puzzles mispricing does however still remain in the high minus low portfolio where there is, as in previous literature Ang(2006)(2009), a significant negative mispricing. The 5-factor model does also have a curious impact on the other variables in the model. The HML factor loses its significance across 2/5 portfolios and its amplitude is far smaller than in the previous models. The SMB of the 5-factor model looks similar to that of the 3-factor model, without controlling for quality. It is significant across the board and rises monotonously from the low to high portfolio indicating the previously documented relationship about the size of the stocks in the portfolios. RMW enters the model significant across the board, being positive in the low portfolio and then falling with each subsequent quintile, this indicates that the low portfolio consists of assets with more robust operating profit and that the high portfolio consists of assets with weaker operating profit assets. CMA follows the pattern described in RMW although the findings tend to be less robust and and smaller in amplitude, where the low portfolio consists of assets with conservative investment strategy and the high portfolio of assets with aggressive investment strategies. In the high minus low portfolio clearly indicates a disparity in the factor loadings of the high and low portfolio where the high portfolio consists of larger firms with weak operating profits and aggressive investment strategies with a higher market risk exposure while being unable to distinguish between the book-to-market characteristic of the portfolio.

Table 5 depicts the final sort test where the Fama-French 5-factor model supplemented by the quality factor. This provides the final sorting test to which there is no analog in previous literature. In it we can clearly see the presence of the mispricing observed in table 3. The α as in table 3 has an inverse relationship to the one typically observed in the idiosyncratic volatility puzzle. The α starts out weakly significant and negative and moves towards being positive and significant in (3) and (4), before finally dropping of to being positive and insignificant in the high portfolio. In the high minus

Table 4: Fama-French 5-factor sort test

The following table shows the result of a sort test conducted on idiosyncratic volatility. First stocks are sorted in accordance with their idiosyncratic volatility into quintiles. This is done on by first regressing each assets daily return over a month on the Fama-French 5-factor model, and then taking the standard deviation of the returns. Once the portfolios have been constructed each portfolio is regressed relative to the Fama-French 5-factor model. The variable of interest here is particularly the α, which would indicate systematic mispricing of the portfolios. Mkt-RF refers to the market beta, HML to the value factor beta, SMB to the size factor beta, RMW to the profitability beta and CMA to the investment beta, where beta is to be interpreted as the risk exposure relative to each factor. Each portfolio consists of stocks residing on the NYSE/ AMEX/ NASDAQ exchanges with prices below 5. The sample period is 1984-2008.All t-statistics are Newey-West corrected.

	Dependent variable: Excess returns					
	(Low)	(2)	(3)	(4)	(High)	(H-L)
α	−0.040	−0.115	0.098	0.062	−0.166	−0.517**
	−0.697	*−1.492*	*0.909*	*0.503*	*−0.695*	*−1.998*
Mkt-RF	0.939***	1.069***	1.138***	1.158***	1.109***	0.168**
	65.452	*47.864*	*35.127*	*33.389*	*16.980*	*2.401*
HML	0.064***	0.098***	−0.059	−0.178**	−0.124	−0.184
	2.884	*2.663*	*−0.776*	*−2.069*	*−1.026*	*−1.454*
SMB	−0.164***	0.117***	0.285***	0.500***	0.712***	0.888***
	−7.365	*3.263*	*4.320*	*5.575*	*7.332*	*8.699*
RMW	0.243***	0.194***	−0.165**	−0.654***	−1.203***	−1.448***
	5.248	*4.228*	*−2.220*	*−7.458*	*−7.202*	*−7.869*
CMA	0.182**	−0.189***	−0.260***	−0.466***	−0.439**	−0.622**
	2.418	*−2.965*	*−3.092*	*−4.059*	*−2.204*	*−2.511*
R^2	0.963	0.947	0.928	0.924	0.880	0.791
Adj R^2	0.963	0.946	0.926	0.923	0.877	0.787

Note: t-statistics in *italics* — *p<0.1; **p<0.05; ***p<0.01

low portfolio there is no discernible α. Infact the amplitude is very small and very insignificant in this portfolio. Mirroring the results in table 3 it can once again be observed that there has been leveling of the market exposure across all 5 portfolios with there being no significant difference in the high minus low portfolio. Compared to table 4 it is however possible to see somewhat of revival of the HML factor making the relationship from table 2 and 3 once again visible. The SMB remains largely unchanged from table 4 still bearing a monotonously falling coefficient. RMW and CMA largely deteriorate in the presence of QMJ, both them suffer from shrinking coefficients in the presence of quality and both lose alot in terms of significance level. RMW remains significant in the extreme portfolios and low while CMA remains significant across the board but with lower t-stats then previously. QMJ enters the regression in much the same way it did in table 3, it has a monotonous falling coefficient as idiosyncratic volatility beomces higher while only being insignificant in (2) portfolio. The low portfolio has a positive coefficient while all others have a negative, especially the (High) portfolio carries a rather high degree of junk stocks being a significant negative coefficient with large amplitude. Looking at the high minus low portfolio it is clear that the net zero portfolio consists of large value stocks with weak operating profits, aggressive investment strategy and tend to be junkier. Of all the coefficients in the high minus low portfolio QMJ carries the largest amplitude more than doubling any other coefficient in absolute size.

Table 5: Fama-French 5-factor sort test with quality control

The following table shows the result of a sort test conducted on idiosyncratic volatility. First stocks are sorted in accordance with their idiosyncratic volatility into quintiles. This is done on by first regressing each assets daily return over a month on the Fama-French 5-factor model and the quality factor, and then taking the standard deviation of the returns. Once the portfolios have been constructed each portfolio is regressed relative to the Fama-French 5-factor model with the addition of the quality factor. The variable of interest here is particularly the α, which would indicate systematic mispricing of the portfolios. Mkt-RF refers to the market beta, HML to the value factor beta, SMB to the size factor beta, RMW to the profitability factor, CMA to the investment factor and QMJ to the quality beta, where beta is to be interpreted as the risk exposure relative to each factor. Each portfolio consists of stocks residing on the NYSE /AMEX/ NASDAQ exchanges with prices below 5. The sample period is 1984-2008. All t-statistics are Newey-West corrected.

	Dependent variable: Excess returns					
	(Low)	(2)	(3)	(4)	(High)	(H-L)
α	−0.142*	−0.071	0.280***	0.350**	0.338	0.095
	−1.825	*−0.986*	*2.928*	*2.413*	*1.353*	*0.318*
Mkt-RF	0.983***	1.049***	1.038***	1.029***	0.900***	−0.088
	39.364	*41.342*	*34.087*	*23.167*	*15.006*	*−1.128*
HML	0.146***	0.039	−0.199**	−0.356***	−0.480***	−0.627***
	4.086	*1.006*	*−2.558*	*−3.739*	*−3.651*	*−3.602*
SMB	−0.136***	0.092**	0.229***	0.449***	0.559***	0.704***
	−4.001	*2.441*	*4.100*	*5.758*	*5.265*	*5.589*
RMW	0.069**	0.290***	0.171	−0.200	−0.412**	−0.474**
	2.094	*3.238*	*1.599*	*−1.504*	*−2.416*	*−2.469*
CMA	0.130**	−0.126*	−0.230**	−0.346***	−0.310*	−0.438**
	2.215	*−1.910*	*−2.578*	*−3.185*	*−1.860*	*−2.280*
QMJ	0.261***	−0.159	−0.515***	−0.611***	−1.151***	−1.426***
	2.970	*−1.643*	*−5.129*	*−3.519*	*−4.521*	*−4.175*
R^2	0.963	0.948	0.937	0.932	0.888	0.813
Adj R^2	0.962	0.947	0.936	0.931	0.885	0.809

Note: t-statistics in *italics* *p<0.1; **p<0.05; ***p<0.01

To summarize these sort test it is clearly observable that both the Fama-French 5-factor sort and QMJ, irrespective of each other and in combination with each other does alot to explain the abnormal return suffered by portfolios sorted on idiosyncratic volatility. While the 5-factor model was unable to eliminate the relationship all together in the extreme high minus low portfolio, it was able to reduce the α's in each individual portfolio to insignificance. A curious effect emerged however with the inclusion of the QMJ factor in that it the low portfolio suddenly started to significantly underperform given its factor exposures. In the high minus low portfolio this effect however disappears again, both in the 3- and 5-factor model. In the 5-factor model it also shows that the middle portfolios, (3) and (4), suddenly outperform the others. Looking at the adjusted R^2 we see a tendency where the initial inclusion of the QMJ does improve it. Moving to the 5-factor model it also improves upon the R^2 except in the high portfolio where it remains close to the QMJ model, in the high-minus-low portfolio however we can see that the QMJ 3-factor model does outperform the 5-factor model in explaining the performance. The 5-factor model with QMJ largely mirrors the adjusted R^2 in the low volatility portfolios but performs better in the higher volatility portfolios, being the best model to explain the high minus low portfolio as well.

3.2 Cross-sectional return test

Theoretically, given a perfect pricing model all the coefficients, factor exposures, estimated in the first step of a Fama-Macbeth regression would suffice to explain the cross-sectional variation among assets. This can be interpreted as all coefficient being significant and the α as being insignificantly different from zero. As such the α of the pricing model is of particular interest in the coming tests. Beyond that the second step will include the respective idiosyncratic volatility over the past month for each asset. This is the main variable of interest is the idiosyncratic volatility which typically enters the analysis with a large negative coefficient, (Ang et al, 2009; Jiang,Xu Yao, 2009; Li, Hou Zhang, 2020; Chichernea, Kassa Slezak, 2018; Han, Hu Lesmond, 2015). Beyond these variables, the variables in table 6 will be only the ones that differ between the models. In the Appendix there will be a separate table containing the full regression results.

Looking first at (FF3) the negative and significant pricing of idiosyncratic volatility is clearly present. It is estimated to be -1.048% and is statistically very significant. There is also a significant positive α indicating that the factor exposures and control variables, alongside idiosyncratic volatility are unable to fully explain the cross-sectional variation in returns. These results align with those found in the papers mentioned above and seemingly confirm the presence of negative pricing of idiosyncratic volatility. The (FF3+QMJ) does little to improve upon these results. While β_{QMJ} enters the model with a significant slight negative yet highly significant price. From the full regression table in the appendix it is also apparent that its inclusion leaves all other parameters unaffected. In this aspect it would seem that the original Fama-French 3-factor model does as well as without the QMJ factor as with it. In the next model the QMJ factor is removed and the 5-factor model is applied instead. Once again this does little to change any of the results, the risk premium of idiosyncratic volatility remains as stable as before and RMW and CMA enter the model insignificant. *alpha* is also largely unchanged and significant.The last model (FF5+QMJ) does just as poorly as the other models when it comes to explaining the idiosyncratic volatility premium. β_{QMJ} Appears in the same fashion as it did in the (FF3+QMJ) model with a small negative and highly significant. β_{CMA} and β_{RMW} do little to improve upon the matter as they both appear largely as they did in the (FF5) model.

One drawback with these Fama-Macbeth cross-sectional regressions highlighted in the methodology chapter is that it gives equal weights to all assets which causes concerns that the noise generated by the disproportional weight put on small-cap assets distort the results. Therefor the previous model is reproduced but now through a weighted-least-squares regression where each cross-sectional coefficient is the weighted coefficient. As with the previous regression the full regression table can be found in the appendix.

The results in (FF3) generally align with the same model in table 6. There is a significant positive α indicating that the model is not sufficient to explain the cross-sectional return of assets, it is however less significant than those observed in table 6. The cross-sectional premium associated with idiosyncratic volatility is also significant and negative indicating the persistence of idiosyncratic volatility puzzle in a value weighted sample. This mirrors the findings of Ang et al (2009) who found very similar results both in size and scope. In the (FF3+QMJ) model quality does lower both size and significance of the idiosyncratic volatility premium, this indicates that β_{QMJ} does actually poses some power in explaining the negative returns associated idiosyncratic volatility. β_{QMJ} Itself however enters the regression low in size and insignificant, as opposed to the same model in table 6, where it enters significantly negative. The next model (FF5) does further improve somewhat on the results by lowering the significance of α and idiosyncratic volatility, while idiosyncratic volatility is still highly significant it has moved to be just on the other side of t-statistic breakpoint. The size of the idiosyncratic volatility is also slightly smaller than the both previous models. β_{RMW} and β_{RMW} both enter with positive coefficients this time but remain insignificant. In the final model (FF5+QMJ) results are improved upon somewhat more with the inclusion β_{QMJ}. While all factor exposures remain insignificant in table 7 there is a clear difference in the size and significance of the idiosyncratic volatility's premium between the (FF3) and (FF5+QMJ) model.

Table 6: Equally-weighted Fama-Macbeth regressions

The following table shows a summary of the result of Fama-Macbeth regression conducted on individual assets. For each month in the sample daily data is used to estimate the first step which are the β coefficients relative to the model of choice. Once the first step has been estimated the β coefficients themselves along with the idiosyncratic volatility over the past month are regressed cross-sectionally upon monthly returns alongside several controls, the procedure is repeated each month and the coefficients are estimated as the average across all cross-sections. The coefficient for each risk-factor and control is available in the full regression table in the Appendix. The coefficients reported in the table below are the main ones of interest. A significant alpha indicates that the exposure to the individual risk factors and controls are unable to explain in full the pricing of an asset. (FF3), (FF3+QMJ), (FF5) and (FF5+QMJ) each refer to different models used to estimate idiosyncratic volatility. They each also differ in the first step. In the (FF3) model Mkt-RF, SMB and HML β are estimated and regressed upon the excess returns in the second step. (FF3+QMJ) applies the same model but with the addition of the QMJ factor. (FF5) applies the same model as (FF3) but includes CMA and RMW, and finally (FF5+QMJ) does the same but with the QMJ factor included. Since no weighting is implemented all assets in a cross section carry the same weight. Each model consists of stocks residing on the NYSE/ AMEX/ NASDAQ exchanges with prices below 5. The sample period is 1984-2008. All t-statistics are Newey-West corrected.

	Dependent variable:			
	Excess returns			
	(FF3)	(FF3+QMJ)	(FF5)	(FF5+QMJ)
α	1.927**	1.928**	1.882**	1.907**
	2.481	*2.486*	*2.366*	*2.417*
IV	−1.048***	−1.036***	−1.093***	−1.124***
	−3.169	*−3.177*	*−3.148*	*−3.184*
β_{QMJ}		−0.132***		−0.131***
		−2.908		*−2.791*
β_{CMA}			−0.021	−0.019
			−0.509	*−0.475*
β_{RMW}			−0.037	−0.046
			−0.938	*−0.845*

Note: t-statistics in *italics* *p<0.1; **p<0.05; ***p<0.01

Table 7: Value-weighted Fama-Macbeth regressions

The following table shows a summary of the result of Fama-Macbeth regression conducted on individual assets. For each month in the sample daily data is used to estimate the first step which are the β coefficients relative to the model of choice. Once the first step has been estimated the β coefficients themselves along with the idiosyncratic volatility over the past month are regressed cross-sectionally upon monthly returns alongside several controls, the procedure is repeated each month and the coefficients are estimated as the average across all cross-sections. In each regression each asset is weighted in accordance with their market-capitalization. The coefficient for each risk-factor and control is available in the full regression table in the Appendix. The coefficients reported in the table below are the main ones of interest. A significant alpha indicates that the exposure to the individual risk factors and controls are unable to explain in full the pricing of an asset. (FF3), (FF3+QMJ), (FF5) and (FF5+QMJ) each refer to different models used to estimate idiosyncratic volatility. They each also differ in the first step. In the (FF3) model Mkt-RF, SMB and HML β are estimated and regressed upon the excess returns in the second step. (FF3+QMJ) applies the same model but with the addition of the QMJ factor. (FF5) applies the same model as (FF3) but includes CMA and RMW, and finally (FF5+QMJ) does the same but with the QMJ factor included. Each model consists of stocks residing on the NYSE/ AMEX/ NASDAQ exchanges with prices below 5. The sample period is 1984-2008. All t-statistics are Newey-West corrected

	Dependent variable:			
	Excess returns			
	(FF3)	(FF3+QMJ)	(FF5)	(FF5+QMJ)
α	1.502**	1.516**	1.497*	1.471*
	1.943	*1.933*	*1.879*	*1.854*
IV	−1.357***	−1.272**	−1.222**	−1.194**
	−2.813	*−2.707*	*−2.599*	*−2.503*
β_{QMJ}		0.063		0.083
		0.948		*1.151*
β_{CMA}			0.065	−0.059
			0.927	*0.835*
β_{RMW}			0.106	0.111
			1.253	*1.287*

Note: t-statistics in *italics* *p<0.1; **p<0.05; ***p<0.01

4 Discussion

In the following section the results will presented in the previous chapter will be summarized and the key findings put into context with the exisiting literature on the subject.

The results from the sort tests above are quite significant and indicate that there is no significant over- or underperformance present in the idiosyncratic volatility sorts once quality has been controlled for. This finding significantly contributes to the findings of Ang et al (2009) Jiang,Xu and Yao (2009) Li, Hou and Zhang (2020) Chicherna, Haimanot and Slezak (2018) Han, Hu and Lesmond (2015), who all document a disparity in the form of a large negative α taking form in the high volatility portfolios. As such one potential explanation to this anomaly is the portfolios makeup of quality and junk stocks. From the sort tests it becomes quite apparent that the quality through its large negative and significant coefficient plays an important role in determining the performance of the portfolios in the idiosyncratic volatility puzzle. RMW and CMA, the new Fama-French factors, play a similar role, a fact that is not entirely surprising since RMW is closely related to the quality factor, as is apparent from the factor correlation matrix in the appendix. Judging from the test conducted in this paper the idiosyncratic volatility puzzle is better served being by the inclusion of the quality factor as opposed to the other Fama-French 5-factors as it alone is able to remove the significance of α in the sort tests.

Moving towards cross-sectional return of assets however the inclusion of quality provides little insight into the cross-sectional pricing of idiosyncratic volatility. In results mirroring those seen in Ang et al (2009) Jiang,Xu and Yao (2009) Li, Hou and Zhang (2020) Chicherna, Haimanot and Slezak (2018), while quality seemingly has some effect on the pricing of idiosyncratic volatility once the cross-section has been value weighted its effect is negligible. As a matter of fact Ang et al (2009) and Han, Hu and Lesmond (2015) are both able to explain the cross-sectional variation far better using variables related to liquidity and sample corrected for liquidity bias. As such the cross-sectional analysis provides no potential answer as to why assets experience a negative premium on the previous periods idiosyncratic volatility.

Another question also arises from the sort test, namely that once the findings in Ang et al (2006)

(2009) has been corrected for using quality the low portfolio underperforms and the higher volatility portfolios are insignificant or significant positive, with the exception of the high volatility portfolio which remains insignificant. Some important qualities are distinguished associated with high and low volatility becomes apparent when controlling for quality. First the high minus low idiosyncratic volatility stocks seemingly co-varies to a higher degree with small value junk stocks with weak operating profits and aggressive investments. Interestingly their exposure to Mkt-RF does not vary significantly across portfolios once quality is controlled for.

All the findings are documented across a longer more recent sample available in the appendix, showing that the findings in this paper is not a finding merely due to the choice of time frame.

The findings in this paper are however able to improved upon, especially the findings in Han, Hu and Lesmond (2015) should guide further research into qualities impact on the idiosyncratic puzzle, as a combination of the 2 approaches may yield more robust answers. Qualities relationship to the puzzle ought to be further investigated by investigating its relationship to aggregate volatility as in Ang et al (2006). As well as cross-sectional Fama-Macbeth analysis conducted on portfolios to determine if its ability to price on a portfolio level differs from its poor performance on an asset level.

All available literature on the subject of the idiosyncratic volatility puzzle uses the Fama-French 3-factor model as basis for its analysis, this is likely to migrate towards analysis conducted by the more recent Fama-French 5-factor model over time. The findings in this paper however seem to indicate that future analysis would be better served by including the quality factor as its relationship to idiosyncratic volatility seems more robust than that of the 5-factor model.

5 Conclusion

In this section conclusions will be drawn regarding how quality impacts the idiosyncratic volatility puzzle.

Quality has curious relationship to the idiosyncratic volatility puzzle. While factor exposure to quality has performed disappointingly poor in explaining individual assets returns in light of the idiosyncratic volatility puzzle, it has provided valuable insights into the question of portfolio performance. Where previous models, the Fama-French 3-factor model, have been unable to explain the performance of portfolios sorted on idiosyncratic volatility, quality is able to explain the performance of the extreme portfolios. The underperformance of high idiosyncratic volatility portfolios are due to their relative junkiness, in fact the 2 portfolios with the highest idiosyncratic volatility in the Fama-French 3-factor and 5-factor models have far larger and significant coefficients in the quality factor than any other factor, except for the excess market return which is more significant.

Together these facts leads this paper to conclude that allthough qualitys' impact on the cross-sectional pricing of idiosyncratic volatility of assets is quite limited, it is an important parameter in explaining performance and as such should be it should be included in all portfolio analysis in the idiosyncratic volatility puzzle. The characteristics entailed in quality; profitability, growth, safety and payout, together help explain the abnormal performance in idiosyncratic volatility. A fact that is not accomplished by any model that leaves this factor out of the equation. Furthermore a direct analog in the form of the RMW factor, which also measures profitability, indicates that quality as a whole does a better job then one of its constituent parts does alone.

6 Refernces

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7 Appendix

Table A1: Correlation Matrix

Correlation matrix depicting the correlation between each assets idiosyncratic volatility, using different estimation models. Where FF3 refers to the idiosyncratic volatility relative to the Fama-French 3-factor model and FF3+QMJ refers to the same model with the addition of the quality factor. FF5 and FF5+QMJ refers to the idiosyncratic volatility relative to the Fama French 5-factor model and the same model but with the addition of the quality factor.

	FF3	FF3+QMJ	FF5	FF5+QMJ
FF3	1	0.998	0.995	0.992
FF3+QMJ	0.998	1	0.994	0.995
FF5	0.995	0.994	1	0.997
FF5+QMJ	0.992	0.995	0.997	1

Table A2: Factor correlation matrix

Depicted below is the correlation between each factor used in the methodology. QMJ stands for quality minus junk quality factor,Mkt-RF for the excess market return, HML for the high minus low book-to-market factor, SMB for the small minus big size factor,RMW for the robust minus weak operating profit and CMA for the conservative minus aggressive investment strategy.

	qmj	Mkt-RF	HML	SMB	RMW	CMA
QMJ	1	−0.562	0.212	−0.453	0.816	0.213
Mkt-RF	−0.562	1	−0.407	0.135	−0.367	−0.475
HML	0.212	−0.407	1	−0.229	0.427	0.687
SMB	−0.453	0.135	−0.229	1	−0.449	−0.091
RMW	0.816	−0.367	0.427	−0.449	1	0.216
CMA	0.213	−0.475	0.687	−0.091	0.216	1

Table A3: Equally weighted Fama-Macbeth regressions

The following table shows a summary of the result of Fama-Macbeth regression conducted on individual assets. For each month in the sample daily data is used to estimate the first step which are the β coefficients relative to the model of choice. Once the first step has been estimated the β coefficients themselves along with the idiosyncratic volatility over the past month are regressed cross-sectionally upon monthly returns alongside several controls, the procedure is repeated each month and the coefficients are estimated as the average across all cross-sections. A significant alpha indicates that the exposure to the individual risk factors and controls are unable to explain in full the pricing of an asset. (FF3), (FF3+QMJ), (FF5) and (FF5+QMJ) each refer to different models used to estimate idiosyncratic volatility. They each also differ in the first step. In the (FF3) model Mkt-RF, SMB and HML β are estimated and regressed upon the excess returns in the second step. (FF3+QMJ) applies the same model but with the addition of the QMJ factor. (FF5) applies the same model as (FF3) but includes CMA and RMW, and finally (FF5+QMJ) does the same but with the QMJ factor included. Since no weighting is implemented all assets in a cross section carry the same weight. Each model consists of stocks residing on the NYSE/ AMEX/ NASDAQ exchanges with prices below 5. The sample period is 1984-2008. All t-statistics are Newey-West corrected.

Table A3: Equally-weighted Fama-Macbeth regressions

	Dependent variable:			
	Excess returns			
	(FF3)	(FF3+QMJ)	(FF5)	(FF5+QMJ)
α	1.927**	1.928**	1.882**	1.907**
	2.481	*2.486*	*2.366*	*2.417*
IV	−1.048***	−1.036***	−1.093***	−1.124***
	−3.169	*−3.177*	*−3.148*	*−3.184*
β_{MKT}	0.259***	0.253***	0.247**	0.246**
	2.816	*2.753*	*2.647*	*2.610*
β_{SMB}	0.096**	0.095**	0.110**	0.107**
	2.052	*2.032*	*2.433*	*2.356*
β_{HML}	−0.011	−0.013	−0.017	−0.019
	−0.183	*−0.223*	*−0.261*	*−0.314*
log(size)	−0.129**	−0.129**	−0.126**	−0.127**
	−2.509	*−2.513*	*−2.402*	*−2.439*
log(B/M)	0.257**	0.260**	0.256**	0.258**
	2.426	*2.472*	*2.434*	*2.468*
1m reversal	0.003**	0.003**	0.003**	0.003**
	2.622	*3.026*	*2.518*	*2.467*
momentum 6m	0.010***	0.010***	0.011***	0.011***
	6.061	*6.223*	*6.050*	*6.110*
spread	0.065***	0.065***	0.059***	0.059***
	3.160	*3.194*	*2.854*	*2.819*
β_{QMJ}		−0.132***		−0.131***
		-2.908		*-2.791*
β_{CMA}			−0.021	−0.019
			−0.509	*−0.475*
β_{RMW}			−0.037	−0.046
			-0.938	*-0.845*

Note: t-statistics in *italics* *p<0.1; **p<0.05; ***p<0.01

Table A4: Value weighted Fama-Macbeth regressions

The following table shows a summary of the result of Fama-Macbeth regression conducted on individual assets. For each month in the sample daily data is used to estimate the first step which are the β coefficients relative to the model of choice. Once the first step has been estimated the β coefficients themselves along with the idiosyncratic volatility over the past month are regressed cross-sectionally upon monthly returns alongside several controls, the procedure is repeated each month and the coefficients are estimated as the average across all cross-sections. In each regression each asset is weighted in accordance with their market-capitalization. The coefficient for each risk-factor and control is available in the full regression table in the Appendix. The coefficients reported in the table below are the main ones of interest. A significant alpha indicates that the exposure to the individual risk factors and controls are unable to explain in full the pricing of an asset. (FF3), (FF3+QMJ), (FF5) and (FF5+QMJ) each refer to different models used to estimate idiosyncratic volatility. They each also differ in the first step. In the (FF3) model Mkt-RF, SMB and HML β are estimated and regressed upon the excess returns in the second step. (FF3+QMJ) applies the same model but with the addition of the QMJ factor. (FF5) applies the same model as (FF3) but includes CMA and RMW, and finally (FF5+QMJ) does the same but with the QMJ factor included. Each model consists of stocks residing on the NYSE/ AMEX/ NASDAQ exchanges with prices below 5. The sample period is 1984-2008. All t-statistics are Newey-West corrected.

Table A4: Value-weighted Fama-Macbeth regressions

	Dependent variable:			
	Excess returns			
	(FF3)	(FF3+QMJ)	(FF5)	(FF5+QMJ)
α	1.502**	1.516**	1.497*	1.471*
	1.943	*1.933*	*1.879*	*1.854*
IV	−1.357***	−1.272**	−1.222**	−1.194**
	−2.813	*−2.707*	*−2.599*	*−2.503*
β_{MKT}	−0.003	0.034	0.006	0.018
	−0.017	*0.195*	*0.033*	*0.099*
β_{SMB}	−0.002	−0.008	−0.005	−0.004
	−0.024	*−0.128*	*−0.086*	*−0.058*
β_{HML}	0.002	−0.016	−0.022	−0.028
	0.025	*−0.172*	*−0.232*	*−0.294*
log(size)	−0.040	−0.044	−0.044	−0.043
	−0.839	*−0.917*	*−0.895*	*−0.891*
log(B/M)	0.091	0.111	0.147	0.160
	0.562	*0.689*	*0.908*	*0.994*
1m reversal	0.005***	0.005***	0.004**	0.004**
	2.813	*2.863*	*2.541*	*2.620*
momentum 6m	0.012***	0.012***	0.012***	0.012***
	4.246	*4.176*	*4.350*	*4.226*
spread	−0.006	−0.009	−0.018	−0.020
	−0.168	*−0.256*	*−0.512*	*−0.574*
β_{QMJ}		0.063		0.083
		0.948		*1.151*
β_{CMA}			0.065	−0.059
			0.927	*0.835*
β_{RMW}			0.106	0.111
			1.253	*1.287*

Note: t-statistics in *italics* — *p<0.1; **p<0.05; ***p<0.01

Table A5: Fama-french 3-factor sort tests, long sample

The following table shows the result of a sort test conducted on idiosyncratic volatility. First stocks are sorted in accordance with their idiosyncratic volatility into quintiles. This is done on by first regressing each assets daily return over a month on the Fama-French 3-factor model, and then taking the standard deviation of the returns. Once the portfolios have been constructed each portfolio is regressed relative to the Fama-French 3-factor model. The variable of interest here is particularly the α, which would indicate systematic mispricing of the portfolios. Mkt-RF refers to the market beta, HML to the value factor beta and SMB to the size factor beta, where beta is to be interpreted as the risk exposure relative to each factor. Each portfolio consists of stocks residing on the NYSE/ AMEX/ NASDAQ exchanges with prices below 5. The sample period is 1984-2019. All t-statistics are Newey-West corrected.

	Dependent variable: Excess returns					
	(1)	(2)	(3)	(4)	(5)	(6)
α	0.229***	−0.020	−0.225**	−0.425**	−0.740***	−1.102***
	3.878	*−0.306*	*−2.556*	*−2.263*	*−3.125*	*−4.110*
Mkt-RF	0.808***	1.078***	1.262***	1.449***	1.694***	0.891***
	24.828	*39.339*	*41.082*	*16.438*	*12.421*	*5.396*
HML	0.175***	0.130***	−0.111**	−0.349***	−0.416**	−0.600***
	2.765	*4.725*	*−2.046*	*−3.025*	*−2.451*	*−2.657*
SMB	−0.228***	−0.054*	0.211***	0.634***	0.882***	1.108***
	−10.800	*−1.934*	*3.939*	*9.273*	*8.292*	*8.972*
R^2	0.915	0.936	0.916	0.866	0.799	0.624
Adjusted R^2	0.914	0.935	0.915	0.864	0.797	0.619

Note: t-statistics in *italics* *p<0.1; **p<0.05; ***p<0.01

Table A6: Fama-French 3-factor sort test with quality control, long sample

The following table shows the result of a sort test conducted on idiosyncratic volatility. First stocks are sorted in accordance with their idiosyncratic volatility into quintiles. This is done on by first regressing each assets daily return over a month on the Fama-French 3-factor model and the quality factor, and then taking the standard deviation of the returns. Once the portfolios have been constructed each portfolio is regressed relative to the Fama-French 3-factor model with the addition of the quality factor. The variable of interest here is particularly the α, which would indicate systematic mispricing of the portfolios. Mkt-RF refers to the market beta, HML to the value factor beta, SMB to the size factor beta and QMJ to the quality beta, where beta is to be interpreted as the risk exposure relative to each factor. Each portfolio consists of stocks residing on the NYSE/ AMEX/ NASDAQ exchanges with prices below 5. The sample period is 1984-2019. All t-statistics are Newey-West corrected.

	Dependent variable: Excess returns					
	(Low)	(2)	(3)	(4)	(High)	(H-L)
α	−0.005	−0.061	−0.020	0.218	0.260	0.133
	−0.098	*−0.668*	*−0.208*	*1.185*	*0.965*	*0.450*
Mkt-RF	0.923***	1.102***	1.169***	1.141***	1.128***	0.209*
	47.005	*33.784*	*41.761*	*20.608*	*11.214*	*1.870*
HML	0.167***	0.134***	−0.112**	−0.361***	−0.426***	−0.601***
	3.403	*4.589*	*−2.121*	*−4.005*	*−2.936*	*−3.509*
SMB	−0.126***	−0.048	0.125**	0.435***	0.488***	0.612***
	−5.789	*−1.361*	*2.484*	*7.204*	*4.591*	*5.347*
QMJ	0.283***	0.080	−0.299***	−0.725***	−1.312***	−1.596***
	5.367	*1.215*	*−5.274*	*−8.069*	*−8.740*	*−9.516*
R^2	0.941	0.925	0.930	0.913	0.864	0.758
Adjusted R^2	0.940	0.924	0.929	0.912	0.862	0.754

Note: t-statistics in *italics* *p<0.1; **p<0.05; ***p<0.01

Table A7: Fama-French 5-factor sort test, long sample

The following table shows the result of a sort test conducted on idiosyncratic volatility. First stocks are sorted in accordance with their idiosyncratic volatility into quintiles. This is done on by first regressing each assets daily return over a month on the Fama-French 5-factor model, and then taking the standard deviation of the returns. Once the portfolios have been constructed each portfolio is regressed relative to the Fama-French 5-factor model. The variable of interest here is particularly the α, which would indicate systematic mispricing of the portfolios. Mkt-RF refers to the market beta, HML to the value factor beta, SMB to the size factor beta, RMW to the profitability beta and CMA to the investment beta, where beta is to be interpreted as the risk exposure relative to each factor. Each portfolio consists of stocks residing on the NYSE/ AMEX/ NASDAQ exchanges with prices below 5. The sample period is 1984-2019.All t-statistics are Newey-West corrected.

	Dependent variable: Excess returns					
	(Low)	(2)	(3)	(4)	(High)	(H-L)
α	0.017	−0.090	0.054	−0.028	−0.196	−0.490**
	0.385	*−1.521*	*0.634*	*−0.287*	*−1.162*	*−2.503*
Mkt-RF	0.924***	1.070***	1.151***	1.197***	1.173***	0.249***
	84.653	*52.702*	*44.421*	*29.736*	*17.333*	*3.376*
HML	0.035	0.095**	−0.023	−0.093	0.033	−0.004
	1.578	*2.482*	*−0.362*	*−1.019*	*0.225*	*−0.024*
SMB	−0.152***	0.086**	0.303***	0.543***	0.708***	0.866***
	−6.744	*2.334*	*5.404*	*7.132*	*7.730*	*8.492*
RMW	0.240***	0.154***	−0.141**	−0.562***	−1.184***	−1.428***
	4.340	*3.215*	*−2.198*	*−7.895*	*−8.875*	*−8.576*
CMA	0.190***	−0.136**	−0.220***	−0.398***	−0.474***	−0.665***
	2.608	*−2.389*	*−2.904*	*−3.651*	*−2.635*	*−2.893*
R^2	0.954	0.939	0.925	0.911	0.850	0.716
Adjusted R^2	0.954	0.938	0.924	0.910	0.848	0.713

Note: t-statistics in *italics* *p<0.1; **p<0.05; ***p<0.01

Table A8: Fama-French 5-factor sort test with quality control, long sample.

The following table shows the result of a sort test conducted on idiosyncratic volatility. First stocks are sorted in accordance with their idiosyncratic volatility into quintiles. This is done on by first regressing each assets daily return over a month on the Fama-French 5-factor model and the quality factor, and then taking the standard deviation of the returns. Once the portfolios have been constructed each portfolio is regressed relative to the Fama-French 5-factor model with the addition of the quality factor. The variable of interest here is particularly the α, which would indicate systematic mispricing of the portfolios. Mkt-RF refers to the market beta, HML to the value factor beta, SMB to the size factor beta, RMW to the profitability factor, CMA to the investment factor and QMJ to the quality beta, where beta is to be interpreted as the risk exposure relative to each factor. Each portfolio consists of stocks residing on the NYSE /AMEX/ NASDAQ exchanges with prices below 5. The sample period is 1984-2019. All t-statistics are Newey-West corrected.

	Dependent variable: Excess returns					
	(Low)	(2)	(3)	(4)	(High)	(H-L)
α	−0.063	−0.033	0.245***	0.219**	0.208	−0.003
	−1.162	*−0.566*	*2.839*	*2.072*	*1.020*	*−0.012*
Mkt-RF	0.963***	1.042***	1.057***	1.075***	0.974***	0.008
	58.943	*54.728*	*50.756*	*34.185*	*18.262*	*0.139*
HML	0.083***	0.061*	−0.137**	−0.241***	−0.209*	−0.296**
	3.183	*1.838*	*−2.199*	*−3.252*	*−1.776*	*−2.265*
SMB	−0.133***	0.073**	0.258***	0.484***	0.612***	0.751***
	−4.991	*2.024*	*5.045*	*7.396*	*7.719*	*7.711*
RMW	0.107***	0.249***	0.178*	−0.151	−0.511***	−0.616***
	3.623	*3.595*	*1.831*	*−1.354*	*−4.179*	*−4.604*
CMA	0.173***	−0.125**	−0.181**	−0.347***	−0.390**	−0.564***
	2.800	*−2.320*	*−2.527*	*−3.437*	*−2.561*	*−3.001*
QMJ	0.199***	−0.141**	−0.477***	−0.616***	−1.006***	−1.215***
	3.460	*−2.081*	*−5.324*	*−5.405*	*−5.582*	*−5.568*
R^2	0.959	0.940	0.935	0.922	0.871	0.761
Adjusted R^2	0.958	0.940	0.934	0.921	0.869	0.758

Note: t-statistics in *italics* *p<0.1; **p<0.05; ***p<0.01