Modeling Scientific Revolutions:
Gärdenfors and Levi on the Nature of Paradigm Shifts
Gärdenfors' and Levi's theories of rational belief changes will be compared to Kuhn's theory of scientific progress. Gärdenfors' proposal to analyze scientific revolutions in terms of big changes in epistemic entrenchment will be criticized for not being able to represent the important constituting function of the paradigms. Parts of Levi's theory that are incompatible with Kuhn's theory will be shown to be either too restrictive or too broad for a model of scientific progress. The paper will end with a comparison of Levi's conceptual frameworks to Kuhn's theory of scientific change. Some characteristics of scientific revolutions will also be discussed.
1. Introduction
This essay is focused on what Gärdenfors (1988, p 7) calls *epistemic theories*, theories about rational belief changes. An epistemic theory uses set theory, logic and other formal systems as tools. Different models have been proposed as suitable models for rational changes of belief. Two well known theories are the AGM-model and Levi's model. Both these will be discussed in this text.

If science is a rational enterprise it should be possible to create an epistemic theory giving the formal description of the rational parts of science. The goal of this essay is to give some ideas that can work as a ground for such a theory.

Many different theories about science have been discussed by philosophers. The one that will be used here is Kuhn's theory of scientific change. The choice of Kuhn's theory is not unproblematic. While Gärdenfors seems to have a Kuhnian conception of science, Levi is critical to Kuhn. His discussion about belief changes is also a discussion in the philosophy of science where theories incompatible with Kuhn's are presented. Arguments against some of Levi's theories about science will be given.

The aim of this text is to give some requirements on a theory about belief changes that wants to work as a model for scientific inquiry. Criteria for an epistemic theory that can describe changes between Kuhnian paradigms will be given. These criteria are found by discussing how the AGM-model and Levi's model accounts for big scientific changes, and by showing where and why these two models fail.

2. Explaining paradigm
Science is often seen as a partly accumulative process where new knowledge is added to the old. According to Kuhn, this view is flawed. Instead periods where science is mainly accumulative are interrupted by *scientific revolutions*. These revolutions question the fundamental parts or the science and ends with a redefinition of the methods available, the problems that are considered central for the scientific field and even the scientists’ conception of the world.

The accumulative periods in science are called *normal science*. Normal science is possible because the scientists share some common beliefs, called *paradigms*. Science without a paradigm exists before any paradigm is accepted and when a scientific revolution takes place as an answer to some unsolved problem for some paradigm.

Before a paradigm is accepted, there is no rule that separates interesting facts from the uninteresting. The fact-gathering is more or less random, and usually restricted to data that already lies ready to hand (Kuhn, 1970, pp 15-16). A theory is created to point out which parts of the big amount of data that is of special interest and particularly revealing. When a theory becomes accepted by the scientific community, it becomes a paradigm.
Normal science is (at least to a great extent) an accumulative enterprise, where new knowledge is added to old. The paradigm puts focus on a small part of the available data, making it possible for the scientists to study the field defined by the paradigm in more detail than it would be possible to do without the paradigm (Kuhn 1970, p 24).

In the postscript to The Structure of Scientific Revolutions, Kuhn (1970, pp 181-191) gives an explanation of what constitutes the paradigm. There might be other parts of the paradigm as well, but Kuhn believes that these are the most important parts. The different components are symbolic generalizations, examples, values and beliefs in particular models (metaphysical and other sorts of models that supply the community with suitable analogies and metaphors). A symbolic generalization is for example $f=ma$, which defines what force, mass and acceleration is. The role symbolic generalization has is not that of a formal definition but rather it helps to explain all the terms used.

In a later text, Kuhn (1990) introduces taxonomic lexicons. Taxonomic lexicons are to a big extent the same as symbolic generalizations and supply the scientist with definitions of important terms and their relation. Without a taxonomic lexicon no description of the world would be possible. Terms in the taxonomic lexicon have two important properties. They are kind-terms taking the indefinite article (a, an or some in English), and they conforms to what Kuhn (1990, p 4) calls the no-overlap principle. No two terms in the taxonomic lexicon may overlap in their referents unless they are related as species to genus.

According to Kuhn (1990, p 5), a taxonomic lexicon might also be called a conceptual scheme, where a conceptual scheme is a particular operating mode of a mental module prerequisite to having beliefs. This mode both supplies and put up bounds on which sets of beliefs it is possible to conceive. Even though the taxonomic lexicon has been introduced as a linguistic structure, Kuhn sees it at least partly as a pre-linguistic module that even animals may have.

According to Kuhn (1970, pp 198-200) different paradigms differ in language, which creates communication-problems between supporters of different paradigms. But the problem can not be solved by creating paradigm-independent definitions of the important terms because Kuhn (2000, p 38) claims that there exist no language into which both theories can be translated without any residue or loss. Kuhn calls this incommensurability.

That it is impossible to translate does not mean that it is impossible for a person to understand two different paradigms. Kuhn (2000, pp 37-40) makes a distinction between translation and interpretation. A translation is a replacement of a word with a more or less equivalent statement word or statement in a different language. For example, a translation of 'Gavagai' might be 'rabbit' or some longer sentence in English, referring exactly to the objects 'Gavagai' refers to. In some cases however, no equivalent term or sentence can be found. Kuhn's (2000, pp 40-41) example is about phlogiston. The phlogiston-theory explained among other things how fire was possible. According to the
theory, something that could burn contained the substance phlogiston. Burning was supposed to be a process where phlogiston moved from the burning object to the surrounding air. The term phlogiston can not, according to Kuhn, be translated into the language used by modern chemists. Phlogiston was used where many different terms are used today. In some cases, like 'phlogisticated air' phlogiston refers to oxygen, in other cases no modern term with the same referent exists at all. A translation of phlogiston would have to give criteria for all the different possible uses of the word.

Some terms like 'element' exist in two different paradigms, but their meaning differ between the paradigms. As some terms have new meanings and some have disappeared from the vocabulary no coherent translation can be done according to Kuhn. That's the way different paradigms are incommensurable.

While working inside a taxonomic lexicon, certain statements can be judged as true or false. But not all statements are candidates for truth or falsity. Kuhn explains how to assert truth and falsity in the following way:

First determine the status of the statement: is it a candidate for true/false? To that question, as you'll shortly see, the answer is lexicon-dependent. And second, supposing a positive answer to the first, is the statement rationally assertable? To that question, given a lexicon, the answer is properly found by something like the normal rules of evidence. (Kuhn 1990, p 9)

So which statements that are candidates for truth or falsity depends on the taxonomic lexicon. If a statement is a candidate either the statement $A$ or its negation $\neg A$ is true.

The question of truth and falsity is not a question of correspondence with an “external, mind-independent world” (Kuhn, 1990, p 6). Rather, it is a question of which of the statements that are better for doing what the scientists are doing. The judgment of truth and falsity is thus dependent on what the scientific enterprise is. According to Kuhn (1990, p 6) the goal of science is much like puzzle-solving. However, the structure of science as explained here exists independent of the goal of science, so most of the reasoning here is independent of what the goal really is.

As a lexicon is a prerequisite for judgment of truth or falsity, lexicons themselves and the paradigms they are part of can not be judged as true or false. But that does not mean that no criteria exist for choices between paradigms. Kuhn (1977, pp 321-322) lists five important criteria for the choice between paradigms. These includes accuracy, consistency with itself and with other currently accepted theories applicable to related aspects of nature, broad scope making it possible to apply the theory to parts far beyond the observations and laws it was initially designed to explain, and simplicity. The theory should also be fruitful when it comes to new scientific findings.

The paradigm is not only the taxonomic lexicon, but according to Irzik and Grünberg (1995, pp 300-301) the taxonomic lexicon and the other parts of the paradigm are intertwined in such a way that a change in one part involves a change in the other. Therefore, a paradigm shift always includes
a change in the taxonomic lexicon, and the characteristics for changes in taxonomic lexicon also apply to paradigm shifts.

It should be noted that even though paradigms are shared among the scientific community and have an important role as shared beliefs and shared linguistic terms, a paradigm also has an important role for an individual inquirer inside the scientific community. Conceptual schemes discussed earlier are an important example of that. In this paper, it is mostly the role of a paradigm for an individual inquirer that will be discussed.

3. The AGM-model

Gärdenfors (1988, pp 7-8) distinguish three factors that form the core of epistemic theories. The first factor is *epistemic states* or *states of belief*. An epistemic state is a representation of a person's beliefs at a certain point of time. Epistemic states are rational idealizations of psychological states.

A second factor is a classification of the different status the elements of the epistemic states can have. Gärdenfors calls these different status *epistemic attitudes*.

The third factor is epistemic inputs. These can be experience, linguistic input and other things that can change a person’s epistemic state.

In the AGM-model, developed by Alchourrón, Gärdenfors and Makinson, epistemic states are represented by *belief sets*. A belief set consists of exactly those sentences that a person accepts as true. As this model is a linguistic model it demands a language $L$. Gärdenfors (1988, p 21) leaves most details about $L$ open, but assumes that it contains the standard sentential connectives. In this essay as in Gärdenfors (1988) book, the following symbols will be used:

- negation: -
- conjunction: &
- disjunction: ∨
- material implication: →

$L$ also contains the constants *truth* $\top$ and *falsity* $\bot$. These constants are used as ideal points, and do not say anything about the external world.

In Gärdenfors' (1988, p 22) model, an inquirer has one of the following epistemic attitude to each sentence $A$ in $L$:

- $A$ is accepted
- $A$ is rejected
- $A$ is indetermined, the inquirer has neither accepted nor rejected $A$. 
If $A$ is rejected then $-A$ is accepted. Thus, rejection can be defined with the help of acceptance.

For an epistemic state to be rational, some further restrictions must be added. Only sets of sentences satisfying the following two rationality criteria are belief sets:

The set of accepted sentences should be *consistent*

Logical consequences of what is accepted should also be accepted (Gärdenfors, 1988, p 22)

Both theses can be motivated pragmatically. Inconsistent sets are of little help as guidance for our actions, and if we can not draw the logical consequences of available information, we will not be able to use the information effectively.

Gärdenfors (1988, pp 24-25) makes some further assumptions about the language $L$. He assumes that it is governed by a logic identified with the consequence relation $\models$. The consequence relation satisfies the following conditions:

- **If $A$ is a truth-functional tautology, then $\models A$.**
- **Modus Ponens.** That is, if $\models A \rightarrow B$ and $\models A$, then $\models B$.
- **Not $\models \bot$.** That is, $\models$ is consistent. (Gärdenfors, 1988, p 24)

The first condition says that a sentence $A$ is logically valid if and only if it is a consequence of the empty set. The two rationality-criteria given earlier can now be formulated in the following way, giving a definition of belief sets:

A set $K$ of sentences is a (nonabsurd) belief set iff (i) $\bot$ is not a logical consequence of the sentences in $K$ and (ii) if $K \models B$, then $B \in K$.

A set of sentences that includes all the logical consequences of the set is said to be in *equilibrium*. As this is part of the definition of belief sets, belief sets are in equilibrium.

As explained before, there are three possible epistemic attitudes to a sentence $A$ in the belief set $K$. If $A \in K$, then $A$ is accepted. If $-A \in K$, then $A$ is rejected. If neither $A \in K$ nor $-A \in K$, then $A$ is indetermined. Given this three attitudes, six different changes are possible. An accepted sentence can become rejected or indetermined, a rejected sentence can become accepted or indetermined, and an indetermined sentence can become accepted or rejected. Gärdenfors (1988, p 47-48) gives three different types of changes that together cover all the possible cases. The first is *expansion*, where either $A$ or $-A$ becomes accepted in the belief set, and where $A$ previously where indetermined. The second type of change is *contraction*, where a previously accepted or rejected sentence becomes indetermined. The last type of change is *revision*, where a previously accepted sentence becomes rejected. Revision can be defined as a sequence of contractions and expansions (Gärdenfors 1988, pp
68-69). If $A$ is to be revised, we can begin by contract $A$ from the belief set $K$, and after the contraction expand with $A$. Thus, only two types of changes are needed to cover all six possible changes.

Expansion is a simple operation in Gärdenfors model (1988, p 47) where the previously indetermined sentence $A$ is accepted as true, together with all the consequences of $A$ and the belief set. Contraction is more problematic. If we contract $A$ from the belief set $K$, sentences which have $A$ as a consequence has to be contracted too. Suppose we have the sentences $B$ and $C$, which together have $A$ as a consequence. Then either $B$ or $C$ or both $B$ and $C$ have to be contracted as well. But how do we choose which of the sentences to contract? Gärdenfors (1988, pp 86-87) introduces epistemic entrenchment as a way to determine a sentence's fate when the current belief set is contracted.¹

Single sentences in the language $L$ have epistemic entrenchment. The epistemic entrenchment of a sentence in the belief set $K$ can be determined independently of what happens to $K$ at contraction. When the belief set $K$ is contracted, the sentences with the lowest epistemic entrenchment in $K$ are the sentences that are given up. The main criterion when determining the epistemic entrenchment of a sentence is how important the sentence is when planning future actions, conducting experiments and so on.

Gärdenfors (1988, pp 91-94) distinguish two different origins of epistemic entrenchment. The ordering of epistemic entrenchment gets its fundamental structure from these two origins. The finer ordering of the epistemic entrenchment is however dependent on pragmatic factors. The pragmatic factors are highly context dependent, and give a complement to the rougher ordering first given by some of the other origins.

The first of the origins is the information-theoretic approach. With this approach, epistemic entrenchment is supposed to be equivalent to informational value. Informational value is then given by some function where a probability-value for the sentence is used as input. In Gärdenfors model however, the sentences in the belief set are hold to be maximally certain, and thus all of them have maximal probability. So this approach is not useful for Gärdenfors.

The second approach is called the paradigmatic approach. Many theories about science hold that some statements in science are never (or seldom) questioned. Kuhn's symbolic generalizations have this property according to Gärdenfors. This creates a hierarchy among the sentences in a scientific theory, where the sentences that are part of the paradigm have the highest epistemic entrenchment. If follows from this that a paradigm-shift involves a big change in the ordering of epistemic entrenchment, and that a paradigm-shift can be detected by a big change.

¹ There exist contraction functions that do not need the notion of epistemic entrenchment. However, epistemic entrenchment can be interesting anyway for example as a measure of a sentences importance in the belief set. Gärdenfors (1988, p 88) considers the notion of epistemic entrenchment to be more fundamental than the contraction function. Thus, epistemic entrenchment can be used to evaluate different contraction functions.
4. Paradigm-shifts as mere changes in entrenchment

How strong is the relation between a paradigm-shift and a big change in the ordering of epistemic entrenchment? Gärdenfors writes:

On my view of epistemic entrenchment a change of paradigm typically involves a radical change of the ordering of epistemic entrenchment, and, vice versa, a substantial change of the degrees of epistemic entrenchment of the theses in a scientific field is a strong indication of what Kuhn calls a "scientific revolution". (Gärdenfors, 1988, p 88)

It is possible to make two different interpretations from this. One interpretation is that while paradigmatic shifts are related to changes in entrenchment but there is something more giving paradigmatic shifts the important features they have. This interpretation will be called the weak interpretation. Another interpretation is that paradigmatic shifts are big changes in entrenchment and nothing else. Levi supports this stronger interpretation of Gärdenfors:

The tenor of his [Gärdenfors'] remarks suggests that he thinks that profound changes in orderings with respect to entrenchment attributable to deep-running changes in corpus or belief state are the stuff of which paradigm switches are made (Levi, 1991, p 145)

Does Gärdenfors support the weak or the strong interpretation? His (1988, p 92) reasoning about paradigms as a way to determine the epistemic entrenchment of sentences suggests that the important feature of Kuhn's theory about paradigms is to give criterion for the evaluation of epistemic entrenchment. This gives support for the stronger interpretation. However, Gärdenfors (1988, p 94) also gives support for the weaker interpretation when he writes that paradigm-shifts can be detected by big changes in the epistemic ordering. If paradigm-shifts were the same as big changes in epistemic entrenchment they would not be detected by such changes, they would be such changes.

Independent of what Gärdenfors really thinks about paradigm-shifts, he does not discuss them in relation to any other notion than epistemic entrenchment. No better structure to represent paradigm-shifts is given. Until some other structure is introduced to solve the problem, we can assume that entrenchment is the only available notion with which we can analyze paradigm-shifts in the AGM-model.

One interesting critique against explaining paradigm shifts as changes in epistemic entrenchment is given by Friedman (2000). His discussion is a comparison between Quine's holistic theory presented in 'Two Dogmas of Empiricism', and Kuhn's theory. According to Quine, the only important difference between statements is their entrenchment:

Certain statements, though about physical objects and not sense experience, seem peculiarly germane to sense experience - and in a selective way: some statements to some experiences, others to others. Such statements, especially germane to particular experiences, I picture as near the periphery. But in this relation of 'germaneness' I envisage nothing more than a loose association reflecting the relative likelihood, in
As epistemic entrenchment is the relative likelihood for one statement rather than another to be revised, Quine's view is the same as the stronger interpretation of Gärdenfors' view.

Quine's holistic theory is developed against Carnap's theory about analytic and synthetic statements. According to Friedman, Kuhn's and Carnap's theories are closely related:

Kuhn's central distinction between change of paradigm or revolutionary science, on the one side, and normal science, on the other, closely parallels the Carnapian distinction between change of language or linguistic framework and rule-governed operations carried out within such a framework. (Friedman, 2000, p 377)

Friedman considers Quine's theory incompatible with both Carnap's and Kuhn's theory. He (2000, p 374) uses an example from the history of science to make his point clear. In the shift to Newtonian physics, Friedman identifies three revolutionary steps. The introduction of a revolutionary mathematical idea with Newton's calculus, a new conception of force and quantity of matter with Newton's three laws of motion, and a new universal law of nature with Newton's universal law of gravitation. All these changes were inspired by the same empirical problem, to give a single mathematical theory of motion that gives an account for both celestial and terrestrial phenomenas. This might seem to give support for Quine's holistic theory. The three parts were introduced together to solve the same problem, and so all parts of the theory, the mathematical as well as the mechanical and gravitational physical, might seem to be treated the same way as they are added to solve the same problem. However, Friedman (2000, p 374-375) argues that the change is a change not only in epistemic entrenchment. Instead some of the changes have a constitutional function without which other parts of the theory would not make sense.

Consider the second law of motion, \( f = ma \), force equals mass times acceleration. Without calculus, the second law of motion would not make sense, as we would not be able to calculate the acceleration. So the mathematical part of the theory provides part of a language which makes it possible to state the theory at all. A similar relationship exists between Newton's law of universal gravitation and his mechanics. The law of universal gravitation states that there is a force of attraction between two masses. The attraction is proportional to the product of the two masses and inversely proportional to the square of the distance between them. The masses experience acceleration towards each other according to the rule. But the acceleration must be relative to some frame of reference. Newton thought the movement was defined relative to absolute space, but practically the acceleration takes place relative to an inertial frame, which is defined as a frame where Newton's laws of motion holds. This means that the law of universal gravitation would not make empirical sense without the laws of motions.
According to Friedman (2000, p 377), the conceptual changes that appears during scientific revolutions can not be explained only with entrenchment. In the section where Kuhn's theory about scientific change were introduced, the role of conceptual schemes was described as something that determines what it is possible to conceive. A shift of paradigm is a shift of conceptual scheme. What is needed is a structure suitable to represent the changes in what it is possible to conceive. As Friedman has shown, entrenchment is not the right notion for this.

5. Levi's model

Because of the AGM-models shortcomings with representing paradigm-shifts, Levi's epistemic theory will be discussed.

In the AGM-model sets of sentences were used to represent an inquirer's belief. Standard set theoretic operations could then be used with the set (for example, the epistemic attitude acceptance can be defined the following way: \( A \) is accepted in \( K \) iff \( A \in K \)). Levi (1991, pp 7-10) has chosen a different approach. The central notion for his system is state of full belief. A person’s state of full belief is the sum of what the person believes at a specific time and the deductive consequences of her beliefs.

When a person changes her beliefs, a different state of full belief is created. All the states of full belief a person can be in is ordered in a conceptual framework. The states in a conceptual framework are potential states of full belief. Note that a change from one potential state of full belief to another does not entail a change of conceptual framework.

The potential states of full belief in a conceptual framework are ordered after strength. A potential state of full belief \( K^* \) is stronger than another potential state of full belief \( K \) if and only if a person in state \( K^* \) believes the same thing as a person in state \( K \) plus something more. The ordering is from stronger to weaker potential states of full beliefs. A weaker state is a consequence of a stronger state.

There are two goals with inquiry in Levi’s model. The first goal Levi (1991, p 10-11) discusses is acquisition of new information that are valuable for us – We strive to maximize informational value.

The other important goal of inquiry according to Levi (1991, p 11) is avoidance of error. The inquirer will try to avoid changing to a state of full belief that is erroneous.

The potential states of full belief that are part of the conceptual framework can be judged in three different ways, corresponding to the AGM-models three different epistemic attitudes. Some states are judged as free of error, some states are judge erroneous, and, at least in most cases, the inquirer is in suspense about the freedom of error or the occurrence of error in some of the states of full belief. The current state and all consequences of the current state are always judged free of error.
That an inquirer should try to avoid error does not mean that an inquirer should try to find the true and complete theory of the world. Levi (1991, pp 61-62) rejects a theory where the goal of inquiry is to get the complete error-free description of the world. He calls that view *messianic realism*. According to Levi, an inquirer should instead be concerned with avoiding error in the next step of inquiry. This milder version of realism is called *secular realism* by Levi.

Levi recognizes three constraints on how an inquirer could judge truth and falsity in potential states of full belief:

I. A potential state of full belief $K$ is judged error-free by an inquirer if and only if it is a consequence of the inquirer's current state of full belief.

II. A potential state of full belief $K$ is judged erroneous if and only if all states having both the current state and $K$ as consequences are judged erroneous.

III. At least one potential state is judged error-free, and at least one potential state is judged erroneous. (Levi 1991, p 11)

I. and II. are constraints on how different states should be judged. III. prevents judgment of freedom from error and occurrence of error to be vacuous.

The only structure of the conceptual framework discussed at this point is that it is ordered according to strength. Levi (1991, pp 12-13) assumes that the order of the potential states of full belief is made in such a way that it defines a consequence relation that is reflexive, antisymmetric and transitive, that is the consequence relation $R$ have the following properties for all potential states of full belief $K_1$, $K_2$ and $K_3$:

\[
K_1 R K_1 \text{ (reflexivity)} \\
\text{if } K_1 R K_2 \text{ and } K_2 R K_1 \text{ then } K_2 = K_1 \text{ (antisymmetry)} \\
\text{if } K_1 R K_2 \text{ and } K_2 R K_3 \text{ then } K_1 R K_3 \text{ (transitivity)}
\]

A set with an order that is reflexive, antisymmetric and transitive is called a *partially ordered set*.

Levi (1991, p 13) imposes some further structural conditions. Given two potential states of full belief $K_1$ and $K_2$ there exist a potential state of full belief $K_3$ such that it is a common consequence

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2. For more information about partially ordered sets, lattices and boolean algebras see notes 3-5, pp 165-166 in Levi's (1991) book. A good online resource is wikipedia, where the following pages are among those of interest:
   - http://en.wikipedia.org/wiki/Lattice_%28order%29
of $K_1$ and $K_2$ and is stronger than or as strong as any other common consequence of $K_1$ and $K_2$. The strongest consequence of the two possible states is called a *join* ($\lor$).

For any two potential states of full belief $K_1$ and $K_2$ there should also be a potential state of full belief $K_3$ such that both $K_1$ and $K_2$ are consequences of $K_3$ and that no other state of full belief that has both $K_1$ and $K_2$ as consequences are weaker than $K_3$. This state is called the *meet* ($\land$) of the two possible states.

A partially ordered set where every possible pair of members has a join and a meet is called a *lattice*. But why should we accept that the conceptual framework is a lattice? Levi (1991, p 13) argues that even though it puts up some formal restrictions on the conceptual framework, it would put up unnecessary roadblocks on inquiry and lead to dogmatism if the framework was not a lattice. Consider two inquirers $X$ and $Y$ that are in different states of full belief $K_1$ and $K_2$, but share a conceptual framework. Sometimes it might be a good idea for the two inquirers to move to a state of full belief $K_3$ which they both believe to be true. From that point they could evaluate $K_1$ and $K_2$ from an unbiased point of view. The existence of a join of $K_1$ and $K_2$ assures that that can be done without any unnecessary loss of informational value.

The existence of a meet can be motivated in a similar way. Sometimes the inquirers $X$ and $Y$ that are in the potential states of full belief $K_1$ and $K_2$ might want to move to a possible state of full belief that have both $K_1$ and $K_2$ as a consequence. But to avoid error, they should move to a possible state of full belief that is as weak as possible while having both $K_1$ and $K_2$ as a consequence. To assume that no such state exists would, as in the case of the existence of a join, put up unnecessary restrictions on inquiry.

With his liberal approach to inquiry, Levi (1991, pp 14-15) motivates some further structure on the conceptual framework. In a conceptual framework, there should be a possible state of full belief $1$ which is the weakest state of full belief. $1$ is a consequence of every other possible state of full belief. A meet of any possible state of full belief $K$ with $1$ will be $K$, that is, $1$ can be defined as $K \land 1 = K$.

The same liberal approach can motivate the existence of a strongest possible state of full belief $0$ that has all other possible states of full belief as consequence. Constraint II says that any state of full belief that has an erroneous state of full belief as consequence should be judged erroneous. Constraint III guarantees that at least one state of full belief in the conceptual framework is considered to be erroneous. As $0$ has all possible states of full belief as consequence, even the erroneous, $0$ is always judged as erroneous (except when the inquirer's current state of full belief is $0$). A join of any possible state of full belief $K$ with $0$ will be $K$, that is, $0$ can be defined as $K \lor 0 = K$.

With the argument that roadblocks in the way of inquiry should be avoided, Levi (1991, pp 14-15) assumes that the join of any two potential states of full belief $K_1$ and $K_2$ includes all and only
the consequences of \( K_1 \) and \( K_2 \). There could exist a lattice where no state of full belief with all the common consequences of \( K_1 \) and \( K_2 \) where available. A join could still exist, but it would not be what we intuitively think of as the join. However, to assume that a potential state of full belief with all the common consequences exists does not rule out any other possible state of full belief. Therefore, Levi assumes its existence to avoid putting roadblocks in the way of inquiry.

The same assumption should be done about the meet of any two potential states of full belief. We can assume that there exist a meet for any two potential states \( K_1 \) and \( K_2 \) and that the meet has only \( K_1 \) and \( K_2 \) and their consequences as its consequences.

Levi does the two last assumptions to ensure that the lattice is *distributive* and comply with the following two assumptions:\(^3\):

For every \( K_1, K_2 \) and \( K_3 \) in the conceptual framework,

\[
(K_1 \lor K_2) \land K_3 = (K_1 \lor K_3) \land (K_2 \lor K_3)
\]

\[
(K_1 \land K_2) \lor K_3 = (K_1 \land K_3) \lor (K_2 \land K_3)
\]

The last assumption Levi (1991, pp 15-16) does with the motivation that no roadblocks should be putted in the way of inquiry is also important for judgments of error. We assume that there is a potential state of full belief \( K^* \) for every potential state of full belief \( K \) such that \( K \land K^* = 0 \). If any state with the properties of \( K^* \) is a consequence of an inquirer's current state of full belief, \( K \) should be judged erroneous, as the meet of the current state of full belief and \( K \) would be 0.

To avoid roadblocks, Levi assumes that there is a weakest state of full belief \( \neg K \) having the properties of \( K^* \) to every state of full belief \( K \) in the conceptual framework. The meet of this weakest state \( \neg K \) and \( K \) should be 1. The complement \( \neg K \) to the potential state of full belief \( K \) that has the properties that \( K \lor \neg K = 1 \) and \( K \land \neg K = 0 \) should be unique\(^4\).

A lattice that is distributive, have a strongest state 0, a weakest state 1, and where every state has a complement is a *boolean algebra*. So by the original assumptions that an inquirer should try to maximize informational value and avoid error, and by the liberal approach to inquiry, Levi has motivated that the conceptual framework should be ordered as a boolean algebra.

\(^3\) When the join of two states includes all and only the consequences of the two states the join is the same as an intersection in set theory. A meet of two states that has only the two states and all their consequences as its consequence is the same as a union. The distributive laws in a boolean algebra can then be motivated in the same way as the distributive laws of set theory.

\(^4\) The law of distribution guarantees that there exists only one complement to any state of full belief. This can be shown the following way. Assume that both \( K_2 \) and \( K_3 \) are complements to \( K_1 \). Given the definition of a complement \( (K_1 \lor K_3) \land K_2 = (K_1 \lor K_2) \land (K_3 \lor K_2) \) can be simplified to \( 1 \land K_3 = 1 \land (K_1 \lor K_2) \) which, given the definition of 1, is the same as \( K_3 = K_2 \lor K_3 \). In the same way, \( (K_1 \lor K_3) \land K_2 = (K_1 \lor K_2) \land (K_3 \lor K_2) \) can be turned into \( K_2 = K_2 \lor K_3 \). From \( K_3 = K_2 \lor K_3 \) and \( K_2 = K_2 \lor K_3 \) we can conclude that \( K_2 = K_3 \).
A boolean algebra where there exist one or more potential states of full belief that are maximally strong without being $0$ is called an *atomic* boolean algebra. The maximally strong potential states would then be called *atoms*. An inquirer would, according to Levi (1991, p 17), be committed to judge only one of the atoms as error-free. The potential states of full beliefs that are consequences of the error-free atom are also considered as error-free, and should be the only potential states of full belief in the conceptual framework considered error-free.

However, a boolean algebra does not need to have any atoms. According to Levi (1991, pp 17-18) a reasonable sophisticated framework will not have any atoms. In an atom-free system, it would be impossible for an inquirer to judge all the error-free potential states of full belief as error-free, as no strongest potential state of full belief weaker than $0$ would exist. There do however exist *subalgebras*, subsets of the boolean algebra with the needed structure for a boolean algebra (a lattice that is distributive, have a strongest state $0$, a weakest state $1$, and where every state has a complement). Some subalgebras will have atoms. An inquirer is committed to judging exactly one of the atoms and its consequences as error-free.

Because the inquirer is concerned to avoid error and maximize informational value, and because roadblocks in the way of inquiry should be avoided, the conceptual framework should be ordered as a boolean algebra. But what determines which states of full belief are parts of the conceptual framework? Levi’s (1991, pp 19-21) answer is that the potential states of full belief in a conceptual framework are the states of full belief the inquirer is *conceptually capable* of adopting at that time. So now the notion of conceptual capacity has to be explained. Levi can think of three different possible explanations. The conceptual capacity of an inquirer can be that specific inquirers intellectual abilities. The conceptual framework would then be the set of potential states of full belief the inquirer can understand. But the argument for ordering the conceptual framework as a boolean algebra has not been that people’s intellectual capacity gives a framework with the structure of a boolean algebra. Indeed, Levi (1991, p 19) believes that very few people would have a conceptual framework generated by their intellectual ability characterizable by a boolean algebra of potential states of full belief. So Levi does not find this definition suitable.

A second possibility would be to define conceptual capacity in a way that includes all intelligent agents. The question of conceptual capacity would be heavily related to questions such as the existence of some limit for human intellect. What is conceptual accessible for one would then be conceptual accessible to all. According to this view, no conceptual changes at all would be possible.

Levi (1991, p 20) proposes a third definition. The goals of an inquirer are as stated earlier to maximize informational value and avoid error. The potential states of full belief that the inquirer is conceptually capable of adopting are the states she considers truth valued in a sense of concern for her.
Levi (1991, p 19) does not see any problem for a now living scientist working with Einsteinian physics to evaluate the ether theory if she wants to, so paradigms do not seem to put up any restrictions on the conceptual framework.

There is no problems for an inquirer to add state of full belief from some one else's conceptual framework to her own according to Levi (1991, p 20) It is only a question of what the inquirer judges as interesting. Which states of full belief the inquirer judge interesting enough to ascribe truth-value depends on the inquirers conceptual values. Levi does not explain what a person gets her conceptual values from.

Now the the goals of inquiry, the structure of the conceptual framework and what determines the set of potential states of full belief have been discussed. But how is inquiry inside a conceptual framework conducted? As in the AGM-model, all changes inside the conceptual framework can be described as a sequence of expansion, moving from one state of full belief to a stronger state of full belief, and contraction, moving from one state of full belief to a weaker state of full belief.

We might want to say something about what is added with an expansion or what is removed with a contraction. In the AGM-model this is easily done because what is added or removed are sentences or sets of sentences. But Levi works with states of full belief instead of sets of sentences. One solution would be to say that after an expansion, an inquirer has come to believe a proposition she did not believe earlier. Propositions have played an important role in many theories about belief-revision. But Levi considers it unnecessary to add another notion to the system. Instead he (1991, pp 21-24) lets potential states of full belief fill the role usually ascribed to propositions.

When we use the statement “X believes that \( h \)”, we are giving a description of X's current state of full belief. What we are saying is that X believes at least “that \( h \)”. Now we can let a potential state of full belief represent “that \( h \)”. X's state of full belief is then at least as strong as the potential state of full belief representing that \( h \). Because potential states of full belief can take the traditional function of propositions, Levi sometimes calls potential states of full belief doxastic propositions. It is possible to make a (not necessarily perfect) mapping from doxastistic proposition to sentences in some language \( L \). That way, we can get a linguistic representation similar to belief sets in the AGM-model from states of full belief.

With our expansions and contractions we strive to get more information of value while avoiding error. Levi (1991, pp 80-81) does not consider information of value to be the same as information or true information. Neither information nor information of value need to be true. As information does not have to be true, the potential state of full belief that carries most information is the inconsistent state \( 0 \), because it is the strongest potential state, having all other potential states of full belief as consequences. But the inconsistent state is always judged erroneous and is therefore avoided if possible.
Information varies with strength according to Levi (1991, p 81). A potential state of full belief that is a consequence of a second potential state of full belief carries less information than the second. All potential states of full belief are not comparable in this way. Two states of full belief where none is a consequence of the other can not be said to have different amounts of information.

But not all information is of value to the inquirer. When trying to find new information the inquirer has a goal. Information the inquirer is not interested in is information, but not valuable for the inquirer. Informational value needs to be separated from information. However, according to Levi (1991, p 82) a potential state of full belief with more information than another state should not be considered as less valuable. It might not be worth the risk of error to expand to a potential state of full belief with some additional relatively uninteresting information compared to some weaker state, but if no risk of error exists, a weaker state of full belief can not be preferred over a stronger state because it contains more informational value. The constraint that a stronger state can not carry less informational value than a weaker state is called the weak monotonicity condition by Levi.

Some examples of what Levi (1991, p 83) includes in informational value is explanatory power, predictive power, simplicity and conformity to some paradigm for an adequate theory. With comparisons of doxastic propositions in terms of explanatory power, simplicity and so on we “attempts to extend the quasi-ordering partially or completely with respect to informational value” (Levi 1991, p 83). So judgment of informational value can be made in cases where there is no relation of consequence between the states in question. In that way, informational value helps us create an ordering of potential states of full belief even when no order can be done in terms of information.

While an inquirer strives after more informational value, she is determined to avoid error. According to Levi (1991, p 90), the risk of error increases with an increase of strength. A stronger state can never be less risky to expand to than a weaker. This puts avoidance of error in opposition with maximization of informational value. While the best method to maximize informational value is to expand to 0, the best method to avoid error is to avoid expansion at all. To stay in the current state of full belief K incurs no risk of error as it is always judged error-free. Levi's (1991, p 92) solution to this problem is to give a formula for evaluating possible expansions. The method starts with recognizing the expansions that adds new information of value to the system and therefore is of interest. Then the formula weights the risk of error with the informational value. The formula gives a value as a result, and if the value lies within some specific range, the expansion is done. Some caution has to be taken when creating such a formula. Expansion into inconsistency could never be legitimate, so the formula must never motivate expanding into 0. The formula must also be balanced so not to risky expansions are made, while expansions are allowed in some cases. After a successful expansion, the new state of full belief becomes the inquirers standard for judgments of error and informational value.
As the current state of full belief and all its consequences are judged free from error, contraction can be done without any risk of error. The problem is instead to motivate why a contraction should be done. Levi (1991, p 117) gives some situations where a contraction would be legitimate. If the current state of full belief shows out to be inconsistent, contraction is necessary. But a contraction can be made for other reasons too. For example, someone might propose some doxastic proposition inconsistent with the inquirer's current state of full belief. The inquirer can then contract to a weaker state where the proposed doxastic proposition can be compared to the former state of full belief. Such a contraction can be motivated if there is a good chance of ending up in a potential state of full belief with higher informational value.

In both expansion and contraction, judgment of error is done relative to the current state of full belief. In inquiry, only judgments of truth or falsity relative to the current state are available:

> What the inquirer judges true or false is relative to what the inquirer fully believes. And when an inquirer is concerned to avoid error, he can proceed only relative to the judgments of truth available to him. (Levi, 1991, p 91)

Before Levi's model will be discussed from a Kuhnian point of view, Levi's own critique of Kuhn will be presented. Levi wants to make his theory incompatible with incommensurability. To do this he adds some restrictions to how changes inside a conceptual framework can be done. He also discusses changes between conceptual frameworks.

Incommensurability is, according to Levi (1991, p 65), when there does not exist some sequence of expansions and contractions, beginning with an initial state of full belief $K_1$ terminating with $K_2$. Levi has created the commensurability thesis to secure that his system is impossible to combine with incommensurability:

**Commensurability Thesis:** Given an initial state of full belief $K_1$ and another state of full belief $K_2$, there is always a sequence of expansions and contractions, beginning with $K_1$, remaining within the space of potential states of full belief and terminating with $K_2$. (Levi 1991, p 65)

As the commensurability thesis is about changes “within the space of potential states of full belief” it only concerns changes inside a conceptual framework. But Levi wants to secure commensurability between conceptual frameworks. He does this by demanding that a change in conceptual framework is from one boolean algebra to another boolean algebra that are either a subalgebra or a superalgebra of the first one:

And if, as I assume, every change in conceptual framework is from one boolean algebra to another that is either a subalgebra or a superalgebra of the first, commensurability is secured over conceptual changes. (Levi, 1991, p 65)
As the name suggests, a superalgebra is a boolean algebra that has the original boolean algebra as a subset. We can formulate this as the super- or subalgebra thesis (SSAT):

**SSAT:** Every change in conceptual framework is from a boolean algebra $K_1$ to another boolean algebra $K_2$ that is either a subalgebra or a superalgebra of $K_1$.

The SSAT will be discussed more in the following sections.

6. **Problems with the definition of conceptual frameworks and changes between them**

Levi has defined his conceptual frameworks to be impossible to combine with Kuhn’s theory about incommensurable paradigms. When introducing conceptual frameworks, Levi notes that theories about ether can be part of the same conceptual framework as Einsteinian physics, and with the SSAT he tries to secure commensurability between conceptual frameworks. In this section, it will be discussed if Levi’s definition of conceptual frameworks and if his SSAT are suitable in a model for scientific activity.

According to Levi, the conceptual framework is determined by the conceptual capacity. The states of full belief an inquirer is conceptually capable of adopting are the states she considers truth-valued in a sense of concern to her (Levi 1991, p 20). What concerns the inquirer is a question of values. As noted in the previous section, Levi does not say much about conceptual values. From this characterization of conceptual capacity, Levi draws the conclusion that “the characterization of an inquirer's conceptual framework is as much a characterization of the inquirer's cognitive value commitments as it is of his conceptual capacities” (Levi 1991, p 21). But something even stronger follows from Levi’s reasoning. The characterization of an inquirer's conceptual framework is a characterization of the inquirer's conceptual values and nothing else. This can be shown by going through Levi’s argument more careful.

The question is what determines which states of full belief are parts of the conceptual framework. As noted earlier, the conceptual framework is not determined by a paradigm. It was also noted that states of full belief from other inquirers can be added to an inquirer’s conceptual framework without problem. The only thing determining the conceptual framework seems to be the conceptual capacity. Levi writes:

> The set of potential states of full belief constituting the conceptual framework for an agent $X$ at $t$ time consists of those states that agent $X$ at $t$ is conceptually capable of adopting at that time. (Levi, 1991, p 19)

The next question is what conceptual capacity is. We should know Levi's answer by now. The states of full belief an inquirer is conceptually capable of adopting is the states she considers as true or false.
in a way that is of interest for her. Levi's definition of conceptual capacity only involves a judgment of states of interest and is thus dependent only on conceptual values. The notion conceptual value is then left unexplained.

Thus there does not seem to be any thing else than the inquirer's conceptual values that determines the inquirer's conceptual framework. Paradigms and intelligence can only play a role if they influence the conceptual values. There are some problems with conceptual values as the only determinant of the conceptual framework. Big conceptual changes in the history of science, are they really characterizable as only changes in conceptual values and nothing else? Are they changes in conceptual framework at all according to Levi? If what Kuhn calls scientific revolutions are not changes of conceptual framework, then what is?

So there are problems with letting conceptual frameworks be determined only by conceptual values. However, the way conceptual frameworks are determined is not the only part of Levi's theory that is impossible to combine with Kuhn's theory of scientific changes. The commensurability thesis and the SSAT are designed to secure commensurability in a conceptual framework and between changes of conceptual framework. According to the SSAT, a change in conceptual framework will be to a sub- or a superalgebra of the earlier. Scientific revolutions would then have to be a conceptual change to a superalgebra, because the potential states of full belief representing the new paradigm has to be added to the conceptual framework. As the new conceptual framework has to be a superalgebra, all the potential states of full belief from the earlier paradigm will still be in the conceptual framework. All the potential states of full belief can then be compared in terms of freedom from error and informational value. Commensurability is secured. But is there any historical support for the SSAT?

Kuhn (1970, pp 98-102) has given strong arguments against the view that earlier paradigms are part of the new paradigms. Even if Newtonian dynamics can be seen as a special case of relativistic dynamics given the right restrictions, terms like space, time and mass still represents Einsteinian space, time and mass even if it is the Newtonian formulas that are used inside the relativistic paradigm.

But Levi's conceptual framework can include many different paradigms, as long as the inquirer considers states of full belief from different paradigms as candidates for truth or falsity. So maybe SSAT says that a scientist after a scientific revolution will consider all the states of full belief from both the old and the new paradigms as candidates for truth and falsity. This interpretation does not require that a new paradigm includes the older paradigm. So consider our inquirer A, whose conceptual framework includes only Newtonian physics. Suppose that he lives in the early 2000\textsuperscript{th} century, and experiences a scientific revolution where Newtonian physics is replaced with Einsteinian
physics. After the scientific revolution, A's conceptual framework should include both Newtonian and Einsteinian physics according to the SSAT. But why would an inquirer who has accepted a relativistic conception of space and time consider sentences about the Newtonian absolute space as candidates for truth or falsity? He surely would understand sentences about absolute space. But that is not the same as judging the sentences as candidates for truth or falsity. As the conceptual framework is determined by the conceptual values, the SSAT says something about the inquirer's values, not what she understands.

Two problems with Levi's theory have been presented here. The first is that his notion of conceptual framework only seems to be restricted by conceptual values, without any further explanation of conceptual values. The second problem is that the sub- or superalgebra thesis creates constraints on changes in the conceptual frameworks that lack historical support. However, some parts of Levi's theory are interesting for a discussion of a formal representation of Kuhnian paradigms and changes between them. That is the topic for the next section.

7. Modeling paradigms and paradigm shifts

Two epistemic theories have been criticized from the point of view of a Kuhnian theory about scientific changes. However, some parts of these theories and the critique against them can be used in an epistemic theory where Kuhnian paradigms and paradigm-shifts are modeled.

In the discussion about the AGM-model it was concluded that changes in epistemic entrenchment was not enough to represent paradigm-shifts. In Levi's model conceptual frameworks play a central role. Can changes between conceptual frameworks be used as a model for paradigm-shifts?

A Levian conceptual framework is a set of states of full belief determined by the conceptual values of the inquirer. For Kuhn the taxonomic lexicon has a similar role to Levi's conceptual values:

What I have been calling a lexical taxonomy might, that is, better be called a conceptual scheme, where the “very notion” of a conceptual scheme is not that of a set of beliefs but of a particular operating mode of a mental module prerequisite to having beliefs, a mode that at once supplies and bounds the set of beliefs it is possible to conceive. (Kuhn, 1990, p 4)

A paradigm can thus be seen as a set of beliefs determined by the lexical taxonomy, and work inside a paradigm can be modeled as moves between states of belief inside the set of beliefs generated by the lexical taxonomy. This set will be called a Kuhnian conceptual framework or just conceptual framework if the context is enough to tell if it is a Kuhnian or a Levian dito. Work inside a Kuhnian conceptual framework is the same thing as normal science.

Beliefs in a Kuhnian conceptual framework can just as in a Levian framework be judged as true or false. As Levi (1991, p 159) points out Kuhn seems to accept the role of informational value in inquiry, especially when it comes to scientific revolutions. So the two goals of inquiry Levi uses to
motivate that the conceptual framework should be structured as a boolean algebra is important for inquiry inside a Kuhnian conceptual framework too. If Levi's argument from the avoidance of error, maximization of informational value and his liberal approach to inquiry to the structure of a boolean algebra is successful it might be possible to use as the structure for Kuhnian conceptual frameworks too.

There are however important differences between the two philosophers' account of judgments of truth and falsity. According to Kuhn a taxonomic lexicon has to be in place before any judgment of truth or falsity can be done. Taxonomic lexicons themselves are not possible to judge as true or false, but the possible candidates for truth and falsity are determined by the taxonomic lexicon.

In Levi's system, truth and falsity is not relative to any taxonomic lexicon. Different paradigms can be evaluated in terms of truth or falsity.

As a conceptual scheme is a precondition for evaluations of truth or falsity for Kuhn, the conceptual scheme itself can not be a candidate for truth or falsity. However, the criteria Kuhn (1977, pp 321-322) gives for choices between paradigms, accuracy, consistency, broad scope and simplicity are included in valuation of what Levi (1991, p 145) calls informational value. So maximization of informational value is important when choosing between paradigms. The different conceptual frameworks can not be considered to have different strength, so the weak monotonicity thesis is not of interest.

Earlier in this text, it has been argued that the super- or subalgebra thesis lacks support in the history of science. But while shifts between Levian conceptual frameworks can correspond to the SSAT, shifts between Kuhnian conceptual frameworks can not. For Levian conceptual changes, the SSAT could be correct in two different cases:

1. If older paradigms are part of newer paradigms as special cases. Kuhn (1970, pp 98-102) has strong arguments against this view.

2. If scientists after a shift to a new conceptual framework always considers states of full belief in the older conceptual framework as candidates for truth or falsity. It was argued against that position.

For Levi the second possibility is a possibility because the conceptual framework is determined by the conceptual values of an inquirer and nothing else. The situation is different with Kuhnian conceptual frameworks, which are defined as the set of beliefs possible to conceive given a specific conceptual scheme. For example, when in a conceptual framework generated by a conceptual scheme that includes the notions of relativity of time, doxastic propositions about absolute time will
not be considered as true or false. To consider states of full belief from an older conceptual framework as true or false, the inquirer must be committed to the conceptual scheme generating the states of belief. A shift between conceptual frameworks is necessary. A model of changes between Kuhnian conceptual changes can therefore not adhere to the SSAT.

8. Conclusion

The only structure available for representations of paradigm shifts in the AGM-model is epistemic entrenchment, but what Friedman calls the “constitutive function” of paradigms can not be represented by epistemic entrenchment alone. A richer model is needed.

Levi's model includes the notion of conceptual frameworks. The AGM-model lacks something similar. However, Levi's model is partly created in opposition to Kuhn's theory. Two parts of Levi's model that makes it incompatible with Kuhn's theory was discussed - that conceptual frameworks can include states of full belief from different paradigms, and the super- or subalgebra thesis. Both these were shown to be problematic. In the case of Levi's definition of conceptual frameworks any source other than the conceptual values of the inquirer was excluded as determiners of the set of potential states of full belief. It was asked if big scientific changes really are characterized as changes in value and nothing more. Or are scientific revolutions conceptual changes at all according to Levi's model? If not, then what are?

The super- or subalgebra thesis was shown to lack historical support. There is no reason for an inquirer to keep all states of full belief from the earlier conceptual framework as candidates for truth or falsity.

From the critique against the AGM-model and Levi's model some conclusions about a representation of Kuhn's theory were drawn. The first is that something more than epistemic entrenchment is needed to analyze paradigms and paradigm-shifts. Kuhn's taxonomic lexicons generate a set of beliefs. Levi's model includes a set of states of full belief.

There are more similarities between Levi's theory and Kuhns. In both, the inquirer strives for maximizing informational value while avoiding errors. Levi uses these both goals to motivate that the set of states of full belief should be ordered as a boolean algebra. It might be possible to use the same structure for a model of Kuhn's theory.

Some other characteristics about a model of Kuhn's theory were presented. As a conceptual scheme is a prerequisite for judgments of truth or falsity, conceptual schemes can not be judged as true or false. Changes in conceptual frameworks are therefore only guided by informational value.

Because a conceptual framework only can contain states of full belief from one conceptual scheme, the SSAT is not applicable on changes of conceptual framework.

In this text some characteristics of an epistemic theory corresponding to Kuhn's theory of scientific changes have been given. Much work remains before such a theory is really given, and not ev-
ery part of Kuhn's theory will be possible to fully formalize. Yet many parts can be formally charac-
terized.

The focus of this work has been paradigm-shifts. Nothing here has indicated that paradigm-shifts
can not be formally represented in a theory about rational belief changes. In fact some characteris-
tics of paradigm-shifts have been given that indicates the contrary.
9. References


