Natural Phenomenon & Market Crashes

The Subprime Mortgage Crisis Analysed with Didier Sornette’s Crash Model

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Abstract

In recent years a lot of criticism against the main assumptions of economic modeling has come to up, this has led to physicists entering the field of economics. By applying models originally developed for physics to economics, especially finance, analyzing methods with a different approach can be used.

Inspired by earthquakes Didier Sornette created a model that analyze financial market crashes. With the theories of power law and log-periodicity he created a model to find the critical point prior to the crash and thereby predict when the market is supposed to crash.

The main objective of this paper is to, with the help of Didier Sornette’s Crash Model, analyze the possibility to predict market crashes. By fitting this model to OMX Stockholm 30 I will analyze if the crash of the Subprime Mortgage Crisis in 2007 could have been predicted.

After fitting the model to the data set I found that the majority of the crash times were close to the real critical point. This leads to the conclusion that the model gives accurate result. To have in mind is that times close to the critical point is used as input variables in the fitting procedure and thereby could have given misleading result. This could be an area for future research in order to increase the reliability in the model.

Keywords: Didier Sornette, market crash, power law, log-periodicity, critical point, econophysics, Subprime Mortgage Crisis, OMXS30.

Acknowledgements

I would like to propose a grateful thanks to my tutor Erik Norrman, Carl Olsson, Sergei Silvestrov and Per Ingelgård for their support in writing this thesis and fitting of the model.
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1 Introduction

1.1 Background

Ever since Benoit Mandelbrot in the early 1960s showed that fluctuations in prices of cotton differs from the expected Gaussian distribution, a lot of criticism against economic models has come up [9]. The Gaussian distribution, where each event happens randomly of the others, is one of the main assumptions for market noise in economic theory. Neither is the assumption of the neoclassical model that agents act in an economic market, which quickly reaches equilibrium, not statistical significant. Though agents more often act out of equilibrium [12 p.16pp]. This creates doubts in economic modeling.

The financial market offers a lot of good-quality statistical data which makes it a grateful field for physicists to work in. This with a combination of the scepticism mentioned above has led to more and more physics entering the field of economics, especially finance. By applying theories and methods originally developed by physicists on economics, a new science named econophysics has been born. Both physics and finance are concerned with systems of many interacting components that obey certain rules [11].

Financial market crashes has shocked the world economy several times the last century. These traumatic events affect millions of people around the world. Yet for several of these crashes no clear cause of the crash has been singled out, for instance “Black Monday” the 19th of October 1987 (see Johansen, A. & Sornette, D. (1995)). The ongoing Subprime Mortgage Crisis which appeared in 2007, was triggered by a dramatic rise in mortgage delinquencies and foreclosures in the United States and later on spread over the globe [2]. Despite this trigger has been singled out, it was after the crash occurred and thereby the disaster could not be avoided. What if there were warnings prior to the crash, could it have been avoided?
1.2 Purpose

The main objective of this paper is to, with the help of Didier Sornette’s Crash Model, analyze the possibility to predict market crashes. At first the model will be introduced and an explanation on how the model works will be held. Second the model will be used to analyze the ongoing financial Subprime Mortgage Crisis. At last a discussion will be held about if the model could have been used to predict the market crash and how it can be used to analyze this and future market crashes.

1.3 Delimitations

To analyze the market crash of the Subprime Mortgage Crisis I will use time series data of OMX Stockholm 30 (OMXS30), which is a market value-weighted index. The index consists of the 30 most traded stocks at Stockholm Stock Exchange [14].
2 Didier Sornette’s Model

2.1 Introduction

Extreme events in complex systems are, according to most physicists, related to large natural catastrophes such as earthquakes, volcanic eruptions, hurricanes and avalanches. Sornette applies this science on the financial market, which is just another complex system with dynamical interacting parts. The financial as well as the geological system suffers of extreme events which erupt in the shape of market crashes and crisis [13 p.15pp]. Sornette uses the concepts of power law, log-periodicity and critical point, which all are founded in physics, to explain why and when stock market crashes.

Most attempts to explain market crashes look at a very short time scale and try to find triggering mechanisms. Sornette suggests a different point of view where he focuses on a longer scale, in other words months and years instead of hours, days and weeks. The underlying cause of the collapse can be found in an increasing build-up of cooperative speculation, which often leads to price acceleration in the market. This phenomenon is more widely known as a “bubble” [13 p.3].

The model has been tested by Sornette and others, for instance Björstedt & Ingelgård. It has been proven to be accurate and give significant warnings of when the market will crash [1, 13].

2.2 The definition of a Market Crash, drawdowns and “outliers”

On “Black Monday” October 19th 1987, the Dow Jones Industrial Average dropped 22.6% in a single day which, in the Gaussian world, has a probability of one event every 520 million years. However, drops like this have occurred several times the last century, not once every million years. This leads to the conclusion of a fatter tail for the distribution. Mandelbrot early proposed the Lévy distribution as a replacement, of which the tail is fatter and the probability
of extreme events is larger. Events like this that lies “out” and beyond of what is probable according to the Gaussian distribution are called “outliers” [9, 13 p.49pp].

To indicate “outliers” one can measure the cumulative loss from the last maximum to the next minimum of the price. This persistent decrease in the price over consecutive days is the definition of a drawdown and is illustrated in Figure 1. Drawdowns and “outliers” can occur at the same time, as well as drawdowns occur alone. In Figure 1 all three drops are defined as drawdowns but only the first drop (“Black Monday” the 19th of October 1987) is also an “outlier” [8].

![Figure 1](image_url)

*Figure 1. Definition of drawdowns, the cumulative loss from the last maximum to the next minimum of the price. The first drop (“Black Monday” the 19th of October 1987) is also an “outlier” [13 p.52].*

As expected by the random walk hypothesis price trajectories from one day to the other should be uncorrelated. This means that positive and negative moves can be described as tossing a coin and ending up with “heads” or “tails”. The probability for such an event is $\frac{1}{2}$, and for two consecutive events for instance negative, negative is $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. If one continue to multiply like this one will notice that for each additional event the probability is divided by two. This is the definition of the exponential distribution. To associate to drawdowns, this implies that for each additional day the market decreases a drawdown is less probable. It is
convenient to observe this distribution on a logarithmic scale where it becomes a straight line. Any deviation from this line will indicate that the distribution differs from the expected exponential distribution [13 p.56].

By looking for drawdowns over consecutive days instead of daily losses indicates that, in special times, the return from one day to another may not be uncorrelated. The probability of a one day loss of 10% is 0.001. A 10% loss over three consecutive days for total 30% occurs with a probability of $0.001 \times 0.001 \times 0.001 = 10^{-9}$. This kind of market crash will thereby occur once every four million years, which obviously isn’t true for the real financial market. The conclusion of this is that the assumption of price changes over days being independent and uncorrelated is not true and consequently a prediction for future events should therefore be possible [13 p.55].

Sornette tested the appearance of “outliers” in different markets. By the method mentioned above, with the exponential distribution as null hypothesis, he showed that several drawdowns did not give significant result for the hypothesis of random walk. This supports that price variations should not be uncorrelated for these events. To support his theory Sornette tested several major world financial indices, the major currencies (in 2001, US$/DM, Yen and CHF), gold and the largest companies in the U.S. market in terms of capitalisation. The conclusion was that 1% to 2% of the largest drawdowns are not all explained by the exponential distribution. Figure 2 shows the distribution of drawdowns for the Dow Jones Industrial Average [8].
In several research articles Sornette defines and searches for “outliers” in different markets. In my point of view an “outlier” is the same as a market crash, which is a drop that lies “out” of what is probable.

### 2.3 Research prior to the Model

In several research articles Sornette et al. have shown that the build-up of speculative bubbles is established in power law acceleration with log-periodic oscillations [3, 4, 5, 7]. This model has been inspired by the analogy with other critical phenomena such as the dependence of the released strain on the time to rupture for various large Californian earthquakes [10].

#### 2.3.1 Power Law

To explain the imitation between agents during different market scenarios Sornette uses the Ising model. When agents interact with each other certain decisions are made, in this case to buy or sell. Agents can thereby imitate each other and make the same decision or the opposite. The imitation strength $K$ measures the degree of agreement and imitation of agents. In the model there exists a critical point $K_c$ that explains when the cluster of agents is of such size that most of them are in agreement. In this point the sensitivity for a small global influence is
substantial [13 p.125pp]. A critical point is defined as an explosion to infinity of a quantity that normally behaves well. In research articles the presence of this critical behaviour has been identified in the financial markets [3].

In natural sciences this behaviour is a so called critical phenomenon. This complex system where the susceptibility goes to infinity is described by a power law [13 p.126]. The mathematical features of a power law are described in section 2.4.

2.3.2 Log-periodicity

The Ising model assumes that the impact on the surrounding of every agent in the system is equal. But in real financial markets this is not true thought the size of agents and their funds differ, e.g. the decision of an institution, with large influence, has greater impact on the surrounding than the decision of an individual trader. By introducing hierarchical levels with currency blocks (US$, Yen & Euro) at the top, at the next level countries, then major banks and so on the model better defines the real market. In this hierarchical network a decision only affects agents at the same level. Suppose an individual trader at the bottom makes a decision to sell which affects its neighbour, their composed decision gain influence to affect agents on the next level and gain more influence and so on. This influence between traders and its hierarchical structure is illustrated in figure 3 [6].

A phenomenon that appears in this hierarchical structure are so called log-periodic oscillations. This means that variables oscillate with increasing frequency when the system approaches a certain critical point. The log-frequencies of the oscillations are determined by
the influence factor which quantify the time a trader waits until observing the decision of its neighbour [13 p.85 & 177pp].

2.4 The mathematical definition

To quantify stock market behaviour when it comes to financial crashes Sornette uses the concept of log-periodic correction of the power law function. The critical phenomenon of the crash is explained by an accelerating power law over time:

\[
F(t) = A + B(t_c - t)^m
\]  

(1)

Where the parameters \(A\) and \(B\) are constants moving the equation approximately up and down. The critical point \(t_c\) designate the time at which the curve declines and the crash initiates. The power of the function is denoted by the parameter \(m\) which, when increased, makes the function steeper.

To explain the endogenous build-up of hierarchical cooperation and imitation in the stock market Sornette applies a log-periodic part to equation (1):

\[
F(t) = A + B(t_c - t)^m + C(t_c - t)^m \cos(\omega \log(t_c - t) + \phi )
\]  

(2)

Where the parameter \(C\) modifies the magnitude of the log-periodic part and when increased the log-periodic oscillations enhance. The angular frequency \(\omega\) describes the period of the cosine-waves and when increased reduce the period. The last parameter of phase displacement \(\phi\) displaces the log-periodic waves along the function. For a graphical explanation of the model and parameter see Björstedt, M. & Ingelgård, P. (2004) [1, 5, 12 p.334pp].
2.5 Appliance problem

After several tests made by Sornette et al. he discovered some weaknesses in the model which could become present if it is introduced to the market. Assume that the prediction of a crash is possible and the model warns for a crash to occur in one or two months from now. This will lead to three possible scenarios:

1. Belief in the model is weak and nobody believes the prediction. Due to the assumption the market will crash. This would be a victory for the “predictors” but anyway they could just be called the “lucky ones” by critics.

2. Belief in the model is strong and everybody accepts the warning. This causes panic hence the market crashes. The prediction was true but the market crashes because of the panic effect and not the predictive power which makes the model questionable.

3. Sufficiently many believe the prediction and make some adjustments to counteract. Hence the crash does not occur and the model disproves itself.

None of the three scenarios is desired. In the first two the crash still occurs and in the last scenario the prediction disproves itself and hence the model appears to be unpredictable [7]. This is a general problem in prediction modelling and is a research area in itself in the field of socioeconomics.
3 Analysis of the Subprime Mortgage Crisis

3.1 Introduction

In this chapter I will analyze the ongoing Subprime Mortgage Crisis with the help of Didier Sornette's crash model, see equation (2). So far many tests on this model have been done by Sornette's team on different kinds of stock markets around the globe, though mostly the major world financial indices [8]. In 2004 the model was first implemented on Swedish stock market data when Affärsvärldens Generalindex (AFGX) was analysed by Björstedt & Ingelgård [1].

For all calculations and evaluations the program MATLAB has been used.

3.2 Stock market data

To analyze the crash of the Subprime Mortgage Crisis I have chosen OMX Stockholm 30 (OMXS30). OMXS30 is a market value-weighted index that consists of the 30 most traded stocks at Stockholm Stock Exchange [14]. The time series data consists of daily last trades from the 1st of January 2003 until the 31st of December 2008. The data set to be analysed is illustrated in figure 4.
To make the data more convenient to analyze I decided to define the data as 250 trading days per year, i.e. 250*6 = 1500 trading days in the whole data set. In the sequel, time is sometimes converted into decimal year units, i.e. 1/250 = 0.004 years per day.

### 3.3 Prediction of future events

Sornette discovered the presence of “outliers” in several major world financial indices which gave the conclusion that the market may not be uncorrelated hence prediction of future events is possible. Evidence of correlation was as well found in the Swedish stock market by Björstedt & Ingelgård by searching for “outliers” [1, 8].

To identify correlation between daily returns in my data set I used a similar method as Sornette. For better accuracy in my result I used a data set with longer data horizon than the one illustrated above. This data set stretches over ten years starting from the 1st of January 1999. At first I identified all drawdowns larger than 1% and for how long they lasted. Subsequently I fitted the distribution of drawdowns (DD) by the exponential law:

\[
-14
\]
\[ N(DD) = N_0 e^{-|DD|DD_c} \]  

(3)

Where \( N_0 \) (in my analysis = 326) denotes the total number of drawdowns and \( DD_c = 0.017 \) is the typical price variation, that is the average negative return divided by two. This exponential equation defines the null hypothesis (no correlation between daily returns) and becomes a straight line on a logarithmic scale. By comparing the cumulative number of drawdowns for specific amplitudes with the null hypothesis one can determine that not all drawdowns can be explained by this, see figure 5 [3].

As seen in figure 5 several drawdowns is not explained by the null hypothesis. This leads to the conclusion that the null hypothesis of no correlation between daily returns is rejected, hence “outliers” are present in OMXS30.
3.4 Fitting the Model

As the fitting procedure has been done before, both by Sornette D. et al. (see Johansen, A. & Sornette, D. (1995)) and Björstedt & Ingelgård (see Björstedt, M. & Ingelgård, P. (2004)), I will use the conclusions made by them to make the fit as good as possible.

3.4.1 Fitting procedure

To make a straightforward seven parameter fit was rejected by Björstedt & Ingelgård and after trying the same method myself I came to the same conclusion. Hence I used another method by minimizing the square variance:

\[ \text{Var}(F) = \frac{1}{N-n} \sum_{i=1}^{N} (y_i - F(t_i))^2 \quad (4) \]

Where \( n \) is the number of free variables in \( F(t) \), \( y_i \) is the actual price at time \( i \) and \( F(t_i) \) is the price evaluated by the model, in equation (2). The three variables \( A, B \) and \( C \) are linear and consequently can be expressed as three linear equations which can be solved analytically. By expressing these explicitly the seven parameters are narrowed down to the four nonlinear variables \( m, t_c, \omega \) and \( \phi \) that are calculated by multiple linear regression of equation (4). Because I am performing a highly nonlinear parameter fit with noisy data there will in general be several local minima. To find the global minima of the variance I used a multidimensional search method called the Gauss-Newton method. This method was successfully used by Björstedt & Ingelgård.

3.4.2 Constraints

Sornette et al. discovered that the best fit of the model appeared when the starting point was at the beginning of the bubble and the endpoint at the highest point prior to the crash. Furthermore the data horizon should be approximately one to two years to give meaningful result [4]. In further testing they also found certain constraining intervals for two of the parameter which are: \( 0.2 < m < 0.8 \) and \( 5 < \omega < 15 \). The phase \( \varphi \) can not be meaningfully limited and \( t_c \) have to be larger than the last point in the sample data [13 p.335pp].

For the Gauss-Newton method to converge the starting points has to be close to the optimal points. Therefore I first tested all possible combinations of \( m, t_c, \omega \) and \( \varphi \) using the constraints
above and created a four-dimensional matrix containing the variance from equation (4) of each fit. By minimizing this matrix I ended up with good starting points for the Gauss-Newton calculation. Using the starting points the Gauss-Newton method searched for local optimal points and best fits for the data set. From these local optimums I tried to find the ones who where best for my analysis. According to Sornette the value of $m$ should be between 0 and 1 for the price to accelerate and remain finite [13 p.335]. I also excluded $B>0$ and results that were obviously wrong, for example imaginary numbers.

3.4.3 Choosing interval
To be able to make the best fit for my data I had to find the beginning of the bubble and the highest peak prior to the crash. By studying the data set in figure 4 the highest peak prior to the crash (as well the largest value in the whole set) is approximately in the middle of 2007. From the figure one can also conclude that the bubble seems to begin sometime in early 2003. I also studied a longer data horizon which gave the same conclusion. According to Sornette, to get the best fit, data horizons of one to two years should be used. On the other hand Björstedt & Ingelgård found the best fit to occur for intervals lasting between 0.8 to 2.4 years. For this test I decided to use data horizons of 0.8 to 2.6.

By finding the maximum value of the data set one can find the starting date 0.8 to 2.6 years before this date, which is approximately from the beginning of 2005 until the middle of 2006. More exactly this is the interval of 2004.956 to 2006.556. Fitting the model with the starting point in this interval with an increment of 0.2 gives me nine different starting points to fit the model. For these nine starting points I fitted the model to the data using input crash points $t_c$ close to (within a week) the maximum value of the set.

3.4.4 Fitting the model to OMXS30
Fitting the model to OMXS30 for the intervals and with the constraints mentioned before gave several warnings for the market to crash. A total of 50 warnings for crash times from 2007.52 until 2008.28 was identified, which are illustrated in appendix A. The most frequent crash dates was between 2007.53 and 2007.55 where 17 of these warnings occurred, see figure 6.
Figure 6. Histogram of crash times for the 50 warnings (50 bins).
4 Discussion and Conclusions

The analysis show several warnings for the market to crash but how do we know that this result is accurate? The majority of the warnings occurred in between of 2007.50 and 2007.60. Since the maximum value prior to the crash, and in my point of view the time of the crash, occurred at 2007.56 these warnings should be considered as acceptable. More exactly, 17 warnings or 34% of the warnings occurred in the interval from 2007.53 to 2007.55. This is approximately from three days up to one week before the real crash date. However, to pinpoint a crash with the deviation of one week is still a good approximation. This leads to the conclusion that Didier Sornette’s Crash Model gives useful warnings and could have been used to predict The Subprime Mortgage Crisis.

One can argue if the same result would be found if the actual time of the crash would not have been used as an input when fitting the model. Suppose another date would have been used, for instance a date far away from the actual time of the crash. Would the majority of the warnings still occur close to the critical point or would it only give nonsense results? If the same crash points would have been found this would strengthen the reliability in this model significantly.

My opinion is that this model could be used by agents to determine how probable a crash is at a certain time. The model can be illustrated as a type of seismograph for the financial market to detect build-up of cooperative behavior prior to market crashes. But to be certain about the result given by the model I would recommend complementary analysis before taking a decision to adjust for a expected market crash.

A weakness of this model is that it uses absolute number when searching for market crashes. For instance is a drop of ten when the index is at 500 twice as large (in percentage) as a drop of ten when the index is at 1000. This means that the model may miss a crash that seems to be small but actually is larger in percentage numbers.
5 Future Research Studies

I have proven Didier Sornette’s Crash Model to be accurate and give significant result, but I also have some doubts in my analysis. Further research is necessary to conclude if my result is significant.

First of all it would be interesting to fit the model with crash times that are far away from the real critical point to see if the result still is the same. If the result is not the same I would have great doubts in the model. Because that would imply that to give accurate result the input crash points in the optimization have to be close to the real crash time. In this case crashes can only be found close to the time of the crash and thereby would be difficult to avoid.

Another research area is to try speculating in future changes of the financial index and thereby get more statistical data for the time series and fit the model to this. Suppose an ascent trend in OMXS30 is supposed to stagnate, then quantify this and add to the data and run the fitting procedure. The new statistical data will change the result but this will also add more uncertainty to the analysis because of the speculation.

In the mathematical sense analyzing a rapid increase in the market is the same as analyzing a critical crash but exactly the opposite. This implies that by inverting the model maybe it could be used to find the time when a declining market is most likely to turn. But this is only correct if the same assumptions prior to the model can be made for the inverted scenario as well.

Another approach would be to analyze different data sets than indices. Since a change in the stock market is the outcome of a change somewhere else in the system maybe other data should be considered, for instance financial ratios. By unravelling other data the build-up for crashes may be detected earlier.
References

Articles


**Literature**


**Internet**

Appendix A

This appendix illustrates the result from my fitting of Dider Sornette’s crash model, equation (2), for OMXS30 with different time intervals. The crash time $t_c$ is chosen from the maximum value prior to the crash (that is 2007.56) with the variation of a week.

**A.1 2004.956 to 2007.524**

![Figure A.1. Sornette’s model fitted to data from 2004.956 to 2007.524](image)

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*Table A.1. The parameters for Sornette’s model to fit data from 2004.956 to 2007.524*
A.2 2005.156 to 2007.524

Figure A.2. Sornette’s model fitted to data from 2005.156 to 2007.524

Table A.2. The parameters for Sornette’s model to fit data from 2005.156 to 2007.524

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**A.3 2005.356 to 2007.524**

![Figure A.3. Sornette's model fitted to data from 2005.356 to 2007.524](image)

Table A.3. The parameters for Sornette's model to fit data from 2005.356 to 2007.524

<table>
<thead>
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**A.4 2005.556 to 2007.524**

*Figure A.4. Sornette’s model fitted to data from 2005.556 to 2007.524*

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*Table A.4. The parameters for Sornette’s model to fit data from 2005.556 to 2007.524*
A.5 2005.756 to 2007.524

Figure A.5. Sornette’s model fitted to data from 2005.756 to 2007.524

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Table A.5. The parameters for Sornette’s model to fit data from 2005.756 to 2007.524
**A.6 2005.956 to 2007.524**

![Figure A.6. Sornette’s model fitted to data from 2005.956 to 2007.524](image)

<p>| | | | | | | |</p>
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*Table A.6. The parameters for Sornette’s model to fit data from 2005.956 to 2007.524*
**A.7 2006.156 to 2007.524**

![Figure A.7. Sornette’s model fitted to data from 2006.156 to 2007.524](image)

**Table A.7. The parameters for Sornette’s model to fit data from 2006.156 to 2007.524**

<table>
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**A.8 2006.356 to 2007.524**

No meaningful result could be found in this data interval.

**A.9 2006.556 to 2007.524**

No meaningful result could be found in this data interval.