Abstract

This thesis focuses on the econometric evaluation of Fisher hypothesis. According to Fisher hypothesis there exist a long run equilibrium relationship between nominal interest rate and inflation rate. We analyze the data for Belgium at the different time periods of 1992 to 2000, 2001 to 2009 and 1992 to 2009. Cointegration technique is used to analyze this long run equilibrium relationship. Using Engle-Granger and Johansen cointegration tests, attempt is made to find cointegration between inflation rate and nominal interest rate. Both Engle-Granger and Johansen tests reject the hypothesis of cointegration between inflation rate and nominal interest rate in all the three sets of data defined above.
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Introduction

The relationship between nominal interest rate and the expected rate of inflation has been a topic of immense interest for a long period of time. Increase in the general level of prices of goods and services in an economy over a period of time is called inflation. The percent increase in prices from one time period to another time period is called inflation rate. The nominal interest rate refers to the rate of interest without having taken account of the rate of inflation or the rate of interest before adjustment for inflation. When we adjust the nominal interest rate for inflation then it is called real interest rate. A standard view, commonly referred to as the Fisher effect, is the idea that a positive one-to-one relationship exists between the (expected) inflation and the nominal interest rate, assuming the real interest rate is stationary (Fisher, 1930). The Fisher equation, often called Fisher hypothesis, states that the nominal interest rate can be affected by either the real interest rate or inflation. Let $r$ denote the real interest rate, $i$ denote the nominal interest rate and $\pi$ denote the rate of inflation, then the Fisher equation is

\[ i = r + \pi \]

The real interest rate is the difference between the nominal interest rate and the rate of inflation.

\[ r = i - \pi \]

It can be called the ex-post real interest rate. According to the quantity theory of money a 1% increase in money growth implies a 1% increase in the rate of inflation. According to the Fisher equation a 1% increase in inflation implies a 1% increase in the nominal interest rate. This one-to-one relationship between the inflation rate and the nominal interest rate is called the Fisher effect. The following equation is the basic form of the Fisher equation estimated in much of the work on real interest.

\[ i_t = \alpha_0 + \alpha_1 \pi_t + e_t \]

Here $\alpha_0$ is a constant term, $\alpha_1$ is the coefficient of inflation rate, $i_t$ is nominal interest rate, $\pi_t$ is the rate of inflation and $e_t$ is an error term and is supposed to be i.i.d process (Bonham, 1991). The existence of this relationship has important implications for policy makers, debtors and creditors, particularly given that it is commonly argued that inflationary expectations may directly influence the nominal interest rate. Because of its great importance in policy making, the Fisher relation has been studied
for many countries. The Fisher effect has been strong in some countries, for example, the United States, Canada and the United Kingdom in the post-war period up until 1979. However, the relationship post-1979 has not been as robust (Crowder and Hoffman, 1996; Mishkin, 1992).

The objective of our study is to see the equilibrium relationship between nominal interest rate and inflation rate by using the cointegration technique. To investigate this question we consider Belgium. We analyze the Fisher hypothesis before (1992-2000) and after (2001-2009) the euro currency introduce in Belgium and in the time period from 1992 to 2009. We use different cointegration tests to analyze this equilibrium relationship in different time periods.

This paper is organized as follows: In the first section we introduce the main concept of Fisher equation and also discuss the data obtained for analysis. Section 2 discusses the main econometric theory used in our analysis. Section 3 contains the graphical representation of our data. In section 4 different unit root tests and cointegration tests are applied to the data and section 5 concludes.

1.1 Data

We select Belgium and collect the monthly data for inflation rate and nominal interest rate to undertake the study of the Fisher hypothesis. The Consumer Price Index (CPI) is used as the measure of Inflation that is defined as the natural log of the ratio of CPI_{t+1} to CPI_t. The data is collected from the official website of Organization for Economic Co-operation and Development (OECD). We collect the data for the time period 1992 to 2009. We divide the data into two equal time periods that is before (from 1992 to 2000) and after (from 2001 to 2009) the introduction of Euro currency to see the equilibrium relation between nominal interest rate and inflation rate in both short periods and also in a long period of time (from 1992-2009).
2. Econometric Theory

This section will discuss various topics of econometric time series which will be used in our analysis. We will start this section with discussing the stationarity and non stationarity of time series data. We will explain unit root in the data and then tests for unit root in a time series. Among the tests we will discuss Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) test for unit root. At the end of this section we will explain cointegration among series and also discuss Engle-Granger and Johansen tests of cointegration.

2.1 Stationarity

In practice a researcher is always interested to know the theoretical mean, variance and autocorrelations of a series when those are unknown. When a time series possesses a constant mean, a constant variance over time and the autocorrelation function that depends only on the length of the expressed lags then such a series is called stationary or covariance stationary. Covariance stationary is also called weakly stationary, second-order stationary or wide-sense-stationary process. In covariance stationary series mean and autocovariances are unaffected by change in the time origin. Formally a time series is covariance stationary if for all t and t-s,

\[ E(y_t) = E(y_{t-s}) = \mu \quad \text{constant & independent of time} \]

\[ \text{var}(y_t) = \text{var}(y_{t-s}) = \sigma^2 < \infty \quad \text{constant & independent of time} \]

\[ \text{cov}(y_t - y_{t-s}) = y_s \quad \text{autocovariances.} \]

Here \( t = 1,2,3,\ldots, T \), and \( s = \text{number of lags} \)

If a series does not possess the above properties then the series is non stationary. There are several methods to check whether series posses the properties of stationarity or not. Analyzing graphs of the series is one of them which provide a rough idea about the stationarity. In this method we mostly use the following two techniques.
• The most common technique is visual analyzing of a time series by observing the line graph for the data. A plot of stationary series fluctuates around its mean. A series with definite upward or downward trend with the passage of time is often representing a non stationary series.

• The second technique to get a rough idea about stationarity is to observe correlogram of autocorrelation function. For a stationary time series, the Auto Correlation Function’s (ACF) tend to zero rather quickly at initial lags values. While for a non stationary series the ACFs show linear decline.

In our analysis we graphically represent our data to see weather our data is stationary or non stationary. For this purpose we plot the data in line graph and also obtain the ACF’s for our time series data. We observe the behavior of these graphs and then decide about the presence or absence of stationarity in the data.

2.2 Unit Root Test

Unit root test help us in investigating whether a given series contains a trend and the nature of trend that is deterministic or stochastic. When any series contains a unit root it tells us about the non stationarity of the series. There are several methods to test for unit root in the series. In our analysis we use the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests to check unit root in our data and then decide about the presence or absence of stationarity in our data.

2.2.1 Augmented Dickey Fuller test

The Augmented Dickey-Fuller test (Dickey and Fuller 1979) is a test for a unit root in a time series. This test is an extension of the Dickey-Fuller test for a larger and more complicated set of time series models. This test provides a formal test for non stationarity in time series data. The ADF test is used to test the presence of unit root in the coefficient of lagged variables in a time series model. If the coefficient of a lagged variable in a time series model shows a value of one then this is an indication of unit root in the series.

To test the presence of a unit root in a time series, Dickey and Fuller considered the estimation of the parameter $\gamma$ from the following models.
\[ \Delta Y_t = \gamma Y_{t-1} + \alpha_2 \Delta Y_{t-1} + \alpha_3 \Delta Y_{t-2} + \cdots + \varepsilon_t \]  
Random walk

\[ \Delta Y_t = \alpha_0 + \gamma Y_{t-1} + \alpha_2 \Delta Y_{t-1} + \alpha_3 \Delta Y_{t-2} + \cdots + \varepsilon_t \]  
Random walk with drift

\[ \Delta Y_t = \alpha_0 + bt + \gamma Y_{t-1} + \alpha_2 \Delta Y_{t-1} + \alpha_3 \Delta Y_{t-2} + \cdots + \varepsilon_t \]  
Random walk with drift and time trend

It is assumed that \( Y_0 = 0 \) and \( \varepsilon_t \sim iid \ N(0,\sigma^2) \)

We test the null hypothesis \( H_0 : \gamma = 0 \), for the above three models.

**Decision rule:**

- **When ADF test statistic value \( > \) test critical value \( \rightarrow \) accept the null hypothesis.**  
  Unit root exists in the series. The series is not stationary.

- **When ADF test statistic value \( < \) test critical value \( \rightarrow \) Reject the null hypothesis.**  
  Unit root do not exist in the series. The series is stationary.

When null hypothesis of unit root is accepted then we difference the data before running a regression for the series. If the null hypothesis is rejected, the series is stationary and can be used without differencing.

### 2.2.2 Phillips-Perron Test

Phillips and Perron (1989) develops a formal procedure to test for unit root in the presence of a structural change at time period \( t = \tau + 1 \). PP proposed nonparametric transformations of the \( \tau \) statistics from the original DF regressions such that under the unit root null, the transformed statistics (the “\( z \)” statistics) have DF distributions.

The model under the null hypothesis is

**\( H_0 \):** \( y_t = \alpha_0 + y_{t-1} + \mu_1 D_p + \varepsilon_t \)

Here \( D_p \) represents a dummy variable such that \( D_p =1 \) if \( t = \tau + 1 \) and zero otherwise.

The model under the alternative hypothesis is

**\( H_1 \):** \( y_t = \alpha_0 + \alpha_2 y_{t-1} + \mu_2 D_L + \varepsilon_t \)

Here \( D_L \) is a level dummy variable such that \( D_L =1 \) if \( t > \tau \) and zero otherwise.

**Decision Rule:**

Accept the null hypothesis of the unit root if the calculated value of the \( t \)-statistics is greater than the critical values (Enders, 2004).
2.3 ACF and PACF

Suppose $Y_1, Y_2, ..., Y_N$ are measurements at time $X_1, X_2, ..., X_N$, the lag $k$ autocorrelation function, ACF, (Box and Jenkins, 1976) is defined as

$$
\tau_k = \frac{\sum_{l=1}^{N-k} (Y_l - \bar{Y})(Y_{l+k} - \bar{Y})}{\sum_{l=1}^{N} (Y_l - \bar{Y})^2}
$$

The ACF is the correlation between two values of the same variable at times $X_i$ and $X_{i+k}$ instead of correlation between two different variables.

Whereas Partial autocorrelation function (PACF) measures the correlation between an observation $k$ periods ago and the current observation, after controlling for observations at intermediate lags (i.e. all lags $< k$).

2.4 Cointegration:

When a non stationary series is differenced ‘d’ times to make it stationary then this series contains ‘d’ unit roots and is said to be integrated of order ‘d’ that is I(d).

Suppose $y_t$ and $x_t$ are I(1) process and

$$
y_t = \beta_0 + \beta_1 x_t + \epsilon_t,
$$

where $\beta_1 \neq 0$ and $\epsilon_t \sim I(0)$

Then $y_t$ and $x_t$ are said to be cointegrated and the regression of $y$ on $x$ is called cointegrating regression.

More generally, Engle and Granger (1987) provide the following definition of cointegration.

The components of the vector $x_t = (x_{1t}, x_{2t}, ..., x_{nt})'$ are said to be cointegrated of order $d, b$, denoted by $x_t \sim CI(d, b)$ if

a) All components of $x_t$ are integrated of order $d$.

b) There exists a vector $\beta = (\beta_1, \beta_2, ..., \beta_n)$ such that linear combination

$$
\beta x_t = (\beta_1 x_{1t} + \beta_2 x_{2t} + \cdots + \beta_n x_{nt})
$$

is integrated of order $(d-b)$, where $b > 0$.

The vector $\beta$ is called the cointegrating vector.

Cointegration among non stationary variables implies that their stochastic trend must be linked.
Engle and Granger (Residual based) cointegration test and Johansen cointegration test are two basic approaches that are commonly used to test for cointegration. Since Fisher equation concerns about the cointegration of nominal interest rate and inflation rate, we check the cointegration between these two variables in three different time period using these two tests.

2.4.1 Engle-Granger (Residual based) test for cointegration

Engle Granger test for cointegration is also called a residual based cointegration test. This test uses the following steps to test for cointegration between variables.

Step 1
In first step we test the variable for their order of integration. If their order of integration is same and not zero we proceed to the next step. In this step we use ADF and PP tests to determine their order of integration.

Step 2
In this step we estimate the long run equilibrium relation by regressing dependent variable on independent one. We then save the residual of this regression equation. Here in our case we first regress nominal interest rate on inflation rate and then inflation rate on nominal interest rate as in this economic theory no information was given about independent and dependent variables. We then save the residual of both the regression equation.

Step 3
In last step we test residual series for unit root. If unit root is present in the residual series we conclude that no cointegration exists among variables. If null of unit root rejected in the residual series we say that the variables are cointegrated. In our case we use the ADF and PP test to see if there exists a unit root in both the residual series. We then decide about cointegration on the basis of results obtain from these two tests.
2.4.2 Johansen Test for cointegration

Johansen’s (1988) methodology takes its starting point in the vector autoregression (VAR) of order \( p \) given by

\[
y_t = \mu + A_1 y_{t-1} + \cdots + A_p y_{t-p} + \varepsilon_t
\]

where \( y_t \) is an nx1 vector of variables that are integrated of order one, commonly denoted \( I(1) \), and \( \varepsilon_t \) is an nx1 vector of innovations. This VAR can be re-written as

\[
\Delta y_t = \mu + \Pi y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + \varepsilon_t
\]

where \( \Pi = \sum_{i=1}^{p} A_i - I \) and \( \Gamma_i = - \sum_{j=i+1}^{p} A_j \)

If the coefficient matrix \( \Pi \) has reduced rank \( r < n \), then there exist \( n \times r \) matrices \( \alpha \) and \( \beta \) each with rank \( r \) such that \( \Pi = \alpha \beta' \) and \( \beta'y_t \) is stationary. Here \( r \) is the number of cointegrating relationships, the elements of \( \alpha \) are known as the adjustment parameters in the vector error correction model and each column of \( \beta \) is a cointegrating vector.

Johansen proposes two different likelihood ratio tests of the significance of these canonical correlations and thereby the reduced rank of the \( \Pi \) matrix: the trace test and maximum eigenvalue test.

1) Trace Statistic

\[
\lambda_{trace}(r) = -T \sum_{i=r+1}^{n} \ln (1 - \hat{\lambda_i})
\]

Where \( \hat{\lambda_i} \) are the estimated values of the characteristic roots and also called Eigen values and \( T \) is the number of usable observation.

2) The Maximum Eigen Value Statistic

\[
\lambda_{max}(r, r + 1) = -T \ln(1 - \hat{\lambda}_{r+1})
\]

The trace test tests the null hypothesis of \( r \) cointegrating vectors against the alternative hypothesis of \( n \) cointegrating vectors. The maximum eigenvalue test, on the other hand, tests the null hypothesis of \( r \) cointegrating vectors against the alternative hypothesis of \( r + 1 \) cointegrating vectors (Hjalmarsson and Österholm, 2007).

Decision Rule:

For both tests if the test statistic value is greater than the critical value we will reject the null hypothesis.
3. Graphical presentation of data

In this section we will graphically present the data for inflation rate and nominal interest rate for different time period. We will start with the data from 1992-2000 for these two series. Next we will discuss data from the time period 2001-2009 and in the last of this section we will discuss the data in long run and will graphically present the data from 1992-2009 for inflation rate and nominal interest rate.

3.1 The period 1992-2000

In our first analysis we use the data for inflation rate and nominal interest rate from the time period 1992 to 2000. Figure 1 and Figure 2 represent inflation rate and nominal interest rate respectively. From these graphs we can see that Inflation Rate shows no trend where as Nominal Interest Rate shows downward trend with passage of time. From these two figures we can see that nominal interest rate is look like non stationary and we are not sure about the stationarity of inflation series.

The ACFs for inflation rate in Figure 3 shows geometric decline which indicates the lack of stationarity in the data. In Figure 4, the ACFs for nominal interest rate shows linear decline which indicates non stationarity in this series. For nominal interest rate we can see that there is only one significant spike in PACFs.
After taking the first difference of the two series we observe that the graphs for inflation rate and nominal interest rate are clearly stationary. Figure 5 and Figure 6 represent inflation rate and nominal interest rate respectively after taking first difference of the original two series. The values of both series fluctuate around its mean.
The correlogram for these two series are clearly stationary when we take the first difference of data. Figure 7 and Figure 8 represents correlogram for inflation rate and nominal interest rate respectively after taking first difference.

3.2 The period 2001-2009

We plot the data of nominal interest and inflation rate for the time period 2001-2009. In the beginning of 2001, Euro currency was introduced in Belgium. The line graphs for these two series are given in Figure 9 and Figure 10. The graph for inflation series shows no specific trend and seems to be stationary. The nominal interest rate has downward trend which tells about the lack of stationarity in the series. Looking to the correlogram for these two series in this time period we can see that the ACFs for nominal interest rate shows linear decline. The ACFs spike for inflation rate tends to zero rapidly after a few lags. The PACFs for nominal interest rate has only one
significant spike and for inflation rate two significant spikes. Figure 11 and Figure 12 represent correlogram for nominal interest and inflation rate respectively.

Figure 9: Graph for inflation rate

Figure 10: Line graph for nominal interest rate

Figure 11: Correlogram for inflation rate

Figure 12: Correlogram for nominal interest rate
We can see from the line graph and correlogram for inflation rate and nominal interest rate that after taking the first difference of the data the two series are clearly become stationary. Figure 13, 14, 15 and 16 shows line graphs and correlograms for these two series after taking the first difference.

**Figure 13: Inflation after 1\(^{st}\) difference**

**Figure 14: Nominal interest after 1\(^{st}\) difference**

**Figure 15: Inflation correlogram after 1\(^{st}\) D**

**Figure 16: Nominal interest correlogram after 1\(^{st}\) D**

<table>
<thead>
<tr>
<th>Autocorrelation</th>
<th>Partial Correlation</th>
<th>AC</th>
<th>PAC</th>
<th>Q-Stat</th>
<th>Prob</th>
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<td><img src="image2.png" alt="Image of correlogram" /></td>
<td><img src="image3.png" alt="Image of correlogram" /></td>
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<td><img src="image5.png" alt="Image of correlogram" /></td>
<td><img src="image6.png" alt="Image of correlogram" /></td>
</tr>
</tbody>
</table>
3.3 The period 1992-2009

Similarly we analyze the data by using graphs and correlograms for inflation rate and nominal interest rate for the time period 1992 to 2009. For this time period the Graph for nominal interest rate are look like non Stationary while we are not sure about the lack of stationarity in the graph of inflation series. For this period of time nominal interest shows downward trend whereas inflation rate fluctuates around its mean value. There seems to be some structural break in the middle of this period which can be seen from Figure 17. In this case examining correlogram for nominal interest rate we can see that the ACFs shows linear decline and only one significant spike of PACFs while for inflation rate the ACF spikes tends to zero very rapidly. The results for nominal interest rate clearly tell us about the presence of non stationarity in the data while we are unclear about the inflation series.
After taking the first difference of the data for inflation and nominal interest rate in this period the two series clearly look stationary. We can see from Figure 21 and Figure 22 that now both the series are stationary. Correlogram in Figure 23 and 24 also confirms the stationarity of both series after taking 1st difference of data.

Figure 21: Nominal interest after 1st difference

Figure 22: Inflation after 1st difference

Figure 23: Nominal interest correlogram after 1st D

Figure 24: Inflation correlogram after 1st D
4. Analysis and Results

In this section we perform different unit root tests and cointegration tests for nominal interest and inflation rate in three different time periods. In our first analysis we perform Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests for these two series to check the stationarity in the data for [1992-2000], [2001-2009] and [1992-2009] time periods. We use Engle Granger and Johansen cointegration tests to see the presence or absence of cointegration between nominal interest and inflation rate in these three time periods.

4.1 Application of Unit Root Test

As define above the necessary condition for cointegration is that both the series should be non stationary. i.e. the two series should be integrated of same order. Previously we observe lack of stationarity in nominal interest rate series while we are unclear about inflation rate series. Now we will apply some statistical tests to test the presence of unit root in to our data. We will use ADF and PP tests to check unit root in our two series for different time periods.

4.1.1 The period 1992-2000

Using the monthly data for both nominal interest and inflation rate we apply ADF and PP tests for the period of 1992 to 2000 and we see that both series are non stationary. For inflation series we use intercept only while for nominal interest rate we use both intercept and trend in our analysis. We use four lags values of both variables in our analysis. We observe in Table 1 that the null hypothesis of unit root is accepted by both ADF and PP tests at 1%, 5% and 10% significance level for both series.

After taking first difference for nominal interest rate and inflation rate we can see from Table 2 that both ADF and PP test confirms the stationarity of our data. For both series ADF and PP test statistics reject the null hypothesis of unit root in our data after taking the first difference.
### Table 1: Unit root tests for stationarity (1992-2000)

<table>
<thead>
<tr>
<th>Test</th>
<th>Test Statistics</th>
<th>C.V 1%</th>
<th>C.V 5%</th>
<th>C.V 10%</th>
<th>Null Hypothesis</th>
<th>Result</th>
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<tbody>
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<td>-3.493129</td>
<td>-2.888932</td>
<td>-2.581453</td>
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<td>PP</td>
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<td>-2.888669</td>
<td>-2.581313</td>
<td>Accept</td>
<td>Non-Stationary</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Nominal Interest Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADF</td>
</tr>
<tr>
<td>PP</td>
</tr>
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</table>

### Table 2: Unit root tests for stationarity (1992-2000) after 1st difference

<table>
<thead>
<tr>
<th>Test</th>
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<th>C.V 1%</th>
<th>C.V 5%</th>
<th>C.V 10%</th>
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<tbody>
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<td>PP</td>
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<td>Reject</td>
<td>Stationary</td>
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<table>
<thead>
<tr>
<th>Nominal Interest Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADF</td>
</tr>
<tr>
<td>PP</td>
</tr>
</tbody>
</table>

#### 4.1.2 The period 2001-2009

Using the monthly data of nominal interest rate and inflation rate we applied both ADF and PP tests for the period 2001 to 2009. For both the series we use intercept and four lag values in the analysis. For inflation series we found that ADF reject the null hypothesis of unit root at 1%, 5% and 10% significance level where as PP reject the null hypothesis at 1%, 5% and accept at 10% significance level. The ADF and PP reject the null hypothesis at 1%, 5% and 10% significance level for nominal interest rate series. The results are given in Table 3.

After taking the first difference of the two series both ADF and PP reject the null hypothesis of unit root. From Table 4 we can see that ADF and PP test statistic values fall in the critical region. Hence we will reject the null hypothesis and conclude that both the series are stationary after taking the first difference.
Table 3: Unit root tests for stationarity (2001-2009)

<table>
<thead>
<tr>
<th>Test</th>
<th>Test Statistics</th>
<th>C.V 1%</th>
<th>C.V 5%</th>
<th>C.V 10%</th>
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<th>Result</th>
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<tbody>
<tr>
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Nominal Interest Rate

<table>
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<th>Test Statistics</th>
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<th>C.V 5%</th>
<th>C.V 10%</th>
<th>Null Hypothesis</th>
<th>Result</th>
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<td>-2.888669</td>
<td>-2.581313</td>
<td>Accept</td>
<td>Non-Stationary</td>
</tr>
</tbody>
</table>

Table 4: Unit root tests for stationarity (2001-2009) after 1st difference

<table>
<thead>
<tr>
<th>Test</th>
<th>Test Statistics</th>
<th>C.V 1%</th>
<th>C.V 5%</th>
<th>C.V 10%</th>
<th>Null Hypothesis</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADF</td>
<td>-5.218129</td>
<td>-4.057528</td>
<td>-3.457808</td>
<td>-3.154859</td>
<td>Reject</td>
<td>Stationary</td>
</tr>
</tbody>
</table>

Nominal Interest Rate

<table>
<thead>
<tr>
<th>Test</th>
<th>Test Statistics</th>
<th>C.V 1%</th>
<th>C.V 5%</th>
<th>C.V 10%</th>
<th>Null Hypothesis</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADF</td>
<td>-7.905081</td>
<td>-3.493129</td>
<td>-2.888932</td>
<td>-2.581453</td>
<td>Reject</td>
<td>Stationary</td>
</tr>
</tbody>
</table>

4.1.3 The period 1992-2009

Augmented Dickey-Fuller and Phillips-Perron tests are performed for nominal interest rate and inflation rate for the time period 1992-2009. For inflation series we use intercept only while for nominal interest rate we use both intercept and trend in our analysis. We use four lags values of both variables in our analysis. The ADF test rejects the presence of unit root in inflation series at 1%, 5% and 10% significance level but PP accepts the null hypothesis at 5% and 10%. For nominal interest rate both the tests reject the null hypothesis at all three significance levels. For this time period we are not completely clear about the non stationarity of inflation series. The results are given in Table 5.
Table 5: Unit root tests for stationarity (1992-2009)

<table>
<thead>
<tr>
<th>Test</th>
<th>Test Statistics</th>
<th>C.V 1%</th>
<th>C.V 5%</th>
<th>C.V 10%</th>
<th>Null Hypothesis</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADF</td>
<td>-2.354129</td>
<td>-3.462574</td>
<td>-2.875608</td>
<td>-2.574346</td>
<td>Accept</td>
<td>Non-Stationary</td>
</tr>
<tr>
<td>PP</td>
<td>-2.998226</td>
<td>-3.460739</td>
<td>-2.874804</td>
<td>-2.573917</td>
<td>Accept</td>
<td>Non-Stationary</td>
</tr>
</tbody>
</table>

Nominal Interest Rate

<table>
<thead>
<tr>
<th>Test</th>
<th>Test Statistics</th>
<th>C.V 1%</th>
<th>C.V 5%</th>
<th>C.V 10%</th>
<th>Null Hypothesis</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADF</td>
<td>-2.502462</td>
<td>-4.001516</td>
<td>-3.430963</td>
<td>-3.139114</td>
<td>Accept</td>
<td>Non-Stationary</td>
</tr>
<tr>
<td>PP</td>
<td>-2.429906</td>
<td>-4.001311</td>
<td>-3.430963</td>
<td>-3.139114</td>
<td>Accept</td>
<td>Non-Stationary</td>
</tr>
</tbody>
</table>

In Table 6 below we can see that both the series are clearly stationary after taking the first difference. Test statistics values for ADF and PP are smaller than the critical values given in the Table 6. Hence we cannot accept the hypothesis of unit root in the two series and conclude that both the series are stationary after taking the first difference.

Table 6: Unit root tests for stationarity (1992-2009) after 1st difference

<table>
<thead>
<tr>
<th>Test</th>
<th>Test Statistics</th>
<th>C.V 1%</th>
<th>C.V 5%</th>
<th>C.V 10%</th>
<th>Null Hypothesis</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADF</td>
<td>-7.166299</td>
<td>-3.462574</td>
<td>-2.875608</td>
<td>-2.574346</td>
<td>Reject</td>
<td>Stationary</td>
</tr>
<tr>
<td>PP</td>
<td>-10.13896</td>
<td>-3.460884</td>
<td>-2.874868</td>
<td>-2.573951</td>
<td>Reject</td>
<td>Stationary</td>
</tr>
</tbody>
</table>

Nominal Interest Rate

<table>
<thead>
<tr>
<th>Test</th>
<th>Test Statistics</th>
<th>C.V 1%</th>
<th>C.V 5%</th>
<th>C.V 10%</th>
<th>Null Hypothesis</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADF</td>
<td>-10.73998</td>
<td>-3.460884</td>
<td>-2.874868</td>
<td>-2.573951</td>
<td>Reject</td>
<td>Stationary</td>
</tr>
<tr>
<td>PP</td>
<td>-10.81905</td>
<td>-3.460884</td>
<td>-2.874868</td>
<td>-2.573951</td>
<td>Reject</td>
<td>Stationary</td>
</tr>
</tbody>
</table>

After applying ADF and PP tests on both series for different time periods we are not completely clear about the stationarity or non stationarity of inflation series in different time periods. If we use the significance level 1% in our analysis then we can say that our both series are non stationary in these three time periods. We will proceed further and look whether the cointegration test can help us to find any relationship between these variables.
4.2 Application of cointegration test

As mentioned earlier, the necessary condition for cointegration is that both the series should be non stationary and integrated of same order. This condition is fulfilled at 1% significance level for both series at different time periods. Now we will proceed further and apply different cointegration methods to test the presence or absence of cointegration between nominal interest rate and inflation rate in different time periods. For this purpose we will use Engle-Granger test and Johansen test to analyze cointegration.

4.2.1 Engle-Granger (Residual based) test for cointegration

We have already seen that nominal interest rate and inflation rate series are non stationary at 1% significance level and integrated of order one. Now we will estimate the long run equilibrium relationship for inflation rate \((i)\) and nominal interest rate \((\pi)\) in the following forms.

\[
i = \alpha_0 + \alpha_1 \pi + e_t
\]

and

\[
\pi = \alpha_0 + \alpha_1 i + e_t
\]

In first step we regress \(i\) on \(\pi\) and then \(\pi\) on \(i\). We run the regression for \(i\) and \(\pi\) before the Euro currency (1992 to 2000), after the introduction of Euro currency (2001 to 2009) and in the long run from (1992 to 2009).

In order to determine if nominal interest rate and inflation rate series are cointegrated, we save the residual sequence from the above two regression equation. If these residual series are stationary, \(i\) and \(\pi\) are cointegrated of order \((1, 1)\). We perform Engle-Granger cointegration test separately for each time period.

4.2.1.1 The period 1992-2000

Since we obtain residual series from a regression equation, there is no need to include intercept term. The parameter of interest in above two equations is \(\alpha_1\). We perform ADF and PP test to check the residual series for unit root. For the regression of \(i\) on \(\pi\) and of \(\pi\) on \(i\) in Table 8 we found that ADF test accept the null hypothesis of unit root in the residual series. In Table 9, PP test also accepts the null hypothesis of unit root for both the regression equations. Hence we can conclude that there is no
cointegration exists between inflation rate and nominal interest rate in this time period.

4.2.1.2 The period 2001-2009

We perform residual based cointegration test for the time period 2001-2009. We can see from Table 8 that when regressing $i$ on $\pi$, ADF test accept the null of unit root at 1% significance level and reject at 5% and 10%. But regression $\pi$ on $i$, ADF test accept the unit root at all significance level. This problem is occurring since for this time period ADF test find stationarity in the inflation series. In Table 9, PP test accepts the null hypothesis of unit root in both cases that is when regressing $\pi$ on $i$ and $i$ on $\pi$. Hence we can conclude that there is no cointegration exists in this time period between inflation rate and nominal interest rate.

4.2.1.3 The period 1992-2009

We can see from Table 8 and Table 9, that in both regression that is when regressing $\pi$ on $i$ and when $i$ on $\pi$, both ADF and PP test accept the null of unit root at 1%, 5% and 10% significance level. Hence we can conclude that there is no long run equilibrium relationship exists between nominal interest rate and inflation rate for this time period.

<table>
<thead>
<tr>
<th>Time period</th>
<th>Dependent variable</th>
<th>Independent variable</th>
<th>ADF test value</th>
<th>C.V 1%</th>
<th>C.V 5%</th>
<th>C.V 10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001-2009</td>
<td>$i$</td>
<td>$\pi$</td>
<td>-3.819871</td>
<td>-4.008</td>
<td>-3.398</td>
<td>-3.087</td>
</tr>
<tr>
<td>2001-2009</td>
<td>$\pi$</td>
<td>$i$</td>
<td>-2.434465</td>
<td>-4.008</td>
<td>-3.398</td>
<td>-3.087</td>
</tr>
<tr>
<td>1992-2009</td>
<td>$\pi$</td>
<td>$i$</td>
<td>-1.908526</td>
<td>-3.954</td>
<td>-3.368</td>
<td>-3.067</td>
</tr>
</tbody>
</table>
We performed Engle-Granger cointegration test for inflation rate and nominal interest rate in different time periods. Using this test we don’t found any evidence about the cointegration between inflation rate and nominal interest rate in these time periods. Now we will apply Johansen cointegration test to the data to see whether it also confirms about no cointegration between inflation rate and nominal interest rate.

4.2.2 Johansen Test of Cointegration

Before applying Johansen test it is important to decide about the Lag length of variables used in the analysis. For this purpose the most common procedure is to estimate a vector autoregressive using the level data. Then we can use the same lag length as in traditional vector autoregressive. Here we are using Eviews-6 statistical software which automatically selects suitable lags length for Johansen cointegration test. We perform Johansen test of cointegration for nominal interest and inflation rate for different time periods. The Johansen test uses two methods, $\lambda_{trace}$ and $\lambda_{max}$ statistic to find cointegration between variables. Here we obtain both the statistics values and decide about the cointegration.

4.2.2.1 The period 1992-2000

We perform Johansen cointegration test for nominal interest and inflation rate for 1992-2000. The $\lambda_{trace}$ and $\lambda_{max}$ both test statistics confirms about no cointegration between these two series in this given time period. Looking at Table 9 we can see that

Table 9: PP test for Residual series

<table>
<thead>
<tr>
<th>Time period</th>
<th>Dependent variable</th>
<th>Independent variable</th>
<th>PP test value</th>
<th>C.V 1%</th>
<th>C.V 5%</th>
<th>C.V 10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001-2009</td>
<td>$i$</td>
<td>$\pi$</td>
<td>-2.886875</td>
<td>-4.008</td>
<td>-3.398</td>
<td>-3.087</td>
</tr>
<tr>
<td>2001-2009</td>
<td>$\pi$</td>
<td>$i$</td>
<td>-2.162840</td>
<td>-4.008</td>
<td>-3.398</td>
<td>-3.087</td>
</tr>
</tbody>
</table>
test statistics values for both $\lambda_{trace}$ and $\lambda_{max}$ are smaller than the critical value at 5% significance level. Hence we cannot reject the null hypothesis of no cointegration and conclude that nominal interest and inflation rate are not cointegrated for the time period 1992-2000.

**Table 9: Johansen cointegration test for 1992-2000**

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Null Hypothesis</th>
<th>Alternative hypothesis</th>
<th>Eigen value</th>
<th>Test statistic</th>
<th>C.V 5%</th>
<th>Decision for null hypoth:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_{trace}$</td>
<td>$r = 0$</td>
<td>$r &gt; 0$</td>
<td>0.054856</td>
<td>8.361262</td>
<td>12.32090</td>
<td>Accept</td>
</tr>
<tr>
<td></td>
<td>$r = 1$</td>
<td>$r &gt; 1$</td>
<td>0.022945</td>
<td>2.437326</td>
<td>4.129906</td>
<td></td>
</tr>
<tr>
<td>$\lambda_{max}$</td>
<td>$r = 0$</td>
<td>$r &gt; 0$</td>
<td>0.054856</td>
<td>5.923936</td>
<td>11.22480</td>
<td>Accept</td>
</tr>
<tr>
<td></td>
<td>$r = 1$</td>
<td>$r &gt; 1$</td>
<td>0.022945</td>
<td>2.437326</td>
<td>4.129906</td>
<td></td>
</tr>
</tbody>
</table>

4.2.2.2 The period 2001-2009

We again perform Johansen cointegration test for nominal interest and inflation rate for the time period 2001-2009. From Table 10 below we can see that the values of $\lambda_{trace}$ and $\lambda_{max}$ statistics are smaller than the critical values. Therefore we cannot reject the null hypothesis of no cointegration. We conclude that there is no cointegration between these two variables in this given time period.

**Table 10: Johansen cointegration test for 2001-2009**

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Null Hypothesis</th>
<th>Alternative hypothesis</th>
<th>Eigen value</th>
<th>Test statistic</th>
<th>C.V 5%</th>
<th>Decision for null hypoth:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_{trace}$</td>
<td>$r = 0$</td>
<td>$r &gt; 0$</td>
<td>0.087803</td>
<td>10.72364</td>
<td>12.32090</td>
<td>Accept</td>
</tr>
<tr>
<td></td>
<td>$r = 1$</td>
<td>$r &gt; 1$</td>
<td>0.010178</td>
<td>1.074180</td>
<td>4.129906</td>
<td></td>
</tr>
<tr>
<td>$\lambda_{max}$</td>
<td>$r = 0$</td>
<td>$r &gt; 0$</td>
<td>0.087803</td>
<td>9.649457</td>
<td>11.22480</td>
<td>Accept</td>
</tr>
<tr>
<td></td>
<td>$r = 1$</td>
<td>$r &gt; 1$</td>
<td>0.010178</td>
<td>1.074180</td>
<td>4.129906</td>
<td></td>
</tr>
</tbody>
</table>
4.2.2.3 The period 1992-2009

We perform Johansen cointegration test to analyze cointegration between nominal interest and inflation rate in the long run for the period 1992-2009. In this case both $\lambda_{trace}$ and $\lambda_{max}$ statistics values are higher than the critical values at 5%. This indicates that there is cointegration exists. It also tells us that there are two cointegration equation exist which is impossible since there are only two variables in the cointegration analysis and we can get at most one cointegration equation if cointegration exists. This problem often occurs when there are stationarity exists in the data. It seems that one of our variable is stationary in this time period. For inflation series if we increase the lag values used in the analysis from 4 to 7 then both ADF and PP reject the null hypothesis of unit root at 5% significance level (Table 12). Hence we can conclude that there is no cointegration exists between inflation rate and nominal interest rate for this given time period.

<table>
<thead>
<tr>
<th>Table 11: Johansen cointegration test for 1992-2009</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>$\lambda_{trace}$ and $\lambda_{max}$ Statistic</strong></td>
</tr>
<tr>
<td>Statistic</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>$\lambda_{trace}$ Statistic</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>$\lambda_{max}$ Statistic</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 12: Unit root tests for inflation series using 7 lag values in the analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Inflation Rate</strong></td>
</tr>
<tr>
<td>Test</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>ADF</td>
</tr>
<tr>
<td>PP</td>
</tr>
</tbody>
</table>

The above results of Johansen test in all three time periods suggest that there is no cointegration exists between inflation rate and nominal interest rate.
5. Conclusion

The main objective of this thesis is to investigate the existence of long run equilibrium relationship between inflation rate and nominal interest rate. This long run equilibrium relation is known as Fisher hypothesis or Fisher equation. We select Belgium and collect the monthly data of both time series variables for two different time periods. The first time period is before the introduction of euro currency in Belgium (1992 to 2000) and the second time period is after the introduction of euro currency in Belgium (2001 to 2009). We analyze long run equilibrium relation between inflation rate and nominal interest rate in both time periods and also in the time period from 1992 to 2009. Using unit root test, we found lack of stationarity in nominal interest series. We also observe that inflation series behaves as stationary series but both ADF and PP test statistic suggest at 1% significance level that this series is also non stationary. Cointegration technique is used to analyze the long run equilibrium relationship between these variables. The Engle-Granger cointegration test (residual based) and Johansen cointegration test are used for this purpose. Both the test failed to find evidence about the long run equilibrium relationship between inflation rate and nominal interest rate in both short and long time period. We conclude that there is no cointegration exists between inflation rate and nominal interest rate in these time periods.
References


