Modelling of African Farm Dynamics Using Bivariate Binary Logistic Regression in WinBUGS

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Abstract

The recent development and implementation of Markov Chain Monte Carlo (MCMC) method is generating application of complicated models over a wide range of sciences. For two correlated binary responses, bivariate binary logistic regression is a suitable way to identify the related covariates and at the same time, their interactions validity can be investigated in terms of logarithm of odds ratio. Present study uses data obtained from a longitudinal survey conducted in 2002-2005 to make a guideline for the agricultural development and food security in Africa. Defining the term Extensification and Intensification as two traditions for the farm dynamics in the selected African states, a simple data analysis code for bivariate binary regression analysis in WinBUGS is developed and comparison is made with the analysis obtained in R under maximum likelihood estimation. Results indicate that some factors for instance ‘availability of new crop technology’, ‘import of Maize’ and ‘stopped intercropping’ shows some negative association with farm dynamics response variable, which concludes that these factors discouraging the production of Maize and areal increase in both Bayesian and maximum likelihood estimation approach. Whereas ‘Change in fertilizer use’, ‘cultivated area increase’ and ‘started selling maize’ shows positive association. This indicates that these factors support the argument of areal and Maize production increase. Farm holders access to modern crop technologies, in combination with commercial incentives to staple crop production emerge as the most important explanation of dynamism. Thus for independent modelling of Extensification and Intensification dynamics, both Bayesian and frequentistic approach mimics the result. But the joint association provides distinctive result in Bayesian approach that concludes that Extensification and Intensification are two diverse way of farm dynamics.

Keywords: WinBUGS, Bivariate binary logistic regression, Extensification, Intensification.
1. Introduction

Food security is fundamental element in human existence. Without food, nothing happens: no economic growth, no science and technology, no music and literature, not even procreation. Unfortunately, the volume of poverty and food insecurity coupled with serious malnutrition and morbidities are knocking many doors of the people in most developing world of sub-Saharan African countries (Asefach and Nigatu, 2007). Despite continued economic growth around the world, food insecurity remains a pressing problem in many parts of Africa (Garrett and Ruel, 1999; Maxwell, 1999; Mougeot, 2005; UN-HABITAT, 2006).

At the turn of the century, sub-Saharan Africa markedly lagged behind the other part of the world in terms of socio-economic development including food production. A widespread poverty, malnutrition and recurrent food scarcities also observed at the same time. For sub-Saharan Africa (SSA), agricultural growth rates over the last decades apparently have barely been at par with population growth, resulting in large imports of staples for the urban-based population (World Bank, 1989; Afrint-I project description).

Diversity is the norm in African farming systems. Even at the individual farm unit, farmers typically cultivate ten or more crops in diverse mixtures that vary across soil type, topographical position and distance from the household compound. Both endogenous factors (household goals, labour, technologies in use and the resource base) and exogenous factors (market development, shifts in demand, agricultural services and policies, the propagation of new technologies and the availability of market and policy information) drive the evolution of individual farms and, collectively the overall farming system. The main staple is maize and the main cash sources are migrant remittances, cattle, small ruminants, tobacco, coffee and cotton, plus sale of maize, pulses and sunflower. Cattle are kept for ploughing, breeding, milk, farm manure, bride wealth, savings and emergency sale. In spite of scattered settlement patterns, community institutions and market linkages in the maize belt are better developed than in other farming systems (IAC Report, 2004).

During the last few decades, a number of studies have been done and remedial measures taken by the policy makers concentrating an attention of the researchers in a great extent. Studies undertake to identify the risk factors and the influential socio-economic variables to have a direction to the farm dynamics as a way to analyze the food crisis following the relevance of
Asian Green revolution for Africa (Machete, 1997; Djurfeldt et al., 2005; Djurfeldt et al., 2009).

In methodological point of view, the recent development of widely accessible computers and the implementation of Markov chain Monte Carlo (MCMC) methods have led to a sudden increase of interest in Bayesian statistics and modelling. This also followed by an extensive research for new Bayesian methodologies generating application of complicated models used over a wide range of sciences. Since 1998 or so, the windows version of BUGS, has earned great popularity among researchers of diverse scientific field for its easy accessibility to use and fit the complicated models.

The present study aims to investigate the prevailing food crisis and farm management related factors in some selected states of Africa. Defining the terms extensification (increasing the cultivated land area by the farm) and intensification (staple food production increment) as dichotomous a bivariate binary logistic regression model developed which identifies the cause-affect variables for intensification or extensification. The combined effect is measured in terms of the logarithm of odds ratios for individual response level and the related factors identify the farm dynamics.

2. Objectives of the study

An important difference between the classical and the Bayesian framework is the introduction of prior information in the form of probability distributions (Dunson, 2001). Moreover, in the Bayesian framework conclusions about parameters are made in terms of a probability statement, i.e., parameter estimates are no longer expressed as point estimates but instead are statistical distributions (Dunson, 2001). Uncertainty associated to parameter estimation is quantified through the use of these probability distributions (Gelman et al., 1996). Therefore one of the objectives for present study is to look up the behaviors of the parameter in a bivariate binary regression model when applied with a Bayesian framework. The results comparison is made with the estimated parameters by likelihood method for the same dataset simultaneously. The specific objectives are as below:

1. To develop an easy accessible bivariate binary regression model in terms of Bayesian inference.
2. To investigate the cause-affect parameters for the extensification and intensification of staple food production in the study region.

3. To make a comparison between the Bayesian method and the method of maximum likelihood estimation for the model parameters.

4. To make a guideline for the prevailing food security problem in the study region in term of farm dynamics.

A simple code written in WinBUGS (version 1.4) has been used to perform all the required computations (see Appendix) in Bayesian approach. Whereas, analysis of data with the VGAM package in R is used for ML estimation.

3. Materials and Methods

Bayesian inference, MCMC and Gibbs sampling method

Bayesian methods have become popular in modern statistical analysis and are being applied to a broad spectrum of scientific fields and research areas. Bayesian data analysis involves inferences from data using probability models for quantities we observe and for quantities about which we wish to learn or in other words analyzing statistical models with the incorporation of prior knowledge about the model or model parameters.

In the Bayesian approach, (Carlin, 2000) in addition to specifying the model for the observed data \( Y = y_1, y_2, y_3, \ldots, y_n \) given a vector of unknown parameters \( \theta \) usually in the form of probability distribution \( f(y | \theta) \), it also suppose that \( \theta \) is a random quantity as well, having a prior distribution \( \pi(\theta | \eta) \), where \( \eta \) is a vector of hyper-parameters. Inference concerning \( \theta \) is then based on its posterior distribution, given by

\[
p(\theta | y, \eta) = \frac{p(y, \theta | \eta)}{p(y | \eta)} = \frac{p(y, \theta | \eta)}{\int p(y, u | \eta) du}
\]

\[
\Rightarrow p(\theta | y, \eta) = \frac{f(y | \theta) \pi(\theta | \eta)}{\int f(y | u) \pi(u | \eta) du}
\]

The prior distribution is a key part of Bayesian inference and represents the information about an uncertain parameter \( \theta \) that is combined with the probability distribution of new data to yield the posterior distribution, which in turn is used for future inferences and decisions involving \( \theta \).
Bayesians use the same statistical models as frequentists. If \( Y = y_1, y_2, y_3, \ldots, y_n \) are i. i. d. with density \( f(y|\theta) \), then the joint distribution of the data is \( f(y|\theta) = \prod_{i=1}^{n} f(y_i|\theta) \). When this is thought of as a function of the parameter rather than the data, it becomes the likelihood. The posterior distribution summarizes the current state of knowledge about all the uncertain quantities (including unobservable parameters and also missing, latent, and unobserved potential data) in a Bayesian analysis. Analytically, the posterior density is the product of the prior density and the likelihood.

For slightly more complex models some posterior quantities could be approximated, but still the list of models for which these approximations work well is rather small. The development and improvement of Monte Carlo techniques has recently made the posterior distributions of very complicated Bayesian models easy to approximate.

Markov chain Monte Carlo (MCMC) methods are simulation-based and enable the statistician or engineer to examine data using realistic statistical models. The main application of the MCMC methods is to generate a sample from a distribution. This sample can then be used to estimate various characteristics of the distribution such as moments, quantiles, modes, the density, or other statistics of interest. These quantities can be written as posterior averages of functions of the model parameters,

\[
p(\theta|y, \eta) = \frac{f(y|\theta)\pi(\theta|\eta)}{\int f(y|u)\pi(u|\eta)du}
\]

It should be noted that the denominator in this equation is a constant of proportionality to make the posterior integrate to one. If the posterior is nonstandard, then this can be very difficult, if not impossible, to obtain. This is especially true when the problem is high dimensional, because there are a lot of parameters to integrate over. Analytically performing the integration in these expressions has been a source of difficulty in applications of Bayesian inference, and often simpler models would have to be used to make the analysis feasible.

Monte Carlo integration estimates of e.g. the mean \( \mathbb{E}(\theta) = \int \theta \cdot p(\theta|y, \eta) d\theta \) by obtaining samples \( \theta_t, t = 1, 2, \ldots, n \) from the posterior distribution \( p(\theta|y, \eta) \) and then calculating the average, \( \mathbb{E}(\theta) = \frac{1}{n} \sum_{i=1}^{n} \theta_{t_i} \). The notation ‘\( t \)’ is used here because there is an ordering or
sequence to the random variables in MCMC methods. When $\theta_i$ independent, then the approximation can be made as accurate as needed by increasing ‘$n$’. Markov chain is a sequence of random variable such that the next value or state of the sequence depends only on the previous one. Thus, it generating a sequence of random variables, $\theta_0, \theta_1, \ldots$ such that the next state $\theta_{t+1}$ with $t \geq 0$ is distributed according to $P(\theta_{t+1} | \theta_t)$ which is called the transition kernel. A realization of this sequence is also called a Markov chain. It is assumed that the transition kernel does not depend on $t$, making the chain time-homogeneous (Wendy et al., 2002).

There are many methods for generating the next model or set of parameter values in an MCMC chain but the most popular by far is the Metropolis-Hastings algorithm. The key to the algorithm is to find a good proposal distribution $q(\theta^* | \theta_t)$ for suggesting a new value $\theta^*$ given the latest value in the chain $\theta_t$. The choice of proposal distribution is essentially arbitrary, but some choices will be much more efficient than others in the sense of giving a chain that settles more quickly to the correct long run probabilities. After generating a random value $\theta_t$, from the chosen proposal distribution, the ratio $\rho = \frac{p(\theta^* | \text{Data}) q(\theta_t | \theta^*)}{p(\theta_t | \text{Data}) q(\theta^* | \theta_t)}$ is therefore calculated. A uniform value, $u$ between 0 and 1, is then generated and if $u < \rho$ the proposed $\theta^*$ is accepted as the next value in the chain, otherwise the next value is a copy of $\theta_t$. A poor choice of proposal distribution can get stuck on one set of parameter values, which will slow down the convergence of the chain (Thompson et al., 2007).

A special case of the Metropolis-Hastings algorithm is Gibbs sampling, which involves cycling through the parameters of the model one at a time rather than treating them as a vector and using the current estimate of the univariate conditional posterior probability distribution as the proposal distribution. Suppose the marginal posterior $p(\theta | y)$ can not be obtained from the joint posterior $p(\theta, \eta | y)$ analytically but the conditional posteriors $p(\theta | y, \eta)$ and $p(\eta | y, \eta)$ have some known form and easy to sample. Gibbs sampler firstly choose starting value for $\eta$, say $\eta^{(0)}$ and then generate via random sampling a single value, $\theta^{(1)}$ from the conditional distribution $p(\theta | y, \eta = \eta^{(0)})$. Next generate $\eta^{(1)}$ from the conditional distribution $p(\eta | y, \eta = \eta^{(0)})$. Then start cycling through the algorithm generating $\theta^{(2)}$ and $\eta^{(2)}$ and so on.
Gibbs sampling can be very inefficient, but by taking parameters one at a time it reduces multidimensional problems to a series of univariate calculations and consequently it is much easier to program (Casella and George, 1992; Thompson et al., 2007). The WinBUGS software provides users with a simple tool to perform these Markov chain Monte Carlo simulations and applying Gibbs sampling to a very flexible class of user specified models.

**WinBUGS procedure**

WinBUGS implements various MCMC algorithms to generate simulated observations from the posterior distribution of the unknown quantities (parameters or nodes) in the statistical model. The idea is that with sufficiently many simulated observations, it is possible to get an accurate picture of the distribution. For any project it requires three files: a program file containing the model specification, a data file containing the data in a specific (slightly strange) format and a file containing starting values for model parameters (optional). It is up to user choice whether the project consists of the above three mentioned files or all these three files in a same file.

WinBUGS enables the user to specify a Bayesian model, either by drawing a directed graph (Lauritzen and Spiegelhalter, 1988) or by using an S-like language. The software then determines the transition kernel for a Markov chain to generate samples from the joint posterior distribution of the unknown quantities in the model. Using a graphical user interface or a script, the user specifies the number of parallel MCMC chains to be run, the number of iterations, the model unknowns to monitor for analysis and reporting, and the types of convergence assessment and output summaries. The final result is numeric and graphical summaries of the estimated univariate marginal posterior distributions of the requested model quantities (Cowles, 2004).

**Bivariate Binary Logistic Regression**

Logistic regression allows one to predict a discrete outcome, such as group membership, from a set of variables that may be continuous, discrete, dichotomous, or a mix of any of these. Generally, the dependent or response variable is dichotomous, such as presence/absence or success/failure. Instances where the independent variables are categorical or a mix of continuous and categorical, also the response variable is dichotomous, logistic regression is
preferred. Applications of logistic regression have also been extended to cases where the dependent variable is of more than two cases, known as multinomial or polytomous logistic regression.

The relationship between the predictor variable and expectation of response variables is not a linear function in logistic regression; instead the logistic regression function is used, which is the \( \text{logit} \) transformation of the probability of occurrence \( \theta \),

\[
\theta = \frac{e^{(\alpha + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_k x_k)}}{1 + e^{(\alpha + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_k x_k)}}
\]

where, \( x_i \) are the predictors, \( \alpha \) = the constant of the equation and, \( \beta \) = the coefficient of the predictor variables.

An alternative form of the logistic regression equation is given by,

\[
\log \left( \frac{\theta}{1-\theta} \right) = \alpha + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_k x_k
\]

The goal of logistic regression is to correctly predict the category of outcome for individual cases using the most parsimonious model. To accomplish this goal, a model is created that includes all predictor variables that are useful in predicting the response variable.

The binary regression model (Collet, 1994), is used to explain the probability of a binary response variable as function of some covariates. Bivariate logistic regression (BLR) differs from ordinary logistic regression in the sense that related response variables are not assumed to be independent of one another. In this regard, bivariate logistic regression is a useful procedure with advantages that include (i) individual modelling of the marginal probability distribution of the bivariate binary responses, and (ii) modelling the odds ratio describing the pairwise association between the two binary responses in relation to several covariates.

Let define two binary dependent variables, \( Y_1 \) and \( Y_2 \), each of which take the value of either ‘0’ or ‘1’. The joint outcome \((Y_1, Y_2)\) for a set of n-paired observation with their occurrence probabilities’ can be outlined as in table-1.
Table 1: Different outcomes of variables with probability of occurrences’

<table>
<thead>
<tr>
<th></th>
<th>$Y_2 = 0$</th>
<th>$Y_2 = 1$</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_1 = 0$</td>
<td>$p_{00}$</td>
<td>$p_{01}$</td>
<td>$1 - p_1$</td>
</tr>
<tr>
<td>$Y_1 = 1$</td>
<td>$p_{10}$</td>
<td>$p_{11}$</td>
<td>$p_1$</td>
</tr>
<tr>
<td>Total</td>
<td>$1 - p_2$</td>
<td>$p_2$</td>
<td>$1$</td>
</tr>
</tbody>
</table>

It is evident from the table that, $p_{rs} = P(Y_1 = r, Y_2 = s)$; $r, s = 0, 1$ are the joint probabilities and $p_j = P(Y_j = 1)$, $j = 1, 2$ be the marginal probabilities for each of the response variables. It is assumed that the observations within pairs are correlated but observations from different pairs are independent.

The bivariate logistic model or bivariate logistic odds-ratio model (BLOM) described by McCullagh and Nelder (1989), Palmgren (1989) and later by Cessie & Houwelingen (1994) is specified by modelling the marginal distribution of each of $Y_j$, and also the odds ratio. The odds ratio $\psi = \left( \frac{p_{11}}{p_{10}} \bigg/ \frac{p_{01}}{p_{00}} \right)$ is used to describe the association between the two responses i.e., $\psi$ can be seen as the ratio of the odds of $Y_1 = 1$ given that $Y_2 = 1$ and the odds of $Y_1 = 1$ given that $Y_2 = 0$, $\psi = 1$ indicating independence between $Y_1$ and $Y_2$. The model is given by,

$$\log \left( \frac{p_j}{1 - p_j} \right) = \beta'_j X \quad \text{for} \quad j = 1, 2$$

$$\log \psi = \beta'_1 X$$

The covariate vector $X$ may include block specific and subunit specific covariates. For covariates values associated with $Y_1$ but not with $Y_2$ the corresponding elements in $\beta_2$ are set to zero, similarly for $\beta_1$ (Palmgren, 1989).

The joint probabilities $p_{11}$ can be obtained in terms of $p_1$, $p_2$ and $\psi$ as,

$$p_{11} = \begin{cases} \frac{1}{2} (\psi - 1)^{-1} \left\{ a - \sqrt{a^2 + b} \right\} & \text{for} \quad \psi \neq 1 \\ p_1 p_2 & \text{for} \quad \psi = 1 \end{cases}$$

Where, $a = 1 + (p_1 + p_2) (\psi - 1)$ and $b = -4 \psi (\psi - 1) p_1 p_2$ (Dale, 1986). The other three joint probabilities $p_{rs}$ can be recovered easily from the marginal’s, $p_{00} = p_1 - p_{11}$, $p_{01} = p_2 - p_{11}$ and $p_{00} = 1 - p_{10} - p_{01} - p_{11}$.
A brief discussion on the model parameter estimation by Maximum likelihood method using Newton-Raphson iteration is done by McCullagh and Nelder (1989), Palmgren (1989) and Cessie & Houwellingen (1994). Later the implementation of such model estimation is discussed by Yee (2008) in VGAM packages in R.

One of the drawbacks of such estimation is that it is using a single root from a quadratic equation of $p_{11}$. The argument behind is that the values of $p_{11}$ can never be negative and odds ratio satisfies, $\psi \geq 0$. But the same assumptions are also true for some of the values of other root.

Therefore it seems better to have an estimate that uses the possible root that satisfies the same assumption as,

$$p_{11} = \begin{cases} \frac{1}{2} (\psi - 1)^{-1} \left\{ a \pm \sqrt{a^2 + b} \right\} & \text{for } \psi \neq 1 \\ p_1p_2 & \text{for } \psi = 1 \end{cases}$$

Where, $a = 1 + (p_1 + p_2)(\psi - 1)$ and $b = -4\psi(\psi - 1)p_1p_2$. The other three joint probabilities can be obtained as usual.

A classical statistical method for the estimation of parameters in the model is based on the maximum likelihood estimate (MLE) and the likelihood ratio (LR) tests. The approximation theory based on large samples usually serves as the basis for deriving classical inference for non-normal data and often requires the use of nonstandard asymptotic theory (Self and Liang, 1987). Such a usual estimation of the model parameter under the proposed equation and its implementation is briefly discussed by Djurfeldt et al., (2009) and Zain et al., (2009).

In a Bayesian approach, parameters are considered random and a joint probability model for both data and parameters is required. The easiest way to circumvent this difficulty is to propose an informative prior, but with small precision, avoiding any complaint about the specification of subjective beliefs (O'Hagan and Haylock, 1997). The joint posterior distribution of the parameters of the proposed models turns out to be analytically intractable, hence simulation-based methods (Tierney, 1994) broadly known as Markov Chain Monte Carlo (MCMC) are required to obtain the point and interval estimates of the parameters. For a zero inflated data simulation studies show that the Bayesian estimation method has better
finite sample performance than the classical method with tighter interval estimates and better coverage probabilities (Ghosh et al., 2006). Therefore, considering a vague normal prior with a very lower precision (higher variance) for the regression co-efficient, the model parameters can be estimated in Bayesian estimation method using WinBUGS software. The proposed models can be written as,

\[ Y_j \sim \text{bernoulli}(p_j), \]
\[
\log \left( \frac{p_j}{1-p_j} \right) = \beta_j^T X_j, \quad \text{for} \quad j = 1, 2
\]
\[ \text{and} \quad \log(\psi) = \beta_j^T X_j, \]
\[ \beta_j \sim \text{MVN}(0, (I \times 10^2)), \quad \text{for} \quad j = 1, 2, 3 \]

An approximated 100(1-\(\alpha\)) percent credible interval for the estimated parameters can be obtained from the percentiles of the posterior distribution.

### 4. Data source and background characteristics

Data for the present study is taken from a longitudinal survey conducted in 2002 (Afrint-I project, 2002-2005) to the group of countries located in the African maize and cassava belt. These two crops are well produced in Africa and the main food grain for some regions in the sub-Saharan region. Eight countries were purposively selected – Ethiopia, Ghana, Kenya, Malawi, Nigeria, Tanzania, Uganda and Zambia. The household sample consists of more than 3000 randomly sampled households in more than 100 sampled villages. The present study is restricted up to the maize growing farmers who constitute 85% of the total sample. The complete responses found from 1533 individuals from 94 villages and 17 regions comprise the final sample size for the study (Table-2).

<table>
<thead>
<tr>
<th>Country</th>
<th>Regions</th>
<th>Villages</th>
<th>Respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Total</td>
</tr>
<tr>
<td>Ethiopia</td>
<td>2</td>
<td>2</td>
<td>68</td>
</tr>
<tr>
<td>Ghana</td>
<td>1</td>
<td>4</td>
<td>84</td>
</tr>
<tr>
<td>Kenya</td>
<td>2</td>
<td>10</td>
<td>243</td>
</tr>
<tr>
<td>Malawi</td>
<td>4</td>
<td>8</td>
<td>272</td>
</tr>
<tr>
<td>Nigeria</td>
<td>2</td>
<td>46</td>
<td>193</td>
</tr>
<tr>
<td>Tanzania</td>
<td>2</td>
<td>10</td>
<td>209</td>
</tr>
<tr>
<td>Uganda</td>
<td>2</td>
<td>5</td>
<td>169</td>
</tr>
<tr>
<td>Zambia</td>
<td>2</td>
<td>9</td>
<td>295</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>17</strong></td>
<td><strong>94</strong></td>
<td><strong>1533</strong></td>
</tr>
</tbody>
</table>

Table-2: Distribution of respondents
In contrast to the crude general measurements of variable changes, such as farming practices, and their relationship to farm dynamism, a more refined approach to observe such changes can be achieved through considering changes in area and yields in relation to the household’s year of establishment, where the latter is used as reference year. In this way, dynamic production patterns (intensification and extensification) may be identified (Djurfeldt et al., 2009). Extensification can be defined either increasing amount of arable land with equal resources or using equal amount of land with decreasing resources. Whereas, intensification of agricultural production refers to using equal amount of land and increasing input of resources or using decreased amount of land with equal input of resources such as labour, technology etc.

For maize, extensification is mainly a subsistence strategy and as such it is constrained both by land scarcity and by labour shortages at the household level. Intensification, on the other hand, is by definition less constrained by the availability of land, but here again labour is a blockage. Like in the classical Javanese case studied by Geertz (1956), intensification unaccompanied by innovations leads to involution. However, intensification in some cases represents a more dynamic type of development, stimulated by commercial incentives (Djurfeldt et al., 2006).

Availability of modern crop technology is another constraint to intensification. In the case of maize, however, uses of chemical fertilizers are not available on terms that are affordable and sustainable for smallholders. Furthermore, intensification tends to be driven more by commercial factors than by demographic ones. Although commercialization is a potent driver, it has not been potent enough, however, to stimulate the sustainable intensification, of maize, rice and sorghum. Thus it is potential to alleviate the African food crisis in both rural and urban areas that have not been tapped (Djurfeldt et al. 2009; Djurfeldt et al, 2006).

A number of characteristics of farm households are likely to impact on farm dynamics. For instance institutional discrimination of female farmers may have some negative impact on their productivity. According to Chayanovian theory, moreover, the life-cycle (especially in a largely subsistence-oriented agrarian economy) and age influence farm trajectories (Chayanov 1986). Consumer-worker ratios are assumed to have an effect similar to that of age, that is production per worker is presumed to rise with the consumer burden of each worker. Likewise, labour-surplus households are expected to be more dynamic than labour-scarce
ones. Similarly, access to social and economic resources leads to expect that high-status households are more dynamic farmers than resource poor households. Finally, it is important to consider whether farm dynamics rely on family or hired labour and a look at the use of hired labour will enable comparison between African family farms, and farms operating by means of hired hands.

**Model Specification**

Correlates for the farm dynamics can be identified in terms of intensification and extensification related factors. The farm holder either extensify the production or intensify the yield of Maize crop compared to the period of farm establishment or may implement the both opportunity. Both the variables considered are dichotomous and for the combined effect, correlates can be identified with the log-linear relationship among the selected variables.

<table>
<thead>
<tr>
<th>Extensification of Land area</th>
<th>Intensification of Yield</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Not-Extensiified</td>
<td></td>
<td>Total</td>
</tr>
<tr>
<td>Not-extended</td>
<td>756 (0.49)</td>
<td>274 (0.18)</td>
<td>1030 (0.67)</td>
</tr>
<tr>
<td>Extended</td>
<td>278 (0.18)</td>
<td>225 (0.15)</td>
<td>503 (0.33)</td>
</tr>
<tr>
<td>Total</td>
<td>1034 (0.67)</td>
<td>499 (0.33)</td>
<td>1533 (1.00)</td>
</tr>
</tbody>
</table>

Odds for Extensification : 0.48
Odds for Intensification : 0.49
Odds Ratio : 2.23

Result from the table indicates that a few farms are following both the intensification and extensification dynamics. Almost half (0.49) of the farms neither extensifying the arable land area nor intensifying the yield of maize showing a stagnated farm dynamics, whereas the proportions for taking either of the farm dynamics are equal (0.18). The odds for extensification and intensification are 0.48 and 0.49 respectively providing an odds ratio 2.23. With usual notation, the odds ratio greater than unity describes a positive association between the events. Therefore it should be a matter of interest to investigate the related factors that are associated with intensification and extensification.

In the present study, bivariate logistics regression is used to provide simultaneous parameter estimation for i) the marginal probability of extensification, ii) marginal probability of intensification and iii) the log-odds ratio describing the probabilities for joint association in the farm dynamics at individual response level. For the i-th individual response (i=1,2,……n)
the variables $Y_{i1}$ and $Y_{i2}$ are the indicator variable for extensification and intensification respectively. The response from an individual farm holder is therefore occurring with four possible probabilities:

$p_{i1}$ : The farm holder response with extensification of arable land for maize crop as well as intensified the production; i.e., the farm holder following both strategies of farm dynamics.

$p_{i0}$ : The farm holder not extensified the arable land but intensified the production of maize crop; i.e., the farm holder follows the intensification strategy only.

$p_{10}$ : The farm holder extensified the arable land but not get intensification in the production; i.e., the farm holder taken the strategy of extensification of land, and

$p_{00}$ : The farm holder neither extensified the arable land nor intensified the production, i.e., the farm remains stagnant in status compared to the previous year.

Table-4: Joint association of probabilities on a cross-classification of the outcomes of extensification and intensification.

<table>
<thead>
<tr>
<th>Extensification of arable land ($Y_1$)</th>
<th>Intensification of Yield ($Y_2$)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Y_2 = 0$</td>
<td>$Y_2 = 1$</td>
</tr>
<tr>
<td>$Y_1 = 0$</td>
<td>$p_{00}$</td>
<td>$p_{01}$</td>
</tr>
<tr>
<td>$Y_1 = 1$</td>
<td>$p_{10}$</td>
<td>$p_{11}$</td>
</tr>
<tr>
<td>Total</td>
<td>$1 - p_2$</td>
<td>$p_2$</td>
</tr>
</tbody>
</table>

Application of bivariate logistic regression provides adjusted estimates of concordance through simultaneous estimation of covariate effects on the odds ratio that describes the pairwise association structure. An additional benefit of bivariate logistic regression is that it may afford greater precision than unadjusted estimates obtained by considering a multinomial distribution. The effects of covariates on the marginal probability of extensification, intensification and the odds ratio are each described with regression equations.

For the i-th ($i = 1, 2\ldots n$) individual response the marginal probabilities for the extensification and intensification is given by the regression model,

$$
\log \left[ \frac{p_j}{1-p_j} \right] = \beta_j'X
$$

$$
= \alpha_j + \beta_{j1}X_1 + \beta_{j2}X_2 + \beta_{j3}X_3 + \ldots + \beta_{jk}X_k ; \quad \text{for } j = 1, 2
$$

This model estimates the probability of extensification or intensification as a function of a number of selected covariates.
A regression model expressing the association between these two responses can be given by,

\[
\log [\psi] = \beta_i' X
\]
\[
= \alpha_3 + \beta_{31} X_1 + \beta_{32} X_2 + \beta_{33} X_3 + \ldots + \beta_{3k} X_k;
\]

The covariates are such selected that they show some significant association with extensification or intensification in bivariate analysis. Such covariates that are dichotomous are- change in fertilizer use, given up intercropping, population pressure: land frontiers reached or not, sex of farm manager, cultivated area increased since reference year, started selling the maize, mechanization since reference year, specialist in maize grower and main income source from food crop, given up crop rotation, hiring farm labour, belongs to poor group and belongs to wealthy group. The covariates that are of continuous type includes- total cultivated area (in hectare), number of household workers, village centrality index, NGO/donor affiliation (measured with some index), per-capita income, consumer-worker ratio, age of arm manager and age-squared.

The models run simultaneously in WinBUGS after defining suitable prior distribution of model parameters. A set of vague normal priors with very lower precision level is used to define the parameters from data. A burn-in of 1000 iterations is allowed, followed by 5,000 iterations where values for the intercept and coefficients are stored. Diagnostic tests for convergence of the stored variables are undertaken, including visual examination of history and density plots. Convergence successfully achieved after 5,000 iterations and the posterior distributions of model parameters are summarized using descriptive statistics.

Maximum likelihood estimator of the model parameters obtained in R using the VGAM packages introduced and developed by Yee and Dirnbock (2009) also has done for the comparison of obtained results.

5. Results and Discussion

It has been assumed that both the extensification and intensification possibly related and associated with a number of co-variates at individual level of response. Therefore a bivariate analysis is done with each of the selected covariate initially to identify the significant related factors. The obtained factors are then regressed in bivariate logistic regression for the
extensification, intensification and the association with different models simultaneously. In Bayesian method, the posterior estimates of beta coefficients are obtained as the mean of simulated betas from different number of samples under consideration after convergence. On the other hand, maximum likelihood estimates of beta co-efficient are obtained through a number of iterations in Newton-Raphson method from the data. In both cases it is convenient to explain the results in terms of odds ratio that is the anti-logarithm (exponential) of the betas’. The respective posterior credible intervals or confidence intervals can be obtained easily in usual manner.

Variables that considered are either binary (dummies), discrete categorical or logged continuous ones. The odds ratio for the binary and discrete categories can be interpreted as the change in relative risk of a certain outcome associated with a change from the lowest category in the independent variable. Values below 1 indicate a negative association and the respective size indicating their contribution to the probability of the outcome. The odds ratio for the continuous variables on the other hand can be interpreted as elasticities that is, certain value of the odds ratio tells us how much a one per cent change in an independent variable implies for the relative risk of the outcome. In such a case, values below 1 also indicate negative elasticities.

Bivariate analysis of data shows that a number of twenty two covariates are significantly associated either with extensification and intensification or one of them at least. From results obtained after multivariate analysis, it is evident that seven variables are significantly associated with extensification, ten variables have some significant association with intensification and none of the selected co-variates associated with the interaction between extensification and intensification in Bayesian method of estimation. On the other hand, ML estimation results in ten distinct variables showing significant association with either extensification or intensification and only two variables have some cause-affect relationship over the joint association of extensification and intensification. The odds ratio with the respective 95% posterior credible intervals and 95% confidence intervals are presented in the table-5.

Graphs in the following pages represent the significant bivariate association for some selected co-variates.
Figure-1: Results from bi-variate analysis for selected categorical variables

<table>
<thead>
<tr>
<th>Extensification</th>
<th>Intensification</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Change in fertilizer use</td>
<td></td>
</tr>
<tr>
<td><img src="image1.png" alt="Bar Chart" /></td>
<td><img src="image2.png" alt="Bar Chart" /></td>
</tr>
<tr>
<td>$\chi^2 &lt; 0.001$, N=1533</td>
<td>$\chi^2 &lt; 0.001$, N=1533</td>
</tr>
<tr>
<td>2. Given up intercropping</td>
<td></td>
</tr>
<tr>
<td><img src="image3.png" alt="Bar Chart" /></td>
<td><img src="image4.png" alt="Bar Chart" /></td>
</tr>
<tr>
<td>$\chi^2 &lt; 0.1$, N=1533</td>
<td>$\chi^2 &lt; 0.001$, N=1533</td>
</tr>
<tr>
<td>3. Sex of Farm Manager</td>
<td></td>
</tr>
<tr>
<td><img src="image5.png" alt="Bar Chart" /></td>
<td><img src="image6.png" alt="Bar Chart" /></td>
</tr>
<tr>
<td>$\chi^2 &lt; 0.001$, N=1533</td>
<td>$\chi^2 &lt; 0.001$, N=1533</td>
</tr>
</tbody>
</table>
4. Cultivated Area increased since reference Year

χ² < 0.001, N=1533

5. Started Selling Maize

χ² < 0.001, N=1533

6. Belongs to wealthy group

χ² < 0.001, N=1533

Farm holders increasing the use of modern fertilizer since reference year are most likely interested with extensification (194%, almost doubled chance) and intensification (282%, almost three-fold higher chance). Bivariate analysis result indicates that farmers who have extended their arable land are using fertilizer in an increasing manner. This is also true that if the land area for cultivation is increased with introducing technological inputs that also causes the higher chance of being intensification of production in Maize. So, farmers using fertilizer are gaining more crops that results intensification in production. Similar results obtained for both ML estimation and Bayesian estimation method.
Table-5: Comparison of estimated odds ratio for the co-variates in Bayesian method and Method of Maximum likelihood.

<table>
<thead>
<tr>
<th>Covariates</th>
<th>Bayesian Estimates Using Gibbs Sampling</th>
<th>Maximum Likelihood Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Extensification</td>
<td>Intensification</td>
</tr>
<tr>
<td>Constants</td>
<td>0.33*</td>
<td>0.60</td>
</tr>
<tr>
<td></td>
<td>(0.16, 0.64)</td>
<td></td>
</tr>
<tr>
<td>Change in Fertilizer use</td>
<td>1.94*</td>
<td>2.82*</td>
</tr>
<tr>
<td></td>
<td>(1.48, 2.55)</td>
<td>(2.18, 3.68)</td>
</tr>
<tr>
<td>Given-up Intercropping</td>
<td>0.76</td>
<td>0.51*</td>
</tr>
<tr>
<td></td>
<td>(0.31, 0.84)</td>
<td></td>
</tr>
<tr>
<td>Population Pressure: Land frontiers</td>
<td>0.89</td>
<td>1.00</td>
</tr>
<tr>
<td>Sex of Farm manager</td>
<td>0.63</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td>(0.47, 0.86)</td>
<td></td>
</tr>
<tr>
<td>Cultivated area increase</td>
<td>4.33*</td>
<td>1.23</td>
</tr>
<tr>
<td></td>
<td>(3.37, 5.60)</td>
<td></td>
</tr>
<tr>
<td>Started Selling of Maize</td>
<td>1.70*</td>
<td>1.69*</td>
</tr>
<tr>
<td></td>
<td>(1.16, 2.51)</td>
<td>(1.16, 2.46)</td>
</tr>
<tr>
<td>Mechanization</td>
<td>1.20</td>
<td>1.12</td>
</tr>
<tr>
<td>Total Cultivated Area</td>
<td>0.98</td>
<td>0.95</td>
</tr>
<tr>
<td>Number of household workers</td>
<td>1.07* (1.00, 1.13)</td>
<td>1.06* (1.00, 1.12)</td>
</tr>
<tr>
<td>Village Centrality index</td>
<td>0.92</td>
<td>1.04</td>
</tr>
<tr>
<td>Introduced new crop technology</td>
<td>0.80* (0.70, 0.91)</td>
<td>0.83* (0.73, 0.95)</td>
</tr>
<tr>
<td>NGO / Donor affiliation</td>
<td>1.06</td>
<td>0.942603</td>
</tr>
<tr>
<td>Import of Maize</td>
<td>0.97* (0.94, 0.99)</td>
<td>0.91* (0.89, 0.94)</td>
</tr>
<tr>
<td>Per-capita income</td>
<td>0.94</td>
<td>1.19* (1.04, 1.38)</td>
</tr>
<tr>
<td>Introduced crop rotation</td>
<td>1.36</td>
<td>1.18</td>
</tr>
<tr>
<td>Hire of labour</td>
<td>1.15</td>
<td>0.95</td>
</tr>
<tr>
<td>Consumer-worker ratio</td>
<td>1.00</td>
<td>1.16* (1.04, 1.32)</td>
</tr>
<tr>
<td>Age of farm manager</td>
<td>2.09</td>
<td>1.66</td>
</tr>
<tr>
<td>Age squared</td>
<td>0.49</td>
<td>0.54</td>
</tr>
<tr>
<td>Belongs to Poor group</td>
<td>1.04</td>
<td>0.77</td>
</tr>
<tr>
<td>Belongs to Wealthy group</td>
<td>1.11</td>
<td>1.84* (1.24, 2.72)</td>
</tr>
<tr>
<td>Specialized in Maize production</td>
<td>1.05</td>
<td>0.98</td>
</tr>
</tbody>
</table>

* Significant at 5% significance level.

Intercropping is the technology of cultivating different crops at a time within the year in a same piece of land that naturally results production increments and minimizes the cost. It is
obvious that farmers who implements intercropping are getting more amount of their crops from same arable land. Although intercropping is not significantly associated with extensification of Maize production, it has some significant impact on intensification. It is evident from the results that farmers who have given up intercropping in the reference year are less likely to intensify the production. The result supported by the ML estimation too, both significant at 5% significance level.

It is previously assumed that institutional discrimination of female farmers may have some negative impact on their farm productivity. Such an argument is not supported by the Bayesian approach of estimation in the present study. But the ML estimation of data results a relative risk for the female farm holders as 0.63 in case of extensification. Therefore, female farmers are less likely to extend the arable land for Maize crops compared to the males. Similar results are also noted by Djurfeldt et al. (2006); female farmers are mostly constrained by their limited access to land and to labour. Thus they are much less likely to have extensified.

Farmers who have increased the cultivated land since reference year are four fold more likely to extend the arable land under Maize crop. The reason behind that, Maize is the staple food in African region and the farmers are using more lands for the cultivation of Maize crop to gain more financial benefit. ML estimation results a positive impact of cultivated area increment over intensification that is not justified in Bayesian approach. Therefore it may conclude that there are some other prevailing factors that drive the farmers to occupy more land under Maize production.

Results indicate that, farmers who started selling of Maize since reference year are 170% more likely to extend the arable land for Maize as well as more likely to have intensification of the production with an equal chance. ML estimation also supports the results. The reason behind is that, farmers who gaining cash amount for Maize are much interested for its cultivation and production increments. Therefore, commercialization of crops seems to be a driven factor for both extensification and intensification.

Labour-surplus households are expected to be more dynamic than labour.-scarce ones. Therefore, number of household workers is showing a significant positive impact on both extensification and intensification. Farms with increasing number of household workers are
more likely to extensify the arable land as well as intensify the production. This is indicating that, labour force is an important driving factor for both extensification and intensification independently.

Availability of modern crop technology in village has been measured in a continuous scale for the present study. Results indicating that an increasing availability of new crop technology is discouraging the extensification or intensification of Maize production. That is, farmers getting the new crop technology available in villages are seems to be interested to produce other cash crops than Maize. Earlier it is concluded that commercialization is a factor for the extensification and intensification for Maize production. If farmers don’t have much opportunity to entrance in the local market for some specific crops, they will definitely loose interest for its production. Again, Results from the ML estimates indicating the factor as significant for the association of extensification and intensification. Although it is not supported by Bayesian estimation, that may be a driven factor for the farm dynamics.

For any community, if local production of main staple food-grain can not meet the consumption, import of that crop is increased. The other way is to encourage local farmers for more production. Results from the present study indicate that the former option is true for the African communities. The average import of Maize as percent of total production is negatively associated with extensification and intensification. As the import increases, farmers are less likely to extend the arable land or intensify the production. The result indicating again the proper marketing policy may be absent in the study region so that the production cost somehow more than the market value for maize.

Per-capita income is a good indicator to the economic behaviors of a state. Results indicate that as per capita income increases in the study region since 2002, peoples with higher per-capita income are more likely to intensify their production. A positive elasticity of 1.19 represents that if per-capita income increased, peoples are able to get benefited from the intensification. Although the result is not investigated by ML estimation, it’s a good indicator that wealthy persons will get more opportunity to increase the production with respect to time.

The availability of household labour is of crucial importance for farm dynamism. It is obvious from the result that a the high positive elasticity of 1.07 for additional household workers in relation to extensification and 1.06 for intensification explains the importance of labour force
in African farm dynamism. Related to this factor is the importance of the consumer burden carried by each worker that is the C/W-ratio, which is significantly and positively related to intensification, with an elasticity of 1.16 in Bayesian estimate and 1.14 in ML estimate. Both these findings reinforce the argument that besides being driven by commercial forces and by scientific-industrial inputs, the African smallholder sector is much dependent on its own labour resources and driven to a large extent by the consumer needs of the household. This is an obvious reflection of the family farm or peasant character of the African smallholder sector earlier obtained by Djurfeldt et al. (2006) and Djurfeldt et al. (2009).

A life-cycle aspect of farm dynamism is expected in the analysis. The relation is visible for extensification only by ML estimates. Positive elasticity for the variables age shows an increasing relative chance for extensification and the statistical significance of age squared points to a curvilinear relation, with the chance of expansion decreasing at older ages.

Economic status of farm holder is another important driver for the intensification of Maize production. Farm holders belong to the wealthy group have 184% more chance to get associated with intensification of Maize production. The above result also supported by the conclusion drawn previously for the association between per-capita income and intensification of Maize production. Which may lead to the fact that that accessing both the land and the inputs needed for an expansive strategy seems to depend upon resources mainly commanded by the rich farmers.

One of the main purposes of present study is to compare the estimated cause-effect co-variates by two distinct methods: usual ML estimation and Bayesian estimation using Gibbs sampling. It is much interesting to find that the results for the joint association of extensification and intensification by two distinct estimation methods are different. The variables ‘intercropping’ and ‘availability of modern crop technology’ showing a cause-effect relationship over the joint association by ML estimation, whereas the Bayesian estimation method proves that extensification and intensification are two distinct way of farm dynamics. The respective wider confidence intervals for the odds ratios for the related co-variates in ML estimation method require much investigation for such an association. The Bayesian approach uses the minimum positive root for the joint association at individual level and identifies that this association does not perfectly have a justification after a number of iterations of the sample observations. This implies that the extensification and intensification defined in the
present study occurs independently of one another and the bivariate association obtained from Table-3, is the result of interaction of variables included in the model rather than the outcome of an interaction between the two processes. Similar result also obtained by Djurfeldt et al. (2009) from the same data when analyses with country dummies included in the model with Malawai as the reference.

6. Conclusion and recommendations

One purpose of the study is to develop an easily accessible code for bivariate binary regression in terms of Bayesian inference and then investigate the estimated parameters with that of ML estimates. In this regard two dichotomous response variables – extensification and intensification defined from the data and try to find out the cause effect co-variates existing at independent level as well as their association. The purpose is achieved on development the WinBUGS code and then comparing the estimates with the results obtained the analysis by R software.

The important purpose of the study is to investigate the farm management and indication of those factors which causes the food scarcity in South African states. It is evident from the results that some factors for instance ‘availability of new crop technology’, ‘import of Maize’, ‘stopped intercropping’ shows some negative association with farm dynamics response variable, which concludes that these factors discouraging the production of Maize and areal increase in both Bayesian and maximum likelihood estimation. Whereas ‘Change in fertilizer use’, ‘cultivated area increase’, ‘started selling maize’ shows positive association. This indicates that these factors support the argument support of areal and Maize production increase. Farm holders access to modern crop technologies, in combination with commercial incentives to staple crop production emerge as the most important explanation of dynamism.

In the present study, joint association between Intensification and Extensification is measure in log odds. It is quite interesting to note that no association has been found by Bayesian estimation between two response factors. Although, having expansion in the size of cultivated area is highly correlated with yield increases, but as noted above, the direction of underlying causal processes is not clear. However, those commercial drivers are currently more important than increasing pressure on the land. For this reason a correlation between the two variables is not unexpected.
Moreover, the factors seem to drive both Intensification and Extensification, help to make food polices against food crises and helps to understand the farm dynamics structure in South African stats. In order to meet the aggregate future food demands, production will have to increase foremost in areas already under cultivation or where commercialization access available and infrastructure networks exists or can be upgraded at a reasonable cost. More specifically in this context, to induce intensive growth of staple food production, remains, not only an unfinished task in sub-Saharan Africa.

The most prominent thing is to note the comparison between two estimation approaches i.e. Bayesian and Maximum likelihood estimation, the strongest point to capture here is that how the posterior distribution changes due to the minor change in the priors. The practical difficulty in this study is to setup the information of the prior distribution. In order to overcome this problem, a much lower precision normal distribution is defined. Consequently, due to the strong or vague definition of prior distribution, the resultant inference almost mimic the results of frequentist approach in both Extensification and Intensification or we can conclude that we are pretty much certain about the posterior distribution. On the other hand, the association show quite different results as unexpected.

The study is not beyond its own limitations. One of such limitation is to identify the best prior to the parameters in the model. Secondly, the available data set includes most of the important co-variates but none can ignore the effect of spatial variations, migration and population factors in the study region on farm dynamism. Further study and research path is opened here. The developed bivariate binary logistic regression model can also be used to analyse data obtained from other field of research like some behavioural sciences and biomedical studies.

References


model{
for (i in 1:N){

# Logistic regression model for extensification
DEP1[i] ~ dbern(P1[i])

# Logistic regression model for intensification
DEP2[i] ~ dbern(P2[i])

# Logistic regression model for joint association in terms of odds ratio

# Calculation of joint probability and marginal probabilities at individual response level
A[i] <- OR[i]-1
B[i] <- (1-OR[i])*(P1[i]+P2[i])-1
C[i] <- (OR[i]*P1[i]*P2[i]+(1-equals(OR[i],1))*(S1[i]*S2[i])+S4[i]*(S1[i]*P1[i]+S2[i]*P2[i]))
D1[i] <- (-B[i]+sqrt((pow(B[i],2)-4*A[i]*C[i])))/(2*A[i])
D2[i] <- (-B[i]-sqrt((pow(B[i],2)-4*A[i]*C[i])))/(2*A[i])
S1[i] <- step(1-D1[i])*step(D1[i])
S2[i] <- step(1-D2[i])*step(D2[i])
S3[i] <- S1[i]*S2[i]
S4[i] <- 1- equals(S3[i],1)
P11[i] <- equals(OR[i],1)*P1[i]+P2[i]-(1-equals(OR[i],1))*(S1[i]*S2[i]+min(D1[i],D2[i]))
P10[i] <- P1[i]-P11[i]
P01[i] <- P2[i]-P11[i]
P00[i] <- 1-(P10[i]+P01[i]+P11[i])
}

# Prior distribution of model parameters
alpha[1] ~ dnorm(0,0.001)
alpha[2] ~ dnorm(0,0.001)
alpha[3] ~ dnorm(0,0.001)

for (j in 1:22){
  beta1[j] ~ dnorm(0,0.001)
  beta2[j] ~ dnorm(0,0.001)
  beta3[j] ~ dnorm(0,0.001)
}

# Set of initial values
list(beta1=c(0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0),beta2=c(0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0),beta3=c(0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0),alpha=c(0,0,0))

# Data
list(N=1533,DEP1,DEP2,INDEP1,INDEP2,INDEP3,INDEP4,INDEP5,INDEP6,INDEP7,INDEP8,INDEP9,INDEP10,INDEP11,INDEP12,INDEP13,INDEP14,INDEP15,INDEP16,INDEP17,INDEP18,INDEP19,INDEP20,INDEP21,INDEP22)