Hedging with Gold Futures: Evidence from China and India

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Abstract

Recently the National Commodity & Derivatives Exchange (NCDEX) and the Shanghai Futures Exchange (SHFE) introduced gold futures trading in India and China respectively. In developing markets like China and India it is important to manage risk. For the sake of choosing the optimal hedge strategy it is sufficient to understand and calculate the optimal hedge ratio. Therefore we divide our analyze on two parts. Firstly, we calculate optimal hedge ratios for hedging spot contracts with futures contracts. Secondly, we evaluate the hedging efficiency of these optimal hedge ratios. We use OLS, VAR and VECM models to estimate constant hedge ratios and VAR-MGARCH to estimate dynamic hedge ratios. It is found that for both China and India VAR-MGARCH model estimates of the time varying hedge ratio give higher volatility reduction compare to the hedge ratios based on models with constant hedge ratio. From the empirical results, we found that Indian gold futures market is effective and Chinese gold futures market is less effective. Overall, gold futures contracts in China and India prove to be a smart and well-needed hedging tool for clever investor.

Keywords: Gold, Futures, Hedge ratio, Hedging effectiveness, China, India.
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1. Introduction

Gold is the most popular investment among all precious metals. Gold has been used as medium of exchange throughout history and a relative standard for currency equivalents. In the 19th century, gold standards were popular in many countries, especially in the European countries. Due to its special properties gold could be used as a hedge against economic, political, social or currency crises.

Any risk averse investor tries to reduce the risk associated with a deal. How can such investor decrease such risk, especially when the price volatility is high? Two years ago, after the beginning of the financial crisis investors started to diversify their assets. At that time gold futures grew rapidly as an instrument of hedging or an ordinary investment. As any other derivatives, future contracts can be used to reduce the risk connected with price movements. Before 2008 trading with gold futures was concentrated in London, Zurich and New-York. But financial crisis gave boost to other futures markets, especially in Asia.

The purpose of this essay is twofold. Firstly, we aim to find out to what extent an investor should hedge his exposure to the risk on the spot gold market with the help of futures contracts. Secondly, we are going to assess the efficiency of the optimal hedging strategy in reducing the total risk.

In our essay we focus on hedging with gold futures contracts in China and India. There are several reasons for such a choice. Firstly, in India and China commodity futures appeared in 2003 and 2008 respectively, and are relatively new. Therefore, there are not so many studies about commodity futures in the chosen countries. Secondly, we choose gold, because India is the biggest producer of gold in the world, and China is the biggest consumer and the second biggest producer of gold in the world.

In spite of the primary position in supplying and demanding gold, China’s gold market is fairly new and reasonably closed for foreign investors. Nowadays, Shanghai Gold Exchange (SGE), which was founded in 2002, is the biggest spot gold exchange in the world. In 2008 Shanghai Futures Exchange (SHFE) was founded, and since then it has been possible to trade futures in China. Despite the fact that trading in gold futures is available only for domestic
investors and has existed only for two years, at the end of 2010 SHFE became the fourth largest gold futures exchange in the world. India’s National Commodity & Derivatives Exchange (NCDEX), the India's largest commodity derivatives exchange, was founded in 2003 and is now among the biggest gold exchanges in the world. NCDEX introduced both spot and futures gold trading in 2003.

Connection between the spot and futures markets is very important for hedging, so there were a lot of studies in this field. The earliest work, which is called “The Economics of Gold Price Movements“, belongs to Peter Abken (1980). It is mentioned there that “an individual’s decision to store gold for future sale requires a prediction of the gold price“. Using monthly data for 1975-1979, he discovered that three-month gold futures prices efficiently assimilate new information, which makes them a good indicator of the future spot prices. Further works on this topic were done by Monroe and Cohn (1986), Ma and Soenen (1988) and Bertus and Stanhouse (2001). The latest work in the area, “Parities and Spread Trading in Gold and Silver Markets: A Fractional Counteraction Analysis”, was presented by Liu and Chou in 2003. They found that significant riskless profit can be made on the basis of the general ECM (Error Correction Model) forecast of the futures prices.

Thus, there a lot of studies investigating the relation between the gold spot and futures prices. There are three major works dedicated to gold futures hedging. Baillie and Myers (1991) used data on six commodities, including gold, for the USA market, and discovered that a constant Ordinary Least Squares (OLS) hedge performs almost as well as a time-varying Generalized Autoregressive Conditional Heteroskedasticity (GARCH) hedge in the sense of reducing variance of a hedged spot portfolio. For the Chinese market Xu, Nordén and Hagströmer (2010) found that “hedging with gold futures reduces the variance of a hedged gold spot position by about 88% in its first two years of existence. Hence, the new Chinese gold futures prove to be an attractive and well-needed hedging vehicle for domestic Chinese gold producers and consumers (who in general are barred from international derivatives markets), immediately following the introduction”. As for the India’s market, there is a paper of Kumar, Singh and Pandey (2008). In their work they examine hedging effectiveness of futures contracts on commodities and financial assets in Indian markets.
They found that using time-varying GARCH gives a little bit better results than the constant variance models, but the latter are effective in reducing variance of hedged spot portfolio.

There is no single opinion about what method yields the hedge ratio which optimally reduces the risk. According to Lien (2002) and Moosa (2003), OLS shows the best performance among the models with constant hedge ratio. On the other hand, according to Ghosh (1993), VECM (Vector Error Correction Model) is the most efficient among the models with constant hedge ratio. However, majority of papers, when it comes to hedge ratio and hedge efficiency, prefer to use bivariate GARCH (Baillie and Myers (1991), Kavussanos and Nomikos (2000), Floros and Vougas (2006)).

The hedge ratio of one usually stands for taking position in futures equivalent to the spot position in volume, but opposite in sign. But this strategy works perfectly (eliminates price risk to zero) if the changes in futures and spot prices move in the same direction. As we know, such perfect case is rare and can be true only for a short time. When we have large time-series of data we need to come up with another method to calculate hedge ratio. The method called “minimum variance hedge ratio” (MVHR) was introduced by Johnson in 1960. According to Johnson, relation between futures and spot prices is not perfect, so risk is the variance of return on a two-asset hedged position. When we are hedging with futures contracts, we need to remember that investor’s hedging decision must be based on finding the optimal hedge ratio with the best hedging efficiency. As the Chinese and Indian markets are relatively new, we expect them to be more volatile compared to developed markets, which is confirmed by Bose (2007). There are several ways to find optimal hedge ratio, which are described in the theoretical part. We think that minimum-variance hedge ratio is the best in the case of new markets such as Chinese and Indian.

Benninga (1984) defines MVHR as the slope coefficient in the OLS regression of changes in spot prices on changes in futures price. In other words, MVHR is the regression coefficient which gives maximum hedging effectiveness. A lot of studies focus on measuring hedging efficiency. They try to find to what extent investors are able to reduce price risk by using futures contracts. Markowitz (1959) measured hedge effectiveness as the reduction in standard deviation of portfolio returns associated with a hedge. But later Johnson
(1960), Stein (1961), Working (1962) and Ederington (1979) measured hedging effectiveness as the percent reduction in variability. We will use the latter method to measure hedging efficiency.

There are two reasons why OLS is criticized for calculating hedge ratio and hedging efficiency. Baillie and Myers (1991) mentioned that hedge ratio calculated using OLS is based on the unconditional distribution of futures and spot prices, which usually is not appropriate, since only conditional distribution takes into account the arrival of new information influencing investment decisions. Secondly, Park and Switzer (1995) point out that OLS is based on hypothesis that relation between futures and spot prices is time invariant, but we know that in reality their joint distribution is time variant. So Park and Switzer use MGARCH model, instead of the constant hedge ratio, to calculate dynamic hedge ratio and hedge effectiveness. It gave positive results – hedge effectiveness was improved compared to the OLS method.

Many recent works on the hedging effectiveness estimate time varying hedge ratios with the help of different GARCH models (MGARCH – Bollerslev (1988), Floros and Vougas, (2006), VAR-MGARCH – Kumar, Singh and Pandey (2008), Bivariate GARCH – Baillie & Myers (1991), Choudhry (2004) etc.). Nevertheless, some recent studies, for example Lien (2002) and Moosa (2003), show that the basic OLS approach dominates GARCH. Therefore, we can make conclusion that the choice of the best model for hedging with futures depends on the specific country and market. So, we choose VAR-MGARCH to estimate dynamic hedge ratios, and OLS to estimate constant hedge ratio and hedging effectiveness.

Our essay contributes to the previous research in couple of ways. Firstly, we examine not the well-known markets, like USA or Japan, but the developing ones. There are almost no papers about hedging efficiency in the Chinese and Indian markets. Secondly, this is the first research analyzing data on these markets after the financial crisis of 2008. Thirdly, we will explore whether the constant or the time varying hedge ratio is more efficient for the Chinese and Indian markets.
The rest of the paper is structured as follows. Section 2 gives the review of previous studies and an overview of the hedge ratios employed by other authors so far. Section 3 presents the theoretical background on hedge ratio and hedging effectiveness. Section 4 describes the data and methodology, which we used for estimating the optimal hedge ratio and assessing hedging efficiency. Empirical results are reported and discussed in Section 5. Finally, Section 6 concludes and suggests practical applications of the obtained results.
2. Literature Review

2.1 Articles Review

In their work Caihong Xu, Lars L. Nordén, and Björn Hagströmer (2010) study the Shanghai Futures Exchange (SHFE) gold futures and evaluate the futures hedging effectiveness. It is the first work which introduces gold futures since the opening of Shanghai Futures Exchange in 2008. They show that hedging with gold futures reduces the variance of a hedged gold spot position by about 88% in its first two years of existence. Also they discover that during world financial crisis escalation reduced the variance till about 70%. They make a conclusion that hedging with gold futures on the Chinese gold market can be a very attractive and useful tool not only for domestic Chinese gold producers but also for consumers and investors.

Brajesh Kumar, Priyanka Singh, and Ajay Pandey (2008) in their work examine hedging effectiveness of futures contracts on financial assets and commodities in the Indian markets. They show that in risk management understanding the optimal hedge ratio is crucial for devising an effective strategy. Also they manage to prove that in most of the cases, VAR-MGARCH model estimates of time varying hedge ratio provide highest variance reduction as compared to hedges based on constant hedge ratio. So, basically, they show that futures can be a great hedging tool when it comes to financial assets and commodities markets.

Suyash N. Bhatt (2010) investigates the hedging effectiveness of the futures market on financial assets and commodities in the Indian markets. Also, he estimates the constant hedge ratio using the Ordinary Least Squares (OLS) model, and the dynamic hedge ratio has been estimated using VAR-MGARCH model. He compares the in-sample performance of these models in reducing portfolio risk. The model providing the highest variance reduction is considered to be the most effective in hedging. Bhatt makes a conclusion that it is crucial to use the optimal amount of hedging instruments and determine the efficiency of hedging.
2.2 Hedge Ratios Review

In this section, alternative theoretical framework on optimal hedge ratios and various criteria for performing hedging effectiveness will be briefly described. The details of models estimation will be presented as well.

The basic financial definition of hedging is to establish a portfolio in spot markets and future markets in an attempt to offset exposure to price fluctuations risk (Sheng-Syan Chen et al. 2002). Thus, the main objective of hedging is to reduce the risks.

We assumed that an investor hold some units of a long spot position and other units of a short futures position. Denote logarithms of spot and futures prices at time t are $S_t$ and $F_t$ respectively. Because we use future contracts to hedge the risk of price changes in spot positions. Thus, the portfolio is a hedge portfolio. The changes in logarithms of prices are returns. The returns on spot and futures positions are $X_{s,t} = S_t - S_{t-1}$ and $X_{f,t} = F_t - F_{t-1}$ respectively. Therefore, the portfolio return is defined as $X_{p,t} = X_{s,t} - \beta X_{f,t}$. Where $\beta$ is the so-called hedge ratio. (Caihong Xu et al. 2010)

The main aim of hedging is to find the optimal hedge ratio $\beta$, which depend on specific model and functions. Through reading lots of hedging literature, we found that the spot position is assumed to be fixed in general. Thus, in this case we only choose the optimum futures position.

We categorized hedge ratios in table 1. And we will discuss them in the following.
Table 1 Hedge ratios list

<table>
<thead>
<tr>
<th>Hedge Ratio</th>
<th>Objective Function</th>
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<tbody>
<tr>
<td>Minimum variance hedge ratio</td>
<td>Minimize variance of $X_{t+1}$</td>
</tr>
<tr>
<td>Sharpe hedge ratio</td>
<td>Maximize $\frac{E(X) - R_f}{\sigma_x}$</td>
</tr>
<tr>
<td>Maximum Expected Utility hedge ratio</td>
<td>Maximize $EU(X_{t+1})$</td>
</tr>
<tr>
<td>Minimum Mean extend-Gini coefficient hedge ratio</td>
<td>Minimize $\Gamma_v(X)$</td>
</tr>
<tr>
<td>Optimal Mean extended-Gini coefficient hedge ratio</td>
<td>Maximize $E(X) - \Gamma_v(X)$</td>
</tr>
<tr>
<td>Minimum Generalized semivariance hedge ratio</td>
<td>Minimize $V_{\delta,\alpha}(X)$</td>
</tr>
<tr>
<td>Minimum Semivariance hedge ratio</td>
<td>Minimize $\text{SemiVar}(X)$</td>
</tr>
<tr>
<td>Optimal mean-generalized semivariance hedge ratio</td>
<td>Maximize $U(X)$</td>
</tr>
<tr>
<td>VAR hedge ratio</td>
<td>$\beta$</td>
</tr>
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</table>

2.2.1 Minimum variance hedge ratio and criterion

The minimum variance hedge ratio is the most widely employed strategy to optimize hedge ratio. It is called MV hedge ratio as well, which based on minimizing the variance of the hedge portfolio. (Johnson, 1960; Ederington, 1979)

Since we will explain more about the MV hedge ratio in the following session, here we just show the final formula MV hedge ratio.

The estimation of hedge effectiveness $\kappa$ is defined in the MV hedge ratio as following:

$$\kappa = \frac{\text{Var}(U) - \text{Var}(H)}{\text{Var}(U)}$$

Where Var(U) is the variance of unhedged portfolio and Var(H) is the variance of the hedged portfolio.

The advantage of the Minimum variance hedge ratio is that it is easy to understand and compute. However, the disadvantage of the MV hedge ratio is that it is not consistent with mean-variance framework due to it ignores the expected return on the hedged portfolio.
2.2.2 Sharpe hedge ratio and criterion

Sharpe hedge ratio is a hedge strategy that considers the risk-return tradeoff. Howard and D’Antonio (1984) consider the optimal level of futures contracts by maximizing the ratio of the portfolio’s excess return to its volatility:

$$\max_\beta \theta = \frac{E(X) - R_f}{\sigma_X}$$

Where $R_f$ is the risk free interest rate and $\sigma^2_X = \text{Var}(X)$.

The optimal hedge ratio is as following:

$$\beta^* = -\frac{(\sigma_s/\sigma_f)\left[(\sigma_s/\sigma_f)\left(\frac{E(f)}{E(s) - R_f}\right) - \rho\right]}{[1 - (\sigma_s/\sigma_f)\left(\frac{E(f)\rho}{E(s) - R_f}\right)\right]}$$

Where $\sigma^2_s = \text{Var}(s)$, $\sigma^2_f = \text{Var}(f)$ and $\rho$ is the correlation coefficient between $s$ and $f$. (Howard and D’Antonio, 1984)

If $E(f)=0$, then $\beta$ reduces to:

$$\beta = \frac{\sigma_s}{\sigma_f} \rho$$

The above formula is the same as that in Minimum variance hedge ratio.

The disadvantage of Sharpe ratio pointed by Chen et al. (2001) is that it is a highly non-linear function, which may lead to minimize instead of maximizing the hedge ratio.

2.2.3 Maximum Expected Utility hedge ratio and criterion

This method considers both the expected return and risk in the derivation of hedge ratios. According to Hsin et al (1994), the mean-variance expected utility function is the following:

$$E_t U(X_{t+1}) = E_t(X_{t+1}) - \gamma \sigma^2_t(X_{t+1})$$

Where $\gamma$ is the level of risk aversion. The predictable component of volatility in the
return is defined as conditional variance. Therefore, in this case, risk is estimated by conditional variance.

Thus, the maximum expected utility hedge ratio at time $t$ is the following:

$$\beta_t^* = \frac{-E_t(f_{t+1}) + 2\gamma \sigma_t(s_{t+1}, f_{t+1})}{2\gamma \sigma_t^2(f_{t+1})}$$

If the futures prices are martingale, $E_t(f_{t+1}) = 0$. Thus, the equation reduces to:

$$\beta_t^* = \frac{\sigma_t(s_{t+1}, f_{t+1})}{\sigma_t^2(f_{t+1})}$$

Which is the conventional minimum hedge ratio.

The criterion applied to evaluate the hedging strategies is based on the utility comparison. Different values of $\gamma$ give various rankings. Therefore, the investor’s risk preference in his choices of hedging strategies should be considered in the criterion. (Wei Chen, 2010).

### 2.2.4 Minimum Mean extended-Gini coefficient hedge ratio and criterion

This method of deriving the optimal hedge ratio is consistent with the concept of stochastic dominance and involves the use of the mean extended-Gini coefficient. (Cheung et al., 1990; Shalit, 1995; Wei Chen 2010). It minimizes the mean extended-Gini coefficient $\Gamma_v(X)$ in the following:

$$\Gamma_v(X) = -u\text{Cov}(X, (1 - G(X))^{v-1})$$

Where $G$ is the cumulative probability distribution and $v$ is the risk aversion parameter. If $0 < v < 1$, it implies risk seekers. And if $v = 1$, it implies risk-neutral investors. If $v > 1$, it implies risk-averse investors. Shalit (1995) claims that if the futures and spot returns are jointly normally distributed, the minimum-MEG hedge ratio would reduce to the MV hedge ratio. In other words, it would be the same as the MV hedge ratio.

Referring to the criterion, according to Wei Chen (2010), the hedge effectiveness $\kappa$ could be denoted as the risk reduction based on MEG coefficient:

$$\kappa = 1 - \frac{\Gamma_v(H)}{\Gamma_v(U)}$$
2.2.5 Optimal Mean extended-Gini coefficient hedge ratio and criterion

This hedge strategy considers maximizing the utility function based on GEM coefficient to take into account the risk return trade-off (Kolb and Okunev, 1993).

The function is defined as the following:

\[ U(X) = E(X) - \Gamma_\nu(X) \]

The hedge ratio obtained from the above formula is denoted as M-MEG hedge ratio. It takes into account the expected return on hedged portfolio. The MGE hedge ratio will reduce to the M-MEG hedge ratio, if the futures prices are martingale (E(R_f) = 0).

Hedging effectiveness is estimated by the magnitude of the utility based on the above function. If the utility is higher, the hedging strategy will be better.

2.2.6 Minimum Generalized semivariance hedge ratio and criterion

This method is suggested by Chen et al., 2001; Lien and Tse, 1998, 2000. It focuses on downside risk, which means the variability of losses. Semivariance is denoted as the variability of returns below the mean. The development of Minimum Generalized semivariance hedge ratio is a milestone in estimating the downside risk.

The optimal hedge is derived by minimizing the GSV as follows:

\[ V_{\delta,\alpha}(X) = \int_{-\infty}^{\delta} (\delta - X)^\alpha dG(X), \ \alpha > 0 \]

Here, G(X) is the probability distribution function of the return on the hedged portfolio. And \( \delta \) and \( \alpha \), which are real numbers, represent the target return and risk aversion, respectively. The above formula suppose investors take into account the investment as risky only if the return is below the target return. \( \alpha < 1 \) represents a risk-seeking investor and \( \alpha > 1 \) means a risk-averse investor. The GSV is consistent with risk perceived by managers since it emphasis on the return below the target return. (Crum et al.1981; Lien and Tse, 2000). If the futures and spot returns are jointly normally distributed and the futures price is a pure martingale process, the GSV hedge ratio would be reduced to the MV ratio as...
Referring to the criterion, the hedge strategy is evaluated by calculation the risk reduction of the downside risk.

\[ \kappa = 1 - \frac{V_{\delta, \alpha}(H)}{V_{\delta, \alpha}(U)} \]

The metrics of the GSV hedge ratio is the following: first of all, it is non-normality. It does not require the normality assumption in the return distribution. Additional, the lower partial moment (GSV) hedge ratios are effective in reducing downside risk and increasing returns. (Eftekhari, 1998). It could be effective with symmetric information. (Wei Chen, 2010).

### 2.2.7 Minimum Semivariance hedge ratio and criterion

This hedge performance is a special case of the minimum GSV hedge ratio. (Wei Chen, 2010). In this case, the target return is the same as the expected return and the parameter of risk aversion is equal to 2. Therefore, the objective function is the following:

\[ \text{SemiVar}(X) = \int_{-\infty}^{\delta} (\delta - X)^2 dG(X), \; \alpha > 0 \]

Here, \( \delta \) is the target return, which equals to the expected return. Again, If the futures and spot returns are jointly normally distributed and the futures price is a pure martingale process, the GSV hedge ratio would be reduced to the MV ratio.

The hedging performance is showed the following:

\[ \kappa = 1 - \frac{\text{SemiVar}(H)}{\text{SemiVar}(U)} \]

### 2.2.8 Optimal mean-generalized semivariance hedge ratio and criterion

The mean-GSV hedge method is an extension of the GSV hedge ratio by considering the mean return in the derivation of the optimal hedge ratio. (Chen et al. 2001). The M-GSV hedge ratio is derived by maximizing the mean-risk utility function as following:
\[ U(X) = E(X) - V_{\delta,\alpha}(X) \]

The M-GSV hedge ratio will be reduced to the MV hedge ratio if the futures prices is a pure martingale process and the futures and spot returns are jointly normal.

The hedging effectiveness is estimated by the magnitude of expected utility based on GSV of the hedged and unhedged positions (Wei Chen, 2010).

### 2.2.9 VAR hedge ratio

This hedge method is widely-used these years to measure the downside risk. VaR (Value at Risk) is becoming the essential and standard risk management tool to make investment decisions and allocations decisions. VaR was defined by Jorion (2000) as an absolute size of losses associated with the hedging strategy.

\[ \min \text{VaR}(X) = Z_\alpha \sigma_x \sqrt{\tau} - E(X)\tau \]

Where \( \text{VaR}(X) \) is the absolute VaR of the hedged portfolio and \( Z \) is the left percentile at \( \alpha \) in the assumption of the standard normal distribution. Note that if two portfolios with the same VaR may have different potential losses. Because VAR provide the return exceeded with \((100 - \alpha)\% \) probability, while do not take into account the magnitude of losses beyond the \((100-\alpha)\)th percentile.

The VaR hedge ratio is the following: (Chun et al., 2006)

\[ \beta = \rho \frac{\sigma_s}{\sigma_f} - E(f) \frac{\sigma_s}{\sigma_f} \sqrt{\frac{1 - \rho^2}{Z_\alpha^2 \sigma_f^2 - E(f)^2}} \]

According to the Cotter and Hanly (2006), the criterion for the hedge performance using the 1% significant level is the following:

\[ \kappa = 1 - \frac{\text{VaR}_{1\%}(H)}{\text{VaR}_{1\%}(U)} \]
3. Theoretical Background

3.1 Futures Contract

Futures contract - a firm agreement between the seller and the buyer for the sale of specific asset at a fixed future date. Contract price, depending on varying market conditions, recorded at the time of the transaction. Because the contract is a specification standard, both parties know exactly what is being traded.

Why do we need futures contracts? Disadvantages and problems associated with the first forward contracts for future delivery have been eliminated in the mid 60-s of the XIX century with the advent of futures contracts. In 1865, it was laid the foundation of all modern futures contracts by introducing grain agreement, standardized according to the following conditions:

- Grain quality;
- The number of grains per contract;
- Date and place of delivery of grain.

As a result, only the changing conditions of the contract remain price. It was determined during the bidding in the room through an open exchange shouting. This meant that the transaction prices have been known to all present traders i.e. became transparent.

Over the next century the number of exchanges that trade futures contracts on a variety of products is constantly growing. By the beginning of the 70-s of XX century, radical changes in politics, economy and principles of regulation led to the emergence on the world commodity and financial markets, floating exchange rates and the development of communications and computer technologies. The combination of these factors increased the volatility of markets, producers / consumers of goods and issuers / buyers of financial instruments have faced an acute need to protect their assets from the risk of price fluctuations.

The need for protection from the risks and seek opportunities to gamble have expanded and strengthened the derivative markets, particularly futures.

Futures contracts have the following common characteristics:
• have the standard features;

• traded on stock exchanges;

• Publicly available, prices are published;

• organize clearing houses.

The role of clearing house can vary depending on the exchange, but in essence it acts as an intermediary between the seller and the buyer of the contract. Clearing house acts as counterparty to both sides, providing them with protection and creating conditions for free trade. Futures contract does not predict future prices. It should also be mentioned that after the expiry of the contract is delivered in accordance with agreed terms and conditions.

Now let’s talk about hedgers. These market participants are trying to protect existing positions from future adverse price movements. So, the producers and consumers of commodities hedge their positions in the cash or physical markets through futures contracts.

In order to hedge market player takes on the futures market position, an equal and opposite to the one he holds in the cash market. Hedge is of two types - short and long, in short hedge open short position in futures, which compensates for the existing long position in the cash market. For example, a fund manager has a portfolio of shares, may hedge its position on the downside by selling index futures contracts. In the long hedge to open long positions on futures to offset an existing short position in the cash market. Refining Company may fix the purchase price by buying futures contracts for crude oil today. Regardless of whether you're coming hedging cash transaction or the current market position, the purpose of hedging is always the same: to compensate for the losses produced in the same market, profits arising in another.

### 3.2 Minimum variance hedge ratios

In this essay, we prefer to employ minimum variance hedge ratio to test the hedge effectiveness in gold future markets, which is the benchmark in the hedge literature, to test the hedge effectiveness because of the following reasons.
First of all, minimum variance hedge ratio is optimum especially for risk averse traders. (Kahl, 1983) Additional, MV hedge ratio is still optimum when future markets are not biased, which has been verified in many empirical studies. (Baillie and Myers, 1991) Finally, another advantage is that it is easy to understand and compute.

In the following session, we would like to explain two hedging models. One is constant hedge ratio model. The other is dynamic hedge ratio model.

3.3 Constant Hedge Ratio Model

Suppose there is a two-period \((t_1, t_2)\) investment decision that an investor faces and the futures contracts are the only way to hedge against risks. An investor buys a unit of the spot and short sells \(\beta\) units of the futures at time \(t_1\). The payoff \(X\) at time \(t_2\) of the hedge portfolio as following:

\[
X = S - \beta f
\]

Where \(S\) and \(f\) are the prices changes between \(t_1\) and \(t_2\) in spot and futures markets respectively.

The variance of the hedge portfolio is as following:

\[
\text{Var}(X) = \text{Var}(S - \beta f) = \text{Var}(S) + \beta^2 \text{Var}(f) - 2\beta \text{Cov}(S, f)
\]

According to the first order condition, we could minimize \(\text{Var}(X)\) to obtain the optimal hedge ratio.

\[
\frac{\partial (\text{Var}(X))}{\partial (\beta)} = 2\beta \text{Var}(f) - 2\text{Cov}(S, f) = 0
\]

\[
\beta^* = \frac{\sigma_{sf}}{\sigma_f^2}
\]

The estimation of hedge effectiveness \(\kappa\) is defined as following:

\[
\kappa = \frac{\text{Var}(U) - \text{Var}(H)}{\text{Var}(U)}
\]

Where \(\text{Var}(U)\) is the variance of unhedged portfolio and \(\text{Var}(H)\) is the variance of the hedged portfolio.
The above is the constant minimum hedge ratio, and we will explain dynamic hedge ratio model in the following session.

### 3.4 Dynamic Hedge Ratio Model

We suppose that an investor buy a unit of the spot and short sell $\beta_t$ units of the futures at time $t$. The following model could present time-varying variances of the spot and futures prices. Thus, the payoff $(X_{t+1})$ at time $t+1$ of the hedge portfolio as following:

$$X_{t+1} = S_{t+1} - \beta_t f_{t+1}$$

Where $S_{t+1}$ and $f_{t+1}$ are the prices changes between $t$ and $t+1$ in spot and futures markets respectively.

The variance of the hedge portfolio is the following:

$$\text{Var}(X_{t+1}|\Omega_t) = \text{Var}(S_{t+1}|\Omega_t - \beta_t f_{t+1}|\Omega_t)$$

$$= \text{Var}(S_{t+1}|\Omega_t) + \beta_t^2 \text{Var}(f_{t+1}|\Omega_t) - 2\beta_t \text{Cov}(S_{t+1}, f_{t+1}|\Omega_t)$$

Where $\Omega_t$ means the conditional on all relevant information available at time $t$.

According to the first order condition, we could minimize $\text{Var}(X_{t+1}|\Omega_t)$ to obtain the optimal hedge ratio.

$$\frac{\partial (\text{Var}(X_{t+1}|\Omega_t))}{\partial (\beta_{t+1}|\Omega_t)} = 2\beta_t \text{Var}(f_{t+1}|\Omega_t) - 2\text{Cov}(S_{t+1}, f_{t+1}|\Omega_t) = 0$$

$$\beta_t^* = \frac{\sigma_{s_{t+1}|\Omega_t}}{\sigma_{f_{t+1}|\Omega_t}}$$

The estimation of hedge effectiveness $\kappa$ is defined as following:

$$\kappa = \frac{\text{Var}(U|\Omega_t) - \text{Var}(H|\Omega_t)}{\text{Var}(U|\Omega_t)}$$

Where $\text{Var}(U)$ is the variance of unhedged portfolio and $\text{Var}(H)$ is the variance of the hedged portfolio.

The difference between expressions in constant hedge model and dynamic hedge model is whether there are conditional moments. Thus, the dynamic hedge models allow the investor
re-evaluate the hedging strategy conditional upon the information is available at every point in time. In other words, the dynamic hedging models are much superior than the constant hedging models.
4. Data And Methodology

4.1 Data Description

The data set in this essay consist of daily spot and daily futures prices in china market and Indian market. The spot gold product applied in this empirical analysis is Au 99.95. We obtain spot gold Au 99.95 prices from Shanghai Gold Exchange (SGE) for China market. And we get spot gold Au 99.95 prices from National Commodity & Derivatives Exchange (NCDE) for Indian market.

The daily gold future prices (for futures with time to maturity three month) for the corresponding sample period are obtained from Shanghai Futures Exchange (SHFE) for China market. And we find the daily gold futures prices (for futures with time to maturity three month) from National Commodity & Derivatives Exchange (NCDE) for Indian market as well. The sample period ranges from 25.02.2009 to 25.02.2011, which consist of 523 trading days. All prices in the data set were converted in US dollars. The two futures contracts trading at this period are analyzed and compared by using the following four hedge ratio models.

4.2 Methodology

There are various methods to estimate hedge effectiveness. In this essay, we decide to employ OLS, Vector Autoregressive regression (VAR), Vector Error Correction model (VEC) to test the constant hedge ratio and Vector Autoregressive Model with Bivariate Generalized Autoregressive Conditional Heteroskedasticity model (VAR-MGARCH) to estimate the dynamic hedge ratio. When we apply OLS, VAR, VECM models, we assume that the joint distribution of spot and futures prices is time invariant. In the other hand, we take into account time varying conditional covariance structure of spot and futures price when we use VAR-MGARCH model to calculate time varying hedge ratio.
4.2.1 OLS Model

This is a method through minimizing the sum of squared vertical distances between the observed responses in the dataset to test the unknown parameters in a linear regression model. The MV hedge ratio is a slope coefficient of the OLS regression. It is the ratio of covariance of spot prices or futures prices and variance of futures prices. The R-square shows the hedging effectiveness of the model. The equation of OLS is the following:

$$R_{st} = \alpha + HR_{ft} + \varepsilon_t$$

Where, $R_{st}$ is the return on spot and $R_{ft}$ is the return on futures. $H$ is the optimal hedge ratio. $\varepsilon_t$ is the error term in the OLS equation. OLS method is widely used in many empirical studies. The most advantage of this method is easy to implement. But it has some disadvantages. First of all, this does not consider the time varying nature and ignores conditioning information. Second, it does not take into account the futures returns as endogenous variable and ignore the covariance between error of spot and futures returns. (Myers and Thompson, 1989; Cecchetti, Cumby, and Figlewski, 1988; Christos and Dimitrios, 2001)

4.2.2 The VAR model

The VAR model is superior than the OLS model, since it gets rid of the problem caused by autocorrelation between errors and treat futures prices as endogenous variable. The VAR model is expressed as follows:

$$R_{st} = \alpha_s + \sum_{i=1}^{k} \beta_{si} R_{st-i} + \sum_{j=1}^{l} \gamma_{j} R_{ft-j} + \varepsilon_{st}$$

$$R_{ft} = \alpha_f + \sum_{i=1}^{k} \beta_{fi} R_{ft-i} + \sum_{j=1}^{l} \gamma_{j} R_{st-j} + \varepsilon_{ft}$$

The distribution of the error term $\varepsilon_{st}$ and $\varepsilon_{ft}$ is independently identically distribute (IID). The MV hedge ratio is represented as:

$$H = \frac{\sigma_{sf}}{\sigma_f}$$
Where, 

\[ \text{Var}(\varepsilon_{St}) = \sigma_s \]
\[ \text{Var}(\varepsilon_{Ft}) = \sigma_f \]
\[ \text{Cov}(\varepsilon_{St}, \varepsilon_{Ft}) = \sigma_{sf} \]

The VAR model does not take into account of the possibility of long-run integration between spot and futures returns and the conditional distribution of spot and futures prices (Christos and Dimitrios, 2001).

4.2.3 The Error Correction Model

The Error Correction Model (VECM) considers the long-term cointegration between spot and futures prices, which eliminate the disadvantage of VAR model (Lien & Luo, 1994; Lien, 1996). If the spot and futures prices are cointegrated of order one, the VECM is presented as follows:

\[ R_{St} = \alpha_s + \beta_S S_{t-1} + \gamma_F F_{t-1} + \sum_{i=2}^{k} \beta_{Si} R_{St-i} + \sum_{j=2}^{l} \gamma_{Fi} R_{Ft-j} + \varepsilon_{St} \]

\[ R_{Ft} = \alpha_F + \beta_F F_{t-1} + \gamma_S S_{t-1} + \sum_{i=2}^{k} \beta_{Fi} R_{Ft-i} + \sum_{j=2}^{l} \gamma_{Si} R_{St-j} + \varepsilon_{Ft} \]

Where, \( S_t \) and \( F_t \) are the nature logarithm of spot and futures prices. The distribution of the error term \( \varepsilon_{St} \) and \( \varepsilon_{Ft} \) is independently identically distribute (IID) as that in VAR model. The way to calculate the MV hedge ratio and hedge effectiveness is similar as that in VAR model (Brajesh Kumar et.al, 2008).

4.2.4 The VAR-MGARCH Model

In general, GARCH model is widely employed to test the time series data. The VAR-MGARCH model takes account of the ARCH effect and estimate time varying hedge ratio. A bivariate GARCH (1,1) model is the following:
\[ R_{St} = \alpha_s + \sum_{i=1}^{k} \beta_{si} R_{St-i} + \sum_{j=1}^{l} \gamma_{Sj} R_{Ft-j} + \varepsilon_{St} \]

\[ R_{Pt} = \alpha_F + \sum_{i=1}^{k} \beta_{Fi} R_{Pt-i} + \sum_{j=1}^{l} \gamma_{Sj} R_{St-j} + \varepsilon_{Pt} \]

\[
\begin{vmatrix}
    h_{ss} \\
    h_{sf} \\
    h_{ff}
\end{vmatrix} =
\begin{vmatrix}
    c_{ss} & \alpha_{11} \alpha_{12} \alpha_{13} & \alpha_{21} \alpha_{22} \alpha_{23} & \alpha_{31} \alpha_{32} \alpha_{33} \\
    c_{sf} & \varepsilon_s^2 & \varepsilon_s \varepsilon_f & \varepsilon_f^2 \varepsilon_{t-1} \\
    c_{ff} & \beta_{11} \beta_{12} \beta_{13} & \beta_{21} \beta_{22} \beta_{23} & \beta_{31} \beta_{32} \beta_{33} & h_{ss} \\
    h_{sf} & h_{sf} & h_{sf} & h_{sf} & h_{sf} \\
    h_{ff} & h_{ff} & h_{ff} & h_{ff} & h_{ff}
\end{vmatrix}
\]

Where, \( h_{ss} \) and \( h_{ff} \) are the conditional variance of the errors \( \varepsilon_{St} \) and \( \varepsilon_{Pt} \). \( h_{sf} \) is the conditional covariance of the errors \( \varepsilon_{St} \) and \( \varepsilon_{Pt} \).

Bollerslev et al. (1988) suggested a restricted version of the above model. The restriction is that only diagonal elements of \( \alpha \) and \( \beta \) matrix are considered and the correlations between conditional variances are supposed to be constant. The conditional variances elements \( (h_{ss}, h_{ff}) \) and the covariance element \( (h_{sf}) \) is showed as (Bollerslev et al., 1988)

\[
\begin{align*}
    h_{ss,t} &= c_{ss} + \alpha_{ss} \varepsilon_{s,t-1}^2 + \beta_{ss} h_{ss,t-1} \\
    h_{sf,t} &= c_{sf} + \alpha_{ss} \varepsilon_{s,t-1} \varepsilon_{f,t-1} + \beta_{sf} h_{sf,t-1} \\
    h_{ff,t} &= c_{ff} + \alpha_{ff} \varepsilon_{f,t-1}^2 + \beta_{ff} h_{ff,t-1}
\end{align*}
\]

The time varying hedge ratio is estimated as:

\[ H_t = \frac{h_{sf}}{h_{ff}} \]
5. Empirical results

5.1 Test Of Unit Root And Cointegration

First of all, we employ ADF and KPSS tests to test whether the spot and futures prices and their first difference are stationary or not. The null hypothesis of ADF test is that the series contains unit root, while the series are stationary used as the null hypothesis of KPSS test. Thus, KPSS is often applied as a confirmatory test of stationarity. The results of unit root test are presented in table 2.

<table>
<thead>
<tr>
<th></th>
<th>Price series</th>
<th>ADF(t stat)</th>
<th>KPSS(LM stat)</th>
<th>Return series</th>
<th>ADF(t stat)</th>
<th>KPSS(LM stat)</th>
</tr>
</thead>
<tbody>
<tr>
<td>China</td>
<td>Spot</td>
<td>-0.283535</td>
<td>2.851863</td>
<td>Spot</td>
<td>-24.23409</td>
<td>0.063467</td>
</tr>
<tr>
<td></td>
<td>Futures</td>
<td>-0.560027</td>
<td>2.837600</td>
<td>Futures</td>
<td>-27.14042</td>
<td>0.058549</td>
</tr>
<tr>
<td>India</td>
<td>Spot</td>
<td>-0.380866</td>
<td>2.875009</td>
<td>Spot</td>
<td>-23.07565</td>
<td>0.053283</td>
</tr>
<tr>
<td></td>
<td>Futures</td>
<td>-0.359407</td>
<td>2.881929</td>
<td>Futures</td>
<td>-23.90629</td>
<td>0.049491</td>
</tr>
</tbody>
</table>

*(**) denotes rejection of the hypothesis at the 5%(1%) level

From the above results, both ADF and KPSS tests show that the spot and futures prices are non-stationary, however the return series are stationary. They have one degree of integration I(1). Then we use Johansen test to test the cointegration between spot and futures prices. The results are showed in Table 3.
### Table 3: Johansen co-integration tests of spot and futures prices

<table>
<thead>
<tr>
<th>Hypthesized</th>
<th>Spot-Future</th>
<th>Trace Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number of CE(s)</td>
<td>Eigenvalue</td>
</tr>
<tr>
<td>China</td>
<td>None</td>
<td>0.121061 **</td>
</tr>
<tr>
<td></td>
<td>At most 1</td>
<td>0.000168</td>
</tr>
<tr>
<td>India</td>
<td>None</td>
<td>0.108187 **</td>
</tr>
<tr>
<td></td>
<td>At most 1</td>
<td>0.000249</td>
</tr>
</tbody>
</table>

*(**) denotes rejection of the hypothesis at the 5%(1%) level

The results from Table 3 indicate that both the spot and futures prices in China market and Indian market are cointegration in the long term and have one cointegration vector.

### 5.2 Hedge Ratio And Hedge Effectiveness: Empirical Results

In this session, we will present the empirical results of hedge ratios and hedge effectiveness by employing the four models (OLS, VAR, VEMC, bivariate GARCH) we mentioned above. We test the time varying hedge ratio for both China gold futures market and Indian gold futures market by using VAR-MGARCH approach. We will compare the hedge ratio and effectiveness estimated from the four models.

#### 5.2.1 OLS Estimates

OLS method is employed to test the constant hedge ratio and hedging effectiveness. The hedge ratio could be obtained from the slope of the regression equation and the hedging effectiveness is given by R-square. The empirical results are given in Table 4.
The hedge ratio $\beta$ for gold futures contracts in China market is 0.357624, which is much smaller than that in India market (0.775235). The hedge effectiveness $R^2$ in China market is 0.252083, which is much smaller than the effectiveness in Indian market (0.677001) as well. It indicates about 25% variance reduction for gold futures contract in China market and 68% variance reduction in Indian market. The results show that the hedge provided by gold futures contracts in Indian markets is effective, however in China market hedge is less effectiveness.

### 5.2.2 VAR Estimates

We use the equation from the VAR model mentioned above to test errors term. And then we employ errors to estimate hedge ratio and hedging effectiveness of both gold futures contracts. Table 5 presents the estimates of the parameters of spot and futures equations. And the optimal hedge ratio and hedge effectiveness are given in Table 6.
Table 5  Estimates of VAR model

a)  Spot prices

<table>
<thead>
<tr>
<th></th>
<th>China</th>
<th>India</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.000937 **</td>
<td>0.000914 **</td>
</tr>
<tr>
<td>$\beta_S$</td>
<td>-0.022102 **</td>
<td>-0.468146 **</td>
</tr>
<tr>
<td>$\beta_{S1}$</td>
<td>0.013000 **</td>
<td>-0.504292 **</td>
</tr>
<tr>
<td>$\beta_{S2}$</td>
<td>-0.100107 **</td>
<td>-0.303256 **</td>
</tr>
<tr>
<td>$\beta_{S3}$</td>
<td>0.035132 *</td>
<td>-0.300559 **</td>
</tr>
<tr>
<td>$\beta_{S4}$</td>
<td>0.016062 **</td>
<td>-0.078020 **</td>
</tr>
<tr>
<td>$\gamma_F$</td>
<td>-0.052894 **</td>
<td>0.468219 **</td>
</tr>
<tr>
<td>$\gamma_{F1}$</td>
<td>0.003308 **</td>
<td>0.391976 **</td>
</tr>
<tr>
<td>$\gamma_{F2}$</td>
<td>0.027261 **</td>
<td>0.289633 **</td>
</tr>
<tr>
<td>$\gamma_{F3}$</td>
<td>-0.043242 **</td>
<td>0.269194 **</td>
</tr>
<tr>
<td>$\gamma_{F4}$</td>
<td>-0.007781 *</td>
<td>0.148929 *</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.019242</td>
<td>0.079165</td>
</tr>
</tbody>
</table>

**(*) denotes significance of estimates at 5%(10%) level
### b) Futures prices

<table>
<thead>
<tr>
<th></th>
<th>China</th>
<th>India</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>0.000594</td>
<td>0.000993</td>
</tr>
<tr>
<td>( \beta_S )</td>
<td>0.366697 **</td>
<td>0.184955 *</td>
</tr>
<tr>
<td>( \beta_{S1} )</td>
<td>0.330562 **</td>
<td>0.000930 *</td>
</tr>
<tr>
<td>( \beta_{S2} )</td>
<td>0.219845 **</td>
<td>0.028556 *</td>
</tr>
<tr>
<td>( \beta_{S3} )</td>
<td>0.153105 **</td>
<td>-0.166065</td>
</tr>
<tr>
<td>( \beta_{S4} )</td>
<td>0.174262 **</td>
<td>-0.005652</td>
</tr>
<tr>
<td>( \gamma_F )</td>
<td>-0.391042 **</td>
<td>-0.195729 **</td>
</tr>
<tr>
<td>( \gamma_{F1} )</td>
<td>-0.232556 **</td>
<td>-0.103268</td>
</tr>
<tr>
<td>( \gamma_{F2} )</td>
<td>-0.187070 **</td>
<td>-0.085167</td>
</tr>
<tr>
<td>( \gamma_{F3} )</td>
<td>-0.168582 **</td>
<td>0.121042</td>
</tr>
<tr>
<td>( \gamma_{F4} )</td>
<td>-0.131893 **</td>
<td>0.088583</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.123770</td>
<td>0.035657</td>
</tr>
</tbody>
</table>

**(*) denotes significance of estimates at 5%(10%) level**
### Table 6  Estimation of hedge ratio and hedging effectiveness

<table>
<thead>
<tr>
<th></th>
<th>China</th>
<th>India</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariance($\varepsilon_F$, $\varepsilon_S$)</td>
<td>6.92E-05</td>
<td>9.05E-05</td>
</tr>
<tr>
<td>Variance ($\varepsilon_F$)</td>
<td>0.000169</td>
<td>0.000111</td>
</tr>
<tr>
<td>Hedge Ratio</td>
<td>0.409467</td>
<td>0.815315</td>
</tr>
<tr>
<td>Variance ($\varepsilon_S$)</td>
<td>9.74E-05</td>
<td>9.63E-05</td>
</tr>
<tr>
<td>Variance(H)</td>
<td>6.91E-05</td>
<td>2.25E-05</td>
</tr>
<tr>
<td>Variance(U)</td>
<td>9.74E-05</td>
<td>9.63E-05</td>
</tr>
<tr>
<td>Hedging Effectiveness</td>
<td>0.290915</td>
<td>0.76621</td>
</tr>
</tbody>
</table>

Hedge ratios both in China and Indian markets estimated from VAR model are higher than OLS method and perform better in reducing variance as well. Hedge ratio calculated by VAR model increased from 0.36 (OLS method) to 0.41 provided by the gold futures contracts in China market and increased from 0.78 (OLS method) to 0.82 in Indian market as well. Additional, hedging effectiveness also increase from 25% (OLS model) to 29% in China market and also increase from 68% (OLS model) to 77% in Indian market. Compare to Indian market, hedge in China gold futures market is still less effective provided by VAR model.

#### 5.2.3 VECM Estimates

The same approach is applied in VECM model as in VAR model. Errors are calculated first and then estimate hedge ratios and hedging effectiveness. Table 7 shows the estimates of the VECM model equation. The results of hedge ratios and hedging effectiveness of gold futures contracts are presented in Table 8.
Table 7  Estimates of VECM model  

a)  Spot prices  

<table>
<thead>
<tr>
<th></th>
<th>China</th>
<th>India</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0,006427</td>
<td>0,033264</td>
</tr>
<tr>
<td>$\beta_S$</td>
<td>-0,092549</td>
<td>0,706434</td>
</tr>
<tr>
<td>$\beta_{S1}$</td>
<td>-0,059186</td>
<td>0,376079</td>
</tr>
<tr>
<td>$\beta_{S2}$</td>
<td>-0,131613</td>
<td>0,214802</td>
</tr>
<tr>
<td>$\beta_{S3}$</td>
<td>-0,069715</td>
<td>0,018060</td>
</tr>
<tr>
<td>$\beta_{S4}$</td>
<td>-0,028061</td>
<td>0,001707</td>
</tr>
<tr>
<td>$\gamma_F$</td>
<td>0,120695</td>
<td>-0,603069</td>
</tr>
<tr>
<td>$\gamma_{F1}$</td>
<td>0,102089</td>
<td>-0,382752</td>
</tr>
<tr>
<td>$\gamma_{F2}$</td>
<td>0,104006</td>
<td>-0,235666</td>
</tr>
<tr>
<td>$\gamma_{F3}$</td>
<td>0,036537</td>
<td>-0,072635</td>
</tr>
<tr>
<td>$\gamma_{F4}$</td>
<td>0,008106</td>
<td>0,013417</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0,538875</td>
<td>0,547718</td>
</tr>
</tbody>
</table>

**(*) denotes significance of estimates at 5%(10%) level
b) Futures prices

<table>
<thead>
<tr>
<th></th>
<th>China</th>
<th>India</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.030293 **</td>
<td>-0.041920 *</td>
</tr>
<tr>
<td>$\beta_F$</td>
<td>0.801857 **</td>
<td>1.665929 **</td>
</tr>
<tr>
<td>$\beta_{F1}$</td>
<td>0.625019 **</td>
<td>1.197107 **</td>
</tr>
<tr>
<td>$\beta_{F2}$</td>
<td>0.413218 **</td>
<td>0.827761 **</td>
</tr>
<tr>
<td>$\beta_{F3}$</td>
<td>0.214592 **</td>
<td>0.348910 **</td>
</tr>
<tr>
<td>$\beta_{F4}$</td>
<td>0.103750 *</td>
<td>0.120002</td>
</tr>
<tr>
<td>$\gamma_S$</td>
<td>-0.686779 **</td>
<td>-1.558092 **</td>
</tr>
<tr>
<td>$\gamma_{S1}$</td>
<td>-0.481415 **</td>
<td>-1.191792 **</td>
</tr>
<tr>
<td>$\gamma_{S2}$</td>
<td>-0.298908 **</td>
<td>-0.875266 **</td>
</tr>
<tr>
<td>$\gamma_{S3}$</td>
<td>-0.169912 **</td>
<td>-0.438590 **</td>
</tr>
<tr>
<td>$\gamma_{S4}$</td>
<td>-0.080095 *</td>
<td>-0.118577 *</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.654946</td>
<td>0.542850</td>
</tr>
</tbody>
</table>

**(*) denotes significance of estimates at 5%(10%) level
<table>
<thead>
<tr>
<th>Table 8</th>
<th>Estimation of hedge ratio and hedging effectiveness</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>China</td>
<td>India</td>
</tr>
<tr>
<td>Covariance (ε_F, ε_S)</td>
<td>6.69E-05</td>
<td>9.15E-05</td>
</tr>
<tr>
<td>Variance (ε_F)</td>
<td>0.000157</td>
<td>0.000111</td>
</tr>
<tr>
<td>Hedge Ratio</td>
<td>0.426115</td>
<td>0.824324</td>
</tr>
<tr>
<td>Variance (ε_S)</td>
<td>9.82E-05</td>
<td>9.67E-05</td>
</tr>
<tr>
<td>Variance (H)</td>
<td>6.97E-05</td>
<td>2.13E-05</td>
</tr>
<tr>
<td>Variance (U)</td>
<td>9.82E-05</td>
<td>9.67E-05</td>
</tr>
<tr>
<td>Hedging Effectiveness</td>
<td>0.290296</td>
<td>0.779997</td>
</tr>
</tbody>
</table>

VECM model is supposed to be the best specified model to estimate constant hedge ratios and hedging effectiveness since it considers the long-run cointegration between spot and futures prices, although this model does not take into account the conditional covariance structure of spot and futures prices. VECM performs better than VAR and OLS models in variance reduction. Compare OLS, VAR with VECM, OLS seems least efficient, although it is the simplest approach to do the test. Hedge ratio calculated by VECM model increased from 0.41 (VAR method) to 0.43 provided by the gold futures contracts in China market and increased from 0.815 (VAR method) to 0.824 in Indian market as well. Additional, hedging effectiveness also increase from 77% (VAR model) to 78% in Indian market and it is similar in China gold futures market both used by VECM and VAR models. Hedge in China gold futures market is less effective provided by VECM model and it is effective in Indian market.

5.2.4 VAR-MGARCH estimates

VAR-MGARCH model is applied to test the time varying hedge ratios and hedge effectiveness. It could consider the time varying volatility and non-linearity in the mean equation. Errors calculated from VAR and VECM model are analyzed as “ARCH effect”
and was found that there exists time varying volatility in the errors term. The estimates of VAR-MGARCH model are presented in Table 9. Table 10 give the statistical properties of time varying hedge ratio for gold futures in China market and Indian market by employing error structure and GARCH (1,1) parameters from VAR-GARCH model equation.

Table 9 GARCH estimates of the VAR-MGARCH (1,1) model

<table>
<thead>
<tr>
<th></th>
<th>China</th>
<th>India</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{ss}$</td>
<td>0.28909***</td>
<td>0.282429***</td>
</tr>
<tr>
<td>$C_{sf}$</td>
<td>0.180873***</td>
<td>0.245351***</td>
</tr>
<tr>
<td>$C_{ff}$</td>
<td>0.200117***</td>
<td>0.224973***</td>
</tr>
<tr>
<td>$\alpha_{11}$</td>
<td>0.136214***</td>
<td>0.290182***</td>
</tr>
<tr>
<td>$\alpha_{22}$</td>
<td>0.110813***</td>
<td>0.231974***</td>
</tr>
<tr>
<td>$\alpha_{33}$</td>
<td>0.138428***</td>
<td>0.19252***</td>
</tr>
<tr>
<td>$\beta_{11}$</td>
<td>-0.0046***</td>
<td>0.004036***</td>
</tr>
<tr>
<td>$\beta_{22}$</td>
<td>0.006153***</td>
<td>0.021676***</td>
</tr>
<tr>
<td>$\beta_{33}$</td>
<td>-0.02612***</td>
<td>0.045079***</td>
</tr>
</tbody>
</table>

**(***) denotes significance of estimates at 5%(10%) level

Table 10 Statistical properties of dynamic hedge ratio from VAR-MGARCH model

<table>
<thead>
<tr>
<th>Hedge ratio</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>China</td>
<td>-0.23741</td>
<td>0.791311</td>
<td>0.432088</td>
<td>0.059468</td>
</tr>
<tr>
<td>India</td>
<td>-0.19459</td>
<td>1.349690</td>
<td>0.879375</td>
<td>0.092923</td>
</tr>
</tbody>
</table>

The mean hedge ratios calculated from VAR-MGARCH model both are higher than other methods. The average optimal hedge ratios for gold futures contracts in China market and Indian market are 0.43 and 0.88 respectively, which are the highest hedge ratios among others estimated by OLS, VAR, VECM models.

The minimum hedge ratios are negative in both markets, which indicate that spot and
futures prices may move in opposite direction (negative covariance) in short term (Tong, 1996). In general, the investor will go long in futures markets to hedge the long spot position. The investor has to modify their futures position more often due to the time varying hedge ratios are less stable and more fluctuations.

Table 11 and 12 shows all of the above hedge ratios and hedge effectiveness calculated by OLS, VAR, VECM and VAR-MGARCH models. The results indicate that hedge ratios estimated from VAR-MGARCH (1, 1) model are the highest and present the greatest variance reduction than other three models. Similar results were proposed in previous studies by Myers (1991) in the US financial and commodity markets and Pandey (2008) in Indian stock and commodity futures markets. However, hedging strategy suggested by VAR-MGARCH model may requires shift in hedging positions frequently and would result in associated transaction costs (Ajay, et.al 2008).

Considering the results from the four models, we can say that using the gold futures contracts as hedging tool in Indian market is effective and in China market it is less effective compare to India market.

Table 11 comparison of optimal hedge ratio estimates by different models

<table>
<thead>
<tr>
<th></th>
<th>OLS</th>
<th>VAR</th>
<th>VECM</th>
<th>VAR-MGARCH</th>
</tr>
</thead>
<tbody>
<tr>
<td>China</td>
<td>0.357624</td>
<td>0.409467</td>
<td>0.426115</td>
<td>0.432088</td>
</tr>
<tr>
<td>India</td>
<td>0.775235</td>
<td>0.815315</td>
<td>0.824324</td>
<td>0.879375</td>
</tr>
</tbody>
</table>

Table 12 comparison of hedging effectiveness for different models

<table>
<thead>
<tr>
<th></th>
<th>OLS</th>
<th>VAR</th>
<th>VECM</th>
<th>VAR-MGARCH</th>
</tr>
</thead>
<tbody>
<tr>
<td>China</td>
<td>0.252083</td>
<td>0.290915</td>
<td>0.290296</td>
<td>0.349731</td>
</tr>
<tr>
<td>India</td>
<td>0.677001</td>
<td>0.76621</td>
<td>0.779997</td>
<td>0.838193</td>
</tr>
</tbody>
</table>
6. Conclusion

Gold futures are universally purchased by investors all over the world, since gold is widely recognized as both a commodity and a financial asset. In the emerging markets like China and India, where gold futures markets have been introduced in recent years and are growing really fast. Indian is the largest gold consuming country in the world and China is the largest gold producing country and second largest gold consuming country. Since Shanghai Futures Exchange (SHFE) and India’s National Commodity & Derivatives Exchange (NCDEX) become more and more important in the gold futures exchanges, it is essential to evaluate the hedge ratio and hedging effectiveness of gold futures contracts in both markets.

In this essay, we present hedge ratios of gold futures from the alternative models (OLS model, VAR model, VECM model and VAR-MGARCH model). Our results indicate that spot and futures prices are cointegrated in long-run. And VECM performs best compared with other constant hedge ratio models, like OLS and VAR models. It provides higher hedge ratio and performs better in variance reduction among the constant hedge ratios. However, VAR-MGARCH gives the highest hedge ratio and performs the best in variance reduction among all of the four hedge ratio models. The result is consistent with the previous study of Myer (1991) and Pandey (2008). However, shifts in the hedging positions might be frequent in the strategy suggested by VAR-MGARCH model and would result in much higher transaction costs.

Considering all mentioned above, we conclude that VAR-MGARCH model provides the best estimate of the optimal hedge ratio both for Chinese and Indian gold futures markets, which is 0.43 and 0.88 respectively. Additionally, hedging on the Chinese gold market allows to reduce the variance of the portfolio by only 34%, compared to 84% on the Indian market.

From our empirical results, we find less evidence indicating that the Chinese market is effective. Thus, we can conclude that Indian gold futures market is effective and Chinese gold futures market is less effective. The new Indian gold futures could be an attractive hedging vehicle. The reasons why Chinese gold futures market is less effective might be the following. First, it founded only three years ago and therefore relatively new. Second, the
market was extremely speculative from the very beginning. Third, the trade prices in the Chinese gold futures market rely mostly on the prices in the spot markets, and latter one is much more mature.
Reference


Wei Chen.(2009). Hedging in China’s Oil futures market. The University of Birmingham.