RISK MANAGEMENT IN CORPORATE LOAN PORTFOLIO

— Default Correlation

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Abstract

Title: Risk management in corporate loan portfolio

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Purpose: The purpose of this paper is to estimate different industries’ default rate correlation over time. The intention is to observe if banks have any diversification opportunities in their corporate loan portfolio and to what extent this may disappear in stressful economic situations.

Methodology: In this paper, the dynamic conditional correlation model is applied to analyze the default correlation structure over time in four Swedish corporate industries. The model applied has the characteristics of being equally flexible as univariate GARCH but not as complex as multivariate GARCH.

Results: The findings of this paper suggest that the default correlation over time is fairly low which gives a bank some diversification opportunity in their corporate loan portfolio. However, in times of economic distress, hence, when default rates are high, the correlations increase significantly. This is discussed to be very unfavorable from a risk management perspective since the correlations break down when needed the most.

Keywords: Dynamic conditional correlation, correlation, risk management, diversification, industry default rate.
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List of Abbreviations

Augmented Dickey-Fuller ................................................................. ADF
Baba-Engle-Kraft-Kroner ......................................................... BEKK
Distance-to-Default ................................................................ DD
Dynamic Conditional Correlation .................................................. DCC
Expected Default Ratio ............................................................... EDF
Expected Loss ............................................................................. EL
Exposure at Default ................................................................. EAD
Generalized Autoregressive Conditional Heteroscedasticity .......... GARCH
Internal Rating Based approach .................................................. IRB
Likelihood Function .................................................................... LF
Log-Likelihood Function ............................................................. LLF
Loss Given Default ..................................................................... LGD
Probability of Default ................................................................ PD
Value-at-Risk .............................................................................. VaR
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1. Introduction

The introductory chapter begins with the background and a discussion of the problem. Next, the formulation of the paper’s question and the purpose are presented for the reader. The chapter ends by presenting a short plan.

Credit lending is the main profit generating activity for the Swedish banks. Credit lending activity involves risks to both the bank who is the lender and its customers. On the one hand there is an obvious credit risk that the banks counterpart does not fulfills its obligations, hence, does not pay back its loan and defaults. On the other hand, a bank with high credit risk increases its own bankruptcy risk, thus, puts the depositor’s money in jeopardy. Since banks are one of the most important cornerstones in today’s society it’s particularly important that they don’t go bankrupt. Credit risk can be argued to be the risk that is most likely to prompt a bank’s failure. According to the financial stability report (2010:2) from the Swedish Central Bank, an average of 60 percent of the asset side for the largest banks in Sweden, consists of credit lending to the public. The high exposures emphasize and quantify the magnitude of potential losses if default rates increases.

Therefore credit risk is of great concern. A bank’s credit risk exposure in corporate lending varies across different individual firms, industries and main sectors. Depending on the economic environment the credit risk and exposure is changing. Thus, understanding and estimating the correlation in the credit portfolio is important. To find good estimates of the correlation is challenging, but it enables the construction of well diversified credit portfolios in which idiosyncratic risk is diminished, where the goal is to only have a systematic risk exposure (Hamerle et al., 2004). Further, Zhou (2001) stress that estimating defaults correlation is the most difficult part of credit portfolio analysis.

There is a wide area of correlation models available. This paper will apply the Dynamic Conditional Correlation model, henceforth DCC. It has its main advantage over other correlation models of being parsimonious, since the number of parameters to be estimated is independent of the number of series (Engel, 2002). Moreover, according to the founder of the model Engel (2000) his model has the characteristics of being equally flexible as univariate GARCH but not as complex as multivariate GARCH.

The DCC model belongs to multivariate GARCH family which is superior to univariate models (Brooks, 2008). However, the model specification in multivariate GARCH models
easily becomes very complex and requires estimation of many parameters. For instance, the original VECH model suffers from not being parsimonious due to its many parameters (Bollerslev et al., 1988). Diagonal VECH on the other hand reduce the number of parameters but instead introduce the difficulty to ensure positive definiteness of the covariance matrices produced (Engle & Kroner, 1995). Finally, the BEKK model which is a restricted VECH ensures positive definite conditional covariance matrices (Ibid). However, it becomes hard to reach convergence in this restricted model since the number of parameters is large (Silvennoinen & Teräsvirta, 2007).

Moreover, Dowd (2005) stress that from financial events in the past, correlations have been seen to appear fairly stable in normal market conditions and then change abruptly and take on extreme values in stressful situations, both high and low. This is a big problem in credit risk portfolio management since it implies that correlations could break down when they are needed the most. Results from Hatchett & Kühn (2006) from investigating credit contagion effects on default probabilities show that corporate interlinkages are significantly magnified in situations of economic stress.

Traditionally risk management has focused on risk stemming from legal or physical causes, such as lawsuits or natural disasters. Credit risk management belongs to the spectrum of financial risk management since it focuses on risks that can be managed using traded financial instruments. Moreover, credit risk management is a structured approach to managing uncertainties through risk assessment. Once the risk is identified, one can start develop different strategies through the use of different resources. Different strategies include risk transferring to a third party, avoiding the risk, reducing the negative effects of the risk, and accepting some or all of the of a particular risk. (Culp, 2001) That banks, companies, and institutional investors actually engage in the credit risk management is visible in the market. The market has witnessed an explosion of the credit default swap market, an increase in different securitization deals and an overall upswing in the interest of credit derivatives. (Culp, 2006)

1.1 Formulation of question

Is there any diversification possibilities in a corporate loan portfolio, and to what extent does the diversification effect change in stressful economic situations?
1.2 Purpose
The main purpose of this paper is to estimate different industries’ default rate correlation over time. The intention is to observe what diversification opportunity a bank has and to what extent this may disappear in stressful situations. This paper applies the DCC model as correlation estimator. A low correlation coefficient between two different industries is considered as a sign of potential portfolio diversification (Markowitz, 1952). The potential diversification effect, i.e. risk reduction possibility, is of most concern in stressful economic situations.

1.3 Plan
The outline of the thesis is as follows: Chapter 2 presents a summary of some previous scientific studies as well as a description of the data and the approach used in this paper. Chapter 3 explains the theory behind credit risk and different correlation models. In chapter 4 the empirical results and analysis of the time-varying correlations are presented and discussed. Finally, concluding remarks of the research are in chapter 5.
2. Method

This chapter first presents a summary of some previous scientific studies in the field. This is followed by a description of the selection of data and time periods followed by the paper’s approach. The chapter ends with some critic on method or relevance.

2.1 Previous scientific studies

Hatchett & Kühn (2006) incorporate the effect of credit contagion in a credit risk model. They find that interlinkages are relatively weak in normal economic situations, whereas they are important in times of economic distress. Their results show that this leads to significantly increase in credit portfolio tail losses. Hence, they come to the conclusion that this has implications on relying in historical loss distribution. Care must be taken not only to average behaviour but also incorporate extreme events.

Carling et al. (2007) tries to formalize that many firms are affected by each other through judicial, financial and business relations. Thus, they test in a survey the impact of incorporate default correlations on a corporate loan portfolio. By incorporating for default risk dependency, the authors improve the drawback of currently available credit risk models such as the KMV, CreditMetric and CreditRisk+. The test is done by compare the generated VaR if incorporate default correlation with the original models portfolio VaR. The data used is real credit data collected from two Swedish banks’ corporate loan portfolios over the period from 1994 to 2000. The authors emphasize the importance of incorporate default risk dependency across loans in a credit portfolio. They come to the conclusion that the risk of a corporate loan portfolio will increase its VaR by 50-200 percent.

Rosenow et al. (2004) divide the German economy into 20 sectors and analyze them by creating a covariance matrix and sample correlation matrix over the defaults. By using eigenvalues they are able to model the different sectors default correlation in a one-factor model. However, when generating short time series and calculating their correlation matrix, large fluctuations are typically observed in the correlation structure. Hence, the one-factor model has large uncertainty in parameter estimation.

Assuming independent sectors is one of the shortcomings of the CreditRisk+. Bürgisser et al. (1999) present a framework where CreditRisk+ is extended to model correlations between
industries. They find this extension important to be able to examine the effects of diversification on active portfolio management.

Zhou (2001) discusses how the problem of portfolio analysis with credit risk has been examined in recent literature. He describes that a portfolio manager is not only concerned about the default of any single party but also the probability of multiple defaults in the portfolio. Further, it is described to exist three effective measurements of credit risk: the probability of default for each individual position over investment horizons, the joint probability of default between every pair of counterparty investment horizons, and the magnitude of financial loss in the event of each possible default. Finally, estimating the default correlation is described to be the most crucial and most difficult part of credit aggregation analysis.

2.2 Handling of data

2.2.1 Selection of data and time periods
To retrieve more accurate correlation estimates the actual data of default rates is used. Another approach would be to have EDF data, which have the properties of being forward looking, however, biased estimates of EDF will result in miscalculated correlations. Further, the default rates are on a monthly basis over the period January 1994 to December 2008 and include the four different industries below. These four specific industries were chosen since they showed to have the highest default ratios. Hence, these industries can be seen to be the most risky ones in a corporate loan portfolio. Therefore they are assumed to be of great importance for a bank’s risk exposure. A more detailed description about what companies that is included in respectively industry specification is found in Appendix A.

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**FOOD** – Food, beverages, and tobacco production

**RETAIL** – Retail trade (excluding vehicles)

**HOTEL** – Hotels and restaurants

**WHOLESALE** – Wholesale trade (excluding vehicles)
To receive the default ratio for each industry also the number of companies is needed. Data of the number of firms is only available on a yearly basis. However through some interpolation, it is solved with the realistic assumption that the number of companies increases linear over the year and is therefore spread out equally on each month. To do the opposite and instead use yearly data is no option considering the complex model that will be estimated. All possible data observation available during the time period is needed to get robust and reliable parameter estimates.

All data series are collected from the reliable source Statistics Sweden. Moreover, after been in contact with the risk management division at one of the major banks in Sweden it proved to be exactly the same data they use themselves.

![Diagram](image)

*Figure 1: Expected interlinkages between the industries.*

Food consists of production companies and is expected to have an interlinkage with the wholesale trade companies to a great extent and to the retail trade to some extent. Hotel and restaurants are not believed to buy their food, beverages and tobacco from the end producers. Instead, hotel and restaurants are believed to be big customers of the wholesale industry. Finally, wholesale companies with a focus on bulk trade are only expected to have an interlinkage with retail trade since they are their main suppliers. Figure 1 above summarizes the expectations and beliefs in an illustrative approach. Bold lines characterize high interlinkage and the dotted line moderate. These expectations will be empirical investigated in the analysis part.

### 2.3 Approach

To receive each industries’ default ratio the actual number of corporate defaults are divided by the number of companies on a monthly basis. To avoid facing the risk of retrieving spurious results and hence get sensible correlations, test for unit roots as well as distribution in all the default rates is made.

This paper will apply the DCC model as correlation estimator with the original conditional variance specification as a GARCH(1,1) process. Where GARCH(1,1) process is found to
perform well on financial data. Figlewski (1997) argue that the specification is sufficiently stable and hold over time. Tarashev (2010) stress the implication of ignoring parameter uncertainty in measuring portfolio credit risk. He shows that it leads to understatement of the portfolio tail risk. Thus, it is of importance to test the validity of the model and therefore a robustness check of the parameter values estimated by the DCC model is performed. This is done by changing the initial parameter values.

Finally a test is performed to see how the correlations are acting in stressful economic situations. The test rank the 30 highest default ratios for every sector and compare if any of this time periods corresponds with any of the 30 time periods with the highest and lowest correlation respectively. The test is performed using “if” matrices to find if any relation is present.

2.4 Critic on method or relevance
Even though the interpolation assumption considering the number of companies is realistic, the unavailability of true monthly data is of some critic. Especially during economic downturn periods there can be expected to be some downward bias in the default rates.
3. Theory

In this section the theories behind credit risk and credit risk models are described. Then, the focus is shifted to the theory behind correlation and correlation models. Finally the theory behind stationarity and the paper’s estimation approach Maximum Likelihood is described.

3.1 Credit Risk
The general definition of credit risk or default risk from a bank’s perspective is the risk of losses resulting from failure of borrowers not fulfilling their obligations. An accepted measurement of credit risk is the probability of default. The PD calculates the likelihood that the borrowers will default. The higher the PD a bank estimates borrowers to have, the higher will the risk be that the bank takes on its books. Another measurement used to get an overview of the credit exposure is the Loss Given Default. LGD gives an estimate of the losses when they actually appear. Finally, when the product of these two measurements is combined with the aggregated exposure at default (EAD) the Expected Loss of the credit portfolio can be retrieved, see formula below. (Allen & Saunders, 2002)

\[ EL = PD \times LGD \times EAD \]

The discussion from the introduction part indeed explains why prudent credit portfolio management is an important area in banks’ operations. For example to keep control of their credit risk and hence, meet the Basel III regulatory capital (Basel Committee on Banking Supervision), while at the same time hold optimal credit portfolios. The purpose of the new improved Basel Capital Accord, which was a result of the recent financial crisis, is to strengthen the resilience of banks and the global banking system in the future. Furthermore, the Basel III allow banks to choose whether to apply their Internal rating based approach (IRB) or the standard method set by Basel III when calculating borrowers risk grade (based on PD) as an input to the Risk Weighted Assets and hence, the economic capital. However, in Sweden, to be able to use the IRB approach it must be sophisticated enough to be confirmed by the Swedish Financial Supervisory Authority. To make sure the banks meet the regulatory tier 1, 2 and 3 pillars. Thus, banks’ use of appropriate credit risk models is a necessity. (Allen & Saunders, 2002)
3.2 Credit Risk Models
There are several models on the market created to measure the variation of credit default risk in loan portfolios. Hamerle et al. (2004) stress that the main challenge for these models is the forecasts of the input parameters PD and the default correlations. For comparison, a short description and pros and cons of some of the most widely accepted credit risk models will be presented below.

3.2.1 The Merton Model
Merton (1974) presents a credit risk model based on the Black-Scholes options pricing formula to calculate the PD. This structural model is widely known and respected within credit risk management. For instance, a recalibrated version of the model is employed by Moodys-KMV™. (Vasic, 2002) The model has a market-value balance-sheet approach, in which the dynamic of the firm value over time follows a drift term and a stochastic process.

\[ \frac{dA_t}{A_t} = \mu_A dt + \sigma_A dZ \]

Where \( \mu_A \) is the drift term, equally to the growth rate of the asset value. \( \sigma_A \) is the stochastic part, thus, the volatility of asset growth, which value is based on the observable equity volatility. The process follows a geometric Brownian motion in continuous time and the diffusion term \( dZ \) follows a normally distributed Wiener-process, with zero mean and a unit variance. It implies that the evolution of the asset value at time \( t \), \( A_t \), can be derived from \( A_0 \) as follows.

\[ A_t = A_0 \exp \left( \left( \mu_A - \frac{\sigma_A^2}{2} \right) t + \sigma_A \varepsilon \sqrt{t} \right) \]

A firm is therefore defined to default if the firm’s asset value falls below the calculated default threshold, \( D \), which is equal to a firm’s debt level. Thus the PD can be calculated as the probability of \( A_t \leq D_t \) as follows

\[ \text{Prob} \ A_t \leq D_t = \text{Prob} \left( A_0 \exp \left( \left( \mu_A - \frac{\sigma_A^2}{2} \right) t + \sigma_A \varepsilon \sqrt{t} \right) \leq D_t \right) \]

Hence, by rearranging the Distance-to-Default can be found, which is measuring the number of standard deviations from default of expected \( A \) for a given time horizon.
An example of the evolution of the asset value and PD is presented in the figure 2 below.

![Figure 2: Distance-to-Default in the Merton model.](image)

Pros with the Merton model is that by using equity returns it gives a forward looking input to the prediction of corporate default. However, evidence has shown that the standard Merton model is incomplete in discriminate between defaults and non-default firms and hence, needs to be enhanced with additional factors. For instance a more realistic approach would be to use the actual interest rates for company’s debt instead of the risk-free. (Stein, 2005)

Another improvement of the model can be done by exchanging the equity returns with expected default frequency data. Sun (2010) comes to the conclusion that this approach has superior default predictive power on a one year horizon.

The models drawback is that it is based on some strong assumptions, such that the asset value follows a normal distribution. Further, since the model is based on market values it complicates the calculations of equity for non-listed firms. Thus, unobservable variables must be derived with balance-sheet equivalence between assets and liabilities. Implying that on the aggregated level uncertainty about the real asset value increases as well as the asset growth volatility is unknown. (Gray & Malone, 2008) Finally, Carling et al. (2007) stress that portfolio credit risk is underestimated since the model does not incorporate correlations.
3.2.2 CreditMetrics™

Another popular approach in credit risk management is the CreditMetrics™ developed by J.P. Morgan. Unlike the Merton model which calculates PD based on market values, CreditMetrics™ instead use a VaR based approach. CreditMetrics™ assess portfolio risk due to changes in debt value caused by possible credit events. Changes in a firm’s credit quality due to upgrades and downgrades in the credit rating, ranging from AAA to CCC on the S&P measurement scale, is included in the measurement of VaR for a given time period. (Gupton et al., 1997)

One important advantage is that the actual distribution of loan values can be used instead of assuming normal distribution (Gupton et al., 1997). An assumption of normal distribution implies that VaR is underestimated due to skewness in the distribution of loan values. As a result of that a borrower either defaults or does not default, the most accurate distribution of PD is often rather a binomial distribution. (Allen & Saunders, 2002)

![Loan distribution diagram](Gupton et al., 1997)

**Figure 3: Loan distribution.**

A general drawback of VaR models is the problem of being a non-additivity risk measure. Also, the research up to date has not dealt with parameter instability or feedback effects for longer time horizons. Finally, it relies on external credit ratings and is historic backward looking. (Sorge & Virolainen, 2004)

3.2.3 CreditRisk+

CreditRisk+ is a macroeconomic model of default risk developed by Credit Suisse First Boston. The model incorporates information about the credit exposures size, maturity, the borrower’s credit quality as well as the macroeconomic condition. (Credit Suisse, 1997) Further, the default rate correlations incorporated in the model are assumed to be driven entirely by macroeconomic factors. (Allen & Saunders 2002) The underlying distributional
assumption in the model and its properties allow the distribution of total portfolio losses to be calculated in a convenient analytic form. Thus, shown in figure 4 below, the model can capture the skewness in default rates by incorporate default rate volatility through its gamma distribution. Where the presence of skewness indicates an increased risk although the number of defaults are the same. (Credit Suisse, 1997)

![Figure 4: Skewness in default rates.](Own production)

Although the CreditRisk+ model is widely accepted on the market, Crouhy et al. (2000) point out some drawbacks. One drawback of the model is that the methodology assumes no market risk. Another drawback is the ignorance of the model is the methodology assumes no market risk. Another drawback is the ignorance of the migration risk, since the model does not incorporate possible changes of the obligors’ credit quality thus the exposure for each obligor is fixed. Furthermore, the model assumes constant interest rate over time and hence, does not incorporate the obligors’ exposure to the variability of future interest rates.

### 3.3 Correlation

Correlation is a measure of linear association between two variables. If two variables are correlated it means that they are treated in a symmetrical way. In other words, there is evidence of a linear relationship between the two variables and that the movements on average are related given by the correlation coefficient. Thus, correlation does not imply that a change in one variable cause changes in the other. (Brooks, 2008) Regarding the correlation coefficient it spans from +1 to -1 and assets are said to be perfectly positively/negatively correlated respectively if taking on these extreme values. (Dowd, 2005)

$$corr(x, y) = \frac{cov(x, y)}{\sigma_x \sigma_y} = \frac{cov(x, y)}{\sigma_x \sigma_y}$$

Where: $cov(x, y) = E[xy] - E[x]E[y]$ or $cov(x, y) = corr(x, y) \times \sigma_x \sigma_y$
A necessary condition for correlation to even exist is that x and y has finite variances and that they are jointly stationary. Further, correlation is a good measure of dependence when the random variables are multivariate elliptical distributed. Even though under elliptical distribution which includes the normal and t-distribution correlation as a measure has some limitations and drawbacks. Firstly, if risks are independent they will have zero correlation. The reverse does not necessarily hold, except in the special case of multivariate normal distribution. Hence, unless multivariate normality zero correlation does not imply that risks are independent. Secondly, correlation is not invariant to transformations of the underlying variables. Meaning, the correlation between x and y are not in general the same as the correlation between ln(x) and ln(y). (Dowd, 2005)

In contrast to correlation there is regression. A regression instead handles the two (or more) variables as dependent y variable and independent x variables. The dependent y variable is assumed to be random or stochastic i.e. to have a probability distribution. The x variables on the other hand are assumed to have fixed values in repeated samples. At last, regression as a tool is considered to be more powerful and flexible compared to correlation. (Brooks, 2008)

3.3.1 Forecasting correlation
Historical estimation of correlation is the most basic estimation method for predicting correlation. The two main implications are that this method gives equal weight on all the included observations and also the choice of the length of \( n \). By giving all included observations equal weight the practitioner incorporate old information equal to new information. Furthermore, practitioners want the \( n \) to be large enough to produce reasonable and robust correlation estimates on one hand. On the other hand they are interested in choosing \( n \) small enough to respond on news in the market. If applying a rolling window, correlations might appear more stable as the sample period to calculate them is increased. But, a long sample period gives little indication of the true correlation which could be very volatile. (Dowd, 2005)

\[
corr(x, y) = \frac{\sum_{i=1}^{n} x_{t-i} y_{t-i}}{\sqrt{\sum_{i=1}^{n} x_{t-i}^2 \sum_{i=1}^{n} y_{t-i}^2}}
\]

3.3.2 Forecasting correlation matrices
More difficult, but also of more importance, to the practitioners than calculate the individual correlation is to calculate and forecast correlation matrices. Difficult since one wants the
estimated correlation matrix to produce portfolio variances that is always positive, strictly positive. In the less restrictive case matrices are allowing for zero variance, non-negative. So regardless of the relative size of the individual positions the former case requires the correlation matrix $\Sigma$ to be positive definite, meaning:

$$w\Sigma w^T > 0 \text{ for any } 1\times m \text{ position size vector } w \neq 0$$

In the latter less restrictive case, the correlation matrix $\Sigma$ must follow the condition:

$$w\Sigma w^T \geq 0 \text{ for any } 1\times m \text{ position size vector } w \neq 0$$

(Dowd, 2005)

Firstly, the properties of positive definiteness are important from a mathematical standpoint since the covariance between two series is now the same irrespective of which series out of two that is taken first. Secondly, it ensures the intuitively result that variances can never be negative. (Brooks, 2008)

Systematic approaches are therefore needed to estimate all the parameters included ensuring that that the produced matrix is positive definite or positive semi-definite. The advantage with a positive definite matrix compared to the less restrictive semi-define is that the former is more convenient to handle. (Dowd, 2005)

### 3.4 Correlation models

#### 3.4.1 The VECH model

A general Multivariate GARCH model is the VECH model introduced Bollerslev et al. (1988). The model parameterizes the vector of all covariances and variances as follows

$$VECH(H_t) = C + \sum_{i=1}^{q} A_i VECH(\varepsilon_{t-i}, \varepsilon'_{t-i}) + \sum_{j=1}^{p} B_j VECH(H_{t-j})$$

$$\varepsilon_{t-i} \sim \mathcal{N}(0, H_t)$$

Where, $H_t$ is a $n \times n$ conditional variance-covariance matrix and $\varepsilon_t$ is an $n \times 1$ innovation (disturbance) vector. $C$ is a $1/2n(n+1) \times 1$ vector. Finally, $A$ and $B$ are both $1/2n(n+1) \times 1/2 \ n(n+1)$ parameter matrices. $VECH(\cdot)$ denotes the column-stacking operator applied to the upper portion of the symmetric matrix. Where the “upper triangular”
portion of a matrix is stacked with each element and thus summarized in a single column vector. So that it represents the conditional variances and covariances at time t. In which they depend on their respectively lagged values as well as lagged squared errors and error cross-products. (Bollerslev et al., 1988)

\[
VECH(H_t) = \begin{bmatrix} h_{iit} \\ h_{ijt} \end{bmatrix}
\]

It can clearly be seen that a construction of the VECH model requires many parameter estimations, thus, it is a difficult model to work with for larger sets of variables. For instance, in the case of estimating the model with two variables it requires an estimate of 21 parameters. In summary the multivariate models are superior over univariate GARCH when it comes to covariance estimates. However, the VECH model suffers from having to estimate many parameters, and hence, do not possess the property of being a parsimonious model.

3.4.2 Diagonal Vech Model
As a result, Bollerslev et al. (1988) also introduced the diagonal Vech model. Where they modified the original VECH model by impose restrictions on the conditional variance-covariance matrix. The restriction implies an assumption of that the A and B matrices in the VECH specification are diagonal. Thus it will reduce the number of necessary parameters which makes it more feasible to estimate when the number of included variables in the system increases. As a comparison of the example above, with two included variables, the number of parameters to be estimated is now only 9 compared to 21 in the original VECH model. However, Engle & Kroner (1995) and Silvennoinen & Teräsvirta (2007) stress the main drawback for these models, which is the difficulty to ensure positive definiteness of the covariance matrices.

3.4.3 The BEKK Model
Engle & Kroner (1995) propose the BEKK model and defines it as a restricted version of the VECH model. The purpose of the model is to overcome the drawback of the VECH model. That is, to make sure that the property of positive definite conditional covariance matrices is always satisfied. This is done in the decomposition of the constant term into a product of two triangular matrices. Thus, the model has the following form

\[
H_t = CC' + A_{11}' \varepsilon_{t-1} \varepsilon_{t-1}' A_{11} + G_{11}' H_{t-1} G_{11}
\]

(Engle & Kroner, 1995)
Where $A$ and $G$ are $n \times n$ matrices of parameters and $C$ is an upper triangular matrix of parameters.

Although, the model’s property of satisfying the necessity of being positive definite, the BEKK model is still a complicated model to estimate due to several matrix inversions. The imposed restrictions to reach parsimoniousness in the BEKK model need a quite large number of parameters to be estimated. Thus, obtaining convergence may therefore be difficult because of the non-linearity. (Silvennoinen & Teräsvirta, 2007)

3.4.4 The DCC model
The DCC model has a main advantage over other correlation models since the number of parameters to be estimated is independent of the number of series. Thus, very large correlation matrices can be estimated. (Engel, 2002) Further, the model is described as parsimonious since it is a two-step model where the first step consists of estimating univariate GARCH models for each asset, and then, using these transformed residuals from the first step to estimate a conditional correlation estimator as a second step. (Engel, 2001) Finally, according to Engel (2000) his model has the characteristics of being equally flexible as univariate GARCH but not as complex as multivariate GARCH.

Returns as the conditional standard deviation times the standardized disturbance:

$$H_{t,i} = E_{t-1}(\sigma_{i,t}^2), \quad \sigma_{i,t} = \sqrt{h_{i,t} \varepsilon_{i,t}}, \quad i = 1,2$$

The expression above clarifies the relation between conditional correlations and conditional variances. Further, epsilon is a standardized disturbance term with mean of zero and variance one.

Instead of the normal correlation formula

$$\rho_{12,t} = \frac{E_{t-1}(\varepsilon_{1,t} \varepsilon_{2,t})}{\sqrt{E_{t-1}(\varepsilon_{1,t}^2)E_{t-1}(\varepsilon_{2,t}^2)}}$$

it is therefore possible to write

$$\rho_{12,t} = \frac{E_{t-1}(\varepsilon_{1,t} \varepsilon_{2,t})}{\sqrt{E_{t-1}(\varepsilon_{1,t}^2)E_{t-1}(\varepsilon_{2,t}^2)}} = E_{t-1}(\varepsilon_{1,t} \varepsilon_{2,t})$$
Hence, the formula above states that conditional correlation is equal to the conditional covariance between the standardized disturbances.

Further, the model assume that returns are conditionally multivariate normal with zero expected return and a covariance matrix $H_t$.

$$r_t | \mathcal{F}_{t-1} \sim N(0, H_t)$$

and

$$H_t \equiv D_t R_t D_t$$

Where $D_t$ is the $k \times k$ diagonal matrix of time varying standard deviations from univariate GARCH models with $\sqrt{h_{i,t}}$ on the $i^{th}$ diagonal, and $R_t$ is the time varying correlation matrix.

A further description of the matrix calculation of $H_t$ is found in Appendix H. The model can be seen as a development of Bollerslev’s (1990) constant correlation estimator where a simple estimate of $R$ is the unconditional correlation matrix of the standardized residuals.

$$H_t = D_t R D_t, \quad \text{where} \quad D_t = \text{diag}\{\sqrt{h_{i,t}}\}$$

The DCC model differs in the way that it allows $R$ to be time varying, i.e. conditional. This makes the model more applicable since the assumption of constant conditional correlation is arguably too restrictive over long time periods (Torben et al., 2005).

In this paper the DCC model follows a GARCH(1,1) specification

$$q_{i,j,t} = \tilde{p}_{i,j} + \alpha (\varepsilon_{i,t-1}\varepsilon_{j,t-1} - \tilde{p}_{i,j}) + \beta (q_{i,j,t-1} - \tilde{p}_{i,j})$$

and multivariate form

$$Q_t = (1-\alpha-\beta)\tilde{Q} + \alpha(\varepsilon_{t-1}\varepsilon_{t-1}) + \beta Q_{t-1}$$

$\tilde{p}_{i,j}$ is the unconditional expectation of the cross product while for the variances $\tilde{p}_{i,i} = 1$ and the conditional estimator which has the satisfying property of being positive definite:

$$\rho_{i,j,t} = \frac{q_{i,j,t}}{\sqrt{q_{i,i,t} q_{j,j,t}}}$$
The unconditional expectation of the numerator in the formula above is $\bar{p}_{ij}$ and each term in the denominator has an expected value of one.

Moreover the DCC model specified will have mean reverting properties as long as

$$\alpha + \beta < 1$$

To guarantee that the model is positive definite the parameters are restricted to be positive

$$\alpha \geq 0 \quad ; \quad \beta \geq 0$$

Finally, the log likelihood estimator used can be expressed as follows and is maximized over the models parameters

$$r_t | \mathcal{F}_{t-1} \sim N(0, H_t)$$

$$L = - \frac{1}{2} \sum_{i=1}^{T} (n \log(2\pi) + \log |H_t| + r_t' H_t^{-1} r_t)$$

$$= - \frac{1}{2} \sum_{i=1}^{T} (n \log(2\pi) + \log |D_t R_t D_t'| + r_t' D_t^{-1} R_t^{-1} D_t^{-1} r_t')$$

$$= - \frac{1}{2} \sum_{i=1}^{T} (n \log(2\pi) + 2 \log |D_t| + \log |R_t| + r_t' R_t^{-1} R_t' r_t)$$

$$= - \frac{1}{2} \sum_{i=1}^{T} (n \log(2\pi) + 2 \log |D_t| + r_t' D_t^{-1} D_t^{-1} r_t - \varepsilon_t' \varepsilon_t + \log |R_t| + \varepsilon_t' R_t^{-1} \varepsilon_t)$$

The programming code used in Eviews is found in Appendix I.

### 3.5 Volatility estimator

#### 3.5.1 GARCH

The symmetric GARCH(1,1) model that was originally introduced by Bollerslev (1986) that is applied in the first step in the DCC model can be written as

$$\sigma_t^2 = \omega + \alpha r_{w,t-1}^2 + \beta \sigma_{t-1}^2$$

The vector GARCH parameters are estimated by maximizing the log likelihood function
Firstly, a GARCH(1,1) process implies that current volatility is an exponentially weighted moving average of past squared returns. Secondly, the covariance stationary GARCH(1,1) process has dynamics that eventually produce reversion in volatility to a constant long-run value, which enables interesting and realistic forecasts.

After some derivation one can rewrite GARCH(1,1) model as

$$\sigma_t^2 = (1 - \alpha - \beta)\sigma^2 + \alpha r_{w,t-1}^2 + \beta \sigma_{t-1}^2$$

Where $$\sigma^2 \equiv \omega/(1 - \alpha - \beta)$$ denotes the long-run, or unconditional variance. This representation shows that the GARCH forecast is constructed as an average of three elements. Equivalently we can also write the model as

$$\sigma_t^2 = \sigma^2 + \alpha(r_{w,t-1}^2 - \sigma^2) + \beta(\sigma_{t-1}^2 - \sigma^2)$$

Which explicitly shows how the GARCH(1,1) model forecasts by making adjustments to the current variance and the influence of the squared return around the long-run, or unconditional variance. Finally, one can also write

$$\sigma_t^2 = \sigma^2 + (\alpha + \beta)(\sigma_{t-1}^2 - \sigma^2) + \alpha \sigma_{t-1}^2(z_{t-1}^2 - 1)$$

Where the last term on the right-hand-side on average is equal to zero. Hence, this shows how the GARCH(1,1) forecasts by making adjustments around the long-run variance with variance persistence governed by $$(\alpha + \beta)$$ and the volatility-of-volatility linked to the level of volatility as well as the size of $$\alpha$$. (Andersen et al., 2005)

### 3.6 Econometric

#### 3.6.1 Stationarity

If a series satisfies the three definitions below it is said to be a weakly stationary process. They state that a stationary process should have a constant mean, a constant variance and a constant autocovariance structure respectively. If the constant mean and constant variance is easy to interpret the constant autocovariance structure requirement can be somewhat more cumbersome. It determine how $$y$$ is related to previous values, and for a stationary process the
covariance between $y_t$ and $y_{t-1}$ should be the same as between $y_{t-10}$ and $y_{t-11}$. (Brooks, 2008)

\[ E(y_t) = \mu \]
\[ E(y_t - \mu)(y_{t-1} - \mu) = \sigma^2 < \infty \]
\[ E(y_t - \mu)(y_{t-2} - \mu) = y_{t-2} - \alpha \forall t_1, t_2 \]

It is important to test the data series for stationarity. Otherwise, non-stationary series could find spurious results between different variables, which means that the relations found are valueless. Further, some asymptotic analysis will be performed in this paper and there standard assumptions are not valid when the series employed are non-stationary. Finally shocks in non-stationary series does not die away. (Brooks, 2008)

Augmented Dickey-Fuller

\[ \Delta y_t = \Psi y_{t-1} + \sum_{i=1}^{p} \alpha_i \Delta y_{t-i} + u_t \]

\[ \text{test statistic} = \frac{\hat{\Psi}}{SE(\hat{\Psi})} \]

$H_0 = \text{Non-stationary}$

What is tested is if the series contains a unit root. This is rejected, hence, $H_0$ is rejected, if the test statistic is more negative than the critical value. One positive aspect with the ADF test is that it ensures that the error term $u_t$ is not autocorrelicated. Some negative aspects are that problem arises determining lag length and that the power of the test is low if the process indeed is stationary but with a root close to the non-stationary boundary. (Brooks, 2008)

3.6.2 Maximum-Likelihood

The Maximum-Likelihood function chooses the parameter values that are most likely to have produced the observed data. A likelihood function (LF) is a multiplicative function of the actual data, however, due to its complexity to maximize the function w.r.t the parameters the logarithms is taken. Thus it will turn LF into an additive function of the sample data, which is known as the Log-Likelihood function (LLF). Maximizing the LLF is done by seek out the parameter-space until “right” values are found, which is equivalently to jointly minimizing
\[ \sum_{t=1}^{T} \log \sigma_t^2 \]

and

\[ \sum_{t=1}^{T} \frac{(y_t - \mu - \phi y_{t-1})^2}{\sigma_t^2} \]

(Brooks, 2008)

Where \( \sigma_t^2 \) are the time-varying conditional variance of the standardized errors. In this paper the iterative technique is applied in order to maximizing the LLF. That implies that some initial parameter values are chosen, and from that the parameter values are updated each iteration until optimum is reached. The drawback of this method is that several local maximums might exist, in which different initial values may thus lead to different outcomes.
4. Empirical results and Analysis

In this section the empirical results are presented and analyzed. The DCC models parameters is tested for significance and in addition a robustness test is performed. Finally, result of how the correlations are changing in distressed situations is presented.

4.1 Default data

Some descriptive statistics is presented below for the four industries. For more extended descriptive statistics about the data see Appendix A and B. In Appendix A it can be seen that the series follows a similar development over time. This is a result of a combination of interlinkages and their common systematic risk exposure, i.e. all firms are somehow exposed to the macroeconomic condition.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Food, Beverage and Tobacco</td>
<td>0.00182</td>
<td>0.00598</td>
<td>0.00000</td>
<td>0.00106</td>
<td>0.000000</td>
</tr>
<tr>
<td>Retail Trade</td>
<td>0.00147</td>
<td>0.00362</td>
<td>0.00060</td>
<td>0.00067</td>
<td>0.000000</td>
</tr>
<tr>
<td>Hotels and Restaurants</td>
<td>0.00208</td>
<td>0.00498</td>
<td>0.00073</td>
<td>0.00099</td>
<td>0.000002</td>
</tr>
<tr>
<td>Wholesale trade</td>
<td>0.00164</td>
<td>0.00400</td>
<td>0.00065</td>
<td>0.00078</td>
<td>0.000000</td>
</tr>
</tbody>
</table>

Table 1: Descriptive statistics of the industry default ratios.

Highest mean of default ratio is found in the hotel and restaurant industry, which should be interpreted as the most risky industry. However, the food industry experiences the highest monthly default ratio. Food is also shown to be the industry with the largest standard deviation during the estimation period, which makes it the most difficult one to predict. On the opposite, the retail industry proved to have the most stable default ratio. It has both the lowest mean and is least volatile which makes it the least risky industry. One explanation to this is believed to depend on the industry’s ability to do price adjustments. Hence, the industry is believed to have an ability to pass through increased prices to their customers.

To avoid facing the risk of retrieving spurious results and hence, be able to draw reliable inference from the estimated output the data must be stationary, as discussed in the theory chapter’s econometric section. The Augmented Dickey-Fuller test of unit root was therefore applied. Results from the ADF tests, when testing for all different specifications, indicate that all the series possess the attribute of being stationary, see Appendix C. Thus, the analysis can
proceed, without the need of transforming the data. This is preferable since, correlation estimates is not invariant when the underlying variables are transformed.

4.2 Results of the DCC model
Since the estimates of the two DCC parameters are significant on the five percent level, all parameters except two, the paper’s empirical result support dynamic conditional correlation. In other words, the assumption about stable correlation structures between the industries is rejected. For further analysis only the food - wholesale correlation pair will be excluded due to highly insignificant. Presented below are the estimated parameter values $T(1)$ and $T(2)$, $\alpha$ and $\beta$ respectively, generated from the DCC model.

<table>
<thead>
<tr>
<th>Correlation</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>Z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>food - wholesale</td>
<td>$T(1)$</td>
<td>0.037477</td>
<td>0.038215</td>
<td>0.980672</td>
</tr>
<tr>
<td></td>
<td>$T(2)$</td>
<td>0.891757</td>
<td>0.115985</td>
<td>7.688586</td>
</tr>
<tr>
<td>food - retail</td>
<td>$T(1)$</td>
<td>0.188763</td>
<td>0.085516</td>
<td>2.207338</td>
</tr>
<tr>
<td></td>
<td>$T(2)$</td>
<td>0.629307</td>
<td>0.152998</td>
<td>4.113163</td>
</tr>
<tr>
<td>food - hotel</td>
<td>$T(1)$</td>
<td>0.105832</td>
<td>0.060394</td>
<td>1.752358</td>
</tr>
<tr>
<td></td>
<td>$T(2)$</td>
<td>0.767569</td>
<td>0.147445</td>
<td>5.205817</td>
</tr>
<tr>
<td>wholesale - retail</td>
<td>$T(1)$</td>
<td>0.162735</td>
<td>0.001134</td>
<td>143.4625</td>
</tr>
<tr>
<td></td>
<td>$T(2)$</td>
<td>0.827909</td>
<td>0.000243</td>
<td>3404.162</td>
</tr>
<tr>
<td>wholesale - hotel</td>
<td>$T(1)$</td>
<td>0.090604</td>
<td>0.027990</td>
<td>3.237042</td>
</tr>
<tr>
<td></td>
<td>$T(2)$</td>
<td>0.891688</td>
<td>0.034410</td>
<td>25.91358</td>
</tr>
<tr>
<td>retail - hotel</td>
<td>$T(1)$</td>
<td>0.132447</td>
<td>0.000827</td>
<td>160.1231</td>
</tr>
<tr>
<td></td>
<td>$T(2)$</td>
<td>0.854294</td>
<td>0.000885</td>
<td>965.1213</td>
</tr>
</tbody>
</table>

* highly insignificant, ** insignificant on 5% level, however, will still be interpreted.

Table 2: Estimated parameter values from the DCC model.

The persistence of shocks in the short-run on the dynamic correlations is greatest between food - retail, closely followed by wholesale - retail, due to their high $\alpha$ values. The short term effect in the correlation pair food - retail is increased since the pair also has the lowest decay term, $\beta$, thus the estimated time-varying correlation becomes even more sensitive to short-run shocks. As a result this indicates that there does not exist a stable correlation structure between the pair and hence, food and retail are relatively less interlinked industries. All other correlation pairs have relatively high $\beta$ values, which indicates that the model put more
weight to long run information. Hence, there is a long run correlation relationship between the different industries. Furthermore, the sum of the parameter values for the last three industry pairs in the table are all close to unity, which indicate persistency in the time-varying correlations. Finally, all model specifications have the property of being strictly mean reverting since the parameters values are jointly less than unity.

To make sure that the paper’s results are reliable, the statistical property of stationary standardized residuals from the GARCH(1,1) estimates are a necessary condition for the DCC model (Engle, 2002). Thus, a test of stationary of the standardized GARCH residuals was made. The ADF test shows that all the standardized residuals from each GARCH series were stationary. Results are shown in Appendix C. In addition, the distribution of the standardized residuals was shown to behave non-normal, with the exception of the food GARCH(1,1). However, this is not a problem since the optimization problem will instead be a quasi-maximum likelihood procedure and will still generate robust estimates.

4.2.1 Output analysis
In Appendix D the behavior of the time-varying conditional correlations are presented graphically. Further, to grasp the series informational value some statistic properties of the DCC output are presented in Appendix E. Of interest are the correlation series maximum and minimum dynamic levels, where a tight spread between these corresponds to a high informational value and vice versa. Intuitively, it’s not optimal with the tightest spread as possible, i.e. no spread, which would result in an unconditional model that cannot capture the time-varying correlations.

The mean correlations between different pairs are fairly low. This should be interpreted as there exist potential diversification possibilities in a corporate loan portfolio. An explanation to this can be that the interlinkages is only one important describing factor and different macrovariables another. Moreover, since the industries are exposed to different macrovariables their default pattern differ some.

Furthermore, the correlations tend to be positive over time with the exception of some short time periods. Negative correlation would otherwise have been very favorable from a risk management’s diversification perspective. In the approach the paper outlined some expectations about the industries’ interlinkages. From the empirical result the paper can verify a positive relationship between high expected interlinkages and relatively high correlation. Between food production and hotel which is expected to be the most unrelated
industries in this paper is also where the lowest mean correlation is found. Correspondingly, the wholesale industry is expected to be highly related to its customers i.e. the retail industry and hotel and restaurants. Results show that this is where the highest mean of correlations are found.

As shown in Appendix D all the estimated DCC correlations show relatively high default correlation during the years 1994-1998. One possible reason for this structural break with decreasing correlations around 1997-1998 can be due to the globalization of the Swedish economy. In the year 1994 Sweden joined the European Union and thus this can be seen as a further development of the economy. This has given the firms the opportunity to diversify their counterparty risk through increased openness to international trade. As a result, the interlinkages and contagion effect of defaults within the industries became temporary weaker.

To be confident about that the DCC model is a more reliable estimator than the rolling window approach a comparison was made. In figure 5 below the default correlation between the wholesale - hotel industry is presented. A full description of the other industries is found in Appendix E. The unconditional estimate is simply the correlation calculated of the whole sample period.

![Figure 5: Correlation comparison of wholesale - hotel DCC and rolling windows.](image)

Furthermore, the two included rolling windows of correlation estimates are of 6 and 12 months respectively. It is visible that the rolling window estimates follows a random pattern. Even for the rolling window of 12 months, which tries to incorporate longer time information, the estimated correlations still show a random pattern with some periods of rapidly changing correlations. This problem can be explained by the well known drawbacks of equal weight
and ghost effects in rolling window estimation. Hence, the results are in line with the criticism among researchers of that using historic data is a naive way to forecast correlation (Dowd, 2005). Since dynamic levels are very wide and the volatility is high the informational value from the rolling window estimations becomes very low.

The implication of using historic rolling windows for a credit portfolio risk management team when forecasting correlations implies that it gives unreliable and stochastic estimates over time. Thus, optimizing such a credit portfolio to the efficient frontier becomes difficult since optimal weights will have to be changed rapidly when there are large changes in correlations. On the other hand, if unconditional estimates are used as input it seems plausible that the corporate credit portfolio risk will be incorrectly estimated over time and as a result not optimal weights in different industries.

4.3 Robustness analysis
As discussed in theory, a downside of optimizing with the maximum likelihood approach is that several local maximums can exist. Therefore a robustness check of the estimated parameters of the DCC model was performed. The test of robustness of the estimated coefficients is evaluated by changing the initial parameter values. In Appendix F the results are presented. For every correlation pair there is the original model specification with initial values of $T(1)=0.2$ & $T(2)=0.7$ and the pre-estimated GARCH parameters as inputs from respective series. Overall, the coefficient values proved to be very rigid from two different robustness tests. Firstly, the average standard deviation between the different conditional correlations series produced is very low. Secondly, the correlations between the correlation series are therefore very high. Even though the test takes into account the changes in both parameter values the series produced showed to have perfect positive correlation.

However, two exceptions exist. First one is between food - wholesale and the second one is between food - retail. Since food’s GARCH produced negative values for one of the initial parameters, the values of $T(1)=0$ & $T(2)=0.999$ were applied subjectively instead. However, the DCC model generated exploding models due to parameter values summarizing to >1 in both cases. In addition, both $\alpha$ parameters are negative.

These two results should be interpreted somewhat differently. In the case of food - wholesale the original model specification didn’t have the problem of finding a local maximum. Instead, the right model specification would be to exclude the $\alpha$, hence there exist no short-run shocks
in this correlation pair. Thus, the DCC model didn’t produce any significant $\alpha$ from any of the initial values tested. This should be interpreted as that it is enough with a long-run model specification for this particular correlation pair, hence, highly interlinked industries.

This differs from the case of food - retail pair which showed perfect proof of problem with a local maximum. In two cases the DCC model find significant and the same coefficient values. In the third it proved to be insignificant and negative due to the subjective chosen initial parameters that made the model find a local maximum.

**4.4 Correlations in economic distress**

As discussed in the introduction there is the problem of correlation breaking down under stressful economic situations. Dowd (2005) describes how the correlations can change abruptly and become either extremely high or extremely low. A test is therefore performed to find out if the same pattern can be found in Swedish industry default data.

The test which ranks the 30 highest default ratios for every industry and compare if any of these time periods corresponds with any of the 30 time periods with the highest or lowest correlation respectively. Results of the test are given in Appendix G. In contrast to Dowd’s statement, empirical results here are proposing that high economic distress, hence, high default rates, will merely result in correlations becoming higher. The positive relationship between the highest default rates and the highest correlations are very strong and significant. On the other hand, very few observations of high default rates correspond to low correlations and can therefore be seen as random.

Further analysis considering the relationship, high default rates and high correlation, reveal that the correlation becomes extremely high. Table 3 below presents the three pairs with the highest average correlation and the corresponding correlation under stressed economic periods. The result show that the default correlations have significant evidence of break down when needed the most. Hence, the paper shows that industries with already relatively high correlation get even higher correlation in stressful situations.

<table>
<thead>
<tr>
<th>Average DCC correlation 1994-2008</th>
<th>wholesale - hotel</th>
<th>retail – hotel</th>
<th>wholesale – retail</th>
</tr>
</thead>
<tbody>
<tr>
<td>0,72</td>
<td>0,67</td>
<td>0,69</td>
<td></td>
</tr>
<tr>
<td>Average DCC correlation under distress</td>
<td>0,89</td>
<td>0,91</td>
<td>0,93</td>
</tr>
<tr>
<td>Increase in percent</td>
<td>24%</td>
<td>36%</td>
<td>35%</td>
</tr>
</tbody>
</table>

*Table 3: Industry pairs of high average correlation.*
In table 4 below the two correlation pairs with relatively low correlation is presented and their respectively increase under stressful economic situations. The pairs’ average correlation under distressed situations remains relatively low, even though the same increase in percent. Intuitively, this is explained by that its initial average value is already low. That the correlation remains relatively low is positive since it will help to limit total portfolio volatility. However, aside from correlation the volatility of the industries’ default rates is crucial in determining its attractiveness. Both correlation pairs with relatively low correlation include food which has both the highest max default rate for a single month and the highest standard deviation. It is therefore expected to have the highest risk/return level because it’s the most difficult industry to manage and predict since big swings are possible. This aspect put some limits considering the food industry’s attractiveness as portfolio diversification opportunity.

<table>
<thead>
<tr>
<th></th>
<th>food - retail</th>
<th>food - hotel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average DCC correlation 1994-2008</td>
<td>0,43</td>
<td>0,41</td>
</tr>
<tr>
<td>Average DCC correlation under distress</td>
<td>0,57</td>
<td>0,55</td>
</tr>
<tr>
<td>Increase in percent</td>
<td>34%</td>
<td>34%</td>
</tr>
</tbody>
</table>

Table 4: Industry pairs of low average correlation.

That default rate correlations merely increase in stressful situations are intuitive and have the explanation of contagion effects and interlinkages which creates domino effects. That interlinkages are significantly magnified under distressed situations is in line with Hatchett & Kühn (2006). The mathematical interpretation becomes that the numerator, the covariance, increase in the correlation formula which result in higher correlations. Moreover, a higher covariance results in a less diversified portfolio. This is unfavorably from a bank’s risk management point of view, since it leads to an increase in risk of the corporate loan portfolio.
5. Conclusion

*In this final chapter conclusions based on the main results are stressed. Moreover, it includes feedback on the thesis purpose and the question is answered. The chapter ends with some suggestions for future research.*

This paper come to the conclusion that Swedish default industry correlations have the unfavorable properties of breaking down when needed the most. Results suggest that industries with already relatively high correlation get even higher correlation in stressful situations. Correlations that are relatively low remains low due to that they merely increase to the same extent as the high correlations pairs in percent. In contrast to Dowd (2005) who describes that correlations can change abruptly and take on extreme values in stressful situations, both high and low. We find that Swedish default industry correlations only has a strong significant relationship with high correlation. The explanation is believed to be strong interlinkages and the contagion effect which causes a domino effect between different industries in defaults. Our result is in line with Hatchett & Kühn (2006) who found that corporate interlinkages are significantly magnified in situations of economic stress.

Furthermore, our empirical results show that there exist potential diversification possibilities in banks corporate loan portfolio. This because the mean correlations between different pairs are found to be fairly low. An explanation was discussed to be that the interlinkage is only one important describing factor and different macroeconomic factors another. Moreover, since the systematic risk factors have different effects on the industries their default pattern will differ some. Finally, this paper put a lot of focus on individual correlation pairs, however, from a risk management perspective one ultimately need to focus on the overall portfolio correlations.

The DCC model applied in this paper verified time-varying correlation structures between all correlation pairs except for food - wholesale. Further, the parameters estimated by the model were found reliable from being tested in a robustness test. The model therefore produces informational value in contrast to the rolling window approach which showed a random pattern. This was explained by the rolling windows drawbacks of equal weight and ghost effects.
5.1 Further research
In particular, it would be of interest to closer analyze the dynamics of industry default correlations in economic distress by adding macrovariables. Furthermore, investigate whether there exist a clear leading/lagging relationship between the industries.
6. References


Data
Statistics Sweden (SCB) data:

Number of firms defaulted:
http://www.ssd.scb.se/databaser/makro/Produkt.asp?produktid=NV1401

Number of active firms:
http://www.ssd.scb.se/databaser/makro/Produkt.asp?produktid=NV0101
Appendix A: Default rates per Industry

Food, Beverage and Tobacco industry, “Livsmedels-, dryckesvaru-, o tobaksind” [SCB: 15+16]
Consist of production companies, such as food and beverage processing. (B2B companies).

Consist of intermediary companies. Hence, sales of another party’s goods to producers and retailers. (B2B companies).

Retail Trade (excl. vehicles), “Detalh, ej med motorf; rep hushållsart o personl art” [SCB: 52]
Consist of companies selling to end consumers without processing, eg. supermarket stores. (B2C)

Hotels and Restaurants , “Hotell och restauranger” [SCB: 55]
Appendix B: Descriptive statistics

<table>
<thead>
<tr>
<th></th>
<th>RETAIL</th>
<th>HOTEL</th>
<th>FOOD</th>
<th>WHOLESALE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.001474</td>
<td>0.002079</td>
<td>0.001817</td>
<td>0.001639</td>
</tr>
<tr>
<td>Median</td>
<td>0.001250</td>
<td>0.001723</td>
<td>0.001684</td>
<td>0.001403</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.003621</td>
<td>0.004980</td>
<td>0.005984</td>
<td>0.003995</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.000595</td>
<td>0.000727</td>
<td>0.000000</td>
<td>0.000654</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.000665</td>
<td>0.000986</td>
<td>0.001057</td>
<td>0.000776</td>
</tr>
<tr>
<td>Skewness</td>
<td>1.274744</td>
<td>0.927765</td>
<td>0.966298</td>
<td>1.179673</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>3.818222</td>
<td>2.867683</td>
<td>4.201338</td>
<td>3.861517</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>53.77770</td>
<td>25.95374</td>
<td>38.83607</td>
<td>47.31543</td>
</tr>
<tr>
<td>Probability</td>
<td>0.000000</td>
<td>0.000002</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>Sum</td>
<td>0.265354</td>
<td>0.374136</td>
<td>0.327024</td>
<td>0.295041</td>
</tr>
<tr>
<td>Sum Sq. Dev.</td>
<td>7.92E-05</td>
<td>0.000174</td>
<td>0.000200</td>
<td>0.000108</td>
</tr>
<tr>
<td>Observations</td>
<td>180</td>
<td>180</td>
<td>180</td>
<td>180</td>
</tr>
</tbody>
</table>
## Appendix C: ADF test

$H_0$: Unit Root (non-stationary)

<table>
<thead>
<tr>
<th>Industry</th>
<th>Reject $H_0$ (1%)</th>
<th>Reject $H_0$ (1%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>retail</td>
<td>Yes</td>
<td>retail GARCH(1,1)</td>
</tr>
<tr>
<td>hotel</td>
<td>Yes</td>
<td>hotel GARCH(1,1)</td>
</tr>
<tr>
<td>food</td>
<td>Yes</td>
<td>food GARCH(1,1)</td>
</tr>
<tr>
<td>wholesale</td>
<td>Yes</td>
<td>wholesale GARCH(1,1)</td>
</tr>
</tbody>
</table>
Appendix D: Graphical Result of Correlations

- Food - Wholesales
- Food - Retail
- Food - Hotel
- Wholesale - Retail
- Wholesale - Hotel
- Retail - Hotel
### Appendix E: Correlation Comparison of different estimation methods

<table>
<thead>
<tr>
<th></th>
<th>food-wholesale</th>
<th>food-retail</th>
<th>food-hotel</th>
<th>wholesale-retail</th>
<th>wholesale-hotel</th>
<th>retail-hotel</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>0.502</td>
<td>0.425</td>
<td>0.411</td>
<td>0.692</td>
<td>0.719</td>
<td>0.669</td>
</tr>
<tr>
<td><strong>Median</strong></td>
<td>0.509</td>
<td>0.456</td>
<td>0.442</td>
<td>0.767</td>
<td>0.722</td>
<td>0.665</td>
</tr>
<tr>
<td><strong>Maximum</strong></td>
<td>0.672</td>
<td>0.811</td>
<td>0.746</td>
<td>0.977</td>
<td>0.947</td>
<td>0.969</td>
</tr>
<tr>
<td><strong>Minimum</strong></td>
<td>0.336</td>
<td>-0.325</td>
<td>-0.134</td>
<td>-0.007</td>
<td>0.254</td>
<td>0.076</td>
</tr>
<tr>
<td><strong>Std. Dev.</strong></td>
<td>0.082</td>
<td>0.225</td>
<td>0.173</td>
<td>0.256</td>
<td>0.173</td>
<td>0.223</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>food-wholesale</th>
<th>food-retail</th>
<th>food-hotel</th>
<th>wholesale-retail</th>
<th>wholesale-hotel</th>
<th>retail-hotel</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>0.189</td>
<td>0.145</td>
<td>0.145</td>
<td>0.476</td>
<td>0.472</td>
<td>0.469</td>
</tr>
<tr>
<td><strong>Median</strong></td>
<td>0.222</td>
<td>0.195</td>
<td>0.208</td>
<td>0.530</td>
<td>0.544</td>
<td>0.557</td>
</tr>
<tr>
<td><strong>Maximum</strong></td>
<td>0.952</td>
<td>0.980</td>
<td>0.881</td>
<td>0.975</td>
<td>0.991</td>
<td>0.981</td>
</tr>
<tr>
<td><strong>Minimum</strong></td>
<td>-0.936</td>
<td>-0.847</td>
<td>-0.828</td>
<td>-0.693</td>
<td>-0.683</td>
<td>-0.965</td>
</tr>
<tr>
<td><strong>Std. Dev.</strong></td>
<td>0.468</td>
<td>0.488</td>
<td>0.480</td>
<td>0.328</td>
<td>0.353</td>
<td>0.354</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>food-wholesale</th>
<th>food-retail</th>
<th>food-hotel</th>
<th>wholesale-retail</th>
<th>wholesale-hotel</th>
<th>retail-hotel</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>0.188</td>
<td>0.134</td>
<td>0.137</td>
<td>0.519</td>
<td>0.506</td>
<td>0.511</td>
</tr>
<tr>
<td><strong>Median</strong></td>
<td>0.156</td>
<td>0.106</td>
<td>0.167</td>
<td>0.531</td>
<td>0.528</td>
<td>0.530</td>
</tr>
<tr>
<td><strong>Maximum</strong></td>
<td>0.812</td>
<td>0.760</td>
<td>0.740</td>
<td>0.895</td>
<td>0.809</td>
<td>0.835</td>
</tr>
<tr>
<td><strong>Minimum</strong></td>
<td>-0.576</td>
<td>-0.651</td>
<td>-0.648</td>
<td>-0.120</td>
<td>0.005</td>
<td>-0.327</td>
</tr>
<tr>
<td><strong>Std. Dev.</strong></td>
<td>0.315</td>
<td>0.338</td>
<td>0.332</td>
<td>0.224</td>
<td>0.182</td>
<td>0.193</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>food-wholesale</th>
<th>food-retail</th>
<th>food-hotel</th>
<th>wholesale-retail</th>
<th>wholesale-hotel</th>
<th>retail-hotel</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unconditional</strong></td>
<td>0.6272</td>
<td>0.6026</td>
<td>0.5997</td>
<td>0.9014</td>
<td>0.9014</td>
<td>0.9037</td>
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</tbody>
</table>
**Appendix F: Robustness analysis**

<table>
<thead>
<tr>
<th></th>
<th>Initial values</th>
<th>Estimated values</th>
<th>Average Std. Error</th>
<th>Correlation with original</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>food - wholesale</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Original</td>
<td>0.200, 0.700</td>
<td>0.037, 0.892</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>food GARCH(1,1) *</td>
<td>0, 0.999</td>
<td>-0.004, 1.006</td>
<td>0.0966</td>
<td>-0.0361</td>
</tr>
<tr>
<td>wholesale GARCH(1,1)</td>
<td>0.126, 0.809</td>
<td>0.037, 0.892</td>
<td>4.3923E-08</td>
<td>1</td>
</tr>
<tr>
<td><strong>food - retail</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Original</td>
<td>0.200, 0.700</td>
<td>0.189, 0.629</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>food GARCH(1,1) *</td>
<td>0, 0.999</td>
<td>-0.001, 1.016</td>
<td>0.2274</td>
<td>0.0528</td>
</tr>
<tr>
<td>retail GARCH(1,1)</td>
<td>0.095, 0.835</td>
<td>0.189, 0.629</td>
<td>2.3478E-07</td>
<td>1</td>
</tr>
<tr>
<td><strong>food - hotel</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Original</td>
<td>0.200, 0.700</td>
<td>0.106, 0.768</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>food GARCH(1,1) *</td>
<td>0, 0.999</td>
<td>0.106, 0.768</td>
<td>7.4135E-09</td>
<td>1</td>
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<tr>
<td>hotel GARCH(1,1)</td>
<td>0.058, 0.894</td>
<td>0.106, 0.768</td>
<td>5.1225E-08</td>
<td>1</td>
</tr>
<tr>
<td><strong>wholesale - retail</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Original</td>
<td>0.200, 0.700</td>
<td>0.163, 0.828</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>wholesale GARCH(1,1)</td>
<td>0.126, 0.809</td>
<td>0.171, 0.814</td>
<td>0.0089</td>
<td>0.9995</td>
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<tr>
<td>retail GARCH(1,1)</td>
<td>0.095, 0.835</td>
<td>0.121, 0.878</td>
<td>0.0344</td>
<td>0.9909</td>
</tr>
<tr>
<td><strong>wholesale - hotel</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Original</td>
<td>0.200, 0.700</td>
<td>0.091, 0.892</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>wholesale GARCH(1,1)</td>
<td>0.126, 0.809</td>
<td>0.091, 0.892</td>
<td>1.5843E-06</td>
<td>1</td>
</tr>
<tr>
<td>hotel GARCH(1,1)</td>
<td>0.058, 0.894</td>
<td>0.091, 0.892</td>
<td>8.9817E-07</td>
<td>1</td>
</tr>
<tr>
<td><strong>retail - hotel</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Original</td>
<td>0.2, 0.7</td>
<td>0.132, 0.854</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>retail GARCH(1,1)</td>
<td>0.095, 0.835</td>
<td>0.131, 0.856</td>
<td>0.0019</td>
<td>0.99999</td>
</tr>
<tr>
<td>hotel GARCH(1,1)</td>
<td>0.058, 0.894</td>
<td>0.133, 0.853</td>
<td>0.0008</td>
<td>0.99999</td>
</tr>
</tbody>
</table>

* Initial GARCH estimate was originally negative and thus set to non-negative.
## Appendix G: Correlations in stressed economic situations

<table>
<thead>
<tr>
<th>High correlation</th>
<th>Low correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>food - wholesale</strong></td>
<td>13 18</td>
</tr>
<tr>
<td><strong>food - retail</strong></td>
<td>8 13</td>
</tr>
<tr>
<td><strong>food - hotel</strong></td>
<td>11 17</td>
</tr>
<tr>
<td><strong>wholesale - retail</strong></td>
<td>16 16</td>
</tr>
<tr>
<td><strong>wholesale - hotel</strong></td>
<td>12 16</td>
</tr>
<tr>
<td><strong>retail - hotel</strong></td>
<td>13 15</td>
</tr>
<tr>
<td><strong>food - wholesale</strong></td>
<td>1 0</td>
</tr>
<tr>
<td><strong>food - retail</strong></td>
<td>3 0</td>
</tr>
<tr>
<td><strong>food - hotel</strong></td>
<td>0 0</td>
</tr>
<tr>
<td><strong>wholesale - retail</strong></td>
<td>0 0</td>
</tr>
<tr>
<td><strong>wholesale - hotel</strong></td>
<td>0 0</td>
</tr>
<tr>
<td><strong>retail - hotel</strong></td>
<td>0 0</td>
</tr>
</tbody>
</table>

## Appendix H: DCC matrix calculation

\[
D_t R_t D_t = \begin{bmatrix} \sqrt{h_{1,t}} & 0 \\ \sqrt{h_{2,t}} & 0 \end{bmatrix} \begin{bmatrix} 1 & \rho_{21} \\ \rho_{21} & 1 \end{bmatrix} \begin{bmatrix} \sqrt{h_{1,t}} \\ \sqrt{h_{2,t}} \end{bmatrix} = \begin{bmatrix} \sqrt{h_{1,t}} \rho_{21} \sqrt{h_{2,t}} \\ 0 \end{bmatrix}
\]

\[
= \begin{bmatrix} h_{1,t} \rho_{21} \sqrt{h_{1,t}} \sqrt{h_{2,t}} \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{\sigma_{1,t}^2}{\sigma_{2,t}^2 \sigma_{1,t} \sigma_{2,t}} \\ \frac{\sigma_{2,t}^2}{\sigma_{2,t}^2 \sigma_{1,t} \sigma_{2,t}} \end{bmatrix}
\]

\[
= \begin{bmatrix} \sigma_{1,t}^2 & \sigma_{1,t} \sigma_{2,t} \\ \sigma_{2,t} \sigma_{1,t} & \sigma_{2,t}^2 \end{bmatrix}
\]
Appendix I: DCC programming code

```plaintext
sample s1 1994M02 2008M12
scalar pi=3.14159
scalar var_z1=@var(z1)
scalar var_z2=@var(z2)
scalar cov_z1z2=@cov(z1,z2)
scalar corr12=@cor(z1,z2)
series var_z1t=var_z1
series var_z2t=var_z2
series cov_z1tz2t=cov_z1z2
coef(2) T
T(1)=0.2
T(2)=0.7

logl dcc
dcc.append @logl logl
'decide var_z1t, var_z2t, cov_z1tz2t
dcc.append var_z1t=@nan(1-T(1)-T(2)+T(1)*(z1(-1)^2)+T(2)*var_z1t(-1),1)
dcc.append var_z2t=@nan(1-T(1)-T(2)+T(1)*(z2(-1)^2)+T(2)*var_z2t(-1),1)
dcc.append cov_z1tz2t=@nan((1-T(1)-T(2))*corr12+T(1)*z1(-1)*z2(-1)+T(2)*cov_z1tz2t(-1),1)
dcc.append pen=(var_z1t<0)+(var_z2t<0)
dcc.append rho12=cov_z1tz2t/@sqrt(@abs(var_z1t*var_z2t))
dcc.append detrDt=@sqrt(garch1*garch2)
dcc.append pen=pen+(detrRt<0)
dcc.append detrRt=@abs(detrRt)
dcc.append logl=(-1/2)*(2*log2*pi)+log(detrRt)+(z1^2+z2^2-2*rho12*z1*z2)/detrRt)-10*pen
smpl s1
dcc.ml(showopts, m=500, c=1e-5)
show dcc.output
graph corr.line rho12
show corr
```

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