The Equity Premium Puzzle post the Financial Crisis

MASTER THESIS

MAY 2011

ÍVAR ALFRED GRÉTARSSON

AND

BERGSVEINN SNORRASON

Thesis supervisor:

Frederik Lundtofte

Lund University

Department of Economics & Business Administration
Master in Finance Program
Sweden
The purpose of this paper is to take a closer look at the equity premium puzzle which is one of the most intriguing puzzles in economics and has been challenging researchers for the last 25 years since Mehra and Prescott first introduced the puzzle. It refers to the empirical fact that risky equity has been outperforming default free debt with about 6 percentage points for the U.S. market. The fact that the historical equity premium has been about 6 percentage points means that we have a puzzle on our hands. The puzzle stems from the fact that the perceived risk related to the equity returns is not high enough to explain these high returns. Investors would need to have implausibly high risk-aversion coefficients to demand such high premiums. However, unless the risk aversion coefficient is large, a high equity premium is impossible since the growth rate of consumption just does not vary enough. Another striking aspect of the puzzle is that, although the risk aversion has been high according to the puzzle, the risk free rate of returns has been extremely low over a long period of time. The equity premium puzzle could, therefore, just as easily be called the risk-free rate puzzle.

The research in this paper is conducted on Icelandic data for the period 1996-2010 and U.S. data for the period 1889-2010. The authors show that the recent financial crisis has had huge impact on the Icelandic equity market while the bigger U.S. equity market has not been as affected. During the last 10 years, however, the default free debt has been outperforming the more risky equity, which has not been the case the last 100 years. Does this mean that the puzzle has vanished? However, when looking at the puzzle from an historical perspective the excessively high equity premium relative to risk-free short term debt still remains a puzzle.

**Keywords:** Equity Premium Puzzle, Survivorship Bias, Non-linear interpolation, Risk aversion, Bond, Equity.
# Table of Contents

## 1 Introduction

1.1 Background .......................................................................................................................... 7  
1.2 Equity Premium .................................................................................................................. 11  
1.3 Problem discussion ............................................................................................................ 12  
1.4 Statement of Purpose ........................................................................................................ 15  
1.5 Limitations ........................................................................................................................ 15  
1.6 Contribution ....................................................................................................................... 15  
1.7 Thesis outline .................................................................................................................... 16

## 2 Theoretical framework

2.1 Literature review ................................................................................................................ 17  
2.2 Theory .............................................................................................................................. 23  
2.3 Attempts to solve the Puzzle ............................................................................................ 31  
  2.3.1 Empirical approach ....................................................................................................... 32  
  2.3.2 Theoretical explanations ............................................................................................... 34

## 3 Empirical Framework

3.1 Data ................................................................................................................................. 37  
3.2 Methodology ..................................................................................................................... 38  
3.3 Results .............................................................................................................................. 41

## 4 Conclusion

45

## References

47

## Referenced Web Resources

52

## A Appendix

53  
  The Nelson – Siegel Model .................................................................................................. 53
List of Figures

Figure 1: Real annual return on the S&P 500 from 1900 to 2009. 8
Figure 2: Real annual return on a relative riskless security from 1900 to 2008. 9
Figure 3: Companies listed on the Icelandic stock exchange from 1991 to 2010. 13
Figure 4: The value of the OMXI from 1993 to 2010. 14
Figure 5: Returns on riskless securities and returns on the S&P500. 44
Figure 6: Returns on riskless securities and returns on the OMXI. 44
List of Tables

Table 1: Consumption Disasters (C) for Iceland and United States from 1945 to 2006 where 2006 is set to 100................................................................. 22

Table 2: GDP Disasters for Iceland and United States from 1945 to 2006 where 2006 is set to 100................................................................. 22

Table 3: Calculated results for Iceland.......................................................... 42

Table 4: Calculated results for U.S. .............................................................. 42
1 Introduction

1.1 Background

In 1985 Mehra and Prescott introduced the Equity Premium Puzzle (EPP) after comparing the average real returns on equity to the average real returns on short term default free debt over the ninety-year period 1889-1978. Over that period, they found that the average real annual yield on the S&P500 Index was 6.98 percent per year while the average yield on the risk-free short term debt was only 0.80 percent per year. They concluded that this difference in return was strikingly large, too large to be explained by any economic models.

Almost 25 years after it was introduced, the equity premium puzzle is not near fully explained in the finance literature, not only has no model been able to fully describe the anomaly of the large excess returns on equity relative to average return on risk free assets,¹ but the puzzle also questions modern economic theories such as the random walk since the empirical fact

¹ Schwert (2003) describes market anomalies in as either market inefficiency or inadequacies in the underlying model.
revealed by Mehra and Prescott shows that stock prices are more likely to follow a trend wise movement rather than a random one.

Why has the rate of returns on stocks been so much higher than the returns on relative risk free assets? The answer might be that because stocks are riskier than Treasury bills, investors will demand a large premium for bearing this additional risk. As can be seen in the tables below, the standard deviation of the returns to stocks has been much larger than that of the returns to T-bills.

Figure 1: Real annual return on the S&P 500 from 1900 to 2009.
Introduction

Background

Figure 2: Real annual return on a relative riskless security from 1900 to 2008.

Damodaran (2010) suggests that the fact that the stock market has had higher returns over the risk-free rate is mainly due to the higher risk involved in equity investments. The equity premium is the excess return in the stock market over the risk-free rate to compensate the investor for holding additional risk in his portfolio. The allocation of assets in the investor’s portfolio is, therefore, composed depending on expected return and how much risk he is willing to accept and hence reflecting the investors risk aversion. If investors become more risk averse it would lead to higher equity premiums and if risk aversion would decline the equity premium would fall.

So, by definition, stocks can be considered much more risky than bills. But is this the case? Mehra and Prescott (2003) used the Capital Asset-Pricing Model (CAPM) to explain the difference in risk between the stocks and bills. According to them, the CAPM model postulates a linear relationship between an asset’s beta, a measure of systematic risk, and its expected return. High beta stocks should, therefore, be able to yield a high expected return on the market, since good times and bad times are captured by the return on the market in the CAPM. The performance of the stock market acts as a proxy for the relevant state of the economy. High beta securities tend to pay more off when the market return is high, i.e. when
times are good and consumption is abundant. It provides less incremental utility than a security that pays off when consumption is low, is less valuable and consequently sells for less. It can, therefore, be concluded that assets with higher betas which pay off in states of low marginal utility will sell for a lower price than similar assets that pay off in states of high marginal utility. Since the rates of return are inversely proportional to asset prices, the lower beta assets will, on average, give a lower rate of return than the former.

Agents prefer to smooth their consumption patterns over time. So assets that pay off a higher amount at times when consumption is already quite high tend to destabilize these consumption patterns, whereas assets that pay off when consumption is low tend to smooth out consumption patterns. Assets that smooth out consumption are considered more valuable and thus should require a lower rate of return to induce investors to hold these kinds of assets.

But are stocks that much riskier than bills so as to justify the high historical equity premium? As pointed out by Mehra (2003), stocks and bonds pay off in approximately the same states of nature or economic scenarios where consumption is similar. They should therefore command approximately the same rate of return. In a paper written by Mehra and Prescott (1980) they concluded, that by using a standard theory to estimate risk-adjusted returns, stocks on average should command at most a 1 per cent return premium over Treasury bonds. The fact that the historical equity premium has been about 6 per cent means that we have a puzzle on our hands. The puzzle stems from the fact that the perceived risk related to the equity returns is not high enough to explain these high returns. Investors would need to have implausibly high risk-aversion coefficients to demand such high premiums. To illustrate this puzzle Mehra and Prescott (1985) used the consumption based model described above in which agents have additively separable utility functions and constant relative risk aversion. The model only contains one parameter, $\alpha$, which can be described as the coefficient of relative risk aversion. So, if consumption were to fall by 1 per cent, then the marginal dollar of income increases by $\alpha$ per cent. Prescott and Mehra (1985) wanted to find out the value of $\alpha$ which is essential in explaining the historic equity premium. They found out that the value was somewhere between 30 and 40, which they concluded to be way too high to be reasonable. The question is, why did they deem this value of risk aversion to be too high? Siegel and Thaler (1997) answered this question by using a simple example of a gamble. Suppose an agent faced a 50 per cent chance of doubling his wealth and a 50 per cent chance of losing half his wealth. Siegel and Thaler (1997) asked how much this agent is really willing to pay to avoid this
gamble. They found out that if his relative risk aversion coefficient is 30, then he is willing to pay 49 per cent of his wealth to avoid a 50 per cent chance of losing half of it. This makes no sense, especially since most empirical estimates suggest that the coefficient of relative risk aversion lies somewhere between 1 and 2, as described earlier in this paper.

Another striking aspect of the puzzle identified by Weil (1989) was that a high value of risk aversion implies that agents will desperately want to smooth their consumption over time, because consumption shortfalls will bring the agent much more pain than surpluses deliver him pleasure. Consequently, as the economy grows and becomes richer over time, agents will have preferences for borrowing from the future, were they are supposed to be richer, in order to improve their current position. Their longing to borrow from the future should, therefore, lead to high real risk-free interest rates. However, the historical real risk-free rates of return have been barely positive. Therefore, as pointed out by Siegel ad Thaler (1997), “the equity premium puzzle could just as easily be called the risk-free rate puzzle”.

1.2 Equity Premium

Equity Premium, which is also called equity risk premium, risk premium, market premium and market risk premium plays an important role in every risk and return model. The expected return reflects the cost of equity and the cost of capital and is, therefore, a central element in financial analysis and corporate valuation. The equity premium is described by Fama and French (2002) as “the difference between the expected return on the market portfolio of common stocks and risk free interest rate”.

The term equity premium is used to describe four different concepts and can, therefore, cause some confusion. Historical Equity Premium (HEP) is the difference between stock market return versus risk-free bond. Expected Equity Premium (EEP) is the expected difference between stock market return versus risk-free bond. Required Equity Premium (REP) is the difference between stock market return versus risk-free bond required by an investor. Finally, the Implied Equity Premium (IEP) assumes that the market is correctly priced and does therefore not need to account for country risk. (Fernandez, 2009) This paper uses the Historical Equity Premium (HEP) since it looks at historical time series.
1.3 Problem discussion

The fact that the equity premium puzzle has yet to be solved more than 25 years since it was first proposed makes it an intriguing topic. Attempts to resolve the puzzle have become a major research impetus in economics and finance. Abel (1990), Benartzi and Thaler (1995), Campbell and Cochrane (1999), Constantinides (1990) and Epstein and Zin (1991) try to resolve the puzzle by proposing alternative assumptions about the preferences of the Mehra and Prescott (1985) model. Reitz (1988) modifies the probability distributions to take rare but disastrous events into account. Brown, Goetzmann and Ross (1995) focus on survivorship bias when it comes to resolving the puzzle while Constantinides and Duffie (1996), Heaton and Lucas (1997) and Storesletten, Telmer and Yaron (1999) look at incomplete markets. Market imperfections are the center of attention in the attempts made by Constantinides, Donaldson and Mehra (2002) and McGrattan and Prescott (2001) to solve the puzzle. None of these attempts has been able to fully resolve the anomalies.

This paper attempts to explain the puzzle by focusing on the survivorship bias theory. Survivorship bias centers around the assumption that the historical risk premium obtained by looking at U.S. data is biased upward because it focuses on the most successful equity market of the twentieth century. Using U.S. data alone to estimate the equity risk premiums will, therefore, result in distorted premiums since they are calculated for a survivor, a stock market that has successfully weathered the vicissitudes of fluctuating financial fortunes. Calculating the equity premiums for a country which is not by definition a survivor should remove such a bias. The current financial crisis makes the focus on survivorship bias all the more relevant since it has caused disturbances and eliminations of equity markets around the globe, making the comparison between survivor and non-survivor countries all the transparent. This paper, therefore, estimates the equity premiums for Iceland using the Mehra and Prescott (1985) model in order to see whether a puzzle exists for a country which can be considered a non-survivor. A comparison is also made between the equity premiums in Iceland and the U.S. The Mehra and Prescott (1985) model was chosen to explain the equity premium puzzle based on the fact that their paper began the whole equity premium puzzle literature. It also offers the reader a simplistic view of how the methodology works. Using a more complicated asset

\[2\text{ Attempts to resolve the puzzle are discussed in more detail later on in the paper.}\]
pricing model than is used in the Mehra and Prescott (1985) model will only make the insights from this methodology more difficult to comprehend.

The Icelandic stock market is young compared to the more mature U.S. stock market. The Iceland Stock Exchange ICEX was officially established in 1985 as a joint venture of several banks and brokerage firms. Trading in government bonds began one year later while the first equities were, however, not listed on the Icelandic stock exchange until 1990. Before the establishment of the Icelandic stock exchange, firms had been trading their equity through publicly owned stock brokers. The small size both in turnover and amount of firms listed on the Icelandic stock exchange makes the market illiquid. As can be seen in the figure below, the highest number of firms listed on the index since being established is 75. At the beginning of 2011 only 6 companies were listed on the index.

![Figure 3: Companies listed on the Icelandic stock exchange from 1991 to 2010.](image)

As can be seen in the graph below, the index grew almost each and every year since trading began, before peaking in the beginning of 2007. At that time the index had climbed to a value of roughly 9000.

---

3 Kauphöll Íslands (2011)
Investors, however, became sceptical of the high stock prices and warned that the companies were overvalued and a sharp drop was inevitable. As expectations started dropping due to these warnings, the index began its decline. The financial crisis added further fuel to the decline and eventually the Icelandic Financial Supervisory Authority suspended trading in shares of six financial companies on the OMIX index on 6th October 2008. Three days later, the government decided to freeze all trading on the exchange in an attempt to prevent further panic spreading throughout the country’s financial market (Rannsóknarskýrslá Althingis, 2010). When the market reopened on the 14th October the market had dropped to a value of 678.4 which corresponds to a plunge of 77% compared to the value before closure. In the months following, the index declined even further before reaching rock bottom in April 2009. At that time the index had dropped to almost 200. Since then the index had begun its revival and in the end of 2010 it stood at 930.

As can be seen in the passage above, Iceland is by no means a survivor since its equity market experienced disturbance in the wake of the financial crisis as trading with stocks was temporarily suspended, and should therefore make an ideal candidate to base the survivorship bias theory on.

In order to evaluate the equity premiums under the survivorship bias theory, a catastrophic event must have occurred. An event that resulted in some kind of disturbance in the equity market, for example, a suspension of trading. Special attention will, therefore, be paid to the
current financial crisis and what affects it has had on the equity premiums in both countries. Theoretically, the equity premium should have decreased in both countries during the recession. So, the question is, does the equity premium still remain a puzzle in the new Millennium, 25 years after it was first mentioned by Mehra and Prescott (1985)?

1.4 Statement of Purpose

The purpose of this paper is mainly twofold

1. To examine whether there exists a premium puzzle in Iceland using the Mehra and Prescott (1985) model
2. To examine whether the recent economic downturn has had a negative effect on the equity premium for both Iceland and the U.S., i.e. has reduced or eliminated the puzzle altogether.

The research questions are, therefore, as follows:

- Does an equity premium puzzle exist in Iceland?
- What effects has the financial crisis had on the equity premium puzzle in both countries?

1.5 Limitations

This paper only examines the equity premiums in the U.S. and Iceland. The choice of countries is motivated by the fact that the U.S. is the biggest economy in the world and truly is a survivor while Iceland is a small country that has experienced disturbances in its equity market, making it a non-survivor. Also, most articles use U.S. data when examining the puzzle. Results cannot be generalized to other countries, i.e. the result for Iceland and the U.S. cannot be generalized to other non-survivor and survivor countries.

1.6 Contribution

Never before has Iceland been used in an article on the EPP, the most likely reason for this is the small size of the market. Also there have been a lot of new events in the global market because of the recent economic downturn that could have led to new findings in solving the puzzle and more importantly, does the puzzle still exist?
1.7 Thesis outline

The plan of this paper is as follows. In the next section of this paper we present the theoretical framework starting with a literature review. We move further on and present a brief overview of previous research regarding equity premium puzzle, relevant theories used, as well as mention previous attempts to solve the equity premium puzzle which is divided into two subcategories with one empirical approach and theoretical explanations.

In the third chapter, the data and methodology used are presented and explained. The third chapter also covers a discussion on the results of the study. Finally we round up by presenting the conclusion of this research.
2 Theoretical framework

2.1 Literature review

This chapter provides a literature review of the Equity Premium Puzzle to give the reader an overview of previous studies on the topic and what has been done in the past trying to solve the puzzle.

In the paper, The Equity Premium, A Puzzle from 1985 by Mehra and Prescott, they surprised many economists and researchers in finance by illustrating that by using standard economic theory to estimate risk adjusted returns, investments in stocks should at most give a one percent real return premium over relatively risk free bonds. However they concluded that they had a puzzle on their hands after finding that during the last 90 years on the U.S. market the premium had been surprisingly higher. The average annual real return to stocks had been 6,98 percent per year but the average annual real return to Treasury bills had only been 0,8 percent per year. Mehra and Prescott were not able to account for the large difference in average yields in any economic model. They put forward the possibility that the reason for the abnormally high premium was maybe not because average return on equity was too high but rather that average risk-free rate is too low. After the publication by Mehra and Prescott (M&P) in 1985 there have been many attempts to explain the puzzle.
In 1992 Siegel extended the time period analyzed by Mehra and Prescott back to 1802 while updating the returns on stocks and bonds to 1990. By dividing the whole period into three sub-periods he found out that recent data overstated the magnitude of how much early stock returns exceeded bond returns. According to him the returns on stocks did not exceed the returns on bonds by nearly the same scale during the early periods. While the short term bond real returns have fallen dramatically over time, the real return on equity has been remarkably constant. His analysis also found that the returns from the bond market during most of the 19th century and after 1980 were far higher than in the period analyzed by Mehra and Prescott. He concluded that the equity premium is, therefore, not nearly as large when viewed over this extended time span.

Again in 1997, Siegel made an attempt to describe the Equity Premium Puzzle and this time with the help of Thaler. They described the Equity Premium Puzzle as an anomaly in the paper “Anomalies, The Equity Premium Puzzle”, since the empirical results are difficult to rationalize. They also used a good example in describing the equity premium that follows: Suppose someone invested 1000 USD in 1925 in Treasury Bills. The investor used a buy and hold strategy until December 31, 1995 where he would be able to pay out 12,720 USD. However, by investing in a portfolio of stocks he would instead have received 842,000 USD or about 66 times as much compared to choosing the risk-free investment. The difference between Treasury bills and the return on stocks is called the equity premium. They pointed out that there are basically two broad approaches which have been used when it comes to explaining the EPP. One approach that researchers have used to explain the puzzle is theoretical explanation. The other approach that researchers use to explain the puzzle is to try to find factors that are able to explain the differences between the returns on stocks and bonds. Ideally, these factors should make either the equity premium smaller or equity returns riskier. These approaches will be described later in the paper. They also point out that although there have been recessions in the past, the equity still greatly outperforms fixed income securities and that the EPP has been high over a long period of time.

Epstein and Zin (1989) developed a class of recursive preferences over intertemporal consumption lotteries. These preference functions allowed the link between the elasticity of intertemporal substitution and the coefficient of relative risk aversion to break down. Further, these preference specifications led to a model of asset returns in which appropriate versions of both the temporal CAPM and the intertemporal consumption-CAPM were tested as a special
case. In this model, the systematic risk of an asset is determined by the covariance with both returns on the market portfolio and consumption growth, while in existing models only one of these factors plays a role. This model had the advantage that it allowed for the opportunity to describe both the high equity premiums and the low bond real rate of returns occurring at the same time. However, their method could only explain about a third of the equity premium.

Campbell and Cochrane (1999) derived a consumption based model by modifying individual preferences in an attempt to solve the puzzle. Their model assumes that utility is affected not only by current consumption but also by a reference level of consumption known as the habitat level, which adapts slowly and nonlinearly to historical consumption. Their model was able to break the link between intertemporal substitution, risk aversion and precautionary savings which are present in the standard power utility model. Their method goes far in explaining a low bond rate of return with a high equity premium. Their model comes close in solving the equity premium puzzle, where the value of risk aversion is only 6.

Fama and French (2002) estimate the equity premium using dividends and earnings growth rates to measure the expected capital gain. Using data for the U.S. from 1951 to 2000, they found that the equity premium was 2.55 per cent in the dividend growth model while the premium was 4.32 percent in the earnings growth model, which are much lower than the equity premium produced by the average returns on the stock market. They suggested that the high average return was due to a decline in discount rates that produces high unexpected capital gains. They concluded that the average stock return during the last half century was a lot higher than expected.

Dzhumashev and Madsen (2009) used a monetary model for a production economy to argue that high historical excess returns to equity were the result of a severe ex post bias in the period from 1915 to 1960, because inflation surprises during this period drove a wedge between ex ante and ex post bond returns. They suggested that this bias was due to unexpected low returns on bonds. They showed that it was difficult to predict inflation during the gold standard period and that after the abandonment of the standard; investors were slow to learn the inflationary consequences of the paper money regime. In fact, historical surveys suggested that inflation expectation were close to zero during this period. All inflation was, therefore, pretty much regarded as unexpected inflation. Unexpected inflation shocks, therefore, drove a wedge between ex post real stock return and ex post bond return. This resulted in the ex post
equity premium increasing without affecting the ex ante premium. Further, they showed that stock returns are neutral to unexpected inflation in steady state. As a result, ex post and ex ante stock returns are identical in steady state. By adjusting the ex post equity premium by the ex post bias reduces the equity premium to 3.3 to 4.4 per cent over the past 132 years.

Kandel and Stambaugh (1991) take a different approach when it comes to trying to solve the equity premium puzzle. In order to model the riskiness of stocks they introduced additional features to the consumption CAPM model. They incorporated spans of both high and low consumption growth into the model, with unpredictable switches between the two. Using this method, they argued that the puzzle doesn’t exist. According to them, perhaps investors are really risk averse, since a high value of $\alpha$ is still needed to explain the historical equity premium. They further argue that although a high level of risk aversion can lead to irrational behavior when taking into account major changes in consumption, it does, however, not imply implausible behavior for small changes in wealth.

In 2003, Mehra takes a retrospective look at the equity premium puzzle and evaluates attempts by a wide variety of economist in trying to seek explanations to solve the puzzle. He also shows that the EPP puzzle is not unique for the U.S. capital markets. Mehra compared EPP in France, Germany, Japan, U.S. and U.K. who together account for more than 85% of the global equity value. UK has an EPP of 4.6 percent (1947-1999), France 6.3 percent (1973-1998), Japan the lowest with 3.3 percent (1970-1999) and finally Germany with 6.6 percent (1978-1997). This shows that even though the EPP is bigger in the U.S. market it is still big enough to be a puzzle in other major markets.

Guveneny (2009) theory for the equity premium puzzle is that stockholders require high equity returns since they are willing to hold equity in their portfolio apart from non-stockholders who are not willing to tolerate the higher risk. Guveneny argues that non-stockholders tend to be low income people who are dependent on labor income. They are therefore more risk averse in their investments and are not able to endure the ups and downs of business cycles. Equity owners are high income people who are able to withstand the ups and downs of the stock market and want to be compensated in the form of high equity premium.
Barro and Ursúa (2008) published a paper on Macroeconomic crises since 1870, looking at real per capita personal consumer expenditures or consumption for twenty-four countries and real per capita GDP for thirty-six. The data used reaches from 1914 to 2006 covering forty-two countries.

They define economic crisis as a cumulative decline in consumption or GDP of at least 10 percent. From this they found 152 crises for GDP and 95 crises for consumption, which can be interpreted as a 3.5 percent chance of a crisis each year with an average duration of 3.5 years. According to their calculations, the OECD countries have an equity premium of around 7 percent on levered equity. They look at both GDP and consumption since during wars GDP has been shown to increase while consumption has decreased, mostly because of vast military spending. The countries used in their sample had an average rate of return of 7.9 percentage for stocks and 0.93 percentage for bills with an average equity premium of 6.99 percentages. Their sample used levered equity since using unlevered equity would have resulted in a lower equity premium. As stock returns tend to fall when more negative than positive news hit the financial markets the negative stock price also tend to be more spread out during crises. One of the findings from the data is that U.S. has been rather immune to crises compared to other countries around the world. In this article they put together a summary of disasters by events or periods. For instance, they show that the Great Depression affected 22 countries where only 9 were OECD and 13 were non-OECD countries. They show that the World War II affected 25 countries, with the most severe decline in consumption and GDP; however it did not negatively affect the U.S. economically.

Even though stock prices tend to fall during crises the risk free rate is the same during crises as during normal times. However during crises the expected real bill returns can be understated because of the inflation. Crises do generate higher than usual inflation rates, during consumption crises the inflation rates were on average 6.6 percent and during GDP crises 6.9 percentages. The two tables below show how the returns on stocks and treasury bills have been affected by consumption and GDP crises in Iceland and the U.S.
### Table 1: Consumption Disasters (C) for Iceland and United States from 1945 to 2006 where 2006 is set to 100.4

<table>
<thead>
<tr>
<th>Country</th>
<th>Disaster period</th>
<th>Decline in consumer expenditure per capita</th>
<th>Stock price decline</th>
<th>Rate of return on bills</th>
<th>Inflation rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iceland</td>
<td>1952 1947</td>
<td>0.25</td>
<td>-</td>
<td>-</td>
<td>0.202</td>
</tr>
<tr>
<td></td>
<td>1969 1967</td>
<td>0.118</td>
<td>-</td>
<td>-</td>
<td>0.108</td>
</tr>
<tr>
<td></td>
<td>1975 1974</td>
<td>0.107</td>
<td>-</td>
<td>-</td>
<td>0.515</td>
</tr>
<tr>
<td></td>
<td>1993 1987</td>
<td>0.176</td>
<td>-</td>
<td>0.06</td>
<td>0.144</td>
</tr>
<tr>
<td>US</td>
<td>1921 1917</td>
<td>0.164</td>
<td>0.584</td>
<td>-0.071</td>
<td>0.139</td>
</tr>
<tr>
<td></td>
<td>1933 1929</td>
<td>0.208</td>
<td>0.631</td>
<td>0.093</td>
<td>-0.064</td>
</tr>
</tbody>
</table>

### Table 2: GDP Disasters for Iceland and United States from 1945 to 2006 where 2006 is set to 100.5

<table>
<thead>
<tr>
<th>Country</th>
<th>Disaster period</th>
<th>Decline in GDP per capita</th>
<th>Stock price decline</th>
<th>Rate of return on bills</th>
<th>Inflation rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iceland</td>
<td>1883 1881</td>
<td>0.125</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>1918 1913</td>
<td>0.221</td>
<td>-</td>
<td>-</td>
<td>0.206</td>
</tr>
<tr>
<td></td>
<td>1920 1919</td>
<td>0.157</td>
<td>-</td>
<td>-</td>
<td>0.114</td>
</tr>
<tr>
<td></td>
<td>1952 1948</td>
<td>0.139</td>
<td>-</td>
<td>-</td>
<td>0.235</td>
</tr>
<tr>
<td>US</td>
<td>1908 1906</td>
<td>0.105</td>
<td>0.365</td>
<td>0.019</td>
<td>0.041</td>
</tr>
<tr>
<td></td>
<td>1914 1913</td>
<td>0.095</td>
<td>0.16</td>
<td>0.034</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>1921 1918</td>
<td>0.118</td>
<td>0.293</td>
<td>-0.057</td>
<td>0.125</td>
</tr>
<tr>
<td></td>
<td>1933 1929</td>
<td>0.29</td>
<td>0.631</td>
<td>0.093</td>
<td>-0.064</td>
</tr>
<tr>
<td></td>
<td>1947 1944</td>
<td>0.165</td>
<td>-0.061</td>
<td>-0.062</td>
<td>0.076</td>
</tr>
</tbody>
</table>

In the paper by DeLong and Magin (2009), on the topic of the past and the future U.S. equity premium, they looked at various explanations for the exceptionally high equity premiums in the past. Since the expected equity premiums are forecasted from a backward-looking model the results will be biased downward. This is mainly due to the fact that more recently firms have not been using dividends as a measure of paying out profits to stockholders but have

---

4 Details and a list of data sources are available on the Internet for GDP and C, (www.economics.harvard.edu/faculty/barro/data_sets_barro)
5 Details and a list of data sources are available on the Internet for GDP and C, (www.economics.harvard.edu/faculty/barro/data_sets_barro)
switched to stock buybacks instead. Firms who pay out less are, therefore, reinvesting more which then implies a faster growth rate of earnings in the future compared to the past. This is then offset by stating that the U.S. economy has been running on luck since 1950 and that the luck will soon cease, resulting in slower growth rate. The conclusion is that while there is no process making the marginal investor less risk averse, or less loss averse, or less myopic the equity premium should still remain in the future. The authors guess that future equity premium will be a bit lower or about 4 percentage compared to the 6 percentage in the past.

2.2 Theory

In order to explain the equity premium puzzle a theoretical model must first be derived. The consumption based model has been seen as an optimal starting point to illustrate the puzzle\(^6\), since it can be used to figure out what the real risks are that drive asset prices and expected returns. Thus, this section starts by deriving the consumption-based model, as described by Cochrane (2000), before explaining the framework Mehra and Prescott (1985) used to showcase the puzzle. This paper uses the Mehra (2003) approach when it comes to deriving the model. The consumption based model shows how much agents should consume, save and what portfolio of assets they should hold. The basic objective of the model is to find the value of any stream of uncertain cash flow, called payoff, \(x_{t+1}\) at time \(t\). As an example, if an agent buys stock today, the payoff in the next period will be the stock price plus dividend, \(x_{t+1} = p_{t+1} + d_{t+1}\). The agent doesn´t know exactly how much he will receive from his investment but he can assess the probability of different possible outcomes.

In order to find out how much the agent will receive on this investment, the value of the payoff to the agent, a convenient mathematic formalism is used. The agent is, therefore, modeled by a utility function defined over current and future values of consumption.

\[U(c_t, c_{t+1}) = u(c_t) + \beta E_t[u(c_{t+1})]\]

Where

\(E_t\) is the expectation operator conditional on information available at time \(t\)

\( \beta \) is the subject time discount factor

\( u \) is an increasing, continuously differentiable concave utility function

\( C_i \) is per capita utility function

The utility function is further restricted to be of the constant relative risk aversion class and to follow a time separable power utility function

\[
   u(c) = \begin{cases} 
   \frac{c^{1-\alpha} - 1}{1 - \alpha} & \alpha > 0, \alpha \neq 1 \\
   \log c & \alpha = 1 
   \end{cases}
\]

Where the parameter \( \alpha \) measures the curvature of the utility function. The curvature shows the agent’s aversion to risk and to intertemporal substitution.

Now, assume that this representative agent can freely buy or sell as much of the payoff \( x_{t+1} \) as he wants at the price \( p_t \). The question is, how much is the agent willing to buy or sell? In order to solve this question, the original consumption level (if the agent bought none of the asset), denoted by \( e \), and the amount of assets he chooses to buy, denoted by \( \xi \), are needed. The problem, therefore, becomes the following:

\[
   \max u(c_t) + \beta E_t[u(c_{t+1})]
\]

Subject to

\[
   c_t = e_t - p_t \xi
\]

\[
   c_{t+1} = e_{t+1} + x_{t+1} \xi
\]

By substituting the constraints into the objective and setting the derivative with respect to \( \xi \) equal to zero, the first-order conditions for optimal consumption and portfolio choice are obtained,
Theoretical framework

$p_t \cdot u'(c_t) = E_t[\beta \cdot u'(c_{t+1}) \cdot x_{t+1}]$

Where the marginal loss in utility of buying an extra unit of the asset at $t$ (the LHS) is equal to the marginal (discounted and expected) gain of the extra payoff $x_{t+1}$ at $t+1$ (the RHS).

The equation can also be expressed as

$p_t = E_t \left[ \frac{\beta}{u'(c_t)} \cdot u'(c_{t+1}) \cdot x_{t+1} \right]$

This is the fundamental asset pricing formula. Given the payoff and agent’s consumption choices, it shows what market price to expect. The formula can be used to price both stocks and riskless one-period bonds. For equity,

$1 = E(mR^e)$

Where $m$ is a stochastic discount factor defined as

$m_{t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)}$

And $R_e$ is the gross return defined as

$R_{t+1} = \frac{x_{t+1}}{p_t}$

For the riskless one-period bond, the relevant pricing expression is

$1 = E(mR^f)$

Where the gross rate of return on the riskless asset is by definition

$R^f = \frac{1}{E(m)}$
The model makes the assumption that \( U(c) \) is increasing so that the stochastic discount factor \( M_{t+1} \) is strictly positive. By making this assumption, the model guarantees that the economy will be arbitrage free and the law of one price will hold.

Expanding the equity price formula will result in

\[
1 = E(mR^e) = E(m)E(R^e) + \text{cov}(m, e)
\]

Substituting for \( R^f \) from the following equation,

\[
1 = E(mR^f)
\]

Results in

\[
E(R^e) = R^f - \text{cov} \left[ \frac{m^*, R^e}{E(m)} \right]
\]

Using the consumption based model the equity premium can, therefore, be easily computed. The expected returns of the asset are equal to the risk-free rate plus a premium for bearing additional risk, which depends on the covariance of the asset returns with the marginal utility of consumption. As explained by Mehra (2003), assets returns that covary positively with consumption, that is to say, assets that pay off in states when consumption is high and marginal utility is low will command a high premium. The reason being, that agents want to smooth their consumption over time and states. Assets that pay off when times are good and consumption is high are less desirable than assets that pay off the same amount when times are bad and consumption is low. Assets that pay off a relatively large amount at times when consumption is already high tend to destabilize the consumption patterns of consumers who wish to smooth their consumption over time and states. Assets, however, that pay off when consumption levels are low enable consumers to smooth their consumption. Assets that pay off when times are good must, therefore, offer a premium to induce agents to hold them.

Mehra and Prescott (1985) posed the question whether the magnitude of the covariance between the assets and the marginal utility of consumption was large enough to justify the large observed equity premium in U.S. equity markets. In order to explain the puzzle this
question must be answered. This paper builds on the methods introduced by Mehra and Prescott (1985) when illustrating the fact that returns earned on a risky security have been much higher than the returns earned on a relatively risk-free bond when taking the covariance between the assets and the marginal utility of consumption into account. It, however, uses the Mehra (2003) approach to derive the model. From this point, the Mehra and Prescott (1985) framework will, therefore, be derived by using the fundamental asset pricing formula previously derived and use the Mehra (2003) approach. First a few assumptions need to be made in order to construct the model. These assumptions are as follows:

The growth rate of consumption, \( w_{t+1} \equiv \frac{c_{t+1}}{c_t} \), is i.i.d.

The growth rate of dividends, \( z_{t+1} \equiv \frac{d_{t+1}}{d_t} \), is i.i.d.

\( w_t \) and \( z_t \) are jointly lognormally distributed.

By making these assumptions, the model can treat \( R^e \) as i.i.d. and also \( w_t \) and \( R^e \) will be jointly lognormally distributed.

By substituting \( u'(c_t) = c_t^{-\alpha} \) into fundamental asset pricing formula

\[
p_t = \beta E_t \left[ (p_{t+1} + d_{t+1}) \frac{u'(c_{t+1})}{u'(c_t)} \right]
\]

will result in the following expression

\[
p_t = \beta E_t[(p_{t+1} + d_{t+1})w_t^{-\alpha}]
\]

Since \( p_t \) is homogeneous of degree 1 in \( d \), it can be expressed as

\[
p_t = \nu d_t
\]

This results in the equation above becoming

\[
\nu d_t = \beta E_t[(\nu d_{t+1} + d_{t+1})w_t^{-\alpha}]
\]
Hence,

\[ \nu = \beta E_t [(w + 1) z_{t+1} w_{t+1}^{-\alpha}] \]

Or

\[ \nu = \frac{\beta E_t (z_{t+1} w_{t+1}^{-\alpha})}{1 - \beta E_t (z_{t+1} w_{t+1}^{-\alpha})} \]

By definition the gross rate of return on equity can be defined as

\[ R^* = \frac{p_{t+1} + d_{t+1}}{p_t} \]

Again, substituting for \( p_t \) will result in

\[ R^* = \left( \frac{\nu + 1}{\nu} \right) \left( \frac{d_{t+1}}{d_t} \right) = \frac{\nu + 1}{\nu} z_{t+1} \]

Or

\[ E_t (R^*) = \frac{\nu + 1}{\nu} E_t (z_{t+1}) \]

Now, since

\[ \frac{\nu + 1}{\nu} = \frac{1}{\beta E_t (z_{t+1} w_{t+1}^{-\alpha})} \]

It can be seen that the expected return on the risky asset is

\[ E_t (R^*) = \frac{E_t (z_{t+1})}{\beta E_t (z_{t+1} w_{t+1}^{-\alpha})} \]

Analogously, the gross return on the riskless asset can be written as
Since the growth rates of both consumption and dividends are assumed to be lognormally distributed, it follows that

\[ R^f = \frac{1}{\beta E_t(w_{t+1})} \]

Taking logs on both sides results in the following

\[ \ln E_t(R^f) = -\ln \beta + \alpha \mu_w - \frac{1}{2} \alpha^2 \sigma^2_w + \alpha \sigma_{w,z} \]

Where

\[ \mu_w = E(ln w) \]
\[ \sigma^2_w = \text{var}(ln w) \]
\[ \sigma_{w,z} = \text{cov}(ln w, ln z) \]

\[ ln w = \text{the continuously compounded growth rate of consumption} \]

Similarly, the returns on the riskless assets are as follows

\[ R^f = \frac{1}{\beta e^{-\alpha \mu_w + 1/2 \alpha^2 \sigma^2_w}} \]

Taking logs on both sides results in the following

\[ ln R^f = -\ln \beta + \alpha \mu_w - 1/2 \alpha^2 \sigma^2_w \]
We therefore have the following formula

\[ \ln E(\frac{R^e}{R^f}) - \ln R^f = \alpha \sigma_{w,z} \]

Where \( \sigma_{w,z} = \text{cov}(\ln w, \ln R^e) \)

The log equity premium in the model used by Mehra (2003) is the product of the coefficient of risk aversion and the covariance of the growth rate of consumption with the return on equity or the growth rate of dividends. The model imposes an equilibrium condition so that \( w = z \). This has the effect that the return on equity must be perfectly correlated with the growth rate of consumption. We therefore get the following expression

\[ \ln E(\frac{R^e}{R^f}) - \ln R^f = \alpha \sigma_w^2 \]

Where the equity premium is now the product of the coefficient of relative risk aversion, \( \alpha \), and the variance of the growth rate of consumption, \( \sigma_w^2 \). The relative risk aversion parameter measures people’s willingness to substitute consumption between successive yearly time periods. There is a wealth of evidence from various studies that the relative risk aversion is a small number. Arrow (1971) argues on theoretical grounds that \( \alpha \) should be approximately one. Friend and Blume (1975) estimate that the relative risk aversion should be in the range of two. Kydland and Prescott (1982) found that the value should be between one and two\(^7\). The puzzle can now be illustrated. The variance of the growth of consumption has only been 0.001275 from the period 1889 to 2009 while the average equity premium has been roughly 6% for the same period. So unless agents are extremely risk averse, such a high equity premium is impossible. The growth rate of consumption just does not vary enough. However, the risk aversion necessary to explain the historic equity premium is about 45-50, which is much too high to be reasonable. There lies the puzzle.

The puzzle can be further illustrated. As noted above, several studies have found the coefficient of risk aversion to be a small number, certainly below 10. By setting the risk

---

\(^7\) For further studies on the low value of the coefficient of risk aversion view the following papers, Altug (1983), Hildreth and Knowles (1982) and Tobin and Dolde (1971). In all these studies the risk aversion was found to be somewhere between zero and two.
2 Theoretical framework

Attempts to solve the Puzzle

aversion coefficient $\alpha$ equal to 10 and $\beta$ to 0,99 what is the risk premium? By plotting these values into the following expression will result in a risk free rate equal to 12,7 percent and a return to equity of 14,1 percent

\[
\ln R^f = -\ln \beta + \alpha \mu_w - 1/2 \alpha^2 \sigma_w^2
\]

\[
= 0,120
\]

\[
R^f = 1,127
\]

\[
\ln E(R^e) = \ln R^f - \alpha \sigma_w^2
\]

\[
= 0,132
\]

\[
E(R^e) = 1,141
\]

which implies a risk premium of 1,4%, which is far lower than the historical premium observed. If lower values were to be chosen for either $\alpha$ or $\beta$, the risk free rate would be even higher and the risk premium would subsequently be even lower, especially when taking into account that most studies indicate that the value of $\alpha$ should be close to two. So, the 1,4% value represents the maximum risk premium that can be achieved, given the bounds on $\alpha$ and $\beta$, in this model. Because the historical equity risk premium has been observed to be more than 6%, there exists a puzzle that risk considerations alone cannot account for.

2.3 Attempts to solve the Puzzle

In recent years there have been many attempts to resolve the equity premium puzzle. None, however, has come close to fully resolving this anomaly. In the subsequent section, an explanation of several efforts to resolve the puzzle will be offered. These attempts at explaining the puzzle can be classified into two broad approaches. The first approach centers around finding factors that demand adaptations to the empirical side of the puzzle. This could, for example, involve finding data that would result in the equity premium being smaller or the
equity returns being riskier. The second approach surrounds the possibility of using different theoretical frameworks to solve the puzzle. The discussion begins by considering the empirical approach which is then followed up by a discussion on the framework approach.

2.3.1 Empirical approach

Survivor bias

Damodaran (2011) states that the historical equity risk premium obtained by looking at data for the U.S. is biased upwards because of a survivor bias, which is induced by picking one of the most successful equity markets of the 20th century. According to Brown, Goetzmann and Ross (1995) more than one-half of the 36 stock markets that were fully functioning at the turn of the 20th century went through substantial disturbances or were obliterated all together. Consequently, estimating the equity risk premium using U.S. data alone will result in distorted premiums since they are calculated for a survivor, a country that has consistently been able to withstand the financial fluctuations throughout these past centuries. Accordingly, the riskiness of the equities are underestimated by relying on U.S. data, since this data does not take into consideration the devastation that might have occurred in the U.S., but didn't. Hence, the true equity risk premium is argued to be much lower. Dimson, March and Stauton (2002) backed up this view by conducting a study of large equity markets over the 20th century. They concluded that the historical risk premium is much closer to 4% than the 6% offered by Mehra and Prescott. This lower risk premium can still be considered too high if it is assumed that investors have reasonable risk aversion coefficients.

One large objection can, however, be raised on the survivor bias of the U.S. equity market. Siegel and Thaler (1997) pointed out that the time period which was used by Mehra and Prescott in their groundbreaking paper did contain economic disasters, the stock market crash in 1929 and the consequent Great Depression. These catastrophes resulted in stocks losing almost 80% of their value and not being able to fully reclaim it until the end of World War II.

Increasing the time period

In his paper, Siegel (1992) investigated whether or not the time period considered by Mehra and Prescott (1985) was special. He prolonged the data on the returns on both real stocks and real bonds in the U.S. back to 1802. Extending the data resulted in an equity premium of 5.3%
which is 1.3% points lower than that reported by Mehra and Prescott. His study also showed that early stock returns did not exceed fixed income returns by nearly the same margin as they did in more recent data. Siegel found evidence that the real returns had fallen substantially in the short term fixed income market over time while the returns on equity had remained remarkably constant. The reason for the decline in the real returns of government bonds is mainly due to higher default risk perceived for young companies in the 19th century and wrong expectations about the inflation in the 20th century.

**Taxes**

McGratten and Prescott (2001) offer one possible explanation for the high equity returns in the period after World War II. According to their paper, the marginal tax rate declined during that period which should cause equity prices to rise. They suggested that a drop in the tax rate on dividends from 50% to 0% over a 40 year period would result in equity prices rising about 1.8% more than the growth rate in GDP. Adding the dividend yield to this expected price appreciation would generate returns similar to the observed equity risk premium. However, as pointed out by Damodaran (2010), the drop in the marginal tax rate was much smaller and can, therefore, not explain the high equity risk premiums observed.

**Mean Reversion and Aversion**

As has been mentioned a couple of times before, the equity premium is an anomaly since the observed risk related to the equity returns is not nearly high enough to describe the measured high returns. However, as pointed out by Siegel and Thaler (1997), if the standard deviations of annual rates of returns are used as a measure of risk it might misrepresent the long term risk if the year to year returns do not follow the random walk process. Siegel (1992) investigated the complication of this assumption by scrutinizing the variability of real returns of both equity and fixed income assets. He concluded that by assuming a random walk the theoretical standard deviation was somewhat higher than the actual standard deviation of 20 year rates of return, since the standard deviation of stock returns decreases quicker than it otherwise would if the returns followed a random walk because the stock returns display mean reversion. This suggests that by looking only at the standard deviation the risk of holding stocks is lower than would be anticipated.
However, the returns on fixed income assets do not follow any mean reversion, unlike stock returns. In contrast to the standard deviation of stock returns, which decreases with the square root of the time horizon, the standard deviation of real returns of fixed income assets are said to follow a process called mean aversion since they decrease less than the square root of the horizon. This results in the standard deviation of annual rates of returns for 20 year horizons to be greater than that of stocks. Based on this reasoning the equity premium puzzle seems to be an even bigger puzzle since fixed income securities appear to be riskier in real terms than stocks (Siegel and Thaler, 1997).

2.3.2 Theoretical explanations

Alternative preference structures

The constant relative risk aversion function used by Mehra and Prescott implies that if investors are risk averse to changes in consumption across different states of nature at some specific point in time, they will be equally risk averse to changes in consumption across time. Some studies maintain that the equity risk premium puzzle stems from using this preference structure to explain individuals’ expected utility function. Several papers have, therefore, introduced different preference structures in an attempt to resolve the puzzle. None has been completely successful.

Epstein and Zin (1989) offered a class of recursive, but not necessarily expected utility, preferences over intertemporal consumption lotteries. The most important feature of these preferences is that they permitted risk attitudes to be disentangled from the degree of intertemporal substitutability. They argued that individuals will always prefer lower and more stable levels of consumption and wealth that they can sustain over a long period of time to higher levels of consumption and wealth that will vary widely from time to time. Although their utility functions were able to explain both the low real interest rates and high equity premiums, this method was only able to explain about a third of the equity risk premium.

Several theories have attempted to resolve to equity premium puzzle by relaxing the time separability of preferences and allowing for adjacent complementarity in consumption. This entails that the overall level of satisfaction derived from a given level of consumption depends, not only on the current level of consumption but also on how it compares to some
benchmark. Financial literature has identified two types of reference consumption levels that can characterized by these time non-separable preference functions.

The first type is based on an external criterion which can be expressed in terms of consumption of some outside reference group, for example the average consumption of the overall economy (Cuadrado, Monteiro and Turnovsky, 2004). This entails comparing your own current consumption level to that of others. Individuals not only get utility from their own consumption but also by knowing that their consumption level is higher than others. This approach is often referred to as “catching up with the Joneses” and has been used by Abel (1990) to examine asset pricing and by Campbell and Cochrane (1995) to resolve the premium puzzle. Their model can explain the equity premium with the coefficient for risk aversion being 6. Although it is quite high it is still much lower than the value reported by Mehra and Prescott (1985).

The second type is an internal criterion which is based on the individuals’ own past consumption level. This approach to modifying the preferences was initiated by Constantinides (1990). His approach involved modifying the utility function so that the utility of consumption depended on the comparison between current and prior consumption. This method has become known as habitat formation. It captures the notion that utility is a decreasing function of past consumption and that marginal utility is an increasing function of past consumption. This approach, therefore, makes individuals more sensitive to reductions in consumption in the short-run since they have become used to maintaining past levels in consumption. Small changes in the consumption level can result in big changes in marginal utility. Individuals, therefore, have a high risk aversion in the short run but a low risk aversion in the long run. This approach was, however, unsuccessful when it came to resolving the equity premium puzzle.

**Myopic loss aversion**

In this explanation there are two assumptions used, the first one is that investors are assumed to be loss averse, meaning that they are more sensitive to losses than to gains. Losses are, therefore, assumed to hurt significantly more than gains yield pleasure. The second is that long term investors are assumed to assess their portfolios regularly. Benartzi and Thaler (1995) describe myopic loss aversion as a combination of the investor’s loss aversion and his
short-sighted investment horizon. Longer investment horizon will make investors more willing to invest in risky equities. Benartzi and Thaler (1995) seek the answer to whether the myopic loss aversion can explain the EPP by performing various tests on the matter. Using simulations they found out that the size of the equity premium was consistent with the previously estimated parameters of prospect theory if investors evaluated their portfolios annually.
3 Empirical Framework

3.1 Data

The data used in this paper consist of five basic series for the period 1889-2010 for the U.S. and for the period 1996-2010 for Iceland. The reason for the limited amount of data on Iceland is due to the fact that the Icelandic Central Bank only offers information on bonds, notes and bills since 1996.

The paper only uses yearly data points. The series are:

- Annual average S&P Composite Stock Price Index
- Annual Dividends from the S&P Series
- Interest on relatively riskless short term debt. For the U.S. the securities used were the ninety-day government treasury bills in the 1931-2009 period, Treasury Certificates for the 1920-1930 period and sixty-day to ninety-day Prime Commercial Paper prior to 1920 For Iceland the securities used were the ninety-day treasury bills and the one year treasury notes
- Consumer Price Index.
- Real per capita consumption

All data for the U.S. is extracted from the web site of Robert Shiller\(^8\). The data on Iceland is extracted from several agencies. The Icelandic Stock Exchanges supplied data on the monthly OMXII15 Stock Price Index, along with data on annual dividends from the OMIX15. The

\(^8\) http://www.econ.yale.edu/~shiller/
interest on the relative riskless short term debt was extracted from the Central Bank of Iceland\textsuperscript{9}. The Consumer Price Index, data on private consumption and data on the population of Iceland was extracted from the Icelandic Statistics Bureau\textsuperscript{10}.

### 3.2 Methodology

In this part of the paper, the previously presented theory and method of illustrating the equity premium puzzle will be examined using the real data discussed above. This is done by solving the following equation

\[
\ln E(R^e) - \ln R^f = \alpha \sigma^2
\]

In order to solve this equation we need the real rate of return on stocks, the real rate of return on a relatively riskless short term debt, the value of relative risk aversion and the variance of the growth rate of consumption. We start by dividing the data series by the CPI to get the real price, dividend and interest, since the data collected is all in nominal terms. The stock price index is multiplied by the value of the CPI index in 2010 and then divided by its current value. The interest rate is multiplied by the current year value of the CPI index and then divided by the next year’s value of the CPI index. This is done so inflation will be taken into account. From this raw data it is possible to calculate the annual measure of growth and returns such as

- Real percentage stock price increase and real dividend yield: These series were used to calculate the average annual real return on the S&P 500 Composite Index over the 120 year period of the study and the average annual real return on the OMXI15 Composite Index over the 16 year study period. The annual return for year \( t \) was computed as
  \[
  (P_{t+1} + D_t - P_t)/P_t.
  \]

- Growth rate of real per capita consumption: This series was used to determine the progress on the growth rate of consumption over the same period for both countries. The growth rate of consumption was computed as \( (C_{t+1} - C_t)/C_t \). In order to find the per capita real consumption the real consumption had to be divided first by the population of each country in question.

\textsuperscript{9} www.sedlabanki.is
\textsuperscript{10} www.hagstofa.is
• The real return on a relatively riskless security: This series was used to calculate the average annual real return on a relatively riskless security over the same period. The annual return for year \( t \) was computed as \( \frac{R^f_{real,t+1} - R^f_{real,t}}{R^f_{real,t}} \). First, however, a term structure had to be derived in order to observe the returns on the relative riskless securities. The term structure describes the relationship between interest rates and time to maturity. The term structure of the interest rates is usually measured by means of a yield curve on zero coupon bonds. The advantage of using zero coupon bonds is that they don’t suffer from the coupon effect. This implies that two bonds that are identical in every respect except for bearing different coupon rates can have different yields to maturity (Caks, 1977). If the Treasury issued a full spectrum of zero coupon bonds every day, then the yield curve could simply be observed and a complete set of yields would be available. The Treasury, however, only issues a limited number of securities that have different maturities and coupons. The entire term structure of interest rates can, therefore, not be observed directly. Instead, a zero coupon yield curve must be constructed from coupon bearing Treasury bonds and notes. The term structure needs to be estimated using approximation methods. The method used in this paper is the model introduced by Nelson and Siegel (1987)\(^{11}\). This model is chosen since it is widely used by central banks, for example the Canadian Central Bank, and other market participants as a method of constructing the term structure of interest rates (Gimenoa and Nave, 2009, and BIS, 2005). The Nelson and Siegel model is both a common and simple function used to describe the term structure. It allows us to specify the term structure as a function of only four parameters according to the following functional form:

\[
y(t) = \alpha_1 + (\alpha_2 + \alpha_3) \frac{\beta}{t} \left(1 - e^{-\frac{t}{\beta}}\right) - \alpha_3 e^{-\frac{t}{\beta}}
\]

Where \( \alpha_1 \) governs the level of the yield curve, \( \alpha_2 \) governs the slope of the curve and \( \alpha_3 \) governs the curvature of the curve. This method constructs the yield curve using a nonlinear interpolation function. In order to do so the yields of different maturing coupon bearing bonds and notes are chosen so that a one year treasury bill can be constructed. Choosing the yields will result in an actual term structure. However, this

\(^{11}\) See appendix 1 for a further discussion on the Nelson and Siegel model.
term structure will have certain yields missing. Most importantly the yield on a one year treasury bill will be missing.

The four parameters in the model will allow us to specify a term structure. By choosing random values for these parameters will result in a predicted term structure. This term structure does not look anything like the term structure that resulted from choosing the yields of different maturing bonds and notes. The next step is to try to make the predicted term structure fit with the actual term structure. This is done in a couple of steps. The first step is to square the difference between the actual yields and the predicted yields. Squaring this difference will result in squared residuals. This is an ordinary least squares approach. The squared residuals are then summed together over the different maturing bonds and notes. To make the predicted term structure best fit with the actual term structure the squared residuals must be minimized. This is an optimization problem where the Excel tool Solver comes in handy. By minimizing the squared residuals the predicted term structure will become similar to the actual term structure and the yield of the one year Treasury bill can be extracted from the predicted term structure. So, the Nelson and Siegel model is used to solve the four parameters that give a predicted term structure that minimizes the sum of the squared residuals. The residual sum of squares is minimized by altering the four parameters in Solver. The predicted term structure produced by Solver will result in a very good fit and, therefore, nonlinearly interpolate the missing yield on the one year Treasury bill.

The equity premium is finally calculated as the difference between the real returns on stocks and the real returns on riskless securities. The variance of the growth rate of consumption is found by taking the variance of the consumption growth rate. Finally, the value of risk aversion is found by solving the following:

\[ \alpha = \frac{\ln E(R^s) - \ln R^f}{\sigma_w^2} \]

It should be noted here that the historical equity premium reported in this paper is stated in terms of an arithmetic mean instead of a geometric mean. The reason being, that the best available evidence indicated that stock returns were uncorrelated over time. When this turns out be the best case, the future expected value of a $1 investment is found by compounding the arithmetic average of the sample return. This should be the correct statistic if one is
interested in reporting the mean value of the investment. However, as Mehra and Prescott (2003) pointed out, the arithmetic average return exceeds the geometric average return and that if both returns are log-normally distributed; the difference between the two is one-half the variance of the returns. Since the annual standard deviation of the equity premium reported by Mehra and Prescott (2003) was 20% this could result in a difference of about 2% between the two measures. They, however, concluded that this was non-trivial since the phenomena under consideration had an arithmetic mean of between 2 and 8%. It might, therefore, come as a shock to some investors when they realize that the equity premium is stated in terms of an arithmetic mean, which is the sum of the annual returns divided by the number of years, since most regulators usually insist that fund performance data be presented in terms of compound annual returns.

3.3 Results

The results of this paper are presented in the tables below. The tables includes the mean of the real returns on a relatively riskless asset, the mean of the real returns on equity, the variance of consumption growth and the relative risk aversion coefficient for both the U.S. and Iceland.

Historically, the equity premium in the U.S. has been roughly 6% compared to a negative risk premium in Iceland, as can be seen in tables 3 and 4. It must, however, be taken into account that the historical values for Iceland only go as far back as 1996, which fails in comparison with the U.S., where the historical values go as far back as 1889. When comparing the equity premiums for the same periods, prior to the financial crisis, it can be seen that the risk premium has been substantially higher in Iceland than in the U.S. From 1996 to 2007 the equity premium in Iceland was almost 11.5% compared to 6% for the U.S. Looking at the period 2000 – 2007 it can be seen that the equity premium in Iceland is almost 13.5% while the equity premium in the U.S. is slightly negative. This implies that the stock market in Iceland grew substantially faster than the U.S. stock market for those periods. Adding the effects of the current financial crisis results in a negative risk premium of almost -9% for Iceland while the U.S. risk premium for the same period was a positive value of 4% for the period 1996 – 2010.
3 Empirical Framework

The value of the relative risk aversion coefficient for both countries, prior to the crisis, is too high to be reasonable for the reasons stated previously in this paper. However, unless the risk aversion coefficient is large, a high equity premium is impossible since the growth rate of consumption just does not vary enough, as can be seen in the tables. Individuals are, however, not that risk averse. As previously pointed out, most empirical studies have estimated that the value of risk aversion lies somewhere between one and two. There lies the puzzle. Adding the effects of the financial crisis, results in a negative risk aversion coefficient for Iceland while the U.S. risk aversion coefficient still remains too high to be reasonable.

It should be noted, however, that the risk aversion coefficients for both countries during the short time periods give an unrealistic picture. This is mainly due to the fact that the variance in consumption is extremely small during short time periods and using these values in the Mehra and Prescott model will, therefore, result in too high risk aversion coefficients when the equity premium is positive. However, for longer time periods it can be concluded that the value of the risk aversion is practical when using the Mehra and Prescott model since the outcomes are similar to those presented by Mehra and Prescott for the time period 1889-1978.

Another point that also deserves mentioning are the negative risk aversion values. According to theory, risk aversion can of course never be negative. However, since the financial crisis resulted in a negative equity risk premium, using the Mehra and Prescott model will give a negative risk aversion coefficient.

Table 3: Calculated results for Iceland.

<table>
<thead>
<tr>
<th>Periods</th>
<th>Return on stocks</th>
<th>Return on riskfree</th>
<th>Equity risk premium</th>
<th>Variance of consumption growth</th>
<th>Risk aversion</th>
</tr>
</thead>
<tbody>
<tr>
<td>1996 - 2007</td>
<td>0,1579</td>
<td>0,0431</td>
<td>0,1148</td>
<td>0,000980</td>
<td>117,1021</td>
</tr>
<tr>
<td>2000 - 2007</td>
<td>0,1749</td>
<td>0,0408</td>
<td>0,1341</td>
<td>0,001300</td>
<td>103,1714</td>
</tr>
<tr>
<td>2000 - 2010</td>
<td>-0,0690</td>
<td>0,0399</td>
<td>-0,1090</td>
<td>0,002769</td>
<td>-39,3511</td>
</tr>
<tr>
<td>1996 - 2010</td>
<td>-0,0464</td>
<td>0,0418</td>
<td>-0,0882</td>
<td>0,002200</td>
<td>-40,0699</td>
</tr>
</tbody>
</table>

Table 4: Calculated results for U.S.

<table>
<thead>
<tr>
<th>Periods</th>
<th>Return on stocks</th>
<th>Return on riskfree</th>
<th>Equity risk premium</th>
<th>Variance of consumption growth</th>
<th>Risk aversion</th>
</tr>
</thead>
<tbody>
<tr>
<td>1996 - 2007</td>
<td>0,07688</td>
<td>0,01672</td>
<td>0,06016</td>
<td>0,00009</td>
<td>642,31968</td>
</tr>
<tr>
<td>2000 - 2007</td>
<td>0,00193</td>
<td>0,00865</td>
<td>-0,00672</td>
<td>0,00013</td>
<td>-50,18629</td>
</tr>
<tr>
<td>2000 - 2010</td>
<td>-0,01142</td>
<td>0,00929</td>
<td>-0,02071</td>
<td>0,00025</td>
<td>-82,89655</td>
</tr>
<tr>
<td>1996 - 2010</td>
<td>0,05664</td>
<td>0,01603</td>
<td>0,04062</td>
<td>0,00030</td>
<td>134,87954</td>
</tr>
<tr>
<td>1889 - 2010</td>
<td>0,07680</td>
<td>0,01977</td>
<td>0,05703</td>
<td>0,00127</td>
<td>44,74235</td>
</tr>
<tr>
<td>1889 - 1978</td>
<td>0,07526</td>
<td>0,01446</td>
<td>0,06170</td>
<td>0,00162</td>
<td>38,05026</td>
</tr>
</tbody>
</table>
The financial crisis has had a much more devastating effect on the equities in Iceland than it has had on the U.S. as can be seen in the figures below. The real returns on the Icelandic equity market fell by more than 250% in the wake of the crisis, while the real returns on the U.S. market fell by almost 40%. There are, however, signs that both markets are recovering from the crisis as the returns on equity are again higher than the returns for relatively risk-free assets. This can be seen figure 4 and 5 below.

According to Mehra (2003), the stock and bond markets should be differently affected by a financial crisis. This paper illustrates that the stock markets in both countries were negatively affected by the crisis, resulting in a giant decline in real returns on the equity markets. One would, therefore, assume that the real returns on the bond markets had also decreased during these periods, which is the case. By viewing both the tables and figures, it is clear that the real returns on the bond markets did not increase due to the crisis. A closer examination will show that the real returns on the bond market actually fell during the crisis. The reason for this decline in interest rates is due to the fact that Government bonds are regarded as a safe investment. Investors generally prefer safe investments like governments bonds in periods of uncertainty. At the height of the financial crisis there was a strong demand for government bonds as people looked for security. The increase in demand resulted in increased bond prices and, therefore, decreased real interest rate returns for both countries. So, in both countries the stock and bond markets were indeed affected differently by the current financial crisis.

The returns on risk free securities are much higher in Iceland than they are in the U.S. The high returns in Iceland would lead one to conclude that there does not exist a risk-free rate puzzle in Iceland while this is not the case in the U.S. The risk free rate in the U.S. has been extremely low during the study period, especially when taking the risk aversion into account. These low rates of returns on riskless securities would lead one to conclude that there does in fact exist a risk-free rate puzzle in the U.S.
Prior to the financial crisis there seems to have existed an equity premium puzzle for Iceland. However, the financial crisis has led to a negative historical equity premium and, therefore, eliminated any signs of a puzzle. Although the equity premium is currently slightly lower in the U.S. than it was prior to the crisis, it still remains a puzzle since the financial crisis was unsuccessful at eliminating the high average equity premiums.
Conclusion

More than 25 years since it was first introduced, the equity premium puzzle has yet to be solved. Looking at historical returns, from 1889 to 1978, on the stock and bond market, Mehra and Prescott (1985) showed that on average the returns on the stock market had been roughly 6% higher than the returns on the bond market. The fact that the historical equity premium has been about 6% means that we have a puzzle on our hands. The puzzle stems from the fact that the perceived risk related to the equity returns is not high enough to explain these high returns. Investors would need to have implausibly high risk-aversion coefficients to demand such high premiums. However, by using a standard theory to estimate the risk-adjusted returns, stocks on average should command at most a 1 per cent return premium over Treasury bonds. There lies the puzzle.

In recent years there have been many attempts to resolve the equity premium puzzle. None, however, has come close to fully resolving this anomaly. These attempts have become a major research impetus in both economics and finance. These attempts at explaining the puzzle can be classified into two broad approaches. The first approach centers around finding factors that demand adaptations to the empirical side of the puzzle. This could, for example, involve finding data that would result in the equity premium being smaller or the equity returns riskier. Among these approaches are survivorship bias, increasing the time period, taxes and mean reversion and aversion. The second approach surrounds the possibility of using different theoretical frameworks to solve the puzzle, such as habitat formation, catching up with the Joneses and Myopic loss aversion.

The purpose of this paper is to evaluate the equity risk premium for a survivor and a non-survivor for the period 1996 – 2010 using the Mehra and Prescott (1985) model and see whether or not a puzzle existed in the wake of the financial crisis. The U.S. was chosen as the
Conclusion

survivor due to its size and ability to withstand financial downturns, while Iceland was chosen due to its small size, immature equity market and the fact that its equity markets experienced disturbance in the wake of the crisis.

Prior to the crisis, the equity premiums in Iceland were almost 11.5% compared to 6% for the U.S. The financial crisis, however, caused the Icelandic equity market to collapse and as a result the returns on stocks plummeted and thereby eliminated any signs of an equity premium puzzle. The U.S., on the other hand, was able to withstand this crisis although their equity markets took a sharp dive. Looking at the period 1996 – 2010 the U.S. had a risk premium of almost 4% while Iceland had a negative premium of almost -9%. Only taking the past ten years into account will result in a negative premium for both countries. This period is, however, too short to make any inferences about whether the financial crisis has eliminated the puzzle for the U.S. The crisis has definitely had a profound effect on the equity premiums in both countries since both of them dropped substantially in the wake of the financial crisis. However, there are clear signs that both markets are recovering from the crisis as the returns on equity are again substantially higher than the returns on bonds. But due to the short life span of the Icelandic market it is safe to conclude that currently there does not exist an equity premium puzzle. Looking at historical data for the U.S. one can conclude that the equity premium puzzle still exists.

So why does there still exist a puzzle in the U.S.? One view might be that the stock markets have just been experiencing more than 200 years of good luck. History has just been kind to the stock markets, especially those in the larger markets. Another view might be that investors are extremely risk averse.

Whatever the reason is, it must, however, be stressed that the analysis which has been conducted over the past 25 years or so is based on historical data, which implies that in the past the equity premium has been too large. The high past returns are by no means a guarantee of high future returns. There is, however, evidence that the equity premium in the U.S. is lower today than it has been in the past. The results suggest that the premium has been decreasing over the past few years. What the future holds, however, remains to be seen.


References


References


Rannsóknarnefnd Alþingis. 2010. “Rannsóknarskýrsla Alþingis”.
Reykjavík: Rannsóknarnefnd


This documentation is based on a template created by Patrik FUHRER and Pedro DE ALMEIDA (Software Engineering Group, University of Fribourg, Switzerland).
Barro, Róbert. 2011, Data retrieved February 21, 2011 from
http://www.economics.harvard.edu/faculty/barro/data_sets_barro

Central Bank of Iceland (Seðlabanki Íslands), 2011. Retrieved February 24, 2011 from
http://www.sedlabanki.is

Kauphöll Íslands (2011). Retrieved February 20, 2011, from


Statistics Iceland (Hagstófa Íslands) 2011. Retrieved February 25 from http://www.hagstofa.is
The Nelson – Siegel Model

The ability to accurately estimate the current term structure of interest rates is extremely important in many areas of finance. Equally as important is the ability to predict the future term structure of interest. It should, therefore, not come as a shock that substantial research effort has been committed to the question of how to optimally estimate, model and forecast the term structure. One of the models that potentially offer answers to these questions is the Nelson and Siegel model (Pooter, 2007).

Nelson and Siegel (1987) proposed to fit the term structure using a simple and parsimonious model which was flexible enough to be able to represent all possible shapes the yield curve undertakes over time. The shapes the yield curve can assume are monotonic, humped and S-shaped. They demonstrated that their proposed model was able to fit to the yields of the U.S. Treasury bill and to forecast the yields of a long-term Treasury bond. The Nelson-Siegel curve, therefore, fits the term structure of interest rates at any given point in time with the simple functional form

\[ Y(\tau) = \beta_1 + \beta_2 \left( \frac{1 - e^{-\lambda\tau}}{\lambda\tau} \right) + \beta_3 \left( \frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau} \right) \]

Where \( Y \) is the yield on a zero coupon bond with \( \tau \) denoting the time to maturity of the security, which is usually ranging from three months up to 30 years. The four key parameters of the model are \( \lambda \), which determines the rate of exponential decay of the loadings for different maturities, and \( \beta_1, \beta_2 \) and \( \beta_3 \). This particular loading structure of the model implies that the betas have the following attributes
$\beta_1$ is responsible for parallel yield curve shifts, since the effect of this parameter is identical for all maturities.

$\beta_2$ represents the slope of the yield curve, since it has maximum effects on shorter maturity yields and minimum effects on longer maturity yields.

$\beta_3$ represents the curvature of the yield curve, since its factor has a hump in the middle of the maturity spectrum and has little effects on both short and long maturities.

These three parameters, therefore, have the interpretation of the yield curve level, slope and curvature.