

Bachelor Thesis

Warped Extra Dimensions in the Randall-Sundrum Model and a Simple Implementation in `PYTHIA 8`

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Abstract

We explore the theory and phenomenology of spatial extra dimensions in the Randall-Sundrum scenario; a possible extension to the Standard Model. We motivate the need for beyond standard model extensions and show how the original Randall-Sundrum model solves the hierarchy problem of particle physics. We also treat an extension to the Randall-Sundrum model where Standard Model fields live in the extra-dimensional bulk and see how it predicts resonant production of Kaluza-Klein excitations of the gauge bosons. In particular, we discuss Kaluza-Klein excited gluons and their subsequent decay into $t\bar{t}$ pairs. Furthermore, we test an implementation of the Randall-Sundrum model in PYTHIA 8 and compare the invariant mass of $t\bar{t}$ pairs coming from Kaluza-Klein gluons with results in the literature.

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Chapter 1

Introduction

Since the beginning of its development in 1960, the Standard Model (SM) of particle physics has allowed physicists to achieve astonishing advances in understanding the fundamental processes occurring in nature. The Standard Model has been remarkably accurate in its predictions and consistent with most experimental results¹. However, it has some shortcomings. One of the perhaps most important is the problem of the small Higgs mass, known as the hierarchy problem, which basically asks the questions why the weak force is a staggering 10^{32} times stronger than gravity. Another important problem comes with the integration of gravity itself. The Standard Model successfully describes the strong, weak and electromagnetic forces, but fails to incorporate the fourth and last known force: gravity. This lack of completeness is very unsatisfactory and leads us to believe that there must be a theory beyond the Standard Model. One popular beyond SM (BSM) extension is supersymmetry, or SUSY for short. Briefly put, SUSY extends on our normal spacetime symmetries and requires that every particle has a supersymmetric partner thus at least doubling the amount of particles needed. Another highly promising BSM extension is that of extra spatial dimensions, which is what we will be focusing on in this paper.

A rough layout of the paper is as follows: In Section 1.1 a brief overview of the Standard Model is given. This is followed by an introduction to the Kaluza-Klein theory in Section 2.1, and large extra dimensions in Section 2.2. Thereafter a thorough review of the Randall-Sundrum model follows in Section 2.3, with highlights in Section 2.3.3 where the hierarchy problem is resolved and in Section 2.3.4 where the original Randall-Sundrum is extended to accommodate SM fields in the extra-dimensional bulk. This extension is then tested in a PYTHIA 8 implementation in Section 3.3.

¹One exception of this is neutrino oscillations, indicating that neutrinos have a small mass. In the SM, neutrinos are massless particles which means that the SM needs to be modified to be consistent with the data.

1.1 The standard model

The Standard Model is a theory describing the known elementary particles and their interactions by combining the theory of electroweak interaction and quantum chromodynamics (QCD). The former being a combination of electromagnetism and weak interaction, and the latter the theory of strong interaction. More precisely, the SM is a relativistic quantum gauge field theory with gauge group $SU(3) \times SU(2) \times U(1)$, where the gauge group $SU(2) \times U(1)$ is the group of electroweak interactions and $SU(3)$ the gauge group of strong interactions. In the SM elementary particles are divided into three main groups: leptons, quarks and bosons.

Both the leptons and the quarks are organized as pairs, or weak isospin $SU(2)_L$ doublets, with each group arranged in three families. The three lepton families each contain a charged lepton and its associated neutrino,

$$\begin{pmatrix} e \\ \nu_e \end{pmatrix}, \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix} \text{ and } \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix},$$

while the three quark families each contain two quarks,

$$\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix} \text{ and } \begin{pmatrix} t \\ b \end{pmatrix}.$$

In addition to electric charge, quarks also carry $SU(3)_C$ color charge. The leptons and the quarks are the building blocks of matter and they belong to the group of particles called fermions, with spin $1/2$. There are in total 24 elementary fermions, 12 leptons and 12 quarks. The “extra” 6 leptons and quarks, which we have not listed here, come from the fact that every particle has its associated antiparticle with opposite electric charge. Table 1 and 2 lists the known elementary fermions and their mass. The third group, the bosons, are particles with integer spin and they are the carriers of the four known forces: electromagnetism, weak force, strong force and gravity. Each force acts with a different strength, given by a corresponding coupling strength α . Table 3 groups the known forces with their associated boson(s).

In order for the SM to have massive gauge bosons and maintain gauge invariance, the $SU(2) \times U(1)$ symmetry has to be spontaneously broken by a scalar field, known as the Higgs field. Massless gauge bosons from the $SU(2)$ group acquire mass by interacting with the Higgs field, while photons, the gauge boson in the $U(1)$ group, remains massless. In fact, it is through the coupling to the Higgs field that all particles acquire their mass.

Table 1.1: List of leptons and their masses. Data taken from the Particle Data Group [1].

Leptons (spin = 1/2)		
Particle/Antiparticle	Symbol	Mass (MeV)
Electron/Positron	$e/\bar{e}$	0.511
Electron neutrino/Electron antineutrino	$\nu_e/\bar{\nu}_e$	$< 2 \times 10^{-6}$
Muon/Antimuon	$\mu/\bar{\mu}$	105.658
Muon neutrino/Muon antineutrino	$\nu_\mu/\bar{\nu}_\mu$	< 0.19
Tau/Antitau	$\tau/\bar{\tau}$	1776.82
Tau neutrino/Tau antineutrino	$\nu_\tau/\bar{\nu}_\tau$	< 18.2

Table 1.2: List of quarks and their masses. Data taken from the Particle Data Group [1].

Quarks (spin = 1/2)		
Particle/Antiparticle	Symbol	Mass
Up/Antiup	$u/\bar{u}$	2.5
Down/Antidown	$d/\bar{d}$	5.0
Charm/Anticharm	$c/\bar{c}$	1.29×10^3
Strange/Antistrange	$s/\bar{s}$	100
Top/Antitop	$t/\bar{t}$	172.9×10^3
Bottom/Antibottom	$b/\bar{b}$	4.19×10^3

Table 1.3: List of forces and their mediating particles.

Forces and Gauge Bosons		
Force	Acts on	Mediating boson
gravity	all particles	graviton (G)
electromagnetism	all electrically charged particles	photon (γ)
weak interaction	quarks, leptons, electroweak gauge bosons	$W^\pm, Z^0$
strong interaction (QCD)	all colored particles (quarks and gluons)	8 gluons (g)

The Standard Model is by all means a highly successful model as it is consistent with most of the current data from particle accelerator experiments, and there is very little experimental evidence for any physics beyond the SM. Despite this, the SM does have a number of theoretical deficiencies which should not be present in a fundamental theory. For example,

- The SM does not contain a quantum description of gravity. In fact, there is no known way of consistently incorporating general relativity in terms of quantum field theory, the framework of the SM.
- The SM does not explain the hierarchy problem, *i.e.* , the huge energy difference between the Planck scale, the scale associated with gravity, and the electroweak scale, the scale at which the symmetry between electromagnetism and the weak interaction is broken.
- The SM requires a large set of (at least 19) arbitrary and unrelated parameters. To explain why neutrinos have mass, it is believed that up to eight more needs to be introduced.
- The SM cannot, without modification, be a candidate for a unified theory in which all forces unify at a certain energy, if such a theory exists. This can be realized by extrapolating gauge coupling measurements, which shows that the electromagnetic, weak and strong forces do not unify at any energy.

In light of this it is believed that the SM is just a low energy manifestation of a more fundamental theory. A number of theories that go beyond the SM have been proposed, such as technicolor, grand unified theories (GUT), supersymmetry (SUSY) and extra dimensions. In this paper our focus will be on the extra-dimensional scenario, in particular the bulk Randall-Sundrum model.

Chapter 2

Extra dimensions

The number of spatial dimensions is a measure of the number of degrees of spatial freedom available for movement in space. Our intuition tells us that we live in a Universe with three such dimensions. However, this might not be quite true. In 1914, Gunnar Nordström introduced the idea of using extra spatial dimensions to unify different forces. He proposed a five-dimensional theory to simultaneously describe electromagnetism and a scalar version of gravity. The idea of extra dimensions was further explored by Theodor Kaluza and Oscar Klein after the advent of general relativity. Kaluza realized in 1921 that the five-dimensional generalization of general relativity can simultaneously describe gravitational and electromagnetic interactions. Klein expanded on the notion of gauge invariance and the physical meaning of the compactification of extra dimensions. What today is called Kaluza-Klein theory utilizes both Kaluza's and Klein's ideas, as we will see in the next section. Although the Kaluza-Klein theory failed in its original intention of unifying all forces (of which all were not even discovered at the time) many models with extra spatial dimensions have subsequently been proposed. However, so far no complete and consistent extra-dimensional model has been constructed. One problem in many extra-dimensional models is that the sizes of the extra dimensions are near the Planck length and therefore practically impossible to probe experimentally. Recent models, however, provide a solution to this problem by proposing extra dimensions large enough to be experimentally accessible for current particle accelerators such as the LHC and the Tevatron.

In this chapter we briefly describe the Kaluza-Klein theory and the ADD theory of large extra dimensions. We then proceed in more detail with the original Randall-Sundrum model with one warped extra dimension. Here we lay out the theoretical framework before proceeding to explain how it resolves the hierarchy problem. Finally we discuss the bulk Randall-Sundrum model which we will be using in our computational analysis at the end of this paper.

2.1 The Kaluza-Klein idea

To get an understanding of some of the ideas and formalism of extra-dimensional theories, it can be useful to begin with the basic idea - the Kaluza-Klein scenario. We will closely follow Ref. [2] for this discussion. The simplest extra-dimensional case we can have is if we introduce just one extra spatial dimension, z . Using relativistic notation, we then write our (4+1)-dimensional space-time as (x^μ, z) , where $\mu = 0, 1, 2, 3$ is the spacetime index corresponding to (t, x^1, x^2, x^3) . If we take z to be compact with compactification radius R , *i.e.* z runs from 0 to $2\pi R$, and identify the extreme points so that $z = 0 = 2\pi R$, our space can be imagined as a four-dimensional cylinder whose three dimensions x^1, x^2 and x^3 are infinite, and the fourth dimension z is a circle of radius R . In this setup, the theory becomes effectively four-dimensional at low energies.

To show this, we assume the space generated by (x_1, x_2, x_3) is homogenous and has a flat metric. Then we can write a complete set of wave functions for a free massless particle on this space as

$$\phi_n = e^{ip_\mu x^\mu} e^{inz/R}, \quad n = 0, \pm 1, \pm 2, \dots, \quad (2.1)$$

where p_μ is the (3+1)-dimensional momentum and $p^z = n/R$ is the momentum in the periodic dimension. With $\phi(x^\mu, z)$ being solutions to the Klein-Gordon equation, and thus fulfilling the five-dimensional equation of motions $(\partial_\mu \partial^\mu + \partial_z \partial^z) \phi(x^\mu, z) = 0$, we obtain

$$p_\mu p^\mu - \frac{n^2}{R^2} = 0. \quad (2.2)$$

This generates an infinite tower of so-called Kaluza-Klein (KK) states¹, with masses $m^2 = n^2/R^2$. Below the energy scale $1/R$, only modes with $n = 0$, *i.e.* the z -independent massless zero-modes, are relevant, making the low energy physics effectively four-dimensional. At energies above $1/R$, however, the KK states become important to consider. No such KK states have been observed at colliders, implying their masses, n/R , must be greater than $\sim \text{TeV}$. This gives a constraint on the extra-dimensional size, R :

$$R \lesssim 10^{-21} \text{ cm}.$$

Searching for experimental confirmations of such tiny dimensions is nearly hopeless.

¹An analogy to this is the quantum mechanical particle in the box scenario, where the particle energy is quantized and increases with n^2 . In the Kaluza-Klein case, the circular fourth dimension z represents the box.

2.2 Large extra dimensions

In 1998, a new take on the extra-dimensional theories was presented by Arkani-Hamed, Dimopoulos and Dvali [3], where they suggest a scenario, which in addition to our four dimensions of spacetime, contains n extra spatial dimensions of finite size R , spanning a space they call “the bulk”. The complete space then has $(4 + n)$ dimensions and can be factorized into $D^4 \times M_n$, where M_n is a n -dimensional compact space with finite volume V_n . Furthermore, they assume all SM particles to live in our familiar four-dimensional domain, forming a hyperspace, or a “3-brane” within the bulk. Gravitons, the hypothetical mediators of gravity, however, are allowed to propagate throughout all $(3 + n)$ spatial dimensions, effectively diluting the gravitational force at distances less than R as it spreads throughout the $(3 + n)$ dimensions. A non-trivial task in this framework is localization of the SM fields, *i.e.* fixing their positions on the brane. One way out of this is to consider our extra-dimensional space M_n to be a n -dimensional torus of radius R and with volume $V_n = (2\pi R)^n$. Then the Einstein action² becomes [1]

$$S_E = \frac{\bar{M}^{2+n}}{2} \int d^4x d^n z \sqrt{-g} R_5, \quad (2.3)$$

where x and z are the common and extra dimensional coordinates respectively, g is the determinant of the metric, R_5 is the five-dimensional Ricci scalar and $\bar{M}$ is the reduced $(4 + n)$ -dimensional Planck mass. The integral can be separated because of the factorization of the spaces. By restricting the metric indices to four dimensions and integrating over z we obtain the four-dimensional reduced Planck mass

$$\bar{M}_{\text{Pl}}^2 = \bar{M}^{2+n} V_n = \bar{M}^{2+n} (2\pi R)^n, \quad (2.4)$$

where the integral in z reduces to the volume $V_n = (2\pi R)^n$ of the extra dimensions. If we take $M = (2\pi)^{n/(2+n)} \bar{M}$ to be the fundamental $(4 + n)$ -dimensional Planck mass, we get

$$\bar{M}_{\text{Pl}}^2 = M^{2+n} R^n = M^2 (MR)^n. \quad (2.5)$$

This implies that $\bar{M}_{\text{Pl}} = M(MR)^{n/2}$ and we can see that the Planck mass is much larger than the fundamental gravity scale, the $(4 + n)$ -dimensional Planck mass M , if the size of the extra dimensions is large compared to the fundamental length M^{-1} , as

$$R = M^{-1} \left(\frac{\bar{M}_{\text{Pl}}}{M} \right)^{2/n}. \quad (2.6)$$

²Action, usually denoted S , describes the dynamics of a physical system in terms of the Lagrangian, $\mathcal{L} = T - V$, where T and V is the kinetic and potential energy respectively, and is defined as the time integral of the Lagrangian:

$$S = \int \mathcal{L} dt$$

In Ref. [3] it is assumed that M is close to the weak scale $M_{\text{EW}} \sim \text{TeV}$. This solves the hierarchy problem – the hierarchy between M_{Pl} and M_{EW} – as it is caused entirely by the large size of the extra dimensions. However, this merely replaces one hierarchy with another, namely, that between the compactification scale $1/R$ and M_{EW} .

The number of extra dimensions is restricted to $n \geq 2$ since the case with $n = 1$ is empirically excluded as $R \sim 10^{13}$ cm and thus gives deviations from Newtonian gravity over solar system distances. For $n = 2$, $R \sim 100 \mu\text{m} - 1 \text{ mm}$ and since Newtonian gravity has been established experimentally down to distances of about $200 \mu\text{m}$ [4] only, this scenario is still possible. More dimensions added result in even smaller values of R .

2.3 The Randall-Sundrum model

The Randall-Sundrum model (RS1) [5] was introduced in 1999 by Lisa Randall and Raman Sundrum as an attempt to explain the hierarchy problem without having to introduce new hierarchies. The model requires the existence of one extra dimension compactified on a circle of radius r_c , with identity between upper and lower half (Fig. 2.1), *i.e.* $(x^\mu, \phi) = (x^\mu, -\phi)$, where ϕ is the angular coordinate for the fifth dimension.

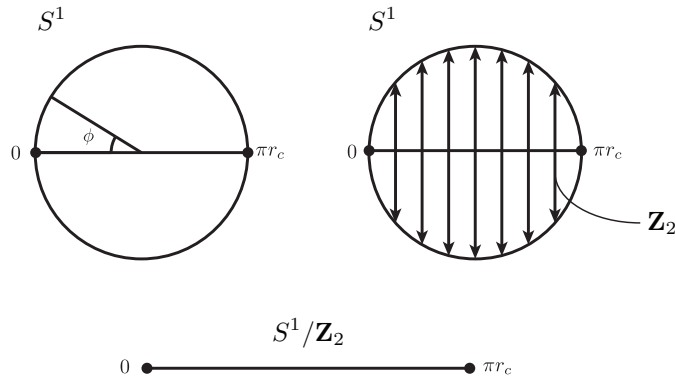


Fig. 2.1: Illustration of the $S^1/\mathbf{Z}_2$ orbifold, *i.e.* the $\mathbf{Z}_2$ orbifold of the circle. Effectively, the circle becomes a line with a fixed point at each end.

2.3.1 Setup

This means we are working in the space $S^1/\mathbf{Z}_2$, where S^1 is the circle group and $\mathbf{Z}_2$ is the multiplicative group $\{-1, 1\}$. The $\mathbf{Z}_2$ group is responsible for the identification $\phi = -\phi$, making the circle in effect become a line with two fixed points; one at $\phi = 0$ and one at $\phi = \pi$. This way of identifying coordinates of a symmetry group is called an orbifold.

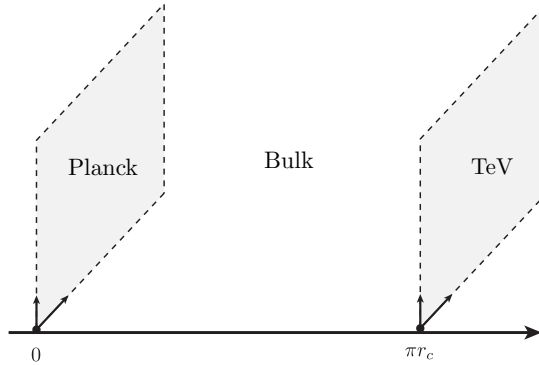


Fig. 2.2: The Randall-Sundrum scenario. Two 3-branes enclosing a five-dimensional bulk.

At each of these points sits a 3-brane, extending in the x^μ directions, enclosing a five-dimensional bulk (Fig. 2.2). The brane sitting at $\phi = 0$ is commonly referred to as the Planck brane, while the brane at $\phi = \pi$ is called the TeV brane. The reason for this, as we will see, is that the energy scales on the branes are in the order of the Planck and TeV scales respectively.

The fundamental action for such a scenario is:

$$S = \int d^4x \int_{-\pi}^{\pi} d\phi \sqrt{-g} \{M^3 R_5 - \Lambda\}, \quad (2.7)$$

where M is the fundamental five-dimensional mass scale, R_5 the five-dimensional Ricci scalar, *i.e.* the trace of the five-dimensional Ricci tensor, g the determinant of the five-dimensional metric and Λ the five-dimensional cosmological constant.

2.3.2 Warped metric

Unlike the ADD model, RS1 considers branes that produce gravitational fields, leading to a warping of the five-dimensional metric. Since the model is supposed to represent actual physics, it is important for it to have four-dimensional symmetry under boosts, rotations and translations, *i.e.* it has to preserve Poincaré invariance in the familiar four dimensions of space. This leads to the following ansatz for the metric:

$$ds^2 = e^{-2\sigma(\phi)} \eta_{\mu\nu} dx^\mu dx^\nu + r_c^2 d\phi^2, \quad (2.8)$$

where $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$ is the four-dimensional Minkowski metric and $e^{-2\sigma(\phi)}$ is the *warp factor*, causing the special geometry of the space.

To see what the warp factor, $\sigma(\phi)$, should be, we need to solve the five-dimensional Einstein equations

$$G_{NM} = R_{NM} - \frac{1}{2} g_{MN} R_5, \quad (2.9)$$

for our metric [Eq. (2.8)]. The indices M and N take the values 0, 1, 2, 3 and 5 and so G_{NM} is the five-dimensional Einstein tensor, R_{NM} the five-dimensional Ricci tensor and g_{NM} the five-dimensional metric. We will not work out the solution to Eq. (2.9) here, as it is just a mechanical operation and thus not important for the discussion. The interested reader might however have a look at Ref. [6] together with [5]. Instead, we go directly to the solution.

The five-dimensional component of G_{MN} is

$$G_{55} = \frac{6\sigma'^2}{r_c^2} = \frac{-\Lambda}{4M^3}. \quad (2.10)$$

Here, σ' is the derivative of $\sigma(\phi)$ with respect to ϕ . Rearranging and integrating gives

$$\sigma(\phi) = \pm k r_c \phi \quad (2.11)$$

with

$$\frac{-\Lambda}{24M^3} \equiv k^2. \quad (2.12)$$

Furthermore, remembering that our space has the symmetry $\phi = -\phi$, we can write

$$\sigma(\phi) = k r_c |\phi|. \quad (2.13)$$

Note that a real solution only exists for $\Lambda < 0$. This means that the space-time in between the two 3-branes is a slice of an anti-de Sitter space³, usually denoted by AdS_5 .

We now have our bulk metric:

$$ds^2 = e^{-2kr_c|\phi|} \eta_{\mu\nu} dx^\mu dx^\nu + r_c^2 d\phi^2. \quad (2.14)$$

2.3.3 Solving the hierarchy problem

We will now see how the physical scales arise naturally from the Randall-Sundrum setup with the metric [Eq. (2.14)] just obtained. With all matter fields confined to the 3-brane at $\phi = \pi$, the action from a fundamental Higgs field is

$$\begin{aligned} S_{Higgs} &= \int d^4x \sqrt{-g_2} \{g_2^{\mu\nu} D_\mu H^\dagger D_\nu H - \lambda(|H|^2 - v_0^2)^2\} \\ &= \int d^4x e^{-4kr_c\pi} \{e^{2kr_c\pi} \eta^{\mu\nu} D_\mu H^\dagger D_\nu H - \lambda(|H|^2 - v_0^2)^2\}. \end{aligned} \quad (2.15)$$

³The anti-de Sitter space has a constant negative curvature and is a solution to Einstein's equations with a negative cosmological constant. For the interested reader, an introduction to AdS geometry is given in Ref. [7].

Normalizing the wave-function, $H \rightarrow e^{kr_c\pi} H$, gives us

$$S_{Higgs} = \int d^4x \{ \eta^{\mu\nu} D_\mu H^\dagger D_\nu H - \lambda (|H|^2 - (e^{-2kr_c\pi} v_0^2)^2) \}. \quad (2.16)$$

Here, $g_2^{\mu\nu} = e^{2kr_c\pi} \eta^{\mu\nu}$ is the metric on the 3-brane positioned at $\phi = \pi$, g_2 is its determinant, H is the Higgs field, v_0 is the Higgs vacuum expectation value and λ is the Higgs self-coupling strength. D_μ is the common gauge covariant derivative, $D_\mu = \partial_\mu - ieA_\mu$. Equation (2.16) is the action of a normal Higgs scalar, but with one important exception: the vacuum expectation value is suppressed, and the physical mass scales are set by a symmetry-breaking scale

$$v \equiv e^{-kr_c\pi} v_0. \quad (2.17)$$

This is a completely general result as the Higgs vacuum expectation value sets all the mass parameters in the Standard Model; any mass parameter m_0 on the TeV brane will correspond to a physical mass

$$m \equiv e^{-kr_c\pi} m_0. \quad (2.18)$$

We now want to determine the effective four-dimensional aspect of the theory which will give us information on how the effective scale of gravity behaves with respect to the extra dimension. Knowing our bulk metric [Eq. (2.14)], we perturbate the action in Eq. (2.7) around it, which leads us to the following schematic form of the effective action:

$$\begin{aligned} S_{\text{eff}} &\supset \int d^4x \int_{-\pi}^{\pi} d\phi 2M^3 r_c e^{-2kr_c|\phi|} \sqrt{-\bar{g}} R_4 \\ &= \frac{M^3}{k} [1 - e^{-2kr_c\pi}] \int d^4x \sqrt{-\bar{g}} R_4, \end{aligned} \quad (2.19)$$

where R_4 is the four-dimensional Ricci scalar made out of the unwarped brane metric $\bar{g}_{\mu\nu}$. We can now read off the value of the effective four-dimensional Planck mass:

$$M_{\text{Pl}}^2 = \frac{M^3}{k} [1 - e^{-2kr_c\pi}]. \quad (2.20)$$

This tells us M_{Pl} , the energy scale of gravity, depends only weakly on the size of the extra dimension, r_c , provided kr_c is moderately large.

We can now understand how a exponential hierarchy between the weak and the gravity scales arise naturally from the theory. From Eq. 2.17 we see that the weak scale is exponentially suppressed along the extra dimension, while Eq. 2.20 tells us that the gravity scale is mostly unaffected by it (see Fig. 2.3). To generate the TeV energy scale, at which SM interactions occur, from the Planck scale at about 10^{19} GeV, the factor $e^{kr_c\pi}$ needs to be of order 10^{15} . Luckily, this is not a problem. The exponential nature

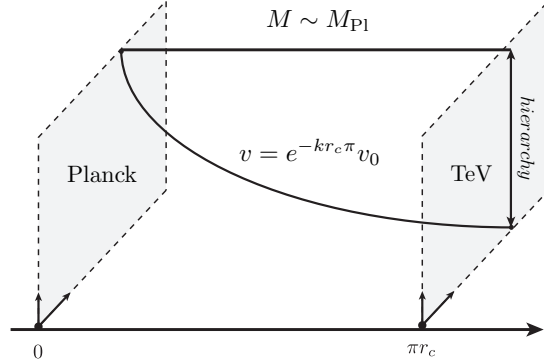


Fig. 2.3: The generation of an exponential hierarchy.

of the warping factor allows for all the fundamental parameters, v_0, k, M and $\mu_c \equiv 1/r_c$ to be set by the TeV scale and no large hierarchies are needed. In fact, the appropriate value for the size of the extra dimension is given by $kr_c \simeq \ln 10^{15}/\pi \simeq 10$ [5]. Thus the Randall-Sundrum model provides a novel solution to the hierarchy problem.

2.3.4 Bulk Randall-Sundrum

While the RS1 model provides a great first experience with a warped extra-dimensional framework and gives a natural solution to the hierarchy problem, a more realistic and theoretically satisfying model would be one with all SM fermions and gauge bosons living in the extra dimension; the Higgs boson, however, is confined to the TeV brane in order to generate spontaneous symmetry breaking [2]. Such models not only take care of the hierarchy problem, they also provide a solution to the flavor puzzle of the SM, explain the fermion mass hierarchy [8, 9] and allow for gauge coupling unification [10, 11, 12]. In this scenario, the SM particles are identified with the zero modes of the five-dimensional fields, and the first signal of this scenario is believed to be the first excited Kaluza-Klein (KK) partner of the SM gluon, called the KK gluon ($g^{(1)}$) [13]. We will follow Ref. [14] together with Ref. [2] to give a brief overview of the formalism describing the gauge and fermion fields in this model. We will also discuss the couplings between the SM fermions and the KK gauge bosons.

The action S_f for a free fermion, Ψ , of mass m is [2]

$$S_f = \int d^4x \int d\phi e^{-4k\pi\phi} \left[V_N^M \left(\frac{i}{2} \bar{\Psi} \gamma^N \partial_M \Psi + h.c. \right) - \text{sgn}(\phi) m \bar{\Psi} \Psi \right], \quad (2.21)$$

where $\gamma^N = (\gamma^\mu, \gamma^5)$ are the gamma matrices, $V_N^M = \text{diag}(e^{k\pi\phi}, e^{k\pi\phi}, e^{k\pi\phi}, e^{k\pi\phi}, -1)$ is a diagonal matrix, $\text{sgn}(\phi)$ is 1 for $\phi > 0$ and -1 for $\phi < 0$, and $h.c.$ is the Hermitian

conjugate term. We can now perform a KK expansion⁴ of the field Ψ ,

$$\Psi_{L,R}(x, \psi) = \sum_{n=0}^{\infty} \psi_{L,R}^{(n)}(x) \frac{e^{k\pi\phi}}{\sqrt{r_c}} \hat{f}_{L,R}^{(n)}(\phi). \quad (2.22)$$

Here, L and R refer to the chirality of the fields and $\hat{f}_{L,R}^{(n)}$ are two orthonormal wave-functions. Due to the orbifolding, $\mathbf{Z}_2$, of the space, we can let the zero-modes of the left-handed function, $\hat{f}_L^{(0)}$, correspond to the SM fields. In particular, we have

$$\begin{aligned} f_L^{(0)} &= \frac{e^{\nu_L k\pi\phi}}{N_0^L} \\ f_R^{(0)} &= \frac{e^{\nu_R k\pi\phi}}{N_0^L}, \end{aligned} \quad (2.23)$$

where $\nu_{L,R}$ is the fermion localization parameter and N_0^L is a normalization factor. These wave functions are exponentially peaked toward the Planck brane for $\nu < -1/2$ and toward the TeV brane for $\nu > 1/2$. The zero-modes acquire mass through the standard Higgs mechanism, and since the Higgs is localized to the TeV brane, the effective Yukawa coupling is

$$\lambda_f = \lambda_0 f_L^{(0)}(\pi) f_R^{(0)}(\pi), \quad (2.24)$$

for a fermion f . λ_0 is the dimensionless five-dimensional Yukawa coupling. By choosing different values of order one for $\nu_{L,R}$, the entire fermion mass hierachy can be generated. In this way, the bulk RS1 model can in a natural way accomodate for both the large top quark mass and the significantly lower electron mass; we simply place the top quark near the TeV brane, giving it a large overlap with the Higgs wavefunction, and the electron near the Planck brane, achiving the opposite effect.

For the gauge bosons in five dimensions, we have the action

$$S_g = \int d^4x d\phi \sqrt{g} - \frac{1}{4} F_{\mu\nu}^a F^{\mu\nu,a}. \quad (2.25)$$

We choose the gauge $A_5 = 0$ and do a KK expansion of $A_\mu(x, \phi)$,

$$A_\mu(x, \phi) = \sum_{n=0}^{\infty} A_\mu^{(n)}(x) \frac{\chi_A^{(n)}(\phi)}{\sqrt{r_c}}. \quad (2.26)$$

The solutions to the wavefunction $\chi_A^{(n)}(\phi)$ are

$$\chi_A^{(n)} = \frac{e^{k\pi\phi}}{N_A^{(n)}} \left[J_1(m_A^{(n)} e^{k\pi\phi}/k) + \alpha_A^{(n)} Y_1(m_A^{(n)} e^{k\pi\phi}/k) \right], \quad (2.27)$$

⁴Basically a generalized Fourier expansion in 5 dimensions.

where J_1 and Y_1 are the standard Bessel functions of the first and second kind, $m_A^{(n)}$ is the mass of the n th KK mode and $N_A^{(n)}$ is a normalization factor. In particular, the zero-mode of the wavefunction is $\chi_A^{(0)} = 1/\sqrt{2\pi}$.

Finally, we have the coupling between a zero-mode fermion f , *i.e.* a ordinary SM fermion, and a KK boson as

$$g_{f\bar{f}A^{(n)}} = g_s \sqrt{2\pi k r_c} \left[\frac{1 + 2\nu}{1 - \epsilon^{2\nu+1}} \right] \int_{\epsilon}^1 dz z^{2\nu+1} \frac{J_1(x_n^A z) + \alpha_n^A Y_1(x_n^A z)}{|J_1(x_n^A) + \alpha_n^A Y_1(x_n^A)|}, \quad (2.28)$$

where g_s is the strong coupling constant in the SM, $\epsilon \equiv e^{-k r_c \pi}$ and x_n^A denote the roots to the Bessel functions. Since the fermions are located at different points in the bulk, they have different values on the localization parameter ν [14]:

$$\begin{aligned} \nu_{t_R} &\approx -0.3, \\ \nu_{Q_{3L}} &\approx -0.4, \\ \nu_{\text{other}} &< -0.5, \end{aligned} \quad (2.29)$$

where t_R is the right-handed top quark and Q_{3L} is the third generation quark doublet. Using these parameters in Eq. (2.28), we obtain the coupling to the first KK excitation of the gluon, $g^{(1)}$, as: $g_{f\bar{f}g^{(1)}} \simeq g_s$ for the Q_{3L} doublet, $g_{f\bar{f}g^{(1)}} \simeq 4g_s$ for the t_R and $g_{f\bar{f}g^{(1)}} \simeq g_s/5$ for the others (light quarks).

2.4 Concluding remarks

While the RS1 framework was explained in detail in this chapter, leading to the resolution of the hierarchy problem, we have refrained ourselves from discussing the phenomenological aspects of the theory, such as experimentally accessible graviton resonances in the TeV region. The reason for this is that our main interest in the RS1 model lies in its framework, not its phenomenology, which is needed to understand the RS1 extension with SM fields in the bulk. Since the original RS1 model has been more and more left behind in favor for its improved extension with bulk SM fields, where lots of progress have been made in recent years, we have chosen to investigate the phenomenology of this extension instead. In particular, our interest has been towards the KK gluon resonance.

While the original RS1 model's main strength is its resolution to the eluding hierarchy problem in addition to being experimentally testable, models with SM fields in the bulk provide additional desirable features, which were mentioned earlier. This does indeed make the bulk RS model one of the most promising BSM extensions.

Chapter 3

Bulk Randall-Sundrum in PYTHIA 8

Although the bulk Randall-Sundrum scenarios provide many phenomenologically interesting production channels, with *e.g.* W' , Z' and composite Higgs, our focus has been on the production of first order Kaluza-Klein gluons through quark-antiquark annihilation since it should provide one of the first indications of an RS framework [14]. We consider KK gluons with a mass of order TeV which are expected to decay primarily into highly energetic top-antitop final states. The phenomenology of first order KK gluons in the RS context has been studied extensively [13, 14, 15, 16, 17, 18], and our reason for additional testing is mostly educational. However, the PYTHIA 8 implementation of this process that we will be using is quite new and it is thus of value to test how well it performs (even though this has been done before in Ref. [19]). Studies of the KK gluon in the context of a bulk RS1 scenario has given rise to restrictions on its mass. For example, the direct experimental search for KK gluon excitations at Tevatron Run II sets a lower bound on the KK gluon mass of $M_{KK} \gtrsim 800$ GeV [20]. Meanwhile, constraints from precision electroweak data give $M_{KK} \gtrsim 2 - 3$ TeV [21], although such a constraint is model dependent. We will be comparing our results with Ref. [14], which investigates generic properties of the KK gluon, and are therefore free to use KK gluons with a broader range of masses.

In this chapter we will first give a brief introduction to the PYTHIA program before going on to discuss the implementation of the above mentioned process, *i.e.* $q\bar{q} \rightarrow g^{(1)} \rightarrow t\bar{t}$, where $g^{(1)}$ is the lowest order KK-gluon. We will then present our results and compare them to results in the literature, and finally we will conclude our work and briefly discuss the outlook and results from ongoing and future experiments.

Note that this chapter is mostly intended to give a brief experimental aspect and act as a counterweight to the otherwise theoretically heavy content of this paper. It is in no way intended as a complete and accurate testing of the PYTHIA 8 program and will therefore be kept relatively short.

3.1 PYTHIA 8

PYTHIA 8 [22] is a high-energy Monte Carlo event generator, capable of generating the complete event of high-energy particle collisions including the normally complex final states, with large multiplicities of hadrons, leptons, photons and neutrinos. Even more so, it can do it using a large set of physical models. PYTHIA 8 is the newest version in the PYTHIA family, and while its predecessors were written in Fortran, PYTHIA 8 has been completely rewritten in C++ to become better suited for the LHC era. Here it is used as a standalone program to generate the particle level final state but in experimental searches PYTHIA must be interfaced with detector specific simulation of the detector response.

3.2 Implementation and parameters

To simulate the production of $t\bar{t}$ final states through $q\bar{q} \rightarrow g^{(1)} \rightarrow t\bar{t}$ in the bulk RS1 scenario, we will be using the PYTHIA 8 process `qqbar2KKgluon*` which assumes that the KK gluon, which has particle id number 5100021, can be produced only by quark–antiquark annihilation [19], *i.e.* $q\bar{q} \rightarrow g^{(1)} \rightarrow q\bar{q}$. The couplings to quarks, both left and right-handed, are general and can be set by the user. In our case, we will be using the coupling strengths obtained in Section 2.3.4, namely: $g_{f\bar{f}g^{(1)}} \simeq g_s$ for the Q_{3L} doublet, $g_{f\bar{f}g^{(1)}} \simeq 4g_s$ for the t_R and $g_{f\bar{f}g^{(1)}} \simeq g_s/5$ for the others. Thus, the coupling to the right-handed top quarks is significantly stronger than to the other quarks and therefore the decay into top–antitop pairs is highly favored with a branching ratio of 92.5%. The KK gluon mass can also be specified by the user, and we will be testing masses of 2, 3, 5 and 7 TeV, in accordance to Ref. [14]. Furthermore, the implementation includes an option to run the simulation with contribution from either just KK gluons, SM gluons or both at the same time.

3.3 Simulation and results

The analysis was done by generating events with the PYTHIA 8 process `qqbar2KKgluon*` which was discussed in the previous section (3.2). Since our only interest in the event

record are KK gluons decaying into $t\bar{t}$ pairs, we select only those events with a simple algorithm which find the t and $\bar{t}$ quarks that have a common KK gluon mother particle. We run our simulation with a center of mass energy $\sqrt{s} = 14$ TeV and assume only KK gluon contribution to the $t\bar{t}$ production. Furthermore, we do not include a SM background in the simulation since we are only interested in the KK gluon production itself and not whether it can be detected through a SM background. Such an analysis, although very important, lies outside the scope of this paper and is left to Refs. [13] and [14].

Our results of the invariant mass distribution of $t\bar{t}$ pairs coming from KK gluons with masses 2, 3, 5 and 7 TeV are shown in Fig. 3.1. It is apparent that the relative differences between peaks are very close to those in Ref. [14] (Fig. 3.2). The lack of SM contribution in our simulation manifests itself in the tails of the resonances, which can be seen in Fig. 3.1 to be suppressed at both the low and high mass end of the spectrum compared to those in Fig. 3.2. However, it is worth noting that the missing SM contribution does not have much affect on the shape of the peaks, except in the 5 TeV peak which has a slightly narrower cutoff on both sides than the corresponding peak in Fig. 3.2. Overall, the PYTHIA 8 implementation of the RS model that we have tested here is in good agreement with previous studies. This is also confirmed by Ref [19], which performs a more thorough testing of the implementation.

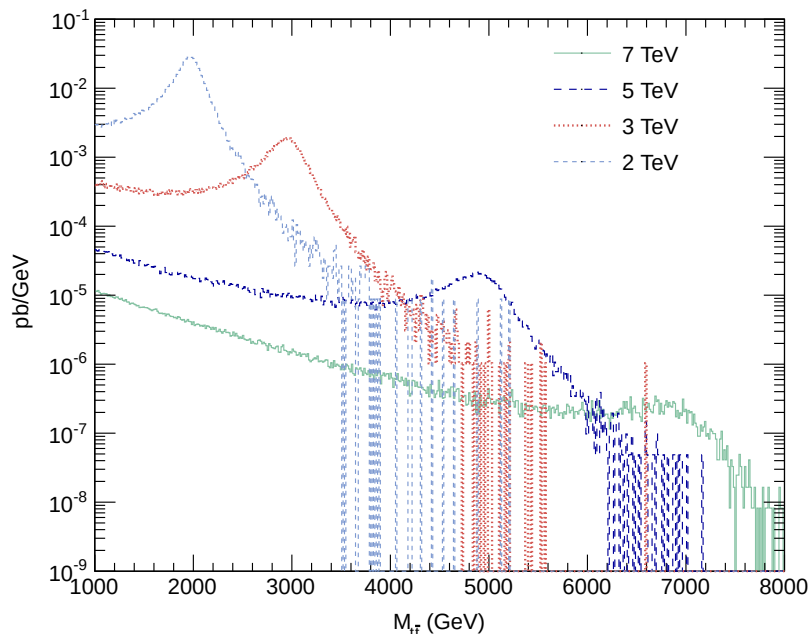


Fig. 3.1: Invariant mass distribution of $t\bar{t}$ pairs coming from KK gluon resonance as simulated in PYTHIA 8.

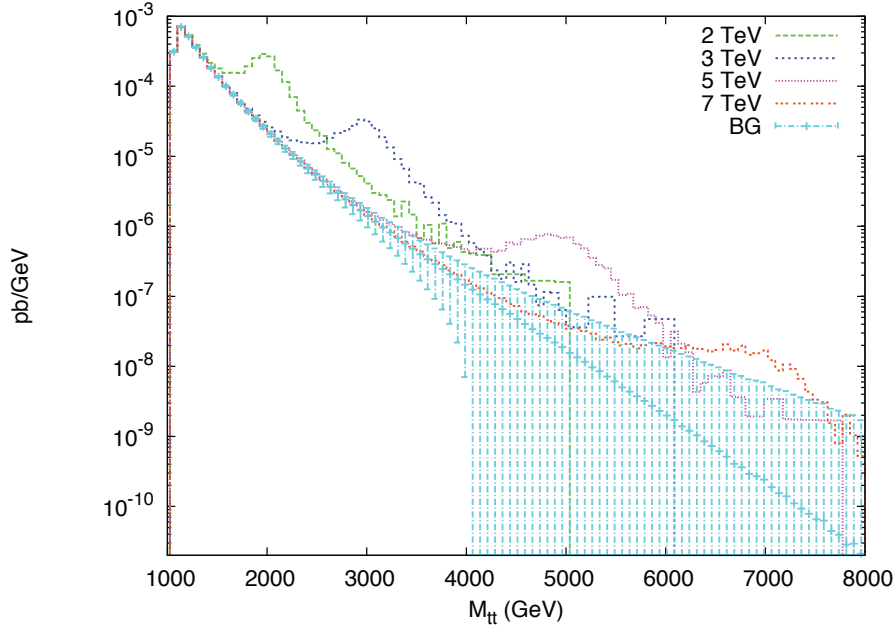


Fig. 3.2: Reference invariant mass distribution of $t\bar{t}$ pairs coming from KK gluon resonance and SM $t\bar{t}$ production [14]

3.4 Outlook

An important, and exciting, aspect of RS models with bulk SM fields is that they are within the range of our current, or soon to come, experimental capacity. In particular, the advent of the LHC will be helpful in probing these theories. One of the first signals of a RS framework is believed to be the heavy KK gluon resonance with mass $M_{KK} \gtrsim 2 - 3$ TeV, decaying into $t\bar{t}$ pairs. Ongoing experiments at the LHC are looking for heavy $t\bar{t}$ resonances and the current bounds excludes KK gluons with mass less than 650 GeV [23]. The search for these resonances has only begun and the current lower bound does not exclude the KK gluons expected from the theory. However, detecting top quarks coming from heavy KK gluons is rather challenging as such energetic top quarks are expected to be highly boosted, *i.e.* not well separated, and therefore hard to identify. In Ref [14] it is shown that top resonances with mass less than 5 TeV are possible to discover if the algorithm to identify tops with high transverse momentum, p_T , can reject the QCD background by a factor of 10.

Chapter 4

Summary and conclusions

In this paper we have taken a look at extensions of the Standard Model involving the existence of extra spatial dimensions. We first gave a brief overview of the Standard Model, before proceeding to introduce the idea of extra spatial dimensions. We used the original Kaluza-Klein theory to further explain the use of extra dimensions. Thereafter, the ADD theory of large extra dimensions was briefly discussed before we proceeded to the main part of the paper: the Randall-Sundrum scenario. In this part we gave an extensive review of the RS1 model, starting with the setup of the framework including an explanation of the warped metric. Using this knowledge, we then explained in detail how the RS1 model solves the hierarchy problem; one of the biggest questions in particle physics. From there we went on to discuss an extension of the original RS1 model where SM fields live in the five-dimensional bulk, as opposed to the original model where SM fields are confined to the TeV brane. In this section we addressed the fermion hierarchy problem which can be explained by the idea of placing fermions at different distances from the TeV brane resulting in smaller or larger Yukawa couplings. This section also discusses the theoretical framework leading to the fermion localization parameters used in the final section of the paper where a more experimental approach is explored. In this part an implementation of the bulk RS model in the high-energy Monte Carlo event generator, PYTHIA 8, was tested. Here we looked at the invariant mass distribution of $t\bar{t}$ pairs originating from KK gluons with masses 2, 3, 5 and 7 TeV respectively, and compared our results with those in Ref. [14]. Our findings show that the RS implementation `qqbar2KKgluon*` in PYTHIA 8 give results which are in good agreement with the literature. Lastly, we gave a brief discussion on the outlook of experimental searches for an RS framework through the detection of KK gluons, which is looking promising although improvements in top identification algorithms seem to be necessary.

This work demonstrates how theory is put into practical use and show how Monte Carlo generators are a great tool to use to guide searches in future experiments with.

Bibliography

- [1] K. Nakamura et al. (Particle Data Group). *J. Phys. G*, 37. 075021 (2010) and 2011 partial update for the 2012 edition.
- [2] H. Davoudiasl, J. L. Hewett, and T. G. Rizzo. Experimental probes of localized gravity: On and off the wall. *Phys. Rev. D*, 63:075004, Mar 2001. doi: 10.1103/PhysRevD.63.075004. URL <http://link.aps.org/doi/10.1103/PhysRevD.63.075004>.
- [3] Nima Arkani-Hamed, Savas Dimopoulos, and Gia Dvali. The hierarchy problem and new dimensions at a millimeter. *Physics Letters B*, 429(3-4):263 – 272, 1998. URL <http://www.sciencedirect.com/science/article/pii/S0370269398004663>.
- [4] Joshua C. Long, Hilton W. Chan, Allison B. Churnside, Eric A. Gulbis, Michael C. M. Varney, and John C. Price. Upper limits to submillimetre-range forces from extra space-time dimensions. *Nature*, 421(6926):922–925, 02 2003. URL <http://dx.doi.org/10.1038/nature01432>.
- [5] Lisa Randall and Raman Sundrum. Large mass hierarchy from a small extra dimension. *Phys. Rev. Lett.*, 83:3370–3373, Oct 1999. URL <http://link.aps.org/doi/10.1103/PhysRevLett.83.3370>.
- [6] Valerii A Rubakov. Large and infinite extra dimensions. *Physics-Uspekhi*, 44(9): 871, 2001. URL <http://stacks.iop.org/1063-7869/44/i=9/a=R01>.
- [7] Ingemar Bengtsson. Anti-de sitter space. 1998. URL <http://www.physto.se/ingemar/Kurs.pdf>. Lecture notes.
- [8] Yuval Grossman and Matthias Neubert. Neutrino masses and mixings in non-factorizable geometry. *Physics Letters B*, 474(3-4):361 – 371, 2000. ISSN 0370-2693. doi: 10.1016/S0370-2693(00)00054-X. URL <http://www.sciencedirect.com/science/article/pii/S037026930000054X>.
- [9] Tony Gherghetta and Alex Pomarol. Bulk fields and supersymmetry in a slice of ads. *Nuclear Physics B*, 586(1-2):141 – 162,

2000. ISSN 0550-3213. doi: 10.1016/S0550-3213(00)00392-8. URL <http://www.sciencedirect.com/science/article/pii/S0550321300003928>.
- [10] Kaustubh Agashe, Roberto Contino, and Raman Sundrum. Top compositeness and precision unification. *Phys. Rev. Lett.*, 95: 171804, Oct 2005. doi: 10.1103/PhysRevLett.95.171804. URL <http://link.aps.org/doi/10.1103/PhysRevLett.95.171804>.
- [11] Lisa Randall and Matthew D. Schwartz. Unification and hierarchy from 5d anti-de sitter space. *Phys. Rev. Lett.*, 88: 081801, Feb 2002. doi: 10.1103/PhysRevLett.88.081801. URL <http://link.aps.org/doi/10.1103/PhysRevLett.88.081801>.
- [12] Walter D. Goldberger and Ira Z. Rothstein. Effective field theory and unification in ads backgrounds. *Phys. Rev. D*, 68: 125011, Dec 2003. doi: 10.1103/PhysRevD.68.125011. URL <http://link.aps.org/doi/10.1103/PhysRevD.68.125011>.
- [13] Kaustubh Agashe, Alexander Belyaev, Tadas Krupovnickas, Gilad Perez, and Joseph Virzi. Cern lhc signals from warped extra dimensions. *Phys. Rev. D*, 77:015003, Jan 2008. doi: 10.1103/PhysRevD.77.015003. URL <http://link.aps.org/doi/10.1103/PhysRevD.77.015003>.
- [14] Ben Lillie, Lisa Randall, and Lian-Tao Wang. The bulk rs kk-gluon at the lhc. *Journal of High Energy Physics*, 2007(09):074, 2007. URL <http://stacks.iop.org/1126-6708/2007/i=09/a=074>.
- [15] Ben Lillie, Jing Shu, and Tim M. P. Tait. Kaluza-klein gluons as a diagnostic of warped models. *Phys. Rev. D*, 76: 115016, Dec 2007. doi: 10.1103/PhysRevD.76.115016. URL <http://link.aps.org/doi/10.1103/PhysRevD.76.115016>.
- [16] Abdelhak Djouadi, Grégory Moreau, and Ritesh K. Singh. Kaluza-klein excitations of gauge bosons at the lhc. *Nuclear Physics B*, 797(1 - 2):1 - 26, 2008. ISSN 0550-3213. doi: 10.1016/j.nuclphysb.2007.12.024. URL <http://www.sciencedirect.com/science/article/pii/S0550321307009704>.
- [17] Rikkert Frederix and Fabio Maltoni. Top pair invariant mass distribution: a window on new physics. *Journal of High Energy Physics*, 2009(01):047, 2009. URL <http://stacks.iop.org/1126-6708/2009/i=01/a=047>.
- [18] Michele Redi and Andreas Weiler. Flavor and cp invariant composite higgs models. 2011. [arXiv:hep-ph/1106.6357v3].

- [19] S. Ask, J.H. Collins, J.R. Forshaw, K. Joshi, and A.D. Pilkington. Identifying the colour of tev-scale resonances. 2011. [arXiv:hep-ph/1108.2396v1].
- [20] Manoranjan Guchait, Farvah Mahmoudi, and K. Sridhar. Tevatron constraint on the kaluza-klein gluon of the bulk randall-sundrum model. *Journal of High Energy Physics*, 2007(05):103, 2007. URL <http://stacks.iop.org/1126-6708/2007/i=05/a=103>.
- [21] Kaustubh Agashe, Antonio Delgado, Michael J. May, and Raman Sundrum. Rs1, custodial isospin and precision tests. *Journal of High Energy Physics*, 2003(08):050, 2003. URL <http://stacks.iop.org/1126-6708/2003/i=08/a=050>.
- [22] Torbjörn Sjöstrand, Stephen Mrenna, and Peter Skands. A brief introduction to pythia 8.1. *HEP05 (2006) 026, Comput. Phys. Comm. 178 (2008) 852*. URL <http://www.sciencedirect.com/science/article/pii/S0010465508000441>.
- [23] Tobias Golling for The ATLAS Collaboration. Search for new physics involving top quarks at atlas. 2011. [arXiv:hep-ex/1109.6734v1].