Evaluating the Origin of Water Vapour in Giant Stars

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Abstract

The advent of infrared astronomy, unlike optical astronomy that was developed over centuries, was delayed until the space age due to our own atmosphere acting as an efficient shield towards this radiation. With a lot of molecules (including water) emitting in the infrared region, it was first in the late 1990s that water no longer was deemed special and was observed in many cooler (K-M) giant stars. Although it was theoretically predicted that stars of these temperatures would form water, it failed to account for water in the warmer K-type stars, thus challenging our knowledge of the atmosphere of these red giants. In time, two scenarios were developed in order to explain the discrepancies between the theoretical and observed spectra. The first being the addition of a warm molecular envelope known as MOLsphere, and the second being a modified temperature profile of the star. The aim of this project was to shed light and put constraints on these scenarios. With the use of synthetic spectra based upon model atmospheres from the MARCS code and high resolution spectra from the instrument TEXES, we had a lot of well resolved spectral lines across many spectral types, presented for the first time in a collected overview fashion (unlike previous work of scattered single observations). Our results show no emission features, but stronger absorption lines for all spectral types than models suggest. Furthermore, we derive the need of a temperature decrease of $\sim 500$ K in the outer regions of the photosphere and couple it to NLTE-cooling, a physical process that can drive the temperature drop in the outskirts of the stellar atmosphere.
Populärvetenskaplig sammanfattning

Att undersöka stjärnor i infrarött är en relativt ny företeelse till skillnad från optisk astronomi (dvs. synligt ljus). Det var först på 60-talet med teleskopet Stratoscope II som man lyckades få spektra från stjärnor i infrarött (våglängder på runt 3 µm), och upptäckte förekomsten av vatten i kalla röda jätter.

Den bakomliggande problematiken till varför modern infraröd astronomi är så ungt är vår egen atmosfär. Jordens atmosfär är ogenomträngligt för en stor del infrarött ljus framförallt på grund av molekyler såsom vatten. Därför var Stratoscope II (och Stratoscope I) kopplade till väderballonger som förde dem upp till stratosfären där man undviker i stort sett allt atmosfäriskt vatten.

I modernare tider bidrog ISO (Infrared Space Observatory), det första dedikerade infraröda rymdteleskopet, med mer lagupplösta spektra som påvisade vattenlinjer hos röda jätter. Det visade sig dock svårt att modellera dessa linjer med vår nuvarande kunskap av dessa stjärnor framförallt för de varmare jätterna (K-typ).

Det var först på 2000-talet som de första högupplösta spektra i infrarött av kalla jätter blev tagna, och detta från marknivå med hjälp utav ett litet gap vid 12 µm i vår atmosfär som tillåtit infrarött ljus att passera.

Med tiden utvecklades två teorier för att förklara skillnaderna mellan teoretiska och observationella spektra. Den första var en ad-hoc lösning i form av en molekyln förblir varm vatten några stjärnradier från stjärnan i fråga, även kallad MOLsphere. Den andra är en modifierad temperaturstruktur (sänkning) i den yttre delen av stjärnan som på så vis skulle ge större absorptionslinjer.

Syftet med det här projektet är att med hjälp av nya högupplösta spektra från en rad jätter, undersöka förekomsten av vatten och därmed kunna lägga restriktioner på de två teorierna. Våra resultat pekar på vatten i starkare absorption (än våra modeller förutsåg) och inte emission från någon av de 10 stjärnorna vi undersökte i 12 µm. Detta täcker ett starkt band på MOLsphere teorin då den förutsäger emissionslinjer från våglängder av 6 µm och högre. I fallet med en modifierad temperaturstruktur, beräknar vi en ungefärlig sänkning av temperatur med 500 K för alla stjärnor, oberoende av deras effektiv-temperatur, kring regionen där vattnet bildas. Detta kopplar vi med ”NLTE-cooling”, en process som fysikaliskt kan förklara varför det är sannolikt med en sådan temperatursänkning i den yttre atmosfären av jätter.
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1 Introduction

Future generations of telescopes such as the European-Extremely Large Telescope (E-ELT) will push the frontier of what is observationally possible and present the possibility of resolving individual stars in galaxies beyond the Local Group (of which the Milky Way is a member). It will be possible to resolve stars in galaxies belonging to the Virgo Cluster located about 16.5 Mpc away from us which, compared to the distance of our neighbouring spiral galaxy Andromeda at roughly 800 kpc, is significantly more distant.

The stars resolved at these distances will naturally be the most luminous ones, namely the red giants. Their spectra will be taken in the infrared, mainly due to requirements set by the adaptive optics, and at low-intermediate resolution \cite{Deep2011}. The strong spectral features that will be pronounced in this region and at this resolution will consist of OH and CO lines.

Although there have been much successes in synthesising spectra based on photospheric models in the optical region, the same cannot be said for infrared wavelengths. Along with CO and OH lines \cite{Tsuji2009}, H$_2$O also shows disagreement with standard photospheric models \cite{Ryde2006} in the IR. The addition of a cool envelope of CO in the outer atmosphere, a “COmosphere”, was suggested \cite{Wiedemann1994} in order to explain the discrepancies between models and observations in a semi-empirical fashion for CO. A question then arises whether the addition of an envelope of warm water in the stellar atmosphere (analogous to the COmosphere) could explain the differences between the models and observations for the H$_2$O lines and where such a layer would be located physically in the atmosphere.

Thus, it is crucial to understand the formation of molecular lines due to e.g., CO, OH and H$_2$O in the atmospheres of red giants. These objects will be probable targets in the future when spectrally exploring individual stars in galaxies outside of the Local Group. The spectral lines of these molecules are among the most prominent ones in the near-IR spectral region, the wavelength region where the stars will be spatially resolved.

1.1 Background

Although we know today that water is an abundant molecule in oxygen-rich atmospheres of late-type stars \cite{Ryde2002}, it was not discovered until the mid 1960s with the world’s first balloon-borne infrared telescope, Stratoscope II \cite{Woolf1964}. It confirmed the presence of water in stars as warm as α Ori (M2 Iab) a supergiant of approximately 3600 K. Even earlier still, before it was observationally confirmed, the presence of water in stellar atmospheres was predicted by \cite{Russell1934}.

The underlying reason of making Stratoscope II balloon-borne was to avoid as much of the Earth’s atmosphere as possible. Consisting of a number
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of different molecules such as H$_2$O and CO$_2$, the atmosphere has reduced transmission at certain regions of the spectra, due to absorption by these molecules. Large regions in the infrared are affected by this absorption with another example being the ozone layer with its ability to absorb ultraviolet light.

Still, the water observed in Stratoscope II was not widely recognized. In the case of µ Cep (a star similar to α Ori), it was used in the discussion of water ice in the interstellar medium (ISM) rather than being associated with water vapour in the star itself (Danielson et al., 1965). With (mostly) ground-based observations failing to detect spectral features of water in non-Mira stars hotter than M7 (Spinrad & Wing, 1969; Johnson & Méndez, 1970), which is about ∼ 3300 K, (Wing & Spinrad, 1970) argued that the molecular bands originally associated with H$_2$O, should instead be attributed to CN.

With the advent of the first dedicated space telescope with near-IR spectrometers, the Infrared Space Observatory (ISO), it became clear that water existed everywhere in space and cosmic water was no longer deemed an oddity. A remarkable result was that water was found in giant stars up to mid-K (∼ 3900 K) spectral classes in low-medium resolution ($R = \lambda/\Delta \lambda \approx 1,600$) (Tsuji et al., 1997; Tsuji, 2000b, 2001). This was unexpected as homogeneous and hydrostatic models of the time only could account for water in giants cooler than late M-type giants. In order to address this issue, the concept of a stationary warm molecular envelope called a MOLsphere (a term coined by Tsuji 2000b) was introduced. Together with the synthetic spectra generated through models, these were used to explain the discrepancies between models and observations (Tsuji et al., 1997; Tsuji, 2000b, a). The idea was supported for red super giants (RSGs) by near- and mid-IR interferometric observations (Perrin et al., 2004, 2005; Ohnaka, 2004; Ohnaka et al., 2009).

Although much of the far-IR region is opaque there are certain gaps in near to mid-IR that are transparent. In conjunction with IR detectors, an opportunity presents itself to observe in infrared without the expense of being airborne or in space. With the help of a high-resolution ($R \approx 10,000$) cryogenic grating spectrometer on the Math-Pierce Telescope at Kitt Peak, Jennings & Sada (1998) thus used the window at 12 µm to identify water lines in Betelgeuse (α Orionis; M2Iab) and Antares (α Scorpii; M1.5Iab). Still, the 12 µm is not entirely transparent and much depends on what airmass (the amount of air that the light travels through) and humidity (more water molecules in the air equals a higher number of absorbers). Fig.1 depicts the atmospheric absorption spectrum of Earth in the 12 micron region for δ Vir which as can be seen, contains features. The vertical comb-like features in the same figure, are the gaps between the orders, these are a consequence of the instrument used (cross-dispersed grating spectrometer) and represents regions with no data.
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Figure 1: The atmospheric absorption of Earth in the 12 µm region as observed by the cross-dispersed grating spectrometer, TEXES. Since the instrument works at ∼ 1600:th order, gaps are produced between the orders. A total of 22 orders are shown in this region.

Continuing with the use of the 12 µm gap, Ryde and collaborators [Ryde et al., 2002] obtained the first high resolution ($R \approx 80,000$) spectrum of a giant star in the mid-infrared, namely Arcturus (K1.5III, $T_{\text{eff}} \approx 4200$ K) using the Texes Echelon Cross Echelle Spectrograph (TEXES) [Lacy et al., 2002]. With clear detection of water lines, it effectively pushed the presence of water towards hotter environments where it was unaccounted for previously. Stars hotter than late M type were not expected to host water vapour in their photospheres (e.g. Tsuji, 2001).

Although MOLspheres offer a possible non-photospheric explanation to the origin of the water vapour, a consequence of a MOLsphere a few stellar radii beyond the photosphere, is that several emission lines should appear longward of approximately 5 µm (see Tsuji, 2000a). However, this interpretation was challenged by Ryde et al. (2006b) since they detected 12 micron lines indisputably in absorption at high resolution ($R \approx 80,000$). Therefore Ryde and collaborators instead utilized models with altered temperature profiles from which the synthetic spectra would explain the puzzle of mismatched water lines (Ryde et al., 2002, 2006a,b). Additional emphasis is put by the group for non-LTE 3D modelling of red giants (and super giants) given their large convective...
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nature. *Local Thermodynamic Equilibrium* (LTE) is a commonly used assumption that greatly simplifies the computation of a synthetic spectra. It basically states that the level populations of the elements are governed by the local temperature (it is explained more thoroughly in the Theory section).

More recent work has displayed the possibility of MOLspheres contaminating the infrared spectra [Tsuji 2008, 2009]. The resulting observed spectra would therefore consist of at least two components, namely the photospheric and the MOLSpheric component. Abundance analyses have to then take into account which component it probes depending the lines utilized for the task. Tsuji (2008, 2009) show that weak CO lines would probe the photospheric abundances whereas intermediate- and higher strength lines show increased deviation from classical micro-turbulent line formation theory. This may be attributed to a separate component (MOLsphere).

With Earth’s atmosphere blocking out large regions in the mid-infrared partly due to the presence of water, it is difficult to confront a model that is able to reproduce the water spectra at high resolution not only at 12 micron but also in regions otherwise opaque for ground observations. With space observatories being of too high expense, the airborne observatory SOFIA offers a solution. The *Stratospheric Observatory For Infrared Astronomy* (SOFIA) consists of a Boeing 747SP aircraft modified to carry a 2.5 meter telescope designed to cruise in the stratosphere above $\sim 99\%$ of Earth’s atmospheric water. In the near future, the Echelon-Cross-Echelle Spectrograph (EXES) [Richter et al. 2010] aboard SOFIA, will with its high resolution capabilities, conduct spectrometry in the mid-infrared regions that are otherwise opaque from the ground. With the use of this unique instrument, the high resolution spectra gathered will provide much needed knowledge for unified models of the stellar atmospheres and put new strong constraints on the nature of the water vapour in red giant stars.

All in all, the outer layers of red giants are complicated and not well understood even for stars that are considered standard and well known such as Arcturus. It seems that the failure to model these are partly due to our poor knowledge of the layers above the photosphere. Additionally, the models are maybe too simplified with the 1D LTE approach. The red super giant Betelgeuse is known to have large-scale convections similar to its size [Buscher et al. 1990]. Extrapolating this to their less extended siblings, the giants, the importance of full 3D NLTE modelling becomes apparent.

**1.2 This project**

With the discussion of water in the photospheres of cool stars centred around a few scattered stars, namely Betelgeuse ($\alpha$ Ori), Antares ($\alpha$ Sco), Arcturus ($\alpha$ Boo) and Herschel’s Garnet Star ($\mu$ Cep), the presence of water in red giants has not been thoroughly investigated across different spectral classes
and at high resolution. In this work we investigate a sequence of ten stars with a range of effective temperatures, effectively spanning mid-M to early-K with the use of high resolution ($R \sim 80,000$) data taken by TEXES at 12 micron. Fig. 2 position these stars in a surface gravity versus effective temperature plot as an overview of the stars used in previous works compared to the data set for this project.

**Figure 2:** An overview of the dataset in a surface gravity versus effective temperature plot, i.e an HR-diagram, with the pluses denoting stars used in this project and the triangles pointing out the stars used prior to this project. With red supergiants being generally more extended than the giants, their surface gravities are lower compared to giants and subgiants. Only one red giant was targeted previously, namely Arcturus ($\alpha$ Boo) at $T_{\text{eff}} \approx 4300$ K. Although not visible, the triangle at $T_{\text{eff}} = 3600$ K is in reality two superimposed ones making it the total count of data points to four in previous works versus ten for this project.

Apart from the benefit of working with high resolution data, another benefit is that the sample populates this critical (for the presence of water) effective-temperature region well enough to examine how the water vapour behaves across these spectral classes. This will enable us to put strong constraints on its origin. Furthermore, assuming the lines are weak, formation region temperatures can be calculated from which conclusions can be drawn on the existence of a modified temperature profile versus a MOLsphere.
1.2.1 MIR spectroscopy

This spectral region mostly consists of lines from H$_2$, H$_2$O, CO, CH$_4$, C$_2$H$_2$ and NH$_3$ (more precisely their rotational & rotational-vibrational modes). Since H$_2$, CH$_4$ and C$_2$H$_2$ lack a permanent dipole, they become unobservable with radio telescopes [Richter et al., 2001] emphasizing the importance of MIR spectroscopy, especially for bio-markers such as methane. The most significant drawback is that the black-body radiation of Earth peaks in MIR [Lacy et al., 2002] which then act as a background source in this spectral region.

2 Basic theory

For pedagogical reasons, the theory section is structured in a directional manner. I start with the core properties of a star such as the formation of the continuum and black body radiation, I then proceed with the photosphere, line formation, radiative transfer and end with the construction of a spectrum by the observer.

2.1 Planck’s radiation law

A concept that is often utilized when working with stars is the theoretical construct of a black body. The structure of it is such that a perfect black body absorbs all incident radiation (i.e reflects none), it is a perfect absorber.

Such a concept is usually visualized using a container that is completely closed except for a very small hole in one wall. Any light that enters the container will certainly have a small probability to get reflected out again but given that the hole is very small, the probability that it gets absorbed inside is much higher. Not only the size of the hole (relative to the area of the walls) correlates with the probability that the photons escape, but also the roughness of the walls and the reflection coefficient.

By now it’s easily understood that this concept doesn’t provide a truly perfect absorber, but can be constructed perfectly enough for us not to observe any difference.

Assuming the light source is constant, the container will, after a given time reach thermal equilibrium, a state where each physical processes are balanced by their opposites and the temperature of the container remains constant and uniform. The radiation leaking from the hole is what one observes. Its size is large enough to be measured but small enough not to disturb the thermal equilibrium. The radiation of a black body in thermal equilibrium is described by Planck’s radiation law:

$$B_\nu(T) = \frac{2\hbar \nu^3}{c^2} \frac{1}{e^{\hbar \nu/kT} - 1}$$  \hspace{1cm} (1)
where \( B_\nu(T) \) is the intensity at frequency \( \nu \) for a black-body temperature of \( T \), \( h \) is Planck’s constant, \( c \) denotes the speed of light and \( k \) is Boltzmann’s constant.

The massive lumps of gas that make up our stars experience a situation similar to the previously mentioned container but here it’s instead heated. The walls of this heated container emits photons and fills the cavity with radiation. These photons are most likely re-absorbed and the hole is again considered insignificant.

Metaphorically the walls here would be the stellar furnace and the region of the star known as photosphere takes the role of the “hole” (or more correctly, the leakage from the photosphere). At the bottom of this photosphere the gas is thick enough to be opaque to radiation (optically thick). The emitted photons don’t travel far before they are re-absorbed again which means a low probability of escape. The equilibrium is maintained by the energy supply that the stellar furnace provides, i.e the escaping energy is replenished by energy from within the star. Thus the material approaches thermal equilibrium and its radiation can be modelled by a black body.

However, no perfect star exists and even the non-spherically symmetric distribution of particles (i.e thickness varies not only with radii but position also) causes the net flow of energy to vary with position, and in turn causes the thermal equilibrium to be non-global. This is apparent with large evolved stars that have large convective cells.

Locally, if the variations are slow enough (which mostly is the case), regions of the star’s surface can be assumed to be in local thermal equilibrium. Furthermore, one can assume that there also exists a chemical, mechanical and radiative equilibrium which is summarised as local thermodynamical equilibrium (LTE).

Fig. 3 visualizes Planck’s radiation law (Eq. 1) for three different temperatures. Increased temperature does not only increase the area under the curve, but also shifts the peak of the curves towards higher frequencies. The relation that describes this shift, or displacement is called “Wien’s law” or more commonly “Wien’s displacement law”.

\[ B_\nu(T) \]
The Planck curve is as stated before, a good starting point and covers the black body aspect of the star. This is however not the entire truth. A stellar spectrum contains a multitude of spectral lines; in emission and absorption superimposed on black body spectra. This is a direct consequence of their upper photospheres and atmospheres being enriched with various elements and the temperature gradient in the atmosphere.

2.2 Stellar atmosphere

The photosphere, also known as the stellar surface is one of a number of layers that make up a star. Our gaze cannot penetrate any deeper layers for reasons explained prior to this section. Above the photosphere are two notable layers, namely the chromosphere and corona.

In the surface layers and above, the dominant mode of energy transport in a typical star is through radiation. Therefore, what one sees and measures, i.e the spectra, is ruled by the knowledge domain of radiative transfer. It’s through radiative transfer that physical parameters of the material that compromises the stars, manifest on the spectra which is discussed in the next chapter.

2.2.1 General radiative transfer

Consider a beam of radiation. As the beam of radiation travels, it can lose intensity in the form of extinction or gain in the form of emission.
Mathematically, the situation is described by considering a beam as it travels in a direction $s$. The difference in specific intensity $I_\nu$ at $s$ and $s+ds$ is then

$$dI_\nu(s) = I_\nu(s + ds) - I_\nu(s) = +dI_\nu(s) + _{-}dI_\nu(s)$$ \hspace{1cm} (2)

The above relation states in other words a sum of what is added and lost in the beam of radiation. The light removed from the beam through extinction processes is

$$-dI_\nu(s) = -\kappa_\nu \rho I_\nu ds$$ \hspace{1cm} (3)

where $\rho$ is the mass density and $\kappa_\nu$ is the mass extinction coefficient with the unit of area per mass. It is also known as “opacity”. The extinction processes in this context refers to two physical processes. The first is where photons are removed through scattering and photon conversion. The second is “true absorption”, i.e the true removal of photons from the beam via photon destruction. The light added by local photon emission as the beam traverses $s+ds$ is

$$+dI_\nu(s) = j_\nu(s) \rho ds$$ \hspace{1cm} (4)

where $j_\nu$ is the emission coefficient with the units erg/(s rad$^2$ Hz g). Likewise, there are two physical processes that contribute to $j_\nu$: Real emission (creation of photons) and the scattering of photons into the direction being considered. Combining Eq. 4 and 3 yields

$$dI_\nu(s) = j_\nu(s) \rho ds - \kappa_\nu \rho I_\nu ds$$

or:

$$\frac{dI_\nu}{ds} = j_\nu - \kappa_\nu \rho I_\nu$$ \hspace{1cm} (5)

This differential equation is a version of the so called transfer equation. When light passes through a slab of material of thickness $D$ from $s = 0$ to $s = D$, it doesn’t react to the distance nor $\kappa_\nu \rho$ alone but a combination of the two:

$$\tau_\nu = \tau_\nu(D) = \int_0^D \kappa_\nu(s) \rho(s) ds$$ \hspace{1cm} (6)

Here $\tau_\nu$ is the optical depth or optical (monochromatic) thickness which unlike the geometrical thickness, is a measure of the “optical” thickness. In other words, the thickness that the radiation sees. With $\kappa_\nu \rho$ having a unit of one over geometrical distance, $\tau_\nu$ becomes unitless. Dividing Eq. 5 with $d\tau_\nu = \kappa_\nu \rho ds$ results in:

$$\frac{dI_\nu}{d\tau_\nu} = \frac{j_\nu}{\kappa_\nu} - I_\nu$$ \hspace{1cm} (7)
A convenient ratio that often occurs in the field of radiative transfer is the ratio of emission to absorption.

\[ S_\nu = j_\nu / \kappa_\nu \]  

(8)

The source function \( S_\nu \) as it is called, has the same units as \( I_\nu \). The naming convention stems from the fact that its a measure of the addition of new photons along the beam. Finally, inserting Eq. 8 in Eq. 7 yields:

\[ \frac{dI_\nu}{d\tau_\nu} = S_\nu - I_\nu \]  

(9)

which is the transfer (differential) equation in its standard form. Though a relatively simple expression, a great amount of effort is spent into computing the source function, especially at the formation of spectral lines (which is discussed later).

As an example, the easiest solution to the transfer equation is in the case of pure extinction (i.e no emission, \( j_\nu = 0 \)),

\[ \frac{dI_\nu}{d\tau_\nu} = -I_\nu \]  

(10)

the solution of which is the exponentially declining function, a simple extinction law,

\[ I_\nu(D) = I_\nu^0 e^{-\tau_\nu(D)} \]  

(11)

where where \( I_\nu^0 \) denotes the beam intensity at the entry point \( s = 0 \) and the parameter \( \exp(-\tau_\nu[D]) \) explains the fraction of photon energy remaining at \( D \). At a certain point, the photon energy reduction becomes significant enough in order to claim it optically thick. This point, at which the distinction between optically thin and thick is made, is set to \( \tau_\nu = 1 \). A slab is called optically thick for \( \tau_\nu > 1 \) and optically thin for \( \tau_\nu < 1 \).

Before a more formal solution of the transfer equation is presented, one also need to account for the photons created in the slab. Consider an intermediate point located inside the slab, \( 0 < s' < D \). Following Eq. 6, the optical path length/thickness experienced by the beam at this point would then be:

\[ \tau_\nu(s') = \int_0^{s'} \kappa_\nu(s) \rho(s) \, ds \]

Similarly, in the convention of Eq. 11, the remaining incident radiation at \( s' \) is:

\[ I_\nu(s') = I_\nu^0 e^{-\tau_\nu(s')} \]  

(12)

An important keyword here is “incident”, i.e the present radiation along the beam at the start line \( I_\nu(s = 0) \).
Now, the local intensity contribution at \( s = s' \) within the slab and along the beam across \( ds \), is given by the emission coefficient (Eq. 4),

\[
dI_\nu(s') = j_\nu(s')\rho(s')\, ds
\]

Combining it with the definition of the source function and:

\[
d\tau_\nu(s') = \kappa_\nu(s')\rho(s')\, ds'
\]

results in:

\[
dI_\nu(s') = S_\nu(s')d\tau_\nu(s')
\]

As was stated before, the source function is a measure of the addition of new photons along the beam, hence its appearance in the above expression.

From the point of \( s = s' \) onwards to the other side (\( s = D \)) the contribution of photons at \( s' \) are reduced by the exponential decay parameter for the “optical” distance \( \tau_\nu(s') \) to \( \tau_\nu(D) \)

\[
[dI_\nu(D)]_{s=s'} = S_\nu(s')\, d\tau_\nu(s')e^{-[\tau_\nu(D) - \tau_\nu(s')]}
\]

With the addition of the attenuated incident radiation \( I_\nu(0) \) and all the contributions within the slab/medium (from \( s = 0 \) to \( s = D \)),

\[
I_\nu(D) = I_\nu(0)e^{-\tau_\nu(D)} + \int_{0}^{\tau_\nu(D)} S_\nu(s)e^{-[\tau_\nu(D) - \tau_\nu(s)]}\, d\tau_\nu(s)
\]

Introducing the dummy variable \( t_\nu \) as arbitrary optical depth points along the line of sight, the basic integral form of the transfer equation is attained

\[
I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_{0}^{\tau_\nu} S_\nu(t_\nu)e^{-[\tau_\nu - t_\nu]}\, dt_\nu
\]

This is the integral form of the transfer equation. An interesting example is in the case of radiation from a homogeneous medium.

A homogeneous medium implies that neither \( j_\nu \) nor \( \kappa_\nu \) varies through the medium. As a consequence, neither does \( S_\nu \). This simplifies Eq. 13 into

\[
I_\nu(D) = I_\nu(0)e^{-\tau_\nu(D)} + S_\nu\left[1 - e^{-\tau_\nu(D)}\right]
\]

again, \( D \) is the geometrical thickness of the medium. For an optically thick case (\( \tau_\nu(D) \gg 1 \)), the exponential factor \( \exp(-\tau_\nu(D)) \approx 0 \). This turns Eq. 15 into:

\[
I_\nu(D) \approx S_\nu
\]
Profoundly, the incident radiation plays no part in the emerging radiation. The intensity becomes only dependent on the source function within the medium and it becomes a black-body radiator (Eq. 1) in the case of LTE.

\[ S_\nu = B_\nu = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1} \]

The source function for this case is given by Planck’s radiation law and is often denoted as \( B_\nu \) instead of \( S_\nu \). Taking the other extreme, an optical thin homogeneous medium, i.e \( \tau_\nu \ll 1 \) yields:

\[ I_\nu(D) \approx I_\nu(0) \quad (17) \]

The emerging intensity in this case is only dependent on the incident one. This follows the logic that going towards having no material in the path, the intensity will not be subject to change.

### 2.2.2 Radiative transfer in a stellar atmosphere

Even though the use of a slab analogy is useful in (astronomical context) studies of the interstellar medium, in the case of spherical stars it is useful to move to a spherical coordinate system. Recalling the transport equation (Eq. 9) and defining the vertical optical depth \( d\tau_\nu = \kappa_\nu \rho dz \) (analogous to differential form of Eq. 6), the transfer equation receives the following form:

\[ \frac{\delta I_\nu}{\kappa_\nu \rho dz} = S_\nu - I_\nu \quad (18) \]

In a Cartesian system centered in the spherical star, the \( z \) axis is chosen toward the observer. Converting to polar coordinates and assuming that \( I_\nu \) has no azimuthal (\( \phi \)) dependence, i.e that there is axisymmetry, Eq. 18 becomes

\[ \frac{\delta I_\nu \cos \theta}{\kappa_\nu \rho} - \frac{\delta I_\nu \sin \theta}{\kappa_\nu \rho r} = S_\nu - I_\nu \quad (19) \]

Where the polar angle \( \theta \) is an angle between \( z \) and the point of interest and \( r \) the radial distance to that same point. For a resolved stellar disk, \( \theta \) analogously maps the radii of the star where the boundaries \( \cos \theta = 0 \) is the center of the disk and \( \cos \theta \to 1 \) is towards the limb of the star.

Eq. 19 is the generic form that is used in radiative transfer calculations in extended atmospheres such as those in supergiants. Main-sequence stars however, most likely have a very thin photosphere when compared to the stellar radius as a whole. As an example, the atmosphere of our own Sun is \( \sim 0.1\% \) of the solar radius.

This conveniently leads to another assumption, namely the plane-parallel approximation which mathematically corresponds to \( \theta \) not depending on \( z \), reducing Eq. 19 to

\[ \cos \theta \frac{dI_\nu}{\kappa_\nu \rho dr} = -I_\nu + S_\nu \quad (20) \]
Substituting in a new geometrical depth variable \( dx = -dr \) together with \( d\tau_\nu = \kappa_\nu \rho dx \), the basic transfer equation for plane-parallel atmospheres is attained.

\[
\cos \theta \frac{dI_\nu}{d\tau_\nu} = I_\nu - S_\nu
\]

where it is common to define \( \mu \equiv \cos \theta \),

\[
\mu \frac{dI_\nu}{d\tau_\nu} = I_\nu - S_\nu
\] (21)

Unlike the slab, the formal solutions of the transfer equation for an atmosphere has different boundary conditions for inward directed radiation \((\theta > 90^\circ, \mu < 0)\) and radiation coming out \((\theta < 90^\circ, \mu > 0)\). Their formal solutions are

\[
I_\nu^- (\tau_\nu, \mu) = -\int_0^{\tau_\nu} S_\nu(t_\nu)e^{-(t_\nu-\tau_\nu)/\mu} dt_\nu/\mu
\] (22)

for \( \mu < 0 \) and

\[
I_\nu^+ (\tau_\nu, \mu) = +\int_{\tau_\nu}^{\infty} S_\nu(t_\nu)e^{-(t_\nu-\tau_\nu)/\mu} dt_\nu/\mu
\] (23)

for \( \mu > 0 \). Again, with \( t_\nu \) serving as a \( \tau_\nu \)-like integration variable. The full intensity at the position \( \tau_\nu \) is the sum of the parts,

\[
I_\nu (\tau_\nu, \mu) = I_\nu^- (\tau_\nu, \mu) + I_\nu^+ (\tau_\nu, \mu)
\] (24)

An important special case occurs at the stellar surface \((\tau_\nu = 0, \mu > 0)\),

\[
I^- (0, \mu) = 0
\]

\[
I^+ (0, \mu) = \int_0^{\infty} S_\nu(t_\nu)e^{-t_\nu/\mu} dt_\nu/\mu
\] (25)

The inward directed intensity is zero at the surface and the emergent intensity at the stellar surface is governed by Eq. 25. This is the equation that needs to be solved in order to compute the spectrum. We then expand the source function using Taylor series, truncating (for simplicity) after the first two terms we receive

\[
S_\nu(\tau_\nu) \approx a_0 + a_1 \tau_\nu
\] (26)

Substituting this in Eq. 25 and using \( \int_0^\infty x^n \exp(-x)dx = n! \) we obtain

\[
I_\nu^+ (\tau_\nu = 0, \mu) \approx a_0 + a_1 \mu
\]

which is the same as the source function at vertical depth \( \mu \), yielding the important *Eddington-Barbier relation* (EBR),

\[
I_\nu^+ (\tau_\nu = 0, \mu) \approx S_\nu(\tau_\nu = \mu)
\] (27)
Recall that $\mu = \cos \theta$. Since the disk of the stars being observed generally is not resolved, the EBR for the emergent flux is:

$$\mathcal{F}_{\nu}^+(0) \approx \pi S_{\nu}(\tau_{\nu} = 2/3) \quad (28)$$

This equation basically states that the flux that emerges from a star comes from the depth of $\tau_{\nu} = 2/3$. In other words, the emergent flux is formed at a depth (i.e. the surface) of $\tau_{\nu} = 2/3$. Although this is not entirely accurate, it gives an important insight into how spectral lines are formed.

As an example, a spectral line is formed when $\kappa_{\nu}$ becomes larger in a narrow range of frequencies. At this range the $\tau_{\nu} = 2/3$ (EBR) level is much higher in the atmosphere where the source function also is lower. Therefore through EBR, the emergent flux is lower and we receive an absorption line.

### 2.2.3 Spectral line behaviour

Examining the behaviour of spectral lines, an easy scenario is that of a weak (or not saturated) line in LTE. It can be shown that the line strength

$$\frac{\mathcal{F}_{L} - \mathcal{F}_{\nu}}{\mathcal{F}_{L}} \approx \text{constant} \frac{\ell_{\nu}}{\kappa_{\nu}} \quad (29)$$

where $\ell_{\nu}$ is the line absorption coefficient, $\kappa_{\nu}$ the continuous absorption coefficient, $\mathcal{F}_{L}$ the continuum flux and $\mathcal{F}_{\nu}$ the surface flux. What the above equation states is that the weak line strength is directly proportional to the ratio of line absorption to continuous absorption. The strength of a line can thus be increased by not only increasing the line absorption but also by decreasing the continuum absorption.

Next, we create a variable that can be used in analysis of a spectral line, namely the **equivalent width**,

$$w = \text{constant} \frac{\kappa_{\nu}}{\kappa_{\nu}} \int_{0}^{\infty} \ell_{\nu} \, \text{d}\nu \quad (30)$$

as the integral over the line profile of Eq. $29$. The line absorption coefficient can be rewritten as

$$\ell_{\nu} \rho = N\alpha \quad (31)$$

where $\rho$ is the mass density, $N$ the number of line absorbers per unit volume and $\alpha$ the absorption coefficient per atom (atomic absorption coefficient). Combining it with Eq. $30$,

$$w = \text{constant} \frac{\kappa_{\nu}}{\kappa_{\nu}} \int_{0}^{\infty} \frac{N\alpha}{\rho} \, \text{d}\nu$$

$$= \text{constant} \cdot N \int_{0}^{\infty} \alpha \, \text{d}\nu \quad (32)$$

24
where $\rho$ was moved outside of the integral and merged with the constant. The atomic absorption coefficient $\alpha$ for natural atomic broadening has the shape of a dispersion profile with its width determined by a damping constant (natural broadening) $\gamma$.

$$\alpha = \frac{2\pi e^2}{mc} \frac{\gamma}{\Delta \omega^2 + (\gamma/2)^2}$$

$$= \frac{e^2}{mc} \frac{\gamma}{\Delta \nu^2 + (\gamma/4\pi)^2}$$

$$= \frac{e^2}{mc} \frac{\lambda^2}{c} \frac{\gamma \lambda^2}{4\pi c} (\Delta \lambda^2 + (\gamma \lambda^2/4\pi c)^2)$$

(33)

where $e$, $m$ and $\omega = 2\pi \nu$ are the elementary charge, electron mass and angular frequency respectively. $\Delta \omega$, $\Delta \nu$ and $\Delta \lambda$ are the distances from the center of the spectral lines in different systems. Equivalent width $w$ is usually measured in units of wavelength (Ångström) thus making it useful to do a unit conversion of the integral in Eq. 32. Integrating $\alpha$ (Eq. 33) yields the total energy taken from a beam of intensity $I$, in the case of a unit $I\lambda$ beam:

$$\int_{-\infty}^{\infty} \alpha \, d\lambda = \int_{-\infty}^{\infty} \alpha \, d\Delta \lambda = \frac{\pi e^2}{mc} \frac{\lambda^2}{c}$$

(34)

Approaching this problem through quantum mechanical means, the need to introduce another variable emerges, namely the oscillator strength, $f$, which differs for each spectral line. Measurements concur with the presence of the oscillator strength therefore

$$\int_{0}^{\infty} \alpha \, d\lambda = \frac{\pi e^2}{mc} \frac{\lambda^2}{c} \cdot f$$

(35)

Substituting it in Eq. [32] we obtain

$$w = \text{constant} \frac{\pi e^2}{mc} \frac{\lambda^2}{c} \cdot f \frac{N}{\kappa_{\nu}}$$

(36)

Now we need to evaluate the number of line absorbers per unit volume, $N$. This quantity is dependent on the abundance of an element $E$ relative to hydrogen $A = N_E/N_H$, the fraction of the element in the $j$:th stage of ionisation $N_j/N_E$, the number of hydrogen molecules per unit volume $N_H$, the statistical weight for the level $g_n$ over the partition function $u(T)$ and the excitation potential for that level $\chi$.

$$N = A \frac{N_j}{N_E} N_H \frac{g_n}{u(T)} e^{-\chi/kT}$$

(37)

Where the partition function is a sizeable function, defined as

$$u(T) = \sum s \, g_s e^{-\chi_s/kT}$$

(38)
Finally, with the help of Eq. 37, Eq. 36 changes into
\[
\log \left( \frac{w}{\lambda} \right) = \log \left[ \text{constant} \frac{\pi e^2 N_j/N_E}{mc^2 u(T)} N_H \right] + \log A + \log g_n f \lambda + \theta_{ex} \chi - \log \kappa_{\nu} \\
= \log C + \log A + \log g_n f \lambda - \theta_{ex} \chi - \log \kappa_{\nu}
\]
where
\[
\theta_{ex} = \frac{5040}{T}
\]
which quantifies the growth of weak lines of the same species. Doppler-dependent effects such as microturbulence and thermal broadening are removed through the division of \( w \) by \( \lambda \). Although the partition function is dependent on temperature, its dependence is weak and the whole expression in the square brackets becomes constant (\( C \)) for a given star and ion.

This equation essentially states that line growth is dependent on the abundance of the species it belongs to, atomic/molecular parameters (\( \log g_n f \) and \( \chi \)), the temperature where the line is formed and the continuous absorption (\( \kappa_{\nu} \)).

Figure 4: Two schematic figures on the behaviour of a spectral line as they grow (a modification of Fig.9.1 in Rutten 2003). To the left is an absorption line at line-center \( \lambda_0 \) in a normalized spectrum. The log-log plot of wavelength-normalized equivalent width as a function of elemental abundance, the curve of growth, is to the right (Eq. 39). Three points of interest are noted, namely when a line is weak (linear 1:1 slope), saturated (flattened curve of growth) and strong (2:1 slope). The effect of microturbulence on the saturation part in the curve of growth is indicated by the dashed curves with an arrow showing the direction of higher microturbulent velocities.

Fig. 4 schematically illustrates the growth of an absorption line (to the left) and the curve of growth of the line together with three points of interest: The linear Doppler-part (1), the saturated part or the shoulder of the curve of growth (2) and the strong line region. Weak lines refer to the initial 1:1 slope of the curve of growth where the line does not have any Lorentzian wings, thus making the Voigt profile well approximated by a Doppler profile.
(hence Doppler-part). When the line turns saturated, the line center gets optically thick and can not grow as opposed to the wings which continue to grow. This manifests itself in the curve of growth as a flattening in the curve. The saturation portion is complicated by several other factors, one of them is the microturbulence $\xi$ which delays saturation for larger velocities (dashed lines in the figure). At the region of strong lines, the core of the line does not change and the growth is focused on the far wings, causing a 2:1 slope. The curve of growth is in this figure mapped as a function of abundance but could as well be $\log g_n f_\lambda$, $\theta_{ex}$ or $\kappa_\nu$ as seen in Eq. [39].

2.3 Line broadening

There are many processes that can cause a spectral line to extend over a range of frequencies/wavelengths rather than having a linewidth of zero. These broadening processes are natural broadening, collisional broadening and Doppler-dependent broadening. The natural broadening is due to the finite lifetimes of the excited states which together with the uncertainty principle imply a spread of energies. In the case of collisional broadening (also known as pressure broadening), the overwhelming numbers of neutral hydrogen makes them the most dominant collisional broadener for non-hydrogen lines in cool-star atmospheres. This effect is known as van der Waals’ broadening and originates from the levels of the transition being disturbed by ‘colliding’ neutral hydrogen in such a way that their energy is altered. Obtained from quantum mechanical formulations, these perturbed energies are often enough not calculated for the species and line of interest. Therefore, certain approximations are utilized such as the Unsöld approximation. Still, the approximations often miscalculate the energies which implies the necessity to introduce an enhancement factor. The van der Waals’ broadening of water is, for instance, an example where the Unsöld approximation with an enhancement factor is used.

Doppler-dependent broadening is separated into thermal and turbulent components. Thermal motions of an ensemble of particles effectively spread out the region of absorption (as the radiation received is Doppler shifted in respect to the receiving particle) or emission (Doppler shift in respect to the observer). The Doppler-width ($\Delta \nu_D$) of the profile due to thermal motions is thus dependent of the temperature $T$ and is

$$\Delta \nu_D = \frac{\nu_0}{c} \sqrt{\frac{2kT}{m}} \quad (40)$$

where $\nu_0$ is the line center, $c$ the speed of light, $k$ is Boltzmann’s constant and $m$ the atomic mass. Moving away from the microscopic scale of thermal particles to a slightly larger scale, the motion of material in the scale of the line formation region (such as small convective motions in a stellar atmosphere), are dubbed microturbulence. These small-scale velocities produce Doppler
shifts that in the classical model of microturbulence, are treated together with the broadening arising from thermal motions. The microturbulence $\xi$ is entered by redefining Eq. [40] as:

$$\Delta \nu_D = \frac{\nu_0}{c} \sqrt{\frac{2kT}{m}} + \xi^2$$

(41)

Turbulent broadening on even larger scales is called macroturbulence, i.e. the presence of turbulent cells large enough such that photons remain in them from the time of creation until the point of escape from the star. If the stellar disk is resolved, each cell produces a complete spectrum that is Doppler-shifted by its corresponding relative motion in respect to the observer. This is however most likely not the case as very few stars have their disks resolved. The macroturbulence does not alter the total absorption of the line. Furthermore, the lines are made shallower through macroturbulence, i.e they are smeared out.

Each of these processes broadens according to a profile which is convolved with the spectral line. Natural and van der Waals' broadening both contribute with Lorentz profiles whereas Doppler-dependent broadening produces a Gaussian, the convolution of these profiles is called a Voigt profile. As mentioned earlier through Fig. [3] the weak lines are degenerate for different microturbulences and have Gaussian profiles as opposed to the intermediate and strong lines. These are non-degenerate for different microturbulences as well as being more affected by the van der Waals' broadening (in cool-star atmospheres) which produces wings and thus the Voigt profile.

3 Theoretical spectra

In this section, we will treat the generation model photosphere, the synthetic spectra and the involving processes and dependencies. The synthetic spectra are created based on a theoretical construct of a photosphere, a model photosphere or atmosphere, usually with certain approximations applied to ease computation which will be explained further in.

The model photospheres used in the evaluation of our observations, were calculated with the latest version of the MARCS code (Gustafsson et al., 2003) in the suite of MARCS-model-photosphere programs originally developed by Gustafsson et al. (1975). The MARCS code computes hydrostatic, spherical model photospheres on the theoretical grounds of LTE (which as mentioned earlier includes chemical equilibrium), homogeneity and the conservation of the total flux, i.e radiative plus convective.

Atomic data of the absorbants in the photosphere are obtained from the VALD database (Piskunov et al., 1995) and R.L Kurucz (1995, private communication). Opacities of numerous diatomic and triatomic molecules with up-to-date dissociation energies and partition functions, are also included.
The stellar model photosphere is computed, nominally, for 54 depth points from Rosseland-optical depths of $\log \tau_{\text{Ross}} = 2.0$ to $\log \tau_{\text{Ross}} = -5.6$. With the stars in our data set, the inner border turns out to be physically located at about 98-99% of the stellar radius, the implication of which is discussed further in Sec. 5 (specifically in the subsection treating angular diameters).

Classical stellar model photospheres are determined fully with the use of a number of stellar parameters. For the MARCS code (and also traditionally), these are the effective temperature of the star, its surface gravity, a measure of the element mixture and an ad-hoc line-width fitting parameter $\xi = \xi_{\text{micro}}$. Even though microturbulence was treated in the previous sections in the physical context of random motions on small scales, the reality is such that the microturbulence serves as an ad-hoc fitting parameter to correct the deficiencies that arise from treating a 3D problem in 1D and not (or badly): Departures from LTE, inhomogeneities, wrong collisional broadening and more.

Using the calculated model photospheres, spectra are synthesized by solving the radiative transfer under the assumption of spherical symmetry, with a resolution calculated for $R \sim 300,000$, even though the final resolution is lower. Since the macroturbulence only serves to smear out all the lines with a certain factor rather than individually, it is applied after the synthesis of the line. Like the microturbulence, serves as an ad-hoc fitting parameter but in this case, also includes the instrument profile, i.e. the smear that is consequence of the instrument in use.

4 Dataset

The observational data was taken with the Texas Echelon Cross Echelle Spectrograph, TEXES (Lacy et al., 2002). This instrument is a ground-based prototype of EXES, a mid-infrared spectrograph designed to be used in the Stratospheric Observatory for Infrared Astronomy, (SOFIA). At the time of observation, TEXES was mounted as a visitor instrument on the 3.0 m NASA Infrared Telescope Facility (IRTF) located at Mauna Kea, Hawaii. Its detector array consists of $256^2$ pixel, Si:As IBC designed for high-resolution spectroscopy by Raytheon Infrared Center for Excellence. The observations were made in TEXES’ “hi-lo” mode, which provides a 0.9 m long, echelon grating achieving a spectral resolving power of $R \sim 60,000$ with either an echelle or a first-order grating as a choice for cross-dispersion.

Before the final spectra are obtained, a calibration sequence is established that pays attention to the telluric absorption spectrum and the black-body temperatures of the surrounding sky, telescope and instrument optics (recalling that a $\sim 300$ K black-body peaks in mid-IR). The reduction of the data is done according to the procedures described by Lacy et al. (2002).
A total of 10 targets were selected and observed at two instances in the year 2000 (31 Oct. & 27 Nov.), two in 2001 (2 & 4 Feb.) and one in 2006 (9 Jul.). These giants and sub-giants are in the early-K to mid-M region as desired according to earlier similar works (see Sec. 1), have bright K-magnitudes ($K \lesssim 0$) due to requirements by the instrument and are all in the solar neighbourhood, i.e a distance less than 100 pc from us. According to the SIMBAD astronomical database, there are around 50 stars that meet these criteria. Thus, a fifth of the giants and subgiants in the solar neighbourhood, are represented in our data set, effectively reducing the probability of having “lucky” targets.

5 Acquiring stellar parameters

In order to create a good theoretical model of a stellar atmosphere we need to breakdown the stellar system into multiple definable variables. For synthesising a model spectrum, the most important of these are $T_{\text{eff}}$, $\log(g)$, [Fe/H], [$\alpha$/Fe] and microturbulence $\xi$. The next section will discuss the methods used to retrieve the stellar parameters for the stars under study in this thesis.

5.1 Effective temperature & angular diameter

Due to the star is being a (near-)black-body radiator, one of the variables which is commonly mentioned and has large implications on the stellar atmosphere is the effective temperature $T_{\text{eff}}$, defined in the following equation:

$$\int_0^\infty \mathcal{F}_\nu d\nu = \sigma T_{\text{eff}}^4$$

where $\mathcal{F}_\nu$ is the flux leaving the stellar surface as a function of wavenumber. Integrating over all wavenumbers we have a measure of power per unit area on the left hand side and Stefan-Boltzmann’s Law on the right hand side. $T_{\text{eff}}$ is then the equivalent temperature of a black-body radiating with the same power per unit area as the star. Assuming this radiator (star) is spherical we multiply with the surface area to acquire the total power emitted which is defined as luminosity.

$$L = 4\pi r^2 T_{\text{eff}}^4$$

Luminosity, due to it’s definition, doesn’t change over distance. One can depict this by imagining a sphere encasing the star with a radius $r$. As $r$ increases the light is spread out over a larger area, therefore the amount of light per area element, flux ($W/m^2$) decreases. Since neither luminosity, effective temperature nor radius (usually) are directly observable, by
convenience we divide with the distance squared to acquire:

\[
\frac{L}{d^2} = \frac{4\pi \sigma r^2 T_{\text{eff}}^4}{d^2}
\]

\[
\frac{L}{4\pi d^2} = \sigma (\tan[\theta/2])^2 T_{\text{eff}}^4
\]

\[
F_{\text{tot}} = \sigma (\theta/2)^2 T_{\text{eff}}^4
\]

where \( F_{\text{tot}} \) is the wavelength integrated flux above Earth’s atmosphere and \( \theta \) the limb-darkened diameter (Mozurkewich et al., 2003). Limb-darkening is a phenomenon in the stellar atmosphere due to the continuum source function decreasing outward that causes the outer regions of the star in the field of view (the stellar disk) to appear darker in comparison with the centre. Simply re-structuring the above equation for \( T_{\text{eff}} \)

\[
T_{\text{eff}} = \left( \frac{4F_{\text{tot}}}{\sigma \theta^2} \right)^{1/4}
\]

there are two observables needed in order to obtain an effective temperature, namely \( F_{\text{tot}} \) and \( \theta \). This is one of the more direct approaches to determine \( T_{\text{eff}} \), i.e small dependence on models, by utilizing angular diameters of stars (Mozurkewich et al., 2003; Davis et al., 2000) or more specifically the limb-darkened angular diameter, which we’ll get to later.

Beginning with \( F_{\text{tot}} \), two important difficulties that present themselves is to account for flux emitted at wavelengths not readily observable from the ground (due to our atmosphere) and correcting the observations for atmospheric & interstellar extinction. Applying photometry at shorter wavelengths and interpolations using best-fit Planck functions at longer wavelengths (> 400 nm), the uncorrected \( F_{\text{tot}} \) could be ascertained.

A majority of the uncertainty for more distant stars arises from the correction of interstellar extinction. However, since all of the stars we use are within 100 pc from the Sun, they aren’t expected to have any significant interstellar extinction. In any case, V-band extinction, \( A_V \) is used for correction where any star regardless of distance, has \( A_V \geq 0.2 \). Methods based on using the color of the star cannot differentiate between circumstellar and interstellar extinction (Mozurkewich et al., 2003). The integrated flux used in these calculations doesn’t need a correction for circumstellar extinction since the absorbed energy in that material re-radiates in the thermal infrared.

Next, the limb-darkened angular diameter \( \theta = \theta_{\text{LD}} \), is related to how the stellar radius is defined which isn’t a trivial parameter. It needs a clear definition and is presented later in section 5.2.1.

As for practically measuring limb-darkening, few methods provide the necessary high angular resolution to perform such measurements. One such
example is the use of gravitational lensing to determine limb darkening coefficients (Albrow et al., 1999). These methods, including microlensing, depend on specific events (other techniques are lunar occultations and light analysis from eclipsing systems) hence resulting in very few stars having their limb darkening measured in this way. This makes the use of interferometry for these measurements a common one (Gray, 2005).

Interferometric measurements produce a visibility curve (right plot in Fig. 5) which is the Fourier-transform of the centre-to-limb intensity profile (left plot in the same figure).

For baselines up to $\sim 20$ m the disk of uniform intensity (dashed) and limb-darkened one (solid) are undistinguishable. The details of limb-darkening are in other words not accurately measured for moderate baselines ($< 30$ m).

Simply assuming the visibility curve in Fig. 5 is our observation of a star and the baseline is less than 20 m, the general approach would be to fit the transform of a uniform disc model to the curve in order to acquire the angular diameter of the equivalent uniform disc, $\theta_{\text{UD}}$. It’s then converted to a limb-darkening corrected angular diameter $\theta_{\text{LD}}$ using a numerical factor $\rho_\theta = \theta_{\text{LD}}/\theta_{\text{UD}}$ (Davis et al., 2000; Mozurkewich et al., 2003). The correction can be as much as $\sim 12\%$ depending on $T_{\text{eff}}$ and surface gravity (Fig. 6).

Fig. 5 displays one method of deducing the limb-darkening conversion factors through photometry as conducted by Mozurkewich et al. (2003) with the photometric data of the stars under study given in Tab. 1.
5 ACQUIRING STELLAR PARAMETERS

Figure 6: (Fig. 2 in Mozurkewich et al. 2003) Limb-darkening corrections as a function of V-K magnitude (essentially temperature). The dashed lines are for main sequence stars and the solid ones represent giants.

<table>
<thead>
<tr>
<th>HR</th>
<th>Name</th>
<th>V</th>
<th>V – K</th>
</tr>
</thead>
<tbody>
<tr>
<td>HR8775</td>
<td>β Peg</td>
<td>2.42</td>
<td>4.63</td>
</tr>
<tr>
<td>HR4910</td>
<td>δ Vir</td>
<td>3.38</td>
<td>4.63</td>
</tr>
<tr>
<td>HR6056</td>
<td>δ Oph</td>
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<td>HR4069</td>
<td>μ UMa</td>
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</tr>
<tr>
<td>HR3705</td>
<td>α Lyn</td>
<td>3.13</td>
<td>3.74</td>
</tr>
<tr>
<td>HR1457</td>
<td>α Tau</td>
<td>0.86</td>
<td>3.67</td>
</tr>
<tr>
<td>HR3748</td>
<td>α Hya</td>
<td>1.97</td>
<td>3.16</td>
</tr>
<tr>
<td>HR5340</td>
<td>α Boo</td>
<td>-0.05</td>
<td>2.95</td>
</tr>
<tr>
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<td>α Ser</td>
<td>2.64</td>
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</tr>
<tr>
<td>HR2990</td>
<td>β Gem</td>
<td>1.14</td>
<td>2.23</td>
</tr>
</tbody>
</table>

Table 1: Photometric data of the stars in the data set (Mozurkewich et al., 2003).

Having applied this correction, we now have a well-defined angular diameter which due to the compact atmosphere assumption is independent of wavelength (section 5.2.1). Coming back to Eq. 44,

$$T_{\text{eff}} = \left( \frac{4 F_{\text{tot}}}{\sigma \theta_{\text{LD}}^2} \right)^{1/4}$$

(45)

this yields together with the integrated flux (discussed earlier), an effective temperature $T_{\text{eff}}$. Table 2 structures the angular diameters, wavelength integrated flux $F_{\text{tot}}$ and $T_{\text{eff}}$ for all of the stars under study together with their uncertainties (Mozurkewich et al., 2003).
5 ACQUIRING STELLAR PARAMETERS

<table>
<thead>
<tr>
<th>HR</th>
<th>Name</th>
<th>$^{1}\theta_{LD}$ (mas)</th>
<th>$^{1}F_{\text{tot}}$</th>
<th>$^{1}T_{\text{eff}}$ (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HR8775</td>
<td>$\beta$ Peg</td>
<td>17.982 ± 0.180</td>
<td>15.22</td>
<td>3448 ± 42</td>
</tr>
<tr>
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<td>$\delta$ Vir</td>
<td>10.709 ± 0.107</td>
<td>6.43</td>
<td>3602 ± 44</td>
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<tr>
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<td>$\delta$ Oph</td>
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<td>3721 ± 47</td>
</tr>
<tr>
<td>HR4069</td>
<td>$\mu$ UMa</td>
<td>8.538 ± 0.085</td>
<td>5.02</td>
<td>3793 ± 47</td>
</tr>
<tr>
<td>HR3705</td>
<td>$\alpha$ Lyn</td>
<td>7.538 ± 0.075</td>
<td>4.10</td>
<td>3836 ± 47</td>
</tr>
<tr>
<td>HR1457</td>
<td>$\alpha$ Tau</td>
<td>21.099 ± 0.211</td>
<td>33.31</td>
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</tr>
<tr>
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<td>9.727 ± 0.097</td>
<td>8.56</td>
<td>4060 ± 50</td>
</tr>
<tr>
<td>HR5340</td>
<td>$\alpha$ Boo</td>
<td>21.373 ± 0.247</td>
<td>48.56</td>
<td>4226 ± 53</td>
</tr>
<tr>
<td>HR5854</td>
<td>$\alpha$ Ser</td>
<td>4.846 ± 0.048</td>
<td>3.38</td>
<td>4558 ± 56</td>
</tr>
<tr>
<td>HR2990</td>
<td>$\beta$ Gem</td>
<td>7.980 ± 0.080</td>
<td>11.82</td>
<td>4858 ± 60</td>
</tr>
</tbody>
</table>

Table 2: Collection of the limb-darkened angular diameters, wavelength integrated flux and calculated effective temperature for the stars in our dataset. $F_{\text{tot}}$ is presented in nWm$^{-2}$.

1) Mozurkewich et al. (2003)

5.2 Surface gravity

The surface gravity denoted $g$, usually presented logged as $\log(g)$ is the gravitational acceleration “felt” by an object with negligible mass (as to not disturb the system) at the surface of another with mass $M$ and radius $r$.

$$g = \frac{GM}{r^2}$$ (46)

where $G$ is the gravitational constant. Next we’ll go through the parameters needed and the calculations involved to retrieve the surface gravities for our stars; the stellar radius $r$ in the following section and the stellar mass $M$ in section 5.2.2.

5.2.1 Stellar radius

The stellar radius $r$ is acquired in our work through the limb-darkened diameter. Since a star is a gaseous sphere (thus not having a sharply defined surface) the definition of the stellar radius becomes a non-trivial matter. Baschek et al. (1991) presents different radius definitions, each of them using another stellar parameter as a base. The different radii are density radius $R_\rho$, mass radius $R_M$, optical-depth radius $R_\tau$, intensity radius $R_I$ and temperature radius $R_T$. Starting in the order given above, the density radius $R_\rho$ is the distance from the centre of the star to a point $r$ at which the main contribution to $\rho(r)$ comes from particles not inherent to the star (for instance interstellar matter).

The mass radius $R_M$ is based on the mass contained within a radius $r$, $M_r$ where the mass fraction $1 - M_r/M$ becomes a small pre-chosen value.
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The optical-depth radius $R_\tau$, which is the most common radius definition used, applies the use of the radial optical depth $\tau(r)$ where the distance to the star centre would be when $\tau(r)$ (as seen from the observer) assumes a pre-chosen value in the order of unity (1 or 2/3). Through the variation of the centre-to-limb intensity of a star, the intensity radius $R_I$ is defined as the distance at which the fraction $I(\hat{r})/I(0)$ ($\hat{r}$ being the stellar coordinate) has a small pre-chosen value. The intensity radius, due to the wavelength dependence of limb-darkening, is a monochromatic quantity. Lastly, the temperature radius $R_T$ is the only one of the mentioned radii that doesn’t revolve around the notion of a pre-chosen numerical value. $R_T$ is the distance from the centre at which the local kinetic temperature $T(r)$ is equal to $T_{\text{eff}}$. Using Eq. 43 would then yield $r = R_T$. Even though $R_T$ has the advantage of being free from pre-chosen values, its dependence on the elusive local kinetic temperature structure (i.e. exact knowledge of the temperature profile) makes its usage scarce.

With the use of angular diameters to acquire $T_{\text{eff}}$, the intensity radii is utilized and should strictly be monochromatic. However, put in context, the atmospheres of most stars are negligible compared to the total dimension of them, which leads to their radii being (almost) independent of wavelength thus giving one $\theta$ (and $T_{\text{eff}}$) ([Baschek et al., 1991; Marengo et al., 2003]. In other words, the limb-darkened angular diameter isn’t significantly dependent on wavelength due to the sheer extent of the stars in comparison with their atmospheres. These group of stars are said to have compact atmospheres.

Although typical main-sequence (MS) stars are considered to belong to this group, observations of the solar disk at different wavelengths show that in a strict sense, not even a solar-type MS star has a compact atmosphere ([Baschek et al., 1991]. This is settled by considering the fact that the outer atmospheres that exists in most stars (e.g. corona, solar winds) has negligible amounts of mass and contributes minimally to the total luminosity. ([Mozurkewich et al., 2003] only bothers with the issue of extended atmospheres in supergiant stars. Due to the stars under study consisting of mainly giants and bright giants, we assume the stars to have compact atmospheres and thus an angular diameter independent of wavelength.

With the limb-darkened angular diameter and a more precise definition of the radius, a final unknown exists in order to obtain the stellar radius, namely the distance to them. This is obtained through the stellar parallax.

By measuring the precise movement and positions of stars one can deduce the distance to them. As an observer moves, objects in the proximity has a higher change in its apparent position than the more distant ones, this is called parallax. Taking it to the astronomical case where Earth is moving in its orbit about Sun, nearby stars are displaced in relation to distant stars.
Through trigonometry this becomes
\[
\tan \pi = \frac{D_{\odot-\oplus}}{D_{\odot-star}}
\]
where \( \pi \) is the parallax angle, \( D_{\odot-\oplus} \) is the Sun-Earth distance (defined as 1 AU) and \( D_{\odot-star} \) is the distance between the Sun and the star in question. For very small \( \pi \),
\[
\tan \pi \to \pi
\]
A *parsec (pc)* is defined as the distance between the Sun and an object with a parallax angle of one arcsecond (1/3600 of a degree). Additionally we can safely assume that the distance Sun-star is \( \approx \) Earth-Star.
\[
D_{\odot-star} [pc] = \frac{1 [AU]}{\pi [as]} \tag{47}
\]
With Eq. (47), measured parallaxes, and the limb-darkened angular diameter \( \theta_{LD} \), the radii can be determined (see Fig. 7). Starting off by again assuming that \( \theta_{LD} \) is very small,

\[
\theta_{LD} = \frac{2r}{D}
\]
Substituting this in Eq. (47) and solving for \( r \),
\[
r = \frac{\theta_{LD}}{2 \cdot \pi} [AU]
\]
or in the more convenient units of solar radii \( R_{\odot} \),
\[
r = \frac{\theta_{LD}}{2 \cdot \pi} 215 R_{\odot} \tag{48}
\]
Table 3 summarizes the radii calculation for our dataset. \( \theta_{LD} \) is taken from Mozurkewich et al. (2003) and the parallax from the latest data reduction of the Hipparcos data (van Leeuwen 2007). As noted the parameter estimates are good with small uncertainties. This follows from the stars being very luminous and in the proximity of the Sun. With \( \mu \) UMa being the most distant one Eq. (47) returns a distance of \( D = 70 \) pc. Put in contrast the radius of our galaxy is in the order of ten-thousands of pc, verifying the proximity of the stars under study and thus the assumption of disregarding interstellar reddening.
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Figure 7: A schematic view of a star and the observer (O). The limb-darkened angular diameter $\theta_{LD}$ denotes the star’s extent, $D$ is its distance from the Sun and $r$ the stellar radius.

<table>
<thead>
<tr>
<th>HR</th>
<th>Name</th>
<th>1$\theta_{LD}$ (mas)</th>
<th>2$\pi$ (mas)</th>
<th>3D (pc)</th>
<th>4$r$ (R$_\odot$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HR8775</td>
<td>$\beta$ Peg</td>
<td>17.982 ± 0.180</td>
<td>16.64 ± 0.15</td>
<td>60.10 ± 0.54</td>
<td>116.22 ± 1.57</td>
</tr>
<tr>
<td>HR4910</td>
<td>$\delta$ Vir</td>
<td>10.709 ± 0.107</td>
<td>16.44 ± 0.22</td>
<td>60.83 ± 0.81</td>
<td>70.06 ± 1.17</td>
</tr>
<tr>
<td>HR6056</td>
<td>$\delta$ Oph</td>
<td>10.471 ± 0.117</td>
<td>19.06 ± 0.16</td>
<td>52.47 ± 0.44</td>
<td>59.08 ± 0.83</td>
</tr>
<tr>
<td>HR4069</td>
<td>$\mu$ UMa</td>
<td>8.538 ± 0.085</td>
<td>14.16 ± 0.54</td>
<td>70.62 ± 2.69</td>
<td>64.85 ± 2.56</td>
</tr>
<tr>
<td>HR3705</td>
<td>$\alpha$ Lyn</td>
<td>7.538 ± 0.075</td>
<td>16.06 ± 0.17</td>
<td>62.27 ± 0.66</td>
<td>50.48 ± 0.73</td>
</tr>
<tr>
<td>HR1457</td>
<td>$\alpha$ Tau</td>
<td>21.099 ± 0.211</td>
<td>48.94 ± 0.77</td>
<td>20.43 ± 0.32</td>
<td>46.37 ± 0.86</td>
</tr>
<tr>
<td>HR3748</td>
<td>$\alpha$ Hya</td>
<td>9.727 ± 0.097</td>
<td>18.09 ± 0.18</td>
<td>55.28 ± 0.55</td>
<td>57.83 ± 0.81</td>
</tr>
<tr>
<td>HR5340</td>
<td>$\alpha$ Boo</td>
<td>21.373 ± 0.247</td>
<td>88.83 ± 0.54</td>
<td>11.26 ± 0.07</td>
<td>25.88 ± 0.34</td>
</tr>
<tr>
<td>HR5854</td>
<td>$\alpha$ Ser</td>
<td>4.846 ± 0.048</td>
<td>44.10 ± 0.19</td>
<td>22.68 ± 0.10</td>
<td>11.82 ± 0.05</td>
</tr>
<tr>
<td>HR2990</td>
<td>$\beta$ Gem</td>
<td>7.980 ± 0.080</td>
<td>96.54 ± 0.27</td>
<td>10.36 ± 0.03</td>
<td>8.89 ± 0.09</td>
</tr>
</tbody>
</table>

Table 3: The calculation of $r$ for our dataset with the usage of parallax $\pi$ and summarized in a table.

1) Mozurkewich et al. (2003)
2) van Leeuwen (2007)
3) $D = \frac{1}{\pi}$
4) $r = \frac{\theta_{LD}}{2\pi} \cdot 215 R_\odot$ (Eq. 48)
5.2.2 Stellar mass & metallicity

Turning our focus to the stellar mass, the now known effective temperature and stellar radius are used to obtain the luminosity using Eq. 43 (Tab. 4).

<table>
<thead>
<tr>
<th>HR</th>
<th>Name</th>
<th>$^{1}T_{\text{eff}}$ (K)</th>
<th>$r$ (R$_{\odot}$)</th>
<th>$^{2}L$ (L$_{\odot}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HR8775</td>
<td>β Peg</td>
<td>3448 ± 42</td>
<td>116.22 ± 1.57</td>
<td>1711 ± 100</td>
</tr>
<tr>
<td>HR4910</td>
<td>δ Vir</td>
<td>3602 ± 44</td>
<td>70.06 ± 1.17</td>
<td>741 ± 44</td>
</tr>
<tr>
<td>HR6056</td>
<td>δ Oph</td>
<td>3721 ± 47</td>
<td>59.08 ± 0.83</td>
<td>600 ± 35</td>
</tr>
<tr>
<td>HR4069</td>
<td>μ UMa</td>
<td>3793 ± 47</td>
<td>64.85 ± 2.56</td>
<td>780 ± 73</td>
</tr>
<tr>
<td>HR3705</td>
<td>α Lyn</td>
<td>3836 ± 47</td>
<td>50.48 ± 0.73</td>
<td>495 ± 28</td>
</tr>
<tr>
<td>HR1457</td>
<td>α Tau</td>
<td>3871 ± 48</td>
<td>46.37 ± 0.86</td>
<td>432 ± 27</td>
</tr>
<tr>
<td>HR3748</td>
<td>α Hya</td>
<td>4060 ± 50</td>
<td>57.83 ± 0.81</td>
<td>814 ± 46</td>
</tr>
<tr>
<td>HR5340</td>
<td>α Boo</td>
<td>4226 ± 53</td>
<td>25.88 ± 0.34</td>
<td>191 ± 11</td>
</tr>
<tr>
<td>HR5854</td>
<td>α Ser</td>
<td>4558 ± 56</td>
<td>11.82 ± 0.05</td>
<td>54.0 ± 2.7</td>
</tr>
<tr>
<td>HR2990</td>
<td>β Gem</td>
<td>4858 ± 60</td>
<td>8.89 ± 0.09</td>
<td>39.5 ± 2.1</td>
</tr>
</tbody>
</table>

Table 4: The calculation of $L$ for our dataset with the use of parallax $r$ and $T_{\text{eff}}$ summarized in a table.

1) Mozurkewich et al. (2003)
2) $L = 4\pi r^2 T_{\text{eff}}^4$

At this point, sufficient data exists to construct an HR-diagram. By plotting the luminosity as a function of $T_{\text{eff}}$ we can ascertain the stellar mass. Through knowledge of stellar structure and evolution coloured by different chemical composition, theoretical models are constructed which are depicted as tracks on the HR-diagram.

Each track would then represent a stellar mass. By knowing the uncertainties in $L$ and $T_{\text{eff}}$ for the stars a window would be apparent of intersecting evolutionary tracks. Hence, their masses can be ascertained. Because the evolution of a star is somewhat dependent on its chemical abundances, the need arises for the stars in our dataset to have their metallicities determined, i.e $[\text{Fe/H}]$.

$$[\text{Fe/H}] = \log_{10} \left( \frac{N_{\text{Fe}}}{N_{\text{H}}} \right)_{\star} - \log_{10} \left( \frac{N_{\text{Fe}}}{N_{\text{H}}} \right)_{\odot}$$

Where $N_X$ is the number density of element X. Note in the above equation, the abundance of Fe over H is a relative measure between the star in question and the Sun.

Albeit there are numerous papers concerning the iron-abundance of our stars, one must have some homogeneity in mind. Different authors utilize different methods, approaches and approximations. Therefore by having a shorter spread of authors and papers the systematic uncertainties associated with the methods are minimized, thus reducing the likelihood of trends perturbing as a cause of this.
This grows difficult when dealing with lesser known stars. Since most of our stars are among the brightest in their constellations, the literature should be extensive. The aim when picking the iron-abundances from the literature, is to have the effective temperature and log($g$) of that reference for that very star, in the proximity of our values. Presented in Tab. 5 are the final iron-abundances for each of the studied stars together with references.

<table>
<thead>
<tr>
<th>HR</th>
<th>Name</th>
<th>[Fe/H]</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>HR8775</td>
<td>β Peg</td>
<td>1−0.11</td>
<td>0.016</td>
</tr>
<tr>
<td>HR4910</td>
<td>δ Vir</td>
<td>1−0.09</td>
<td>0.016</td>
</tr>
<tr>
<td>HR6056</td>
<td>δ Oph</td>
<td>5−0.03</td>
<td>0.019</td>
</tr>
<tr>
<td>HR4609</td>
<td>μ UMa</td>
<td>3−0.00</td>
<td>0.020</td>
</tr>
<tr>
<td>HR3705</td>
<td>α Lyn</td>
<td>2−0.26</td>
<td>0.011</td>
</tr>
<tr>
<td>HR1457</td>
<td>α Tau</td>
<td>1−0.00</td>
<td>0.020</td>
</tr>
<tr>
<td>HR3748</td>
<td>α Hya</td>
<td>2−0.12</td>
<td>0.015</td>
</tr>
<tr>
<td>HR5040</td>
<td>α Boo</td>
<td>4−0.60</td>
<td>0.005</td>
</tr>
<tr>
<td>HR5854</td>
<td>α Ser</td>
<td>2−0.03</td>
<td>0.021</td>
</tr>
<tr>
<td>HR2990</td>
<td>β Gem</td>
<td>2−0.07</td>
<td>0.017</td>
</tr>
</tbody>
</table>

Table 5: A table of the iron-abundances from the literature, selected not only by proximity to temperature and log($g$) but also the relevance of the articles.

1) Smith & Lambert (1985)
2) McWilliam (1990)
3) Mallik (1998)
4) Leep et al. (1987)
5) Koleva & Vazdekis (2012)
6) $Z = 10^{0.977[Fe/H]−1.699}$

Bertelli et al. (2008, 2009) discuss two different sets of evolutionary tracks which utilizes isochrones by Girardi et al. (2000) that are in the range of the studied stars metallicity-wise. These were $Z = 0.008$ and $0.017$. Note that $Z$ here is the total metallicity corresponding to all metals and not the iron-abundance $[Fe/H]$. To convert it, the following formula was implemented by Bertelli et al. (1994):

$$Z = 10^{0.977[Fe/H]−1.699}$$ (50)

With the iron-abundances we can proceed with the calculation of $Z$ using Eq. 50.

The set of evolutionary tracks used for a given star, were simply the ones with the closest $Z$ value. Additionally for more accurate tracks, the second most abundant element in the stars, helium ($Y$) was set to $Y = 0.260$.

Fig. 8 displays our dataset in an HR-diagram together with the evolutionary tracks, with Fig. 8b showing $Z = 0.017$ tracks and Fig. 8a $Z = 0.008$. The red color marks stars with that corresponding metallicity whereas green color tracks the stars on the asymptotic giant branch (AGB). Stars with less mass appear further down because of their lower luminosity and effective temperature.
Starting from the bottom track (black dashed) in Fig. 8b is $M = 1.20, 1.40, 1.60, 1.80, 1.95, 2.10, 2.30, 3.00$ and $4.00 \, M_\odot$ for the uppermost track. Likewise in Fig. 8a, starting from the bottom $M = 1.20, 1.40, 1.60, 1.80, 1.90, 2.20, 3.00$ and $4.00 \, M_\odot$. The AGB tracks (green dashed) start with $M = 1.2 \, M_\odot$ with a step size of $0.2 \, M_\odot$ to $1.9 \, M_\odot$. From here, the AGB tracks in the $Z = 0.008$ plot continue with $2.2$ and $3.00 \, M_\odot$ whereas it is in the $Z = 0.017$ plot by $2.1$, $2.3$ and finally $3.0 \, M_\odot$.

While the tracks are similar in masses, there is a clear expected shift in luminosities and temperatures with the metal-poorer stars being generally brighter than stars with higher metal content at the same temperatures. Another interesting point in Fig. 8a & 8b occurs around $T_{\text{eff}} \sim 4550$ K, the zigzagging of the tracks that depict the RGB (red giant branch) to AGB (asymptotic giant branch) phase.

The asymptotic giant branch is aptly named so due to its closely aligned path to the red giant track in an HR diagram. If some of the stars in the dataset in fact were in the AGB they would, as depicted in Fig. 8a & 8b, only have the slight mass change of $\Delta M_\odot \sim 0.05 \, M_\odot$ in a worst case scenario. Therefore it is largely ignored as a factor of error since the main contributor lies in the resolution of the evolutionary tracks and how well the mass estimates are made.
Figure 8: HR-diagrams depicting the stars being studied for the two different metallicities $Z = 0.008$ and $Z = 0.017$. Red colored stars point out stars with the metallicity of interest. The lines are evolutionary tracks by Bertelli et al. (2008, 2009) which are based on Girardi et al. (2000) isochrones, the green ones are evolutionary tracks of AGB stars. Since this plot serves to convey the general idea of the mass estimation, approximately half of the lines were removed, to reduce clutter. Lower mass evolutionary tracks have lower $T_{\text{eff}}$ and luminosity and therefore appear further down in the above plots. For both metallicities the masses of the black dashed lines start with $M = 1.2 \, M_\odot$ with a step-size of roughly $0.2 \, M_\odot$ to $2.3 \, M_\odot$ followed by $M = 3.00$ and $4.00 \, M_\odot$. At roughly $T_{\text{eff}} \sim 4550$ K, the RGB to AGB transition is depicted. In the case of the green dashed lines they start with $M = 1.2 \, M_\odot$ with a step size of $0.2 \, M_\odot$ to $1.9 \, M_\odot$. From here $Z = 0.008$ continues with $2.2$ and $3.00 \, M_\odot$ whereas $Z = 0.017$ has $2.1$, $2.3$ and $3.0 \, M_\odot$. 
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5.2.3 Surface gravity - calculation

Now, with the obtained stellar mass values from the evolutionary tracks combined with the stellar radius, the calculation of \( \log(g) \) for our dataset can finally be done using Eq. 46 (Tab. 6).

<table>
<thead>
<tr>
<th>HR</th>
<th>Name</th>
<th>( ^1 T_{\text{eff}} ) (K)</th>
<th>( ^2 L ) (L( \odot ))</th>
<th>( ^3 M ) (M( \odot ))</th>
<th>( \log(g) ) (cgs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HR8775</td>
<td>( \beta ) Peg</td>
<td>3448 ± 42</td>
<td>1711 ± 100</td>
<td>1.68 ± 0.13</td>
<td>0.53 ± 0.04</td>
</tr>
<tr>
<td>HR4910</td>
<td>( \delta ) Vir</td>
<td>3602 ± 44</td>
<td>741 ± 44</td>
<td>1.25 ± 0.05</td>
<td>0.84 ± 0.03</td>
</tr>
<tr>
<td>HR6056</td>
<td>( \delta ) Oph</td>
<td>3721 ± 47</td>
<td>600 ± 35</td>
<td>1.35 ± 0.15</td>
<td>1.02 ± 0.06</td>
</tr>
<tr>
<td>HR4069</td>
<td>( \mu ) UMa</td>
<td>3793 ± 47</td>
<td>780 ± 73</td>
<td>1.80 ± 0.10</td>
<td>1.07 ± 0.06</td>
</tr>
<tr>
<td>HR3705</td>
<td>( \alpha ) Lyn</td>
<td>3836 ± 47</td>
<td>495 ± 28</td>
<td>0.90 ± 0.10</td>
<td>0.98 ± 0.06</td>
</tr>
<tr>
<td>HR1457</td>
<td>( \alpha ) Tau</td>
<td>3871 ± 48</td>
<td>432 ± 27</td>
<td>1.45 ± 0.15</td>
<td>1.27 ± 0.06</td>
</tr>
<tr>
<td>HR3748</td>
<td>( \alpha ) Hya</td>
<td>4060 ± 50</td>
<td>814 ± 46</td>
<td>2.75 ± 0.25</td>
<td>1.35 ± 0.05</td>
</tr>
<tr>
<td>HR5340</td>
<td>( \alpha ) Boo</td>
<td>4226 ± 53</td>
<td>191 ± 11</td>
<td>1.10 ± 0.10</td>
<td>1.65 ± 0.05</td>
</tr>
<tr>
<td>HR5854</td>
<td>( \alpha ) Ser</td>
<td>4558 ± 56</td>
<td>54.0 ± 2.7</td>
<td>1.60 ± 0.10</td>
<td>2.50 ± 0.03</td>
</tr>
<tr>
<td>HR2990</td>
<td>( \beta ) Gem</td>
<td>4858 ± 60</td>
<td>39.5 ± 2.1</td>
<td>2.20 ± 0.10</td>
<td>2.88 ± 0.03</td>
</tr>
</tbody>
</table>

Table 6: The calculation of luminosity, stellar mass and surface gravity for our data.

1) Mozurkewich et al. (2003)
2) \( L = 4\pi r^2 T_{\text{eff}}^4 \)
3) Deduced from evolutionary tracks (Bertelli et al., 2008, 2009).

Additionally, in tab. 7 a comparison is made with the parameters created using the web interface PARAM 1.1. It does a Bayesian estimate of the stellar parameters \( M \), \( \log(g) \) and \( r \) amongst others (da Silva et al., 2006) with the use of Girardi et al. (2000) isochrones, through prior knowledge of the probabilities associated with the placement of a star in the HR-diagram. The inputs are \( T_{\text{eff}} \), [Fe/H], V-magnitude and parallax. With the use of PARAM, we receive a perspective of our parameters and verify them to a certain extent.

To highlight the dissimilarities between PARAM 1.1 and the stellar parameter based on Mozurkewich et al. (2003), we construct a figure (Fig. 9). Here the difference \( \Delta \log(g) = \log(g_{\text{MOZ}}) - \log(g_{\text{PARAM}}) \) (and its propagated errors) are plotted as a function of \( \log(g_{\text{MOZ}}) \). The indices MOZ and PARAM simply denote our parameters and PARAM respectively.

The agreement between these two methods are generally good with the exception of \( \beta \) Peg, the reason of which might be due to the differences in methodology along with the fact that it is the most extreme star in our dataset (highest luminosity, lowest surface gravity). Restating Eq. 43,

\[
L = 4\pi r^2 T_{\text{eff}}^4
\]

and keeping its structure in mind, the luminosity calculation was done by knowing the limb-darkened diameter, wavelength integrated flux and par-
allax. PARAM creates luminosity values with the use of photometry, more specifically the V magnitude along with a bolometric correction (B.C). Through the $T_{\text{eff}}$ (which is an input in the interface, therefore the same as the one provided by Mozurkewich et al. [2003]) the stellar radius is calculated. Along side this calculation, the log($g$) is obtained with the use of $T_{\text{eff}}$ and the luminosity from isochrones.

To conclude, the main difference between the two methods is the introduction of the B.C. The errors associated with it might cause the luminosity calculated by PARAM to deviate, which in turn propagates to the other parameters.

Black bodies of cooler temperatures have increasingly more of their luminosities located outside the V-magnitude range. As such, the bolometric correction for a star with temperatures in the range of $\beta$ Peg, will be large along with its errors.

### Table 7: A comparison of parameters made with the web interface PARAM 1.1

(da Silva et al. [2006]). For the required inputs in PARAM 1.1, the parameters provided by Mozurkewich et al. (2003) were used along with the ones derived in this thesis.

<table>
<thead>
<tr>
<th>HR</th>
<th>M (M$_{\odot}$)</th>
<th>log(g) (cgs)</th>
<th>r (R$_{\odot}$)</th>
<th>M (M$_{\odot}$)</th>
<th>log(g) (cgs)</th>
<th>r (R$_{\odot}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HR8775</td>
<td>1.68 ± 0.13</td>
<td>0.53 ± 0.04</td>
<td>116.22 ± 1.57</td>
<td>2.036 ± 0.283</td>
<td>0.19 ± 0.11</td>
<td>181.69 ± 24.31</td>
</tr>
<tr>
<td>HR4910</td>
<td>1.25 ± 0.05</td>
<td>0.84 ± 0.03</td>
<td>70.06 ± 1.17</td>
<td>1.006 ± 0.082</td>
<td>0.67 ± 0.09</td>
<td>73.54 ± 7.86</td>
</tr>
<tr>
<td>HR6056</td>
<td>1.35 ± 0.15</td>
<td>1.02 ± 0.06</td>
<td>59.08 ± 0.83</td>
<td>1.194 ± 0.173</td>
<td>0.89 ± 0.10</td>
<td>62.34 ± 6.30</td>
</tr>
<tr>
<td>HR4069</td>
<td>1.80 ± 0.10</td>
<td>1.07 ± 0.06</td>
<td>64.85 ± 2.56</td>
<td>1.554 ± 0.140</td>
<td>0.98 ± 0.08</td>
<td>64.16 ± 5.14</td>
</tr>
<tr>
<td>HR3705</td>
<td>0.90 ± 0.10</td>
<td>0.98 ± 0.06</td>
<td>50.48 ± 0.73</td>
<td>0.988 ± 0.067</td>
<td>1.02 ± 0.06</td>
<td>49.11 ± 3.05</td>
</tr>
<tr>
<td>HR1457</td>
<td>1.45 ± 0.15</td>
<td>1.27 ± 0.06</td>
<td>46.37 ± 0.86</td>
<td>1.289 ± 0.266</td>
<td>1.19 ± 0.10</td>
<td>46.04 ± 3.21</td>
</tr>
<tr>
<td>HR3748</td>
<td>2.75 ± 0.25</td>
<td>1.35 ± 0.05</td>
<td>57.83 ± 0.81</td>
<td>2.236 ± 0.184</td>
<td>1.20 ± 0.08</td>
<td>59.80 ± 3.31</td>
</tr>
<tr>
<td>HR5340</td>
<td>1.10 ± 0.10</td>
<td>1.65 ± 0.05</td>
<td>25.88 ± 0.34</td>
<td>0.994 ± 0.078</td>
<td>1.61 ± 0.05</td>
<td>24.76 ± 1.05</td>
</tr>
<tr>
<td>HR5854</td>
<td>1.60 ± 0.10</td>
<td>2.50 ± 0.03</td>
<td>11.82 ± 0.05</td>
<td>1.281 ± 0.235</td>
<td>2.40 ± 0.10</td>
<td>11.39 ± 0.52</td>
</tr>
<tr>
<td>HR2990</td>
<td>2.20 ± 0.10</td>
<td>2.88 ± 0.03</td>
<td>8.89 ± 0.09</td>
<td>1.891 ± 0.371</td>
<td>2.79 ± 0.10</td>
<td>8.82 ± 0.34</td>
</tr>
</tbody>
</table>
5 ACQUIRING STELLAR PARAMETERS

Figure 9: The difference $\Delta \log(g) = \log(g_{\text{MOZ}}) - \log(g_{\text{PARAM}})$ plotted as a function of $\log(g_{\text{MOZ}})$. The parameters based on Mozurkewich et al. (2003) and PARAM 1.1 (da Silva et al., 2006) are indexed MOZ and PARAM respectively.

5.3 Micro-turbulent velocities

The microturbulence, as discussed earlier, is a difficult parameter (see McWilliam 1990) that becomes important especially for lines that are near-saturated and beyond. With giants being more extended, they tend to have higher microturbulent velocities compared to main sequence stars. Two sources of microturbulent velocities were used and is summarized for the studied stars in Tab. 8. The first source was a tool used by the Gaia-ESO survey to calculate microturbulences for stars within a range of $T_{\text{eff}}$’s and surface gravities which fit our dataset. The second was based on values set by Tsuji (1985) for some of our stars. Comparing the two sources, a certain offset of around $\Delta \xi \sim 1.5$ is notable. Due to this spread of velocities, we use a homogeneous value of $\xi = 2.0 \text{ kms}^{-1}$ for all the stars in study.
As previously mentioned, the iron-abundance $[\text{Fe}/\text{H}]$ is a scaling from solar abundance, and the metallicity $Z$ represents the collective metal content of a star. Ideally one would like to do an element-wise abundance analysis, however with the difficulties these present, a step in the right direction is to adjust the levels of at least the $\alpha$-elements in a star apart from a $[\text{Fe}/\text{H}]$ scaling. In our Sun, these elements collectively make up the majority of the metal content thus emphasising the importance of paying close attention to these elements. Additionally, $\alpha$-elements are important for the calculation of the model atmospheres and the synthetic spectra due to the electron pressure. Several of these elements are major e$^-$ donors in cool-star atmospheres.

The name originates from the $\alpha$-process in which the stellar furnace utilizes $\alpha$-particles ($^4\text{He}$ atoms) to create heavier elements through nuclear...
fusion. This name is however merely a convenience. It is in fact referring to that some metals with even-$Z \leq 22$ namely: (C, N), O, Mg, Si, S, Ca, and Ti, are enhanced in abundance relative to iron in certain stars, including other reaction chains than the $\alpha$-process (McWilliam, 1997) with C and N being the main exceptions there.

We utilize a schematic view of the alpha element abundance (Fig. 10) from Tinsley (1979). This view is also backed up by the work of Kirby et al. (2008), additionally it follows our metal opacities and is therefore ideal to use. The arrows indicate the sensitivities in this view. The initial mass function (IMF) is simply the distribution of stellar masses at $t = 0$. If this distribution was shifted towards high-masses (a larger number of stars massive enough to go supernovae), the enrichment due to SN II would cause the $[\alpha/\text{Fe}]$ to increase. A higher star formation rate (SFR) would imply a higher production of iron before the first SN 1a (supernovae type 1a) thus offsetting the knee in the figure towards higher iron-abundances. The negative slope mimicking a depletion is in fact due to the SN 1a outputting a higher fraction of iron in comparison to alpha elements.

The solar abundances from which the alpha abundance and metallicity are scaled, are taken from Grevesse & Sauval (1998).
the figure (Fig. [10]) with the following relation.

\[
[\alpha/Fe] = \begin{cases} 
0.3 & \text{if } [\text{Fe/H}] < -1 \\
-0.3 \cdot [\text{Fe/H}] & \text{if } -1 \leq [\text{Fe/H}] < -0.2 \\
0 & \text{if } [\text{Fe/H}] \geq -0.2
\end{cases}
\]  

(51)

5.5 Summary: Stellar parameters

Finally, in Tab. [10] the parameters are structured and summarized. These are the ones that are used in constructing model atmospheres and synthetic spectra.

<table>
<thead>
<tr>
<th>HR</th>
<th>Name</th>
<th>Spec. Type</th>
<th>(T_{\text{eff}} [\text{K}])</th>
<th>(\log(g)) (cgs)</th>
<th>([\text{Fe/H}])</th>
<th>(\xi ,[\text{kms}^{-1}])</th>
<th>(2[\alpha/Fe])</th>
</tr>
</thead>
<tbody>
<tr>
<td>HR8775</td>
<td>(\beta) Peg</td>
<td>M2.5II</td>
<td>3448 ± 42</td>
<td>0.54 ± 0.03</td>
<td>(-1.11)</td>
<td>2.0 ± 0.1</td>
<td>0.00</td>
</tr>
<tr>
<td>HR4910</td>
<td>(\delta) Vir</td>
<td>M3.0III</td>
<td>3602 ± 44</td>
<td>0.84 ± 0.03</td>
<td>(-0.09)</td>
<td>2.0 ± 0.1</td>
<td>0.03</td>
</tr>
<tr>
<td>HR6056</td>
<td>(\delta) Oph</td>
<td>M0.5III</td>
<td>3721 ± 47</td>
<td>1.02 ± 0.06</td>
<td>(0.00)</td>
<td>2.0 ± 0.1</td>
<td>0.00</td>
</tr>
<tr>
<td>HR4069</td>
<td>(\mu) UMa</td>
<td>M0.0III</td>
<td>3793 ± 47</td>
<td>1.07 ± 0.06</td>
<td>(0.00)</td>
<td>2.0 ± 0.1</td>
<td>0.00</td>
</tr>
<tr>
<td>HR3705</td>
<td>(\alpha) Lyn</td>
<td>K7.0III</td>
<td>3836 ± 47</td>
<td>0.98 ± 0.06</td>
<td>(-0.26)</td>
<td>2.0 ± 0.1</td>
<td>0.08</td>
</tr>
<tr>
<td>HR1457</td>
<td>(\alpha) Tau</td>
<td>K5.0III</td>
<td>3871 ± 48</td>
<td>1.27 ± 0.06</td>
<td>(0.00)</td>
<td>2.0 ± 0.1</td>
<td>0.00</td>
</tr>
<tr>
<td>HR3748</td>
<td>(\alpha) Hya</td>
<td>K3.0II</td>
<td>4060 ± 50</td>
<td>1.35 ± 0.05</td>
<td>(-0.12)</td>
<td>2.0 ± 0.1</td>
<td>0.00</td>
</tr>
<tr>
<td>HR5340</td>
<td>(\alpha) Boo</td>
<td>K1.5III</td>
<td>4226 ± 53</td>
<td>1.67 ± 0.03</td>
<td>(-0.60)</td>
<td>2.0 ± 0.1</td>
<td>0.18</td>
</tr>
<tr>
<td>HR5854</td>
<td>(\alpha) Ser</td>
<td>K2.0III</td>
<td>4558 ± 56</td>
<td>2.50 ± 0.03</td>
<td>(0.03)</td>
<td>2.0 ± 0.1</td>
<td>0.00</td>
</tr>
<tr>
<td>HR2990</td>
<td>(\beta) Gem</td>
<td>K0.0III</td>
<td>4858 ± 60</td>
<td>2.88 ± 0.03</td>
<td>(-0.07)</td>
<td>2.0 ± 0.1</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 10: Comprehensive table with the final parameters of the stars which are used when generating model atmospheres.

1) Mozurkewich et al. [2003]
2) Eq. [51]
3) Smith & Lambert [1985]
4) McWilliam [1990]
5) Mallik [1998]
6) Leep et al. [1987]
7) Koleva & Vazdekis [2012]

6 Results

Here we present the data on all the stars in the form of their observed spectra and their synthetic spectra, the signal-to-noise measurements of the observational spectra, the molecular data on the water lines of interest in the 12 micron region and finally the equivalent widths as a function of effective temperature. These results are then further analysed in the Discussion section (Section [7]) of this thesis.

Firstly, the TEXES data along with the synthetic spectra based on the stellar parameters acquired, are plotted for all stars in the spectral region of 806.5 to 821.5 cm\(^{-1}\) (Fig. [12][20]). The black solid line is the observed spectra,
the red marks the synthetic spectra based on a standard model atmosphere and the blue is the telluric spectra, i.e. the absorption spectrum of Earth’s atmosphere. All the data was Doppler-shifted to lab wavelengths and the lack of data is indicated by a missing solid black line. Naturally, the observational data presented has had its telluric component removed, although there are still artifacts in the data that can be traced to the telluric spectra. This makes it invaluable to present it together with the observational data. The blue and green ticks above the data indicate positions of water- and OH-lines respectively as they appear in the model spectra. Dotted-lines denote water lines of interest with measured equivalent widths. Emission lines due to Rydbergs transitions are also identified and marked (Sundqvist et al., 2008).

The comb-like gaps that appear in the data are a consequence of the TEXES instrument being a cross-dispersed grating spectrometer working at high orders (∼ 1600:th) which causes gaps between the orders since they do not overlap in wavelength (which is not the case for optical echelle spectrometers) resulting in a non-continuous spectra, hence its appearance as a rectangular absorption line. Although the data consists mostly of absorption lines, some emission features are detected, most notably in α Boo (Arcturus) in the form of Si, Al and Mg (Sundqvist et al., 2008). The quality of the data is not only ascertained through the instrument and telescope capabilities, but stellar brightness, exposure times and atmospheric conditions also play a vital role in attaining good spectra. The signal-to-noise S/N is sensitive to these factors and is presented for each observational spectrum in Tab. 17. Although a high S/N is preferable, no attempt was made to reach a target S/N for all spectra. From these figures one can deduce that synthetic spectra based on standard MARCS model atmospheres, does not fit for the H$_2$O nor the OH lines at 12 µm.

Molecular data on the relevant lines in the 808-822 cm$^{-1}$ region are collected in Tab. 15 (data partly from Ryde et al., 2002). The transitions that give rise to the spectral lines are quantified through their lab frequencies, the excitation energy of the lower level $E''_{\text{exc}}$, the transition probability log $gf$ and the quantum numbers of the levels in play. In molecular spectroscopy, these quantum numbers are ordered with the upper level (primed quantum numbers) followed by the lower level (double primed quantum numbers) and are separated into rotational and vibrational parts. The rotational quantum state of the molecule is defined by $J(K_a, K_c)$, where $J$ is the total angular momentum quantum number and $K$ is the projection of $J$ onto the principal axis of the molecule in question (which means $-J \leq K \leq +J$), in this case H$_2$O. The vibrational quantum number consists of three values which together defines the type of vibration (such as rocking, wagging, stretching etc.). Naturally, the transition between the energy levels occur on both a rotational and a vibrational level, however the transitions provided in Tab. 15 are all purely rotational. It is therefore sufficient to provide the
vibrational quantum number for one level only. Also, all the lines are within the three first vibrational states.

From the molecular data we can conclude that there are two lines at the low excitation level of $E_{\text{exc}}^\prime \sim 0.4 - 0.5$ eV and transition probabilities of $\log gf \sim -2.8$ while the rest lie at higher values ($E_{\text{exc}} \sim 1.0 - 1.4$ eV) with line strengths of around $\log gf \sim -1.5$.

In Tab. 11-14 our measured equivalent widths of the 12 H$_2$O lines are given. Using the IRAF software system, the observed equivalent widths were ascertained by fitting a Gaussian profile to the spectral line. If no widths are measured due to its placement in a gap, a noisy region or simply because of lack of data, it is given as ‘−’ (unknown) as opposed to a measured equivalent width of 0 mÅ. The equivalent widths presented with the synthetic spectra have a cut-off at 2.5 mÅ, the model equivalent widths below this cut-off are therefore given as ‘−’. As expected, the observed equivalent widths are inversely proportional to the effective temperature. A higher temperature implies a smaller abundance of water and therefore smaller equivalent widths. This is visualised in Fig. 21 for the 815.9 cm$^{-1}$ water line which is plotted with a neighbouring OH-line, moreover it can be seen that the water line vanishes at around 4300 to 4500 K. The observed equivalent widths are much stronger than their model equivalents for all stars. A measure of the line strengths are given for each line and star (Tab. 11-14) and together with the curve of growth of water constructed from stellar model atmospheres (Fig. 11), we deduce that the observed equivalent widths are around the region of saturation (dotted line in Fig. 11).
Figure 11: The curve of growth of H_2O as predicted by stellar model atmospheres with line strength as a function of oscillator strength. A spectral line with a line strength at the linear part in the beginning, commonly known as the Doppler-part, is called weak. The flattening region (start approximated by the dotted line at \( \log W/\nu = -5.5 \)) is when the spectral line becomes saturated and the linear part at higher line strengths, which is not mapped in this particular curve of growth, represents the growth of strong lines. Note the choice of abscissa is a range of \( \log(gf) \) for a typical line, this is equivalent to changes in \( \log A \) as seen in Eq. 39.

Additionally, \( \beta \) Gem lacks observational water in the same way as for \( \alpha \) Ser, which is why it is omitted in Fig. 12 through 20.

The spectral lines, although picked to be as isolated as possible, are still blended especially towards the cooler stars in our sample. Since the tool-set provided to calculate synthetic spectra, can supply accurate non-blended equivalent widths, a precaution is implemented based on the observed spectra to take line blending into consideration. Through the fitted Gaussian profiles of the observed spectral lines (while ascertaining their equivalent widths), a measure of the spread of the lines are obtained, the full width half maximum (FWHM). It specifies, as the name implies, the breadth of the line halfway to the line center. The point of full blend is set for features with line centers within the FWHM (as retrieved in the observed spectra) of the targeted line.

Therefore, by examining the presence of other spectral lines within the FWHM of a target line and summing up their non-blended equivalent widths, we calculate a value of the model equivalent widths more akin and comparable to the observed widths. Tab. 16 illustrate the blending properties of the 12 water lines for the studied stars through the use of the FWHM of the
corresponding observed line. The level of blending is indicated by the color of the cell where green corresponds to an unblended line (set as 100-90% of the total equivalent width within the observed FWHM being contributed by the target line), orange marks an intermediate blend of 90-70% and red cells indicate strong blend where less than 70% of the equivalent width originates from the line of interest. White cells point out lack of data, i.e. the target line had an equivalent width less than the cut-off at 2.5 mÅ.

Uncertain placements of the continuum together with the spread of profile fits for each spectral line, introduces uncertainties for the measured equivalent widths of the observed spectra. However, this measurement related uncertainty is absent with the precise model equivalent widths provided by the tool-set. The FWHM has a certain spread albeit too small (or the uncertainty of the FWHMs does not noticeably change the model equivalent widths) to make any real difference. Instead, new synthetic spectra are created using atmospheric models with the effective temperatures $T_{\text{eff}} \pm \sigma_{T_{\text{eff}}}$, as given in Tab. 10 from which the calculated equivalent widths are set as the uncertainty. Naturally, one can argue that other stellar parameters also play a vital role in the determination of the uncertainty for the model equivalent widths. Therefore an analysis was made for two typical stars in the dataset, $\alpha$ Hya and $\delta$ Oph, to see the effect on the equivalent widths when varying $\log(g)$ and $\xi$. The equivalent widths for the 12 micron lines increases by 10-20% in strength (around 0.1 dex) for a change in $\log(g)$ of 0.1 dex. The effect of different microturbulent velocities is discussed in the next section.

Fig. 22 and 23 illustrate the the equivalent widths as a function of the effective temperature together with their uncertainties, plotted for the observed and model equivalents widths respectively. Each line in these figures are color-coded and have their excitation energies along with the line strengths provided in the legend. The uncertainties in Fig. 23 are given in such a way that they aim at the $W_{\text{mod}}$ given $T_{\text{eff}} \pm \sigma_{T_{\text{eff}}}$ the dotted lines. Here the difference in the equivalent widths associated with the standard deviation of the effective temperature, are in the order of 20% (0.08 dex) for the coolest stars ($\beta$ Peg) and $\sim$ 100% (0.30 dex) for hottest star with any trace of water in our sample ($\alpha$ Ser), in comparison with the standard equivalent width. The high difference for the hot stars is an effect of their effective temperature being in a region where water barely starts becoming prominent, causing a comparison between small and small values. In both plots the presence of water increases with comparable slopes and as deduced previously, the waterlines starts disappearing at around effective temperatures above 4300 K in the observed spectra (Fig. 22). Interestingly, the synthetic spectra based on the standard atmospheric models cannot account for water in stars already at around 4200 K.

The observed and synthetic spectra (Fig. 12-20), the obtained equivalent widths (Tab. 11-14) and the illustration of these (Fig. 22-23) all display the inability of the standard MARCS model atmosphere to create $H_2O$ (or OH)
lines that fit the data. Furthermore, the expected water line emission due to the presence of MOLspheres (see Tsuji 2000a; Ryde et al. 2006b) is not detected.
Figure 12: The observations of $\beta$ Peg, $\delta$ Vir, and $\delta$ Oph (black line) in the region of 806.5 to 811.5 cm$^{-1}$ together with the synthetic spectra of their respective standard models (red line) and the atmospheric absorption of Earth (blue line) which is shifted down for visibility. The blue ticks above the spectra indicate spectral features of H$_2$O whereas green marks OH (the OH lines are further distinguished with the green “OH” text). Dotted lines mark the spectral features of interest which are ultimately used in the calculation of the formation region temperatures. The lack of observational data is indicated by a missing solid black line.
Figure 13: The observations of $\mu$ UMa, $\alpha$ Lyn, and $\alpha$ Tau (black line) in the region of 806.5 to 811.5 cm$^{-1}$ together with the synthetic spectra of their respective standard models (red line) and the atmospheric absorption of Earth (blue line) which is shifted down for visibility. The blue ticks above the spectra indicate spectral features of H$_2$O whereas green marks OH (the OH lines are further distinguished with the green “OH” text). Dotted lines mark the spectral lines of interest which are ultimately used in the calculation of the formation region temperatures. The lack of observational data is indicated by a missing solid black line.
Figure 14: The observations of $\alpha$ Hya, $\alpha$ Boo, and $\delta$ Ser (black line) in the region of 806.5 to 811.5 cm$^{-1}$ together with the synthetic spectra of their respective standard models (red line) and the atmospheric absorption of Earth (blue line) which is shifted down for visibility. The blue ticks above the spectra indicate spectral features of H$_2$O whereas green marks OH (the OH lines are further distinguished with the green “OH” text). Dotted lines mark the spectral lines of interest which are ultimately used in the calculation of the formation region temperatures. Moreover, emission features of neutral Si and Al are marked (see Sundqvist et al., 2008). The lack of observational data is indicated by a missing solid black line.
Figure 15: The observations of $\beta$ Peg, $\delta$ Vir, and $\delta$ Oph (black line) in the region of 811.5 to 816.5 cm$^{-1}$ together with the synthetic spectra of their respective standard models (red line) and the atmospheric absorption of Earth (blue line) which is shifted down for visibility. The blue ticks above the spectra indicate spectral features of H$_2$O whereas green marks OH (the OH lines are further distinguished with the green "OH" text). Dotted lines mark the spectral features of interest which are ultimately used in the calculation of the formation region temperatures. Moreover, the emission feature of neutral Ca at 814.969 cm$^{-1}$ is marked (see Sundqvist et al., 2008). The lack of observational data is indicated by a missing solid black line.
Figure 16: The observations of $\mu$ UMa, $\alpha$ Lyn, and $\alpha$ Tau (black line) in the region of 811.5 to 816.5 cm$^{-1}$ together with the synthetic spectra of their respective standard models (red line) and the atmospheric absorption of Earth (blue line) which is shifted down for visibility. The blue ticks above the spectra indicate spectral features of H$_2$O whereas green marks OH (the OH lines are further distinguished with the green “OH” text). Dotted lines mark the spectral lines of interest which are ultimately used in the calculation of the formation region temperatures. Moreover, the emission feature of neutral Ca at 814.969 cm$^{-1}$ is marked (see Sundqvist et al., 2008). The lack of observational data is indicated by a missing solid black line.
Figure 17: The observations of α Hya, α Boo, and δ Ser (black line) in the region of 811.5 to 816.5 cm$^{-1}$ together with the synthetic spectra of their respective standard models (red line) and the atmospheric absorption of Earth (blue line) which is shifted down for visibility. The blue ticks above the spectra indicate spectral features of H$_2$O whereas green marks OH (the OH lines are further distinguished with the green “OH” text). Dotted lines mark the spectral lines of interest which are ultimately used in the calculation of the formation region temperatures. Moreover, the emission feature of neutral Ca at 814.969 cm$^{-1}$ is marked (see Sundqvist et al., 2008). The lack of observational data is indicated by a missing solid black line.
Figure 18: The observations of $\beta$ Peg, $\delta$ Vir, and $\delta$ Oph (black line) in the region of 816.5 to 821.5 cm$^{-1}$, together with the synthetic spectra of their respective standard models (red line) and the atmospheric absorption of Earth (blue line), which is shifted down for visibility. The blue ticks above the spectra indicate spectral features of H$_2$O whereas green marks OH (the OH lines are further distinguished with the green "OH" text). Dotted lines mark the spectral features of interest which are ultimately used in the calculation of the formation region temperatures. Also, one emission feature in the form of neutral Mg is visible at 818.1 cm$^{-1}$ (see Sundqvist et al., 2008). The lack of observational data is indicated by a missing solid black line.
Figure 19: The observations of $\mu$ UMa, $\alpha$ Lyn, and $\alpha$ Tau (black line) in the region of 816.5 to 821.5 cm$^{-1}$ together with the synthetic spectra of their respective standard models (red line) and the atmospheric absorption of Earth (blue line) which is shifted down for visibility. The blue ticks above the spectra indicate spectral features of H$_2$O whereas green marks OH (the OH lines are further distinguished with the green "OH" text). Dotted lines mark the spectral lines of interest which are ultimately used in the calculation of the formation region temperatures. Also, one emission feature in the form of neutral Mg is visible at 818.1 cm$^{-1}$ (see Sundqvist et al., 2008). The lack of observational data is indicated by a missing solid black line.
Figure 20: The observations of α Hya, α Boo, and δ Ser (black line) in the region of 816.5 to 821.5 cm\(^{-1}\) together with the synthetic spectra of their respective standard models (red line) and the atmospheric absorption of Earth (blue line) which is shifted down for visibility. The blue ticks above the spectra indicate spectral features of H\(_2\)O whereas green marks OH (the OH lines are further distinguished with the green “OH” text). Dotted lines mark the spectral lines of interest which are ultimately used in the calculation of the formation region temperatures. Also, one emission feature in the form of neutral Mg is visible at 818.1 cm\(^{-1}\) (see Sundqvist et al., 2008). The lack of observational data is indicated by a missing solid black line.
<table>
<thead>
<tr>
<th>$\nu_{\text{lab}}$ [cm$^{-1}$]</th>
<th>$E''_{\text{exc}}$ [eV]</th>
<th>log $gf$</th>
<th>$J'(K_a', K_c')$</th>
<th>$J''(K_a'', K_c'')$</th>
<th>$v_1 v_2 v_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>808.63210</td>
<td>1.140</td>
<td>−1.40</td>
<td>23(11,12)</td>
<td>22(10,13)</td>
<td>(010)</td>
</tr>
<tr>
<td>814.67491</td>
<td>1.385</td>
<td>−1.18</td>
<td>20(16,5)</td>
<td>19(15,5)</td>
<td>(020)</td>
</tr>
<tr>
<td>814.67491$^a$</td>
<td>1.385</td>
<td>−1.66</td>
<td>20(16,5)</td>
<td>19(15,5)</td>
<td>(020)</td>
</tr>
<tr>
<td>815.30059</td>
<td>0.498</td>
<td>−2.51</td>
<td>18(7,12)</td>
<td>17(4,13)</td>
<td>(000)</td>
</tr>
<tr>
<td>815.897$^a$</td>
<td>1.396</td>
<td>−1.00</td>
<td>21(21,0)</td>
<td>20(20,1)</td>
<td>(010)</td>
</tr>
<tr>
<td>815.900</td>
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<td>−1.48</td>
<td>21(21,0)</td>
<td>20(20,1)</td>
<td>(010)</td>
</tr>
<tr>
<td>816.45026</td>
<td>0.398</td>
<td>−3.21</td>
<td>17(5,13)</td>
<td>16(2,14)</td>
<td>(000)</td>
</tr>
<tr>
<td>816.68703</td>
<td>1.014</td>
<td>−1.35</td>
<td>24(12,13)</td>
<td>23(11,12)</td>
<td>(000)</td>
</tr>
<tr>
<td>817.15695</td>
<td>1.206</td>
<td>−1.18</td>
<td>21(16,5)</td>
<td>20(15,6)</td>
<td>(010)</td>
</tr>
<tr>
<td>817.20881</td>
<td>1.014</td>
<td>−1.83</td>
<td>24(12,12)</td>
<td>23(11,13)</td>
<td>(000)</td>
</tr>
<tr>
<td>818.4238$^b$</td>
<td>1.029</td>
<td>−1.66</td>
<td>22(16,6)</td>
<td>21(15,7)</td>
<td>(000)</td>
</tr>
<tr>
<td>818.4247$^b$</td>
<td>1.029</td>
<td>−1.19</td>
<td>22(16,7)</td>
<td>21(15,6)</td>
<td>(000)</td>
</tr>
<tr>
<td>819.04574</td>
<td>1.319</td>
<td>−1.30</td>
<td>21(13,8)</td>
<td>20(12,9)</td>
<td>(020)</td>
</tr>
<tr>
<td>819.93233</td>
<td>1.050</td>
<td>−1.42</td>
<td>25(11,14)</td>
<td>24(10,15)</td>
<td>(000)</td>
</tr>
<tr>
<td>820.19016</td>
<td>1.360</td>
<td>−1.04</td>
<td>21(20,1)</td>
<td>20(19,2)</td>
<td>(010)</td>
</tr>
<tr>
<td>820.19016$^a$</td>
<td>1.360</td>
<td>−1.52</td>
<td>21(20,1)</td>
<td>20(19,2)</td>
<td>(010)</td>
</tr>
<tr>
<td>820.58322</td>
<td>1.245</td>
<td>−1.15</td>
<td>21(17,5)</td>
<td>20(16,5)</td>
<td>(010)</td>
</tr>
<tr>
<td>820.58322$^a$</td>
<td>1.245</td>
<td>−1.62</td>
<td>21(17,5)</td>
<td>20(16,5)</td>
<td>(010)</td>
</tr>
</tbody>
</table>

Table 15: Molecular data for the relevant H$_2$O lines in the 808–822 cm$^{-1}$ region constructed partly from Ryde et al. (2002) data.

$^a$ Uncertain assignment of the quantum numbers for the states of the transition.

$^b$ Assigned from Ryde et al. (2002) which in turn are based on assignments by Tsuji (2000b).
Table 11: The equivalent widths of water features in mÅ for the entire dataset at 12 µm. Since some level of blending exists for all lines, both model and observational equivalent widths were measured (the other option being that the equivalent widths for the models being obtained theoretically). Each line also had a measure of their observational line strength calculated \( \log W_{\text{obs}} / \nu \), where \( W_{\text{obs}} \) is the observed equivalent width and \( \nu \) the wavenumber.
<table>
<thead>
<tr>
<th>HR</th>
<th>Name</th>
<th>$T_{\text{eff}}$ (K)</th>
<th>$815.900$ cm$^{-1}$</th>
<th>$816.450$ cm$^{-1}$</th>
<th>$816.687$ cm$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>mod.</td>
<td>obs.</td>
<td>$\log W_{\text{obs}}$</td>
<td>mod.</td>
</tr>
<tr>
<td>HR8775</td>
<td>$\beta$ Peg</td>
<td>3448 ± 42</td>
<td>713</td>
<td>826</td>
<td>-5.17</td>
</tr>
<tr>
<td>HR4910</td>
<td>$\delta$ Vir</td>
<td>3602 ± 44</td>
<td>446</td>
<td>555</td>
<td>-5.34</td>
</tr>
<tr>
<td>HR6056</td>
<td>$\delta$ Oph</td>
<td>3721 ± 47</td>
<td>285</td>
<td>466</td>
<td>-5.42</td>
</tr>
<tr>
<td>HR4069</td>
<td>$\mu$ UMa</td>
<td>3793 ± 47</td>
<td>186</td>
<td>421</td>
<td>-5.46</td>
</tr>
<tr>
<td>HR3705</td>
<td>$\alpha$ Lyn</td>
<td>3836 ± 47</td>
<td>147</td>
<td>481</td>
<td>-5.40</td>
</tr>
<tr>
<td>HR1457</td>
<td>$\alpha$ Tau</td>
<td>3871 ± 48</td>
<td>140</td>
<td>421</td>
<td>-5.46</td>
</tr>
<tr>
<td>HR3748</td>
<td>$\alpha$ Hya</td>
<td>4060 ± 60</td>
<td>32.2</td>
<td>165</td>
<td>-5.87</td>
</tr>
<tr>
<td>HR5340</td>
<td>$\alpha$ Boo</td>
<td>4226 ± 53</td>
<td>6.62</td>
<td>146</td>
<td>-5.92</td>
</tr>
<tr>
<td>HR5854</td>
<td>$\alpha$ Ser</td>
<td>4558 ± 56</td>
<td>–</td>
<td>0.00</td>
<td>$-\infty$</td>
</tr>
<tr>
<td>HR2990</td>
<td>$\beta$ Gem</td>
<td>4858 ± 60</td>
<td>–</td>
<td>0.00</td>
<td>$-\infty$</td>
</tr>
</tbody>
</table>

Table 12: The equivalent widths of water features in mÅ for the entire dataset at 12 µm. Since some level of blending exists for all lines, both model and observational equivalent widths were measured (the other option being that the equivalent widths for the models being obtained theoretically). Each line also had a measure of their observational line strength calculated $\log W_{\text{obs}}$, where $W_{\text{obs}}$ is the observed equivalent width and $\nu$ the wavenumber.
Table 13: The equivalent widths of water features in m˚A for the entire dataset at 12 µm. Since some level of blending exists for all lines, both model and observational equivalent widths were measured (the other option being that the equivalent widths for the models being obtained theoretically). Each line also had a measure of their observational line strength calculated $\log \frac{W_{\text{obs}}}{\nu}$, where $W_{\text{obs}}$ is the observed equivalent width and $\nu$ the wavenumber.
Table 14: The equivalent widths of water features in mÅ for the entire dataset at 12 µm. Since some level of blending exists for all lines, both model and observational equivalent widths were measured (the other option being that the equivalent widths for the models being obtained theoretically). Each line also had a measure of their observational line strength calculated $\log \frac{W_{\text{obs}}}{\nu}$, where $W_{\text{obs}}$ is the observed equivalent width and $\nu$ the wavenumber.

<table>
<thead>
<tr>
<th>HR</th>
<th>Name</th>
<th>$T_{\text{eff}}$ (K)</th>
<th>$819.046 \text{ cm}^{-1}$</th>
<th>$819.932 \text{ cm}^{-1}$</th>
<th>$820.583 \text{ cm}^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>mod.</td>
<td>obs.</td>
<td>$\log \frac{W_{\text{obs}}}{\nu}$</td>
</tr>
<tr>
<td>HR8775</td>
<td>$\beta$ Peg</td>
<td>3448 ± 42</td>
<td>420</td>
<td>655</td>
<td>-5.27</td>
</tr>
<tr>
<td>HR4910</td>
<td>$\delta$ Vir</td>
<td>3602 ± 44</td>
<td>256</td>
<td>537</td>
<td>-5.36</td>
</tr>
<tr>
<td>HR6056</td>
<td>$\delta$ Oph</td>
<td>3721 ± 47</td>
<td>158</td>
<td>313</td>
<td>-5.59</td>
</tr>
<tr>
<td>HR4069</td>
<td>$\mu$ UMa</td>
<td>3793 ± 47</td>
<td>102</td>
<td>343</td>
<td>-5.55</td>
</tr>
<tr>
<td>HR3705</td>
<td>$\alpha$ Lyn</td>
<td>3836 ± 47</td>
<td>79.5</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>HR1457</td>
<td>$\alpha$ Tau</td>
<td>3871 ± 48</td>
<td>75.6</td>
<td>209</td>
<td>-5.77</td>
</tr>
<tr>
<td>HR3748</td>
<td>$\alpha$ Hya</td>
<td>4060 ± 60</td>
<td>16.2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>HR5340</td>
<td>$\alpha$ Boo</td>
<td>4226 ± 53</td>
<td>4.37</td>
<td>118</td>
<td>-6.02</td>
</tr>
<tr>
<td>HR5854</td>
<td>$\alpha$ Ser</td>
<td>4558 ± 56</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>HR2990</td>
<td>$\beta$ Geminorum</td>
<td>4858 ± 60</td>
<td>-</td>
<td>0.00</td>
<td>$-\infty$</td>
</tr>
</tbody>
</table>
Table 16: An overview on the blending of the water lines. The blends are calculated by examining the contribution of line equivalent widths in the vicinity of the target spectral line for the model spectra. Vicinity in this context has been determined as the FWHM of the measured observational line, thus deeming the other spectral lines within this area as completely blended with the line of interest. Green marks an unblended line where 100 - 90\% of the total equivalent width contribution to the equivalent width by the targeted line. Orange is 90-70\% whereas red marks a strong blend where less than 70\% of the equivalent width originates from the line of interest, for the given FWHM. White spaces indicate lack of data, i.e the target line had a equivalent width less than 2.5 \text{mÅ}.

![Blending Diagram]

Table 17: The signal-to-noise per pixel ratios for all stars.

<table>
<thead>
<tr>
<th>HR</th>
<th>Name</th>
<th>S/N</th>
</tr>
</thead>
<tbody>
<tr>
<td>HR8775</td>
<td>(\beta) Peg</td>
<td>140</td>
</tr>
<tr>
<td>HR4910</td>
<td>(\delta) Vir</td>
<td>110</td>
</tr>
<tr>
<td>HR6056</td>
<td>(\delta) Oph</td>
<td>90</td>
</tr>
<tr>
<td>HR4069</td>
<td>(\mu) UMa</td>
<td>100</td>
</tr>
<tr>
<td>HR3705</td>
<td>(\alpha) Lyn</td>
<td>100</td>
</tr>
<tr>
<td>HR1457</td>
<td>(\alpha) Tau</td>
<td>90</td>
</tr>
<tr>
<td>HR3748</td>
<td>(\alpha) Hya</td>
<td>120</td>
</tr>
<tr>
<td>HR5340</td>
<td>(\alpha) Boo</td>
<td>290</td>
</tr>
<tr>
<td>HR5854</td>
<td>(\alpha) Ser</td>
<td>70</td>
</tr>
<tr>
<td>HR2990</td>
<td>(\beta) Gem</td>
<td>80</td>
</tr>
</tbody>
</table>
Figure 21: Superimposed observed spectra of the 815.90 cm$^{-1}$ H$_2$O line along with a neighbouring OH-line. The H$_2$O line strength is inversely proportional to the temperature, with the 815.90 cm$^{-1}$ line disappearing most notably at around 4300 to 4500 K. With less oxygen bound to water for higher temperatures, the OH line strength increases before it follows the same trend as the water line.
Figure 22: Observed equivalent widths in mÅ as a function of effective temperature in K. Each line is color-coded according to the legend where their excitation energies and the line strengths also are provided. The uncertainties in the effective temperature is given by Tab. 10 whereas different placements of the continuum and profile fits make up the uncertainty on the observed equivalent widths.
Figure 23: The model equivalent widths in mÅ as a function of effective temperature in K. Each line is color-coded according to the legend where their excitation energies and the line strengths also are provided. The uncertainties (solid line) are obtained by calculating the equivalent widths for model atmospheres with effective temperatures at ±1σ as provided in Tab. [10]. This type of uncertainty is not be confused with measurement uncertainties that originates from a spread of different profile fits and continuum placements otherwise associated with the observed spectra.
7 Discussion

There are a number of different scenarios that can explain the discrepancies in the synthetic equivalent widths versus the ones measured in the observed spectra. First off, we increase the oxygen abundance in the atmospheric models and observe its effects on the spectra. Next, we examine the basis of altered stellar temperature profiles and how these enhance the water lines in the models. The solution involving MOLspheres is then discussed briefly followed by the importance of NLTE effects on the water lines.

7.1 Higher oxygen abundance

A semi-empirical and efficient way of altering line strengths is to modify the abundances of the elements involved. Fig. 24 plots the observed spectra, the synthetic spectra based on a standard atmospheric model (dubbed standard model) in red and an oxygen enhanced model. Picking one of the studied stars, δ Oph, the aim was to make the modelled 815.900 cm$^{-1}$ water line to fit the observational equivalent. With an increased oxygen abundance of 0.6 dex, the line successfully fits, however it also introduces other synthetic absorption lines (e.g. water features beyond the gap at 817.5 cm$^{-1}$) that are stronger than the noise and should thus have been detected in the observed spectra. Moreover, an enrichment of $\Delta \log \epsilon_O = 0.6$ dex (i.e a factor of 4) is an unrealistic increase and not well motivated. However, there is recent work by [Tsuji 2008, 2009] that points out difficulties for the classical micro-turbulent model coupled with 1D model atmospheres, to describe OH and CO lines at near-saturation and stronger (log $W/\nu \gtrsim -4.75$). Therefore, these intermediate-strength (less strong but saturated) lines need to be treated with an adjustable microturbulence and not be based upon the micro turbulent velocities of the weak lines (log $W/\nu \lesssim -4.75$). The adjustable microturbulence would then manifest itself as an increased abundance in the order of $\Delta \log A \approx 0.5$ to 1 dex. Conclusively, this is physically interpreted as the infrared spectra being contaminated by a second component, namely a MOLsphere in the outer atmosphere of stars [Tsuji 2008, 2009]. The question arises whether these results taken for OH and CO, can be applied to H$_2$O and at what line strengths this becomes an issue if any.
DISCUSSION

Figure 24: The observed spectra of $\delta$ Oph together with the standard model and an oxygen enhanced model (by 0.6 dex). The aim was to enhance the oxygen enough to make the 815.900 H$_2$O line fit. Although the equivalent widths of the H$_2$O lines (marked with a blue tick above the spectra) do increase, the large addition of oxygen is unrealistic and would have shown in chemical analyses. Furthermore, as a consequence of the increased oxygen abundance, new lines are apparent in the synthetic spectra at 817.5 to 818 cm$^{-1}$. These are however non-existent in the observed spectra.

7.2 Microturbulent sensitivity

As was stated earlier, the water lines in the studied stars are in the region between weak and saturated (Tab. 11-14, Fig. 11). The sensitivity of the synthetic lines with different microturbulent velocities is thus worthwhile to be examined further, specifically due to the saturated lines.

We create different synthetic spectra by varying the microturbulences in one of the hotter stars, $\alpha$ Hya, and one of the colder, $\delta$ Oph. By computing the equivalent widths for microturbulent velocities in the known spread (see Tab. 8) and comparing it with the ones obtained from the standard value ($\xi = 2.0$ km/s = $\xi_{2.0}$), we receive a sense of how much impact the different microturbulences have on the equivalent widths. This is depicted in Fig. 25 where the percentage of the difference in equivalent widths between two microturbulences is plotted against the line strength as acquired in the standard model with $\xi_{2.0}$. This figure is for $\delta$ Oph, the coolest star in this test. With $\alpha$ Hya being hotter and therefore hosts a majority of weak lines, the differences between different micro turbulences would not be noted in
equivalent width (see Fig. 11). The black symbols note the change between \( \xi_{1.0} \) and \( \xi_{2.0} \) whereas the red are \( \xi_{2.0} \) to \( \xi_{3.0} \). The changes in the equivalent width, are up to 40\% for some lines in the extreme (in this case) scenario between \( \xi_{1.0} \) and \( \xi_{3.0} \).

Figure 25: The percentage of the difference in equivalent widths for \( \delta \) Oph between two microturbulences (A, B), plotted against the line strength as acquired in the standard model with \( \xi_{2.0} \). The choice of A, B are presented in the legend and are picked to be the known spread of \( \xi \) (see Tab. 8).

However, the equivalent widths cannot parametrize the spectral line alone. This becomes apparent when lines get smeared out and retain their areas (equivalent widths) but may appear completely different in a spectrum. Thus, Fig. 26 illustrate the effects on the synthetic spectra for different velocities for both \( \delta \) Oph and \( \alpha \) Hya in a characteristic water line. This figure displays the role of microturbulence in an excellent manner for the weak Doppler-part of the curve of growth (\( \alpha \) Hya) and the saturated part (\( \delta \) Oph).
Figure 26: The synthetic spectra of α Hya and δ Oph superimposed together with the synthetic spectra from, in total, three different microturbulent velocities (1.0, 2.0 and 3.0 km/s). This water line, although not selected in the list of studied lines, is as representative as any other line in the 12 micron region.

First off, the weak lines are affected in both the wings and the line center (smeared out), thus retaining the area and an unchanging equivalent width versus different microturbulences, in consent with Fig. 11. This can be seen as the fact that the microturbulence also acts as the macroturbulence in the way that it smears the lines out. The macroturbulence is convolved upon the Doppler broadening factor (Eq. 41) containing the microturbulence $\xi$.

In the case of δ Oph, at the point of saturation, $\xi$ is able to change the area depending on its velocity (hence the non-degeneracy in Fig. 11 for different micro-turbulent velocities at this region) and the growth, as seen in Fig. 26, is focused on the wings and not in the line center.

A certain concern arises for the weak lines where the macro- and micro-turbulence have similar effects. In other words, lowering $\xi$ and increasing the macroturbulence having the same end results. Knowing that the spectral resolution of our observations are

$$R = \frac{\lambda}{\Delta \lambda} \sim 60,000$$
and from the definition of Doppler shift, we receive
\[
\frac{\Delta \lambda}{\lambda} = \frac{v}{c} \quad \frac{1}{R} = \frac{v}{c} \quad \Rightarrow v \sim 5 \text{ km/s}
\]

A instrument profile of at least 5 km/s is thus convolved with the observed spectra. Therefore, the synthetic spectra need to account for a macroturbulence of this value or greater and is the one chosen in the synthetic spectra presented in Fig. 12 to 20. Conclusively, this constraint removes the possibility of the weak lines to be fitted with the observations. Additionally, and perhaps a more solid observation, is that the line center does not get deeper for the saturated lines. The standing issue is that the synthetic lines are too “shallow”, therefore it is not plausible that different microturbulences will provide a solution to the problem.

### 7.3 Altered temperature profiles

Nevertheless, another attempt at a solution is to adjust the temperature profiles (e.g. Ryde et al., 2002) which then alters the regions that allow the formation of water molecules. The formation region temperatures can be obtained via the equivalent widths with the use of the weak line approximation (Eq. 39), which is restated here:

\[
\log \left( \frac{W}{\lambda} \right) = \log C + \log A + \log g f \lambda - \frac{5040}{T} \chi - \log \kappa
\]

Subtracting the lines \( W_1 \) and \( W_2 \) (each with their own excitation energy \( \chi \) and \( \log g f \) values) using the above equation, we will eventually end up with the following expression:

\[
T = \frac{5040(\chi_1 - \chi_2)}{\log g f_1 - \log g f_2 - \log W_1/W_2}
\]

where we assume that \( \kappa \) is constant in the spectral region under study. The formation region temperature \( T \) has the units of K and is as as the name implies, a measure of the temperature at which the lines are formed. Like stated before, Eq. 39 or 52 assumes that this is a single value (causing \( \log C \) to be a constant in the first place) and is thus not valid for the case of a temperature gradient and an “elongated” spectral forming region. Since this very well might be the case, a justification of the usage of the formula, is required.

Proceeding with calculating the optical depths of the different water lines using Eq. 1 in Ryde et al. (2006b),

\[
\tau_{i0} \approx 0.02654 \frac{N_{\text{col}}}{\sqrt{\pi} \Delta \nu T} \left( 1 - e^{-h \nu_0/kT} \right) \frac{g f_{1u}}{U(1500 \text{ K})} e^{-\chi/kT}
\]
where $N_{\text{col}}$ is the column density of water vapour, $\Delta \nu_D$ the Doppler line width, $g$ is the statistical weight and $f$ the oscillator strength. The index $l$ denotes the lower level whereas $u$ is the upper one. $U$ is the partition function of water vapour at the temperature of 1500 K and $\chi$ is the excitation energy of the lower level.

The main interest here is to get a handle at which depths these lines are formed and not to know the absolute value. Rewriting Eq. 54

$$\tau_{\nu_0}^l \propto C \cdot g_l f_{lu} e^{-\chi/kT} \quad (55)$$

When computed for the water lines of interest and assuming a formation region temperature of 2000 K (taken from Ryde et al. 2002), the lines seem to be formed at an optical depth of a factor of $\sim 2$ of each other (Tab. 18). The size of the region is in other words small, which leads to the question on how the temperature gradient is in this formation region. A large temperature gradient in a short region is equivalent to a small gradient in a large region. Both situations causes Eq. 53 to be physically unsound.

<table>
<thead>
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<th>$\nu_{\text{lab}}$ [cm$^{-1}$]</th>
<th>$\tau_{\nu_0}^l \cdot 10^4/C$</th>
</tr>
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</tr>
<tr>
<td>814.675</td>
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<tr>
<td>820.583</td>
<td>0.516105</td>
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</tbody>
</table>

Table 18: The water lines of interest and the computed optical depth proxy (Eq. 54). Since the interest lies in comparing and not the absolute values, a factor of $10^4$ was applied for clarity.

To acquire a sense of where the formation would be, temperature wise, Eq. 53 is calculated using both the synthetic and the observed equivalent widths for the following groups:

$$(\nu_1, \nu_2) = (815.301, 816.687)$$
$$= (815.301, 817.157)$$
$$= (816.450, 816.687)$$
$$= (816.450, 816.687)$$
where $\nu = \nu_{\text{lab}}$ identifies the spectral lines used, given in Tab. 15. These were picked in a way so that there were large differences not only between $\chi_1$ and $\chi_2$, but also for $\log gf$, such that their differences were not in the same order as their individual uncertainties. The temperatures land at around $T = 2500 \text{ K}$ for the observed spectra. For Arcturus (Ryde et al., 2002) this $T$ together with the span of optical depths imply a $\Delta T \sim 20 \text{ K}$. This validates the assumption that the water lines are formed at the same optical depth and thus temperature. Furthermore, the slope of the temperature profile is approximately the same for a wide range of $T_{\text{eff}}$ and $g$ (Gray 2005, Chap. 9), making this ansatz sensible for all stars.

Backtracking, using Eq. 53, it can be shown in the case that the temperature profile of the atmospheric model is identical to that of the star, we will obtain the following for a single spectral line:

$$\log \frac{W_{\text{obs}}}{W_{\text{mod}}} = \log \frac{W_{\text{obs}}}{\lambda} - \log \frac{W_{\text{mod}}}{\lambda} = \Delta \log A \quad (56)$$

where $\Delta \log A$ is the difference in abundance (H$_2$O) between the star and the model star.

$$\Delta \log A = \log [A_{\text{obs}}(T_{\text{obs}})] - \log [A_{\text{mod}}(T_{\text{mod}})] \quad (57)$$

An important note is that the abundance is dependent on the temperature for molecules in contrast to atomic abundances, or more specifically the temperature at the line-formation region since it is that abundance that is visible in the spectra. The temperature dependence of the molecular abundance equilibrium comes from a complicated interplay of other molecular abundances containing the same elements together with their dissociations etc. In other words, the building blocks are not dependent on the temperature, whereas how you build with them is. Combining the data that makes up Fig. 22 and 23 we receive Fig. 27 that illustrate the $W_{\text{obs}}/W_{\text{mod}}$ trend as a function of effective temperature together with the propagated uncertainties.
Figure 27: This plot combines Fig. 22 and Fig. 23 into a measure of likeness as a function of temperature. Like the legend implies, the lines have different excitation energies and in conjunction with Eq. 57, one can deduce that a different temperature profile might be a solution and/or a NLTE correction that is dependent on excitation energy.

Eq. 56 implies that the lines would merge to one point in Fig. 27 ($W_{\text{obs}}/W_{\text{mod}}$ being only dependent on the abundance differences in the model and the star) and correcting it would simply be done by adjusting the abundance of $\text{H}_2\text{O}$ in the model (since $\log A_{\text{obs}}$ naturally is set by the star already). By raising the oxygen abundance in the models (as was done previously), we supplied more $\text{O}$ in order to the increase $\text{H}_2\text{O}$ abundance. This resulted in an unsatisfactory solution.

On the other hand, if the temperature scaling is incorrect in the model, the resulting equation for one spectral line is

$$
\log \frac{W_{\text{obs}}}{\lambda} - \log \frac{W_{\text{mod}}}{\lambda} = \Delta \log A - \frac{5040}{T_{\text{obs}}} \chi + \frac{5040}{T_{\text{mod}}} \chi
$$

$$
= \Delta \log A - 5040 \chi \left( \frac{1}{T_{\text{obs}}} - \frac{1}{T_{\text{mod}}} \right) \quad (58)
$$

$\Delta \log A$ is kept in order to remind the reader of the temperature dependence it has as stated by Eq. 57. As a consequence of the faulty temperature profile, the differences of the lines are expected to be a function of excitation energy. The direction of which depends on sign of $(1/T_{\text{obs}} - 1/T_{\text{mod}})$. Moreover, with a correct temperature profile, $\Delta \log A \rightarrow 0$. 
Using Eq. 53, $T_{\text{mod}}$ is computed and the difference

$$T_{\text{diff}} = T_{\text{mod}} - T_{\text{obs}}$$

(59)
is thus obtained. This difference is larger than zero, therefore $T_{\text{mod}} > T_{\text{obs}}$. With this knowledge, one can deduce that the lines (due to Eq. 58) will align themselves in a way that lower excitation energies $\chi$ means a higher value of $\log(W_{\text{obs}}/W_{\text{mod}})$. Fig. 27 (together with Tab. 15) is depicting this exact situation, hence a different temperature scaling might be a promising solution.

The equivalent width ratios used to calculate $T_{\text{obs}}$ and $T_{\text{mod}}$ are motivated with the same reasoning as with the computation of the optical depths, i.e. avoiding the situation where the differences between the excitation energies and $\log(gf)$ are in the same order as their uncertainties. The list of the 18 ratios used can be seen in Tab. 19 together with their excitation energies and oscillator strength ratios.

<table>
<thead>
<tr>
<th>$\nu_1/\nu_2$ (in cm$^{-1}$)</th>
<th>$E''<em>{\text{exc},1}/E''</em>{\text{exc},2}$</th>
<th>$\log(gf)_1/\log(gf)_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>815.301/816.687</td>
<td>0.498/1.014</td>
<td>-2.51/-1.35</td>
</tr>
<tr>
<td>815.301/817.157</td>
<td>0.498/1.206</td>
<td>-2.51/-1.18</td>
</tr>
<tr>
<td>816.450/816.687</td>
<td>0.398/1.014</td>
<td>-3.21/-1.35</td>
</tr>
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<td>816.450/817.157</td>
<td>0.398/1.206</td>
<td>-3.21/-1.18</td>
</tr>
<tr>
<td>816.450/815.900</td>
<td>0.398/1.396</td>
<td>-3.21/-1.00</td>
</tr>
<tr>
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<td>0.498/1.396</td>
<td>-2.51/-1.00</td>
</tr>
<tr>
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<td>0.498/1.014</td>
<td>-2.51/-1.83</td>
</tr>
<tr>
<td>816.450/817.208</td>
<td>0.398/1.014</td>
<td>-3.21/-1.83</td>
</tr>
<tr>
<td>816.450/819.932</td>
<td>0.398/1.050</td>
<td>-3.21/-1.42</td>
</tr>
<tr>
<td>816.450/818.420</td>
<td>0.398/1.029</td>
<td>-3.21/-1.19</td>
</tr>
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<td>815.301/819.932</td>
<td>0.498/1.050</td>
<td>-2.51/-1.42</td>
</tr>
</tbody>
</table>

Table 19: 18 spectral line pairs of H$_2$O features together with the molecular data of the dividends and divisors, used in the calculation of the observational and model formation region temperatures ($T_{\text{obs}}$ and $T_{\text{mod}}$ respectively).

With the use of Eq. 53 and Eq. 59, the formation region temperatures and their difference $T_{\text{diff}}$ are calculated respectively. These are collected and presented Tab. 20 with the addition of the mean and the standard deviation.
of the calculated temperatures for each star using all the available ratios (with data). The formation region temperatures are plotted in Fig. 28 as a function of the line strength of the highly excited line in the corresponding ratio. These figures map which temperatures are obtained using saturated equivalent widths where the dotted line (if present) indicates the region at which the highly excited line in the ratio, is saturated according to the curve of growth derived from model atmospheres (Fig. 11). The saturation of the line with low excitation energy (in the corresponding ratio), is colour coded as green or red for weak or saturated respectively.

With the desire of attaining formation region temperatures based on non-saturated equivalent widths, a temperature mean (horizontal line in these figures) should preferably be calculated using temperatures at weak line strengths, i.e to left of the dotted line (if present) and coloured green. As these figures illustrate what a more correct temperature could be, the \( <T_{\text{obs}} > \) and \( <T_{\text{mod}} > \) are, as stated before, obtained using all equivalent width ratios with available data for that particular star. Although some of these ratios are saturated, the inclusion of them are motivated by having sound statistics. Since hotter stars have weaker lines, they are expected to form a horizontal line in these figures (denoting a single formation region temperature). This is however not the case for the observed formation region temperatures in our hottest star with a sensible amount of water features, \( \alpha \) Boo (Fig. 28a). Here, the effect of having small equivalent widths and thus sensitive to noise, becomes apparent. Therefore, there is a trade-off between simply having not too weak lines but still non-saturated enough to be considered “weak”. The issue is further convolved with the fact that certain stellar spectra have somewhat low S/N in comparison to the high-resolution capabilities of TEXES, the consequences of which can be seen in \( \alpha \) Hya (Fig. 28b) where it not only leads to fewer measured observed lines (and thus less sound statistics) due to them being buried in the noise, but the lines are also harder to measure.

As expected, the weak line approximation does not hold for most of the lines in the cool stars because of the high line strengths there, turning the flat trend to scatter for both synthetic and observational data (28h). However, interestingly enough in the intermediate stars (notably in Fig. 28c, 28f and 28g), there seems to be a saturation effect for the models, i.e scattered, whereas the observational data remains non-saturated (on a flat trend). With the curve of growth being synthesized from model atmospheres, it reflects the temperature profile of them and not the modified version (e.g the rescaled Arcturus model in Ryde et al. 2002). With the outer regions of the temperature profile being lowered, the Doppler-part in the curve of growth is extended, causing line strengths otherwise saturated, to be weak.
7 DISCUSSION

(a) $\alpha$ Boo

(b) $\alpha$ Hya
(e) $\alpha$ Tau

(d) $\alpha$ Lyn
7 DISCUSSION

(e) $\mu$ UMa

(f) $\delta$ Oph
Figure 28: The formation region temperatures $T_{\text{form}}$ of the observational and model/synthetic data plotted against the line strengths of $W_2$ (see Tab. 19), $\log W_2/\nu$ in the equivalent width ratios. The dotted line marks the line strengths where the water lines become saturated (for $W_2$) whereas the green or red color denotes weak or saturation respectively of $W_1$ in the ratio. Therefore, the ideal values are located at line strengths less than the dotted line ($< -5.5$) and color coded green.
The mean of the difference between the formation region temperature of the observations and model atmosphere \(< T_{\text{diff}} >\), is illustrated in Fig. 29 along with error bars. There does not seem to be any trend with effective temperature other than a flat one. In other words, the temperature profiles of the model atmospheres needs to be corrected with a constant value at the line formation region of water and is not dependent on what effective temperature the star have. This goes hand in hand with what was stated earlier, namely that the temperature profiles are approximately same across a wide range of \(T_{\text{eff}}\) and \(g\). \(T_{\text{diff}}\) gives an order of this scaling which is around 500 K, which is comparable to the 300 K obtained for \(\alpha\) Boo (Ryde et al., 2002).

Figure 29: The mean of the difference between synthetic and observational formation region temperature \(< T_{\text{diff}} >\) against the effective temperatures of the stars.

7.4 MOLspheres

The presence of an outer molecular envelope uncoupled to the photosphere, i.e. MOLsphere, would cause optically thick (saturated) lines beyond \(\sim 5\mu m\) to be in emission (Fig.1 in Ryde et al., 2006b). Backtracking, it was deduced from the results that no emission lines originating from water was detected in any of the stars at 12 micron. Tsuji (2006, see notes) discussed the possibility of the 12 \(\mu m\) water lines in absorption for the supergiant \(\mu\) Cep (M2Ia) being an exception due to local phenomena due to dust unlike the otherwise detected emission at 6 \(\mu m\) and 40 \(\mu m\) for this star by ISO at \(R \approx 1,600\) (Tsuji, 2000a). Although this is possible, the mechanism should
then be valid in the 12 $\mu$m region for giants as well as subgiants because of the lack of emission in our data. However, the advent of high-resolution infrared spectroscopy outside of the 12 $\mu$m (e.g. EXES) will put definite constraints to this discussion.

Still, interferometric studies of water vapour around giant stars (Ohnaka, 2004) show an increase in apparent size as the water lines get probed, which can be interpreted as the presence of a MOLsphere.

### 7.5 NLTE effects

The need to consider NLTE effects is discussed by Ryde et al. (2002, 2006a, b). The important aspect of the LTE breakdown in the upper photosphere, is omitted in classical hydrostatic, homogeneous model photospheres of these red giants and subgiants. Although neither the ad-hoc solution of MOLsphere nor the altered temperature profile provide a definitive solution to the water problem across many spectral regions (6, 12 and 40 $\mu$m for instance), the treatment of NLTE effects might.

Ongoing work by J. Lambert suggests that the treatment of NLTE in the upper photospheres of giants, leads to a drop of temperature, i.e. NLTE cooling. Fig. 30 (Lambert, 2013, in prep.) displays this cooling effect and its impact when using a new self-consisting computation and comparing it to a standard LTE model photosphere. The cooling effect is about four times as strong in the outer regions of the star (low optical depth), resulting therefore in a lower temperature in the outer photospheres of these stars which certainly has its implications on the resulting spectra.
Figure 30: (Lambert, 2013, in prep.) Cooling rate (per unit volume and time) of the supergiant α Ori plotted against optical depth $\tau_\nu$. The curve marked as “ETL” in the legend corresponds to a classical model photosphere computed in LTE whereas “NETL” shows the new, self-consistent computation. The NLTE cooling effect is about four times stronger than predicted in the classical case for the outer regions of the photosphere (low optical depths resulting in lower temperatures.)
<table>
<thead>
<tr>
<th></th>
<th>( T_{\text{obs}} ) [K]</th>
<th>( T_{\text{mod}} ) [K]</th>
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</thead>
<tbody>
<tr>
<td>[cm(^{-1})]</td>
<td>( \beta ) Peg</td>
<td>( \delta ) Vir</td>
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<td>815.301/816.687</td>
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<td>2162</td>
</tr>
<tr>
<td>816.450/819.932</td>
<td>2766</td>
<td>2581</td>
</tr>
</tbody>
</table>

Continued on next page
\[ T_{\text{diff}} = T_{\text{mod}} - T_{\text{obs}} \text{[K]} \]

| \(815.301/816.687\) | 293 | 380 | 544 | 387 | 625 | 542 | 630 | 585 | – | – |
| \(815.301/817.157\) | 206 | 204 | 468 | 221 | 475 | 598 | – | 451 | – | – |
| \(816.450/816.687\) | 176 | 242 | 653 | 483 | 467 | 662 | – | – | – | – |
| \(816.450/817.157\) | 130 | 130 | 600 | 344 | 337 | 695 | – | – | – | – |
| \(816.450/815.900\) | 384 | 509 | 799 | 724 | 554 | 735 | – | – | – | – |
| \(815.301/815.900\) | 580 | 711 | 751 | 700 | 668 | 662 | 371 | 574 | – | – |
| \(815.301/817.208\) | 482 | 1109 | 1821 | 727 | 720 | 976 | – | 220 | – | – |
| \(816.450/817.208\) | 244 | 688 | 1580 | 762 | 547 | 1013 | – | – | – | – |
| \(816.450/819.932\) | -124 | 27 | 1006 | 531 | 206 | 622 | – | – | – | – |
| \(816.450/818.420\) | 224 | 317 | 1021 | 737 | 486 | 840 | – | – | – | – |
| \(814.450/814.675\) | 452 | 716 | 1260 | 714 | – | – | – | – | – | – |
| \(816.450/808.633\) | – | 357 | 1238 | 682 | 601 | – | – | – | – | – |
| \(815.301/808.633\) | – | 553 | 1351 | 638 | 754 | – | – | 931 | – | – |
| \(815.301/814.675\) | 708 | 1003 | 1331 | 686 | – | – | – | 234 | – | – |
| \(815.301/814.675\) | 356 | 472 | 1063 | 714 | 636 | 777 | 703 | 1049 | – | – |
| \(815.301/820.583\) | – | 673 | 1201 | 739 | 634 | – | – | 879 | – | – |
| \(816.450/820.583\) | – | 458 | 1146 | 759 | 498 | – | – | – | – | – |
| \(815.301/819.932\) | -230 | 60 | 1033 | 445 | 376 | 489 | – | 1046 | – | – |

**Table 20:** The observed (\(T_{\text{obs}}\)) and model (\(T_{\text{mod}}\)) formation region temperatures calculated with the use of Eq. 53. The difference between the model and observed temperatures is also computed and dubbed \(T_{\text{diff}}\). Since we assume that all lines are formed at one region and temperature, the mean and standard deviation of all the lines for a given star was obtained and presented in the bottom of this Table.
8 Conclusions & Future prospects

From the two previous sections, we concluded that the synthetic spectra based on standard atmospheric models fail to describe the observed spectra. The lack of emission lines in the 12 micron region across all stars in the sample, puts heavy constraints (if not even ruling out) a classical MOLsphere for K-M giants/subgiants, unless all the stars in our sample suffer from the warm alumina grains argued by Tsuji (2006) for the super giants \( \alpha \) Ori and to some extent for \( \mu \) Cep. Also, probing different microturbulences and abundances did not improve the synthetic water lines but rather worsened it.

The solution is probably a combination of non-LTE modelling and effects due to inhomogeneity (with subgiants and giants being somewhat extended and prone to develop inhomogeneities).

With these conclusions as basis, the future prospects involve both observational and theoretical aspects that certainly can improve the results and bring about a global solution to the water problem, unlike the current state where different regions and schools (theoretical, interferometric or spectroscopic) have results supporting both competing ideas, i.e alternate temperature profile as a consequence of non-LTE effects or the addition of a MOLsphere.

It is certainly worthwhile, although complicated and computationally heavy, to map the non-LTE (NLTE) effects in these stars. The results of this thesis is in support of the idea that the K-M subgiants/giants need to be modelled with reduced temperatures in the outer photosphere at the line formation region of water. Work by Lambert (2013, in prep.) connects the temperature reduction with the physical process, NLTE-cooling, which indicate strong reduction in temperature around the line formation region of water. The next step would be to quantify this predicted decrease in temperature and compare it with the values obtained in this study.

The inhomogeneities of the water in stars, i.e the presence of water not being radially symmetric but rather formed in pockets, are examined by creating 3D models of a star in a box (closed boundaries) and is ongoing work by B. Freytag and collaborators.

With the difficulty of modelling these water lines, there is the positive implication of the problem having an unique solution. To further constrain it observationally is to acquire high resolution spectra in the otherwise opaque (from the ground) regions of 6 \( \mu \)m and 40 \( \mu \)m. Depending on whether the water lines in these regions are in emission or absorption, it puts severe constraints on either the idea of alternate temperature profiles or MOLspheres respectively (see Ryde et al., 2002). These spectra are realized in the coming years with the EXES instrument aboard SOFIA (Richter et al., 2001; Richter et al., 2010).

In an even more distant future, the E-ELT will provide the possibility of
achieving low-resolution MIR spectra of giants and supergiants not only in the Virgo Cluster (Deep et al., 2011) but also our own giants in the Milky Way, in a much greater detail and hopefully resolution.
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