Experimental Particle Physics Bachelor's Thesis

The Mass of the Higgs Boson

A Search for the Higgs Boson in the Four-Muon Decay Channel

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Abstract

Within the Standard Model of particle physics, there was only one undiscovered fundamental particle. This was the Higgs boson. In July 2012, the announcement was made that a Higgs like particle had been discovered by the ATLAS Collaboration and the CMS Collaboration at CERN.

The aim of this Bachelor’s thesis was to use data collected by the ATLAS detector facility to estimate the mass of the Higgs boson. The estimation was done with consideration to one decay channel, namely when the Higgs boson decays into two Z bosons, and each Z boson subsequently decays into two muons of opposite charge.

Setting constraints on the four-muon system kinematical data yielded a selection of possible Higgs candidates. A histogram was created of the invariant masses of the possible Higgs bosons. The masses of the intermediate step Z bosons were also put in a histogram.

Next, an estimation of the background processes was made by forming new four-muon systems, by combining Z bosons from different events. This yielded a mass spectrum that was later normalized to the same integral as the original resulting mass spectrum. Finally, the estimated background was subtracted from the original results, to yield a histogram without background.

Both the results with and without the estimated background subtracted displayed a prominent peak in the mass range 120 GeV/c^2 – 125 GeV/c^2. When the background was subtracted, the amplitude of this peak was about twice the amplitude of any other bin, and it was one of the few bins that did not include the “zero events registered” value within its error bars. This is a strong indication that a particle with a mass within this range was detected, and the range is exceptionally close to the mass of the Higgs boson reported by the ATLAS and CMS Collaborations; 126 GeV/c^2.
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1 Introduction

Particle physics is the most fundamental of the sciences; it describes the very smallest constituents of matter and energy. In particle physics there is a model that describes the fundamental particles and how they interact. This is called the Standard Model (SM) of particle physics. All present research within particle physics is either done to measure the parameters of the Standard Model to a high precision or to find an anomaly that suggests physics beyond this model.

It is the fundamentality of the field that is appealing to the author of this thesis. He has always been interested in finding the deepest cause for every process, and to know the interactions that, on the most fundamental level, govern the world he inhabits. This has inevitably led him into the field of particle physics, and this thesis is a step in the direction of a future there.

When choosing the subject of the project in the autumn of 2012, the author approached the Division of Particle Physics at Lund University, and discussed a number of different subjects with several different researchers. Finally, a subject was settled upon.

The subject of this thesis is to estimate the mass of the Higgs boson from experimental results. Until very recently, the one remaining major conundrum within the SM was the theorized, but never detected, Higgs boson. The Higgs boson is needed in the model to yield mass to all included particles. In July 2012 the ATLAS Collaboration and the CMS Collaboration at CERN, Geneva, announced discovery of a particle with properties similar to the Higgs boson.

The data used for the analysis in this project will be the same data used for the discovery of the Higgs boson at ATLAS. There will also be additional data, collected at ATLAS after the announced discovery. In the Large Hadron Collider, which collides protons in the ATLAS detector, the Higgs boson is to be produced through a number of channels at beam energies of √s = 7 TeV and √s = 8 TeV. It is then predicted to decay in several different decay channels, whereof one will be taken into account and analyzed in this work. The channel in question is where the Higgs boson decays to two Z bosons, each of which decays into two muons (one with negative charge and one with positive). Initially, two Z-bosons will be reconstructed from the four-muon-system. Next, the four muons are used to reconstruct a Higgs boson, and a mass spectrum of possible Higgs candidates will be obtained by calculating the invariant mass of the four-muon-system.

The four muons, however, cannot be chosen arbitrarily. There are a number of different restrictions and criteria that must be met, both for the individual muons, the reconstructed Z-bosons and for the entire system. These criteria have several purposes. Firstly, they are there to minimize the measurement errors, as some intervals of values might be unreliable. Secondly, they minimize the amount of background production of a pair of Z-bosons that is taken into account in the statistics.

As a part of the work in this thesis, a macro in the ROOT framework will be created, where all selections and calculations will be done.

All of the scientific details presented above will be further described and elaborated upon below.
2 Background

2.1 The Standard Model of Particle Physics

The Standard Model (SM) [1] of particle physics is the model which, to the best of human ability, describes the universe on the most fundamental level. All experimental particle physics of today is based upon either enforcing or extending the SM. The model describes two main classes of particles; bosons and fermions. The fermions (half integer spin) are what are usually known as matter particles, as they include the electron and the quarks making up the proton and the neutron. The bosons (integer spin) are the force carriers, or the particles propagating the different interactions.

<table>
<thead>
<tr>
<th>Type (Fermion)</th>
<th>Flavour</th>
<th>Anti-particle</th>
<th>Mass [2], $m/(\text{MeV}/c^2)$</th>
<th>Charge, $Q/(\text{Unit Charge})$</th>
<th>Colour Multiplicity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Up-Type (Quarks)</td>
<td>$u$ (Up)</td>
<td>$\bar{u}$</td>
<td>2.3</td>
<td>2/3</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>$c$ (Charm)</td>
<td>$\bar{c}$</td>
<td>1 280</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$t$ (Top)</td>
<td>$\bar{t}$</td>
<td>174 000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Down-Type (Quarks)</td>
<td>$d$ (Down)</td>
<td>$\bar{d}$</td>
<td>4.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$s$ (Strange)</td>
<td>$\bar{s}$</td>
<td>95</td>
<td>$-1/3$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$b$ (Bottom)</td>
<td>$\bar{b}$</td>
<td>4 180</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Charged Leptons (Leptons)</td>
<td>$e$ (Electron)</td>
<td>$\bar{e}$</td>
<td>0.511</td>
<td>$-1$</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>$\mu$ (Muon)</td>
<td>$\bar{\mu}$</td>
<td>106</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\tau$ (Tauon)</td>
<td>$\bar{\tau}$</td>
<td>1 780</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Neutrinos (Leptons)</td>
<td>$\nu_e$ (Electron Neutrino)</td>
<td>$\bar{\nu}_e$</td>
<td>$\leq 1 \times 10^{-6}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\nu_\mu$ (Muon Neutrino)</td>
<td>$\bar{\nu}_\mu$</td>
<td>$\leq 1 \times 10^{-6}$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\nu_\tau$ (Tauon Neutrino)</td>
<td>$\bar{\nu}_\tau$</td>
<td>$\leq 1 \times 10^{-6}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.1 – The Fermions of the Standard Model

Above the fermions of the Standard Model are displayed, along with some of their discerning properties.

Note: the displayed neutrinos ($\nu_e$, $\nu_\mu$, $\nu_\tau$) are the flavour eigenstates (or weak eigenstates) of the neutrinos. These do not correspond to the mass eigenstates of the neutrinos ($\nu_1$, $\nu_2$, $\nu_3$), but are linear combinations of them. The mass limits given above for the neutrinos are on the mass eigenstates, not the flavour eigenstates.

The fermions are sorted into two main categories, the leptons and the quarks. The separation between them is due to their colour multiplicity. The colour multiplicity of a particle describes the number of colour states the particle could have (much like spin multiplicity describes the number of possible spin states for a particle). A particle with colour multiplicity 1 can only have one colour state, which is white. A particle with colour multiplicity 3 has three different possible colour states; red (r), green (g), blue (b). The
fermions are also separated into three generations, each generation with identical properties (disregarding the mass) as the two others. Every particle also has an antiparticle, sharing all the properties of the corresponding particle, except for the electric charge, which is of equal size but opposite in sign, and the possible colour charges, which are the “anti-” states of the possible colours (antired, antigreen, antiblue).

The bosons of the standard model are of two kinds. There are the gauge bosons, which are the quanta of the gauge fields, and there is one scalar boson.

<table>
<thead>
<tr>
<th>Type, Boson</th>
<th>Flavour</th>
<th>Anti-particle</th>
<th>Associated Interaction</th>
<th>Mass $[\text{2}], m/(\text{GeV}/c^2)$</th>
<th>Charge, $Q/(\text{Unit Charge})$</th>
<th>Colour Multiplicity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gauge Boson</td>
<td>$\gamma$ (Photon)</td>
<td>[Itself]</td>
<td>Electromagnetic</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>$Z$ (Z-boson)</td>
<td>[Itself]</td>
<td>Weak</td>
<td>91.2</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>$W^+$ (W-boson)</td>
<td>$W^-$</td>
<td>Weak</td>
<td>80.4</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>$g$ (Gluon)</td>
<td>[Itself]</td>
<td>Strong</td>
<td>0</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>Scalar Boson</td>
<td>$H$ (Higgs-boson)</td>
<td>[Itself]</td>
<td>[None]</td>
<td>[Unknown]</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

The photons of the SM only interact with electrically charged objects, i.e. the $W$-bosons and all fermions except the neutrinos. The $W$-bosons interact with fermions of left-handed chirality, and the $Z$-bosons interact with fermions of left-handed chirality or non-zero electric charge.

The gluons of the SM are the carriers of the strong force. They only interact with objects of non-unitary colour multiplicity. This means that they interact with quarks and not leptons. As the gluons themselves have a non-unitary colour multiplicity, they also interact with each other, and therefore the strong force obtains an extra component in its potential. Thus the interaction gains asymptotic freedom, and the quarks (the only colour charged fermions) are confined to only exist within colour neutral combination. The possible combinations are the following: a three quark state (red green blue), a three anti-quark state (antired antigreen antiblue) or a quark-anti-quark state (red antired, green antigreen or blue antiblue).

The Higgs boson of the standard model interacts with everything that has mass. This means it interacts with all bosons and fermions except for the gluon and the photon. This also renders it the only particle that can interact with the neutrino of right handed chirality (provided that they do exist).

2.2 The Higgs Boson

The standard model Higgs boson is the boson responsible for giving particles their mass [1]. This happens through the so called mass mechanism (the mass mechanism is also known as e.g. the Higgs mechanism or the Brout-Englert-Higgs mechanism, but there is a suggestion that it be redubbed as the mass mechanism).
The mass mechanism in the SM breaks the electro-weak symmetry, and from the four degrees of freedom, three massive gauge bosons and one massive scalar boson emerge. These are the $Z$, $W^+$ and $W^-$ bosons and the Higgs boson, respectively. The fermions also gain their mass from the mass mechanism, but in a different manner. Quantified, it is the interaction between a particle and the Higgs boson that yields the mass to that particle. In the case of the Higgs boson, it is a self interaction that yields mass. The Higgs boson treated here is that of the SM. There are several non-SM models that incorporate more than one Higgs boson, but these are not treated here.

As it is the interaction with the Higgs boson that yields mass, it is intuitively likely (and correct) that the stronger a particle interacts, the higher will its mass be. This is seen in the SM Lagrangian, where the interaction terms between the Higgs boson and the fermions are proportional to its mass. This means that, without knowing the mass of the Higgs, it would interact most strongly with the top quark, as that is the heaviest SM particle, and second to most strongly with the $Z$-boson.

The width, $Γ$, of a vertex is a measure of how probable the interaction within that vertex. The width of a decay of a specific initial state through a specific decay channel is given as the inverse of the lifetime, $τ$, of that particular decay channel. In natural units:

$$Γ = \frac{1}{τ}$$

Thereby, the width is a measurement of the probability of the vertex. The branching ratio, $Br$, of a certain particle is also connected to the width. The branching ratio is a measure of how often a specific particle decays to a given final state, over how often that particular initial particle decays into anything at all. It is defined below.

$$Br(\text{initial } → \text{final}) = \frac{Γ_{\text{initial } → \text{final}}}{Γ_{\text{initial } → \text{anything}}} \quad Γ_{\text{initial } → \text{anything}} = \sum_{\text{all final states}} Γ_{\text{initial } → \text{final}}$$

Approximations of the widths of the vertices containing a Higgs boson are given below:

$$Γ_{\text{Hff}} \approx \frac{α_2 m_f^2 m_H}{4m_W^2} \quad Γ_{\text{HWW}} \approx \frac{α_2 m_H^3}{16m_W^2} \quad Γ_{\text{HZZ}} \approx \frac{α_2 m_H^3}{32m_W^2}$$

Here $m_f$, $m_H$, $m_W$ and $m_Z$ denote the masses of the fermion in question, the Higgs boson, the $W$ boson and the $Z$ boson, respectively. This approximation is done when $m_H^2$ is significantly larger than $m_W^2$. As the interaction width depends so heavily on the particle mass, only the heaviest particles will be seen interacting with the Higgs boson. See Table 2.3 for a comparison of the interaction widths of different fermion masses.

<table>
<thead>
<tr>
<th>$Γ_{\text{Hff}}/m_H$</th>
<th>$Br$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$100 \text{ meV}/c^2$</td>
<td>$100 \text{ eV}/c^2$</td>
</tr>
<tr>
<td>$≈ m_e$</td>
<td>$≈ m_e$</td>
</tr>
<tr>
<td>$1.30 \times 10^{-26}$</td>
<td>$1.30 \times 10^{-20}$</td>
</tr>
<tr>
<td>$≈ 1 \times 10^{-24}$</td>
<td>$≈ 1 \times 10^{-18}$</td>
</tr>
</tbody>
</table>

$9.99999 \times 10^{-25}$ | $9.99999 \times 10^{-19}$ | $9.99999 \times 10^{-13}$ | $9.99999 \times 10^{-7}$ | $9.99999 \times 10^{-1}$

Table 2.3 – The Interaction Widths in a Hff-vertex for Different Fermion Masses

Above some interaction widths for different fermion masses are displayed. The widths are divided by the unknown mass of the Higgs boson for convenience.
As the neutrino-Higgs vertex has such a small width, its branching ratio will be virtually zero. And since the right handed neutrino would only interact with the Higgs boson, this means that the right handed neutrino will most probably not be observed barring some new physics.

Intuitively, the most favourable channel for observation of the Higgs boson would be the one with the highest branching ratio, i.e. the channel with the highest (allowed) final state mass (see Table 2.3). This, however, is not necessarily true, as heavy particles tend to decay further and the final state products might not be easily detectable. This is further elaborated upon in Section 2.7.

2.3 The Large Hadron Collider

At the writing moment, the Large Hadron Collider (the LHC) [4][5] is the largest of the particle accelerators in the CERN accelerator complex, and in the world. It will therefore be the accelerator that yields the highest energy to the particles being accelerated by it. In the LHC machine, there are two synchrotron storage rings for protons, one going in each direction. The LHC also accelerates ions of Lead (Pb), and the p-p, Pb-p and Pb-Pb collisions take place and are detected at four detector facilities around the ring. These are: the ATLAS detector, the ALICE detector, the CMS detector and the LHCb detector.

![Figure 2.1 – The Large Hadron Collider](image)

*Above is an overview of the LHC rings. The proton beams are injected at the bottom of the image, and they cross at the four sites marked on the ring (CMS, LHCb, ATLAS, ALICE).*

The protons travel in the rings in bunches of (initially) $1.1 \times 10^{11}$ protons per bunch. In each beam there are 2 808 bunches, i.e. 5 616 bunches in the LHC. The protons are accelerated by 8 radio frequency (RF) cavities per beam. Here, an electric field accelerates the protons to yet higher energies until they reach the desired energy. The RF cavities also ensure that the bunching of the protons is upheld. There are 9 593...
magnets around the LHC, that keep the beam on its trajectory. 1232 of these are the main dipole magnets, used to change the direction of the beam (i.e. turning it in its nearly circular path). The remaining magnets are quadrupoles, sextupoles, octupoles, decapoles etc. that correct the beam trajectories and focuses the beam in the transverse plane. For the equipment to be able to perform as required, both the RF cavities and the magnets must be superconducting. This is achieved by cooling these components using a cooling system with superfluid helium.

The luminosity of a beam is a measure of the performance of the accelerator. It states how many particles pass through a given area during a given time, and is usually presented in the unit cm$^{-2}$ s$^{-1}$. The luminosity of the LHC is $10^{34}$ cm$^{-2}$ s$^{-1}$, and the number of collisions per second is $\sim 10^9$.

The collider has a circumference of 26,659 m and the protons travel the ring with a frequency of 11,245 Hz. This yields a speed of the protons of 0.99996 c. And as a beam might last up to 10 hours, a given proton bunch will have travelled about $10^{13}$ m before being dumped.

2.4 The ATLAS Detector System

The ATLAS detector [6] is one of the four detectors around the ring of the LHC. It is built in the approximate shape of a cylinder, with symmetric sections. It is 44 m long and has a 25 m diameter, with the collision point in the centre. The detector system consists of many concentric semi-cylindrical detectors around the collision point, and end cap detectors in the transverse plane to the beam line. The main parts of the detector system are described below.

![Figure 2.2 – The ATLAS Detector](image)

The ATLAS detector [7] is displayed above, with each part named.
2.4.1 The Inner Detector (ID)
The ID is the 7 m long, 1.15 m diameter, detector situated right in the centre of the large detector system. It encompasses the collision point in all directions (except in the beam pipe), and is enclosed in one of the magnets of the ATLAS detector, the solenoid magnet. The solenoid magnet provides a magnetic field of about 2 T parallel to the beam line, which makes the particle trajectories bend, and thereby provides a momentum measurement. The ID itself consists of several layers.

- The Pixel Detector. The innermost layer, the pixel detector, consists of three concentric barrels of small radii, around the collision point. These barrels are pixelated, and therefore each particle travelling through the pixel detector leaves three spatial points of their track. These are used to determine the location of each interaction point, and to detect particles that might live long enough to travel some mm, but not further (for example hadrons containing b-quarks).
- The Semiconductor Tracker (SCT). This detector is mounted just outside the pixel detector, and consists of eight layers of semiconductor detectors. These are also pixelated, though not as finely as the pixel detector, and contribute with measurements on vertex location, impact parameter and particle momentum.
- The Transition Radiation Tracker (TRT). This outermost part of the ID consists of many layers of 4 mm diameter straws. These each contain a gold plated wire, and a gas mixture. The TRT is built in such a way as to be able to discriminate between tracking hits and transition radiation hits. It can also measure the drift time of a particle track, and thus gives a spatial resolution of 170 μm.

2.4.2 The Calorimeters
Outside of the solenoid magnet are the calorimeters, which themselves are encompassed by the 8 toroid magnets. The toroid magnets provide a magnetic field where the field lines wrap in concentric circles around the beam line. This bends the particle trajectories to yield further momentum measurements. The calorimeter system consists of two parts. Both consist of interlayered absorbing metal plates with a sensing element. In the absorbing plates the particles lose energy and interact to create particle showers. The showers are detected by the detecting material.

- The Electromagnetic (EM) Calorimeter. The inner of the calorimeter is the electromagnetic calorimeter, where electrons and photons are detected. Here the detector material is liquid argon.
- The Hadronic Calorimeter. The outer calorimeter detects the hadrons, most of which pass through the EM calorimeter. Here the detector parts are plastic scintillators.

2.4.3 The Muon Spectrometer (MS)
As the muons are the only produced particles that traverse the entire detector without being stopped (except for neutrinos) they have their own detection system on the very outside of the detector. The main parts are described below.

- The Thin Gap Chambers and the Resistive Plate Chambers. These are used for triggering detection and to measure the angle of the track in the transverse plane.
- The Monitored Drift Tubes. This part of the detector is similar to the TRT of the ID, but with different tube diameters. Here the curvature of a track is measured, yielding further momentum measurements of the track.
- The Cathode Strip Chamber. This is the outermost part of the detector, where precision spatial coordinates of the track are measured to a resolution of 60 μm.
2.5 Data Retrieval

At the ATLAS detector site, proton bunches cross $4 \times 10^7$ times per second, yielding $1 \times 10^9$ pp-collisions per second (23 collisions per crossing). As this is an overwhelming amount of data, certain triggers must be used to reduce the amount of data. The three-level TDAQ trigger system [8][9] is briefly described below:

<table>
<thead>
<tr>
<th>Part of TDAQ</th>
<th>Method</th>
<th>Incoming event rate $R/(s^{-1})$</th>
<th>Outgoing event rate $R/(s^{-1})$</th>
<th>Reduction factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 1</td>
<td>Hardware, special purpose processors</td>
<td>40 000 000</td>
<td>100 000</td>
<td>400</td>
</tr>
<tr>
<td>Level 2</td>
<td>Software, computing farms</td>
<td>100 000</td>
<td>3 000</td>
<td>33.3</td>
</tr>
<tr>
<td>Event Filter</td>
<td>Software, computing farms</td>
<td>3 000</td>
<td>200</td>
<td>15</td>
</tr>
<tr>
<td>Entire TDAQ</td>
<td>Both hardware and software</td>
<td>40 000 000</td>
<td>200</td>
<td>200 000</td>
</tr>
</tbody>
</table>

*Table 2.4 – The TDAQ Three Level Trigger System*

Above, the TDAQ trigger system is described. Included are the method used for triggering on each level, the incoming and outgoing event rates, and the reduction factor of each level.

In Level 1, the events processed in custom made electronics. They are sorted through by certain triggers, and are either deemed to be interesting or thrown away. These triggers could include e.g. isolation requirements and thresholds on the transverse momentum of the particles. The Level 1 system has a processing time of $\sim 2 \mu s$, which includes the time it takes for the information to go from the detector to the trigger system.

Only certain information of the tracks is passed on to Level 2; e.g. pseudorapidity, azimuthal angle, transverse momentum thresholds. Here, the interesting parts of the data are investigated using higher level triggers, and candidates for the muons, electrons, photons etc are created. The processing time is $\sim 10$ ms.

In the third level of the TDAQ, the event filter, the complete event is treated, using the full granularity of the detector. Here, tracks are fitted and vertices are reconstructed, and accepted events are stored in the database. The processing time is $\sim 1$ s.

After the trigger system, the data is recorded. The raw data that comes out is about 1.6 Mb per event, i.e. 320 Mb per second. This is reduced to 1 Mb per event (200 Mb/s) after reconstructing the tracks and calorimeter showers etc., and finally to 0.1 Mb per event (20 Mb/s) of physics data.

As the subject of this project is to search for a signal of the Higgs boson in the four-muon decay channel, the only tracks of interest detected by the ATLAS detector system are the muon tracks. These tracks are detected by the ID and the MS, and the combination of the two tracks into one is done using the STACO algorithm [8].

The STACO algorithm is a method to combine two tracks from independent measurements (in this case the ID and the MS), into one track. This requires the parameter vectors of the two tracks ($P_1$ and $P_2$) and their covariance matrices ($C_1$ and $C_2$), and yields the combined track ($P$) and its covariance matrix ($C$). The combination is done in the following way:

$$ (C_1^{-1} + C_2^{-1}) \times P = (C_1^{-1} \times P_1) + (C_2^{-1} \times P_2) \quad C = (C_1^{-1} + C_2^{-1})^{-1} $$
As a measure of how well the two tracks fit together, the $\chi^2$ is calculated in the following way:

$$\chi^2 = \left( (P - P_1)^T \times C_1^{-1} \times (P - P_1) \right) + \left( (P - P_2)^T \times C_2^{-1} \times (P - P_2) \right)$$

The combination of the two tracks is only accepted if the value of $\chi^2$ is below a maximum allowed threshold value $\chi^2_{\text{max}}$. If several different tracks are present in the MS and ID, the combinations are made in the following order: the combination with the lowest value of $\chi^2$, second to lowest $\chi^2$, third to lowest $\chi^2$, etc. until there are no more combinations with a $\chi^2$-value below $\chi^2_{\text{max}}$.

Tracks that are only present in the ID or the MS (i.e. could not be combined in the above described fashion) are still used in the data, but only containing information from one detector, not two.

2.6 Production of the SM Higgs Boson at the LHC

Below is a listing of the main production channels for the Higgs boson at the LHC [10][11].

- **Gluon fusion.** This is where two gluons fuse, via a heavy quark loop, into a Higgs boson. The Higgs boson cannot be directly produced by the gluons, as they are massless and do not couple to the Higgs boson.

- **Vector boson fusion.** Here two vector bosons (either $W^+$ and $W^-$ or two $Z$ bosons) fuse to create a Higgs boson.

- **Higgs Strahlung.** This is the case where a vector boson emits a Higgs boson. Either the initial or the final vector boson must be off shell, to preserve the energy of the system.
- Top-Antitop Association. This is where a Higgs is created in association with the creation of a top-quark-anti-top quark pair.

The cross sections for these processes are displayed below, in Figure 2.3.

**Figure 2.3 – The Production Cross Sections of the SM Higgs Boson**

Above are the cross sections for different production processes of the Higgs boson at the LHC [10]. These are put as a function of the mass of the Higgs boson. The different notations in the figure correspond to the different processes in the following way: $pp \rightarrow H$ is the gluon fusion, $pp \rightarrow q\bar{q}H$ is the vector boson fusion, $pp \rightarrow WH$ is the Higgs strahlung process in the $W$ boson case, $pp \rightarrow ZH$ is the Higgs strahlung process in the $Z$ boson case, $pp \rightarrow t\bar{t}H$ is the top-antitop associated production processes. The figure above describes the cross sections for $\sqrt{s} = 8$ TeV. The difference between this figure and the corresponding one for $\sqrt{s} = 7$ TeV is negligible for all intents and purposes of this thesis.
2.7 **Decay of the SM Higgs Boson at the LHC**

As can be seen in Section 2.2, the preferred decay of a Higgs boson is highly dependent on the mass of the Higgs boson. Some possible decays follow [10].

- Higgs to fermions. Here, the Higgs boson decays immediately into a fermion-antifermion pair.

\[ H \rightarrow f \rightarrow f \bar{f} \]

- Higgs to photons. Here the Higgs boson decays into two photons via a heavy quark loop.

\[ H \rightarrow q \rightarrow \gamma \gamma \]

- Higgs to gluons. This is the same kind of process as the Higgs to photons process, but with gluons instead of photons.

\[ H \rightarrow q \rightarrow g \rightarrow g \]

- Higgs to W bosons. This is where the Higgs boson decays into a pair of W bosons. These, in turn, each decay into either a charged-lepton-neutrino pair, or a pair of quarks.

\[ H \rightarrow W \rightarrow f \rightarrow f \bar{f} \]

\[ H \rightarrow W \rightarrow f'' \rightarrow f''\bar{f}'' \]
- Higgs to Z bosons. This is where the Higgs boson decays into a pair of Z bosons. These, in turn, each decay into a charged lepton pair, a neutrino pair or a quark-antiquark pair.

In Figure 2.4 below, the branching ratios to different decay channels can be seen.

![Figure 2.4](image_url)

*Figure 2.4 – The Decay Branching Ratios of the Higgs Boson*

The branching ratios for the different Higgs boson decays, as a function of the mass of the Higgs boson [10].
In the detector, only the final products of the decay can be detected. These final states are not necessarily the ones given above; the particle may have decayed further. The final products of each given particle follow \[3\].

- Photons usually do not change.
- Gluons hadronize into hadronic showers.
- Tauons and quarks also give rise to hadronic showers (and a tauon neutrino, in the tauon case).
- Muons, electrons and neutrinos do not decay.

This means that the main channels to be investigated in the search for the SM Higgs boson at the ATLAS detector must both have a fairly high branching ratio and have reasonably detectable final decay products. Hadronic showers are easy to detect, but difficult to use in an analysis, as they consist of a lot of separated particles. This yields large errors, and (in the case of an event with several showers) identification difficulties. In addition to this, pp-collisions have a large background of hadronic showers, which further complicates the analysis. The decay channels of the Higgs boson that are listed above will be listed again below, now with their final products and status as search channels at the LHC \[3\].

- Higgs to fermions. This is not a main search channel. It is included in the search, but not to a large extent. As neutrinos will not be detected, they are not included. Both an electron pair and a muon pair have small branching ratios, so they are not seen very often. The remaining fermions create hadronic showers, which is difficult to perform an accurate analysis of.
- Higgs to photons. Even though this has a fairly small branching ratio, it is used as a main search channel, as the signal is so clear. The photons remain photons, and are easy to detect in the electromagnetic calorimeter.
- Higgs to gluons. This is not used as a main search channel, as the gluons hadronize.
- Higgs to W bosons. In the cases where the final products hadronize, this is not used as a main search channel. But when the final products are \(e^+e^-, \mu^+\mu^-, \nu_e\nu_e\) or \(\nu_\mu\nu_e\), the channel is used as a main search channel. This is despite the fact that the kinetic information of the neutrino must be calculated, as the neutrinos cannot be detected.
- Higgs to Z bosons. Like in the case of Higgs to W bosons, this is not used as a main search channel in the cases where the decay products hadronize. Neither is it used when the decay products are neutrinos. It is, however, used as a main search channel when the decay products are pairs of muons or electrons. They are readily detectable, and the branching ratio is fairly high.

The subject of this work is the last one of the ones described above, or more precisely, the decay:

\[
H \rightarrow ZZ \rightarrow \mu^+\mu^-\mu^+\mu^- \]

This is the channel used in this analysis for two main reasons. Firstly, the branching ratio of Higgs to two Z bosons is fairly high, and so is the Z boson to muon pair branching ratio. Secondly, the muons from the reaction are too fast to decay in the detector, and are readily detected in both the ID and the MS, where very clear kinematic information is left. This makes the \(H \rightarrow 4\mu\) channel very advantageous for observations.
2.8 Background ZZ Production

As stated above, only the final products of any decay can be detected. This means that any process with identical final state as the desired one will also be included in the analysis. This, in turn, means that undesired processes will be included in the data; there will be a certain background [12]. As this thesis deals with the final state of two negative muons and two positive muons, the background would be any other process yielding this final state. As the location of a vertex to great accuracy can be determined, the background muons would also have to originate from the same collision, and from two Z bosons. The leading order production channels of two Z-bosons follow (the next order channels give contributions of \((10^{-9})\)).

- Quark-Antiquark Fusion. This is where a quark and an antiquark of the same flavour fuse to form two Z bosons.

\[
\begin{align*}
\text{q} & \rightarrow Z \\
\bar{\text{q}} & \rightarrow Z \\
\text{q} & \rightarrow Z \\
\bar{\text{q}} & \rightarrow Z
\end{align*}
\]

- Gluon Fusion. This is where two gluons fuse into two Z bosons, via a quark loop.

\[
\begin{align*}
\text{g} & \rightarrow \text{q} \\
\text{g} & \rightarrow \bar{\text{q}} \\
\text{g} & \rightarrow \text{q} \\
\text{g} & \rightarrow \bar{\text{q}}
\end{align*}
\]

For a proper analysis, each background process must be thoroughly examined. This is usually done with Monte Carlo (MC) simulations, which is beyond the scope of this project, mainly due to the time frame of the project.

In addition to the double Z boson background production, there are other processes that will produce the four-muon final state desired in this project, which thereby add to the background. One such process is where only one Z boson is created, and four muons emerge \((pp \rightarrow Z \rightarrow 4\mu)\). Other background processes include the misidentification of hadronic jets as muons and heavy quark decays that produce muons.

2.9 The ROOT Framework

The ROOT framework [13][14] is an object oriented framework used for every high energy physics (HEP) computational purpose. It is also extensively used within other areas, such as astronomy, biology, medicine and finance. The ROOT framework is built with C++ and has a built in C++ interpreter, CINT.

The main reason for the widespread use of ROOT is the large number of HEP related applications it has built in. For example, ROOT has its own data storage format, especially built to handle large amounts of data, and retrieve different data sets with minimum effort. Additionally, ROOT has tools for creating e.g. histograms, graphics and user interface, and the fact that it provides a framework yields a consistency to programs written by different users.

All analysis performed for this project was carried out using the ROOT framework.
3 Method

The full code used for this project is included in Section 9.

3.1 Start-Up and Definition of Muon Quadruplets

At the start of this Bachelor’s work, the author received data files from the ATLAS Collaboration. The files themselves contained all of the kinematical data necessary for the calculation of the invariant mass of the system (collected by the STACO algorithm). The invariant mass, $m_{4\mu}$, was calculated as follows.

$$m_{4\mu} = \sqrt{E_{\text{tot}}^2 - p_{x,\text{tot}}^2 - p_{y,\text{tot}}^2 - p_{z,\text{tot}}^2}$$

$$E_{\text{tot}} = E_{\mu_1} + E_{\mu_2} + E_{\mu_3} + E_{\mu_4}$$

$$p_{x,\text{tot}} = p_{x,\mu_1} + p_{x,\mu_2} + p_{x,\mu_3} + p_{x,\mu_4}$$

$$p_{y,\text{tot}} = p_{y,\mu_1} + p_{y,\mu_2} + p_{y,\mu_3} + p_{y,\mu_4}$$

$$p_{z,\text{tot}} = p_{z,\mu_1} + p_{z,\mu_2} + p_{z,\mu_3} + p_{z,\mu_4}$$

Here, the indexed $\mu_i$ represents the $i$:th muon of the system.

The first task of the work for this thesis was to learn how to handle the .root format, as the received files were in that format. After this was done, the author could start investigating the files for suitable events. For a first calculation, each event containing four muons or more was considered suitable. In an event containing four muons, the calculation of the four-muon invariant mass was straightforward. In the case of more than four muons, however, this was not as simple. A selection among the muons had to be made, as only four were to be included.

As the mass of the Higgs boson was unknown, the most logical way to determine which four muons to choose was to define them in terms of $Z$ boson pairs [3]. This meant that each pair of muons that was defined to come from the same $Z$ boson first and foremost had to be uncharged. Second, the recreated $Z$ bosons (i.e. muon pairs) were defined to be either the leading $Z$ boson ($Z_1$) or the sub-leading $Z$ boson ($Z_2$). The corresponding muon pairs were the leading muon pair ($\mu_1$ and $\mu_2$) and the sub-leading muon pair ($\mu_3$ and $\mu_4$). The leading and sub-leading $Z$ bosons were defined by their invariant masses (formula displayed below).

$$m_{Z1} = \sqrt{(E_{\mu_1} + E_{\mu_2})^2 - (p_{x,\mu_1} + p_{x,\mu_2})^2 - (p_{y,\mu_1} + p_{y,\mu_2})^2 - (p_{z,\mu_1} + p_{z,\mu_2})^2}$$

$$m_{Z2} = \sqrt{(E_{\mu_3} + E_{\mu_4})^2 - (p_{x,\mu_3} + p_{x,\mu_4})^2 - (p_{y,\mu_3} + p_{y,\mu_4})^2 - (p_{z,\mu_3} + p_{z,\mu_4})^2}$$

The leading muon pair was defined to be the pair of oppositely charged muons with the invariant mass closest to the tabular mass of the $Z$ boson ($m_Z = 91.1876$ GeV/$c^2$ [2]). The sub-leading muon pair was defined to be the pair of oppositely charged, remaining, muons that had the highest invariant mass.

With all of this information, the quadruplets of muons possibly originating from a Higgs boson were established.
3.2 Applying Selection Criteria

To achieve the most reasonable results, however, certain criteria must be fulfilled by the muon quadruplet. The selection criteria used here are the ones presented in the paper [3]. These must be present for several reasons, e.g. to reduce the amount of background double-Z production events in the analysis or to reduce the amount of other particles misidentified as muons in the detection.

The first one was mentioned above, and ensures that the muons might actually come from a Z boson. This states that each defined pair of muons must be uncharged in total (i.e. that the muons in the pair must be of opposite charge).

Next, there were triggers put on the collection of data. These concerned the transverse momentum of the muons (the momentum in the plane perpendicular to the beam pipe). In each quadruplet at least one muon must have fulfilled the single muon trigger, or one pair must have fulfilled the di-muon trigger. The triggers were defined differently for the different collision energies available. For 7 TeV, the single muon trigger threshold was set at $p_T \geq 18 \text{ GeV/}c$, while for 8 TeV it was set at $p_T \geq 24 \text{ GeV/}c$. For the di-muon trigger, the 7 TeV thresholds were set at $p_T \geq 10 \text{ GeV/}c$ for both muons in the pair, and for 8 TeV data there were two possible di-muon triggers; the first being that both muons exceeded $p_T \geq 13 \text{ GeV/}c$ and the second being that one muon exceeded $p_T \geq 18 \text{ GeV/}c$ and another exceeded $p_T \geq 8 \text{ GeV/}c$.

In addition to the triggers on the transverse momentum, there were additional criteria on the transverse momenta. First, all muons in the quadruplet must fulfil $p_T > 6 \text{ GeV/}c$. Second, the highest $p_T$ of the group must exceed 20 GeV/$c$, the second highest must exceed 15 GeV/$c$ and the third highest must exceed 10 GeV/$c$. These criteria on the transverse momenta render the trigger values obsolete, since any quadruplet fulfilling them automatically fulfils the single-muon trigger (and only one of the triggers needed to be fulfilled).

In addition to the definition of the leading and sub-leading Z bosons above, they have additional constraints. The leading muon pair is required to have an invariant mass, $m_{12}$, between 50 GeV/$c^2$ and 106 GeV/$c^2$. The sub-leading muon pair is required to have an invariant mass, $m_{34}$, below 115 GeV/$c^2$. The sub-leading muon pair also has a lower mass limit, which depends on the invariant mass of the four muon system, $m_{4\mu}$ (which would be the mass of the possible Higgs boson, $m_H$). This is defined in the following way: If $m_{4\mu}$ is below 120 GeV/$c^2$, then $m_{34}$ is required to be above 17.5 GeV/$c^2$, if $m_{4\mu}$ is above 190 GeV/$c^2$ then $m_{34}$ must be above 50 GeV/$c^2$, and between those values of $m_{4\mu}$, $m_{34}$ varies linearly.

The next criterion concerns the pseudorapidity, $\eta$, of the muons. The pseudorapidity is a measure of the angle of the particle track in relation to the plane transverse to the beam pipe (the polar angle, $\theta$). The definition of the pseudorapidity follows:

$$\eta = -\ln \left( \tan \left( \frac{\theta}{2} \right) \right)$$

The ID covers the pseudorapidity range $|\eta| < 2.5$, while the MS covers the range $|\eta| < 2.7$. Therefore the criterion of pseudorapidity is $|\eta| < 2.7$ for all muons in the quadruplet.

The tracks of the muons in the quadruplet are required to be separated from each other as well. That this is upheld is ensured by a restriction on $\Delta R$, defined as follows.

$$\Delta R = \sqrt{\Delta \eta^2 + \Delta \phi^2}$$

Here, $\Delta \eta$ is the difference in pseudorapidity of the two muons, and $\Delta \phi$ is the difference in azimuthal angle.
Here, \( \eta \) is the pseudorapidity and \( \phi \) is the azimuthal angle around the beam line of the track. \( \Delta R \) is required to fulfil \( \Delta R > 0.1 \) for any pair of muons \( i \) and \( j \) of the quadruplet. This separation is required as track information of one muon might otherwise be mistakenly mixed with the track information of another muon.

The following criteria concern the origin of the muon. The longitudinal impact parameter, \( z_0 \), defined the distance of the reconstructed track of the muon to the primary vertex of the collision in the direction of the beam line. The transverse impact parameter, \( d_0 \), is defined as the distance between the beam line and the reconstructed track in the transverse plane. The restriction on both of these parameters is that the muon track must have a point where \( z_0 < 10 \) mm and \( d_0 < 1 \) mm. The primary vertex of the event is the reconstructed vertex with the highest \( p_T^2 \). The transverse momenta included in this sum are the ones associated to that particular vertex, and at least three of them must fulfil \( p_T > 0.4 \) GeV/c. This criterion is necessary to avoid identifying possible cosmic rays as muons from the collision.

The muons relevant for this analysis have a less broad distribution of the transverse impact parameter than muons from certain background processes do (heavy flavour decays, \( Z + \) jets etc.). This fact is used to reduce the background in the data, and applied by the condition that the transverse impact parameter divided by its uncertainty is lower than 3.5 (\( d_0/\sigma_{d0} < 3.5 \)).

Next, to reject backgrounds where \( J/\Psi \) is produced and decays, all possible muon pairs of the same flavour and opposite charge must have an invariant mass of \( m_{\mu\mu} > 5 \) GeV/c\(^2\) (i.e. well above the mass of the \( J/\Psi \) meson).

The next two criteria concern the isolation of the muons in the collision (or their tracks). In both, a cone of size \( \Delta R = 0.2 \) is setup around the muon track, and any track from another muon of the quadruplet is disregarded. In the first criterion, the normalized track isolation discriminant is defined as the sum of \( p_T \) in the cone excluding the muon \( p_T \), and divided with the muon \( p_T \). This sum only includes tracks with \( p_T \) higher than 1 GeV/c and which are compatible with the muon vertex. The normalized track isolation discriminant is required to have a value lower than 0.15 for each muon in the quadruplet. The next of the two isolation criteria is called the calorimetric isolation discriminant, and is defined as the sum of the \( E_T \) registered in calorimetric cells inside the cone (excluding the muons) and divided by the \( p_T \) of the muon. The calorimetric isolation discriminant is required to have a value less than 0.30 if the muon was detected both by the ID and the MS, and less than 0.15 if it was detected only by the MS (and not the ID).

As all these criteria are applied, an efficiency for combined signal reconstruction and selection of the SM Higgs boson at \( m_H = 125 \) GeV/c\(^2\) of 37% for the 7 TeV data and 36% for the 8 TeV data is yielded.

With all of these criteria in place, a histogram of the possible masses of the Higgs boson could be created (see Figure 4.2). Histograms were also created of the masses of the \( Z \) bosons (Figure 4.3), and a two dimensional histogram of how their masses correlated was also made (Figure 4.4).
3.3 Recombining Z Bosons for Background Estimation

A complete and rigorous estimation of the background of double Z boson production and of misidentification of other particles as muons would require extensive MC simulations. This was not possible within the time frame of this thesis, but an alternative method to estimate the background was applied in its stead. This method is described below.

The leading and sub-leading muon pairs from each event were stored. Each reconstructed Z boson was then paired with a Z boson from another event, and the invariant mass of the new system was calculated. If the newly formed system fulfilled all of the above applied criteria (Section 3.2), the invariant mass of the system was stored in a histogram. This was done for every possible pair of Z bosons from different events. No regard was taken to whether the Z boson was defined as leading or sub-leading in its individual event, as a Z boson that was defined as leading might have been defined as sub-leading in combination with a different Z boson than originally.

The resulting histogram was renormalized to have the same integral as the original results (the mass spectrum resulting from the data), and used as a background estimator to the results (Figure 4.5 and Figure 4.6). This is a valid estimation as the new histogram contains the invariant mass of a system of a pair of Z bosons produced independently of each other.
4 Results and Analysis

4.1 ATLAS – Official Results

Below is a histogram displaying the official results from the Higgs group at ATLAS [15]. This also contains the background estimation that was performed there using MC simulations. The data set used to retrieve these results was slightly different from the one used in this project. This will not, however, nullify a comparison of the two works.

![ATLAS Preliminary Results](image)

**Figure 4.1 – The official results plot from the ATLAS collaboration**

Above is the histogram containing the official results from ATLAS. This is the figure corresponding to the histogram in Figure 4.2 of this project. The figure also contains an estimation of the background (red and purple) and the simulated signal for a Higgs boson of mass 125 GeV/c² (light blue).
4.2 Four-Muon Invariant Mass

Below, the histogram containing the masses of the possible Higgs bosons (i.e. the invariant masses of all muon quadruplets that passed the selection criteria) is displayed. The histogram contains 135 entries, i.e. 135 events pass the selection criteria for the four-muon system.

![Histogram of four-muon invariant mass](image)

*Figure 4.2 – The invariant mass of the four-muon system*

Above, the histogram of the possible masses of the Higgs boson is displayed. All entries are the invariant masses of quadruplets of muons that satisfy all selection criteria described in Section 3.2, i.e. all muons that might have originated from a Higgs boson via the double Z boson decay. The errors displayed are statistical.

As can be seen in Figure 4.2, there are several clear peaks in the histogram, i.e. there are several invariant masses of muon quadruplets that are frequently occurring in the data. The most prominent ones are narrow ones at 85-95 GeV/$c^2$ and 120-125 GeV/$c^2$ and a wide one at around 180-240 GeV/$c^2$. These are all also present in the official results published by ATLAS (Figure 4.1). The peak at 85-95 GeV/$c^2$ is also present in the background estimation of Figure 4.1, and is explained by resulting from the following reaction.

\[
pp \rightarrow Z \rightarrow \mu^+\mu^-\mu^+\mu^-
\]

As a single Z boson is produced, the peak is expected to be at the mass of one Z boson, which is observed.

The broad peak at around 180-240 GeV/$c^2$ is also visible in both the ATLAS results and their background estimation, and is explained by the background processes from Section 2.8 where two Z bosons are produced without connection to a Higgs boson. This peak is in the region around twice the mass of a Z boson, as Z bosons with no mass restriction in their production (i.e. not through a resonance of well defined mass) are most likely to be produced on shell. Therefore these double Z systems will be produced with invariant mass about equal to or larger than $2m_Z$.

The last clear peak of Figure 4.2, at 120-125 GeV/$c^2$, is also visible in the ATLAS results. It is not, however, predicted by the background measurements. In Figure 4.1 there is a blue area signifying the signal
a Higgs boson of mass $m_H = 125 \text{ GeV}/c^2$ would display. This is something that would explain the resulting peak at that particular four-muon invariant mass, i.e. this seems to be the peak searched for in this project.

Below, a histogram containing the invariant masses of the $Z$ bosons is displayed.

![Histogram of Z boson invariant masses](image)

**Figure 4.3 – The invariant masses of the two Z bosons**

Above, the masses of the recreated $Z$ bosons is displayed. The leading $Z$ boson mass (the leading muon pair invariant mass) is represented by the colour blue and the sub-leading $Z$ boson mass (the sub-leading muon pair invariant mass) is represented by the colour red. The errors displayed are statistical.

The histograms containing the muon pair invariant masses (i.e. the $Z$ bosons) (Figure 4.3) seem to be fairly straightforward. Both reconstructed $Z$ bosons have clear peaks around the real $Z$ boson mass at $91 \text{ GeV}/c^2$. The leading muon pair has an invariant mass more highly concentrated around $m_Z$ than the sub-leading muon pair. This is only to be expected as that was the definition of the two different pairs (see Section 3.1). Both $Z$ bosons have clear rises at the lower masses of their allowed mass intervals. This might indicate that those particular events did not originate from two $Z$ bosons, but are a part of some other background process.
Below is a two dimensional histogram containing the correlation of the masses of the Z bosons.

![Two dimensional histogram]

*Figure 4.4 – The correlation of the masses of the recreated Z bosons*

The histogram above displays the correlation between the mass of the leading Z boson to the sub-leading Z boson from the same event. The x-axis shows the mass of the leading Z boson; the y-axis shows the mass of the sub-leading Z boson. The size of the box indicates the number of events with that combination of masses (of the leading and sub-leading Z bosons).

The excess of events at the lower limits of the intervals (seen in Figure 4.3) can also be observed in Figure 4.4, in which the masses of the reconstructed Z bosons of each event are correlated to each other. Here, clear excesses can be seen at masses of \( m_{12} \approx 50 \text{ GeV/c}^2 \) and \( m_{34} \approx 20 \text{ GeV/c}^2 \), regardless of the mass of the other reconstructed Z boson. The clearest feature of this figure is, however, the overwhelming peak at \( m_{12} \approx m_{34} \approx m_Z \), as this is the point where both reconstructed Z bosons would be on shell. This excess of events follows the mass of the sub-leading muon pair to smaller masses, as \( m_{12} \) remains the same, which displays the preferred case of having one of the Z bosons on shell. These effects are predictable, given the definitions of the muon pairs (Section 3.1). From these definitions it is also clear that Figure 4.4 should not display any entries above the diagonal \( m_{12} = m_{34} \) as both masses are below \( m_Z \). At masses above \( m_Z \), there is no such connection, as \( m_{12} \) strives to be as close to \( m_Z \) as possible, while \( m_{34} \) strives to be as high as possible.
4.3 Four-Muon System with Recombined Z Bosons

Figure 4.5 contains the same histogram as Figure 4.2. Superimposed on this is a histogram containing the four-muon invariant mass of the recombined system (see Section 3.3). The recombined spectrum acts as a background estimator.

The histogram above displays the invariant mass of the four muon system, the result from Figure 4.2 (green), along with the invariant mass of the four muon system using Z bosons from different events (red). The latter has been renormalized to the same integral as the former and is included as an estimation of the background. The errors displayed are statistical. The green histogram has larger errors than the red histogram, as there are more statistics for the red histogram than the green one.

Figure 4.5 displays results of combining Z bosons from different events to retrieve a four muon system. From the figure, it is clear that the overall shapes of the recombined mass spectrum and the original results (from Figure 4.2) are very similar. Most prominently, the estimated background displays a broad peak around $2m_Z$. It also displays a smaller, but clear peak at $m_Z$, and also a less pronounced peak at 120-125 GeV/c².

The red histogram, the recombined Z boson “events”, is used as a background estimator in Figure 4.6 as the main background source in this project is the double Z production. The recombination simulates events with two entirely separate Z bosons being produced, either through the same process or two different processes, and then decaying to muons. This recombination is a fairly good background estimator since the two new Z bosons are entirely uncorrelated. This means that there will be no preferred value for the invariant mass of the system (such as there is for resonances, e.g. the Higgs mass). However, as this only takes into account the background processes containing two Z bosons, it is not a good background estimator for other background processes (such as the one Z boson resonance). This is the reason for the estimated background peak at $m_Z$ being so much smaller than the produced results. The same is true for the peak at 120-125 GeV/c². As can be seen, small peaks are still visible at these invariant masses. These are present as such a large number of events produced these masses before, hence they will be a “preferred result” among the limited set of combinations of Z bosons available.
Below, an estimate of background events has been subtracted from the results in Figure 4.2. The background was estimated by combining Z bosons from different events into four-muon systems.

\[ \text{Figure 4.6} \quad \text{The invariant mass of the four-muon system, subtracted estimated background} \]

The histogram displaying the invariant mass of the four muon system, with the background subtracted. The errors displayed are a combination of the statistical errors of the results and the background estimation.

Figure 4.6 displays the results of using this as a background estimator and subtracting the results. As can be observed in this figure, the line representing “zero observed events” is within the error bars of nearly every histogram bin with the background subtracted. There are bins which do not have the zero line within their error bars (e.g. three consecutive bins around 170 GeV/c²) where a two Z boson event might have been expected as the cause. Here, it is assumed that the small number of events (135) is the reason for this unexpected result, as the statistics could be considered insufficient. There are also a number of bins where the bin content had been zero and the estimation was not, which mostly rendered the zero line outside of the error bars at those bins. This is also assumed to be because of the low statistics. The only bins relevant for discussion here are the ones at \( m_Z \) and at 120-125 GeV/c². These do not have the zero line within their error bars, as the “two uncorrelated Z boson background” is not a good estimator at those masses (see previous paragraph). In the figure, it is clearly seen that the number of events in the bin at 120-125 GeV/c² is close to double that of the (absolute value of the) number of events in any other bin (with the estimated background subtracted). And as it also does not include the zero line within its error bars, there is a strong suggestion that this is the bin containing the mass of the Higgs boson.
Below is a two dimensional histogram containing the mass of the reconstructed Higgs boson on one axis, and which Z bosons were used for that mass on the other axis.

![Figure 4.7 – The correlation between four-muon invariant masses and the initial Z bosons](image)

Above is the histogram displaying which event gave rise to which mass of the reconstructed four-muon system. The y-axis displays the reconstructed four-muon invariant masses. The x-axis shows which event the Z boson involved in creating a four-muon system of that mass came from. As each event contained two reconstructed Z bosons, the x-axis is twice the number of recorded events long. The interval [1;135] represents the leading Z bosons, and the interval [136;270] represents the sub-leading Z bosons. The colour code indicates how many Higgs bosons of that mass were created by a Z boson from that particular event.
Below, the histogram from *Figure 4.7* is zoomed in on relevant regions. The mass region 500 GeV/c² to 600 GeV/c² is relevant as *Figure 4.5* displays peaks at 515-520 GeV/c² and 555-560 GeV/c², at the same masses that had non-zero bin content in *Figure 4.2*. The event region is relevant, as one specific event seems to have been the source of each of these mass entries. These events are present in the event region on the x-axis. *Figure 4.8* displays the region for the leading Z boson, but a very similar histogram is present for the corresponding event numbers for the sub-leading Z boson.

As can be seen throughout *Figure 4.5* it, the four-muon system with Z bosons from different events tend to display peaks at the same masses as the original results displayed. This is especially apparent in the region of invariant masses between 500 GeV/c² and 600 GeV/c². This correlation is investigated in *Figure 4.7* and *Figure 4.8*, where a two dimensional histogram displays how often a given Z boson (x-axis) yields a specific invariant mass of the total system (y-axis). The x-axis is divided in such that the first 135 bins are the leading Z bosons of the original events, and the second 135 bins are the sub-leading Z bosons. *Figure 4.8* is a zoomed in region of *Figure 4.7*. It is clear that both of the mass peaks in question (515-520 GeV/c² and 555-560 GeV/c²) each originate from a specific event, as both the leading and sub-leading Z bosons of the pairs contain bin values of 35-40, while the other bins at that mass contain values of 0-10. If either Z boson from either of the relevant events is used in the recombination, there is a strong probability of it yielding the corresponding invariant mass (regardless of what other Z boson it is combined with). This is assumed to be a result of some deviancy from the norm of those events, some sort of peculiarity uncommon among the other events. As this peculiarity arguably could have emerged to cause a bias towards any high invariant mass, this is assumed to not be of relevance to the analysis. The curious peak anomalies displayed in *Figure 4.5* are concluded to be an effect of insufficient statistics, in contrary to an intrinsic bias towards those masses.
5 Possible Expansion of this Project

This project is not as rigorously performed as the data analysis performed by the ATLAS collaboration, with the decisive factor being the time available. One main part of the analysis that had to be left out of this project was to perform extensive MC simulations of pp-collision events to estimate the background of processes with final states identical to the desired one. MC calculations of this type were performed for the ATLAS analysis, and as is described in Section 4.2, these estimations were partly used as comparison to the results of this project.

Additionally, corrections to the results are made in the ATLAS analysis, which were not included in this project. Many corrections were already applied to the data set as it was received by the author, such as in the reconstruction of tracks and compensating for flaws in detectors. There were, however, some corrections that were to be done afterwards. Among these are corrections for the final state radiation (FSR). This correction would systematically heighten each invariant mass of the four-muon system, but estimations of FSR would also require extensive MC simulations.

Next, MC simulations could also be made to estimate the signal a possible Higgs boson of a certain mass would yield in a mass spectrum such as Figure 4.2. This kind of simulation would be done in combination with the background simulations; in order to see which Higgs boson mass would yield the signal that fitted in the results best.

Finally, in [3] it is also pointed out that applying a Z-mass constrained kinematic fit to the muon pairs would improve the resolution of the four-muon invariant mass. As \( m_\mu < 190 \text{ GeV}/c^2 \) this would be applied to the leading muon pair, and at higher masses this would be applied to both the leading and the sub-leading muon pairs. This fit was performed in the ATLAS analysis, but not in the analysis of this project.

As can be seen in this section, most of the parts of the analysis that were left out of this project have to do with MC simulations, and all of them were left out due to issues of time availability. If this project would be expanded upon, these would be some areas to naturally pursue.

Alternatively, a different route could be taken in a possible expansion of the project. This would be to investigate the properties of the invariant mass spectrum with recombined Z bosons. A topic of such an analysis could be to investigate what exactly differentiates the events which gave rise to the high-mass peaks from the other events. Another topic could be to, through MC simulations, create a spectrum of double Z boson event invariant masses, recombine the Z bosons to create the corresponding spectrum, and investigate the differences between the two spectra.

There can certainly be many more routes for expansion of this project, but no more will be presented here.
6 Conclusions

6.1 The Search for Higgs

In the results directly retrieved from the four-muon data, the most prominent peak was seen at 120-125 GeV/$c^2$ (see Figure 4.2). This peak was also not explainable by background processes, and is hence a strong candidate for the mass of the Higgs boson.

The same peak was also one of the few that did not have the “zero events observed”-line within its errors (see Figure 4.6). In the same figure, it was the one furthest away from the zero line, i.e. the bin with the largest difference between observed events and estimated background. Here, the number of observed events (estimated background subtracted) was close to double that of the corresponding number of any other bin. These facts also point to it being a strong candidate for the Higgs mass.

Given these results, this work strongly indicates that the Higgs mass is between 120 GeV/$c^2$ and 125 GeV/$c^2$. Taking into account the lack of FSR corrections, it could be somewhat higher, and a fit to the estimated signal of a Higgs boson would yield an even sharper result.

Finally, a remark can be made on the fact that the analysis performed by the ATLAS collaboration yielded an invariant mass spectrum very similar to the resulting invariant mass spectrum of the analysis of this project (compare Figure 4.1 and Figure 4.2). The mass range of the Higgs boson concluded in this thesis was also very close to the mass of the Higgs boson given in [3], which was 126 GeV/$c^2$. 
7 Acknowledgements

The author would first and foremost like to give many thanks to Oxana Smirnova for her help and guidance during the course of this project. The discussions between the author and Oxana were both enlightening and humoristic, and this made each step of the project more interesting and enjoyable. Without her, this project would not be at the level it is.

The author would also like to thank Anthony Hawkins, who gave of his time to explain and give aid at the beginning of the project.

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Finally, the author would like to thank the Division of Particle Physics at Lund University, for giving him the opportunity to pursue his interest in fundamental particle physics, and allowing him to perform this project for them.
8 References


Appendix

Below follows the code for the macro constructed for the analysis of this project.

9.1 The Code

```cpp
#include "TROOT"
#include "TFile.h"
#include "TTree.h"
#include " TBranch.h"
#include <iostream>
#include <vector>
#include <cmath>

// Function for calculating Z boson mass
double massZ(double Ea, double pxa, double pya, double pza, double Eb, double pxb,
             double pyb, double pzb){
  double mZ=sqrt((Ea+Eb)*(Ea+Eb) - (pxa+pxb)*(pxa+pxb) - (pya+pyb)*(pya+pyb) -
                (pza+pzb)*(pza+pzb));
  return mZ;
}

// Function for calculating Higgs boson mass
double massH(double Ea, double pxa, double pya, double pza, double Eb, double pxb,
              double pyb, double pzb, double Ec, double pxc, double pyc, double pzc,
              double Ed, double pxd, double pyd, double pzd){
  double mH=sqrt((Ea+Eb+Ec+Ed)*(Ea+Eb+Ec+Ed) - (pxa+pxb+pxc+pxd)*(pxa+pxb+pxc+pxd) -
                 (pya+pyb+pyc+pyd)*(pya+pyb+pyc+pyd) -
                 (pza+pzb+pxc+pzd)*(pza+pzb+pxc+pzd));
  return mH;
}

// Z bosons must be uncharged
bool noZcharge(int qa, int qb, int qc, int qd){
  bool bothZuncharged=true;
  if( (qa+qb) != 0 ){bothZuncharged=false;};
  if( (qc+qd) != 0 ){bothZuncharged=false;};
  return bothZuncharged;
}

// The transverse momenta must be high enough
bool ptcriterion(double pta, double ptb, double ptc, double ptd){ // Both criteria
  int overlower[]={0, 0, 0, 0};
  double lowerlim[]={6000, 10000, 15000, 20000};
  double ptlist[]={pta, ptb, ptc, ptd};
  for(int iter=0; iter<4; iter++){
    for(int jter=0; jter<4; jter++){
      if(ptlist[iter]>=lowerlim[jter]){overlower[jter]++;
    }
  }
  bool outer=true;
  for(int iter=0; iter<4; iter++)
  
      if(overlower[iter]<4-iter){outer=false; } }
```
return outer;
}

// The Z boson mass is constricted
bool acceptableZmass(double mZ1, double mZ2, double mH){
    // without this there are 159 acceptable entries
    if(mH<120000){double minmZ2=17500;}
    if(mH>=120000 && mH<=190000){double minmZ2=((mH*3.25-267500)/7);}
    if(mH>190000){double minmZ2=50000;}

    bool Zmassesok=true;
    if(mZ1<50000 || mZ1>106000 || mZ2<minmZ2 || mZ2>115000){Zmassesok=false;}
    return Zmassesok;
}

// The pseudorapidity criterion
bool withineta(double etaa, double etab, double etac, double etad){
    bool etaoK=true;
    if((etaa*etaa) > 7.29 ){etaoK=false;};
    if((etab*etab) > 7.29 ){etaoK=false;};
    if((etac*etac) > 7.29 ){etaoK=false;};
    if((etad*etad) > 7.29 ){etaoK=false;};
    // abs(eta) < 2.7
    return etaoK;
}

// The separation requirement
bool deltaR(double etaa, double etab, double etac, double etad, double phia, double phib, double phic, double phid){
    double et[]= {etaa, etab, etac, etad};
    double ph[]= {phia, phib, phic, phid};
    bool outdR=true;
    for(int iter=0; iter<4; iter++){
        for(int jter=iter+1; jter<4; jter++){
            double dR=sqrt(( (et[iter]-et[jter]) * (et[iter]-et[jter]) ) + ( (ph[iter]-ph[jter]) * (ph[iter]-ph[jter]) ) ));
            if( dR < 0.1){outdR=false;};
        }
    }
    return outdR;
}

// Each muonpair must have high enough invariant mass
bool allpossiblemassmumu(double Ea, double px, double py, double pza, double qa, double Eb, double pxb, double pby, double pzb, double qb, double Ec, double pxc, double pyc, double pzc, double qc, double Ed, double pxd, double pyd, double pzd, double qd){
    double Eh[]={Ea, Eb, Ec, Ed};
    double pxh[]={px, pxb, pxc, pxd};
    double pyh[]={py, pby, pyc, pyd};
    double pzh[]={pza, pzb, pzc, pzd};
    double qh[]={qa, qb, qc, qd};

    bool allmok=true;
    for(int iter=0; iter<4; iter++){
        for(int jter=iter+1; jter<4; jter++){
            if(qh[iter]+qh[jter]==0){
            }}
        }
    return allmok;
}
if(massZ(Eh[iter], pxh[iter], pyh[iter], pzh[iter],
    pyh[jter], pzh[jter]) <= 5000){allmok=false;
}
}
return allmok;

// An isolation requirement
bool etconecriterion30(double etca, double etcb, double etcc, double etcd, double
    pta, double ptb, double ptc, double ptd){
double etc[]={etca, etcb, etcc, etcd};
double pt[]={pta, ptb, ptc, ptd};
bool etcout=true;
for(int iet=0; iet<4; iet++) if(etc[iet]/pt[iet]>0.3){etcout=false;};
return etcout;
}

// Another isolation requirement
bool etconecriterion15(double etca, double etcb, double etcc, double etcd, double
    pta, double ptb, double ptc, double ptd, double staa, double stab, double
    stac, doublestad){
double etc[]={etca, etcb, etcc, etcd};
double pt[]={pta, ptb, ptc, ptd};
double sta[]={staa, stab, stac, stad};
bool etcout=true;
for(int iet=0; iet<4; iet++) if(etc[iet]/pt[iet]>0.15 &&
    sta[iet]!=0){etcout=false;};
return etcout;
}

// Yet another isolation requirement
bool ptconecriterion(double ptca, double ptcb, double ptcc, double ptcd, double
    pta, double ptb, double ptc, double ptd){
double ptcq[]={ptca, ptcb, ptcc, ptcd};
double pt[]={pta, ptb, ptc, ptd};
bool ptcout=true;
for(int ipt=0; ipt<4; ipt++) if(ptcq[ipt]/pt[ipt]>0.15){ptcout=false;};
return ptcout;
}

// The transverse impact parameter restriction
bool d0criterion(double d0a, double d0b, double d0c, double d0d){
double d0vec[]={d0a, d0b, d0c, d0d};
bool d0out=true;
for(int id0=0; id0<4; id0++) if((d0vec[id0]*d0vec[id0]) > 1 ){d0out=false;};
// abs(d0)<1
return d0out;
}

// The longitudinal impact parameter restriction
bool z0criterion(double z0a, double z0b, double z0c, double z0d){
double z0vec[]={z0a, z0b, z0c, z0d};
bool z0out=true;
for(int iz0=0; iz0<4; iz0++) if((z0vec[iz0]*z0vec[iz0]) > 100 ){z0out=false;};
// abs(z0)<10
return z0out;
// The cut on the transverse impact parameter restriction
bool d0cutcriterion(double d0a, double d0b, double d0c, double d0d, double cd0a, double cd0b, double cd0c, double cd0d)
{
    double d0vec[]={d0a, d0b, d0c, d0d};
    double cd0vec[]={cd0a, cd0b, cd0c, cd0d};
    bool cd0out=true;
    for(int id0=0; id0<4; id0++)
    {
        if( ((d0vec[id0]*d0vec[id0]/cd0vec[id0])) > 12.25 )
        {
            cd0out=false;
        }
    }
    // abs(d0/sigmad0)<3.5
    return cd0out;
}

// THE ACTUAL MACRO IS BELOW
void InvMassHiggs()
{
    // Creating some variables and vectors
    int counter=0;
    std::vector<float> *etcone7, *ptcone7, *nucone7, *d0pv7, *z0pv7, *c0vdpv7, *etcone8, *ptcone8, *nucone8, *d0pv8, *z0pv8, *c0vdpv8;
    std::vector<int> *standalone7, *standalone8;
    int n7, eve7, run7, n8, eve8, run8;

    // The constant tabmZ is the tabular mass of the Z boson, from PDG
double tabmZ=91187.6;

    // Creating some histograms
    TH1F *hH, *hZ1, *hZ2, *hHrecomb, *hHsubrec;
    TH2F *hZcorr, *hZHcorr;
    // Creating branch pointers
    TBranch *pxbr7=0, *pybr7=0, *pzbr7=0, *Ebr7=0, *nbr7=0, *runbr7=0, *chargebr7=0, *ptbr7=0, *etabr7=0, *phi7=0, *pxbr8=0, *pybr8=0, *pzbr8=0, *Ebr8=0, *nbr8=0, *runbr8=0, *chargebr8=0, *ptbr8=0, *etabr8=0, *phi8=0, *etconebr7=0, *ptconebr7=0, *nuconebr7=0, *d0pvbr7=0, *z0pvbr7=0, *c0vdpvbr7, *etconebr8=0, *ptconebr8=0, *nuconebr8=0, *d0pvbr8=0, *z0pvbr8=0, *c0vdpvbr8, *standalonebr7, *standalonebr8;

    // Defining histograms
    hH=new TH1F("InvMass of 4mu","The Invariant Mass of the Four Muon System;m_{4µ} (GeV);Events / 5 GeV",220,0,1100); //entries lie []
    hH->Sumw2();
    hZ1=new TH1F("InvMass of Z1","The Invariant Mass of the Muon Pairs;m_{12}, m_{34} (GeV);Events / 5 GeV",24,0,120); //entries lie []
    hZ1->Sumw2();
    hZ2=new TH1F("InvMass of Z2","The Invariant Mass of the Muon Pairs;m_{12}, m_{34} (GeV);Events / 5 GeV",24,0,120); //entries lie []
    hZ2->Sumw2();
    hHrecomb=new TH1F("InvMass of 4mu, Recombined Z","The Invariant Mass of the Four Muon System, Z-Bosons from Different Events;m_{4µ} (GeV);Events / 5 GeV",220,0,1100); //entries lie []
    hHrecomb->Sumw2();
    hHrecombrenorm=new TH1F("InvMass of 4mu, Recomb Z, Renorm","The Invariant Mass of the Four Muon System, Z-Bosons from Different Events, Renormalized;m_{4µ} (GeV);Events / 5 GeV",220,0,1100); //entries lie []
hHrecomnorm->Sumw2();
hHrecomnorm=new TH1F("InvMass of 4mu, Renorm","The Invariant Mass of the Four Muon System, Renormalized;m_{4\mu} (GeV);Events / 5 GeV",220,0,1100); //entries lie []
hHrecomnorm->Sumw2();
hZcorr=new TH2F("InvMass of Z, Correlation","The Invariant Masses of Z1 and Z2, and their Correlation;m_1 (GeV);m_2 (GeV)", 20,15,115,20,15,115);
hZcorr->Sumw2();
hHsubrec=new TH1F("InvMass, Without Background","The Invariant Mass of the Four Muon System, Estimated Background Subtracted;m_{4\mu} (GeV);Events / 5 GeV",120,50,650);
hHsubrec->Sumw2();
hHsub1=new TH1F("To Use for hHsubrec, 1","The Invariant Mass of the Four Muon System, Estimated Background Subtracted;m_{4\mu} (GeV);Events / 5 GeV",120,50,650);
hHsub1->Sumw2();
hHsub2=new TH1F("To Use for hHsubrec, 2","The Invariant Mass of the Four Muon System, Estimated Background Subtracted;m_{4\mu} (GeV);Events / 5 GeV",120,50,650);
hHsub2->Sumw2();
hZHcorr=new TH2F("InvMass of H, to Z-channel","The Invariant Mass of H, Correlated with the Z1 and Z2 it came from;Z boson, Nr;m_{4\mu} (GeV)", 270,0,270,220,0,1100);
hZHcorr->Sumw2();

// Opening the ROOT files
TFile *pointer7=TFile::Open("C:/root/macros/muonroots/mu1225.root");
TFile *pointer8=TFile::Open("C:/root/macros/muonroots/mu1344.root");
// The 7 or 8 refers to the number of TeV the file comes from
cout << "\nFiles opened.\n";

// Connecting Pointers to the branches of the tree
TTree *colltree7=(TTree*)pointer7->Get("CollectionTree");
TTree *phystree7=(TTree*)pointer7->Get("physics");
TTree *colltree8=(TTree*)pointer8->Get("CollectionTree");
TTree *phystree8=(TTree*)pointer8->Get("physics");
cout << "\nTrees built.\n";
colltree7->SetMakeClass(1);
colltree8->SetMakeClass(1);
colltree7->SetBranchAddress("EventNumber", &eve7, &evebr7);
colltree7->SetBranchAddress("RunNumber", &run7, &runbr7);
phystree7->SetBranchAddress("mu_staco_px", &px7, &pxbr7);
phystree7->SetBranchAddress("mu_staco_py", &py7, &pybr7);
phystree7->SetBranchAddress("mu_staco_pz", &pz7, &pzbr7);
phystree7->SetBranchAddress("mu_staco_E", &E7, &Ebr7);
phystree7->SetBranchAddress("mu_staco_n", &n7, &nbr7);
colltree7->SetBranchAddress("mu_staco_charge", &q7, &chargebr7);
colltree7->SetBranchAddress("mu_staco_pt", &pt7, &ptbr7);
colltree7->SetBranchAddress("mu_staco_eta", &eta7, &etabr7);
colltree7->SetBranchAddress("mu_staco_phi", &phi7, &phibr7);
colltree7->SetBranchAddress("mu_staco_etcone20", &etcone7, &etconebr7);
colltree7->SetBranchAddress("mu_staco_ptcone20", &ptcone7, &ptconebr7);
colltree7->SetBranchAddress("mu_staco_nucone20", &nucone7, &nuconebr7);
colltree7->SetBranchAddress("mu_staco_d0_exPV", &d0pv7, &d0pvbr7);
colltree7->SetBranchAddress("mu_staco_z0_exPV", &z0pv7, &z0pvbr7);
phystree7->SetBranchAddress("mu_staco_cov_d0_exPV", &covd0pv7, &covd0pvbr7);
phystree7->SetBranchAddress("mu_staco_isStandAloneMuon", &standalone7, &standalonebr7);
phystree8->SetBranchAddress("mu_staco_px", &px8, &pxbr8);
phystree8->SetBranchAddress("mu_staco_py", &py8, &pybr8);
phystree8->SetBranchAddress("mu_staco_pz", &pz8, &pzbr8);
phystree8->SetBranchAddress("mu_staco_E", &E8, &Ebr8);
phystree8->SetBranchAddress("mu_staco_n", &n8, &nbr8);
colltree8->SetBranchAddress("EventNumber", &eve8, &evebr8);
colltree8->SetBranchAddress("RunNumber", &run8, &runbr8);
phystree8->SetBranchAddress("mu_staco_charge", &q8, &chargebr8);
phystree8->SetBranchAddress("mu_staco_pt", &pt8, &ptbr8);
phystree8->SetBranchAddress("mu_staco_eta", &eta8, &etabr8);
phystree8->SetBranchAddress("mu_staco_phi", &phi8, &phibr8);
phystree8->SetBranchAddress("mu_staco_etcone20", &etcone8, &etconebr8);
phystree8->SetBranchAddress("mu_staco_ptcone20", &ptcone8, &ptconebr8);
phystree8->SetBranchAddress("mu_staco_nucone20", &nucone8, &nuconebr8);
phystree8->SetBranchAddress("mu_staco_d0_exPV", &d0pv8, &d0pvbr8);
phystree8->SetBranchAddress("mu_staco_z0_exPV", &z0pv8, &z0pvbr8);
phystree8->SetBranchAddress("mu_staco_cov_d0_exPV", &covd0pv8, &covd0pvbr8);
phystree8->SetBranchAddress("mu_staco_isStandAloneMuon", &standalone8, &standalonebr8);

cout << "Branch addresses set.\n";

// Collecting the number of entries in each dataset
Long64_t nent7=phystree7->GetEntries();
Long64_t nent8=phystree8->GetEntries();
cout << "Number of entries retrieved.\n";

// Vectors for recombination of Z bosons are created
int mu1place[200], mu2place[200], mu3place[200], mu4place[200], entryno[200],
sqrts[200];
for(int iset=0; iset<200; iset++){mu1place[iset]=-1; mu2place[iset]=-1;
mu3place[iset]=-1; mu4place[iset]=-1; entryno[iset]=-1; sqrt2[iset]=-1;};
cout << "The mu-place arrays have been initiated.\nThe loops are next.\n\n";

// BELOW IS THE CALCULATION OF THE FOUR-MUON INVARIANT MASS
for(int numTeV=7; numTeV<=8; numTeV++){ // Looping over both 7 TeV and 8 TeV
  if(numTeV==7){Long64_t nent=nent7;}
  if(numTeV==8){Long64_t nent=nent8;}

  for (Long64_t i=0;i<nent;i++) { // Looping over all events of each data set
    if(numTeV==7){nbr7->GetEntry(i); int nn=n7;} // The number of muons in entry i
    if(numTeV==8){nbr8->GetEntry(i); int nn=n8;} // The number of muons in entry i

    if (nn>=4){ // For all entries with enough muons (four or more)
      if(numTeV==7){ // Collecting the 7 TeV data
        Ebr7->GetEntry(i); pxbr7->GetEntry(i); pybr7->GetEntry(i); pzbr7->
        GetEntry(i);
evebr7->GetEntry(i); runbr7->GetEntry(i); chargebr7->GetEntry(i); ptbr7->
>GetEntry(i);
etabr7->GetEntry(i); phibr7->GetEntry(i);
etconebr7->GetEntry(i); ptconebr7->GetEntry(i); nuconebr7->GetEntry(i);
d0pvbr7->GetEntry(i); z0pvbr7->GetEntry(i); covd0pvbr7->GetEntry(i);
standalonebr7->GetEntry(i);
*nucone, *d0pv, *z0pv, *covd0pv;
std::vector<int> *standalone;
E=E7; px=px7; py=py7; pz=pz7; q=q7; pt=pt7; eta=eta7; phi=phi7;
etcone=etcone7; ptcone=ptcone7; nucone=nucone7; d0pv=d0pv7; z0pv=z0pv7;
covd0pv=covd0pv7; standalone=standalone7;
int eve=eve7, run=run7;
}
if(numTeV==8){ // Collecting the 8 TeV data
Ebr8->GetEntry(i); pxbr8->GetEntry(i); pybr8->GetEntry(i); pzbr8->
>GetEntry(i);
etabr8->GetEntry(i); phibr8->GetEntry(i);
etconebr8->GetEntry(i); ptconebr8->GetEntry(i); nuconebr8->GetEntry(i);
d0pvbr8->GetEntry(i); z0pvbr8->GetEntry(i); covd0pvbr8->GetEntry(i);
standalonebr8->GetEntry(i);
*nucone, *d0pv, *z0pv, *covd0pv;
std::vector<int> *standalone;
E=E8; px=px8; py=py8; pz=pz8; q=q8; pt=pt8; eta=eta8; phi=phi8;
etcone=etcone8; ptcone=ptcone8; nucone=nucone8; d0pv=d0pv8; z0pv=z0pv8;
covd0pv=covd0pv8; standalone=standalone8;
int eve=eve8, run=run8;
}

int mu1=-1, mu2=-1, mu3=-1, mu4=-1;
double latemZ1diff=1000000, latemZ2=0;

// The following loops determine which muons are to be defined in the leading
and sub-leading muon pairs

for(int j=0; j<n; j++){ // Loop for mu1
for(int k=j+1; k<n; k++){ // Loop for mu2
latemZ2=0; // Must be reset after every set of potential mu1 and mu2
double mmZ1=massZ(E->at(j), px->at(j), py->at(j), pz->at(j), E->at(k), px->
>at(k), py->at(k), pz->at(k));
for(int m=0; m<n; m++){ // Loop for mu3
for(int n=m+1; n<n; n++){ // Loop for mu4
if ((m!=j && m!=k) && (n!=j && n!=k)) { // Checking that no muon is used more
than once

double mmZ2=massZ(E->at(m), px->at(m), py->at(m), pz->at(m), E->at(n), px->
>at(n), py->at(n), pz->at(n));
double mmH=massH(E->at(j), px->at(j), py->at(j), pz->at(j), E->at(k), px->
>at(k), py->at(k), pz->at(k), E->at(m), px->at(m), py->at(m), pz->at(m),
E->at(n), px->at(n), py->at(n), pz->at(n));
bool noQ=|no2charge(q->at(j),q->at(k),q->at(m),q->at(n));
bool ptcrit=ptcriterion(pt->at(j),pt->at(k),pt->at(m),pt->at(n));
bool accZm=acceptableZmass(mmZ1,mmZ2,mmH);
bool etain=withineta(eta->at(j),eta->at(k),eta->at(m),eta->at(n));
}
bool dRcrit=deltaR(eta->at(j),eta->at(k),eta->at(m),eta->at(n),phi->at(j),phi->at(k),phi->at(m),phi->at(n));
bool posmumu=allpossiblemassmumu(E->at(j), px->at(j), py->at(j), pz->at(j), q->at(j), E->at(k), px->at(k), py->at(k), q->at(k), E->at(m), px->at(m), py->at(m), q->at(m), E->at(n), px->at(n), py->at(n), q->at(n));
bool etc30=etconecriterion30(etcone->at(j),etcone->at(k),etcone->at(m),etcone->at(n),pt->at(j),pt->at(k),pt->at(m),pt->at(n));
bool etc15=etconecriterion15(etcone->at(j),etcone->at(k),etcone->at(m),etcone->at(n),pt->at(j),pt->at(k),pt->at(m),pt->at(n),standalone->at(j),standalone->at(k),standalone->at(m),standalone->at(n));
bool ptco=ptconecriterion(ptcone->at(j),ptcone->at(k),ptcone->at(m),ptcone->at(n),pt->at(j),pt->at(k),pt->at(m),pt->at(n));
bool d0crit=d0criterion(d0pv->at(j),d0pv->at(k),d0pv->at(m),d0pv->at(n));
bool z0crit=z0criterion(z0pv->at(j),z0pv->at(k),z0pv->at(m),z0pv->at(n));
bool d0cutcrit=d0cutcriterion(d0pv->at(j),d0pv->at(k),d0pv->at(m),d0pv->at(n),covd0pv->at(j),covd0pv->at(k),covd0pv->at(m),covd0pv->at(n));

// The following loop assigns the leading and sub-leading muon pairs, if all criteria are met
if( noZq && pctrit && etain && dRcrit && posmumu && accZm && etc30 && etc15 && ptco && d0crit && z0crit && d0cutcrit){
  if( ( sqrt((mmZ1-tabmZ)*(mmZ1-tabmZ)) < latemZ1diff ) ) { // Assigning leading muon pair
    mu1=j;
    mu2=k;
    latemZ1diff=sqrt((mmZ1-tabmZ)*(mmZ1-tabmZ));
  }
  if( ((m!=mu1 && m!=mu2) && (n!=mu1 && n!=mu2)) && ( mmZ2 > latemZ2 ) ) { // Assigning sub-leading muon pair
    mu3=m;
    mu4=n;
    latemZ2=mmZ2;
  }
}

// All the fun criteria that must be fulfilled
}

// No muon is used more than once
}
// Loop for mu4
}
// Loop for mu3
}
// Loop for mu2
}
// Loop for mu1

// If the event met all criteria, this loop calculates and stores the masses of H, Z1 and Z2
if(mu1>=0 && mu2>=0 && mu3>=0 && mu4>=0){ // If all muons are determined
  double mZ1=massZ(E->at(mu1), px->at(mu1), py->at(mu1), pz->at(mu1), E->at(mu2), px->at(mu2), py->at(mu2), pz->at(mu2));
  double mZ2=massZ(E->at(mu3), px->at(mu3), py->at(mu3), pz->at(mu3), E->at(mu4), px->at(mu4), py->at(mu4), pz->at(mu4));
  double mHiggs=massH(E->at(mu1), px->at(mu1), py->at(mu1), pz->at(mu1), E->at(mu2), px->at(mu2), py->at(mu2), pz->at(mu2), E->at(mu3), px->at(mu3), py->at(mu3), pz->at(mu3), E->at(mu4), px->at(mu4), py->at(mu4), pz->at(mu4));

  // Which muon is which is stored for the recombination
}
mu1place[counter]=mu1; mu2place[counter]=mu2; mu3place[counter]=mu3;
   mu4place[counter]=mu4; entryno[counter]=i; sqrts[counter]=numTeV;

// Various histograms are filled
hH->Fill(mHiggs/1000);
hHrenorm->Fill(mHiggs/1000);
hHsub1->Fill(mHiggs/1000);
hZ1->Fill(mZ1/1000);
hZ2->Fill(mZ2/1000);
hZcorr->Fill(mZ1/1000,mZ2/1000);

   counter++;
   cout << "-:"

} // If all muons are determined within the event, the histogram is filled,
   and stuff is printed

} // Closing the loop that ensures that the event has 4 or more muons
} // Closing the loop over all entries in the data set
} // Closing the loop over 7 TeV and 8 TeV

   cout << "\n\n" << counter <<" Higgs candidates have been processed.\n
The recombination is next.\n\n";

// BELOW IS THE RECOMBINATION OF Z-BOSONS

   int countrecomb=0;

   // The recombination of Z bosons from different events is performed below
for(int it12=0; it12<counter; it12++){ // Stepping through all Za (the leading
   muon pairs from above)
for(int it34=it12+1; it34<counter; it34++){ // Stepping through all Zb (the sub-
   leading muon pairs from above)

// Collecting the 7 TeV data for Za
if(sqrts[it12]==7){
   Ebr7->GetEntry(entryno[it12]); pxbr7->GetEntry(entryno[it12]); pybr7-
      ->GetEntry(entryno[it12]); pzbr7->GetEntry(entryno[it12]);
   etabr7->GetEntry(entryno[it12]); phibr7->GetEntry(entryno[it12]); chargebr7-
      ->GetEntry(entryno[it12]);
   std::vector<float> *E12, *px12, *py12, *pz12, *eta12, *phi12, *q12; E12=E7;
   px12=px7; py12=py7; pz12=pz7; eta12=eta7; phi12=phi7; q12=q7;
}

// Collecting the 8 TeV data for Za
if(sqrts[it12]==8){
   Ebr8->GetEntry(entryno[it12]); pxbr8->GetEntry(entryno[it12]); pybr8-
      ->GetEntry(entryno[it12]); pzbr8->GetEntry(entryno[it12]);
   etabr8->GetEntry(entryno[it12]); phibr8->GetEntry(entryno[it12]); chargebr8-
      ->GetEntry(entryno[it12]);
   std::vector<float> *E12, *px12, *py12, *pz12, *eta12, *phi12, *q12; E12=E8;
   px12=px8; py12=py8; pz12=pz8; eta12=eta8; phi12=phi8; q12=q8;
}

// Collecting the 7 TeV data for Zb
if(sqrts[it34]==7){
   Ebr7->GetEntry(entryno[it34]); pxbr7->GetEntry(entryno[it34]); pybr7-
      ->GetEntry(entryno[it34]); pzbr7->GetEntry(entryno[it34]);
   etabr7->GetEntry(entryno[it34]); phibr7->GetEntry(entryno[it34]); chargebr7-
      ->GetEntry(entryno[it34]);
   std::vector<float> *E12, *px12, *py12, *pz12, *eta12, *phi12, *q12; E12=E7;
   px12=px7; py12=py7; pz12=pz7; eta12=eta7; phi12=phi7; q12=q7;
}

// Collecting the 8 TeV data for Zb
if(sqrts[it34]==8){
   Ebr8->GetEntry(entryno[it34]); pxbr8->GetEntry(entryno[it34]); pybr8-
      ->GetEntry(entryno[it34]); pzbr8->GetEntry(entryno[it34]);
   etabr8->GetEntry(entryno[it34]); phibr8->GetEntry(entryno[it34]); chargebr8-
      ->GetEntry(entryno[it34]);
   std::vector<float> *E12, *px12, *py12, *pz12, *eta12, *phi12, *q12; E12=E8;
   px12=px8; py12=py8; pz12=pz8; eta12=eta8; phi12=phi8; q12=q8;
etabr7->GetEntry(entryno[it34]); phibr7->GetEntry(entryno[it34]); chargebr7-
>GetEntry(entryno[it34]);
std::vector<float> *E34, *px34, *py34, *pz34, *eta34, *phi34, *q34; E34=E7;
px34=px7; py34=py7; pz34=pz7; eta34=eta7; phi34=phi7; q34=q7;
}

// Collecting the 8 TeV data for Zb
if(sqrts[it34]==8){
  Ebr8->GetEntry(entryno[it34]); pxbr8->GetEntry(entryno[it34]); pybr8-
>GetEntry(entryno[it34]); etabr8->GetEntry(entryno[it34]); phibr8->GetEntry(entryno[it34]); chargebr8-
>GetEntry(entryno[it34]);
std::vector<float> *E34, *px34, *py34, *pz34, *eta34, *phi34, *q34; E34=E8;
px34=px8; py34=py8; pz34=pz8; eta34=eta8; phi34=phi8; q34=q8;
}

// Assigning muons
int ma1, ma2, ma3, ma4, mb1, mb2, mb3, mb4;
ma1=mu1place[it12]; ma2=mu2place[it12]; ma3=mu3place[it12]; ma4=mu4place[it12];
mb1=mu1place[it34]; mb2=mu2place[it34]; mb3=mu3place[it34];
mb4=mu4place[it34];

// Combining the leading muon pair of event A with the sub-leading lepton pair
// of event B
bool dRcrit=deltaR(eta12->at(ma1),eta12->at(ma2),eta34->at(mb3),eta34-
>at(mb4),phi12->at(ma1),phi12->at(ma2),phi34->at(mb3),phi34->at(mb4));
bool posscombcrit=allpossiblemassmumu(E12->at(ma1), px12->at(ma1), py12-
>at(ma1), pz12->at(ma1), q12->at(ma1), E12->at(ma2), px12->at(mb2), py12-
>at(mb2), pz12->at(mb2), q12->at(mb2), E34->at(mb3), px34->at(mb3), py34-
>at(mb3), pz34->at(mb3), q34->at(mb3), E34->at(mb4), px34->at(mb4), q34->at(mb4));
if(dRcrit & posscombcrit){
double mHiggs=massH(E12->at(ma1), px12->at(ma1), py12->at(ma1), py12-
>at(ma1), pz12->at(ma1), q12->at(ma1), E12->at(ma2), px12->at(ma2), py12-
>at(ma2), pz12->at(ma2), q12->at(ma2), E34->at(mb3), px34->at(mb3), py34-
>at(mb3), pz34->at(mb3), q34->at(mb3), E34->at(mb4), px34->at(mb4), q34->at(mb4));
hHrecomb->Fill(mHiggs/1000);
hHrecombnorm->Fill(mHiggs/1000);
hHsub2->Fill(mHiggs/1000,-1);
hZHcorr->Fill(it12,mHiggs/1000);
hZHcorr->Fill(it34+counter,mHiggs/1000);
countrecomb++;
}

// Combining the sub-leading muon pair of event A with the leading lepton pair
// of event B
bool dRcrit=deltaR(eta12->at(ma3),eta12->at(ma4),eta34->at(mb1),eta34-
>at(mb2),phi12->at(ma3),phi12->at(ma4),phi34->at(mb1),phi34->at(mb2));
bool posscombcrit=allpossiblemassmumu(E12->at(ma3), px12->at(ma3), py12-
>at(ma3), pz12->at(ma3), q12->at(ma3), E12->at(ma4), px12->at(ma4), py12-
>at(ma4), pz12->at(ma4), q12->at(ma4), E34->at(mb1), px34->at(mb1), py34-
>at(mb1), pz34->at(mb1), q34->at(mb1), E34->at(mb2), px34->at(mb2), q34->at(mb2));
if(dRcrit & posscombcrit){
double mHiggs=massH(E12->at(ma3), px12->at(ma3), py12->at(ma3), py12-
>at(ma3), pz12->at(ma3), q12->at(ma3), E12->at(ma4), px12->at(ma4), py12-
>at(ma4), pz12->at(ma4), q12->at(ma4), E34->at(mb1), px34->at(mb1), py34-
>at(mb1), pz34->at(mb1), q34->at(mb1), E34->at(mb2), px34->at(mb2), q34->at(mb2));
hHrecomb->Fill(mHiggs/1000);
hHrecombnorm->Fill(mHiggs/1000);
hHsub2->Fill(mHiggs/1000,-1);
hZHcor->Fill(it12+counter,mHiggs/1000);
hZHcor->Fill(it34,mHiggs/1000);
countrecomb++;
}

// Combining the leading muon pair of event A with the leading lepton pair of event B
bool dRcrit=deltaR(eta12->at(ma1),eta12->at(ma2),eta34->at(mb1),eta34->at(mb2));
bool posscombcrit=allpossiblemassmumu(E12->at(ma1), px12->at(ma1), py12->at(ma1), pz12->at(ma1), q12->at(ma1), E12->at(ma2), px12->at(mb2), py12->at(mb2), E34->at(mb1), px34->at(mb1), py34->at(mb1), q34->at(mb1), E34->at(mb2), px34->at(mb2), py34->at(mb2), q34->at(mb2));
if(dRcrit && posscombcrit){
double mHiggs=massH(E12->at(ma1), px12->at(ma1), py12->at(ma1), E12->at(ma2), px12->at(ma2), py12->at(ma2), E34->at(mb1), px34->at(mb1), py34->at(mb1), E34->at(mb2), px34->at(mb2), py34->at(mb2));
hHrecomb->Fill(mHiggs/1000);
hHrecombnorm->Fill(mHiggs/1000);
hHsub2->Fill(mHiggs/1000,-1);
hZHcor->Fill(it12,mHiggs/1000);
hZHcor->Fill(it34,mHiggs/1000);
countrecomb++;
}

// Combining the sub-leading muon pair of event A with the sub-leading lepton pair of event B
bool dRcrit=deltaR(eta12->at(ma3),eta12->at(ma4),eta34->at(mb3),eta34->at(mb4),phi12->at(ma3),phi12->at(ma4),phi34->at(mb3),phi34->at(mb4));
bool posscombcrit=allpossiblemassmumu(E12->at(ma3), px12->at(ma3), py12->at(ma3), E12->at(ma4), px12->at(ma4), py12->at(ma4), q12->at(ma4), E34->at(mb3), px34->at(mb3), py34->at(mb3), q34->at(mb3), E34->at(mb4), px34->at(mb4), py34->at(mb4), q34->at(mb4));
if(dRcrit && posscombcrit){
double mHiggs=massH(E12->at(ma3), px12->at(ma3), py12->at(ma3), E12->at(ma4), px12->at(ma4), py12->at(ma4), E34->at(mb3), px34->at(mb3), py34->at(mb3), E34->at(mb4), px34->at(mb4), py34->at(mb4));
hHrecomb->Fill(mHiggs/1000);
hHrecombnorm->Fill(mHiggs/1000);
hHsub2->Fill(mHiggs/1000,-1);
hZHcor->Fill(it12,mHiggs/1000);
hZHcor->Fill(it34+counter,mHiggs/1000);
countrecomb++;
}
}

// Ending the loop over all Zb
cout << "-:-";

// Ending the loop over all Za
cout << "\n\n" << countrecomb << " recombined Higgs candidates have been processed.\nThe histogram will be drawn.\n";
// BELOW HISTOGRAMS ARE DRAWN

hHrecombnorm->Scale(counter); hHrenorm->Scale(countrecomb);

int drawer=0;
// 1 -> hH The invariant mass of all four-muon systems
// 2 -> hZ1 The invariant mass of the leading muon pair
// 3 -> hZ2 The invariant mass of the sub-leading muon pair
// 4 -> hHrecomb The invariant mass of the four muon system, when Z-bosons are recombined from different events
// 5 -> hZ1 & hZ2 The histogram combining hZ1 and hZ2
// 6 -> hHrenorm & hHrecomb The histogram combining renormalized versions of hH and hHrecomb, so they have the same integral
// 7 -> hHsubrec The renormalized hH minus the renormalized hHrecomb, the latter acting like a background estimation
// 8 -> hZcorr The 2D-histogram of the correlation between the Z-boson masses
// 9 -> hZHcorr The 2D-histogram of the correlation of which Z bosons gave which mH
// 0 -> hH & hHrecomb The histogram combining the renormalized background estimation with the original result

if(drawer == 1){ hH->Draw(); }
if(drawer == 2){ hZ1->Draw(); }
if(drawer == 3){ hZ2->Draw(); }
if(drawer == 4){ hHrecomb->Draw(); }
if(drawer == 5){ hZ1->SetLineColor(4); hZ2->SetLineColor(2); hZ1->Draw(); hZ2->Draw("same"); leg_hist = new TLegend(0.2,0.6,0.48,0.75); leg_hist->SetHeader("Leading and Sub-Leading Z-boson"); leg_hist->AddEntry(hZ1,"Leading Z-boson"); leg_hist->AddEntry(hZ2,"Sub-Leading Z-boson"); leg_hist->Draw(); }
if(drawer == 6){ hHrecombnorm->SetLineColor(2); hHrenorm->SetLineColor(8); hHrenorm->Draw(); hHrecombnorm->Draw("same"); leg_hist = new TLegend(0.4,0.7,0.75,0.88); leg_hist->SetHeader("Results, Renormalized and Recombined"); leg_hist->AddEntry(hHrenorm,"The Results, Z-bosons from Same Event, Renormalized"); leg_hist->AddEntry(hHrecombnorm,"The Results, Z-bosons from Different Events, Renormalized"); leg_hist->Draw(); }
if(drawer == 7){ hHsub2->Scale(counter); hHsub1->Scale(countrecomb); hHsubrec->Add(hHsub1,hHsub2); hHsubrec->Draw(); }
if(drawer == 8){ hZcorr->Draw("box"); }
if(drawer == 9){ hZHcorr->Draw("colz 0"); }
if(drawer == 0){ hHrecomb->SetLineColor(2); hH->SetLineColor(8); hH->Draw(); hHrecomb->DrawNormalized("same",counter); leg_hist = new TLegend(0.4,0.7,0.75,0.88); leg_hist->SetHeader("Results, Renormalized and Recombined"); leg_hist->AddEntry(hH,"The Results, Z-bosons from Same Event"); leg_hist->AddEntry(hHrecomb,"The Results, Z-bosons from Different Events, Renormalized"); leg_hist->Draw(); }

cout << "The histogram has been drawn.\n"; 

} // The macro is finished