Lund University

Master Thesis in Economics

In the long run, we're all mean reverting

Wavelet analysis on the fate of real exchange rates

Author: David Berger, 870803-1532

Supervisor: Fredrik NG Andersson

Place, Date: Lund, June 2013

Abstract

This paper analyzes to what extent PPP holds in the long run. Firstly, standard unit root tests are used to test for stationarity. These results are then compared to the ones provided by a wavelet based OLS and an approximate ML estimator. Using these to determine the integration order of an ARFIMA(p,d,q) process, the results support the PPP hypothesis and indicate that real exchange rates are mean reverting and subject to long swings. Unit root tests are therefore inept for analyzing real exchange rates.

Keywords: Purchasing Power Parity, Long Run Memory, ARFIMA(p,d,q), Unit root tests, Wavelets

Contents

“We have normality, I repeat we have normality.” She turned her microphone off - then turned it back on, with a slight smile and continued: “Anything you still can’t cope with is therefore your own problem.” ~ The Hitchhiker’s Guide to the Galaxy

1 Introduction

In general, one distinguishes between absolute and relative purchasing power parity (PPP). They are both strongly linked to the law of one price and imply that two identical goods, bought in different markets, will have an identical price when expressed in the same currency. Otherwise it would be possible to buy the same good in one country at a lower price, and then sell it in another one at a higher price (arbitrage). The concept of PPP has been put forward in the early twentieth century by the Swedish economist Gustav Cassel who proposed that “[a]s long as as anything like free movement of merchandise and a somewhat comprehensive trade between the two countries takes place; the actual rate of exchange cannot deviate very much from this purchasing power parity” (Cassel, 1918, pg. 413). More formally, “this can be expressed as

$$RE = \frac{SP}{P^*}, \tag{1.1}$$

where RE denotes the real exchange rate, P and P^* domestic and foreign price levels respectively and S the nominal exchange rate” (Berger, 2012, pg. 1) expressed in British notation.

Absolute PPP implies that RE is equal to unity and is assumed to hold continuoulsy in monetary models of exchange rate determination, such as the flexible price model. However, it is unrealistic to make this assumption because transaction costs, taxes, uncertainty, price discrimination, etc. are not identical across markets. It is therefore more reasonable to assume that RE is equal to a constant rather than unity, known as relative PPP. Hence, “if PPP were to hold, the real exchange rate would be stationary over time” (Berger, 2012, pg. 1).

Whether or not this is the case has been subject to a very lively academic debate. In general, prolonged departures from PPP have been observed empirically and monetary models such as Dornbusch's overshooting model, the Frankel real interest rate differential model, and the portfolio balance model allow for these deviations. There is a broad consensus that PPP does not hold in the short run (Artus, 1978; Dornbusch, 1980; Frenkel, 1981; Taylor, 2002) and therefore follows a random walk process. However, the previously mentioned monetary models heavily rely on PPP to hold and be stationary in the long run. Otherwise "almost everything we say about monetary policy is wrong" (Alvarez, Atkeson & Kehoe, 2007, pg. 339). From a policy perspective, this is evidently very important because the government and central bank would then not be able to actively steer, intervene, and influence monetary policy.

In order to test for stationarity, one "traditionally" employs unit root tests, such as the (Augmented) Dickey Fuller ((A)DF), Phillips-Perron (PP), and Kwiatkowski, Phillips, Schmidt and Shin (KPSS). Using these, most empirical studies do not reject that RE contains a unit root (Adler & Lehmann, 1983; Meese & Singleton, 1983; Corbae & Oularies, 1986; Barnhart & Szakmary, 1991; Fujihara & Mougoue, 1994; Taylor, 1995; Lopez, Murray & Papell, 2004; Belaire-Franch & Opong, 2005). However, traditional unit root tests have low power with respect to roots close to unity (Abuaf & Jorion, 1990; Diebold & Rudebusch, 1991; Hassler & Walters, 1994; Andersson, 2012) which may provide evidence against unit root tests rather than PPP (Abuaf & Jorion, 1990; Engel & Hamilton, 1990).

Engel and Hamilton (1990) show that exchange rates display long swings and the same conclusion can be drawn when looking at Figure 1 below.

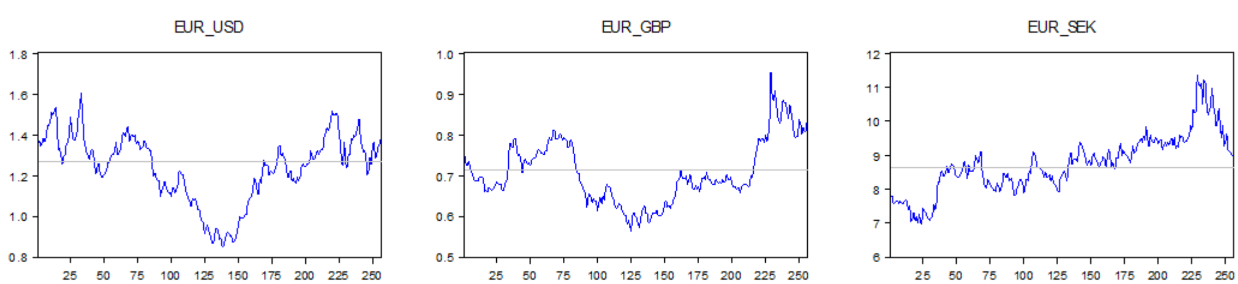

Figure 1: Real Exchange Rate: EUR/USD, EUR/GBP, EUR/SEK, January 1990 - April 2011

Figure 1 shows the monthly real EUR/USD, EUR/GBP, and EUR/SEK exchange rates[1], which all appear to be subject to long swings but mean reverting. Hence, RE may be fractionally integrated. A fractionally integrated process (FIP) is considered to be mean reverting and subject to long swings as long as the order of integration, $1/2 \leq d < 1$ (Granger & Joyeux, 1980).

It is the aim of this paper to contribute to the debate to what extent PPP holds in the long run and to estimate the order of integration using wavelet analysis. Furthermore, this paper will extent the results and analysis provided by Berger (2012). Additional autoregressive (AR) and moving average (MA) terms will be added to the initial FIP in order to model RE as an ARFIMA(p,d,q) process. A FIP therefore is a special case of an ARFIMA(p,d,q) model, which is also considered to be mean reverting as long as $1/2 \leq d < 1$. It should be noted that most studies, for computational reasons, only include AR(1) and/or MA(1) components (Baillie, Chung & Tieslau, 1996; Jensen, 2000; Sena, Reisen & Lopes, 2006) and this paper will therefore refrain from using higher order AR/MA terms. The parameter of interest, d, will be estimted using a wavelet based OLS estimator put forward by Jensen (1999) and Percival and Walden's (2000) approximate maximum likelihood estimator (AMLE). Despite a slow mean-reversion process (swings of approximately ten years can be observed in the case of some currency pairs, e.g. USD/SEK, EUR/GBP, USD/NOK), the results show strong support for the PPP hypothesis in the long run, irrespective of data frequency and base currency. This indicates that standard monetary models can be applied and while it may take a long time for monetary policy to have an effect, the important aspect is that it has an effect at all.

The subsequent parts of this paper are organized as follows: section two outlines the theoretical foundations behind ARFIMA(p,d,q) processes, including unit root and wavelet theory, while section three presents and discusses the data and empirical results. Finally, a conclusion is drawn in the last part of the paper.

---

[1]The gray line indicates the average real exchange rate.

2 Integrated processes

This section draws on Berger (2012), where the "data generating process (DGP) of an integrated process is given by

$$(1-L)^d x_t = \varepsilon_t, \tag{2.1}$$

where ε_t is white noise ($var(\varepsilon) = \sigma_\varepsilon^2$), d the fractional integration order" (Berger, 2012, pg. 3), and L the lag operator such that $Lx_t = x_{t-1}$. Substituting $d = 1$ into equation (2.1) yields a random walk

$$x_t = x_{t-1} + \varepsilon_t, \tag{2.2}$$

which is considered to be non-stationary, contain a unit root, and equation (2.2) only becomes stationary when taking the first difference. If a process needs to be differenced d times in order to become stationary, it is said to be $I(d)$. Theoretically, one can find processes that need to differenced more than once in order to become stationary but this is rarely the case within economics (Gujarati, 2004).

2.1 Unit Roots

In order to formally test if equation (2.1) contains a unit root, various tests can be employed. The most common tests are the ADF, PP, and KPSS. Other tests such as the ones provided by Elliot, Rothenberg, and Stock Point Optimal (ERS) and Ng and Perron (NG) are included in software packages like EViews but since the first three tests are the most prominent ones, they will be discussed in more detail below.

The ADF builds upon the "standard" Dickey-Fuller (DF) test. A general AR(1) process

$$x_t = \rho x_{t-1} + y_t'\delta + \varepsilon_t$$

is stationary if $|\rho| < 1$ and modified by the DF in such a way that x_{t-1} is subtracted from both sides

$$\Delta x_t = \alpha x_{t-1} + y_t'\delta + \varepsilon_t. \tag{2.3}$$

Given that $\alpha = \rho - 1$, the following hypotheses are tested:

$$\begin{cases} H_0: & \alpha = 0, \ (non - stationary) \\ H_1: & \alpha < 0. \ (stationary) \end{cases}$$

However, equation (2.3) only works for an AR(1) process. In order for ε_t to be white noise and to consider higher order lags, AR(p), the ADF adds p lagged difference terms

$$\Delta x_t = \alpha x_{t-1} + y_t'\delta + \beta_1 \Delta x_{t-1} + ... + \beta_p \Delta x_{t-p} + \nu_t.$$

Practically, the first issue with the ADF is how to correctly specify the lag length. However, through the use of information criteria, most software packages will automatically select the correct one. More significantly, the ADF has low power with respect to determining roots close to the non-stationary boundary (Abuaf & Jorion, 1990; Diebold & Rudebusch, 1991; Hassler & Walters, 1994; Andersson, 2012). Hence, it may indicate a unit root while the original DGP is actually mean reverting. Similar restrictions apply to the PP test, which is asymptotically equivalent to the ADF (University of Washington, 2012) and which modifies the t-ratio of the α coefficient in order to control for serial correlation.

The KPSS is a Lagrange Multiplier test and differs from the above tests as it utilizes reverse hypotheses. The idea is that a series can be decomposed into the sum of a random walk, a stationary error, and a determinsitic trend

$$x_t = \alpha t + r_t + \varepsilon_t,$$

where

$$r_t = r_{t-1} + u_t.$$

According to Kwiatkowski, Phillips, Schmidt and Shin (1992), this implies the following hypotheses:

$$\begin{cases} H_0: & \sigma_u^2 = 0, \ (stationary) \\ H_1: & \sigma_u^2 > 0. \ (non-stationary) \end{cases}$$

The KPSS can therefore be seen as complementary to the ADF/PP tests (Hobijn, Franses & Ooms, 2004).

When looking at equation (2.3), the question becomes whether or not to include a constant and/or trend in the above tests. According to Enders (2010), one should start with both and thereafter it is sensible to only include a constant when testing the first difference. Lastly, a general limitation of all the above tests is that they only consider the special case of $d \in \mathbb{Z}$, whereas the general case $d \in \mathbb{R}$ is not addressed.

2.2 ARFIMA(p,d,q)

As mentioned in the previous section, one major shortcoming of the tests presented is that they assume $d \in \mathbb{Z}$. An ARFIMA(p,d,q) allows for $d \in \mathbb{R}$ and is given by

$$\Phi(L)(1-L)^d(x(t)-\mu) = \Theta(L)\varepsilon(t), \tag{2.4}$$

where $\Phi(L)$ and $\Theta(L)$ represent the AR(p) and MA(q) terms

$$\Phi(L) = 1 + \phi_1 L + \phi_2 L^2 + \cdots + \phi_p L^p,$$

$$\Theta(L) = 1 + \theta_1 L + \theta_2 L^2 + \cdots + \theta_q L^q.$$

The spectral density function (SDF), which decomposes the DGP's variance with respect to frequency, of equation (2.4) is given by

$$S_x(f) = \frac{\sigma_\varepsilon^2}{[4sin^2(\pi f)]^d} \times \Omega, \tag{2.5}$$

where f represents the frequency and Ω the SDF of the ARMA(p,q) process[2]. However, the long run dynamics are not affected by the AR/MA components (Brockwell & Davis, 1991) and the ARFIMA(p,d,q) can therefore be reduced to an ARFIMA(0,d,0) in the long run. The Fourier transform is the most prominent tool in order to display a series in the frequency rather than the time domain,

$$F(f) = \sum_{t=-\infty}^{\infty} x_t e^{-i2\pi ft}.$$

Hence, the function is now displayed based on oscillating sine and cosine functions. Furthermore, it is only necessary to consider frequencies over a unit interval[3] and it is also possible to transform the signal back to the time domain[4]. However, "the time domain has been entirely dropped and wavelet analysis can therefore be used to provide time and frequency resolution" (Berger, 2012, pg. 8). This is a major advantage as "[b]y decomposing a time series into time-frequency space, one is able to determine both the dominant modes of variability and how those modes vary in time" (Torrence & Compo, 1998, pg. 61). Wavelet analysis has further advantages with respect to non-stationary and inhomogeneous systems (Abramovich, Bailey & Sapatinas, 2000).

2.2.1 Discrete Wavelet Transform (DWT)

A wavelet is a small wave that has finite energy and satisfies the admissibility condition $\int \psi(t)dt = 0$. Hence, ψ is an oscillating function that rapidly decreases as $t \to \pm\infty$. The first and most commonly used wavelet is the Haar wavelet, named after the Hungarian

[2] $\Omega = \frac{|\Theta(e^{-2\pi if})|^2}{|\Phi(e^{-2\pi if})|^2}$.

[3] For proof, see Percival and Walden (2000).

[4] For proof, see Andersson (2008).

mathematician Alfréd Haar. For different scales, it is defined as

$$\psi^H_{\lambda,u}(t) = \begin{cases} -\frac{1}{\sqrt{2\lambda}} & u-\lambda < t \leq u; \\ \frac{1}{\sqrt{2\lambda}} & u < t \leq u+\lambda; \\ 0 & otherwise. \end{cases} \tag{2.6}$$

Figure 2 depicts the Haar wavelet for different scales, λ, and translations, u.

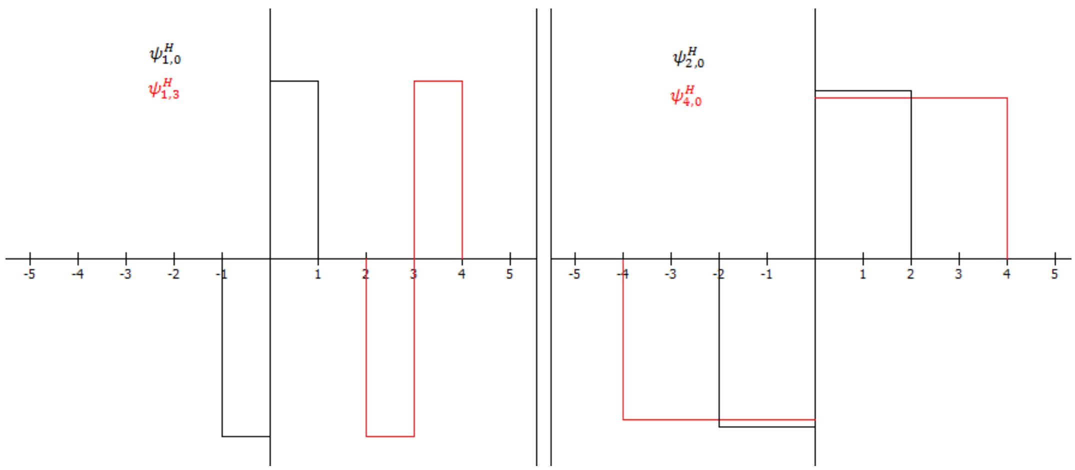

Figure 2: Haar wavelet, different scales and translations

From the above, one can coherently see that u shifts the wavelet along the x-axis while λ shows how averages of y_t "over many different scales are changing from one period of length λ to the next" (Percival & Walden, 2000, pg. 10). It is now possible to use a frequency interpretation as the first scale captures frequencies $\frac{1}{4}$ to $\frac{1}{2}$, the second frequencies from $\frac{1}{8}$ to $\frac{1}{4}$ and so forth.

"The DWT uses dyadic scales, $\lambda = 2^{j-1}$ and $j = 1, 2, ..., J$, and therefore only works for observations that can be expressed as a power of two[5]" (Berger, 2012, pg. 10). For the Haar wavelet, the DWT can be obtained through the pyramid algorithm. This algorithm was proposed by Mallat (1989) and is the most efficient way of calculating the DWT (Andersson,

[5]This is not strictly true. For a more detailed discussion on what is called the maximum overlap discrete wavelet transform (MODWT), see Percival and Walden (2000).

2008). Several pyramid-like steps are utilized and the first stage "simply consists of transforming the time series $\mathbf{X}$ of length $[T = 2^J]$ into the $[T/2]$ first level wavelet coefficients $\mathbf{W_1}$ and the $[T/2]$ first level scaling coefficients $\mathbf{V_1}$" (Percival and Walden, 2000, pg. 80). This process is iterated until $\mathbf{W_j}$ only contains one entry. By substituting $\lambda = 1$ into equation (2.6), the respective wavelet and scaling coefficients are obtained. Evidently, these differ depending on the choice of wavelet[6]. The complete wavelet transform is given by

$$w = \mathbf{\Phi} x, \tag{2.7}$$

where

$$\mathbf{\Phi} = \begin{bmatrix} \mathbf{\Phi_1} \\ \mathbf{\Phi_2} \\ \vdots \\ \mathbf{\Phi_J} \\ \mathbf{\Gamma_J} \end{bmatrix} = \begin{bmatrix} \mathbf{B_1} \\ \mathbf{B_2 A_1} \\ \vdots \\ \mathbf{B_J A_{J-1}} \times \ldots \mathbf{A_2 A_1} \\ \mathbf{A_J A_{J-1}} \times \ldots \mathbf{A_2 A_1} \end{bmatrix}.$$

$\mathbf{B_J}$ and $\mathbf{A_J}$ contain the respective wavelet and scaling coefficients

$$\mathbf{B} = \begin{bmatrix} h_1 & h_0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_1 & h_0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h_1 & h_0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & & & \vdots & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & h_1 & h_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & h_1 & h_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & h_1 & h_0 \end{bmatrix},$$

[6] See Berger (2012) for a more thorough discussion on why the Haar wavelet is utilized.

$$\mathbf{A} = \begin{bmatrix} g_1 & g_0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & g_1 & g_0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & g_1 & g_0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & & & \vdots & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & g_1 & g_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & g_1 & g_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & g_1 & g_0 \end{bmatrix},$$

and $\mathbf{\Gamma_J}$ is the zero frequency component which captures all (linear) determinstic components.

2.2.2 Wavelet-based OLS estimation

Given that the long run dynamics are not affected by Ω, the SDF in equation (2.5) can be estimated, according to Jensen (1999), using a log-linear relationship between the wavelet variance and its corresponding scale. More specifically,

$$ln\,R(j) = ln\,\sigma^2 - d\,ln 2^{2j}, \tag{2.8}$$

where

$$\bar{R}(j) = \frac{1}{2^j} \sum_{k=0}^{2^j-1} w_{j,k}^2. \tag{2.9}$$

The long-run order of integration, d, will thus determine the slope.

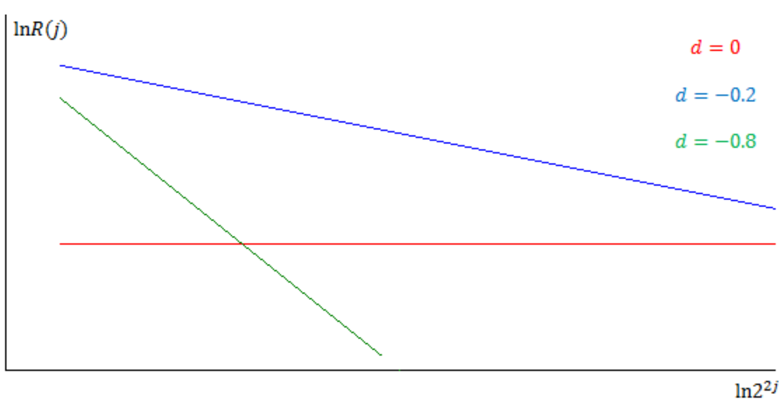

Figure 3: Wavelet OLS, different slopes of d

Figure 3 shows that a higher order of integration corresponds with a steeper slope and vice versa. It can be seen from equations (2.8) and (2.9) that $\mathbf{\Gamma_J}$ is not included in the estimation and one does therefore not need to be concerned about stochastic or deterministic trends.

2.2.3 Wavelet-based AMLE estimation

The AMLE is obtained using high-frequency, non-boundary coefficients ($\mathbf{W}'_{\mathbf{nb}} : \ j < J$), from the wavelet transfom given by equation (2.7) and has been proposed by Percival and Walden (2000). Excluding the boundary coefficients evidently leads to a loss of information but 89% of the transform coefficients are non-boundary coefficients for $T = 256$ (Andersson, 2012). The AMLE likelohood function is given by

$$L(d, \sigma_\varepsilon^2 \mid \mathbf{W}'_{\mathbf{nb}}) = \frac{exp(-[\mathbf{W}'_{\mathbf{nb}}]^T \sum_{\mathbf{W}'_{\mathbf{nb}}}^{-1} \mathbf{W}'_{\mathbf{nb}}/2)}{(2\pi)^{T/2} \mid \sum_{\mathbf{W}'_{\mathbf{nb}}} \mid^{1/2}},$$

where the order of integration affects the covariance matrix $\sum_{\mathbf{W_{nb}}}$, which is approximately a diagonal matrix with diagonal elements

$$C_j = 2^{j+1} \int_{1/2^{j+1}}^{1/2^j} \frac{\sigma_\varepsilon^2}{[4sin^2(\pi f)]^d} df. \tag{2.10}$$

Assuming that the shocks in equation (2.1) are normally distributed, the estimated order of integration provided by the AMLE is also normally distributed $\forall d$. Furthermore, equation (2.10) is derived assuming that the errors have a flat spectrum throughout the entire frequency band and are therefore not correlated (Brockwell & Davis, 1991). If this were the case, the AMLE would be affected by short run dynamics (Ω) and AR/MA components would need to be taken into account. In order to check if these can be dropped, it is sensible to split the data into a high and low frequency component (Andersson, 2012) and see if these provide different estimates of the integration order.

3 Empirics

Bearing in mind the theoretical part, section 3.1 will outline the used dataset before the long run memory parameter is empirically determined in part 3.2.

3.1 Data

As in Berger (2012), the data used in this paper has been obtained using Thompson Financial Datastream 5.0, *except* from the Swedish/American CPI and the USD/SEK nominal exchange rate. The latter has been accessed through `www.oanda.com` (2013) and the former through Statistics Sweden (2013)/U.S. Bureau of Labor Statistics (2013).

Three seperate groups have been formed, each using a different base currency (Group 1: USD, Group 2: GBP, Group 3: EUR). Each base currency is then plotted against a basket of currencies (SEK, JPY, NOK) and the remaining two base currencies. In accordance with equation (1.1), the real exchange rate is calculated by multiplying the nominal exchange rate with the ratio of domestic, P, and foreign, P^*, price levels respectively, where the consumer price index (CPI) is used as a price level proxy. Given that each country uses a different base year, the CPI values have been transformed using January 2005 (=100) as a base. RE is then calucated for 256 monthly observations from January 1990 to April 2011. For the first group, this looks as follows.

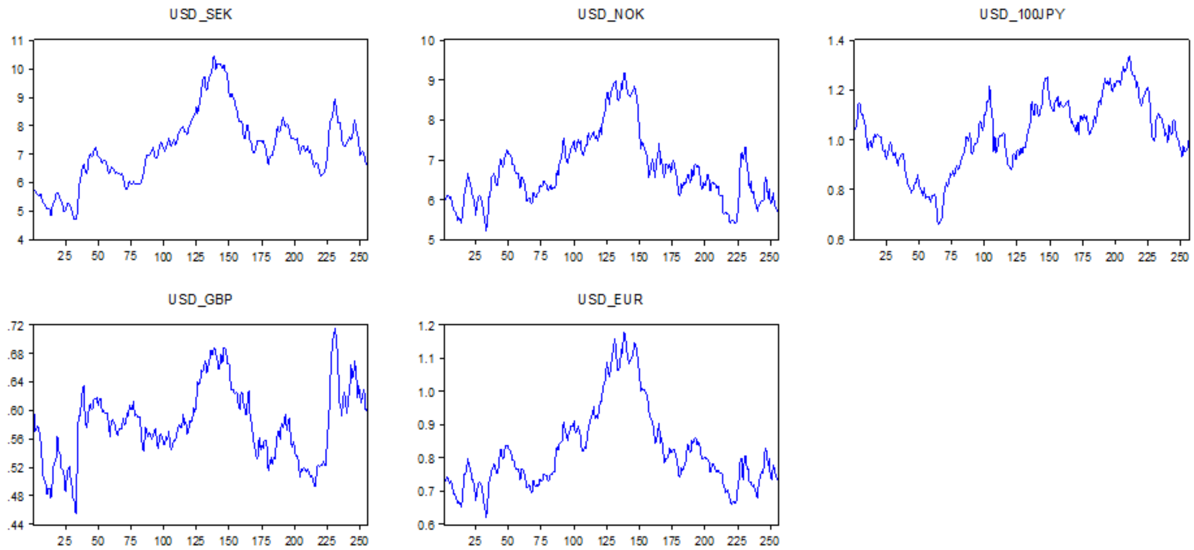

Figure 4: USD vs Basket of currencies, January 1990 - April 2011

At observation 125 (approximately), corresponding to May 2000, Figure 4 shows a strong appreciation of the USD against most other currencies. This coincides with the burst of the dot-com bubble, causing the NASDAQ Composite to lose 78% of its value (Investopedia, 2013). Historically, the USD has been regarded as a safe haven (Engel & Hamilton, 1990), causing it to appreciate in uncertain times. Hence, the USD also appreciates against the SEK/NOK/GBP at observation 225 (September 2008), corresponding to the current financial crisis. It is noteworthy that the latter appreciation is not seen against the EUR and JPY. One possible explanation for this is the fact that both currencies, especially the EUR, are widely used as reserve currencies (International Monetary Fund, 2013) and may therefore also be regarded as a safe haven, albeit to a lesser extent.

The second group uses the GBP as a base currency, as indicated in Figure 5 below.

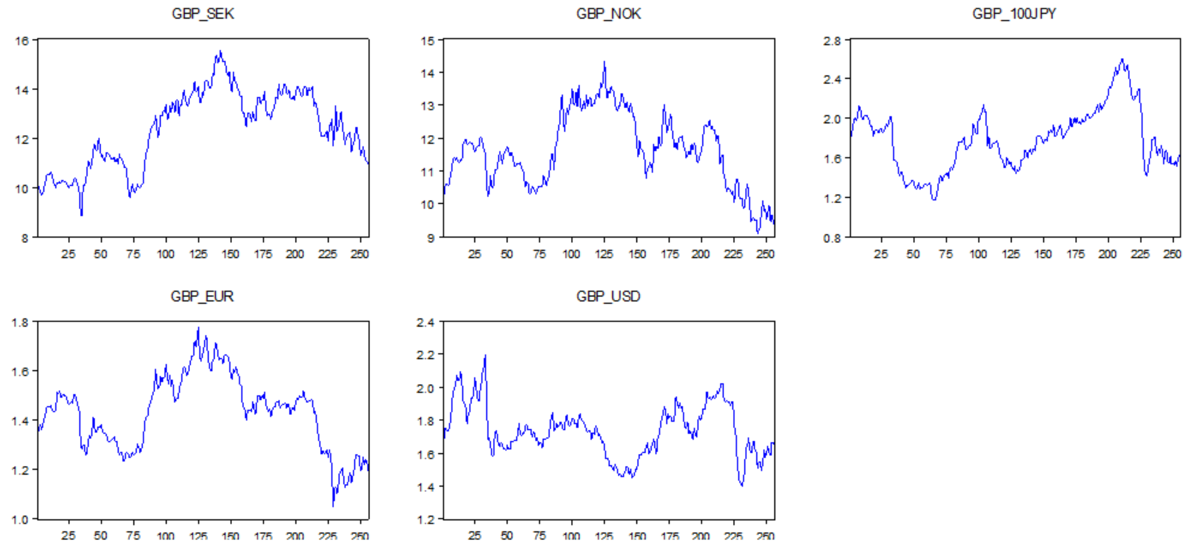

Figure 5: GBP vs Basket of currencies, January 1990 - April 2011

A sharp drop of the GBP against all other currencies is seen around observation 225 (September 2008). Given that the financial sector accounts for a significant share of Britain's GDP (Bank of England, 2011), it is evidently greatly affected by the current financial crisis. A study by the Department for Business Innovation & Skills (2010, pg. 9) concluded that "evidence suggests that the UK lags behind its main competitors such as the United States and Germany", promting a rather controversial article in The Guardian (2011) which called this fact the greatest de-industrialisation of any major nation. Despite its continuing depreciation, the GBP is still considered to be overvalued and thereby undermining Britain's competitiveness (The Economist, 2013).

The EUR is the last base currency and *RE* is plotted agains the remaining currencies in Figure 6 below.

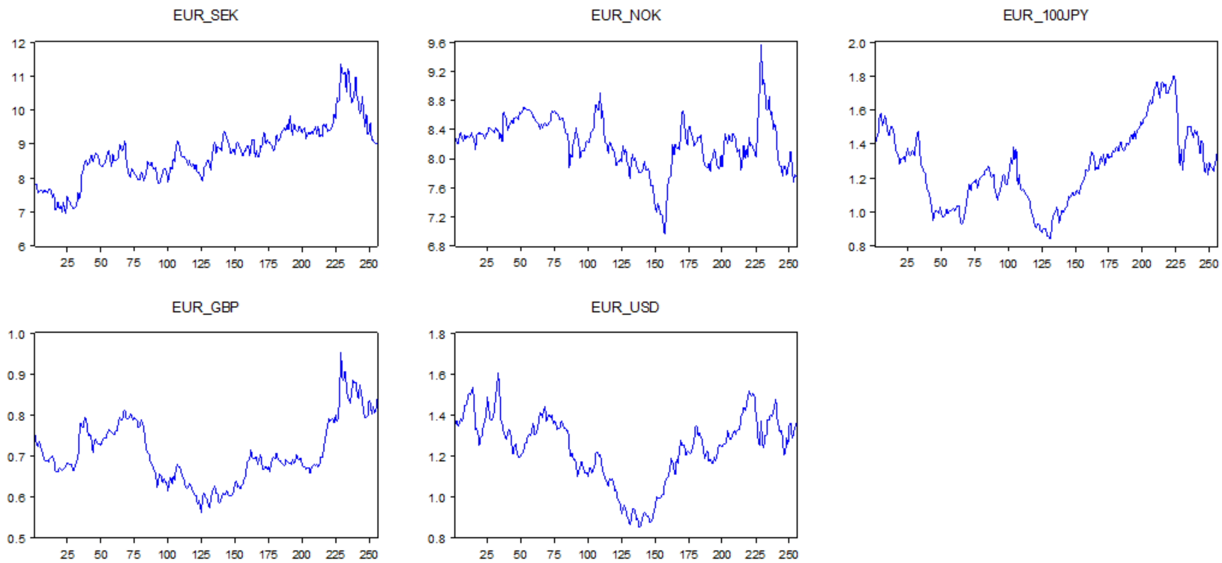

Figure 6: EUR vs Basket of currencies, January 1990 - April 2011

It can be seen that the EUR remained stable against all other currencies prior to its introduction in 1999, corresponding to observation 110. This can partially be explained by the fact that the European curriencies plotted above were part of the ECU basket of currencies[7] and thus part of the ECU exchange rate. After the EUR was introduced it depreciated against most other currencies, indicating an initial loss of confidence among investors. Once this initial period passed, the EUR gained in value and only started to depreciate (with the exception of the GBP) as a result of the current financial crisis.

3.2 Results

Each of the three groups has two different set of results. Firstly, the unit root tests are reported and, as has been outlined in section 2.1, one needs to decide whether or not to include a constant and/or trend when testing for unit roots. The (1) indicates a constant and trend while (2) represents testing the first difference including a constant. Lastly, the long run order of integration is determined using a wavelet based OLS and AMLE estimator.

[7] On 1 January 1999, the EUR replaced the ECU at a value of 1EUR = 1ECU.

Hereby, the high frequencies have been excluded (3-6) in order to check if the model is free from short run dynamics and may therefore be represented by an ARFIMA(0,d,0).

	ADF (1)	ADF (2)	PP (1)	PP (2)	KPSS (1)	KPSS (2)	I(d)
USD/SEK	-1.81	-10.60***	-1.64	-10.65***	0.32***	0.14	1
USD/NOK	-1.76	-15.03***	-1.83	-15.06***	0.37***	0.12	1
USD/JPY	-2.22	-15.69***	-2.41	-15.69***	0.17**	0.08	1
USD/GBP	-2.64	-14.00***	-3.04	-14.02***	0.14*	0.04	1
USD/EUR	-1.46	-14.30***	-1.57	-14.22***	0.36***	0.15	1

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

Table 1: Base Currency USD: Results, Unit Root Tests

Table 1 clearly indicates that RE is a non-stationary process integrated of order one. This conclusion holds for all currency pairs of the first group and one could therefore rightly say that PPP does not hold in the long run. However, it is noteworthy that the KPSS test seems to be a bit more sensitive with respect to accurately determining roots close to the non-stationary boundary, as indicated by the slightly lower significance levels of USD/JPY and USD/GBP.

	d (OLS)	d (AMLE): 1-6	d (AMLE): 3-6
USD/SEK	0.95	0.93	0.97
USD/NOK	0.91	0.94	0.92
USD/JPY	0.93	0.90	0.89
USD/GBP	0.68	0.65	0.70
USD/EUR	0.84	0.89	0.86

Table 2: Base Currency USD: Results, Wavelet Estimation

A different picture emerges when looking at Table 2. RE does not appear to be affected by short run dynamics and can be considered mean reverting and subject to long swings ($1/2 \leq d < 1$), irrespective of data frequency and estimator. Hence, PPP holds in the long run but it takes a long time for the series to come back to its mean. The order of integration is generally lower for "large" currencies (with the exception of the USD/JPY rate) than for smaller ones. This indicates that smaller currencies tend to shadow larger ones.

A similar conclusion can be drawn when the GBP is used as a base currency.

	ADF (1)	ADF (2)	PP (1)	PP (2)	KPSS (1)	KPSS (2)	I(d)
GBP/SEK	-1.45	-13.76***	-1.18	-17.32***	0.39***	0.23	1
GBP/NOK	-1.96	-16.59***	-1.81	-16.69***	0.35***	0.28	1
GBP/JPY	-1.70	-14.32***	-2.04	-14.39***	0.14*	0.07	1
GBP/USD	-2.74	-14.09***	-3.11	-14.07***	0.15**	0.04	1
GBP/EUR	-1.84	-15.02***	-1.25	-16.54***	0.33***	0.19	1

*** p<0.01, ** p<0.05, * p<0.1

Table 3: Base Currency GBP: Results, Unit Root Tests

Initially, one would conclude that all series are non-stationary and therefore contain a unit root. Again, the KPSS appears to more accurately determine roots close to the non-stationary boundary (GBP/JPY and GBP/USD), as opposed to the ADF and PP.

	d (OLS)	d (AMLE): 1-6	d (AMLE): 3-6
GBP/SEK	0.92	0.88	0.91
GBP/NOK	0.92	0.90	0.95
GBP/JPY	0.78	0.79	0.81
GBP/USD	0.63	0.72	0.70
GBP/EUR	0.95	0.94	0.97

Table 4: Base Currency GBP: Results, Wavelet Estimation

Regardless of data frequency and estimator, all series can be represented by an ARFIMA(0,d,0) and are actually mean reverting and subject to long swings, as indicated by Table 4. Hence, one can conclude that the PPP hypothesis holds in the long run. Again, small currencies (SEK, NOK) tend to shadow larger ones (USD, JPY) as they show a higher order of integration, with the exception of the EUR. This can possibly be explained by the fact that the GBP used to be part of the ECU, prior to the establishment of the EUR, and therefore tracks part of its own past realizations.

Lastly, the EUR displays very similar results.

	ADF (1)	ADF (2)	PP (1)	PP (2)	KPSS (1)	KPSS (2)	I(d)
EUR/SEK	-1.45	-13.62***	-1.56	-10.81***	0.23***	0.09	1
EUR/NOK	-1.87	-14.87***	-1.76	-13.56***	0.27***	0.19	1
EUR/JPY	-2.03	-15.21***	-2.16	-15.24***	0.28***	0.11	1
EUR/USD	-1.71	-14.31***	-1.82	-14.25***	0.39***	0.11	1
EUR/GBP	-1.65	-15.24***	-1.64	15.24***	0.34***	0.18	1

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

Table 5: Base Currency EUR: Results, Unit Root Tests

Standard unit root tests provide no support for the PPP hypothesis and RE is said to contain a unit root. As opposed to the two previous base currencies, the KPSS does not provide different results when the EUR is used. However, this can actually be seen as evidence against these tests rather than PPP, as indicated by Table 6.

	d (OLS)	d (AMLE): 1-6	d (AMLE): 3-6
EUR/SEK	0.88	0.92	0.90
EUR/NOK	0.86	0.81	0.84
EUR/JPY	0.90	0.87	0.92
EUR/USD	0.92	0.95	0.94
EUR/GBP	0.97	0.97	0.98

Table 6: Base Currency EUR: Results, Wavelet Estimation

Clearly, all series are stationary, mean reverting and the PPP hypothesis appears to hold in the long run. However, it is interesting to note that the difference in the order of integration between the currency pairs is much less pronounced if the EUR is used as a base currency, as opposed to the GBP and USD, indicating that the EUR is more integrated with e.g. the SEK and NOK.

4 Conclusion

"Da steh ich nun, ich armer Tor! Und bin so klug als wie zuvor; Heiße Magister, heiße Doktor gar, und ziehe schon an die zehen Jahr, herauf, herab und quer und krumm, meine Schüler an der Nase herum – und sehe, daß wir nichts wissen können!"[8] ~ *Faust*

Using wavelet-based long run memory estimators, this paper has shown support for the PPP hypothesis to hold in the long run. This has significant ramifications with respect to monetary models, which are therefore applicable. Hence, "it may take a very long time for monetary policy to have an effect but the important aspect is that it has an effect at all" (Berger, 2012, pg. 20). RE appears to be slowly mean reverting and subject to long swings. As such, it can be represented by an ARFIMA(p,d,q) process which, in the long run, can be reduced to an ARFIMA(0,d,0) model. Such a process cannot be represented by an AR(p) model and standard unit root tests are therefore not applicable once $1/2 \leq d < 1$. If they are applied regardless, they tend to falsely suggest a unit root. It should be noted that the KPSS appears to be a little more sensitive with respect to accurately determining roots close to unity and it is therefore very viable to use it as a complentary test in addition to the ADF/PP. However, the general conclusion remains unchanged and more advanced techniques are required. Real exchange rates tend to be mean reverting and through arbitrage it should then, theoretically, be possible to exploit this mean reverting behaviour in order to make profit. However, given that this process takes a very long time, this may practically not be applicable.

[8]
And here, poor fool! with all my lore
I stand, no wiser than before:
I'm Magister - yea, Doctor - hight,
And straight or cross-wise, wrong or right,
These ten years long, with many woes,
I've led my scholars by the nose,-
And see, that nothing can be known!

5 References

Abramovich, F., Bailey, T. C., Sapatinas, T. (2000). Wavelet Analysis and Its Statistical Applications. *Journal of the Royal Statistical Society. Series D (The Statistician), 49*(1), 1-29.

Abuaf, N., Jorion, P. (1990). Purchasing Power Parity in the Long Run. *The Journal of Finance, 45*(1), 157-174.

Adler, M., Lehmann, B. (1983). Deviations from Purchasing Power Parity in the Long Run. *The Journal of Finance, 38*(5), 1471-1487.

Alvarez, F., Atkeson, A., Kehoe, P. J. (2007). If Exchange Rates are Random Walks, Then Almost Everything We Say About Monetary Policy is Wrong. *American Economic Review, 97*(2), 339-345.

Andersson, F. N. G. (2012). *Exchange Rates Dynamics Revisited: A Panel Data Test of the Fractional Integration Order*. Under Review.

Andersson, F. N. G. (2008). *Wavelet Analysis of Economic Time Series*. Lund Economic Studies, 149.

Artus, J. R. (1978). Methods of Assessing the Long Run Equilibrium Value of an Exchange Rate. *Journal of International Economics, 8*, 277-299.

Baillie, R. T., Chung, C., Tieslau, M. A. (1996). Analysing Inflation by the Fractionally Integrated ARFIMA-GARCH Model, *Journal of Applied Econometrics, 11*, 23-40.

Bank of England. (2011). *Measuring financial sector output and its contribution to UK GDP*. Retrieved 2013-05-16, from: `http://www.bankofengland.co.uk/publications/Documents/quarterlybulletin/qb110304.pdf`

Barnhart, S. W., Szakmary, A. C. (1991). Testing the Unbiased Forward Rate Hypothesis: Evidence on Unit-Roots, Co-Integration, and Stochastic Effects. *Journal of Financial and Quantitative Analysis, 26*, 245-267.

Belaire-Franch, J., Opong, K. K. (2005). Some Evidence of Random Walk Behaviour of EUR Exchange Rates using Ranks and Signs. *Journal of Banking and Finance, 29*, 1631-1643.

Berger, D. (2012). Does PPP hold in the long run? An empirical approach using wavelets. *LUP Student Papers*.

Brockwell, P. J., Davis, R. A. (1991). *Time Series: Theory and Methods*. New York: Springer Series in Statistics.

Cassel, G. (1918). Abnormal Deviations in International Exchanges. *The Economic Journal, 28*(112), 413-415.

Corbae, D., Ouliaris, S. (1986). Robust Tests for Unit Roots in the Foreign Exchange Market. *Economic Letters, 22*, 375-380.

Department for Business Innovation & Skills. (2010). *Manufacturing in the UK: An economic analysis of the sector*. Retrieved 2013-05-16, from: `http://www.bis.gov.uk/assets/biscore/business-sectors/docs/m/10-1333-manufacturing-in-the-UK-an-economic-analysis-of-the-sector`

Diebold, F. X., Rudebusch, G. D. (1991). On the Power of Dickey-Fuller Tests against Fractional Alternatives. *Economic Letters, 35*, 155-160.

Dornbusch, R. (1980). Exchange Rate Economics: Where Do We Stand?. *Brookings Papers, 1*, 143-206.

Enders, W. (2010). *Applied Econometric Time Series*. New Jersey: John Wiley & Sons, Inc.

Engel, C., Hamilton, J. D. (1990). Long Swings in the Dollar: Are They in the Data and Do Markets Know It?. *The American Economic Review, 80*(4), 689-713.

Frenkel, J. A. (1981). Flexible Exchange Rates, Prices and the Role of News: Lessons from the 1970's. *Journal of Political Economy, 89*, 665-705.

Fujihara, R. A., Mougoue, M. (1994). Temporal Aggregation and Unit Roots in Nominal Foreign Exchange Rates. *Review of Quantitative Finance and Accounting, 4*, 291-303.

Granger, C. W. J., Josyeux, R. (1980). An Introduction To Long-Memory Time Series Models And Fractional Differencing. *Journal of Time Series Analysis, 1*(1), 15-29.

Gujarati, D. N. (2004). *Basic Econometrics*. New York: McGraw-Hill.

Hassler, U., Walters, J. (1994). On the Power of Unit Root Tests against Fractional Alternatives. *Economic Letters, 45*, 1-5.

Hobij, B., Franses, P. H., Ooms, M. (2004). Generalizations of the KPSS-test for stationarity. *Statistica Neerlandica, 58*(4), 483-502.

International Monetary Fund. (2013). *Currency Composition of Official Foreign Exchange Reserves (COFER).* Retrieved 2013-05-18, from:
`http://www.imf.org/external/np/sta/cofer/eng/cofer.pdf`

Investopedia. (2013). *Market Crashes: The Dotcom Crash.* Retrieved 2013-05-18, from:
`http://www.investopedia.com/features/crashes/crashes8.asp#axzz1vVFOuCQl`

Jensen, M. J. (2000). An alternative maximum likelihood estimator of long-memory processes using compactly supported wavelets. *Journal of Economic Dynamics & Control, 18*, 361-387.

Jensen, M. J. (1999). Using Wavelets to Obtain a Consistent Ordinary Least Squares Estimator of the Long-memory Parameter. *Journal of Forecasting, 18*(1), 17-32.

Kwiatkowski, D., Phillips, P. C. B., Schmidt, P., Shin, Y. (1992). Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root? *Journal of Econometrics, 54*, 159-178.

Lopez, C., Murray, C. J., Papell, D. H. (2004). State of the Art Unit Root Tests and Purchasing Power Parity. *Journal of Money, Credit, and Banking, 37*(2), 361-369.

Mallat, S. G. (1989). Multiresolution Approximations and Wavelet Orthonormal Bases of $L^2(R)$. *IEEE Transactions on Pattern and Analysis and Machine Intelligence, 11*, 674-693.

Messe, R. A., Singleton, K. J. (1983). Empirical exchange rate models of the seventies. Do they fit out of sample? *Journal of International Economics, 14*, 3-24.

Oanda. (2013). Average Exchange Rates. Retrieved 2013-05-15, from:
`http://www.oanda.com/lang/de/currency/average?amount=1&start_month=1&start_year=1990&end_month=4&end_year=2012&base=USD&avg_type=Month&Submit=1&exchange=SEK&interbank=0&format=HTML`

Percival, D. B., Walden, A. T. (2000). *Wavelet Methods for Time Series Analysis.* Cambridge: Cambridge University Press.

Sena, M. R., Reisen, V. A., Lopes, S. R. C. (2006). Correlated Erros in the Parameters Estimation of the ARFIMA Model: A Simulated Study. *Communications in Statistics - Simulation and Computation, 35*, 789-802.

Statistics Sweden. (2013). *Consumer Price Index (CPI).* Retrieved 2013-05-15, from: http://www.scb.se/Pages/TableAndChart____272152.aspx

Taylor, A. M. (2002). A Century Of Purchasing Power Parity. *The Review of Economics and Statistics, 84*(1), 139-150.

Taylor, M. P. (1995). The Economics of Exchange Rates. *Journal of Economic Literature, 33*(1), 13-47.

The Economist. (2013). *UK economy: Is the pound undervalued or overvalued?.* Retrieved 2013-05-30, from: http://www.cmegroup.com/education/files/eiu-2013-04-22.pdf

The Guardian. (2011). *Why doesn't Britain make things any more?.* Retrieved 2013-05-16, from: http://www.guardian.co.uk/business/2011/nov/16/why-britain-doesnt-make-things-manufacturing

Torrence, C., Compo, G. P. (1998). A Practical Guide to Wavelet Analysis. *Bulletin of the American Meteorological Society, 79*(1), 61-78.

University of Washington. (2012). *Unit Root Tests.* Retrieved 2012-05-19, from: http://faculty.washington.edu/ezivot/econ584/notes/unitroot.pdf

U.S. Bureau of Labor Statistics. (2013). *Consumer Price Index.* Retrieved 2013-05-15, from: ftp://ftp.bls.gov/pub/special.requests/cpi/cpiai.txt