Credit Value Adjustment

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Abstract

In this thesis the topic Counterparty Credit Risk in OTC derivative transactions is described and the pricing component arising from it, i.e., the Credit Value Adjustment (CVA), is discussed. The unilateral CVA and DVA are derived in the case where one party engaging in a transaction is assumed to be defaultable and bilateral CVA is derived in the case where both parties in a transaction are assumed to be defaultable. In this context hedging aspects are also examined and risk-neutral pricing of CVA is discussed.

The set-up of a numerical tool for CVA computations is then described and a simple tool for computing CVA for single interest rate swaps is developed. In connection the input data needed for the computations is discussed and a method for constructing proxy CDS spread curves, where there is a lack of quoted CDS spreads referenced to a particular counterparty in the market, is described.

As a second part the relation between CVA from a regulatory perspective, driven by the CVA capital charge introduced in the third Basel accord, CVA from an accounting perspective, driven by IFRS, and CVA from a market perspective, as a potentially tradeable asset, is discussed. In connection to this the standardised and the advanced approaches to computing the CVA capital charge are examined and the similarities and differences to the market CVA are clarified. The implications of implementing CVA in the pricing of OTC derivatives within a bank are finally discussed.

Keywords: Basel III, Bilateral CVA, Counterparty credit risk, Credit value adjustment, CVA, CVA capital charge, DVA, OTC derivatives.
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<td>BIS</td>
<td>Bank of International Settlements</td>
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<tr>
<td>BCBS</td>
<td>Basel Committee on Banking Supervision</td>
</tr>
<tr>
<td>BCVA</td>
<td>Bilateral CVA</td>
</tr>
<tr>
<td>CDF</td>
<td>Cumulative Distribution Function</td>
</tr>
<tr>
<td>CDS</td>
<td>Credit Default Swap</td>
</tr>
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<td>CCDS</td>
<td>Contingent Credit Default Swap</td>
</tr>
<tr>
<td>CSA</td>
<td>Credit Support Annex</td>
</tr>
<tr>
<td>CVA</td>
<td>Credit Value Adjustment</td>
</tr>
<tr>
<td>DVA</td>
<td>Debt Value Adjustment</td>
</tr>
<tr>
<td>EAD</td>
<td>Exposure-At-Default</td>
</tr>
<tr>
<td>EE</td>
<td>Expected Exposure</td>
</tr>
<tr>
<td>EMIR</td>
<td>European Markets and Infrastructure Regulation</td>
</tr>
<tr>
<td>EONIA</td>
<td>Euro Overnight Index Average</td>
</tr>
<tr>
<td>EURIBOR</td>
<td>Euro Interbank Offered Rate</td>
</tr>
<tr>
<td>FRA</td>
<td>Forward Rate Agreement</td>
</tr>
<tr>
<td>FV</td>
<td>Future Value</td>
</tr>
<tr>
<td>FVA</td>
<td>Funding Value Adjustment</td>
</tr>
<tr>
<td>IFRS</td>
<td>International Financial Reporting Standards</td>
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<tr>
<td>IRS</td>
<td>Interest Rate Swap</td>
</tr>
<tr>
<td>LGD</td>
<td>Loss-Given-Default</td>
</tr>
<tr>
<td>LIBOR</td>
<td>London Interbank Offered Rate</td>
</tr>
<tr>
<td>MtM</td>
<td>Mark-to-Market</td>
</tr>
<tr>
<td>OIS</td>
<td>Overnight Indexed Swap</td>
</tr>
<tr>
<td>OTC</td>
<td>Over-The-Counter</td>
</tr>
<tr>
<td>P&amp;L</td>
<td>Profit-and-Loss</td>
</tr>
<tr>
<td>PFE</td>
<td>Potential Future Exposure</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability Density Function</td>
</tr>
<tr>
<td>PV</td>
<td>Present Value</td>
</tr>
<tr>
<td>R</td>
<td>Recovery rate</td>
</tr>
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<td>SMEs</td>
<td>Small and Medium-sized Enterprises</td>
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<tr>
<td>UCVA</td>
<td>Unilateral CVA</td>
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<tr>
<td>UDVA</td>
<td>Unilateral DVA</td>
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<td>VaR</td>
<td>Value-at-Risk</td>
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1 Introduction

1.1 Background

The recent financial crisis, with its start in the U.S. housing market in 2007, revealed new behaviour of the financial system under stress and assumptions that were valid pre-crisis turned out not to be valid anymore. Risks, that in the past had been deemed insignificant and overlooked, took most market players by surprise and enormous losses followed as a consequence. The crisis quickly spread from the financial markets to the real economy and it turned out to be one of the most pervasive crises in recent times with a global recession and the European sovereign debt crisis following.

In the past banks and other participants on the financial markets have engaged in repo and Over-The-Counter (OTC) derivative transactions with each other under the assumption that large financial institutions are as good as free of default risk. The possibility that a large financial institution would go bankrupt was assumed to be negligible and a too big to fail mentality prevailed (Dash, 2009). Additionally, liquidity was cheap and banks could conveniently satisfy their funding needs in the interbank market. The LIBOR rate was generally thought of as a proxy for the risk-free rate and used by banks for discounting future cash flows and deriving forward rates.

In the beginning of the crisis credit quality started to deteriorate, liquidity tightened and it became uncertain if all market players would survive. The interbank market for credit dried up and quoted rates on the LIBOR market started to incorporate large risk premia, making it costly for banks to obtain funding. Furthermore, banks saw their trading books with OTC derivative positions lose substantially in Mark-to-Market (MtM) value and their credit quality decline. Suddenly the risk of default became real and the solvency or liquidity of even prestigious institutions with high credit ratings was questioned. As a result many financial institutions suffered large losses in MtM values and in a few cases it lead to defaults with the most spectacular example being that of the investment bank Lehman Brothers. Many other institutions could only survive due to government support or from being bought up by other institutions.

The crisis has had persistent consequences. Lessons learned by market participants include the concept of bank failure, that liquidity is costly and that funding can be expensive with the possibility of the market for it even drying out (Bianchetti and Carlicchi, 2012). Because of these market changes it has become necessary to take liquidity risk, funding risk and counterparty credit risk into consideration when dealing with OTC derivative transactions. As a result the traditional set-up with a single risk-free yield curve based on LIBOR used for both discounting and computing forward rates has been abandoned for a multi-curve framework consisting of the risk-free curve, which is based on the OIS rate and used for discounting, and a set of forward curves based on LIBOR rates with different tenors to reflect the different liquidity and credit risk associated with the tenor. Another result is the introduction of value adjustments, which are used to adjust the price of an OTC derivative for the counterparty credit risk.

Counterparty credit risk has come into the focus of both regulators and financial institutions themselves in the post-crisis era. The third Basel accord presented by the
1.2 OTC Derivatives

OTC derivatives are contingent claims that are privately negotiated between two parties without the involvement of an exchange. OTC contracts can be highly specialised and formulated exclusively for the needs of the involved parties. In contrast to OTC derivatives, exchange-traded derivatives are highly standardised and the contract terms need to follow the specifications of the exchange. The exchange provides a common market place where transparency is high and it facilitates liquidity, which makes it easy for participants to trade and unwind positions before maturity, if they like. Additionally, an exchange guarantees the cash flows that are agreed on in the contract terms. This makes the risk of not receiving the promised cash flows relatively low, as it depends on the survival of the exchange and not on the survival of the single counterparty with which the contract is traded (Pykhtin and Zhu, 2007). For OTC contracts there is no third party that guarantees that the payments agreed on are made and therefore the parties fully bear the involved credit risk\(^1\): each party is fully exposed to the risk that the other party will not fulfil its contractual obligations due to default.

The OTC derivative markets have grown tremendously over the last years with outstanding notional amounts and gross market values having quadrupled over the past decade. The development is illustrated in Figure 1, which is adopted from statistics issued by Bank of International Settlements (2013). The statistics show the aggregated size of the OTC derivative markets in the G10 countries and Switzerland on a semi-annual basis. From December 2011 and onwards the Australian and Spanish markets are also included. In Table 1 the gross market values for December 2012 are reported per type of underlying. The table shows that interest rate derivatives are, with about 75 % of the total gross market value, by far the most traded contracts. Within this category Interest Rate Swaps (IRS) have a share of over 90 %, corresponding to a share of approximately 70 % of the whole OTC derivative markets.

\(^1\)Risk mitigants, such as collateralised trading and trading over centralised clearing houses, can reduce counterparty credit risk. These mitigants are further discussed in Section 2.2.
1.2 OTC Derivatives Credit Value Adjustment

(a) Notional amounts outstanding (in trillions of USD) in the markets for OTC derivatives from June 1998 until December 2012.

(b) Gross market values (in trillions of USD) in the markets for OTC derivatives from June 1998 until December 2012.

Figure 1: The graphs are based on statistics on the OTC derivative markets in the G10 countries, Switzerland, Australia and Spain published semi-annually (Bank of International Settlements, 2013).

Table 1: Gross market values of OTC derivative contracts in December 2012 in the G10 countries, Switzerland, Australia and Spain categorised according to type of underlying (Bank of International Settlements, 2013).

<table>
<thead>
<tr>
<th>Underlying</th>
<th>Gross market value (billions of USD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>All contracts</td>
<td>24,740</td>
</tr>
<tr>
<td>Foreign exchange</td>
<td>2,304</td>
</tr>
<tr>
<td>Interest rate</td>
<td>18,833</td>
</tr>
<tr>
<td>Equities</td>
<td>605</td>
</tr>
<tr>
<td>Commodities</td>
<td>358</td>
</tr>
<tr>
<td>Credit default swaps</td>
<td>848</td>
</tr>
<tr>
<td>Unallocated</td>
<td>1,792</td>
</tr>
</tbody>
</table>
1.3 Counterparty Credit Risk

Gregory (2010) defines counterparty credit risk as the specific form of credit risk that arises from OTC derivative transactions and security financing transactions. It is the risk that a counterparty will default before the maturity of a contract and hence not be able to fulfil its obligations to the other party, as specified by the terms of the contract. The risk is larger in OTC derivative transactions than security financing transactions due to the size of the OTC derivative markets and the potential complexity of the products (Gregory, 2010).

According to Pykhtin and Zhu (2007) counterparty credit risk mainly differentiates itself from other forms of credit risk in two ways. The first cause is the bilateral nature of the credit risk: A derivative position has a positive market value for one party and a corresponding negative value for the counterparty, but during the life of the contract the market value can change such that the first party has a negative market value and the counterparty the positive value. This means that the credit risk is present on both sides of the contract. For comparison, take an instrument for which the market value cannot change sign, such as a bond. The party having the long position faces the risk of the issuer defaulting and hence bears credit risk. The issuer, on the other hand, does not face credit risk because his financial position does not depend on the survival of the bond holder. In the case of a derivative, such as an IRS, both parties face credit risk because when the floating rate is above the swap rate the contract has a positive market value for the fixed payer and when the floating rate is below the swap rate the floating payer has a positive value in his books. The second particularity of counterparty credit risk is the variability in exposure, which is a measure of how much capital is at risk: The credit risk of the bond position can be quantified by determining the exposure, which is the Present Value (PV) of the bond, and weighting it with the probability of the issuer defaulting. The single determinant of the PV and the exposure of the bond is the discount rate. It is generally quite stable as long as interest rates are not very volatile. The exposure for a derivative position is equal to the PV but capped at zero from below, such that it never takes negative values. The exposure can be highly volatile depending on the underlying risk factors and the degree of leverage of the derivative in question.

1.4 Key Drivers of Credit Value Adjustment

There are mainly three drivers behind the current development of incorporating counterparty credit risk in the valuation and pricing of derivatives. According to Dorval and Schanz (2011) these drivers can be divided into the three categories regulation, accounting and pricing. These three perspectives all aim at valuing counterparty credit risk and provide more or less related definitions of the price of this risk - the CVA. This is illustrated in Figure 2, which aims to show how CVA affects a bank in several ways: through regulation via Basel III, through accounting via International Financial

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2 This is not the case for all derivatives. For some derivatives, such as vanilla options, the same party always has the positive market value and its counterparty the negative market value.
1.4 Key Drivers of Credit Value Adjustment

Credit Value Adjustment Reporting Standards (IFRS), if applicable, and through pricing via market-observed prices.

Figure 2: The key drivers that are defining CVA and providing the framework to which banks are confined.

The Basel III document states that “a bank must add a capital charge to cover the risk of MtM losses on the expected counterparty risk (such losses being known as credit value adjustments, CVA) to OTC derivatives” (Basel Committee on Banking Supervision, 2011). The document specifies a standardised and an advanced approach for computing the capital charge and it strictly defines how and to which extent the capital charge may be reduced by hedging the CVA³.

From the accounting side IFRS requires that banks measure the fair value of OTC derivatives in their books, which includes adjusting the value according to the credit quality (Schubert, 2011). This means that a bank needs to adjust the book value of an OTC derivative to account for the risk of counterparty default as well as for a potential default of the own institution (Kengla and De Jonghe, 2012). This implies that the CVA has to be determined as well as the Debt Value Adjustment (DVA), which analogously to CVA is the market price of the counterparty credit risk due to default of the own institution, which the counterparty is facing⁴.

The third key driver of CVA is pricing, which amounts to incorporating the counterparty credit risk in the price of an OTC derivative. As Sokol (2012) remarks, this has the implication that there is no longer a single fair value of a financial derivative, instead the fair value depends on with which counterparty the derivative is traded and the portfolio of existing transactions with that counterparty. The price of a derivative thus has to be determined by adjusting the risk-neutral price by subtracting the CVA and possibly adding the DVA, which makes prices not only dependent on market parameters, but also on the credit quality of the counterparty and even the credit quality of the own institution in the case of DVA. The case where the credit quality of both counterparties are taken into account the term bilateral CVA is used for the combined CVA and DVA⁴.

The three drivers lie the basis at which level a bank may choose to handle the concept of CVA. Dorval and Schanz (2011) define four possible strategies:

³The Basel accords and the CVA capital charge in particular are discussed in Section 5.1.
⁴DVA and the related bilateral CVA are more formally introduced in Section 3.1.
1. **Measuring** CVA and calculating the associated capital charge such that the bank is compliant with regulation and accounting standards.

2. **Advising** the trading department on CVA-related risks and how to charge counterparties for CVA.

3. **Hedging** through aggregating the CVA at a designated CVA desk that performs hedging and charges the other trading desks correspondingly.

4. **Trading** through letting the CVA desk take speculative positions on CVA in order to generate a profit (or loss).

The first strategy defines the minimum level that all banks need to incorporate in order to be compliant with the regulation. On this level CVA is measured as dictated by regulators. Banks that are relatively small on OTC derivatives may choose to implement CVA at this level. Banks with more complex trading activities and larger OTC derivative portfolios may choose to implement CVA at a higher level in order to charge counterparties for CVA, hedge the risk or for speculative purposes. On these levels a simulation engine is needed for the measurement of CVA.

This thesis is mainly concerned with the pricing perspective, although the link between the pricing component, the regulatory CVA and CVA in accounting will also be discussed.
2 Managing Counterparty Credit Risk

The price of a derivative has traditionally been computed using risk-neutral pricing yielding the fair price that has been offered to other market participants without adjustment for the credit quality of the counterparty (Carver, 2012b). Counterparty credit risk has been handled by means such as using risk limits to avoid having too large exposure to a single counterparty or only trading with counterparties with high enough credit quality, as discussed by Gregory (2010). The author further points out that the counterparty risk seldom has been incorporated in the price of a derivative. With the introduction of CVA this is accomplished and it results in that there no longer is one single price of a derivative, but the price depends on the counterparty in question, the current portfolio of contracts with that counterparty, as well as the existence of netting and collateral agreements, as Carver (2012b) describes.

The CVA of a derivative is defined to be the difference between the risk-free value, when assuming no counterparty credit risk, and the true value, that is,

\[
CVA \triangleq PV_{\text{risk-free}} - PV,
\]

where \(PV\) denotes the present value of the derivative contract\(^5\). Putting it another way, CVA is the market price of counterparty credit risk (Pykhtin and Zhu, 2007).

2.1 Quantification and Metrics

In order to be able to quantify counterparty credit risk, parameters that are relevant as determinants have to be established. According to Gregory (2010), the determinants include

- The contract in question and especially how its value varies due to changes in market and credit risk factors.
- The credit-worthiness of the counterparty.
- Netting agreements and other contracts that are contained in the same netting set.
- Collateral that supports the contract.
- Hedging aspects, i.e., is the risk possible to hedge and at which cost?

When determining CVA the above aspects need to be taken into account. The parameters that determine CVA are defined in the following.

\(^5\)A formal definition of CVA in the pricing context is given in Section 3.
2.1 Quantification and Metrics

Credit Value Adjustment

2.1.1 Present Value

When computing CVA the PV is needed as of the time of valuation and at future time points, in which case it will be a Future Value (FV)\(^6\). Depending on the type of contract the value is determined by market and credit risk factors and can be highly variable. The PV at an arbitrary time \( t \geq 0 \) is given by the expected value of the discounted payouts beyond that time point under the risk-neutral measure. The PV at time \( t \), denoted by \( V(t, T) \), is hence

\[
V(t, T) = \mathbb{E}_Q^{\mathcal{F}_T} \left[ \sum_{u \in (t, T]} C(u)D(t, u) | \mathcal{F}_t \right], \tag{2}
\]

where \( C(u) \) is a payout at the time \( u \), which may be dependent on events in the interval \((0, u]\) (such as a barrier being reached), \( T \) is the time of maturity of the contract, \( u \) assumes discrete time points in the interval \((t, T]\) where payouts occur and \( D(t, u) \) is the discount factor between the times \( t \) and \( u \). Furthermore, \( \mathcal{F}_t \) is the market filtration and \( Q, \mathcal{M} \) the risk-neutral measure\(^7\).

2.1.2 Exposure

The exposure is the amount that would be lost if the counterparty would default on the contract. The exposure depends on whether the contract is an asset or a liability of the investor. If the contract has a positive PV it is an asset of the investor that is to be received from the counterparty. In case of counterparty default this value will not be paid out in its full amount and the exposure is hence equal to the PV. If the contract has a negative PV it is a liability of the investor and hence the investor has the obligation to pay the value to the counterparty. In case of counterparty default this amount is still due and is to be paid to the creditors of the defaulted counterparty. Thus, the exposure is equal to the PV if it is positive and is zero otherwise, i.e.,

\[
E(t) = \max\{V(t, T); 0\} \tag{3}
\]

As was established previously and highlighted in (2) the PV of a contract is a risk-neutral expectation. This means that the exposure is also a risk-neutral expectation and this is sometimes pointed out by referring to it as the *expected exposure*.

An illustration of PV and exposure is shown in Figure 3. From the figure it is seen that the exposure has a profile similar to the pay-off of a call option. Exposure is a key determinant of CVA and because of the similarity between exposure and an option pay-off CVA can be represented as a short call option on the PV of the underlying contract\(^8\) (Stein and Lee, 2010).

\(^6\)The FV is the PV at a future date and the difference between the two are only a matter of discounting/capitalisation. In this thesis PV will generally be used to denote values of contracts and in the case of a value at a future date, say \( t \), the term PV at time \( t \) will be used.

\(^7\)The probabilistic model of the market will be discussed further in Section 3.2.

\(^8\)The similarities between CVA and an option is further discussed in Section 3.10.
2.1 Quantification and Metrics

Credit Value Adjustment

2.1.3 Probability of Default

The probability of default is a probabilistic description of the credit-worthiness of a counterparty. It is expressed as a Probability Density Function (PDF), which assigns probability mass to time points, or more commonly, by the associated Cumulative Distribution Function (CDF). In this thesis $P_D(t)$ will refer to the CDF of default. $P_D(t)$ for a counterparty that has not yet defaulted has the properties of being a monotonically increasing function of time with $P_D(0) = 0$ and $\lim_{t \to \infty} P_D(t) = 1$. In words, the probability of the counterparty defaulting in the instance of valuation is zero and the probability then increases over time to reach the value one as time tends to infinity.

$P_D(t)$ forms a term structure of default probability from which the appreciated short- and long-term credit-worthiness of a counterparty can be read, as Gregory (2010) describes: A counterparty with low credit quality will have a term structure that is steep in the beginning and then flattening due to the high probability of default in the near future. A counterparty with high quality but with the prospect of deteriorating will have a flat curve in the beginning followed by a steep increase later on. Finally, a counterparty with high credit quality and stable outlooks will have a curve that is slowly increasing.

The default probability curve should be fetched from market prices of Credit Default Swap (CDS) contracts referenced to the counterparty in question, in which case the curve will be market implied and hence risk-neutral (Brigo et al., 2013). Since traded CDSs are restricted to only a few reference entities which typically do not include small

![Exposure as a function of PV for a fixed point in time. The exposure is equal to the PV if it is positive and zero otherwise.](image)

Figure 3: Exposure as a function of PV for a fixed point in time. The exposure is equal to the PV if it is positive and zero otherwise.
and medium-sized enterprises (SMEs) the default probability curve will often have to be estimated in another way. An estimate is typically derived from bond spreads as suggested by Berd et al. (2003) or by some proxy variables that are mapped to factors which can be weighted together to form the default probability curve. Such proxy variables could for example be external credit ratings or default probabilities of similar entities (Kengla and De Jonghe, 2012).

A sample term structure of default is seen in Figure 4. The curve shows the market-implied term structure of default of Credit Suisse Group over a period of ten years derived from CDS quotes from the third of September 2013 delivered by Markit\textsuperscript{9}.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.png}
\caption{Term structure of default of Credit Suisse Group derived from CDS spreads as of the third of September 2013 delivered by Markit.}
\end{figure}

\textsuperscript{9}Markit Financial Information Services: \url{http://www.markit.com/en/}
2.1.4 Recovery Rate and Loss-Given-Default

In case of a default there will generally be a recovery value paid out to the creditors of the defaulted party. The holders of OTC derivative contracts are typically in the same class as senior bond holder and will thus be among those creditors receiving the highest recoveries (Gregory, 2010). The recovery rate, denoted by $R$, is the ratio of the exposure that would be recovered in the case of default. The recovery rate can equivalently be expressed as the loss-given-default, $LGD$, which is the ratio of the exposure that would actually be lost. Hence,

$$LGD = 1 - R$$

The recovery rate is an important figure in the computation of CVA, since it potentially has a large effect on the lost amount.

2.2 Risk Controlling and Mitigation

Controlling counterparty credit risk has historically been carried out through the use of credit lines that set limits on the exposure to specific counterparties that are not allowed to be overridden, limiting trading to counterparties of high credit quality as well as diversifying such that credit risk is not concentrated to a small group of counterparties.

When it comes to reducing the risk, and thereby the CVA, this can mainly be done in one of four ways:

- Moving trading from traditional OTC markets to centralised clearing houses.
- Using netting agreements, which regulate how the exposure of different contracts with the same counterparty may be used to offset each other.
- Supporting the contracts with collateral.
- Hedging positions such that the exposure is reduced.

2.2.1 Centralised Clearing Houses

A centralised clearing house, also called a central counterparty, acts as a third party that stands between the two transacting parties, which are both members of the clearing house, and guarantees the contractual obligations in case that one of the parties defaults. In this manner the risk is transferred to the clearing house and the credit risk concentrated to one entity, which in the best case will enjoy diversification effects. In the worst case it may, on the other hand, increase systemic risk, which will be the case if the credit quality of the members of the house are heavily correlated (Gregory, 2010).

Centralised clearing houses are becoming more common and the Basel Committee explicitly states that one of the aims with the Basel III document is to “provide additional incentives to move OTC derivative contracts to central counterparties” (Basel Committee on Banking Supervision, 2011). Additionally the Dodd-Frank Act in the U.S. and the European Markets and Infrastructure Regulation (EMIR) both introduce
mandatory central clearing of a range of OTC derivative products, such as highly liquid and standardised IRS and CDS contracts, between financial counterparties (Gilmore and Ryan, 2013). The development of central clearing of IRSs during the years 2006 until 2011 is shown in Figure 5, where it is seen that the notional amount of IRSs that are traded through a centralised clearing house has more than five-folded over that period.

![Figure 5: Outstanding notional amounts in trillions of USD of OTC-traded IRSs that are cleared over a centralised clearing house. The graph is adopted from Bank of International Settlements (2011).](image)

2.2.2 Netting Agreements

Netting agreements provide the possibility to net exposures of different contracts with the same counterparty against each other. Without a netting agreement all contracts are entities of their own and a positive value of one contract is not allowed to be offset by the negative value of another to decrease the overall exposure. Thus, in a default scenario the positive value of one contract will be lost, whereas the negative value of another will still be due. If a netting agreement is in place, the values of the contracts included in the agreement are aggregated and the exposure is determined from the combined PV of the netting set. Let $S$ denote a netting set containing $N$ trades. The exposure of the netting set is given by

$$E^S(t) = \max \left\{ \sum_{i=1}^{N} V_i(t,T) ; 0 \right\},$$

whereas the exposure for the same trades without netting would be given by the sum of the exposures on each trade, determined from (3). The effect of netting is seen from the inequality

$$\max \left\{ \sum_{i=1}^{N} V_i(t,T); 0 \right\} \leq \sum_{i=1}^{N} \max \left\{ V_i(t,T); 0 \right\},$$

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2.2 Risk Controlling and Mitigation

Credit Value Adjustment

where the expression on the left-hand side represents the exposure of a portfolio where netting is allowed and the right-hand side represents the exposure of the same portfolio without netting.

An example of netting in case of a portfolio containing two instruments is shown in Figure 6. The graphs on the left-hand side of the figure show realisations of the PV processes over an interval of ten years of each position, respectively. The instrument depicted in the upper graph has a positive PV during the first five years and thereafter it becomes negative. The other instrument, in the lower graph, has a positive PV over the whole period. The two graphs on the right-hand side of the figure show the exposure profile of the portfolio. The upper graph represents the case in which netting is not allowed and hence the exposure is formed through aggregating only the positive parts of the PVs of the positions. The lower graph represents the case where the positions are allowed to be netted and one can see that the exposure becomes zero after approximately six and a half years, which is when the negative PV of the first position dominates the positive PV of the second one.

![Figure 6: The effect of netting on a portfolio containing two instruments. The graphs on the left-hand side show realisations of the PV processes over ten years of the two positions, respectively. The upper graph on the right-hand side depicts the exposure profile of the portfolio in the case where netting is not allowed, whereas the lower graph depicts the exposure profile when netting is allowed.](image)

Brigo et al. (2013) discuss the effectiveness of netting and conclude that it depends on the correlation between the contracts contained in the netting set. If the contracts are strongly negatively correlated the effect of netting will be high because as one contract assumes a high positive value other contracts will assume high negative values with large probability. The opposite case, where the contracts are strongly positively correlated, represents the situation where netting only provides a weak benefit.

The presence of netting agreements is important for CVA. Since netting has the potential to reduce the exposure of a portfolio, and hence reduce the counterparty credit...
risk, it will also reduce the price of this risk, i.e., the CVA.

2.2.3 Collateralisation

Collateralisation reduces credit risk by supporting a transaction with a pool of collateral assets, which can take the form of cash, securities or physical assets. The collateral assets are posted to the pool by the party having the negative PV and in case of default the other party will assume ownership of the assets as compensation for non-payments.

The exchange of collateral is regulated in a credit support annex (CSA), which commonly includes

- **Threshold**, which is the limit of the PV above which collateral needs to be posted.
- **Minimum transfer amount**, which is the minimum difference between PV and collateral value that requires posting of additional collateral.
- **Eligible assets**, i.e., which assets are allowed as collateral and which haircuts are to be applied to different assets.
- **Frequency** of marking-to-market and collateral rebalancing.

Collateral can reduce the counterparty credit risk very effectively and leaves only *gap risk*, which is the combined risk that the PV of the supported contract changes between the last date of collateral posting and the default event and that the collateral changes in value between the default event and the time at which the collateral receiver assumes the ownership of the collateral and, depending on asset, can liquidate it. OTC derivative transactions that are collateralised have thus only counterparty credit risk in the form of gap risk and it is therefore for uncollateralised OTC derivative transactions that the pricing of this risk, i.e., the determination of CVA, is of most importance. However collateralisation also gives rise to costs in the form of capital costs for the assets posted as collateral. These costs are accounted for by the Funding Value Adjustment (FVA). FVA is a large topic on its own and will therefore not be discussed in this thesis, although it has connections to CVA. The interested reader is referred to Kenyon and Stamm (2012), who give a thorough discussion of the topic.

In the presence of collateral the exposure of a contract, or netting set, needs to be adjusted to reflect the available collateral pool. Let $Coll(t)$ denote the value of the collateral that would be available to the investor at time $t$ if the counterparty was to default at that point in time. The exposure of a contract under collateral agreement becomes

$$E_{coll}(t) = \max\{V(t, T) - Coll(t); 0\}$$  \hspace{1cm} (7)

When forming exposure at future time points the collateral $Coll(t)$ becomes a stochastic process, which depends on the specific asset used for collateral, the value process of the contract, $V(t, T)$, and on the terms specified in the CSA.
2.2 Risk Controlling and Mitigation

2.2.4 Hedging

Hedging market risks has been pursued a long time in order to reduce Profit-and-Loss (P&L) volatility and to eliminate unwanted risks while keeping others. Hedging of credit risk has become more common the last couple of years. This has been facilitated by the growth of the credit derivatives market as reported by Bank of International Settlements (2006, 2008, 2010, 2013). The basic instrument for hedging credit risk is the CDS, which is a contract referenced to a credit entity, such as a company, a sovereign or a bond, and pays the holder of the contract in case of default of the reference entity. Counterparty credit risk is a combination of market risk and credit risk, which makes the risk complicated to hedge, especially in the case where the two risk types are highly dependent.

Cesari et al. (2009) discuss how counterparty credit risk can be hedged using credit derivatives and concludes that it may be complicated in practice due to the fact that CDSs are traded on relatively few entities and the market for the contracts is not necessarily liquid. Another problem is to match the exposure at the time of counterparty default with the notional of the CDS, which is very difficult if the exposure is highly variable. A solution to this is the Contingent Credit Default Swap (CCDS), which has the property of the notional being indexed to the value of another contract. The CCDS is a highly specialised contract and each transaction is hence tailor made, which makes the market quite illiquid and CCDSs are traded on even fewer reference entities than CDSs\textsuperscript{10}.

\textsuperscript{10}The specific hedging of CVA and its complications will be more thoroughly discussed in Section 3.12.
3 Pricing Counterparty Credit Risk

Being the price of counterparty credit risk, CVA is in itself a financial instrument which has to be priced from market data. According to Gregory (2012) the price of a financial contract can be defined in one of two ways:

1. **The actuarial price**, which is the fair value represented by the expected value of future payouts adjusted by a risk premium that accounts for unexpected losses.

2. **The risk-neutral price**, which is the expected value of future payouts of the contract under a risk-neutral measure and the cost of a perfect hedging strategy.

The actuarial approach makes use of the physical measure, i.e., historical probabilities, and is used to determine CVA as an insurance premium that is charged and kept as a reserve against future losses due to counterparty default. Additionally a risk premium is added to the CVA charge in order to account for unexpected losses, which may occur in e.g. a stressed market environment. One way of determining the premium is by using a quantile measure, similar to Value-at-Risk (VaR)\(^{11}\).

In the risk-neutral approach CVA is the cost of a perfect hedging strategy that nullifies all sensitivities associated with CVA, i.e., a hedging strategy that makes the OTC derivative books immune to changes in CVA. If the market is incomplete a perfect hedging strategy might not exist and the CVA is determined as the cost of a feasible hedging strategy plus a component that accounts for the hedging error\(^{12}\). The risk-neutral approach is the one that will be taken throughout this section under the assumption that the market is complete\(^{13}\).

Thus, let CVA of a contract be defined as the market price of counterparty credit risk on that contract given by the risk-neutral expectation of the loss that is due to counterparty default between inception and maturity weighted with the risk-neutral probability of the counterparty defaulting. In choosing the risk-neutral approach one has also implicitly chosen to make use of market-implied data for the computations, if these data exist (Brigo et al., 2013). Thus input data such as volatilities and probability of default need to be fetched by observing current prices on the market, i.e., be implied by the market.

In this section CVA will mainly be treated for contracts that are not supported by collateral. It is no critical simplification to exclude collateral, since it does not add much complexity to the derivation\(^{14}\). The derivations are to a large extent based on those of Brigo et al. (2013).

\(^{11}\)In the actuarial approach CVA is determined as a capital reserve, which is the basis of CVA in the regulatory perspective driven by Basel III.

\(^{12}\)The actuarial approach and the risk-neutral approach are further discussed in Section 6 in the context of how CVA can be managed within a bank.

\(^{13}\)An account of risk-neutral pricing is given in Section 3.3. Hedging of CVA and market completeness are discussed in Section 3.12.

\(^{14}\)When collateral is included in the numerical computation of CVA the complexity is increased since it becomes necessary to model the collateral value over time.
3.1 DVA and Bilateral CVA

Before deriving the formula for CVA the Debt Value Adjustment (DVA) and Bilateral CVA (BCVA) will be introduced. The CVA by itself is an adjustment for unilateral counterparty credit risk, which relies on the assumption that the counterparty is risky but that the investor is default risk-free. When large banks started to charge their corporate clients for counterparty credit risk in the beginning of the 2000’s, the unilateral CVA was the approach taken (Gregory, 2010). With the financial crisis it was seen that large banks were not default-remote which lead to questioning of the unilateral default assumption; banks started to charge CVA between themselves and large corporates realised that they too faced counterparty credit risk arising from transactions with risky banks. When two parties charge each other unilateral CVA they adjust the price in opposite directions and in order to be able to agree on the price the DVA and bilateral CVA were introduced.

DVA is analogous to CVA and is the price of counterparty credit risk from the perspective of the counterparty, i.e., the price of the risk that the investor defaults before maturity of a derivative contract and fails to fulfill his obligations to the counterparty. The DVA of the investor is hence the CVA of the counterparty and vice versa. CVA and DVA always have opposite signs and whereas CVA decreases the value of a derivative, DVA increases the value.

The bilateral CVA is the combination of CVA and DVA and is thus the adjustment applied to the counterparty credit risk-free derivative price under the assumption that both parties may default. The adjustment is equal for both counterparties, such that when added to the risk-free price both parties end up with the same risky derivative price. A complication of bilateral CVA, which will be seen in the derivation, is that one needs to take into account the order of default and the bilateral CVA does in general not equal the sum of the unilateral CVA and the unilateral DVA.

The introduction of DVA is not uncontroversial, since it has a counter-intuitive implication (Carver, 2012a): Just as the CVA sets a value on the default of a counterparty, DVA sets a value to the own default. This means that as the credit quality of the investor deteriorates the value of his OTC derivative positions increase and he makes MtM profits. The reason is that in the case of default he would gain from not having to pay all his liabilities and with a lower credit quality this default rebate will be bigger. However, it is questionable whether this rebate really has an economic value, because default is a binary transition and once a firm has defaulted it does not gain from paying a low compared to a high recovery rate from its remaining assets. As Gregory (2012) writes: “An institution can obviously realise the BCVA component by going bankrupt but, like an individual trying to monetise their own life insurance, this is not relevant”.

3.2 The Probabilistic Model

In order to derive CVA the probabilistic framework needs to be specified and notation introduced. First, define the probabilistic model of the market by letting \((Ω, G, G_t, Q)\) be the probability space, where \(Ω\) is the sample space, \(G\) the \(σ\)-algebra with \((G_t)_{t \geq 0}\) being the complete market filtration up to time \(t\) and \(Q\) the risk-neutral measure under
which all discounted prices of tradeable assets are martingales. The sample space can be viewed as a product space containing all possible outcomes in the market and credit risk world, i.e., \(\Omega = \mathcal{M} \times \mathcal{C}\), where \(\mathcal{M}\) is the set of all possible market risk factor outcomes and \(\mathcal{C}\) is the set of all possible credit risk factor outcomes\(^{15}\). The filtration is an enlarged filtration defined as \(\mathcal{G}_t = \mathcal{F}_t \vee \mathcal{H}_t\), where \(\mathcal{F}_t\) is the credit risk-free market filtration and \(\mathcal{H}_t\) the default filtration, i.e., the complete market filtration consists of the credit risk-free market information and explicit default time monitoring. \(\mathcal{H}_t\) is the right-continuous default filtration generated by the default event: \(\mathcal{H}_t = \sigma(\{\tau \leq u\} : u \leq t)\), where \(\tau\) is the time of default of the counterparty. The risk-neutral measure \(Q\) is a measure of the form \(Q : \mathcal{G}_t \mapsto [0,1]\). Let \(Q_M\) be the risk-neutral measure on the space of market risk \(\mathcal{M}\). It is a measure of the form \(Q_M : \mathcal{F}_t \mapsto [0,1]\). The prices of derivatives free of default risk are computed with respect to the credit risk-free market filtration \(\mathcal{F}_t\) under the risk-neutral measure \(Q_M\), whereas derivative prices subject to default risk are computed with respect to the complete market filtration \(\mathcal{G}_t\) under the risk-neutral measure \(Q\). Let \(E^{Q_M}[\cdot | \mathcal{F}_t]\) and \(E^Q[\cdot | \mathcal{G}_t]\) be expectations conditional on the credit risk-free market filtration \(\mathcal{F}_t\) and on the complete market filtration \(\mathcal{G}_t\) under the corresponding risk-neutral measures, respectively.

As previously, the derivative contract is defined by a stream of payouts, where we denote a payout at time \(t\) by \(C(t)\), and a maturity date denoted by \(T\). A payout \(C(t)\) may be contingent on events in the interval \((0,t]\), such as a barrier being reached, cf. barrier options. Furthermore, let \(D(s,t)\) denote the discount factor used to transfer a payout at time \(t\) back to the earlier time \(s\). The counterparty credit risk-free price of the derivative contract at time \(t < T\) is denoted by \(V(t,T)\) and given by the risk-neutral expectation of the discounted future payouts as in (2). Let \(\tau\) denote the time of default of the counterparty. It is a stopping time, i.e., \(\tau : \Omega \mapsto \mathbb{R}^+\) with distribution function \(P_D(t)\). In case that the counterparty defaults it is assumed that a fraction of any outstanding liabilities will be recovered with a rate \(R\), which will generally be a function of time, i.e., \(R = R(t)\). Let \(\mathbb{1}_{\{A\}}\) be the indicator function, which takes the value 1 in case that the event \(A\) occurs and zero otherwise.

### 3.3 Risk-Neutral Pricing and Market Completeness

Björk (2009) gives an account of risk-neutral pricing, which is the basis of this section. The mathematical derivation of CVA is done as an expectation under the risk-neutral measure, denoted by \(\mathbb{Q}\), under which all discounted asset price processes are martingales. The existence of such a risk-neutral measure is equivalent to the market not allowing arbitrage opportunities and the measure is unique if and only if the market is complete, according to the fundamental theorem of asset pricing. Market completeness requires that there exists a perfect hedging portfolio for every contingent claim. In the case that the market is incomplete the risk-neutral pricing approach cannot guarantee that unambiguous prices are obtained, but the prices will depend on the particular measure chosen. Only contingent claims that can be perfectly hedged in an incomplete market

\(^{15}\)The product space \(\Omega\) will be explained in more detail in Section 3.6.
3.4 Unilateral CVA

The Unilateral CVA (UCVA) is the price of counterparty credit risk assuming that the counterparty can default before contract maturity, but the investor is default risk-free. Two cases are possible: The counterparty can survive until maturity, in which case all payouts will occur according to the contract terms, and the present value at time zero for the investor is

$$PV_{\tau > T} = V(0, T)$$  \hspace{1cm} (8)

In the other case the counterparty defaults at some point before maturity, in which case all payouts before the default time are paid according to the contract. The payouts that would have been due after the default will not be made and the outcome for the investor depends on the exposure at the time of default of the counterparty. If the PV is positive this value is lost, except for a possible recovery value. If the present value is negative it is still due to the creditors of the defaulted counterparty. Hence, in the case of default at time $\tau$, before maturity, the present value at time zero for the investor is

$$PV_{\tau \leq T} = V(0, \tau) + (R \max \{V(\tau, T); 0\} + \min \{V(\tau, T); 0\}) D(0, \tau)$$  \hspace{1cm} (9)

The present value of the contract with UCVA is given by the risk-neutral expectation of the two present values (8)-(9). Thus,

$$PV = \mathbb{E}^Q\left[1_{\{\tau > T\}} V(0, T) + 1_{\{\tau \leq T\}} V(0, \tau) + 1_{\{\tau \leq T\}} (1 - R) \max \{V(\tau, T); 0\} \right]$$

$$= \mathbb{E}^Q\left[1_{\{\tau > T\}} V(0, T) + 1_{\{\tau \leq T\}} V(0, \tau) + 1_{\{\tau \leq T\}} (1 - R) \max \{V(\tau, T); 0\} \right]$$

$$- \mathbb{E}^Q\left[1_{\{\tau > T\}} + 1_{\{\tau \leq T\}} V(0, T) - 1_{\{\tau \leq T\}} (1 - R) \max \{V(\tau, T); 0\} \right]$$

$$= \mathbb{E}^Q \left[ V(0, T) \big| \mathcal{F}_0 \right] - \mathbb{E}^Q \left[ 1_{\{\tau \leq T\}} (1 - R) \max \{V(\tau, T); 0\} \right]$$

The terms on the last line are identified to be the risk-free present value of the
3.5 Unilateral CVA for a Single Cash Flow

contract and the CVA. Thus, the CVA is indeed an additive adjustment term, which was stated in (1). The CVA term is further developed:

\[
\text{CVA} = \mathbb{E}^Q \left[ \mathbb{1}_{\{\tau \leq T\}} \left(1 - R\right) \max \{V(\tau, T); 0\} D(0, \tau) | \mathcal{G}_0 \right]
\]

\[
= \mathbb{E}^Q \left[ \int_0^T \left(1 - R\right) \max \{V(s, T); 0\} D(0, s) \delta(s - \tau) ds | \mathcal{G}_0 \right]
\]

\[
(\star) = \mathbb{E}^Q \left[ \mathbb{E}^{Q,M} \left[ \int_0^T \left(1 - R\right) \max \{V(s, T); 0\} D(0, s) \delta(s - \tau) ds | \mathcal{F}_t \cup \{\tau = t\} \right] \times \mathbb{1}_{\{\tau = t\}} \mathbb{1}_{\{t \leq T, \mathcal{G}_0\}} \right]
\]

\[
= \mathbb{E}^Q \left[ \mathbb{E}^{Q,M} \left[ \left(1 - R\right) \max \{V(t, T); 0\} D(0, t) | \mathcal{F}_t \cup \{\tau = t\} \right] \mathbb{1}_{\{\tau = t\}} | t \leq T, \mathcal{G}_0 \right]
\]

\[
= \int_0^T \mathbb{E}^{Q,M} \left[ \left(1 - R\right) \max \{V(t, T); 0\} D(0, t) | \mathcal{F}_t \cup \{\tau = t\} \right] dP_D(t),
\]

where \(\delta(\cdot)\) is the Dirac delta function and (\star) follows from the law of total expectation.

Finally we have arrived at an expression for the UCVA

\[
\text{UCVA} = \int_0^T \mathbb{E}^{Q,M} \left[ \left(1 - R\right) \max \{V(t, T); 0\} D(0, t) | \mathcal{F}_t \cup \{\tau = t\} \right] dP_D(t) \quad (10)
\]

### 3.5 Unilateral CVA for a Single Cash Flow

In order to illustrate the formula for UCVA consider a simple transaction involving one single cash flow, following Morini and Prampolini (2011). An investor has agreed to lend an amount of money to a default risky counterparty. Let \(K\) be the nominal amount, which the counterparty will repay the investor at maturity, \(T\). Let \(f(t, T)\) be the instantaneous forward rate with maturity \(T\) determined at time \(t\). Discount factors are determined from the instantaneous forward rate, e.g. the discount factor from time \(t\) back to time \(s\) determined at time \(u\) is

\[
D(u; s, t) = \exp \left\{ - \int_s^t f(u, v) dv \right\} \quad (11)
\]

The counterparty credit risk-free PV of the cash flow is determined through discounting the nominal, e.g. the risk-free PV at time zero is

\[
PV_0 = KD(0; 0, T), \quad (12)
\]

which is the amount that the investor would lend the counterparty at time zero if the counterparty was free of default risk.

In case that the counterparty would default a recovery amount will be paid and it is determined by the recovery rate, \(R\), which is assumed to be constant. Let \(s\) be the CDS
spread of the counterparty and assume that this spread applies to all tenors of CDSs. Furthermore, assume a reduced-form intensity model for the CDS spread\textsuperscript{16}, such that the CDF of default is given by

\[ P_D(t) = \mathbb{Q}\left(\{\tau \leq t\}\right) = 1 - \exp\left\{-\frac{s}{1-R}t\right\} \]

The UCVA for the cash flow at time zero is determined from (10) as

\[
\text{UCVA}_0 = \int_0^T \mathbb{E}^{\mathbb{Q},\mathcal{M}} \left[ (1-R)KD(t; t, T)D(t; 0, t) \right] \frac{s}{1-R} \exp\left\{-\frac{s}{1-R}t\right\} dt \\
= \int_0^T \mathbb{E}^{\mathbb{Q},\mathcal{M}} \left[ (1-R)K \exp\left\{-\int_t^T f(t, s) ds\right\} \exp\left\{-\int_0^t f(t, s) ds\right\} \right] \times \\
\frac{s}{1-R} \exp\left\{-\frac{s}{1-R}t\right\} dt \\
= (1-R)K \int_0^T \mathbb{E}^{\mathbb{Q},\mathcal{M}} \left[ \exp\left\{-\int_t^T f(t, s) ds\right\} \right] \frac{s}{1-R} \exp\left\{-\frac{s}{1-R}t\right\} dt \\
= (1-R)K \int_0^T \mathbb{E}^{\mathbb{Q},\mathcal{M}} \left[ D(t; 0, T) \right] \frac{s}{1-R} \exp\left\{-\frac{s}{1-R}t\right\} dt \\
= (1-R)KD(0; 0, T) \left[ -\exp\left\{-\frac{s}{1-R}t\right\} \right]_0^T \\
= (1-R)KD(0; 0, T) \left( 1 - \exp\left\{-\frac{s}{1-R}T\right\} \right) \\
= (1-R)KD(0; 0, T)P_D(T) \]

Thus, the amount that the investor will lend to the counterparty at time zero in exchange for the nominal \(K\) at maturity is

\[
\text{PV}_0 - \text{UCVA}_0 = KD(0; 0, T) - (1-R)KD(0; 0, T)P_D(T) \\
= KD(0; 0, T) \left( 1 - (1-R)P_D(T) \right) \\
= KD(0; 0, T) \left( 1 - (1-R)P_D(T) \right) \\
= KD(0; 0, T) \left( 1 - (1-R)P_D(T) \right) \]

It is rather an exception than a rule that CVA can be calculated analytically and in practice one has to rely on numerical methods\textsuperscript{17}.

\textbf{3.6 CVA and the Product Space of Market and Credit Risk}

One of the complexities of CVA is that it is determined by both market and credit risk factors. In the risk-neutral pricing approach this was highlighted by the risk-neutral expectation being conditional on the complete market filtration \((\mathcal{G}_t)_{t \geq 0}\), which provides measurability of both market- and credit-related events. The CVA is hence a quantity

\textsuperscript{16} The reduced-form intensity model of CDS spreads is described in Section 4.3.3.

\textsuperscript{17} In Section 4 it is described how CVA is computed numerically via simulation methods.
determined in the product space of market- and credit risk, which was denoted by \( \Omega = \mathcal{M} \times \mathcal{C} \). The space \( \Omega \) contains the full dependence structure between the two subspaces, which might be very complicated. The CVA formula (10) relies on the Tonelli-Fubini theorem (Seppäläinen, 2012) to transform one single expectation conditional on the complete market filtration \( \mathcal{G}_t \) to two iterated expectations conditional on the filtrations \( \mathcal{F}_t \) and \( \mathcal{H}_t \), respectively. An illustration of the CVA formula in the context of the Tonelli-Fubini theorem is shown in Figure 7, which provides a visualisation of the product space as a three dimensional space with the horizontal axis representing the credit risk subspace and the two perpendicular axes representing the market risk subspace. In (10) the product space is foliated by holding the time of default fixed at time points in the interval \((0, T]\). In the figure the foliation is illustrated by the two planes.

CVA is calculated through evaluating the expectation inside the integral for each subspace in the foliation and then integrating these expectations weighted with the probability of the corresponding default time. Through this procedure the calculation of CVA is reduced to calculating the market value of a netting set at future times, where each future time point is assumed to be the time of default, and the integral is taken over each possible default time. Thus, the computation of CVA is reduced to determining the PV at future time points with infinitesimal distance between them.
3.7 Unilateral DVA

The Unilateral DVA (UDVA) is the price of counterparty credit risk assuming that the investor may default before maturity of the contract, but the counterparty is default risk-free. Let $\tau_I$ denote the default time of the investor. Similarly to the default time of the counterparty the time of default of the investor is a stopping time $\tau_I : \Omega \rightarrow \mathbb{R}^+$ with distribution function $P^I_D(t)$. The recovery rate in the case that the investor defaults is denoted by $R_I$ and is generally time-dependent, i.e., $R_I = R_I(t)$. Just as for the UCVA there are two possible cases: The investor can survive until maturity and the payouts occur according to the contract, in which case the present value at time zero for the investor is

$$\text{PV}_{\tau_T > T} = V(0, t)$$

In the second case the investor defaults before maturity and all payouts before the default time occur according to the contract terms. If the contract has a positive value for the investor at default time this value will be received in its full amount by the creditors of the investor. If, on the other hand, the contract has a negative value at the time of default only the recovery amount, determined by the recovery rate, is due to the counterparty. Hence, in the case of default at time $\tau_I$, before maturity, the present value at time zero to the investor is

$$\text{PV}_{\tau_T \leq T} = V(0, \tau_I) + (\max\{V(\tau_I, T); 0\} + R_I \min\{V(\tau_I, T); 0\}) D(0, \tau_I)$$

After computing the present value as the risk-neutral expectation of the present values (16)-(17) and identifying the DVA term one finds an expression of the same form as the one for the UCVA:

$$\text{UDVA} = \int_0^T \mathbb{E}^Q_M[(1 - R_I) \min\{V(t, T); 0\} D(0, t)|\mathcal{F}_t \vee \{\tau_I = t\}] dP^I_D(t)$$

3.8 Bilateral CVA

In the case of Bilateral CVA (BCVA) both the investor and the counterparty are assumed to be able to default before maturity of the contract. Three cases are possible:

1. Both the counterparty and the investor survive until maturity and all payouts occur according to the contract terms,

2. The counterparty defaults at time $\tau$, before maturity and before the investor,

3. The investor defaults at time $\tau_I$, before maturity and before the counterparty.

In the first case the present value at time zero from the investor’s perspective is

$$\text{PV}_{\tau_T > T \land \tau_I > T} = V(0, T)$$

In the second case all payouts before $\tau$ occur according to the contract. If the value of the contract is positive at default for the investor this amount is lost, except for a
recovery amount, and if it is negative it is still due. The present value at time zero for the investor is

\[
PV_{\tau \leq T \land \tau < \tau_I} = V(0, \tau) + \left( R \max \{V(\tau, T)\} + \min \{V(\tau, T)\} \right) D(0, \tau) \tag{20}
\]

In the third case all payouts before \(\tau_I\) occur according to the contract. If the value of the contract is positive at default for the investor this amount is still due to be received and if it is negative only the recovery fraction needs to be paid. Thus, the present value at time zero from the perspective of the investor is

\[
PV_{\tau_I \leq T \land \tau < \tau_I} = V(0, \tau_I) + \left( \max \{V(\tau_I, T)\} + R_I \min \{V(\tau_I, T)\} \right) D(0, \tau_I) \tag{21}
\]

The present value of the contract with BCVA is the risk-neutral expectation of the three present values (19)-(21), thus

\[
\begin{align*}
\text{PV} &= \mathbb{E}^Q \left[ 1_{\{\tau > T \land \tau_T > T\}} V(0, T) + 1_{\{\tau \leq T \land \tau < \tau_T\}} \left( V(0, \tau) + \left( R \max \{V(\tau, T), 0\} + \min \{V(\tau, T), 0\} \right) D(0, \tau) \right) + 1_{\{\tau_T \leq T \land \tau < \tau_I\}} \left( V(0, \tau_I) + \left( \max \{V(\tau_I, T), 0\} + R_I \min \{V(\tau_I, T), 0\} \right) D(0, \tau_I) \right) \bigg| \mathcal{F}_0 \right] \\
&= \mathbb{E}^Q \left[ 1_{\{\tau > T \land \tau_T > T\}} V(0, T) + 1_{\{\tau \leq T \land \tau < \tau_T\}} \left( V(0, \tau) + \left( V(\tau, T) - (1 - R) \max \{V(\tau, T), 0\} \right) D(0, \tau) \right) + 1_{\{\tau_T \leq T \land \tau < \tau_I\}} \left( V(0, \tau_I) + \left( V(\tau_I, T) - (1 - R_I) \min \{V(\tau_I, T), 0\} \right) D(0, \tau_I) \right) \bigg| \mathcal{F}_0 \right] \\
&= \mathbb{E}^Q \left[ 1_{\{\tau > T \land \tau_T > T\}} + 1_{\{\tau \leq T \land \tau < \tau_T\}} + 1_{\{\tau_T \leq T \land \tau < \tau_I\}} \right] V(0, T) - 1_{\{\tau \leq T \land \tau < \tau_T\}} (1 - R) \max \{V(\tau, T), 0\} D(0, \tau) - 1_{\{\tau_T \leq T \land \tau < \tau_I\}} (1 - R_I) \min \{V(\tau_I, T), 0\} D(0, \tau_I) \bigg| \mathcal{F}_0 \right] \\
&= \mathbb{E}^{Q, M} \left[ V(0, T) \big| \mathcal{F}_0 \right] - \mathbb{E}^Q \left[ 1_{\{\tau \leq T \land \tau < \tau_T\}} (1 - R) \max \{V(\tau, T), 0\} \big| \mathcal{F}_0 \right] - \mathbb{E}^Q \left[ 1_{\{\tau_T \leq T \land \tau < \tau_I\}} (1 - R_I) \min \{V(\tau_I, T), 0\} \big| \mathcal{F}_0 \right]
\end{align*}
\]

By identifying the terms on the right-hand side of the last equality sign in the derivation one finds that the present value of a derivative when both the investor and the counterparty are assumed to be defaultable consists of the counterparty credit risk-free present value less a CVA term and a DVA term. The CVA term is always positive and will thus decrease the value of the derivative, whereas the DVA term is always nega-
tive, which will increase the value of the derivative. The final adjustment depends on whether the investor or the counterparty faces the largest exposure and who is most likely to default first. The BCVA term is

\[ \text{BCVA} = \mathbb{E}^Q \left[ \mathbb{1}_{\{\tau \leq T \wedge \tau < \tau_I\}} (1 - R) \max\{V(\tau, T), 0\} D(0, \tau)|\mathcal{G}_0 \right] + \]

\[ \mathbb{E}^Q \left[ \mathbb{1}_{\{\tau_I \leq T \wedge \tau < \tau_I\}} (1 - R_I) \min\{V(\tau_I, T), 0\} D(0, \tau_I)|\mathcal{G}_0 \right] \] (22)

It is important to note that the CVA and DVA parts of the BCVA differ from the UCVA and UDVA. The difference comes from the fact that when both the investor and the counterparty are defaultable the order of default comes into play and with that the dependence between the two needs to be taken into account. Because of dependence the defaults need to be modelled using a bivariate distribution.

### 3.9 CVA in the Presence of Collateral

Collateral acts as a guarantee that reduces exposure according to (7). The derivation of CVA for collateralised derivative positions is analogous to the case without collateral with the difference that the present value is reduced by the value of the collateral. In the case of UCVA this leads to

\[ \text{UCVA}_{\text{coll}} = \int_0^T \mathbb{E}^{Q,M} \left[ (1 - R) \max\{V(t, T) - \text{Coll}(t); 0\} D(t, T)|\mathcal{F}_t \vee \{\tau = t\} \right] dP_D(t) \] (23)

### 3.10 CVA as an Option

By looking at the expressions for UCVA, UDVA and BCVA in (10), (18) and (22) the similarities to option pay-offs are identified by the presence of the \[ \max\{\cdot\} \] and \[ \min\{\cdot\} \] functions. In fact, the UCVA is an American call option on the residual PV at the time of default with strike zero and where the exercise is contingent on the default event. The investor is short this option at an amount equal to \[ LGD \] and the counterparty is long the same quantity of the option. By analysing formula (10) more closely one sees that the UCVA is the time average of the discounted option pay-off weighted with the probability that the option will be exercised in each instance and taking the, possibly time-dependent, \[ LGD \] into account. Analogously, UDVA is an American call option that the investor is long and the counterparty short. Similarly to the Merton option on a firm’s assets (Black and Scholes, 1973) the DVA describes how a firm can gain from a default by having its losses capped by the market value of the firm. Thus, the DVA is a benefit to the investor that increases the value of a derivative contract through providing him with the option of not having to pay the full liability that he has taken on. The BCVA, obviously, contain both options with the intricacy that they are contingent on each other, meaning that when one option has been exercised the other one becomes worthless.
Kenyon and Stamm (2012) discuss the consequences of the option-like character of CVA and conclude that its price depends on the volatility of the PV of the underlying contract and is as such model-dependent. Derivative contracts where the counterparty credit risk-free value is model-independent become model-dependent once counterparty credit risk is taken into account. For example, the risk-free price of an IRS depends only on the term structure prevailing at the time of valuation and is hence model-independent. When CVA is added the term structure needs to be dynamically modelled in order to incorporate the optionality correctly.

3.11 Discretisation and Simplifying Assumptions

In order to be able to compute CVA numerically the CVA formulae (10), (18) and (22) need to be discretised. This is done by dividing the time to maturity into a number of periods, so called time buckets, and assuming that the exposure, the recovery rate and the probability of default are constant over each bucket. Gregory (2010) discusses two simplifications that are commonly made:

1. The exposure is independent of the probability of default.
2. The recovery rate is independent of exposure and the time of default and commonly even assumed to be constant.

Both assumptions are controversial, but the first one can be outright wrong. The concepts of right-way and wrong-way risk are used to describe correlation between credit quality and exposure and are important to consider before making the first assumption (Pykhtin and Zhu, 2007). Right-way risk describes a situation where credit quality and exposure are positively correlated, that is, when exposure has the tendency to decrease as credit quality deteriorates. Wrong-way risk describes the opposite situation, in which credit quality and exposure are negatively correlated, such that exposure has the tendency to increase as credit quality worsens. Pykhtin and Zhu (2007) exemplifies the two cases by a swap transaction between an oil company and a bank, where one party pays fixed and the other pays floating crude oil price. Since the prospects of an oil company are negatively affected by a lower oil price its credit quality will be deteriorating in the long term. If the oil company pays floating and the bank fixed, the exposure of the bank decreases with a lower oil price and the risk is right-way. If, on the other hand, the oil company pays fixed and the bank floating, the exposure of the bank increases with a lower oil price and the risk is wrong-way. Pykhtin and Zhu (2007) write “While right/wrong-way risk may be important for commodity, credit and equity derivatives, it is less significant for FX and interest rate contracts. Since the bulk of banks’ counterparty credit risk has originated from interest-rate derivative transactions, most banks are comfortable to assume independence between exposure and counterparty credit quality.” Though the statement is a generalisation it can provide some comfort to banks with their trading activities mainly concentrated to the money and foreign exchange markets.

The second assumption is less discussed and it is generally very hard to determine how the recovery rate is related to exposure, time of default and time in general. Usually
the true recovery rate is known quite some time after a default has occurred, depending on the length of the bankruptcy proceedings. It has therefore become standard to assume a constant recovery rate, which is also commonly done in the valuation of CDSs (Brigo et al., 2013). However, the recovery rate plays an important role when determining CVA, since it scales the exposure linearly and has the possibility to provide a large discount for the investor in the case of default. Recovery rates can vary very much even within the same industry. In a study by Emery and Ou (2009) the recovered values on senior unsecured bonds of financial institutions that defaulted in 2008 are presented. The recovery rates range from less than 5 %, such as for the Icelandic banks Kaupthing, Glitnir and Landsbanki, and above 70 %, such as for the American bank GMAC. Computing CVA with a recovery rate of 5 % or 70 % certainly results in vastly different numbers.

Following Gregory (2010), the discretised version of the UCVA with \( N \) time periods under the assumption of a constant recovery rate can be written as

\[
CVA \approx (1 - R) \sum_{i=1}^{N} \mathbb{E}E^*(t'_i)D(0,t'_i)\Delta P_D(t_i),
\]

where \( t'_i \) denotes the midpoint of the interval \((t_{i-1}, t_i]\), \( \mathbb{E}E^*(t'_i) \) is the expected exposure at time \( t'_i \) conditional on counterparty default in that instance and \( \Delta P_D(t_i) = P_D(t_i) - P_D(t_{i-1}) \) is the probability of default occurring in the interval \((t_{i-1}, t_i]\). Under the assumption of independence between exposure and probability of default \( \mathbb{E}E^*(\cdot) \) is exchanged with \( \mathbb{E}E(\cdot) \), which is the expected exposure without conditioning on default time. In the discretised formula the discount factor has been separated from the exposure and moved out of the expectation with the result that the expected exposure is discounted using discount factors derived from the empirical yield curve as of the time of valuation. This is theoretically correct as long as the expected exposure is computed under the T-forward measure\(^{18}\) \( Q^T \), under which forward prices are martingales (Brigo and Mercurio, 2006).

The discretisation is illustrated in Figure 8. The smooth curve (continuous) shows the development of the exposure over time weighted with the instantaneous default probability curve (PDF) and the (possibly dependent\(^{19}\)) loss-given-default. The CVA is given by integrating this curve over time from valuation date until maturity. The step function (stroked) is the discretised version of the smooth curve. The time has been divided into ten buckets, over which the exposure and loss-given-default are assumed constant. The step function is the result of weighting the exposure over each bucket with the probability of default in each bucket and the loss-given-default. An approximation of the CVA is given by summing up the areas under the rectangles formed by the step function.

\(^{18}\)The T-forward measure was first introduced in Jamshidian (1987).

\(^{19}\)The loss-given-default can be dependent on probability of default, exposure and time.
3.12 Hedging CVA and Market Completeness

After having derived formulae for CVA the hedging aspect should be more thoroughly discussed. As was noted earlier the CCDS contract can potentially fully hedge counterparty credit risk. However it is a rather synthetic instrument and though it has the possibility to transfer counterparty credit risk to an external party, the external party will have problems to hedge the risk he has assumed via the CCDS. The instrument is really only a means of transferring risk and when one also considers the small size of the CCDS market this approach to hedging becomes even less viable.

The approach that will be taken instead is to disentangle the CVA into different components and hedge each component by itself. Following Cesari et al. (2009) the components of CVA that need to be hedged are

- **Credit spread** represented in the CVA formula as the probability of default.
- **Underlying market risk** consisting of market risk factors that determine the exposure.
- **Cross-dependency** between credit spread and underlying market risk factors.

Since the CVA of a netting set is a quantity dependent on the exposure and the probability of counterparty default over the whole term of the netting set the above hedges has to be matched over the **term structure**, which amounts to constructing hedges consisting of several instruments with maturities and different notional values distributed along the time horizon. In the following the three components will be discussed in the context of hedging the CVA of an arbitrary netting set with some counterparty.
3.12 Hedging CVA and Market Completeness

3.12.1 Credit Spread

The credit spread is generally the main driver of CVA volatility and it is typically hedged using CDS contracts. A CDS hedges credit migrations, i.e., changes in the probability of default, and to some extent recovery risk. Credit migrations have the effect of widening or tightening the credit spread with the consequence of an increasing or decreasing CVA, respectively, and with the value of the CDS changing in the same direction. The recovery risk is difficult to hedge since it can be highly uncertain, as was previously discussed. However, by matching the seniority of the bond tracked by the CDS with the seniority of the netting set the risk is reduced. OTC derivatives are pari passu with senior bond holders and a CDS hedge should therefore contain CDSs on senior bonds (Gregory, 2010).

The hedge of the credit spread component of CVA amounts to constructing a portfolio of CDSs with different maturities that matches the exposure and term structure of the netting set. In the best case there is a liquid market of so called single-name CDSs, i.e., CDSs with the counterparty as reference entity, and otherwise the hedge can be set up using CDSs on indices. The benefit of using single-name CDSs is that idiosyncratic risk can be hedged, which includes any default event. In the case of CDSs on indices only the systematic risk can be hedged and the effectiveness depends on the correlation between the counterparty and the index. Also, a default of the counterparty cannot be hedged using index-linked CDSs. Despite this, it is uncommon to use single-name CDSs for hedging because of hedging costs. However, for a derivative book with several counterparties an index hedge can work well since diversification across the counterparties comes into force.

The credit hedge can be set up as a static or as a dynamic hedge. The two strategies are illustrated in Figure 9.

The static credit hedge is constructed from the Potential Future Exposure (PFE) profile, which is a quantile measure similar to Value-at-Risk (VaR) and quantifies the exposure that will not be exceeded at a given confidence level. Following Gregory (2012) the PFE at the confidence level $\alpha$ is defined as the $\alpha$-quantile of the PV-distribution, i.e.,

$$PFE_\alpha(t) = \inf \{ e \in \mathbb{R} : F_{V(t,T)}(e) \geq \alpha \},$$

(25)

where $V(t,T)$ is defined in (2) and $F_{V(t,T)}$ is its CDF. In the example the hedge has been built from the PFE profile at the confidence level 90 %, which means that in 90 % of all cases the hedge will represent over-hedging and in 10 % of the cases it will be under-hedging. As a result this strategy is rather expensive.

The dynamic hedging strategy is constructed from the Expected Exposure (EE) profile and as with any dynamic hedging strategy it needs to be adjusted frequently in order to account for changes in credit spread and EE profile along the term structure. Because of the interaction between the term structure of default and the EE profile in the determination of CVA it is difficult to say how a change in credit spread affects the CVA. It depends on the shape of the EE profile and where the default probability is concentrated. For example, if the credit spreads change such that the default probability...
3.12 Hedging CVA and Market Completeness

3.12.1 Credit Value Adjustment

Figure 9: A comparison of a static and a dynamic hedging strategy of the credit part of CVA. Both hedges are constructed from eight CDSs, depicted by staples, with maturities spread along the term of the netting set. The static hedge is constructed from the PFE profile at the confidence level 90%, shown as the stroked line, and will cover the credit loss in 90% of all cases. The dynamic hedge is constructed from the EE profile and is shown as the continuous line.

increases in the short term and decreases in the long term the CVA will be increasing if the EE profile reaches its maximum in the short term and decreasing if the profile reaches its maximum further in the future. The hedge should be able to capture the CVA changes as long as it consists of sufficiently many CDSs with different tenors along the term of the netting set. The key to a successful dynamic credit hedge is the ability to rebalance the hedge frequently enough such that it tracks large changes in the CVA but not so frequently such that it becomes too expensive.

The ultimate success when constructing any of the two types of credit hedge portfolios is dependent on the existence of a liquid CDS market with contracts of the right seniority and with a sufficient range of maturities. Gregory (2010) concludes that the most practical and cheapest approach for banks is to hedge the credit risk in CVA using index-linked CDSs and to construct overall hedges across groups of counterparties.

3.12.2 Underlying Market Risk

The exposure component of CVA is driven by market risk factors, e.g. interest rates, FX rates, equity prices and volatilities. Because CVA is dependent on the future development of the exposure not only changes in spot and forward prices of the factors are relevant, but also the volatility of each factor. This is a result of the option-like character of CVA. For example, if the implied volatility of a risk factor increases but the price of it is constant the EE profile will shift upwards because of the increased uncertainty of the future exposure.
Hedging changes in CVA that are due to changes in market risk factors is very similar to hedging changes in the counterparty credit risk-free PV due to changes in the same factors: Hedges of spot and forward price changes are accomplished using delta hedges, i.e., through taking positions directly in the underlying risk factor, and possibly gamma hedges, i.e., through taking positions in derivatives whose values are dependent on the value of the risk factor. Hedging volatility changes is accomplished using vega hedges, which can consist of taking positions in variance swaps, if a liquid market exists, or in derivatives that are sensitive to the same volatility. The size of a hedge position is determined by the sensitivity of the CVA to the particular risk factor. The sensitivity will generally need to be determined numerically as a difference quota through computing the CVA with a shifted risk factor price.

Just as in the case of credit spread hedging, market risk hedges have to match the term structure of the netting set. Consider for example the delta hedge of the interest rate for the CVA of a netting set containing IRSs in one currency: The delta hedge could consist of positions in Forward Rate Agreements (FRA) with different notional values and maturities spread along the term structure. The corresponding vega hedge could be performed via a portfolio of swaptions with maturities spread over the term structure.

The similarity in hedging strategies for market risks between the counterparty credit risk-free PV and the CVA raises the question whether CVA hedging should be performed by a designated CVA desk or by the trader taking the position in the first place. Since the CVA is a cost its value decreases for changes in market risk factors that make the counterparty credit risk-free PV increase. This has the implication that the hedges have opposite signs and if the two hedging activities are split the role of the CVA desk will be to unwind parts of the hedging positions that the traders originally have taken (Gregory, 2010).

### 3.12.3 Cross-Dependency

One of the complexities of CVA is that it is determined by both market and credit risk factors. By hedging credit spread changes and market risk changes separately dependencies between the two risk types are ignored. These dependencies are called cross-gammas and are in general very difficult to hedge because the true dependence is hard to estimate and there is often a lack of instruments in the market with the relevant sensitivities (Gregory, 2010). Failure to hedge cross-gammas can, in the case of high correlation, lead to severe over- or under-hedging even if the credit and the market risk components have been perfectly hedged in isolation.

In the cases of right-way or wrong-way risk cross-gamma hedging is performed through adjusting the notional amounts of the CDSs in the credit hedge portfolio such that they take exposure changes that are due to changes in credit spread into account. The CDS notional are determined through computing the sensitivity of the CVA with respect to credit spread changes under the assumption that the exposure also changes due to correlation. Since correlation does not reflect dependence in general this approach is rather coarse.
3.12 Hedging CVA and Market Completeness

3.12.4 A Note on Hedges and CVA

A portfolio that hedges CVA will contain a range of instruments and potentially other OTC derivatives. Thus, the hedge portfolio may itself give rise to CVA and it is therefore necessary to carefully consider how the hedging is performed. In the best case hedging can be performed over an exchange, such that the hedge instruments do not add counterparty credit risk to the books. If this is not possible the hedging transactions can be entered with counterparties where collateral agreements exist, over a centralised clearing house or, at least, with a counterparty with a high credit quality, such that the counterparty credit risk originating from the original OTC derivative is exchanged for a lower risk coming from the hedge counterparty.

3.12.5 Hedging DVA and Bilateral CVA

If an institution would like to hedge its DVA it would theoretically be performed analogously to the hedging of CVA, i.e., through hedging credit spread, underlying market risk and cross-dependency separately. To hedge the credit spread component of DVA the institution would sell CDSs referenced to itself. However, it is highly doubtful whether it is possible to find a counterparty willing to buy CDSs referenced to the seller (Carver, 2012a). The alternative to selling CDSs on oneself is to short the own bonds. Another possibility is to use index-linked CDSs, which is the most viable alternative for hedging the own credit spread and the efficiency depends on how correlated the credit spread of the own institution is with the credit index. This strategy does not hedge idiosyncratic risk in general and a default event in particular. An open question is whether it is really meaningful for an investor to hedge his own default.

To hedge BCVA an institution would combine the CVA and the DVA hedges. The credit spreads would be hedged via portfolios of CDSs and the underlying market risk hedge would be combined into one portfolio. The hedges of market risks for CVA and DVA consist of positions in the same instruments with the difference that one portfolio contains a long position and the other a short position. The hedging positions of the BCVA would hence be smaller compared to a hedge of UCVA, which shows that DVA reduces the sensitivity of CVA to market risks. BCVA adds another level of complexity to the hedging through introducing an additional risk component that arises from the cross-dependency between the credit spreads of the counterparty and the investor. Like the cross-gamma of credit spread and exposure, the cross-gamma of the credit spreads is very difficult to hedge. Gregory (2010) suggests a hedging strategy consisting of hedging the counterparty’s credit spread via index-linked CDSs and adjust the positions using the betas of the own institution and the counterparty against the credit index. This strategy will take the correlation between the credit spreads of the two parties into account, but not hedge the credit spreads directly, which means that changes in the credit spreads are not fully hedged.
3.12.6 Implications for Market Completeness and Pricing

In Section 3.3 the reliance of risk-neutral pricing on market completeness was mentioned. Since market completeness requires hedgeability it is sensible to discuss the pricing approach in connection with hedging. It has been seen that CVA hedging can be problematic, but is possible, at least in theory, when the dependence structure between credit spread and exposure is nil or fully described by linear correlation. In reality the hedgeability is rather a question of the existence of hedging instruments and their liquidity. Risk-neutral pricing relies on the non-existence of arbitrage opportunities, such that if any mispriced contract would exist on the market some market participant would immediately be able to realise a profit through taking a position in the mispriced contract and constructing a replicating portfolio effectively nullifying the risk in the positions. It is questionable if this is practically possible in the case of counterparty credit risk with the implication that the computation of CVA cannot be entirely reduced to a risk-neutral pricing problem (Gregory, 2012). In this case CVA is determined as the cost of an imperfect hedging strategy with an additional component to reflect the hedging error.
4 Implementing CVA in the Pricing of OTC Derivatives

In this section it is discussed how CVA can be computed for single derivatives and netting sets using numerical methods and hence be implemented in a pricing tool. As was noted in Section 3.10 CVA increases the complexity of the pricing for all kinds of derivatives through adding model-dependence to the valuation and will generally require that the valuation of even the simplest derivative contracts is carried out using simulation methods.

This section focuses on UCVA, i.e., it is once again assumed that the counterparty is risky and the investor is default risk-free. In the end it is discussed how the method can be adjusted to compute BCVA, i.e., to also allow for investor default.

4.1 Computing CVA via Simulation Methods

Simulation methods are commonly used for the pricing of exotic derivative contracts and this is also the case for calculating CVA\(^{20}\). Simulation methods have the advantage of being very general and the limitation of the kind of financial instruments that can be handled is set only by the ability to find a suitable model of the market. The disadvantages of simulation methods are that they are computationally intense and time-consuming.

In this section it is described how CVA is priced numerically for a netting set in a so called CVA engine. The netting set contains an arbitrary number of OTC derivative contracts that are uncollateralised\(^{21}\).

Simulation methods in derivative pricing relies on Monte Carlo simulation of the future state of the market, such that a large number of market scenarios are generated. Each scenario is a description of how the market may develop from today until the time horizon of the simulation. The pay-off of a derivative contract is determined in each scenario and the price today is given by discounting the expected pay-off, which is given by the average of the pay-offs in all scenarios. The same approach is taken when pricing CVA: the pay-off of the CVA option is determined in each scenario, the average pay-off over all scenarios is determined and discounted back to today. Because the counterparty might default at any time from today until maturity of the netting set the pay-off in each scenario is an expected value, given by weighting the exposure at each time bucket with the probability of default over that bucket.

The steps for computing CVA of a netting set with simulation methods are described in the following. The steps are

1. Identification of risk factors and construction of market scenarios.
2. Valuation of the derivatives in the netting set at future dates.

\(^{20}\)Recall that CVA has the structure of an American option with exercise contingent on an external event, see Section 3.10.

\(^{21}\)The simulation method can be extended to collateralised transactions, in which case the collateral process is incorporated in the model and the exposure computed as in (7).

4. Weighting the exposure profiles with the probability of counterparty default and loss-given-default, and average over all scenarios.

5. Discounting of the weighted expected exposure profile and summation.

The steps will be illustrated using an example of one single IRS in the currency CHF, that pays the fixed rate annually, receives the floating rate quarterly and matures in ten years. The method is completely analogous for a netting set containing several contracts, but a single contract was chosen in order to keep the example more manageable.

4.1.1 Identification of Risk Factors and Scenario Generation

The first step before starting the simulation is to identify the risk factors that are relevant for the contracts in the netting set. Risk factors are parameters that determine the prices of the contracts and examples for some common derivatives include interest rates, foreign exchange rates, equity prices, credit spreads and volatilities. In order to be able to simulate the values of the risk factors the (multivariate) distribution of risk factor changes are required. The most common method is to model each risk factor using a stochastic process and to model the dependence between a set of factors by incorporating correlation between the stochastic drivers of each risk factor process. To exemplify a market model of this kind, consider a portfolio containing an interest rate derivative and an equity derivative with the same counterparty. Risk factors for this portfolio can include the short interest rate, the equity price and the credit spread of the counterparty. After choosing a Hull-White 1-factor process for the term structure of interest rates (Hull and White, 1990), a Geometric Brownian Motion for the equity price (Black and Scholes, 1973), a Cox-Ingersoll-Ross process for the credit spread (Cox et al., 1985) and correlations between the processes the following hybrid model is obtained:

\[
\begin{align*}
    dr_t &= (\theta(t) - \alpha r_t) dt + \sigma_r dW^r_t \\
    dE_t &= \mu E_t dt + \sigma_E E_t dW^E_t \\
    d\lambda_t &= k(\theta - \lambda_t) dt + \sigma_\lambda \sqrt{\lambda_t} dW^\lambda_t \\
    dW^r_t dW^E_t &= 0.05 dt \\
    dW^r_t dW^\lambda_t &= 0.03 dt \\
    dW^E_t dW^\lambda_t &= 0.08 dt,
\end{align*}
\]

(26)

where the short rate is denoted by \( r_t \), the equity price by \( E_t \) and the default intensity by \( \lambda_t \). The parameters \( \theta(t), \alpha, \mu, k \) and \( \vartheta \) determine drifts and mean reversions, and the parameters \( \sigma_x, x \in \{ r, E, \lambda \} \) determine volatilities with the subscripts referring to the short rate, the equity price and the default intensity, respectively. \( W^x_t, x \in \{ r, E, \lambda \} \) are standard Brownian Motions, with superscripts referring to the short rate, the equity price and the default intensity, respectively.
Correlation generally cannot capture the true dependence structure for different risk factors and as a result the method described above can generate scenarios in which the market is not arbitrage-free. In an arbitrage-free market there are certain restrictions on how different risk factors may relate to each other and the more risk factors are added to the market model, the more restrictions are added to which market scenarios are feasible (Cesari et al., 2009).

An example: assume that the netting set contains contracts on the currencies USD, EUR and SEK, and that the accounting currency is CHF. The risk factors in the market model include six FX rates and when generating arbitrage-free market scenarios the relationship between these FX rates needs to be handled carefully. For example, given the rates USD/CHF and USD/SEK the rate SEK/CHF is completely determined and restrictions are also put on the rates EUR/USD, EUR/CHF and EUR/SEK.

The criterion that the market scenarios are arbitrage-free is important, because if too many scenarios that allow arbitrage opportunities are generated the scenarios do not represent the true distribution of market states and as a result the distribution of derivative values will not reflect reality. It is therefore important to make sure that the market model only generates arbitrage-free scenarios or that each generated scenario is tested for arbitrage opportunities before it is used for pricing.

Since CVA depends on both market risk factors and credit risk factors the market model should ideally incorporate risk factors of both kinds. The dependence structure between the two dimensions can be captured through, for example, introducing correlation or a copula model.

Once the risk factors and their distribution have been specified the market model is discretised and calibrated to market data, such that it reflects the current state of the market. Then a large number, typically thousands or more, of scenarios are created. The scenarios are constructed through drawing random risk factor changes and propagating the market on a discrete time grid, \([t_0, t_1, ..., t_N]\), where \(t_0\) is the date of valuation, at which the market state is known, and \(t_N\) should be chosen to be the latest maturity date of the derivatives in the netting set or any later date. A fine time grid makes the simulation more accurate, but makes the computations more time-consuming.

For the netting set that we consider as an example the CHF yield curve was identified as risk factor and the market was modelled using the Vašíček short rate model (Vašíček, 1977). The short rate was simulated over ten years on a daily basis and the number of scenarios was set to 1’000. Cesari et al. (2009) write that 10’000 scenarios usually provide satisfactory results and is a good balance between accuracy and computation time for most models used in practice.

### 4.1.2 Valuation of Derivatives at Future Dates

When a set of feasible market scenarios has been constructed the development of the PV of each derivative is determined. This is done through performing full valuations

\[22\] Linear correlation can generally not capture the true dependence structure between the two dimensions and the use thereof is in most cases a simplification.
of each derivative at future dates until the maturity date of that derivative. Valuation
dates can be chosen to be the same as the grid dates, on which the simulation was
carried out, or any subset of those dates. Again, a finer valuation grid renders a more
accurate result but requires more computer performance. From the valuation dates time
buckets are created, such that one valuation date falls into each bucket. The valuation
dates should be chosen with respect to the derivatives in the netting set such that all
important dates are captured. For example, at least one valuation should occur between
each payout of a derivative. It is especially important to choose a fine grid in the case of
path-dependent derivatives, such as barrier options, where dates between payout dates
also may have a great influence on the PV. In commercial software it is common to use
a scheme with daily valuations the first two weeks, weekly valuations from two weeks
until three months, monthly valuations from three months until two years and thereafter
quarterly valuations. It is also common to use a dynamic date scheme, such that the
original time grid is adjusted so that two valuation dates are chosen around each payout.
One of the dates should lie close before the payout and the other date close after. This
ensures that large changes in the PV are captured and that the value in each time bucket
constitutes a representative time-average.

In Figure 10 five sample realisations of the PV process of the IRS are shown.

![Figure 10: Five sample realisations of the PV process of an IRS. The paths show how
the value of the swap develops over time in five different market scenarios.](image)

4.1.3 Construction of Exposure Profiles of the Netting Set

The next step in the calculation is to determine the exposure profile of the netting set
in each scenario. To this end, the PV paths of the netting set are constructed through
adding up the PV paths for all derivatives scenario-wise. Next, exposure profiles are
determined through capping the obtained PV paths at zero from below. An exposure profile shows the pay-off of the CVA option as a function of time in a specific scenario.

4.1.4 Weighting Exposure Profiles, Probability of Default and Loss-Given-Default

After the exposure profiles have been obtained the probability of default is computed over each time bucket and the exposure in each bucket is multiplied by the default probability over that bucket. If the term structure of counterparty default is incorporated in the market model this step has to be done scenario-wise. The probability-weighted exposure in each bucket in each scenario is then multiplied by the loss-given-default. The loss-given-default can also be included in the market model through defining it as a risk factor. When the weighted exposure profiles have been constructed the average in each time bucket is computed such that a single weighted exposure profile is obtained. This profile shows the expected loss on the netting set over time weighted with the probability of the loss being realised.

If one or both of the term structure of default and the loss-given-default are held constant, that is, not incorporated in the market model, one usually averages over scenarios before multiplying with these quantities in order to avoid performing the same operations several times.

In the example of the IRS neither the term structure of default nor the loss-given-default were modelled and in order to avoid multiplying the exposure in each scenario with these quantities the expected exposure was computed before the weighting was carried out. In Figure 11 the expected exposure profile of the swap is shown on the left-hand side and on the right-hand side the same profile is shown after weighting it with the default probability of Credit Suisse Group obtained from the default-curve shown in Figure 4.

![Figure 11: On the left-hand side the expected exposure profile of an IRS is shown. The curve shows the average over all market scenario. On the right-hand side the expected exposure profile has been weighted with the probability of counterparty default.](image)
4.1 Computing CVA via Simulation Methods

4.1.5 Discounting of the Weighted Exposure Profile and Summation

The last step of the CVA computation is to discount the weighted exposures from each time bucket back to today and then summing the terms. The result is the CVA, in other words, the expected loss on the netting set over its lifetime weighted with the probability that counterparty default occurs. An illustration of this procedure is shown in Figure 12.

Figure 12: Illustration of how CVA is computed numerically through discounting the expected exposure weighted with the probability of default and loss-given-default from each discrete time bucket and summing them up.

4.1.6 Extension to Bilateral CVA

In the case of Bilateral CVA (BCVA) the simulation method works similarly to the unilateral case. The main complexity that is added is that the dependence between the credit quality of the counterparty and the credit quality of the investor must be incorporated. This amounts to finding the joint distribution of default times of the two parties through e.g. the joint CDF of default denoted by $P^{(\tau, \tau_I)}(t, t_I)$. From this distribution probabilities of the kind $Q\left(\{t_{i-1} < \tau \leq t_i, \tau_I > t_i\}\right)$ can be computed, i.e., the joint probability of one party defaulting and the other party surviving during a certain time interval.

From the formula of BCVA (22) one realises that it is not only the exposure that is needed for the computation, but also its counterpart negative exposure, which is the exposure that the counterparty faces. Its definition is completely analogous to that of exposure, i.e.,

$$NE(t) = \min\{V(t, T); 0\}$$  \hspace{1cm} (27)

The steps for computing bilateral CVA of a netting set are

1. Identify risk factors and construct market scenarios.
2. Value each derivative in the netting set at future dates.
3. Construct the exposure profiles and the negative exposure profiles of the netting set.
4. (a) Weight the exposure profiles with the joint probability of counterparty default and investor survival, i.e., $Q\left(\{t_{i-1} < \tau \leq t_i, \tau_I > t_i\}\right)$ and loss-given-default of the counterparty. Construct the weighted expected exposure profile as the arithmetic average over all scenarios.

(b) Weight the negative exposure profiles with the joint probability of investor default and counterparty survival, i.e., $Q\left(\{\tau > t_i, t_{i-1} < \tau_I \leq t_i\}\right)$ and loss-given-default of the investor. Construct the weighted expected negative exposure profile as the arithmetic average over all scenarios.

5. Discount the weighted expected exposure profile and the weighted expected negative exposure profile and sum up all discounted values.

4.2 Computing CVA via Replication with Options

In some special cases CVA can be approximated via a replication strategy consisting of options with zero strike and a range of maturities. The underlying asset of the option is a basket of the derivatives in the netting set for which the CVA is to be computed. If one single derivative is of interest, such as an IRS, the method is a viable alternative. An option on an IRS is called a swaption, which is a traded instrument, and depending on model choice there even exist closed-form expressions for its price. For other derivatives and for netting sets in general it is complicated to determine the values of the options in the replication strategy and one would rather determine the CVA directly via simulation methods, as discussed previously.

When using the option approximation method the lifetime of the derivative in question is divided into discrete buckets. The options in the replication strategy are chosen such that each option matures in one bucket. The exposure of the derivative in each bucket is approximated by the PV of the respective option. The exposure in each bucket is then weighted with the probability of default over that bucket and the loss-given-default. Finally the weighted exposures are discounted and summed, as was illustrated in Figure 12.

This method of computing CVA has the advantage of being less computationally intense. The obvious disadvantage is the limitation to derivatives for which there are simple option price formulae and the de facto inability to compute CVA for netting sets.

4.3 Implementation of a CVA Tool for Interest Rate Swaps

A tool for the computation of CVA for single IRSs was implemented, following the procedure described in Section 4.1. The default probability curves were not modelled, but kept constant as of the date of valuation. As risk factor the term structure was identified and it was simulated using two different short rate models. In the following there is an exposition on the valuation of IRSs, a description of the rate models and an account of a method for constructing proxy CDS spread curves in the absence of traded CDSs referenced to a particular counterparty. As conclusion the results obtained are presented for different set-ups of rate models and default probability curves.
4.3 Implementation of a CVA Tool for Interest Rate Swaps

4.3.1 Valuation of Interest Rate Swaps

A CVA tool needs to incorporate valuation models for the assets that it supports. Björk (2009) gives an account of the valuation of IRSs settled in arrears, which has been the basis of this exposition. In this section the price formula for a payer swap will be stated, i.e., a swap that pays a fixed rate and receives a floating rate. The price of a receiver swap is merely a matter of changing signs.

An IRS is tied to a floating rate, such as the 6 month LIBOR rate. At inception the swap rate, which is the fixed rate, is determined such that the PV of the swap is zero. Define two sets of dates:

\[ \{ t^\text{float}_i \}_{i=0,...,N} \]

is the set of dates that are relevant for the floating payments. Floating payments occur at the dates \( t^\text{float}_1, ... t^\text{float}_N \) and the rate over the period \( (t^\text{float}_i, t^\text{float}_{i+1}) \) is fixed at date \( t^\text{float}_i - \text{offset} \), where the offset is typically two banking days. Thus the first floating rate is fixed at date \( t^\text{float}_0 - \text{offset} \).

\[ \{ t^\text{fix}_j \}_{j=0,...,M} \]

is the set of dates that are relevant for the fixed payments. Fixed payments occur at the dates \( t^\text{fix}_1, ... t^\text{fix}_M \). Furthermore, let \( K \) be the nominal amount, \( r^\text{fix} \) the swap rate, \( \delta^\text{fix}_j \) the day count over the interval \( (t_{j-1}, t_j) \) according to the fixed rate convention, \( L(t^\text{float}_i, t^\text{float}_{i+1}) \) the floating rate over the time interval \( (t^\text{float}_i, t^\text{float}_{i+1}) \), \( \delta^\text{float}_i \) the day count over the interval \( (t_{i-1}, t_i) \) according to the floating rate convention and \( D(s,t) \) the discount factor that takes a payout at time \( t \) back to time \( s \). The price for the swap at time \( t \) is

\[
\Pi(t) = \sum_{i=\min \{ k \in \{1,...,N\} : t \leq t_k \}}^{N} KL(t_{i-1}, t_i) \delta^\text{float}_i D(t, t_i) - \sum_{j=\min \{ k \in \{1,...,M\} : t \leq t_k \}}^{M} K r^\text{fix}_j \delta^\text{fix}_j D(t, t_j) \tag{28}
\]

4.3.1.1 A Note on Discounting in the Presence of CVA

In the pre-crisis times, before counterparty credit risk was consistently incorporated in the pricing of OTC derivatives, interest rate and foreign exchange products were valued and discounted using swap curves, which were derived from the LIBOR market. The LIBOR rate is the reference rate for unsecured transactions given by an average of deposit rates that a number of member banks offer each other and is quoted for terms ranging from one day to one year. The LIBOR rate was generally seen as a good proxy of the risk-free interest rate, which was a result of the credit and liquidity risk in the interbank market being considered low and therefore ignored to a large extent as Bianchetti and Carlicchi (2012) explain. Alongside LIBOR the OIS rate is another reference rate for unsecured transactions based on overnight indexing. This rate is the average of the rates at which overnight transactions have been executed among a number of member banks. Because of the short-term horizon this rate is considered as the best proxy of the risk-free rate, especially in the post-crisis era. In the past the spreads between LIBOR
4.3 Implementation of a CVA Tool for Interest Rate Swaps Credit Value Adjustment

and OIS rates have been small but from 2007 and onwards the spreads have become larger, which can be seen in Figure 13, adopted from Bianchetti and Carlicchi (2012). The figure depicts the historical time series of the EURIBOR 6M rate, the EONIA 6M rate and the spread between them. According to the authors the spread has its origin from the different credit and liquidity risk that the rates represent.

As a result of the yield spread between LIBOR and OIS the LIBOR rate is no longer considered as a good proxy of the risk-free rate and the principles behind the valuations of interest rate and foreign exchange contracts have changed. In the case of OTC derivatives with cash collateral where margining is done daily it is market consensus to use the OIS curve for discounting because this is exactly the rate that the collateral will earn. Clarke (2010) provides a justification of this based on an arbitrage argument. In the case of uncollateralised OTC derivatives different opinions have been uttered regarding discounting. Bianchetti and Carlicchi (2012) suggest that a counterparty-specific funding curve should be used, whereas Hull and White (2013) claim that the correct way is to derive discount factors from the OIS curve because credit risk is incorporated through CVA and using LIBOR rates for discounting would result in credit risk being accounted for twice. In this thesis the swap curve is simulated through time and used for determining future PVs and these values are subsequently discounted using the OIS curve.\footnote{Since the T-forward measure is used for simulation the discount factors can be derived from the empirical OIS curve at valuation date with the advantage that the spread between the swap curve and the OIS curve does not need to be incorporated in the market model.}

Figure 13: Historical time series of the EURIBOR 6M rate, EONIA 6M rate and the spread between them. Adopted from Bianchetti and Carlicchi (2012).
4.3 Implementation of a CVA Tool for Interest Rate Swaps Credit Value Adjustment

4.3.2 Term Structure Models of the Short Rate

Short rate models are a specific type of term structure models, in which it is assumed that the short rate, denoted by \( r(t) \), is the single stochastic process driving the evolution of the term structure. However, since the short rate does not provide a full description of the term structure, a specification of the term structure equation is needed. If the term structure equation has the form

\[
p(t, T) = \exp \left\{ A(t, T) - B(t, T)r(t) \right\},
\]

where \( p(t, T) \) is the time \( t \)-price of a zero-coupon bond maturing at time \( T \), and \( A(t, T) \) and \( B(t, T) \) are deterministic functions, the model is called an affine term structure model. Two such models were implemented in the CVA tool, namely the Vašíček and the Hull-White 1-factor models.

4.3.2.1 Vašíček

The Vašíček model was first presented in Vašíček (1977) and has the property of being mean reverting to a long-term mean rate. The evolution of the short rate under the risk-neutral measure is described by the SDE

\[
dr(t) = k(\theta - r(t))dt + \sigma dW(t),
\]

where \( k \) is the speed of mean reversion, \( \theta \) is the long-term mean rate, \( \sigma \) is the constant volatility and \( W(t) \) is a standard Wiener process under \( Q_\mathcal{M} \). In the Vašíček model the deterministic functions in the term structure equation (29) are given by

\[
B(t, T) = \frac{1}{k} \left\{ 1 - e^{-k(T-t)} \right\}
\]

\[
A(t, T) = (B(t, T) - (T - t)) \left[ \theta - \frac{\sigma^2}{2k^2} \right] - \frac{\sigma^2 B^2(t, T)}{4k}
\]

For the purpose of computing expected exposure in the discretised CVA formula (24) the short rate needs to be simulated under the T-forward measure \( Q_\mathcal{T} \). Following Bohner (2011) the Vašíček-dynamics under this measure can be written as

\[
dr(t) = (k\theta - \sigma^2 B(t, T) - kr(t))dt + \sigma dW^T(t),
\]

where \( W^T(t) \) is a \( Q_\mathcal{T} \)-Wiener process. After discretising the formula using the Euler scheme one obtains

\[
r_{t+1} = r_t + (k\theta - \sigma^2 B(t, T) - kr_t) \Delta t + \sigma \sqrt{\Delta t} \varepsilon_t,
\]

\[24\]The term structure equation is found by inverting the empirical yield curve. For a detailed description of this procedure see Björk (2009).
where $\varepsilon_t \sim \mathcal{N}(0, 1)$.

### 4.3.2.2 Hull-White 1-Factor

The Hull-White 1-factor model was first presented in Hull and White (1990) and is an extension of the Vašíček model. It has the property of being able to fit the empirical term structure at the calibration date exactly, which is accomplished through introducing a time-dependent mean rate. The dynamics under the risk-neutral measure is described by

$$dr(t) = (\theta(t) - kr(t))dt + \sigma dW(t),$$

where $k$ is the speed of mean reversion, $\frac{\theta(t)}{k}$ is the mean rate at time $t$, $\sigma$ is the constant volatility and $W(t)$ is a standard Wiener process under $Q_M$. The name 1-factor comes from the fact that this model, as well as the Vašíček model, has one single stochastic term driving the short rate.

In the Hull-White 1-factor model the deterministic functions in the term structure equation (29) are given by

$$B(t, T) = \frac{1}{k} \left\{ 1 - e^{-k(T-t)} \right\}$$

and

$$A(t, T) = \ln \frac{p^*(0, T)}{p^*(0, t)} \left\{ B(t, T) f^*(0, t) - \frac{\sigma^2}{4k} B^2(t, T)(1 - e^{-2kt}) \right\},$$

where $p^*(0, s)$ is the market price at time zero of a zero-coupon bond maturing at time $s$ and $f^*(0, s)$ is the instantaneous forward rate maturing at time $s$ determined at time zero, both derived from the empirical term structure.

The short rate dynamics under the T-forward measure $Q^T$ is, following Bohner (2011), given by

$$dr(t) = (\theta(t) - \sigma^2 B(t, T) - kr(t))dt + \sigma dW^T(t),$$

where $W^T(t)$ is a $Q^T$-Wiener process. Applying the Euler scheme yields a discretisation given by

$$r_{t+1} = r_t + (\theta_t - \sigma^2 B(t, T) - kr_t) \Delta t + \sigma \sqrt{\Delta t} \varepsilon_t,$$

where $\varepsilon_t \sim \mathcal{N}(0, 1)$.

### 4.3.2.3 A Note on Model Calibration

In order for a rate model to be used in pricing it needs to be calibrated such that it represents the dynamics of the term structure prevailing in the market. The Vašíček and the Hull-White 1-factor models have three parameters each that determine the behaviour of the short rate.

In the Hull-White model the parameter $\theta(t)$ is determined such that the initial term structure given by the model exactly matches the empirical term structure at the date of calibration. The mean reversion $k$ and the volatility $\sigma$ are calibrated to interest rate options such as caps and floors or swaptions using some optimisation algorithm.
such that when these contracts are priced using the model the prices obtained are as close as possible to the market-observed prices (Brigo and Mercurio, 2006). In the implementation of the CVA tool the volatility of the Hull-White model could be fetched directly from the front office system.

4.3.3 Default Probabilities

The term structure of default probabilities of a counterparty is an essential input parameter in any calculation of CVA. In the implementation of a CVA tool there is a choice of modelling the term structure dynamically or assuming a static default curve.

In theory the term structure should be modelled in order to capture the true dynamics of default probabilities. Brigo et al. (2013) give account of default modelling using reduced-form intensity models, which is commonly used in the context of CDS pricing. In this setting the default time $\tau$ is considered as the first jump of a Poisson process with a stochastic intensity. The probability that default occurs before some future time $t$, for a counterparty that has not yet defaulted, is given by

$$Q(\{\tau \leq t\}) = 1 - \mathbb{E}\left[e^{-\int_0^t \lambda(u)du}\right],$$

(40)

where $\lambda(u)$ is the default intensity. Brigo et al. (2013) note that the second term on the right-hand side in the equation can be recognised as the time-zero price of a zero coupon bond in a short rate model with the short rate equal to $\lambda$. Thus in order to model the default intensity the spectrum of positive short rate models is at disposal.

If a static default term structure is used, the same reduced-form model is still fundamental. The difference is that instead of specifying a short rate model for the default intensity it is defined as a deterministic function of time.

The calibration of the model is done through observing CDS spreads of the particular counterparty and deriving the default intensity via the relation

$$\lambda(t) = \frac{S(0,t)}{LGD},$$

(41)

where $\lambda(t)$ is the forward instantaneous default probability, i.e., intensity, $S(0,t)$ is the spread at time zero of a CDS maturing at time $t$ and $LGD$ is the loss-given-default. O’Kane and Turnbull (2003) provide an account of CDS valuation and the construction of default intensity curves from CDS quotes. When deriving default probabilities from CDS spreads two quantities need to be estimated and practice is to assume a constant $LGD$ rate and then compute the default intensity from (41). Assuming a constant $LGD$ rate is in line with how CDSs are priced and the correctness of doing this is rather a question of how good current CDS models are. The calibration gives intensities at a set of discrete time points, which is determined by the maturities of CDSs that are available. The intensity function is then constructed over the desired time horizon through interpolation. In the case where a short rate model has been specified the calibration must also incorporate the volatility of CDS spreads. This volatility has to be derived from derivatives with CDSs as underlying asset, such as CDS options.
4.3 Implementation of a CVA Tool for Interest Rate Swaps Credit Value Adjustment

An advantage of modelling the default term structure vis-à-vis using a static default curve is that dependence between default probability and market risk factors can be incorporated. This is necessary in the presence of right- and wrong-way risk. In the implementation in this thesis the default term structure was chosen to be static with the drawback that the CVA tool is unable to capture dependence between exposure and probability of default. If dynamic modelling is chosen one faces the problem of calibrating the model to the volatility of CDSs. There exist very few CDS options in the market and it is therefore difficult to calibrate the model. The choice of a static term structure is in most cases rather a consequence of the lack data to use in the calibration of a dynamic model than an outspoken wish to use the simpler model.

4.3.3.1 A Note on Default Probabilities in the Case of DVA and Bilateral CVA
When bilateral CVA, and DVA in particular, is computed it becomes necessary not only to model the default curve of the counterparty, but also the own term structure of default needs to be incorporated. The investor himself is most likely the market participant who has the best conditions for coming up with a good proxy of his own probability of default. However, it is questionable if a firm will frankly use its best proxy and through its bilateral CVA reveal its own views on its health. There will always be incentives for a firm to underestimate its default probability and thus be able to close transactions at lower than fair prices. Especially for firms with low credit quality there are incentives to underestimate the own probability of default in order to signal in the market that they are credit-worthy counterparties.

4.3.3.2 Cross Section of CDS Spreads
In the previous it has been assumed that there are market-quoted prices of CDS contracts over the relevant time horizon. However, in many cases there will not be any CDS contracts referenced to a particular counterparty and when there are, there might only be a few maturities that are liquid. In the absence of quoted CDS spreads there is a need to construct proxy curves that reproduce default term structures representing generic counterparties, which can be mapped to groups of actual counterparties. Chourdakis et al. (2013) present one such possibility. They describe a method of constructing proxy CDS spreads from quoted CDS spreads using a cross section method where the proxy spread is described by five factors. The first factor is a global one, which is explaining for all CDS spreads. The other factors are industry sector, geographical region of the obligor, credit rating of the obligor and seniority of the obligor. The proxy spread of an obligor $o$ is given by

$$S_{\text{proxy}}^o = M_{\text{global}}\cdot M_{\text{sector}(o)}\cdot M_{\text{region}(o)}\cdot M_{\text{rating}(o)}\cdot M_{\text{seniority}(o)} \quad (42)$$

In the implementation of the CVA tool the cross section method was used with factors chosen as in the original paper, with the difference that the set of industry sectors was reduced to Governments, Financials and Others. The data set of CDS spreads was
delivered by Markit\textsuperscript{25}. The remaining factors were chosen directly from the factors used by Markit: The set of region factors was \textit{Europe, North America, Japan, Asia excluding Japan, Australasia, Africa and Middle East} and \textit{Latin America}. The credit rating factors were a subset of the ratings issued by S&P, more exactly AAA, AA, A, BBB, BB, B and CCC. The seniority factors were \textit{SNRFOR} (Senior unsecured debt) and \textit{SUBLT2} (Sub-ordinated or lower Tier-2 debt). The number of factors thus amounted to 20.

The cross section was performed using the ordinary least squares method with a logarithmic regression model. Assuming $n$ credit spreads in the data set, the regression model was described by

$$\begin{pmatrix}
y_1 \\
y_2 \\
\vdots \\
y_n
\end{pmatrix} = \begin{pmatrix}
\tilde{a}_1 \\
\tilde{a}_2 \\
\vdots \\
\tilde{a}_n
\end{pmatrix} \begin{pmatrix}
x_1 \\
x_2 \\
\vdots \\
x_{20}
\end{pmatrix} + \begin{pmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\vdots \\
\varepsilon_n
\end{pmatrix},$$

(43)

where, for $i \in [1, n]$ and $j \in [1, 20]$, $y_i = \log S_i$ is the logarithm of the $i$th CDS spread, $\tilde{a}_i$ is a vector of length 20 containing binary values with the value one in the columns which correspond to factors that are relevant for the $i$th CDS spread, $x_j = \log M_j$ is the logarithm of the $j$th factor and $\varepsilon_i$ is the error term for the $i$th CDS spread.

The cross section was performed for CDS quotes in the currencies CHF, EUR and USD and for maturities in the set \{0.5, 1, 2, 3, 4, 5, 7, 10, 15, 20, 30\} years. From the factor estimates a set of default probability curves was constructed. The curve set included curves for unsecured debt in the three currencies for all sectors, all credit ratings, regions constrained to \textit{Europe} and \textit{North America} and seniority constrained to \textit{Senior unsecured debt}.

A sample term structure of default from the cross section is shown in Figure 14. The continuous curve shows default probabilities of \textit{Financials} with credit rating A in \textit{Europe} in the currency CHF over a period of 10 years originating from the cross section of CDS spread data. The stroked line shows the term structure of default of Credit Suisse Group, which at the time had been issued the credit rating A from S&P, derived from CDS spread data. The CDS data contained spread quotes from the market as of the third of September 2013. The graphs show that the proxy curve overall shows a lower default probability and that the term structure of default of Credit Suisse Group has a flatter shape. Especially it is seen that the proxy curve predicts a short-term default with a considerably lower probability than the curve derived directly from CDS quotes on Credit Suisse Group. One should bear in mind that the proxy curve is derived from an average of a set of relevant spreads in the CDS data set and therefore it must not necessarily coincide with a default curve of a real obligor, but rather be an average of the default curves from obligors in the same category.

\textsuperscript{25}Markit Financial Information Services: \url{http://www.markit.com/en/}
4.3 Implementation of a CVA Tool for Interest Rate Swaps  Credit Value Adjustment

![Figure 14: The term structure of default of Senior unsecured debt from Financials with credit rating A in Europe in the currency CHF over 10 years derived from a cross section of CDS spreads (continuous curve) and the term structure of default of Credit Suisse Group derived from CDS spreads (stroked curve). All CDS data was fetched from the market the third of September 2013.](image)

In the data set of CDS spreads delivered by Markit there are also quotes of the recovery rate for each obligor and seniority. For the proxy curves an additional cross section was performed in order to obtain a recovery rate for each curve. The cross section was performed using the same factors as for the CDS spreads. The model obtained was the same as the one in (43) with the difference that the model was not logarithmic.
4.3 Implementation of a CVA Tool for Interest Rate Swaps Credit Value Adjustment

4.3.4 Results

The output from the CVA tool for IRSs is a matrix containing margins in basis points for swaps with different maturities traded with counterparties with different credit ratings. The margin is used to adjust the counterparty credit risk-free par swap rate such that the swap trades at par in an environment with counterparty credit risk. For a receiver swap the margin is added to the swap rate and for a payer swap it is subtracted from the swap rate. An extract of the matrix from the 21st of August 2013 is shown in Table 2. The matrix contains margins for payer swaps paid in arrears with the floating rate tied to the three month CHF LIBOR paid quarterly and the fixed rate paid annually. Five swaps with different tenors are represented in the matrix and the CVA margin is reported in seven cases for each. In each case the swaps have been traded with a generic counterparty with a credit rating that is different from the other cases. The swaps had start date equal to the valuation date plus offset, such that the first floating rate was fixed at valuation date. The swap rate for each swap was determined such that the swap was fair in a counterparty credit risk-free setting. The term structure of interest rates was modelled using the Hull-White 1-factor model with mean rate parameter calibrated to the CHF 3M-LIBOR curve at valuation date and the volatility parameter calibrated to swaptions in the front office system. The term structures of default of the generic counterparties were calculated using the cross section method for Senior unsecured debt from Financials in Europe in the currency CHF. The cross section method was also used to determine the recovery rates for the generic counterparties. The matrix shows the pattern one would expect, namely that the CVA is strictly increasing with increasing tenor and strictly increasing with decreasing credit rating.

Table 2: Extract of the CVA matrix for payer swaps from the 21st of August 2013. The swaps were valued at inception for different tenors and with generic counterparties with different credit ratings. The Hull-White 1-factor model was used for modelling the yield curve and the term structures of default were computed using the cross section method. The CVA figures represent margins in basis points to be subtracted from the fair swap rate.

<table>
<thead>
<tr>
<th>Rating</th>
<th>Recovery rate</th>
<th>1</th>
<th>3</th>
<th>6</th>
<th>10</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>0.3996</td>
<td>0.00081056</td>
<td>0.02244601</td>
<td>0.29991398</td>
<td>1.12711568</td>
<td>1.89317385</td>
</tr>
<tr>
<td>AA</td>
<td>0.4005</td>
<td>0.00278994</td>
<td>0.05861815</td>
<td>0.52432972</td>
<td>1.55360361</td>
<td>2.40004953</td>
</tr>
<tr>
<td>A</td>
<td>0.4007</td>
<td>0.00357254</td>
<td>0.07828167</td>
<td>0.70259169</td>
<td>2.03411012</td>
<td>3.06842297</td>
</tr>
<tr>
<td>BBB</td>
<td>0.4007</td>
<td>0.00633005</td>
<td>0.14020505</td>
<td>1.16170190</td>
<td>3.20720758</td>
<td>4.71883212</td>
</tr>
<tr>
<td>BB</td>
<td>0.3991</td>
<td>0.01577213</td>
<td>0.32065931</td>
<td>2.46767354</td>
<td>6.40233893</td>
<td>9.26249676</td>
</tr>
<tr>
<td>B</td>
<td>0.4015</td>
<td>0.02671870</td>
<td>0.59604967</td>
<td>4.33278545</td>
<td>10.5049745</td>
<td>14.3677912</td>
</tr>
<tr>
<td>CCC</td>
<td>0.3937</td>
<td>0.06753167</td>
<td>1.15163113</td>
<td>6.42012869</td>
<td>14.0182219</td>
<td>30.0440879</td>
</tr>
</tbody>
</table>
4.3 Implementation of a CVA Tool for Interest Rate Swaps Credit Value Adjustment

4.3.4.1 Comparison of CVA for Different Short Rate Models

To study the impact of the term structure model a second CVA matrix was computed with the yield curve modelled using the Vašíček model. Apart from the term structure model, the set-up was the same as for the first CVA matrix. In Table 3 the CVA for payer swaps with a generic counterparty with credit rating A are reported as a margin in basis points. It is seen that the results obtained for the two different rate models are very different: The Hull-White 1-factor model leads to drastically higher CVA figures compared to the Vašíček model. For receiver swaps the case is the opposite, although the difference between the models is smaller. The reason for this is mainly the difference in mean rate in the two models. In the Hull-White model the instantaneous forward rates from the empirical yield curve (CHF 3M-LIBOR) at the date of calibration are effectively used as predictors of the short rate in the future. The short rate will hence have a time-dependent drift that is determined by the empirical yield curve. The CHF 3M-LIBOR curve that prevailed at the 21st of August 2013 is shown in Figure 15 and after inspection it becomes clear that with this curve the short rate in the Hull-White framework is predicted to rise from a low level of approximately 0.01% to levels above 2%. In the Vašíček model, on the other hand, the mean rate is kept constant at the level that prevailed at the calibration date, i.e., the short rate in the Vašíček framework is predicted to stay at the low level of approximately 0.01%.

Table 3: Comparison of the CVA for payer swaps computed using the short rate models Vašíček and Hull-White 1-factor.

<table>
<thead>
<tr>
<th>Term structure</th>
<th>Recovery rate</th>
<th>1</th>
<th>3</th>
<th>6</th>
<th>10</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vašíček</td>
<td>0.4007</td>
<td>0.00117857</td>
<td>0.01658097</td>
<td>0.08123319</td>
<td>0.21486672</td>
<td>0.39152369</td>
</tr>
<tr>
<td>Hull-White 1F</td>
<td>0.4007</td>
<td>0.01553132</td>
<td>0.17197248</td>
<td>1.08785600</td>
<td>2.57577422</td>
<td>3.69645555</td>
</tr>
</tbody>
</table>

Figure 15: Term structure from CHF 3M-LIBOR as of the 21st of August 2013.
Whether forward rates are good predictors of the future short rate or not is a question that is discussed in finance from time to time. However, even if this is not the case, the Hull-White model is concluded to be a better choice than the Vasicek model since it has the desired property of being able to reproduce the empirical yield curve at the calibration date. Additionally, in the current interest rate environment with interest rates at historically low levels it seems likely that interest rates will rise in the long term rather than stay at the low level, which is the assumption of the Vasicek model.

**4.3.4.2 Comparison of CVA for Different Sources of Default Probabilities**

In order to examine the impact of the term structure of default two additional CVA matrices were computed with different sources of default probabilities. The set-up was the same as for the first CVA matrix, but instead of using default probabilities from generic counterparties calculated using the cross section method two other sources were used. The full set of default probability curves was:

- Default probability curve of Credit Suisse Group, which at the time was rated A by S&P, implied from CDS spreads in CHF.
- Proxy curve computed using the cross section method for Senior unsecured debt from Financial in Europe in CHF with credit rating A.
- Default probability curve of A-rated companies derived from historical rating transition data from S&P.

In Table 4 the CVA for payer swaps with default probabilities from the three different sources is reported as a margin in basis points and in Figure 16 the corresponding term structures of default are shown. The first two rows in the CVA matrix correspond to the cases when the default term structure is market implied and the last row shows the CVA when the default term structure is derived from historical data. One clearly notes the large difference between using market-implied and historical data and it is concluded that using historical data when market data is not available leads to vastly different results. The two market-implied default curves lead to CVA margins with the same order of magnitude and as one would expect from looking at Figure 14, where the proxy curve is plotted together with the curve for Credit Suisse Group, the proxy curve gives slightly smaller CVA figures, especially for swaps with short tenors.
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Table 4: Comparison of the CVA for payer swaps computed using three different sources of default probability curves. The sources were Market implied with a curve derived from CDS spreads referenced to Credit Suisse Group, Cross section with a curve computed using the cross section method of CDS spreads and Historical with a curve derived from historical rating transition data from S&P.

<table>
<thead>
<tr>
<th>PD source</th>
<th>Recovery rate</th>
<th>1</th>
<th>3</th>
<th>6</th>
<th>10</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market implied</td>
<td>0.4000</td>
<td>0.01553132</td>
<td>0.17197248</td>
<td>1.08785600</td>
<td>2.57577422</td>
<td>3.69645555</td>
</tr>
<tr>
<td>Cross section</td>
<td>0.4007</td>
<td>0.00357254</td>
<td>0.07828167</td>
<td>0.70259169</td>
<td>2.03411012</td>
<td>3.06842297</td>
</tr>
<tr>
<td>Historical</td>
<td>0.4008</td>
<td>0.00001140</td>
<td>0.00017334</td>
<td>0.00116258</td>
<td>0.00367062</td>
<td>0.00650515</td>
</tr>
</tbody>
</table>

Figure 16: The graphs show term structures of default for A-rated entities with data from the three different sources: a) CDS spreads in CHF on Credit Suisse Group, b) a cross section of CDS spreads and c) historical rating transition data. Note the different scales on the vertical axes.
5 The Regulatory and Accounting Perspectives

The main focus has up to this point been the market perspective of CVA, in which CVA is a financial instrument that is priced and charged for when engaging in OTC derivative transactions. However, market practice is not the only set of rules defining the scope in which banks operate and, as mentioned earlier, in the context of CVA the regulatory and the accounting perspectives are also relevant.

5.1 The Basel Accords and the CVA Capital Charge

The Basel accords, with the first version introduced in 1988, define a framework for determining a minimum capital requirement that banks need to hold (Larson, 2011). The accords introduced the concept Risk-Weighted Assets (RWA), which is a weighting of the off-balance sheet exposures with a factor that depends on the riskiness of each asset class. RWA is used for calculating the Capital Adequacy Ratio (CAR), which is a measure of how much capital a bank holds in relation to its RWA, in other words, how well the bank’s balance sheet can absorb losses.

With the second Basel accord, introduced in 2004, the framework was extended and the original capital requirements were refined to include market risk and operational risk alongside credit risk, which was the focus of the first Basel accord. Counterparty credit risk was captured to the extent that reserves had to be held against the default risk of a counterparty (Fares and Genest, 2013). Derivatives were included in the RWA and their contribution was determined through one of several methods, depending on the internal modelling-capabilities of the bank in question. The capital charge on OTC derivatives under Basel II only kept reserves against counterparty default, but, as was mentioned in the introduction of this thesis, most of the losses that banks suffered during the financial crisis originated from credit migrations but not defaults. In other words, most losses were due to changes in CVA, which Basel II did not capture. With the third Basel accord the Basel Committee thus introduced a capital charge relating to CVA volatility with the aim of building up reserves against MtM losses and furthermore incentivise collateralised trading and moving trading to centralised clearing houses (Kenyon and Stamm, 2012).

5.1.1 The CVA Capital Charge

The CVA capital charge is a Value-at-Risk (VaR)-like quantity, in the sense that it is defined as a quantile of the distribution of CVA change over a given time period at the confidence level 99 % (Douglas and Pugachevsky, 2012). Unlike market VaR, which is concerned with the quantile loss, the CVA capital charge is concerned with the quantile gain, i.e., increases in CVA. The principle behind the calculation is to determine the distribution of the change in CVA for all netting sets over the relevant time horizon. The capital charge is defined as the 99 % quantile of this distribution. Basel III offers two possibilities for banks to compute the capital charge: a standardised method and an advanced method, which is based on internal models that have to be approved by
regulators. Under either of the two approaches the capital charge is calculated through regarding the credit spreads of the counterparties as the only factors driving the change in CVA.

When hedging was discussed in Section 3.12 it was concluded that the factors determining CVA were market risk factors and credit risk factors. It was argued that a complete CVA hedge would incorporate pure market risk hedges, pure credit risk hedges, in the form of CDSs, and additionally one would adjust the hedge positions to account for cross-gammas. Hedging has per definition the effect of decreasing P&L volatility and thus one would expect that through hedging CVA the related capital charge would also be reduced. This is partly the case and Basel III strictly states which hedges are allowed and these only include credit risk hedges, which is in line with the assumption that credit spreads are the only driving factors behind the change in CVA. The hedges that are allowed are restricted to CCDSs, single-name CDSs and index-linked CDSs.

5.1.1.1 The Standardised Method

Under the standardised method the Basel Committee on Banking Supervision (2011) defines the CVA capital charge by the formula

\[
\text{CVA-Charge} = 2.33 \sqrt{h} \left\{ \left( \sum_{i=1}^{n} \frac{1}{2} w_i (M_i EAD_i^{\text{total}} - M_i^{\text{hedge}} B_i) - \sum_{j=1}^{m} w_j M_j^{\text{ind}} B_j^{\text{ind}} \right)^2 + \sum_{i=1}^{n} \frac{3}{4} w_i^2 (M_i EAD_i^{\text{total}} - M_i^{\text{hedge}} B_i)^2 \right\}^{1/2},
\]

(44)

where

- \( h = 1 \) year is the capital horizon,
- \( n \) is the number of counterparties,
- \( w_i \) is a weight for counterparty \( i \), which depends on its external credit rating (the weights are found in Table 5),
- \( M_i \) is the effective maturity of the portfolio of transactions with counterparty \( i \), as defined in Basel Committee on Banking Supervision (2011),
- \( EAD_i^{\text{total}} \) is the Exposure-At-Default of the portfolio of all transactions, i.e., the sum of all netting sets, with counterparty \( i \), which can be computed using two different methods defined in Basel Committee on Banking Supervision (2011),
- \( M_i^{\text{hedge}} \) is the maturity of single-name CDS hedge referenced to counterparty \( i \),
- \( B_i \) is the notional purchased of single-name CDS hedge referenced to counterparty \( i \),
- \( m \) is the number of index hedges,
• \( w_j \) is a risk weight of index hedge \( j \), which depends on the average index spread,
• \( M_{j}^{\text{ind}} \) is the maturity of index-linked CDS hedge \( j \) and
• \( B_{j}^{\text{ind}} \) is the notional purchased of index-linked CDS hedge \( j \).

Table 5: The risk weights \( w_i \) to be used in the standardised formula (44) depending on external credit rating of the counterparty (Basel Committee on Banking Supervision, 2011).

<table>
<thead>
<tr>
<th>Rating</th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight</td>
<td>0.7%</td>
<td>0.7%</td>
<td>0.8%</td>
<td>1.0%</td>
<td>2.0%</td>
<td>3.0%</td>
<td>10.0%</td>
</tr>
</tbody>
</table>

By reverse engineering and identifying the terms in (44) the structure of the formula has been analysed by Pykhtin (2012) and Douglas and Pugachevsky (2012) among others. They arrive at the conclusion that the CVA capital charge under the standardised method is in fact the 99 % quantile of the change in value of a portfolio containing normally distributed assets with a specific variance matrix over the term one year. This will be shown in the following:

First equation (44) is rewritten using \( \vartheta_i = w_i(M_i EAD_{i}^{\text{total}} - M_i^{\text{hedge}} B_i) \) and \( \vartheta_{\text{ind}} = w_{\text{ind}} M_{\text{ind}} B_{\text{ind}}, \) i.e.,

\[
\text{CVA-Charge} = 2.33 \sqrt{h} \left\{ \sum_{i=1}^{n} \vartheta_i^2 + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{i-1} \vartheta_i \vartheta_j + \vartheta_{\text{ind}}^2 - \vartheta_{\text{ind}} \sum_{i=1}^{n} \vartheta_i \right\}^{1/2},
\]

where the notation has been eased by assuming that the index hedges originates from one single credit index. This represents no loss of generality since this hedge can be viewed as a hedge on an index of indices.

Second the change in total CVA, across all counterparties, with reduction for credit hedges included, that is due to changes in credit spreads, is linearised, yielding

\[
\Delta \text{CVA} = \sum_{i=1}^{n} \left( \Delta \text{CVA}_i - \Delta \text{Hedge}_i \right) - \Delta \text{Hedge}_{\text{ind}} \approx \sum_{i=1}^{n} \left( \frac{\partial \text{CVA}_i}{\partial s_i} - \frac{\partial \text{Hedge}_i}{\partial s_i} \right) \Delta s_i - \frac{\partial \text{Hedge}_{\text{ind}}}{\partial s_{\text{ind}}} \Delta s_{\text{ind}}
\]

where \( \text{CVA}_i \) is the CVA originating from the portfolio of all transactions with counterparty \( i \), \( \text{Hedge}_i \) is the value of hedges referenced to counterparty \( i \), \( \text{Hedge}_{\text{ind}} \) is the value of index hedges, \( s_i \) is the信用spread of counterparty \( i \) and \( s_{\text{ind}} \) is the credit spread of the credit index.

Next, it is assumed that the credit spread at time \( t \) of each counterparty \( i \) is log-normally distributed with mean \( s_i^0 \) and volatility \( \omega_i \), i.e., \( \ln(s_i^t) \sim \mathcal{N}(s_i^0, \omega_i \sqrt{t}) \), and...
that the correlation between all pairs of credit spreads are equal and constant, i.e.,
\[ \rho = \text{corr}(s^i_t, s^j_t) \] for \( i \in [1, n], j \in [1, n] \) and \( i \neq j \). The credit spread of the index at time \( t \) is also assumed to be log-normally distributed with mean \( s^0_{\text{ind}} \) and volatility \( \omega_{\text{ind}} \), i.e.,
\[ \ln (s^\text{ind}_t) \sim \mathcal{N}(s^0_{\text{ind}}, \omega_{\text{ind}}^2 \Delta t) \]
and the correlation between the index and each counterparty credit spread is \( \rho_{\text{ind}} = \text{corr}(s^\text{ind}_t, s^i_t) \) for \( i \in [1, n] \). The assumption of log-normally distributed credit spreads has the implication that each credit spread is a Geometric Brownian Motion and after applying Itô’s lemma to the credit spread dynamics the variation is found to be
\[ \Delta s^i = s^i_0 \Delta t - s^i_0 \left( \exp \left\{ -\frac{1}{2} \omega^2_i \Delta t + \omega_i \sqrt{\Delta t} \epsilon_i \right\} - 1 \right), \quad (47) \]
where \( \epsilon_i \sim \mathcal{N}(0, 1) \). After applying Taylor’s formula to the exponential function and ignoring terms of order higher than linear the variation becomes
\[ \Delta s^i \approx s^i_0 \left( -\frac{1}{2} \omega^2_i \Delta t + \omega_i \sqrt{\Delta t} \epsilon_i \right) \approx s^i_0 \omega_i \sqrt{\Delta t} \epsilon_i, \quad (48) \]
where in the second approximation it has been assumed that \( \omega^2_i \Delta t \) is close to zero.

For notational relief, let \( \sigma_i = s^i_0 \omega_i \). The linearisation and the normality assumption implies that a small change in credit spread is approximately normally distributed, i.e.,
\[ \Delta s^i \sim \mathcal{N}(0, \sigma_i \sqrt{\Delta t}) \]
By observing equation (46) and having in mind that credit spread changes are approximately normally distributed random variables and as such it is normally distributed as well. After introducing the notation \( \xi_i = \frac{\partial \text{CVA}_i}{\partial s^i_t} - \frac{\partial \text{Hedge}_i}{\partial s^i_t} \) and \( \xi_{\text{ind}} = \frac{\partial \text{Hedge}_{\text{ind}}}{\partial s^\text{ind}_t} \), the expectation and variance of the change in CVA can be written
\[ \mathbb{E}[\Delta \text{CVA}] = 0 \quad \text{(49)} \]
\[ \text{Var}[\Delta \text{CVA}] = \sum_{i=1}^{n} \xi_i \sigma_i^2 \Delta t + 2 \rho \sum_{i=1}^{n} \sum_{j=1}^{i-1} \xi_i \xi_j \sigma_i \sigma_j \Delta t + \]
\[ (\xi_{\text{ind}} \sigma_{\text{ind}})^2 \Delta t - 2 \rho_{\text{ind}} \xi_{\text{ind}} \sigma_{\text{ind}} \Delta t \sum_{i=1}^{n} \xi_i \sigma_i \]
\[ \quad \text{(50)} \]
The VaR at level 99 % of the total CVA change, i.e., the 99 % quantile of the total CVA change, is
\[ \text{CVA-VaR}_{99\%} = \Phi^{-1}(0.99) \sqrt{\Delta t} \left\{ \sum_{i=1}^{n} (\xi_i \sigma_i)^2 \Delta t + 2 \rho \sum_{i=1}^{n} \sum_{j=1}^{i-1} \xi_i \xi_j \sigma_i \sigma_j \Delta t + \right. \]
\[ (\xi_{\text{ind}} \sigma_{\text{ind}})^2 \Delta t - 2 \rho_{\text{ind}} \xi_{\text{ind}} \sigma_{\text{ind}} \Delta t \sum_{i=1}^{n} \xi_i \sigma_i \right\}^{1/2}, \quad (51) \]
where $\Phi(\cdot)$ is the standard normal CDF with $\Phi^{-1}(0.99) \approx 2.33$. By comparing the CVA capital charge under the standardised method in (45) with the CVA-VaR in (51) it can be concluded that the capital charge indeed is the 99 % quantile of the total change in CVA after assuming that credit spreads are the only factors driving the changes and that they are normally distributed with a specific variance structure. A comparison of the terms in the two equations is shown in Table 6. The table shows the components in the CVA-VaR as derived above in the left column and the components of the standardised capital charge in the right column.

Table 6: Comparison of the terms in the CVA-VaR as derived above (51) (left column) and the Basel III CVA capital charge under the standardised method (44) (right column).

<table>
<thead>
<tr>
<th>Sensitivity w.r.t. single-name CDS spread</th>
<th>Quantile of the CVA change</th>
<th>Basel III capital charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\partial CVA_i}{\partial s_i}$</td>
<td>$\frac{\partial \text{Hedge}_i}{\partial s_i}$</td>
<td>$M_i EAD_i^{\text{total}} - M_i^{\text{hedge}} B_i$</td>
</tr>
<tr>
<td>Sensitivity w.r.t. index CDS spread</td>
<td>$-\frac{\partial \text{Hedge}<em>{\text{ind}}}{\partial s</em>{\text{ind}}}$</td>
<td>$-M_{\text{ind}} B_{\text{ind}}$</td>
</tr>
<tr>
<td>Volatility of CDS spread change</td>
<td>$s_i^0 \omega_i \sqrt{\Delta t}$</td>
<td>$w_i \sqrt{h}$</td>
</tr>
<tr>
<td>Correlation single-name spreads</td>
<td>$\rho$</td>
<td>0.25</td>
</tr>
<tr>
<td>Correlation single-name and index spreads</td>
<td>$\rho_{\text{ind}}$</td>
<td>0.5</td>
</tr>
</tbody>
</table>

The sensitivities in the first two rows of Table 6 deserve a short explanation: With inspiration from the discretised CVA formula (8) approximate the CVA on netting set level with only one time bucket, i.e.,

$$CVA \approx (1 - R) \mathbb{E}(t') D(0,t') P_D(t'), \quad (52)$$

where $t'$ is some time point between valuation date and maturity date of the netting set chosen such that the approximation is as close as possible to the true CVA. Also assume that the default intensity of the counterparty is constant over time. By combining the probability of counterparty default from the default intensity (40) and the default intensity from the CDS spread and loss-given-default (41) the default probability can be computed as

$$P_D(t') = 1 - \exp \left\{ - \frac{s}{1-R} t' \right\} \approx \frac{s}{1-R} t', \quad (53)$$

where $s$ is the CDS spread of the counterparty. The approximation has been found through performing a Taylor development of the exponential function and ignoring terms.
of order higher than linear. By substituting (53) into (52) one arrives at
\[ \text{CVA} \approx E(t')D(0, t')s', \]  
and the sensitivity of CVA with respect to the CDS spread is
\[ \frac{\partial \text{CVA}}{\partial s} \approx E(t')D(0, t')s', \]  
from which it can be understood that the sensitivity is a product between the discounted exposure and a time quantity. It is very similar to the corresponding sensitivity used in the formula for the standardised CVA capital charge (44): \( M_i EAD_i \), which is also the product between an exposure and a time quantity. A similar connection between the sensitivities for the CDS hedges can be found by relying on the same assumptions and similar approximations for the value of a CDS. An exposition can be found in Pykhtin (2012).

### 5.1.1.2 The Advanced Method
A bank calculating the CVA capital charge under the advanced method is required to have a simulation engine approved by the regulator. The capital charge is calculated as three times the sum of two VaR numbers at the confidence level 99% during the period 10 days. The VaR is calculated once with model parameters calibrated to the current market environment and once with model parameters calibrated to a stressed historical market environment. The capital charge is hence

\[ \text{CVA-Charge} = 3 \left( 10\text{days}q_{99\%}^\text{Current market} \Delta \text{CVA} + 10\text{days}q_{99\%}^\text{Stressed market} \right) \]  

The quantile calculations have to be performed with a specific formula for CVA, which the Basel Committee on Banking Supervision (2011) has defined as

\[ \text{CVA} = \text{LGD} \sum_{i=1}^{T} \left( \frac{\text{EE}(t_{i-1})D(t_{i-1}) + \text{EE}(t_i)D(t_i)}{2} \times \max \left\{ \exp \left\{ - \frac{s_{i-1}t_{i-1}}{\text{LGD}} \right\} - \exp \left\{ - \frac{s_{i}t_{i}}{\text{LGD}} \right\} \right\} \right), \]  

where

- \( t_i \) is the \( i \)th valuation time,
- \( T \) is the maturity of the longest maturing contract with the counterparty,
- \( s_i \) is the credit spread of the counterparty at time \( t_i \), or a proxy based on credit rating, industry sector and region if spreads are not available,
- \( \text{LGD} \) is the market-assessed loss-given-default,
Credit hedges are taken into account by reducing the expected exposure, i.e., if one assumes that one single single-name CDS is used for hedging the expected exposure in (57) becomes
\[ EE^H(t) = EE(t) - N_{CDS}Z(t), \tag{58} \]
where \( EE^H(t) \) is the hedged expected exposure at time \( t \), \( N_{CDS} \) the notional of the CDS and \( Z(t) \) a discount factor.

The formula (57) is in principle the same as the discretised version of the unilateral CVA formula that was derived in the pricing approach (24); the calculation is done through dividing the time into buckets, finding the expected exposure in each bucket, weighting it with the probability of default and loss-given-default. From the formula it is also understood that it relies upon the assumption that exposure and credit spread are independent, i.e., the CVA capital charge under Basel III does not incorporate right- and wrong-way risk.

5.1.2 Regulation and DVA

As was discussed previously DVA is commonly incorporated in the valuation of OTC derivatives as a necessary component that makes it possible for two parties to arrive at a common price at which they can transact. From the regulatory perspective, on the other hand, DVA is not recognised. A regulatory DVA would represent perverse incentives because one way of decreasing a bilateral CVA capital charge, and hence capital costs, would be through worsening the own credit quality; if the own default is likely to occur before the defaults of counterparties the investor has a lower need of capital buffers. Therefore by becoming riskier, as indicated by a wide credit spread of the own institution, capital would be released and by becoming less risky, as indicated by a narrow credit spread of the own institution, more capital would need to be reserved.

5.2 IFRS and Fair Value Accounting

From the accounting perspective CVA has been driven by financial accounting standards such as GAAP in the U.S. and the International Financial Reporting Standards (IFRS) (Dorval and Schanz, 2011). IFRS states that financial derivatives are subject to fair value accounting, which means that their book values should reflect the market values including adjustments for credit quality of the parties involved in the contract. According to Fares and Genest (2013) fair value accounting for OTC derivatives means that the derivatives are marked-to-market at a value that is defined by market participants. This has the implication that when the credit spread of a counterparty changes the value of an OTC derivative with that counterparty changes because other market actors value the contract differently because of the change in appraised financial strength. The case when the credit spread of the investor changes is completely analogous resulting in that
5.3 CVA in the Different Contexts

The different definitions of CVA in the three contexts put restrictions on how CVA can be integrated in a bank and has also implications for hedging. Because of the strict rules of how the CVA capital charge is calculated, the exclusion of DVA and the restriction of which hedge instruments that are allowed as dictated by the Basel III document there is a potentially significant discrepancy between CVA from a regulatory perspective and CVA from the pricing and accounting perspectives.

If a bank uses the standardised approach no simulation is performed and the exposure is measured through Exposure-At-Default \((EAD)\), which is determined using one of two methods. The basis of the \(EAD\) in both methods is the current PV, which is adjusted using add-ons and multipliers that can be found in look-up tables. Additionally, volatilities for CDS spreads are provided as static risk weights in Table 5, whereas market-implied data is needed for pricing. The standardised method is hence a rather coarse calculation and the capital charge can differ very much from the CVA that is priced in the market. The implication is that it will be difficult to implement an integrated CVA tool and it is likely that one tool will be used for pricing OTC derivatives and a (much) simpler tool will be used for capital charge calculations. The obvious disadvantage is the lack of connection between the two calculations, which will lead to a low degree of transparency and it will be difficult to see the connection between transactions and the additional capital that has to be reserved.

If a bank instead uses the advanced approach the definitions come closer and it will be easier to integrate the CVA calculations in a common engine. However, there will be differences in the calculations arising from which parameters are used as input. The regulatory calculations are not allowed to be performed by simply fetching market-implied parameters, but also need to be performed using historical ones from a stressed market condition. It might be possible to make use of the same software in the calculations of regulatory and market CVA, but the calculations have to be done separately with different model parameters.

The discrepancy has also implications for the hedging strategy a bank may choose. A hedging strategy that is minimising the P&L volatility will generally not be the hedging strategy that minimises the capital charge. One reason is the restriction of hedge instruments that are allowed from the regulatory perspective. For example, in Section 3.12.3 it was discussed how cross-dependency between market risk and credit risk was incorporated in a hedging strategy through adjusting the nominals of the CDS positions. Since market risk is not recognised as a factor driving CVA in the Basel III document an adjustment of the CDS nominals will lead to sub-optimal hedging from a regulatory perspective. Another reason to the difference in hedging strategies is the exclusion of accounting standards require that both CVA and DVA are incorporated in the book-keeping of OTC derivatives.

Because fair value accounting requires that the book value of a derivative reflects the value at which it could be traded in the market the valuation method used for accounting can preferably coincide with the valuation method used for pricing.
DVA from the regulatory side. DVA is likely to be incorporated in pricing by many market actors in order to be able to agree on prices in the market. Since DVA can be hedged, at least to some extent, it introduces another source of discrepancy between the perspectives. One last reason to why hedging strategies differ is the parameters used in the calculation. Since the CVA capital charge cannot simply be calculated using market-implied parameters the calculations will always differ to some extent and lead to different sensitivities of CVA with respect to credit spread changes, which means that the notionals of credit hedges will differ.
Credit Value Adjustment

6 CVA Integration

In this section CVA will be discussed in an organisational context with the aim of showing an approach to implementing CVA in a bank such that it is integrated consistently between departments. In doing this the market perspective of CVA will be connected to the regulatory and accounting perspectives. Throughout this section the notion risk-free will be used to describe a financial contract that does not bear counterparty credit risk, but is only sensitive to market risks. Similarly, the notion fair will be used to describe a financial contract that is transacted at a price such that the PV is zero at inception.

As a first naïve step it is assumed that the trading department in a bank starts to charge CVA from its counterparties without other departments taking notice of the change in pricing approach. Assume that a trader at the bank enters an IRS with a risky counterparty. Before the introduction of CVA-charging he would have calculated the risk-free fair swap rate and the counterparty would have been offered this rate plus a smaller margin, that would account for administrative costs, hedging costs and locking in a profit for the provision of liquidity. For simplicity it is assumed that this margin is zero. The swap would have represented a fair bet on the interest rate and had a risk-free PV of zero at inception. The position would be delivered to the risk department and the accounting department, which both would arrive at a first-day P&L of zero.

After the introduction of CVA-charging the trader would calculate the risky fair swap rate, which is the sum of the risk-free fair swap rate and a CVA premium. For a swap the premium is typically charged as a spread on the swap rate that over the life of the swap adds up to the CVA determined at the inception of the transaction. In a market environment with counterparty credit risk the swap has once again a PV of zero, but when valuing the swap in a risk-free setting the CVA margin would make the swap unfair and generate a PV different from zero at inception. When the position is delivered to the risk and accounting departments the swap would therefore have a first-day P&L, which incorrectly would be booked as a profit or loss. If these figures were used to determine the CVA capital charge there would be a double counting of CVA, resulting in a higher capital charge and hence capital reserves higher than what is required. Double counting means that the CVA capital charge is determined from the swap rate, which includes a CVA margin, and hence the capital charge will contain a component that is due to the CVA that the counterparty is charged.
In order to handle CVA consistently and avoid double counting the trading department needs to separate the CVA from the OTC derivative contracts. Each derivative can be divided into a risk-free contract and a contract that carries the counterparty credit risk. This division is illustrated in Figure 17, where it is shown how an OTC derivative, that does not belong to a netting set, can be split into a corresponding risk-free contract and a short CCDS position. For a derivative belonging to a netting set the division is similar, but a difference arises since the CCDS contract is referenced to the whole netting set and not to the single derivative. At any point in time during the lifetime of the OTC derivative the PV equals the PV of the equivalent risk-free contract less the CVA, i.e.,

\[ PV = PV^{\text{risk-free}} - CVA \] (59)

Figure 17: An OTC contract with a risky counterparty can be split into a risk-free contract and a short CCDS referenced to the risky counterparty and the risk-free contract.

When CVA-charging is introduced the CVA premium is matched such that the initial PV of the OTC derivative equals the risk-free PV:

\[ PV = PV^{\text{risk-free}} - CVA_{\text{incr}} + CVA^p = PV^{\text{risk-free}}, \] (60)

where CVA_{\text{incr}} denotes the incremental CVA, which is the CVA contribution of a new transaction to the netting set to which it belongs\(^{26}\), and CVA^p denotes the CVA premium received from the counterparty.

An illustration of how the above-mentioned division of OTC derivatives is reflected in the booking of transactions is shown in Figure 18. This set-up will be present, explicitly or implicitly, independently of the level of sophistication that a bank chooses in the treatment of CVA. The figure depicts how OTC derivatives with risky counterparties are booked in a CVA book. At inception the price that the counterparty is charged

\(^{26}\)For a discussion of incremental CVA and CVA allocation to trade level see Pykhtin and Rosen (2010).
Credit Value Adjustment

consists of the fair price in a risk-free setting plus the CVA premium. In the trade moment there will be an additional internal transaction that transfers the risk-free part of the contract to the trading book. Through this procedure the counterparty credit risk is kept in the CVA book together with the initial CVA premium and the trading book is consequently immunised to changes in the credit quality of counterparties.

Figure 18: Illustration of how CVA can be managed within the trading department of a bank. The CVA book contains the original OTC derivative positions with risky counterparties and additionally the reverse transactions traded at their fair price in a risk-free setting with the trading book. Through this procedure the trading book is immunised to counterparty credit risk. The trading book is hedged in a conventional manner using market hedges and depending on the approach to CVA management the CVA book may be hedged using credit and market hedges.

How the CVA book is managed will differ between banks depending on which approach they take to handling CVA. Banks with small OTC derivative portfolios may take a passive stance with an insurance approach to CVA. Banks with large OTC derivative books may choose a more active approach, in which a CVA desk with active hedging is set up. The CVA desk may even aim for generating profits through taking CVA-related positions and trying to find arbitrage opportunities in the market. The two approaches are based on Gregory (2012) and are described in the following.

In the passive approach the CVA book has the function of an insurance. No hedging of CVA is done and the counterparty credit risk is hence kept in the CVA book. The CVA premia that the counterparties are charged are kept as reserves and used to cushion against losses that are due to deteriorating credit quality of counterparties. In the case
of a default the OTC derivatives with the counterparty are closed with a resulting loss in the CVA book. At the same time the corresponding risk-free derivatives, that were traded between the CVA book and the trading book, are closed at their market values, which have not been affected by the default.

Since the CVA book acts as insurance and no attempt to hedging of counterparty credit risk is taken, the actuarial approach to pricing should be taken. The CVA would be computed as the expected loss due to counterparty default under the real-world measure and a component would be added to account for unexpected losses. A feasible approach is to determine the CVA-VaR at the desired confidence level. This CVA-VaR is similar to the one discussed in the regulatory context, but instead of only defining credit spreads as risk factors also market risk factors should be included. It is also important to bear in mind that in this context risk-neutral probabilities should not be used, but the model should be calibrated to historical data.

A problem of the passive approach is model backtesting. Defaults occur relatively seldom and backtesting thus needs to be done over a long time horizon, typically several years such that business cycles are taken into account. This makes it difficult to determine whether reserves are insufficient or perhaps unnecessarily high.

In the passive approach the CVA book may be managed by the trading department, but it is more likely to be the responsibility of the risk department.

In the active approach the CVA book constitutes an additional trading book, which differs from the traditional trading book by having counterparty credit risk exposures as additional dimension. The aim is to hedge the CVA to the largest possible extent such that the P&L volatility due to changes in CVA is minimised. At the same time hedging costs need to be taken into account such that hedging is kept at an economically viable level. The CVA premia that counterparties are charged are used for hedging and therefore the CVA is calculated using the risk-neutral pricing approach. Because of the complexity of CVA-hedging, which mainly arises due to cross-gammas, and the possible lack of tradeable hedge instruments the hedging error must also be taken into account and charged for.

The management of the CVA book in the active approach is the responsibility of the trading department, since it does not differ very much from any other trading book.

A typical bank will choose an approach that lies somewhere between the two extremes that were described above. The extent to which hedging is done will typically be determined by the size of the CVA book, the risk-appetite of the bank and the existence of tradeable hedge instruments.

\[27\] The actuarial approach was briefly discussed in the introduction of Section 3.
References


