Asymmetric Conditional Variance in Housing Prices
- Testing for Downward Rigidity

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Abstract:

The purpose of this paper is to analyze the volatility dynamics of housing markets in the United Kingdom and the United States in order to determine if market inefficiencies cause downward rigidity in housing prices. We hypothesize that the potential downward rigidity will prevent prices from appropriately adjusting downwards, and that positive shocks in housing prices will increase the conditional variance in the following period more than a corresponding negative shocks. We analyze the conditional variance in the housing prices by employing various autoregressive conditional heteroscedastic (ARCH) models. Our results for the United Kingdom and the United States show that shocks in housing prices have a positively asymmetric effect on the conditional variance in the following period, and that the E-GARCH model is better than the GJR-GARCH model at modeling this asymmetry.

key words: housing prices, asymmetric, price rigidity, E-GARCH, GJR-GARCH
# Index

1. INTRODUCTION .................................................................................................................. 1

2. THEORY ............................................................................................................................... 4
   LOSS-AVERSION .............................................................................................................. 4
   EQUITY CONSTRAINT ...................................................................................................... 5
   MARKET INEFFICIENCIES AND DOWNWARD RIGIDITY .............................................. 5

3. LITERATURE REVIEW ........................................................................................................ 8

4. DATA ................................................................................................................................... 12
   SEASONAL ADJUSTMENT ............................................................................................... 14
   STATIONARITY ................................................................................................................ 14

5. METHODOLOGY .................................................................................................................. 16
   THE ARCH AND GARCH MODELS .............................................................................. 16
   THE GJR-GARCH MODEL ............................................................................................. 17
   THE E-GARCH MODEL ................................................................................................... 18

6. PRE-ESTIMATION ............................................................................................................... 19
   DETERMINING THE MEAN EQUATION ........................................................................ 19
   TESTING FOR ARCH AND GARCH EFFECTS .............................................................. 20

7. RESULTS .............................................................................................................................. 21
   ARCH AND GARCH MODELS ....................................................................................... 21
   GJR-GARCH(1,1) MODEL ............................................................................................. 23
   E-GARCH(1,1) MODEL .................................................................................................. 24
   NEWS IMPACT CURVE ................................................................................................. 25
   ADDITIONAL DIAGNOSTIC CHECKS ........................................................................... 26
   ROBUSTNESS ................................................................................................................ 27

8. CONCLUSION ....................................................................................................................... 30

REFERENCES ......................................................................................................................... 31

APPENDIX A: TIME SERIES PLOTS ....................................................................................... 34

APPENDIX B: DIAGNOSTIC CHECKS ....................................................................................... 35
1. Introduction

Housing markets exhibit special characteristics that in many ways make them different from other financial markets. Previous research has found that they display a positive correlation between prices and trading volume, high volatility in prices, and an observed reluctance of sellers to reduce asking prices in down markets (Genesove and Mayer 2001, p.1233; Engelhardt 2003, p.171). Recent research has also shown that standard asset market models have failed to give satisfying explanations for these puzzling findings (Haurin et.al. 2013, p.2), and surprisingly little research have been directed towards analyzing the causes and effects of these market inefficiencies. Some authors, such as Genesove and Mayer (2001), have argued that inefficiencies in housing markets could be caused by a reference dependent disposition effect, where investors and homeowners become unwilling to realize capital losses. This irrational behavior among investors and homeowners would reduce the supply in the housing markets during downturns, which would effectively prevent housing prices from adjusting downwards and therefore causing downward rigidity in housing prices.

The main purpose of this paper is to analyze if unexpected shocks in housing prices exhibit an asymmetric response in the conditional variance, which could be an indication of downward rigidity in housing prices. This will be achieved by answering the following question:

Is the conditional variance in the United States’ and the United Kingdom’s housing prices positively asymmetric?

The theoretical assumption is that an unexpected downturn in housing prices would lead to a reduction in the supply and trading volume of housing, which would prevent the prices from appropriately adjusting downwards and therefore reducing the volatility in the housing prices. Evidence of a positively asymmetric effect on the conditional variance would thus be interpreted as support of the hypothesis: that there is downward rigidity in the housing market.

There exists a body of articles that have employed different autoregressive conditional heteroscedastic (ARCH) models to model the conditional variance in housing prices, but
only a handful of these articles have analyzed the potential existence of asymmetric conditional variance in the housing market. Most notable among these are two articles by I-Chun Tsai et.al. on housing price volatility in the United Kingdom, which find evidence of asymmetric volatility for both regional and national aggregates (2009 and 2013). Influential policymakers such as Ben Bernanke have argued that too little is known about the behavior of housing prices, suggesting that central banks should not respond to increases in housing prices (1999 and 2001). The potential importance of Tsai’s findings are sweeping, since the existence of downward rigidity in housing prices could add new insights to the understanding of the reoccurring phenomenon of housing bubbles, and therefore affect the conception of how to conduct appropriate monetary policy. The significance of Tsai et.al.’s articles are so far limited to their findings for the United Kingdom, since other authors have found evidence that downward rigidity could be a national phenomenon caused by inappropriate government intervention and regulation (Tsai 2013, p.405). The downward rigidity in prices and the associated conditional asymmetric variance could therefore be a national phenomenon that is limited to the United Kingdom.

This paper will build upon the work of Tsai et.al. in order to test their hypothesis on the most recent housing price data for the United Kingdom and the United States. The hypothesis will be tested by employing various ARCH/GARCH models, which allow positive and negative shocks to have asymmetric effects on the conditional variance in the following period. Results indicative of significantly higher conditional variance following positive shocks than negative shocks will be interpreted as evidence of downward rigidity in housing prices, and vice versa. Previous research by Tsai et.al. has focused exclusively on data for the United Kingdom, and we want to make a contribution to the existing literature by including data for the United States into our analysis. Evidence supporting the existence of asymmetric effects and downward rigidity in both the United Kingdom’s and the United States’ housing prices would imply that price defensiveness in the housing markets is not only limited to the United Kingdom, which would add support to the generalizability of Tsai et.al.’s previous findings.

The dataset used in our analysis is mainly comprised of three time series on housing price indexes. The data for the United Kingdom, all housing prices and new housing
prices, consists of two quarterly measures from the Nationwide Building Society. Our data for new housing prices in the United States is measured at a monthly frequency and collected from the United States Census Bureau[1]. Each series is logged and first differenced in order to create a stationary return of housing price series that allows for ARCH/GARCH estimations without spurious estimates.

As stated, we aim to determine the existence of downward rigidity in housing prices by analyzing the volatility dynamics in the data and by test testing for asymmetric responses in the conditional variance. For this purpose, we use two different asymmetric GARCH models, the GJR-GARCH and E-GARCH models. Ordinary ARCH/GARCH models with symmetric volatility estimates will also be estimated and compared with the results in the asymmetric models.

This paper will be outlined as follows. Section 2 will outline the theoretical model, which explains the relationship between inefficiencies in the housing market, downward rigidity in housing prices and the symptomatic asymmetric conditional variance in housing prices. In section 3, a brief literature review will introduce the reader to the work of Tsai et.al. and the rest of the existing literature on housing price dynamics (the literature review will contain technical terms associated with ARCH/GARCH modeling and readers who are unfamiliar with the terminology would probably prefer to start off by reading the methodology in section 5). Section 4 and 5 will describe the data and the methodology used in the analysis. Section 6 conducts a couple of pre-estimation tests before we perform the regressions. The results from our analysis will then be analyzed in section 7, followed by a concluding summary in section 8.

[1] Data on existing housing prices was only available for shorter measurement periods.
2. Theory

The housing market is dominated by homeowners that trade in the homes they live in. Furthermore, transaction costs, tax considerations and carrying costs prevent professional investors from exploiting potential profit opportunities. For these reasons, it is generally believed that housing markets are less efficient than other financial markets (Case and Shiller 1989, p.125; Ihlanfeldt and Mayock 2012, p.91). These inefficiencies could lead housing prices to deviate from their fundamental value, and the existence of downward rigidity could effectively prevent prices from adjusting downwards, causing bubbles to build up and eventually burst. Hence, well-functioning housing markets are important factors in ensuring macroeconomic stability, and we will therefore take a closer look at the causal mechanism that explains the relationship between market inefficiencies and downward rigidity.

The existence of downward rigidity in housing prices could occur as a consequence of several factors, but we will focus on two potential explanations. The theory of loss-aversion is based on behavioral finance and assumes that homeowners and investors behave irrational, while the theory of equity constraints assumes a correlation between housing prices, wealth and mobility. These theories are mutually compatible and they help to explain why market inefficiencies could lead to downward rigidity in housing prices.

Loss-aversion

The theory of loss-aversion is based on a much-celebrated paper by Daniel Kahneman and Amos Tversky, which analyzed the psychology of decision-making under risk. According to the theory, investors view their investments in relation to a reference point, which is often assumed to be the initial purchasing price or an expected higher future value. Psychological experiments have shown that investors are willing to accept a higher level of risk in the domains below the reference point than in the domains above the reference point (Kahneman and Tversky 1979, p.268-269), which means that investors will ride losses and sell winnings (Shefrin and Statman 1985, p.785). It is therefore possible that higher future expectations will cause homeowners and investors to ride their losses, since housing markets typically tend to increase quite rapidly over
time (Thoma 2013, p.46). Victoria Dobrynskaya notes that this asymmetry in the reference-dependent utility will cause investors to behave defensively and therefore to set their reservations prices higher than the equilibrium price (2008, p.21).

**Equity constraint**

The theory of equity constraints takes a different approach to downward rigidity, stating that downward rigidity results as a consequence of decreasing household wealth during downturns in the housing market. It is well known that the housing value is the largest component of household wealth, and changes in the housing price could therefore have a large impact on the household mobility as well as the transaction frequency in the housing market. To illustrate, imagine a family that purchases a $100,000 house by using $20,000 of their own capital and by borrowing an additional $80,000 from the bank. Furthermore, imagine if the housing market were to decrease by 20% immediately after the purchase. The housing value would now equal the $80,000 loan, which means that the household’s net wealth has been reduced from $20,000 to zero. The wealth reduction will reduce the household mobility and prevent homeowners from moving, since banks demand a down-payment as a security in order to grant new loans (Akkoyn et.al. 2012, p.3; Stein 1995, p.380). Hence, the equity constraint theory predicts that homeowners facing a loss will set their reservation prices higher than the equilibrium price.

**Market inefficiencies and downward rigidity**

Both the loss-aversion and equity constraint theory predict that homeowners facing a loss will have reservation prices that are higher than the equilibrium price, which means that they must face a longer time on the market (Genesove and Mayer 2001, p.1253). Furthermore, the price is not supposed to be correlated with the transaction frequency in a well-functioning market (Akkoyn et.al. 2012, p.2-3), but the spread between the asking price and the equilibrium price prevents the demand and supply from clearing, which according to our theory should reduce the transaction frequency. These two predictions are empirically verified using data for the United States’ housing market. As demonstrated in Figure 1 and 2, the ‘growth in the monthly housing price index’ is positively correlated with the ‘number of new one-family houses sold’, and negatively correlated with the ‘median number of months on the market’.
Moreover, the increasing spread between the reservation price and equilibrium price during downturns will cause the market to work slower. Research has shown that the volatility in housing markets adjust back to the normal level of volatility much more quickly after positive shocks in housing prices than negative shocks (Miller and Peng
Mille and Peng note that this effect could be partially explained by the reluctance of homeowners to reduce the housing stock by demolishing their homes when the demand for housing decreases, while construction companies are able to construct new housing units within quarters to meet increases in the demand (ibid.). The supply can therefore increase quickly, but will decrease slowly. This observation is consistent with the theories above and gives further evidence to the claim that housing markets are less efficient during downturns.

Finally, if the spread between the reservation price and the equilibrium price results as a consequence of price defensiveness, then the volatility in the housing price would decrease as the market becomes less efficient and slower. Future expectations could also affect investment decisions so that homeowners and investors ride their losses in hope of increasing prices in the future. Our theories therefore predict that the conditional variance in the housing prices data will respond asymmetrically to unexpected shocks. Evidence in favor of this prediction will therefore be interpreted as support of the hypothesis, that there is downward rigidity in housing prices.
3. Literature review

As stated earlier, this paper owes its theoretical framework to two articles by I-Chun Tsai on the volatility dynamics of housing prices in the United Kingdom. These articles are perhaps the only articles that analyze the existence of downward rigidity through asymmetric ARCH/GARCH modeling and they therefore constitute a central part of this literature review. The first article by I-Chun Tsai on asymmetric volatility in housing prices was coauthored with Ming-Chi Chen in 2009 and used a GJR-GARCH model to analyze the asymmetric ARCH/GARCH effects. A step-by-step procedure was used to test for stationary, to select a mean equation, to test for the presence of ARCH/GARCH effects and to determine the variance equation. The final results showed that the conditional variance parameter is positive and significant and that the AR(1)-GJR-GARCH(1,1) had the best fit among all the models considered. The article is well structured and its results make a convincing argument about the relationship between downward rigidity in housing prices and the asymmetric variance. However, two imperfections are worth noting. The mean equation is only determined in terms of autoregressive (AR) components, and no moving average (MA) components are considered in the testing process. Furthermore, the models are unable to resolve the apparent autocorrelation in the data and even the final AR(1)-GJR-GARCH(1,1) model has highly significant autocorrelation coefficients in the correlogram (Tsai and Chen 2009, p.88). These two shortcomings are most likely connected and it is possible that the inclusion of moving average component in the mean equation could have reduced the autocorrelation substantially. Despite its flaws, this article provides the to date best analysis of asymmetric volatility in housing prices and we will draw upon this paper in order to further improve the modeling of housing price volatility.

The second article by Tsai on downward rigidity in housing prices in the United Kingdom uses the same methodology and data as in the previous article. After having concluded that the volatility is asymmetric, Tsai continues her analysis to show that the price defensiveness of homeowners and investors in the United Kingdom also causes monetary policy to have an asymmetric effect on housing prices. Monetary supply is typically negatively correlated with housing prices, meaning that the central bank can influence housing prices by adjusting the interest rate (Andersson 2011, p.4; Mishkin 2001, p.5; Tobin 1969, p.17), but Tsai shows in her article that this relationship is
asymmetric. By using an asymmetric error correction model Tsai is able to show that there is a significant long-term relationship for positive changes in the monetary supply and housing prices, but the long-term relationship is insignificant for negative changes, which implies that housing prices fail to adjust downwards when the monetary supply decreases (Tsai 2013, p.412). In her conclusion Tsai argues that a tightening of the monetary policy will not influence housing prices downwards and that policy makers should take this effect into account in order to avoid the build-up of housing price bubbles. Moreover, this article gives a more comprehensive account of the theoretical underpinnings that explains why downward rigidity in prices and asymmetric volatility occurs. Investors and homeowners are assumed to be irrational and unwilling to realize capital losses. During downturns the reserve price of sellers is higher than the expected price of the buyer, therefore lowering the transaction frequency and lengthening the selling time.

Next to these two articles by Tsai et.al. we found five other articles that have analyzed the conditional variance in housing prices by including asymmetric GARCH components into their models. The difference between these articles and Tsai’s is that they lack the theoretical explanation for why the conditional variance in housing prices would behave asymmetrically, yet most of them still find evidence of positive and significant asymmetric conditional variance. Studies in psychology have shown that there exists a ‘confirmation bias’ in academic publications and that academic writers, consciously or unconsciously, interpret evidence in ways that are consistent with their current beliefs (Nickerson 1998, p175). This means that these five studies, which do not hypothesize a certain outcome in terms of the asymmetric parameter, are less likely to be biased and more reliable in terms of their actual findings. In terms of their findings three of them find positive and significant asymmetric parameters, two of them have insignificant parameters, and none of them find evidence of negative parameters.

Geoff Willcocks has written two articles on the conditional variance in regional housing prices in the United Kingdom. The data used in his first article is taken from ‘Communities and Local Government - Office of Deputy Prime Minister’ and consists of mixed-adjusted quarterly data for the period 1968Q2-2007Q2. This data generates 157 observations and Willcocks notes that it is considerably less than the recommended minimum for ARCH/GARCH estimations. Despite the low number of observations
Willcocks finds a positive and significant asymmetric parameter in his E-GARCH estimations (Willcocks 2009, p.405, 412). In his second article Willcocks uses the same quarterly data as Tsai et.al, collected from the Nationwide Building Society for the period 1973Q4-2009Q2. This time the number of observations is 142, which is even less than in his previous article. The asymmetric parameter is insignificant in all regions and Willcocks therefore concludes that the asymmetric effect on the conditional variance is modest at best (Willcocks 2010, p.346).

In two articles by William Miles the asymmetric parameter is found to be significant and positive in most major cities in the United States, while insignificant in all regions of the United Kingdom. One of the articles analyzes the volatility dynamics of housing prices in different cities in the United States using quarterly data with a total of 110 observations. The results for the GJR-GARCH model indicate that larger cities with better functioning housing markets tend to exhibit a significant and positive asymmetry (Miles 2008, p.86). In the second article he performs the same analysis using data form the Nationwide Building Society for regional housing prices in the United Kingdom. The measurement period used in his analysis (1973Q4-2009Q4) is only two observations longer than the period used in Willcocks (2010), meaning that the total number of 144 observations is still relatively small. His results for the asymmetric parameter in the GJR-GARCH model of regional housing prices volatility in the United Kingdom are also insignificant (Miles 2011, p.95), and therefore confirm Willcocks previous findings.

Lastly, an article by Bruce Morley and Dennis Thomas used an E-GARCH-in-Mean model on the Financial Time's monthly regional housing price data in the United Kingdom, running from 1995M2-2008M7. The results showed that the asymmetric coefficient was positive and significant in six out of ten regions in the United Kingdom (Morley and Thomas 2011, p.738). They note that these findings contradict the standard volatility behavior of financial data and theorize that it could be explained by the speculative nature of the national housing market and large interest rate cuts in 2001. However, they control for interest rates in the mean equation and the asymmetric effect remained positive and significant in six out of ten regions, even though the interest rate variable in the mean equation was significant in most of the regions (ibid. p.740).
The existing research on asymmetric conditional variance in housing prices is summarized in Table 1 below. A majority of the papers covered in this literature review support the hypothesis of positive asymmetry and only a few of them reject it. Also, bear in mind that the low number of observations used in most of the estimations will bias the estimates towards having insignificant asymmetric parameters.

<table>
<thead>
<tr>
<th>Asymmetric parameter</th>
<th>Positive</th>
<th>Insignificant</th>
<th>Negative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tsai, (2013)</td>
<td>Miles, (2011)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Willcocks, (2009)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thomas &amp; Morley, (2011)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Miles, (2008)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Findings in the existing literature.
4. Data

We will use aggregate nationwide housing price data for both the United Kingdom and the United States in our analysis. Measures of housing prices were also available for regional and metropolitan areas, but we will focus on the aggregate data since it has the advantage of longer measurement periods and presumably less measurement errors.

The number of observations plays an important role in ARCH/GARCH estimations and 250 observations is often said to be the recommended minimum requirement in order for the models to accurately estimate the conditional variance (Willcocks 2010, p.343). The choice of data has therefore been made in order to maximize the number of observations by using the longest available measurement periods at the highest available measurement frequency.

We will use two series of data for the United Kingdom, ‘all housing prices’ (UKall) and ‘new housing prices’ (UKnew), which are provided by the Nationwide Building Society’s housing price database. The Nationwide Building Society’s methodology description states that data was estimated through hedonic regressions, generating a mix-adjusted estimate of the ‘typical’ housing price. This measure is not to be confused with the average housing price, since an average price is usually higher because a small amount of more expensive houses sold tend to bias the average price to be higher than the price of a ‘typical’ house. The methodology description also states that the original data had a slightly seasonal pattern with higher prices in the spring and summer. This pattern was removed by an unstated method, and the non-seasonally adjusted data was unfortunately not available. The UKall and UKnew are both measured at a quarterly rate and available for the period 1952Q4-2014Q1. These are the same time series as used in Tsai et.al. (2009 and 2013), but this study will differ from Tsai et.al.’s articles in the sense that we will include the most recent data and consider the entire sample period, instead of using restricted subsamples[2]. Using the entire sample, we obtain a number of 246 observations, which is fairly close to the recommended minimum. Thus, our data provides the to date highest number of observations used in a study of asymmetric volatility of housing price returns in the United Kingdom.

[2] Tsai et.al. has used restricted samples in their articles, which reduces the number of observations: (2009), 1955Q4-2005Q4: 200 observations. (2013), 1986Q3-2011Q4: 99 observations.
The data used in our analysis of the United States’ housing price return is a monthly measure of the ‘median sales price of new houses’ (USnew), collected from the United States Census Bureau. The data has not been seasonally adjusted and the sample period (1967M1-2014M4) consists of a total of 568 observations. The measurement period for the US data is shorter than the period for the UK data, but the high measurement frequency still generates more than twice as many observations. The use of high frequency data has the advantage of generating more observation, but the overall sample period tends to be shorter and the actual month-to-month increase in housing prices could be considerably smaller than the measurement error. We have found no previous studies of ARCH/GARCH estimations on monthly housing price data for the United States’ and this study might therefore be the very first study to use monthly data to analyze the existence of asymmetric volatility in the United States’ housing price returns.

Each housing price series is transformed into the return series by logging and first-differencing the data according to:

$$ y_t = \ln(i_t) - \ln(i_{t-1}) $$

where $y_t$ is the log return in period $t$ and $i_t$ is the housing price index.

The three series exhibits a diverse set of properties that makes a suitable dataset to test the hypothesis, that there is downward rigidity in housing prices. The hypothesis makes no distinction between new and existing housing sales, but our data for UKall and UKnew should be able to capture if there is a discernable difference. The existence of asymmetric volatility in the housing price return could be an isolated national phenomenon but the cross-country data allows us to determine whether an eventual relationship is generalizable to other countries or not. Furthermore, the housing price indexes could be constructed using different estimation methods. Our dataset contains ‘typical housing price’ measures for the United Kingdom and a ‘median housing price’ measure for the US. The hypothesis should be insensitive to the estimation methodology employed in deriving the housing price indexes and we therefore have no reason to prefer one of the estimation methodologies to the other. Also, the monthly measurement frequency for the US is higher than the quarterly measures for the UK, which not only generates more observations, but also allows us to capture the asymmetric conditional
variance if the duration of the asymmetric effect on the conditional variance is shorter
than one quarter. Together, these variations in the properties of the dataset make up a
challenging test of the hypothesis. Furthermore, the data was seasonally adjusted and
then tested for stationarity. The rest of this section will explain this process in greater
detail.

**Seasonal adjustment**
The UKnew and UKall data was only available as seasonally adjusted series, while the
USnew was non-seasonally adjusted. It is important to remove seasonality in the data in
order to make sure that the autocorrelation in the series are not affected by the seasonal
pattern, but research has also shown that seasonal adjustment could have a negative
effect on the ARCH/GARCH estimations. Eric Ghysels, Clive W.J. Granger and Pierre L.
Siklos has through Monte Carlo studies shown that typical seasonal adjustment filters,
such as the United States Census Bureau’s X-11 and X-12 filters, can have a strong
downward bias on the conditional variance, thereby making actual ARCH/GARCH effects
small and insignificant (1997, p.16). Ghysels et.al. also finds that filters that smoothen
the series to remove potential outliers will reduce the conditional variance. The effects
of seasonal adjustment filters on the estimation of volatility models are still relatively
unexplored and a failure to account for these effects could adversely affect the
estimations.

Removing outliers by smoothening the data will reduce the volatility in the series, which
could adversely affect the ARCH/GARCH estimations. We will therefore remove the
seasonal pattern by regressing quarterly dummies on the UKnew and UKall series and
monthly dummies on USnew series, excluding the intercept. The predicted residuals
from the regression are then used as the new seasonally adjusted series.

**Stationarity**
The stationarity requirement has to be fulfilled in order to avoid spurious regressions.
Previous studies have found that housing price series are typically characterized by
difference stationarity (Willcocks 2010, p.344), and the logged series are therefore
tested for the presence of a unit root in both levels and first-differences. For this
purpose, a Dickey-Fuller GLS test is used while the appropriate lag-length is automatically specified according to the Schwarz information criterion.

The test is unable to reject the null-hypothesis of unit root in levels in all three series, which means that all series are non-stationary in levels. Repeating the test in first-differences we find that both series for the United Kingdom are difference stationary at the 1% level, while the series for the United States is still non-stationary, not even at the 10% significance level. The non-stationarity in the United States housing price return series creates a problem for our estimations and we therefore plot the data in order to find a solution to the problem. It is apparent from looking at the data that the beginning of the series contains a few outliers and that the volatility in the beginning of series is considerably higher than the rest of the series (see Figure A.3 in appendix), meaning that the break in the volatility could cause the series to be non-stationary (Hillebrand 2005, p.135-136; Simonato 1992, p.136). By dropping the first 192 observations from our sample, we obtain a series that is now stationary at the 1% significance level and without outliers.

Finally, all the relevant descriptive statistics of the transformed data and the stationarity tests are summarized in Table 2. Note that the mean in each series is positive, which means that there is a tendency for housing prices to increase over time (this can also be seen by analyzing the time series plots in Appendix A.). Also, the number of observations were reduced by one after first-differencing.

<table>
<thead>
<tr>
<th>Variables:</th>
<th>UKall</th>
<th>UKnew</th>
<th>USnew</th>
<th>USnew</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>245</td>
<td>245</td>
<td>567</td>
<td>376</td>
</tr>
<tr>
<td>Mean</td>
<td>0.016</td>
<td>0.018</td>
<td>0.0031</td>
<td>0.0019</td>
</tr>
<tr>
<td>Std dev.</td>
<td>0.024</td>
<td>0.024</td>
<td>0.0033</td>
<td>0.0016</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.618</td>
<td>0.648</td>
<td>1.319</td>
<td>0.512</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>5.208</td>
<td>6.413</td>
<td>5.614</td>
<td>5.17</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.057</td>
<td>-0.065</td>
<td>-0.0081</td>
<td>-0.0039</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.117</td>
<td>0.141</td>
<td>0.017</td>
<td>0.0092</td>
</tr>
<tr>
<td>DF-GLS test</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Level</td>
<td>-1.553</td>
<td>-1.312</td>
<td>-0.572</td>
<td>-0.807</td>
</tr>
<tr>
<td>First diff.</td>
<td>-4.793***</td>
<td>-5.060***</td>
<td>-2.485</td>
<td>-4.378***</td>
</tr>
</tbody>
</table>

Note: H0 in DF-GLS test, variable is not stationary

Table 2: Descriptive statistics of the logged, first-differenced and seasonally adjusted housing price data.

Source: Nationwide Building Society's and the United States Census Bureau.
5. Methodology

Our housing price data for the United Kingdom and the United States do not seem to have constant means and all series seem to exhibit phases of tranquility followed by periods of high volatility. The volatility dynamics in these series are analyzed using ARCH-family models, and the asymmetric conditional variance is captured using two different asymmetric-GARCH models, the GJR-GARCH and E-GARCH model. Each model is briefly explained in the following.

The ARCH and GARCH models

Robert Engle (1982) showed that it was possible to simultaneously model a time-dependent mean and variance, by allowing the mean to follow an ARMA process while the conditional variance is determined by past realizations of the squared error term. This came to be known as the Autoregressive Conditional Heteroscedastic (ARCH) model. Tim Bollerslev (1986) later extended the ARCH model into the Generalized Autoregressive Conditional Heteroscedastic (GARCH) model by allowing the conditional variance to depend, not only on past realizations of the squared error term, but also on past realizations of the conditional variance itself. The ARCH model is thus a special case of the GARCH model, which we specify below.

If we let $y_t$ denote the return of the housing price in period $t$, then the return depends on the error process according to the GARCH($p,q$) model:

$$y_t = a_0 + \sum_{i=1}^{p'} a_i y_{t-i} + \sum_{i=0}^{q'} b_i \varepsilon_{t-i} + \varepsilon_t$$

$$\varepsilon_t | \psi_{t-1} \sim \mathcal{N}[0, h_t]$$

$$h_t = \alpha_0 + \sum_{i=0}^{p} \beta_i h_{t-i} + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2$$

where $q$ and $p$ are lag lengths of GARCH lags and $h_t$ is the heteroscedastic conditional variance. The first equation is the mean equation (ARMA), this equation specifies the mean $y_t$ as being dependent on the mean in previous periods and past error terms. The second equation assumes that the error term, conditional on the available information set from previous periods $\psi_{t-1}$, is normally distributed around a zero mean and that its variance depends on $h_t$. The third equation is the variance equation (GARCH), which
specifies the variance $h_t$ as being determined by the variance in previous periods and the square of past error terms.

It is important to note that the mean equation models the homeowners’ and investors’ expectations. The error term $\epsilon_t$ is therefore assumed to capture the unexpected housing price shocks that translate into increases in the conditional variance.

**The GJR-GARCH model**

In the ARCH and GARCH models the variance is assumed to be symmetric, meaning that positive and negative shocks of the same magnitude will have identical impacts on the conditional variance in the following period. But according to our theory we would expect the variance to be positively asymmetric and positive shocks should hence have a larger impact than negative shocks on the conditional variance in the next period. Glosten, Jaganathan and Runkle (1993) showed that this asymmetry in the conditional variance could be modeled simply by introducing an interaction dummy variable for positive values of the error term in the previous period. The GJR-GARCH model could therefore be specified as:

$$
\begin{align*}
    y_t &= a_0 + \sum_{i=1}^{p'} a_i y_{t-i} + \sum_{i=0}^{q'} b_i \epsilon_{t-i} + \epsilon_t \\
    \epsilon_t | \psi_{t-1} &\sim N[0, h_t] \\
    h_t &= \alpha_0 + \sum_{i=1}^{p} \beta_i h_{t-i} + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \gamma \epsilon_{t-1}^2 D_{t-1}
\end{align*}
$$

where the mean equation and distributional assumptions of the error term are identical to the GARCH model, while the variance equation has been slightly altered to allow for asymmetry. The dummy variable is determined so that $D_{t-1} = 1$, when $\epsilon_{t-1} > 0$; otherwise $D_{t-1} = 0$. The $\gamma$ parameter measures the asymmetric effect and is therefore our parameter of interest. Our theory stipulated that downward rigidity would translate into positive asymmetry in the conditional variance, which implies that positive and significant values for $\gamma$ will be interpreted as evidence in support of our hypothesis.
The E-GARCH model

Another popular asymmetric GARCH model that we will employ is the exponential-GARCH (E-GARCH) model by Daniel Nelson (1991). This model was originally introduced to overcome the problem of non-negative parameter restrictions in the GJR-GARCH model, but it also has some interesting properties that allow the impact response function to have a kinked shape (more on this in section 7). The model is specified as:

\[
y_t = a_0 + \sum_{i=1}^{p'} a_i y_{t-i} + \sum_{i=0}^{q'} b_i \varepsilon_{t-i} + \varepsilon_t
\]

\[
\varepsilon_t | \psi_{t-1} \sim N[0, \ln(h_t)]
\]

\[
\ln(h_t) = \alpha_0 + \sum_{i=1}^{p} \beta_i \ln(h_{t-i}) + \sum_{i=1}^{q} \alpha_i \frac{\varepsilon_{t-i}}{\sqrt{h_{t-i}}} + \sum_{i=1}^{q} \gamma_i \left[ \frac{|\varepsilon_{t-i}|}{\sqrt{h_{t-i}}} - \sqrt{2/\pi} \right]
\]

where the variance of the error term is now determined in a non-linear form by \( \ln(h_t) \). The E-GARCH model uses the standardized residuals from the previous period [i.e. \( \varepsilon_{t-1} \) divided by \( \sqrt{h_{t-1}} \)] to model the persistence in the variance. The absolute value of the error term in the variance equation forces the variance to behave asymmetrically by offsetting negative values in the third term of the equation (Nelson 1991, p.351). Again, the \( \gamma \) parameter shows the asymmetric effect and the interpretation will be the same as in the GJR-GARCH model.
6. Pre-estimation

We have already established in section 4 that all three series are difference-stationary. However, there are a couple of procedures that we need to cover before we can model the volatility dynamics. In the following, we will determine the appropriate mean equation for each series and then test for the existence of volatility clustering (ARCH/GARCH effects).

Determining the mean equation

The mean equation is specified in each series using the Box-Jenkins test procedure. By using the return of housing prices as our dependent variable and lags of the return and lagged error terms as our independent variables we try to minimize the Akaike Information Criterion (AIC) and Schwartz Bayesian Criterion (SBC). Both autoregressive and moving average components are hence included in the testing procedure, but due to the loss of degrees of freedom, only a maximum of four lags are considered.

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UKall mean equation selection, by AIC

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UKnew mean equation selection, by AIC

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USnew mean equation selection, by AIC

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USnew mean equation selection, by SBC

Table 3: Mean equation selection, AIC and SBC.
As shown in Table 3, the AIC and SBC suggest different specifications. The SBC introduces a heavier punishment for introducing more variables into the specification and therefore always selects a more parsimonious model. The AIC seems to favor the same ARMA models as the SBC for lower lag lengths, but as the lag length increases the AIC decreases rapidly. We will however use the mean equations selected by the SBC, since the SBC has better small sample properties, and since the AIC is biased towards selecting an over-parameterized model (Tsai and Chen 2009, p.84). Furthermore, diagnostic checks using the autocorrelation function and the partial autocorrelation function suggest that additional AR and MA lags are redundant.

Testing for ARCH and GARCH effects

The last step before we continue to estimate our ARCH and GARCH models is to determine if the error term from the mean equation exhibits volatility clustering. By using a Lagrange multiplier test, we test if the squared residuals obtained by regressing the mean equations in each series are autocorrelated. Table 4 summarized the LM-tests for each series. All three series show evidence of volatility clustering, which means that the variance in each series is time dependent. Having established that we have ARCH and GARCH effects in our variance equation allows us to finally estimate the conditional variance.

<table>
<thead>
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<th>Lag length:</th>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TR^2</td>
<td>3.51</td>
<td>6.90</td>
<td>7.62</td>
<td>8.31</td>
</tr>
<tr>
<td>p-value</td>
<td>0.06</td>
<td>0.03</td>
<td>0.05</td>
<td>0.08</td>
</tr>
<tr>
<td><strong>UKnew</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TR^2</td>
<td>2.12</td>
<td>9.36</td>
<td>11.74</td>
<td>11.71</td>
</tr>
<tr>
<td>p-value</td>
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<td>0.00</td>
<td>0.02</td>
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<tr>
<td><strong>USnew</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TR^2</td>
<td>3.21</td>
<td>3.10</td>
<td>6.79</td>
<td>7.29</td>
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<tr>
<td>p-value</td>
<td>0.07</td>
<td>0.03</td>
<td>0.06</td>
<td>0.08</td>
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</table>

H0, no ARCH/GARCH effects

Table 4: LM-test for ARCH/GARCH effects.
7. Results

The main purpose of our analysis is to analyze the existence of asymmetric effects in the conditional variance of the housing price series. We will however start off by estimating a few symmetric variance models in order to select the appropriate lag length in the ARCH and GARCH components. The asymmetric components in the GJR-GARCH and E-GARCH are then considered in the succeeding sections.

ARCH and GARCH models

The estimates from three different symmetric models, the ARCH(1), ARCH(2) and GARCH(1,1), are shown in Table 5. A quick glance at the estimates of the mean equations shows that all ARMA coefficients are highly significant, except for the intercept in the UKall series. The DF-GLS test in section 4 suggested that the restricted USnew series was stationary at the 1% significance level, but the near unity estimates for $a_1$ raises some concern for stationarity problems. However, the negative moving average component, as measured by the $b_1$ in the UKnew and USnew estimations, is at least indicative of a mean reverting process.

The estimates in the variance equation are positive and significant for all parameters in all models for the UKnew and USnew, while some of the estimates for the UKall series are insignificant in the ARCH(1) and ARCH(2) models. The lagged variance component in the GARCH(1,1) model, as measured by the $\beta_1$ parameter, is highly significant for all series and it is therefore reasonable to believe that this component should belong in the model. Furthermore, the GARCH(1,1) is also the only model where the $\alpha_1$ parameter is significant for all three series.

In the bottom of the table we find some diagnostic statistics that will help us to compare the models. The AIC and SBC statistics are included to measure the fit of each model and lower numbers are again indicative of a better fit. According to both the AIC and the SBC, the ARCH(2) has the best fit for the UKall series, while the GARCH(1,1) has the best fit for the UKnew and USnew. The Q(20) statistic shows the autocorrelation for the twentieth lag in the correlogram for the standardized residuals, while the $Q^2(20)$ statistic shows the autocorrelation for the squared standardized residuals. All models seem to have resolved the problem of autocorrelation in the residuals for the UKnew.
### Table 5: Empirical results from the symmetric ARCH/GARCH models.

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<td>0.0152***</td>
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<tr>
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<td>(-0.10)</td>
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<td>(7.29)</td>
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<tr>
<td>$a_1$</td>
<td>0.749***</td>
<td>0.911***</td>
<td>0.960***</td>
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<tr>
<td></td>
<td>(21.03)</td>
<td>(29.34)</td>
<td>(40.92)</td>
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<tr>
<td>$b_1$</td>
<td>-</td>
<td>-0.811***</td>
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<tr>
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<td>(-15.83)</td>
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### ARCH

<p>| | | | | | | | | | |</p>
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<td>30.201*</td>
<td>34.497**</td>
<td>17.513*</td>
<td>26.483*</td>
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* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

and USnew, but there is still some autocorrelation left in all models for the UKall. This is a problem worth stressing, since it makes our parameter estimates less reliable and could therefore invalidate our results. As noted in the literature review, this was also a major problem with both of Tsai’s articles where the Q(20) statistic for the UKall series was even as high as 131.08 (Tsai and Chen 2009, p.86). Potential solutions to this problem could be to include a moving average component into the mean equation, as we did for the UKnew and USnew, or to include control variables. Additional diagnostic checks were also performed to analyze the severity and extent of the autocorrelation (see appendix B). Finally, if we continue to the Q²(20) statistic we see that all models are able to resolve the autocorrelation in the squared standardized residuals. Again, the ARCH(2) has the lowest Q²(20) statistic for the UKall series and the GARCH(1,1) has the lowest statistic for the UKnew and USnew series.

Before we proceed to test for asymmetry in the conditional variance we need to determine the best fitting model for each series. Based on the diagnostic statistics, the GARCH(1,1) has the best fit for the UKnew and the USnew, while the ARCH(2) has the
best fit for the UKall. However, the ARCH(2) for the UKall series includes one insignificant parameter and further comparisons of the autocorrelation function and partial autocorrelation function for the ARCH(2) and GARCH(1,1) models showed that the GARCH(1,1) might even be better at resolving the autocorrelation in the predicted residuals. We seem to have reached an impasse in the model selection process and have therefore decided to use the GARCH(1,1) model for the UKall series, despite the suggested diagnostic statistics. The diagnostic statistics are after all not that different, and having identical models for all three series also simplifies the coming analysis somewhat.

Additional lags beyond the GARCH(1,1) has been tested for, but the GARCH(1,1) still has the lowest AIC and SBC statistics.

**GJR-GARCH(1,1) model**

The estimates of the mean equation coefficients in the in the GJR-GARCH(1,1) model are similar to the estimates in previous models. Looking at the variance equation in Table 6, the ARCH and GARCH coefficients for the UKall and USnew series are now closer to zero and insignificant. More interestingly, the asymmetric $\gamma$ parameter is positive for all three series, but only significant for the USnew series.

Comparing the diagnostic statistics, the AIC and SBC suggest that the symmetric GARCH(1,1) is better at modeling the UKall and UKnew series, while the GJR-GARCH(1,1) gives a better fit for the USnew series. The changes in the $Q(20)$ and $Q^2(20)$ statistics are minor and does not affect the significance level.

The significant and positive asymmetric parameter for the USnew series suggests that there is downward rigidity in the housing market for new single-family homes in the United States. The overall impression is that there might be a positive asymmetry in the variance for all series and that the GJR-GARCH(1,1) model is incapable of capturing this effect.
The results from the E-GARCH(1,1) model are shown in Table 7 and all the parameters in the variance equation are now significant. Furthermore, the asymmetric parameter is positive and significant at the 1% level in all series and we can therefore conclude that there is downward rigidity in all three housing price series.

All the diagnostic statistics, except the $Q^2(20)$ statistic, indicate that the E-GARCH(1,1) model is better than the GJR-GARCH(1,1) at modeling the volatility dynamics. However, the GARCH(1,1) still has a lower AIC and SBC statistics for the UKall and UKnew series and therefore a better overall fit. The fact that the GARCH(1,1) has a better fit in two of the series does not imply that the conditional variance is not asymmetric, only that the simple GARCH(1,1) is a better model overall.
Table 7: Empirical results from the E-GARCH model.

News impact curve

It is difficult to interpret the size of the asymmetric parameters in the tables and we have so far only looked at the sign of the coefficients and their significance levels. One way to compare the asymmetric response in the conditional variance is to plot the news impact curve. The news impact curve shows the predicted conditional variance across different values for the lagged standardized residuals, $z(t-1)$ (Engle and Ng 1993, p.1751).

Figures 3 through 5 compare the news impact curves for the GJR-GARCH(1,1) and E-GARCH(1,1) models. The size of the asymmetry and the shape of the predicted conditional variance are easier to interpret graphically. Both models have their lowest predicted conditional variance centered on the zero shock value. The GJR-GARCH has a smoother shape while the E-GARCH has a kink at its center. Furthermore, the figures demonstrate in a more comprehensive way that the predicted conditional variance is higher for a positive shock than a corresponding negative shock.
Additional diagnostic checks

The predicted standardized residuals from the asymmetric regressions were also analyzed using Bartlett’s white-noise test and Shapiro-Wilk’s test for normality. The standardized residuals for all series are within the 95% band in Bartlett’s white-noise test (see appendix B), which means that our results should not be caused by autocorrelation.

The histograms in appendix B display the distribution of the standardized residuals in comparison to the normal distribution. The only models that had normally distributed standardized residuals according to the Shapiro-Wilk’s test were the GJR-GARCH and E-GARCH for the USnew series. To have non-normally distributed standardized residuals...
is a problem, but it is a matter of degree. It is therefore important to take this problem into consideration when we draw our conclusions in section 8.

**Robustness**

A major problem with our ARCH/GARCH estimations is the lack of control variables in the mean equations. Standard macroeconomic variables are highly likely to have some explanatory power on the housing price return and should therefore be controlled for, but our choice of data for the housing prices are in most cases considerably longer than the measurement periods for other available control variable. Other articles have solved the problem of shorter measurement periods in the control variables by restricting the starting data of the sample period (Tsai et.al. 2009, p.409; Tsai 2013, p.87), our analysis however aimed at using the longest available sample periods in order to obtain the highest possible amount of observations for our ARCH/GARCH estimations.

In most cases, our housing prices data had longer measurement periods than the relevant control variables. Data for most of the relevant macroeconomic control variables for the United Kingdom where available as quarterly data from the first quarter of 1955 and onward, while the same variables for the United States where only available from January 1990 and onward. Using shorter measurement periods for the control variables implies that the number of observations would be reduced, which could cause the ARCH/GARCH effects to become insignificant even if the control variables themselves have no effect on the housing price return at all. The validity of inferences based on robustness checks with a reduced number of observations is also limited since the Akaike and the Schwarz information criterions are not comparable between estimations with different numbers of observations.

We did however, despite the limited validity of the inference, perform a series of robustness checks on both the GJR-GARCH and the E-GARCH models using the available data. The estimates will not be presented here because of the sheer number of estimations that results from using different ARCH/GARCH models, combinations of control variables and lags of control variables. A brief summary of the results will instead be given.
The variables included in the robustness checks were the growth in the logged gross national product (UK only), the growth in the logged consumer price index, 3-month and 10-year interest rates (UK only), the dollar/sterling exchange rate, 30-year conventional mortgage rates (US only) and an arbitrarily set crisis-dummy variable for the financial crisis (US only). The interest rate was used as a substitute for mortgage rate in the United Kingdom, which is also the reason for why the interest rate was not included in the robustness checks for the United States. Also worth noting is that the consumer price index, the interest rates for the United Kingdom and the mortgage rate for the United States were the only variables that had longer measurement periods then the available housing price data. All the data for the control variables was gathered from the Federal Reserve Bank of St. Louis’ database.

We started off by performing the robustness checks on the GJR-GARCH model for all three housing price return series. The growth in the logged gross national product had a positive sign and was significant at the 5%-level in the UKall estimations, the growth in the logged consumer price index was positive and significant at the 10%-level in the UKnew estimations, while the rest of the control variables where insignificant. In the USnew estimations, mortgage rate, the lag of the mortgage rate and the crisis-dummy for the financial crisis were significant at the 1%-level, and the rest of the variables were insignificant. When we included both the mortgage rate and the crisis-dummy into the mean equation the strength of the AR(1) coefficient was reduced below 0.9, which implies that more control variables could potentially reduce the near unity coefficient. None of the estimations changed the sign or the significance of the asymmetric parameter. This means that the asymmetric parameter remained positive but insignificant in all the GJR-GARCH estimations after controlling for other relevant macroeconomic variables.

The robustness checks for the E-GARCH estimations had a tendency to break down as we introduced more and more control variables in the mean equation. The estimations encountered flat log likelihoods and therefore failed to converge, which meant that we received no output from most of our regressions. Nevertheless, we were able to include single variables at the time into the mean equations.
The only variable that had an effect on the asymmetric parameter was the growth in the logged gross national product. It changed the sign on the asymmetric effect in the UKnew, reducing the parameter to close to zero and made the asymmetric effect insignificant, but the control variable itself was not significant. In the estimations of the USnew series, mortgage rate and the arbitrary break dummy were again significant at the 1%-level, but none of them had an effect on the sign or the significance of the asymmetric parameter.

The overall impression from our limited robustness checks is that the asymmetric parameter remains insignificant and positive in the GJR-GARCH, while the parameter for the E-GARCH model remains significant and positive. These robustness checks have therefore confirmed our previous results.
8. Conclusion

This paper has shown that there exists an asymmetric effect in the conditional variance and that an unexpected positive shock in the housing price return is typically followed by a proportionately higher increase in the conditional variance than a negative shock. The theory stipulated that this would occur as a consequence of price defensiveness among homeowners and investors, and we therefore conclude that the housing price series analyzed in this paper are characterized by downward rigidity. Our recommendation to policymakers is to take this effect into account, in order to avoid the build-up of future housing price bubbles.

Our main contribution to the existing literature on downward rigidity in housing prices was to show that Tsai’s hypothesis of asymmetric conditional variance was applicable to data for the United States, which means that downward rigidity in housing prices could be generalizable to other countries besides the United Kingdom. Our data for all housing prices and new housing prices shows that this downward rigidity exists in markets for existing houses as well as in markets for newly constructed houses. We have also proved that these results are insensitive to the choice of measurement frequency and estimation method used in generating the housing price data.

In the future, more data will be available for more countries and we will be able to perform more reliable tests of the hypothesis. A lot of housing price indexes are measured at a monthly frequency and this high frequency data will rapidly generate high numbers of observations, making them appropriate for ARCH/GARCH estimations. We would therefore advise future researchers to look for the longest available high frequency data. More research is also needed on regional and local housing markets, even though data for lower aggregate levels are less reliable. A major shortcoming in our analysis was the problem encountered when we tried to include additional control variables into the mean equation. We hope that future researchers will look into this and find a way to resolve this problem.
References


Appendix A: Time Series Plots

Figure A.1 Growth in logged housing prices, UKall. Source: Nationwide Building Society.

Figure A.2: Growth in logged housing prices, UKnew. Source: Nationwide Building Society.

Figure A.3: Growth in logged housing prices, USnew. Source: United States Census Bureau.
Appendix B: Diagnostic Checks

Bartlett’s white-noise test for GJR-GARCH, UKall.

Histogram for GJR-GARCH, UKall.

Bartlett’s white-noise test for E-GARCH, UKall.

Histogram for E-GARCH, UKall.

Bartlett’s white-noise test for GJR-GARCH, UKnew.

Histogram for GJR-GARCH, UKnew.
Bartlett's white-noise test for E-GARCH, UKnew.  
Histogram for E-GARCH, UKnew.

Bartlett's white-noise test for GJR-GARCH, USnew.  
Histogram for GJR-GARCH, USnew.

Bartlett's white-noise test for E-GARCH, USnew.  
Histogram for E-GARCH, USnew.