Prudent Valuation & Model Risk Quantification

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Abstract

This paper is a master’s thesis by Erik Kivilo and Carl Olofsson at the Faculty of Engineering (LTH) at Lund University. The research and writing took place during the fall of 2014 under the supervision of Magnus Wiktorsson at the Division of Mathematical Statistics at Lund University, and Per Thåström at EY, Copenhagen.

The thesis concerns prudent valuation of fair-valued financial instruments under current regulation on capital requirements for credit institutions and investment firms within the EU. The aim is to explain the concept of prudent valuation, and develop statistical methods for the calculation of additional valuation adjustments (AVAs) required by the regulations. As there has been little focus on model risk in previous regulations, the main objective is to quantify model risk AVA in a way that is compliant, using current research on prudent valuation and model risk.

The method suggested in this paper captures the instantaneous valuation uncertainty related to model risk as defined in the regulation. The first step of the method is to define a group of plausible models and calibration approaches for the instrument type. Each combination of model and calibration is then assigned a probability weight based on the number of parameters in the model, and a measure of fit that includes all available market data. When prices for the instrument have been calculated for all the different models and calibrations, the probability weights are used to form a cumulative price probability distribution for the instrument. The method is to our understanding in line with the current regulation, and should according to us hold some advantages compared to other proposed methods.
Preface

The topic of this thesis was suggested by the Quantitative Advisory Services, QAS, team at EY, Copenhagen. We would like to thank Per Thåström at EY, and Magnus Wiktorsson at Lund University, for their valuable contributions. Their knowledge and expertise in the subjects of finance and statistics has been a great asset throughout this thesis. We would also like to thank the entire QAS team at EY for the resources provided, and a great time.
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<td>Additional Valuation Adjustment</td>
</tr>
<tr>
<td>AIC</td>
<td>Akaike Information Criteria</td>
</tr>
<tr>
<td>BIS</td>
<td>Bank for International Settlements</td>
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<tr>
<td>BN-S</td>
<td>Barndoff-Nielsen-Shephard Model</td>
</tr>
<tr>
<td>BCBS</td>
<td>Basel Committee for Banking Supervision</td>
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<td>BIC</td>
<td>Bayesian Information Criteria</td>
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<td>B&amp;S</td>
<td>Black &amp; Scholes</td>
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<td>BM</td>
<td>Brownian Motion</td>
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<td>CRR</td>
<td>Capital Requirements Regulation</td>
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<td>COC</td>
<td>Close-out Costs</td>
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<td>CET</td>
<td>Common Equity Tier</td>
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<td>CP</td>
<td>Concentrated Positions</td>
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<td>CIR</td>
<td>Cox-Ingersoll Ross</td>
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<tr>
<td>CVA</td>
<td>Credit Valuation Adjustment</td>
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<tr>
<td>CDF</td>
<td>Cumulative Distribution Function</td>
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<tr>
<td>DIG</td>
<td>Digital Barrier</td>
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<td>DP</td>
<td>Discussion Paper</td>
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<td>DIB</td>
<td>Down-and-In Barrier</td>
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<td>DOB</td>
<td>Down-and-Out Barrier</td>
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<td>ET</td>
<td>Early Termination</td>
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<td>EM</td>
<td>Equivalent Martingale</td>
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<tr>
<td>EBA</td>
<td>European Banking Authority</td>
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<td>EC</td>
<td>European Commission</td>
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<td>FDIC</td>
<td>Federal Deposit Insurance Corporation</td>
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<td>FV</td>
<td>Fair Value</td>
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<tr>
<td>Acronym</td>
<td>Full Form</td>
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<td>---------</td>
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</tr>
<tr>
<td>FSA</td>
<td>Financial Services Authority</td>
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<tr>
<td>FVA</td>
<td>Funding Valuation Adjustment</td>
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<td>FVO</td>
<td>Fair Value Option</td>
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<td>FAC</td>
<td>Future Administrative Costs</td>
</tr>
<tr>
<td>HFT</td>
<td>Held for Trading</td>
</tr>
<tr>
<td>HEST</td>
<td>Heston Stochastic Volatility Model</td>
</tr>
<tr>
<td>HESJ</td>
<td>Heston Stochastic Volatility Model with Jumps</td>
</tr>
<tr>
<td>IPV</td>
<td>Independent Price Verification</td>
</tr>
<tr>
<td>IFRS</td>
<td>International Financial Reporting Standards</td>
</tr>
<tr>
<td>IFC</td>
<td>Investing and Funding Costs</td>
</tr>
<tr>
<td>LS</td>
<td>Least Squares</td>
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<tr>
<td>LIBOR</td>
<td>London Interbank Overnight Rate</td>
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<td>MPU</td>
<td>Market Price Uncertainty</td>
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<tr>
<td>MG</td>
<td>Martingale</td>
</tr>
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<td>ML</td>
<td>Maximum Likelihood</td>
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<td>MSE</td>
<td>Mean Square Error</td>
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<td>MOF</td>
<td>Measure of Fit</td>
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<td>MR</td>
<td>Model Risk</td>
</tr>
<tr>
<td>MC</td>
<td>Monte Carlo</td>
</tr>
<tr>
<td>NUP</td>
<td>Net Unrealised Profit</td>
</tr>
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<td>NIG</td>
<td>Normal Inverse Gaussian</td>
</tr>
<tr>
<td>NV</td>
<td>Notional Value</td>
</tr>
<tr>
<td>OCC</td>
<td>Office of the Comptroller of the Currency</td>
</tr>
<tr>
<td>OR</td>
<td>Operational Risk</td>
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<tr>
<td>OU</td>
<td>Ornstein Uhlenbeck</td>
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<tr>
<td>Acronym</td>
<td>Explanation</td>
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<td>OIC</td>
<td>Other Comprehensive Income</td>
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<td>OIS</td>
<td>Overnight Indexed Swap</td>
</tr>
<tr>
<td>OTC</td>
<td>Over-the-Counter</td>
</tr>
<tr>
<td>PDE</td>
<td>Partial Differential Equation</td>
</tr>
<tr>
<td>PL</td>
<td>Profit &amp; Loss</td>
</tr>
<tr>
<td>PV</td>
<td>Prudent Valuation</td>
</tr>
<tr>
<td>QAS</td>
<td>Quantitative Advisory Services</td>
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<tr>
<td>QIS</td>
<td>Quantitative Impact Study</td>
</tr>
<tr>
<td>RTS</td>
<td>Regulatory Technical Standards</td>
</tr>
<tr>
<td>RNVF</td>
<td>Risk Neutral Valuation Formula</td>
</tr>
<tr>
<td>RWA</td>
<td>Risk Weighted Assets</td>
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<tr>
<td>TREA</td>
<td>Total Risk Exposure Amount</td>
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<td>UCS</td>
<td>Unearned Credit Spreads</td>
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<tr>
<td>UIB</td>
<td>Up-and-In Barrier</td>
</tr>
<tr>
<td>UOB</td>
<td>Up-and-Out Barrier</td>
</tr>
<tr>
<td>VG</td>
<td>Variance Gamma</td>
</tr>
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</table>
Chapter 1

Introduction

1.1 Background

1.1.1 The Financial Crisis

During the recent financial crisis of 2007-2008 several vulnerabilities were exposed in the global financial system. The quest for profit in the banking system had led to excessive leverage and deteriorating level and quality of the capital base (Basel III, paragraph 4, 2010). Some governments were forced to lend a helping hand to save the banks, or suffer the collapse of their countries entire economies (Basel III, paragraph 4, 2010). In order to limit the impact and the risk of future crisis the governing organs of the world set out to create stricter requirements and better reporting standards across the financial sector.

1.1.2 Capital Requirements

A key feature in the regulations that came forth in the aftermath of the crisis was a call for stricter capital requirements for the global banking system. These new legal conditions forces banks to hold a larger portion of loss absorbing capital in relation to their total assets weighted by their risk, TREA, (Basel III, paragraph 6, 2010).

\[
\frac{\text{loss absorbing capital}}{TREA} \geq \text{requirement} \quad (1.1)
\]
Loss absorbing capital refers to both tier 1 capital which absorbs losses while remaining solvent and tier 2 capital which is capital that absorbs losses during a wind up (Basel III, paragraph 49, 2010). The assets, which have been weighted based on their risk, is the total risk exposure amount, TREA, previously known as risk-weighted assets, RWA.

![Figure 1.1: Balance sheet and capital](image)

It is of critical importance that a bank’s risk exposures are backed by a sufficient capital base. With the new regulations, banks are now required to follow a more careful approach in assessing the value of their loss absorbing capital, with focus on the common equity which is the highest quality of capital (Basel III, paragraph 48, 2010). One important element in this process is **Prudent Valuation**.

### 1.1.3 Prudent Valuation

The regulation on prudent valuation require banks to calculate an additional valuation adjustment, AVA, to account for cost and risk factors which have not been accounted for in the book value of the loss absorbing capital. The AVA should capture uncertainty in the loss absorbing capabilities of the fair valued assets and liabilities that qualify as capital of highest quality, CET1 capital. The AVA is based on a prudent valuation of the assets and
liabilities, which is a better measure than the book value for assessing the loss absorbing capabilities of the capital (CRR, 2013, article 105(1)). This prudently valued capital is then used as base when determining capital to TREA ratio for capital requirements purposes.

\[
\frac{CET1 \text{ capital} - AVA}{TREA} \geq 4.5\%
\]  

(1.2)

1.2 Purpose

The purpose of this thesis is to break down the regulation on prudent valuation, and develop statistical methods for AVA calculations. As there has been little focus on model risk in previous regulations, the main concern of this thesis is to quantify model risk in a manner resulting in a fair and compliant model risk AVA. The methods in this thesis should have a practical application for all institutions subject to these regulations, and provide help when implementing a compliant framework for prudent valuation.

1.3 Outline

The first part, Chapter 2, of this thesis concerns the financial concepts needed for the understanding of model risk and prudent valuation. Chapter 3, 4 & 5 break down the regulation on prudent valuation. In chapter 6, the statistical methods for prudent valuation developed by us are presented. An example of the method for model risk AVA is given in chapter 7. Lastly, chapter 8 is a discussion on prudent valuation and our approach to model risk. The appendix contains mathematical theory and additional figures and data.
Chapter 2

Financial Theory and Regulation

2.1 Banking Risks

There are many risks facing banks. Risks are usually defined by the adverse impact on profitability of several distinct sources of uncertainty (Bessis, 1995). The type of risks that a bank faces are among others; credit-, liquidity-, interest rate-, foreign exchange- and solvency risk. Although risks are often seen as a negative aspect of a bank’s business it is vital for the bank’s future profitability. The risk-reward trade-off is constantly present and is the foundation upon which banks conduct their business.

2.1.1 Model Risk

Ever since the industry started to use more sophisticated models to price derivatives there has been an increase in the complexity possible of a derivative. With computers we are able to price derivative in a just manner which have led to an increase in derivative instruments in the market. With all these new derivative instruments we are able to reduce the market risk greatly since we can use a more detailed product for our hedging strategy. But a risk that has sprung up because of this is the model risk. Models rely on imperfect assumptions and estimates which carries a risk.

A model is defined as ”a quantitative method, system, or approach that applies statistical, economic, financial, or mathematical theories, techniques, and assumptions to process input data into quantitative estimates” (OCC, 2011). Models are simplifications of reality which give rise to an uncertainty that is model risk. Model risk can be related to several factors:
• input data;
• design of model; and
• misuse of the model.

There are therefore different ways of quantifying model risk. The framework for prudent valuation for model risk AVA clarifies that this adjustment is concerned with the first and second item in the list above.

There are different ways to handle model risk; for example institutions can take no model risk into account, it can use worst-case approaches or it can use Bayesian statistics approaches.

2.1.2 Counterparty Credit Risk

During the financial crisis more losses originated from deterioration of counter party credit worthiness, than from actual counter party defaults. However, in Basel II there was only a charge for the risk of counter party default, not for credit deterioration before default. Therefore, Basel III introduced the Credit Valuation Adjustment, CVA, as a capital charge for losses associated with a deterioration in the credit worthiness of a counter party (Basel III, 2010, paragraph 14(b)).

2.2 Accounting

The main purpose of financial reports are to provide financial information about a company that can help investors, management, regulators and creditors in their analysis. Three important documents should be prepared annually to demonstrate the financial state of the company. These are the income statement, the cash flow statement and the balance sheet. These follows accounting standards that differ for different countries. There is an international standard of accounting called International Financial Reporting Standard, IFRS, that will be assumed to be the applicable in this thesis.

2.2.1 Balance Sheet

The purpose of the balance sheet is to provide financial information to interested parties so they can evaluate the financial state of the company. It is
divided into three parts, called the elements of financial statements: assets, liabilities and equity. See figure 2.1 for an illustrative example of a balance sheet. The basic equation must always balance, hence the name balance sheet:

\[
\text{Assets} = \text{Liabilities} + \text{Equities} 
\]  

(2.1)

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities and Equity</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Short Term Assets</strong></td>
<td><strong>Short Term Liabilities</strong></td>
</tr>
<tr>
<td>Cash</td>
<td>Commercial Paper</td>
</tr>
<tr>
<td>Receivables</td>
<td>Line of Credit</td>
</tr>
<tr>
<td>Treasury Bills</td>
<td>Other</td>
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<tr>
<td><strong>Other Cash Equivalent Assets</strong></td>
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<td><strong>Long Term Assets</strong></td>
<td><strong>Long Term Liabilities</strong></td>
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<td>Loans</td>
<td>Deposits</td>
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<td>Debt Securities</td>
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<td><strong>Non Financial Assets</strong></td>
<td><strong>Hybrid Securities</strong></td>
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<td>Real Estate</td>
<td>Equity Capital</td>
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<tr>
<td>Foreign Subsidiaries</td>
<td>Shareholder’s Equity</td>
</tr>
<tr>
<td>Goodwill</td>
<td>Retained Earnings</td>
</tr>
<tr>
<td></td>
<td>Preferred Shares</td>
</tr>
</tbody>
</table>

Figure 2.1: General outline of a Balance Sheet

The balance sheet provides the most important information about the company and is also the document that this thesis is focused on.

**Assets**

An asset is defined according to the IFRS as "a resource controlled by the entity as a result of past events and from which it is expected that future economic benefit are expected to flow to the entity" (IFRS, paragraph 4.4(a), 2013). An asset is recognised in the balance sheet when it is probable that the company will reap economic benefit from the item and that is has a cost or value that can be measured with reliability (IFRS, paragraph 4.44, 2013). Assets can be classified into subgroups, one of which is financial assets.
Financial Assets

This paper focuses on financial assets. Financial assets are measured either by using amortised cost or fair value (IFRS 7, paragraph 8, 2013). The basis for whether to use amortised cost or fair value to measure the value of a financial asset depends on:

- "the entity’s business model for managing the financial assets; and
- the contractual cash flow characteristics of the financial asset.” (IFRS 9, paragraph 4.1.1, 2013)

A financial asset should be recognised using fair value unless it is measured by amortised cost. Amortised cost is used when these criteria are met:

- "The asset held within a business model whose objective is to hold assets in order to collect contractual cash flows; and
- The contractual term of the financial asset give rise on specified dates to cash flows that are solely payments of principal and interest on the principal amount outstanding.” (IFRS 9, paragraph 4.1.2, 2013)

Liabilities

A liability is defined according to the IFRS as "a present obligation of the entity arising from past events, the settlement of which is expected to result in an outflow from the entity of resources embodying economic benefits" (IFRS, paragraph 4.4(b), 2013). A liability is recognised in the balance sheet when the outflow of economic benefit is probable and that the item has a cost or value that can be measured with reliability (IFRS, paragraph 4.46, 2013). Liabilities can be classified into subgroups.

Financial Liabilities

This paper is interested in financial liabilities. Financial assets are measured either by using amortised cost or fair value (IFRS 7, paragraph 8, 2013). The standard method for financial liabilities is amortised cost, but fair value is used by certain exceptions specified in paragraph 4.2.1 in IFRS 9 (2013). The exception that this thesis is interested in is derivatives that are liabilities which should be measured at fair value if the intent is to hold for trading. Certain hybrid instruments with embedded derivatives could be measured at fair value if it initially measured at fair value.
Equities

Equity is defined according to the IFRS as "the residual interest in the assets of the entity after deducting all its liabilities" (IFRS, paragraph 4.4(b), 2013). Equity should therefore be thought of as solely a residual term.

Off-Balance Sheet Transactions

Off-balance sheet transactions are contingencies. For banking purposes, contingencies also include guarantees given to customers and confirmed credit lines (Bessis, 1995). The contingencies do not pose any immediate risk exposure since there is no outflow of funds at the starting date. The outflow of funds occur only when certain conditions when exercised by the counterparty. The typical types are derivatives such as swaps, options and futures. Their notional values are off-balance sheet items but their fair values are recorded in the balance sheet (FDIC, 2012).

2.2.2 Fair Value

Fair value is a measurement of an asset or a liability. The other method is called amortised cost and should not be subject to the scope of this thesis. The method of fair value recognition is a market-based measurement and not entity-specific (IFRS 13, paragraph 2, 2013). The fair value measurement is defined as "the price that would be received to sell an asset or paid to transfer a liability in an orderly transaction between market participants at the measurement date" (IFRS 13, paragraph 9, 2013).

This measurement can either be based observed quoted prices for the identical asset or it could use other valuation techniques which should maximize observable inputs and minimize the use of unobservable inputs.
Valuation Techniques

An institution should use the technique which is appropriate for the given situation. It should maximise the use of observable inputs and minimise the use of unobservable inputs (IFRS 13, paragraph 61, 2013). There are three widely used techniques:

1. Market approach;
2. Cost approach; and
3. Income approach.

This thesis focuses mainly on the Income approach.

Fair Value Hierarchy

In order to increase comparability across measurements, IFRS has established three levels of inputs. The highest priority is given to quoted market prices (level 1 inputs) and the lowest to unobservable inputs (level 3).
Level 1 Inputs

Level 1 inputs are defined as quoted prices on an active market that the institution can access at the measurement date for an identical asset or liability (IFRS 13, paragraph 76, 2013).

Level 2 Inputs

These are other observable inputs that are not the quoted market price. These could be:
1. quoted prices for not identical assets in active markets;
2. quoted prices for identical assets in non-active markets; and
3. not quoted prices for identical assets in active markets (e.g. implied volatilities, credit spreads). (IFRS 13, paragraph 82, 2013)

Level 3 Inputs

The last type of input is unobservable inputs. This should be used when there is little to no market activity and should therefore be based on assumptions.

2.2.3 Mark-to-Market Position

This is a valuation methodology which uses readily available prices in orderly transactions that are sourced independently. Orderly transactions refers to the fact that the transaction was not forced or happened out of distress.

2.2.4 Mark-to-Model Position

Marking-to-model is a methodology in which a position is valued by using a benchmark, extrapolating or otherwise calculated from market input. It is done when there is no readily available market data for the position.

2.3 Capital

2.3.1 Tier 1 Capital

Tier 1 capital is the capital that allows an institution to continue its activities and remain solvent. Tier 1 capital has a subgroup with what is the purest
form of capital, common equity tier 1 capital (Basel III, paragraph 49, 2010).

**Common Equity Tier 1 Capital**

CET1 capital consists of the following: capital instruments, share premium accounts related to capital instruments, retained earnings, accumulated other comprehensive income, other reserves and funds for general banking risk (CRR, article 26, 2013). CET1 capital is the highest level of capital that allows an institution to remain solvent. It is of most subordinated interests in order for it to be of maximum loss absorbency. The more exact definition of what conditions a capital instrument needs to meet in order to be classified as CET1 capital is described in the CRR (article 28, 2013).

**2.3.2 Tier 2 Capital**

In case of insolvency this type of capital ensures that depositors and creditors are being repaid (Basel III, paragraph 49, 2010).

**2.4 Total Risk Exposure Amount**

TREA was previously known as RWA, risk weighted assets, and is a measure of a bank’s assets exposure, weighted by its risk (CRR, article 92(3), 2013). Different assets fall under different categories, from the safest, e.g. cash, to more risky, e.g. loans.

**2.5 Capital Requirements**

During the recent financial crises of 2007-2008 the banking sector held insufficient level of high quality capital. What was also discovered was the inconsistency across jurisdictions on the definitions of capital. The new regulations put forward by the Basel committee have a strong focus on common equity which is the capital of highest quality. Basel III puts the following restrictions on this type of capital which has been accepted by the EU (Basel III, paragraph 94(b), 2010):

\[
\frac{CET1 \text{ capital}}{\text{TREA}} \geq 4.5\% 
\]

(2.2)

\[
\frac{\text{Tier 1 capital}}{\text{TREA}} \geq 6\%
\]

(2.3)
\[
\frac{\text{Own funds}}{\text{TREA}} \geq 8\% \quad (2.4)
\]

2.6 Regulation

2.6.1 Basel III

Basel III is an international regulatory standard for financial institutions. It is the third document produces by the Basel Committee for Banking Supervision, BCBS, located at the Bank for International Settlements, BIS, in Basel, Switzerland. It was first published on December 2010 as a reform measure to strengthen the regulation, supervision and risk management after the financial crisis of 2007-2008.

![Phase-In timetable of Basel III](image)

This thesis is interested in the deductions from CET1 capital.

2.6.2 Capital Requirements Regulation, CRR

The capital requirements regulation, CRR, is a regulation that was published by the European Commission, EC, in June 2013. It is a regulation that is directly applicable to all EU member states on the subject ”on prudential requirements for credit institutions and investment firms” which is an implementation of certain aspects of the framework presented in Basel III (CRR, 2013). It was implemented in January 2014, but will be phased-in, similar to the recommendations from Basel III. This thesis will give full detail to article 105 on ”Requirements for prudent valuation”. They delegated the development of a regulatory technical standard to the European Banking Authority, EBA (CRR, article 105(14), 2013).
2.6.3 Regulatory Technical Standards, RTS

On March 31, 2014, EBA published its final draft regulatory technical standard, RTS, on prudent valuation. This sets out the technical details concerning the prudent valuation adjustments to fair-valued positions in the trading book and banking book. It can be summarised in two approaches, the simplified approach and the core approach. More on this in the next chapter.
Chapter 3

Introduction to Prudent Valuation

3.1 Objective of Prudent Valuation

The requirements of prudent valuation apply to all credit institutions and investment firms in the EU (CRR, article 34 & 105, 2013). The purpose of prudent valuation is to find an additional valuation adjustment, AVA, to an institutions CET1 capital. The AVA should be deducted from the CET1 capital to achieve a prudent capital base for the capital requirements. The AVA for a specific position should capture risks and costs associated with valuation and liquidation of the position that have not already been captured in the fair value of the position.

3.2 Timeline

The process of shaping the prudent valuation regulation started in the end of 2012 with a discussion paper published by the EBA. The discussion paper was followed by a consultation paper and a quantitative impact study in 2013. The CRR set the framework for prudent valuation, and the RTS defines how it is to be implemented. The final draft of the RTS still awaits approval by the EC. The approval is expected in the end of 2014, or beginning of 2015. This means that the CRR is in force but not yet the RTS.
3.3 Documentation, Systems & Control

3.3.1 Documentation

The institutions should provide guidelines on (RTS, article 18(1), 2014):

- The methods for quantifying AVAs;
- The hierarchy of market data sources used in the methods;
- The requirements for zero AVAs;
- The expert based methods to quantify AVAs;
- The methods for determining if the position is a concentrated position with assumed exit horizon; and
- The fair-valued assets and liabilities for which a change in accounting valuation partially impacts the CET1 capital according to article 4(2) and 8(1) (RTS, 2014).

In addition to the above, institution should also keep records of their calculations so that it can be analysed later. This documentation should be reviewed at least annually and needs to get approval from senior management (RTS, article 18(3), 2014).
3.3.2 Systems & Control

The information from the AVA calculations should be provided to senior management so that they get an understanding of the uncertainty in the institutions fair-values positions (RTS, article 18(2), 2014). Institutions need to have an independent control unit which should authorise the calculation initially and monitor it subsequently (RTS, article 19(1), 2014).

Furthermore, institutions need to have the following controls for the governance of fair-valued positions (RTS, article 19(2), 2014):

- Management need to sign off on all changes in the calculations;
- A statement of the institution’s appetite for exposure; and
- Independence between risk taking and control units.

The internal audit review, mentioned above, should include the following (RTS, article 19(3), 2014):

- A institution wide product inventory;
- Methods for valuation for each product;
- A validation process that ensures that the point above is actual practice in both risk taking- and control units;
- Defined thresholds that indicates whether valuation models are still robust;
- A formal IPV process;
- A new product approval process; and
- A new deal review process to ensure that market data are used to assess whether the valuations remains prudent.
3.4 Impact of Prudent Valuation

Before writing the final draft RTS (2014), the EBA conducted a quantitative impact study, QIS, to estimate the impact of the regulation. They asked 59 banks across 15 jurisdictions to calculate the AVA deductions. The following was obtained:

<table>
<thead>
<tr>
<th>All amounts in €m</th>
<th>AVA €m</th>
<th>% of CET1</th>
<th>% of FV balance sheet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>53</td>
<td>0,18%</td>
<td>0,10%</td>
</tr>
<tr>
<td>Medium</td>
<td>634</td>
<td>0,83%</td>
<td>0,10%</td>
</tr>
<tr>
<td>Large</td>
<td>12743</td>
<td>1,57%</td>
<td>0,07%</td>
</tr>
<tr>
<td>Total</td>
<td>13430</td>
<td>1,46%</td>
<td>0,07%</td>
</tr>
</tbody>
</table>

Figure 3.2: Impact of the prudent valuation framework by size of institution (RTS, 2014)

Where the following are used:

- Small banks - sum of absolute value of fair valued assets and liabilities are less than 15 billion EUR;
- Medium banks - sum of absolute value of fair valued assets and liabilities are between 15 billion and 100 billion EUR; and
- Big banks - sum of absolute value of fair valued assets and liabilities are more than 100 billion EUR.

3.5 Definitions

3.5.1 Valuation Positions

Valuation position refers to the division of relevant fair valued assets and liabilities into suitable portfolios, where the instruments are sensitive to the same underlying risk factors. A valuation position is a portfolio consisting of related instruments (RTS, paragraph 2(a), 2014).

3.5.2 Valuation Input Parameters

The valuation input parameters are market observable or non-observable parameters that influence the fair value of a valuation position, e.g. market prices if the position is traded, or market derived parameters such as implied volatilities if the position is valued using a model. If the position simply
consists of one specific traded instrument, the valuation input is the market price of the instrument (RTS, paragraph 2(b), 2014).

3.5.3 Valuation Exposures

Valuation exposure means the amount of a valuation position, instrument or portfolio, which is sensitive to movement in a valuation input. If the position consists of one specific traded instrument the valuation exposure is simply the amount invested in the asset. If the valuation position is a portfolio of instruments the valuation exposure is the netted amounts sensitive to the valuation input parameters. This means that long and short positions in the same instrument are netted to form exposures, and instruments sharing a hedging relationship are netted to form exposures (RTS, paragraph 2(c), 2014).

3.6 Identification of relevant Valuation Positions

The first step in the process of prudent valuation is to identify which positions that fall within the scope of the regulation. The RTS makes clear that the prudent valuation standard should be applied to all financial instruments and commodities measured at fair value in both the trading and the banking book (CRR, article 34 & 105, 2013). Assets measured using other accounting standards, such as amortized cost, are therefore not subject to prudent valuation. This discussion is described more in detail in chapter 2, under the section fair value.

For fair-valued assets and liabilities for which a change in accounting valuation has a partial or zero impact on common equity tier 1 (CET1) capital, their values shall only be included in proportion to the impact of the relevant valuation change on CET1 capital (RTS, article 4(2) & 8(1), 2014). This is in line with the objective of prudent valuation to adjust the CET1 capital base for regulatory purposes.
3.7 Determination of Approach

The additional valuation adjustment can be calculated in one of two ways. The first method is the simplified approach which can be used by smaller firms to limit the operational burden of the calculations. It is also allowed since smaller firms is thought of as to have a limited valuation uncertainty given the size of their fair value portfolios (RTS, article 8, 2014). This approach may be applied when the sum of the absolute values of fair valued assets ($A$) and liabilities ($L$) in the balance sheet is less than 15bn euros,

$$15 \text{bn EUR} > |A| + |L|$$

(RTS, article 4(1), 2013), where exactly matching offsetting positions are excluded, and positions that have a partial impact on CET1 capital are included proportionally.

Institutions that meet the requirements for the Simplified Approach could choose to use the core approach instead if they see it fit.
The second method is the core approach which is a more advanced method that requires more work, but results in an AVA which better reflects the uncertainty in the value of the institution’s financial positions. Institutions exceeding the 15bn euros limit are obliged to calculate the AVA using the core approach.

Also, if a group breaches the threshold of 15bn euros on a consolidated basis it needs to calculate AVA using the core approach for all its entities. Moreover, if an institution which uses the Simplified Approach breaches the threshold level for two consecutive quarters it should notify relevant authority, and plan to implement the core approach within the following two quarters (RTS, article 4(1), 2013).

3.8 The Simplified Approach

The simplified approach for calculating AVA is simple and crude. It is based on a percentage of their fair value portfolio. Institutions should simply calculate the AVA by taking 0.1% of the sum of absolute values of the fair valued assets and liabilities adjusted for offsetting positions, and positions that have a partial impact on CET1 capital (RTS, article 5, 2013):

$$AVA = 0.001(|A| + |L|)$$  \hspace{1cm} (3.2)
Chapter 4
The Core Approach

4.1 Introduction to the Core Approach

The core approach is a more advanced method of Prudent Valuation that requires calculations of AVAs on different levels, for a variety of cost and risk factors. The cost and risk factors that the AVAs should capture are:

- Market price uncertainty (MPU);
- Close-out costs (COC);
- Model risk (MR);
- Unearned credit spreads (UCS);
- Investing and funding costs (IFC);
- Concentrated positions (CP);
- Future administrative costs (FAC);
- Early termination (ET); and
- Operational risk (OR).

Since the core approach calculates AVAs on different levels, and allows for diversification benefits for some cost and risk factors (categories) the aggregation is not a simple summation.
The calculation of total AVA in the core approach has the following steps:

1. Identify all valuations positions in the trading and the banking book;

2. Determine whether the valuation position can be shown to have zero AVA;

3. Calculate AVAs ($AVA_{category_{position}}$) for all cost and risk factors for all valuation positions;

4. Could all categories be accounted for?
   
   (a) If yes, calculate category level AVA;
      
      i. For MPU, COC and MR:
         
         $$AVA_{category} = 0.5 \sum AVA_{category_{position}}$$  \hspace{1cm} (4.1)

      ii. For the remaining categories:
         
         $$AVA_{category} = \sum AVA_{category_{position}}$$  \hspace{1cm} (4.2)

   (b) If no, use the fall-back approach for said valuation position.

5. Calculate total AVA:

   $$Total\ AVA = \sum AVA_{category} + \sum AVA_{fall-back_{position}}$$  \hspace{1cm} (4.3)
Figure 4.1: Process of determining AVA under the core approach
4.2 Zero AVA Positions

An institution may calculate a zero AVA where the fair value including adjustments can be shown to already be at a prudent level. Evidence to support this could be one of two types (DP on RTS, 2012):

- There is strong evidence of actual trades or readily-tradable quotes at the balance sheet date and time for sizes of trade that indicate the position could be closed in its entirety at the fair value on the balance sheet.

- The balance sheet valuation is already suitably prudent. Where there is little evidence to support the price level, incontrovertible evidence would be required to show that this is the case (for instance 100% provisioning).

4.3 The Fall-back Approach

When there is insufficient data to calculate an AVA, and an expert-based approach is not applicable, for one or more cost or risk factors, the AVA for the position must be assessed using a fall-back approach. In this approach the total AVA for the position should be calculated as (RTS, article 7(2b), 2014):

- Derivative positions:
  \[ AVA_{\text{position}} = NUP + 0.1 * NV \]  
  (4.4)

- Non-derivative positions:
  \[ AVA_{\text{position}} = NUP + 0.25 * |FV - NUP| \]  
  (4.5)

where net unrealised profit, NUP, is the positive change in fair value since trade inception and NV is the notional value.
4.4 Example of Calculation of Total AVA

In this example there are six valuation positions; x, y, z, u, v and w. The last three has one or more cost and risk factors which could not be estimated. Therefore the fall-back approach is applied. For position x there are readily available quotes which allows for mark-to-market. Position y and z are marked-to-model. The operational risk is calculated using the advanced measurement approach which leads to a zero AVA for operational risk. The data in this example is not based on any real world data and should only be thought of as illustrative for summation for total AVA.

<table>
<thead>
<tr>
<th>Categories</th>
<th>Positions</th>
<th>Total Category AVA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>x</td>
<td>y</td>
</tr>
<tr>
<td>MPU</td>
<td>12</td>
<td>3</td>
</tr>
<tr>
<td>COC</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>MR</td>
<td>0</td>
<td>22</td>
</tr>
<tr>
<td>UCS</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>IFC</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>CP</td>
<td>0</td>
<td>12</td>
</tr>
<tr>
<td>FAC</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>ET</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>OR</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Sum Total Category AVA</td>
<td>67</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4.2: Example of summation of category level AVA

<table>
<thead>
<tr>
<th>Fall-Back Approach</th>
<th>Positions</th>
<th>Sum Fall-Back AVA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>u</td>
<td>v</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>3</td>
</tr>
</tbody>
</table>

Figure 4.3: Example of summation of fall-back AVA

<table>
<thead>
<tr>
<th>Sum Total Category AVA</th>
<th>Sum Fall-Back AVA</th>
<th>Total AVA</th>
</tr>
</thead>
<tbody>
<tr>
<td>67</td>
<td>17</td>
<td>84</td>
</tr>
</tbody>
</table>

Figure 4.4: Example of summation of Total AVA

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4.5 Valuation Input Parameter Reduction

For the purpose of calculating AVAs for market price uncertainty, close-out costs and model risk, which are calculated on a valuation exposure level, a reduction of valuation input parameters can sometimes be applied. The valuation exposures should then be remapped appropriately on the new set of parameters. A reduction of parameters is allowed if the following conditions hold (RTS, article 9(4) & 10(5), 2014):

- The total valuation exposure amount does not change.
- The reduced set of parameters can be mapped to market tradable instruments
- The profit and loss volatility of the valuation exposure for the last 100 trading days is not decreased by 10% or more.

\[
\frac{\sigma_{P&L}^{\text{unreduced}} - \sigma_{P&L}^{\text{reduced}}}{\sigma_{P&L}^{\text{unreduced}}} \leq 0.1
\]  

(4.6)

If a valuation input parameter reduction is carried out the resulting combination of parameters and exposures can only be used to calculate AVAs for risk factors that are treated at exposure level. This method cannot be used for the rest of the costs and risks because they need to be calculated based on actual positions held in the banking or trading book.
Chapter 5

AVA Calculations for Specific Cost and Risk Factors

5.1 Introduction

The general idea for the AVA calculations for each category is to add an additional valuation adjustment on top of the fair value adjustments to achieve a prudent value corresponding to a 90% confidence level for the worst exit valuation with respect to that cost/risk factor. For a long position the prudent value will be lower than the fair value (10% quantile), and for a short position the prudent value will be higher than the fair value (90% quantile). In this thesis the position is assumed to be a long position, unless otherwise specified. The equation and the figure below presents the general idea.

\[ PV = FV - AVA \] (5.1)
5.2 Market Price Uncertainty AVA

The following section is based on article 9 (RTS, 2014). The AVA for market price uncertainty should reflect the uncertainty in present market prices on instruments used for the valuation of a position or exposure.

5.2.1 MPU in Mark-to-Market Positions

If the position is mark-to-market the MPU AVA should capture the uncertainty in the market price of the position. In the case that the position is a portfolio of traded instruments the MPU AVA should consider the MPU of all the instruments in the portfolio. The uncertainty in the market price of a traded position comes from not knowing exactly at what price the position can be sold right now. The more liquid the market for the position is, and the more quotes of similar size exists, the smaller the risk associated with market price uncertainty.
5.2.2 MPU in Mark-to-Model Positions

For a mark-to-model position the market price uncertainty comes from using uncertain market price data when calibrating the model parameters. In this case the market price uncertainty AVA should reflect all uncertainty in the model’s pricing of the position that comes from the market price uncertainty of the model input data.

5.2.3 Zero MPU AVA

The AVA for market price uncertainty can only be set to zero if both of the following two conditions are met:

- The position can be traded:
  There exists evidence of a traded price for the position or a price can be determined using data from a liquid two-way market, as defined in Article 338 (CRR, 2013); and

- The price is certain:
  The market data does not indicate any uncertainty in the market price of the position.

5.2.4 Calculation of MPU AVA

Case of Sufficient Bid-Ask Data or Mid-Prices

In this case there exists a sufficient range of market bid-ask data or market mid-prices to calculate a MPU AVA. The method is as follows:

1. Construct a range of plausible mid-prices for the position;

2. Estimate a prudent point within this range where there is 90% confidence that the mid-price would be at that point or better if the position was to be exited; and

3. Calculate the market price uncertainty AVA by subtracting the estimated prudent mid-price from the estimated mid-price used in the fair valuation of the position.

In this case an AVA for close-out costs is required.
Case of Sufficient Exit Prices

In this case there exists a sufficient range of market exit prices to calculate a MPU AVA. The method is as follows:

1. Construct a range of plausible exist prices for the position;

2. Estimate a prudent point within this range where there is 90% confidence that the position can be exited at that price or better; and

3. Calculate the market price uncertainty AVA by subtracting the estimated prudent exit price from the estimated exit price used in the fair valuation of the position.

No AVA for close-out costs is required in this case.

Insufficient Data Case

Use an expert based approach (RTS, article 9(5), 2014).

5.3 Close-out Costs AVA

The following section is based on article 10 (RTS, 2014). The AVA for close-out costs, COC, should capture the costs associated with not being able to exit a position at mid-price.

5.3.1 Zero Close-out Costs AVA

The close-out costs AVA can be assessed to be zero in the following two cases:

- Market price uncertainty AVA based on exit price:
  If the AVA for market price uncertainty has been calculated based on exit price the close-out costs have already been accounted for in that AVA, and the close-out costs AVA is therefore zero; and

- Exiting at mid-price:
  If there is evidence of a 90% confidence that sufficient liquidity exists to exit the position at mid-price the close-out costs AVA can be assessed to be zero.
5.3.2 Calculation of Close-out Costs AVA

Case of Bid-Ask Data

1. Construct a range of plausible bid-ask spreads for the position;

2. Estimate a prudent point within this range where there is 90% confidence that the spread will be of that size or smaller when exiting the position; and

3. Calculate the close-out cost AVA by taking 50% of the estimated prudent bid-offer spread.

Insufficient Data Case

Apply expert-based approach (RTS, article 10(6), 2014).

5.4 Model Risk AVA

The following section is based on article 11 (RTS, 2014). A model risk AVA should be calculated whenever a position is valued using a mark-to-model approach. Model risk increases when there is a greater dispersion of the prices for the range of possible models and model calibrations that could be used by market participants. The model risk AVA should capture the uncertainty associated with the choice of model, and the calibration approach for the parameters used in the model. When market data is used as input for the calibration any risk that originates from uncertainty in the market price of the input data should be included in the market price uncertainty AVA, and not in the model risk AVA. A method for calculation of market price uncertainty AVA for a marked-to-model position is presented in chapter 6.

5.4.1 Calculation of Model Risk AVA

In our understanding the range of possible models and calibrations can be formed by varying the following examples below (for equity derivative pricing). The significance of each factor depends on the derivative so hopefully some of them can be considered negligible, and it might come down to three to five factors that needs to be varied in order to construct a range of models and calibrations.

- Pricing approach (e.g. Interpolation/extrapolation from similar instruments, modelling the underlying asset);
• Interpolation method (e.g. Linear, splines);
• Distribution assumptions (e.g. Normal, student t);
• Model for underlying (e.g Heston, BN-S);
• Type of calibration data (e.g. Vanilla options, volatility surface);
• Size of calibration set (e.g. Full data set, limited or altered data);
• Calibration time horizon/window (e.g. 1 day, 1 week); and
• Method for parameter estimation (e.g. ML, LS).

Range of Models and Calibrations

In the case were a sufficient range of plausible models and calibrations can be identified and implemented the RTS states that the following should be applied:

1. Construct a range of plausible valuations for the position;

2. Estimate a prudent point within this range where there is 90% confidence that the position can be exited at that price or better; and

3. Calculate the model risk AVA by subtracting the estimated prudent price from the model price used in the fair valuation of the position.

Methods for these steps are presented in chapter 6.

Case of Insufficient Data - Expert-based Approach

When a set of plausible valuation models cannot be determined or implemented a expert-based approach should be applied to estimate the model risk AVA. The RTS states that the expert-based model should consider the following:

• Complexity of products relevant to the model;
• Diversity of possible mathematical approaches and model parameters;
• The degree to which the market for relevant products is one way;
• The existence of unhedgeable risks in relevant products; and
• The adequacy of the model in capturing the pay-off of the products in the portfolio.
5.5 Unearned Credit Spreads AVA

The following section is based on article 12 (RTS, 2014). The Unearned credit spreads AVA should capture any uncertainty in any adjustment for counterparty credit risk. If a position is subject to a credit valuation adjustment, CVA, this AVA should capture the uncertainty in that CVA. The uncertainty that originates from uncertainty in the input should be allocated to the MPU AVA, uncertainty from bid-ask spreads to the COC AVA, and uncertainty from model risk to MR AVA. Below are examples for what to include here, based on our understanding of the RTS.

**UCS AVA to MPU AVA**

- MPU in CDS spreads used in CVA calculations;
- MPU in default probabilities;
- MPU in recovery rates; and
- MPU in models for exposure at default.

**UCS AVA to COC AVA**

- Possibly COC for credit default swaps. However, as there is no actual trade it is unclear whether or not close out costs are relevant.

**UCS AVA to MR AVA**

- Model risk in CVA calculation;
- Model risk in default probability estimations;
- Model risk in recovery rate estimations; and
- Model risk in EAD estimations.
5.6 Investing and Funding Costs AVA

The following section is based on article 13 (RTS, 2014). The investing and funding costs AVA should capture the uncertainty in a value of a position that comes from uncertainty in the discounting rate. The uncertainty that originates from uncertainty in the input should be allocated to the MPU AVA, uncertainty from bid-ask spreads to the COC AVA, and uncertainty from model risk to MR AVA.

"This AVA should be calculated with reference to the discount rate being applied to derivative transactions, term matched with the trade payments, covering the contractual maturity of the trades. Where these trades have embedded optionality, the expected maturity based upon optimal behaviour for the option holder may be used. Expected maturity should be assessed prudently, given uncertainty in option models, or market data inputs.” (FAQ on PV, 2013)

Our interpretation of this is that the IFC AVA should capture uncertainty in the funding valuation adjustment, FVA, which is still debated, and not yet fully understood and implemented on the market. It is with this interpretation the following examples are presented.

**IFC AVA to MPU AVA**

- MPU in OIS, LIBOR, bonds, swaps, and other interest rate derivatives used for discounting.

**IFC AVA to COC AVA**

- Possibly COC for OIS, LIBOR, bonds, swaps, and other interest rate derivatives used for discounting.

**IFC AVA to MR AVA**

- Choice of calibration data for the rates (LIBOR, OIS, etc.); and
- Choice of time horizon/window for calibration and prediction.
5.7 Concentrated Positions AVA

The following section is based on article 14 (RTS, 2014). Concentrated positions AVA should capture the risk associated with holding a relatively large position in relation to the market liquidity. Holding a concentrated position carries a risk because there is uncertainty in how fast the position can be exited, and how the exit strategy affects the market price.

5.7.1 Calculation of Concentrated Positions AVA

The following three-step approach should be applied to calculate the AVA:

1. Identify concentrated positions based on the following:
   - Size of valuation position relative to the liquidity of the market;
   - The institutions ability to trade in the market; and
   - Average daily market volume and typical daily trading volume of the institution.

2. For each concentrated position for which a market price for that size is unavailable, an institution should estimate a prudent exit period for the position.

3. If the prudent exit period exceeds ten days an institution should estimate an AVA based on the following:
   - The volatility of the market price of the position;
   - The volatility of the bid offer spread for the position; and
   - The impact of the hypothetical exit strategy on market prices.

5.8 Future Administrative Costs AVA

The following section is based on article 15 (RTS, 2014). Future administrative costs AVA is the costs associated with administrative and future hedging costs over the expected life of the valuation position. This is discounted by the risk-free rate. The administrative costs is all incremental staffing and fixed costs incurred in managing the portfolio. Future administrative costs AVA could be zero in the case that market price uncertainty and close-out costs incorporates all costs associated with fully exiting a position.
5.9 Early Termination AVA

The following section is based on article 16 (RTS, 2014). Early termination AVA considers the potential losses from non-contractual early terminations of client trades. This AVA is calculated historically by taking the percentage of client trades which have terminated early and the losses that corresponds to those.

5.10 Operational Risk AVA

The following section is based on article 17 (RTS, 2014). Operational risk AVA is the risk associated with potential losses as a result of operational risk in valuation processes.

5.10.1 Zero Operational Risk AVA

If an institution has applied the advanced measurement approach, AMA, for operational risk and can say that it has fully accounted for all operational risk associated to valuation processes then the AVA can be set to zero.

5.10.2 Calculation of Operational Risk AVA

If operational risk is not calculated using the AMA the AVA for Operational Risk is calculated as 10% of the AVAs for market price uncertainty and close-out costs:

\[ AVA_{OR}^{position} = 0.1 \times (AVA_{MPU}^{position} + AVA_{COC}^{position}) \] 

(5.2)
Chapter 6
Statistical Methods for Prudent Valuation

6.1 Estimation of Prudent Point within Data Range

For the estimation of a prudent point, \( pp \), within a given data set, which is required in most of the AVA calculations, the RTS states: "institutions shall estimate a point within the range where they are 90% confident" (RTS, article 9(5), 2014). This means that the prudent point is never allowed to be outside the data range. As the distribution of the data generally is unknown, we propose to assign equal probability to all data points and use linear interpolation between closest ranks around the relevant percentile, \( P \) (10% or 90%). The data is sorted in ascending order, \( v_1 \leq \ldots \leq v_N \), for \( N \) points of data. For interpolation the following is needed:

\[
 k + d = (N - 1) \times P
\]  

(6.1)

where \( k \) is the integer part, and \( d \) is the decimal part of \( (N - 1) \times P \). The formula for calculating the prudent point, \( pp \), is then:

\[
 pp = v_{k+1} + d(v_{k+2} - v_{k+1})
\]  

(6.2)
6.2 AVA Calculations for Mark-to-Model Positions

When an instrument does not have a tradable market price, institutions will use a model to calculate a price for the instrument. A model uses other information about the market such as prices for similar instruments, or vanilla instruments on the same underlying asset, as input for the valuation. The RTS (article 11(1), 2014) states that valuation uncertainty from market derived input parameters (such as prices for similar instruments) must be separated from valuation uncertainty related to the model and model calibration. Uncertainty in market derived parameters should be captured in MPU AVA, and uncertainty related to the model and model calibration should be captured in the MR AVA. Therefore a method to calculate MPU AVA for mark-to-model positions is presented in the section below, although focus will be on the method for model risk as this is the main concern of this thesis.

The two different types of input give rise to two main types of models:

- Type 1: Models interpolating/extrapolating from market prices.
- Type 2: Stochastic models for the underlying asset using the RNVF.

As any uncertainty in the price that is related to uncertainty in the discounting rate is captured in the IFC AVA the methods suggested below assumes discounted pay-off. Similarly, as any uncertainty related to counterparty credit risk is captured in the UCS AVA the CVA impact on the price is ignored.

6.2.1 Variable Declaration

$X$: Stochastic variable representing the discounted payoff of the instrument for which we seek a prudent value.

$\Pi(X)$: Price of the instrument with pay off $X$.

$z = (z_1, ..., z_R)$: Prices for similar instruments that may be used in type 1 models.
Type 1 Models

This type of model prices the instrument by interpolation or extrapolation from market prices of similar traded instruments. The interpolating/extrapolating function, $f$, of the model is what determines the output price. These models are in general more straight-forward and less advanced than the type 2 models, however, for exotic derivatives with more complex pay-offs the type 2 models are necessary. The price for an instrument under a type 1 model can be expressed as a function of the input prices used:

$$\Pi(X|\text{Type 1}) = f(z)$$  \hspace{1cm} (6.3)

Type 2 Models

In this type of model the price of the underlying asset is modelled by a stochastic process and the instrument is priced using the RNVF. The pricing model can be calibrated using traded vanilla options on the same underlying asset. Options with different strikes and different maturities are used to derive an implied volatility surface. This surface is then used to tune the model parameters so that the model fits the implied volatility structure, and produce prices for the vanilla options. As the pay off is discounted the price is:

$$\Pi(X|\text{Type 2}) = \mathbb{E}^Q[X]$$  \hspace{1cm} (6.4)

6.3 Market Price Uncertainty

6.3.1 Type 1 Models

If there is no MPU in the traded input instruments the MPU AVA for the marked-to-model position should be set to zero. Otherwise, calculations need to be carried out to see how the uncertainty in the prices of the input instruments affects the model output price.

If the relation between the input parameters and the output price is simple enough (e.g. linear) a joint 90% confidence interval for the parameters can be used. If the relation is more complex, an AVA for market price uncertainty can be calculated using a Monte Carlo method.
Monte Carlo Method

Given that there is a number of plausible market prices for each input instrument we suggest to use a Monte Carlo approach to assess the impact of the MPU in input instruments on the model valuation.

In every simulation a price for the instrument is produced based on different random combinations of prices of the input parameters from their vectors of candidate prices. Every simulation follows these steps:

1. For all input parameters a price from the parameters vector of candidate prices is chosen at random; and
2. The model is then applied and produces a price based on the randomly chosen parameter values.

The prudent value of the instrument with respect to MPU is found by constructing a one-sided 90% confidence interval.

In the regular derivative pricing process one could use the average, expectation, or similar measure for the set of plausible market prices for a specific input instrument. For the purpose of prudent valuation one should instead randomly choose a price from the vector of plausible prices for a specific input instrument. Randomly choosing prices for all input instruments in this way will yield a random, plausible realization. Pricing the target instrument using the new parameter values will result in a random plausible price for the target instrument. Repeating this process a large number of times will produce a large set of random plausible model valuations for the mark-to-model instrument. The prudent value with respect to MPU for a long position is the 10% percentile of the valuations, and the PV for a short position is the 90% percentile.
Implementation of the Monte Carlo Method

Using N number of simulations and assuming R number of input instruments. Assuming that the values in the vectors of plausible prices for the input instruments are arranged in ascending order.

N: number of simulations.
n: the n:th simulation, where n=1,...,N.
R: number of input instruments.
K: maximum number of plausible prices for input instrument.
P(row, column): matrix of prices for the input instruments.
P(r,k): the k:th plausible price for the r:th input instrument, were r=1,...,R and k=1,...,K.
\( \Pi_{MC}^n(X) \): the price produced in the n:th simulation.
\( \Pi_{PV}^{MPU}(X) \): prudent value of the target instrument with respect to MPU.
\( \Pi_{FV}(X) \): the model price of the target instrument used in fair value calculations.

Monte Carlo Steps

1. Simulate a set of N valuations, \( \Pi_{MC}^1(X),...,\Pi_{MC}^N(X) \), for the target instrument. For each simulation \( n = 1,...,N \):

   (a) Price all input instruments by, for every input instrument, \( r = 1,...,R \), randomly choosing a price from the plausible prices, \( P(r,k) \), \( k = 1,...,K \), for that instrument.

   (b) Calculate the price of the target instrument under these parameter values, \( \Pi_{MC}^n(X) \).

2. Obtain the MPU prudent value of the target instrument, \( \Pi_{PV}^{MPU}(X) \), by taking the relevant percentile of the simulated prices, \( \Pi_{MC}^1(X),...,\Pi_{MC}^N(X) \).

3. Calculate the MPU AVA as the absolute value of the difference between the model price used in fair valuation and the MPU prudent value:

   \[
   MPU \ AVA = \Pi_{FV}(X) - \Pi_{PV}^{MPU}(X)
   \]

6.3.2 Type 2 Models

In this case it is the effect of the MPU of the calibration options on the model output price that is to be decided. As the calibration of these models allow for more parameter freedom within the bid-ask spread any MPU in the calibration options will have very little effect on the model’s output price.
6.4 Model Risk - Range of Models Approach

This method addresses article 11(3) in the RTS on prudent valuation (2014) and applies to the case when a set of plausible valuation models and calibrations can be determined and implemented. The range originates from different combinations of choices related to the following factors:

- Pricing approach;
- Interpolation method;
- Distribution assumptions;
- Model for underlying;
- Type of calibration data;
- Size of calibration set;
- Calibration time horizon/window; and

6.4.1 Candidate Models

The first step in assessing the model risk for an instrument is to identify a set of plausible candidate pricing models and calibrations that other market participants use to price the instrument. This means that model- and calibration alternatives/combinations that exceed the natural bounds for the instrument, or are illogical, out-dated, or not deemed to have a significant usage among market participants, should be excluded. The resulting set of model- and calibration combinations will from now on be referred to as the set of candidate models.

\[ M = M_1, ..., M_J \]: The set of candidate models (model- and calibration combinations).

Natural Bounds for the Valuation

Natural bounds can be market prices for the following:

- Market price of a sub- or super performing instrument; and
- Market price of sub- or super performing portfolio.
For example if a prudent value for a non traded call option is sought, the market price of a traded call with higher/lower strike is a natural bound to the price of the non traded call. (Assuming the call options are identical in all aspects other than the strike price.)

6.4.2 Variable Declaration

$X$: Stochastic variable representing the discounted pay off of the derivative for which we seek a prudent value.

$\Pi(X|M_j)$: Price of the target instrument under model $M_j$.

$\Pi_{Proxy}^n(X)$: Price for the target instrument derived from proxy data. $n = 1, \ldots, N$.

$Z = Z_1, \ldots, Z_R$: Stochastic variable representing the discounted pay off of traded similar derivatives on the same underlying.

$\Pi^{Market}(Z_r)$: Market price for derivative with pay off $Z_r$.

$\Pi(Z|M_j)$: Price for derivative with pay off $Z$ in model $M_j$.

$C_i$: Stochastic variable representing the discounted pay off of calibration instrument $i$, $i = 1, \ldots, I$.

$\Pi^{Market}(C_i)$: Market price of calibration instrument $i$.

$\Pi(C_i|M_j)$: Price of calibration instrument, $i$, under model $M_j$.

$\Pi^{PV, MR}(X)$: Prudent value of the target instrument with respect to model risk.

6.4.3 Model Probability Weights

The next step is to assign every model in $\mathcal{M}$ a probability weight using Akaike Information Criterion, AIC, and Akaike weights. This idea comes from a paper by Detering and Packham (2013). However, we propose to expand the measure of fit to go beyond the mean-squared-error to calibration data, to also consider pricing error to similar instruments, proxy data and historical data. The effect of these probability weights will be that a model with an over-all better fit to data will be assigned a higher probability weight,
at the same time as the complexity of the model (number of parameters) is considered.

In the case where there is no proxy data, no traded similar instruments, or any historical data, the mean squared error to calibration instrument prices, $MSE$, is the measure of fit, $MOF$:

$$MOF(M_j) = MSE(M_j) = \frac{1}{I} \sum_{i=1}^{I} |\Pi^{Market}(C_i) - \Pi(C_i|M_j)|^2 \quad (6.5)$$

If there exists prices for similar derivatives on the same underlying, $\Pi^{Market}(Z_1),...,\Pi^{Market}(Z_R)$, these can be considered to a factor $\alpha = \alpha_1,...,\alpha_R$. The factors should be the same for all models. The more similar the derivative is to the target instrument, the higher the factor should be in relation to the magnitude of the MSEs and any other error terms in the MOF. The error terms $\epsilon^Z$ are calculated as:

$$\epsilon^Z_r = \Pi^{Market}(Z_r) - \Pi(Z_r|M_j) \quad (6.6)$$

Also, if there are prices, $\Pi^{Proxy}_1(X),...,\Pi^{Proxy}_N(X)$, based on proxy data (proxy prices) available, the error to proxy prices, $\epsilon^{Proxy}$, can also be considered to a factor $\beta = \beta_1,...,\beta_N$, that reflects the relevance of the proxy data. The more relevant the proxy data is, the higher the factor should be in relation to the magnitude of the MSEs and any other error terms in the MOF. The error terms are:

$$\epsilon^{Proxy}_n = \Pi^{Proxy}_n - \Pi(X|M_j) \quad (6.7)$$

Lastly, if there exists any historical pricing errors, $\epsilon^{Hist}_1,...,\epsilon^{Hist}_G$, for the instrument and the specific model, these can be considered to a factor $\gamma = \gamma_1,...,\gamma_G$, that reflects the relevance of the historical price. The more recent the data is, the higher the factor should be in relation to the magnitude of the MSEs and any other error terms in the MOF.

Adding all these errors to the $MOF$:

$$MOF(M_j) = MSE(M_j) + \sum_{r=1}^{R} \alpha_r |\epsilon^Z_r|^2 + \sum_{n=1}^{N} \beta_n |\epsilon^{Proxy}_n|^2 + \sum_{g=1}^{G} \gamma_g |\epsilon^{Hist}_g|^2 \quad (6.8)$$

The AIC value for model $M_j$ is calculated as follows (Detering & Packham, 2013):
\[ AIC(M_j) = n[1 + \ln(2\pi) + \ln(MOF(M_j))] + 2K(M_j) \] (6.9)

where \( K(M_j) \) is the number of parameters in model \( M_j \).

The best AIC value, \( AIC_{\text{min}} \), is simply the smallest AIC value archived in the set of candidate models \( \mathcal{M} \):

\[ AIC_{\text{min}} = \min_{M_j \in \mathcal{M}} AIC(M_j) \] (6.10)

The likelihood of model \( M_j \), \( L(M_j) \), is based on the difference \( AIC(M_j) - AIC_{\text{min}} \):

\[ L(M_j) = \exp \left\{ - \frac{(AIC(M_j) - AIC_{\text{min}})}{2} \right\} \] (6.11)

The probability weight for a model is calculated as its Akaike weight:

\[ P(M_j) = \frac{L(M_j)}{\sum_{M_i \in \mathcal{M}} L(M_i)} \] (6.12)

### 6.4.4 Price Probability Distribution and Prudent Value

By arranging the models in \( \mathcal{M} \) according to \( \Pi(X|M_j), j = 1, \ldots, J \), with the model producing the lowest price first, a cumulative probability can be introduced.

\( CDF(M_j) \): Cumulative probability up to including model, \( M_j \).

\[ CDF(M_j) = \sum_{i=1}^{j} P(M_i) \] (6.13)

Prudent valuation aims for a confidence degree of 90%. For a long position this means the 10%:th lowest price (10% percentile), and for a short position the 10%:th highest price (90% percentile).

**Breach Point**

\begin{align*}
\text{Long position:} & \quad j^* = \arg \min_{j \in (1, J)} CDF(M_j) > 0.1 \\
\text{Short position:} & \quad j^* = \arg \min_{j \in (1, J)} CDF(M_j) > 0.9
\end{align*} (6.14) (6.15)
Prudent Value without Interpolation

Long position: \( \Pi_{MR}^{PV}(X) = \begin{cases} \Pi(X|M_{j^*}), & j^* = 1 \\ \Pi(X|M_{j^* - 1}), & j^* > 1 \end{cases} \) (6.16)

Short position: \( \Pi_{MR}^{PV}(X) = \Pi(X|M_{j^*}) \) (6.17)

Interpolation

\( \Delta \Pi = \Pi(X|M_{j^*}) - \Pi(X|M_{j^* - 1}) \): Price difference

\( \Delta \Pi = CDF(M_{j^*}) - CDF(M_{j^* - 1}) \): Probability difference

Prudent Value for Long Position using Interpolation

\[ \Pi_{MR}^{PV}(X) = \begin{cases} \Pi(X|M_{j^*}), & j^* = 1 \\ \Pi(X|M_{j^* - 1}) + (0.1 - CDF(M_{j^* - 1})) \frac{\Delta \Pi}{\Delta \Pi}, & j^* > 1 \end{cases} \] (6.18)

Prudent Value for Short Position using Interpolation

\[ \Pi_{MR}^{PV}(X) = \begin{cases} \Pi(X|M_{j^*}), & j^* = 1 \\ \Pi(X|M_{j^* - 1}) + (0.9 - CDF(M_{j^* - 1})) \frac{\Delta \Pi}{\Delta \Pi}, & j^* > 1 \end{cases} \] (6.19)
6.5 Alternative Approach

The approach to measure model risk suggested above is, to our understanding of the regulation, the most suitable for measuring model risk. However, a measure proposed by Detering and Packham (2013) was also taken under consideration. This measure takes the optimal hedge into account, and has a time horizon similar to that of the value at risk measure.

\( Q_{M_j} \): Probability measure for a specific model and calibration.

\( \Phi = (\phi, u_1, ..., u_I) \): Optimal hedging strategy to the payoff \( X \) given a specific probability measure.

\( L_t^{Q_{M_j}}(X, \Phi) \): Loss process of hedging the payoff \( X \) with the strategy \( \Phi \) using measure \( Q_{M_j} \).

The loss from hedging under the measure \( Q_{M_j} \) is described by the loss distribution function:

\[
L_t^{Q_{M_j}}(X, \Phi) = - (\mathbb{E}^{Q_{M_j}}[X - \sum_{i=1}^{I} u_i H_i] + \int_{0}^{t} \phi \, dS - \mathbb{E}^{Q_{M_j}}[X - \sum_{i=1}^{I} u_i H_i | F_t])
\]  (6.20)

Using this loss function the AVA for model risk could be calculated using a value at risk approach under a specific time horizon. For a long position with discounted pay-off \( X \), valued under model \( Q_{M_j} \) the MR AVA could be calculated as:

\[
AVA = Var_{0.1}(L_t^{Q_{M_j}}(X, \Phi)) = \inf\{l \in \mathbb{R} : \mathbb{P}(L_t^{Q_{M_j}}(X, \Phi) > l) \leq 0.9 \} \leq 0.9 \]  (6.21)

To evaluate the loss distribution in time \( t > 0 \), standing in time 0, \( F_t \) is needed. As it is unknown, a possible approach is to use \( Q \), the set of plausible measures, to say something about the possible future progression of the underlying asset. Further discussion on this topic is in Chapter 8.
Chapter 7

Example for Model Risk AVA

In this example the data is from a paper written by Wim Schoutens, Erwin Simons and Jurgen Tistaert (2003). The example is about model risk AVA for barrier options, which in general are highly sensitive to model risk, due to the fact that different models put different probabilities on the barrier being breached. There will sometimes be large price dispersion, resulting in high AVAs. The size of the AVAs in this example is therefore not in any way to be seen as representative for AVAs in general. However, as this example is based on actual prices produced by accepted models it shows how big an impact Model Risk AVA could have for exotic derivatives. For the vanilla call option related to the barrier options in this example the largest MR AVA (had it not been marked-to-market) would have been smaller than 0.6% (see figure 10.2 in the appendix).

This example addresses one of the factors contributing to model risk: choice of model for underlying. However, it could easily be extended to cover all factors by simply expanding the set of candidate models to include various calibration approaches as well. Throughout the example we assume long positions. As the AVA is calculated for a single instrument the diversification benefit (50% reduction) is not taken into account. All tables and diagrams show AVA before diversification benefits.

In the first two sections the investigated instruments and models are described. The third section shows how the probability weights for the models are derived using the root-mean-square-errors obtained by Schoutens et al. (2003). The fourth section covers the calculation of Model Risk AVA for a specific instrument using the suggested Range of Models Approach. In the fifth section Model Risk AVA among a range of related barrier options is compared to investigate the barrier-level’s effect on the AVA.
7.1 Setup

7.1.1 Instruments

T: Duration of the instruments

$S_t$: Price of underlying asset at time $t$. $0 < t < T$

$M_T$: Maximum price for the underlying in $(0, T)$

$m_T$: Minimum price for the underlying in $(0, T)$

$K$: Strike price

$H$: Barrier level

$X$: Pay off

- Digital barrier option (DIG)
  \[X_{DIG} = 1|_{M_T^S \geq H}\]

- Down-and-out barrier call option (DOB)
  \[X_{DOB} = (S_t - K)^+|_{m_T^S > H}\]

- Down-and-in barrier call option (DIB)
  \[X_{DIB} = (S_t - K)^+|_{m_T^S \leq H}\]

- Up-and-out barrier call option (UOB)
  \[X_{UOB} = (S_t - K)^+|_{M_T^S < H}\]

- Up-and-in barrier call option (UIB)
  \[X_{UIB} = (S_t - K)^+|_{M_T^S \geq H}\]

(Schoutens et al., 2003).
7.1.2 Models

The set of candidate models are the seven models presented by Schoutens et al. (2003). The models are described in more detail in the appendix.

- Heston stochastic volatility (HEST);
- Heston stochastic volatility with jumps (HESJ);
- The Barndorff-Nielsen-Shephard (BN-S);
- Normal inverse gaussian Lévy process with CIR stochastic clock (NIG-CIR);
- Normal inverse gaussian Lévy process with Gamma-OU stochastic clock (NIG-OUΓ);
- Variance Gamma Lévy process with CIR stochastic clock (VG-CIR); and
- Variance Gamma Lévy process with Gamma-OU stochastic clock (VG-OUΓ).
7.1.3 Probability Weights

Using the presented root square error for the models as measure of fit, the AIC can be calculated, and the probability weights follows.

<table>
<thead>
<tr>
<th>Model</th>
<th>NIG-OUT</th>
<th>VG-CIR</th>
<th>VG-OUT</th>
<th>HEST</th>
<th>HESJ</th>
<th>BN-S</th>
<th>NIG-CIR</th>
</tr>
</thead>
<tbody>
<tr>
<td>rmse</td>
<td>3,27</td>
<td>2,38</td>
<td>3,44</td>
<td>3,03</td>
<td>2,81</td>
<td>3,52</td>
<td>2,35</td>
</tr>
<tr>
<td>mse</td>
<td>10,72</td>
<td>5,68</td>
<td>11,80</td>
<td>9,17</td>
<td>7,90</td>
<td>12,36</td>
<td>5,52</td>
</tr>
<tr>
<td># parameters</td>
<td>7</td>
<td>7</td>
<td>7</td>
<td>5</td>
<td>8</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>AIC</td>
<td>50,47</td>
<td>46,02</td>
<td>51,14</td>
<td>45,38</td>
<td>50,33</td>
<td>47,47</td>
<td>45,82</td>
</tr>
<tr>
<td>Likelihood</td>
<td>0,08</td>
<td>0,73</td>
<td>0,06</td>
<td>1,00</td>
<td>0,08</td>
<td>0,35</td>
<td>0,80</td>
</tr>
<tr>
<td>Prob. weights</td>
<td>0,03</td>
<td>0,23</td>
<td>0,02</td>
<td>0,32</td>
<td>0,03</td>
<td>0,11</td>
<td>0,26</td>
</tr>
</tbody>
</table>

Table 7.1: Calculation of probability weights for the models using AIC and Akaike weights.

Figure 7.1: Probability weights for the models. Derived using Akaike Information Criterion, AIC, and Akaike weights.
7.2 Model Risk AVA for a Specific Instrument

In this section the range of models approach is applied to a down-and-in barrier call option with a strike price equal to the stock price, and a barrier level of 95% of the stock price.

7.2.1 Price Probability Distribution

Sorting the models according to their pricing of the instrument allows for cumulative probabilities:

<table>
<thead>
<tr>
<th>Model</th>
<th>Price</th>
<th>Probability</th>
<th>Cumulative Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>VG-OUΓ</td>
<td>190,98</td>
<td>0,018</td>
<td>0,018</td>
</tr>
<tr>
<td>NIG-OUΓ</td>
<td>209,51</td>
<td>0,025</td>
<td>0,043</td>
</tr>
<tr>
<td>VG-CIR</td>
<td>218,51</td>
<td>0,234</td>
<td>0,278</td>
</tr>
<tr>
<td>NIG-CIR</td>
<td>228,1</td>
<td>0,259</td>
<td>0,536</td>
</tr>
<tr>
<td>BN-S</td>
<td>279,61</td>
<td>0,114</td>
<td>0,650</td>
</tr>
<tr>
<td>HESJ</td>
<td>336,25</td>
<td>0,027</td>
<td>0,677</td>
</tr>
<tr>
<td>HEST</td>
<td>336,35</td>
<td>0,323</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 7.2: Sorted prices from lowest to highest with their respective probability and the cumulative probability.
Figure 7.2: Cumulative probability for the produced prices for the Down-and-In barrier call option.

In this example the confidence bound 0.1 (long position) is exceeded for the third smallest price, 218.51, produced by the VG-CIR model. This means that there is two prices below the 0.1 confidence bound, and \( j^* = 3 \) so the second calculations in equation (6.16) and in equation (6.18) should be applied.

### 7.2.2 Prudent Value without Interpolation

If interpolation is not used the prudent value with respect to model risk is the price below (long position) the 0.1 confidence bound:

\[
PV = 209.51
\]

### 7.2.3 Prudent Value with Interpolation

Using interpolation the prudent value is calculated as:

\[
PV = 209.51 + (0.1 - 0.043) \frac{218.51 - 209.51}{0.278 - 0.043} = 211.69
\]
7.2.4 Comparison to Equal Probability Weights Approach

As long as the set of candidate models is smaller or equal to eleven, not using probability weights (assigning equal weights) will always result in the prudent value being the lowest price (for long positions) produced in the set of models. As there in this example are seven models, not using weights would result in the prudent value being the lowest price produced in the set of models, 190.98, leading to larger AVAs. As in this example, for any case where the set of candidate models is smaller than twelve, not using probability weights will result in larger AVAs. Further discussion on this is in Chapter 8.

Figure 7.3: Prices and AVAs (using interpolation) for the Down-and-In barrier call option in the different models.
7.3 Model Risk AVA for Different Barrier Options

In this section a larger portion of the data from Schoutens et al. (2003) is used to show how the barrier level affects the AVA for the different types of one-touch barrier call options and the digital barrier option. For each option type and every barrier level the AVA is calculated for every model. The AVA for a specific option type and barrier using a specific model is then expressed as a percentage of the value of the option in that model. A percentage average AVA is then plotted in diagrams to show how the AVA relates to the barrier level for the five different types of barrier options.

7.3.1 Resulting Diagrams

![Average AVA for Digital Barrier](image)

Figure 7.4: Average AVA for the digital barrier option in percent for different barrier levels. H: Barrier, S(0): Stock price
Figure 7.5: Average AVA for the Down-and-In barrier call option in percent for different barrier levels. H: Barrier, S(0): Stock price

Figure 7.6: Average AVA for the Up-and-In barrier call option in percent for different barrier levels. H: Barrier, S(0): Stock price
Figure 7.7: Average AVA for the Down-and-Out barrier call option in percent for different barrier levels. H: Barrier, S(0): Stock price

Figure 7.8: Average AVA for the Up-and-Out barrier call option in percent for different barrier levels. H: Barrier, S(0): Stock price
7.3.2 Analysis of the Diagrams

For options that give pay off if the barrier is breached (up-and-in, down-and-in, and the digital) the AVAs seems to increase with the distance from $S_0$ to the barrier. For options that lose the pay off if the barrier is breached (up-and-out and the down-and-out) the trend seems to be the opposite. There is also a big difference in the size of the AVA between the different types of barrier options. The digital and the UIB has an average AVA of only 2%, while the DIB and the UOB have an average of over 30%. These differences between different types of barrier options and barrier levels will be an important aspect to account for if an expert-based approach is applied instead of a range of models approach.

7.3.3 Comparison to the Fall-Back Approach

The fall-back approach for derivative positions calculates the total AVA of the position as the sum of net unrealised profit (NUP) and 10% of the notional value. To compare this to the results in this example the NUP is set to zero, and the resulting total AVA is then simply 10% of the price. Comparing this to the results in the example above there are two things to consider. The first one is that the fall-back approach calculates the total AVA for a position, not only MR AVA. The other is that MR AVA is subject to diversification benefits of 50%. To account for these things the AVAs presented in this example should be considered to 50%, keeping in mind that the fall-back approach calculates the total AVA of the position.

It should also be noted that some barrier options are extremely sensitive to model risk, as can be seen on the large price dispersion for some of the options, the point being that the size of the AVAs in this example in no way are representative for AVA size in general. Considering all of this the 10% total AVA of the fall-back approach is to be compared to MR AVAs that are in average 7.5%, varying from zero to 46% (DIB, H/S(0)=0.5 under Heston, see appendix figure 9.2) after diversification benefits. Considering that other AVAs will be added to the 7.5% it may, from a P&L point of view, be beneficial to calculate AVA using the fall-back approach for exotic derivative types that are highly sensitive to model risk (or any other factor) if it can be properly motivated and allowed.
Chapter 8

Discussion

8.1 Interpolation

Using interpolation leads to lower AVAs as the prudent value comes closer to the expected value when interpolating. The prudent point must be within the range, not an actual observed data point (RTS, article 11(3), 2014). So to our understanding there is nothing in the regulation that would prevent us from interpolating in the range of plausible valuations.

8.2 Competing Approaches to Model Risk

In our research, starting in August 2014, we have identified primarily two other relevant approaches to model risk quantification. The first one that we encountered was proposed by Detering and Packham (2013), which focuses on the model risk related to financial derivatives. The paper not only accounts for model risk in the valuation, but also takes the model risk related to hedging the derivative over time into account. For more details see Chapter 6.

The second approach was proposed by Numerix (Bianchetti, 2014), which is an American company that develops software for risk analysis, during a web-based seminar on 2014-11-12. Their seminar confirmed our general understanding of the prudent valuation, and also to some extent our suggested methods. They proposed the same type of measure for model risk as we do (instantaneous and focusing only on valuation uncertainty), however, they did not incorporate probability weights.
The major differences between our approach and the two competing approaches to model risk are related to:

- Measure for model risk (time horizon and hedging over time)
- Use of probability weights for the candidate models

8.3 Measure for Model Risk

The model risk measure proposed by Detering and Packham (2013) takes the optimal hedge into account, and has a time horizon similar to that of the value at risk measure. Our opinion is that this approach is not in line with the RTS (2014), and the purpose of prudent valuation. The RTS (article 11(3), 2014) states that "where possible, institutions shall calculate the model risk AVA by determining a range of plausible valuations produced from alternative appropriate modelling and calibration approaches". What this implies is that the AVA is only to capture uncertainty in the valuation, not uncertainty coming from hedging the position over time. This makes sense as the purpose of prudent valuation is not to calculate capital charges for future unexpected losses, but rather to account for uncertainty in the instantaneous loss absorbing capabilities of CET1 capital. This is why the RTS (2014) constitutes an AVA for market price uncertainty, not market risk. The loss distribution measure is suitable for calculating Model Risk in the context of market risk, but not in the context of market price uncertainty.

As for the hedge, we argue that the only part of a hedging strategy that affects the model risk AVA is the optimal model independent static hedge. This is due to arbitrage arguments where any part of the derivative’s value that comes from a static, and model independently hedgable pay off is not subject to model risk. If the model independently hedgable part of the pay off is not marked-to-market for some reason, one can simply consider $X$ to be the discounted unhedgeable part of the pay off. Any dynamic part of the hedge depends on the model and does therefore not reduce model risk.
8.4 Model Probability Weights

In the approach for model risk proposed by Numerix (Bianchetti, 2014) the models are not assigned probability weights. Instead, Numerix propose that the measure of fit should help decide if a model is to be included in the candidate set in the first place. While we find this reasonable, it may result in a larger need for professional judgement, which could complicate the implementation of the systems for prudent valuation.

The use of an probability weights based on an information criterion in our approach is motivated by the argument that it may not always be easy, or implementable, to say which models have a significant usage on the market, and which models are better or worse, for a specific type of instrument. We therefore allow for a larger set of candidate models, and then rely on a good measure of fit in combination with an information criterion to assign small probability weights to irrelevant models. The information criterion punishes models with poor fit, and high number of parameters. This reduces the impact of a poorly chosen set of candidate models on the AVA, as models that should not be in the candidate set are given a small probability weight.

In the approach that we propose the probability weights were derived using Akaike Information Criterion. One could argue that Bayesian Information Criterion (BIC) may have been more theoretically sound, however we argue that the number of parameters were already punished too hard in the AIC, and BIC would result in even more parameter punishment.

8.4.1 Measure of Fit in the AIC

In our method we decided to extend the measure of fit (MOF) beyond mean-squared-error in fit to calibration surface (MSE) (Detering & Packham, 2013), to also include fit to prices of similar instruments, prices derived from proxy data, and historical pricing data. This was motivated by our opinion that relevant pricing errors should be more important than fit to calibration surface. Another motivation was that any market data available should be used where relevant (RTS, article 3(2), 2014). By adding these terms to the MOF we include as much relevant data as possible in the process to find the prudent value. The result will be that models pricing the calibration instruments accurately, but not the actual instrument of interest, will get a higher MOF value, and smaller probability weights. This leads to more accurate cumulative probabilities for the prices, which results in a more fair prudent value.
8.5 Implementation of Approaches to Model Risk AVA

A big challenge when implementing any of the methods for Model Risk AVA will be to decide which models and calibration approaches to include in the candidate set for different types of financial derivatives and/or underlying. In our example with the barrier options we chose only to consider different models for the underlying, tuned to the same calibration set. Numerix (Bianchetti, 2014) in their example calculated Model Risk AVA for a Bermudan swaption. They used five different models for the short-rate, and three sets of calibration instruments, resulting in 15 different valuations. However, there is nothing restricting the calculations to one, or two-dimensions. One should consider which factors have most impact on the valuation, and include a variety of plausible approaches related to those factors. The set of candidate models and calibration approaches should then be some set of combinations of the plausible approaches to each factor.

Even though our approach requires the calculation of model weights, we feel that it would be easier to implement than the approach suggested by Numerix, as our approach require less professional judgement in the choice of candidate set. The approach suggested by Detering & Packham (2013), would to our understanding be far more complicated to implement.

8.6 Conclusion

Motivated by the discussions above we conclude that our approach to model risk is both fair, and compliant with the RTS (2014). The model risk AVA should capture instantaneous uncertainty in the valuation of a position, and we argue that this is unrelated to the hedging of the position, as model risk from hedging requires a non-zero time horizon. The use of a modified AIC to derive probability weights for the models in the candidate set is motivated by less need for professional judgement, and a more fair cumulative price probability. We also conclude that interpolation within the range of valuations is allowed, and should be applied.
8.7 General Discussion on Prudent Valuation Regulation

In our opinion the regulation concerning prudent valuation, i.e. Basel III (2010), the CRR (2013) and the RTS (2014), lacks precision in describing the exact nature of prudent valuation. We understand the difficulty in specifying a framework that will capture all aspects of the calculations, and be implementable for institutions all across the EU, but we still feel that there are some parts of the regulation that could have been improved. The regulation could for example have been a lot more precise as for how to calculate AVAs for some of the specific cost and risk factors (e.g. IFC AVA). However, as the final draft is not yet approved, there is still a chance that some of this will be clarified before the regulation fully comes into force.

As the regulation stands today we feel that there is large room for local supervisory judgement, i.e. different implementations of the regulation on a national level by the local FSA. This could lead to different local standard practices of prudent valuation across the member states. This in return would result in unfair competition among players within the EU.

The cost of the prudent valuation regulation for a specific institution is hard to assess. To start with, in order to be compliant with the regulation institutions will have to pay for the development of new methods and the implementation of new systems. There will also be the potential cost of holding additional capital in order to be compliant. We therefore think that the regulators are fair to allow smaller institutions, with a limited amount of fair valued assets and liabilities in the CET1 capital, to use a simplified approach, thus limiting the operational burden of the regulations.

In addition to the cost of this regulation we also believe that there might be some changes to the valuation techniques, and the trading strategies of institutions. It is possible that institutions to a larger extent will try to avoid holding illiquid positions.
8.8 Further Research

For an institution to be able to fully implement prudent valuation more research must be carried out to determine the exact nature of some of the AVAs. The IFC AVA should to our understanding be based on the FVA, which to our knowledge is not yet fully developed and understood. Other aspects of the regulation, such as reliability of different data sources and the expert-based approaches, might also require further investigation.

8.9 Final Thoughts

The regulation on prudent valuation requires institutions to revisit the valuation of CET1 capital instruments with a more risk aware methodology. This process requires the classical valuation systems to meet with the risk management systems to accurately re-value the instruments with respect to the cost- and risk factors in prudent valuation. This should result in a more fair CET1 capital requirement for any specific institution, and may also lead to a better understanding within the institution of the risks related to the instruments in the CET1 capital.
Bibliography

Articles


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Books


Official documents


European Banking Authority (EBA), (2012), *Discussion paper (DP) relating to draft regulatory technical standards on prudent valuation under article 100 of the draft capital requirements regulation*, Brussels, Belgium.


**Other**

Chapter 9

Appendix

This appendix can be read without going into great detail, and is not necessary for the understanding of our method for the calculation of AVA. The section is here for the interested reader who wishes to understand the models used for pricing financial instruments and why they differ in model risk AVA. At the end of the appendix additional figures and data can be found.

9.1 Probability Theory

9.1.1 Definitions

Definition 1 (Sample Space)
The sample space, $\Omega$, is defined to contain all possible outcomes (Åberg, 2010).

Definition 2 (Family)
A family, $F$, is a group of subsets of the sample space $\Omega$ (Åberg, 2010).

Definition 3 (Algebra)
Let $\Omega$ be a non empty set. A family $F$ of subsets of $\Omega$ is called an algebra if (Åberg, 2010):

1. $\Omega \in F$
2. $A \in F \rightarrow A^c \in F$
3. $A, B \in F \rightarrow A \cup B \in F$. 

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Definition 4 (σ-Algebra)
An algebra $\mathcal{F}$ is called a σ-algebra if, for any countable set of index, $\mathcal{I}$ and $A_i \in \mathcal{F}$ for $i \in \mathcal{I}$ we have (Åberg, 2010):

$$\bigcup_{i \in \mathcal{I}} A_i \in \mathcal{F}$$

Definition 5 (Measurable Space)
A set of $\Omega$ with a σ-algebra $\mathcal{F}$ is called a measurable space, and is denoted by $$(\Omega, \mathcal{F})$$ (Åberg, 2010).

Definition 6 (Measurable Function)
Let $$(\Omega, \mathcal{F})$$ be a measurable space. A function $X$ from $\Omega$ to $\mathbb{R}$ is called a measurable function if the inverse image of a set $B \in \mathcal{B}(\mathbb{R})$ is in $\mathcal{F}$, that is (Åberg, 2010):

$$X^{-1}(B) = \{\omega \in \Omega : W(\omega) \in B\} \in \mathcal{F}$$

Definition 7 (Probability Measure)
Let $\mu$ be a measure on the measurable space, $$(\Omega, \mathcal{F})$$, then it is called a probability measure if

$$\mu(\Omega) = 1$$

The usual notation of a probability measure is $\mathbb{P}$ (Åberg, 2010).

Definition 8 (Probability Space)
The triplet $$(\Omega, \mathcal{F}, \mathbb{P})$$ is called a probability space (Åberg, 2010).

9.2 BM and its Stochastic Integral

9.2.1 Definitions

Definition 9 (Stochastic Process)
A stochastic process $X$ is defined as a collection of random variables on the same probability space $$(\Omega, \mathcal{F}, \mathbb{P})$$ (Åberg, 2010).

$$X = \{X_t : X_t \text{ random variable on } (\Omega, \mathcal{F}, \mathbb{P}), \ t \in T\}$$
Definition 10 (Brownian Motion)
Let $W = \{W_t : t \in \mathbb{R}_+\}$ be a stochastic process. $W$ is a BM if it satisfy the following properties (Åberg, 2010):

1. $W_0 = 0$

2. The increments are independent and stationary. That is, for $0 \leq h \leq s \leq t$, $(W_s - W_{s-h})$ is independent of $(W_{t+h} - W_t)$

3. The distribution of an increment is $(W_{t+h} - W_t) \in \mathcal{N}(0, h)$

4. $W_t$ has continuous paths.

Definition 11 (Filtration)
Consider a stochastic process $X$ on the time interval $[0,T]$. The filtration of $X$ at time $t$ is (Åberg, 2010):

$$F^X_t = \sigma\{X_s : s \leq t\}$$

The filtration can be interpreted as the information gained as time evolve, namely on what has happened for the traded asset on the market.

Definition 12 (Adapted Process)
Let $\mathcal{F}_t$ be a filtration. The process $Y_t$ is called adapted to $\mathcal{F}_t$ if $Y_t$ is measurable with respect to $\mathcal{F}_t$ for any $t$ (Åberg, 2010).

Definition 13 (Markov Process)
Let $X$ be a stochastic process in continuous time such that for any $s < t$

$$\mathbb{E}[f(X_t)|F^X_s] = \mathbb{E}[f(X_t)|X_s]$$

for any measurable function $f$. Then $X$ is a Markov Process (Åberg, 2010).

Definition 14 ($\mathcal{L}^2$-Martingale)
Let $X$ be a stochastic process adapted to the filtration $\mathcal{F}_t$ with the following conditions (Åberg, 2010):

1. $\mathbb{E}[X_t^2] < \infty \quad \forall \ 0 \leq t < \infty$

2. $\mathbb{E}[X_t|\mathcal{F}_s] = X_s \quad \forall \ 0 \leq s \leq t < \infty$.

Then $X_t$ is a $\mathcal{L}^2$-Martingale with respect to the filtration $\mathcal{F}_t$. 

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9.3 Derivative Pricing

Stochastic models for pricing derivatives usually model the price of the underlying asset as a stochastic process \( \{S_t\}_{t \in [0,T]} \) defined on a probability space \((\Omega, \mathcal{F}, \mathbb{P})\). It then determines the price by either solving the PDE or by using the MG-approach, which is the one used in this thesis. Before describing the models used in this thesis, a brief introduction of derivative pricing will be presented. Starting with the work of Black and Scholes (1973).

9.3.1 Partial Differential Equation, PDE

Black and Scholes described the market as consisting of a stock, \( S_t \) and a bank account \( B_t \), with the following dynamics:

\[
\begin{align*}
    dS_t &= \mu S_t dt + \sigma S_t dW_t \\
    dB_t &= rB_t dt
\end{align*}
\]

where \( S_0 = s \) and \( B_0 = 1 \) (Black, Scholes, 1973). We can then express a derivative of type European call option as a PDE:

**Theorem 1 (B&S PDE)** Consider a derivative of European style, given by the pay-off function \( \Gamma(S_T) \). Let \( F(t,x) \in C^{1,2}([0,T],\mathbb{R}_+) \) be the solution to the PDE:

\[
\begin{align*}
    \partial_t F(t,x) + rx\partial_x F(t,x) + \frac{\sigma^2 x^2}{2} \partial_{xx}^2 F(t,x) - rF(t,x) &= 0 \\
    F(T,x) &= \Gamma(x)
\end{align*}
\]

Then the price of the derivative in B&S market is given by \( \Pi_t = F(t,S_t) \) (Åberg, 2010).

**Theorem 2 (Transforming the PDE to discounted expectation)** Consider a derivative of European style with pay-off function \( \Gamma(S_T) \). If the price can be written as \( \Pi_t = F(t,S_t) \), where \( F(t,x) \in C^{1,2}([0,T],\mathbb{R}_+) \) then the price in B&S market is given by (Åberg, 2010):

\[
\Pi_t = e^{-rt}E[\Gamma(X_T|F_t^X)]
\]

where \( X_t \) is given by the SDE:

\[
\begin{align*}
    dX_u &= rX_u du + \sigma X_u dW_u \\
    X_t &= s
\end{align*}
\]
**Theorem 3 (B&S formula)** The price $C_{BS}$ of an European call option in B&S market is:

$$C_{BS}(t) = s\Phi[d_1] - e^{-rt}K\Phi[d_2]$$

where

$$d_1 = \frac{\ln\left(\frac{s}{K}\right) + (r + \frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}}$$

and

$$d_2 = d_1 - \sigma\sqrt{\tau}$$

This formula is called the B&S formula (Åberg, 2010).

The price of an European call option can be analytically calculated in the market by using this formula. It initially priced options very accurately but now it does not longer price options as accurately and is now mostly used as a benchmark by market participants. More sophisticated models have been developed which takes other factor of the stock market into account.

### 9.3.2 The Martingale Approach

This approach relies on a MG property of the discounted price process. Equation 9.1 can be rewritten as

$$\frac{\Pi(t)}{B_t} = \mathbb{E}\left[\frac{\Gamma(X_T)}{B_T} \Bigg| \mathcal{F}_t\right]$$

Which is the definition of a MG according to definition 14 if we assume that the price is bounded. However, $X_t$ is not the initial model for the stock, which is:

$$dS_t = \mu S_u du + \sigma S_u dW_u$$

$$S_t = s$$

and if $\mu \neq r$, the discounted price process $\frac{S_t}{B_t}$ is no MG. The MG approach rest on the change of the underlying probability measure (Åberg, 2010).

**Definition 15 (EM Measure)**

An equivalent MG-measure $Q$ is a probability measure with the following two conditions (Åberg, 2010):

1. The discounted price processes $\frac{S_i}{B_0}$ are $Q$-MG for all $i \in \{0, ..., n\}$.
2. The measures $Q$ and $P$ are equivalent.

The EM-measures are the link between the economic and mathematical properties of the market.
The Fundamental Theorems

Theorem 4 (No-Arbitrage Condition) If there is at least one EM-measure \( Q \) there are no Arbitrage opportunities (Åberg, 2010).

Theorem 5 (Completeness Condition) Assume there is EM-measures \( Q \) in the market. Then the market is complete if and only if \( Q \) is unique (Åberg, 2010).

Theorem 6 (The Risk Neutral Valuation Formula) Let there be an EM-measure \( Q \) and let \( h \) replicate the derivative with pay-off function \( \Gamma \). Then the price at time \( t \) (\( \Pi(t) \)) is given by (Åberg, 2010):

\[
\Pi(t) = S_0(t)E^Q\left[\frac{\Gamma(S_T)}{S_0(T)}\bigg|\mathcal{F}_t\right]
\]  

(9.2)

9.4 Derivative Pricing Models

The completeness criteria is not fulfilled in the market for complex derivatives. The problem with the existence of more than one risk-neutral probability measure is that different \( Q \)s often derive different prices for the same derivative. This does not apply for vanilla options where the payoff functions are simple enough to allow for them to be priced consistently. However, for more complex derivatives, such as the path-dependent types, this inconsistency in pricing among the different models leads to uncertainty about the value of the derivative. Below we will present a number of models use to calculate the price of a derivative which will be used in our example.

9.4.1 Equity Models

Heston Stochastic Volatility Model, HEST

The Heston model assumes that the variance in the B&S market follows a CIR process.

\[
dS_t = (r - q)S_t dt + \sigma_t S_t dW_t
\]

\[
d\sigma_t^2 = \kappa(\eta - \sigma_t^2)dt + \theta \sigma_t^\sigma_t \tilde{W}_t
\]

(9.3)

(9.4)

where \( \sigma_0 \geq 0 \) and \( W = \{W_t, \ t \geq 0\} \) and \( \tilde{W} = \{\tilde{W}_t, \ t \geq 0\} \) are two correlated standard BMs \( Cov[dW_t\tilde{W}_t] = \rho dt \) (Heston, 1993).
**Heston Stochastic Volatility Model with Jumps, HESJ**

This model is an extension of the HEST with an extra term which models jumps in the asset price. It is assumed that jumps occur as a Poisson process and that the percentage jump-sizes are lognormally distributed (Bakshi, G., Cao, C., Chen, Z., 1997).

\[
\begin{align*}
dS_t &= (r - q - \lambda \mu_J)S_t dt + \sigma_t S_t dW_t + J_t dN_t, \quad S_0 \geq 0
\end{align*}
\]  

(9.5)

where \(N = \{N_t, \ t \geq 0\}\) is an independent Poisson process with intensity \(\lambda > 0\). \(J_t\) is the percentage jump size that is identically and independently distributed over time, with unconditional mean \(\mu_J\). The standard deviation of \(\log(1 + J_t)\) is \(\sigma_J\):

\[
\log(1 + J_t) \sim \text{Normal} \left( \log(1 + \mu_J) - \frac{\sigma_J^2}{2}, \sigma_J^2 \right)
\]

(9.6)

The SDE of volatility process remains unchanged. \(J_t\) and \(N\) are independent of both \(W\) and \(\tilde{W}\).

**The Barndorff-Nielsen-Shephard Model, BN-S**

The BN-S has a similar structure to HEST with exception that the volatility is modeled by an Ornstein Uhlenbeck, OU, process. Volatility can only jump upwards and they decays exponentially. A co-movement between the jump in volatility and the jump downwards in stock price is also incorporated into the model. Specifically the Gamma-OU process is chosen (Barndorff-Nielsen, & Shephard, 2001).

\[
d\sigma_t^2 = -\lambda \sigma_t^2 dt + dz_{\lambda t}
\]

(9.7)

where \(\lambda > 0\) and \(z = \{z_t, \ t \geq 0\}\) which is a compound-Poisson process:

\[
z_t = \sum_{n=1}^{N_t} x_n
\]

(9.8)

where \(N = \{N_t, \ t \geq 0\}\) is a Poisson process, \(\mathbb{E}[N_t] = at\) and \(\{x_n, \ n = 1, 2, \ldots\}\) which is i.i.d. sequence with exponential law with mean \(1/b\) (Schoutens et al., 2003).

The \(\mathbb{Q}\)-dynamics of the log-price \(Z_t = \log S_t\) is:

\[
dZ_t = (r - q - \lambda k(-\rho) - \frac{\sigma^2}{2})dt + \sigma_t dW_t + \rho dz_{\lambda t}, \quad Z_0 = \log S_0
\]

(9.9)

where \(W = \{W_t, \ t > 0\}\) is a BM independent of \(z\) where \(k(u) = \log \mathbb{E}[\exp(-uz_1)]\) is the cumulant function of \(z_1\).
Levy Models with Stochastic Time

Levy models use stochastic time to model stochastic volatility. Periods with high volatility can be looked at as if time runs faster and vice versa (Clark, 1973). The asset price is modeled by the exponential of the Levy process suitably time changed. This paper discussed the Normal Inverse Gaussian, NIG, and the Variance Gamma, VG, distributions.

The other process is a stochastic clock. The time is modeled by $y = \{y_t = t > 0\}$ which needs to be positive and the elapsed time in units $t$ is then the integrated process $Y = \{Y_t = \int_0^t y_s ds\}$. This paper is going to be concerned with two processes for $y$; the CIR process and the Gamma-OU process. This gives us 4 models, NIG-CIR, NIG-OU, VG-CIR and VG-OUT presented below (Schoutens et al., 2003).

**NIG Levy Process** $NIG(\alpha, \beta, \delta)$ with $\alpha > 0$, $-\alpha < \beta < \alpha$ and $\delta > 0$. So an increment over $[s, s + t]$ follows a $NIG(\alpha, \beta, \delta)$-law.

**VG Levy Process** $VG(C, G, M)$ with $C > 0$, $G > 0$ and $M > 0$. So an increment over $[s, s + t]$ follows a $VG(C, G, M)$-law.

**CIR Stochastic Clock** The CIR process is the rate of time change that solves the SDE:

$$dy_t = \kappa(\eta - y_t)dt + \lambda y_t^{1/2}dW_t$$  \hspace{1cm} (9.10)

where $W = \{W_t, t \geq 0\}$ is a standard BM.

**Gamma-OU Stochastic Clock** Rate of time change is the solution to the SDE:

$$dy_t = -\lambda y_t dt + dz_t$$  \hspace{1cm} (9.11)

where the process $z = \{z_t =, t \geq 0\}$ is a compound Poisson process.

9.5 Figures and Data
Figure 9.1: AVAs for the options and models. In the calculations for each AVA interpolation has been applied according to the suggested method.
Figure 9.2: AVAs as percent of the price for the options and models. In the calculations for each AVA interpolation has been applied according to the suggested method.
Figure 9.3: Barrier option prices (Schoutens et al., 2003, p. 25)