CLASSIFICATION OF NON-STATIONARY HEART RATE VARIABILITY USING AR-MODEL PARAMETERS

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Abstract

This thesis explores the connection between the heart rate variability and both stress and age. Two methods are used to classify the heart rate variability data, the autoregressive model and the Markov chain model. The autoregressive model is further expanded to become an autoregressive model with extraneous input using the respiratory signal as input signal. The Markov chain models are compared with their stationary distribution using the Kolmogorov-Smirnov test. Autoregressive parameters are compared using confidence intervals. The results indicate statistically significant deviations between age groups for both the autoregressive models and the Markov model. The stress related results were not as clear as the age related results, however some deviations were obtained for both models, indicating some stress related influence on the HRV.
1 Introduction

1.1 Background

The human heart has been under research for a very long time. Heartbeat recordings from as early as 1872 has been reported. The method used to obtain heartbeat recordings is called electrocardiography or ECG. Augustus Waller was one of the pioneers within the area and created one of the first electrocardiograph machines that recorded the heart beat in real time, [Waller, 2004]. The information obtained from electrocardiography has developed over the years and today even the smallest heart reaction is picked up in an ECG. The various deflections have been assigned letters as references. Figure 1 shows an example of the regular deflections.

1.2 The human heart

The human heart is the muscle that keeps our blood flowing. The time difference between two consecutive heartbeats is referred to as the RR interval (RRI), where R is illustrated in figure 1. The contractions are fairly evenly distributed and the number of contractions within one minute is referred to as the pulse. When more oxygen is needed in the body the pulse increases. The pulse is commonly used as a measurement of the heart activity when exercising. When the body is relaxed the pulse is referred to as resting pulse.

1.3 Heart rate variability

Although the heart contractions described in the previous section are fairly regular, there are small time differences. These time differences create a varying signal with the pulse as mean. The variations in the signal are referred to as the heart rate variability (HRV) and it contains much information about the autonomic nervous system (ANS). The HRV has been proven to have a connection to craniological diseases as well as other non cardiologic diseases, [Peng et al., 2015] [Rajendra Acharya, U et al., 2006]. Furthermore it is known that the HRV is affected by the age of a patient. The variance of the RRI signal has a tendency to decrease with an increased age. HRV frequencies are usually separated between high frequency (HF) and low frequency (LF) bands. The low frequency band lay around 0.1 Hz and the high frequency band is around 0.25 Hz [Rajendra Acharya, U et al., 2006]. The HF band is related to the respiratory frequency, during expiration the heart rate decreases and during inspiration the heart rate increases. This results in an oscillation in the RRI signal and is referred to as the respiratory sinus arrhythmia (RSA), [Widjaja et al., 2014].

1.4 Aim

The aim of this thesis is to develop statistical models for the heart rate variability to search for statistical significant deviations between stressed persons and persons that are not stressed as well as investigating age related deviations. Two main statistical models will be explored. The first model is an autoregressive model (AR-model). The second model is a Markov chain model. The Markov
Figure 1: Heart beat with letters showing regular deflections

chain will be fitted to the AR parameters in hope to evaluate their movement. The AR model will be further developed to become an autoregressive model with extraneous input (ARX-model). The input signal will be the respiratory signal. The aim is to reduce the influence of the RSA and obtain AR parameters that are more accurate to other HRV variations. As a final approach the mean and variance of the RRI signal will be calculated and used to compare groups of persons. The raw data handled in this thesis has been obtained from 52 participating persons. Each person has been asked to sit and relax, and breath in the same tempo as a metronome. During the test the tempo of the metronome increased resulting in increased respiratory frequency from the patients. The patients were relaxed with resting pulse during the test and no other influences were present. During the test each heartbeat was measured as well as the respiratory signal. Figure 2 shows an RRI series and respiratory series from one of the participating persons. Note that the RRI process is a non-stationary and non-zero mean process. Figure 3 illustrates how the respiratory frequency increases over time.

The frequency has a clear increasing trend shown in figure 3.
Figure 2: Raw data from person number 3. HRV signal on top, respiratory signal at bottom.

Figure 3: Frequency plot for person number 3
2 Theory

2.1 Auto-regressive Model

The autoregressive (AR) model is a representation of a stationary process. It states that each value can be derived linearly from its previous values as well as a random Gaussian variable with mean zero, the noise. The number of previous values necessary to represent the process properly is called the order of the AR process and is referred to as p. The formula for the autoregressive model is

\[ X_t = \sum_{i=1}^{p} a_i \cdot X_{t-i} + \varepsilon_t \]  

where \( a_i \) are called the AR parameters and \( \varepsilon_t \) is the random Gaussian variable distributed according to

\[ \varepsilon_t \in N(0, \sigma^2). \]

This model can be applied to a process to generate parameter values that represents the process. With the obtained parameters it is possible to generate new processes with the same properties as the modeled process. This is for instance used in the area of prediction.

To find the correct parameters, \( a_i \), several methods are available. Amongst others, the Yule-walker equations can be used. They are obtained according to

\[ r_x(0) + a_1 r_x(1) + \ldots + a_p r_x(p) = \sigma^2 \]
\[ r_x(k) + a_1 r_x(k-1) + \ldots + a_p r_x(k-p) = 0 \]  

where \( r_x \) represents the covariance function of the process and is defined as

\[ r_x(s, t) = C[x(s), x(t)] = E[x(s)x(t)] - m_x(s)m_x(t). \]

Further \( \sigma^2 \) represents the standard deviation of the process. These equations results in \( p+1 \) unknown parameters and \( p+1 \) equations. The solution is therefore easily obtainable, [Lindgren et al., 2003]. A second way of obtaining the coefficients is via the least squares method. The least squares method uses linear regression to obtain a function with the AR parameters as variables. This equation contains the Gaussian noise variable and by minimizing the noise the AR parameters are obtained. The auto-regressive formula, (1), can be rewritten as

\[ X = U\vartheta + E, \]

with \( X \) and \( U \) defined as

\[ X = \begin{pmatrix} x_{p+1} \\ x_{p+2} \\ \vdots \\ x_n \end{pmatrix} \]

\[ U = \begin{pmatrix} x_{p} & x_{p-1} & \cdots & x_1 \\ x_{p+1} & x_{p} & \cdots & x_2 \\ \vdots & \vdots & \ddots & \vdots \\ x_{n-1} & x_{n-2} & \cdots & x_{n-p} \end{pmatrix} \]
and \( \theta = (a_1, a_2, ..., a_p) \).

The noise vector \( E \) is defined according to

\[
E = \begin{pmatrix}
\varepsilon_{p+1} \\
\varepsilon_{p+2} \\
\vdots \\
\varepsilon_{n-1} \\
\varepsilon_n
\end{pmatrix}
\] (8)

and contains the noise estimation for all time steps.

Rearranging equation (5) results in

\[
E = X - U \theta.
\] (9)

This is a vector and we minimize the sum of the squared errors,

\[
Q(\theta) = (X - U \theta)'(X - U \theta)
\] (10)

The resulting parameters of the model are found as [Lindgren et al., 2003]

\[
\hat{\theta} = (U'U)^{-1}U'X.
\] (11)

To find the proper order, \( p \), of terms in the AR model the error is considered. Two methods will be used, the final prediction error (FPE) and the Akaike’s information Criterion (AIC), [Boardman et al., 2002].

\[
FPE = \sigma^2 \frac{(N + p + 1)}{(N - p - 1)};
\] (12)

\[
AIC = \log(\sigma^2) + 2 \frac{p + 1}{N}
\] (13)

where \( N \) stands for the number of samples in the process. Our model needs to be consistent for all processes. Therefore the same order will be used for all data. An inconsistent order would result in inconsistent autoregressive parameters or suboptimal performance. The results from the AIC and FPE will therefore be added to create an over all estimate of the error.

### 2.2 ARX Model

To further develop the autoregressive model we expand the model to contain the respiratory signal as an input signal. The AR model then becomes an Autoregressive model with extraneous input (ARX). The representation in equation 1 is expanded to

\[
X_t = \sum_{i=1}^{p} a_i \cdot X_{t-i} + \sum_{i=1}^{q} b_i \cdot Y_{t-i} + \varepsilon_t,
\] (14)

where \( Y_t \) stands for the samples of the respiratory input signal. To obtain the parameters the least squares method described in section 2.1 is used. The order used to represent the input signal process is referred to as \( q \). Since the model is expanded the matrix and vectors need to be adjusted to include the respiratory signal.
\[ U = \begin{pmatrix} x_p & x_{p-1} & \cdots & x_1 & y_q & y_{q-1} & \cdots & y_1 \\ x_{p+1} & x_p & \cdots & x_2 & y_{q+1} & y_q & \cdots & y_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \ddots & \vdots \\ x_{n-1} & x_{n-2} & \cdots & x_{n-p} & y_{n-1} & y_{n-2} & \cdots & y_{n-q} \end{pmatrix} \quad (15) \]

\[ X = \begin{pmatrix} x_{p+1} \\ x_{p+2} \\ \vdots \\ x_n \end{pmatrix} \quad (16) \]

The \( \theta \) vector becomes \( \theta = (a_1, a_2, \ldots, a_p, b_1, b_2, \ldots, b_q) \). Together with (11) the noise is minimized and the ARX parameters are obtained.

### 2.3 Auto-regressive model comparison

Since the data is time varying, mostly because of the increasing respiratory frequency, it is necessary to fit our AR models over several smaller intervals. This will result in several values for each autoregressive parameter. These parameters will be representative of the varying process. The intervals will step forward with a specific length that will be referred to as stepsize and the length of the window will be referred to as the windowsize. Figure 4 illustrates the two constants.

It is not certain that all parameter has time dependence, however by dividing the data into smaller parts and calculating the AR parameters separately over each interval, the local parameter precision increases in comparison with calculating the parameters over the entire data sequence.

In the search of deviance between groups the results will be presented with confidence intervals. Confidence intervals are calculated as

\[ I_\lambda = E[\lambda] \pm \sigma_{0.05} \frac{\sqrt{V(\lambda)}}{\sqrt{n}}. \quad (17) \]

Where \( \sigma_{0.05} = 1.96 \) gives a 95 per cent confidence interval and \( n \) is the number of underlying data samples. \( \lambda \) needs to be distributed according to the normal distribution.

\[ \lambda \in N(c, d) \quad (18) \]

for constants \( c \) and \( d \).

To determine if a parameter is stationary a linear polynomial will be fit to the parameter sequence. The formula for a linear polynomial is

\[ a(t) = \alpha t + \beta, \quad (19) \]

where \( t \) represents time. By replacing \( \lambda \) in (17) with the calculated \( z \) values, confidence intervals are obtained. If the confidence interval overlaps zero the parameter can be seen as stationary. If the parameter is stationary, comparison can be made by calculating the confidence intervals for the AR parameter. If the parameter shows time dependence it is not possible to compare the AR parameter with confidence intervals. Instead the \( z \) values will be compared by replacing \( \lambda \) in (17) with \( z \). Henceforth \( z \) will be referred to as slope.
The aim is to compare groups of subjects. Since parameters are generated for each individual and their corresponding process, we need to define a way to obtain our group confidence intervals. Each subject in a group has a calculated mean value and variance for the parameter to be compared. By calculating the group parameter mean and group mean variance, the group confidence interval can be obtained and compared.

2.4 Markov-Chain Model

The second statistical model explored in this thesis is the Markov chain model. A Markov chain is based around states. The chain jumps from one state to another with different probabilities. This motion creates a probability matrix that can be investigated. An example can be seen in figure 5 where the corresponding probability matrix defined as

$$ P = \begin{bmatrix} 0 & 0.5 & 0.5 \\ 0.7 & 0 & 0.3 \\ 0.7 & 0.2 & 0.1 \end{bmatrix}. $$ (20)

An important feature of a Markov chain is that it obeys the Markov chain property, [Rydén and Lindgren, 2000]

$$ P(S_i = s_i | S_{i-1} = s_{i-1}, ..., S_0 = s_0) = P(S_i = s_i | S_{i-1} = s_{i-1}), $$ (21)

saying that the probability that the chain moves between two specific states is only depending on the current state of the chain. All states previously visited are forgotten and irrelevant. The Markov model will be based around the AR and ARX models derived in sections 2.1 and 2.2. Each AR/ARX parameter changes over time and becomes a process that can be used to estimate the Markov chain and its probabilities.

To create a state space for each parameter, the maximum and minimum value calculated for each parameter will be noted and the resulting interval will be divided into L equal parts. To compare the Markov chains the so-called stationary distribution, $\pi$, of the Markov chain will be calculated according to

$$ \pi P = \pi, $$ (22)
where $P$ is the probability matrix and $\pi = [\pi_1, \pi_2, ..., \pi_L]$, [Rydén and Lindgren, 2000]. As $\pi$ is a distribution function, 
\[ \sum \pi_i = 1, \] (23) needs to be satisfied. Obtaining the stationary distribution is done by calculating 
\[ \lim_{n \to \infty} \pi = P(0)P^n, \] (24) provided that a stationary distribution exists. The initial distribution, $P(0)$, has no effect on the result as $n$ goes towards infinity and can therefore be chosen arbitrarily. With the stationary distribution it is possible to obtain the cumulative density distribution (CDF) of the stationary probability distribution. The CDF is derived according to 
\[ F_a(i) = \sum_{i=1}^{n} \pi_a(1:i). \] (25)
To determine deviations between Markov chain the hypothesis
\[ H_0 : F_A(x) = F_B(x) \] (26) will be tested. The hypothesis can be either confirmed or rejected with help of the Kolmogorov-Smirnov test, [Young, 1977]
\[ D_{\text{Estimate}} = \sup_n |F_{1,n}(x) - F_{2,n'}(x)| \] (27)\[ D_{\text{Threshold}} > c(\alpha)_{0.01} \sqrt{\frac{n + n'}{mn'}} \] (28)\[ c(\alpha)_{0.01} = 1.95. \]
Figure 6 illustrates how $D_{\text{Estimate}}$ is obtained as the furthest distance between two CDFs. The $D_{\text{Threshold}}$ value calculated in (28) will further be referred to as the threshold necessary to reject the hypothesis. The accuracy of the Markov chain model increases as the amount of data available to model increases. It is therefore of interest to generate a large amount of data from the AR/ARX models.
2.5 Mean and variance comparison

As a final approach the mean and variance will be compared for the different groups. An estimate of the mean is obtained by calculating

$$\hat{\mu}_x = \frac{\sum_{i=1}^{N} x_i}{N} \quad (29)$$

and the variance is obtained by calculating

$$\hat{\sigma}_x^2 = \frac{1}{N} \sum_{i=1}^{N} (x_i - \hat{\mu}_x)^2. \quad (30)$$

where $N$ stands for the number of samples in the process. The mean and variance of a specific group can be calculated with confidence intervals in a similar way as described in section 2.3.

3 Data preparation & Model Validation

The age of a person has a known influence on the variance of the RR intervals. Figure 7 shows the variance as a function of the age of the person and it is clear that the variance seems larger for persons below an age of 30. To be able to search for age differences an age that divides the young and old persons must be chosen. During the simulations two different age separators will be used, age 30 and age 40.

3.1 Data preparation

The sampling frequency during the tests were 4 Hz and there are 961 samples from each person. This leads to a test duration of approximately 4 minutes. The information known from each participant is age and a professionally determined stress level labeled with 1 for no stress and 2 for stress. The table below shows this information for the first 6 participants. No other information has been given of the participating persons.
The test persons are spread out in age spanning from ages 21 to 61. Half of the subjects have been diagnosed as stressed persons. The raw HRV signal seen in Figure 2 is a non-stationary stochastic process with non-stationary covariance and it is therefore necessary to convert it to a stationary stochastic process before applying the autoregressive models to the data. The task is to subtract the mean from the process so the mean becomes zero without losing any vital information. This will be done by dividing the sequence into smaller parts, and then subtracting each part by its mean. The method is usually referred to as overlapped segment averaging (OSA) and is defined according to

\[
HRV^m_i = \frac{\sum_{j=i-k}^{i+k} HRV_j}{2k}.
\]  

(31)

The choice of \( k \) in (31) is decided by looking at the correlation between the HRV sequence and the new normalized sequence. This correlation is plotted in figure 8. A \( k \)-value of 50 seems to reflect a good correlation and small loss of information for most of the data sets. The correlation constant is obtained according to

\[
\rho = \frac{C[x_r, x]}{\sqrt{V[x_r]V[x]}},
\]  

(32)
Figure 8: Plot of the length of $k$, against correlation between normalized and original data vectors for several persons.

where $x_r$ represents the raw RRI data and $x$ represents the normalized data. The correlation coefficient is always between -1 and 1. A correlation of 1 would be the result of two vectors with the same changes, due to the averaging a correlation coefficient of 1 is not expected. However, since only the OSA has been applied to the data a correlation of 0.7 or higher would be acceptable. Low correlation is most likely the result of large fluctuations in the LF band described in section 1.3.

The criteria for a strictly stationary process are that the distribution of the process is to be unchanged depending on time shifts over the process. Since the distribution of our data is unknown we will have to settle for the criteria of the weakly stationary process. The criteria for a weakly stationary process are that the expected value, at any given time-sequence of the process, should be zero. The covariance function should also be finite and depend only on the time difference $\tau = t - s$. This means that for a weakly stationary process the covariance function can be defined as a function of distance in time. The covariance between two points in one part of the process should be equal to the covariance between two equally spaced points in an other part of the process.

3.2 AR/ARX

In figure (9) it can be seen that an order of 11 results in the smallest total error from both methods. This tells us that an order of 11 linear regressive terms in the AR model is sufficient to model the processes. The figure shows the error for the first 20 terms. The error for a higher number of terms is not reduced and therefore not shown in the figure. Observe that the y-axis is not representative of the actual errors as it is the sum of all errors from the test data. As the sampling frequency is 4 Hz our model takes the past 2.75 seconds into account. The order for the respiratory signal is determined in the same way as the HRV signal, by using (12) and (13). Two terms are sufficient to represent the respiratory signal. However, since the respiratory signal is seen as an input signal to the HRV, an
increased number of terms might be of value. The reason for this is that the error estimation does not consider the covariance between the two data vectors. Looking at the raw data in figure 2, it is obvious that there is a strong delayed correlation between the two data vectors especially with a time difference of 15-20 samples. This is confirmed by looking at the covariance function described in (4). With this in mind the order of the respiratory signal is set to 20 in the ARX model. Simulations with an AR order of 4 were performed and the resulting noise were consistently higher than the noise obtained with order 11 for all persons. This indicates that our order valuation is reasonable. Further the windowsize and the stepsize described in section 2.3 needs to be set. The stepsize will have a large impact on the covariance of each parameter in time. A small stepsize will result in a high covariance and correlation between the values of the parameter in time, and will also increase the amount of parameter data generated. Since we want to model the AR parameter development over time, this is not necessarily a drawback. The stepsize seems to have small impact on the results during simulations and will therefore be set to 1. This will generate a more data than if the windowsize were set to a larger number. More data results in more accurate confidence interval. The windowsize presented in section 2.3 will be set to 200 and 400. The AR model and the ARX model are very similar and to justify the ARX model simulations are needed to see that it gives any further information other than what we get from the AR model. The additional 20 ARX parameters show a strong wave like motion most likely corresponding to the respiratory wave shape in the HRV signal. It is reasonable to believe that these coefficients absorb the respiratory motion from the HRV signal and that the first 11 coefficients therefore are more accurate with aspect to the pure HRV deviations. The first 11 ARX parameters are not equal to the 11 AR parameters.
therefore the model has some effect on the resulting coefficients and seems like an interesting model to examine. Furthermore, the resulting noise from (10) is very similar between the models, however significantly lower with the ARX model, about one third. Figure 11 shows the error for both the ARX model (left) and the AR model (right) over 18 time windows. The noise has a clear trend of increasing over time indicating increased error with higher respiratory frequency. Figure 10 shows the average noise for each of our subjects. A trend seems to be that the noise from the AR/ARX model seems to be lower for persons of higher age than persons of younger age (left). The conclusion can be drawn that the high ridges seen in Figure 11 is the noise obtained for persons of lower age. The same conclusion cannot be drawn between stress groups (right).

To be able to compare the parameters using confidence intervals as described in section 2.3 the AR parameters will have to be stationary. If the parameter is not stationary, it cannot be assumed to distributed according to the normal distribution. The parameter $\alpha$ described in (19) were calculated for each parameter and person. By calculating the confidence interval for $\alpha$ it is possible to determine if the development over time is present or not. If the confidence
interval of $\alpha$ passes through zero, the parameter process is referred to as stationary, otherwise it is seen as non-stationary. The non-stationary processes will not be compared with confidence interval as described in section 2.3. They will instead be evaluated depending on their $\alpha$ values. The confidence intervals for $\alpha$ can be seen in the table below. Parameters that does not pass through zero are marked as grey and their $\alpha$ values will be compared instead.

### 3.2.1 Autoregressive Roots

To further investigate the autoregressive models the roots are shown in figure 13. One clear difference between the roots is that with a windowsize of 400 the roots move significantly less. This is very reasonable since strong, short variations in the signal is dampened with a larger windowsize. Simulations with a windowsize of 100 were performed. The roots were very inconsistent and did not seem to capture the process in a proper way. Windowsizes of 200 and 400 seems to generate more accurate results and show reasonable time development. Figure 12 shows the same roots as figure 13, however only for the first modeled window. By comparing these figures the movement of the roots can be seen.

### 3.3 Markov-Chain Model

The number of states in the Markov chain is determined as described in section 2.4 by the letter L. Several values were tested and a value of 20 generates results that have good enough resolution to separate differences between Markov chains with good precision and will therefore be used during simulation.

### 3.4 Chi-Square test

Due to the amount of AR parameters generated by our models, some results are certain to be obtained when using a confidence interval of 95 per cent. To make sure that these results are not generated by the sample variance, they will be tested according to the $\chi^2$- test. This will test the hypothesis if the AR parameters generated by one group is the same as the AR parameters generated by another group. This test will give further strength to any results obtained.
Figure 12: First roots for AR/ARX model fitted to person 3
Figure 13: All roots for AR/ARX model fitted to person 3
By averaging all the AR parameters according to

$$\bar{\theta}^A = \frac{1}{N_A} \sum_{i=1}^{N_A} \theta_i^A$$

$$\bar{\theta}^B = \frac{1}{N_B} \sum_{i=1}^{N_B} \theta_i^B,$$

a mean value for each parameter is obtained. We assume that each parameter is distributed according to

$$\theta_i^A \sim N(\mu_i^A, \sigma_i^A)^2)$$

$$\theta_i^B \sim N(\mu_i^B, \sigma_i^B)^2).$$

The new averaged parameters then have the distribution

$$\bar{\theta}^A \sim N(\mu^A, \sqrt{\sigma^A^2/N_A})$$

$$\bar{\theta}^B \sim N(\mu^B, \sqrt{\sigma^B^2/N_B}).$$

The hypothesis we want to try is

$$H_0 : \mu^A = \mu^B.$$ (36)

By defining $\Delta \theta$ as

$$\Delta \theta = \mu^A - \mu^B,$$ (37)

we obtain a parameter distributed according to

$$\Delta \theta \sim N(\mu^A - \mu^B, \sqrt{\frac{\sigma^A^2}{N_A} + \frac{\sigma^B^2}{N_B}).}$$ (38)

Since the hypothesis we test is $\Delta \theta = 0$, we can rewrite $\Delta \theta$ to be distributed as the standard normal distribution according to

$$\Delta \theta \times \left(\frac{\sigma^A^2}{N_A} + \frac{\sigma^B^2}{N_B}\right)^{-1} \sim N(0, 1),$$ (39)

and the following will hold.

$$(\Delta \theta)^2 \sim \chi^2(2).$$ (40)

Using the equations above we obtain

$$t = (\Delta \theta)' \left(\frac{\sigma^A^2}{N_A} + \frac{\sigma^B^2}{N_B}\right)^{-1} \Delta \theta \sim \chi^2(2).$$ (41)

The $\chi^2$-test is then performed as

$$t > \chi^2_{0.05}(2) \quad \mu^A \neq \mu^B \quad \chi^2_{0.05}(2) = 5.99.$$ (42)
4 Results

4.1 AR-Model

Figure 14 shows the confidence intervals for the stationary parameters from the comparison between age groups using the AR model. Figure 15 shows the confidence intervals for the slope parameter for the non-stationary parameters using the AR model. For the stationary parameters, parameter 1 shows a clear deviation for both intervals and both age separators. Parameter 9 also shows a clear deviation when the two age groups are split at 30. The result is weakened when the groups are separated at 40. Parameter 10 also shows a trend to be higher for persons above 30, however this result is not seen when the groups are separated by 40. When looking at the distributions of the parameters, they have a clear similarity to the normal distribution so our previous assumptions are reasonable.

For the non-stationary parameters, parameter 3 shows deviance between age groups when they are separated by 30, however not by 40. A trend within the results seems to be that when dividing the age groups by 30 the results tend to be clearer. This might have something to do with the variance shown in figure 7. Figure 16 shows the confidence intervals for the stationary parameters when comparing stress groups and 17 shows the confidence intervals for the slope coefficient using the AR model. Parameter 10 shows a deviation when using window size 400, however it is not as strong for a window size of 200. No other trends seem to be present.

4.2 ARX-Model

Figure 18 shows the confidence intervals for the stationary parameters and 20 shows the slope parameters when comparing age groups using the ARX model. The results are very similar to the results of the AR model. Parameter 1 shows a clear deviation in the same way as for the AR model. Parameter 9 has a clear deviation when the age groups are separated by 30 but not by 40. Parameter three however was not stationary for the AR model in contrast to the ARX model. It generated a result when the age groups were separated by 40 and a window size of 400 was used. The non-stationary parameters show that parameter 5 generates a result when the groups are separated by 30 and a window size of 200 is used. The result is similar for age 40, however not as strong when the window size is 400. Figure 19 shows the stationary parameters when comparing stress groups using the ARX model and figure 21 the equivalent non-stationary parameters. No deviance can be seen for the slope parameters. For the stationary parameters, parameter 10 shows deviance when the window size is set to 400 and the result is strong with window size 200 as well. Parameter 11 also shows some deviation with window size 400.

4.3 Markov-Chain Model

The tables in equation 43, 44, 45 and 46 shows the result from the Kolmogonov-Smirnov test shown in equation 27. Since there are two different window sizes, the necessary $D_{Threshold}$ value to reject the hypothesis that the two distributions are the same is different. For a window size of 200 a $D_{Threshold}$ value 0.1 is
Figure 14: Confidence intervals for stationary parameters. AR model. Blue represents persons below the variable Age and red above Age.
Figure 15: Confidence intervals for slope coefficient. AR model. Red represents persons below the parameter Age and red, persons above Age.

Figure 16: Confidence intervals for parameter 1-11 with AR models. Blue represents not stressed persons and red stressed persons.
Figure 17: Confidence intervals for slope coefficient. AR model. Blue represents stressed persons, red represents not stressed persons.

Figure 18: Confidence intervals for stationary parameters. ARX model. Blue represents persons below the variable Age and red above Age.
Figure 19: Confidence intervals for stationary parameters. ARX model. Blue represents not stressed persons and red stressed persons.

Figure 20: Confidence intervals for slope coefficient. ARX model. Red represents persons below the parameter Age and red persons above Age.
necessary, for a window size of 400 a $D_{\text{Threshold}}$ value of 0.1160 is necessary. If the calculated $D_{\text{Estimate}}$ value is above the threshold the element is marked as light grey. If the value is larger than 0.2, indicating a large deviation, the value is marked as dark grey. When comparing the age groups several values pointing towards rejecting the hypothesis were found. Parameter 1 shows, in similarity with the parameter comparison, a clear difference. The difference seems to be stronger in the ARX model than the AR model. Parameter 4 also shows consistent results with window size 400. For the AR model parameter 11 generates large $D_{\text{Estimate}}$ values with window size 400. The ARX model generates large $D_{\text{Estimate}}$ values for parameter 10 in the same manner. An interesting trend in the Markov results are that, in contrast from the AR/ARX parameter comparisons, the results are stronger when the age groups are separated by 40. For the comparison between stress groups some significant results are generated. For the AR model parameters, parameter 2 and 11 are above the $D_{\text{Threshold}}$ value necessary to reject that the two distributions are the same with a window size of 400. For the ARX model parameters, parameter 9 and 10 are above the necessary $D_{\text{Threshold}}$ value when window size 400 is used. When a window size of 200 is used, parameter 4 and 9 are above the critical value.

Figure 21: Confidence intervals for slope coefficient. ARX model. Blue represents stressed persons, red represents not stressed persons.
### AR-Model

<table>
<thead>
<tr>
<th>Age</th>
<th>30</th>
<th>30</th>
<th>40</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>Window size</td>
<td>200</td>
<td>400</td>
<td>200</td>
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### ARX-Model

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### AR-Model

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<tr>
<td>11</td>
<td>0.0633</td>
<td>0.1271</td>
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</tbody>
</table>
### 4.4 General parameter comparison

#### 4.4.1 Age comparison

When looking at confidence intervals of the mean of the RR intervals no significant deviations can be found. However, the variance intervals can be seen in equation 48 and a clear difference can be seen between the age groups. With an age divider of 30 the difference is larger than for 40. The difference between the mean variance of elder persons and younger persons are calculated to 0.0291 and 0.0373 for age 40 and 30 respectively. By subtracting this from the data obtained when looking at stress levels the age correlation might be reduced and more of the stress information is evaluated.

<table>
<thead>
<tr>
<th>Age=30</th>
<th>Young</th>
<th>Old</th>
</tr>
</thead>
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<tr>
<td>(0.0764 0.1114)</td>
<td>(0.0461 0.0671)</td>
<td></td>
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</table>

<table>
<thead>
<tr>
<th>Age=40</th>
<th>Young</th>
<th>Old</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.0722 0.1001)</td>
<td>(0.0435 0.0704)</td>
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</table>

#### 4.4.2 Stress Comparison

The confidence intervals of the calculated variance is shown in the table below. No deviations can be seen from the raw data. No deviations can be seen with the subtracted mean variance from the age comparison either.

<table>
<thead>
<tr>
<th>Stressed</th>
<th>Not stressed</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.0002 0.0106)</td>
<td>(-0.0022 0.0070)</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Age=30</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(-0.0129 0.0000)</td>
<td>(-0.0136 -0.0015)</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Age=40</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(-0.0089 0.0018)</td>
<td>(-0.0096 0.0001)</td>
</tr>
</tbody>
</table>
4.5 Chi-Square test

The results from the $\chi^2$-test shows that the hypothesis in (36) can be rejected for parameter one when comparing age groups. The other parameters do not show the same results. The results generated by the stress groups do not show any deviance between the parameters. The results might therefore be generated by sample variance.

5 Conclusions

5.1 AR-Model

The autoregressive model showed clear results between age groups as well as stress groups. The results between age groups were stronger than the results generated between the stress groups and when using an age separator of 30 the results were clearer. The stress comparison also generated significant results. Parameter 10 had a deviation especially for the larger windowsize.

5.2 ARX-Model

When the ARX model was fitted to the data with the respiratory signal as input signal, the approximated noise was smaller compared to the AR model. The reduced noise indicates a better fit and possibly more reliable results. The results however were very similar to the AR model. The stationarity of the AR and ARX parameters were not the same for the two models. The ARX model showed non-stationarity for parameters 4 to 8 while the AR models non-stationary parameters were more spread out. This might indicate that the ARX model shows more consistency that the AR model. When comparing the stress groups the same parameter as in the AR model indicated deviation, parameter 10. The results were however stronger for the ARX model.

5.3 Markov-Chain Model

Several D values were larger than the threshold when comparing age groups and in accordance with the Kolmogorov-Smirnov test these parameters are not generated from the same distribution. However since our data is generated from measurements from different persons this seems reasonable. The $D_{\text{Threshold}}$ value calculated in the Kolmogorov-Smirnov test, the threshold, should perhaps be set higher in this case due to this fact. Some deviations could be seen between the stress groups. The largest obtained $D_{\text{Estimate}}$ value was generated by parameter 9 with a windowsize of 400 for the ARX model. The same parameter generated results for a windowsize of 200 as well, indicating a consistent result.

5.4 General parameter comparison

The variance of the raw data showed significant differences between age groups. No stress related deviations could be seen. This was in line with what was expected.
5.5 General conclusions

Unfortunately the ARX model does not seem to have absorbed much of the respiratory input signal that was predicted. Even though the ARX model showed promising with reduced noise the results were very similar to the AR model when comparing the parameters. However the ARX model showed a greater consistency in non-stationary parameter range as well as some of the Markov results. The overall results are interesting. The known fact that age has an influence on HRV is clear throughout the results. Some parameters show clear and consistent deviations between age groups for all setup values and for both models. The fact that the HRV has a known tendency to change with increased age and that the age related results were many and strong, indicates some validity to our models. The stress related results are not as many as the age related results. However some clear results were obtained. The results between stress groups were consistently stronger when a window size of 400 was used. If these results are strong enough to state that clear deviations between stress groups are present in general is doubtful, however, they show promise that the HRV might be related to stress and more clear results might be obtained when classifying the HRV process using other models. Unfortunately the $\chi^2$-test indicated that most of our autoregressive parameter results might be due to the sampling variance, however the results generated still show trends and should not be overlooked.
Acronyms

HRV  Heart rate variability
RRI  RR-interval (Time between heartbeats)
RSA  Respiratory sinus arrhythmia
AR   Autoregressive
ARX  Autoregressive model with extraneous input
FPE  Final prediction error
AIC  Akaike’s information criterion
CDF  Cumulative Distribution Function
PDF  Probability Distribution Function
References


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