Model of Hydro Power Plant-
New Algorithm
for Turbine Governors

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May 2005
Abstract
The process of hydro plant is on first sight easy to understand and well documented. Development of new control strategies could be accordingly based on that knowledge and trial and error approach at the commissioning. This traditional approach shows to be time consuming and expensive. Plant models available are often obsolete as they are simplified for the narrow linear range of the working area and not directly executable. Intuitive knowledge of the original control developers is practically not available. Real plants are run under stringent economical demands and thus not available for testing.

The main goal of this Master Thesis was to test modern strategies for control development using a “test bench” for test and verification. The “test bench” was developed as a Hydro Plant library in the modelling language Modelica, under the simulation shell Dymola (www.dynasim.se).

Applicability of that test bench was thereafter tested through evaluation of a gain scheduling approach based on an online system identification using a PRBS. The results were good, proving that the Hydro Plant library is an efficient development tool. The model of an entire hydro plant allows tests of new control laws, verification of new designs of plant control components and last but not least making commissioning more efficient.

Keywords

Classification system and/or index terms (if any)

Supplementary bibliographical information

ISSN and key title
0280-5316

Language
English

Number of pages
82

Recipient’s notes

Security classification

The report may be ordered from the Department of Automatic Control or borrowed through University Library, Box 3, SE-221 00 Lund, Sweden Fax +46 46 222 42 43
Acknowledgements
First we would like to thank our supervisors Jan Tuszyński and Tore Hägglund for making this thesis possible. Jan Tuszyński’s extensive knowledge in modelling and hydro power plants has been invaluable to us. We would also like to thank the people at Dynasim AB and Henrik Lindsjö at Waplans Mekaniska Verkstad AB for input and help during the work.
1 Introduction
1.1 Background
There are about 1600 hydro power stations in Sweden, with a total capacity of 16 GW, producing about 46% of our electric energy. Hydro power has low environmental impact, it is a renewable energy source and the basic technology is still the same as used hundreds of years ago.

The main goal of this Master Thesis was to test modern strategies for control development using a “test bench” for test and verification. The “test bench” was developed as a Hydro Plant library in the modelling language Modelica, under the simulation shell Dymola (www.dynasim.se). Applicability of that test bench was thereafter tested through evaluation of a gain scheduling approach based on an online system identification using a PRBS. The results were good, proving that the Hydro Plant library is an efficient development tool. The model of an entire hydro plant allows tests of new control laws, verification of new designs of plant control components and last but not least making commissioning more efficient.

1.2 Scope and objectives
This thesis builds on two main subjects:
• Development of the Hydro Plant library for the dynamic model of Hydro Power systems
• Model based development and implementation of the new turbine governor algorithms

1.3 Structure and overview of the process modelled

![Block diagram of hydro power plant](image)

The Hydro Plant library includes the following components:
• Reservoir
• Penstock
• Guide Vanes and Turbine
• Generator
• Power System

The control system covers the following:
• Turbine Governor
• Guide Vane and Runner Actuator

The turbine governor represents the lowest level of the control system. The higher level control systems are:

• Water Level Control
• Turbine Group Control
• Various types of optimizers

1.4 Terms and Definitions

1.4.1 Abbreviations
DAE: Differential-algebraic Equation
MCB: Main Circuit Breaker
MV: Media Vector
ODE: Ordinary Differential Equation
PID: Proportional, Integral, Derivative
PRBS: Pseudo Random Binary Signal

1.4.2 Symbols

<table>
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<td>Heat capacity</td>
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<td>F</td>
<td>Force</td>
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<tr>
<td>G</td>
<td>Media momentum (M*v)</td>
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<td>h</td>
<td>Specific enthalpy</td>
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<td>q</td>
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<td>T</td>
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<td>τ</td>
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<td>U</td>
<td>Internal energy of the media</td>
<td>Nm</td>
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<tr>
<td>u</td>
<td>Specific internal energy</td>
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<td>Δ</td>
<td>Relative roughness</td>
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</tr>
<tr>
<td>η</td>
<td>Dynamic viscosity</td>
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</tr>
<tr>
<td>ρ</td>
<td>Density of the media</td>
<td>kg/m³</td>
</tr>
<tr>
<td>ω</td>
<td>Angular velocity</td>
<td>rad/s</td>
</tr>
</tbody>
</table>
2 Theoretical background

2.1 Basic equations of Thermo Hydraulic Systems

The simulation of thermodynamic systems is based on three equations concerning mass, energy and momentum conservation in a control volume. The equations can be expressed as partial differential equations for an infinitely small control volume (local form) and then be transformed to suit physical shapes (global form). The equations for the global form can be expressed as the following differential equations:

\[
\begin{align*}
\frac{dM}{dt} &= m_{dot_i} - m_{dot_o} = \sum m_{dot} \\
M: & \quad \text{Total mass in the control volume \ [kg]} \\
m_{dot_i}, m_{dot_o}: & \quad \text{Mass flow in and out of the volume \ [kg/s]} \\
\end{align*}
\]

Equation (2.1a) can be rewritten as a derivative of density \( \rho \), recognising that \( M = \rho V \):

\[
\frac{dM}{dt} = \frac{d(\rho V)}{dt} = \frac{d\rho}{dt} V + \rho \frac{dV}{dt} = \sum m_{dot}
\]

assigning,

\[
\sum m_{dot} = \left( \sum m_{dot} - \rho \frac{dV}{dt} \right)
\]

and,

\[
\frac{d\rho}{dt} = \frac{\sum m_{dot}}{V}
\]

\[
V: \quad \text{Volume of the control volume \ [m}^3]\]

\[
\rho: \quad \text{Density of the media \ [kg/ m}^3]\]

\[
\frac{dU}{dt} = m_{dot_i} h_i - m_{dot_o} h_o + q + W_s - p \frac{dV}{dt};
\]

(2.2) Energy conservation:

assigning \( \Sigma e^i = \Sigma (m_{dot} h) + q + W_s \)

\[
\frac{dU}{dt} = \Sigma e^i - p \frac{dV}{dt};
\]

\[
U: \quad \text{Internal energy of the media in the control volume \ [Nm]} \\
h_i, h_o: \quad \text{Specific total enthalpy in and out from the control volume \ [Nm/kg]} \\
\Sigma e^i: \quad \text{Sum of all incoming (+) and leaving (-) energy flows \ [Nm/s]} \\
q: \quad \text{Incoming (+) and exiting (-) heat flow from ambient environment \ [Nm/s]} \\
W_s: \quad \text{External mechanical energy flow \ [Nm/s]} \\
p: \quad \text{Media pressure in the control volume \ [N/m}^2]\]

It is assumed that the total enthalpy includes both the static and dynamic components i.e.:
\[ h = C \cdot T + \frac{v^2}{2} \]

(2.3) Momentum conservation:

\[
\frac{dG}{dt} = m \cdot \dot{v}_i - m \cdot \dot{v}_o + (A_i p_i - A_o p_o - F_f) + A \rho g(z_i - z_o)
\]

\(G\): Media momentum in the control volume [kgm/s]
\(v_i, v_o\): Media velocity in, out of the control volume [m/s]
\(p_i, p_o\): Control volume intake, outlet pressure [N/m²]
\(A_i, A_o\): Control volume intake, outlet area [m²]
\(A\): Control volume average (representative) cross section area [m²]
\(m \cdot \dot{v}_i, m \cdot \dot{v}_o\): Mass flow on the inlet, outlet of the control volume [kg/s]
\(g\): Gravitational acceleration constant [m/s²]
\(z_i, z_o\): Elevation of control volume inlet, outlet [m]
\(F_f\): Friction force [N]

2.2 Media state \([p, T(h)]\) calculation

2.2.1 Media density as a function of states \([p,T(h)]\)

Media density is based on the states \(p\) and \(T\):

\[
\frac{d\rho}{dt} = \left| \frac{\partial \rho}{\partial T} \right|_p \frac{dT}{dt} + \left| \frac{\partial \rho}{\partial p} \right|_T \frac{dp}{dt}
\]

where,

\[
\left| \frac{\partial \rho}{\partial T} \right|_p = \alpha_T \text{ and } \left| \frac{\partial \rho}{\partial p} \right|_T = \alpha_p
\]

In case of ideal gases, \(\alpha_p\) and \(\alpha_T\) can be calculated from the gas law \(p = \rho \cdot R \cdot T\), and accordingly:

\[
\alpha_T = -\frac{\rho}{T}
\]

(2.4b)

and,

\[
\alpha_p = \frac{\rho}{p}
\]

In case of gas mixtures and gases with non-ideal properties, \(\alpha_{pg}\) and \(\alpha_{Tg}\) will be taken from gas property tables.

When dealing with liquids it is more convenient to use coefficient of elasticity \(\beta\), defined as follows:
\[ \frac{\Delta p}{\rho} = \frac{\Delta p}{\beta} = \frac{\Delta V}{V} \]

(2.5) accordingly

\[ \alpha_p = \frac{\rho}{\beta} \]

\(\beta\) has the same unit as \( p \) [N/m\(^2\)] and has the property of being volume additive, which means that the mixture of gas and liquid total volume change (caused by the pressure change or vice versa) is \( \Delta V_{\text{tot}} = \Delta V_{\text{lq}} + \Delta V_{\text{gas}} \). Now assuming: \( V_{\text{gas}} = \varepsilon V_{\text{tot}} \), and at the small \( \varepsilon, V_{\text{lq}} \approx V_{\text{tot}}: \)

\[ \Delta V_{\text{tot}} = \frac{V_{\text{lq}}}{\beta} \Delta p + \frac{V_{\text{gas}}}{p} \Delta \rho; \quad (\text{as } \beta = p \text{ for ideal gases}) \]

\[ \frac{\Delta V_{\text{tot}}}{V_{\text{tot}}} = \left( \frac{1}{\beta} + \frac{\varepsilon}{p} \right) \Delta p \]

(2.6) and finally,

\[ \frac{1}{\beta_{\text{tot}}} = \left( \frac{1}{\beta} + \frac{\varepsilon}{p} \right) \]

Control volume changes, caused by pressure, can be added in a similar way. Coefficients \( \beta (\alpha_p) \) and \( \alpha_T \) should be calculated from the liquid properties tables\(^1\).

From equation (2.4a) the following basic form binding media states is derived:

\[ \frac{dp}{dt} = \alpha_T \frac{dT}{dt} + \alpha_p \frac{dp}{dt} \]

(2.7) Substituting \( \frac{dp}{dt} \) with equation (2.1b) results in the expression:

\[ \frac{\sum m_{\text{dot}}^1}{V} = \alpha_T \frac{dT}{dt} + \alpha_p \frac{dp}{dt} \]

(2.8) Equation (2.8) is valid for both gases and liquids.

### 2.2.2 States \( p \) and \( T \) for liquids

Equation (2.8) represents dependence between time derivatives of \( T \) and \( p \). Solving both requires a second equation, which can be taken from the energy conservation (2.2).

Internal energy \( U \) can be written as:

\[ U = M \cdot u \]

\( U: \) Energy of the media in the control volume [Nm]

\( M: \) Mass of the media in the control volume [kg]

\( u: \) Specific internal energy [Nm/kg]

\(^1\) See section 3.5
By assuming that all internal energy of the liquid depends on the temperature only, \( u \) can be defined as:

\[
C = \frac{du}{dt}
\]

Assuming constant \( C \) in the working range,

\[
u = u_0 + C \cdot T
\]

The derivative of \( U \):

\[
\frac{dU}{dt} = \frac{d(\rho V C T)}{dt} = \frac{d\rho}{dt} V C T + \frac{dV}{dt} \rho C T + \frac{dT}{dt} \rho V C
\]

Substituting \( \frac{dU}{dt} \) in equation (2.2), with equation (2.11) yields:

\[
\rho V C \frac{dT}{dt} = \frac{d\rho}{dt} V C T - \frac{dV}{dt} \rho C T + \Sigma e^1 - \frac{dV}{dt} p
\]

assigning: \( \Sigma e = \Sigma e^1 - (\rho C T + p) \frac{dV}{dt} \)

\[
(a) \quad \frac{dT}{dt} = \frac{\Sigma e}{\rho V C} - \frac{d\rho}{dt} \frac{T}{\rho}
\]

Final formulation for \( \frac{dT}{dt} \) is accordingly:

\[
(2.13) \quad \frac{dT}{dt} = \frac{1}{M C} (\Sigma e - \Sigma m_{\text{dot}^1} T C)
\]

We now use (2.8) for \( \frac{dp}{dt} \) calculation where \( \alpha_p \) is substituted with \( \frac{\rho}{\beta} \):

\[
(2.14) \quad \frac{dp}{dt} = \beta \left( \frac{\Sigma m_{\text{dot}^1}}{M} - \frac{\alpha_p}{\rho} \frac{dT}{dt} \right)
\]

Equations (2.13) and (2.14) can now be used universally for calculation of \( \frac{dT}{dt} \) and \( \frac{dp}{dt} \) in all kinds of control volumes containing liquids with the properties \( \rho, \beta, \alpha_t \) and \( C \) taken from liquid property tables. States of a particular volume are calculated from \( \Sigma e \) and \( \Sigma m_{\text{dot}^1} \), which define communication with all volumes interconnected.

### 2.2.3 Special cases of \( \frac{dp}{dt} \) calculation
Equation (2.14) calculates \( \frac{dp}{dt} \) based on the assumption of media compressibility, \( \alpha_p > 0 \).

This assumption is generally valid, but there are cases when the compressibility is negligible and the pressure can be easily calculated from other dominating factors.

Such cases are open reservoirs or containers where the liquid shares total volume with a dominating volume of gas (\( \varepsilon > 0.1 \) in equation (2.6)).

In case of reservoirs the pressure of the liquid depends of the liquid level only. Assuming \( \frac{d\rho}{dt} = 0 \) in equation (2.1a):

\[
\rho \frac{dV}{dt} = \Sigma m_{\dot{\text{m}}},
\]

or,

\[
\frac{dH}{dt} = \frac{\Sigma m_{\dot{\text{m}}}}{\rho A_r};
\]

(2.15)

\( H \): Surface level of the reservoir \([m]\)

\( A_r \): Representative area of the reservoir \([m^2]\)

As the compressibility is negligible the pressure can now be calculated using:

(2.16) \( p = \rho \cdot g \cdot H \)

The temperature derivative is still calculated using equation (2.13).

2.3 Conservation of momentum for mass flow calculation

2.3.1 Mass flow acceleration and forces involved

Equation (2.3) is used for calculating the mass flow through a volume. This equation is Newton’s second law:

(2.17) \( M \frac{d^2x}{dt^2} = \frac{d(M \cdot v)}{dt} = \Sigma F \)

\( M \): Mass of all media contained in the module \([kg]\)

\( v \): Velocity of the mass \([m/s]\)

Recognizing that \( M = V \cdot \rho \) and \( v = m_{\dot{\text{m}}}/(A \cdot \rho) \) equation (2.17) can be written as:

(2.18) \( \frac{d(M \cdot v)}{dt} = \frac{d(V \cdot \rho \cdot m_{\dot{\text{m}}})}{dt} \cdot \frac{1}{\rho \cdot A} = L \cdot \frac{d(m_{\dot{\text{m}}})}{dt} = \Sigma F = F_{\text{jet}} + F_p + F_f + F_g \)

\( V \): Volume enclosed in the connecting element, or \( A \cdot L \) \([m^3]\)

\( L \): Length of liquid column \([m]\)

\( \Sigma F \) represents all forces acting on the flowing mass, these are:
The “Jet Force” expressed as:

\[ F_{jet} = \dot{m}_i \cdot v_i - \dot{m}_o \cdot v_o \]

or, if assuming constant flow through the control volume,

(2.19a)

\[ F_{jet} = \frac{\dot{m}_i \dot{m}_o}{\rho} \left( \frac{1}{A_i} - \frac{1}{A_o} \right) \quad [N] \]

The “Jet Force” will differ from zero only if \( \dot{m}_i \) along the control volume axis differs from \( \dot{m}_o \), or/and \( A_i \) differs from \( A_o \). The jet force is discussed in detail in section 2.4.

The next force affecting the mass flow is the force due to pressure difference at the inlet and outlet:

(2.19b)

\[ F_p = p_i A_i - p_o A_o \quad [N] \]

\( A_i, A_o \): Cross-section area of the inlet and outlet \([m^2]\]

\( p_i, p_o \): Pressure at the inlet and outlet \([N/m^2]\)

This force is normally the dominating force generating the mass flow.

The third force is the friction force, \( F_f \) (section 2.3.2), representing the energy lost by the moving liquid.

The last force is the gravitational force:

(2.19c)

\[ F_g = A \cdot \rho \cdot g \cdot (z_i - z_o) \quad [N] \]

\( z_i, z_o \): Height of the inlet/outlet over reference level \([m]\)

\( A \): Cross-section area of the inlet and outlet \([m^2]\)

The block diagram in Figure 2 shows the basic principles of how \( \dot{m}_o \) is calculated.
2.3.2 Friction calculation

The friction acting on the flowing media is [1]:

\[ F_f = p_{Loss} \cdot A \]

where,

\[ p_{Loss} = \lambda(Re, \Delta) \frac{L}{2D} \eta \cdot v \cdot \frac{L}{2D} \eta = \lambda_2(Re, \Delta) \frac{L}{2D} \eta \]

\[ \lambda: \text{friction coefficient} \]
\[ \lambda_2: \text{used friction coefficient} \]
\[ Re: \text{Reynolds number} \]
\[ = \frac{vD\rho}{\eta} = \frac{D}{A\eta} \cdot m_{dot} \]
\[ L: \text{length of pipe} \]
\[ A: \text{cross-section area of pipe} \]
\[ D: \text{diameter of pipe} (4*A/wetted) \]
\[ \Delta: \text{relative roughness} \]
\[ \rho: \text{density} \]
\[ \eta: \text{dynamic mean viscosity} \]
\[ v: \text{mean velocity} \]

\( \lambda_2 \) is introduced as \( \lambda_2 = \lambda \cdot Re \cdot |Re| \) allowing \( \lambda_2 = 0 \) when \( m_{dot} = 0 \) in the region \( Re < 2000 \). Doing this prevents division by zero at \( m_{dot} = 0 \). The characteristics of \( \lambda_2 \) can be seen in Figure 3.
The characteristics of the pressure loss function are divided into three regions: laminar flow, turbulent flow and the transition region in between.

In the first region, \( \text{Re} \leq 2000 \), the flow is laminar and the exact solution of the 3-dim. Navier-Stokes equations (momentum and mass balance) are used under the assumptions of steady flow, constant pressure gradient and constant density and viscosity (= Hagen-Poiseuille flow):

\[
\lambda_2 = 64 \cdot \text{Re} \quad \text{or} \quad p_{\text{Loss}} = \frac{64k\sqrt{D}}{A\eta}m_{\text{dot}}
\]

When \( \text{Re} \geq 4000 \) the turbulent region is entered where the Colebrook-White equations yield an implicit relationship between \( \text{Re} \) and \( \lambda \):

\[
\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{2.51}{\text{Re}\sqrt{\lambda}} + 0.27\Delta\right)
\]

By using the relationship \( \lambda_2 = \lambda \cdot \text{Re} \cdot |\text{Re}| \), \( \text{Re} \) can be expressed as:

\[
\text{Re} = -2\sqrt{\lambda_2} \log\left(\frac{2.51}{\text{Re}\sqrt{\lambda_2}} + 0.27\Delta\right) \cdot \text{sign}(\lambda_2)
\]

\( \lambda_2 \) is computed through an approximation of the inverse of the Colebrook-White equation:

Figure 3: \( \lambda_2(\text{Re}, \Delta) = \lambda \cdot \text{Re} \cdot |\text{Re}| \) (x-axis: \( \log(\text{Re}) \), y-axis: \( \log(\lambda_2) \))
The transition region between laminar and turbulent flow $\text{Re}_{\text{lam}} \leq \text{Re} \leq 4000$, is more or less chaotic. Depending on the relative roughness, $\Delta$, the transition from laminar flow can vary. In the case when the relative roughness equals zero the transition takes place at $\text{Re}=2000$ but for higher values of $\Delta$ the transition will start earlier. To calculate where the transition region starts the following approximation is used:

\begin{equation}
\text{Re}_{\text{lam}} = 745 \cdot e^{k_3}
\end{equation}

where,

\begin{equation}
k_3 = \begin{cases} 
1 & \text{if } \Delta \leq 0.0065 \\
0.0065 & \text{if } \Delta > 0.0065
\end{cases}
\end{equation}

In this region $\lambda_2$ is approximated by third degree polynomial:

\begin{equation}
\lambda_2 (\text{Re}) = a_1 \text{Re}^3 + a_2 \text{Re}^2 + a_3 \text{Re} + a_4
\end{equation}

The coefficients $a_1 \ldots a_4$ can be found as it is known where the transition region start and end as well as $\frac{d\lambda_2}{d\text{Re}}$ at these points.
2.4 Basic Components of the Flow Transfer

Closed volumes are modelled through interconnected control volumes and connecting modules. The thermodynamic state of a specific control volume is determined by pressure $p$, and temperature $T$. From these quantities other media properties like density and elasticity can be derived. The mass flow and energy exchange between volumes is carried out in the connecting modules using the state and media properties of the adjacent volumes.

2.4.1 General structure

The process modelled can be described as a transfer of liquid media through a chain of two types of components (Figure 4):

- Volume, container
- Connecting module, transporting media between the volumes

This basic structure builds on the following assumptions:

- State of the media $[p, T, \rho, ..]$ is calculated in volumes only
- Media is transported in connecting modules only

Those assumptions cannot be fulfilled as equations (2.13) and (2.14) for media state calculations, and equation (2.18) for mass flow calculation require the following:

- $h_i, h_o$: total enthalpies of the incoming media must be calculated in connecting modules, i.e. out of the volume components
- $m_{\text{dot}}_i, m_{\text{dot}}_o$: the jet force (equation 2.19a) depends on the external flow coming to the entrance of the connecting module (i.e. $m_{\text{dot}}_i$ if flow $> 0$, $m_{\text{dot}}_o$ if flow $< 0$); i.e. externally of the connecting modules

The total media enthalpy contains both the internal energy of the media ($C^*T$) and kinetic energy $\frac{v^2}{2}$ (equation 2.2b). The internal energy part can be easily changed in the connecting modules by adding heat transferred from the environment, or heat losses of the represented machinery ($\approx$-1-efficiency). Estimation of the incoming kinetic energy and jet force is more complicated.
2.4.2 Problem of the media flow in the volumes

As it is assumed generally that the mass flow calculated of the momentum conservation (equation 2.18) represents connecting module flow outlet, it is easy to calculate outlet enthalpy. Calculation of inlet enthalpy requires the rate of incoming flow that is, the flow which “hits” the inlet of the connecting module.

The same “hitting” flow is needed for impact calculation of the incoming media (m_dot*v) required in equation (2.18).

“Hitting flow”, meaning that there is a flow component in the volume, is exemplified in Figure 5:

- There are two flows, m_dot(+)_1,2, entering the volume (generally m_dot(+)_i).
- Entering (+) flows are added into single vector component, Sum_mIn, flow

\[ \text{Sum}_\text{mIn} = K_{al} \cdot \sum m_\text{dot}(+)_i; \]

- There are two flows, m_\text{dot}(-)_1,2, “hitting” connecting modules emptying the volume (generally m_\text{dot}(-)_i). Flows m_\text{dot}(-)_i will be distributed as a portion of the Sum_mIn defined as follows:

\[ m_\text{dot}(-)_i = \text{rec}_i \cdot \text{Sum}_\text{mIn}; \]

- Factor rec_i represents flow recovery factor showing how well Sum_mIn vector is aligned with the axis of the connecting module “i”. For all rec factors:

\[ \sum_{i=1}^{n} \text{rec}_i \leq 1; \text{ with each, } \text{rec}_i \geq 0; \]

\[ \text{Figure 5: Flows entering the volume} \]

The above approach is approximate only, but sufficiently supportive for the modelling presented in this thesis. While estimating rec_i factors the following was considered:

- Factor K_{al} represents addition of vectors and would be 0 for opposite (+) flows of equal strength, or 1 for parallel flows.
- Each component rec_i corresponds in theory to \( \cos(\alpha_i) \), where “\( \alpha_i \)” is an angle between Sum_mIn and the axis of the i-th connecting module.
- Special forms of the volume should be considered, allowing rec_i at \( \alpha_i = 90^\circ \), being > 0.
In case of segmented pipe (penstock) where division in volumes and connecting modules is purely theoretical, factors $K_{al}$ and $rec$ are 1, which means that both kinetic energy and impact force are simply transferred to the next segment.

2.5 Basic Concept of the Control System

2.5.1 PID [10]

A typical turbine governor is based on PID structure.

The P part or proportional part produces a control signal that is proportional to the error.

$$P(t) = K_p \cdot e(t)$$

(2.28)

where,

$$e(t) = y_{ref}(t) - y(t)$$

$y_{ref}(t)$: Set point

$y(t)$: Measured value

The problem with this type of controller is that it will lead to a stationary error. To eliminate the stationary error integral action is introduced:

$$I(t) = \frac{K_p}{T_i} \int e(t)dt$$

(2.29)

Where $T_i$ is called integral time and decides how the integral part is weighted.

Although the I-part eliminates the stationary error, this is achieved at the expense of stability margin. This is because the pole placed in origo, causing an extra 90° of phase lag, reducing the amplitude and phase margins. To counter that phase shift a third term, the derivative action term is added.

One big disadvantage with proportional and integral action is the fact that they do not anticipate what is going to happen in the future. A way to achieve this is to extrapolate the error curve along its tangent, and letting the control act on this predicted error $e_p$ defined as:

$$e_p(t) = e(t) + T_d \frac{de(t)}{dt}$$

(2.30)

This leaves the D-part defined as:

$$D(t) = K_p \cdot T_d \frac{de(t)}{dt}$$

(2.31)

Where $T_d$ is of dimension time and is called derivative time. It can be interpreted as the prediction time horizon. The big advantage of using derivative action is, as mentioned above, the improved damping of an oscillatory system. There are however drawbacks with the D-part, the parameter $T_d$ must be chosen carefully if it shall not do more harm than good. It is also sensitive to high frequency noise, so filtering the error signal is important.

Combining all three parts results in a control law of the PID controller on the form:
\[ u(t) = P + I + D = K_p \left[ e(t) + \frac{1}{T_i} \int e(t) dt + T_d \frac{de(t)}{dt} \right] \]

### 2.5.2 Modifications of PID and Discretization

When implementing a PID controller in practice, some modifications should be made for better performance. To start with the maximum gain of the derivative part is limited by approximating the transfer function \( sT_d \), which is equivalent to the D-part Laplace transformed, with:

\[ sT_d \approx \frac{sT_d}{1 + \frac{sT_d}{N}} \]

This right hand side of the equation approximates the derivative well at low frequencies but the gain is limited to \( N \) at high frequencies. \( N \) is called maximum derivative gain and is typical in the range of 10-20.

One more modification to be made on the derivative part is to not let it act on the error but instead just on the measured variable, \( y \). The reason for this is that a step change in the set point otherwise drives the control signal to its limits, causing large overshoots in the step response. This yields the following expression for the D-part:

\[ D(s) = -\frac{sT_d}{1 + \frac{sT_d}{N}} \cdot Y(s) \]

The integral action part was also modified. In almost every system there are limitations of the output of the controlled process. The combination of saturation of the process output and a controller with integral action causes so called integrator windup. There are several ways to avoid this, for example conditional integration or inhibiting the integration whenever the output saturates. These implementations will be discussed below.

PID controllers used on actual hydro power plants are of the discrete type which means that the above described algorithm must be translated into discrete time. The basic idea is to translate the existing continues-time control design into discrete-time as good as possible. To do this, the most common approach in control purposes is to try to approximate the Laplace operator, \( s \), with some function of the shift operator or \( z \)-operator. The most common are Forward difference, Backward difference and Tustin’s approximation.

When using forward difference the following approximation of the Laplace operator is used:

\[ s = \frac{z - 1}{h} \]

\(^2\) See equation (3.32)
Equation (3.35) is equivalent with replacing all derivatives with forward difference:

\[ \frac{dx}{dt} \approx \frac{x(t + h) - x(t)}{h} \]  

(2.36)

When using forward difference it is possible to obtain an unstable discrete-time system from a stable continuous-time system.

In the backward difference method the following approximation to the Laplace operator is used:

\[ s = \frac{z - 1}{z \cdot h} \]  

(2.37)

This means that all derivatives are approximated by the backward difference:

\[ \frac{dx}{dt} \approx \frac{x(t) - x(t - h)}{h} \]  

(2.38)

When using backward difference the complex left hand plane will be mapped into the unit circle. This means that a stable time-continuous system will also be stable in the discrete-time system.

The last discussed method is the Tustin approximation which approximates the \( s \) operator with:

\[ s = \frac{2}{h} \frac{z - 1}{z + 1} \]  

(2.39)

The Tustin approximation maps the complex left hand plane exactly on the unit circle.

**Discretization of P-part:** As the P-part doesn’t include any derivative part the discretization means replacing the continuous variables with their samples versions. Rewriting equation (2.28) yields,

\[ P(t_k) = K_p \cdot e(t_k) \]  

(2.40)

where subscript \( k \) denotes the sampling instances.

**Discretization of I-part:** From equation (2.29) it follows that:

\[ \frac{dI(t)}{dt} = \frac{K_p}{T_i} e(t) \]  

(2.41)

By using forward difference the equation can be written as,
\[
\frac{I(t_{k+1}) - I(t_k)}{h} = \frac{K_p}{T_i} e(t_k)
\]

where \(h\) is the sampling time.

From equation (2.42) the final expression is derived:

\[
I(t_{k+1}) = I(t_k) + \frac{K_p \cdot h}{T_i} e(t_k)
\]

**Discretization of D-part:** The D-part expressed with the \(s\) operator can be seen in equation (2.31). This equation can be rewritten to time-continuous system:

\[
\frac{T_d}{N} \frac{dD(t)}{dt} + D = -K_p T_d \frac{dy(t)}{dt}
\]

This equation is then approximated by backward difference:

\[
\frac{T_d}{N} \frac{D(t_k) - D(t_{k-1})}{h} + D(t_k) = -K_p T_d \frac{y(t_k) - y(t_{k-1})}{h}
\]

Equation (2.45) is then rewritten to yield the final expression of the D-part:

\[
D(t_k) = \frac{T_d}{T_d + N \cdot h} D(t_{k-1}) - \frac{K_p \cdot T_d \cdot N}{T_d + N \cdot h} \left( y(t_k) - y(t_{k-1}) \right)
\]
3 Hydro Plant Library. Modelica Implementation

The purpose of this chapter is to give a brief description of the Hydro Plant package. For a more in-depth look the library documentation should be used [14].

3.1 Main aspects of the Modelica language

The Modelica language was developed to be used in physical modelling and to make it easy to exchange models and model libraries. The reuse issue is solved by making the language well suited to use an object-orientated approach when creating models. Many other general-purpose tools such as for example SIMULINK are essentially based on a modelling methodology using input-output blocks with causal interactions. This means that the models are expressed as an interconnection of sub models written on explicit state-space form:

\[
\frac{dx}{dt} = f(x,u) \\
y = g(x,u)
\]

Where \( u \) is the input and \( y \) is the output. It is uncommon that a natural decomposition into subsystems lead to such a model which makes efforts in terms of analysis and analytical transformations to obtain the problem in this form unavoidable. By using this form the blocks/models will have a unidirectional data flow and clearly defined inputs and outputs. When building up a model of a large system, the non-causal approach used in Modelica can in many cases be more suitable.

It is no longer necessary to decide in advance what the inputs and outputs of the model must be. Instead, the models can contain the constitutive relationships appropriate for the component, for example Ohm’s law. These constitutive relationships are then augmented with conservation laws yielding a complete system of equations.

As mentioned above, Modelica supports object-orientated modelling where the behaviour of the lowest level may be expressed in terms of ordinary differential equations (ODE) without requiring it to be written on state space form. Instead, the models can be written using algebraic equations, so called differential-algebraic equation system (DAE), without the restrictions against the presence of algebraic loops or implicit non-linear equations.

3.2 Basic Principles of the Library

The main components of the library are containers and connecting modules interconnected as described in section 2.4. That basic interconnection is applied in a way illustrated in Figure 6. It can be seen that the media state, calculated in nodes of volumes, stored in the MV vector, is always directed out from the volumes into the connecting modules where the mass flow and enthalpy are calculated. Depending on the direction of the mass flow, mass is either added or subtracted from the adjacent volumes. This enables bidirectional flow throughout all chains of the media transfer.
3.3 Structure of the Hydro Power Library

Basic structure of the library is shown below. The hydro plant package contains three main types of classes which are low level classes, high level classes and functions.

The low level models have been implemented with code reuse in mind and thus enabling higher level models to extend them. By making their function general it is possible to use them in almost any higher level class. A typical example of a lower level class is a connector, which is the interface through which communication between different objects take place.

The high level classes extend one or more lower level classes and have a more specific function. The possibility to reuse these classes is often only doable in specific and limited cases. An example of a high level class is the model of a closed volume.

Function calls takes place within both low- and high level classes and returns calculated results based on given inputs.

3.4 Basic Library

The basic library contains all the basic components used for the more complex components.

The icons library contains all icon representations used throughout the library and will not be further discussed here.

Global constants, types and conversion units are kept in the GlobalHP package. The content of the package is inherited by all other classes and thereby reachable from all classes.

MathBlocks is a package containing basic mathematical blocks used throughout the library. One important component is the PRBS. The PRBS is built up from unit delays with pre set start values outputting a pseudo random signal at each sample time.
3.4.1 Connectors

Connectors are used to send and receive variables to/from adjacent components. In each connector it is defined what kind of signals it can handle and if the signal is an input or output. Connectors contained in the package are:

**Mass port**: The mass port connector handles the variables mass flow and enthalpy. Every component which needs to be able to send or receive mass flow and/or enthalpy has to use the mass flow connector.

**Water Media Vector**: The connector contains media property vector $MV$ and the value of $Sum_mIn$ (see section 2.4.2). The vector, which is of size ten, is designed to hold media properties such as density, elasticity, heat capacity and so forth.

**Signal**: The signal connector is an all purpose connector containing a vector, $signal$, of size $n$ and type real.

**Boolean**: This connector is designed to handle signals of Boolean type. Number of inputs or outputs of the vector, $signal$, is $n$.

3.4.2 Basic calculations

The basic calculation package contains all functions. Some of the more important ones will be discussed here.

**pTLiquid**: function for calculation of the pressure and temperature derivatives in a given closed volume. Equations used for calculations can be seen in section 2.2.2 above. Needed inputs to the function are:

- Sum of all mass flows [kg/s]
- Sum of energy [J]
- Active part of the container volume [m$^3$]
- Water media vector$^4$

From these inputs the pressure and temperature derivatives are outputted.

**pTLiquidRes**: function for calculation of the pressure and temperature derivatives in a given open volume. The temperature derivatives are calculated in the same way as for the closed volume but for the pressure derivative equation (2.16)$^5$ is used.

**Mass Flow Calculation**: This function calculates the mass flow derivative between two adjacent volumes. The equation used for this can be seen in section 2.3$^6$. Inputs needed for this calculation are:

---

$^3$ See equation (2.13) and (2.14)
$^4$ See section 3.5 and 3.4.1
$^5$ See section 2.2.3
$^6$ See equation (2.18)
- Mass flow from adjacent volumes [kg/s]
- Pressures at the adjacent volumes [bar]
- Density of the media in the adjacent volumes [kg/m³]
- Area of the adjacent volumes intake/outtake [m²]
- Friction [N]
- Height difference between the adjacent volumes [m]

### 3.5 Property Calculations

This package contains functions which are used to calculate media properties given a set of inputs. In the current implementation of the library the only represented media is water. Media properties calculated are:

- Density [kg/m³]
- Specific heat capacity [Nm/kgK]
- Elasticity coefficient [bar]
- $\alpha_T$ [kg/Km³]
- Dynamic viscosity [kg/ms]

Inputs needed, for calculating these properties, are:

- Pressure [bar]
- Temperature [K]
- Amount of gas in media [p.u]
- Container elasticity [m³/bar]
- Volume of the media [m³]

The specific heat capacity is assumed constant as it only changes marginally around the studied temperatures and pressures relevant during simulation. The same assumption is applied to the dynamic viscosity.

The density is calculated as a function of pressure and temperature. By using data from media tables, where the pressure varies between 1 and 60 bar and the temperature between 3 and 40°C, a dataset is acquired.

By fitting the data in a least-square sense, as two second degree polynomials, results in a function on the form:

$$ \rho = (a_1 T + a_2) p^2 + (b_1 T^2 + b_2 T + b_3) p + c_1 T^2 + c_2 T + c_3 $$

Where the coefficients $a_{1,2}$, $b_{1,3}$ and $c_{1,3}$ are the result of the fitting of the data.

Figure 7 shows a plot of the estimated function together with the original data. As can be seen the error is marginal.

---

7 See section 2.2.1
The elasticity coefficient can be calculated through \(^8\):

\[
\beta = \frac{\rho}{\partial \rho / \partial p}
\]

Using the result in equation (3.1) together with (3.2) leads to:

\[
\beta = \frac{\rho}{2(a_1 T + a_2) + b_1 T^2 + b_2 T + b_3}
\]

Equation (2.6) will allow inclusion of gas contents and container elasticity:

\[
\beta_m = \beta \frac{p}{p + \varepsilon \beta + \frac{dV}{dp} \cdot \frac{\beta \cdot p}{V}}
\]

\[
\frac{dV}{dp} : \text{Container elasticity} \quad [m^3/\text{bar}]
\]

As can be seen from equation (3.4) a small amount of gas in the water will substantially decrease the elasticity coefficient.

\(^8\) See section 2.2.1 equation (2.5)
Finally, $\alpha_T$ is calculated using the expression:\n
\begin{equation}
\alpha_T = \left[ \frac{\partial \rho}{\partial T} \right]_p = a_i p^2 + 2b_1 T \cdot p + b_2 p + 2Tc_1 + c_2 = a_i p^2 + p \cdot (2b_1 T + b_2) + 2Tc_1 + c_2
\end{equation}

### 3.6 Hydro Components

Two main packages represent hydro components:

- **Containers**, with basic models representing media volumes in closed and opened containers
- **Connecting Modules**, with models representing media conduits.

#### 3.6.1 Containers

There are two types of volumes represented here. These are *Open Volumes* and *Closed Volumes*. There are also variations of the two types allowing connection of two or more connecting modules.

Containers use at least two mass ports and two water media vector connectors\(^9\) to communicate with its surroundings. The water media vector are output connectors and the mass ports are inputs.

![Figure 8 Two closed volumes and connecting module. The communication between the volumes is carried out through the connecting module. The bottom connectors are the water media vector connectors and the top connectors are the mass ports.](image)

**Closed Volume**: The closed volume represents independent container or an enclosed water segment of a long media conduit. The state of the volume is decided by its pressure and temperature, both calculated using the function $p_{TLiquid}^{11}$. 

Input value, sum of mass flow, $\Sigma m_{\dot{}}$ is represented by equation (2.1b) The $\frac{dV}{dt}$ term is crucial for varying volume of the container which for hydro plants is in most cases constant.

Input sum of energy, $\Sigma e$ represented in equation (2.12), is used under the assumption that the heat exchange $q$ and the mechanical work lost on the media $W_s$ are negligible. This assumption leads to the fact that the dominating factor deciding the energy input is $\Sigma (m_{\dot{}} h)$.

---

\(^9\) See section 2.2.1 equation (2.4a)  
\(^{10}\) See section 3.4.1  
\(^{11}\) See section 3.4.2
The enthalpy, \( h \), and mass flow, \( m_{dot} \), are calculated in the connected conduits and obtained through the mass port connectors.

Input, water media vector, is acquired through a function call to Water Properties\(^{12}\).

The sum of all incoming mass flows is calculated in the model as an output value of the mass flow connector:

\[
(3.6) \quad \text{Sum}_m \text{In} = \max(0, m_{dot}^l) + \max(0, m_{dot}^r)
\]

See discussion on recovery factor in section 2.4.2.

**Open Volume:** The open volume represents a non enclosed volume with free surface, on the border between the media and the gas volume above.

Assuming dominating gas volume above the surface allows calculation of the pressure on media surface of the gas compression, and neglecting media compressibility. Pressure of the media will accordingly depend solely of the surface pressure and media level as expressed in equations (2.15) and (2.16)\(^{13}\).

Media temperature calculation is basically the same as for the closed volume, but requires consideration of varying media volume in the container. The open volume energy content is now expressed as:

\[
(3.7) \quad \sum e = m_{dot}^L \cdot h_L + m_{dot}^R \cdot h_R - \rho \cdot C \cdot T \cdot \frac{dV}{dt}
\]

The last term of the equation indicates that energy contained in the open volume will be dispersed when the media volume expands, or in other words if media mass increases / decreases more/less energy will be needed to heat it up.

As it will be discussed later on, the shape of the open volume influences media flows between the control volumes/segments. The shape became standardized as shown in Figure 9\(^{14}\).

---

\(^{12}\) See section 3.5

\(^{13}\) See section 2.2.3

\(^{14}\) The shape is contained in a separate subclass to the open volume component which makes it easy to implement other shapes
The volume of the component is calculated by the following equation:

\[
V = \left( \frac{H \cdot (T_w - B_w)}{2 \cdot H_{\text{max}}} + B_w \right) \cdot H \cdot L
\]

\( L \): Length of the volume (segment) (as shown in Figure 10) \([\text{m}]\)

\( H \): Media level (when \( H = H_{\text{start}} \), calculates \( V_{\text{start}} \)) \([\text{m}]\)

\( H_{\text{max}} \): Max height of the volume (until overflow) \([\text{m}]\)

\( T_w \): Top width (restricted to \( \geq B_w \)) \([\text{m}]\)

\( B_w \): Bottom width \([\text{m}]\)

### 3.6.2 Connecting Modules

As with the containers there are two kinds of connecting modules, *Mass Flow* and *Mass Flow Open*. The former models the connection between closed volumes and the latter between open volumes. Also contained in this package are connecting modules which deal with the transition from an open volume to a closed as well as the other way around. This becomes important when modelling the complete water ways in a hydro plant. The transition will take place when connecting a reservoir to a penstock.

Connecting modules communicate with both ends control volumes through two pairs of connectors, output connector of *mass port* and input connector of *media vector*.\(^{15}\)

\(^{15}\) See section 3.4.1, and compare with Figure 8
It is worth to note that the sum of all mass flows through the connecting modules will at all times be zero (i.e. media mass can not be stored).

**Mass Flow Closed Module:** The mass flow through the connecting modules is calculated in most general cases by using equation (2.18)\(^{16}\).

Parameters are pre-defined before simulation to decide the characteristic of the jet force\(^{17}\).

The forces due to the gravitational force and pressure can both be calculated since the pressure difference, area of the inlet/outlet, density of the incoming flow and height difference between the volumes are all known.

The friction force is calculated using the equations in section 2.3.2. All parameters required are defined in the model including the roughness of the closed volume walls, physical dimensions, etc. Media density and viscosity are provided by the connected volumes through the media vector (MV) connectors.

Assuming mass flow from left to right through the connector the enthalpy is calculated using:

\[
\begin{align*}
  h_R &= C_L \cdot T_L + \frac{1}{2} \left( \frac{m_{\text{dot}}}{\rho_L \cdot A_R} \right)^2 \\
  h_L &= C_L \cdot T_L + \frac{1}{2} \left( \frac{sum_{m_{\text{in}}} \cdot rec}{\rho_L \cdot A_L} \right)^2 
\end{align*}
\]

(3.9)

\text{and,}

\[
\begin{align*}
  h_R &= C_R \cdot T_R + \frac{1}{2} \left( \frac{sum_{m_{\text{in}}} \cdot rec}{\rho_R \cdot A_R} \right)^2 \\
  h_L &= C_R \cdot T_R + \frac{1}{2} \left( \frac{m_{\text{dot}}}{\rho_R \cdot A_L} \right)^2 
\end{align*}
\]

(3.10)

The second term of the equations is the kinetic energy of the mass flow. In this setup, when the mass is moving from the volume on the left side of the connection module to the volume on the right side, a \text{rec} and \text{Kwl} factor of one would mean that no kinetic energy is lost in the transition.

When the mass flow is directed from the right volume into the left volume the following equation is used:

\[
\begin{align*}
  h_R &= C_R \cdot T_R + \frac{1}{2} \left( \frac{sum_{m_{\text{in}}} \cdot rec}{\rho_R \cdot A_R} \right)^2 \\
  h_L &= C_R \cdot T_R + \frac{1}{2} \left( \frac{m_{\text{dot}}}{\rho_R \cdot A_L} \right)^2 
\end{align*}
\]

(3.10)

\text{and,}

\[
\begin{align*}
  h_R &= C_R \cdot T_R + \frac{1}{2} \left( \frac{m_{\text{dot}}}{\rho_R \cdot A_L} \right)^2 \\
  h_L &= C_R \cdot T_R + \frac{1}{2} \left( \frac{sum_{m_{\text{in}}} \cdot rec}{\rho_R \cdot A_R} \right)^2 
\end{align*}
\]

**Mass Flow Open Module:** When calculating the friction and mass flow between open volumes the previously used equations have to be modified.

\(^{16}\) See section 2.3  
\(^{17}\) See section 2.4.2
Figure 10 below shows a section of two adjacent volumes between which the flow will be calculated.

The parameters provided and known media levels ($H_{\text{left}}$ and $H_{\text{right}}$) will allow calculation of the shared and non shared area between the water columns, based mainly on the *Min/Max Common Height* of the contact section\textsuperscript{18}. The problem lies therein that the two volumes might have different dimensions when looking at the cross section. For example the bottom width or top width might differ which will lead to that the two volumes will have different inclination of the walls.

The derivative of the mass flow is calculated using:

\[
\frac{d(m_{\text{dot}})}{dt} = \frac{F_{\text{fric}} + A_s \cdot \rho \cdot g \cdot (H_L - H_R + Z_L - Z_R)}{L} + \text{slope} \cdot k_f
\]

$L$: Length of the mass flow connector [m]

$F_{\text{fric}}$: Friction force (directed opposite to $m_{\text{dot}}$) [N]

$A_s$: Shared area between the volumes [m$^2$]

$H_{L,R}$: Height of the water column over the reference level [m]

$Z_{L,R}$: Height over reference level [m]

$k_f$: Percent of usable flow from the sliding mass [p.u]

\textsuperscript{18} Compare Figure 12
The first term in the \( \frac{\text{der}(m \dot{m})}{dt} \) equation is the friction force added with the pressure difference which is multiplied with the shared area. This is similar to equation (2.18)\(^{19} \) used when calculating the mass flow for closed volumes.

The second term deals with the mass flow between the segments where they are not connected by water (see \( \Delta H \) in the figure below). The first term in equation (3.11) will not deal with this part as it is not included by the shared area. Instead this part is seen as a sliding mass, and slope is calculated through:

\[
\frac{d(M \cdot v)}{dt} = M \cdot g_a
\]

where,

\[
v = \frac{Q}{A_{ns}} = \frac{m \dot{m}}{\rho \cdot A_{ns}}
\]

(3.12) yielding,

\[
\text{slope} = \frac{d(m \dot{m})}{dt} = g_a \cdot \rho \cdot A_{ns}
\]

where,

\[
g_a = \frac{(H_L - H_R)}{\sqrt{L^2 + (H_L - H_R)}} \approx \frac{(H_L - H_R)}{L} g
\]

\( A_{ns} \): Non shared area between the volumes \( [m^2] \)

The calculation of the friction force is also done differently when dealing with open volumes. One important assumption is that the flow is always assumed laminar which simplifies the friction calculations.

Enthalpy to the volumes is calculated in the same way as for the closed volume component.

### 3.7 Hydro subsystems

This package contains the models describing the waterways of a hydro power plant. The waterways can be divided into three categories, the reservoir, the penstock and the surge tank.

A reservoir is basically a water reserve from where water is taken to produce electrical energy in the plant. To transform the potential energy stored in the reservoir to electrical power the water has to be transported to the mechanical components of the power plant through the penstock.

\(^{19}\) See section 2.3
Often a surge tank is connected to the penstock. The surge tank is used to relieve the rest of the penstock from pressure transients, moving along the penstock, resulting e.g. from the water hammer phenomena.

### 3.7.1 Reservoir

Not all hydro power plants have reservoirs; some are placed directly in the stream of a river and use the kinetic energy of the flowing water as primary energy source. The advantages of hydro power plants that use reservoirs are however many; for one it increases the control possibilities of the power output.

The reservoir can be viewed as stored energy that can be converted into electrical power when needed. It makes it possible to store water from periods with much precipitation, and even save water from wet years for generating electricity during dry years.

Each hydro power plant has an assigned max water level allowed in the reservoir for environmental reasons. If this level is exceeded the hydro plant is normally fined. Concept of the energy storage means that the power plants will try to be as close to this level as possible. Problems arise when the water reservoir level is close to the max allowed level and the power demand is low. In situations like this the plant will be forced to let certain amount of water through the gates. It is a waste of money for the plant and there are environmental consequences of dumping large amount of water into the river. Water level in the reservoir should be planned according to the expected power demand, the profit for produced electricity, the expected flow rate with which the reservoir water level rises and so on.

At a pumped storage hydro plant there are both an upper and a lower water reservoir and the hydro power plant is designed to be able to operate in reversed mode. Pumping water up to the higher reservoir means energy storage during times (night) when the power demand is low and cheap energy production available (e.g. nuclear power plants). The efficiency loss in the energy conversions makes these facilities net energy consumers, but since they can be brought online to operate rapidly when needed, and when energy price is high, they are considered a valuable resource.
The model of the reservoir is built by automatically connecting Open Volumes and Mass Flow Open connectors. Main parameters of the model are the following:

- Number of sections in the reservoir which automatically decides the number of mass flow connectors.
- Middle shore-to-shore width of the reservoir
- Side shore-to-shore width of the reservoir
- Length of the reservoir
- Bottom levels of each section over the reservoir reference level
- Max water level in each volume
- Start level of the water in the reservoir
- Start temperature of the water
- Roughness coefficient of the walls used for friction calculations.
- Ratio between the top and bottom width
- Parameter deciding the contour of the reservoir, as explained below
- Percentage of usable shared area

The outlet/inlet from/to the reservoir, where the connection between the reservoir and penstock takes place, is normally assumed in the leftmost and rightmost volume.

The method of sectioning the reservoir is basically division in finite elements, and accordingly a high number of sections in the reservoir will improve simulation accuracy and produce more life like behaviour. High number of sections increases complexity and simulation speed will suffer. Given the parameters above the reservoir will be automatically dimensioned to fit this data, as shown in Figure 12.

Each section of the figure represents a volume which means that the reservoir in the figure is built out of twenty volumes. To decide the shape of each section the following contour equation is used:

\[(3.13) \quad \left(\frac{x}{a}\right)^k + \left(\frac{y}{b}\right)^k = 1\]

\[x: \quad \text{Position on the x-axis of the volume} \quad [m]\]
\[y: \quad \text{Calculated variable deciding the width of each volume} \quad [m]\]
\[a: \quad \frac{\text{ReservoirLength}}{2} \quad [m]\]
\[b: \quad \frac{\text{MiddleWidth} - \text{SideWidth}}{2} \quad [m]\]
\[k: \quad \text{Contour factor deciding the shape of the reservoir}\]

\textit{Origo of the x-y coordinates is in the middle of the reservoir.}
Figure 12 Figure shows the parameters, concerning the dimension of the reservoir, set prior to simulation. Top left figure shows the reservoir from above the bottom left figure from the long side and the top right from the short side.

The contour equation is applied to each volume and the width is calculated for both the bottom width and top width of each volume. The difference between the two is that when calculating for the bottom width the ratio between the bottom and top width is used. This will result in volumes where the “walls” will tilt.

Figure 13: 3-dimensional figure of the reservoir
Contour factor $k$ allows a wide range of reservoir “coasts”. Examples, seen in Figure 14, are calculated for reservoir width set to 200m, the side width to 10m and the length set to 1000m. Note that the x-axis shows number of volume segment, not the length.

Figure 14: different coast shapes depending of factor $k$

Figure 15 presents a simulation result of the reservoir. The reservoir contains 5 volume segments; $k$ is 1 and the start water level 25m in all volumes except for the leftmost where it is 33m.

Figure 15 Simulation of the reservoir
The plot shows how the start level difference in the reservoirs produces a wave that travels the length of the reservoir. Interesting to note here is that the wave will have lower amplitude in the middle volume than in the other ones. This is especially obvious when comparing with the wave amplitude in the rightmost volume. The reason for this behaviour is the chosen shape of the reservoir from which it can be seen that the rightmost volume will have a relatively small volume compared with the middle volume which will have the largest volume. The wave will continue to travel the length of the reservoir, back and forth, with decreasing amplitude due to the friction.

3.7.2 Penstock
The water from the reservoir is transported in a penstock to the turbine where the energy conversion takes place. The maximum possible head of the penstock increases the energy while the length should be as short as possible. The length depends on circumstances as the nature of the ground and environmental requirements.

As the form of the penstock will influence the output power of the turbine it is important to provide a proper penstock model. The model includes the effect of the water inertia which makes the power actually decrease initially when opening the guide vane and vice versa when closing. This is due to the pressure drop needed for water column acceleration.

Elasticity (water compressibility) described above gives rise to travelling waves of pressure and flow in the water travelling along the penstock at (under ideal conditions) the velocity of sound in water. Pressure wave and water acceleration results in the water hammer phenomena which in the worst case can damage or destroy penstocks, guide vanes, valves and even the turbine. To reduce the effect of this phenomenon, surge tanks are required leading the pressure wave there instead of letting it travel through the penstock.

The penstock is implemented in Dymola as an automatic build up model of closed volumes and mass flow connectors. The following parameters are required:

- Total length
- Height
- Diameter at the start and end of the pipe
- Amount of gas in water
- Start pressure at the top of the pipe (should equal bottom pressure of reservoir)
- Start temperature
- Roughness of the pipe
- Recovery factor
- Number of volumes of the pipe. This will also decide the number of mass flow connectors

A high number of the penstock sections will improve simulation accuracy and produce more life like behaviour but simulation speed will suffer.

The penstock model implemented enables almost any shape and configuration of water ways both on the inlet and outlet side of the turbine.

A test of the penstock model is displayed below with the simulation setup according to Figure 16.
This simulation shows the static properties of the penstock. The water flow is stabilizing at zero as it may only flow between the sections of the penstock.

The head and length of the penstock is 30m respectively 50m. The start pressure at the top is 1.8 bar which indicates that there is about 20m water head above the inlet to the pipe. Two simulations were performed, one where the penstock was built up from 3 volumes and the other where it was built up from 5 volumes. The result can be seen in Figure 17.

The bottom and top pressure of the pipe are the same in both simulations as expected. The difference is that in the more accurate setup the pressures are divided between more volumes and thus giving more information and accuracy.

The next test is a simulation of a penstock connected to two reservoirs, see Figure 18.
In this simulation water will flow through the pipe out into the bottom reservoir. The result of the simulation is shown Figure 19.

Figure 19 Simulation of a penstock with 3 respectively 5 volumes connected to reservoirs. The plots show the pressures.

Here an important difference between the simulation using three sections and the simulation using five sections can be seen. In the five section simulation the interaction between the volumes in the pipe shows a more complex behaviour since in this simulation the pipe is better described with more segments.

As can be seen in the figure the pressures are now lower than the static pressures displayed in Figure 17. The reason for the pressure difference is that in the latter simulation there is both static and dynamic pressure. The fact that the mass is moving will lower the static pressure, caused by the friction acting on the media.

Since the pressures behaved quite different depending on the number of elements in the pipe, it was tested if the stationary mass flows became the same with 3 and 15 sections. The pipe dimensions were the same in both setups.
Figure 20 Mass flows for a pipe with 3 and 15 volumes

Figure 20 shows the result of this test. As can be seen the mass flow through the two simulation setups are equivalent as expected.

At the start of every simulation the mass flow is initialized to zero. From Figure 21 it can clearly be seen that the mass accelerates. This acceleration of the mass won’t happen simultaneously in all segments of the pipe. What will happen is that the mass in the section closest to the exit of the pipe will start to move out of the pipe as a result of the pressure difference between the section and the connected reservoir. When this happens, the equilibrium in the pipe is disturbed and mass from the adjacent pipe section will start to flow into the volume. This will propagate through all sections of the pipe and will result in water being taken from the top reservoir. The mass flow will continue to accelerate until a new equilibrium is found between the pressure difference, friction, gravitational force and the jet force\textsuperscript{20}.

Figure 21 shows results of this test during the first moments of the simulation.

As can be seen the mass flow first rises in the last volume in the reservoir which causes the mass flow to rise through all other volumes in the pipe.

\textsuperscript{20} See section 2.4.2
Next the water hammer effect will be tested. The water hammer effect will takes place when the guide vane is closed rapidly, causing pressure waves and mass to move up the penstock.

The simulation setup can be seen in Figure 22. Please note that a surge tank isn’t used in this setup.

![Figure 22 Simulation setup](image)

The mass flow connector between the pipe and reservoir is now replaced with a valve modelling the guide vanes. In the test the valve will be open during the first 15 seconds of simulation after this it will close in one second leaving a five percent opening. The results can be seen in Figure 23.

![Figure 23 Affect on the mass flow due to a water hammer](image)

A can be seen from Figure 23 mass flow drops rapidly and starts to oscillate. The negative mass flow indicates that the mass of moving up the pipe.

Now let’s see what happens with the pressures, note that the time scale is different from Figure 23.
Figure 24 Pressure wave due to water hammer

Figure 24 shows how the pressure rises in the volume closest to the valve just when it closes. This pressure rise then propagates to the adjacent volumes up in the pipe creating a wave.

Figure 25 shows the event over a longer time perspective.

Figure 25 Pressure wave due to the water hammer. The top plot shows pressure of a volume close to the top of the pipe, middle plot shows the pressure in a volume in the middle of the pipe and the bottom plot shows the pressure in the volume closest to the valve.

The figure shows that the pressure rise is greater further down the pipe than at the top. The reason for this is that the bottom volume is affected by the whole water column.

Next the opposite phenomena will be tested. The valve will be almost closed during the first five seconds of simulation, after this it will be fully open. The result can be seen below.
Figure 26 The top plot shows pressure of a volume close to the top of the pipe, middle plot shows the pressure in a volume in the middle of the pipe and the bottom plot shows the pressure in the volume closest to the valve.

From Figure 26 it can be seen that in all volumes the pressure drops when the valve opens. The pressure drop is greater in the volume closest to the valve because this volume is affected by the whole water column. The pressure drop is an effect of the water inertia and will have the consequence that the power output of a connected turbine will actually fall before it rises when the guide vane is opened.

Further it can be concluded that the pressure will stabilize to a lower value after the valve is opened. The cause for this is the dynamic pressure as explained above.

Traditionally the phenomenon of water column accelerating in a penstock was described by the water inertia time defines as:

\[
T_w = \frac{Q}{g \cdot H_b} \sum \frac{L}{A}
\]

\(H_b\): Water head \([m]\)

\(L\): Length of the penstock section \([m]\)

The water inertia time is the time it takes for the water to accelerate to 66% of its final flow value.
A simulation was performed with the following parameters:

- \( L = 150 \text{m} \)
- \( A = 0.78 \text{m}^2 \)
- \( H_b = 50 \text{m} \)

By using these parameters in equation (3.14) and finding the volume flow from the figure on the left yields a \( T_w \) of 2.35s which corresponds well with the simulated result.

### 3.8 Mechanical Components

This package contains models for an actuator which is based on a model called Newton base. The purpose of the models is to simulate the dynamics of a moving mass.

#### 3.8.1 Newton Base

The model uses Newton’s second law which states that the force applied to a mass, is equal to the rate of change of momentum of the mass:

\[
\sum F = m \cdot a
\]

- \( m \): Mass of the body \([\text{kg}]\)
- \( a \): Acceleration of the body \([\text{m/s}^2]\)
- \( \Sigma F \): Sum of all forces acting on the body \([\text{N}]\)

The forces acting on the body are defined as:

\[
\sum F = F_{\text{bal}} + F_{\text{spring}} + F_{\text{ext}} + F_{\text{ang}} - v \cdot K_{\text{dmp}}
\]

where,

\[
F_{\text{spring}} = -K_{\text{spring}} \cdot x_{\text{spring}}
\]

and,

\[
F_{\text{ang}} = \sin\left(\frac{\theta \cdot \pi}{180}\right) \cdot g
\]

- \( F_{\text{bal}} \): Constant balancing force (e.g. balancing weight of the mass) \([\text{N}]\)
$F_{spring}$: Spring force     \([N]\)

$K_{spring}$: Spring constant     \([N/m]\)

$x_{0spring}$: Initial spring compression     \([m]\)

$F_{ext}$: External force acting on the mass     \([N]\)

$K_{dmp}$: Damping coefficient     \([Ns/m]\)

$v$: Velocity of the mass     \([m/s]\)

$F_{ang}$: Added force in case of non horizontal position of the mass     \([N]\)

$g$: Gravitational acceleration     \([m/s^2]\)

Given equation (3.15) and (3.16) it is possible to calculate both the velocity and position of the moving mass.

The main aspect of the Newton Base model is handling mass movement limited in both position and velocity.

### 3.8.2 Actuator

The actuator model is based on Newton Base with an added feedback loop. It is used for modelling guide vanes.

Figure 27 shows a block diagram of the model. The actual position of the mass is given by the output signal $Y$ while $Y_{ref}$ represents the desired position.

![Figure 27 Schematic figure of the actuator model](image)

The input signal to Newton base is:

\[(3.17) \quad u = K \cdot (Y_{ref} - Y)\]

$Y_{ref}$: Reference signal     \([m]\)

$Y$: Current position of the mass     \([m]\)

$K$: Factor amplifying error signal

Figure 28 shows a test run of the model. The mass can move between 0 and 1 meter and the speed is limited to 0.2 m/s.
3.9 Mechanical Subsystems

The actuator Kaplan model describes the actuators connected for a Kaplan turbine. In the Turbines section 3.9.2., different types of turbines are discussed and a mathematical model of turbines is derived.

3.9.1 Actuator Kaplan

The actuator used for a Kaplan turbine controls both the guide vane opening and the pitch of the turbine blades of the runner\textsuperscript{21}.

Two Actuator\textsuperscript{22} models were used, as seen in Figure 29.

---

\textsuperscript{21} See 3.9.2  
\textsuperscript{22} See 3.8.2
The reference signal from the turbine governor, $Y_{ref}$, is sent to the guide vane which outputs the actual guide vane opening, $Y$. The $Y$ opening is then used as an input to the Kaplan combination cam to find $\alpha_{ref}$, which is the required blade angle. The $\alpha_{ref}$ is then executed by the second Actuator $\alpha$. The actual angle of the blades, $\alpha$ and $Y$, are then used in Table A allowing implementation of a turbine characteristic algorithm for calculation of the representative turbine flow area $A_{out}$.

Figure 30 shows a typical Kaplan combination cam. In most practical cases the cam is dependent of the actual water head, which was not implemented in this thesis.

3.9.2 Turbines
Modern hydropower turbines originate from wooden water-wheels being used for several hundreds of years. Although much have changed in material and design aspects, the principle to convert potential energy of the water into rotational kinetic energy of the turbine is still the same. The efficiency keeps getting better and is now a bit over 90%, even peeking up to as much as 96% in rare cases and under good conditions. There are three basic design types common amongst modern hydro-turbines:
- Pelton turbine
- Francis turbine
- Kaplan turbine

Turbines can be divided into two categories, impulse and reactive. Pelton is the most common type of impulse turbine while both Francis and Kaplan are examples of reactive turbines. Pelton turbine is best suited for high heads, for medium heads Francis works best while exploitation of the low heads is possible with Kaplan turbines.

Below follows a brief description of each turbine type. The turbine implemented in Hydro Plant Library is general allowing simulation of all types of turbines.

**Pelton turbine:** In a Pelton turbine the water is lead through one or more fixed nozzles, in each of which the pressure from the water head is converted to kinetic energy in form of a water jet (\(m*v\) part of the momentum equation 2.3). This jet force of water is then directed on the buckets of the runner where they loose almost all of their kinetic energy at atmospheric pressure. Figure 31 illustrates the principle design. The runner has buckets placed all around its periphery and it is at these the jet is aimed making the runner move. The splitter ridge (basic idea invented by Lester A. Pelton) divides the bucket in two and makes the water leave in a more controlled and efficient way. As the wheel rotates in air, it must be placed above the tailrace water level which means loss of static head.

![Figure 31: Impulse turbine (Pelton) [1]](image-url)
**Francis Turbine:** In contrast to an impulse turbine the runner in a Francis turbine is completely submerged in water. Water enters the volute, flows through the guide vanes and then enters the runner which changes the momentum of the water, producing a reaction on the turbine. The guide vanes are so arranged that the potential energy of the water is converted into a rotary motion. The water flows radial towards the centre, and in its way are the curved vanes which the water impinges making the runner rotate. In contrast to the impulse turbine where only a few of the runner buckets at any given time are used for the energy conversion, the runner blades in a reaction turbine all divide the strain, making them able to deal with larger mass flows of water.

![Figure 32: Francis turbine](image)

**Kaplan Turbine:** Kaplan turbines are of reaction type but here the main flow direction is parallel to the axis of rotation. Characteristic for Kaplan turbines are also that the blade pitch is adjustable. The control is designed so the blade angle varies with the guide vane opening to reach a maximum efficiency for a given operating condition. Because of this feature, the Kaplan turbine has a somewhat flat efficiency curve over a large range of flows. Kaplan turbines are not suited for large water pressures but can handle very large volume flows and operate well at low heads.
The turbine model of the Hydro Plant Library is implemented based on a single valve principle where the flow of the water is throttled by “A” factor defining the turbine throughput (as defined in box “Table A”, Figure 29).

The mechanical power available from a hydraulic turbine is given by:

\[
P_m = Q \cdot \Delta p \cdot \eta_T
\]

- \( P_m \): Turbine output power \([W]\)
- \( Q \): Water volume flow through the turbine \([m^3/s]\)
- \( \Delta p \): Pressure difference of water between inlet to outlet of turbine \([N/m^2]\)
- \( \eta_T \): Turbine mechanical efficiency

As \( \Delta p \) is available from the inlet and tail volumes of the turbine the volume flow through the turbine has to be found. Flow characteristics of the turbine will be based on the nominal turbine parameters:
\[
Turbine_{\text{Char}} = \frac{Q_N}{A_N \cdot \sqrt{\frac{\Delta p_N}{\rho_N}}}
\]

where,

(3.19) \[\dot{Q}_N = \frac{P_N}{\Delta p_N \cdot \eta_{T-N}}\]

and,

\[\Delta p_N = H_N \cdot \rho_N \cdot g\]

\(Q_N\): Nominal volume flow through the turbine \([m^3/s]\)
\(\Delta p_N\): Nominal pressure drop over the turbine \([Pa]\)
\(H_N\): Nominal water head \([m]\)
\(\rho_N\): Nominal density of the water \([kg/m^3]\)
\(P_N\): Nominal power of the turbine \([W]\)
\(A_N\): Nominal area of the turbine opening \([m^2]\)

In equation (3.19) \(P_N\), \(\eta\), \(\rho_N\) and \(H_N\) are all assumed known and set as simulation parameters. \(A_N\) will be normally estimated of the \(Q_N\), basing on the estimated design flow rate “v” through the turbine.

Having the characteristics of the turbine, the actual volume flow through the turbine can be calculated:

(3.20) \[Q = A \cdot \eta_{Vol} \cdot Turbine_{\text{Char}} \cdot \sqrt{\frac{\Delta p}{\rho}}\]

\(A\): Area of the turbine throughout \([m^2]\)

Equation (3.20) accounts for turbine losses through efficiency factors; first through nominal mechanical efficiency factor \(\eta_{T-N}\) and secondly through water flow efficiency \(\eta_{Vol}\). Both factors can be assumed constant and given by parameters, or alternatively given through efficiency maps of the particular turbine as shown in Figure 35.

For Kaplan turbines there is additional efficiency factor due to the error between \(\alpha\) and \(\alpha_{ref}\), called the combination error. The relationship between the combination error and combination efficiency loss can be seen in Figure 34.

Turbine will be accordingly treated as a connecting module with standard connectors introduced above.

---

23 See section 3.9.1
Figure 34 Relationship between the combination error and efficiency loss

Figure 35 Efficiency curve due to frequency deviation from nominal and opening of the guide vane
3.10 Electrical Components

In this section developed electrical components are discussed. The Grid Load component together with the Grid Production and Main Circuit Breaker (MCB) will together build up the power grid. The generator is studied and the theory behind the model is discussed. The final model is the synchronizer which has the function to synchronize the frequency of the studied power plant with the frequency of the grid before the MCB is closed.

3.10.1 Grid Production

The Grid Production model is the representation of all other production units connected to the grid.

The grid production units are divided into different groups depending on their response time. This enables simulation of different behaviour depending on the type of power plants connected to the grid. For example a nuclear power plant has a much longer response time than a gas turbine power plant.

The error signal to the plant turbine governors is described by\(^{24}\):

\[
e = \Delta f + e_p \cdot \Delta P
\]

\(\Delta f\): Frequency deviation between the nominal and actual frequency [Hz]
\(\Delta P\): Power deviation between reference and actual group power output [W]
\(e_p\): Speed regulation factor [unit]

By assuming the error to be zero equation (3.21) can be written:

\[
P_{\text{ref}} = P - \frac{\Delta f}{e_p}
\]

This power reference is then used as the input to the ramp function. A ramp function is used because a hydro plant responds much like a ramp to step changes in power reference. A plot of a test run of the used ramp function can be seen in Figure 36.

---

\(^{24}\) See section 3.12
As can be seen from the figure the rate of change will always be the same independent of the size of reference change.

As the power grid modeling requires testing of various disturbances there are means provided to add or to reject connected power plants from a specific group.

3.10.2 Grid Load
The load model represents the power demand of the grid.

The load of the grid is divided into three groups: one resistive, one frequency dependent and one quadratic frequency dependent. The response to step changes of the load is simulated by using first order transfer functions. The time constants of the transfer functions decide the response time. As the load on a grid is constantly changing a normal distributed random number generator was added to the load of each group. In addition to this there is also the option to add a disturbance at a given time. The input to the transfer functions looks as follows:

\[
TF_i = \text{Load}_{\text{Tot}} \cdot \text{Part}_i \cdot (\text{Disturbance}_i + \text{RNG}_i) \cdot f^{i-1}
\]

(3.23)

where,

\[
i = 1...3
\]

\[
f: \quad \text{Frequency of the grid} \quad [\text{nominal}]
\]

\[
\text{Load}_{\text{Tot}}: \quad \text{Total load on the grid} \quad [\text{W}]
\]

\[
\text{Part}_i: \quad \text{Percentage of the load belonging to group } i \quad [\text{p.u}]
\]

\[
\text{Disturbance}_i: \quad \text{Added or subtracted load units of group } i \quad [\text{units}]
\]

\[
\text{RNG}_i: \quad \text{Added or subtracted random value to group } i \quad [\text{units}]
\]

Before simulation start the total size of the load is set as well as how the load is divided between the three groups.
3.10.3 Main Circuit Breaker (MCB)
The MCB connects the plant generator to the grid. As connecting the generator running at
other rate (frequency) then that of the grid would cause generator damage the frequency
deviation is supervised. If the frequency deviation, at a given time, is outside of the
specified limit the connection to the grid is prevented otherwise the breaker closes if so
required.

The MCB is as well the main component of the generator protection which would open the
breaker when generator failure is imminent.

3.10.4 Generator
The turbine shaft in hydro power plants is normally connected directly to a generator shaft.
The torque transformed that way runs the generator producing electricity to the power grid.

The frequency generated by the generator is determined by the angular velocity of the
turbine the generator model was based on the Newtonian formulation including inertias of
both the generator and the turbine runner.

The fundamental equations used in the model for the generator are:

\begin{align}
\tag{3.24}
P &= \omega \cdot T \\
P: & \quad \text{Power} \quad [W] \\
\omega: & \quad \text{Angular velocity} \quad [\text{rad/s}] \\
T: & \quad \text{Torque} \quad [\text{Nm}] \\
\tag{3.25}
\omega_e &= \frac{n_p}{p_n} \\
\omega_e: & \quad \text{Electrical angular velocity} \quad [\text{rad/s}] \\
\omega_m: & \quad \text{Mechanical angular velocity} \quad [\text{rad/s}] \\
p_n: & \quad \text{Number of pole couples in the generator} \quad [\text{units}] \\
\tag{3.26}
T_{acc} &= J \frac{d\omega_m}{dt} = T_m - T_e \\
T_{acc}: & \quad \text{Accelerating torque} \quad [\text{Nm}] \\
J: & \quad \text{Representative moment of inertia} \quad [\text{kg m}^2] \\
\omega_m: & \quad \text{Mechanical angular velocity} \quad [\text{rad/s}] \\
T_m: & \quad \text{Mechanical torque} \quad [\text{Nm}] \\
T_e: & \quad \text{Electromagnetic torque} \quad [\text{Nm}] \\
\end{align}
By multiplying $\omega$ on both sides of equation (3.26) it can be written:

$$J \cdot \omega_m \frac{d\omega_m}{dt} = \omega_m \cdot (T_m - T_c)$$

(3.27)

now,

$$J \frac{d\omega_m}{dt} = \frac{P_m - P_e}{\omega_m}$$

$P_m$: Power generated by generator \[W\]

$P_e$: Power deficit/surplus from grid as $P_L - P_G$ (Load-Generated) \[W\]

From equation (3.27) it can be seen that the angular momentum, and then also the frequency, will rise when the generator connected produces more power than needed by the grid. And likewise, when the connected generator can’t produce enough power to balance the net, the frequency will fall.

Important to see here is that all generators that are connected to the grid have the same frequency. If the load of the grid is larger than what is produced by the power plants connected to it the frequency will drop. Since a large part of the electric components are designed for a specific grid frequency (in Sweden 50 Hz) there must be balance between produced and used power, to prevent the frequency from falling or rising. The grid load is constantly changing due to constant load variations and availability of the generating capabilities.

Efficiency losses and damping for the generator is also taken into account during simulation through:

$$P_m = P_t \cdot \eta - \omega^2 \cdot \frac{K_{dmp}}{\omega_N^2}$$

(3.28)

$P_t$: Power generated \[W\]

$\eta$: Efficiency of generator \[p.u\]

$K_{dmp}$: Damping factor of generator as p.u of nominal power \[p.u\]

There are two different stages of the generator modelling: unsynchronized / no-load turbine – generator, and generator in load operation, i.e. connected to the grid. In the second stage the whole dynamics of the grid will be represented by the turbine – generator requiring that representative moment of inertia will include all rotating units of the grid (reduced to the same angular speed).

In traditional generator representation the generator acceleration time is approximated using:

$$T_a = \frac{J \cdot n_N^2}{90 \cdot P_m}$$

$n_N$: Nominal speed \[rpm\]
3.10.5 Synchronizer
Before a power plant is connected to the power grid the frequency of the plant generator has to be synchronized with the frequency of the grid. This is done by the Synchronizer.

Figure 37 below shows a simplified block diagram of how the model works.

If the frequency deviation is above or below the values specified in the hysteresis the integral of the pulse will be added or subtracted from the generator frequency until the frequency deviation is acceptable. This means that the turbine governor is “fooled” to accept the actual grid frequency as the reference.

The Synchronizer model is activated only if the Main Circuit Breaker is open. Closed MCB disconnects the synchronizer reference from the turbine governor.

3.11 Electrical Subsystems
Electrical subsystems contain:
- Generator and Synchronizer
- Power Grid

Figure 38 below shows a block diagram of the internal connections of the Power Grid and Generator and Synchronizer models.
The main aspect of the grid system is ensuring balance between the grid load and grid production, at the given reference frequency. Note that the frequency deviation is a measure of the load-generation unbalance.

There are two frequencies identified; local generator frequency and grid frequency. There will only be a difference between these two when the simulated generator is disconnected from the grid, that is, when the MCB is open.

### 3.12 Control system

The controller in hydro power plant has two tasks. The first is control of the frequency and the second one is control of the power output. These two tasks can in some cases conflict with each other. As mentioned earlier if more power is produced than there is load demand the frequency will rise and vice versa. One conflict that can arise from this is when the power reference is set to high. The controller will then be told to increase the power output but this in turn will lead to an increase of the frequency which tells the controller to lower the power output. As frequency is the most important factor to consider, from the perspective of the entire power grid, the speed regulation factor is introduced. The speed regulation factor is a weight factor between the frequency and power error. This means that a low value of the speed regulation factor tells the controller to put more emphasis on the frequency error and vice versa. This leads to the following expression for the error signal to the controller:

\[
e = \Delta f + ep \cdot \Delta P
\]

- \(\Delta f\): Frequency deviation \([p.u]\)
- \(\Delta P\): Power deviation \([p.u]\)
- \(ep\): Speed regulation factor \([p.u]\)
In hydro power plants, the PID (Proportional Integral Derivative) controller is normally used for the turbine governor, and is accordingly implemented in this thesis.

3.12.1 Implementation of the control system in Dymola
The turbine governor is of standard PID structure with a feed forward part added to the control signal.
The error signal is a combination of frequency error and power error, the weight factor between those two is as mentioned before\(^{25}\) the so called speed regulation.

![Figure 39 Structure of implemented turbine governor (PID controller)](image)

From Figure 39 it can be seen that the measured frequency and power are used as the controller feedback. The obtained control signal is then used as the guide vane actuator position reference.

The I-part uses both conditional integration and a limiter to avoid integrator windup.

The primary function of the integrator is the remove any stationary errors. If the error of the process is large there is really no need to let the integrator be active. Letting it be active often leads to a fast growing I-part which can lead to overshoots in the process. With this in mind the integrator is only allowed to be active if the current position of the actuator is

\(^{25}\) See 3.12 equation (3.24)
close enough to the set point. If this isn’t the case that integrator will hold its current value independently of the error signal to the PID controller. Figure 40 shows how the I-part holds its current value when the error in the actuator is too large.

In addition to the conditional integration a variable limiter was also implemented. The limits of the integrator are decided by the feed forward signal. This prevents the output from the I-part to grow too large which often is enough to prevent integrator windup.

From Figure 39 it can also be seen that the derivative part of the controller only acts on the measured frequency signal and not the error. The reason for this is described above\textsuperscript{26}. When the PID is used to control a hydro power plant this precaution is really not necessary as the frequency set point will never change.

The power reference is sent to a ramp function to produce the feed forward signal. The ramp function will rise or fall at the same rate independently of the rate of the input reference\textsuperscript{27}. The ramp time, which is the time it takes for the ramp to go from zero to its max value, should correspond to the response time of the turbine and generator.

The outputs from the P, I and D-parts are added with the feed forward signal to produce the control signal which in turn is sent to the guide vane.

\textsuperscript{26} See section 2.5.2
\textsuperscript{27} See Figure 36 section 3.10.1
4 Test Bench of the Control Development

This section will contain some tests for verification of the developed library. The model used is a representation of a complete hydro power plant connected to a power grid.

4.1 Model description

The model simulated can be seen in Figure 41.

---

Some important parameters set, used in all simulations, are:

- Water head: 50m
- Penstock diameter: 8m
- Nominal power of the generator: 75MW
- Generator pole pairs: 32
- Inertia of the generator: 5000000 kgm²
- Power grid: 5000MW

When working under no load, the PID parameters were found by using the following rules of a thumb used traditionally at the commissioning of real hydro plants:
\[
K_p = 0.4 \frac{T_a}{T_w}
\]

\[
\frac{1}{T_i} = \frac{0.2 \cdot K_p}{T_w}
\]

\[
T_d = 0.45 \cdot T_a
\]

See section 3.7.2 and 3.10.4 for explanation of \(T_w\) and \(T_a\).

### 4.2 Model validation

#### 4.2.1 No Load and Load Response

This test will illustrate a hydro power plant acting under no load and when connected to the power grid. The result can be seen in Figure 42. At simulation start the load of the grid is increased which causes the frequency to drop.

When there is no load present the governor will only have the frequency error as input signal which will have the effect that the frequency of the hydro plant generator is controlled to equal the nominal frequency. This behaviour can be seen during the first 110s of simulation. The reason for the generator frequency drop at the start of the simulation is that the guide vane is closed at simulation start.

After 110s a synchronization phase is initialized, aiming to match the generator frequency to the frequency of the power grid.

When 300s has passed and the frequency of the generator is synchronized to the grid frequency, the MCB is closed, the power reference is set to 50MW and new PID parameters are applied. As expected the power output increases until nominal frequency is reached.
Figure 42 Top plot shows the frequency of the generator and the power grid. The bottom plot shows the produced power and power reference.

Figure 43 shows that after 300s the pressure drops in the volume adjacent to the guide vane opening as a result of the acceleration of the water in the penstock\textsuperscript{28}.

\subsection*{4.2.2 Load Rejection}

The events when disconnecting the load as the hydro power plant is working under stationary conditions will be shown here. The simulation result can be seen in Figure 44.

After 500s of simulation the MCB is opened causing the frequency of the generator to rise. When this happens the guide vane is closed rapidly causing a large combination error\textsuperscript{29} which can be seen in the bottom plot.

\textsuperscript{28} As discussed in section 3.7.2
\textsuperscript{29} See section 3.9.2, Figure 34
Figure 44 Load rejection
5 Implementation and Tests of the New Control Development

5.1 Objectives

When tuning PID parameters or developing alternative control schemes, it is of big importance to know the nature of the power grid and the power plant simulated. As the plant is concerned there is pretty good knowledge of the plant physics. It is more complicated for the grid. If the power grid is large in relation to the power plant it can be assumed that the power plant will not affect the grid frequency noticeably and the PID parameters can be chosen to be somewhat more aggressive. But in the other case when the simulated power plant outputs a noticeable part of the whole grid power, precautions need to be taken. In this case overshoots and oscillations in the local plant will affect the grid frequency and the PID parameters should be chosen with stability in mind instead of speed. The problem gets more complicated as the grid changes continuously; it can be stiff when large or soft when small. Information about grid size and characteristics is theoretically available but mainly centrally in the grid dispatch centres. The problems addressed here concerns how to get this knowledge locally at the power plant.

Figure 45 shows a simulation of a grid of 10000 MW which drops some power plants after 2 seconds leaving the grid with a power deficit of 50 MW. Our power plant will connect to the grid after 150 seconds and has the power reference set to 50MW. The P, I and D parameters are set to 2.5, 3 and 0.5 respectively.

![Figure 45 Simulation of a grid of 10000 MW connected to a power plant producing 70 MW. Top plot shows the grid frequency and the bottom plot shows the power output from the simulated power plant.](image-url)
As can be seen from the figure the connected power plant will only cause an overshoot of about 0.0007 of the grid frequency when connected.

Using the same PID parameters a second simulation is performed with the difference that the grid is now of the size 1000 MW. The result can be seen in Figure 46.

The overshoot of the grid frequency, after connecting our power plant, is now 0.015 which is a drastic difference from the simulation of the grid of the size 10000MW.

![Figure 46 Simulation of a power grid of 800 MW connected to a power plant producing 70 MW. Top plot shows the grid frequency and the bottom plot shows the power output from the simulated power plant.](image)

By setting the PID parameters more cautious this overshoot can be reduced. Figure 47 shows the same simulation setup used in previous simulation but with the PID parameters set to 1.8, 2 and 10. As can be seen the overshoot is reduced in size yielding a better step response.
5.2 Gain scheduling approach
Knowing this relationship some different strategies were examined to automatic change the PID parameters depending on the nature of the grid.

The idea was that without previous knowledge of the grid the PID controller should be able to tune the parameters.

5.2.1 Grid identification through power – frequency correlation
The first approach tried was to calculate the correlation between the produced power and the frequency of the grid over a certain time. The idea was that when the power output from our studied power plant changes the frequency change of the grid would also change in the same direction and from this the correlation could be calculated. The correlation was thought to be high when the power output was high relative to the grid size and vice versa. Knowing this correlation some PID parameter sets could be found working well under a certain value of the correlation. Some problems were encountered using this strategy.

A big problem with systems like this is that it is hard to tell what is affecting what. If a studied power plant changes its power output and the frequency rises or falls a certain amount there is no way to tell if the studied power plant is the only cause for this. This would only be the case if all other power plants and the grid itself were static. There is always the possibility that the load has changed or that another power plant connected to the grid has changed its power output. The worst case scenario, which isn’t uncommon at
all, happens when the studied power plant increased its power output but at the same time the load increased. This results in a negative correlation if the load increase is bigger than the power increase which would be interpreted as power increase leads to lower grid frequency.

Furthermore problems arise from the fact that the power and frequency signals will always oscillate somewhat. We are interested to find out in which direction the frequency will move and how much it will rise or fall, but as the data collected from the signals are instantaneous it is possible that the data points used for the correlation are picked up when the frequency is oscillating in the opposite direction as the frequency is moving as a whole. In Figure 48 a portion of a frequency signal is seen. It can clearly be seen that the signal is rising during the observed time span. In the middle plot a line is added to symbolize the mean value of the signal. The last plot shows what can go wrong when using the above described correlation method. As can be seen from the plot the three data points sampled give the impression that the signal is decreasing which, if the power input is rising, will lead to negative correlation between the frequency and power.

Another problem occurs when the frequency of the grid is below or above its nominal value. This will not only trigger the studied power plant to react but all power plants connected to the grid which makes it impossible to know how much the studied power plant affected the frequency change.

5.2.2 Active grid identification using PRBS
To improve the results of the passive correlation study, a PRBS was added to the governor control signal. It was clearly an improvement but not good enough so another approach was tried.

A big problem, as explained above, is clearly that it is hard to tell what is affecting what. To minimize this problem the system can be studied over a longer duration of time and a mean
value calculated. This should minimize the affect of the situations when the correlation turns out to be negative.

In this second experiment a PRBS was added to the control signal. Each time the PRBS went active the integral of the power output change was measured, yielding the amount of extra energy put into the grid. Figure 49 shows this result of this setup in a simulation. From the top plot it can be seen that the PRBS sends its first pulse after 70 seconds. The middle plot shows how the power output increases as a result of this pulse and the bottom plot is the integral of the power increase, or in other words, the amount of energy inserted into the grid due to the PRBS. After the PRBS becomes inactive the integral will hold its value until a new signal occurs.

Figure 49 Top plot shows the control signal added with the PRBS signal. Middle plot shows how the power output increases when the PRBS becomes active. The bottom plot shows the integral of the power increase.

To find out how this extra added energy affect the grid the same procedure as the one described above is applied to integrate the grid frequency.

By sending a number of PRBSs and calculating the mean value of the quota between the integral of the frequency and power change some of the problems discussed above are eliminated or minimized.

It is still not possible to know if the frequency change of the grid is exclusively a result of the simulated power plant but as the mean value is used it is affordable to have some results that are not entirely true. The problem with negative correlation is also made very unlikely as now the total surface is studied instead of just a few samples of the signal.
The mean value is during simulation continuously calculated by using the following equation:

\[
\text{Mean} = \frac{1}{n} \sum_{k=1}^{n} \left( \frac{\int_{\text{PRBS}_{\text{Start}}^k}^{\text{PRBS}_{\text{End}}^k} (f - f_{\text{PRBS}_{\text{Start}}}^k) \, dt}{\int_{\text{PRBS}_{\text{Start}}^k}^{\text{PRBS}_{\text{End}}^k} (P - P_{\text{PRBS}_{\text{Start}}}^k) \, dt} \right)
\]

\(\text{(5.1)}\)

where,

- \(n\): Total number of PRBS pulses applied to control signal
- \(f\): Frequency of the grid \([\text{Hz}]\)
- \(P\): Power produced \([\text{W}]\)
- \(\text{PRBS}_{\text{Start}}^k\): Start time of pseudo random binary signal \(k\) \([\text{s}]\)
- \(\text{PRBS}_{\text{End}}^k\): End time of pseudo random binary signal \(k\) \([\text{s}]\)
- \(P_{\text{PRBS}_{\text{Start}}}^k\): Power at the start of pseudo random binary signal \(k\) \([\text{W}]\)
- \(f_{\text{PRBS}_{\text{Start}}}^k\): Frequency at the start of pseudo random binary signal \(k\) \([\text{Hz}]\)

The expected result is that the larger the grid is compared with the studied hydro plant the smaller the mean value will be. Some simulations are shown here to verify this theory.

In the first simulation a 20000 MW grid is connected to the studied hydro power plant. The PRBS becomes active after 70 seconds and the mean value according to equation \((5.1)\) starts to be calculated. Each PRBS pulse is 10 seconds long. There is approximately a 65 MW deficit on the grid. The result can be seen in Figure 50.
Figure 50 Test of the identification method with a 20000MW grid. Top plot shows resulting mean value, middle plot shows the grid frequency and the bottom plot shows the power output from the studied hydro plant.

As can be seen from the top plot the mean value ends up around 0.35. After 3 pulses the mean value is already stabilized as the following PRBSs doesn’t change the mean value noticeably.

A second simulation is performed with a smaller power grid to see if the theory that the mean value gets higher holds.

The same simulation setup as above is used with the exception that the size of the power grid is set 1000 MW. The result can be seen in Figure 51.

Figure 51 Test of the identification method with a 1000MW grid. Top plot shows resulting mean value, middle plot shows the grid frequency and the bottom plot shows the power output from the studied hydro plant.

In this simulation the mean value ended up around 1.8 which is a drastic difference from the previous result of 0.35.
The result from this can be used with gain scheduling by finding “good” PID parameters for different grid setups and then calculate the mean value corresponding to the grid. This is done for a couple of grid sizes and then first order linear interpolation is to find the relationship between the parameters and power grid size. To change the parameters during simulation the grid is identified by calculating the mean value and the PID parameters are then changed accordingly.

5.3 Results
Two simulations are performed using the setup seen in Figure 41

In the first simulation grid identification isn’t used and the initial PID parameters are poorly set. After 800s there is a load disturbance in the grid causing the frequency to drop. When this happen the power reference is changed to make up for the load increase. With the current PID parameters there is an overshoot in both the frequency and power step response. See Figure 52.

![Figure 52 Simulation of a 3000MW grid and connected hydro power plant](image)

The second simulation is similar to the first during 550s. After this the PRBS is activated and the grid identification commences. After 700s the identification is completed and new PID parameters are set just before the load disturbance of the grid. The result can be seen in Figure 53.
The identification phase identified the grid as small relative to the power plant and PID parameters were accordingly set cautious. In Figure 53 the improvement can be seen through the fact that the overshoot during the step response, of both the frequency and power, is eliminated.

![Simulation showing improvements when using gain scheduling and identification](image)

**5.4 Proposal for improvement**

During the identification through PRBS the only part to be identified is the actual status of the grid. Identification described in this report was carried out in open loop configuration, or so called manual mode of turbine governor operation. As open loop can be potentially dangerous during unexpected transients in the grid, it would be desirable to carry out the identification procedure in close loop. It is theoretically possible as the structure and characteristic of the turbine governor is known, allowing construction of a filter removing the local control loop effect from the grid response.
6 Conclusions
Throughout this thesis a hydro plant library has been developed. The library is proven an effective development tool and well suited to test new control laws, verification of new designs and making commissioning more efficient.

Some additional tests of the library are required concerning implementation of complex water ways with a long penstock, surge tanks and split it into several sub penstocks each sub connected to a separate turbine. This has been done to some extent but never verified and tested.

The efficiency data of the turbine and guide vane models are coarsely approximated. The library is however designed to make it easy to replace these loss factors when data, more specific to a certain turbine or guide vane, becomes available.

The turbine governor implemented is a standard controller widely used and simulations show that the behaviour is identical to a “real” turbine governor.

The approach used when identifying the connected grid seems to be working fine but some issues still need to be resolved. Mainly aspects of grid identification in open and closed control loop, already discussed in proposals for further development.

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30 See Figure 11
7 Literature and References


[14] CD with Hydro Plant Library and detailed documentation (may be acquired from the authors)