Empirical Research on Value-at-Risk Methods of Chinese Stock Indexes

by

Xibei Chen

Master’s Programme in Finance

Supervisor: Birger Nilsson
Abstract

The Chinese stock market has been established for more than 20 years. Although it is not as mature as the highly developed western securities markets, it has a huge influence on the global economy. It is significant to study the risks of the Chinese stock market, especially the risk of stock indexes.

Affected by the economic globalization today, more and more financial derivatives and financial instruments appear which may lead to the increase of related risk so that the demand of research on the risk of the financial market is also getting higher and higher. Risk measurement is a key in risk management, and its measurement methods are constantly evolving. Value at Risk (VaR) method is one of the effective methods to measure the financial risk, which is widely used in domestic and foreign financial institutions. Compared with traditional models, it has much more accuracy and reasonability and is much easier to implement.

As the two main indexes in Chinese stock market, the Shanghai Composite stock index and the Shenzhen Component index are selected as the research objectives. And the loss series of the two indexes are tested through normality test, unit root test, autocorrelation test and ARCH effect test. The outcomes of these tests indicate these loss series are skewed and stationary with the effect of ARCH. Hereby, the GARCH-type models are suitable to be used to estimate VaR. The TGARCH model and the EGARCH model under the hypothesis of Student’s t-distribution and generalized error distribution are employed for the six test periods from 2011 to 2016. And it can be concluded with backtesting that all these four models (the VaR-TGARCH-t model, the VaR-TGARCH-GED model, the VaR-EGARCH-t model and the VaR-EGARCH-GED model) are appropriate for the two indexes despite the fact several models fail the Kupiec test for the period 2015-2016. For the Shenzhen Component index, the VaR-TGARCH-t model may fit it most because all numbers of violations for the six test periods fall in the confidence intervals.

Keywords: Value at Risk, TGARCH, EGARCH, Student’s t-distribution, GED, backtesting, Kupiec test
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1. Introduction

1.1 Background

After that China published the reform and opening-up policies and becomes a member of the World Trade Organization, China’s economic development has already had an inextricable link with the world economy. Since October 1, 2016, the Chinese yuan has officially been added to Special Drawing Rights (SDR). The Chinese yuan become the third largest reserve currency with the initial weight of 10.92%, surpassing the yen and the pound and following the dollar and the euro (International Monetary Fund, 2016). The event is an important milestone in the internationalization of the RMB, which is also the evidence that China’s economy will deepen its influence on the global economic environment. As the world's second largest economy, the transformation and upgrading of China's economy can be said to be one of largest economic restructuring in the world's economic history (Xinhua Finance Agency, 2016). Under the background of the current global economic downturn, financial market turmoil and emerging markets vulnerable to develop, its fiscal and financial risks are further increased during the transformation.

At the same time, the stock market of China has been developing at an amazing speed, thus attracting many investors to focus their attention on the stock market of China. Briefly, China’s stock market is still not too open and unstable to the monetary policy and the variation of the investment environment. However, even though there are some drawbacks for the Chinese stock market, it is still worthwhile to conduct relevant research because China plays a significant role and has a great influence on the global investment environment.

There are two stock exchanges in China, namely, the Shanghai Stock Exchange and the Shenzhen Stock Exchange, but both are not completely open to foreign investors. Large-cap stocks are generally listed on the Shanghai Stock Exchange, while the Shenzhen Stock Exchange is dominated by small-cap stocks. There have been 1281 and 1975 listed companies on the Shanghai Stock Exchange and the Shenzhen Stock Exchange respectively until the end of May 23, 2017 (Shanghai Stock Exchange, 2017; Shenzhen Stock Exchange 2017). The Shanghai Composite Index (the SSE Composite Index) and the Shenzhen Component Index (the SZSE Component Index) are two main stock indexes in the stock market.
market. The Shanghai Composite Index is compiled by the Shanghai Stock Exchange, with all the stocks listed on the Shanghai Stock Exchange as the basis for the calculation of the weighted average stock price index according to the issuance volume, which can reflect the changes in the stock prices of listed stocks on the Shanghai Stock Exchange. The Shenzhen Component Index is calculated based on the stock prices of the 500 representative listed companies from the Shenzhen Stock Exchange, which can be a comprehensive reflection of the trend of the market change on the Shenzhen Stock Exchange. The top 10 weighted stocks of the SSE Composite Index are PetroChina (PetroChina Company Limited), ICBC (INDUSTRIAL AND COMMERCIAL BANK OF CHINA LIMITED), Sinopec Corp. (China Petroleum and Chemical Corporation), China Life (CHINA LIFE INSURANCE COMPANY LIMITED), Bank of China (BANK OF CHINA LIMITED), CHINA SHENHUA (China Shenhua Energy Company Limited), PING AN OF CHINA (PING AN INSURANCE (GROUP) COMPANY OF CHINA, LTD.), CMB (China Merchants Bank Co.,Limited), BANKCOMM (BANK OF COMMUNICATIONS CO.,LTD.) and CHALCO (ALUMINUM CORPORATION OF CHINA LIMITED). It is obvious that the stocks in the financial and energy sectors have the highest weight of the SSE Composite Index. The changes in the energy and finance industries take the major responsibility of the trend and volatility of the index. With regard to the SZSE Component Index, the number of samples of the index was expanded from 40 to 500 in 2015. Through the reorganization, the weight of the financial real estate sector went down from 31% to 16% and the proportion of the consumer sector decreased from 28% to 17%. However, the information technology sector became the first largest component accounting for 18% (The Trade, 2015). It is clear that the proportion of new economic sectors has increased significantly but the traditional industries have fewer proportions.

The trend of the Chinese stock market is hard to predict. Since June 2015, a turbulence began due to the popping of the stock market bubble and ended in early February 2016(The Conversation,2016). Therefore, the research on the distribution, volatility and risk of China's stock market indexes has attracted more and more attention from the whole world.

The expansion of the financial instruments has led to the increasing volatility and the expanding risk of the financial market. Some serious financial crises like the Asian financial crisis, the US subprime mortgage crisis and the European debt crisis exacerbated the instability of the capital market. These extreme events had a direct impact on the stock
indexes, exchange rates, commodity prices and other market trends. Financial risks can not only seriously affect the operation of financial institutions and industrial and commercial enterprises, but also pose a serious threat to the stability of a country and the global economy.

There are many types of financial risks, such as market risk, liquidity risk, operational risk, credit risk and legal risk. The market risk is the most common and special risk which is the basis of other types of financial risks. Financial risk management includes risk identification, risk measurement, and risk management implementation, assessment and adjustment. The core of risk management is risk measurement because risk measurement directly determines the effectiveness of risk management. At present, the management of financial risk has become a new area of research and the Value-at-Risk(VaR) method and it has been widely used as a mainstream instrument of financial market risk measurement.

1.2 Research Purpose

At present, compared with the western countries, the system of risk management in Chinese stock market is immature. With the emergence of innovative financial instruments, investors can choose to further widen the investment channels, and financial control measures will be increasingly relaxed, which makes the investors face the corresponding expanding risks. The employment of Value-at-Risk methods for risk management has been very popular all over the world. There are many domestic financial institutions and scholars in China who have begun to carry out research on Value-at-Risk methods and apply them to Chinese security market. However, the different VaR models to measure risk may have different consequences, which may lead to the misunderstanding of the financial market for the investors in China or even in other countries. Therefore, it is necessary to find a Value-at-Risk method which is most suitable for the Chinese stock market. It is of great significance to accurately forecast and control the risk of Chinese stock market and promote its healthy development.

1.3 Outline of the Thesis

The full text has been divided into five chapters. This paper is organized as follows. The first section is the introduction which has expounded the background of Chinese stock market and risk management and research purpose of the paper.
In Section 2, the literature review about the development of the theory and application of Value-at-Risk and the history of the evolution of GARCH-type models are briefly described.

The third section is about the theory of Value-at-Risk and GARCH-type models and the methodology which is used in the empirical research. In this part, the definition and characteristics of VaR and the methods used in VaR estimation are introduced. The main methods to calculate VaR are historical simulation method, Monte Carlo simulation method and the variance-covariance method and the paper highlights the variance-covariance method, under three different loss distributions (normal distribution, Student’s t-distribution and generalized error distribution). As for the GARCH-type models, the ARCH model, the GARCH model, the TGARCH model and EGARCH model are elaborated. The principle of the Kupiec test method is also discussed. Then, the paper shows the data sources and the principle of choosing the data.

The fourth section is empirical analysis. Two closing prices of the SSE Composite Index and the SZSE Component Index are selected. The paper analyses the trend of the stock indexes at first. Then a series of tests like the normality test, the stationary test, the autocorrelation test and the ARCH effect test are conducted on the loss series for six evaluation windows from 2001 to 2016. It turns out that the loss distribution is leptokurtic, asymmetric and stationary and it has the ARCH effect. So the GARCH-type models are employed to the calculation of the volatility of the Value-at-Risk estimations. According to the different distributions of the disturbance terms, four kinds of models are established which are the VaR-TGARCH-t model, the VaR-TGARCH-GED model, the VaR-EGARCH-t model and the VaR-EGARCH-GED model to calculate VaR of two Chinese stock indexes. According to the Kupiec test, the number of violations of each model is in the range of acceptance except for the period 2015, so all the four models are appropriate to be adopted in the Chinese stock market.

The fifth section is the conclusion. It has pointed out the shortage of the paper and a possible future research direction.
2. Literature review

Value-at-Risk (VaR) is a measure of the maximum loss for a given portfolio, time period and confidence interval in risk management. It became a unique concept after the stock market crash of 1987. Since the 1990s, many scholars have begun to focus on the research of VaR method. J.P Morgan (1994) introduced Risk Metrics as a VaR-based risk measurement system, which formed a unified standard of risk measurement. Hendrics (1996) did empirical research on one thousand foreign exchange portfolios with parametric method, Historical Simulation, Monte Carlo simulation method. Hull and White (1998) proposed a new historical simulation method by calculating the ratio of the current volatility to historical volatility and then adjusting the historical data. Dowd and Kevin (1999) pointed out that the traditional methods of risk measurement with the assumption of the normal distribution are not appropriate and will produce many problems.

Besides focus on the traditional methods, some scholars combined VaR with the extreme value theory. Fisher and Tippett (1976) used the generalized extreme value distribution to fit the distribution of financial returns, laying the theoretical basis of the application of extreme value theory to VaR. Hendrics (1997) first introduced the extreme value theory into the modeling of market risk. He observed the extreme changes in the US stock market and analyzed the data of the New York Stock Exchange from 1885 to 1990. Kupiec (1995) found a technique that can estimate the tail values of the distribution of potential gains and losses. The study showed that the tail of the US stock market returns followed the Frechet distribution of the extreme value theory, and even during the Great Depression, the state of the tail did not change, indicating that in the long term the results of the calculation is also very stable.

Many scholars specifically described some aspects of VaR, but only Jorion (1997) started to systematically study VaR. He focused on risk measurement methods, the regulation and
development of VaR, the application of the VaR system to measure the risk of transaction and investment processes and the implementation of a safe risk management system. Jorion (1997) described and introduced the definition and calculation of VaR in detail. There are four methods to evaluate financial products by using VaR: historical simulation method, Delta-Normal method, Monte Carlo simulation method and stress test method. Pichler and Selitsch (1999) proposed a new method of calculating VaR from the perspective of mathematics. They used Taylor series to analyze the changes in financial returns and analyzed the mathematical characteristics of VaR, because it was hard for these traditional VaR methods to capture abnormal volatility in the financial markets. Li (1999) proposed a semi-parametric method with a fourth-order moment statistic without any assumption about the normal distribution of the loss sequence. It is only necessary to calculate the mean, variance, skewness and kurtosis of the yield sequence to calculate VaR at a certain confidence level. He employed Risk Metrics Model and semi-parametric model to analyze twelve major currency exchange rates from December 17, 1989, to February 8, 1999, and found that the semi-parametric model was more robust than the Risk Metrics model and this method overcame the problem of loss sequence distribution. Kaplanski and Kroll (2001) established a VaR-based equilibrium pricing model and proposed VaR-Beta (VB) to measure the risk of a single asset at equilibrium. They argued that VaR-Beta had a greater ability to interpret than traditional beta. Yiu (2004) discussed the optimal portfolio under VaR constraints and argued that the volume of optimal investment in risky assets was reduced by the restriction of VaR. Palaro et al. (2004) used Conditional Copula to Estimate Value at Risk. Castellacci et al. (2003) calculated the VaR of the nonlinear portfolio from the point of view of the accuracy and efficiency. The main conclusion is that the Delta-Gamma VaR normal model may not be as accurate as Delta VaR. The VaR method considering the non-linear value of the portfolio has a significant advantage over the Monte Carlo method. In 2002, Engle proposed DCC model (Dynamic Conditional Correlation model) in order to study the dynamic correlation. The DCC model is composed of a flexible one-dimensional GARCH model and a correlation coefficient model with simple parameters, which can be used to study the nonlinear time-dependent correlation between variables.

The history of the GARCH-type model is also not very long. In 1982, Engle creatively put forward the ARCH model, which was used to analyze the volatility of the inflation index of the United Kingdom. Then in 1986, Bollerslev improved the ARCH model and proposed a generalized ARCH (GARCH) model. Many studies have shown that the simple GARCH (1,1)
model, GARCH (1,2) model, GARCH (2,1) model in most cases can fully reflect the fluctuation of financial data. Neilsonl (1990) found that there was a leverage effect in the study of financial time data, that is, the positive and negative fluctuations in the price of financial assets had different effects on the subsequent time series data. Therefore, the EGARCH model was proposed. Zakoian et al. (1994) proposed the TGARCH model, that is, the auxiliary variables are set in the GARCH model to achieve the purpose of distinguishing positive and negative effects. In 1995, Engle and Kroner proposed the BEKK model in order to ensure the positive definite property of the conditional covariance. McNeil and Frey (2000) argued that the financial time was always heteroscedastic, so the GARCH model was used to remove the conditional heteroskedasticity of the series, and then the extreme value theory was applied to the model residual term to further improve the estimation effect.

Taking into the validity of GARCH-type models to estimate the fluctuation of the financial market, plenty of researchers started to combine VaR calculation with the GARCH-type models. In 2006, Cathy W.S. et al. studied the different VaR methods based on seven types of GARCH model. The results show that the VaR estimation based on GARCH model has better prediction effect. Kam and Philip (2006) applied the extreme value method to the EGARCH model to study the extreme risk of the stock market, which not only eliminated heteroskedasticity but also considered leverage effect. So and Yu (2006) concluded that Value-at-Risk can be estimated better with the GARCH model. Cuthbertson and Nitzche (2008) used multivariate GARCH models to calculate the dynamic hedging ratio of stock index futures in empirical analysis and point out that the exponential fluctuation can effectively verify the market changes and calculate system risk. In view of the persistence of volatility, the GARCH-type models can be used to describe the uncertainties among different variables. Through these applications, it can be found that the GARCH-type models and VaR methods have already been very important tools in financial markets. Li et al.(2016) used jump-diffusion models (GARCH-JUMP, ARJI, ARJI-TREND, and GARJI) to the impacts of the jumps, the asymmetric information, and the permanent component of volatility in the Chinese stock market and its futures market.

This paper calculates VaR based on the TGARCH model and the EGARCH model under Student’s t-distribution and generalized error distribution. Compared with the research on the risk before, several rolling windows are used to calculate the daily-VaR. The size of the rolling window is ten years and each year from 2011 to 2016 is one evaluation window. For
each evaluation window, the number of violations of each model can be calculated and tested if the actual frequency of violations deviates too much from the predicted frequency of violations. According to the empirical outcomes, the optimal model can be selected.
3. Methodology

In the paper, the fluctuation and risk of stock indexes are studied by combining the theoretical analysis and the empirical analysis. Some statistical software such as Eviews 9.5 and Excel are used in the data analysis. The loss distribution from 2001 to 2016 is analyzed by the ADF unit root test, normal distribution test, autocorrelation test, ARCH-LM test and other methods. VaR can be calculated based on the GARCH-type models under different types of distributions. Considering the leptokurtosis of the distribution of losses, Student’s t-distribution and generalized error distribution will be used instead of normal distribution. The Kupiec test is applied to test if the observed frequency of VaR violations is very different from the predicted ones. Moreover, the rolling window as the estimation window is adopted to estimate VaR which means when moving forward, the oldest loss observation should be dropped and the latest observation is added to the new sample.

3.1 Introduction to Value-at-Risk

Since the eighties and nineties of last century, the wind of the globalization quickly swept the world, which greatly promoted the international trades and the flow of capital between countries but also was followed by the financial risks. A key factor in risk measurement is the study of the volatility of financial assets, which is also important in option pricing and asset allocation. At the same time, Value-at-Risk as a measurement tool plays a significant role in the allocation of resources, information disclosure and performance evaluation.

3.1.1 Definition of Value-at-Risk (VaR)

Value-at-risk is the smallest loss \( l \) such that the probability of the loss \( l \) is less than a future portfolio loss \( L \), is not larger than \( 1 - \alpha \). The function is defined as:

\[
VaR_\alpha(L) = \min \{ l: \Pr (L > l) \leq 1 - \alpha \} \tag{1}
\]

Under the assumption of a continuous loss distribution, Value-at-Risk (VaR) refers to the maximum possible loss of a financial asset (or portfolio) for a specific period at a certain level of confidence. The following equation defines VaR mathematically:
For a continuous loss distribution, the two aforementioned definitions are equivalent.

3.1.2 Characteristics of VaR

1. VaR method can be applied effectively in normal fluctuation of the market and it can not be a good measure of risk when extreme conditions arise.

2. VaR is a comprehensive risk measure based on an integrated framework that takes into account all possible market risks. Under the conditions that the confident level and the period are fixed, the greater the value of VaR is, the larger the risk is.

3. VaR approach can be used to measure risk caused by different risk factors and different portfolios.

4. In the case of normal fluctuations of the financial market, when the time span is shorter, the distribution of loss is closer to the normal distribution and the value of VaR is more accurate and more effective.

5. The two basic parameters that affect the value of VaR are the holding period and the confidence level.

3.1.3 Methods of calculating VaR

The methods of calculating VaR have two categories: non-parametric and parametric approaches. The nonparametric approaches do not need to know the distribution of losses, while the parametric approaches require the distribution of known losses. Non-parametric methods mainly include historical simulation method and Monte Carlo simulation method and the parametric methods mainly refers to the variance-covariance methods. This paper focuses on the research of influences of the Variance-Covariance approach on Chinese stock market.
The Variance-Covariance approach is a parametric method. The calculation process of the parametric method can be regarded as an estimate of the variance-covariance matrix of the portfolio returns.

There are some differences in methods of calculating VaR when the distributions are different.

1. Normal distribution
The theory of normal distribution plays an important role in statistics and finance. The normal distribution is fully described by two parameters, the mean \( \mu \) and the standard deviation \( \sigma \).

The probability density function for a normal distribution with mean and volatility is:

\[
f(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{x-\mu}{\sigma} \right)^2 \right]
\]  

VaR can be calculated as follows:

\[
VaR_{\alpha}(L) = \mu + \sigma z_{\alpha}
\]

where \( z_{\alpha} \) denotes the \( \alpha \)-quantile for the standard normal distribution and \( \sigma \) is usually calculated by GARCH-type models.

2. Student’s t-distribution
It is well known that the kurtosis of the Student’s t-distribution is larger than the standard normal distribution, which can accommodate the heavy tail of the actual financial time series very well.

The probability density function for a Student’s t-distribution is:

\[
f(x) = \frac{\Gamma \left( \frac{\nu+1}{2} \right)}{\sigma \sqrt{\left( \nu-2 \right) \pi \nu} \Gamma \left( \frac{\nu}{2} \right)} \left[ 1 + \frac{1}{\nu-2} \left( \frac{x-\mu}{\sigma} \right)^2 \right]^{-(\nu+1)/2}
\]

\( \Gamma \) is the gamma function.
where the parameter \( \nu \) is often called the degrees of freedom which controls kurtosis. \( \nu \) can be estimated by the method of maximum likelihood based on the probability density function. However, for \( \nu > 4 \), the relation between sample kurtosis \( k \) and the parameter \( \nu \) can be shown as:

\[
k = 3 + \frac{6}{\nu - 4} \quad \text{or} \quad \nu = \frac{4k - 6}{k - 3}
\]

(6)

The formula for calculating VaR under an assumption of t-distributed losses is:

\[
VaR_\alpha (L) = \mu + \left( \frac{\nu - 2}{\nu} \right) \sigma t_{\alpha, \nu}
\]

(7)

where \( t_{\alpha, \nu} \) is the \( \alpha \)-quantile for the distribution.

3. Generalized Error Distribution (GED)

GED distribution is flexible and similar to Student’s t-distribution. It can reflect the characteristics of the loss tail of the financial market through the adjustment of the parameters.

Its probability density function is:

\[
f(x) = \frac{\nu \Gamma(\nu/3)^{1/2}}{2 \Gamma(1/\nu)^{3/2}} \exp \left( -|x|^{\psi} \left( \frac{\Gamma(\nu/3)}{\Gamma(1/\nu)} \right)^{1/2} \right)
\]

(8)

where \( \psi \) denotes a tail-fatness parameter. When \( \psi = 2 \), \( R \) represents a standard normal distribution. When \( \psi > 2 \), the distribution has thin tails and vice versa.

To arrive at the calculation of VaR under the generalized error distribution, the equation below can be used:

\[
VaR = \mu_t + g_\alpha \sigma_t
\]

(9)

Where \( g_\alpha \) represents the quantile at confidence level \( \alpha \) which can be obtained through Eviews.
The advantages and the Variance - Covariance method:
First, the variance - Covariance method is relatively easy to implement, the calculation steps are simple.
Second, this approach selects the appropriate distribution to calculate the variance.

The disadvantages of the Variance - Covariance method:
First, the accuracy of VaR depends on the choice of loss distribution, which may lead to model risk.
Second, for the sake of simplicity of the calculations, the distribution of assumptions may not be confirmed with the actual conditions.

3.2 GARCH-type models

With the known equations of the calculations of VaR, the volatilities of losses need to be forecasted first. Instead of using the sample variance of the estimated period directly, the GARCH-type models will be employed since the kind of models can account for time-varying conditional volatility.

In the actual analysis, there exist some problems such as heteroskedasticity, leptokurtosis and ‘volatility clustering’ in the financial time series. In order to solve these problems, in 1982 Engle proposed ARCH model (autoregressive conditional heteroskedasticity model) in the study of the British inflation rate. Its core idea is that the conditional variance of the residual term in the time series model depends on its previous value. In 1986, Bollerslev made a further improvement in the form of variance on the basis of the ARCH model and proposed the GARCH model. Since then, GARCH was developed into a variety of models, such as TGARCH model, EGARCH model, PARCH model, IGARCH model and so on.

3.2.1 ARCH model
ARCH models do not assume that the variance is constant. The conditional mean model is:

$$y_t = \beta_1 + \beta_2 x_{2t} + \cdots + \beta_k x_{kt} + u_t$$  (10)
A general definition of the conditional variance of $u_t$:

$$\sigma_t^2 = Var(u_t | \Omega_{t-1}) = E[(u_t - E(u_t))^2 | \Omega_{t-1}] \tag{11}$$

It is usually assumed that $E(u_t) = 0$, so

$$\sigma_t^2 = Var(u_t | \Omega_{t-1}) = E[(u_t)^2 | \Omega_{t-1}] \tag{12}$$

An ARCH(1) model is shown as below:

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 \tag{13}$$

All (G)ARCH models contain (at least) two equations – one for the mean (‘1st moment’) and one for the variance (‘2nd moment’).

3.2.2 GARCH model

Generalized ARCH (GARCH) models allow the conditional variance to be dependent upon previous own lags. A GARCH (1,1) model is shown as follows:

$$\sigma_t^2 = \omega + \alpha u_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{14}$$

where $\omega$ is a constant term, $u_{t-1}^2$ is the ARCH term which represents yesterday’s squared error from the conditional mean equation and $\sigma_{t-1}^2$ is the GARCH term which represents yesterday’s forecast variance. Moreover, $(\alpha + \beta) < 1$ is required for volatility to mean-revert.

A GARCH(p,q) model is:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i u_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \tag{15}$$

The advantages of GARCH model compared with ARCH model:
First, it takes into account all possible ‘ARCH effects’, while at the same time more parsimonious.
Second, it is less likely to violate non-negativity constraints (since usually there are fewer estimated parameters).

3.2.3 TGARCH model

GARCH models cannot account for ‘leverage effects’. In order to deal with the leverage effect of volatility, Zakoian (1994) proposed the Threshold GARCH (TGARCH) with an additional term to account for possible asymmetries. The principle of the model is that the effect of shocks greater than the threshold is different from the effect of shocks below the threshold.

The variance equation of TGARCH is:

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta \sigma_{t-1}^2 + \gamma u_{t-1} I_{t-1}$$ (16)

where $I_{t-1} = 1$ if $u_{t-1} < 0$; $I_{t-1} = 0$, otherwise.

For the leverage effect, it will be seen that $\gamma > 0$. The impact of negative information is greater than the impact of positive information.

3.2.4 EGARCH model

Nelson (1991) proposed the exponential GARCH model (EGARCH) which can indicate asymmetries of volatility.

The variance equation is:

$$\ln(\sigma_t^2) = \omega + \beta \ln(\sigma_{t-1}^2) + \gamma \frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \alpha \left[ \frac{|u_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right]$$ (17)

where $\ln(\sigma_{t-1}^2)$ is a regular GARCH term, $\gamma \frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}}$ is last period’s shock and $\frac{|u_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}}$ replaces regular ARCH term accounting for the absolute value of volatility shock in the last period.
There is thus no need to artificially impose non-negativity constraints on the model parameters. If the relationship between volatility and returns is negative, \( \gamma \), will be negative. According to Nelson assumption, the errors follow a generalized error distribution.

It is known that the rational use of the GARCH-type models can effectively improve the accuracy of the estimated VaR value than the traditional methods.

### 3.3 Backtesting VaR

In order to ensure the validity of the models, backtesting will be used. The Kupiec test is the standard frequency test for VaR. The test compares the actual number of VaR violations with the expected number of VaR violations. A VaR violation occurs when an actual loss is larger than the Value-at-Risk estimate for a given day. If the actual number of violations is too far away from the predicted frequency of violations, the model should be rejected. Non-violation is marked with zero (0) and a violation with one (1). The number of violations \( X \) is equal to the sum of all ones. The Kupiec test is, therefore, a binomial test.

Denoting the number of observations in the evaluation period by \( N \) and the expected relative frequency of violations by \( p = 1 - \alpha \), the probability of observing \( x = 0, 1, 2, \ldots, N \) violations is:

\[
\Pr(X = x) = \frac{N!}{x!(N-x)!} p^x (1 - p)^{N-x} \tag{18}
\]

where \( X \) is a binomially distributed stochastic variable which denotes the number of violations.

The predicted number of violation is \((1 - \alpha) \times N\). Under the condition that \( x \geq (1 - \alpha) \times N \), if the probability \( \Pr(X \geq x) \) calculated is less than the significance level, the underlying VaR-model is rejected, otherwise not rejected. Under the condition that \( x \leq (1 - \alpha) \times N \), if the probability \( \Pr(X \leq x) \) calculated is less than the significance level, the underlying VaR-model is rejected, otherwise not rejected. These two conditions are in a one-sided test.
The paper constructs a confidence interval for the observed frequency of violations which is a two-sided test. If the actual number of violation falls outside the confidence interval, the underlying VaR-model is rejected. The lower bound $x_{low}$ and the upper bound $x_{high}$ for the number of violations can be calculated with the cumulative binomial probability.

### 3.4 Data sources

The choice of data is critical and the main factors are the frequency and the period. As for the frequency of the data, this paper chooses the daily data to model VaR because too low frequency will lead to inaccuracy and instability and too high-frequency data may cause some problems about noise. In this paper, a large enough sample size is selected for the calculation of VaR. The Shanghai Composite Index (SSE Composite Index) and the Shenzhen Component Index(SZSE Component Index) are used which can be obtained from Sina Finance.

In order to ensure the stability and accuracy of the model, in this paper daily closing prices which range from January 2, 2001, to December 30, 2016 are chosen to calculate the 1-day VaR. The underlying observations are returns on the index and these returns are converted to losses by assuming a portfolio with a value of 100. The size of the evaluation window is one year and there are six evaluation windows from 2011 to 2016. For each test period, the size of the estimation window is ten years and these estimation windows are all rolling(moving) windows. This means for the second VaR estimate the very first loss observation need to be dropped but the latest is added in.
4. Analysis and Discussion

This paper studies the current situation of Chinese stock market by recognizing the volatility of the financial market and the characteristics of risks based on the GARCH-type models and the theory of VaR to achieve the purpose of optimizing the allocation of resources. As is known from the theory introduced before, the calculation of the VaR estimates based on the GARCH-type models is much more accurate than the traditional ones. In the empirical part, the characteristics of the indexes and the loss observations are tested first and the TGARCH model and the EGARCH model based on Student’s t-distribution and GED distribution respectively are applied to calculate VaR.

4.1 Characteristics of two indexes

Based on the closing prices from 2010 to 2016, Eviews was used to make the trends of the prices of the Shanghai Composite Index and the Shenzhen Component Index.

(a) Trend of the SSE Composite Index

(b) Trend of the SZSE Component Index

*Figure 4.1 The trends of the SSE Composite Index and the SZSE Component Index*

It can be seen from Figure 4.1 that each index has obvious fluctuations and especially each index had a sharp decline in 2008 due to the global financial crisis and fell in 2015 because of some complicated reasons like a big bubble in several concept stocks.
4.2 Characteristics of Loss

(a) Loss of the SSE Composite Index

(b) Loss of the SZSE Component Index

Figure 4.2 Loss of the indexes

From Figure 4.2, it can be seen that the volatility was rough in 2008 and 2015 and if volatility is lower (higher) than average the current holding period, then volatility is likely to be lower (higher) than average the next holding period as well. This indicates the well-known phenomena of volatility clustering and heteroskedasticity.

4.2.1 Testing for non-normality

Table 4.1 Descriptive Statistics

<table>
<thead>
<tr>
<th></th>
<th>the SSE Composite Index</th>
<th>the SZSE Component Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-0.024011</td>
<td>-0.036617</td>
</tr>
<tr>
<td>Maximum</td>
<td>8.840626</td>
<td>-0.040705</td>
</tr>
<tr>
<td>Minimum</td>
<td>-9.857043</td>
<td>9.289852</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>1.647229</td>
<td>1.839634</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.213448</td>
<td>0.210499</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>7.317999</td>
<td>6.148646</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>3040.618</td>
<td>1629.73</td>
</tr>
<tr>
<td>p-value</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>
As can be seen from Table 4.1, the skewness is not 0 and the kurtosis is larger than 3 for the two distributions, which reflects the asymmetry and leptokurtosis of the distributions. Moreover, the p-value is smaller than 0.05 at the significant level 5% which indicates that the Jarque-Bera statistic is significant and the null hypothesis of normality should be rejected. So it is better to use Student’s t-distribution and generalized error distribution instead of normal distribution.

4.2.2 Testing for unit roots

It is very important to determine if a series is stationary or not, because the stationarity or otherwise of a series can have a strong effect on its behavior and properties. The augmented Dickey-Fuller (ADF) test is conducted in the paper., the null hypothesis for the Augmented Dickey-Fuller test is defined as the presence of a unit root, and the alternative is stationarity.

Table 4.2 Results of the ADF test

<table>
<thead>
<tr>
<th>Augmented Dickey-Fuller test statistic</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shanghai</td>
<td>-60.95483</td>
<td>0.0001</td>
</tr>
<tr>
<td>Shenzhen</td>
<td>-59.36741</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

Clearly, it can be from Table 4.2 that the p-values of the test statistics for the two indexes are smaller than the critical values, so the null hypothesis of a unit root should be rejected that is the two series are both stationary.

4.2.3 Testing for autocorrelation

It is necessary to detect if there exists autocorrelation in the residuals. The Durbin-Watson (DW) test may be the most common test for autocorrelation. It has an assumption about the relationship between a residual and the immediately preceding one. The test equation is:

\[ u_t = \rho u_{t-1} + v_t \]  \hspace{1cm} (19)

where \( v_t \sim N(0, \sigma_v^2) \).
The null hypothesis of the DW test is $\rho = 0$, i.e., there is no evidence of a relationship between the errors at time $t - 1$ and $t$.

The test statistic can be calculated as:

$$DW = \frac{\sum_{t=2}^{T} (u_{t} - u_{t-1})^2}{\sum_{t=2}^{T} u_{t}^2}$$  \hspace{1cm} (20)

**Table 4.3 Consequences of the DW test**

<table>
<thead>
<tr>
<th>Index</th>
<th>Durbin-Watson statistic</th>
<th>Log likelihood</th>
<th>F-statistics</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>the SSE Composite Index</td>
<td>1.998963</td>
<td>-7431.163</td>
<td>3715.491</td>
<td>0.0000</td>
</tr>
<tr>
<td>the SZSE Component Index</td>
<td>1.997194</td>
<td>-7856.023</td>
<td>3524.489</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

If the value of the DW statistics is close to zero, then there is a positive autocorrelation. If the value is close to 4, then there is a negative autocorrelation. If the value is closer to 2, it means no autocorrelation. As can be seen from Table 4.3, the Durbin-Watson statistics are 1.998963 and 1.997194 respectively. So there is no autocorrelation in the residuals of the two loss series.

It is shown from Part B of the Appendix that the autocorrelation coefficients and the partial autocorrelation coefficients for the two indexes are very close to zero, so the ARMA(0,0) is selected as the conditional mean model.

4.2.4 Testing for ARCH effect

Before estimating a GARCH-type model, it is important to test for ARCH effects at first to make sure that this kind of models is appropriate for the dataset.

**Table 4.4 Results of the ARCH test**

<table>
<thead>
<tr>
<th>Index</th>
<th>Statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>the SSE Composite Index</td>
<td>F-statistics</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>Obs*R-squared</td>
<td>0.0000</td>
</tr>
<tr>
<td>the SZSE Component Index</td>
<td>F-statistics</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>Obs*R-squared</td>
<td>0.0000</td>
</tr>
</tbody>
</table>
It can be seen from Table 4.4 that both the F-statistic and the Obs*R-squared are very significant, suggesting the null hypothesis should be rejected i.e. there exists the presence of ARCH in the losses of the two indexes. In other words, there is a significant heteroskedasticity and volatility clustering in these time series. Therefore, the GARCH-type models should be established to fit the two characteristics.

4.3 Calculating VaR based on the GARCH-type models

The years from 2011 to 2016 are taken as six test periods with the size of one year for each evaluation window. For example, for the test period 2011, the observed losses are from the period 2001 to 2010 and for the period 2012, the estimation window is from 2002 to 2011.

4.3.1 Consequences of the empirical research on the GARCH-type models

According to the analyzed results above, this paper conducts the TGARCH model and the EGARCH model based on Student’s t-distribution and generalized error distribution to compute the volatility for the SSE Composite index and the SZSE Component index. Through backtesting to the VaR estimates based on the GARCH-type models, the best VaR models can be selected for Chinese stock market.

The GARCH-type models are estimated with the data from the estimation window. The unexpected losses (residuals) are obtained by subtracting the sample mean (mu) estimated as the average of the loss observations of the estimation window. The initial value for variance($\sigma^2_0$) is set as the sample variance.

The parameters of different GARCH-type models can be generated by Eviews. All equations of the conditional variances have been listed in Part C of the Appendix.

4.3.2 Consequences of VaR

The sample mean for each model is calculated with the rolling window technique i.e. the first
sample mean in the test period is the average of the loss observations of the estimation window, but when calculating the second sample mean, the first loss observation needs to be dropped and the first observation in the evaluation window should be added in. The calculation of the degrees of freedom also adopts this method. The respective 0.99-quantiles for the three different distributions can be calculated in Excel by T.INV() and Eviews.

The values of VaR can be obtained by the formulas as below:

\[
VaR_\alpha (L) = \mu + \sqrt{\frac{v-2}{v}} \sigma t_{\alpha,v} \quad \text{(Student's t-distribution)} \\
\]

and

\[
VaR = \mu \epsilon + g_\alpha \sigma \epsilon \quad \text{(GED distribution)}
\]

4.3.3 Backtesting

The Kupiec’s two-sided test is employed to do backtesting of VaR. Unlike the one-sided test, the probability of the number of violations does not need to be generated in the two-sided test. Instead, the upper and lower bounds which represent the maximum and minimum numbers of VaR violations allowed should be known. To get a more accurate confidence interval, a confident level of 95% is selected.

Table 4.5 Results of backtesting

(a)Results of backtesting of the SSE Composite Index

<table>
<thead>
<tr>
<th>Year</th>
<th>TGARCH-t</th>
<th>TGARCH-GED</th>
<th>EGARCH-t</th>
<th>EGARCH-GED</th>
<th>Number of observations</th>
<th>Lower</th>
<th>Upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>1</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>244</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>2012</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>243</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>2013</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>238</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>2014</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>245</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>2015</td>
<td>8</td>
<td>7</td>
<td>7</td>
<td>7</td>
<td>244</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>2016</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>6</td>
<td>244</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>Total</td>
<td>17</td>
<td>20</td>
<td>18</td>
<td>21</td>
<td>1458</td>
<td>8</td>
<td>22</td>
</tr>
</tbody>
</table>
(b) Results of backtesting of the SZSE Component Index

<table>
<thead>
<tr>
<th>Year</th>
<th>TGARCH-t</th>
<th>TGARCH-GED</th>
<th>EGARCH-t</th>
<th>EGARCH-GED</th>
<th>Number of observations</th>
<th>Lower</th>
<th>Upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>244</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>2012</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>243</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>2013</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>238</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>2014</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>245</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>2015</td>
<td>5</td>
<td>6</td>
<td>5</td>
<td>6</td>
<td>244</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>2016</td>
<td>6</td>
<td>7</td>
<td>7</td>
<td>7</td>
<td>244</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>Total</td>
<td>17</td>
<td>19</td>
<td>18</td>
<td>20</td>
<td>1458</td>
<td>8</td>
<td>22</td>
</tr>
</tbody>
</table>

As can be seen from Table 4.5, the actual numbers of violation of the period 2015-2016 for these two indexes are larger than the previous test period. The reason behind is the 2015-16 Chinese stock market turbulence as mentioned in the introduction part. Although the turmoil has gradually disappeared, Chinese stock market is still in the state of the downturn.

For the SSE Composite Index, except that the number of violations for each model is not in the confidence interval in 2015, all these four models are appropriate to estimate VaR for the other period. For the SZSE Component Index, only the TGARCH-t model can meet the requirement of the confidence interval in 2016, but for the other years, all these four models can be applied to the index.

All the four models fit the total numbers of the violations of the two indexes very well. As a result, the four underlying VaR models can be applied to Chinese stock market.
5. Conclusion

5.1 Practical Implications

This paper mainly studies the application of risk measurement model to Chinese stock market. Through the combination of theoretical analysis and empirical analysis, this paper attempts to find reliable models to analyze the fluctuation characteristics of financial time series of two major stock indexes in China. The risk of the stock indexes is estimated by the VaR-EGARCH model and the VaR-TGARCH model under Student’s t-distribution and generalized error distribution considering asymmetry, leverage effect, heteroskedasticity and volatility clustering. The validity of these models is examined by the Kupiec frequency test. Through the empirical study of Chinese stock indexes, the following conclusions can be reached:

1. The calculation of VaR based on the GARCH-type models can be applied to Chinese stock market. For the case of 99% confidence level, these four models effectively reflect the risk of the loss series except in 2015 and 2016. However, the VaR-TGARCH-t model passes all backtesting of the SZSE stock indexes for different test periods. It clear that the big bubble in Chinese stock market is the main source of risk. Two main component of the bubble is the overestimated housing prices and the proportion of the real estate sector in the SSE Composite Index is very large, which may be one main reason that leads to its probability of violation is higher than the SZSE Stock Index in 2015.

2. In risk management, the volatility model and the distribution hypothesis of the loss series are very sensible, and it can be beneficial to characterize the loss series and provide a good theoretical basis and a technical guarantee for the financial risk management. According to the results of the empirical research, the GARCH-type models are very appropriate for the phenomenon of volatility clustering. The coefficients of the parameters of leverage effect in the TGARCH model are always negative, which indicates under the condition of the same expected return, bad news will increase the future variance than good news.
5.2 Future Research

The risks in Chinese stock market are various and the two indexes are not enough to depict the complexity of Chinese stock market. Further research on Chinese stock market is necessary.

1. This paper only selected the Shanghai Composite Index and the Shenzhen Component Index as the research objects instead of applying the models to different industries or different kinds of funds or portfolios. To do further research on Chinese stock market in detail, it is sensible to compare various types of models conducted in different industries, especially in the real estate sector.

2. Extreme circumstances are not taken into consideration and market risk is the only source of risk to be considered in this paper. The stress test and the scenario analysis could be employed in the future research, and it is essential to study the effects of operational risk, credit risk, and liquidity risk.

3. In recent years, high-frequency data research has become a new trend in the financial field. Most of the financial time series such as stock prices, interest rates and transaction volume are analyzed by high-frequency data. For the further accuracy, the data of higher frequency should be analyzed. However, the technology of extract effective data is still not too mature.
Key references


The Conversation (2016). China’s stock market is in for a turbulent 2016
http://theconversation.com/chinas-stock-market-is-in-for-a-turbulent-2016-52731 [Accessed 1 June 2017]


Appendix

Part A: Statistical histogram of the loss distributions

(a) Statistical histogram of the loss distribution of the SSE Composite Index

(b) Statistical histogram of the loss distribution of the SZSE Component Index

Part B: Correlograms of the ACF and the PACF

(a) Correlogram of the ACF and the PACF of the SSE Composite Index
(b) Correlogram of the ACF and the PACF of the SZSE Component Index

Part C: The outcomes of the TGARCH and EGARCH models for 2011-2016

(a) The TGARCH models and EGARCH models of the SSE Composite index:

\[
\sigma_t^2 = 0.044057 + 0.110218 \cdot u_{t-1}^2 - 0.055248 \cdot u_{t-1}^2 (u_{t-1} < 0) + 0.907251 \cdot \sigma_{t-1}^2
\]

\[
\text{TGARCH-t}
\]

\[
\text{TGARCH-GED}\quad \sigma_t^2 = 0.043752 + 0.109561 \cdot u_{t-1}^2 - 0.051667 \cdot u_{t-1}^2 (u_{t-1} < 0) + 0.903983 \cdot \sigma_{t-1}^2
\]

\[
\text{EGARCH-t}\quad \ln(\sigma_t^2) = -0.1134468 + 0.175218 \cdot \frac{|u_{t-1}|}{\sqrt{\sigma_{t-1}^2}} + 0.035298 \cdot \frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}} + 0.983131 \cdot \ln(\sigma_{t-1}^2)
\]

\[
\text{EGARCH-GED}\quad \ln(\sigma_t^2) = -0.118726 + 0.180447 \cdot \frac{|u_{t-1}|}{\sqrt{\sigma_{t-1}^2}} + 0.034942 \cdot \frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}} + 0.982405 \cdot \ln(\sigma_{t-1}^2)
\]

\[
\text{TGARCH-t}\quad \sigma_t^2 = 0.037074 + 0.084247 \cdot u_{t-1}^2 - 0.036850 \cdot u_{t-1}^2 (u_{t-1} < 0) + 0.923173 \cdot \sigma_{t-1}^2
\]

\[
\text{TGARCH-GED}\quad \sigma_t^2 = 0.035569 + 0.083451 \cdot u_{t-1}^2 - 0.033606 \cdot u_{t-1}^2 (u_{t-1} < 0) + 0.921626 \cdot \sigma_{t-1}^2
\]

\[
\text{EGARCH-t}\quad \ln(\sigma_t^2) = -0.095553 + 0.144467 \cdot \frac{|u_{t-1}|}{\sqrt{\sigma_{t-1}^2}} + 0.020607 \cdot \frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}} + 0.987187 \cdot \ln(\sigma_{t-1}^2)
\]

\[
\text{EGARCH-GED}\quad \ln(\sigma_t^2) = -0.099091 + 0.147584 \cdot \frac{|u_{t-1}|}{\sqrt{\sigma_{t-1}^2}} + 0.020252 \cdot \frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}} + 0.987197 \cdot \ln(\sigma_{t-1}^2)
\]

\[
\text{TGARCH-t}\quad \sigma_t^2 = 0.024584 + 0.061142 \cdot u_{t-1}^2 - 0.013389 \cdot u_{t-1}^2 (u_{t-1} < 0) + 0.927945 \cdot \sigma_{t-1}^2
\]

\[
\text{TGARCH-GED}\quad \sigma_t^2 = 0.02548 + 0.063158 \cdot u_{t-1}^2 - 0.015097 \cdot u_{t-1}^2 (u_{t-1} < 0) + 0.9354086 \cdot \sigma_{t-1}^2
\]

\[
\text{EGARCH-t}\quad \ln(\sigma_t^2) = -0.087195 + 0.128875 \cdot \frac{|u_{t-1}|}{\sqrt{\sigma_{t-1}^2}} + 0.011241 \cdot \frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}} + 0.990113 \cdot \ln(\sigma_{t-1}^2)
\]

\[
\text{TGARCH-t}\quad \sigma_t^2 = 0.044057 + 0.110218 \cdot u_{t-1}^2 - 0.055248 \cdot u_{t-1}^2 (u_{t-1} < 0) + 0.907251 \cdot \sigma_{t-1}^2
\]
### (b) The TGARCH models and EGARCH models of the SZSE Component index

<table>
<thead>
<tr>
<th>Year</th>
<th>Model</th>
<th>Equation</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>TGARCH-t</td>
<td>( \sigma_t^2 = 0.0044671 + 0.099144 \cdot u_{t-1}^2 - 0.045106 \cdot u_{t-1}^2 (u_{t-1} &lt; 0) + 0.9015432 \cdot \sigma_{t-1}^2 )</td>
<td>( \sigma_t^2 = 0.048978 + 0.101591 \cdot u_{t-1}^2 - 0.044557 \cdot u_{t-1}^2 (u_{t-1} &lt; 0) + 0.909776 \cdot \sigma_{t-1}^2 )</td>
</tr>
<tr>
<td></td>
<td>TGARCH-GED</td>
<td>( \ln(\sigma_t^2) = -0.104055 + 0.159301 \cdot \frac{</td>
<td>u_{t-1}</td>
</tr>
<tr>
<td></td>
<td>EGARCH-t</td>
<td>( \ln(\sigma_t^2) = -0.104055 + 0.159301 \cdot \frac{</td>
<td>u_{t-1}</td>
</tr>
<tr>
<td>2012</td>
<td>TGARCH-t</td>
<td>( \sigma_t^2 = 0.046338 + 0.084183 \cdot u_{t-1}^2 - 0.034295 \cdot u_{t-1}^2 (u_{t-1} &lt; 0) + 0.920711 \cdot \sigma_{t-1}^2 )</td>
<td>( \sigma_t^2 = 0.043069 + 0.084647 \cdot u_{t-1}^2 - 0.037247 \cdot u_{t-1}^2 (u_{t-1} &lt; 0) + 0.924073 \cdot \sigma_{t-1}^2 )</td>
</tr>
<tr>
<td></td>
<td>TGARCH-GED</td>
<td>( \ln(\sigma_t^2) = -0.091381 + 0.138260 \cdot \frac{</td>
<td>u_{t-1}</td>
</tr>
<tr>
<td>Year</td>
<td>Model</td>
<td>Equation</td>
<td></td>
</tr>
<tr>
<td>------</td>
<td>----------------</td>
<td>--------------------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>2013</td>
<td>TARCH-GED</td>
<td>$\ln(\sigma^2_t) = -0.093190 + 0.11246 * \frac{</td>
<td>u_{t-1}</td>
</tr>
<tr>
<td></td>
<td>TARCH-t</td>
<td>$\sigma^2_t = 0.035795 + 0.066047 * u^2_{t-1} - 0.023107 * u^2_{t-1} (u_{t-1} &lt; 0) + 0.936412 * \sigma^2_{t-1}$</td>
<td></td>
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<tr>
<td></td>
<td>TARCH-GED</td>
<td>$\sigma^2_t = 0.038776 + 0.066863 * u^2_{t-1} - 0.023927 * u^2_{t-1} (u_{t-1} &lt; 0) + 0.934073 * \sigma^2_{t-1}$</td>
<td></td>
</tr>
<tr>
<td>2014</td>
<td>TARCH-t</td>
<td>$\sigma^2_t = 0.038971 + 0.060900 * u^2_{t-1} - 0.022703 * u^2_{t-1} (u_{t-1} &lt; 0) + 0.939956 * \sigma^2_{t-1}$</td>
<td></td>
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<td></td>
<td>TARCH-GED</td>
<td>$\sigma^2_t = 0.044372 + 0.065106 * u^2_{t-1} - 0.025294 * u^2_{t-1} (u_{t-1} &lt; 0) + 0.934771 * \sigma^2_{t-1}$</td>
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<tr>
<td></td>
<td>TARCH-t</td>
<td>$\ln(\sigma^2_t) = -0.072768 + 0.110431 * \frac{</td>
<td>u_{t-1}</td>
</tr>
<tr>
<td></td>
<td>TARCH-GED</td>
<td>$\ln(\sigma^2_t) = -0.076853 + 0.117613 * \frac{</td>
<td>u_{t-1}</td>
</tr>
<tr>
<td>2015</td>
<td>TARCH-t</td>
<td>$\sigma^2_t = 0.032381 + 0.056119 * u^2_{t-1} - 0.013625 * u^2_{t-1} (u_{t-1} &lt; 0) + 0.942498 * \sigma^2_{t-1}$</td>
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<td>TARCH-GED</td>
<td>$\sigma^2_t = 0.035835 + 0.059679 * u^2_{t-1} - 0.015642 * u^2_{t-1} (u_{t-1} &lt; 0) + 0.938312 * \sigma^2_{t-1}$</td>
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<tr>
<td></td>
<td>TARCH-t</td>
<td>$\ln(\sigma^2_t) = -0.074822 + 0.113374 * \frac{</td>
<td>u_{t-1}</td>
</tr>
<tr>
<td></td>
<td>TARCH-GED</td>
<td>$\ln(\sigma^2_t) = -0.078257 + 0.118698 * \frac{</td>
<td>u_{t-1}</td>
</tr>
<tr>
<td>2016</td>
<td>TARCH-t</td>
<td>$\sigma^2_t = 0.041256 + 0.070470 * u^2_{t-1} - 0.020458 * u^2_{t-1} (u_{t-1} &lt; 0) + 0.930088 * \sigma^2_{t-1}$</td>
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<tr>
<td></td>
<td>TARCH-GED</td>
<td>$\sigma^2_t = 0.039412 + 0.067468 * u^2_{t-1} - 0.016597 * u^2_{t-1} (u_{t-1} &lt; 0) + 0.930966 * \sigma^2_{t-1}$</td>
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<tr>
<td></td>
<td>TARCH-t</td>
<td>$\ln(\sigma^2_t) = -0.089524 + 0.137801 * \frac{</td>
<td>u_{t-1}</td>
</tr>
<tr>
<td></td>
<td>TARCH-GED</td>
<td>$\ln(\sigma^2_t) = -0.090055 + 0.137203 * \frac{</td>
<td>u_{t-1}</td>
</tr>
</tbody>
</table>