Relating gas turbine performance to combined cycle efficiency

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"The two fundamental laws of thermodynamics are, of course, insufficient to determine the course of events in a physical system. They tell us that certain things cannot happen, but they do not tell us what does happen."

- Alfred J. Lotka, (1922)
Abstract

**keywords**: combined cycle efficiency, second law efficiency, exergy, HRSG, irreversibility, heat engine, thermodynamics, T-q diagram.

The gas turbine has been around for over a century, providing power for a variety of applications. The efficiency, i.e. the amount of power produced per kilogram of fuel provided, has increased steadily over the years and is today greater than 44 percent in a state of the art gas turbine. For electricity production the efficiency can be increased further by combining the gas turbine with a steam turbine. The energy in the hot exhaust gases can be used to boil water into steam, which can then be used to drive a steam turbine producing additional electricity. In a combined cycle the efficiency can reach 63-64 percent with current technology. These power plants are expensive to operate primarily due to the fuel prices, which heavily drives the need for even higher efficiency. This is why companies operating older power plants often consider upgrading their components.

When an upgrade is considered there is often an information gap between the OEM (original equipment manufacturer) and the buyer, in this case the plant owner. If the plant is not delivered as a turnkey by a single OEM, parts and components are purchased from different OEMs specialized on the specific equipment, leading to a situation where the OEM have limited knowledge about the environment in which their equipment operates. Commonly the solution to this is to create models and apply extensive heat and work balance equations to calculate the impact on the plant following an upgrade of a component. This is a time consuming and complicated task which also makes it unnecessarily expensive since an expert has to be involved to perform the calculations. In this report a method of predicting the combined cycle efficiency change is presented. It is a compact formulation which has the potential to speed up the process considerably. This could provide a tool for the OEM to quickly be able to provide an answer to the customer, regarding the potential efficiency increase.
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Nomenclature

1P\textsubscript{NRH} Single pressure no reheat
2P\textsubscript{NRH} Two pressure no reheat
3P\textsubscript{NRH} Three pressure no reheat
\bar{T} Mean temperature
\dot{m} Mass flow
\eta_{\text{2nd}} Second law efficiency
\eta_{\text{BC}} Bottoming cycle efficiency
\eta_{\text{cc}} Combined cycle efficiency
\eta_{\text{C}} Carnot efficiency
\eta_{\text{HRSG}} HRSG efficiency
\eta_{\text{SC}} Steam cycle efficiency
\eta_{\text{th,rev}} Internally reversible efficiency
\eta_{\text{th}} Thermal efficiency
BC Bottoming Cycle
BC Bottoming cycle
c_p Specific heat at constant pressure
CCPP Combined cycle power plant
Cond Condenser
CV Control volume
CW Cooling water
E Exergy
$exh$  Exhaust
$f$  Fuel
$GT$  Gas turbine
$H$  Enthalpy
$HP$  High pressure
$HRSG$  Heat recovery steam generator
$I$  Irreversibility
$K$  Kelvin
$LHV$  Lower heating value
$LP$  Low pressure
$NOx$  Nitrogen Oxides
$OEM$  Original Equipment Manufacturer
$p$  Pressure
$Q$  Heat
$R$  Gas constant
$S$  Entropy
$SIT$  Siemens Industrial Turbomachinery
$ST$  Steam turbine
$stm$  Steam
$T−q$  Temperature heat transfer diagram
$TH$  Temperature High
$TL$  Temperature Low
$V$  Volume
$W$  Work
1. Introduction

Finspång in Östergötland is known for its long history of industrial development and production. For more than three centuries iron canons were produced here and delivered abroad and to the Swedish armed forces. When the canon production closed down in the early 20th century, the Ljungström brothers, Birger and Fredrik, saw the possibilities of the available factories and favorable location. They decided to locate their newly founded production company Svenska Turbinfabriks AB Ljungström (STAL) in Finspång. In the year 1913 they started building steam turbines in the empty workshops [1]. Their new radial steam turbine construction were both smaller and more efficient than any other turbine on the market at the time. Over 100 years later, a considerable number of power producing machines have been sold worldwide. Since the start, the company has changed many times and in 2003 Siemens took over as new owners and founded Siemens Industrial Turbomachinery AB (SIT). As of today the products being developed and sold in Finspång consist of a number of medium sized gas turbines for industrial and electrical power production.

1.1 Background

The use of a single gas turbine (GT) for electrical power production leaves a lot of potential energy unused, as a lot of heat is lost in the hot exhaust gases. To take care of this loss it is common to use a steam turbine (ST) driven by steam produced by the heat from the gas turbine. The combination of one or more gas turbines with one or more steam turbines is known as a combined cycle power plant (CCPP). The steam is produced in a heat recovery steam generator (HRSG), basically consisting of a number of heat exchangers with water on the cold side and exhaust gas on the hot side. Depending primarily on the exhaust mass flow, exhaust temperature and the optimization of the HRSG a specific amount of energy can be recovered from the hot exhaust gases. Since no extra fuel is burned, i.e. the heat input to the combined cycle is the same as for the simple cycle GT, the over all thermal efficiency $\eta_{cc}$ can be increased substantially.

In order to maximize the useful work produced per unit fuel supplied, it is desirable to use the highest possible heat addition temperature and the lowest possible heat rejection temperature. This is easily concluded from Carnot’s
Figure 1.1: Carnot efficiency as a function of $T_H$, $T_L$ is kept constant at 273 K.

Theorem. If the temperature at which heat is supplied is approaching infinity, and or the temperature at which heat is rejected approaches zero the Carnot efficiency approaches unity, as seen in figures 1.1 and 1.2. The heat rejection temperature is directly proportional to the Carnot efficiency, i.e. the incremental decrease of the temperature always results in the same increase in efficiency. This is not the case with the heat addition temperature, the gain in efficiency levels out at a certain temperature, and the further increase of the temperature will not result in a significant increase in efficiency.

In reality, where the machines operates, there are numerous limitations that dictate the possibilities in choosing these temperatures. On the cold side it is usually the plant location that determines the availability of heat sink i.e. a river or sea close to the power plant. It is not economically viable to lower the temperature readily available at the site, therefore there is not much that can be done about the heat rejection temperature. On the hot side the primary obstacles deterring an increase in temperature is limits relating to material characteristics. Under operation the gas turbine rotor is exposed to extreme conditions. The available materials needed to withstand the environment are limited. Most metal alloys melting points are well below the temperatures that a state of the art gas turbine first rotor blades are exposed to. With better materials the temperature can be increased for better cycle efficiency, however nitrogen oxide (NOx) formation is strongly depending on temperature which effectively limits the temperature due to emission regulations.

For a given location with a given temperature in the heat sink, the only way of reducing the environmental impact and cost is to increase the thermal efficiency of the plant. The gas turbine exhaust gases are always warmer than the ambient temperature. Theoretically it is then possible to drive a heat engine using the temperature difference between the hot exhaust gases and the cold heat sink. By utilizing the otherwise rejected heat and producing more useful work without increasing the fuel input the thermal efficiency of the plant is
Figure 1.2: Carnot efficiency as a function of $T_L$. $T_H$ is kept constant at 1523 K.

increased. The Rankine cycle is well suited to be combined with the Brayton cycle using the exhaust gases as a source for the heat needed to generate steam. CCPP can reach an overall thermal efficiency of above 63% in state of the art power plants today.

1.2 Objectives

The objective of this project is to develop a method that can be used in a first attempt to adequately and quickly determine the change in combined cycle efficiency of a power generating plant, when variation in key plant parameters occurs. The objectives is also to present an explanation of the underlying mechanisms at work in the processes using fundamental thermodynamics with focus on the application of the second law efficiency approach.

1.3 Approach

The approach taken in this project is to relate the thermal efficiency to the Carnot efficiency i.e. using the second law efficiency. This makes it possible to produce a compact formulation that can be used to predict the change in combined cycle efficiency, based on knowledge about the combined cycle efficiency prior to the change in gas turbine performance. The second law approach evaluates the different components performance compared to their theoretical maximum, thus revealing the true potential of the component. [3] For a specific power plant incorporating a combined cycle the total efficiency is commonly well known to the plant owner. However it is not straight forward to predict how the efficiency will vary if the GT performance is changed, e.g. if an upgrade
of the GT is considered. To determine this there is usually a need for extensive and time consuming heat balance calculations. In this project a simpler and more compact way to reach the same goal is presented and evaluated. The second law efficiency greatly depends on the temperatures in the cycle. As the exhaust temperature increases it will lead to an increase in both Carnot efficiency and thermal efficiency, and the fact that the second law efficiency is the ratio between these two efficiencies advocates the thesis that it is possible to assume a constant second law efficiency with satisfying accuracy. The approach is evaluated when a change in exhaust temperature and or mass flow occurs. Four correction factors are introduced into the equation to reduce the errors in the predictions. The combined cycle efficiency is dependent on a large number of parameters. Depending on the specific situation the knowledge of the power plant may differ, e.g. when an upgrade of GT performance is proposed by an original equipment manufacturer (OEM) to a site owner. The OEM may have limited information about the site data, but would like to be able to predict the overall efficiency increase said GT upgrade would result in. It is in such a situation the approach proposed in this report could be considered.

1.4 Previous work and literature study

Previous work closely related to the subject have been carried out by Gülen. In reference [3] a second law approach is used to evaluate the efficiency of a Rankine bottoming cycle of a combined power plant. In the exergy analysis a control volume is placed around the bottoming cycle (BC) and the exergy supplied to the system is simply the exergy in the exhaust gases from the topping cycle. The exergy related to heat transfer out of the RBC is split between the condenser, HRSG and the fuel preheater. The exergy destruction inside the control volume is quantified in the HRSG, in the steam turbines and in the condenser. There are also minor parts of exergy destruction related to electrical and mechanical losses in the generator, shafts and bearings. Gülen derived suitable expressions for the component irreversibilities in the bottoming cycle, these equations are applied herein to clarify how different cycle parameters will affect the combined cycle efficiency.

El Masri [4] concluded that the second law based approach to analyze combined cycle power production can be useful for pinpointing and quantifying losses when optimizing a new plant.

1.5 Tools

1.5.1 IPSEPro

The program that is used in this report is IPSEPro-PSE from SimTech Simulation Technology GmbH. It is a heat and mass balance calculating and simulating program that quickly solves equations using matrix categorization and
then applying the Newton-Raphson method root-finding algorithm. The Process Simulation Environment (PSE) is used together with the model developing kit IPSEPro-MDK to build a representation model of a two pressure no reheat (2PNRH) combined cycle. This is done by starting from standard models of each component and then recode the components so that every component have the required variables e.g. irreversibilities or ambient temperature. This model is then used to test the sensitivity of different components when varying important input data. The results are then plotted together.

1.5.2 Newton-Raphson method

The working principle of the Newton-Raphson method is calculating the root to a function on the form $f(x) = 0$ by providing an initial estimation $x_n$, and then comparing the function value to a tangent line to find a better estimation $x_{n+1}$. The process is repeated until the difference between the estimations is below a given tolerance i.e. the system has converged.

$$g(x) = f'(x_n)(x - x_n) + f(x_n) \tag{1.1}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{1.2}$$

The estimation $x_{n+1}$, found by intersecting the function $g(x)$ and the x-axis, is generally closer to the root than $x_n$. An example of this can be seen in figure 1.3.

![Figure 1.3: Representation of Newton-Raphson method](image)

Figure 1.3: Representation of Newton-Raphson method
2. **Theory**

2.1 **Heat engine**

The thermodynamic definition of a heat engine is a device that can convert thermal energy into mechanical energy which can be used to produce mechanical work. The heat engine has been around since the antiques [5], but the concept was first used on a wider front as a power source driving useful mechanical loads in the era of the industrial revolution. The heat engine is often confused with the theoretical cycle it operates according to, however the heat engine is a physical machine and a thermodynamic cycle is the theory behind it.

The process of an arbitrary heat engine is driven by taking thermal energy from a heat source and use it to heat up a working substance. The working substance can then be lowered down to a lower temperature level while mechanical energy is extracted in the process. The state of the working substance is then changed by transferring heat to a heat sink. It is the difference in state of the working substance that is exploited to create a net work output from the engine.

During the process there are losses of energy to the surroundings because of heat transfer through the system walls and also internally in the engine because of friction. Naturally these irreversibilities lowers the efficiency of the engine.

There are a number of successful applications of different heat engines. The Otto engine and the Diesel engine are just two of the most well-known machines often used for mechanical drive of cars and ships. They both differ considerably from the most common power generating engines, but all heat engines share the aforementioned characteristics i.e. they receive heat from a high temperature source, they convert part of the heat to work and they reject the waste heat to a sink and they are all cycle operated.

2.2 **Carnot cycle**

The theoretical upper limit of a heat engine was first defined by the French engineer and physicist Sadi Carnot in 1824 [6]. Carnot stated that any heat engine operating between the same two temperatures can never achieve an efficiency higher than the Carnot efficiency, (eq. 2.1) i.e. the Carnot efficiency dictates the limit of possible work produced by providing the engine with a certain amount of heat. Carnot’s principle became the foundation on which the second law
of thermodynamics were built upon. It can be said that the Carnot engine is a physically equivalent of the second law. The Carnot Cycle is a theoretical version of an internally reversible heat engine.

\[
\eta_{\text{Carnot}} = \eta_{\text{th,rev}} = 1 - \frac{T_L}{T_H}
\]  

(2.1)

The closed cycle consists of four different steps. When these four steps have all occurred the working medium will have returned to its original state, ergo the cycle is complete.

2.2.1 Reversible isothermal expansion

The working medium is first expanded at a constant temperature \( T_H \) thereby doing work on the surroundings. When a gas is expanding the temperature drops an increment \( dT \), but since the process is very slow (quasi-static) the heat provided by the energy source will have enough time to heat up the working medium equally \( dT \) degrees, so that the temperature is kept constant. The total amount of heat transfered to the gas during this step is \( Q_H \), i.e. the energy input from the heat source.

2.2.2 Reversible adiabatic expansion

At this point the energy input is removed, and is not in direct contact with the working medium any more. The process is adiabatic i.e. no heat is transfered out of the system. The working medium continues to do work on the surrounding by expanding, thereby slowly lowering the temperature to \( T_L \).

2.2.3 Reversible isothermal compression

At the third step the working medium is connected to a heat sink with the temperature \( T_L \). An external force is applied to the system, compressing the gas so that the volume decreases and the pressure increases. The heat generated in the compression step is transfered to the heat sink so that the temperature is kept constant at \( T_L \).

2.2.4 Reversible adiabatic compression

In the final step the working medium is no longer connected to the heat sink. The temperature increase from the continuing compression will then adiabatically rise the temperature back to \( T_H \). The working medium have now returned to the first state and the cycle can be repeated.
2.3 First and second laws of thermodynamics

2.3.1 First law

Originally the first law of thermodynamics was empirically developed over many years of practice. It was not officially stated until the year 1850 when German physicist and mathematician Rudolf Julius Emanuel Clausius restated Sadi Carnot’s principle, the Carnot cycle [7]. His statement is known as the “thermodynamic approach” to the nature of the relationship between heat and work.

"In a thermodynamic process involving a closed system, the increment in the internal energy is equal to the difference between the heat accumulated by the system and the work done by it."

This can be expressed as the difference in internal energy of a closed system which is always equal to the heat provided to the system minus the work produced by the same system.

$$\Delta U = Q - W \quad (2.2)$$

I can also be interpreted in the way that the total energy of an isolated system is constant. [6] It is possible to change its form by transforming the energy from one form into another, but it is not possible to create or destroy energy.

2.3.2 Second law

"There have been nearly as many formulations of the second law as there have been discussions of it."

Philosopher and physicist P.W. Bridgman, (1941)

The first law of thermodynamics is dealing with the conservation of energy for a system. The second law puts further limitations on the system by introducing a new state variable, entropy, and stating that the entropy of a closed system is over time always either the same or increasing. The only case when the entropy is constant is for a system undergoing a reversible process. However, such a process is physically impossible to achieve since irreversibilities are always present in the system. To define the absolute entropy for a system in a certain state the third law of thermodynamics is needed. However the absolute value is not as interesting as the change in entropy. The change in entropy for a system going from one state to another is defined in eq. 2.3.

$$\Delta S_{sys} = S_2 - S_1 = \int_{T_1}^{T_2} \frac{\delta Q}{T} + S_{gen} \quad (2.3)$$

The $S_{gen}$ term is the entropy generated because of system irreversibilities. This term is zero in the reversible case. Key is that the entropy increase is always related to a reference temperature.
\[
\oint \frac{\delta Q}{T} \leq 0 \quad (2.4)
\]

The relation famously known as Clausius inequality was first developed by Clausius in the 1850s. Clausius proposed that "A transformation whose only final result is to transfer heat from a body at a given temperature to a body at a higher temperature is impossible."

### 2.4 Brayton cycle

The Brayton cycle has its name from American mechanical engineer George Brayton \[8\] and is the theoretical model that describes a constant pressure heat engine such as a gas turbine. A real gas turbine must be run as an open cycle because of the exhaust gas composition, however the model is a closed cycle process reusing the exhaust gases in the compressor intake. This enables closed cycle calculations and analysis considering the engine as a closed system.

![Figure 2.1: Representation of the Brayton cycle working principle \[9\]](image)

Since there are many applications using the Brayton cycle a lot of research and development have been carried out, resulting in a number of different machines using different layout to make the cycle better for a specific task. Interrupting the expansion in the turbine and reheating the working medium to a higher temperature will increase the work output from the cycle. This is because of the fact that the work required to compress or expand the working medium is proportional to the specific volume of the working medium. Since the working medium can be considered an ideal gas, and the reheating is carried out under constant pressure, following the equation of state (eq. 2.5) it is easily shown...
that an increase in temperature will increase the specific volume of the working medium.

\[ pv = RT \] (2.5)

Reheating is also favorable in the sense that the maximum temperature of the cycle can be kept at a level at which the nitrogen oxide (NOx) emissions are complying with the regulations. While at the same time extracting more work from the cycle.

Intercooling of the working medium between the compression stages will have similar effect on the cycle performance, following the same logic. However, intercooling and reheating will not improve the thermal efficiency of the cycle since intercooling will reduce the temperature at which heat is added and reheating will increase the temperature at which heat is rejected. Following equation (2.1) one can see that the Carnot efficiency will decrease. For single cycle use reheat and intercooling is always used together with regeneration, i.e. the exhaust gas heat is used to increase the temperature of the air prior to compression. Lowering the need for heat input to the system.

2.5 Rankine cycle

The most common steam cycle using water steam as working medium is modeled by the Rankine cycle. The cycle consist of four steps similar to the Carnot cycle. However the Rankine cycle is a "real" cycle in the sense that it takes into consideration the real characteristics of the working medium.

![Figure 2.2: Rankine cycle working principle](image)

In the first step the condensed water is pumped up to a desired pressure. This occurs with the water in fluid form. Following the same logic as in the section for the Brayton cycle, it is very favorable to increase the pressure with a low specific volume of the working medium. As seen in (eq.2.6) the specific work required to increase the pressure of the fluid is directly proportional to the
specific volume and pressure difference. In the Rankine cycle all the compression occurs when the working medium is in liquid form, this is why the compression work is small.

\[ w_{\text{pump,in}} = v(P_2 - P_1) \] (2.6)

The water is then heated under constant pressure by adding heat from a heat source. The heat source may consist of a boiler burning a suitable fuel, or of a heat exchanger using the heat from a hot medium. At a temperature dictated by the pressure the water will reach its saturation temperature and begin to evaporate. When the all of the working medium is fully evaporated the steam is usually superheated to a even higher temperature by continuing to add heat from the heat source. The steam is then expanded in a steam turbine and mechanical work can be extracted from the cycle by letting the pressure and temperature drop. The fluid exiting the steam turbine is usually in the mixed zone, with high enough steam fraction to prevent corrosion on the last turbine stage. The mixture is then condensed back to liquid at constant pressure transferring heat to a cold heat sink and the cycle is completed. The available heat sink greatly affects the potential efficiency of the cycle. It may consist of a nearby lake or an air cooled cooling tower.

### 2.6 Combined cycle

As the name suggests the combined cycle consists of a combination of two thermodynamic cycles, the Brayton cycle for the gas turbine system and the Rankine cycle for the steam cycle system. As previously mentioned the reason for combining the cycles is that the Brayton cycle exhausts gas at a temperature often above 500°C, which makes it possible to drive the steam cycle using the energy in the exhaust gas. The reference to the Brayton cycle as the "topping cycle" and the Rankine cycle as the "bottoming cycle" is due to the cycles relative positioning on the temperature entropy diagram, where the Brayton cycle is situated above the Rankine cycle.

The combined cycle efficiency is the total work output divided by the heat input of the topping cycle. This can be derived into eq. 2.7, where it is apparent that an increase in GT efficiency not always leads to an increase in combined cycle efficiency.

\[ \eta_{\text{CC}} = \eta_{\text{GT}} + \eta_{\text{BC}} (1 - \eta_{\text{GT}}) \] (2.7)

\[ \eta_{\text{BC}} = \eta_{\text{HRSG}} \eta_{\text{SC}} \] (2.8)
2.7 Second law efficiency

2.7.1 Exergy

When considering a heat engine in a specific location with a defined heat source and heat sink, the difference between total energy and useful energy is the lost energy or waste energy of the system. The theoretical upper limit of the useful energy that can be produced by the heat engine is the exergy of said system. This is illustrated in figure 2.4. If all the processes in the system are reversible, then the exergy equals the actual work produced by the system. This is impossible in a real application, since there are always irreversibilities in the system. But the concept gives an indication of how well the system is performing in its environment, and the room left for improvements.

\[ \eta_{th} = \frac{W_{\text{net,out}}}{Q_{in}} = 1 - \frac{Q_L}{Q_H} \]  

(2.9)

\[ \eta_{2nd} = \frac{\eta_{th}}{\eta_{th,rev}} \]  

(2.10)

2.8 Carnot efficiency vs thermal efficiency

Applying an exergy balance on a Carnot cycle results in

\[ E^{\text{in}} = W_C \]  

(2.11)
Since

$$E_{C}^{in} = \int (1 - \left(\frac{T_L}{T_H}\right))dQ \quad (2.12)$$

$$E_{C}^{in} = \eta C Q \quad (2.13)$$

and there is no thermal exergy leaving the cycle since the temperature at the exit is at the low reference temperature.

For the irreversible "real" Brayton cycle the exergy balance equation yields

$$E_{B}^{in} = W_B + E_{B}^{out} + \sum I_{CV}^{B} \quad (2.14)$$

Which makes the difference between eqs. (2.11) and (2.14)

$$E_{C}^{in} - E_{B}^{in} = W_C - W_B - [E_{C}^{out} + \sum I_{CV}^{C}] \quad (2.15)$$
Leading to

\[ W_B = W_C - I_{in}^Q - I_{out}^Q - \sum I_{CV}^Q \]  \hspace{1cm} (2.16)

Where

\[ I_{in}^Q = E_{in}^C - E_{in}^B = \int \left( \frac{T_L}{T} - \frac{T_L}{T_H} \right) dQ_B \]  \hspace{1cm} (2.17)

and

\[ I_{out}^Q = E_{out}^B = \int \left( 1 - \frac{T_L}{T} \right) dQ_A \]  \hspace{1cm} (2.18)

The irreversibilities \( I_{out}^Q \) and \( I_{in}^Q \) are related to the temperature addition to and rejection from the Brayton cycle taking place at a temperature difference rather than at a constant high temperature as is the case in the Carnot cycle. And due to heat rejection to the atmosphere at a temperature above the reference temperature. Thus the Brayton cycle efficiency is always lower than the corresponding Carnot efficiency following equation 2.19. [10] An analogous analysis can be performed both for the Rankine bottoming cycle as well as the whole combined cycle.

\[ \eta_C - \eta_B = \frac{W_C}{Q_B} - \frac{W_B}{Q_B} = \frac{(I_{in}^Q + I_{out}^Q)}{Q_B} + \sum I_{CV}^Q \]  \hspace{1cm} (2.19)

### 2.9 Logarithmic mean temperature

Throughout this project all mean temperatures used are defined using the logarithmic mean. The first law gives the energy balance, i.e. the incremental flow of heat and work of a process is balanced by the internal energy.

\[ du = dq - dw \]  \hspace{1cm} (2.20)

According to the second law, for a reversible process the entropy generation is equal to the incremental heat divided by the temperature.

\[ ds \geq \frac{dq}{T} \]  \hspace{1cm} (2.21)

\[ Tds \geq dq \]  \hspace{1cm} (2.22)

\[ dw = pdv \]  \hspace{1cm} (2.23)

Definition of enthalpy

\[ h = u + pv \]  \hspace{1cm} (2.24)

Combining equation 2.20 to 2.24 gives the second law equation, or the Gibbs equation.
\[ Tds - pdv = d(h - pv) \]  
(2.25)

\[ Tds - pdv = dh - pdv - vdp \]  
(2.26)

\[ dh = Tds + vdp \]  
(2.27)

**Derivation of \( \bar{T} \)**

\[ \bar{T} = \frac{1}{\Delta s_{overall}} \sum_{i=1}^{N} \Delta h_i \]  
(2.28)

\[ \Rightarrow \bar{T} = \frac{h_3 - h_2}{s_3 - s_2} \]  
(2.29)

From the Gibbs equation of entropy (2.25) one can derive the expression for the incremental difference in entropy.

\[ ds = \frac{dh}{T} = c_p \frac{dT}{T} \]  
(2.30)

Where \( c_p \) is the specific heat at constant pressure. The volume times pressure difference part is zero for a theoretically isobaric heat addition. Integration of eq. 2.30 gives the overall difference in entropy.

\[ \Delta s = s_3 - s_2 \]  
(2.31)

\[ \Delta s = c_p \int_2^3 \frac{1}{T} dT \]  
(2.32)

\[ \Delta s = c_p \ln \left( \frac{T_3}{T_2} \right) \]  
(2.33)

Inserted in Equation 2.29 gives the \( \bar{T} \), logarithmic mean temperature depending only on the two temperatures \( T_3 \) and \( T_2 \).

\[ \bar{T} = \frac{c_p(T_3 - T_2)}{c_p \ln \left( \frac{T_3}{T_2} \right)} \]  
(2.34)

\[ \Rightarrow \bar{T} = \frac{T_3 - T_2}{\ln \left( \frac{T_3}{T_2} \right)} \]  
(2.35)
2.10 Exergy balance of the combined cycle

2.10.1 Exergy input

\[
0 = \dot{m}_{GT} \cdot (e_{exh} - e_{stck}) \pm \sum_i \left(1 - \frac{T_0}{T_i}\right) \cdot \dot{Q}_i - \dot{W}_{BC} - \dot{I}
\]  
(2.36)

When applying this exergy balance to the model created in IPSE the dominating exergy input to the system is the mass flow related exergy leaving the GT exhaust.

\[
\dot{E}_{in} = \dot{m}_{exh} \cdot e_{exh}
\]  
(2.37)

Where

\[
e_{exh} = (h_{exh} - h_0) - T_0(s_{exh} - s_0) + \frac{V^2}{2} + gz
\]  
(2.38)

The subindex 0 refers to the dead state. The kinetic and potential exergy is disregarded as it is negligible. This exergy input is known from GT calculations.

The other exergy input consists of the pumping work in the system. Since the pumps are driven by electrical motors and not by steam turbines using steam produced in the system, the total energy input to the pumps equals the exergy input. However this exergy is subtracted from the generator output when determining the cycle net output, and it is therefore unnecessary to take any further measures to quantify the pumping exergy input.

2.10.2 Exergy out

The exergy leaving the BC is the net work output from the generator, the exergy related to heat transfer out of the condenser and exergy related to mass transfer out of the stack. The largest part is the net work output from the generator. The stack exergy is strongly depending on the stack temperature and exhaust mass flow.

\[
\dot{E}_{stck} = \dot{m}_{exh} \cdot \left((h_{stck} - h_0) - T_0(s_{stck} - s_0)\right)
\]  
(2.39)

The heat that is transferred away from the BC in the condenser could have been used to drive a heat engine and produce work. This work equals the exergy transferred, and is determined according to the equation below.

\[
\dot{E}_{cond} = \dot{Q}_{cond} \cdot \left(1 - \frac{T_0}{T_{CW}}\right)
\]  
(2.40)

Where the cooling water mean temperature, $T_{CW}$, is defined using the same method as for the heat exchanger mean temperatures. The heat rejected from the condenser is large in any steam cycle. However the exergy lost due to heat transfer out of the bottoming cycle is not very large. This is because of the relatively low temperature difference between the cooling mass flow entering the condenser and leaving the condenser.
2.10.3 HRSG irreversibilities

For the complete HRSG the expression for the irreversibilities related to heat transfer between the cold steam mass flow and the hot exhaust gases can be expressed as eq. 2.44. In the exergy balance the heat supplied to the HRSG is expressed as the available part of the heat output from the hot exhaust gases, eq.2.42. Using the definition of exergy destruction in rate form the exergy destruction in each of the heat exchangers in the HRSG can be determined.

\[
\dot{E}_{dest} = \dot{S}_{gen} \cdot T_0 = \frac{\dot{Q}}{T} \cdot T_0 \tag{2.41}
\]

\[
\dot{Q} = \dot{Q}_{in} \cdot \left(1 - \frac{T_L}{T_H}\right) \tag{2.42}
\]

As the temperature varies in the different parts of the HRSG an expression for the mean steam and exhaust temperature is used to approximate the inlet and outlet conditions of each heat exchanger.

The mean temperature is defined as:

\[
\bar{T} = \frac{h_2 - h_1}{s_2 - s_1} \tag{2.43}
\]

\[
\dot{I}_{HRSG} = \dot{Q}_{in} \cdot \frac{T_0}{\bar{T}_{stm}} \left(1 - \frac{T_{stm}}{T_{exh}}\right) \tag{2.44}
\]

This approach can be adapted to the individual heat exchanger in the model for a better prediction of the total exergy destruction rate in the HRSG.

2.10.4 ST irreversibilities

The exergy destruction in the steam turbines is calculated using standard definitions. This is an easy procedure when the steam mass flow and entropy before and after each turbine is known. The entropy difference is determined using the ST isentropic efficiency. When another pressure level is introduced in the model the calculation gets more extensive as more states needs to be calculated, but the procedure is the same. When all the irreversibilities are calculated they are added together as the total ST irreversibility.

\[
\dot{I}_{ST} = \dot{m} \cdot T_0 \cdot (s_{out} - s_{in}) \tag{2.45}
\]

2.10.5 Pumping irreversibilities

Since the pumping work in the bottoming cycle is low and the pumps in the evaporators are not increasing the pressure but are only circulating the fluid, the entropy generation related to pumping is also low. And since the exergy destruction is strongly linked to the entropy generation, it to is low.
The pumps in the model have an isentropic efficiency of 0.9.

### 2.10.6 Exergy balance

Looking back to the exergy balance equation 2.36 When the exergy destruction and rejection is subtracted from the exergy input the remaining exergy matches the net work output very well. For the 2PNRH there is a 0.3 percent difference which is small considering that the temperatures used in the exergy destruction model are mean temperatures.
3. **HRSG theory**

3.1 HRSG pressure levels

The heat recovery steam generator is an integral part of the combined cycle. The optimization of the HRSG is heavily affecting the overall plant efficiency. The increased performance of the HRSG that comes from adding another pressure level originates from the better possibility to transfer heat to the steam from the exhaust gases and by that lower the stack temperature and increase the steam mean temperature. [11] By increasing the number of pressure levels the influence of variation in exhaust temperature decreases. This is because of the relative shift of LP economizer pinch point to the left in the T-Q diagram.

3.2 T-Q diagram

To visualize the HRSG performance it is common to use a diagram with energy transfer on the horizontal axis and temperature on the vertical axis, known as a T-Q diagram.

![Figure 3.1: Schematic representation of a single pressure HRSG](image)
The GT exhaust is represented by the upper line in the diagram and the mass flow is from hot to cold as the heat energy transfer to the colder steam mass flow occurs. The steam mass flow is represented by the lower line. When the water is evaporated the temperature of the fluid is constant, and the temperature of the exhaust gas is decreasing, this leads to an increasing temperature difference between the two lines. Theoretically this temperature difference could be used to drive a Carnot engine thus increasing the energy produced by the system and thereby also increasing the efficiency of the plant. In the evaporator the difference in temperature between the cold side of the steam and the cold side of the exhaust gases is known as the pinch point, illustrated in figure 3.1. This is an important design parameter when optimizing the HRSG. If there were no temperature difference between the two lines in the T-Q diagram then the HRSG would operate at its theoretical maximum efficiency. However there are physical limitations that makes constructing such a HRSG impossible in reality. The most obvious reason for this being the evaporation of steam under constant temperature, effectively making the lines diverge. A common way to counter this effect is to use several pressure levels in the HRSG. By splitting the steam into different parts generated at different pressures it is possible to reduce the mass flow in each evaporator, which makes the constant temperature line shorter. By increasing the number of pressure levels the efficiency of the HRSG is increased, however each extra pressure level comes with its own evaporator and often economizer and super heater as well, leading to a substantial cost increase with each extra level. Combined with the diminishing effect on the gain in efficiency by exceeding three pressure levels, HRSG:s with two or three pressure levels is commonly used.

Another physical limit is the fact that to reduce the pinch points the heat transfer area must be increased. Thus leading to increased cost both due to more material being used and also due to larger HRSG footprint.

### 3.3 Stack temperature

The stack temperature is of vital importance in the thermodynamic evaluation of the HRSG and thus the combined cycle. It basically dictates the portion of the heat provided in the exhaust gases that is wasted to the surroundings. In a HRSG the stack temperature is either determined by the pinch point in the LP economizer [12] or by reaching the minimum allowed stack temperature due to condensation of water from the exhaust gases. Damage to the economizer tubes due to condensation is generally prevented by keeping the temperature above a certain limit. However, this limit will not be active at full load nor in most of the operational window unless the fuel contains large amount of sulfur. If this is the case, the stack temperature will be lowered by an increase in exhaust temperature. In the T-q diagram 3.2 the line representing the exhaust gases will pivot around the fixed economizer pinch point.

As shown by the graph the stack temperature is reduced following an increase in exhaust temperature. From the figure it is also clear that the decrease in stack
temperature is always less than the increase in exhaust gas temperature. Since the stack temperature is a result of the LP evaporator cold side temperature it is the position of this point in the T-q diagram that dictates this difference. When a second pressure level is introduced in the HRSG the optimum LP boiler pressure level is being pushed down. This lower LP pressure leads to the pinch point in the boiler being pushed to the left in the T-q diagram. In a three pressure HRSG the decrease in stack temperature following an increase in exhaust temperature would relatively be even smaller, since the LP pressure is generally lower for a three pressure plant and hence the evaporation temperature is lower.

Figure 3.2 presents a good visualization on the increase in steam mean temperature when the exhaust temperature increases. It can be seen that when an increase in exhaust temperature part of the LP steam is transferred to the HP boiler, i.e. the HP boiler line in the T-q diagram is longer. Since the HP boiler operates on a higher temperature this mass flow transfer leads to an increase in overall steam mean temperature of the steam cycle, explaining the increasing BC efficiency. This is since the increased steam mean temperature effectively reduces the irreversibilities in the cycle, thus increasing the efficiency.
4. Methodology

In this section the methodology of the project is presented in order to describe what has been done to accomplish the objectives. The models created in IPSE are described in detail along with the assumptions and decisions made during the creation process. The correlation between second law efficiency and combined cycle efficiency is derived. The component irreversibilities dependence on changes in exhaust temperature and mass flow is evaluated. The constant second law efficiency approach is introduced, derived and improved upon by developing the correction factors.

4.1 Model description

In this project numerous thermodynamic calculations were carried out based on heat and work balance equations. This would not have been possible without specific models of the systems that were examined. IPSEpro provides the possibility to build highly customizable models, including the possibility to rebuild the different components, alter the governing equations and to program desired features during the project. The resulting data from the cycle calculations could then be exported to produce graphs and tables that can be presented.

Since the result of this project is going to be used to evaluate power plants using different number of pressure levels in their bottoming cycles the equations and approximations will have to be altered to fit the number of pressure levels in the specific power plant. When the number of pressure levels increase it gets harder to get good approximations. This is mainly because of the mass flow being split into different streams in the HRSG and ST. Because of this, the combined cycle efficiency is first determined for the single pressure bottoming cycle, and then developed to work for two and three pressure levels.

4.1.1 1PNRH

The simplest model used in this project is the single pressure no reheat (1PNRH) bottoming cycle. The combined cycle model used is a 1x1 configuration, i.e there is only one gas turbine delivering heat to the steam cycle. This configuration will stay the same throughout the project. In this model the HRSG consists of three heat exchangers. One economizer, one evaporator and one super heater.
Figure 4.1: The 1PNRH model built in IPSEpro
There are three pumps in the model, one to circulate the evaporator, one to pump the condensate to the deaerator and one to increase the pressure of the deaerated water before it enters the economizer. As previously mentioned the pump work is very small compared to the generator output. This means that the total pumping irreversibilities is small as well.

Although only one pressure level exists in the model, there are two steam turbines. This is because of the bleed mass flow needed to the deaerator.

In this simple model no pressure losses are taken into consideration. The HRSG is considered adiabatic, i.e. no heat transfer to the surroundings are modeled.

4.1.2 2PNRH

The two pressure no reheat cycle built in IPSE is similar to the one pressure cycle in every way except for the additional low pressure boiler and added high pressure economizer. The model has also been extended to include fuel preheating, extracting a small water mass flow after the high pressure economizer. With this model it is possible to capture the irreversibility decrease in the components since steam mass flow is transferred between the pressure levels. The model is also expanded to include a temperature controller on the super heater to be able to control the temperature of the hot steam entering the steam turbine. This is done by spraying water from the HP economizer stream to the stream leaving the HP boiler.

4.1.3 3PNRH

The three pressure model was built in IPSE for the purpose of calculating the correction factors applicable to a combined cycle incorporating three pressure levels.

4.1.4 Reheat

Reheat means heating up the steam leaving the first steam turbine to the same temperature as before the first steam turbine, thereby increasing the mean steam temperature of the BC and by that also increasing the efficiency. To accomplish this another heat exchanger is introduced into the HRSG.

4.2 Exergy balance evaluation

To understand the exergy transfer in the combined cycle it is important to differentiate between exergy destruction internally in the system and exergy losses related to energy being transferred out of the system. This differentiation is illustrated in figure 2.5. There are internal irreversibilities causing exergy destruction in the heat exchangers, condenser, steam turbines and pumps. These components must be evaluated respectively and then added together. There is
also a miscellaneous loss to take care of small losses related to pipe friction and valves. The model created in IPSEpro is used for finding an appropriate expression for the temperature or temperatures at which the exergy destruction in the system occurs and how this temperature varies when the exhaust temperature and mass flow is altered. It is therefore important to evaluate the exact exergy destruction in the model, in order to know if the approximation is sufficiently accurate. The model is evaluated using second law analysis of the different parts. An exergy balance equation is used and the irreversibility rate from each part of the system is summed up to the total cycle irreversibility.

4.3 Irreversibility change with exhaust temperature

The bottoming cycle irreversibilities depends on the GT exhaust temperature. This can be seen in the equations in the presented theory. The mean steam temperature increases which leads to decreased irreversibility. Following an increase in exhaust temperature the different parts of the bottoming cycle will be individually changed, however the common trend is that an increased GT exhaust temperature will result in lower bottoming cycle irreversibilities. The graphs below represent the irreversibility of the major parts of the bottoming cycle for a combined cycle with dual pressure HRSG.

![Graphs showing irreversibility change with exhaust temperature](image)

Figure 4.2: Exhaust temperature in [°C] on the x-axis and irreversibility in [kW] on the y-axis
It is apparent that the pumping irreversibilities are to small to be paid much attention in the analysis of the overall performance. It is also clear that the HRSG irreversibility change is largest, strongly linked to the mass flow being shifted from the LP to the HP boiler.

4.3.1 Why use the second law efficiency?

In figure 4.3 the second law efficiency, Carnot efficiency and bottoming cycle efficiency is plotted as a function of exhaust temperature. The Carnot efficiency increases since the hot temperature is the exhaust temperature, simply following equation 2.1. The bottoming cycle efficiency is also increased as the exhaust temperature increases. This is because of the higher heat addition temperature in the bottoming cycle caused by the larger fraction of steam lifted to the high pressure boiler. The second law efficiency is the ratio between the bottoming cycle efficiency and the Carnot efficiency eq. 2.10. Since both the bottoming cycle efficiency and Carnot efficiency increases the second law efficiency will remain close to constant. This correlation is something that can be used when relating the GT performance to the combined cycle efficiency.

However, due to material limitations in the steam turbine the superheater temperature reaches its limit when the exhaust temperature rises, this leads to the need for spray cooling to comply with this limit. The spray mass flow is extracted after the HP economizer, at around 330 °C and is mixed with the superheated steam. This inevitably leads to a reduced second law efficiency since the BC efficiency does not increase as much as the BC Carnot potential. This can be seen in figure 4.3.

![Figure 4.3: Efficiency variation with exhaust temperature](image)

5. Derivation of correlation

5.1 Derivation of combined cycle efficiency equation

The combined cycle efficiency is the ratio of power generated per heat input.

\[ \eta_{CC} = \frac{P_{GT} + P_{BC}}{Q_{in}} \]  \hspace{1cm} (5.1)

The only heat input to the system is the fuel entering the GT combustion chamber. This is quantified as the mass flow of fuel multiplied by the lower heating value LHV.

\[ Q_{in} = \dot{m}_{fuel} \cdot LHV \]  \hspace{1cm} (5.2)

The power produced by the bottoming cycle can be related to the heat input and the steam cycle efficiency. The heat input to the bottoming cycle equals the heat leaving the topping cycle.

\[ P_{BC} = \dot{m}_{GT} \cdot c_{p}(T_{exh} - T_{stck}) \cdot \eta_{BC} \]  \hspace{1cm} (5.3)

The thermal efficiency of the bottoming cycle can be expressed using the second law efficiency and the Carnot efficiency.

\[ \eta_{BC} = \eta_{2nd} \left( 1 - \frac{T_{L}}{T_{H}} \right) \]  \hspace{1cm} (5.4)

Combining the above equations one can rewrite the combined cycle efficiency as follows. Where the Carnot heat engine represents the bottoming cycle operating between the gas turbine exhaust temperature and the condenser temperature.

\[ \eta_{CC} = \frac{P_{GT} + \dot{m}_{GT} \cdot c_{p}(T_{exh} - T_{stck}) \cdot \eta_{2nd} \cdot \left( 1 - \frac{T_{cond}}{T_{exh}} \right)}{\dot{m}_{f} \cdot LHV} \]  \hspace{1cm} (5.5)
5.2 Evaluation of combined cycle efficiency equation

Based on GT calculations most of the parameters in equation 5.5 is known. The GT power, mass flow and exhaust temperature is known from these calculations. The condenser temperature is treated as the ambient temperature plus a constant depending on the cooling possibilities i.e. water cooling or air cooling. The specific heat is found from gas tables since the composition is known from GT calculations. The unknown parameters are the stack temperature and the second law efficiency, these are the parameters that need to be treated separately to get a good approximation. This is explained in the following sections. The specific heat also needs some commenting.

5.2.1 Gas turbine exhaust temperature

![Figure 5.1: Variation in GT exhaust temperature](image)

In figure 5.1 the prediction of the combined cycle efficiency using equation 5.5 is presented when the exhaust temperature is varying in the range between 545 and 565 °C. This interval is chosen since a 20 degree temperature increase is considered reasonable to expect when an upgrade of the GT is implemented. The reference point is where the exhaust temperature is 545, the second law efficiency is calculated for that loadpoint and is then kept constant to illustrate how this impacts the combined cycle efficiency. Information about the calculation procedure can be found in section 5.4. The gray line represents equation 5.5 and the black line is plotted with values from the heat and work balance calculations in which the second law efficiency is free to vary for each point calculated. It can be concluded that correlation can predict the combined cycle efficiency with an accuracy in the order of 0.04 percentage points i.e. 0.02 percentage points per 10 degrees increased exhaust temperature.
5.2.2 Gas turbine exhaust mass flow

In figure 5.2 the variation on the x-axis is the increase in exhaust mass flow. The gray line is equation 5.5 with the constant second law efficiency and the black line is the heat and work balance calculation. Similarly to the exhaust temperature variation the equation follows the calculation data with a slight under-prediction for larger increases in exhaust mass flow. It is evident that there is room for an increased accuracy in the prediction from the equation, which leads to the introduction of the correction factors for the second law efficiency and stack temperature.

5.2.3 Specific heat

The specific heat in the exhaust gas is depending on the gas composition, primarily on the carbon dioxide content. This is in turn depending on the GT fuel to air ratio. In equation 5.5 the specific heat will be unknown due to its dependence on the stack temperature. When applying the equation the specific heat is given by a gas table where the inputs will be the exhaust pressure, temperature and composition. To get a good estimation a mean specific heat is used. The mean value is found by estimating the stack temperature as the corrected constant stack temperature (presented in section 5.3.1). This estimation leads to an error, however it is small because the stack temperature will have limited effect on the value from the gas table as seen in figure 5.3. Furthermore, the stack temperature will not vary much compared to the exhaust temperature.
A sensitivity analysis was carried out to verify this. The corrected stack temperature was increased by 10 percent to determine the impact on the specific heat, the result from a 10 percent over-prediction of the corrected stack temperature was less than 0.0008 decrease of specific heat as can be seen in figure 5.3. The same number for an under-prediction of the corrected stack temperature by 10 percent. This is with good margin to the actual corrected stack temperature.

5.3 Derivation of correction factors

To attain a working correction factor it is imperative to find out how the second law efficiency and stack temperature deviates from their original values, and to separately allocate the change of these parameters to the change in GT mass flow and exhaust temperature. This has to be done carefully since all parameters are strongly depending on each other. This is well described by Gülen in ref. [3]. The method applied in this project is to use the IPSE model and the model development kit to state an equation that represents the combined cycle efficiency using only varying parameters, i.e. no constants or approximations. Another equation is then stated in which the second law efficiency is kept constant. The first correction factor is inserted in the equation. Then a variation of the GT parameter for said correction factor is made in the IPSE model. The values of both the exact equation and the equation with the constant second law efficiency and correction factor are collected in a vector. The value of the correction factor will then vary with the GT parameter to correct the efficiency. The values are plotted against the GT parameter and since it is linear, both for the exhaust gas temperature and the mass flow, a linear equation can be created from the values. The interesting part is the coefficient in front of the variable, because it is a measure of how much the parameter influences the efficiency.

The sensitivity of the second law efficiency correction factors was tested by
changing the steam cycle parameters. The dominating parameter affecting the correction factors was found to be the number of HRSG pressure levels, this has been pointed out earlier in the report. The change in pinch points for the evaporators made no significant impact on the correction factors at all.

5.3.1 Correction factors

The correction factors are derived using the IPSE-Pro models of the system. There are four factors in the equation, one each for correction of the second law efficiency when exhaust mass flow and temperature is changed, and one each for correction of the stack temperature when the same change is made. All four correction factors are determined using the same method. Since both the second law efficiency and the stack temperature varies close to linearly when the exhaust parameters are changed, they are a dimensionless number that follows a linear equation with the change in the exhaust parameter as a variable.

\[ x_{\eta \Delta m} = 1 - 0.00009176 \cdot \Delta m_{GT} \]  \hspace{1cm} (5.6)

This correction factor is introduced to compensate for the change in second law efficiency when the exhaust mass flow is changed.

\[ x_{\eta \Delta T} = 1 - 0.0003285 \cdot \Delta T \]  \hspace{1cm} (5.7)

This correction factor is introduced to compensate for the change in second law efficiency when the exhaust temperature is changed. These factors will correct the decrease in second law efficiency when a change of GT mass flow and or temperature occurs. It can be seen that the decrease is rather small. The coefficient basically says that to obtain a good combined cycle efficiency following a change in exhaust temperature and mass flow it is necessary to lower the second law efficiency by 0.0003285 times the change in exhaust temperature and 0.00009176 times the change in exhaust mass flow. This again agrees with the thesis that the second law efficiency is essentially independent of these parameters. Due to the steam turbine temperature limit the live steam is usually sprayed down to a set temperature by bypassing some of the HP boiler mass flow to the HP super heater. If the change in exhaust temperature were to decrease, this spray mass flow would be reduced down to a certain point after which it would be shut of completely. At this point the correction factor for exhaust temperature effect on second law efficiency is no longer needed.

\[ x_{T \Delta \dot{m}} = 1 + 0.00037427 \cdot \Delta m_{GT} \]  \hspace{1cm} (5.8)

This correction factor is introduced to compensate for the change in stack temperature when the exhaust mass flow is changed. In figure 3.2 an increase in exhaust temperature can be seen, and how this affects the stack temperature and second law efficiency.

\[ x_{T \Delta T} = 1 - 0.00022203 \cdot \Delta T \]  \hspace{1cm} (5.9)
This correction factor is introduced to compensate for the change in stack temperature when the exhaust temperature is changed. These factors will take care of the change in stack temperature when a change in GT mass flow and or temperature occurs. All four factors are inserted in equation 5.5. Which leads to the final form of the equation. 5.10.

\[
\eta_{CC} = \frac{P_{GT} + \dot{m}_{GT} \cdot c_p(T_{exh} - xT\Delta T \cdot xT\Delta \dot{m} \cdot T_{stck}) \cdot x\eta\Delta T \cdot x\eta\Delta \dot{m} \cdot \eta_{2nd} \cdot \left(1 - \frac{T_{exh}}{T_{exh}}\right)}{\dot{m}_f \cdot LHV}
\]

\[\text{(5.10)}\]

The divergence of the lines in fig. 5.1 depends on the second law efficiency being kept constant. As stated before, the second law efficiency is nearly constant in the equation. The correction factor is introduced in order to get an even better prediction of the combined cycle efficiency. The correction factors presented in this part of the report is suitable for a dual pressure HRSG, in table the correction factors for other systems can be found. The stack temperature and both correction factors related to it is the corrected stack temperature.

\[T_{stck,corr} = xT\Delta T \cdot xT\Delta \dot{m} \cdot T_{stck}\]

\[\text{(5.11)}\]

The second law efficiency with the related correction factors is the corrected second law efficiency.

\[\eta_{2nd,corr} = xT\Delta \dot{m} \cdot x\eta\Delta \dot{m} \cdot \eta_{2nd}\]

\[\text{(5.12)}\]

### 5.4 Calculation procedure

Starting from equation 5.5 the data from a power plant where the combined cycle efficiency is known is inserted. If the stack temperature is not known an estimate is made based primarily on the number of pressure levels in the HRSG. It is suitable to hard-code this estimation into the software used when implementing this method, making it simple for the user since there is no need for experience to make the estimation. As a rule of thumb a good estimation would be 110 °C for a 1 pressure HRSG, 95 °C for a 2 pressure HRSG and 80 °C for a 3 pressure HRSG. The specific heat is found from gas tables, this is done using the known exhaust gas composition, exhaust pressure and temperature. The mean value is used for the specific heat, i.e. the value of the specific heat is calculated both for the exhaust temperature and the stack temperature and then averaged. The equation can then be solved for the second law efficiency. When this is done equation 5.10 is applied with the upgraded GT performance data along with the correction factors. The correction factors are calculated based on the difference in exhaust gas temperature and exhaust mass flow, all according to their respective equation presented previously in the report. Both the stack temperature and the second law efficiency is kept constant and is adjusted by the correction factors. This results in a new combined cycle efficiency which
can be compared to the old value to determine the difference. No iteration is needed.

5.5 Spray cooling of live steam

In cases where the exhaust temperature decreases after the upgrade it is favorable to leave the correction of the second law efficiency with regard to exhaust temperature out of the equation.

In figure 5.4 the value of the correction factor is presented as the exhaust temperature decreases. When the exhaust temperature decreases it will eventually pass the point where spray cooling is no longer needed since the steam temperature is below the steam turbine limit without cooling. This is why there is a change in inclination. If the correction factor is not removed from the equation it will lead to an over-prediction, which is clearly seen in the figure. Following the assumption that the spray mass flow is designed to be low at the design point in the HRSG, it is likely that the spray mass flow reaches zero after a small decrease in exhaust temperature. This leads to the suggestion that said correction factor should be removed for any decrease in exhaust temperature, even if this is not necessarily true for all HRSG designs.
6. Validation

The equation was validated against data from calculations of an upgrade project of a Siemens combined cycle consisting of two gas turbines and one steam turbine. The prediction of the combined cycle efficiency difference when compared to the data was found to be in the low hundredth of a percent, which makes for a close enough approximation for a first quick calculation. To put the results in context the approach proposed in this report was compared to the approach of keeping the BC thermal efficiency constant in the prediction of combined cycle efficiency change. The thermal efficiency was calculated from eq. 2.7 using the same data from Siemens upgrade project. This showed that the prediction from the second law approach is closer to the actual efficiency change by over 30 percentage points. By that the objectives can be considered to be achieved. To perform a further statistically reliable validation, more site data would be needed.

Validation with Siemens gas turbine fleet was carried out. Four different scenarios were examined using existing GT models. An increase in turbine inlet temperature by 25 and 50 degrees, an increase of compressor mass flow by 5 percent and an increase in diffuser recovery by 10 percent. The prediction result is presented in the table below, as deviation from the combined cycle efficiency calculated by the model.

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<th>TIT + 50 C</th>
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In the cases where the exhaust temperature decreases the correction factor
for the second law efficiency related to exhaust temperature difference was not included in the equation, as explained in section 5.5. The error from predicting the combined cycle efficiency without the correction factors are presented. It can be seen that the correction factors makes the prediction considerably more precise in cases where the exhaust temperature changes.

The comparison in precision in the table is from equation 2.7 where the thermal efficiency is kept constant to predict the combined cycle efficiency after an upgrade. It can be seen that the second law approach is far more precise in the prediction.
7. Discussion and analysis

The results from the project i.e. the derived combined cycle efficiency equation and especially the understanding of what affects its precision implies that it is indeed possible to implement the approach when predicting the combined cycle efficiency change.

The ambition of this project was to find a way to quickly predict the combined cycle efficiency increase. With focus on the simplicity of the method in order to increase the speed of the calculation process and to make it easier to perform an estimation on the combined cycle efficiency increase without the need for extensive experience from heat and work balance calculations. Initially the potential approach of using exergy destruction calculations to predict the combined cycle efficiency was evaluated. However, it was concluded that such an approach would not effectively reduce the complexity of the calculations, i.e. it presented no substantial reduction of equations and heat and work balance calculations needed to obtain the result compared to the conventional procedure. Thus the exergy balance approach was rejected in favor of the more straightforward equation derived in this report.

When the first equation (eq. 5.5) describing the combined cycle efficiency prediction was derived it was evaluated by comparing the predictions of the equation with heat and work balance calculations. The heat and work balance models described in section 4.1 were created to this purpose. Different parameters in the model was changed to find out which parameters made the largest impact on the prediction. It was concluded that when the GT exhaust temperature and mass flow was changed, it made the prediction less accurate when the stack temperature and second law efficiency was kept constant. The sensitivity of the estimation of GT exhaust gas mean specific heat was examined since the stack temperature after the upgrade is unknown and this affects the mean specific heat in the equation. With knowledge about this sensitivity the use of the corrected stack temperature in the gas table when estimating the specific heat is the best approach, considering generality.

One challenge was to find a good balance between applicability and generality. Based on the fact that a large number of parameters affect the combined cycle efficiency it is extremely difficult to produce a compact formulation that can be used for the many different bottoming cycles that are in operation world wide. Every assumption made in the model that is used to produce the correction factors inevitably leads to sacrifices in the applicability of the equation and
since the end result, i.e. the prediction of the combined cycle efficiency, has to be fairly close to the correct value in order to be at all useful it makes the whole project rather delicate.

7.1 Correction factors

When a change in GT performance is considered e.g. an upgrade of the GT is proposed which will result in a change of the parameters involving the bottoming cycle, there are limitations in the system that dictates the maximum possibility of changing certain key parameters. That is, to use the combined cycle efficiency equation for a prediction in efficiency change, the equation needs to be altered further since the input parameters are not individually independent of each other, this leads to the need for the correction factors to compensate for both stack temperature and second law efficiency being kept constant. The correction factors will compensate the slight alterations in second law efficiency and stack temperature when a change in exhaust mass flow or temperature is made. To begin with, two correction factors were introduced into the equation, one for the stack temperature and one for the second law efficiency. It was then concluded that the change in exhaust temperature and mass flow affects the stack temperature in opposite ways, an increase in exhaust mass flow will increase the stack temperature and an increase in exhaust temperature will decrease the stack temperature. This leads to the need for another correction factor for the stack temperature. Since the prediction must be applicable to different GT upgrades, and since there is no general connection between the change in exhaust parameters there has to be a separate correction factor for the change in exhaust temperature and mass flow to correct the constant second law efficiency. This led to the final expression for the equation used to predict the efficiency change, eq. 5.10.

7.2 Stack temperature decrease - first law

The increase in exhaust temperature will result in a decrease of stack temperature. This will increase the combined cycle efficiency, which is easy to understand applying the first law of thermodynamics. The heat captured by the HRSG increases, leading to an increased power output from the ST without increasing the fuel consumption of the machine. This can be seen in figure 3.2, the total heat input into the HRSG increases both because of the exhaust temperature being higher and the stack temperature being lower. The fuel input to the combined cycle is unchanged hence the efficiency increases.
7.3 Steam mean temperature increase - second law

From a second law point of view the increase in combined cycle efficiency comes from the increased steam mean temperature, i.e. the increased exhaust temperature results in a larger HP mass flow, effectively shifting the LP boiler steam to the HP level. This effect is also illustrated in fig. 3.2. The HP boiler operates on a higher temperature compared to the LP boiler, thus increasing the mean steam temperature. The higher mean steam temperature effectively reduces the irreversibilities of the system and consequently reduces the total internal losses, leading to an increased overall efficiency of the combined cycle.

Regarding the reasoning behind the use of a second law approach, this is primarily because it presents a powerful tool when comparing different alternatives of components in a power plant. While the results are always in agreement with the first law of thermodynamics, the major advantage of using the second law approach is that it provides a good representation of which components to target when improving the power plant. Furthermore the second law approach also contributes to understanding the mechanisms of the bottoming cycle and how the components reacts to changes in the cycle, since it comprehensively quantifies the irreversibilities. Furthermore, the equations describing the irreversibilities presented in the theory that are derived by Gülen in ref [3] can individually or added together be used to understand how the change in various parameters will affect the total internal losses in the systems.

7.4 Different number of pressure levels

The need for a set of correction factors derived for the correct number of pressure levels is clear when following the reasoning about applicability. The stack temperature does not vary in the same way for a dual pressure HRSG as for a triple pressure one, as can be seen in figure 3.2. This is the main reason behind the need for different correction factors when considering steam generators with different number of pressure levels.

7.5 HRSG configuration

The correction factors are developed using a model with a defined HRSG configuration. When applying the equation 5.10 to a power plant this assumes that the HRSG configuration is similar to the one created in the model. If the HRSG of the plant differs from the modeled one that could lead to a source of error in the prediction. The material limitations of the steam turbine leads to the need for spray cooling of the HP super heated steam to control the temperature. As previously mentioned this affects the second law efficiency since the increase is hindered due to the temperature control. This aspect is something that need to be considered by the user.
7.6 Sources of error

The correction factors are straight line equations, simply to keep them compact. This ultimately leads to an introduction of error since the model calculated values marginally deviates from this straight line. This is once again a balance between compactness and precision of the equation. To capture the deviations from the straight line equation would require the introduction of a significantly higher order polynomial equation.

There are several controllers in the model used to create the correction factors. The HP SH spray being of major importance since it introduces nonlinearities into the system. There is also a bleed steam controller to the deaerator. If the change in GT performance is changed passed the limits in these controllers the system responds nonlinearly making the linear correction factors less precise leading to a decrease in prediction accuracy.
8. Conclusions

With knowledge about the site prior to an upgrade i.e. known stack temperature and combined cycle efficiency the equation can predict the efficiency change after the upgrade within a couple of hundredth of a percent following an exhaust temperature increase by 15 °C. This is without the need for any iteration. When the correction factors were updated to fit the pressure levels of the validation site the prediction error was reduced by more than half, down to just above one hundredth of a percent on the combined cycle efficiency.
9. Future work

9.1 Software implementation

The scope of this project is limited to the presentation of a method of predicting the change in the combined cycle efficiency for a power generating plant. To make use of the proposed method it will have to be implemented in a suitable software.

9.2 Improvement of correction factors

Improvement of the correction factors to better predict efficiency change for a specific HRSG configuration. This is preferable to consider in the case that more information exists about the specific site targeted.

9.3 HRSG optimization

The results presented in this report could also be of use in the process of designing a new HRSG and efficiently optimizing it for combined cycle efficiency of the plant.
Bibliography


