Mobile Positioning in MIMO System Using Particle Filtering

Bizjajeva, Svetlana; Rydén, Tobias; Edfors, Ove

Published in:
2007 IEEE 66th Vehicular Technology Conference, vols 1-5

DOI:
10.1109/VETECF.2007.175

2007

Mobile Positioning in MIMO System Using Particle Filtering

Svetlana Bizjajeva
Centre for Mathematical Sciences
Lund University
Lund 221 00, Sweden
Email: svetik@maths.lth.se

Tobias Rydén
Centre for Mathematical Sciences
Lund University
Lund 221 00, Sweden
Email: tobias@maths.lth.se

Ove Edfors
Department of Electroscience
Lund University
Lund 221 00, Sweden
Email: Ove.Edfors@es.lth.se

***Abstract*—This paper represents the results of a simulation study on positioning of a mobile unit in MIMO settings. We used two different approaches for modeling the mobile movement, combined with a simple geometrical model for the MIMO channel. Three different particle filters were implemented for the position estimation. The results show that all three filters are able to achieve estimation accuracy required by Federal Communication Commission. The dimensionality of the particle filter state space is independent of the number of antenna elements, and it is possible to increase the number of antennas and use more sophisticated channel models without changing the filtering algorithms.**

***Index Terms*—MIMO, mobile positioning, channel modeling, particle filtering, simulations.**

I. Introduction

Wireless systems are now used worldwide to help people and machines to communicate with each other irrespectively of their location. In a global perspective, wireless stands to be a method most people will use to connect to the Internet. New generation wireless communication systems (4G) should be able to provide clients with all the benefits associated with the World Wide Web: multimedia, e-commerce, unified messages, peer-to-peer network etc. To increase system performance is thus very important.

Another goal of 4G systems is to allow switching between networks of systems that gives "the best" connection at the moment. Mixing various connections from satellites to local area networks may result in a crowded frequency spectrum and requires a signaling strategy that is spectrally efficient.

Using multiple antennas at both transmitter and receiver can solve these issues. The MIMO (multiple-input multiple-output) technology, proposed by Paulraj and Kailath in [1], increases the spectral efficiency of a system. It enables high capacities suited for Internet and multimedia services and also dramatically increases range and reliability. In the last few years, MIMO systems have emerged as one of the most promising approaches for high data-rate wireless systems. For more details about the MIMO technology see, for example, [2].

The positioning of a mobile unit in MIMO settings is a challenging problem. During the last decade various location technologies have been invented using either cellular network-based, mobile-based, or hybrid approaches. A comprehensive overview of different positioning methods can be found in [3]. Most known and widely used is the satellite Global Positioning System (GPS), which is based on measurements of time difference of arrival. The propagation time of signals is measured simultaneously from satellites at known locations and the distance between a satellite and a user receiver is obtained by multiplying the propagation time with the speed of light, assuming the line of sight (LOS). In most applications however, the LOS signal is succeeded by multi-path components that arrive to the receiver with a short delay. This introduces significant errors in the LOS path time of arrival and gain estimation, especially in urban environments with many reflections from buildings and other objects. On the contrary, MIMO systems can use the information from multipath components to improve the accuracy of the estimation.

The key concept for the positioning in MIMO settings is the selection of an appropriate model for the propagation channel. With the proper channel model the location problem can be solved using sequential Monte Carlo methods, also called particle filtering, see [4]. In this paper we will investigate the performance of particle filtering in MIMO system settings.

II. State-space model and particle filtering

The positioning problem in a MIMO setting consists in the estimation of the receive antenna coordinates at time t given the signal strength measurements at the receiver end up to this time, if the transmitted signal is known. In a state-space model framework this problem corresponds to computation of the filtering probability density function and estimation of the expected values of the state variables.

Consider a discrete state-space model with additive noise,

$$\begin{cases} z_{t+1} &= f(z_t) + \varepsilon_t, \\ u_t &= h(z_t) + e_t, \end{cases} \tag{1}$$

where the process noise ε_t and the measurement noise e_t are independent random variables with known probability density functions $p_\varepsilon(\varepsilon_t)$ and $p_e(e_t)$, respectively. Arbitrary, often non-linear functions $f(z_t) : \mathbb{R}^n \to \mathbb{R}^n$ and $h(z_t) : \mathbb{R}^n \to \mathbb{R}^m$ describe the evolution of the state variables, z_t, and the measurements, u_t, over time.

Suppose that the measurements up to time t, $u_{0:t}$, are avaliable. Then the filtering probability density for the state variables, $p(z_t|u_{0:t})$, is derived using Bayes' formula,

$$p(z_t|u_{0:t}) = \frac{p(u_t|z_t)p(z_t|u_{0:t-1})}{p(u_t|u_{t-1})}, \tag{2}$$

where

$$p(u_t|u_{t-1}) = \int p(u_t|z_t)p(z_t|u_{0:t-1})\, dz_t.$$

This density can be used to estimate the expected values of the state variables according to

$$I(g(z_t)) = \mathsf{E}_{p(z_t|u_{0:t})}(g(z_t)) = \int g(z_t)p(z_t|u_{0:t})\, dz_t. \tag{3}$$

The integrals involved in (2) and (3) can be analytically evaluated only in a limited number of cases. The most important special case is the linear Gaussian state-space model, when the Kalman filtering technique is applicable. Many popular algorithms for the non-linear/non-Gaussian case, like the extended Kalman filter and Gaussian sum filter, rely on analytical approximations of the integrals [5]. The great computational power of modern computers however allows using numerical methods based on Monte Carlo integration. A complete description of sequential Monte Carlo methods can be found in [6]. In the next paragraphs we shall briefly explain the basic steps in the derivation of the particle filtering algorithm.

The particle filter approximates the density by a large set of M samples (particles), $\{z_{t|t}^{(i)},\ i = 1 \dots M\}$, where each particle has an associated normalized weight, $\tilde{w}_t^{(i)}$, such that $\tilde{w}_t^{(i)} \geq 0$ for all i and $\sum_{i=1}^M \tilde{w}_t^{(i)} = 1$. An empirical estimate of the filtering probability density function is then given by

$$\hat{p}(z_t|u_{0:t}) \approx \sum_{i=1}^{M} \tilde{w}_t^{(i)} \delta(z_t - z_{t|t}^{(i)}),$$

where $\delta(\cdot)$ is the Dirac delta function. Further, an estimate of the integral (3) is the weighted sample mean,

$$\hat{I}_M(g(z_t)) = \sum_{i=1}^{M} \tilde{w}_t^{(i)} g(z_{t|t}^{(i)}).$$

The particles are initialized at random points of the state space. The filter updates the particle locations and weights each time a new observation is avaliable. Firstly, the particle location is obtained by passing the current particles through the system dynamics:

$$z_{t+1|t}^{(i)} = f(z_{t|t}^{(i)}) + \varepsilon_{t+1}^{(i)}, \quad \text{where } \varepsilon_{t+1}^{(i)} \sim p_\varepsilon(\varepsilon_{t+1}).$$

The unnormalized weights $\omega_t^{(i)}$ are usually updated sequentially in time, and the updating coefficients are equal to the values of the conditional density of the observations, evaluated at the observed values,

$$\omega_{t+1}^{(i)} = \omega_t^{(i)} p(u_{t+1}|z_{t+1|t}^{(i)}).$$

This updating mechanism has the serious drawback that normalized weights tend to degenerate with time, in the sense that after few steps of the algorithm all but one of the normalized weights are very close to zero. As a result, a large computational effort is spent on updating trajectories with very small contribution to the final estimate. To avoid the degeneracy problem, a resampling procedure is introduced. This step consists of resampling with replacement among the predictive particles, according to the updated and normalized weights,

$$\mathbb{P}(z_{t+1}^{(i)} = z_{t+1|t}^{(j)}) = \tilde{w}_{t+1}^{(j)},\ i = 1 \dots M,$$

where normalized weights are given by

$$\tilde{\omega}_t^{(i)} = \frac{\omega_t^{(i)}}{\sum_{j=1}^M \omega_t^{(i)}}.$$

After resampling all weights are set to $1/M$.

The weight of a particle reflects how likely the obtained measurement is, given the present state. Particles with large weights have high probabilities to be drawn from the true distribution and thus have high probabilities of being resampled. At the same time, particles with low weights appear to come from the wrong distribution and have to be discarded. There are several resampling algorithms proposed in the literature, namely, simple random resampling, stratified resampling, systematic sampling and residual sampling, see [7] and [8].

If the state-space model (1) contains a linear Gaussian sub-structure, the estimates can be improved by using a marginalized particle filter [9]. Consider a state-space model, that is linear in all states and with additive Gaussian noise for some states. The state vector z_t can then be split into two parts,

$$z_t = (z_t^k,\ z_t^p)',$$

where z_t^k corresponds to the states with Gaussian dynamics and z_t^p corresponds the rest of the states. Similarly we split the vector of errors $\varepsilon_t = (\varepsilon_t^k,\ \varepsilon_t^p)'$, where $\varepsilon_t^k \sim N(0, \Sigma_\varepsilon)$ and $\varepsilon_t^p \sim p_\varepsilon(\varepsilon_t^p)$.

Then the model (1) can be rewritten in the following way:

$$\begin{cases} z_{t+1}^k &= A_t^k z_t^k + A_t^p z_t^p + \varepsilon_{t+1}^k, \\ z_{t+1}^p &= B_t^k z_{t+1}^k + B_t^p z_t^p + \varepsilon_{t+1}^p, \\ u_t &= h_t(z_t^p) + C_t z_t^k + e_t. \end{cases} \tag{4}$$

This split of the state variables and errors allows us to split the filtering probability density into two components,

$$p(z_t|u_{0:t}) = p(z_t^k|z_t^p,\ u_{0:t})p(z_t^p|u_{0:t}).$$

Here the first term can be evaluated analytically by the Kalman filter and the second term can be estimated using particle filtering. Such a combination reduces the computational complexity of the algorithm [10] and allows to obtain better estimates with the same number of particles. More about this splitting technique, also called Rao-Blackwellization, can be found in [6].

III. Models for positioning in MIMO settings

A. Movement models

The transmit antenna is assumed kept at a fixed point with a fixed orientation. The receiver moves along an arbitrary trajectory and the receiving antenna turns randomly at some time points. The position of the receiver at time t is expressed by the abscise $x_{R,t}$ and the ordinate $y_{R,t}$ of the reference point.

In the Cartesian approach, these quantities are related to the horizontal and vertical velocities $\dot{x}_{R,t}$ and $\dot{y}_{R,t}$. These velocities, in turn, are assumed to follow a simple linear Markovian model with random accelerations. The evolution of these four states is described by the system of equations

$$\begin{cases} \dot{x}_{R,t+1} = \dot{x}_{R,t} + \Delta t \cdot a_{x,t+1}, \\ \dot{y}_{R,t+1} = \dot{y}_{R,t} + \Delta t \cdot a_{y,t+1}, \\ x_{R,t+1} = x_{R,t} + \Delta t \cdot \dot{x}_{R,t+1}, \\ y_{R,t+1} = y_{R,t} + \Delta t \cdot \dot{y}_{R,t+1}, \end{cases} \tag{5}$$

where $a_{x,t}$ and $a_{y,t}$ are independent random variables with $N(0,\, \sigma_x^2/\Delta t)$ and $N(0,\, \sigma_y^2/\Delta t)$ distributions, respectively.

In the polar approach the coordinates of the receiver are related to the speed of the receiver, $v_{R,t}$, and the direction of movement, $\phi_{R,t}$. Both the velocity and the direction are assumed to follow simple linear Markovian models with random acceleration and turns. This gives the evolution equations

$$\begin{cases} v_{R,t+1} = v_{R,t} + \Delta t \cdot a_{t+1}, \\ \phi_{R,t+1} = \begin{cases} \varepsilon_{t+1} & \text{with probability } \delta, \\ \phi_{R,t} + \sqrt{\Delta t} \cdot \varepsilon_{t+1} & \\ \qquad \text{with probability } 1-\delta, \end{cases} \\ x_{R,t+1} = x_{R,t} + \Delta t \cdot v_{R,t+1} \cos\left(\phi_{R,t+1}\right), \\ y_{R,t+1} = y_{R,t} t + \Delta t \cdot v_{R,t+1} \sin\left(\phi_{R,t+1}\right). \end{cases} \tag{6}$$

The acceleration, a_t, and the turns, ε_t, are assumed to be sequences of independent random variables with $N(0,\, \sigma_a^2/\Delta t)$ and $U[-\pi,\, \pi]$ distributions, respectively.

The dynamics of the receiving antenna orientation was modeled as

$$\psi_{R,t+1} = \begin{cases} \nu_{t+1} & \text{with probability } \delta', \\ \psi_{R,t} + \sqrt{\Delta t} \cdot \nu_{t+1} & \\ \qquad \text{with probability } 1-\delta', \end{cases} \tag{7}$$

where the random turns ν_t are independent variables from a $U[-\pi,\, \pi]$ distribution.

B. MIMO propagation channel model

Recently, many different models for MIMO propagation channels have been proposed. An overview of these models can be found in [11], and the research on this topic is continued. In our simulation study we will model the channel using the geometrical approach introduced in [12]. The basic idea of this approach is to place scatterers at random and then emulate the propagation process from the transmitter to the receiver, taking into account the effect of scattering.

Consider a MIMO system with N_T transmit elements and N_R receive elements. At time t the relationship between input and output can be expressed as

$$u_t = H_t v_t + e_t, \tag{8}$$

where v_t is the N_T-vector of the transmitted signal, u_t is the N_R-vector of the received signal and e_t is an additive Gaussian noise term. The channel is described by the deterministic $N_R \times N_T$ channel response matrix H_t. A single element of this matrix, $h_{nm,t}$, is the impulse response from the m-th transmit to the n-th receive antenna element. Suppose there are N_S scatterers around the receiver and the transmitter. Then the impulse response is determined by

$$h_{nm,t} = \sum_{s=1}^{N_s} A_{s,t} \exp(j\phi_s) \exp(j\zeta_{ms}) \exp(j\eta_{ns,t}). \tag{9}$$

Here $A_{s,t}$ denotes the amplitude damping for the path between the transmitter, scatterer s and the receiver,

$$A_{s,t} = d_{s,t}^{-\nu} \cdot \alpha_s, \tag{10}$$

with the total traveling distance $d_{s,t}$, random damping α_s at the scatterer and propagation coefficient ν. Note that we assume no LOS and single scattering for all paths.

The phase shift is composed of three components: random phase shift ϕ_s at scatterer s, phase shift ζ_{ms} at the m-th transmit antenna element and phase shift $\eta_{ns,t}$ at the n-th receive antenna element. These are derived from simple geometrical relationships and given by

$$\begin{aligned} \zeta_{ms} = \frac{2\pi}{\lambda} \cdot d_{T,m} \cdot |\sin(\theta_{T,s})| \\ \cdot \operatorname{sign}(d_{T,s} - d_{T,ms}), \end{aligned} \tag{11}$$

$$\begin{aligned} \eta_{ns,t} = \frac{2\pi}{\lambda} \cdot d_{R,n} \cdot |\sin(\theta_{R,s,t} + \frac{\pi}{2} - \psi_{R,t})| \\ \cdot \operatorname{sign}(d_{R,ns,t} - d_{R,s,t}), \end{aligned} \tag{12}$$

where λ denotes the wave length and the angular parameters and the distances are explained in Figure 1.

IV. Particle filtering algorithms

Equations (5)–(7)–(8) and (6)–(7)–(8) define a state-space model with five states. The position of the receiver is involved in the measurement equation in a highly non-linear way. It influences both the amplitude damping and the phase shifts, since the coordinates of a reference point

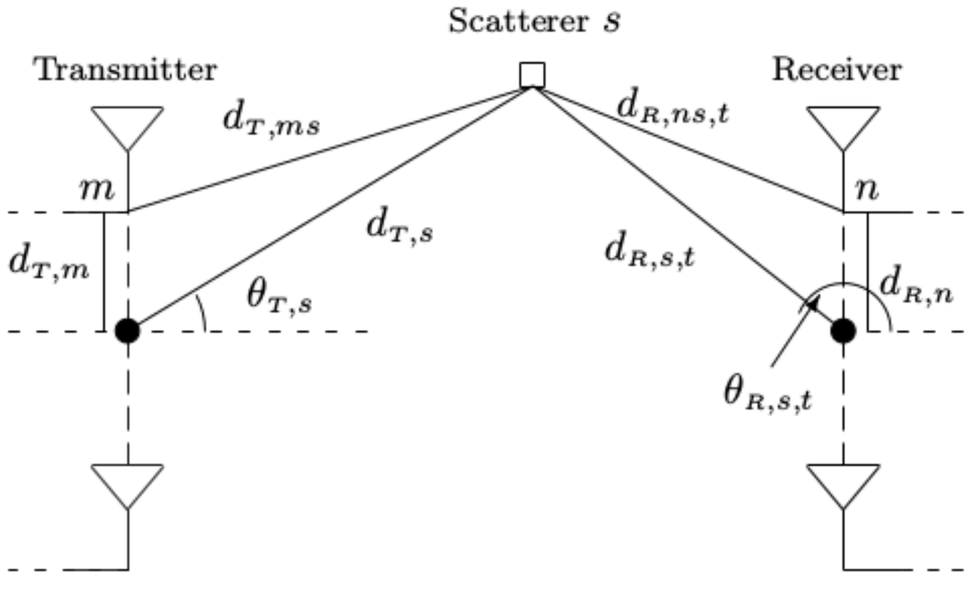

Figure 1. The distances and angular parameters used in the phase shift derivations.

at the receiver are used in the calculation of all distances in (10), (11), and (12). In addition, the direction of arrival, $\theta_{R,s,t}$, depends on the position of the receiver at time t. In the Cartesian model the velocities, however, have linear Gaussian dynamics and do not appear in the measurement equation.

The distribution of the measurement noise e_t is the N_T-dimensional complex Gaussian distribution with zero mean and covariance matrix $\mathbf{\Sigma}_e$.

The particle filter weights equal the conditional density of the observations, given the states:

$$\begin{aligned} w_t^{(i)} &= p(u_t|z_{t|t-1}) = p_e(u_t - h(z_t)) \\ &= \frac{1}{\pi^{N_T}|\mathbf{\Sigma}_e|}\times \\ &\times \exp\left\{-(\overline{u_t - H_{i,t}v_t})'\mathbf{\Sigma}_e^{-1}(u_t - H_{i,t}v_t)\right\} \end{aligned} \tag{13}$$

The estimates of the position of the receiver are simply the weighted sample means over M particles, evaluated with the normalized weights:

$$(\hat{x}_{R,t},\ \hat{y}_{R,t}) = \left(\sum_{i=1}^{M}\tilde{w}_{t,i}x_{R,i,t},\ \sum_{i=1}^{M}\tilde{w}_{t,i}y_{R,i,t}\right). \tag{14}$$

The estimate of the direction of the receiver is the circular mean direction [13]:

$$\begin{aligned} S &= \sum_{i=1}^{M}\sqrt{\tilde{w}_{t,i}}\cos\psi_{R,i,t} \\ C &= \sum_{i=1}^{M}\sqrt{\tilde{w}_{t,i}}\sin\psi_{R,i,t} \\ \hat{\psi}_{R,t} &= \begin{cases} \tan^{-1}(S/C),\ \text{if}\, S>0, C>0 \\ \tan^{-1}(S/C)+\pi,\ \text{if}\, C<0 \\ \tan^{-1}(S/C)+2\pi,\ \text{if}\, S<0,\ C>0. \end{cases} \end{aligned} \tag{15}$$

To measure the degeneracy of the algorithm we used the effective sample size estimate [14], defined as

$$\hat{N}_e = \frac{1}{\sum_{i=1}^{N}[\tilde{\omega}_t^{(i)}]^2}. \tag{16}$$

Resampling was carried out if $\hat{N}_e$ fell below 60% of the total number of particles.

The filtering algorithm is specified as follows:

1) *Initialization*, $t = 0$
   a) For $i = 1$ to M, sample

$$(x_{R,i,0},\ y_{R,i,0})' \sim U\left([a,b]\times[a',b']\right)$$

$$\psi_{R,i,0} \sim U[0,\ 2\pi]$$

$$\dot{x}_{R,i,0} \sim U[c,\ d],\ \dot{y}_{R,i,0} \sim U[c,\ d]$$

or

$$v_{R,i,0} \sim U[c,\ d],\ \phi_{R,i,0} \sim U[0,\ 2\pi]$$

   b) Set $t = 1$
2) *PF time update*
   For $i = 1$ to M, move current particles according to (6) or (5) and turn the antenna according to (7).
3) *PF measurement update*
   a) For $i = 1$ to M evaluate the channel matrix $H_{i,t}$, update the weights, normalize, calculate the effective sample size estimate. Estimate the position and the direction according to (14).
   b) If the effective sample size estimate is less than $0.6M$, resample with replacement M particles according to the normalized weights using systematic sampling. Set all weights equal to $1/M$.
4) Set $t = t + 1$ and iterate to step 2.

The algorithm for the marginalized particle filtering combines the Kalman filter for the velocities in the Cartesian model $(\dot{x}_{R,t},\ \dot{y}_{R,t})$ with the particle filter for the coordinates and the direction of the antenna.

1) *Initialization*, $t = 0$
   a) For $i = 1$ to M, sample

$$(x_{R,i,0},\ y_{R,i,0})' \sim U\left([a,b]\times[a',b']\right)$$

$$\psi_{R,i,0} \sim U[0,\ 2\pi]$$

$$\dot{x}_{R,i,0} \sim U[c,\ d],\ \dot{y}_{R,i,0} \sim U[c,\ d]$$

   b) Set $t = 1$
2) *Resampling (PF measurement update)*
   a) For $i = 1$ to M evaluate the channel matrix $H_{i,t}$, update the weights, normalize, calculate the effective sample size estimate. Estimate the position and the direction according to (14).
   b) If the effective sample size estimate is less than $0.6M$, resample with replacement M particles according to the normalized weights using the systematic sampling procedure. Set all weights equal to $1/M$.
3) *PF time update and KF update*
   a) KF measurement update:

$$(\dot{x}_{R,i,t|t},\ \dot{y}_{R,i,t|t}) = (\dot{x}_{R,i,t|t-1},\ \dot{y}_{R,i,t|t-1}),$$

$$P_{t|t} = P_{t|t-1}.$$

b) PF time update:
For $i = 1$ to M, move current particles according to (6) or (5) and turn the antenna according to (7).

c) KF time update:

$$\dot{x}_{R,i,t+1|t} = \frac{1}{\Delta t}\left(x_{R,i,t+1|t} - x_{R,i,t|t}\right)$$
$$\dot{y}_{R,i,t+1|t} = \frac{1}{\Delta t}\left(y_{R,i,t+1|t} - y_{R,i,t|t}\right)$$
$$P_{t+1|t} = P_{t|t} = \Delta t \begin{pmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{pmatrix}$$

4) Set $t = t + 1$ and iterate to step 3.

V. Simulations

The transmitting antenna is linear with $N_T = 3$ elements distanced by the half wave length, and the wave length $\lambda = 3/20$ m corresponding to 2 GHz frequency. The transmitter is located at the origin, $(x_T,\ y_T)' = (0, 0)'$, and the orientation of the transmit antenna is 90°.

The receiving antenna is also linear with three elements, $N_R = 3$, distanced by the half wave length. The receiver starts moving at (100, 0) with speed 6 km/h and direction 0°. The initial orientation of the antenna is 45°. The receiver moves along the trajectory with turns of size 90° and 45° during 3 minutes, and turns the antenna by 90° or 45° at some time points.

There are 45 scatterers, placed randomly within the area $[0, 300] \times [0, 250]$ m. The amplitude damping at each scatterer is simulated from a Rayleigh distribution with mean -6 dB, and the phase shifts are uniformly distributed between 0 and 2π. All these are fixed over time, and used as the filter input.

The standard deviation for signal noise is set to 10^{-8} for each antenna element in order to keep signal to noise ratio between 10 and 30 dB. Noises on different antenna elements are assumed to be independent, which gives the diagonal covariance matrix

$$\Sigma_e = \begin{pmatrix} 10^{-16} & 0 & 0 \\ 0 & 10^{-16} & 0 \\ 0 & 0 & 10^{-16} \end{pmatrix},$$

for the calculation of weights by (13).

The sampling rate is 100 times per sec, so $\Delta t = 0.01$ sec and $T = 18,000$. Propagation coefficient ν is set to 3.5.

Three different filters are applied: common particle filters with either Cartesian (5) or polar (6) model for the mobile movement (Filters 1 and 2, respectively) and the marginalized particle filter, based on the Cartesian model (Filter 3). All filters are run using 500 particles, with $\sigma_x^2 = \sigma_y^2 = 3$, $\sigma_a^2 = 3$ in common particle filters, and with $\sigma_x^2 = \sigma_y^2 = 2$ in the marginalized particle filter. The probabilities δ and δ' in the models of the direction of movement (6) and of the antenna orientation (7) are set to 0.01. In all three filters initial positions are sampled within the area $[95, 105] \times [-5, 5]$ m, and initial speeds are sampled from $U[1, 6]$ km/h.

To estimate the over-time performance for each filter we calculated the RMSE, based on R runs for the position estimates,

$$RMSE_t = \sqrt{\frac{1}{R}\sum_{r=1}^{R}\left[(x_t - \hat{x}_{t,r})^2 + (y_t - \hat{y}_{t,r})^2\right]},$$
$$t = 1, \ldots, T,$$

and the arithmetic sample mean over R runs for the angle estimates.

Figures 2 and 3 show the true trajectory and the true direction of the receiver together with the results for three different filters. All filters seem to perform quite well and are able to follow the track, with the estimation error not more than 25 m. Filter 1 has the largest over-time mean estimation error in the position, 4.9 m, compared to 3.5 m and 3.4 m for filters 2 and 3, respectively.

The estimation error in the orientation of the receiver is quite large at the turning times, but at the next time point decreases to less than 25° for all three filters. Over-time mean errors for all three filters are around 4°.

Figure 4 displays the RMSE for these filters based on 68 runs. Over-time performance for all filters corresponds with the results from one run. Filter 1 has largest over-time mean RMSE of 16.3 m, whereas for the filters 2 and 3 over-time mean RMSE is 5.4 m and 3.9 m, respectively. The mean estimation error in the antenna orientation is displayed in Figure 5. All three filters have similar precision of estimation, with the mean error staying below 20° most of the time.

Table I shows the Federal Communication Commission (FCC) performance requirements for the mobile location, expressed in error probability. For example, at least 67% of the positioning errors should be smaller than 100 m. To compare our results with these requirements, we have calculated the positioning error at each time point for 68 runs of three different filters, and then evaluated 67 and 95 percentiles for each time point. Maximal over-time values for three filters are given in Table II. Comparing these two tables, we see that the estimation accuracy for all three filters fits the FCC requirements for both network-based and mobile-based positioning.

We have also applied these three filters in more sophisticated situations, where the receiver moves along the circular or sinusoidal track and the receiving antenna makes a full round during the movement.

Results are displayed in Figure 6 and show good performance of all three filters in the estimation of both the position and the antenna orientation.

VI. Conclusions

Three different particles filters were applied for mobile positioning in a MIMO settings: Filter 1, based on the Cartesian model (5) for the states variables; Filter 2, based

Table I
FCC REQUIREMENTS FOR MOBILE- AND NETWORK-BASED POSITIONING, EXPRESSED IN ERROR PROBABILITY.

Error %	Mobile-based	Network-based
67	50 m	100 m
95	150 m	300 m

Table II
THE MAXIMAL PERCENTILES FOR 68 RUNS OF THREE DIFFERENT FILTERS,WITH THE MAXIMUM IS TAKEN OVER ALL TIME POINTS.

Error %	Filter 1	Filter 2	Filter 3
67	17 m	16 m	12 m
95	34 m	24 m	25 m

on the polar model (6) for states, and Filter 3, marginalized particle filter. Results, averaged over 68 independent runs of these filters, show good performance, satisfying the FCC performance requirements for the mobile location in network-based positioning as well as in mobile-based positioning. The marginalized particle filter, being the combination of Kalman filter and particle filter, shows the best performance with over-time mean RMSE of 3.9 m and 95% of positioning errors below 25 m.

In our simulation we used a very simple channel model with a small number of transmit and receive antennas. It is possible to increase the number of antennas and use more sophisticated channel models (including e.g. effects of multiple scattering) without changing the filtering algorithms. In addition, the dimensionality of the particle filter state space is independent of the number of antenna elements, as well as of the number of scatterers.

Note however, that the positioning with particle filters in these settings requires large computational power. There are two reasons for that. First, the calculations involve high-dimensional matrices, with one dimension equal to the number of particles. Second, all tested filters have high resampling rate about 98%, which means that the filters degenerate and need to resample at almost every step. The practical solution to the first problem can be to reduce the number of particles. It will increase the positioning error, but at the same time decrease the computation time. A solution for the second problem is somewhat more difficult. In our filtering algorithms we sampled particles according to the system dynamics. In other words, we chose the prior distribution of states as the sampling distribution This choice gives a simple expression for the calculation of weights, but filter may perform badly if the likelihood is peaked. As a solution, one can use so-called auxiliary particle filter, discussed by Pitt and Shephard in [4].

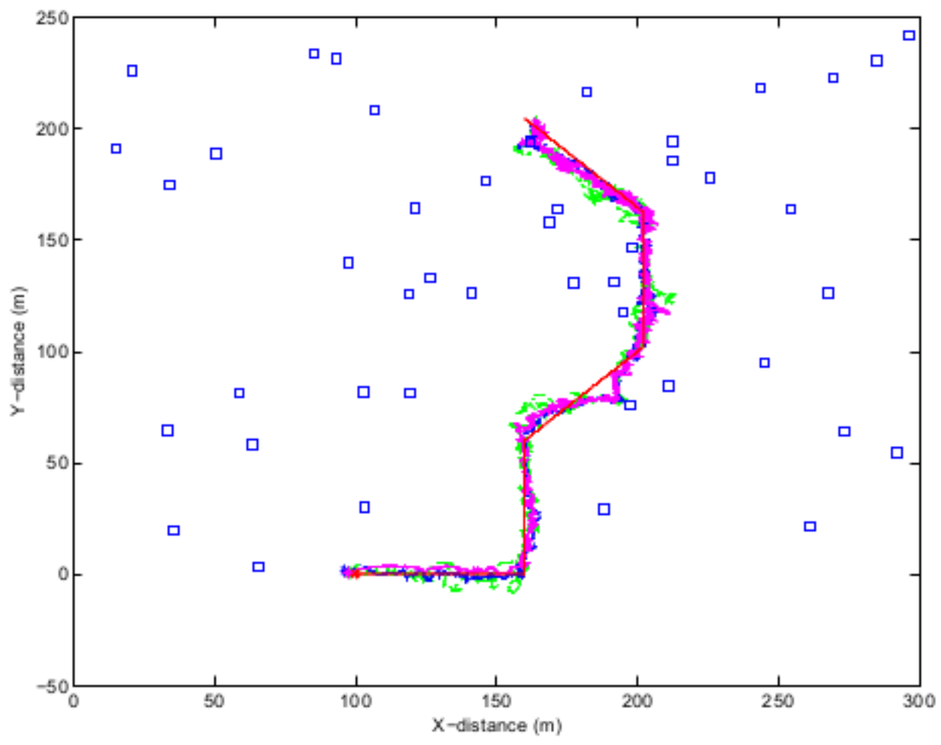

Figure 2. True trajectory and the filtering results based on one run. Stars corresponds to the initial position of the mobile (red) and to the starting positions of three filters (green: Filter 1, blue: Filter 2 and magenta: Filter 2. Squares show positions of scatterers. Green line shows the results for Filter 1, blue: Filter 2, magenta: Filter 3.

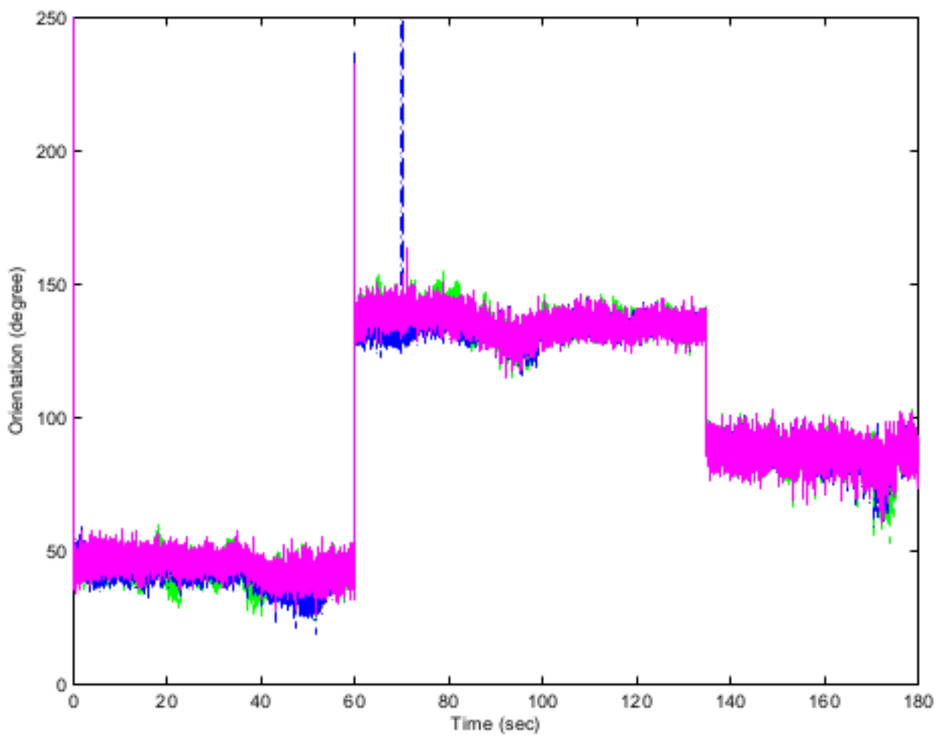

Figure 3. True orientation of antenna (red line) and the filtered one for three different filters. Over-time mean errors are 4.4°, 4.1° and 3.8° for Filter 1 (green line), Filter 2 (blue line) and Filter 3 (magenta line), respectively.

REFERENCES

[1] A. Paulraj and T. Kailath, "Increasing capacity in wireless broadcast systems using distributed transmission/directional reception," US Patent, 5 345 599, 1994.

[2] A. Paulraj, R. Nabar, and D. Core, *Introduction to Space-Time Wireless Communications*. Cambridge University Press, 2003.

[3] J. Syrjärinne, "Studies of modern techniques for personal positioning," Ph.D. dissertation, Tampere University of Technology, 2001.

[4] A. Doucet, N. de Freitas, and N. Gordon, Eds., *Sequental Monte Carlo Methods in Practice*. Springer-Verlag New York, 2001.

[5] B. Anderson and J. Moore, *Optimal Filtering*. Englewood Cliffs, 1979.

[6] A. Doucet, S. Godsill, and C. Andrieu, "On sequential Monte Carlo sampling methods for Bayesian filtering," *Statistics and Computing*, no. 10, pp. 197–208, 2000.

[7] G. Kitagawa, "Monte Carlo filter and smoother for non-Gaussian nonlinear state space model," *Journal of Computational and Graphical Statistics*, vol. 5(1), pp. 1–25, 1996.

[8] J. Liu and E. Chen, "Sequential Monte Carlo methods for dynamic systems," *Journal of American Statistical Association*, vol. 93, pp. 1032–1044, 1998.

[9] T. Schön, R. Karlsson, and F. Gustafsson, "The Marginalized Particle Filter in Practice," in *Proceedings of IEEE Aerospace Conference*.

[10] R. Karlsson, T. Schön, and F. Gustafsson, "Complexity Analysis

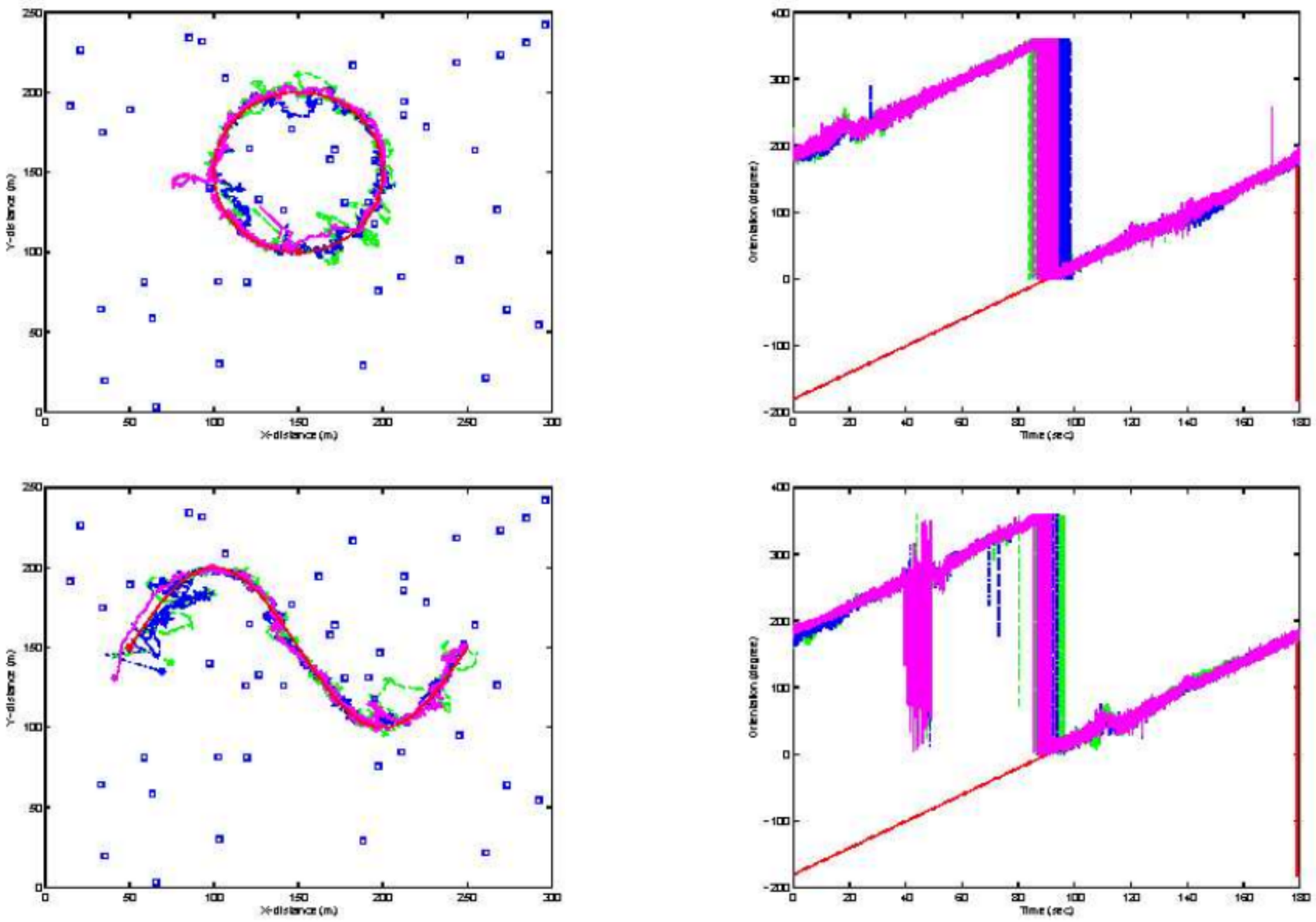

Figure 6. True trajectory (left) and true orientation of the antenna (right) together with the filtering results for the circular (upper panel) and the sinusoidal (lower panel) movement. Green lines show the results for Filter 1, blue: Filter 2, magenta: Filter 3. Over-time mean errors in the positioning do not exceed 10 m. Over-time mean errors in estimation of the antenna orientation are around 10°.

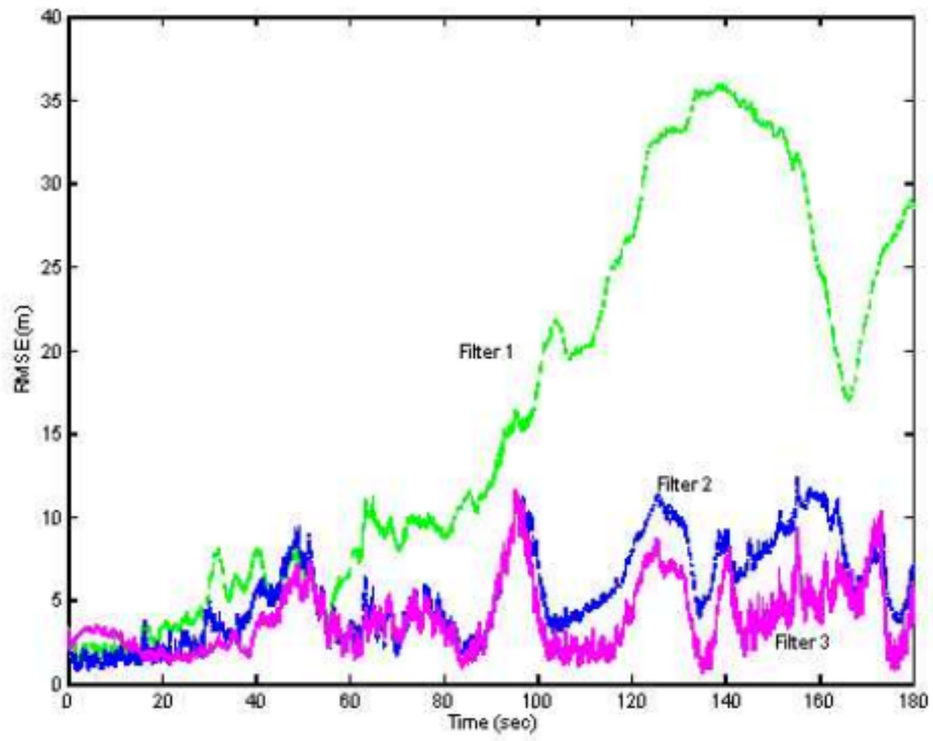

Figure 4. RMSE for the position estimate with three filters, based on 68 runs. Color code: green – Filter 1, blue – Filter 2, magenta – Filter 3.

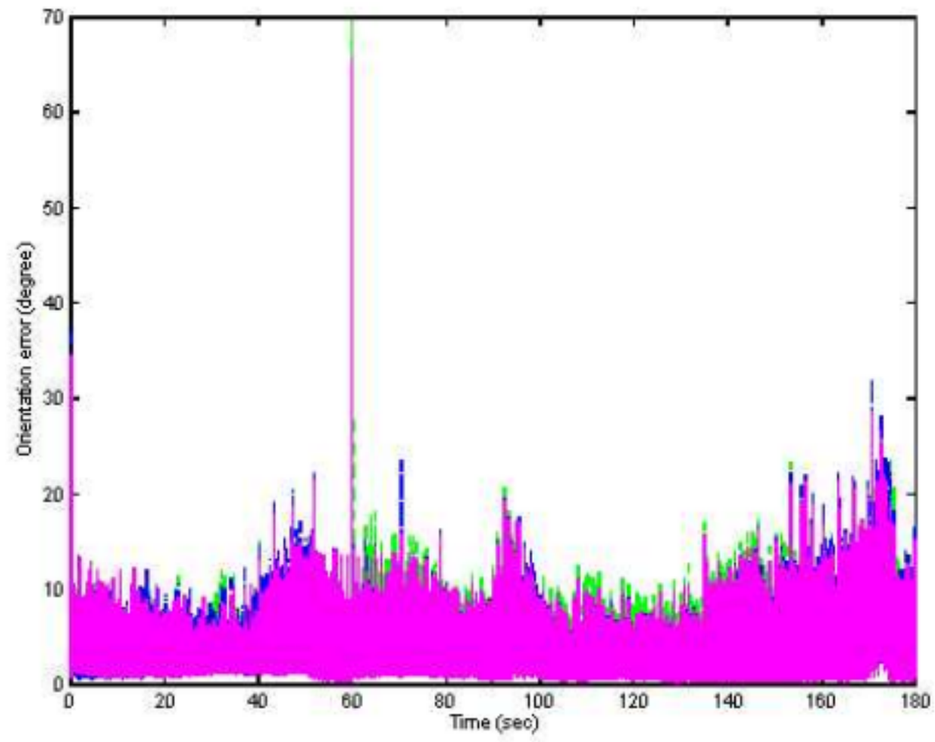

Figure 5. Mean error in the estimation of the antenna direction. Over-time mean is 5° for Filter 1 (green line), 12° for Filter 2 (blue line) and 4° for Filter 3 (magenta line).

of the Marginalized Particle Filter," *IEEE Transactions on Signal Processing*, vol. 53(11), pp. 4408–4411, 2005.

[11] K. Yu and B. Ottersten, "Models for MIMO propagation channels: a review," *Wireless Communications and Mobile Computing*, vol. 2, pp. 653–666, 2002.

[12] A. F. Molisch, "A generic model for MIMO Wireless Propagation Channels in Macro- and Microcells," *IEEE Transactions on Signal Processing*, vol. 52, no. 1, pp. 61–71, Jan 2004.

[13] N. Fisher, *Statistical Analysis of Circular Data*. Cambridge University Press, 1993.

[14] A. Kong, J. S. Lui, and W. H. Wong, "Sequential imputations and Bayesian missing data problems," *Journal of the Americal Statistical Association*, vol. 89, pp. 278–288, 1994.