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Channel Estimation in the Uplink of an OFDM System

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Abstract – This paper deals with the design of linear pilot based channel estimators in the uplink of an OFDM-based multiuser system. We evaluate such estimators and pilot patterns, showing by means of examples the importance of careful pilot positioning.

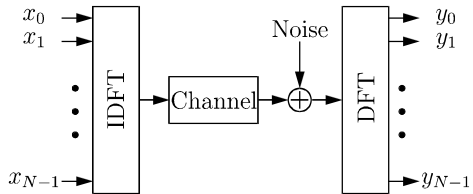


Figure 1: Baseband OFDM system model.

1 Introduction

The OFDM technique [1] has proved its potential in wired systems and in broadcast low-rate wireless systems employing differential modulation schemes [2, 3]. Coherent OFDM transmission [4], requires estimation and tracking of the fading channel.

The aim of this paper is to present and evaluate a channel estimation strategy for multi-user wireless communication systems based on OFDM. In the downlink of such a system all users can use pilot symbols transmitted by the base station on all subcarriers and the channel estimation resembles the broadcast case. In the uplink, however, the base station needs to track one physical channel for each user and can only exploit the pilot symbols transmitted by this particular user. The design of such a system involves two important steps. First, the allocation of subcarriers to the users is crucial with regard to the channel estimation. Second, both the pattern and amount of pilots is important.

The OFDM modulation technique allows a channel estimator to use both time and frequency correlation between channel attenuations. In [5] time and frequency correlation has been used separately - a combined scheme using two separate FIR-Wiener-filters, one in the frequency direction and the other in the time direction.

In this paper, we present and analyse a class of block-oriented channel estimators in the up-link of an OFDM system. This class consists of linear estimators using pilot symbols transmitted by the user terminal. The material presented in this paper is an excerpt of a more comprehensive case study of an OFDM-based multi-user system [6, 7, 8].

2 System model

Figure 1 illustrates the baseband OFDM system model we use in the sequel. We consider the transmission of complex numbers, taken from some signal constellation. Specifically, we concentrate on 4-PSK.

The data x_k are modulated on N subcarriers by an inverse discrete Fourier transform (IDFT) and the last L samples are copied and put as a preamble (cyclic prefix) [9] to form the OFDM symbol. This data vector is transmitted over the channel, whose impulse response

is shorter than L samples. The cyclic prefix is removed at the receiver and the signal is demodulated with a discrete Fourier transform (DFT). The insertion of a cyclic prefix avoids ISI and preserves the orthogonality between the tones, resulting in the simple input-output relation [1]

$$y_k = h_k x_k + n_k, \quad k = 0, \dots, N-1, \quad (1)$$

where h_k is the channel attenuation at the k th subcarrier and n_k is additive white complex Gaussian noise. In spite of the loss of transmission power and bandwidth associated with the cyclic prefix, the simple channel estimation and equalization generally motivate its use.

3 User allocation and channel correlation

As a frame-work for the oncoming analysis, we adopt the following scheme: Each user is assigned K parallel rectangular transmission blocks (size $n_t \times n_f$ data symbols in each block). After the transmission of n_t OFDM symbols, the K transmission blocks are relocated in frequency, similar to a frequency hopping system. Figure 2 shows the relevant parameters in the context of the user allocation structure.

The larger the transmission blocks are, the more channel correlation can be used by the channel estimator. However, in a real system, we want to exploit as much as possible of the available channel diversity. Therefore the optimal block size should be determined by evaluation of a coded and interleaved system. Even if channel estimation performs best for large transmission blocks, the diversity gain obtained by choosing smaller blocks, thus allowing an increased hopping frequency, may improve the overall performance. Another aspect associated with the choice of the block size is the granularity in the bit rate assigned to each user. Since one transmission block is the smallest unit allocated, the block size will usually determine the flexibility in the individual assignment of data rates to mobile terminals. A full analysis of all these aspects is beyond the scope of this paper.

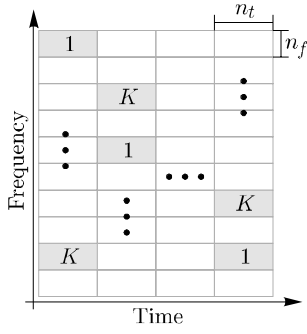


Figure 2: Channel allocation scheme in time and frequency. Each user is assigned K simultaneous transmission blocks of size n_t symbols and n_f subcarriers.

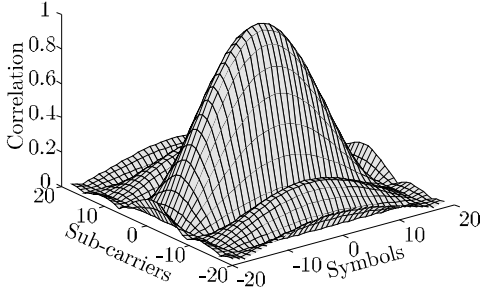


Figure 3: Time-frequency correlation (magnitude) for a uniform channel. $N = 1024$, $L = 128$, $f_d = 2\%$.

Before we start the channel estimation analysis we introduce two channel types and their corresponding statistical properties.

Example 1 Assume that the channel consists of independent impulses, each fading according to Jakes' model [10] with a maximum relative Doppler frequency f_d and that the channel has a uniform power-delay profile of the same length as the cyclic prefix. Further, assume that the time delay of each impulse is uniformly distributed over the cyclic prefix and independent of its fading amplitude. Then the correlation between channel attenuations separated by Δ_t symbols and Δ_f sub-carriers is

$$r(\Delta_t, \Delta_f) = r_t(\Delta_t) r_f(\Delta_f) \quad (2)$$

where the correlation depending on the time separation is [10]

$$r_t(\Delta_t) = J_0 \left(2\pi f_d \Delta_t \frac{N+L}{N} \right) \quad (3)$$

and the correlation depending on the frequency separation is

$$r_f(\Delta_f) = \begin{cases} 1 & \text{if } \Delta_f = 0 \\ \frac{1 - e^{j2\pi L \Delta_f / N}}{j2\pi L \Delta_f / N} & \text{if } \Delta_f \neq 0 \end{cases} \quad (4)$$

Figure 3 displays this time-frequency correlation for $N = 1024$, $L = 128$, and $f_d = 2\%$.

Example 2 We modify the channel from Example 1 so that it has an exponentially decaying power-delay profile and a lower Doppler frequency. The correlation

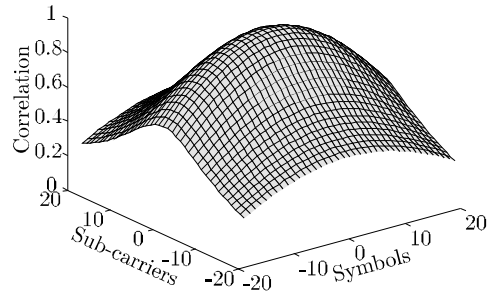


Figure 4: Time-frequency correlation (magnitude) for a channel with exponentially decaying power-delay profile. $N = 1024$, $L = 128$, $f_d = 1\%$, $\tau_{rms} = 16$.

between channel attenuations separated by symbols and sub-carriers is given by (2) where $r_t(\Delta_t)$ is the same as in the previous example and

$$r_f(\Delta_f) = \frac{1 - e^{-L(\frac{1}{\tau_{rms}} + j2\pi\Delta_f/N)}}{\left(\frac{1}{\tau_{rms}} + j2\pi\frac{\Delta_f}{N}\right) (1 - e^{-L/\tau_{rms}})} \quad (5)$$

The parameter τ_{rms} in (5) determines the decay of the power-delay profile. Figure 4 displays this time-frequency correlation for $N = 1024$, $L = 128$, $f_d = 1\%$, $\tau_{rms} = 16$. Note that the Doppler frequency is lower than in Example 1.

The coherence time Δ_{t_0} and coherence bandwidth Δ_{f_0} of the channels are given by the -3dB-crossings of the two factors $r_t(\Delta_t)$ and $r_f(\Delta_f)$, respectively. For the channel of Example 1 these widths become

$$\Delta_{t_0} \approx \frac{0.24N}{(N+L)f_d} = 10.7 \quad (6)$$

and

$$\Delta_{f_0} \approx \frac{0.6N}{L} = 4.8. \quad (7)$$

For the sake of channel estimation, we wish to shape each block (size $n_t \times n_f$) such that the correlation between channel attenuations within each block is exploited in a good way in the channel estimation. Intuitively, the transmission block should be shaped so that it resembles the shape of the peak of the correlation function as much as possible. Let us express the shape of the channel correlation function by the ratio

$$C \triangleq \frac{\Delta_{t_0}}{\Delta_{f_0}} = \frac{0.4L}{(N+L)f_d} = 0.4 \frac{\tau_c}{f_d}, \quad (8)$$

where $\tau_c = L/(N+L)$ is the length of cyclic prefix relative to the total symbol length. We choose to shape the transmission block so that it reflects the shape of the correlation, i.e., $\frac{n_t}{n_f} \approx C$. Notice that this does not affect the absolute size of each block.

Example 3 Consider a system with $N = 1024$ sub-carriers and $L = 128$ samples in the cyclic prefix. The channel is assumed to have a uniform power-delay spread in the interval $[0, L]$ and a maximal relative Doppler frequency $f_d = 2\%$. Thus $\tau_c = 1/9$ and, from expression (8), $C = 0.4 \frac{1/9}{0.02} \approx 2.2$. Based on

this we choose transmission blocks that are about twice as long in the time direction as wide in the frequency direction (measured in the number of subcarriers and OFDM symbols).

4 Channel estimation

Consider one block consisting of n_f subcarriers during n_t OFDM symbols, *i.e.*, a total of $n_t \times n_f$ transmitted data symbols. Assume that P of these data symbols are known pilot symbols with unit energy. Finally, assume that the channel covariance matrix is known. In the next section we will see that this last assumption is not critical.

Without loss of generality, we base our estimators on the least-squares (LS) estimate of the channel attenuations at the pilot positions. This LS estimate is given by [11]

$$\hat{\mathbf{h}}_{l_s, P} = \mathbf{X}_P^{-1} \mathbf{y}_P \quad (9)$$

where subscript P denotes that we address pilot positions, \mathbf{X}_P is a $P \times P$ diagonal matrix whose diagonal entries are the known pilot symbols and \mathbf{y}_P is a $P \times 1$ vector containing the received pilot symbols. Using (1), we get

$$\hat{\mathbf{h}}_{l_s, P} = \mathbf{h}_P + \tilde{\mathbf{n}}_P \quad (10)$$

where $\tilde{\mathbf{n}}_P = \mathbf{X}_P^{-1} \mathbf{n}_P$ is a noise vector. Thus, the covariance properties of \mathbf{h}_P are transferred to $\hat{\mathbf{h}}_P$ and its auto-covariance matrix becomes

$$\mathbf{R}_{\hat{\mathbf{h}}_{l_s, P} \hat{\mathbf{h}}_{l_s, P}} = \mathbf{R}_{\mathbf{h}_{l_s, P} \mathbf{h}_{l_s, P}} + \sigma_n^2 \mathbf{I}. \quad (11)$$

We now consider the class of linear estimators

$$\hat{\mathbf{h}} = \mathbf{W} \hat{\mathbf{h}}_{l_s, P}, \quad (12)$$

estimating all channel attenuations \mathbf{h} within the block, using a weighting matrix \mathbf{W} . In particular we are interested in the linear estimator that minimizes the mean-squared error. The linear minimum mean-squared error (LMMSE) estimator is given by the weighting matrix [11]

$$\mathbf{W} = \mathbf{R}_{\mathbf{h} \hat{\mathbf{h}}_{l_s, P}} \mathbf{R}_{\hat{\mathbf{h}}_{l_s, P} \hat{\mathbf{h}}_{l_s, P}}^{-1}, \quad (13)$$

where $\mathbf{R}_{\mathbf{h} \hat{\mathbf{h}}_{l_s, P}}$ denotes the cross-covariance matrix between all attenuations \mathbf{h} and the LS estimates of the attenuations at the pilot positions $\hat{\mathbf{h}}_{l_s, P}$.

The error associated with the linear estimator (12) has covariance matrix

$$\mathbf{R}_{\mathbf{e}\mathbf{e}} = \mathbf{R}_{\mathbf{h}\mathbf{h}} - \mathbf{R}_{\mathbf{h} \hat{\mathbf{h}}_{l_s, P}} \mathbf{W}^H - \mathbf{W} \mathbf{R}_{\hat{\mathbf{h}}_{l_s, P} \mathbf{h}}^H + \mathbf{W} \mathbf{R}_{\hat{\mathbf{h}}_{l_s, P} \hat{\mathbf{h}}_{l_s, P}} \mathbf{W}^H. \quad (14)$$

Since the true channel correlation properties usually are not known to the receiver, a mismatch between the design correlation and the true correlation is expected.

In the special case where we employ the true channel correlation (no mismatch) in the design of \mathbf{W} expression (14) reduces to

$$\mathbf{R}_{\mathbf{e}\mathbf{e}} = \mathbf{R}_{\mathbf{h}\mathbf{h}} - \mathbf{W} \mathbf{R}_{\hat{\mathbf{h}}_{l_s, P} \hat{\mathbf{h}}_{l_s, P}}^H.$$

Although the mean-squared error is easily calculated by the above formulae, the bottom line performance

measure in communication system is its average bit-error rate (BER). Evaluating a coded system with interleaving is beyond the scope of this paper, but we will use the above formulae in conjunction with the analytical expressions from [12] to evaluate the performance in terms of uncoded BER.

5 Performance examples

In this section we show a few examples on linear channel estimators, based on the previously described concept, and their performance in terms of uncoded 4-PSK bit-error rate (BER). We have chosen to exemplify by assuming that the exponentially decaying channel in Example 2 is the true channel of the system, and initially, that there is no correlation mismatch, and all transmitted symbols are pilots. Further, we have chosen to set the block size to 5 tones and 11 OFDM symbols, which coincides with the coherence time and coherence bandwidth of Example 1. The SNR of the system is set to $E_b/N_0 = 20$ dB.

Example 4 *In an ideal channel estimation situation we know the true channel correlation and all data transmitted is known to the receiver (pilots). Figure 5 shows the 4-PSK BER, over one transmission block, under these ideal conditions.*

We can not expect to know the channel correlation beforehand and we therefore present an example where there is mismatch in both frequency correlation and time correlation.

Example 5 *As an example design mismatch we apply an estimator designed for the channel in Example 1 to the channel in Example 2. The BER is shown in Figure 6.*

Comparing the BER of Example 5 with the BER of Example 4, a mismatched estimator design does not necessarily have any great effect on the performance of the system.

We continue our series of examples by removing the ideal pilot condition and evaluate the performance when only a small fraction of the symbols are pilots. We will use two different pilot patterns to illuminate the impact of pilot positioning. The two chosen pilot patterns are displayed in Figure 7.

The pilot densities of the two patterns are 7.3% and 9.1%, respectively. That is, about 93% and 91% of the positions are available for data transmission. We use the correlation mismatch from Example 5, and apply the two slightly different pilot patterns.

Example 6 *Using pilot pattern 1 in the channel estimation, the average BER is more than four times higher, Figure 8, compared to the ideal situation with all pilots, Figure 5.*

Example 7 *Using pilot pattern 2, instead of pilot pattern 1, in the channel estimation we reduce the BER considerably. The resulting BER is displayed in Figure 9.*

Comparing the last two examples, we see that only small changes in the pilot pattern may result in a BER reduction by a factor two.

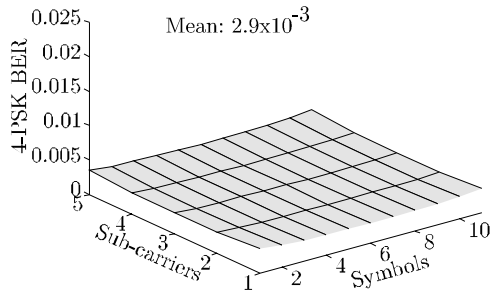


Figure 5: Uncoded 4-PSK BER without correlation mismatch and all symbols as pilots.

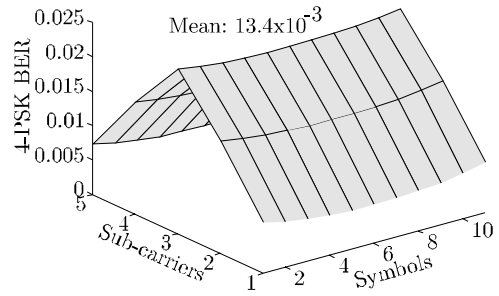


Figure 8: Uncoded 4-PSK BER with correlation mismatch and pilot pattern 1.

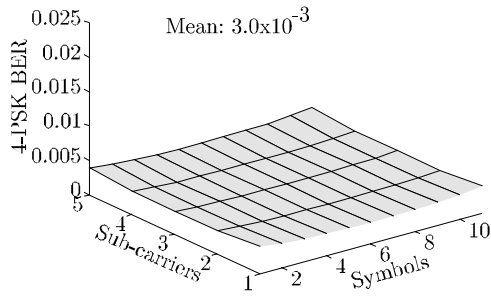


Figure 6: Uncoded 4-PSK BER with correlation mismatch and all symbols as pilots.

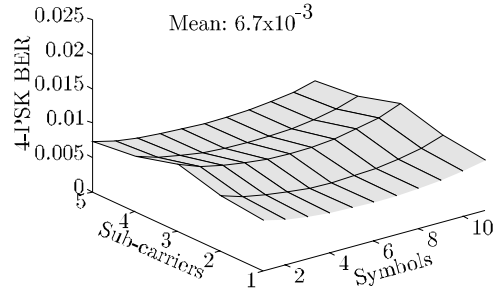


Figure 9: Uncoded 4-PSK BER with correlation mismatch and pilot pattern 2.

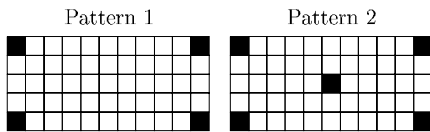


Figure 7: Pilot patterns used in Example 6 and Example 7. Pilot positions marked by ■.

6 Conclusions

The channel estimator, under the examined conditions and parameter choices, can be designed for a uniform power-delay profile and a high Doppler frequency, without a great loss in estimator performance. Further, we have shown that one of the critical tasks in the design is the positioning of pilot symbols. In the experiments an increase in pilot density from 7.3% to 9.1% reduced the uncoded 4-PSK BER by a factor two.

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