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# On Interleaver Design for Serially Concatenated Convolutional Codes<sup>1</sup>

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**Abstract**—Serially concatenated convolutional codes are considered. The free distance of this construction is shown to be lower-bounded by the product of the free distances of the outer and inner codes, if the precipices of the interleaver are sufficiently large. It is shown how to construct a convolutional scrambler with a given precipice.

## I. INTRODUCTION

An interleaver is a single input, single output, causal device which produces the output sequence  $\mathbf{y} = \dots y_{-1}y_0y_1\dots = \dots x_{\pi(-1)}x_{\pi(0)}x_{\pi(1)}\dots$ , that is, a permutation of the input sequence  $\mathbf{x} = \dots x_{-1}x_0x_1\dots$ . The invertible function  $\pi$  denotes the permutation on the input sequence indices, i.e., the output symbol  $y_j$  at depth  $j$  is the  $\pi(j)$ th symbol  $x_{\pi(j)}$  of the input sequence. The *interleaver delay* is given by  $\delta = \max_j \{j - \pi(j)\}$ .

The set of *separations* [1]  $(s, t)$  of an interleaver with permutation  $\pi$  is given by

$$\{(s, t) \mid |\pi(j) - \pi(j')| < s \Rightarrow |j - j'| \geq t, \forall j \neq j'\}$$

That is, two symbols positioned within an interval of length  $s$  in the input sequence are guaranteed to be separated by at least  $t - 1$  positions in the output sequence. Clearly, if the interleaver has the separation  $(s, t)$ , then the corresponding deinterleaver has the separation  $(t, s)$ . Furthermore, the *precipice*  $(s, t)_p$  is a separation  $(s, t)$  such that neither  $(s+1, t)$  nor  $(s, t+1)$  do exist in the set of separations. In general, an interleaver can have several precipices.

We use the concept of *convolutional interleaving* to describe the interleaver by a *convolutional scrambler* [2].

**Definition 1** An infinite matrix  $\mathbf{S} = (s_{ij})$ ,  $i, j \in \mathbb{Z}$ , that has one 1 in each row and one 1 in each column and that satisfies  $s_{ij} = 0$ ,  $i > j$  is called a *convolutional scrambler*.

The interleaved sequence is then given by  $\mathbf{y} = \mathbf{xS}$ .

## II. SERIALY CONCATENATED CONVOLUTIONAL CODES

Consider a serial concatenation of two convolutional encoders with a convolutional scrambler in between.

**Theorem 1** Let  $d_{\text{free}}$  be the free distance of a serially concatenated convolutional code. If the interleaver has at least one precipice  $(s, t)_p$  that satisfies the inequalities

$$s \geq \min\{(j_{\text{free}}^{\text{co}} + 1)c_o, (j_{\text{free}}^{\text{rco}} + 1)c_o\}$$

$$t \geq j_{2\text{free}}^{\text{bi}} b_i$$

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then

$$d_{\text{free}} \geq d_{\text{free}}^o d_{\text{free}}^i$$

where  $d_{\text{free}}^o$  and  $d_{\text{free}}^i$  denote the free distance of the outer and inner convolutional codes, respectively, and  $j_{\text{free}}^{\text{co}}$ ,  $j_{\text{free}}^{\text{rco}}$ , and  $j_{2\text{free}}^{\text{bi}}$  are derived from the active distances [3].

## III. THE $(q, r)$ CONVOLUTIONAL SCRAMBLER

A  $(q, r)$  convolutional scrambler is a convolutional scrambler  $\mathbf{S}_{(q,r)} = (s_{ij})$  with

$$s_{ij} = 1, \quad j = i + R_r(iq), \quad q + 1 < r$$

where  $\gcd(q+1, r) = 1$  and  $R_{(q+1)}(r) = 1$ . The period of this scrambler is  $T = r$  and the delay is  $\delta = r - 1$ .

**Theorem 2** Given a  $(q, r)$  convolutional scrambler, then

$$(s, t) = \left( \frac{r-1}{q+1}, q+1 \right)$$

is a precipice.

**Example 1** Consider the  $(3, 13)$  convolutional scrambler. It has period  $T = 13$  and delay  $\delta = 12$ . There is one precipice at  $(s, t)_p = (3, 4)$ . Thus, all symbols within a segment of size three in the input sequence are separated by at least three bits in the output sequence.

The  $(q, r)$  convolutional scrambler provides the possibility to realize a convolutional scrambler for a given precipice  $(s, t)_p$  by letting  $q = t - 1$  and  $r = st + 1$ . This gives an interleaver with interleaver delay  $\delta = st$ , which is the minimal required interleaver delay for the considered precipice.

**Example 2** Consider a serially concatenated convolutional code generated with one inner rate  $R_i = 1/2$  encoder with  $d_{\text{free}}^i = 5$  and one outer rate  $R_o = 2/3$  encoder with  $d_{\text{free}}^o = 3$ , both with overall constraint length  $\nu = 2$ . This construction gives a free distance of  $d_{\text{free}} = 15$ . The required precipice is  $(s, t)_p = (13, 12)$ , hence, an interleaver with delay of  $\delta = 156$  is required. This corresponds to 78 information bits.

## REFERENCES

- [1] J. L. Ramsey, "Realization of optimum interleavers", *IEEE Trans. on Inform. Theory*, vol. IT-16, pp. 338-345, Sep. 1970.
- [2] R. Johannesson and K. Sh. Zigangirov, *Fundamentals of Convolutional Coding*, IEEE Press, Piscataway, N.Y., 1999. ISBN 0-7803-3483-3.
- [3] S. Höst, R. Johannesson, K. Sh. Zigangirov, and V. V. Zyablov, "Active distances for convolutional codes", *IEEE Trans. on Inform. Theory*, vol. IT-45, pp. 658-669, March 1999.