New tailbiting encoders

Ståhl, Per; Anderson, John B; Johannesson, Rolf

Published in:
[Host publication title missing]

DOI:
10.1109/ISIT.1998.708994

1998

New Tailbiting Encoders

Per Ståhl[1]
Dept. of Information Technology
Lund University, Box 118
S-221 00 Lund, Sweden
Email per.stahl@it.lth.se

John B. Anderson[2]
ESCE Dept.
Rensselaer Poly. Inst.
Troy, NY 12180-3590
Email anderson@ecse.rpi.edu

Rolf Johannesson
Dept. of Information Technology
Lund University, Box 118
S-221 00 Lund, Sweden
Email rolf@it.lth.se

Abstract — **In a tailbiting trellis encoder, the starting state of the encoder is set to the state at which the machine will be at the end of the encoded frame. By this means the probability of decoding error at the end of the frame can be kept low without the addition of termination bits and the consequent rate loss. We report on an extensive search for short and moderate-length convolutional encoders for tailbiting trellis representations of block codes at rates 1/4, 1/3, 1/2, and 2/3. The short tailbiting representations found are typically as good as the best known block codes.**

I. Introduction

Consider a binary-symbol trellis representation of a block code of L trellis stages, a total of K information symbols, block length N, 2^b branches per trellis node, and c symbols per branch; the number of codewords is $M = 2^K = 2^{bL}$ and its rate is $R = K/N = b/c$ data bits per channel bit. For tailbiting trellis representations of the block codes, the codeword set is limited to those encoder output sequences which end in the same encoder states at which they began.

II. The Code Search

An effective strategy for finding good generators for tailbiting (TB) trellis representations is to search exhaustively for the best generators. An early such study is [1]; we search to much higher m than they do and correct some apparent errors. For TB circles out to $K = 50$ information bits, we have found the optimal generators by exhaustive search out to memory 5 at rate $R = 1/4$, $m = 6$ at rate $R = 1/3$, $m = 8$ at rate $R = 1/2$, and $m = 3$ at rate $R = 2/3$. Although these encoders suffice for most applications, good codes with longer memory are sometimes needed. A way to gain an idea of the capabilities of long memory TB encoders is to choose longer generators at random and keep the best found after, say, several days of computer search. We found this method quite effective. A complete listing of all these generators appears in [2].

The code trellis paths fall into two cases.

Case (i): (Intra minimum distance, d_{intra}) The neighbor path touches the allzero path at least once; all paths considered are within the subset of paths leaving state 0. Finding the minimum weight such path is the same as finding the free distance for a convolutional code. We call this the *intra minimum distance* for the tailbiting trellis representation.

Case (ii): (Inter minimum distance, d_{inter}) The neighbor path never touches the allzero path. We call this the *inter minimum distance* for the tailbiting trellis representation.

The minimum distance d for the tailbiting trellis representation is $d = \min\{d_{\text{intra}}, d_{\text{inter}}\}$. If the tailbiting circle is long enough d_{intra} equals the minimum distance d. Because d_{inter} might be less than d_{intra} and, hence, determine the minimum distance, TB trellis representations of block codes sometimes have quite different optimal generators than do terminated convolutional codes. The TB condition can forbid troublesome words in a code set and lead to very strong codes.

III. Results and Observations

Optimal generators for short TB trellises are very different from those of convolutional codes. Consider a fixed memory m. Long TB trellises have both the same generators and the same distance as the best memory-m ordinary codes. For TB lengths L in between (about $4m$ in the rate $R = 1/2$ case), the generators become the same as or at least roughly similar to the ordinary best free distance generators of memory m, but the achieved distance falls short.

Several especially interesting codes were found during the searches. A number of rate $R = 1/3$ (15,5,7) TB trellis representations with memory 3 were found which are equivalent to the (15,5,7) BCH code. This 8-state TB trellis is the trellis description of this BCH code with the fewest states [3]. Among the optimal rate $R = 1/2$ codes, the (24,12,8) code found is the extended Golay code. Some of our TB trellis representations beat those given in [1]. For example, one of our rate $R = 1/4$ TB trellis representations, viz., (48,12,17), ties the shortening of the (49,13,17) code. In [1] the best rate $R = 12/48$ code has $d = 16$.

IV. Conclusion

We have constructed a new list of optimal short- and moderate-length encoders for tailbiting trellis representations of block codes. These encoders are as good as the best known encoders at their rate and block length, and at the same time the codes encoded by them have important advantages over block and convolutional codes.

References

[1] B. D. Kudryashov and T. G. Zakharova, "Block codes from convolutional codes," *Problemy Peredachi Informatsii*, vol. 25, pp. 98–102, Oct.-Dec. 1989.

[2] P. Ståhl, J. B. Anderson and R. Johannesson, "Optimal and Near-Optimal Encoders for Short and Moderate-length Tailbiting Trellises," Submitted to the *IEEE Trans. on Inform. Theory*, Jan., 1998.

[3] A. Lafourcade and A. Vardy, "Lower bounds on trellis complexity of block codes", *IEEE Trans. Inform. Theory*, vol. 41, pp. 1938–1954, 1995.

[1]This research was partly supported by the Foundation for Strategic Research–Personal Computing and Communication under Grant PCC-9706-09.

[2]This research was supported in part by the Swedish Research Council for Engineering Sciences under the "Visiting Professors Programme" 1996/97.