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Active Distances and Cascaded Convolutional Codes[1]

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Abstract — **A family of active distances for convolutional codes is introduced. Lower bounds are derived for the ensemble of periodically time-varying convolutional codes.**

I. Introduction

The "extended distances" were introduced by Thommesen and Justesen [1] for unit memory (UM) convolutional codes. We present (non-trivial) extensions to encoder memories $m \geq 1$ and call them *active distances* since they stay "active" in the sense that we consider only those codewords which do not pass two consecutive zero states [2].

II. Active Distances

Consider the ensemble of binary, rate $R = b/c$, periodically time-varying convolutional codes encoded by a polynomial generator matrix of memory m and period T,

$$G = \begin{pmatrix} G_0(t) & \cdots & G_m(t+m) & & \\ & G_0(t+1) & & \cdots & G_m(t+m+1) \\ & & \ddots & & & \ddots \end{pmatrix} \qquad (1)$$

in which each digit in each of the matrices $G_i(t+T)$ for $0 \leq i \leq m$ and $0 \leq t \leq T-1$, is chosen independently and equally likely to be 0 and 1.

Let $\mathcal{U}^r_{[t_1-m,t_2+m]}$ be the set of information sequences $\boldsymbol{u}_{t_1-m} \cdots \boldsymbol{u}_{t_2+m}$ such that the first m and the last m subblocks are zero and they do not contain $m+1$ consecutive zero subblocks.

Let $\mathcal{U}^c_{[t_1-m,t_2]}$ be the set of information sequences $\boldsymbol{u}_{t_1-m} \ldots \boldsymbol{u}_{t_2}$ such that the first m subblocks are zero and they do not contain $m+1$ consecutive zero subblocks.

Let $\mathcal{U}^s_{[t_1-m,t_2]}$ be the set of information sequences $\boldsymbol{u}_{t_1-m} \ldots \boldsymbol{u}_{t_2}$ such that at least one subblock is nonzero and they do not contain $m+1$ consecutive zero subblocks.

Next we introduce the truncated time-varying generator matrix

$$G_{[t,t+j]} = \begin{pmatrix} G_m(t) & & \\ \vdots & \ddots & \\ G_0(t) & & G_m(t+j) \\ & \ddots & \vdots \\ & & G_o(t+j) \end{pmatrix}. \qquad (2)$$

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Definition 1 *Let C be a time-varying convolutional code encoded by a time-varying, polynomial generator matrix. Then the* jth order active row distance *is*

$$a^r_j \stackrel{\text{def}}{=} \min_t \min_{\mathcal{U}^r_{[t-m,t+j+m]}} w_H(\boldsymbol{u}_{[t-m,t+j+m]} G_{[t,t+j+m]}), \qquad (3)$$

the jth order active column distance *is*

$$a^c_j \stackrel{\text{def}}{=} \min_t \min_{\mathcal{U}^c_{[t-m,t+j]}} w_H(\boldsymbol{u}_{[t-m,t+j]} G_{[t,t+j]}), \qquad (4)$$

and the jth order active segment distance *is*

$$a^s_j \stackrel{\text{def}}{=} \min_t \min_{\mathcal{U}^s_{[t-m,t+j]}} w_H(\boldsymbol{u}_{[t-m,t+j]} G_{[t,t+j]}). \qquad (5)$$

For a convolutional code encoded by a time-varying, noncatastrophic, polynomial generator matrix we define its free distance as $d_{\text{free}} \stackrel{\text{def}}{=} \min_j a^r_j$.

III. Cascaded Codes

Consider a scheme with two convolutional codes in cascade.

Theorem 1 *There exist cascaded convolutional codes in the ensemble of periodically time-varying cascaded convolutional codes whose active distance satisfies*

$$\delta^r_l \stackrel{\text{def}}{=} \frac{a^r_j}{mc} \geq (l+1)h^{-1}(1 - \frac{l}{l+1}R) - O(\frac{\log_2 m}{m}) \qquad (6)$$

for $l \geq l^r_0 = O(\frac{1}{m})$,

$$\delta^c_l \stackrel{\text{def}}{=} \frac{a^c_j}{mc} \geq lh^{-1}(1-R) - O(\frac{\log_2 m}{m}) \qquad (7)$$

for $l \geq l^c_0 = O(\frac{\log_2 m}{m})$, *and*

$$\delta^s_l \stackrel{\text{def}}{=} \frac{a^s_j}{mc} \geq lh^{-1}(1 - \frac{l+1}{l}R) - O(\frac{\log_2 m}{m}) \qquad (8)$$

for $l \geq l^s_0 = \frac{R}{1-R} + O(\frac{\log_2 m}{m})$.

By minimizing the lower bound on the active row distance we obtain nothing but the main term in Costello's lower bound on the free distance, viz., $\frac{R}{-\log_2(2^{1-R}-1)}$.

References

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