Intermodulation noise realted to THD in dynamic nonlinear wideband amplifiers

Sjöland, Henrik; Mattisson, Sven

Published in:
IEEE Transactions on Circuits and Systems - 2, Analog and Digital Signal Processing

1998

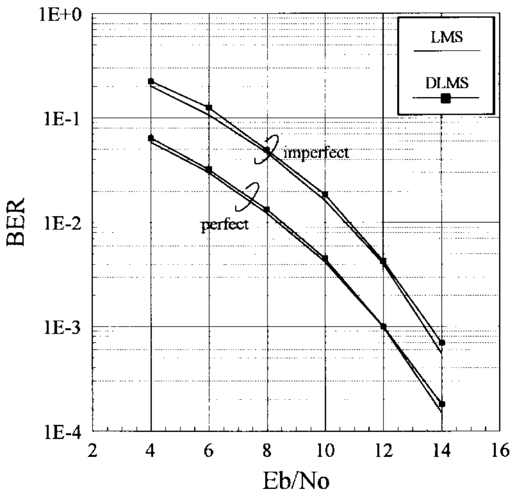

Fig. 7. BER comparison of the DLMS and LMS algorithms over channel 2 ($\beta = 0.01$).

performance was similar to the DLMS algorithm. In these cases the reduced step size and more severe channel distortion resulted in very little performance variation between the three algorithms.

In Fig. 7, the performance of the LMS and DLMS algorithms are compared in terms of the bit-error rate (BER) over channel 2 for a (6,6) DFE (500 training symbols were allocated). Perfect decision feedback and imperfect decision feedback were compared. The performance loss in this case is only marginal, since the delay in adaptation is not very large. This is expected since, for both algorithms, convergence was reliably achieved within the 500-symbol training period.

IV. Conclusion

In this paper pipelined transversal filter-based DFE's employing the DLMS training algorithm have been described. An order-recursive DFE structure was developed which allows a DFE of arbitrary length to be constructed by cascading a series of identical processing modules. Alternative filtering structures were chosen for the FFF's and FBF's in order to minimize the global communication. The performance of the new DFE's were compared using simulated channels to introduce ISI and were found to be only marginally inferior to those for the conventional DFE. However, the pipelined DFE's more than double the throughput rate of conventional structures and are very suitable for VLSI implementation. A pipelined version of the DNLMS algorithm was also proposed for a DFE, which removes the dependency of the convergence speed on the input signal power.

Acknowledgment

The authors would like to thank the other members at the Centre for Communications, University of Bristol, and the reviewers of this paper for their helpful suggestions and for pointing out some of the references.

References

[1] A. Nix, M. Li, J. Marvill, T. Wilkinson, I. Johnson, and S. Barton, “Modulation and equalization considerations for high performance radio LAN's (HIPERLAN),” in *Proc. PIMRC*, vol. 3, The Hague, The Netherlands, Sept. 1994, pp. 964–968.

[2] S. P. Smith and H. C. Torng, “A fast inner product processor based on equal alignments,” *J. Parallel Distrib. Comput.*, vol. 2, no. 4, pp. 376–390, 1985.

[3] G. Long, F. Ling, and J. G. Proakis, “The LMS algorithm with delayed coefficient adaptation,” *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 37, pp. 1397–1405, Sept. 1989.

[4] M. D. Meyer and D. P. Agrawal, “A high sampling rate delayed LMS filter architecture,” *IEEE Trans. Signal Processing*, vol. 40, pp. 727–729, Nov. 1993.

[5] J. G. Proakis, *Digital Communications*, 2nd ed. New York: Macmillan, 1989.

[6] N. J. Bershad, “Analysis of the normalized LMS algorithm with Gaussian inputs,” *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-34, pp. 793–806, Aug. 1986.

[7] S. Haykin, *Adaptive Filter Theory*, 2nd ed. Englewood Cliffs, NJ: Prentice-Hall, 1991.

[8] R. Perry, D. Bull, and A. Nix, “An adaptive DFE for high data rate applications,” in *IEEE Proc. Vehicular Technology Conf. (VTC 1996)*, vol. 2, Atlanta, GA, Apr. 1996, pp. 686–690.

[9] J. Thomas, “Pipelined systolic architectures for DLMS adaptive filtering,” *J. VLSI Signal Processing*, vol. 12, pp. 223–246, June 1996.

[10] H. Herzberg, R. Haimi-Cohen, and Y. Be'ery, “A systolic array realization of an LMS adaptive filter and the effects of delayed adaptation,” *IEEE Trans. Signal Processing*, vol. 40, pp. 2977–2803, Nov. 1992.

[11] H. Samueli, B. Daneshrad, B. C. Wang, and H. T. Nicholas, “A 64-tap CMOS echo canceller/decision feedback equalizer for 2B1Q HDSL transceivers,” *IEEE J. Select Areas Commun.*, vol. 9, pp. 839–847, Aug. 1991.

[12] K. K. Parhi, C-Y. Wang, and A. P. Brown, “Synthesis of control circuits in folded pipelined DSP architectures,” *IEEE J. Solid-State Circuits*, vol. 27, pp. 29–43, 1992.

[13] N. R. Shanbhag and K. K. Parhi, *Pipelined Adaptive Digital Filters*. Norwell, MA: Kluwer, 1994.

[14] A. Gatherer and T. H.-Y. Meng, “A robust adaptive parallel DFE using extended LMS,” *IEEE Trans. Signal Processing*, vol. 41, pp. 1000–1005, Feb. 1993.

[15] A. P. Clark and S. F. Hau, “Adaptive adjustment of receiver for distorted digital signals,” *Proc. Inst. Elect. Eng.*, vol. 131, no. 5, pp. 526–536, Aug. 1984.

Intermodulation Noise Related to THD in Dynamic Nonlinear Wide-Band Amplifiers

Henrik Sjöland and Sven Mattisson

***Abstract*—In this brief it is shown that the power of the intermodulation noise of a wide-band amplifier with dynamic nonlinearities can be estimated by the total harmonic distortion (THD) with a sinusoid input signal of appropriate amplitude and frequency. The THD is, as opposed to the intermodulation noise, easy to measure and use as a design parameter. This brief is an extension of our paper [1], which treats static nonlinearities.**

***Index Terms*—Distortion, intermodulation, wide-band amplifiers.**

I. Introduction

In [1] it was shown that the intermodulation noise power due to a static nonlinearity can be estimated by a total harmonic distortion

Manuscript received September 5, 1996; revised July 7, 1997. This paper was recommended by Associate Editor V. Porra.

The authors are with the Department of Applied Electronics, Lund University, S-22100 Lund, Sweden (e-mail: hsd@tde.lth.se).

Publisher Item Identifier S 1057-7130(98)05062-9.

(THD) measurement if the input amplitude is selected appropriately. If the variance of the wide-band signal is σ, the input amplitude A is to be selected as

$$A = 2.5\sigma. \tag{1}$$

Equation (1) was derived assuming a Gaussian input signal and similar distortion contribution from even- and odd-order nonlinearities. If odd-order nonlinearities dominate, the amplitude is to be selected higher, and if even-orders dominate, it has to be lower. The variance σ is chosen to keep the clipping distortion power below the required level [1].

In the derivation of (1) a novel approach was used. Using the probability densities of a sinusoid and a Gaussian, the error (distortion) was calculated assuming that the amplifier characteristic could be represented by a low-order polynomial. Clipping distortion was treated separately. By comparing the distortion in the sinusoid case with the Gaussian, (1) could be derived. This approach is now augmented to allow dynamic nonlinearities. The result is that we can also determine an appropriate frequency for the THD measurement.

The error of the amplifier output depends on the signal and its time derivatives and their history. This is too complex to use directly. To be able to handle each derivative by itself and ignore their history, we average the error contribution from each time derivative.

Let x be the input; the average output error is then given by

$$\bar{\varepsilon} = g_0(x) + g_1(x') + g_2(x'') + g_3(x^{(3)}) + \cdots \tag{2}$$

where $g_0(x)$ models the static nonlinearity. Note that all g_n functions give mean errors. As in [1], we assume these mean-error functions to be soft before clipping.

Amplifiers can be designed to be very linear at a certain frequency, but highly nonlinear at other frequencies. Such amplifiers can not be modeled well by (2) because they rely on cancellation of derivative terms, thus making it inappropriate to handle each derivative by itself. The required balance conditions are typically narrow-band and are thus not used in wide-band amplifiers.

II. Derivatives with Gaussian and Sinusoidal Input

Assume the input signal to be Gaussian with constant spectral density $R_x(f)$ from zero up to $f_{\max}$. Above $f_{\max}$, we assume the spectral density to be zero. If we take an nth-order derivative of the Gaussian input signal, it is also Gaussian. The spectral density is given by

$$R_{x^{(n)}} = (2\pi f)^{2n} R_x(f). \tag{3}$$

The variances of the derivatives can now be calculated by integrating the spectral densities

$$\begin{aligned}\sigma_{x^{(n)}}^2 &= \int_0^{f_{\max}} (2\pi f)^{2n} R_x(f)\, df \\ &= \frac{\sigma_x^2}{f_{\max}} (2\pi)^{2n} \cdot \frac{f_{\max}^{2n+1}}{2n+1} = \sigma_x^2 \cdot \frac{(2\pi f_{\max})^{2n}}{2n+1}.\end{aligned} \tag{4}$$

If we take a derivative of a sinusoid, the result will also be a sinusoid but with a different phase and amplitude. The phase does not affect the distribution, so we just have to consider the amplitude, where the amplitude of the nth derivative is

$$A_n = A \cdot (2\pi f)^n. \tag{5}$$

We want to find the frequency f_n, where the distortion due to the nth derivative is similar for the sinusoid and the wide-band signal. As the derivatives of a sinusoid are sinusoids and the derivatives of a Gaussian signal are Gaussian, we can use (1) that relates the distortion with a sinusoid input signal to that with a Gaussian one

$$\left.\begin{aligned} A_n = 2.5\sigma_{x^{(n)}} &\Leftrightarrow \{(4,5)\} \Leftrightarrow A^2 (2\pi f_n)^{2n} \\ &= 2.5^2 \sigma_x^2 \cdot \frac{(2\pi f_{\max})^{2n}}{2n+1} \\ A &= 2.5\sigma_x \end{aligned}\right\} \Rightarrow f_n = \frac{f_{\max}}{\sqrt[2n]{2n+1}}. \tag{6}$$

Equation (6) is the main result in this brief, and it gives the frequency to use in the THD test. Depending on the order of the dominating derivative, the frequency is to be selected differently. The amplitude is determined by the static nonlinearity.

III. Slew-Rate Clipping

Equation (1) and, hence, also (6), are only valid if the nonlinearity g_n is soft. As in [1], we therefore handle clipping separately. Clipping can occur in any derivative, but as slew-rate (SR) clipping dominates in most cases, we concentrate on that. The static clipping has already been examined in [1].

The maximum value of the time-derivative of the output is called the SR. If the demanded derivative exceeds SR, there will be large distortion called SR clipping [2].

Let D be the derivative of the input signal normalized with SR

$$D = \frac{x'}{\mathrm{SR}}. \tag{7}$$

To estimate the SR clipping distortion power we use the integral

$$\begin{aligned} &P_{\mathrm{SR\,cliptot}} \\ &\quad = \int_{-\infty}^{-1} p(D) P_{\mathrm{SR\,clip}}(D)\, dD + \int_1^{\infty} p(D) P_{\mathrm{SR\,clip}}(D)\, dD \\ &\quad = \{\text{Gaussian, Symmetry}\} \\ &\quad = 2\int_1^{\infty} \frac{1}{\sigma_D\sqrt{2\pi}} e^{-\frac{D^2}{2\sigma_D^2}} \cdot P_{\mathrm{SR\,clip}}(D)\, dD. \end{aligned} \tag{8}$$

If we compare (8) with [1, eq. (9)], which describes static clipping, we see a similarity. The exponential function determines the order of magnitude in both equations. The result is that if the function $P_{\mathrm{SR\,clip}}(D)$ is not much different from the polynomial of [1, eq. (9)], σ_D is to be selected approximately equal to σ_x for the same amount of SR clipping as static clipping distortion

$$\sigma_x \approx \sigma_D, \sigma_D \cdot \mathrm{SR} = \sigma_x \cdot 2\pi \cdot \frac{f_{\max}}{\sqrt{3}} \Rightarrow \mathrm{SR} \approx 2\pi \cdot \frac{f_{\max}}{\sqrt{3}}. \tag{9}$$

Equation (9) gives approximately the required SR capability of a wide-band amplifier when the input signal is Gaussian and has a constant spectral density. The amplifier has to be capable of producing a sinusoid with the maximum amplitude required at $f_{\max}$ divided by the square root of three, without SR clipping. This is independent of the required dynamic requirements of the amplifier, as the power of the signal will be selected small enough to keep the static clipping below the required level.

It remains to show that $P_{\mathrm{SR\,clip}}(D)$ behaves as stated. Fig. 1 illustrates a typical SR clipping scenario.

The error-voltage-time product due to the demanded SR $(d > \mathrm{SR})$ for the time interval ΔT centered around P is

$$\mathrm{VT}_{\mathrm{err}} = \Delta T \cdot (d - \mathrm{SR}) \cdot T(d). \tag{10}$$

The average error voltage due to the demanded SR d then becomes

$$\begin{aligned} V_{\mathrm{SR\,clip}}(d) &= \frac{\mathrm{VT}_{\mathrm{err}}}{\Delta T} = (d - \mathrm{SR}) \cdot T(d) \Rightarrow P_{\mathrm{SR\,clip}}(D) \\ &\approx V_{\mathrm{SR\,clip}}(D)^2 = (D-1)^2 \cdot \mathrm{SR}^2 \cdot T(D)^2. \end{aligned} \tag{11}$$

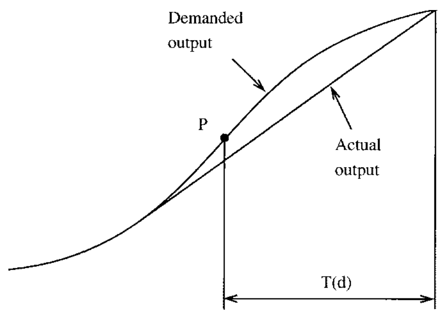

Fig. 1. An SR clipping scenario.

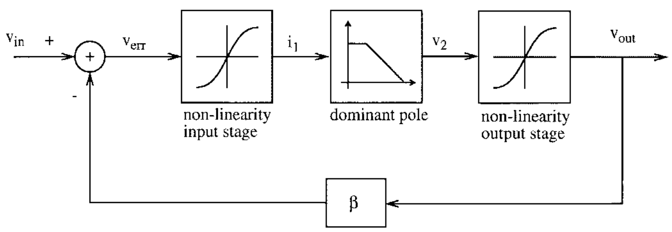

Fig. 2. Amplifier model for the numerical experiment.

$P_{\mathrm{SR\,clip}}(D)$ behaves almost as the polynomial in [1, eq. (9)]. The only difference is the $\mathrm{SR}^2 T(D)^2$ term that has a value of about one and always between zero and four.

IV. Results

The amplitude of the THD test is determined by the static nonlinearity [1], and the frequency is determined by (6). When the first-order derivative (SR) dominates the dynamic distortion, the frequency is to be selected as $f_{\max}/\sqrt{3}$. At this frequency, the amplifier must also be able to reproduce a sinusoid of full amplitude without SR clipping (9).

The THD test gives an estimate of the intermodulation distortion with a wide-band Gaussian input signal when both dynamic and static nonlinearities can be significant.

A drawback with the method presented is that the amplifier must have a small signal bandwidth much larger than the operating bandwidth in order to get an accurate THD measurement. If this is not the case, a two-tone test would be preferable. The two tones are to be selected at frequencies close to the THD test frequency. To avoid clipping we suggest the amplitudes to be $A/2$. Trigonometric equations [3] can be used to relate the intermodulation components of the two-tone test to the harmonics of a THD test without bandwidth reductions.

V. Numerical Experiment

To validate the method we performed a numerical experiment in MATLAB [4]. We used the amplifier model of Fig. 2, which models the behavior of a typical dominant pole amplifier with feedback.

The parameters were selected so that the model should behave as an audio power amplifier. The dominant pole was located at 1 kHz. The direct current (dc) gain of the dominant-pole stage was 200 kΩ, the transconductance of the input stage was 20 mS, the voltage gain of the output stage was one, and β was 1/20, resulting in a dc loop gain of 200 and a 200-kHz bandwidth. The maximum current of the input stage was ±2 mA, resulting in an SR of 2.5 V/μs referred to the output. Before clipping, the nonlinearity of the input stage was third-order compressive with a third-order intercept point of 288 mV referred to v_{err}. The maximum output voltage was set to ±20 V by output stage clipping. Before clipping, the nonlinearity of the output stage was also third-order compressive, but with an intercept point of 56 V referred to v_2.

The amplifier can just handle an output signal, at 20 kHz, of 20 V before SR clipping occurs, resulting in the SR clipping distortion being smaller than the output stage clipping distortion for a wide-band Gaussian signal with $f_{\max}$ up to 20 kHz $* \; 1.73 = 34.6$ kHz.

A Gaussian signal with constant spectral density between dc and $f_{\max} = 20$ kHz and $\sigma = 0.2$ was generated and sent through the amplifier model with the nonlinearities present and an identical model without the nonlinearities. The power of the difference between the outputs, which is equal to the power of the intermodulation noise from the nonlinear amplifier, was then calculated.

A sinusoid with the amplitude $2.8\sigma = 0.56$ V and the frequency 20 kHz$/1.73 = 11.56$ kHz was then generated and fed to the amplifier model. The amplitude was selected higher than (1) because the third-order nonlinearity dominates [1]. We got the THD figure by using a fast Fourier transform (FFT) on the output signal.

To make the MATLAB program simple, we used Forward Euler as integration method and generated an input signal with sufficiently small time steps to make the integration numerically stable.

The distortion related to maximum amplitude was 0.041% for the Gaussian signal and 0.074% for the sinusoid. To demonstrate the importance of correct frequency, we also tested a sinusoid with the same amplitude but different frequencies. The THD was 0.099% at 20 kHz and 0.014% at 2 kHz. This indicates that the test frequency is approximately correct. The estimation of the intermodulation was, in this case, pessimistic, but less than a factor of two too large.

VI. Conclusion

In this brief the statistical approach for estimating intermodulation noise in static nonlinearities of [1] has been augmented to include dynamic nonlinearities. The method results in a simple relation between THD and intermodulation distortion, which was validated by a numerical experiment. The static nonlinearity determines the amplitude of the signal in the THD test, and the dynamic nonlinearity determines the frequency.

References

[1] H. Sjöland and S. Mattisson, "Intermodulation noise related to THD in wide-band amplifiers," *IEEE Trans. Circuits Syst. I*, vol. 44, pp. 180–183, Feb. 1997.

[2] E. M. Cherry, "Transient intermodulation distortion—Part I: Hard nonlinearity," *IEEE Trans. Acoust, Speech, Signal Processing*, vol. ASSP-29, pp. 137–146, Apr. 1981.

[3] B. Westergren and L. Råde, *BETA Mathematics Handbook*. Lund, Sweden: Studentlitteratur, 1993.

[4] *MATLAB Reference Guide*, The MathWorks, Inc., Natick, MA, 1994.