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## APPENDIX

We base the assumption  $\gamma(s, x) \ll \rho(s, x)$ , made in Section III, on (24) and (31)–(33), and evaluate  $\gamma(s, x)$  and  $\rho(s, x)$ , using Table I for  $L, \rho_F, \mu, D(x)$  (Peterson),  $S_V(x)$  (Zwislocki),  $\sigma(x)$  (Allen),  $\phi(x)$  (Allen) and  $R_V = 56 \text{ kg/m}^3 \text{ s}$  (needed only to evaluate  $\gamma(s, x)$ ) [7]. We set  $s = j2\pi f$  in (24) and (31) and plot  $|\gamma(f, x)|$  and  $|\rho(f, x)|$  versus frequency (in the range 0–30 kHz) and displacement along the cochlea. These plots (Figs. 7 and 8) show that, at any position along the cochlea,  $|\gamma(f, x)|$  and  $|\rho(f, x)|$  are similar to the frequency response of a highly tuned filter. The resonant frequency is higher near the base of the cochlea and decreases toward the apex. The lowest value of  $|\rho(f, x)|$  in the stop-band region is 40 while the highest value of  $|\gamma(f, x)|$  is 1.5, which makes  $\rho$  at least 25 times larger than  $\gamma$ . At resonance, the smallest peak of  $|\rho(f, x)|$  is approximately 600, whereas the highest peak of  $|\gamma(f, x)|$  is 6, which makes  $\rho$  at least 100 times larger. Therefore, in light of these results, it is reasonable to ignore  $\gamma(s, x)$  in (25).

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## Intermodulation Noise Related to THD in Wide-Band Amplifiers

Henrik Sjöland and Sven Mattisson

**Abstract**—In this brief, it is shown that the power of the intermodulation noise of a wide-band amplifier with a Gaussian input signal, can be estimated by the total harmonic distortion (THD) with a sinusoid input signal of appropriate amplitude. The THD is, as opposed to the intermodulation noise, easy to measure and use as a design parameter. A novel method based on probability density functions is used. The method is demonstrated by a practical example, and a mathematical experiment is made to validate it.

## I. INTRODUCTION

Intermodulation occurs when two or more signals at different frequencies passes a nonlinearity. The intermodulation products (noise) of a wide-band signal are very complex, since the signal contains an infinite number of frequency components. When designing a wide-band amplifier, it is necessary to have an estimate of the linearity required to keep the intermodulation below a certain level. The complex intermodulation, therefore, has to be related to something that is easier to use as a design parameter. The total harmonic distortion THD is a common measure of nonlinearity, and is in this paper related to the intermodulation power. THD is defined as the ratio of the total energy in the harmonics and the energy in the base signal, when a single sinusoid is applied, see for example [1, pp. 18–27].

We assume that the input signal amplitude is normal distributed. This is true for Gaussian noise and it is a good approximation when several similar uncorrelated random signals are added, such as a number of radio channels of similar power. Another approximation made is that all the intermodulation noise power will be in the working band of the wide-band amplifier. If the bandwidth is large enough this is a good approximation.

Let  $x$  be the input and  $f(x)$  the output of an amplifier. To model clipping we assume:

$$f(x) = \begin{cases} f(1) & x > 1 \\ x + a_2x^2 + a_3x^3 \cdots & |x| \leq 1 \\ f(-1) & x < -1 \end{cases} \quad (1)$$

The characteristic is normalized such that the gain and the largest input signal amplitude before clipping equals one, see Fig. 1.

## II. TIME INTEGRAL FORMULATION

Assume that THD is measured for an input signal  $x = A \sin(t)$ . Let the nonlinearity be represented by

$$g(x) = a_2x^2 + a_3x^3 + \cdots \quad (2)$$

The total power due to the nonlinear terms  $g(x)$  is then

$$P_T = \frac{1}{2\pi} \int_0^{2\pi} g[x(t)]^2 dt. \quad (3)$$

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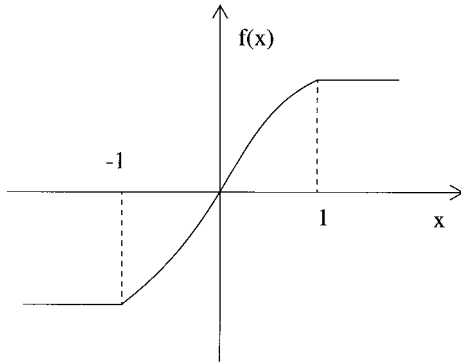


Fig. 1. The characteristic.

In Table I it is shown that  $P_T$  beside the harmonics also contains power at the fundamental and dc.

Let the input signal be  $x = s(t)$ . The intermodulation power is then given by

$$P_{IM} = \lim_{T \rightarrow \infty} \frac{1}{T} \cdot \int_0^T g[s(t)]^2 dt. \quad (4)$$

### III. PROBABILITY DISTRIBUTION FORMULATION

$P_T$  and  $P_{IM}$  can also be calculated using probability distributions. Let  $p(x)$  denote the probability density function of the input signal. The distortion power can then be calculated as the expected value of the squared difference between the ideal and actual output:

$$\begin{aligned} E\{[x - f(x)]^2\} &= \int_{-\infty}^{\infty} [x - f(x)]^2 p(x) dx \\ &= \int_{-\infty}^{-1} [x - f(-1)]^2 p(x) dx \\ &\quad + \int_1^{\infty} [x - f(1)]^2 p(x) dx \\ &\quad + \int_{-1}^1 [x - x - g(x)]^2 p(x) dx \\ &= P_{clip} + \int_{-1}^1 g(x)^2 p(x) dx \end{aligned} \quad (5)$$

$$\begin{aligned} P_{clip} &\approx \int_{-\infty}^{-1} (x + 1)^2 p(x) dx \\ &\quad + \int_1^{\infty} (x - 1)^2 p(x) dx \end{aligned} \quad (6)$$

$P_{clip}$  is the distortion due to clipping. This distortion mainly depends on the input signal distribution, and is not affected by the nonlinearity before clipping. It thus has to be treated separately and will not be related to the THD measurement, that is made with nonclipping signal levels.

Fig. 2 shows the plot of  $p_s(x)$ , the probability distribution of  $x = A \sin(t)$

$$\begin{aligned} p_s(x) &= \frac{1}{\pi} \cdot \frac{d}{dx} \arcsin\left(\frac{x}{A}\right) \\ &= \frac{1}{\pi A} \cdot \frac{1}{\sqrt{1 - \left(\frac{x}{A}\right)^2}}, \quad |x| < A. \end{aligned} \quad (7)$$

In Fig. 3 the Gaussian distribution is shown for comparison

$$p_g(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-x^2/2\sigma^2}. \quad (8)$$

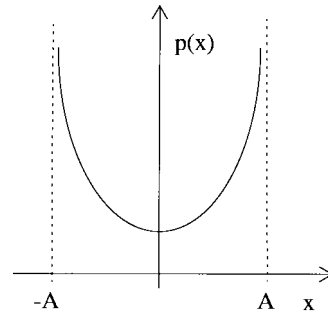
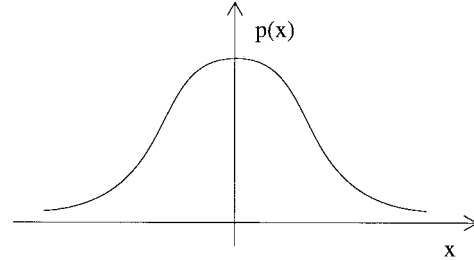

 Fig. 2. The probability density of  $A \sin(t)$ .


Fig. 3. The probability density of the normal distribution.

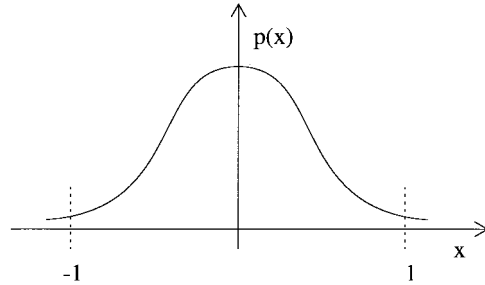


Fig. 4. The parts outside the dotted lines corresponds to clipping distortion.

### IV. THE POWER OF THE CLIPPING DISTORTION WITH GAUSSIAN INPUT

Fig. 4 illustrates that the tails of the normal distribution is the cause of the clipping distortion.

Since  $p_g(x) = p_g(-x)$  and the clipping limits are symmetrical  $+1$  and  $-1$ :

$$\begin{aligned} P_{clip} &\approx \{\text{symmetry}\} \\ &= 2 \int_1^{\infty} (x - 1)^2 p_g(x) dx \\ &= \frac{2}{\sigma\sqrt{2\pi}} \int_1^{\infty} (x^2 - 2x + 1) e^{-x^2/2\sigma^2} dx. \end{aligned} \quad (9)$$

We evaluated (9) numerically for different values of  $\sigma$ , and related it to maximum sinusoid power before clipping

$$\begin{aligned} \sigma = \frac{1}{4} &\Rightarrow P_{clip} \approx -61 \text{ dB} \\ \sigma = \frac{1}{5} &\Rightarrow P_{clip} \approx -85 \text{ dB} \\ \sigma = \frac{1}{6} &\Rightarrow P_{clip} \approx -113 \text{ dB} \\ \sigma = \frac{1}{7} &\Rightarrow P_{clip} \approx -144 \text{ dB} \end{aligned}$$

The lower the variance, the lower the probability of clipping, and hence the distortion power due to clipping. Depending on the demands a sufficiently low value of the variance (and thereby signal power) must be chosen.

V. THE POWER CONTRIBUTION OF A  $X^n$ -TERM

For the case of a sinusoidal input with the amplitude  $A$ , the total power contribution of  $x^n$  is calculated using the probability distribution of the sinusoid (7). The integral is [2, formula 82, p. 146]:

$$\begin{aligned}
 P_{T_n} &= \int_{-A}^A x^{2n} p_s(x) dx \\
 &= \frac{1}{\pi A} \int_{-A}^A \frac{x^{2n}}{\sqrt{1 - \left(\frac{x}{A}\right)^2}} dx \\
 &= \left\{ t = \frac{x}{A} \right\} \\
 &= A^{2n} \cdot \frac{1}{\pi} \int_{-1}^1 \frac{t^{2n}}{\sqrt{1-t^2}} dt \\
 &= A^{2(n-1)} \cdot A^2 \cdot \frac{1}{\pi} \cdot \frac{2n-1}{2n} \int_{-1}^1 \frac{t^{2(n-1)}}{\sqrt{1-t^2}} dt \\
 &= A^2 \cdot \frac{2n-1}{2n} \cdot P_{T_{n-1}}
 \end{aligned}
 \tag{10}$$

$$\begin{aligned}
 P_{T_0} &= \frac{1}{\pi} \int_{-1}^1 \frac{1}{\sqrt{1-x^2}} dx \\
 &= \frac{1}{\pi} \cdot \text{asin}(x) \Big|_{-1}^1 \\
 &= 1
 \end{aligned}
 \tag{11}$$

where the recursive formula (10), (11) gives

$$P_{T_n} = A^{2n} \cdot \frac{2n-1}{2n} \cdot \frac{2n-3}{2n-2} \cdot \frac{2n-5}{2n-4} \cdot \dots \cdot \frac{1}{2}
 \tag{12}$$

For the case of Gaussian input the integral instead becomes

$$\begin{aligned}
 P_{IM_n} &= \int_{-1}^1 x^{2n} p_g(x) dx \\
 &= \int_{-1}^1 x^{2n} \frac{1}{\sigma\sqrt{2\pi}} e^{-(x^2/2\sigma^2)} dx.
 \end{aligned}
 \tag{13}$$

VI. RESULTS

We want to find the amplitude  $A$  of a sinusoid input signal, so that the THD power generated is the same as the intermodulation power generated by a wide-band signal of a given variance. The variance is limited by the maximum acceptable clipping distortion (9). By measuring the THD with the input amplitude  $A$  we can then estimate the intermodulation distortion, and thereby we simplify the intermodulation distortion measurement.

As opposed to (13) it is possible to integrate (14) analytically [2, formula 42, p. 164]. The result (15) is close to the THD/IM-quotient when  $A$  is less than about 0.5, since the clipping then has negligible effect on the intermodulation.  $H(n)$  is the harmonic power content, see Table I.

$$\begin{aligned}
 P_{IM_n(\text{unbound})} &= \int_{-\infty}^{\infty} x^{2n} \frac{1}{\sigma\sqrt{2\pi}} e^{-(x^2/2\sigma^2)} dx \\
 &= \dots \\
 &= (2n-1)(2n-3) \cdot \dots \cdot 1 \cdot \sigma^{2n}
 \end{aligned}
 \tag{14}$$

$$\Rightarrow \left( \frac{\text{THD}}{\text{IM}} \right)_n = \left( \frac{A}{\sigma} \right)^{2n} \cdot \frac{1}{2n \cdot (2n-2) \cdot (2n-4) \cdot \dots \cdot 2} \cdot H(n)
 \tag{15}$$

We seek a value of  $A/\sigma$  that make the THD/IM-quotient as close to one as possible for different  $n$  values. In Fig. 5 THD/IM versus  $A/\sigma$  for different  $n$  is plotted for  $A$  fixed to 0.25. From this figure it is clear that (16) is a reasonable choice. A plot of THD/IM versus  $A$  and  $n$  when the input amplitude is selected according to (16) is

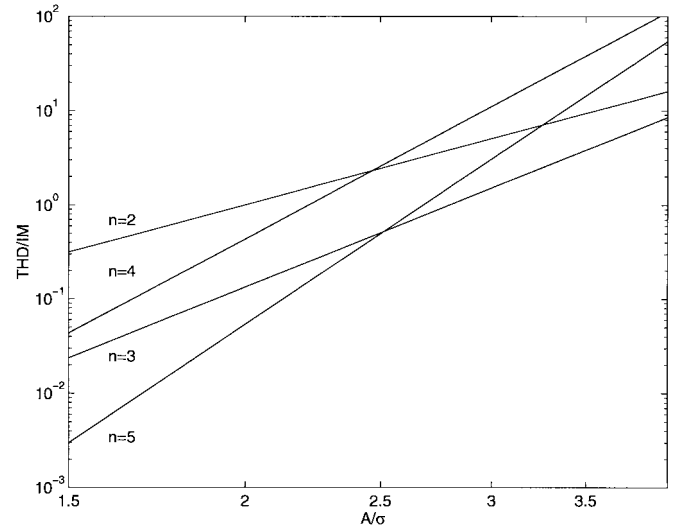


Fig. 5. THD/IM-quotient versus  $A/\sigma$  for fixed  $A = 0.25$ .

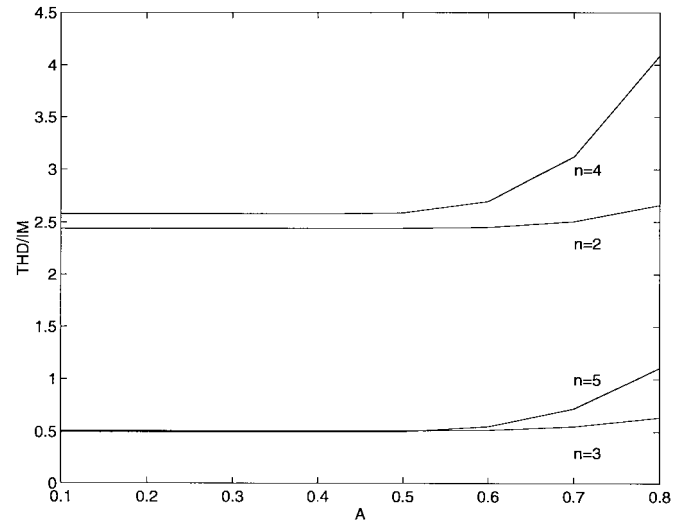


Fig. 6. THD/IM-quotient versus  $A$  and  $n$  for  $A/\sigma = 2.5$ .

shown in Fig. 6. The bend upward at  $A > 0.5$  is due to the clipping limits reducing (13).

$$A = 2.5\sigma.
 \tag{16}$$

If there is as much contribution from even as from odd order nonlinearities, the amplitude should be selected according to (16). The THD/IM-quotient of the dominating orders are to be as close to one as possible, so if the odd orders dominate, the amplitude should be slightly increased, and if the even orders dominate, the amplitude should be slightly decreased. This is due to the smaller harmonic content  $H(n)$  of the odd orders in a THD test.

VII. PRACTICAL PROBLEMS AND EXAMPLE

If a nonGaussian signal dominates the input, the resulting distribution would not be Gaussian. Typically, it is possible, though, to perform the numerical integrations and find a suitable value of  $A$  for almost any practical distribution, using the methods described in this brief. To keep  $P_{clip}$  down,  $p(x)$  must be low at the clipping limits. For a sinusoid on the other hand,  $p(x)$  is infinite at the peaks. Probably every practical input signal therefore gives less distortion (excluding clipping) than a sinusoid of maximum amplitude before clipping.

TABLE I  
DISTRIBUTION OF DISTORTION POWER DUE TO DIFFERENT ORDERS OF NONLINEARITY

order	trigonometric expression [2]	harmonic power	fundamental power	DC power
2	$\sin^2 t = -0.5 (\cos 2t - 1)$	50%	0	50%
3	$\sin^3 t = -0.25 (\sin 3t - 3\sin t)$	10%	90%	0
4	$\sin^4 t = 0.125 (\cos 4t - 4\cos 2t + 3)$	65%	0	35%
5	$\sin^5 t = 1/16 (\sin 5t - 5\sin 3t + 10\sin t)$	20%	80%	0

TABLE II  
IM AND THD RELATED TO MAXIMUM FUNDAMENTAL VOLTAGE BEFORE CLIPPING,  $\sigma = V_{\max}/4$  AND  $A = 2.5 \sigma$

type	$A_{\text{loop}}$	IM	THD	type	$A_{\text{loop}}$	IM	THD
FET	10	0.26%	0.32%	BJT	24	0.26%	0.33%
	100	0.031%	0.039%		210	0.031%	0.039%

The nonlinearity of most amplifiers depends on frequency. A suitable test frequency must be found for the THD-measurement. Another problem is to find how the intermodulation noise is distributed in the frequency plane. This paper does not treat this distribution, but just the total power of the intermodulation products.

The harmonics belonging to different order  $x^n$ -terms can add in or out of phase, affecting THD but not intermodulation. The expectation value of the THD power is, however, the powers of the different orders added together, as assumed in this brief. Unless the circuit relies on term-cancellation, this is not likely to become a problem.

A wide-band IF-amplifier is to be designed. The noise power in the operating band is 60 dB below maximum sinusoidal output power, and we want the intermodulation power to be below this.

The input clipping limit is  $V_{\max} = 17.5$  mV. We assume  $\sigma = V_{\max} \cdot 1/4$ , giving a clipping distortion of about -61 dB. Then we choose the input amplitude  $A$  for the THD-test. In this example, the odd order terms dominate. At  $A = 12$  mV the THD/IM quotients for the odd orders is near one [(16) suggests 10.9 mV]. We pessimistically chose the highest operating frequency, 20 MHz, as the test frequency. This is no problem as the bandwidth of the amplifier is about 200 MHz, resulting in a correct measurement of the first nine harmonics.

In the case when the intermodulation products are uniformly spread, the THD is to be below -60 dB  $-20 \log (12/17.5) = -57$  dB = 0.14%. (The  $20 \log (12/17.5)$ -term relates the harmonic power to  $V_{\max}$  instead of  $A$ .) However, in the case when the signal consists of 100 radio channels, and all intermodulation products pessimistically are assumed to be in the same channel, the THD requirement will be  $-57$  dB  $-10 \log 100 = -77$  dB = 0.014%.

## VIII. MATHEMATICAL EXPERIMENT

It is very difficult to measure intermodulation noise power, as it is very hard, or impossible, to separate the noise and signal components. Because of this a mathematical experiment was made to validate our method.

The program MatLab was used to study two different nonlinearities with negative feedback applied. An ideal FET (square law) and an ideal BJT (exponential) were examined. The harmonic distortion of an ideal transistor with negative feedback is treated in [3] and [4]. The clipping limits were set such that the fundamental peak current was 75% of the quiescent current. The reason for not choosing 100% is that the exponential characteristic never reaches zero. In addition, if the limit is set high, the high-order terms might become dominant for large signals, and  $A$  might have to be adjusted. The results in Table II, show that for both transistor types the THD-estimate was quite close.

## IX. CONCLUSIONS

In this brief, a new statistical approach for estimating intermodulation noise has been presented. The method results in a simple relation between THD and intermodulation distortion, that was validated by a mathematical experiment.

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