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Abstract

A simple and intuitive metric of multiplexing efficiency is proposed for evaluating the performance of MIMO antennas in the spatial multiplexing mode of operation. The metric is particularly useful for antenna engineers whose goal is to achieve the optimum antenna system design. Experimental results involving prototype mobile terminals highlight the effectiveness of our proposal.

1   Introduction

Despite intense academic research in multiple-input multiple-output (MIMO) technology for over two decades [1], and its recent adoption in major wireless standards, performance characterization of multiple antenna terminals is a subject of current interest. Depending on the signal-to-noise ratio (SNR) of the received signal, different MIMO modes are required to optimise the system performance. For the low SNR regime, diversity techniques are applied to mitigate fading and the performance gain is typically expressed as diversity gain (in decibel, dB) [2]. Such a measure is convenient for antenna designers, since performance improvement is translated into a tangible power gain, or equivalently, an increase in coverage area.

On the other hand, higher SNR facilitates the use of spatial multiplexing (SM), *i.e.*, the transmission of parallel data streams, and information theoretic capacity in bits per second per Hertz (bits/s/Hz) is the performance measure of choice [3]. However, capacity is a system level metric that is not intuitive to antenna engineers who would prefer a power related measure, such as the diversity gain. Moreover, since SM is the primary mechanism for increasing the spectral efficiency of MIMO systems, it is important to consider it explicitly in antenna design.

This report introduces *multiplexing efficiency* as a power related metric for the SM mode of operation and derives its approximate closed form expression. An example application is given for two realistic mobile terminal prototypes.

2   Multiplexing Efficiency Metric

For a $M \times M$ MIMO channel $\mathbf{H}$, the ergodic channel capacity without channel information at the transmitter (*i.e.*, equal transmit power allocation) can be expressed as [3]

$$\bar{C} = \mathbb{E}\left\{ \log_2 \det\left(\mathbf{I}_M + \frac{\rho_{\mathrm{T}}}{M}\mathbf{H}\mathbf{H}^H \right) \right\}, \tag{2.1}$$

where the signal-to-noise ratio (SNR) ρ_{T} is defined by $\rho_{\mathrm{T}} = P_{\mathrm{T}}/\sigma_{\mathrm{n}}^2$. P_{T} denotes the transmit power and σ_{n}^2 is the noise power at the receiver.

Since the interest of this report is in antenna design, the reference propagation environment of i.i.d. Rayleigh fading channel $\mathbf{H}_{\mathrm{w}}$ is assumed. Without loss of generality, the case of receive antennas is examined. Then, the MIMO channel is given by

$$\mathbf{H} = \mathbf{R}^{1/2}\mathbf{H}_{\mathrm{w}}, \tag{2.2}$$

where $\mathbf{R}$ denotes the receive correlation matrix which fully describes the effects of the antenna on the channel, *i.e.*, it characterizes the efficiency, efficiency imbalance and correlation among the receive antennas. Specifically, $\mathbf{R} = \boldsymbol{\Lambda}\tilde{\mathbf{R}}$, where $\tilde{\mathbf{R}}$ is a normalized correlation matrix with diagonal elements of 1, and $\boldsymbol{\Lambda}$ denotes a diagonal matrix of antenna efficiencies η_i given as

$$\boldsymbol{\Lambda} = \begin{bmatrix} \eta_1 & & & \\ & \eta_2 & & \\ & & \ddots & \\ & & & \eta_M \end{bmatrix}. \tag{2.3}$$

In order to obtain a reliable estimate of the multiplexing capability of the antennas, it is noted that at high SNRs, (2.1) can be written as [3]

$$\bar{C} \approx \mathbb{E}\left\{\log_2 \det\left(\frac{\rho_{\mathrm{T}}}{M}\mathbf{H}_{\mathrm{w}}\mathbf{H}_{\mathrm{w}}^H\right)\right\} + \log_2 \det(\mathbf{R}), \tag{2.4}$$

where the first term denotes the capacity of the ideal i.i.d. Rayleigh channel at high SNR, which is achieved with ideal antennas in uniform 3D angular power spectrum (APS), *i.e.*, $\mathbf{R} = \mathbf{I}_M$. Ideal antennas are 100% efficient and completely orthogonal to one another in radiation patterns (either in space and/or polarization).

Since $\log_2 \det(\boldsymbol{\Lambda}) \leq 0$ and $\log_2 \det(\tilde{\mathbf{R}}) \leq 0$ (see also [3]) in (2.4), non-ideal antenna effects will result in a constant degradation in the ergodic capacity over SNR, relative to that of the i.i.d. channel. However, the absolute capacity gap does not lend itself to a convenient interpretation, and its relative impact on the achieved capacity changes with SNR. On the other hand, translating this gap into a power related measure solves these problems. In this context, the multiplexing efficiency (in dB) is defined as the SNR (or power, assuming the noise σ_{n}^2 is the same) penalty in the real multiple-antenna prototype to compensate for the antenna effects and restore the ergodic capacity of the i.i.d. channel $\bar{C}_0$ (*i.e.*, $\bar{C} = \bar{C}_0$ in (2.1) when $\mathbf{H} = \mathbf{H}_{\mathrm{w}}$), or equivalently

$$\eta_{\mathrm{mux}} = \rho_0 - \rho_{\mathrm{T}} \leq 0 \ [\mathrm{dB}], \tag{2.5}$$

where ρ_0 is the reference SNR (taken at the high SNR regime, *e.g.*, $20\,\mathrm{dB}$) used with the i.i.d. Rayleigh channel to achieve the capacity $\bar{C}_0$, whereas ρ_{T} is the SNR required by the real antennas to achieve $\bar{C}_0$ in the same channel.

In practice, it is convenient to have a closed form expression for multiplexing efficiency in terms of antenna efficiency, efficiency imbalance and correlation, in the same manner as diversity gain. Therefore, it is undesirable to calculate ergodic capacity from a large number of realizations of the channel matrix, as is commonly the case. Using Jensen's inequality, the ergodic capacity is upper bounded by C_{ub} [4]

$$\bar{C} \leq \bar{C}_{\mathrm{ub}} = \log_2 \det\left(\mathbf{I}_M + \frac{\rho_{\mathrm{T}}}{M}\mathbb{E}\{\mathbf{H}\mathbf{H}^H\}\right) = \log_2 \det\left(\mathbf{I}_M + \frac{\rho_{\mathrm{T}}}{M}\mathbf{R}\right). \tag{2.6}$$

For i.i.d. channels, $\mathbf{H} = \mathbf{H}_{\mathrm{w}}$ and $\frac{1}{M}\mathbb{E}\{\mathbf{H}\mathbf{H}^H\} = \mathbf{I}_M$, thus $\bar{C}_{\mathrm{ub}}$ reduces to

$$\bar{C}_{0,\mathrm{ub}} = M \log_2 (1 + \rho_0). \tag{2.7}$$

Consequently, η_{mux} can be approximated using these capacity bounds by substituting $\rho_{\mathrm{T}} = \rho_0/\eta_{\mathrm{mux}}$ (*i.e.*, linear scale) into (2.6) and then equating the expression with (2.7), *i.e.*, solving for η_{mux} in the following:

$$(1 + \rho_0)^M = \det\left(\mathbf{I}_M + \frac{\rho_0/\eta_{\mathrm{mux}}}{M}\mathbf{R}\right). \tag{2.8}$$

In general, the result is a polynomial in η_{mux} of order M. Therefore, fast and efficient numerical root-finding algorithms may be used and the solutions filtered with the requirement $0 \leq \eta_{\mathrm{mux}} \leq 1$. Furthermore, for a polynomial of up to degree four, a closed form solution for the roots is known.

3   Case Study: 2×2 MIMO

For two receive antennas, the antenna efficiency and normalized correlation matrices in $\mathbf{R} = \mathbf{\Lambda}\tilde{\mathbf{R}}$ are given by

$$\mathbf{\Lambda} = \left[\begin{array}{cc} \eta_1 & 0 \\ 0 & \eta_2 \end{array}\right], \tilde{\mathbf{R}} = \left[\begin{array}{cc} 1 & r \\ r^* & 1 \end{array}\right]. \tag{3.1}$$

where r is the complex correlation. The procedure outlined above in (2.8) is used to obtain a polynomial in η_{mux} as

$$(2 + \rho_0)\eta_{\mathrm{mux}}^2 - (\eta_1 + \eta_2)\eta_{\mathrm{mux}} - \eta_1\eta_2(1 - |r|^2)\rho_0 = 0. \tag{3.2}$$

The quadratic formula can be then used to derive a closed form expression to the approximate multiplexing efficiency, given as

$$\eta_{\mathrm{mux}} = \frac{2(1 - |r|^2)\eta_1\eta_2\rho_0}{\sqrt{(\eta_1 + \eta_2)^2 + 4\eta_1\eta_2(1 - |r|^2)(2 + \rho_0)\rho_0} - (\eta_1 + \eta_2)}. \tag{3.3}$$

Recall that the gap in capacity as indicated in (2.4) is a constant and does not depend on SNR, which implies that at a sufficiently high SNR, η_{mux} should also be a constant. Indeed,

$$\lim_{\rho_0 \to \infty} \eta_{\mathrm{mux}} = \sqrt{\eta_1\eta_2(1 - |r|^2)} \tag{3.4}$$

which shows that the multiplexing efficiency is determined by the geometric mean (or the arithmetic mean in dB scale) of the antenna efficiencies together with a correlation induced term. This implies that the impact of correlation and efficiency imbalance on η_{mux} can be studied separately, as shown in Fig 1. The equation (3.4) also shows that when the antenna efficiencies are the same, the multiplexing efficiency contains the same efficiency value.

In Fig 1(a), it is observed that the multiplexing efficiency is relatively insensitive to low to moderate values of correlation, with the decrease in efficiency of lower than $1\,\mathrm{dB}$ for correlation of up to 0.6. However, as the correlation increases beyond 0.6, the multiplexing efficiency decreases more severely. This observation is consistent with the rule of thumb that the influence of correlation on diversity gain becomes
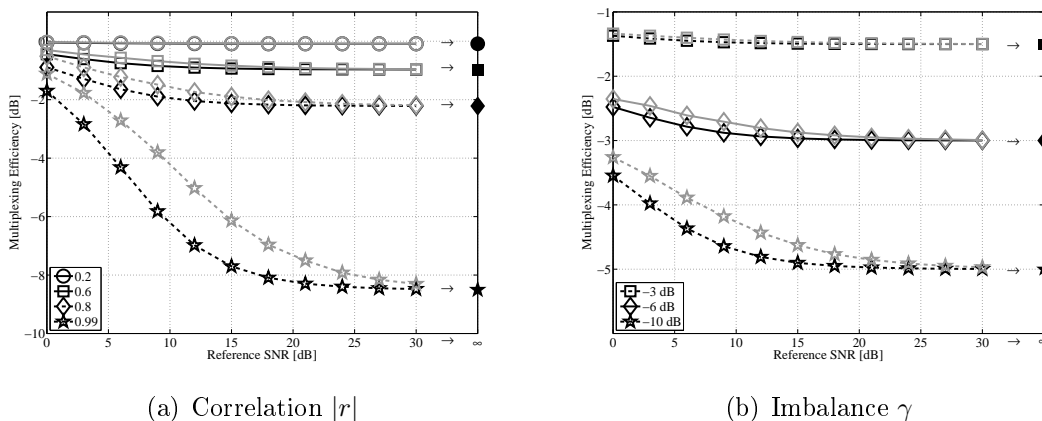
(a) Correlation $|r|$ (b) Imbalance γ

Figure 1: Multiplexing efficiency due to (a) antenna correlation ($\eta_1 = \eta_2 = 1$) and (b) efficiency imbalance ($r = 0$ and $\eta_1 = 1, \eta_2 = \gamma$ in $\mathbf{\Lambda}$). The darker curves denote η_{mux} derived from the capacity upper bounds (3.3) and the lighter curves denote the exact results obtained from Monte Carlo simulation, respectively. The limits of η_{mux} from (3.4) are also shown with filled markers.

significant for correlation of above 0.7. In addition, the rate of convergence to the limiting value with SNR decreases significantly when the correlation is increased. Nevertheless, convergence is achieved at 30 dB SNR even for the highly unlikely extreme correlation of 0.99. This indicates that the approximate closed form expression of η_{mux} in (3.3) is accurate for practical prototypes (as is confirmed by the later examples) at commonly used reference SNR values, *e.g.*, $\rho_0 = 20$ dB. In any case, Fig 1(a) also reveals that the approximate solution of (3.3) is a conservative estimate, which gives a lower bound to the exact η_{mux}. Fig 1(b) confirms that at sufficiently high SNR and with $|r| = 0$, the multiplexing efficiency is the arithmetic average of the individual antenna efficiencies (in dB). Last but not the least, results in Fig 1 confirm that at sufficiently high SNR, η_{mux} converges to a constant as indicated by (3.4).

4   Numerical Results

To illustrate the effectiveness of the proposed metric for characterizing MIMO capability, two realistic mobile terminal prototypes are evaluated (see Fig 2(a)). Each

Table 1: Performance characteristics of prototypes P1 and P2.

		P1	P2
Correlation $|r|$		0.80	0.19
Efficiency	η_1	-4.7 dB	-3.9 dB
	η_2	-5.2 dB	-4.2 dB
Multiplexing Efficiency η_{mux}		-7.2 dB	-4.2 dB
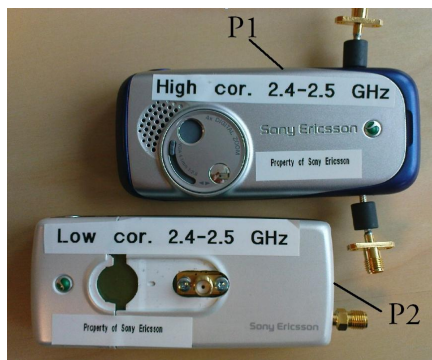
(a) Prototype photo
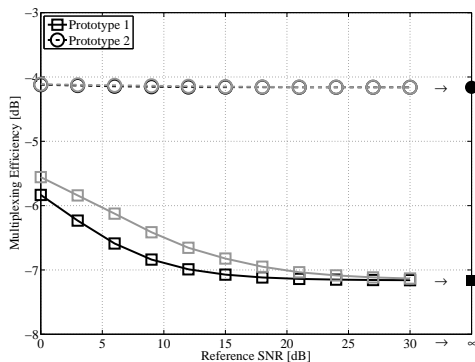
(b) Multiplexing efficiency metric

Figure 2: (a) Photo of the two terminal prototypes P1 and P2. (b) Multiplexing efficiency of P1 and P2. The darker curves denote the approximate η_{mux} and the lighter curves denote the exact results obtained from simulation, respectively. The limits of η_{mux} from (3.4) are also shown with filled markers.

of the test prototypes is fully equipped as a normal mobile terminal (with plastic casing, display screen, circuit board/components, etc.) and has two well-matched antennas operating in the 2.45 GHz frequency band. The antennas for prototype "P1" is intentionally equipped with a dual-feed PIFA to achieve high correlation (for the purpose of testing) whereas prototype "P2" is designed with spatially separated ceramic chip antennas for low correlation. The characteristics of the antenna prototypes including measured efficiency and magnitude of the pattern correlation under uniform 3D APS are summarized in Table 1. As can be seen in Table 1, P1 suffers from much higher correlation, lower efficiency, and slightly higher efficiency imbalance as compared to P2.

The multiplexing efficiency of P1 and P2 are shown in Fig 2(b). It is observed that the multiplexing efficiency of P2 is at -4 dB, which is mainly attributed to practical limitations in antenna efficiency for fully-equipped terminal prototypes. On the other hand, P1 has a significantly lower multiplexing efficiency of -7 dB. Referring to Table 1, (3.4) and Fig 1(a), the lower average antenna efficiency contributes to a loss of 1 dB and the correlation coefficient of 0.8 is responsible for a further 2 dB loss in multiplexing efficiency.

5 Conclusion

In this paper, multiplexing efficiency is proposed as a simple and intuitive metric for evaluating the effectiveness of MIMO antenna terminals operating in the SM mode. Instead of comparing the ergodic capacity, the metric quantify the performance in terms of absolute efficiency. An example highlights its utility to antenna engineers

in identifying and addressing critical design parameters, which will likewise be useful for testing MIMO terminals with different antenna characteristics.

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