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# Essays on Financial Market Volatility

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To Anna and Andy



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Ai Jun Hou  
Lund, April 2011

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# Chapter 1

## Introduction and Summary

This thesis examines financial market volatility and volatility spill-over between financial markets. It consists of three papers and focuses on adapting and proposing models for the estimation and forecasting of financial market volatility. Different applications of the estimated and forecasted volatility are demonstrated in each paper. The next sections give a brief introduction to the parametric and nonparametric volatility models, as well as the estimation methods used in this thesis. A short summary of each paper follows.

### 1.1 The volatility models

Volatility is defined as the degree to which the price of an equity or other financial assets tends to move or fluctuate over a period of time. It plays a central role in the valuation of such financial derivatives as options and futures and can, in fact, have a large effect on portfolio selection and risk management. Therefore, volatility modeling and forecasting is central to finance; it has been one of the most active areas of research in empirical finance and time series econometrics during the past two decades.

Most researchers agree that volatility is predictable in many asset markets (see, e.g., Bollerslev et al., 1992), although they differ on how it should be modeled. The evidence from the contemporary finance literature for predictability has led to a variety of approaches. The initial developments were tightly parametric, but the recent literature has moved in less parametric, and even fully nonparametric, directions. The common empirical observation is that financial market volatility is time varying and persistent, shows clustering, responds asymmetrically to shocks, and is different across assets, asset classes, and countries. (see, e.g., Bollerslev et al., 1992)

#### 1.1.1 The parametric models

In financial time series, one often observes that big shocks tend to be followed by big shocks in either direction, and small shocks tend to follow small shocks. This is referred to as volatility clustering. In order to model such patterns, the ARCH model (Engle, 1982) and the GARCH model (Bollerslev, 1986) allow the variance to depend upon

its history. Since those models were introduced, the financial econometrics literature has focused considerable attention on time-varying volatility and development of new tools for volatility measurement, modeling, and forecasting based on the ARCH and the GARCH model. One of the most interesting extensions of the ARCH and GARCH models are the “asymmetric” volatility models that consider the asymmetric response to shocks.

Volatility’s asymmetric phenomenon, where it increases more after a negative than after a positive shock of the same magnitude, is another common empirically observed characteristic of financial markets. This implies a negative correlation between return innovations and future expected conditional variances. Two economic theories explain asymmetric volatility: the leverage effect and time-varying risk premia (volatility feedback). The leverage effect (see, e.g., Black, 1976 Christie, 1982) indicates that an increase in financial leverage leads to an increased volatility level. Volatility rises when stock prices go down and decreases when stock prices go up. As an alternative explanation of the larger increase in volatility after a negative shock, many researchers (see, e.g., French et al., 1987 Campbell and Hentschel, 1992 Wu, 2001) state that news that volatility will be higher in the future will induce risk-averse investors to sell positions today until expected return rises to compensate for the risk, necessitating an immediate stock-price decline to allow for higher future return. Hence, the leverage hypothesis claims return shocks lead to changes in conditional volatility, whereas the time-varying risk premium theory contends that return shocks are caused by changes in conditional volatility.

The GJR-GARCH model of Glosten et al. (1993) is specifically designed to accommodate such asymmetries. Within this model, the asymmetry is identified and determined by a dummy that depends on the sign (negative and positive) of the corresponding return innovations in the conditional variance equation. A similar motivation underlies the EGARCH model in Nelson (1991). Although the log-transform complicates the calculation of conditional variance forecasts, it conveniently avoids having to impose nonnegativity on the parameters of the variance equation.

Alternatively, as discussed above, the asymmetries in the return–volatility relationship may also be attributed to volatility feedback. This feature is captured by the ARCH/GARCH-in-Mean type formulation (Engle et al., 1987), in which the functional form of the conditional mean depends explicitly on the conditional variance. A number of papers have employed this framework to capture the empirically observed asymmetry in equity-return volatility (see, e.g., Campbell and Hentschel, 1992 Bekaert and Wu, 2000).

Another important empirical finding is the strong volatility persistence showing in most daily and weekly financial returns. To capture this, Engle and Bollerslev’s (1986) IGARCH model directly imposes unity on the sum of the return-innovation coefficients and the lagged variance. However, the imposition of a unit-root in the conditional variance arguably exaggerates the true dynamic dependencies. Several alternative long-memory, or fractionally integrated ARCH-type formulations have also been estimated and analyzed more formally in the literature (see, e.g., Baillie et al., 1996 Ding et al., 1993 Zumbach, 2004). Possible explanations for the apparent long-memory dependencies based on the aggregation of volatility components have been explored by many researchers (see, e.g., Andersen and Bollerslev, 1997 Engle and Lee, 1999 Liu, 2000).

Meanwhile many researchers argue that the high persistence in volatility and lower accuracy in the volatility forecast are due to structural breaks (see, e.g., Engle and Bollerslev, 1986 Diebold and Inoue, 2001). Lamoreux and Lastrapes (1990) show that the model with switched parameter values, such as Hamilton’s (1989) Markov switching model, may provide a more accurate tool for modeling volatility. Hamilton and Susmel (1994) indicate also that a Markov switching process can provide a better statistical fit to the data than the traditional GARCH model.

### 1.1.2 The nonparametric models

The term *nonparametric* (Li and Racine, 2007) refers to statistical techniques that do not require a researcher to specify a functional form for the estimated object. Rather than assuming the functional form of an object is known up to a few unknown parameters, the nonparametric model substitutes less-restrictive assumptions, such as differentiability and moment restrictions, on the estimated object. Since nonparametric techniques make fewer assumptions about the estimated object than do parametric techniques, nonparametric estimators tend to be slower to converge to the objects being studied than *correctly specified* parametric estimators. In addition, unlike their parametric counterparts, the convergence rate is typically inversely related to the number of variables involved, which is sometimes referred to as the “curse of dimensionality.” However, it is often the case that, even for moderately sized data sets, nonparametric approaches can reveal structure in the data that might be missed when using parametric functional specifications. Therefore, nonparametric methods are more appropriate when i) we know very little about the functional form or the distributions of the object being estimated, ii) the number of variables is not too large, and iii) we have reasonably large data set.

Further, *semiparametric* refers to statistical techniques that do not require a researcher to specify a parametric functional form for some part of the estimated object but do require parametric assumptions for other parts.

Nonparametric and semiparametric methods have attracted great interests from statisticians in the past few decades (see, e.g., Silverman, 1986 Härdle, 1990 Scott, 1992 Wand and Jones, 1995 Fan and Gijbels, 1996 Härdle et al., 2004 Fan and Yao, 2005). The parametric procedures for volatility modeling rely on explicit functional-form assumptions regarding the expected volatility. The nonparametric procedures are generally free from such functional-form assumptions and afford estimates of volatility that are flexible yet consistent (Andersen et al., 2005). The advantages of the nonparametric model include, for example, disregarding the functional form of the volatility and the strong assumptions of the distribution of the residuals in the conditional mean equation. Bülman and McNeil (2002) introduce a nonparametric GARCH model in which the latent volatility process is a nonparametric function of the lagged return residuals and the lagged volatility.

In this thesis, we apply Bülman and McNeil’s (2002) nonparametric GARCH model to volatility estimating and forecasting. As mentioned above, the curse of dimensionality is a common problem in nonparametric smoothing. The additive semiparametric model is a common tool to reduce nonparametric functions’ dimension as a remedy to the curse of dimensionality. Therefore, we use additive regression to decompose the whole

nonparametric function of Bühlman and McNeil's (2002) nonparametric GARCH model into several additive structured nonparametric functions.

## 1.2 The estimation methods

In this section, we introduce the estimation methods used for the volatility estimation and forecasting in this thesis.

### 1.2.1 Bayesian-based Markov chain Monte Carlo method

The maximum likelihood method is commonly used for parametric estimation. With this method, a model is estimated by maximizing the likelihood function of the data, and the statistical inference is made based on the fitted models. However, some complicated models, such as the Markov switching model, are a mixture over all possible state configurations. This makes model estimation infeasible with the maximum likelihood method. With the advances in the computing facilities, the Bayesian-based Markov chain Monte Carlo (MCMC) method has been widely used in financial econometrics and financial modeling nowadays. We use the MCMC method for the estimation of the regime-switching model used in the third paper (see Chapter 4 in details).

The conditional distribution and the prior distribution play essential roles in the MCMC method. For example, consider an inference problem with parameter vector  $\theta$  of an unknown model and with the data set,  $X$ . The distribution  $f(\theta | X)$  of parameters given the data is called the posterior distribution, and it is proportional to the product of the likelihood function  $f(X | \theta)$  and the prior distribution  $p(\theta)$ . In practice, because the posterior is often either unknown or complicated to access directly, one draws the parameters from the prior distributions, which is highly dependent on the researcher's knowledge about the parameters of the model.

For a univariate posterior draw, if the prior and posterior distributions belong to the same family of distributions, the prior distribution is called a conjugate prior distribution, and it can dramatically simplify the MCMC drawn. Some well-known conjugate priors can be found in the Bayesian statistics of DeGroot (1990).

For a joint posterior drawn, German and German's (1984) Gibbs Sampling (or Gibbs Sampler) is the most common method when the likelihood function is hard to obtain. For example, if one needs to randomly drawn from the joint distribution of  $f(\theta_1, \theta_2 | X)$ , and the individual conditional distributions ( $f_1(\theta_1 | \theta_2, X)$  and  $f_2(\theta_2 | \theta_1, X)$ ) are available. One can first draw a random number from each of the conditional distributions,  $\theta_{1,0}$  and  $\theta_{2,0}$ , and set it as iteration 0. Then iteration 1 is based on continuously drawn information, obtaining  $\theta_{1,1} = f_1(\theta_1 | \theta_{2,0}, X)$  and  $\theta_{2,1} = f_2(\theta_2 | \theta_{1,1}, X)$ . Next, the researcher uses the new parameters as starting values and repeats the draw to obtain  $\theta_{1,2}$  and  $\theta_{2,2}$ . Repeating the iterations for  $m$  times yields a sequence of  $(\theta_{1,1}, \theta_{2,1}), \dots, (\theta_{1,m}, \theta_{2,m})$ . Under some regularity conditions,  $(\theta_{1,m}, \theta_{2,m})$  converges to the targeted joint draw of  $f(\theta_1, \theta_2 | X)$ .

Besides the above method, in this thesis, we have also used a special type of Gibbs Sampler to draw the model parameters in the third paper: Tanner's (1996) Griddy Gibbs

sampler. This method is very applicable when the posterior distribution is univariate. The main idea is to form a simple approximation of the inverse CDF of the posterior density, then draw a uniform random number and transfer the observation via the approximated inverse CDF to obtain a random draw for the parameters (see details in Chapter 4).

### 1.2.2 The additive semiparametric regression

We use the additive approach to reduce the dimension of the nonparametric function for the first two papers in the thesis. The method is from Hastie and Tibshirani (1990). We consider a estimation of  $s_0, s_1(\cdot), \dots, s_p(\cdot)$  in the additive structure,

$$E(Y | x) = s_0 + \sum_{j=1}^p s_j(X_j), \quad (1.1)$$

where  $E s_j(X_j) = 0$  for every  $j$ . If we assume that the model,  $Y = s_0 + \sum_{j=1}^p s_j(X_j) + \varepsilon$  is in fact correct, and assume also that we know  $s_0, s_1(\cdot), \dots, s_{j-1}(\cdot), s_{j+1}(\cdot), \dots, s_p(\cdot)$ , and further define the partial residual as

$$R_j = Y - s_0 - \sum_{k \neq j} s_k(X_k), \quad (1.2)$$

then  $E(R_j | X_j) = s_j(X_j)$  and minimizes  $E(Y - s_0 - \sum_{k=1}^p s_k(X_k))^2$ . As we do not know  $s_k(\cdot)$ s, we can find a way to estimate  $\hat{s}_j(\cdot)$  given the estimates  $\{\hat{s}_i(\cdot), i \neq j\}$ . The resulting iterative procedure is the *backfitting* algorithm.

For example, assume we need to estimate three nonparametric functions,  $E[Y | X] = s_0 + s_1(X_1) + s_2(X_2)$ , where  $X_1$  and  $X_2$  are the explanatory variables. We first set the initialization iteration and let  $s_0^0 = E(Y)$ ,  $s_1^0 \equiv s_2^0(\cdot) \equiv 0$ . The initial nonparametric functions will be  $(s_0^0, 0, 0)$ . We then nonparametrically regress  $Y - s_0^0$  on  $X_1$  to get the functions  $s_1^1$  and regress  $Y - s_0^0 - s_1^1(X_1)$  on  $X_2$  to get  $s_2^1$ . The same procedure is applied to get  $s_0^1$ . The nonparametric functions are now  $(s_0^1, s_1^1, s_2^1)$ . We then calculate  $RSS = E(Y - s_0^1 - s_1^1(X_1) - s_2^1(X_2))^2$ . We go to the next iteration and repeat the same procedure to get the nonparametric functions  $(s_0^2, s_1^2, s_2^2)$ , repeating this procedure to iteration  $m$  to get the nonparametric functions  $(s_0^m, s_1^m, s_2^m)$  such that  $RSS = E(Y - s_0^m - s_1(X_1)^m - s_2^m(X_2))^2$  fails to increase, yielding the final smoothed functions.

## 1.3 Summary of thesis

There are three papers in this thesis. They examine the volatility in the equity and short-term interest-rate markets, and the spillover from the short term interest rate market to the equity market.

### 1.3.1 Summary of paper 1

The first paper, titled "Asymmetry effects in Chinese stock market volatility: A generalized additive nonparametric approach" examines Chinese stock market volatility and return volatility asymmetry.

Given the unique characteristics of the Chinese markets and the fact that the typical Chinese investor is more prone to speculation and less sophisticated than those from more mature markets (Tan et al., 2008), the Chinese stock volatility behaves differently from that of other markets. Therefore, the conventional volatility models, such as the GARCH-family approaches, that rely heavily on volatility specification and known distributions of the returns, might insufficiently characterize the volatility of the Chinese markets. This paper therefore applies Bühlman and McNeil's (2002) nonparametric smoothing technique to examine the volatility of the Chinese stock markets. Further, we develop a new technique that applies the iterative estimation algorithm of Bühlman and McNeil's (2002) NP model to Hastie and Tibshirani's (1990) Generalized Additive Model. The motivation of this adjustment is to avoid the curse of dimensionality, to provide a more accurate volatility forecast than the parametric models, and to become more computationally efficient than the original nonparametric model.

The results from this paper suggest that the leverage effect exists in the Chinese stock markets: Bad news does affect the return volatility more than good news. However, as implied by the news impact curve from the GAM NP model, a limited amount of good news is needed to keep the market calm. Further, compared with the superior performance of the nonparametric model in the in-sample volatility estimation and out-of-sample forecast, the GJR and EGARCH models tend to overestimate the volatility process in turbulent periods and yield larger estimation errors. Our results suggest that the nonparametric model is a more appropriate tool to use in estimating the Chinese stock-return volatility than the parametric GARCH models, such as the GJR and EGARCH models. We recommend the use of the nonparametric model in estimating and investigating the return volatility in the Chinese stock markets and other emerging stock markets that have features similar to those of the Chinese stock markets.

### 1.3.2 Summary of paper 2

The second paper, titled "Modeling and forecasting short-term interest rate volatility: A semiparametric approach," proposes semiparametric procedures to estimate the short-term interest-rate volatility. This paper is coauthored with Sandy Suardi.

This paper proposes a semiparametric procedure to estimate the volatility of the weekly three-month U.S. Treasury bills. The new approach accommodates asymmetry, levels effect and serial dependence in the conditional variance, and is based on the Bühlman and McNeil's (2002) nonparametric procedure. The potential usefulness of the semiparametric approach for estimating short-rate volatility is examined by comparing its forecast performance with a variety of one-factor short-rate diffusion models. Results from our Monte Carlo simulation illustrate the robustness of the semiparametric approach when estimating short-rate volatility with misspecification in the short-rate drift function and

the underlying innovation distribution. Moreover, the in-sample forecast performance of the semiparametric approach is superior to the parametric models considered. The empirical application to three-month U.S. Treasury bill yields suggests that the semiparametric estimation procedure provides superior in-sample and out-of-sample volatility forecasts compared to the widely used diffusion volatility models of Brenner et al. (1996), which feature asymmetric and level-dependent conditional variance. Although the semiparametric approach does not specify asymmetry in the volatility process, this procedure improves upon the fit and the predictive power of the volatility estimates. We do not find any evidence of nonlinearities in short-rate drift and conditional skewness in the short-rate change distribution. Finally, we demonstrate that the semiparametric approach, which yields a greater degree of accuracy in modeling short-rate change volatility, has pertinent implications for pricing long-dated and path-dependent interest-rate derivatives. Using the simulation method, we show that the semiparametric modeling approach gives rise to significantly different probability distributions of future interest-rate levels compared with parametric short-rate models. The confidence intervals of future interest-rate levels are narrower than for any of the parametric models considered, thereby leading to less price variability in interest-rate derivatives.

### 1.3.3 Summary of paper 3

The third paper, titled “The return variance of the EMU equity markets and spillover effects from short-term interest rates,” examines equity-return volatility and the spillover effects from short-term interest rates in the EMU area.

The empirical study is carried out by estimating an extended Markov switching GJR-in-mean (EMS GJR-M) model with a Bayesian-based Markov chain Monte Carlo methodology. Our results suggest that two regimes exist in the EURO area stock markets, a high-mean low-variance (bull) market and a low-mean high-volatility (bear) market. Most of the Euro countries have the same regime-switching status between the bull and bear markets. The correlation between the first two moments of returns is not stable over time, but varies between the bull and the bear markets. Our results also suggest that bad news from unexpected stock returns (negative residuals from returns) has an asymmetrically larger effect on the returns and the volatility than good news has. Such an impact is larger in the bear market than in the bull market. Surprisingly, as implied in the news-impact surface, we find that changes in short-term interest rates only significantly affect stock market volatility in the bear period in most of the EMU countries. In particular, the effect of an increase in interest rates is asymmetrically larger than that of a decrease in interest rates. Portfolio performance, based on the out-of-sample forecast results of various models, indicates that the EMS GJR-M model outperforms the MS GJR-M (Markov switching GJR-in-mean), the single switching GJR-M (GJR-in-mean), and the GJR models. Further, the models with regime switching yield better portfolio performance than those without it, emphasizing the importance of the interest-rate impact and the regime specification when modeling volatility. Ignoring such state-dependent asymmetric effects from short-term interest rates on stock returns and their volatility will lead to invalid inferences, biased volatility forecasts.



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## Chapter 2

# Asymmetry Effects in Chinese Stock Market Volatility: A Generalized Additive Nonparametric Approach

### 2.1 Introduction

The Chinese stock markets have grown rapidly since the establishment of the Shanghai Stock Exchange (SSE) in December 1989 and the Shenzhen Stock Exchange (SZSE) in April 1991. Specially, with the recent boom in China's economy, China's stock markets have been attracting an enormous amount of attention from policy makers, investors, and academics. Chinese stock markets are interesting and deserve attention also because they exemplify many unique characteristics that differ from well-developed Western financial markets. One of the unique characteristics is that the Chinese stock markets are the only equity markets covered by the International Finance Corporation that have completely segmented trading between domestic and foreign investors (see Chui and Kwok, 1998 Yang, 2003). The A-share market is only open to Chinese domestic investors while the B-share market was only open to foreign investors before February 2001.<sup>1</sup> Many studies (see Chui and Kwok, 1998 Yang, 2003) also address the fact that the Chinese stock markets are tightly controlled by the government: The markets are at most partially privatized, and the state maintains state shares in varying amounts. The presence of market segmentation and heavy government regulations give rise to mispricing and information asymmetry, making the market clearly imperfect and incomplete (Chan et al., 2007). Further, stock trading is still new to most domestic participants. The A shares are dominated by domestic individual investors who typically lack sufficient knowledge and

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<sup>1</sup>In order to increase the mobility of B shares and to strengthen foreign fund investment on the capital market, with a view of paving the way towards China accession to the WTO, the Chinese government lifted the restriction of people in the territory of China investing in B shares on February 19, 2001. However, even after the rule changes, B shares cannot exceed 25% of a company's total shares to ensure that Chinese stock markets are not overly influenced by foreign investment, and domestic investors can trade and own B shares only if they have foreign currency.

experience in investments (China Securities and Futures Statistical Yearbook, 2004).

Given the unique characteristics of the markets and given that the typical Chinese investor is more prone to speculation and less sophisticated than those from more mature markets (Tan et al., 2008), Chinese stock volatility behaves very differently from that of other markets. Therefore, conventional volatility models, such as the GARCH-family approaches, that rely heavily on volatility specification and known distributions of returns, might insufficiently characterize the volatility of the Chinese market. Bühlman and McNeil (2002) propose a nonparametric GARCH model (hereafter NP model), in which the hidden volatility process is a function of the lagged volatility and lagged value of the innovations from returns and is estimated by an iterative nonparametric algorithm. This model is more attractive than the parametric GARCH-family models because it requires neither a specification of the functional form of the hidden volatility process nor that of the distribution of the innovations.

In this paper, we investigate the Chinese stock return volatility and the asymmetric effect of shocks on return volatility<sup>2</sup> by applying the NP model. Moreover, we contribute methodologically to the literature by suggesting a generalized additive model with the nonparametric approach (hereafter GAM NP model) that applies the iterative estimation algorithm of the NP model to the generalized additive model of Hastie and Tibshirani (1990). The motivation for such an adjustment is that the GAM NP model can avoid the curse of dimensionality, which is a common problem for the nonparametric estimation of a multidimensional regression.<sup>3</sup> Further, as will be shown in the Monte Carlo simulation and the empirical investigation, this newly proposed GAM NP model can deliver a more accurate volatility estimate than the parametric GARCH-family models and becomes computationally more efficient than the NP model. Also novel in our approach is that we extend the news impact curve from Engle and Ng (1993) to the nonparametric context and use it to measure and examine the asymmetric effect of shocks on volatility.

Currently, GARCH-family models are the most common in the investigation of the Chinese stock-return volatility and the asymmetric effect of market news on volatility. For example, Yeh and Lee (2000) use the GJR model proposed by Glosten et al. (1993) to examine Chinese stock market volatility from May 22, 1992, to August 27, 1996. They find that investors in China chase after good news indicating that the impact of good news (positive unexpected returns) on future volatility is greater than that of bad news (negative unexpected returns). By estimating both the GJR and the EGARCH model, Friedmann and Sanddorf-Köhle (2002) report that bad news increases volatility more than good news in A-share and composite indices, whereas good news increases volatility more than bad news in B-share indices based on a sample beginning on May 22, 1992, and ending on September 16, 1999. The good-news-chasing-investor phenomenon in China makes the Shanghai and Shenzhen stock markets relatively unique and different from many other stock markets in the world. Lee et al. (2001) provide the same result

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<sup>2</sup>The asymmetric effect often refers to the volatility increasing more after a negative shock than after a positive shock of the same magnitude (see Black, 1976 Christie, 1982).

<sup>3</sup>Under the curse of dimensionality, the optimal rate of convergence of a nonparametric estimation of a multidimensional regression decreases with increasing dimensionality (Linton and Mammen, 2005). For the multidimensional smoothing, efforts must be made to alleviate the problem (Härdle et al., 2004).

as Friedmann and Sanddorf-Köhle (2002) with the EGARCH model and daily return data from December 12, 1990, to December 31, 1992. Zhang and Li (2008) investigates the asymmetry effect of bad news on Chinese stock volatility with a partial adjustment process. They find that the asymmetry effect begins to appear in May 1996. Dividing the total sample into two periods, Huang and Zhu (2004) produce results from the EGARCH and GJR models showing that the asymmetry effect only exists in the period between February 2001 and September 2003.

In view of the different findings from past research regarding the leverage effect of Chinese stock-return volatility, we examine Chinese stock market volatility and the asymmetric effect of market news on the volatility using data from January 2, 1997, to August 31, 2007. Several questions will be addressed in the investigations: Do Chinese stock market volatilities react asymmetrically to shocks as in most mature stock markets in the world? Are investors in the Chinese stock markets still chasing after good news? Do volatilities in the Shanghai and in the Shenzhen stock markets react similarly to the market news? The answers to these questions have important implications for market practitioners forecasting stock returns and volatility, and for risk managers formulating optimal strategies for portfolio selection and risk management.

The results from this paper suggest that the leverage effect exists in the Chinese stock markets: Bad news does affect return volatility more than good news. However, as implied by the news impact curve (NIC) from the GAM NP model, a small amount of good news is needed to keep the market calm. Further, compared with the superior performance of the GAM NP in the in-sample estimation and the out-of-sample forecast, the GJR and EGARCH models tend to overestimate the volatility process in turbulent periods and yield larger estimation errors. Our results suggest that the nonparametric smoothing approach is a more appropriate tool for estimating Chinese stock-return volatility than the parametric GARCH models.

The rest of the paper is organized as follows. In section 2.2, we present the non-parametric models and the model estimation algorithm. Section 2.3 performs the Monte Carlo simulation to evaluate the performance of the parametric and nonparametric models. Section 2.4 compares the performance of the nonparametric models with various GARCH-family models and examines the asymmetric effects of shocks on the volatility. Section 2.5 concludes.

## 2.2 Modeling time-varying volatility

In this section, we introduce the NP and GAM NP models and the model-estimation algorithm used to estimate Chinese stock market volatility. As we will evaluate and compare the performance of the nonparametric models with the parametric models, we first introduce the parametric GARCH-family models.

### 2.2.1 Parametric GARCH-family models

Bollerslev (1986)'s GARCH model has been the most widely used model for the volatility estimation since it was first proposed. As pointed out by Bera and Higgins (1993), most of the applied financial works show that GARCH (1,1) provides a flexible and parsimonious approximation to the conditional variance dynamics and is capable of representing the majority of financial series. The GARCH (1,1) model is written as

$$\begin{aligned} R_t &= \mu + X_t, & X_t &= \sigma_t z_t, & z_t &\sim N(0, 1), \\ \sigma_t^2 &= \omega + \alpha_1 X_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \end{aligned} \quad (2.1)$$

where  $\omega > 0$ ,  $\alpha_1, \beta_1 \geq 0$ ,  $(\alpha_1 + \beta_1) < 1$ , and  $X_{t-1}$  may be treated as a collective measure of news about equity returns arriving to the market over the previous periods.

In the simple GARCH (1,1) approach, good news and bad news—positive and negative shocks—have the same impact on the conditional variance. Many studies have found evidence of asymmetry in stock-price behavior: Negative surprises seem to increase volatility more than positive surprises do. To allow asymmetric effects in the volatility, Glosten et al. (1993) add an additional term in the conditional variance and formulate the so-called GJR model. The GJR (1,1) is specified as

$$\begin{aligned} R_t &= \mu + X_t, & X_t &= \sigma_t z_t, & z_t &\sim N(0, 1), \\ \sigma_t^2 &= \omega + \beta_1 \sigma_{t-1}^2 + (\alpha_1 + \gamma I) X_{t-1}^2, \end{aligned} \quad (2.2)$$

where  $\omega > 0$ ,  $(\alpha_1 + \gamma) \geq \alpha_1 \geq 0$ ,  $\beta_1 \geq 0$ ,  $(\alpha_1 + 0.5\gamma + \beta_1) < 1$ .  $I$  is an indicator for negative  $X_{t-1}$ . That is,  $I = 1$  for  $X_{t-1} < 0$ , and  $I = 0$  for  $X_{t-1} \geq 0$ . The structure of this model indicates that a positive  $X_{t-1}$  contributes  $\alpha_1 X_{t-1}^2$  to  $\sigma_t$ , whereas a negative  $X_{t-1}$  has a larger impact of  $(\alpha_1 + \gamma) X_{t-1}^2$  with  $\gamma > 0$ . Therefore, if parameter  $\gamma$  is significantly positive, then negative innovations generate more volatility than positive innovations of equal magnitude.

Another volatility model that accounts for asymmetric impacts on the conditional variance is Nelson's (1991) exponential GARCH model (EGARCH). The EGARCH(1,1) is specified as

$$\begin{aligned} R_t &= \mu + X_t, & X_t &= \sigma_t z_t, & z_t &\sim N(0, 1), \\ \log \sigma_t^2 &= \omega + \beta_1 \log \sigma_{t-1}^2 + \alpha_1 \left\{ \frac{|X_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - E \left[ \frac{|X_{t-1}|}{\sqrt{\sigma_{t-1}^2}} \right] \right\} + \gamma \frac{X_{t-1}}{\sqrt{\sigma_{t-1}^2}}. \end{aligned} \quad (2.3)$$

Here the coefficient  $\gamma$  signifies the leverage effect of shocks on the volatility. The key advantage of the EGARCH model is that the positive restrictions need not be imposed on the variance coefficients.  $\gamma$  must be negative for evidence of asymmetric effects.

In this paper, we leave the functional form of the variance process unspecified and attempt to estimate it as a nonparametric mean. We show that the nonparametric model can capture the asymmetry effect from the unexpected news and outperforms the more

common parametric GARCH-family models.

### 2.2.2 The generalized additive nonparametric model

Compared with the parametric models, a nonparametric model enjoys advantages of relaxing the specification of the variance process and the error-distribution assumptions. One example is the NP model from Bühlman and McNeil (2002):

$$\begin{aligned} R_t &= \mu + X_t, & X_t &= \sigma_t z_t, \\ \sigma_t^2 &= f(X_{t-1}, \dots, X_{t-p}, \sigma_{t-1}^2, \dots, \sigma_{t-q}^2), \end{aligned} \quad (2.4)$$

where the stationary stochastic process  $\{X_t; t \in \mathbb{Z}\}$  is adapted to the filtration  $\{F_t; t \in \mathbb{Z}\}$  with  $F_t = \sigma(\{X_s; s < t\})$ .  $\{z_t; t \in \mathbb{Z}\}$  is an i.i.d. innovation with zero mean and unit variance and a finite fourth moment.  $z_t$  is also assumed to be independent of  $\{X_s; s < t\}$ .  $f: \mathbb{R} \times \mathbb{R}_+ \mapsto \mathbb{R}_+$  is a strictly positive valued function.  $\sigma_t$  is the time-varying volatility and  $\sigma_t^2$  is the conditional variance of  $\text{var}[X_t | F_{t-k}]$ , where  $\{1 \leq k \leq \max(p, q)\}$ . Bühlman and McNeil (2002) have shown that the nonparametric function  $f$  can be estimated by regressing  $X_t^2$  on the lagged variables  $X_{t-1}$  and  $\sigma_{t-1}^2$  using a nonparametric smoothing technique.

However, the proposed model cannot avoid the common problem of a multidimensional nonparametric smoothing, the curse of dimensionality. In order to overcome this difficulty, Hastie and Tibshirani (1990) propose the generalized additive model, which enables the dependent variable to depend on an additive predictor through a nonlinear function. We apply Hastie and Tibshirani's (1990) generalized additive procedure to the NP model which gives rise to the GAM NP model:

$$\begin{aligned} R_t &= \mu + X_t, & X_t &= \sigma_t z_t, \\ \sigma_t^2 &= \mu + f(X_{t-1}) + g(\sigma_{t-1}^2), \end{aligned} \quad (2.5)$$

where  $f: \mathbb{R} \mapsto \mathbb{R}_+$  is a positive-valued function satisfying  $f(x) = f(-x)$ —i.e.,  $f(x) = \alpha|x|$ ,  $0 < \alpha < 1$ —and  $g: \mathbb{R}_+ \mapsto \mathbb{R}_+$  is a positive nondecreasing function satisfying  $g(\sigma^2) = \beta\sigma$ ,  $0 < \beta < 1$ .

We observe that the model in equation (2.5) can be written with the following transformation.

$$\begin{aligned} X_t^2 &= \mu + f(X_{t-1}) + g(\sigma_{t-1}^2) + V_t, \\ V_t &= (\mu + f(X_{t-1}) + g(\sigma_{t-1}^2))(z_t^2 - 1) \end{aligned} \quad (2.6)$$

Clearly,  $V_t$  is a martingale difference series with  $E[V_t | F_{t-1}] = 0$  and  $\text{cov}[V_s, V_t | F_{t-1}] = 0$  for  $s < t$ .



From equation (2.6), it follows that

$$\begin{aligned} E[X_t^2 | F_{t-1}] &= \mu + f(X_{t-1}) + g(\sigma_{t-1}^2), \\ \text{var}[V_t^2 | F_{t-1}] &= (\mu + f(X_{t-1}) + g(\sigma_{t-1}^2))^2 (E[z_t^A] - 1), \end{aligned} \quad (2.7)$$

This suggests that we can estimate the conditional variance with a nonparametric regression of a generalized additive model. The regression is performed according to the additive structure of  $\sigma_t^2$  using the back-fitting algorithm, which was first introduced by Friedman and Stuetzle (1981) and generalized by Hastie and Tibshirani (1990). This tool is now widely used for nonparametric estimation in iterative procedures. We estimate the conditional variance by the generalized additive model according to the following formula.

$$\hat{\sigma}_t^2 = \hat{\mu} + \hat{f}(X_{t-1}) + \hat{g}(\hat{\sigma}_{t-1}^2) \quad (2.8)$$

### 2.2.3 Estimation algorithm

Assume we have a data sample  $\{X_t^2 : 1 \leq t \leq n\}$  satisfying the process of (2.5).<sup>4</sup>

1. In the first step, we set  $m = 1$  (the current iteration) and calculate a first estimate of volatility  $\{\hat{\sigma}_{t,0}^2 : 1 \leq t \leq n\}$  as the initial estimation by fitting the data with the GARCH (1,1) model with a maximum-likelihood estimate.
2. We regress  $\{X_t^2 : 2 \leq t \leq n\}$  on the lagged returns,  $\{X_{t-1} : 2 \leq t \leq n\}$  and  $\{\hat{\sigma}_{t-1,m-1}^2, 2 \leq t \leq n\}$ , through a nonparametric smoothing procedure with the back-fitting algorithm to obtain estimates  $\hat{f}_m$  and  $\hat{g}_m$ .
3. In the third step, we calculate  $\{\hat{\sigma}_{t,m}^2 = \hat{\mu}_m + \hat{f}_m(X_{t-1,m-1}) + \hat{g}_m(\hat{\sigma}_{t-1,m-1}^2) : 2 \leq t \leq n\}$  as specified in (2.8).
4. We proceed to increment the iteration  $m$  and return to the second step until  $m = M$ , where  $M$  is the prespecified total number of iterations.
5. Finally, we average the last  $k$  of such estimates to obtain the final smoothed volatility,  $\hat{\sigma}_{t,\text{final}}$ , and perform the final nonparametric regression with the back-fitting algorithm by regressing  $\{X_t^2 : 2 \leq t \leq n\}$  against  $\{X_{t-1} : 2 \leq t \leq n\}$  and  $\hat{\sigma}_{t-1,\text{final}}^2$  to get the final estimates  $\hat{f}_{\text{final}}$  and  $\hat{g}_{\text{final}}$ . The final estimated volatility can be calculated by  $\hat{\sigma}_{t,\text{final}}^2 = \hat{\mu}_{\text{final}} + \hat{f}_{\text{final}}(X_{t-1}) + \hat{g}_{\text{final}}(\hat{\sigma}_{t-1,\text{final}}^2)$ .

## 2.3 Monte Carlo simulation

We use Monte Carlo simulation to estimate and examine a standard GARCH model and a GARCH model with an asymmetry effect. The purpose of the Monte Carlo simulation is to show that with a significantly large asymmetric effect, the GAM NP model can offer

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<sup>4</sup>Readers interested in the justifications and proofs of this algorithm are referred to Bülman and McNeil (2002)

better estimates of the unobserved volatility than can the parametric GARCH-family models and can perform as well as the NP model (performing even better in many cases). We generate  $n = 1000$  observations and 50 realizations for each random process. For the nonparametric models, the number of iterations is set to  $M = 8$ , and a final smoothing is performed by averaging the previous four iterations ( $K = 5$ ) according to the algorithm presented in the previous section. The performance of each model is evaluated using the mean of the Mean Squared Error (MSE) and the mean of the Mean Absolute Error (MAE) from each iteration. The MSE and the MAE are calculated according to the formulas

$$\begin{aligned} MSE(\hat{\sigma}_{s,m}) &= \frac{1}{n-20} \sum_{t=21}^n (\hat{\sigma}_{t,m} - \sigma_t)^2 \quad \text{and} \\ MAE(\hat{\sigma}_{s,m}) &= \frac{1}{n-20} \sum_{t=21}^n |\hat{\sigma}_{t,m} - \sigma_t|, \end{aligned} \quad (2.9)$$

where  $\hat{\sigma}_{t,m}$  is the estimated volatility at time  $t$  from each iteration  $m$  and  $\sigma_t$  is the true volatility at time  $t$ . The first 20 values are excluded from the calculation because the volatility estimates at the first few points may be unreliable.

The data are simulated from the variance process, which follows GARCH and Threshold GARCH (TGARCH) models specified as follows.

$$\sigma_t^2 = 7 + 0.1\sigma_{t-1}^2 + 0.66X_{t-1}^2, \quad (2.10)$$

$$\sigma_t^2 = 7 + 0.1\sigma_{t-1}^2 + (0.66I_{\{X>0\}} + 0.2I_{\{X\leq 0\}}) X_{t-1}^2 \quad (2.11)$$

In the variance process of equation (2.11), the asymmetry effect between the positive and negative shocks is built into the ARCH effect, along the lines of models suggested by Glosten et al. (1993) and Fornari and Mele (1997). We simulate the process given by equation (2.11) with  $t$ -distributed residuals with four degrees of freedom and estimate it with both Gaussian and  $t$ -distributed errors. Figure 2.1 plots the true volatility surfaces of the processes specified in equations (2.10) and (2.11). It can be easily seen from Figure 2.1 that if the true volatility is under the GARCH specification of process given by equation (2.10) (the left plot), the volatility surface is very smooth. However, with the asymmetry effect of the process given by equation (2.11), there is a significant discontinuity in the volatility surface. In this case, we show that the nonparametric model can smooth the segmented volatility surface quite well and therefore outperforms the parametric models. For the purpose of comparison, we fit the simulated process given by equation (2.11) with the EGARCH, GJR and NP models, and compare their fit with that of the GAM NP and NP models.

- Figure 2.1 about here -

In Figure 2.2, we plot the estimated volatility surfaces of the eight iterations and the final smoothing of the GAM NP model from one randomly chosen iteration. We can clearly observe that the smoothing has been well performed already after the first

iteration and the surface has been perfectly smoothed at the final stage of smoothing. This indicates that the estimation algorithm is recovering the essential features of the volatility surface and demonstrates the convergence of the smoothing method.

- Figure 2.2 about here -

Table 2.1 compares the performance of the GARCH, EGARCH, GJR, GAM NP and NP models. Table 2.2 presents the goodness-of-fit simulation results from the nonparametric models. It is evident from these two tables that the MSE and MAE of the nonparametric models are much lower than those of the parametric GARCH models. For example, it can be seen from Table 2.2 that the MSE and the MAE are 0.555 and 0.615 for the GARCH model with Gaussian errors before smoothing. The MAE and the MAE start to decrease in each iteration and reach 0.221 (0.261) and 0.339 (0.405) at the final stage of smoothing for the GAM NP (NP) model. Although the EGARCH and GJR (TGARCH) models capture the asymmetric effects partially, they cannot match the nonparametric models' goodness of fit. For example, it can be seen from Table 2.1 that the MSE and the MAE of the EGARCH model with Gaussian errors are 0.3 and 0.43, respectively, while those of the GJR model are 0.39 and 0.507, respectively. More interestingly, the goodness of fit of the GAM NP model indicates that it performs even better than the NP model: The MSE (MAE) of the GAM NP model (with normal fit) is 15.4% (16.4%) lower than that of the NP model. We also notice that the choice of the distribution for the parametric GARCH models clearly matters. There is evidence that the EGARCH and GJR models with  $t$ -distributed innovations perform better than the ones with Gaussian innovations, but this is not the case for nonparametric estimations. The NP and GAM NP models provide nearly identical results with both Gaussian and  $t$  errors. Figure 2.3 plots the estimated volatility process compared with the true volatility, which is an arbitrary selection of 100 observations from a simulated realization of the process given by equation (2.11). The left-hand plot shows the true volatility (solid line) compared with parametric GARCH (1,1) estimates with  $t$  innovations (dotted line) and the right-hand plot shows the true volatility (solid line) with the GAM NP estimate obtained after a final smooth (dotted line). It is clearly shown in the figure that the GAM NP model yields volatility estimates that match the true volatility movements better than those of the GARCH model. In particular, the sharp spikes observed at the 40th and 90th observations of the true volatility are well captured by the GAM NP model but not by the GARCH model.

- Tables 2.1, 2.2 and Figure 2.3 about here -

From the Monte Carlo simulation, we conclude that the GAM NP model provides more accurate volatility estimation and captures more of the asymmetric effect of shocks compared with the parametric GARCH models and the NP model.

## 2.4 Chinese stock market volatility

The Chinese stock market is relatively young, yet it is developing quickly. By the end of 2007, there were 860 listed companies in the SSE with the total market value of RMB 29.09 trillion, of which A shares represented RMB 26.85 trillion and B share represented RMB 1.3 trillion. In the SZSE, there were 670 listed companies with a total market capitalization of RMB 5.73 trillion, of which A shares represented RMB 5.61 trillion and B shares represented RMB 0.12 trillion.

As discussed by many reports, the Chinese stock market is highly controlled by the government. The Chinese Securities Regulatory Commission (CSRC), as a ministry-rank unit of the State Council, performs almost all supervisory, regulatory, and enforcement function over the security market. Chinese firms need the approval from CSRC to be listed and sell their equity. The approval process is affected by many nonmarket factors, and it is not unusual for a company to wait several years to receive listing permission. Furthermore, many of the listed companies are former state-owned enterprises (SOEs). When the SOEs go public, no less than 50% of the shares will be kept by the state.<sup>5</sup> In addition, most companies will also hold retained shares for legal persons and internal employees of the companies. The state-retained shares, legal-person shares, and employee shares account for 60%–70% of equity and only the other shares are publicly tradable. Another characteristic of the Chinese stock markets is the market segmentation. The Chinese equity markets have two classes of ownership-restricted shares: A shares, which can be owned and traded by Chinese citizens, and B shares, which can be owned and traded by foreigners and, after February 2001, local Chinese residents who hold foreign currencies.<sup>6</sup> Despite their identical payoffs and voting rights, A shares are much more liquid than B shares.<sup>7</sup> The unique characteristics of the Chinese markets make them clearly imperfect and incomplete (Chan et al., 2007).

Some reports have given comprehensive reviews of the Chinese stock markets. For those interested in learning more about this emerging market, Wang et al. (2004), Chan et al. (2007) and Green (2004) are three very good references.

### 2.4.1 The data

The data used in this paper include the daily closing prices of the two primary Chinese indices, the Shanghai Stock Exchange Composite Index (SHCI) and the Shenzhen Stock Exchange Component Index (SZCI) from January 2, 1997, to August 31, 2007. The SHCI

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<sup>5</sup>Shares classified as A shares are designated for domestic investors and B, H and N shares are designated for overseas investors. A shares are further divided into state shares, legal-person shares, tradable A shares, and employee shares. State shares are those owned by the central government and local governments. Legal-person shares are those held by domestic legal entities and institutions such as other stock companies, state-private mixed enterprises, and nonbank financial institutions. Both state shares and legal-person shares are not tradable on the stock exchanges.

<sup>6</sup>The B-share market is the result of Chinese regulation. Generally, companies allowed to list shares have to fulfill a greater number of restrictions when issuing B shares than when listing A shares.

<sup>7</sup>A shares traded on average for 420% more than the corresponding B shares. In addition, A shares turned over at a much higher rate—500% versus 100% per year for B shares (see Mei et al., 2009).

has been published since 1991 and includes all Shanghai-listed companies weighted by capital stocks. The SZCI has been published since 1995 and is a value-weighted index of 40 stocks listed on the Shenzhen Stock Exchange. As key market regulations, such as the raising/down limit, were not well established until the end of 1996, we chose to analyze the data starting from January 1, 1997. The daily prices are downloaded from <http://www.sohu.com>.

All data are converted to their daily log returns, and multiplied by 100 as follows,

$$r_t = 100(\log(P_t) - \log(P_{t-1})). \quad (2.12)$$

In order to assess and compare the predictive performance of the nonparametric models with various parametric models, the data is further divided into an in-sample group (from January 1, 1997, to August 31, 2006) and an out-of-sample group (from September 1, 2006, to August 31, 2007). The whole sample has 2,573 observations and the last 243 are used for the out-of-sample forecasts. We use the expanding window for the out-of-sample forecasts. We first do the in-sample estimation using the data from January 1, 1997, to August 31, 2006, and use the parameters from the in-sample estimation to forecast the overnight volatility for the next day. Then we add one more data from the second day (September 1, 2006) and redo the estimation, using the parameters from this estimation to forecast the volatility for the following day. We repeat this estimation-and-forecast procedure until the end of the out-of-sample forecast period.

Further, we calculate the realized volatility as the proxy for the true volatility for the out-of-sample forecast. The realized volatility is calculated using the high-frequency (5-minute) data as<sup>8</sup>

$$RV_t = \sum_{i=1}^n r_{i,t}^2, \quad (2.13)$$

where  $n$  is the total number of high-frequency intervals (i) in day  $t$ . This method is used extensively in the literature (see, e.g., French et al., 1987 Day and Lewis, 1992 Pagan and Schwert, 1990 Andersen et al., 2001, 2000) The high-frequency data are obtained from <http://www.wstock.net>.<sup>9</sup>

Table 2.3 provides the statistical summary of the returns of both indices. Clearly, the mean of both series is close to zero, exhibits high kurtosis and is negatively skewed. In particular, the skewness in the Shanghai stock market is much higher than that in the Shenzhen stock market. The Jarque–Bera test further confirms that the return distributions are not normal. The augmented Dickey–Fuller test suggests that they are stationary time series. The two series are highly positively correlated at 0.926.

Figure 2.4 plots the index price and returns of the SHCI and the SZCI. The returns largely mirror each other and look very volatile. Both series also display strong volatility

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<sup>8</sup>Due to the data availability, we use the realized volatility as a proxy for the true volatility only for the out-of-sample forecasts.

<sup>9</sup>This is the website of a Chinese investment company named Huasheng and is only available in Chinese. However, the site can be well translated into English by Google's translation system.

clustering. These are typical characteristics of financial time series. Further, there are several peaks and troughs in the return series. The first peak occurred on May 12, 1997, where the SHCI/SZCI hit a record high 6103.62/1500 points. After going through a stable two-year period, it experienced a sharp decline before rising and reaching its second peak on July 1, 1999. Thereafter the stock indices began to increase in a relatively stable fashion, reaching its third peak in 2000–2001. It then declined again until the first half of 2005. However, after that the stock market began to rise rapidly and continued to accelerate upwards until it reached another historical high on August 31, 2007. It can be seen, therefore, that the period 2005 to 2007 is the most volatile period in the SHCI and SZCI.

- Table 2.3 and Figure 2.4 about here -

### 2.4.2 The in-sample estimation results from various models

We first fit the series from January 1, 1997, to August 31, 2006, with the standard GARCH(1,1) model. Considering the existence of the asymmetry effects of shocks on the return volatility in the Chinese stock markets, we also fit the data with the EGARCH and GJR models. For all these models, the innovations are assumed to be both Gaussian and Student- $t$  distributed. The estimated parameters and Ljung–Box Q-statistics tests of the standardized residuals are presented in Table 2.4. Note that all parameters of the conditional volatility are significant at the 5% level. The coefficient of lagged variance  $\beta$  shows very high volatility persistence. The sum of  $\alpha$  and  $\beta$  from the GARCH model is close to 1, offering evidence of volatility clustering. The  $p$ -values of the Ljung–Box Q-statistic test at the lag 20 of the standardized residual series from all models fail to suggest the autocorrelation at a 5% significance level. Thus, all models appear to be adequate in describing the linear dependence in the return and volatility series.

In the Shanghai stock market, the estimated leverage parameters  $\gamma$  of the EGARCH and GJR models with Gaussian ( $t$ ) distributed innovations are  $-0.036$  ( $-0.063$ ) and  $0.06$  ( $0.095$ ), respectively. In the Shenzhen stock market, the values of  $\gamma$  for these two models of Gaussian ( $t$ ) innovation are  $0.028$  ( $-0.035$ ) and  $0.036$  ( $0.055$ ). All these parameters are significant at the 5% level with the exception of the  $\gamma$  from the EGARCH model with Gaussian errors in the Shenzhen market. The significance of the parameters indicates the existence of the asymmetry effect in the Chinese stock markets. That is, bad news (negative shocks) has a larger impact on return volatility than good news (positive shocks). Notably, the asymmetric effect is higher in the SHCI than in the SZCI. It is also worth noting that the leverage effect estimated from models fitted with  $t$ -distributed innovations is higher than that with normally distributed innovations. The existence of the asymmetry effect as in other mature stock markets in the world may be a positive sign for market efficiency and completeness. It also suggests that the Chinese stock market is integrating with other world stock markets.

- Table 2.4 about here -

Next we use the NP and the GAM NP approaches to smooth the Chinese stock

volatility surface based on the volatility and innovations obtained from the GARCH(1,1) model. We evaluate the performance of various models by calculating four loss functions and comparing the results from the GAM NP model with the parametric models. For reference, we also estimate the NP model from Bülman and McNeil (2002) and compare its result with the newly proposed GAM NP model. The goodness-of-fit measures are,

1. *MSE1*: *MSE1* is calculated as  $\frac{1}{n} \sum_{t=1}^n (X_t^2 - \hat{\sigma}_t^2)^2$ , which is the mean squared error between the squared innovation  $X_t^2$  and the squared estimated volatility  $\hat{\sigma}_t^2$ . As  $X_t^2 = \sigma_t^2 + V_t$ , where  $V_t$  is the martingale series with zero mean, the mean squared error between both can be a good indicator to illustrate the goodness of fit.
2. *MAE1*: *MAE1* is calculated as  $\frac{1}{n} \sum_{t=1}^n |X_t^2 - \hat{\sigma}_t^2|$ , which is the Mean Absolute Error between the squared innovation  $X_t^2$  and the squared estimated volatility  $\hat{\sigma}_t^2$ .
3. *MSE2*: *MSE2* is calculated as  $\frac{1}{n} \sum_{t=1}^n (\hat{\sigma}_t - \sigma_t)^2$ , which is the Mean Squared Error between the estimated volatility,  $\hat{\sigma}_t$ , and the true volatility proxy,  $\sigma_t = \sqrt{y_t^2}$ , where  $y_t$  is the daily return at time  $t$ .
4. *MAE2*: *MAE2* is calculated as  $\frac{1}{n} \sum_{t=1}^n |\hat{\sigma}_t - \sigma_t|$ , which is the Mean Absolute Error between the estimated volatility,  $\hat{\sigma}_t$ , and the proxy for the true volatility,  $\sigma_t = \sqrt{y_t^2}$ , where  $y_t$  is the daily return at time  $t$ .

Besides checking the goodness of fit of the models, we also use the DM test suggested by Diebold and Mariano (1995) to check the significance of the improved predictability of the nonparametric models,

$$DM = \frac{E(d_t)}{\text{var}(d_t)} \sim N(0, 1), \quad (2.14)$$

where  $d_t = (e_{A,t} - e_{B,t})^2$  and  $e_{A,t}$  and  $e_{B,t}$  are prediction errors of two rival models, A and B, respectively.  $E(d_t)$  and  $\text{var}(d_t)$  are the mean and variance of the time series of  $d_t$ , respectively.

The goodness-of-fit results of various models are presented in Table 2.5. It is clear that the GARCH model performs the worst according to all goodness-of-fit measures. Compared with the GARCH model, the EGARCH model improves the volatility estimation by capturing the leverage effects. For the GJR model, it slightly improves the result from the GARCH estimation in the Shanghai Stock Exchange (SSE), while in the Shenzhen Stock exchange (SZSE), it is even worse off than the GARCH model. This is perhaps not surprising because the asymmetric effect is not as strong in the Shenzhen stock market as it is in the Shanghai stock market. However, this may indicate that the EGARCH model can capture more leverage effect than the GJR model can in the Chinese stock markets. When looking at the nonparametric models, we first find that the distributions of errors do not matter in the estimation, because all loss functions from the GAM NP and the NP model with  $t$  distributions do not differ from the ones with Gaussian distributions. We observe also that the GAM NP model outperforms all the parametric models and the NP model. The NP model outperforms the GARCH and the

GJR models, but not the EGARCH model with  $t$ -distributed errors according to all of the goodness-of-fit measures except the  $MSE1$ .

We then perform the DM test to investigate the significance of the improvement of the nonparametric model. The DM test is performed under the null hypothesis that the improvement of the model in the column (the GAM NP and the NP model) upon the model in the row (the parametric models) is not significant. The DM test results reported in Table 2.6 show that both the GAM NP model and the NP model significantly outperform the GARCH and GJR models at the 5% significance level according to almost all of the selected goodness-of-fit measures. The improvement of the GAM NP model upon the EGARCH model with  $t$ -distributed errors is only marginal. Further, we find that the NP model significantly underperforms the EGARCH with  $t$ -distributed errors in the SSE, but this underperformance is nearly insignificant in the SZSE. Although the improvement upon the EGARCH model with  $t$ -distributed errors is at the marginal level, with the advantages of no need to assume the functional form of the variance process and the distribution of errors, the GAM NP can still be an appropriate tool for examining the return volatility in the Chinese equity markets. Specially, in order to show the forecast ability of these models, we need to examine their out-of-sample performances.

- Table 2.5 and Table 2.6 about here -

### 2.4.3 The out-of-sample forecast improvements of the nonparametric models

The out-of-sample period is from September 1, 2006, to August 31, 2007. Besides the true volatility proxy of,  $\sqrt{y_t^2}$ , used the in in-sample estimation, the realized volatility, calculated from the 5-minute high-frequency data, is also used as the true volatility proxy. Further, to demonstrate the importance of our results and the application of the GAM NP model in practice, we calculate the 90%-forecasted return intervals which are based on the one-day ahead out-of-sample forecasts.

The performance of the out-of-sample volatility forecasts of various models are summarized in Table 2.7. We find from this table that the nonparametric models perform much better than the parametric models in delivering a lower forecast error. For example, in the Shanghai stock market, the MSE (MAE) of the GAM NP model (with normal fit) is 10% (7%), for the  $|y_t|$  volatility proxy, and 13% (9%), for the implied volatility proxy, lower than the one from the GARCH model. Similarly, in the SZSE, there is an approximately 5% reduction in the MSE and MAE for both volatility proxies. Compared with the GJR model, the EGARCH model appears to be a better parametric model in capturing the asymmetric effect of market news in the out-of-sample forecast. We notice that the GJR model in many cases performs even worse than the GARCH model. The poor performance of the GJR model in the out-of-sample volatility forecast has also been reported by Wei (2002). The author shows that the GJR model has higher forecast errors than a random-walk model when examining the Chinese stock markets' return volatility.

- Table 2.7 about here -



As in the in-sample estimation section, we do the DM test to investigate the significance of the nonparametric model's improvement. Table 2.8 shows the results of the DM test under the null hypothesis that the improvement of the model in the column (the GAM NP and the NP model) upon the model in the row (the parametric models) is not significant. This table shows that the nonparametric models significantly outperform the GARCH, GJR and EGARCH models in the Shanghai stock market according to almost all of the measures. However, in the Shenzhen market, compared with the EGARCH model, the improvements of the nonparametric model are almost significant at the 5% level when the volatility proxy is squared returns and are only at the marginal level when volatility proxy is the realized volatility. One of the reasons can be that the asymmetry effect in SZSE is not as high as in the SSE. The performance of the GAM NP model is nearly as good as the NP model. However, during the estimation, we experienced a significant reduction of computing time when estimating the GAM NP model compared to the NP model. Further, with the advantage of avoiding the curse of dimensionality, the GAM NP model can be an attractive tool for multidimensional nonparametric smoothing.

- Table 2.8 about here -

After obtaining the out-of-sample forecasted volatility, we use the forecasted values to build up the 90% return interval. The return intervals are calculated according to  $\hat{r}_t = \hat{\mu} \pm q_k \sqrt{\hat{\sigma}}$ , where  $q_k$  is the percentage of the quantile of normally distributed errors and  $\hat{\mu}$  and  $\hat{\sigma}$  are the forecasted conditional mean and volatility. It is worth noting that the lower bound of the interval is approximately the 5% daily value-at-risk (VaR) measure when the initial value of the investment is 1 Yuan.

Figure 2.5 plots the 90% intervals of the forecasted returns based on the forecasted conditional mean and the volatility from the EGARCH, GJR and GAM NP models for the SHCI and the SZCI. Interestingly, the intervals built upon the forecasted conditional mean and variance from various models do not differ that much when the market is relatively stable. When extreme events occur in the market, however, both the EGARCH and the GJR model provide a much wider return interval than the GAM NP model does. The most obvious example is the sudden drops in the SHCI and SZCI indices on February 27, 2007,<sup>10</sup> where the return from the EGARCH and GJR models is overestimated in the upper bound and underestimated in the lower bound. As mentioned earlier, the lower bound of the interval is the 5% daily VaR measure when the initial value of the investment is 1 Yuan. Hence, when the market becomes extremely volatile, the 5% VaR based on the parametric model is overestimated in both Shanghai and Shenzhen stock markets.

- Figure 2.5 about here -

This result is generally in line with Engle and Ng's (1993), Yeh and Lee's (2000), and Friedmann and Sanddorf-Köhle's (2002) studies. In particular, Engle and Ng (1993)

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<sup>10</sup>In the absence of any sign of circumstances, this "Black Tuesday" came and dumped the SSE and the SZSE. The SHCI and the SZCI declined by 8.84% and 9.29%, and hit the record of the biggest daily drop within the last ten years.

provide evidence that the volatility predicted by the EGARCH model is much higher than that predicted by the other models. Yeh and Lee (2000) argue that the application of the GJR model to daily Chinese returns leads to overshooting in the estimated conditional variance in periods of high volatility. Friedmann and Sanddorf-Köhle (2002) examine asymmetry by extending the news impact curve of Engle and Ng (1993) to the conditional news impact curve and argue that the overshooting of the volatility predictions from the GJR model is due to an acceleration of the news impact in the periods of high volatility. They also found that the EGARCH can overestimate volatility in a manner similar to the GJR model.

In summary, the GAM NP and the NP models perform much better than the parametric model in describing the volatility characteristics and capturing the rise and fall of the volatility in the Chinese stock markets. Because the EGARCH and GJR models tend to overestimate the volatility in turbulent periods and therefore yield larger estimation errors in general, they are less appropriate tools for estimating the Chinese stock volatility than the nonparametric models.

#### 2.4.4 Analyzing asymmetry via the news impact curve

In the previous section, the estimation results from the EGARCH and GJR models have shown that the asymmetry effect of unexpected news exists in the Chinese stock markets. We now further examine the asymmetry effects from the perspective of the nonparametric model. We use the news impact curve (NIC) proposed by Engle and Ng (1993) to demonstrate the asymmetry of shocks estimated from the GAM NP model. The NIC relates today's returns to tomorrow's volatility and works as a major tool for measuring how new information is incorporated in volatility estimates. Holding constant the information dated  $t - 2$  and earlier, it displays the implied impact of the functional relationship between conditional variance at time  $t$  and the shock term (error term) at time  $t - 1$ . Engle and Ng (1993) define the NIC as the expected conditional variance of the next period conditional on the current shocks,  $\epsilon_t$ .

$$E(\sigma_{t+1}^2 | \epsilon_t), \quad (2.15)$$

For the NIC of the GAM NP model, we extend the original news impact curve to the nonparametric context:

$$\sigma_t^2 = f(X_{t-1}) + g(\sigma^2), \quad (2.16)$$

where  $\sigma$  is the conditional volatility,  $X_{t-1}$  are the shocks from news, and  $f$  and  $g$  are the estimated nonparametric functions from the GAM NP model. The relationship between the shocks and the conditional volatility is therefore described in the nonparametric functions of  $f$ .

- Figure 2.6 about here -

The news impact curves of the EGARCH, GJR, and GAM NP models in the Shanghai and Shenzhen markets are plotted in Figure 2.6. The parameter values used for

constructing the NIC of the EGARCH and GJR models are from Table 2.4.  $f$  and  $g$  are the estimated nonparametric functions from the in-sample estimation. It is obvious that all models suggest the existence of asymmetric effects in stock returns because the NICs of all models are not symmetric about zero. Typically, negative news drives volatility up more than good news. In these models, any news today drives up volatility tomorrow. For example, in the SHCI, the asymmetric effect is clearly shown with all curves displaying an approximately  $20^\circ$  slope for “good news” and a  $40^\circ$  slope for “bad news.” We observe less asymmetric effect of bad news relative to good news in the Shenzhen stock market.

The NIC of the EGARCH and GJR models have their minimum shocks at  $X_t = 0$  implying no news is good news. In contrast to the parametric models, the NIC of the GAM NP model has its minimum larger than zero, 0.5 in the *SSE* and 1.5 in the *SZSE*. In this model, the NIC is a right-shifted asymmetric parabola. This phenomenon is consistent with the TGARCH model NIC from Engle and Ng (1993) and Christian (2007). This may suggest that, in the Chinese stock markets, a minimum amount of good news is required for the markets to remain as calm as possible. In this case, no news implies a higher volatility than in the tranquil market period. This further suggests that although the model implies the existence of a leverage effect, the typical good-news-chasing behavior of the Chinese stock investors found by Yeh and Lee (2000) has not changed. One of the reasons for Chinese investors’ good-news-chasing behavior explained by Yeh and Lee (2000) is that due to the lack of institutional investors, the trading values of the Shanghai and Shenzhen stock markets are completely generated by individual investors who have no access to inside information and irrationally act on noise as if it were information that would give them an edge. This typically reflects the investors’ behavior in Shenzhen.<sup>11</sup> The fast-growing stock market and its development produce more noise, making the investors more likely to speculatively and impetuously chase “good news.”

Given the fact that GAM NP better explains the volatility of the Chinese stock markets, we can see from the NIC that both the EGARCH and the GJR model overestimate the volatility reaction to the shocks between 2 and  $-2$ . However, the parametric models underestimate the volatility reaction to the extremely large shocks (the GAM NP has the highest variance in both directions when news is larger than 2 and smaller than  $-2$ ). Further, the GAM NP model has the best performance in capturing more asymmetric effects of shocks because the slopes of the two sides of the GAM NP model’s NIC are both steeper than the EGARCH and the GJR models.

As a result, compared with the EGARCH and the GJR model, the GAM NP model can provide us with better volatility estimates which capture the asymmetric effects of market news. The GAM NP model is more flexible in reflecting the actual market’s conditions as implied by the news impact curve. The findings from this paper have important implications for portfolio selection, asset pricing, and risk management. For

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<sup>11</sup>Within the last 20 years, owing to China’s economic liberalization under the policies of reformist leader Deng Xiaoping, Shenzhen became China’s first, and arguably one of the most successful Special Economic Zones, moving from a small village to a major financial center and China’s second busiest port.

instance, as implied by the news impact curves, there are significant differences in the predicted volatility incorporated with asymmetric effects of market news in the GAM NP model and other models. This may lead to a significant difference in current option price, portfolio selection, and dynamic hedging strategies. Only the most appropriate model can provide us with the best estimate of return volatility.

## 2.5 Conclusion

Using more recent data, this paper updates previous studies on Chinese stock-return volatility by examining the return volatility and the asymmetric effect of market news on the volatility in the Chinese stock markets using a nonparametric approach. Further, in order to avoid the curse of dimensionality, the back-fitting algorithm from the generalized additive model of Hastie and Tibshirani (1990) is applied to the nonparametric smoothing technique from Bühlman and McNeil (2002). Compared with the parametric GARCH models commonly used for capturing volatility asymmetry, the nonparametric models perform much better in capturing the asymmetry effect and in describing the characteristics of Chinese stock-return volatility.

With respect to the predicted return volatility's asymmetric reaction to good news and bad news, we find that the return volatility responds more strongly to bad news in the Chinese stock markets in our sample period. We extend the news impact curve to the nonparametric setting to further examine the asymmetry effect implied by the GAM NP model. Interestingly, the evidence based on the news impact curve of the GAM NP model suggests that the good-news-chasing behavior of the Chinese domestic investor continued. Additionally, the markets behave such that they require a certain amount of good news in order to remain as calm as possible.

When all the models are employed to obtain the overnight out-of-sample forecast, the nonparametric models yield the lowest forecast errors and outperform the parametric models by capturing the observed spikes in the volatility of returns. In contrast, the EGARCH and the GJR models tend to overestimate the volatility and returns in the high-volatility periods. The forecasted returns are therefore more accurate from the nonparametric model especially when the market is very volatile. There are many emerging stock markets attracting investors from all over the world. These markets may be as imperfect and incomplete as the Chinese stock markets have been. We recommend the use of the GAM NP and the NP model in estimating and investigating the return volatility in the Chinese stock markets and other emerging stock markets with features similar to those of the Chinese stock markets.

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# Tables

**Table 2.1: Simulation results from the parametric and nonparametric models**

Model	Normal		Student- $t$	
	MSE	MAE	MSE	MAE
GARCH	0.5554 (0.0466)	0.6154 (0.0201)	0.5553 (0.0464)	0.6155 (0.0201)
GJR	0.3901 (0.0424)	0.5070 (0.0272)	0.3896 (0.0420)	0.5066 (0.0270)
EGARCH	0.3004 (0.0445)	0.4295 (0.0296)	0.2976 (0.0378)	0.4273 (0.0230)
NP	0.2614 (0.0477)	0.4051 (0.0373)	0.2614 (0.0477)	0.4051 (0.0373)
GAM NP	0.2215 (0.0581)	0.3387 (0.0459)	0.2215 (0.0582)	0.3387 (0.0459)

*Note:* This table shows the estimated mean of the Mean Squared Errors (MSE) and the mean of the Mean Absolute Errors (MAE) for the estimated samples of  $n = 1000$  and 50 realizations for the GARCH, GJR, EGARCH, and nonparametric models. In the case of the  $t$  distributed errors, there are four degrees of freedom. The standard errors of the MSE and the MAE are in parentheses.



**Table 2.2: Simulation results from the GAM NP and the NP model**

Model	Normal				Student- $t$			
	NP model		GAM NP model		NP model		GAM NP model	
	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
GARCH	0.555 (0.047)	0.615 (0.020)	0.555 (0.047)	0.615 (0.020)	0.555 (0.046)	0.616 (0.020)	0.555 (0.046)	0.616 (0.020)
Iteration 1	0.347 (0.043)	0.460 (0.027)	0.286 (0.048)	0.393 (0.031)	0.347 (0.043)	0.460 (0.027)	0.286 (0.048)	0.393 (0.031)
Iteration 2	0.281 (0.045)	0.414 (0.032)	0.237 (0.053)	0.349 (0.041)	0.281 (0.045)	0.414 (0.032)	0.237 (0.053)	0.349 (0.041)
Iteration 3	0.267 (0.045)	0.406 (0.034)	0.225 (0.056)	0.339 (0.044)	0.267 (0.045)	0.406 (0.034)	0.225 (0.056)	0.339 (0.043)
Iteration 4	0.262 (0.044)	0.405 (0.033)	0.221 (0.055)	0.338 (0.044)	0.262 (0.044)	0.405 (0.033)	0.221 (0.055)	0.338 (0.044)
Iteration 5	0.264 (0.046)	0.406 (0.035)	0.221 (0.058)	0.337 (0.045)	0.264 (0.046)	0.406 (0.035)	0.221 (0.057)	0.337 (0.045)
Iteration 6	0.263 (0.047)	0.406 (0.037)	0.222 (0.058)	0.339 (0.045)	0.263 (0.047)	0.406 (0.037)	0.222 (0.058)	0.339 (0.045)
Iteration 7	0.262 (0.048)	0.405 (0.037)	0.224 (0.061)	0.341 (0.049)	0.262 (0.048)	0.405 (0.037)	0.224 (0.061)	0.341 (0.049)
Iteration 8	0.263 (0.048)	0.406 (0.037)	0.224 (0.060)	0.340 (0.047)	0.263 (0.048)	0.406 (0.037)	0.224 (0.060)	0.340 (0.047)
Final	0.261 (0.048)	0.405 (0.037)	0.221 (0.058)	0.339 (0.046)	0.261 (0.048)	0.405 (0.037)	0.221 (0.058)	0.339 (0.046)

*Note:* This table shows the estimated mean of the Mean Squared Errors (MSE) and the mean of the Mean Absolute Errors (MAE) at each iteration for the simulated sample of  $n = 1000$  and 50 realizations for the GAM NP and NP models. The first iteration of the nonparametric models is based on the result of the GARCH model. In the case of the  $t$  distributed errors, there are four degrees of freedom. The standard errors of the MSE and the MAE are in parentheses.

**Table 2.3: Data description**

	Shanghai Composite Index (SHCI)		Shenzhen Component Index (SZCI)	
Size	2573		2573	
Mean	0.068		0.067	
Median	0.070		0.048	
Min	−9.334		−9.935	
Max	9.401		9.530	
Std. Dev.	1.576		1.738	
Skewness	−0.203		−0.090	
Kurtosis	8.331		7.524	
JB test	3064.2	(0.001)	2198.0	(0.001)
ADF test	−50.972	(0.001)	−49.107	(0.001)
Correlation	0.926			

*Note:* This table reports summary statistics for the Shanghai Composite Index (SHCI) and the Shenzhen Component Index (SZCI) return series from January 1997 to August 2007. The JB test is the Jarque–Bera test for normality and the ADF test is the augmented Dickey–Fuller test for stationarity. The  $p$  values for the Jarque–Bera test and the augmented Dickey–Fuller test are reported in parentheses.

**Table 2.4: In-sample estimations of the GARCH, EGARCH, and GJR models**

	Shanghai Composite Index						Shenzhen Component Index					
	GARCH		EGARCH		GJR		GARCH		EGARCH		GJR	
	Normal	Student- $t$	Normal	Student- $t$	Normal	Student- $t$	Normal	Student- $t$	Normal	Student- $t$	Normal	Student- $t$
$\mu$	0.000 (0.021)	0.014 (0.023)	-0.011 (0.024)	0.009 (0.023)	-0.018 (0.025)	0.004 (0.023)	-0.021 (0.026)	-0.029 (0.026)	0.040 (0.029)	-0.034 (0.025)	-0.034 (0.028)	-0.037 (0.026)
$\omega$	0.095 (0.013)	0.093 (0.023)	0.038 (0.005)	0.027 (0.009)	0.082 (0.012)	0.090 (0.022)	0.067 (0.010)	0.097 (0.024)	0.080 (0.010)	0.027 (0.009)	0.062 (0.010)	0.093 (0.024)
$\alpha_1$	0.139 (0.011)	0.117 (0.018)	0.242 (0.018)	0.239 (0.029)	0.098 (0.011)	0.077 (0.018)	0.100 (0.007)	0.102 (0.016)	0.276 (0.018)	0.216 (0.027)	0.082 (0.008)	0.080 (0.016)
$\beta_1$	0.829 (0.013)	0.848 (0.021)	0.964 (0.006)	0.957 (0.010)	0.845 (0.012)	0.844 (0.021)	0.879 (0.007)	0.865 (0.019)	0.932 (0.009)	0.967 (0.009)	0.882 (0.007)	0.863 (0.019)
$\gamma$			-0.036 (0.008)	-0.063 (0.016)	0.060 (0.014)	0.095 (0.028)			0.028 (0.010)	-0.035 (0.014)	0.036 (0.011)	0.055 (0.024)
DoF		4.638 (0.455)		4.87 (0.486)		4.725 (0.458)		4.848 (0.517)		4.952 (0.539)		4.882 (0.518)
Q(20)	24.32	24.43	24.91	25.48	25.12	25.53	27.29	27.17	26.10	27.55	28.28	28.97

*Note:* This table shows the estimated coefficients of the parametric GARCH, GJR and EGARCH models for the Shanghai Composite Index and the Shenzhen Component Index return series. The sample period is from January 1997 to August 2006. The data are on a daily basis and have 2,330 observations. All returns are scaled by 100. The GARCH, GJR and EGARCH models are estimated according to equations 2.1, 2.2 and 2.3. The standard errors are reported in parentheses. The last row reports the test statistics of the Ljung-Box Q-test for residual autocorrelation of all models at lag 20. The critical value for 20 lags at the 5% significance level is 31.4104.

**Table 2.5: Goodness of fit for in-sample forecasts**

Model	Distribution	SHCI				SZCI			
		<i>MSE1</i>	<i>MAE1</i>	<i>MSE2</i>	<i>MAE2</i>	<i>MSE1</i>	<i>MAE1</i>	<i>MSE2</i>	<i>MAE2</i>
GARCH	Normal	39.123	2.677	1.310	0.879	48.509	3.081	1.472	0.934
	Student- <i>t</i>	38.908	2.655	1.297	0.878	48.380	3.062	1.461	0.934
EGARCH	Normal	38.037	2.576	1.246	0.859	48.232	3.061	1.462	0.944
	Student- <i>t</i>	37.883	2.564	1.240	0.858	47.956	3.013	1.431	0.926
GJR	Normal	38.884	2.663	1.299	0.875	48.564	3.082	1.471	0.933
	Student- <i>t</i>	38.787	2.652	1.293	0.873	48.457	3.068	1.462	0.934
NP	Normal	37.846	2.595	1.261	0.870	47.734	3.037	1.439	0.930
	Student- <i>t</i>	37.851	2.596	1.261	0.870	47.735	3.037	1.438	0.929
GAM NP	Normal	37.700	2.553	1.236	0.857	47.858	3.015	1.429	0.924
	Student- <i>t</i>	37.708	2.553	1.238	0.856	47.874	3.012	1.429	0.924

*Note:* This table shows the goodness of fit of all models for the in-sample forecast for the Shanghai Composite index (SHCI) and the Shenzhen Component Index (SZCI) using four different measures.  $MSE1 = \frac{1}{n} \sum_{t=1}^n (X_t^2 - \hat{\sigma}_t^2)^2$  is the mean squared error between the squared innovation,  $X_t^2$ , and the squared estimated volatility,  $\hat{\sigma}_t^2$ .  $MAE1 = \frac{1}{n} \sum_{t=1}^n |X_t^2 - \hat{\sigma}_t^2|$  is the mean absolute error between the squared innovation,  $X_t^2$ , and the squared estimated volatility,  $\hat{\sigma}_t^2$ .  $MSE2 = \frac{1}{n} \sum_{t=1}^n (\hat{\sigma}_t - \sigma_t)^2$  is the mean squared error between the estimated volatility and the true volatility proxy,  $\sigma_t = |y_t|$ , where  $y_t$  is the daily return at time  $t$ .  $MAE2 = \frac{1}{n} \sum_{t=1}^n |\hat{\sigma}_{t,m} - \sigma_t|$  is the mean absolute error between the estimated volatility and the true volatility proxy,  $\sigma_t = |y_t|$ , where  $y_t$  is the daily return at time  $t$ .  $n$  is the total observations in the in-sample estimation.

**Table 2.6: The DM test results for the in-sample forecasts**

Model	Distribution	GAM NP				NP			
		$MSE1$	$MAE1$	$MSE2$	$MAE2$	$MSE1$	$MAE1$	$MSE2$	$MAE2$
GARCH	Normal	3.209	5.504	4.512	4.231	2.389	3.710	2.977	2.003
	Student- $t$	2.874	4.905	4.022	4.024	2.224	3.000	2.399	1.889
SHCI EGARCH	Normal	1.0464	1.653	1.026	0.378	0.550	-1.443	-1.472	-3.882
	Student- $t$	0.796	0.859	0.506	0.168	0.118	-2.710	-2.353	-4.99
GJR	Normal	2.856	5.51	4.389	3.820	1.954	3.379	2.545	1.209
	Student- $t$	2.562	5.329	4.294	3.812	1.700	2.888	2.212	0.832
GARCH	Normal	2.373	4.988	4.648	3.199	2.165	3.268	3.566	1.468
	Student- $t$	2.271	4.187	3.940	3.613	2.000	2.281	2.805	2.140
SZCI EGARCH	Normal	0.897	2.701	2.759	4.860	1.049	1.324	1.864	3.593
	Student- $t$	0.445	0.125	0.266	0.565	0.883	-2.171	-0.963	-1.269
GJR	Normal	2.419	5.506	5.029	3.347	2.160	3.476	3.589	1.332
	Student- $t$	2.223	5.155	4.681	3.910	1.865	2.638	2.828	1.975

*Note:* This table shows the DM test results of various models for the in-sample forecasts of the Shanghai Composite Index (SHCI) and the Shenzhen Component Index (SZCI). The reported values are the test statistics of the DM test under the null hypothesis that the improvement of the model in the column (the GAM NP and the NP model) upon the model in the row (the parametric models) is not significant. The nonparametric models in the columns are the benchmark models. The significance level is 5%.

**Table 2.7: The Goodness of fit for the out-of-sample forecasts**

Model	Distribution	Shanghai Composite Index				Shenzhen Component Index			
		Benchmark I		Benchmark II		Benchmark I		Benchmark II	
		MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
GARCH	Normal	2.13	1.129	0.596	0.578	2.602	1.257	0.731	0.653
	Student-t	2.088	1.114	0.587	0.575	2.559	1.241	0.709	0.642
EGARCH	Normal	2.026	1.087	0.573	0.546	2.531	1.228	0.696	0.644
	Student-t	1.983	1.064	0.577	0.542	2.49	1.218	0.693	0.62
GJR	Normal	2.138	1.123	0.639	0.587	2.607	1.256	0.724	0.643
	Student-t	2.12	1.109	0.625	0.581	2.563	1.238	0.744	0.655
NP	Normal	1.905	1.047	0.515	0.526	2.411	1.195	0.686	0.614
	Student-t	1.903	1.045	0.517	0.525	2.403	1.192	0.689	0.614
GAM NP	Normal	1.909	1.056	0.516	0.526	2.468	1.205	0.690	0.620
	Student-t	1.908	1.055	0.517	0.525	2.442	1.207	0.689	0.619

*Note:* This table shows the goodness of fit for various models for the out-of-sample forecast in the Shanghai and Shenzhen markets. The out-of-sample period is from September 2006 to August 2007. Benchmark I uses  $\hat{\sigma}_t = \sqrt{(y_t)^2}$  as the true volatility proxy. Benchmark II uses the realized volatility,  $\hat{\sigma}_t = \sqrt{\sum_{i=1}^n r_{i,t}^2}$ , as the true volatility proxy, where  $n$  is the total number of high frequency intervals,  $i$ , in day  $t$

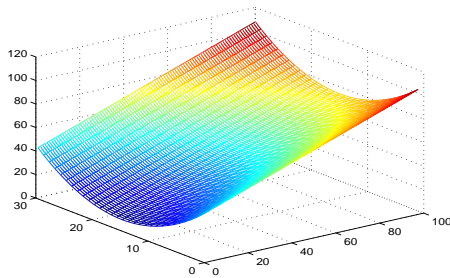
**Table 2.8: The DM test results for out-of-sample forecasts**

Volatility proxy			Shanghai Composite Index				Shenzhen Component Index			
Benchmark	Model	Distribution	GAM NP		NP		GAM NP		NP	
			MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
I	GARCH	Normal	4.47	5.04	4.50	5.50	3.32	4.57	4.00	4.77
		Student- $t$	3.73	4.44	3.96	5.07	2.97	3.82	3.87	4.16
	EGARCH	Normal	2.28	2.46	2.92	3.57	1.98	2.02	2.22	2.21
		Student- $t$	2.19	1.89	2.51	2.01	1.87	1.99	2.11	1.99
	GJR	Normal	4.53	4.55	4.68	5.28	3.70	4.79	4.14	4.54
		Student- $t$	4.32	3.87	4.45	4.55	3.74	4.08	4.10	3.78
II	GARCH	Normal	2.49	3.24	2.78	3.44	3.98	3.12	4.36	3.85
		Student- $t$	2.20	3.03	2.59	3.31	3.41	2.61	3.88	3.25
	EGARCH	Normal	2.05	2.27	2.26	2.26	0.32	1.68	0.65	1.95
		Student- $t$	3.17	2.14	3.15	2.13	0.22	0.80	0.35	1.06
	GJR	Normal	3.35	3.77	3.68	4.16	2.77	3.56	4.61	3.98
		Student- $t$	3.76	4.14	4.00	4.49	2.30	3.23	4.17	3.39

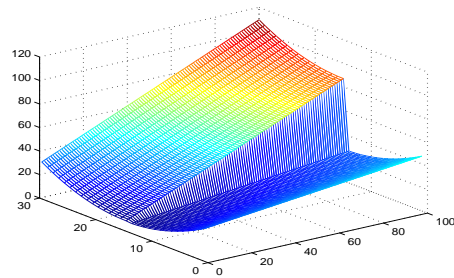
*Note* : This table shows the DM test results for the goodness of fit for various models for the out-of-sample forecast in Shanghai and Shenzhen markets. The reported values are the test statistics from the DM test under the null hypothesis that the improvement of the model in the column upon the model in the row is not significant. The Benchmark I uses  $\hat{\sigma}_t = \sqrt{(y_t)^2}$  as the true volatility proxy. The Benchmark II uses the realized volatility  $RV_t = \sqrt{\sum_{i=1}^n r_{i,t}^2}$  as the true volatility proxy.

# Figures

**Figure 2.1:** Volatility surfaces from simulated processes



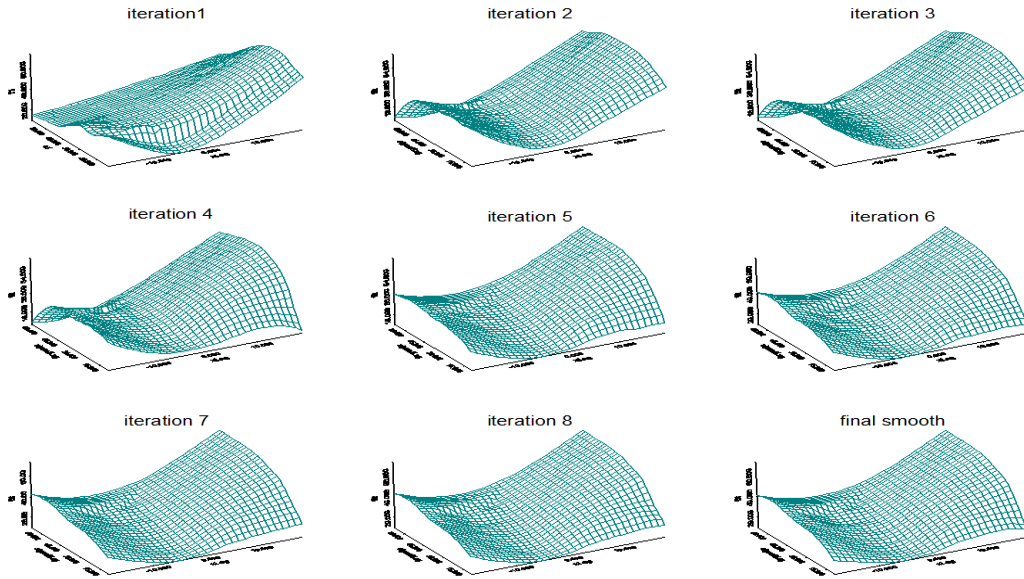
(a) Symmetric: process (2.10)



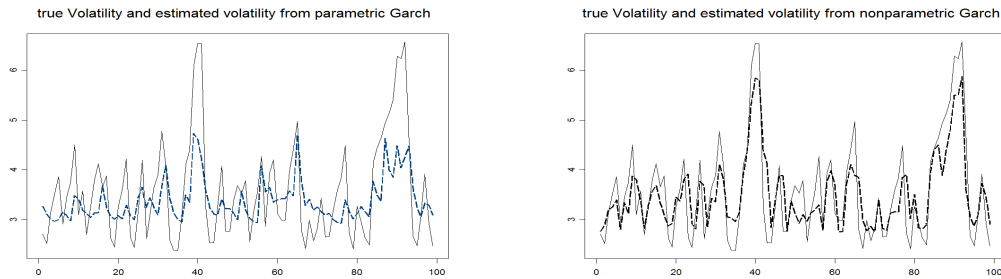
(b) Asymmetric: process (2.11)

*Note:* This figure plots the volatility surfaces from the processes specified in equations 2.10 and 2.11.



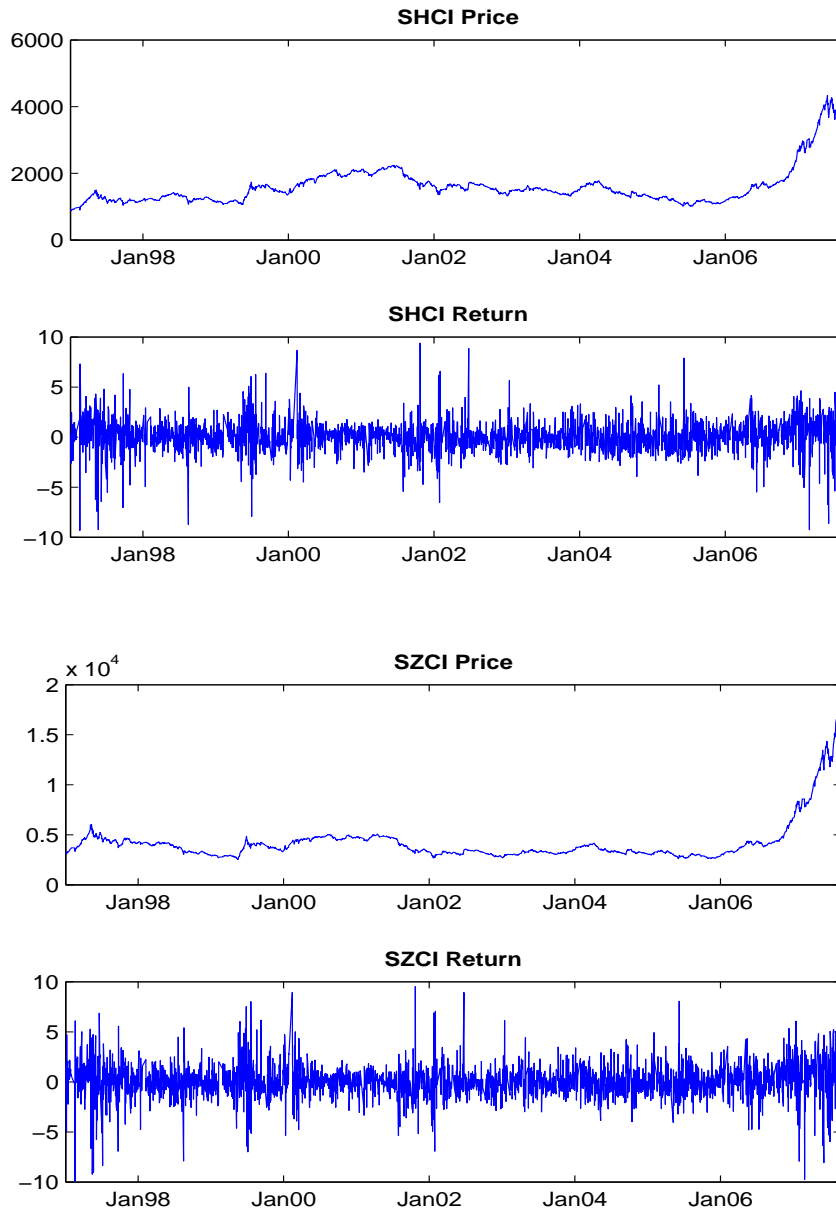
**Figure 2.2:** Smoothed volatility surface from each iteration

*Note:* This figure plots the volatility surface of the GAM NP model at each iteration of a randomly chosen realization. The last plot is the final smoothed volatility surface. In each plot, the  $z$ -axis is the smoothed volatility from each iteration. The  $x$ -axis is the lagged return,  $X_{t-1}$ . The  $y$ -axis is the lagged smoothed sigma,  $\hat{\sigma}_{t-1}$ , from the previous iteration.

**Figure 2.3:** Estimated and the true volatility

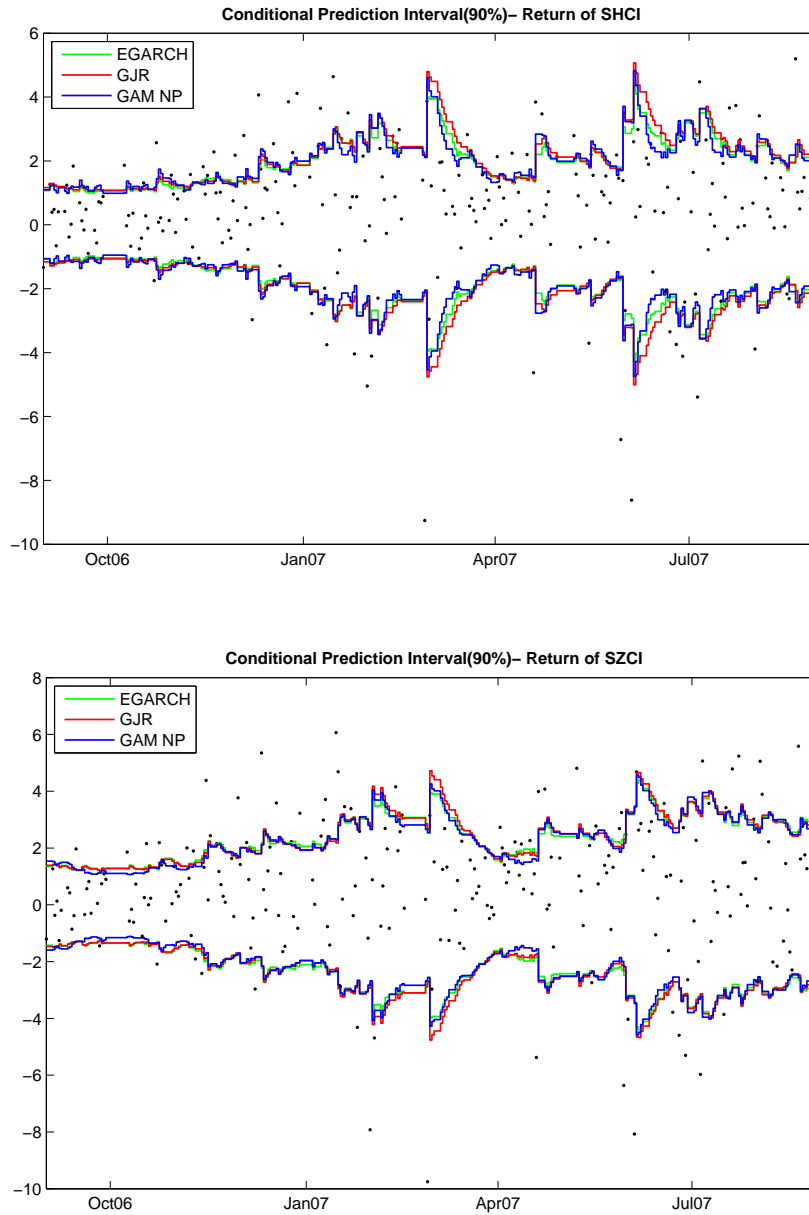
*Note:* This figure plots the true volatility and the estimated volatility from the GARCH and the GAM NP model for a randomly chosen iteration of 100 points. The solid line is the true volatility and the dotted line is the estimated volatility. The left plot is the true and estimated volatility from the GARCH model. The right plot is the true and estimated volatility from the GAM NP model.

**Figure 2.4:** Price and return for Shanghai Composite Index and the Shenzhen Component Index

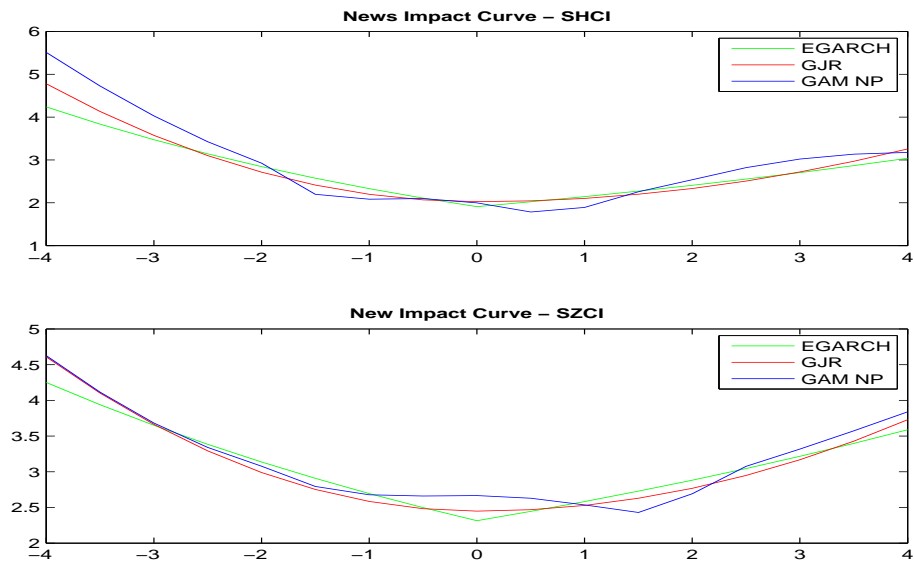


*Note:* This figure plots the price and the return series of the Shanghai Composite Index (SHCI) and the Shenzhen Component Index (SZCI) for the entire sample period from January 1997 to August 2007. The first two plots are the price and return series of the SHCI and the last two plots are the price and the return series of the SZCI.

**Figure 2.5:** The 90 % conditional prediction interval



*Note:* This figure plot the 90% conditional prediction interval for the return of the Shanghai Composite Index and the Shenzhen Component Index. The returns intervals are calculated based on the out-of-sample forecast results for these two series. The out-of-sample forecast starts on September 01, 2006 and ends on August 31, 2007.

**Figure 2.6:** News impact curves

*Note:* This figure plot the news impact curve from the GAM NP, EGARCH and GARCH models. The  $x$ -axis represents the lagged market news, and the  $y$ -axis represents the volatility calculated based on equation 15 (for parametric models) and equation 16 (for the GAM NP model).

## Chapter 3

# Modeling and Forecasting Short-Term Interest Rate Volatility: A Semiparametric Approach

### 3.1 Introduction

There is an extensive literature on the modeling of the short-term interest rate as this rate is fundamental to the pricing of fixed-income securities. The short-term rate is also a necessary input, for example, for the optimal portfolio choice, hedging strategies, and other investment decisions. Further, the short rate influences the macro economy; therefore, it is a target instrument for monetary policy makers. One of the earliest papers that formally compares a number of single-factor models is Chan et al. (1992) (we refer the proposed model from this paper as CKLS hereafter). Based on U.S. data, their study controversially rejects the commonly adopted square root diffusion model of Cox et al. (1985), whereby the volatility of short-rate changes is proportional to the square root of the interest-rate levels. Instead, their model shows that volatility is more sensitive to interest-rate levels, specifying an exponent for the commonly known level effect in the region of 1.5. A more recent study by Brenner et al. (1996) (we refer the proposed model from this paper as BHK hereafter) shows that models that parameterize volatility as a function of only interest-rate levels tend to over emphasize the sensitivity of volatility to levels and do not take into consideration the serial correlation in conditional variances. They propose a new class of models which allows volatility to depend on both interest-rate levels and information shocks. There is by now a general consensus in the literature that short-rate models that account for both the levels effect and serial correlation in the volatility processes perform better than models that parameterize only the levels effect or the serial dependence in the conditional variances (Bali, 2000).

Unlike for the diffusion process, analysis of short-rate models provides mixed empirical evidence of mean reversion and it remains highly controversial about whether the possible mean reversion is linear or nonlinear. While a large proportion of research reports a linear drift (see, e.g., CKLS and the models nested by it), others argue to the contrary (Ait-

Sahalia, 1996*b,a*, Conley et al., 1997 Jones, 2003), finding nonlinear mean reversion. Using a semi-nonparametric approach, Aït-Sahalia (1996*b,a*) constructs a general specification test of a short-rate model and rejects a linear drift in favor of models that imply no mean reversion for levels of the short rate between certain threshold levels and strong mean reversion for extreme levels of the short rate. Stanton (1997) and Jiang (1998) estimate a model of the short rate nonparametrically using different data sets from Aït-Sahalia (1996*b*) and find evidence in support of nonlinearities in the drift function. Arapis and Gao (2006) also use nonparametric methods to show that the short-rate drift is nonlinear. Bali and Wu (2006) document evidence that the speeds of mean reversion for short-term interest rates at extremely high interest rates such as in the Volcker (1979–1982) regime are different than at normal times. They attribute the nonlinearity in short-rate drift to the differences in the degree of mean reversion at different interest-rate levels. Christiansen (2010) allows for extreme value mean reversion by including the smallest short rate during the previous year in the mean equation and finds that the US short rates exhibit extreme value mean reversion. Be that as it may, the robustness of the nonlinear drift function in short-rate models has been questioned by some authors. Pritsker (1998) examines the finite-sample properties of Aït-Sahalia's nonparametric test, showing that upon adjusting for the high persistence in interest rates, the nonlinearity in the drift function becomes statistically insignificant. Chapman and Pearson (2000) perform simulation exercises and show that the evidence supporting the nonlinear drift function could be an artifact of the nonparametric estimation procedure rather than a true feature of the data-generating process. Using Bayesian estimation methods, Jones (2003) shows that the determination of short-rate drift specification is dependent on the assumption of the prior distribution. In particular, under the assumption of a flat prior distribution and the imposition of stationarity in interest-rate dynamics, he identifies a nonlinear drift. However, when he implements an approximate Jeffrey's prior, there is no mean-reverting evidence. Durham (2004) finds that the significance of nonlinearity in the drift function depends on the specification of the diffusion process, a finding which agrees with Bali (2007). Takamizawa (2008) uses cross-sectional relations to estimate Aït-Sahalia's (1996*a*) model, but he finds that there is generally no nonlinear mean reversion.

This paper considers an alternative method for modeling short-rate volatility. We apply a semiparametric smoothing technique to the generalized autoregressive conditional heteroskedasticity (GARCH) model of short-rate volatility. This involves estimating a parametric form of the short-rate drift function followed by estimating the hidden volatility process nonparametrically. Because the literature is divided on the appropriate drift specification, we estimate both linear and nonlinear drift functions of the short rate. The estimation of a parametric drift specification qualifies this approach as a semiparametric method (Jiang and Knight, 1997). To estimate the latent volatility process, we use the algorithm developed by Bühlman and McNeil (2002) and apply it to Hastie and Tibshirani's (1990) generalized additive model. Bühlman and McNeil (2002) argue that estimating the volatility process with the nonparametric approach is less sensitive to model misspecification and does not require *a priori* knowledge of the innovation distribution. This feature makes the application of the nonparametric method attractive for estimating a

short-rate diffusion process given that short rates are known to possess distributions that depart from normality. We specify the latent volatility process as a general additive function of the lagged value of the conditional variance, innovations, and interest-rate levels. This specification is consistent with a class of single-factor short-rate diffusion models where the volatility of short-rate changes is serially dependent on past volatility, squared innovations, and interest-rate levels. In addition, the additive structure of the hidden volatility facilitates the use of a backfitting algorithm to estimate the diffusion process.

The potential usefulness of the proposed semiparametric approach for estimating short-rate volatility is examined by comparing its forecast performance with a variety of one-factor short-rate diffusion models. Results from our Monte Carlo simulation illustrate the robustness of the semiparametric approach when estimating the short-rate changes' sensitivity to misspecification in the short-rate drift function and the underlying innovation distribution. Moreover, the forecast performance of the semiparametric approach is superior to that of the parametric models considered in the simulated data. The empirical application to three-month U.S. Treasury bill yields suggests that the semiparametric estimation procedure provides in-sample and out-of-sample volatility forecasts superior to the short-rate volatility models of BHK, which feature asymmetric and level-dependent conditional variance. Although the semiparametric approach does not specify the asymmetric feature of the volatility process, this procedure improves upon the fit and the predictive power of the volatility estimates. We do not find any evidence of nonlinearities in short-rate drift or conditional skewness in the short-rate-change distribution. Finally, we demonstrate that the semiparametric approach, which yields a greater degree of accuracy in modeling short-rate-change volatility, has pertinent implications for pricing long-dated and path-dependent interest-rate derivatives.<sup>1</sup> Using simulation methods, we show that the semiparametric modeling approach gives rise to significantly different probability distributions of future interest-rate levels compared with parametric short-rate models. The confidence intervals of future interest-rate levels are narrower than for any of the parametric models considered, thereby leading to a less price variability for interest-rate derivatives.

The rest of the paper is organized as follows. Section 3.2 describes the short-rate models and the semiparametric smoothing technique. Section 3.3 outlines the design of the Monte Carlo experiment to examine the in-sample predictive power of the semiparametric approach and its forecast property when subject to possible misspecifications in the drift function and the innovation distribution. This section also reports the results of the simulation study. Section 3.4 applies the semiparametric technique to the U.S. short-term interest rates to evaluate its in-sample and out-of-sample forecast performance relative to other short-rate models. Implications of this forecast improvement on pricing interest-rate derivatives are also discussed. Section 3.5 concludes.

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<sup>1</sup>These interest-rate derivatives include index amortizing rate swaps, CMO swaps, swaptions, mortgages, and adjustable-rate preferred securities.



## 3.2 The short-rate models and the semiparametric approach

### 3.2.1 The short-rate models

The generalized continuous-time short-rate specification of CKLS is,

$$dr = (\mu + \lambda r) dt + \phi r^\delta dW, \quad (3.1)$$

where  $r$  denotes the level of the short-term interest rate,  $W$  is a Brownian motion, and  $\mu$ ,  $\lambda$ , and  $\delta$  are parameters. The drift component of short-term interest rates is captured by  $\mu + \lambda r$  while the variance of unexpected changes in interest rates equals  $\phi^2 r^{2\delta}$ . The parameter  $\phi$  is a scale factor and  $\delta$  controls the degree to which the interest-rate level influences the volatility of short-term interest-rate changes. The CKLS model nests many of the existing interest-rate models. For example, when  $\delta = 0$  then (3.1) reduces to Vasicek's (1977) model, while  $\delta = 1/2$  yields the Cox et al. (1985) model, see CKLS *inter alia* for further details. There is a dearth of literature focusing on the univariate CKLS model. Czellar et al. (2007) study different estimation techniques for the CKLS short-rate model. Bali and Wu (2006) investigate extensions of the mean specification of the CKLS model. On the other hand, Nowman and Sorwar (2005) use the CKLS model to price bonds and contingent claims.

It is common to consider the Euler–Maruyama discrete time-approximation to (3.1):

$$\Delta r_t = \mu + \lambda r_{t-1} + \varepsilon_t. \quad (3.2)$$

Let  $\Omega_{t-1}$  represent the information set available at time  $t - 1$ , and let  $E(\varepsilon_t | \Omega_{t-1}) = 0$ . Suppose  $h_t$  represent the conditional variance of the short-term interest-rate changes; then  $E(\varepsilon_t^2 | \Omega_{t-1}) \equiv h_t = \phi^2 r_{t-1}^{2\delta}$ . It can be seen that the only source of conditional heteroskedasticity in (3.2) is through the level of the interest rate. BHK relaxes the assumption of a constant  $\phi^2$  by allowing it to vary according to the information arrival process. One common approach to capturing the effect of unanticipated news is the GARCH(1,1) model:

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 h_{t-1}. \quad (3.3)$$

The innovation  $\varepsilon_t$  denotes a change in the information set from time  $t - 1$  to  $t$  and can be treated as a collective measure of unanticipated news. In (3.3), only the magnitude of the innovation is important in determining  $h_t$ . BHK extends (3.2) to allow information from unanticipated news and the one-period lagged interest-rate levels to govern the dynamics of short-rate volatility in the following way.

$$\begin{aligned} \Delta r_t &= \mu + \lambda r_{t-1} + \varepsilon_t, \\ \varepsilon_t &= \sqrt{h_t} z_t, \quad z_t \sim t(v) \text{ and} \\ h_t &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 h_{t-1} + b r_{t-1}^{2\delta} \end{aligned} \quad (3.4)$$

Equation (3.4) is known as the GARCH-X process. Under the restriction  $\alpha_0 = \alpha_1 = \alpha_2 = 0$ , (3.4) collapses to (3.2) where  $b = \phi^2$  and volatility depends on interest-rate levels

alone. Furthermore, when  $b = 0$ , there is no levels effect. The GARCH-X model does not permit short-rate volatility to respond asymmetrically to interest-rate innovations of different signs. BHK relaxes the assumption of a symmetric GARCH-X process by modeling the conditional variance specification as

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 h_{t-1} + b r_{t-1}^{2\delta} + \alpha_3 \xi_{t-1}^2, \quad (3.5)$$

where  $\xi_{t-1} = \min(0, \varepsilon_{t-1})$ . BHK refers to this model as the AsyGARCH-X. For simplicity, we refer to the symmetric (asymmetric) GARCH-X as the GARCHX (AGARCHX) model. For the purpose of this paper, we only consider the additive levels effect as the (A)GARCHX model is consistent with the generalized additive nonparametric GARCH model discussed in the next subsection.<sup>2</sup> In practice, when estimating the (A)GARCHX model, it is common to scale the interest-rate-level term in the variance equation with a factor (1/10) such that the levels dependence in the conditional variance is captured by  $b(r_{t-1}/10)^{2\delta}$  (see Brenner et al., 1996).

The linear drift in equation (3.2) implies that the strength of mean reversion is the same for all levels of the short rate. Even though there is no *a priori* economic intuition that would suggest the existence of a nonlinear drift, empirical research has shown that there is evidence of nonlinear drift in short-term interest rates. That is, mean reversion is stronger for extreme low or high levels of short rates. Ait-Sahalia (1996b) advocates the use of a flexible functional form to approximate the true unknown shapes of the short-rate process. He estimates a short-rate model,

$$dr_t = \left( \mu + \lambda_1 r_t + \lambda_2 r_t^2 + \frac{\lambda_3}{r_t} \right) dt + \sqrt{\beta_0 + \beta_1 r_t + \beta_2 r_t^{\beta_3}} dW_t, \quad (3.6)$$

and finds that the test rejects a linear drift in favor of models that imply no mean reversion for levels of the short rate between 4% and 22% and strong mean reversion for levels outside that range. Conley et al. (1997) adopt the same drift parameterizations as Ait-Sahalia but keeps the constant elasticity variance diffusion used by CKLS:

$$dr_t = \left( \mu + \lambda_1 r_t + \lambda_2 r_t^2 + \frac{\lambda_3}{r_t} \right) dt + \sigma r_t^\gamma dW_t. \quad (3.7)$$

They find that the drift function displays mean reversion only for rates below 3% or above 11%. Bali (2007) also estimates Ait-Sahalia's (1996b) nonlinear drift specification in (3.6) but with a diffusion process that follows a GARCH(1,1) model and is dependent on interest-rate levels. Bali and Wu (2006) estimate a variant of the drift specification in (3.6) which includes a fifth-order polynomial. Because of the extensive research that adopts Ait-Sahalia's (1996b) nonlinear drift specification and the possible influence this nonlinear drift might exert on the conditional volatility of interest-rate changes, we also estimate

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<sup>2</sup>BHK also considers the multiplicative levels effect in which  $\phi^2$  in  $E(\varepsilon_t^2 | \Omega_{t-1}) \equiv h_t = \phi^2 r_{t-1}^{2\delta}$  follows a GARCH(1,1) process.

a discrete-time approximation of Aït-Sahalia's (1996 *b*) nonlinear drift specification,

$$\Delta r_t = \mu + \lambda_1 r_{t-1} + \lambda_2 r_{t-1}^2 + \frac{\lambda_3}{r_{t-1}} + \varepsilon_t, \quad (3.8)$$

and the conditional variance of the short-term interest-rate changes that follows equation (3.5).

Empirical studies on short-term interest rates have shown that the standardized residuals obtained from the GARCH models exhibit leptokurtosis. The assumption of normality is easily rejected by the Jarque–Bera test when applied to short-rate data. Consequently, the Student's  $t$  distribution is commonly employed to capture the thicker tails in the empirical distribution of short rates. There are, however, other nonnormal distributions that have been used to characterize the distribution of short-rate changes. In particular, much attention has been paid in modeling the skewness of the distribution. Bali (2007) adopts the skewed generalized error distribution of Theodossiou (1998) as well as Hansen's (1994) skewed  $t$  distribution to capture the skewness in the empirical distribution of the three-month U.S. Treasury bill yield. Following Bali (2007), we employ both the Student's  $t$  and Hansen's skewed  $t$  distributions in the Monte Carlo experiment and empirical application. For the skewed  $t$  distribution, we define the residuals in equation (3.4) as

$$\varepsilon_t = \sqrt{h_t} z_t, \quad z_t \sim \text{Hansen's } t(v, \eta). \quad (3.9)$$

The parameters  $\eta$  and  $v$  control the direction of asymmetry and kurtosis of the distribution. Hansen's skewed  $t$  distribution is defined by

$$f(z_t; \Theta, v, \eta) = \begin{cases} bc \left( 1 + \frac{1}{v-2} \left( \frac{bz_t+a}{1-\eta} \right)^2 \right)^{-\frac{v+1}{2}} & \text{if } z_t < -\frac{a}{b} \\ bc \left( 1 + \frac{1}{v-2} \left( \frac{bz_t+a}{1+\eta} \right)^2 \right)^{-\frac{v+1}{2}} & \text{if } z_t \geq -\frac{a}{b}, \end{cases} \quad (3.10)$$

where  $z_t = \frac{\varepsilon_t}{\sqrt{h_t}}$ ,  $\Theta$  is the set of parameters associated with the drift and diffusion specifications of the short-rate model, and the constants  $a$ ,  $b$  and  $c$  are given by

$$a = 4\eta c \left( \frac{v-2}{v-1} \right), \quad b^2 = 1 + 3\eta^2 - a^2, \quad c = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{\pi(v-2)}\Gamma\left(\frac{v}{2}\right)}.$$

For  $\eta = 0$ , Hansen's distribution reduces to the traditional standardized  $t$  distribution, while for  $\eta = 0$  and  $v = \infty$ , it reduces to a normal density. The density (3.10) is defined for  $2 < v < \infty$  and  $-1 < \eta < 1$ .

### 3.2.2 The generalized additive semiparametric GARCH model

Consider the short-rate model,

$$X_t = \sigma_t Z_t \quad (3.11)$$

$$\sigma_t^2 = f_1(X_{t-1}) + f_2(\sigma_{t-1}^2) + f_3(r_{t-1}), \quad (3.12)$$

where  $\{Z_t; t \in \mathbb{Z}\}$  is an i.i.d innovation with zero mean, unit variance, and finite fourth moment;  $X_t = \Delta r_t - (\mu + \lambda r_{t-1})$  for a linear drift; and  $X_t = \Delta r_t - (\mu + \lambda_1 r_{t-1} + \lambda_2 r_{t-1}^2 + \frac{\lambda_3}{r_{t-1}})$  for a nonlinear drift. Let  $f_1 : \mathfrak{R} \rightarrow \mathfrak{R}_+$ ,  $f_2 : \mathfrak{R}_+ \rightarrow \mathfrak{R}_+$ , and  $f_3 : \mathfrak{R}_+ \rightarrow \mathfrak{R}_+$  be strictly positive-valued functions. The conditional variance and volatility are denoted by  $\sigma_t^2$  and  $\sigma_t$ , respectively. Further, assume that  $X_t$  and  $r_t$  are stationary stochastic processes and  $\{X_t : t \in \mathbb{Z}\}$  is adapted to the  $\sigma$ -filtration  $\{F_t; t \in \mathbb{Z}\}$  with  $F_t = \sigma(\{X_s; s \leq t\})$ . The assumption of stationarity in  $r_t$  is empirically verified by performing Seo's (1999) unit root test on the three-month U.S. Treasury bill rate. To ensure comparability with the CKLS and BHK short-rate models, we first estimate the linear drift function specified in equation (3.2). However, given the vast literature on short-rate models with nonlinear drift functions, we also investigate Aït-Sahalia's (1996b) nonlinear-drift specification given by equation (3.8). The exact form of the functions  $f_1$ ,  $f_2$ , and  $f_3$  in (3.12) is left unspecified, but it can be estimated by a nonparametric method in which  $X_t^2$  is regressed on the lagged variables  $X_{t-1}$ ,  $\sigma_{t-1}^2$ , and  $r_{t-1}$ . To show that this procedure is applicable for estimating the unobserved variable  $\sigma_t^2$ , we rewrite the model (3.11 and 3.12) as

$$\begin{aligned} X_t^2 &= f_1(X_{t-1}) + f_2(\sigma_{t-1}^2) + f_3(r_{t-1}) + V_t \\ V_t &= [f_1(X_{t-1}) + f_2(\sigma_{t-1}^2) + f_3(r_{t-1})] (Z_t^2 - 1), \end{aligned} \quad (3.13)$$

where  $V_t$  is a martingale difference series with  $E(V_t) = E(V_t | F_{t-1}) = 0$  and  $\text{cov}(V_s, V_t) = \text{cov}(V_s, V_t | F_{t-1}) = 0$  for  $s < t$ . Taking the conditional expectations of  $X_t^2$  in (3.13) yields

$$E(X_t^2 | F_{t-1}) = f_1(X_{t-1}) + f_2(\sigma_{t-1}^2) + f_3(r_{t-1}), \quad (3.14)$$

and its conditional variance can be shown to be

$$\text{var}(X_t^2 | F_{t-1}) = [f_1(X_{t-1}) + f_2(\sigma_{t-1}^2) + f_3(r_{t-1})]^2 [E(Z_t^4) - 1]. \quad (3.15)$$

To estimate the latent variable  $\sigma_t^2$  in (3.12), we adopt the estimation algorithm of Büلمان and McNeil (2002). For a given data sample, we first calculate the volatility estimate,  $\hat{\sigma}_{t,0}$ , by estimating the linear drift specification (3.2) and the conditional variance (3.3) using the method of maximum likelihood. Further, we verify that the semiparametric approach is robust to possible nonlinear drift by calculating the volatility estimate  $\hat{\sigma}_{t,0}$  with a nonlinear drift specification. Note that  $\hat{\sigma}_{t,0}$  is used as the initial volatility estimate. In the first iteration, we regress  $\{X_t^2; 2 \leq t \leq n\}$  against  $\{X_{t-1}; 2 \leq t \leq n\}$ ,  $\{\hat{\sigma}_{t-1,0}^2; 2 \leq t \leq n\}$ , and  $\{r_{t-1}; 2 \leq t \leq n\}$  using a nonparametric smoothing procedure

with a backfitting algorithm to obtain an estimate  $\hat{f}_{i,1}$  of  $f_i$  for  $i = 1, 2$  and  $3$ .<sup>3</sup> The regression is performed with regression weights  $\{\hat{\sigma}_{t,0}^{-2}; 2 \leq t \leq n\}$  as this yields improved estimates of  $\sigma_t^2$  (Bülman and McNeil, 2002). Having estimated  $f_{i,1}$ , we then calculate  $\hat{\sigma}_{t,1}^2 = \hat{f}_{1,1}(X_{t-1}) + \hat{f}_{2,1}(\hat{\sigma}_{t-1,0}^2) + \hat{f}_{3,1}(r_{t-1})$ . In the next iteration, we perform another regression to obtain  $\hat{f}_{i,2}$  and  $\hat{\sigma}_{t,2}^2$  which yields improved estimates of the conditional variance  $\hat{\sigma}_{t,2}^2$ . This iterative process is performed for a prespecified number of iterations,  $m$ . As shown by Bülman and McNeil (2002) and according to our estimation experience, which is documented in the simulation results, the improvement over the parametric GARCH estimation of volatility can be attained in a small number of iterations, usually the first four iterations. The algorithm can be improved by averaging over the final  $K$  estimates:

$$\hat{\sigma}_{t,*} = \frac{1}{K} \sum_{m=M-K+1}^M \hat{\sigma}_{t,m}. \quad (3.16)$$

Note that we average over the volatility rather than the conditional variance since  $\hat{\sigma}_t$  is our proxy for volatility. In the final smoothing, we regress  $X_t^2$  against  $X_{t-1}$ ,  $\hat{\sigma}_{t-1,*}^2$ , and  $r_{t-1}$  to obtain  $\hat{f}_i$  and  $\hat{\sigma}_t^2 = \hat{f}_1(X_{t-1}) + \hat{f}_2(\hat{\sigma}_{t-1,*}^2) + \hat{f}_3(r_{t-1})$ . In our empirical application and simulation experiments, we obtain the final smoothing based on  $K = 4$  for eight iterations ( $m = 8$ ).

### 3.3 Monte Carlo study

#### 3.3.1 Experimental design

The purpose of the simulation experiment is to illustrate the superior volatility-forecast performance of the semiparametric procedure compared with parametric short-rate models. In addition, we show that the semiparametric method yields volatility forecasts that are invariant to the underlying distribution of the short rate and its drift specification.

The data generating process (DGP) for interest rates with a linear drift follows the AGARCHX model (3.2) and (3.5). Specifically, the DGP with a linear drift is

$$\begin{aligned} \Delta r_t &= 0.06 - 0.008r_{t-1} + \varepsilon_t, \\ \varepsilon_t &= \sigma_t z_t, \quad z_t \sim t(4), \\ \sigma_t^2 &= 0.24 + 0.1026\varepsilon_{t-1}^2 + 0.5595\xi_{t-1}^2 + 0.3282\sigma_{t-1}^2 + 0.015(r_{t-1}/10), \end{aligned} \quad (3.17)$$

where  $\xi_{t-1} = \min(0, \varepsilon_{t-1})$ . The use of Student's  $t$  distribution for the interest-rate innovation is consistent with the widely observed nonnormal short-term interest-rate distribution. Moreover, for the purpose of examining the effects of nonlinear drift functions on

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<sup>3</sup>For a discussion of the backfitting algorithm, refer to Friedman and Stuetzle (1981) and Hastie and Tibshirani (1986).

forecasts generated by the semiparametric approach, we consider the DGP,

$$\begin{aligned}\Delta r_t &= 0.06 + 0.008r_{t-1} - 0.01r_{t-1}^2 + 0.0002/r_{t-1} + \varepsilon_t, \\ \varepsilon_t &= \sigma_t z_t, \quad z_t \sim t(4), \\ \sigma_t^2 &= 0.24 + 0.1026\varepsilon_{t-1}^2 + 0.5595\xi_{t-1}^2 + 0.3282\sigma_{t-1}^2 + 0.015(r_{t-1}/10),\end{aligned}\tag{3.18}$$

where  $\xi_{t-1} = \min(0, \varepsilon_{t-1})$ . The parameter values used in the DGPs are typical of short-rate empirical research. We discard the initial 50 observations to mitigate the effect of start-up values yielding samples of 1000 observations, drawn with 50 replications. The small number of replications does not bias the results in any way. In fact, this is consistent with the number of replications performed in the simulation experiment conducted by Bülman and McNeil (2002). Upon generating the data, we estimate the parametric models of short-term interest rates with linear and nonlinear drifts, with symmetric and asymmetric GARCHX models, and with three different innovation distributions, namely normal, Student's  $t$ , and Hansen's (1994) skewed  $t$  distributions. In addition, we estimate the latent volatility using the method of the generalized additive semiparametric GARCH model discussed in the previous section. For both DGPs, we fit linear and nonlinear drift specifications before applying the nonparametric smoothing technique to the volatility estimates. The parametric models are estimated by maximizing the log-likelihood function using the Broyden, Fletcher, Goldfarb, and Shanno algorithm with the Bollerslev and Wooldridge (1992) robust standard error.

To compare the goodness of fit of the in-sample volatility estimates for the different models, we compute the mean of the Mean Absolute Error (MAE) and the mean of the Mean Squared Error (MSE) of each realization. The MSE and the MAE are calculated as:

$$MAE(\hat{\sigma}_{\cdot,m}) = \frac{1}{1000-r} \sum_{t=r+1}^{1000} |\hat{\sigma}_{t,m} - \sigma_t| \quad \text{and} \quad MSE(\hat{\sigma}_{\cdot,m}) = \frac{1}{1000-r} \sum_{t=r+1}^{1000} (\hat{\sigma}_{t,m} - \sigma_t)^2,\tag{3.19}$$

where  $r = 50$  because the semiparametric estimate of volatility at the first fifty time points are omitted as these estimates may be unreliable, and  $m$  applies only to the semiparametric approach and refers to the specific number of iterations. These measures are computed at each iteration of the semiparametric procedure to show the degree of improvement in the goodness of fit of the volatility estimates. For the 50 independent realizations, we average our volatility estimation error statistics to provide an estimate of mean of the MSE and the mean of the MAE, as well as the standard errors for the MSE and MAE estimates.

### 3.3.2 Simulation results

Figures 3.1(a) and (b) plot volatility estimates from the DGP with linear and nonlinear drifts, respectively. Column three of Figures 3.1(a) and (b) shows volatility plots of the semiparametric method while columns one and two show volatility plots of the paramet-

ric GARCHX and AGARCHX models. Both the true and estimated volatility are plotted together to provide a visual impression of the fit. To conserve space, we only report an arbitrarily selected sample of 100 observations from one of the replication results. The plot of the volatility estimates produced by the semiparametric method is based on the final smoothed  $\sigma_t$  estimate. A cursory look at Figures 3.1(a) and (b) suggests that the semiparametric approach yields volatility estimates that match the true simulated volatility better than the parametric models' estimates. This result is robust to the innovation-distribution assumption. The GARCHX and AGARCHX models fail to produce volatility estimates that can adequately capture the variation in the true volatility even though in some instances they capture the spikes relatively well. Another interesting observation shows in Figures 3.1(a) and (b) is that the parametric volatility estimates tend to be higher than the actual volatility level. This is not the case with the semiparametric estimates; they trace the level of the true volatility well. When comparing the volatility estimates produced by the GARCHX and the AGARCHX models, we find that a model with asymmetric conditional variance produces estimates that better depict the actual volatility. This result may not be surprising as the DGP possesses this asymmetric feature in the conditional variance. There are some indications that the volatility estimates generated by the same parametric model but with different innovation-distribution assumptions are distinct. This distinction is less noticeable with estimates produced by the semiparametric approach.

- Figures 3.1(a) and (b) about here -

Tables 3.1(a) and (b) show the estimation error results for the in-sample volatility estimates of various short-rate models for DGPs with linear and nonlinear drifts, respectively. For both DGPs, there is evidence that the standard GARCH model yields the largest MSE and MAE. This result is robust to the innovation-distribution assumption and the drift specification that is estimated. On the other hand, amongst the different parametric GARCH models, the AGARCHX model produces the lowest MSE and MAE. There is evidence that fitting the correct conditional variance specification and using the appropriate innovation distribution give rise to significant improvement in the MSE and MAE. In the case of the linear-drift DGP with a linear drift fit, the improvement in the MSE between the GARCH and AGARCHX models is about 16% for the normal distribution, 26% for the Student's  $t$  distribution, and 16% for the skewed  $t$  distribution. On the other hand, for a nonlinear drift DGP with a linear fit, the improvement in the MSE between the GARCH and AGARCHX models is about 12% for the normal distribution, 14% for the Student's  $t$  distribution, and 9% for the Skewed  $t$  distribution. We also observe that fitting an erroneous drift specification tends to increase the MSE and MAE of the in-sample fit.

-Tables 3.1(a) and (b) about here -

The estimation error of the semiparametric approach for both DGPs indicates that the MSE and MAE are substantially smaller than for the parametric models. For the

linear drift DGP, between the best-fitting AGARCHX model with linear drift and the semiparametric approach with linear drift, the improvement in the MSE (MAE) is about 21% (6%) for both normal and skewed  $t$  distributions, and 20% (6%) for the Student's  $t$  distribution. Similarly, for the nonlinear drift DGP with a nonlinear fit, the improvement is about 9% (4%) for normal distribution, 7% (4%) for Student's  $t$ , and 12% (3%) for skewed  $t$  distribution. It can be inferred, therefore, that while there is gain to be made from using a semiparametric approach over parametric GARCH models in estimating latent volatility, the benefit is more substantial for the case of a short-rate model with a linear drift. An interesting observation about the semiparametric approach, which contrasts the parametric models, is that the MSE and MAE produced by the final smoothed semiparametric approach tend to be very close to each other for the three different innovation distributions, as well as the different drift specifications. This result is interesting as it suggests that the semiparametric approach yields volatility estimates that are robust to the innovation-distribution assumption and possible misspecification of the short-rate drift function—a feature that is lacking in the parametric models.

Last but not least, according to Bühlman and McNeil (2002), the apparent improvement in the volatility estimates produced by the semiparametric technique should show up in the first four iterations of the smoothing procedure. Indeed we observe that the reduction in the estimation error (relative to a GARCH model) is largest at the first iteration of the procedure. However, this reduction is more substantial in the case of the linear-drift model than the nonlinear-drift model.

## 3.4 Empirical application

### 3.4.1 Data description

The empirical investigation is based on 1,892 weekly observations on the 3-month U.S. Treasury bill rate, sampled from February 9, 1973 to May 8, 2009. The data are obtained from the Federal Reserve Bank of St. Louis (FRED) database. This period includes a shift from historically high interest rates in the late 1970s to early 1980s during the Volcker monetary regime to low interest-rate levels in the latter part of the sample period. The interest-rate data and the first differenced series are presented in Figure 3.2. Summary statistics for the data set are provided in Table 3.2.

- Figure 3.2 about here -

From Figure 3.2 it is clear that there is indeed a tendency for the volatility in the interest-rate series to be positively correlated with current interest-rate levels. At the start of the sample period, the association between the interest rate and its volatility is visible. This feature becomes more apparent for the 1979–1983 period during which both the level and volatility of the rate are high. The level effect is not as obvious after the Volcker monetary regime. These empirical features tally with those reported in Brenner et al. (1996). The time-varying nature of the volatility in the sample is indicative that unexpected “news” might be equally important in explaining the volatility of interest



rates, in addition to the level effect.

- Table 3.2 about here -

The time-varying nature of the volatility that is evident in Figure 3.2 is associated, in turn, with an empirical distribution for the first-differenced data that exhibits excess kurtosis. The relevant kurtosis statistic reported in Table 3.2 is significantly greater than the value of 3 associated with the normal distribution. The negative skewness coefficient is also significantly less than the value of zero associated with the symmetric normal distribution. This is reflective of a “leverage” effect of sorts, whereby interest-rate falls are associated with higher volatility than increases of the same magnitude. The first-differenced data exhibits strong correlation as shown by the Ljung–Box test statistic which overwhelmingly rejects the null hypothesis of no serial correlation at the 10th lag orders. The interest-rate series clearly possesses conditional heteroskedasticity as indicated by application of a formal 10th-order LM test for ARCH to the residuals from an AR(10) regression of the interest-rate data. The Jarque–Bera test strongly rejects the null of normality in the interest-rate series.

The stationarity property of the interest-rate data is less clear cut. There is a lot of controversy in the literature surrounding the unit root property of interest rates. Short-rate diffusion models estimated by Marsh and Rosenfeld (1983), Chan et al. (1992), and Aquila et al. (2003) *inter alia* based on U.S. data document evidence that short-term interest rates behave like a random-walk process. In contrast, Brenner et al. (1996) and Ball and Torous (1999) amongst others show supporting evidence that the U.S. short-rate means revert. As is widely known, the standard Dickey–Fuller test is subject to typically moderate-size distortion in the presence of a neglected GARCH effect in the series (see Kim and Schmidt, 1993 Haldrup, 1994). To circumvent the problem of neglected GARCH effects in unit-root testing, Seo (1999) suggests the unit-root test equation and the GARCH process should be estimated jointly when the series examined exhibits GARCH effects. We pursue this testing approach to ensure that the unit-root test result is robust to the presence of GARCH effects. As is evident from Table 3.2, the mean level of interest rate is 5.8252. This suggests that the unit-root tests should include an intercept in the mean equation.

Seo (1999) augments the standard Dickey-Fuller testing equations as follows.

$$\begin{aligned}\Delta y_t &= \alpha + \beta y_{t-1} + \varepsilon_t \\ \sigma_t^2 &= \phi_0 + \phi_1 \varepsilon_{t-1}^2 + \phi_2 \sigma_{t-1}^2 \\ \varepsilon_t &= \sigma_t v_t \quad , \quad v_t \sim N(0, 1)\end{aligned}\tag{3.20}$$

The mean equation in (3.20) differs slightly from Seo’s (1999) approach in which the intercept is excluded. Seo (1999) considers the use of a preliminary regression to demean or detrend the series prior to testing the series for a unit root. Cook (2008), however, presents an approach where the deterministic terms are explicitly included in the testing equation such as in (3.20). Moreover, he simulates a new set of critical values involving different  $\phi_0$ ,  $\phi_1$ , and  $\phi_2$  parameter values that are more typical in empirical research.

The unit-root hypothesis is examined via the maximum likelihood  $t$ -ratio for  $\beta$ , which is denoted as  $t_\beta$ . Seo (1999) shows that the asymptotic distribution of  $t_\beta$  is a mixture of the nonstandard Dickey–Fuller distribution and the standard normal. The extent to which the distribution moves towards the standard normal from the Dickey–Fuller depends upon the strength of the GARCH effect which is determined by a nuisance parameter,  $\rho$ . The null hypothesis of a unit root is rejected if  $t_\beta$  is less than the critical value at the conventional significance levels.

In addition to applying the Seo (1999) test, we also perform the augmented Dickey–Fuller (ADF) test and the higher powered GLS-based Dickey–Fuller test (Elliott et al., 1996). The optimal lag length, or degree of augmentation, of the testing equation is determined using the modified Akaike Information Criterion (MAIC) proposed by Ng and Perron (2001) following initial consideration of a maximum lag length given by  $\text{int}[12(T/100)]^{0.25}$ , where  $T$  is the total sample size. Hayashi (2000) provides a justification of this upper bound. The appropriate degree of augmentation for both tests is found to be 25. The results obtained from the application of these tests, denoted as  $\tau_\mu$  and  $\tau_\mu^{\text{GLS}}$ , are given in Table 3.2. Using the 5% critical values obtained from Fuller (1996) and Pantula et al. (1994), the derived test statistics, respectively, show the unit-root null hypothesis is not rejected by either of the tests. However, the interest-rate series clearly possesses conditional heteroskedasticity as indicated by the application of a formal 10th order LM test for ARCH to the residuals from the ADF test. Given the presence of conditional heteroskedasticity, Seo’s (1999) approach outlined above is followed to test the unit-root hypothesis. Accordingly, an ADF testing equation with 18 lags is estimated jointly with a GARCH(1,1) process using maximum likelihood estimation and the Bernt–Hall–Hall–Hausman algorithm. The test statistic using Bollerslev and Wooldridge’s (1992) standard errors is denoted as  $t_\beta(\text{BW})$ .<sup>4</sup> We simulate the 5% critical value for the estimated GARCH parameters of  $\{\hat{\phi}_1, \hat{\phi}_2\} = \{0.14, 0.85\}$  along with the effective sample size of 1,892 observations since neither Seo’s (1999) nor Cook’s (2008) studies provide critical values that can be applied to our results.<sup>5</sup> The simulated critical value at the 5% level of significance is  $-1.9073$  for the nonrobust standard errors and  $-1.8891$  for the Bollerslev–Wooldridge robust standard errors. The calculated test statistic for  $t_\beta$  and  $t_\beta(\text{BW})$  are  $-2.4301$  and  $-2.5147$ , respectively. These results imply that the unit-root hypothesis can be rejected comfortably in both cases. On the basis that Seo’s (1999) test incorporates the GARCH effects into the testing framework, we are more inclined to believe in its robust inference. That is, the weekly 3-month U.S. Treasury bill interest rates are stationary.

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<sup>4</sup>Cook (2008) shows that the maximum likelihood estimation of the Seo (1999) unit-root test equations could employ Bollerslev and Wooldridge’s (1992) robust standard errors. The  $t$ -test statistic for the slope coefficient  $\beta$  with robust standard error is given by  $t_\beta(\text{BW})$ .

<sup>5</sup>The appendix provides details of the simulation to obtain 1%, 5%, and 10% critical values for  $t_\beta$  and  $t_\beta(\text{BW})$ .

### 3.4.2 Empirical results

The data-description statistics indicate that an appropriate model of short-rate volatility should account for its time-varying nature, its asymmetric response to shocks of different signs and its dependence on interest-rate levels. For this reason, we estimate the GARCHX and AGARCHX models for the diffusion process. As for the drift specification, we estimate both linear and nonlinear drifts to determine the presence of nonlinearities. Given the evidence of unconditional skewness in short-rate changes, we also estimate the models with three different distribution assumptions, namely normal, Student's  $t$  and skewed  $t$ . All the models are estimated with Bollerslev and Wooldridge's (1992) quasi-maximum likelihood method, which gives robust standard errors. The in-sample and out-of-sample volatility forecasts of these parametric models are then compared with those of the semiparametric model. To produce the one-period-ahead out-of-sample volatility forecasts, we exclude the last 100 observations from our sample and estimate the parametric and semiparametric models recursively over the remainder of the data. In other words, each time we produce a one-period-ahead volatility forecast, we estimate the model using all the data up until the period prior to that forecast. The estimation results for the parametric models with linear and nonlinear drifts are reported in Tables 3.3(a) and (b), respectively.

- Tables 3.3(a) and (b) about here -

It can be seen in Table 3.3(a) that the coefficients of the linear drift function are only statistically significant at the 5% significance level for the models fitted with a Student's  $t$  distribution. The estimate for the coefficient  $\lambda$ , which captures the degree of mean reversion, is very small, implying that the degree of mean reversion is weak. The estimates of the interest-rate-level sensitivity parameters ( $b$  and  $\delta$ ), the coefficients of last period's unexpected news ( $\alpha_1$ ), the last period's volatility ( $\alpha_2$ ), and the coefficient of the asymmetric response of current volatility to last period's bad news ( $\alpha_3$ ), are found to be highly significant. Taken together, these results suggest that there is overwhelming evidence of GARCH, levels, and asymmetric GARCH effects in the diffusion process. In terms of maximized log-likelihood values, the AGARCHX with Student's  $t$  distribution performs better than the other models. There is evidence that, independent of the underlying distribution, models that account for both asymmetric GARCH and levels effects perform better than models that do not account for asymmetric GARCH effects. The simple GARCH model performs the worst in terms of the log-likelihood values. This model fails to capture the asymmetry and level dependence in the short-rate volatility process. Moreover, the Ljung-Box test of the 12th-order serial correlation in the squared standardized residuals rejects the null of no serial correlation, implying that the GARCH model does not adequately characterize the volatility dynamic of short-rate changes. The skewness parameter,  $\eta$ , of the skewed  $t$  distribution turns out to be statistically insignificant at all conventional significance levels. Furthermore, the  $\eta$  estimate for the three short-rate models is virtually zero, implying that a Student's  $t$  distribution is adequate to characterize the short-rate distribution. Our finding supporting the use of

the Student's  $t$  instead of skewed  $t$  distribution is consistent with Bali's (2007) results.

In Table 3.3(b), we show the estimation results for nonlinear-drift short-rate models. Regardless of the error-distribution assumption, the coefficients  $\lambda_2$  and  $\lambda_3$ , which govern the nonlinear dynamics in the drift function, are statistically insignificant. Our results, which support the lack of evidence of nonlinearity in the 3-month T-bill data, concur with Bali's (2007) earlier findings showing that the incorporation of the GARCH effects into the volatility process gives rise to no evidence of nonlinearity in the drift specification. The skewness parameter,  $\eta$ , of the skewed  $t$  distribution again turns out to be statistically insignificant for all models, suggesting there is no evidence for skewness asymmetry in the short-rate-change distribution. Comparing models with linear and nonlinear drifts across similar distribution assumptions indicates a substantial reduction in the log-likelihood value, thereby suggesting that a short-rate model with linear drift is the preferred specification. Based on this result, we do not consider the in-sample and out-of-sample forecast performance of short-rate models with nonlinear drift and a skewed  $t$  distribution.

We use four different metrics to evaluate the in-sample and out-of-sample volatility-forecast performance of the semiparametric approach compared to its parametric counterparts. In addition to the MAE and MSE measures given in equation (3.19), we also use the Akaike Information Criterion (AIC), which is a penalized negative log-likelihood criterion adjusted for the degree of parameters that are estimated, and Bali's (2007)  $R_{\text{vol}}^2$  measure. For the four metrics, we proxy the unobserved true volatility,  $\sigma_t$ , with  $|r_t - r_{t-1}|$ . The AIC is computed as

$$AIC = 2K + T \left[ \ln \left( \frac{2\pi \cdot RSS}{T} \right) + 1 \right], \quad (3.21)$$

where  $K$  is the number of estimated parameters,  $T$  is the sample size, and  $RSS = \sum_{t=1}^T (\sigma_t - \hat{\sigma}_t)^2$ , where  $\sigma_t$  is the true volatility proxy,  $\hat{\sigma}_t$  is the model estimated volatility. The  $R_{\text{vol}}^2$  measure essentially computes the total variation in the true volatility proxied by  $|r_t - r_{t-1}|$  that can be explained by the estimated conditional volatilities. This is obtained from the coefficient of determination of an OLS regression of the form

$$\sigma_t = a_0 + a_1 \sigma_t^f + e_t, \quad (3.22)$$

where  $\sigma_t$  and  $\sigma_t^f$  are the volatility proxy of  $|r_t - r_{t-1}|$  and the forecasted volatility, respectively. It should be highlighted that the  $R_{\text{vol}}^2$  measure is a crude measure and is subject to the following caveat. As pointed out by Andersen and Bollerslev (1998), the idiosyncratic component of daily interest-rate changes is large, thus the use of realized interest-rate changes may not fully capture day-by-day movements in volatility. To circumvent this problem, we use a range-based volatility proxy by adopting the Garman and Klass's (1980) extreme-value estimator to construct a minimum-variance unbiased estimator that utilizes the opening, closing, high, and low prices. Due to the paucity of high-frequency data, the use of the GK extreme-value estimator is deemed as a compromise to the preferred realized-volatility measure derived from high-frequency

data (see Andersen et al., 2001).<sup>6</sup> Our choice of the GK estimator is also motivated by the findings of Bali and Weinbaum (2005), who perform a horse race among all the extreme-value estimators featured in the literature. They show that, in practice, the GK estimator is the least biased and most efficient estimator compared with other extreme-value estimators. The GK minimum variance and unbiased estimator is  $\hat{\sigma}_{\text{GK}}^2 = \frac{1}{n} \sum_{t=1}^n \left\{ 0.511 \left( \ln \frac{H_t}{L_t} \right)^2 - 0.019 \left[ \ln \left( \frac{C_t}{O_t} \right) \ln \left( \frac{H_t L_t}{O_t} \right) - 2 \ln \left( \frac{H_t}{O_t} \right) \ln \left( \frac{L_t}{O_t} \right) \right] - 0.383 \left[ \ln \left( \frac{C_t}{O_t} \right) \right]^2 \right\}$ ,  $n \geq 1$ , where  $O_t$ ,  $C_t$ ,  $H_t$ , and  $L_t$  denote, respectively, the opening, closing, high and low prices on day  $t$  and  $n$  is the number of days in the sample. The IRX (the value of the 13-week Treasury Index) index data are obtained from the Yahoo/finance web site.<sup>7</sup>

- Tables 3.4(a) and (b) about here -

Tables 3.4(a) and (b) report the results of the four metrics for evaluating the in-sample forecast performance of the models using the volatility benchmarks  $|r_t - r_{t-1}|$  and  $\hat{\sigma}_{\text{GK}}$ , respectively. Focusing on the results with volatility proxy  $|r_t - r_{t-1}|$ , we find that the AGARCHX model with the Student's  $t$  distribution performs the best compared with other parametric models. Not only does it deliver the lowest MSE and MAE, it also gives the lowest AIC and highest  $R_{\text{vol}}^2$ . The GARCH model, which does not take into account the level dependence and asymmetric response in the conditional variance of short-rate changes, performs the worst. However, there is evidence that the semiparametric model yields a superior in-sample volatility forecast as judged by the four metrics. When compared with the best-fitting AGARCHX Student's  $t$  model, the reduction in the MSE and MAE based on the final smoothed semiparametric method is about 3% and 1%, respectively. The AIC shows a marked improvement in the fit, falling from  $-498.52$  to  $-601.92$ , while the  $R_{\text{vol}}^2$  increases by about 10%. Looking at the four metrics, we also find that in each iteration of the semiparametric smoothing procedure there is a significant improvement in the volatility forecast performance compared with the AGARCHX Student's  $t$  model. Interestingly, we find that for the semiparametric approach, fitting a nonlinear drift function erroneously to obtain an initial volatility estimate does not give rise to an inferior forecast performance. The difference in forecast performance results for the linear and nonlinear drifts with the semiparametric approach is negligible, implying that the choice of the drift function is immaterial to the forecast performance of the semiparametric approach. This result corroborates the simulation results in which we find that neglecting to fit the correct drift function in the semiparametric approach does not bear any influence on its volatility-forecast performance. Another important finding is that the choice of the innovation distribution, whether it is normal, Student's  $t$ , or skewed  $t$ , does not have a considerable impact on the forecast performance of the semipara-

<sup>6</sup>Implied volatility can be obtained from the value of the 13-week Treasury index (IRX), which is based on the discount rate of the most recently auctioned 13-week U.S. T-bill. However, high-frequency IRX data are only available from November 3, 1997. Our sample period, on the other hand, commences from February 9, 1973.

<sup>7</sup>The URL for the IRX data is <http://finance.yahoo.com/q/hp?s=%5EIRX+Historical+Prices>. We thank the anonymous referee for directing us to this data source.

metric approach.<sup>8</sup> Taken together, these results highlight the robust forecast property of the semiparametric approach to possible misspecifications of the drift function and the innovation distribution. In Table 3.4(b), we show that these results are qualitatively unchanged even with the use of a more accurate volatility benchmark (i.e.  $\hat{\sigma}_{\text{GK}}^2$ ) to assess the forecast performance of the semiparametric approach relative to parametric models.

- Figure 3.3(a) and (b) about here -

Given the extensive results reported in Tables 3.4(a) and (b), we summarize these findings by presenting them in Figures 3.3(a) and (b). To interpret the plot, the four shaded bars represent the metric value of the parametric models: AGARCHX-T, AGARCHX-N, GARCHX-T, and GARCHX-N in that order. The line plotted across the x-axis is a locus of the metric value for the GARCH model (represented by the first mark on the x-axis), the metric value for the eight iterations of the semiparametric approach (represented by the second to ninth marks on the x-axis), and the final smoothed stage of the semiparametric approach (represented by the tenth mark on the x-axis). It is evident from the plot that the semiparametric approach yields the best results based on all four forecast-performance measures. The results are consistent whether we use the crude or more accurate volatility benchmark. To visually illustrate the superior performance of the semiparametric approach compared with the AGARCHX Student's  $t$  model, we plot in Figures 3.4(a) to (d) the in-sample volatility estimates of the two models for an arbitrarily selected period January 1, 1997–January 1, 2000. Figures 3.4(a) and (b) employ  $|r_t - r_{t-1}|$  as the true volatility proxy while Figures 3.4(c) and (d) are based on the more accurate volatility proxy given by  $\hat{\sigma}_{\text{GK}}^2$ .

- Figures 3.4(a), (b), (c) and (d) about here -

By comparing the plots of the volatility estimates between the best-fitting parametric AGARCHX Student's  $t$  model and the semiparametric model, we can see that the latter model is capable of capturing movements of the short-rate volatility process better than the former model. The AGARCHX Student's  $t$  model tends to yield an overly smoothed volatility estimate of the true volatility process proxied by  $|r_t - r_{t-1}|$  ( $\hat{\sigma}_{\text{GK}}^2$ ) in Figure 3.4(b) (Figure 3.4(d)). There are two important features about the way the volatility estimates obtained from the semiparametric approach improve upon the estimates of the AGARCHX Student's  $t$  model. First, there are peaks or spikes in the volatility of the short-rate changes that are well captured by the semiparametric model, but not by the AGARCHX Student's  $t$  model. For example, the peak observed on January 1, 1998 is clearly captured by the semiparametric approach, but not by the AGARCHX Student's  $t$  model. Second, the volatility estimates produced by the semiparametric approach tend to match the rise and fall in interest rates better than the AGARCHX Student's  $t$  model. The most obvious of this point is the drop in interest rates between the two peaks that happened prior to January 1, 1999 (see Figure 3.4(a)). While the semiparametric

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<sup>8</sup>To conserve space, we do not report the results for the semiparametric approach with a skewed  $t$  distribution. These results are available from the authors upon request.

approach does not fully capture the drop in rates, it does a better job at capturing the fall in interest rates than the AGARCHX Student's  $t$  model.

- Tables 3.5(a) and (b) about here -
- Figures 3.5(a), (b), (c) and (d) about here -

Turning to the out-of-sample volatility-forecast performance of the semiparametric model, we find that the volatility estimates obtained from the final smoothed stage have better predictive power than those produced by the parametric models. Using the volatility benchmark  $\hat{\sigma}_{\text{GK}}^2$  (see Table 3.5(b)), the improvement in the volatility-forecast estimation error measured by the MSE and the MAE is 14% and 11%, respectively, between the final smoothed semiparametric approach and the AGARCHX Student's  $t$  model. On the other hand, the reduction in the forecast estimation error based on the crude volatility proxy  $|r_t - r_{t-1}|$  is more conservative: The MSE and MAE fall by 6% and 5%, respectively (see Table 3.5(a)). Figures 3.5(a) to (d) provide plots of the volatility-forecast estimates of the two contending models. Unlike the in-sample volatility estimates, we fail to find that the semiparametric approach is capable of capturing the observed peaks in interest-rate volatility, particularly with the volatility benchmark  $\hat{\sigma}_{\text{GK}}^2$  and the sharp spike at the start of the forecast horizon (see Figure 3.5(c)). However, this does not diminish the out-of-sample forecast performance of the semiparametric approach compared with the AGARCHX Student's  $t$  model. The latter model continues to provide an overly smoothed out-of-sample volatility forecast of interest rates. In contrast, the semiparametric approach yields volatility forecasts that better capture fluctuations in the short rate, thus leading to a smaller estimation error than the AGARCHX Student's  $t$  model.

### 3.4.3 Implications for pricing interest-rate derivatives

Given that the volatility processes of the semiparametric and parametric models are distinct, it is very likely that the two classes of models will generate different probability distributions of future interest-rate levels. Predictions of future interest rates are essential for pricing long-dated, path-dependent interest-rate derivatives such as the index amortizing rate (IAR) and swaps, amongst others. For the purpose of illustration, we consider the IAR swaps. The notional value of the IAR swaps is reduced over time according to an amortization schedule based on the level of a reference interest rate on a particular fixed date in the future (usually every three or six months). The value of this swap is contingent on the probability distribution of the reference rate on each reset date. Since the amount of principal that remains on any reset date depends on past interest-rate levels, the IAR swaps are considered "path-dependent" securities. In other words, fluctuations in interest rates and hence the accuracy in modeling short-rate volatility matters for the pricing of the IAR swaps. For a detailed discussion of the IAR swaps refer to Galaif (1993).

To examine how an improvement in the estimation accuracy of short-rate volatility could affect the pricing of interest-rate derivatives, we follow BHK and perform the

following experiment. We simulate the semiparametric model and the parametric models 5,000 times using the 3-month U.S. Treasury bill rate estimation results with June 8, 2007, as the starting date. The interest-rate level is 4.67% on this date. Following BHK, we focus on the volatility process and employ the mean equation  $r_t - r_{t-1} = -0.0015$  given that the average weekly change in the interest rate over the estimated sample period is  $-0.0015$ .<sup>9</sup> Figure 3.6 graphs the 5th, 25th, 50th, 75th and 95th percentiles of the 5,000 simulated paths for each horizon up to 100 weeks for the different short-rate models. The solid lines represent the confidence intervals for simulated interest rates based on the parametric models. The ordering from the outermost to innermost lines represents the resulting interest-rate distributions for the AGARCHX-T, GARCHX-T, AGARCHX-N and GARCHX-N models. The dotted lines denote the short-rate distribution of the semiparametric model.

- Figure 3.6 about here -

Visual inspection of Figure 3.6 suggests that there are several interesting results. First, the distribution assumption in the parametric models does not seem to matter for derivative prices. The interest-rate distributions are very similar when comparing between the same type of model with different distribution assumptions. Second, like BHK, we find that whether we model asymmetries in the parametric models or not is immaterial for the paths of future interest rates; therefore, this will not greatly affect interest-rate derivative prices. Third, amongst the different models considered, the confidence intervals of future short-rate levels generated by the semiparametric model are narrower, particularly at the 5% and 95% levels. In other words, the semiparametric model predicts a narrower confidence band of extreme interest-rate movements than the parametric models. For other confidence levels considered, we find that the future levels of short-term interest rates are comparable with the parametric models.

Based on these results, what can be said about the pricing of certain path-dependent interest-rate derivative such as the IAR swaps mentioned above, mortgages and collateralized mortgage obligations? Given that parametric models produce larger upper tails, the average predicted amortization will be less for such models than the semiparametric model. In other words, the predicted lives of these securities and their cash flows will increase. Accordingly, these securities would be overpriced by the parametric models relative to the semiparametric model. On the other hand, the larger lower tails of the parametric models would imply that these securities would be underpriced compared to the semiparametric model. Our results for the parametric models are consistent with those of BHK who find that the conditional-variance model specification does not influence the pricing of interest-rate derivatives. In particular, they show that whether a model specifies an asymmetric conditional variance or an additive or multiplicative levels effect in the variance specification does not yield significant differences in the pricing of interest-rate derivatives. Likewise, we demonstrate that the asymmetric specification of the diffusion process and the distribution assumption for parametric models do not affect the pricing of interest-rate derivatives. More importantly, we find that the narrower

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<sup>9</sup>Although the mean-reverting slope coefficient is significant, the coefficient estimate is very close to zero. Therefore, ignoring the mean-reverting dynamics in the simulation is a reasonable simplification.



confidence intervals of future interest-rate levels produced by the semiparametric model relative to any of the parametric models suggests that our method would yield less price variation for long-dated and path-dependent interest-rate derivatives.

Although the semiparametric model does not give rise to a simple analytical solution for the pricing of derivatives, the estimation process sets up naturally for Monte Carlo evaluation. Thus, like the BHK models, the semiparametric model can be easily applied to the valuation of securities that already require Monte Carlo evaluation. These securities include those interest-rate derivatives discussed above.

### 3.5 Conclusion

In this paper an application of a semiparametric GARCH approach to modeling short-term interest-rate volatility has been proposed. The semiparametric smoothing technique uses a general additive function of lagged innovations, volatilities, and past interest-rate levels with a backfitting algorithm to estimate the unobserved diffusion process. While the volatility model is estimated semiparametrically, it resembles the widely used short-rate volatility models of BHK, which feature interest-rate-level dependence and an asymmetrical dynamic in the conditional variance. Consequently, we compare the performance of the semiparametric approach with this class of single-factor short-rate diffusion models in terms of its ability to characterize short-rate volatility. Our simulation study shows that the semiparametric model provides a superior fit of the in-sample volatility estimates to a GARCH model that exhibits asymmetry and the levels effect. The volatility forecast performance of the semiparametric procedure, unlike the parametric GARCH models, is also robust to potential misspecification in the short-rate drift and the innovation distribution. The empirical application to weekly 3-month U.S. Treasury bill rates between 1973 and 2009 further illustrates improvement in the in-sample and out-of-sample predictive power of the semiparametric model over BHK's models. Finally, we show that the greater degree of accuracy in modeling short-rate volatility offered by the semiparametric model is important for pricing long-dated and path-dependent interest-rate derivatives.

For future research, we intend to apply this technique to Black et al.'s (1990) two-factor model of with stochastic volatility, which was developed and estimated by Bali (2003). The two factors of the model are the short-term interest rate and the volatility of interest-rate changes. This would involve performing a nonparametric estimation on both the drift and diffusion of the short-rate process. The application of this technique to the two-factor arbitrage-free model could be used to assess the importance of modeling short-rate-change volatility accurately and its implications on default-free bond pricing.

# Appendix

To simulate the critical values for the Seo (1999) test, the following DGP is employed.

$$\begin{aligned}y_t &= y_{t-1} + \varepsilon_t, \quad t = 1, \dots, T \\ \sigma_t^2 &= \phi_0 + \phi_1 \varepsilon_{t-1}^2 + \phi_2 \sigma_{t-1}^2 \\ \varepsilon_t &= \sigma_t v_t, \quad v_t \sim N(0, 1)\end{aligned}\tag{3.23}$$

We set the parameters  $\phi_1 = 0.14$ ,  $\phi_2 = 0.85$ , and  $\phi_0 = 1 - \phi_1 - \phi_2$ . These values are taken from estimates of our ADF testing equation with 18 lags which is estimated jointly with a GARCH(1,1) process.  $T$  is set to 1,892 to match our sample size. Once the data are simulated, we perform Seo's (1999) test by estimating

$$\begin{aligned}\Delta y_t &= \alpha + \beta y_{t-1} + \varepsilon_t \\ \sigma_t^2 &= \phi_0 + \phi_1 \varepsilon_{t-1}^2 + \phi_2 \sigma_{t-1}^2 \\ \varepsilon_t &= \sigma_t v_t, \quad v_t \sim N(0, 1),\end{aligned}\tag{3.24}$$

with the maximum likelihood method using the Bernt–Hall–Hall–Hausman algorithm. The resulting  $t$ -test for the null hypothesis of a unit-root process in  $y_t$  (i.e.,  $\beta = 0$ ), which is denoted as  $t_\beta$ , is computed. In addition, we compute the robust  $t$ -test,  $t_\beta(\text{BW})$ , using Bollerslev and Wooldridge's (1992) robust standard error. The experiment is repeated 25,000 times and each time the test statistic values for  $t_\beta$  and  $t_\beta(\text{BW})$  are saved. The resulting series of  $\hat{t}_\beta$  and  $\hat{t}_\beta(\text{BW})$  are sorted and the 1%, 5%, and 10% critical values are obtained accordingly. The critical values at the 1%, 5%, and 10% significance levels for  $t_\beta$  are  $-2.4280$ ,  $-1.9073$ , and  $-1.6701$ , and for  $t_\beta(\text{BW})$  are  $-2.4196$ ,  $-1.8891$ , and  $-1.6454$ , respectively.

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# Tables

Table 3.1: Estimates of Mean Squared and Mean Absolute Volatility Estimation Error for Simulated Data

(a) DGP - linear drift

Models	Normal				Student's $t$				Skewed $t$			
	MSE	Std. err.	MAE	Std. err.	MSE	Std. err.	MAE	Std. err.	MSE	Std. err.	MAE	Std. err.
Estimated with a linear drift function												
GARCH	0.3262	0.1210	0.3794	0.0638	0.3558	0.1357	0.4044	0.0716	0.3229	0.1193	0.3772	0.0559
Iteration 1	0.2316	0.1065	0.3561	0.0664	0.2310	0.0968	0.3561	0.0638	0.2308	0.0962	0.3558	0.0635
Iteration 2	0.2036	0.0861	0.3336	0.0617	0.2028	0.0944	0.3337	0.0629	0.2027	0.0951	0.3333	0.0631
Iteration 3	0.2065	0.0886	0.3359	0.0598	0.2030	0.0839	0.3336	0.0595	0.2033	0.0847	0.3336	0.0595
Iteration 4	0.2126	0.0961	0.3411	0.0627	0.2038	0.0855	0.3349	0.0607	0.2057	0.0845	0.3370	0.0593
Iteration 5	0.2085	0.0855	0.3402	0.0603	0.2057	0.0876	0.3371	0.0597	0.2062	0.0881	0.3379	0.0602
Iteration 6	0.2095	0.0846	0.3408	0.0605	0.208	0.0864	0.3389	0.0607	0.2083	0.0876	0.3392	0.0624
Iteration 7	0.2155	0.0934	0.3445	0.0628	0.2094	0.0864	0.3398	0.0602	0.2098	0.0868	0.3399	0.0604
Iteration 8	0.2137	0.0906	0.3426	0.0623	0.2117	0.0916	0.3404	0.062	0.2119	0.0918	0.3401	0.0619
Final smoothing	0.2158	0.0929	0.3442	0.0617	0.2123	0.0909	0.3415	0.0611	0.2125	0.0911	0.3417	0.0614
AGARCHX	0.2738	0.1193	0.3674	0.0693	0.265	0.1011	0.3631	0.0652	0.2705	0.1052	0.3640	0.0511
GARCHX	0.3194	0.1234	0.3789	0.0711	0.2945	0.1217	0.3704	0.0686	0.3116	0.1143	0.3755	0.0532
Estimated with a nonlinear drift function												
GARCH	0.3286	0.1234	0.3785	0.065	0.3552	0.1352	0.4025	0.0717	0.3224	0.1324	0.3826	0.06
Iteration 1	0.231	0.1071	0.3552	0.0677	0.2306	0.0972	0.3551	0.0644	0.2378	0.0984	0.3638	0.0661
Iteration 2	0.2038	0.0871	0.3331	0.0623	0.2039	0.0951	0.3343	0.0644	0.213	0.0961	0.3436	0.0642
Iteration 3	0.2081	0.0897	0.3368	0.0611	0.2031	0.0835	0.3335	0.0597	0.2154	0.0888	0.3454	0.0626
Iteration 4	0.2147	0.0996	0.3413	0.0639	0.2053	0.0856	0.3362	0.0611	0.2143	0.086	0.346	0.0613
Iteration 5	0.2082	0.0857	0.3397	0.0613	0.2069	0.0892	0.3374	0.0604	0.2176	0.0918	0.3484	0.0636
Iteration 6	0.2117	0.0852	0.3423	0.0618	0.2098	0.086	0.3397	0.0605	0.2192	0.0884	0.3499	0.0628
Iteration 7	0.2143	0.0907	0.3438	0.0632	0.2095	0.0864	0.3392	0.0599	0.2211	0.0901	0.3502	0.0629
Iteration 8	0.2173	0.0958	0.345	0.0641	0.2135	0.0921	0.3414	0.0621	0.2233	0.0942	0.3512	0.0644
Final smoothing	0.2167	0.0927	0.3449	0.0629	0.2134	0.0909	0.3418	0.0613	0.2239	0.094	0.3522	0.0639
AGARCHX	0.2846	0.1237	0.3689	0.0714	0.2739	0.1223	0.3672	0.0692	0.2814	0.1248	0.3681	0.0663
GARCHX	0.3205	0.1260	0.3813	0.0725	0.3131	0.1255	0.3811	0.0710	0.3207	0.1269	0.3859	0.0701



(b) DGP - Nonlinear drift

Models	Normal				Student's $t$				Skewed $t$			
	MSE	Std. err.	MAE	Std. err.	MSE	Std. err.	MAE	Std. err.	MSE	Std. err.	MAE	Std. err.
Estimated with a linear drift function												
GARCH	0.1094	0.0708	0.1893	0.0741	0.1077	0.072	0.1877	0.067	0.1042	0.0714	0.1907	0.0667
Iteration 1	0.0808	0.0651	0.1791	0.0501	0.0809	0.0649	0.1791	0.0500	0.0805	0.0637	0.1788	0.0497
Iteration 2	0.0704	0.0532	0.1669	0.0461	0.0685	0.0524	0.1651	0.0449	0.0692	0.0529	0.1649	0.0443
Iteration 3	0.0756	0.0568	0.1747	0.0498	0.0780	0.0758	0.1739	0.0522	0.0764	0.0722	0.1735	0.0509
Iteration 4	0.0710	0.0517	0.1693	0.0431	0.0729	0.0581	0.1703	0.0415	0.0716	0.055	0.1695	0.0439
Iteration 5	0.0772	0.0605	0.1740	0.0473	0.0739	0.0558	0.1725	0.0460	0.0770	0.0596	0.1746	0.0523
Iteration 6	0.0782	0.0680	0.1748	0.0533	0.0700	0.0521	0.1684	0.0427	0.0721	0.0569	0.1691	0.0448
Iteration 7	0.0755	0.0568	0.1746	0.0467	0.0756	0.0614	0.1724	0.0466	0.0750	0.0583	0.1721	0.0458
Iteration 8	0.0782	0.0623	0.1750	0.0471	0.0754	0.0577	0.1752	0.0486	0.0742	0.0577	0.1732	0.0467
Final smoothing	0.0772	0.0603	0.1750	0.0472	0.0756	0.0591	0.1734	0.0464	0.0753	0.0595	0.1725	0.0462
AGARCHX	0.0967	0.0695	0.1851	0.0652	0.093	0.0645	0.1825	0.0607	0.0944	0.0673	0.1838	0.0625
GARCHX	0.0991	0.0705	0.1872	0.0643	0.0956	0.0671	0.186	0.0618	0.0968	0.068	0.1867	0.0632
Estimated with a nonlinear drift function												
GARCH	0.0998	0.1057	0.1865	0.0724	0.0843	0.0646	0.1823	0.0647	0.0963	0.0693	0.1909	0.0653
Iteration 1	0.0917	0.0961	0.1816	0.0591	0.0846	0.0782	0.1786	0.0518	0.083	0.0564	0.1843	0.0502
Iteration 2	0.0856	0.0948	0.1727	0.0602	0.0688	0.0519	0.163	0.0422	0.0719	0.0572	0.1686	0.048
Iteration 3	0.0888	0.097	0.1772	0.0591	0.0754	0.0555	0.1716	0.0478	0.0726	0.0528	0.1721	0.0469
Iteration 4	0.0852	0.0907	0.1751	0.0559	0.0709	0.0501	0.1682	0.0432	0.0751	0.0579	0.1739	0.0494
Iteration 5	0.0844	0.0899	0.1738	0.0527	0.0779	0.0597	0.1733	0.0481	0.0798	0.0628	0.1778	0.0499
Iteration 6	0.0841	0.0907	0.176	0.0551	0.074	0.0538	0.1721	0.0462	0.0777	0.0603	0.1764	0.049
Iteration 7	0.0839	0.0884	0.1766	0.0543	0.0737	0.0557	0.1713	0.0454	0.0768	0.0597	0.1751	0.0476
Iteration 8	0.0869	0.0906	0.1783	0.055	0.0747	0.0542	0.1732	0.0458	0.0789	0.0532	0.1769	0.0492
Final smoothing	0.0862	0.0907	0.1766	0.0535	0.0747	0.055	0.1731	0.0463	0.079	0.0551	0.177	0.0493
AGARCHX	0.0945	0.1022	0.1846	0.0619	0.0806	0.0612	0.1804	0.0521	0.0901	0.0635	0.1825	0.0534
GARCHX	0.0977	0.1025	0.1852	0.0637	0.0838	0.0637	0.1817	0.058	0.0942	0.0668	0.1843	0.0599

*Note:* The results are for simulated data from the DGP in equation (3.17) for the linear drift and equation (3.18) for the nonlinear drift with the conditional variance following equation (16). The sample size is 1,000 and the number of replications is 50. The rows labeled as iterations and final smoothing are the results for the semiparametric approach.

**Table 3.2: Summary Statistics for the U.S. Short Rates**

Variable	Mean	SD	Skewness	Kurtosis	JB test	Q(10)
$r_t$	5.8252	3.0448	0.7665	1.0476	271.82	18017.36
					[0.00]	[0.00]
$\Delta r_t$	-0.0028	0.2319	-0.6466	17.9637	25557.59	185.53
					[0.00]	[0.00]
Variable	ARCH(10)	$\tau_\mu$	$\tau_\beta^{GLS}$	$\tau_\beta$	$\tau_\beta(\text{BW})$	
$r_t$	35.5992	-1.4902	-1.5440	-2.4301	-2.5147	
	[0.00]					

*Note:* The JB test represents the Jarque–Bera test of normality. Q(10) is the Ljung–Box test of serial correlation of order 10. ARCH(10) is the test for ARCH effect up to order 10 for the resulting residual of an AR(10) regression on the short rate.  $\tau_\mu$  and  $\tau_\beta^{GLS}$  are the ADF and the GLS-based Dickey-Fuller test statistics and their 5% critical values are -2.8629 and -1.95, respectively.  $\tau_\beta$  and  $\tau_\beta(\text{BW})$  are the test statistics for Seo's (1999) test with the latter using Bollerslev and Wooldridge's (1992) robust standard errors. The simulated critical values of  $\tau_\mu$  and  $\tau_\beta^{GLS}$  are -1.9073 and -1.8891 at the 5% significance level.

Table 3.3: Short-Term Interest-Rate Model Estimates (February 2, 1973–June 8, 2007)

(a) Linear drift specification

Models	Normal			Student's $t$			Hansen's Skewed $t$		
	GARCH	GARCHX	AGARCHX	GARCH	GARCHX	AGARCHX	GARCH	GARCHX	AGARCHX
$\mu$	0.0042 (0.0063)	0.0039 (0.003)	0.0033 (0.0031)	0.0037 (0.0039)	0.0018 (0.0024)	0.0014 (0.0026)	0.0026 (0.0019)	0.0041 (0.0058)	0.0015 (0.0045)
$\lambda$	-0.0014 (0.0016)	-0.0002 (0.0008)	-0.0004** (0.0002)	-0.0002* (0.0001)	-0.0003** (0.0001)	-0.0003 (0.0001)	-0.0007* (0.0003)	-0.0004 (0.0008)	-0.0006 (0.0008)
$\alpha_0$	0.0001 (0.0003)	0.0001 (0.0002)	0.0003* (0.0001)	0.0000 (0.00001)	0.00002 (0.00002)	0.00002 (0.00004)	0.0002 (0.0005)	0.0003* (0.0001)	0.0000 (0.0001)
$\alpha_1$	0.1055* (0.0252)	0.3031* (0.0695)	0.2157* (0.0757)	0.1181* (0.0258)	0.2415* (0.0455)	0.2643* (0.0478)	0.1825* (0.0731)	0.2016* (0.068)	0.2389* (0.0412)
$\alpha_2$	0.8915* (0.0252)	0.6719* (0.1069)	0.6643* (0.1857)	0.8815* (0.0258)	0.7110* (0.0575)	0.7054* (0.0518)	0.8107* (0.0359)	0.7589* (0.1205)	0.7182* (0.1033)
$\alpha_3$	—	—	0.0337* (0.0137)	—	—	0.0082* (0.0011)	—	—	0.0028* (0.001)
$\beta$	—	0.0076* (0.0014)	0.0006* (0.0001)	—	0.0010* (0.0003)	0.0005* (0.0001)	—	0.0012* (0.0004)	0.0003* (0.0001)
$\delta$	—	0.4417* (0.0861)	0.5231* (0.1494)	—	2.6622* (0.331)	3.14407* (0.7811)	—	2.3817* (0.493)	2.9637* (0.5182)
$v$	—	—	—	5.2401* (0.4109)	4.4148* (0.4034)	4.4212* (1.5303)	4.9613* (0.3153)	4.1945* (0.5738)	4.3182* (0.5016)
$\eta$	—	—	—	—	—	—	-0.0005 (0.0015)	-0.0001 (0.0021)	-0.0002 (0.0014)
LL	2810.19	2868.07	2871.29	2841.77	2912.65	2946.31	2135.35	2139.57	2146.18
$Q(\varepsilon_t/\sigma_t)$	194.8024 [0.0000]	168.6053 [0.0000]	170.8604 [0.0000]	189.8495 [0.0000]	170.431 [0.0000]	172.7072 [0.0000]	153.6514 [0.0000]	161.4817 [0.0000]	169.2258 [0.0000]
$Q(\varepsilon_t^2/\sigma_t^2)$	22.1335 [0.0361]	11.0381 [0.5256]	11.5038 [0.4863]	20.394 [0.0599]	11.728 [0.4614]	12.002 [0.4455]	21.3148 [0.0459]	10.3564 [0.5848]	10.0106 [0.6151]

(b) Nonlinear drift specification

Models	Normal			Student's $t$			Hansen's Skewed $t$		
	GARCH	GARCHX	AGARCHX	GARCH	GARCHX	AGARCHX	GARCH	GARCHX	AGARCHX
$\mu$	0.0513*** (0.0269)	0.036 (0.0247)	0.0356 (0.0236)	0.0158 (0.0247)	0.0034 (0.0031)	0.0018 (0.0024)	0.0103 (0.0256)	0.0023** (0.0011)	0.0214 (0.0668)
$\lambda_1$	-0.0009 (0.0007)	0.0047** (0.0017)	0.0044** (0.0021)	-0.0019 (0.0026)	0.0011 (0.0007)	0.0016* (0.0004)	0.0015 (0.0037)	0.0013 (0.0035)	0.0006 (0.0014)
$\lambda_2$	-0.0003 (0.0005)	0.0000 (0.0005)	0.0000 (0.0005)	0.0000 (0.0006)	-0.0001 (0.0002)	-0.0002 (0.0006)	0.0007 (0.0005)	-0.0000 (0.0001)	-0.0004 (0.0003)
$\lambda_3$	-0.0004 (0.0291)	-0.0002 (0.0181)	-0.0002 (0.0195)	-0.0001 (0.0171)	-0.0002 (0.0042)	-0.0001 (0.0035)	-0.0002 (0.0725)	-0.0001 (0.0395)	-0.0002 (0.0497)
$\alpha_0$	0.0001 (0.0003)	0.0004** (0.0002)	0.0005* (0.0002)	0.00014** (0.00006)	0.0003* (0.0001)	0.0002** (0.0001)	0.0002** (0.0001)	0.0003* (0.0001)	0.0001* (0.0000)
$\alpha_1$	0.1645* (0.0082)	0.3180* (0.0514)	0.2157* (0.0757)	0.1657* (0.0337)	0.2643* (0.0478)	0.2583* (0.0507)	0.1367* (0.0413)	0.1714* (0.0385)	0.2010* (0.0294)
$\alpha_2$	0.8255* (0.1088)	0.6619* (0.1311)	0.6726* (0.1342)	0.8213* (0.0415)	0.7054* (0.0518)	0.7217* (0.0507)	0.8415* (0.2106)	0.8107* (0.1359)	0.7718* (0.1547)
$\alpha_3$	—	—	0.0298* (0.0107)	—	—	0.0147* (0.0035)	—	—	0.0108* (0.003)
$\beta$	—	0.0593* (0.0065)	0.0009* (0.0001)	—	0.0006* (0.0001)	0.0069* (0.0012)	—	0.0018** (0.0009)	0.0035* (0.0009)
$\delta$	—	0.0540* (0.0059)	3.236* (0.3405)	—	4.5966* (0.743)	0.3979* (0.0671)	—	4.2613* (0.918)	0.2859* (0.0101)
$\nu$	—	—	—	4.9327* (0.3879)	4.4043* (0.3871)	4.3314* (0.4398)	4.2138* (0.131)	4.1017* (0.6133)	4.0981* (0.5819)
$\eta$	—	—	—	—	—	—	-0.0008 (0.0561)	-0.0003 (0.0318)	-0.0005 (0.1036)
LL	2443.13	2474.76	2491.27	2334.8	2355.82	2369.91	1753.42	1814.79	1830.25
$Q(\varepsilon_t/\sigma_t)$	165.1314 [0.0000]	128.3772 [0.0000]	131.0561 [0.0000]	141.2809 [0.0000]	137.175 [0.0000]	148.9106 [0.0000]	123.5912 [0.0000]	146.2058 [0.0000]	151.8333 [0.0000]
$Q(\varepsilon_t^2/\sigma_t^2)$	21.0615 [0.0495]	17.9813 [0.1163]	18.021 [0.1151]	19.5918 [0.0752]	15.3428 [0.2233]	14.7311 [0.2565]	20.1113 [0.0650]	12.4285 [0.4119]	12.0091 [0.4449]

Note: The GARCHX (AGARCHX) model refers to the symmetric (asymmetric) GARCH model with additive level effects given by equation 3.4 (3.5). LL denotes the log-likelihood value,  $Q(\varepsilon_t/\sigma_t)$  and  $Q(\varepsilon_t^2/\sigma_t^2)$  are the Ljung-Box test statistics for serial correlation in the standardized and squared-standardized residuals up to order 12, respectively. \*, \*\* and \*\*\* denote significance at 1%, 5% and 10% significance levels.

**Table 3.4: The Goodness of Fit of In-Sample Volatility Estimates of Parametric and Semiparametric Models of U.S. Short Rates over the Period February 9, 1973–June 8, 2007**

(a) Volatility Benchmark  $|r_t - r_{t-1}|$

	Normal				Student's $t$			
	MSE	MAE	AIC	$R_{\text{vol}}^2$	MSE	MAE	AIC	$R_{\text{vol}}^2$
Linear drift								
GARCH	0.0441	0.1632	-465.2797	0.3539	0.0444	0.1645	-451.8839	0.3564
Iteration 1	0.041	0.1599	-584.3063	0.3982	0.0411	0.1601	-583.19	0.3989
Iteration 2	0.0413	0.1595	-572.9455	0.3941	0.0415	0.1601	-564.1346	0.3937
Iteration 3	0.0409	0.1583	-589.5497	0.3929	0.041	0.1584	-586.7339	0.3926
Iteration 4	0.0408	0.1578	-593.7955	0.3913	0.0408	0.1579	-592.4436	0.391
Iteration 5	0.0408	0.1576	-593.4775	0.3901	0.0409	0.1577	-589.1935	0.389
Iteration 6	0.0407	0.1571	-598.9764	0.3903	0.0407	0.1572	-596.3312	0.3893
Iteration 7	0.0407	0.1571	-599.0108	0.3906	0.0408	0.1573	-593.5321	0.3892
Iteration 8	0.0406	0.1567	-603.5222	0.3909	0.0406	0.1568	-601.1995	0.3901
Final smoothed	0.0406	0.1569	-601.8026	0.3908	0.0407	0.1572	-601.9206	0.3899
AGARCHX	0.0433	0.1609	-468.8008	0.3545	0.0421	0.1584	-498.5179	0.3573
GARCHX	0.0439	0.1617	-459.0652	0.3540	0.0428	0.1596	-486.4507	0.3566
Nonlinear drift								
GARCH	0.0441	0.1633	-463.5183	0.3534	0.0444	0.1645	-453.066	0.3567
Iteration 1	0.0411	0.1602	-581.6097	0.3981	0.0411	0.1601	-583.1312	0.3989
Iteration 2	0.0414	0.1597	-571.1824	0.3943	0.0415	0.1601	-563.9604	0.3942
Iteration 3	0.0409	0.1584	-588.7446	0.3932	0.041	0.1584	-586.7507	0.3928
Iteration 4	0.0408	0.158	-593.2638	0.3919	0.0408	0.1579	-592.6804	0.3911
Iteration 5	0.0409	0.1578	-590.8762	0.3901	0.0409	0.1577	-589.8344	0.3893
Iteration 6	0.0407	0.1573	-597.7895	0.3905	0.0408	0.1574	-594.9734	0.3897
Iteration 7	0.0407	0.1573	-597.6123	0.3905	0.0408	0.1573	-593.8119	0.3896
Iteration 8	0.0406	0.157	-601.9312	0.3914	0.0406	0.1569	-600.9026	0.3903
Final smoothed	0.0407	0.1572	-600.0607	0.3907	0.0407	0.1572	-600.0972	0.3901

(b) Volatility Benchmark  $\hat{\sigma}_{\text{GK}}^2$

Linear drift	Normal				Student's $t$			
	MSE	MAE	AIC	$R_{\text{vol}}^2$	MSE	MAE	AIC	$R_{\text{vol}}^2$
GARCH	0.0263	0.1366	-1325.280	0.4139	0.027	0.1387	-1281.644	0.4128
Iteration 1	0.0249	0.1340	-1417.338	0.4309	0.0251	0.1344	-1409.191	0.4307
Iteration 2	0.0245	0.1321	-1448.451	0.4322	0.0248	0.1328	-1428.123	0.4312
Iteration 3	0.0242	0.1311	-1464.101	0.4284	0.0244	0.1313	-1455.203	0.427
Iteration 4	0.0243	0.1308	-1462.552	0.4259	0.0244	0.1311	-1452.146	0.4245
Iteration 5	0.0243	0.1307	-1462.171	0.4264	0.0244	0.1308	-1454.157	0.4246
Iteration 6	0.0241	0.1303	-1470.949	0.4251	0.0243	0.1304	-1462.523	0.4223
Iteration 7	0.0242	0.1303	-1469.129	0.4257	0.0243	0.1306	-1459.599	0.4241
Iteration 8	0.0241	0.1299	-1475.989	0.4262	0.0242	0.1302	-1466.137	0.4242
Final smoothed	0.0241	0.1301	-1474.438	0.4263	0.0242	0.1304	-1464.826	0.4240
AGARCHX	0.0268	0.134	-1293.64	0.4189	0.0261	0.1335	-1334.653	0.4219
GARCHX	0.0266	0.1342	-1302.411	0.4167	0.0258	0.1327	-1352.352	0.4225
Nonlinear drift	MSE	MAE	AIC	$R_{\text{vol}}^2$	MSE	MAE	AIC	$R_{\text{vol}}^2$
GARCH	0.0266	0.1369	-1310.771	0.4129	0.027	0.1387	-1283.112	0.4129
Iteration 1	0.025	0.1343	-1411.852	0.4313	0.025	0.1344	-1409.544	0.4307
Iteration 2	0.0245	0.1323	-1444.576	0.4325	0.0248	0.1329	-1427.342	0.432
Iteration 3	0.0243	0.1313	-1460.787	0.4287	0.0244	0.1314	-1455.954	0.4275
Iteration 4	0.0243	0.131	-1459.072	0.4265	0.0244	0.1311	-1453.408	0.4249
Iteration 5	0.0243	0.1309	-1458.577	0.427	0.0244	0.1308	-1455.779	0.4251
Iteration 6	0.0242	0.1305	-1469.125	0.4258	0.0243	0.1305	-1461.697	0.4234
Iteration 7	0.0242	0.1304	-1468.508	0.4263	0.0243	0.1307	-1459.126	0.4244
Iteration 8	0.0242	0.1303	-1469.766	0.4268	0.0242	0.1303	-1466.36	0.4247
Final smoothed	0.0241	0.1303	-1470.881	0.4257	0.0242	0.1304	-1464.585	0.4240

Note: The rows labeled as iterations and final smoothed are the results for the semiparametric approach. The prefix "A" denotes asymmetric while the suffix "X" denotes level effects.

**Table 3.5: The Out-of-Sample Volatility-Forecast Performance of Parametric and Semiparametric Models of U.S. Short Rates over the Period June 15, 2007–May 8, 2009**

a. Volatility Benchmark $ r_t - r_{t-1} $						
	GARCH	Final Smoothed	AGARCHX-T	GARCHX-T	AGARCHX-N	GARCHX-N
MSE	0.0349	0.0318	0.0339	0.0344	0.03499	0.0352
MAE	0.1431	0.1225	0.1295	0.1319	0.1346	0.1359

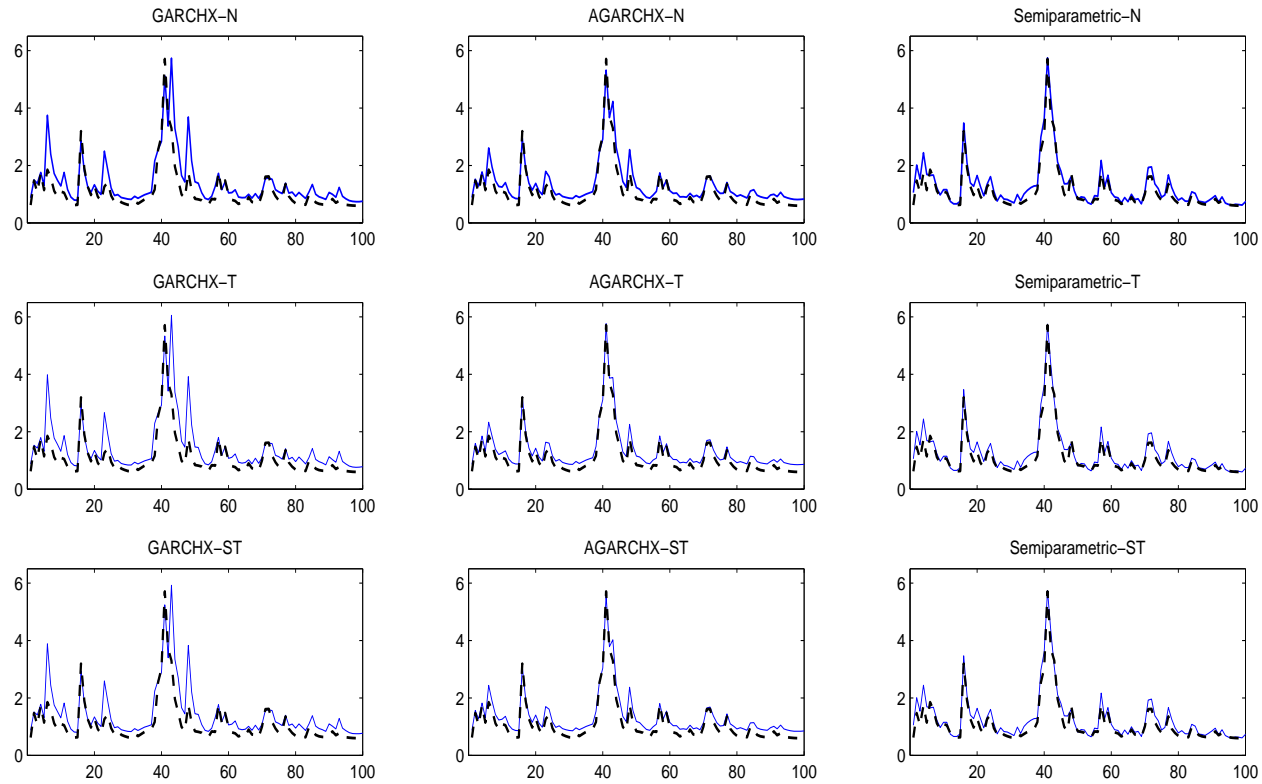
b. Volatility Benchmark $\hat{\sigma}_{GK}^2$						
	GARCH	Final Smoothed	AGARCHX-T	GARCHX-T	AGARCHX-N	GARCHX-N
MSE	0.0498	0.0395	0.0458	0.0466	0.0477	0.0486
MAE	0.1744	0.1402	0.1581	0.1643	0.1666	0.1678

*Note:* See note to Table 3.4. “T” and “N” denote Student’s  $t$  and normal distributions, respectively.

# Figures

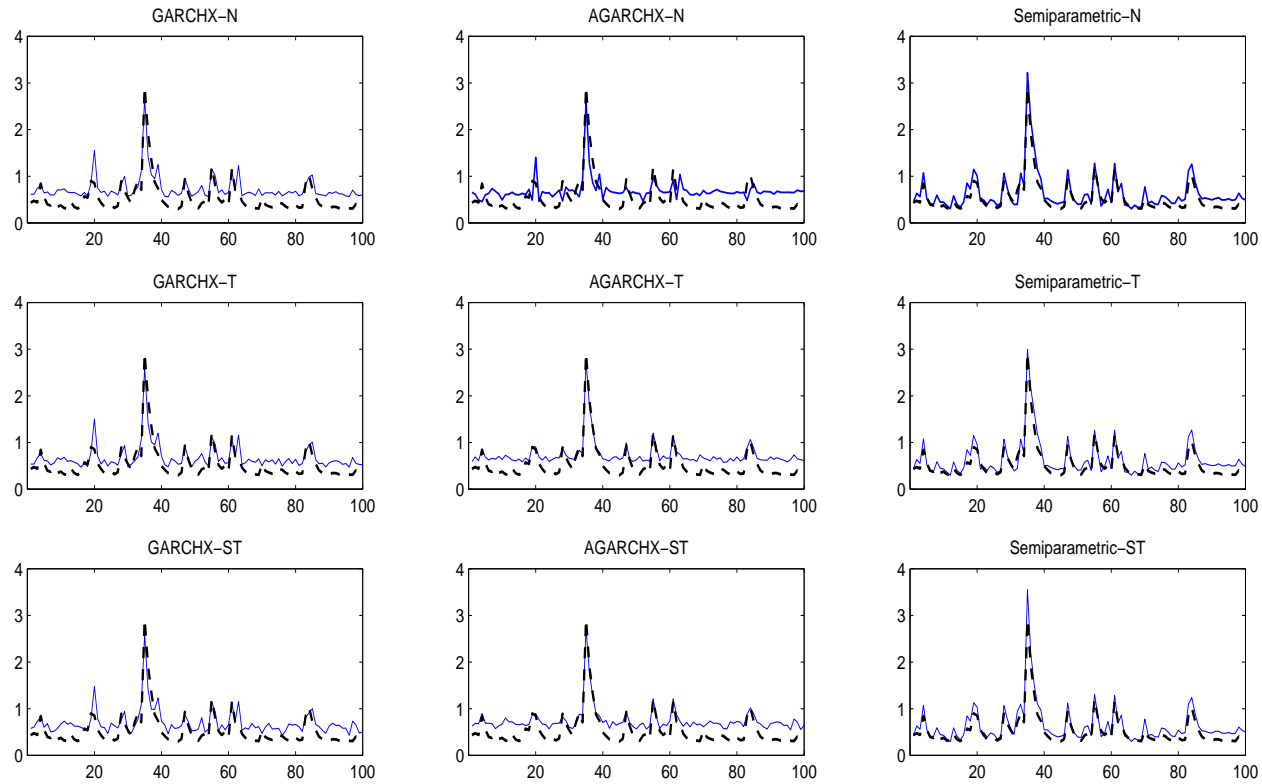


Figure 3.1: (a) Volatility Estimates of Various Models for Simulated Data with Linear Drift



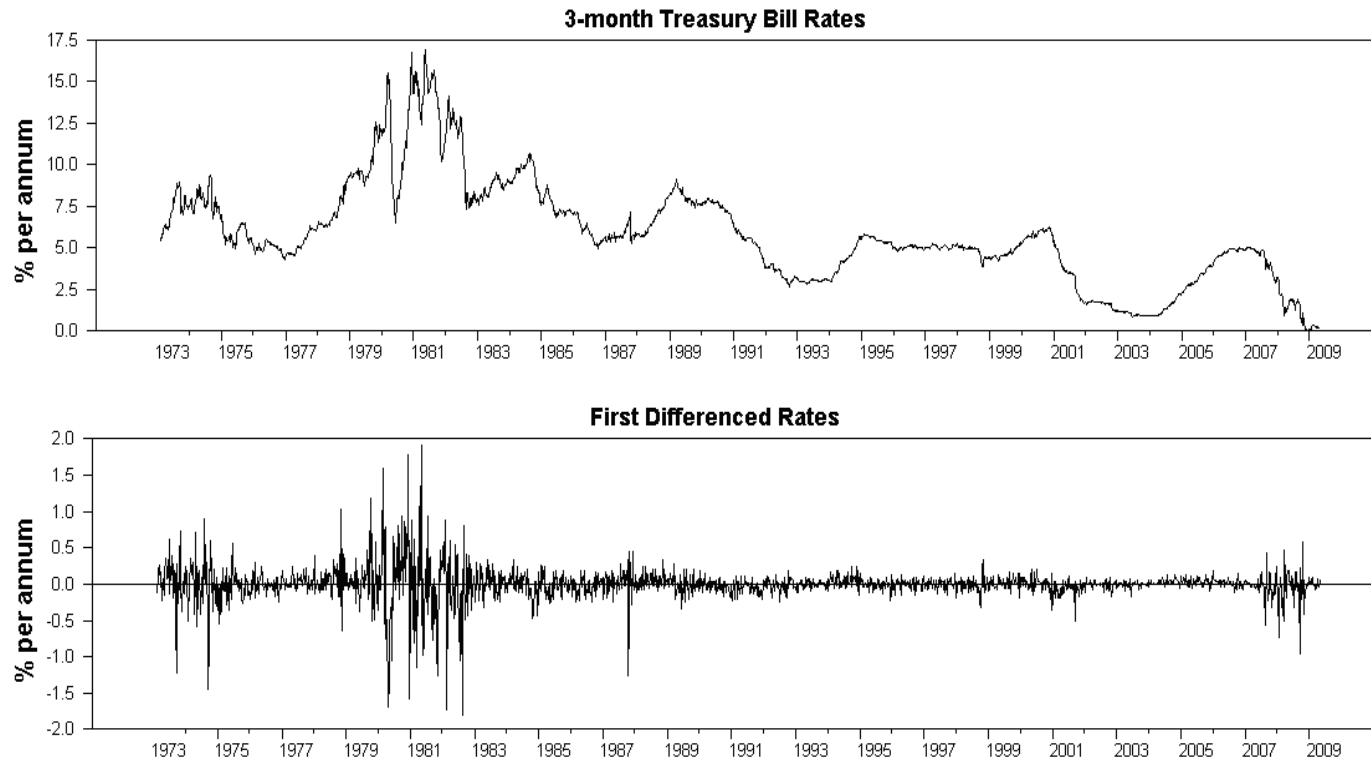
*Note:* The dotted line represents simulated true volatility, while the solid line represents the estimated volatility derived from estimating a model with a linear drift. For the semiparametric approach, the final smoothed volatility is presented. N, T and ST denote normal, Student's  $t$ , and skewed  $t$  distributions, respectively.

Figure 3.1: (b) Volatility Estimates of Various Models for Simulated Data with Nonlinear Drift



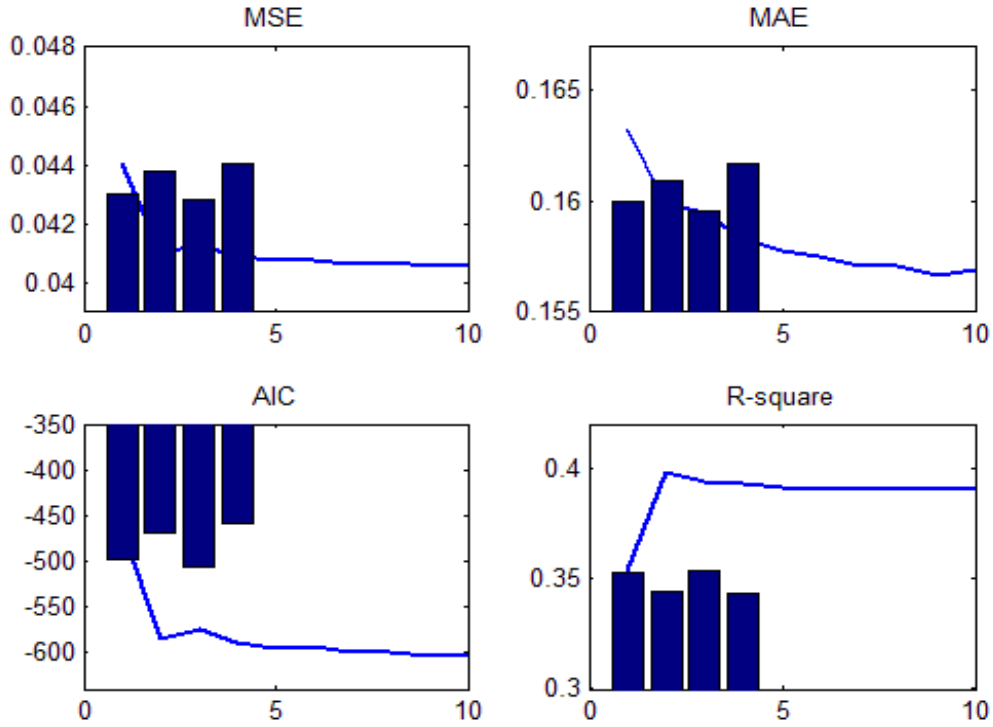
*Note:* The dotted line represents simulated true volatility, while the solid line represents the estimated volatility derived from estimating a model with a nonlinear drift. For the semiparametric approach, the final smoothed volatility is presented. N, T and ST denote normal, Student's  $t$  and skewed  $t$  distributions, respectively.

Figure 3.2: The U.S. Short Rates: Levels and First Differences

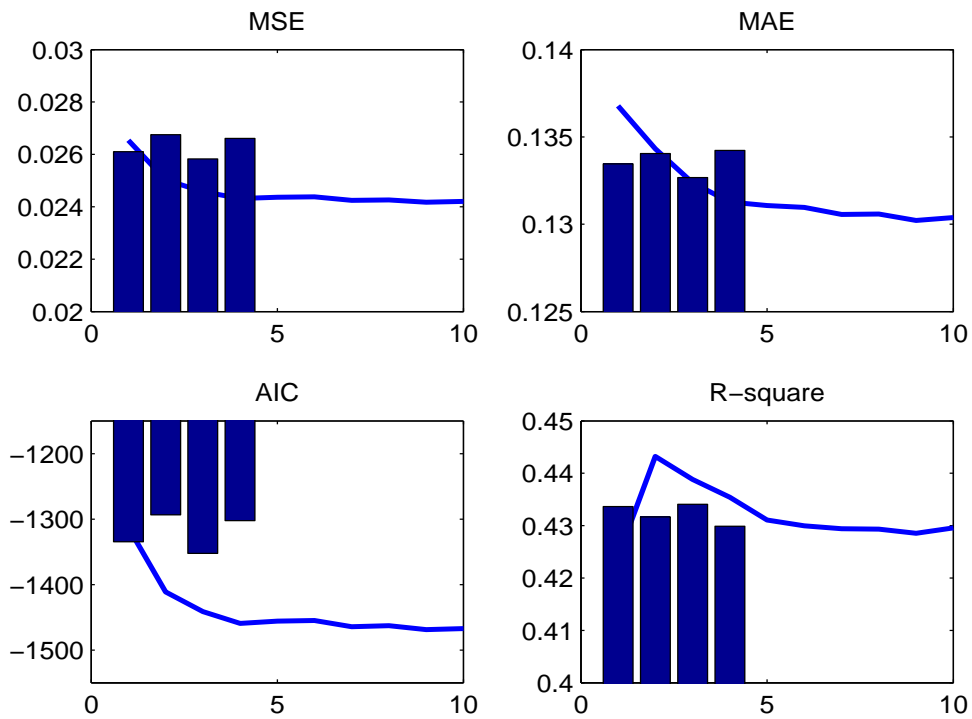


*Note:* This figure plots the level and the first difference of the three-month Treasury bill rates. The first plot is the level, and the second is the first difference.

Figure 3.3: Plots of MSE, MAE, AIC and  $R^2_{vol}$  for the In-Sample Volatility-Forecasting Performance of the Parametric and Semiparametric U.S. Short-Rate Models

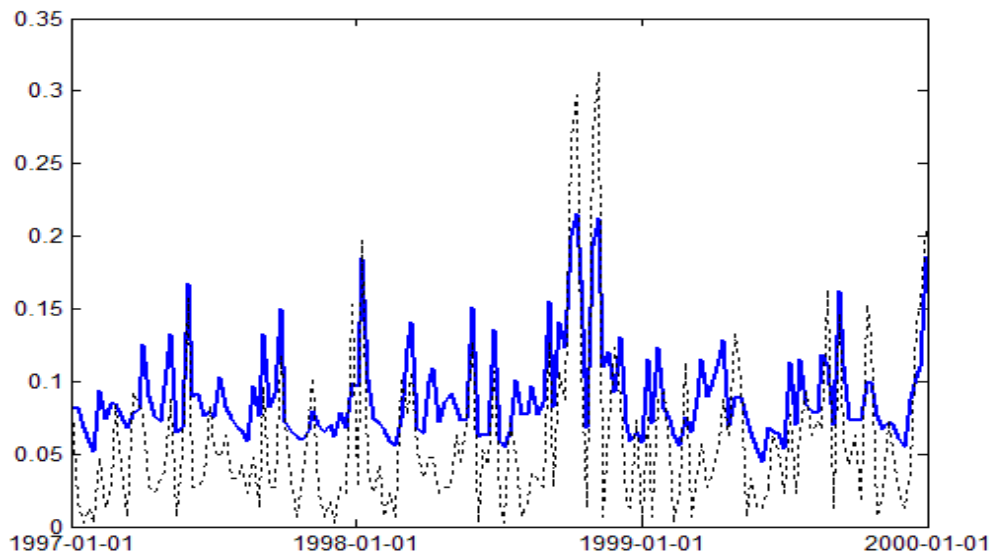


(a) Volatility Benchmark  $|r_t - r_{t-1}|$

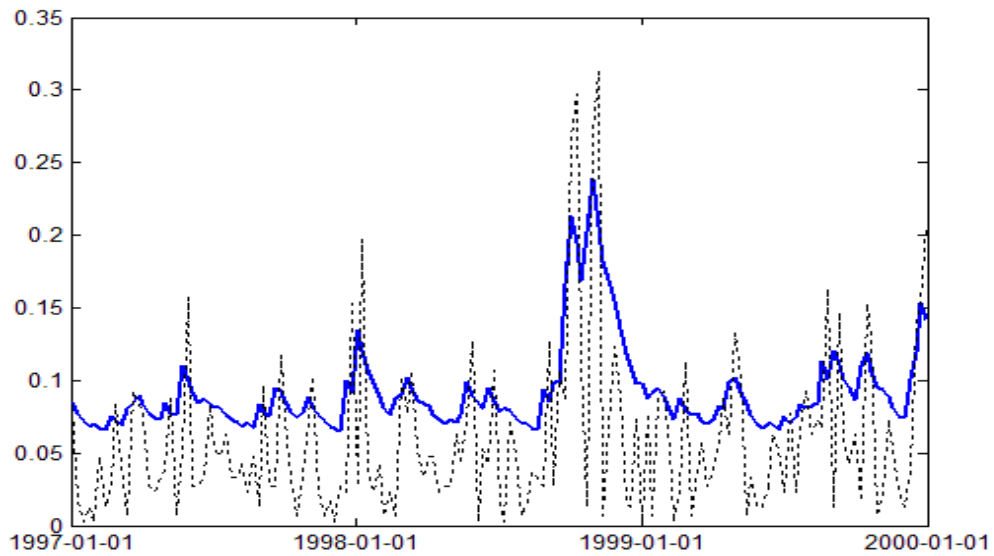
(b) Volatility Benchmark  $\hat{\sigma}_{GK}^2$ 

*Note:* The shaded bars represent the metric value for the parametric models in the following order: AGARCHX-T, AGARCHX-N, GARCHX-T, GARCHX-N. The 1 to 10 marks on the x-axis are to be interpreted in the following way. The first mark represents the metric value for the parametric GARCH model. The second to ninth marks represent the metric values for the eight iterations that are performed in the semiparametric procedure, while the tenth mark denotes the metric value for the final smoothing stage.  $R^2$  denotes  $R_{vol}^2$ . The results are for the sample period February 9, 1973–June 8, 2007.

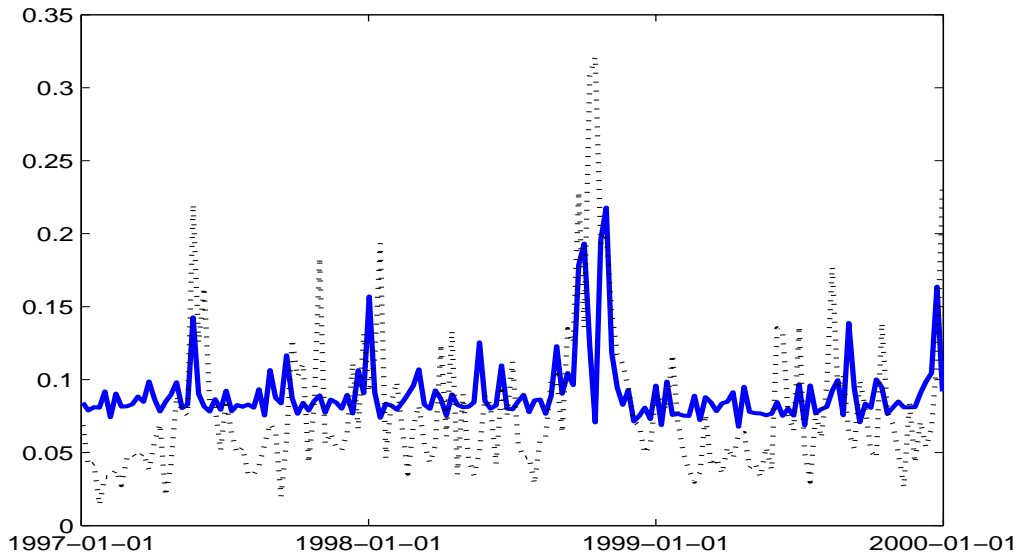
Figure 3.4: Plots of In-Sample Volatility Estimates for the U.S. Short Rates over the Sample Period January 1, 1997–January 1, 2000



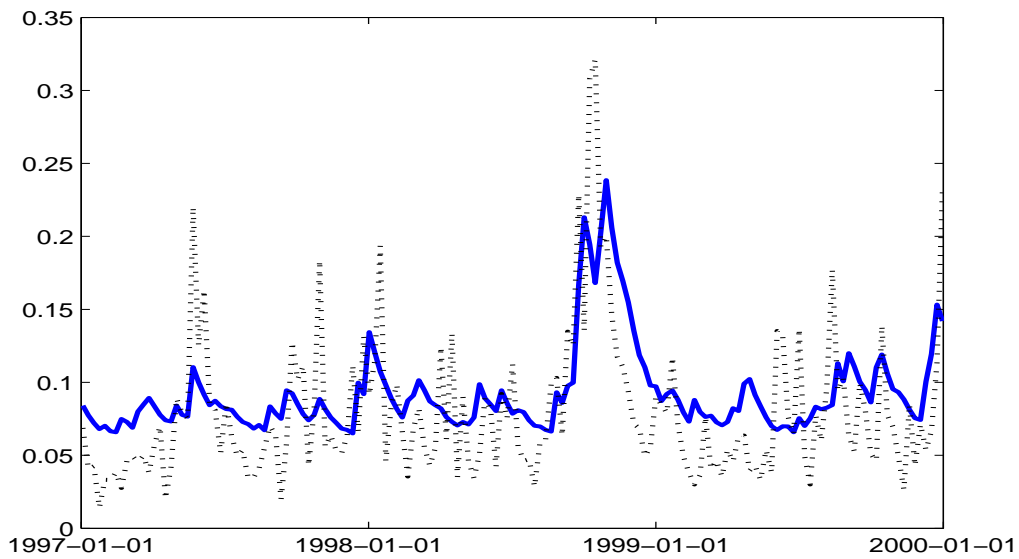
(a) Semiparametric approach



(b) AGARCHX-T model



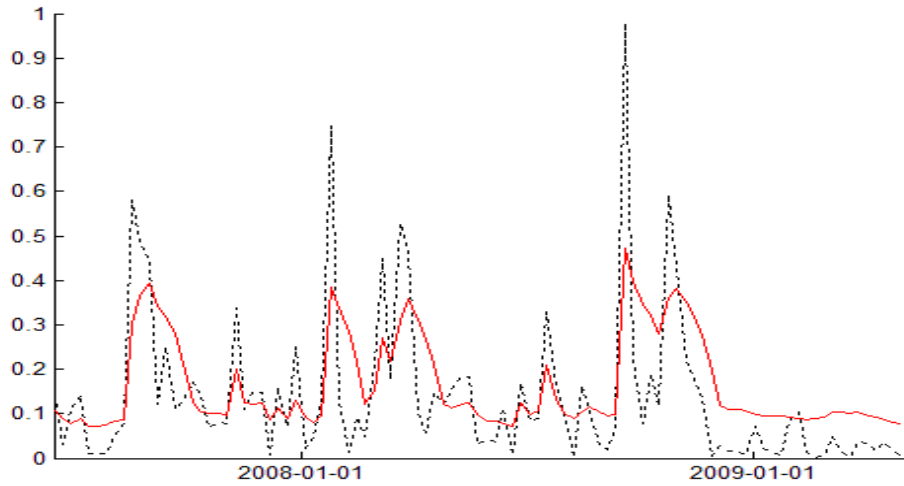
(c) Semiparametric approach



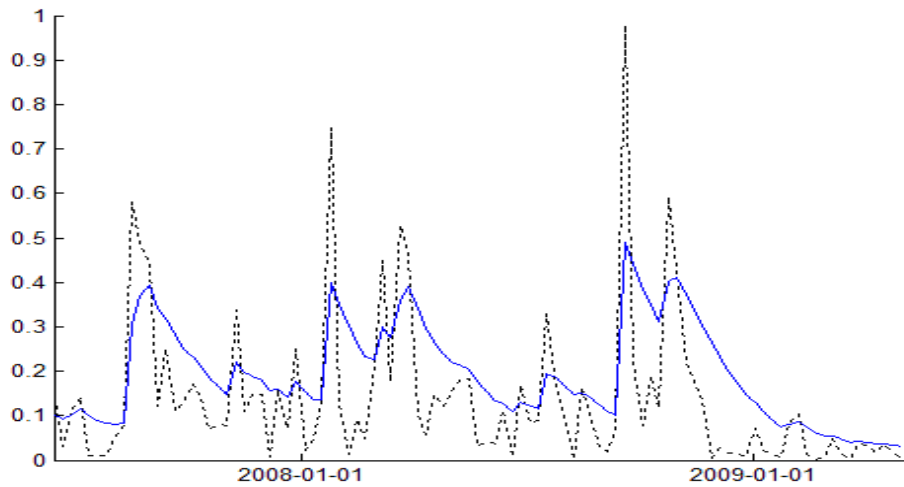
(d) AGARCHX-T model

*Note:* The dotted line in (a) and (b) is the true volatility proxied by  $|r_t - r_{t-1}|$ . The dotted line in (c) and (d) is the true volatility proxied by  $\hat{\sigma}_{GK}^2$ . The line marked in bold is the volatility estimate  $\hat{\sigma}_t$ .

Figure 3.5: Plots of Out-of-Sample Volatility Forecasts for the U.S. Short Rates over the Sample Period June 15, 2007–May 8, 2009

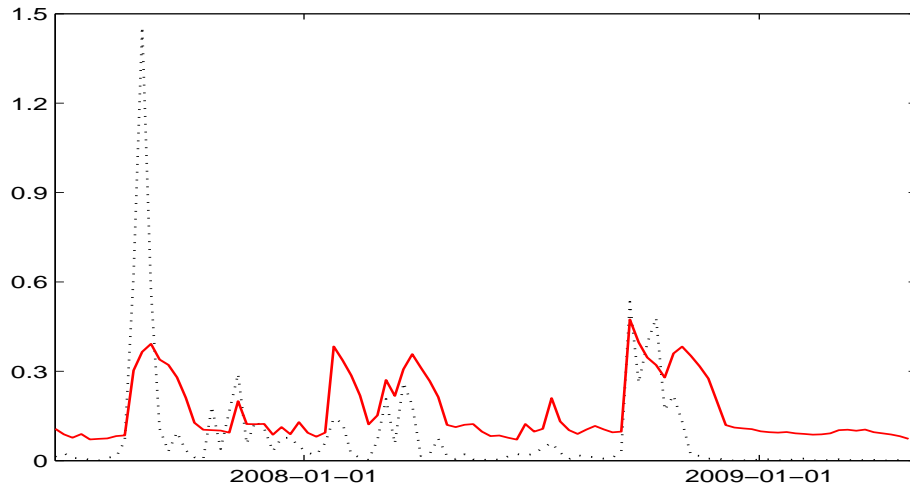


(a) Semiparametric approach

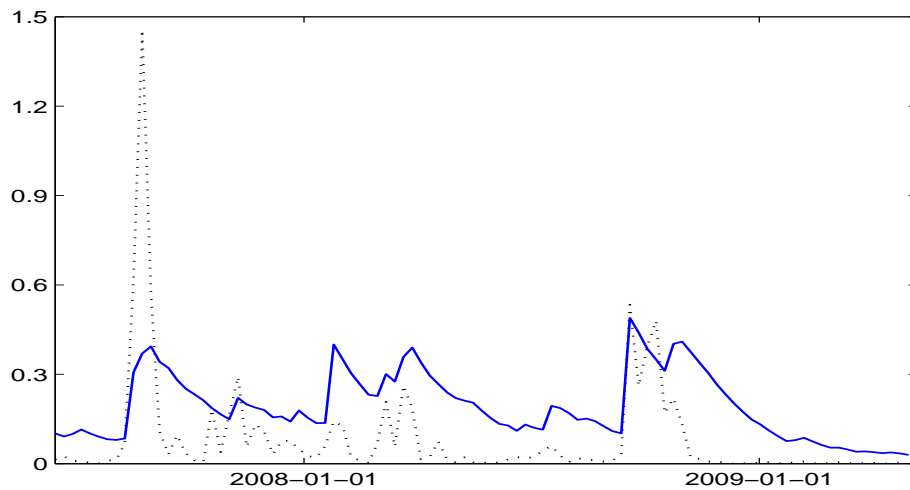


(b) AGARCHX-T model





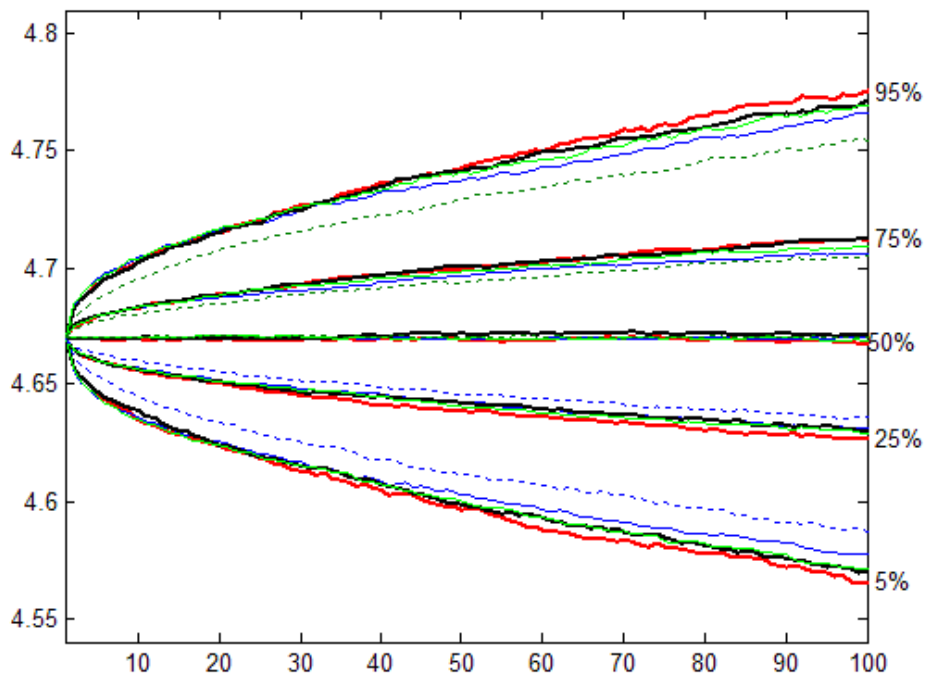
(c) Semiparametric approach



(d) AGARCHX-T model

*Note:* The dotted lines in (a) and (b) are the true volatility proxied by  $|r_t - r_{t-1}|$ . The dotted line in (c) and (d) is the true volatility proxied by  $\hat{\sigma}_{\text{GK}}^2$ . The line marked in bold is the volatility estimate  $\hat{\sigma}_t$ .

**Figure 3.6: Confidence Intervals for Simulated Interest Rates from BHK and Semiparametric Models**



*Note:* The solid lines (reading from the outermost to the innermost lines) are confidence intervals for simulated interest rates from the AGARCHX-T, GARCHX-T, AGARCHX-N and GARCHX-N models. The dotted lines are confidence intervals for the semiparametric model.



## Chapter 4

# EMU Equity Markets' Return Variance and Spillover Effects from Short-Term Interest Rates

### 4.1 Introduction

The last decades have witnessed policymakers using the stock market as the intermediate channel to stabilize inflation and output. However, much of the effect of monetary policy comes through the influence of short-term interest rates on other asset prices including bond and stock prices that, in turn, significantly influence real economic activities. Since the Monetary Policy Committees in the UK started to use short-term interest rates as the tool for achieving its inflationary target in 1997, there is an increasing trend of using the short-term interest rate rather than the money supply as intermediate targets for monetary policy in the world. Recently, the unexpected shocks from the money market, such as the Russian debt crisis in 1998 and the subprime mortgage crisis of 2007, have shown how the domino effect of short-term interest-rate shocks can affect the financial market globally. Henry (2009) argues that with huge fluctuations in the short-term money market, firms seeking funding in the short-term rate, and lending in long-term relatively illiquid securities, become insolvent simply because they cannot access sufficient cash to finance their short-term activities and not because they are unviable in the medium to long term. Therefore, it is important for policymakers and analysts to understand how short-term interest-rate changes affect stock prices and for them to pay close attention in pursuit of their final objectives.

Many researchers have examined the impact of interest rates on stock prices, but the relationship between the short-term interest rate and stock prices is still controversial. Earlier studies employed Treasury bill rates as a proxy for the expected inflation to examine the relationship between interest rates and stock returns (see, e.g., Nelson, 1976 Fama and Schwert, 1977 Fama, 1981 Shanken, 1990). These studies find a negative relationship between stock returns and Treasury bill rates. Domian et al. (1996) mainly use yields on one-month Treasury bills to examine the relationship between stock returns and interest-

rate changes. The results from this study show asymmetric relations; that is, drops in interest rates are followed by large positive stock returns while increases in interest rates have little effect. By present-value models, the negative relation between interest rates and stock prices stems from the fact that an interest-rate increase (decrease) causes an increase (decrease) in expected future discount rates which should cause stock prices to fall (rise) and long-term interest rates to rise (fall). However, certain empirical attempts have provided evidence in favor of a positive relationship between interest rates and stock prices (see, e.g., Asprem, 1989 Shiller and Beltratti, 1992 Barsky, 1989). Barsky (1989) explains the positive relationship in terms of a changing risk premium. For instance, a drop in interest rates could be the result of increased risk or precautionary saving as investors substitute away from risky assets – e.g., stocks – into less risky assets – e.g., bonds. Shiller and Beltratti (1992) argue in favor of such a positive relationship on the grounds that changes in interest rates could carry information about certain changes in future fundamentals.

Meanwhile, since the seminal Bernanke and Blinder (1992), the impact of changes in different interest-rate instruments used as the proxy for monetary policy on the stock market has been examined in the financial literature (see, e.g., Thorbecke, 1997 Bomfim, 2003 Rigobon and Sack, 2002 Bernanke and Knutner, 2005 Davig and Gerlach, 2006 Basistha and Kurov, 2008 Henry, 2009). In particular, using the three-month Eurodollar rate as a proxy of monetary policy, Rigobon and Sack (2002) show that increases in the short-term interest rate negatively impact stock prices and significantly positively impact market interest rates, with the largest effect on rates with shorter maturities.<sup>1</sup> Another important issue considered in the interest-rate literature is that the effect of interest rates is different in bull and bear markets. As defined in Maheu and McCurdy (2000) and Perez-Quiros and Timmermann (2000), bull markets display high returns coupled with low volatility (a stable regime), and bear markets have a low return and high volatility (a volatile regime). Some empirical studies have established that the effect of interest rates on conditional returns is larger in a volatile regime than in a stable regime. For example, using a Markov-switching model, Chen (2007) investigates how monetary policy, measured by interest-rate instruments, affects stock returns, concluding that such an impact is asymmetrically large in the bear periods. Henry (2009) uses a Markov-switching EGARCH model to examine the impact of short-term interest-rate surprises on the volatility of returns in the UK stock market. Using a Markov-switching model, Perez-Quiros and Timmermann (2000) study the relationship between changing credit market conditions, including short-term interest rates, and stock market. They all find a similar asymmetric effect of interest rates on stock returns in the bear market. Meanwhile, a different conclusion is found in the Markov-switching framework. In contrast to the previous work suggesting interest rates significantly impact stock markets, Ang and Bekaert (2002) confirm that the evidence to support the effect of interest rates on returns does not exist, even if the regime-switching characteristics are added into the empirical

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<sup>1</sup>Ellingsen and Söderström (2001) have also used changes in the three-month interest rate as a measure of policy innovations for estimating the term structure's response. Favero et al. (1999) examine the transmission of monetary policy in Europe, using the three-month Euro rate as a proxy for that policy.

model.

This paper investigates the spillover effect of interest rate impacts on stock returns and the volatility of returns in the Euro area in different regimes. We extend the current literature in several aspects. First, departing from most previous work, which primarily examines the effect of interest rates on stock prices and returns, we analyze the potential impact of changes in short-term interest rate on both stock returns and the volatility of returns. Because the conditional variance is considered to be a proxy for risk in the financial and economic fields, it has important influence on monetary policymaking, asset-allocation decisions, and risk management. Merton (1980) suggests that one should use accurate variance estimates in accounting for the risk level when estimating expected returns. Optimal inference about the conditional mean of asset returns requires that the conditional variance be correctly specified. The investigation of interest rates' impact on both stock returns and the volatility of returns is of importance to financial-market participants making effective portfolio selection and formulating risk-management strategies.

Second, we contribute to the current literature by investigating the asymmetric effect of the increased interest rates on returns and the volatility of returns in bull and bear markets in the Economic and Monetary Union (EMU) stock markets. Although there is substantial evidence for the asymmetrical effect of interest rates on stock returns in bear and bull markets, no research has been done to examine whether increases and decreases in interest rates have the same effect in different market states. Further, reviewing Sellin's (2001) survey, it is clear that most of the studies focus mainly on the effect of interest rates on U.S. financial markets. In contrast, the impact of short-term interest-rate movements on stock markets in the EMU area has received surprisingly little attention in the recent literature. We examine the impact of the interest rates on the stock markets in the EMU countries.

Third, our empirical work updates the current literature by investigating the spillover effect of the money market on stock returns and the volatility of stock returns by extending the Markov-switching GJR GARCH in Mean model (MS GJR-M). We extend the MS GJR-M model by adding interest-rate movements directly to the variance process of the MS GJR-M model, and formulate the Extended Markov-switching GJR GARCH-M model (EMS GJR-M). We use the changes (not the level) of the short-term interest rate because we want to examine how the fluctuations in the short-term interest rates affect the EMU equity market, meanwhile the first difference square is a commonly used proxy for the short-term interest-rate variance. By setting the first-difference squares to the conditional variance of the equity return, we can investigate the spillover effect of the short-term interest-rate market on the EMU equity market. There are several advantages of the proposed model in this paper. First, a regime switching model can capture structural breaks in the volatility in terms of bull and bear markets.<sup>2</sup> Second, given the widespread evidence of the asymmetrical effect of unexpected shocks on stock

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<sup>2</sup>Lamoureux and Lastrapes (1990), Hamilton and Susmel (1994) and Cai (1994) argue that ignoring these structural shifts in the volatility process causes GARCH models to overestimate the persistence of volatility.

volatility (see, e.g., Glosten et al., 1993 Engle and Ng, 1993), the MS GJR-M model has sufficient flexibility to characterize the persistent and asymmetrical response (leverage effect) of the volatility to shocks. Meanwhile the time varying risk premia theory (see, e.g., French et al., 1987 Campbell and Hentschel, 1992) states that the volatility asymmetry is due to the volatility feedback; that is, if volatility is increased, so is the risk premium in case of a positive trade-off between risk and return. Hence, the discount rate is also increased, which in turn, for an unchanged dividend yield, lowers the stock price. Therefore, the MS GJR-M model captures the volatility feedback via a GARCH in Mean (GARCH-M) process from Engle et al. (1987). Finally, adding interest-rate movements and distinguishing increases in interest rates enable us to investigate three types of asymmetric effects in the variance process, i.e., the asymmetric effect of unexpected shocks (negative/positive news) from the stock market, the asymmetric effect of unexpected shocks (interest rate increases/decreases) from the interest-rate market, and the asymmetric effect of unexpected shocks in different market states. We investigate these asymmetric effects by modifying the news impact curve (NIC) as suggested by Engle and Ng (1993) to the news impact surface, in which the variance process depends on the shocks from stock returns and from interest-rate changes in different market states. We estimate the MS GJR-M and EMS GJR-M models with the Markov Chain Monte Carlo (MCMC) method instead of the traditional maximum-likelihood method. Because of the structure of the proposed model, the conditional variance depends on all past history of the state variables. The evaluation of the likelihood function for a sample path of length  $T$  and  $k$  states requires the integration over all  $k^T$  possible paths, rendering the maximum likelihood estimation infeasible. To the best of our knowledge, this is the first time that a MS GJR-M model has been estimated in the literature.

Our results suggest that two regimes exist in the EURO area stock markets, a high-mean low-variance (bull) market and a low-mean high-variance (bear) market. Most of the Euro countries have the same regime switching status between the bull and bear markets. The correlation between the first two moments of returns is not stable over time, but varies between the bull and the bear markets. Our results suggest also that bad news from unexpected stock returns (negative residuals from returns) has an asymmetrically larger effect on the returns and the volatility than good news. Such an impact is larger in the bear market than in the bull market. Surprisingly, as implied in the news impact surface, we find that the change in short-term interest rates only significantly affects the stock market volatility in the bear period in most of the EMU countries. In particular, the effect of an increase in interest rates is asymmetrically larger than that of a decrease in interest rates. Portfolio performance, based on the out-of-sample forecast results of various models, indicates that the EMS GJR-M model outperforms other models, including the MS GJR-M model and a single switching GJR-M model. The models with regime switching yield better portfolio performance than the ones without it, emphasizing the importance of the interest-rate impact and the regime specification when modeling volatility. Ignoring such state-dependent asymmetric effects from short-term interest rates on stock returns and their volatility will lead to invalid inferences, biased forecasts and consequently inefficient portfolio selection and risk management due to the

biased volatility estimates.

This paper proceeds as follows. Section 4.2 presents the extended Markov-switching GJR GARCH-M model. Section 4.3 demonstrates the model-estimation algorithm. Section 4.4 describes the data used and reports the empirical results. Section 4.4 also performs the asset allocation based on the out-of-sample forecasts result from various models. Section 4.5 concludes.

## 4.2 The model

In this section, we present the model used and proposed in this paper.

### 4.2.1 The Markov-switching GJR GARCH-M model

There is a substantial literature describing the volatility of stock returns. Since Engle (1982) introduced the ARCH (autoregressive conditional heteroskedasticity) model and Bollerslev (1986) introduced the GARCH (generalized autoregressive conditional heteroskedasticity) model, these types of volatility modeling techniques have been extended and applied extensively to characterize the volatility of stock returns. One common observed characteristic of the volatility is the volatility asymmetry, where the volatility increases more after a negative shock than after a positive shock of the same magnitude. Two economic theories explain the asymmetric volatility pattern: The leverage effect and the volatility feedback. The volatility feedback (see Campbell and Hentschel, 1992) indicates that the news that future volatility will be higher will induce the risk-averse investors to sell their positions today until the expected return rises up to compensate for the risk. This feature can be captured by the GARCH in Mean (GARCH-M) type formulation (see Engle et al., 1987),<sup>3</sup> in which the conditional mean depends explicitly on the conditional variance. The GARCH-M model also allows us to explore the intertemporal relation between risk and return. Another extension of the standard GARCH model, the EGARCH (Nelson, 1991) and the GJR GARCH (Glosten et al., 1993), capture asymmetry in the conditional variance by the so called leverage effect (Black, 1976). The leverage effect indicates that the increases in the financial leverage lead to an increased volatility level. We choose to use both the GARCH-M and the GJR model to capture the asymmetry in the volatility.

A standard GARCH model with the GJR specification and the GARCH-M effect, which we refer to as the GJR-M( $p, q$ ) model, has the following form,

$$\begin{aligned} r_t &= \beta\sqrt{h_t} + \epsilon_t, & \epsilon_t &= \sqrt{h_t}z_t, & z_t &\sim N(0, 1), \\ h_t &= \alpha_o + \sum_{i=1}^p (\alpha_i + \gamma_i d_i) \epsilon_{t-i}^2 + \sum_{j=1}^q \beta_j h_{t-j}, \end{aligned} \quad (4.1)$$

---

<sup>3</sup>The GARCH-M was primarily motivated by Merton's (1973) Intertemporal Capital Asset Pricing Model (ICAPM)



where  $\epsilon_t$  may be treated as a collective measure of news about equity prices arriving to the market over the last period, and  $\alpha_o > 0, \alpha_i \geq 0, \beta_j \geq 0, \alpha_i + \beta_j + 0.5\gamma_i < 1$ .  $d_i$  is an indicator for negative  $\epsilon_{t-i}$ :

$$d_i = \begin{cases} 1 & \text{if } \epsilon_{t-i} < 0, \\ 0 & \text{if } \epsilon_{t-i} \geq 0. \end{cases}$$

It can be seen from the model that a positive  $\epsilon_{t-i}$  contributes  $\alpha_i \epsilon_{t-i}^2$  to  $\sigma_i$ , whereas a negative  $\epsilon_{t-i}$  has a larger impact  $(\alpha_i + \gamma_i) \epsilon_{t-i}^2$ . Therefore, if parameter  $\gamma_i$  is significantly positive, then negative innovations generate more volatility than positive innovations of equal magnitude.

While estimating financial and macroeconomic series, some economists find that both ARCH and GARCH models may encounter high persistence in volatility and lower accuracy in predicting performance. Diebold and Inoue (2001) argues that the high persistence is caused by structural breaks in the volatility process during the estimation period. Lamoureux and Lastrapes (1990) point out that models with switched parameter values, such as the Markov-switching model of Hamilton (1989), may provide a more appropriate tool for modeling volatility. Hamilton and Susmel (1994) propose a model with sudden discrete changes in the volatility-governing process. They found that a Markov-switching process provides a better statistical fit to the data than a GARCH model without switching.

Therefore, this paper employs a two-state MS GJR-M model to capture the GARCH-M effect (volatility feedback) in the conditional mean, the leverage effect and structural breaks in the conditional variance. The MS GJR-M (1,1) model is defined as follows.

$$\begin{aligned} r_t &= \beta_i \sqrt{h_{i,t}} + \epsilon_{i,t}, & \epsilon_{i,t} &= \sqrt{h_{i,t}} z_t, & z_t &\sim N(0, 1), \\ h_{i,t} &= \alpha_{io} + \alpha_{i1} h_{t-1} + \alpha_{i2} \epsilon_{t-1}^2 + \alpha_{i3} d_i \epsilon_{t-1}^2, \end{aligned} \quad (4.2)$$

where  $z_t \sim N(0, 1)$ ,  $i = 1, 2$  represents the state and  $\alpha_{io} > 0, \alpha_{i1} \geq 0, (\alpha_{i2} + \alpha_{i3}) \geq 0, (\alpha_{io} + \alpha_{i2} + 0.5\alpha_{i3}) < 1$ .  $d_i$  is an indicator for negative news from the last period and in different state  $i$ . Following Hamilton (1989, 1990), we assume that the state vector,  $S_t$ , follows a first-order Markov process with the hidden transition probabilities matrix,

$$\Pi = \begin{pmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{pmatrix},$$

where,

$$\begin{aligned} \pi_{11} &= P(S_t = 1 | S_{t-1} = 1) = 1 - e_1, \\ \pi_{12} &= P(S_t = 2 | S_{t-1} = 1) = e_1, \\ \pi_{21} &= P(S_t = 1 | S_{t-1} = 2) = e_2, \\ \pi_{22} &= P(S_t = 2 | S_{t-1} = 2) = 1 - e_2, \end{aligned} \quad (4.3)$$

where  $0 < e_i < 1$ , for  $i = 1, 2$ . A small  $e_i$  means that the return series has a tendency to stay in the  $i$ th state with the expected duration.

For the model in 4.2 to be identifiable, we assume that  $\beta_2 > \beta_1$  so that State 2 is

associated with higher conditional returns. If  $\alpha_{1j} = \alpha_{2j}$  for all  $j$ , the model becomes a simple GJR in Mean model. If  $\beta_i\sqrt{h_t}$  is replaced by  $\beta_i$ , then the model in 4.2 reduces to a Markov-switching GJR model.

Parameter  $\beta$  is the risk premium. A positive  $\beta$  indicates that the return is positively related to the volatility. Parameters in the GARCH components satisfy conditions similar to those of GARCH models. If the parameters have significant differences between regimes, then there exists a bull market and a bear market in stock returns.

#### 4.2.2 The extended Markov-switching GJR GARCH-M model with the interest-rate effect

Holding the transition probability matrix constant, we measure the impact of the interest-rate differential on the stock market by extending the MS GJR-M model to the EMS GJR-M model. This model is formulated by adding the interest-rate changes to the variance process:

$$\begin{aligned} r_t &= \beta_i\sqrt{h_{i,t}} + \epsilon_{i,t}, & \epsilon_{i,t} &= \sqrt{h_{i,t}}z_t, & z_t &\sim N(0,1), \\ h_{i,t} &= \alpha_{i0} + \alpha_{i1}h_{t-1} + \alpha_{i2}\epsilon_{t-1}^2 + \alpha_{i3}d_i\epsilon_{t-1}^2 + \alpha_{i4}\chi_{t-1}^2 + \alpha_{i5}f_i\chi_{t-1}^2, \end{aligned} \quad (4.4)$$

where parameters in the variance process satisfy conditions similar to those in the MS GJR-M model. The interest-rate differential,  $\chi_t = (I_t/I_{t-1})$ , captures changes in short-term interest rates, where  $I_t$  is the interest-rate level at time  $t$ . The indicator for positive changes, increases, in interest rates satisfies

$$f_i = \begin{cases} 1 & \text{if } \chi_{t-1} > 0, \\ 0 & \text{if } \chi_{t-1} \leq 0. \end{cases}$$

For this model to be well defined, we use the squared first difference of interest rates to examine their impact on the conditional variance. As we are estimating the conditional variance, which is the squared conditional volatility in the GJR model, we use the squared differences of interest rates in order to keep the interest-rate differentials and the estimated volatility at the same scale. Meanwhile the first difference square is a commonly used proxy for the short-term interest-rate variance. By setting the first-difference squares to the conditional variance of the equity return, we can investigate the spillover effect of the short-term interest-rate market on the EMU equity market. Further, in this specification, we can examine different asymmetrical effects on the volatility of stock returns. Besides the asymmetric effects from market news, we can also examine if an increase in interest rates asymmetrically affects the stock market in the bear and bull markets. Hence, a negative  $\chi_{t-1}$  (drops in interest rates) contributes  $\alpha_{i4}\chi_{t-1}^2$  to  $\sigma_i$ , whereas a positive  $\chi_{t-1}$  (increases in interest rates) has a impact  $(\alpha_{i4} + \alpha_{i5})\chi_{t-1}^2$  if  $\alpha_{i5}$  is significantly different from zero. The coefficients  $\alpha_{14}$  and  $\alpha_{15}$  measure the effect of movements in the interest rate on the conditional variance in the bear market, while  $\alpha_{24}$  and  $\alpha_{25}$  measure the impact of interest-rate fluctuations on volatility in the bull market.

One alternative study of interest rate's impact can be done by allowing the trans-

mission matrix to be time varying. However, it is still an open question whether the specification of a time-varying transition probability is suitable for all financial data. Some studies report that the regime-switching model with the time-varying transition probability performs worse compared with the regime-switching model with a fixed transition probability.<sup>4</sup> Therefore, we choose to analyze the MS GJR-M and EMS GJR-M models with a fixed transition probability.

### 4.3 Model Estimation

In this section, we describe the estimation algorithm for a MCMC method. This estimation algorithm will be tested with a Monte Carlo simulation.

#### 4.3.1 Markov chain Monte Carlo estimation method

The evaluations of the likelihood function of Models 4.2 and 4.4 are complicated as they are a mixture over all possible state configurations. This may lead to computational difficulties with the maximum likelihood estimation. We estimate the model with a Bayesian-based MCMC method. A Bayesian statistical model consists of a parametric statistical model,  $f(x|\theta)$ , and a prior distribution on the parameters,  $p(\theta)$ . The optimal Bayes estimator under quadratic loss is simply the posterior mean:  $\hat{\theta} = E[\theta|Y = y] = \int \theta p(\theta|y) d\theta$ . Therefore, we need to compute the posterior density of our model parameters. The posterior density is determined by the prior density and the likelihood.

$$\begin{aligned} p(\theta|y) &= \frac{f(y, \theta)}{f(y)} \\ &= \frac{f(y|\theta)p(\theta)}{\int f(y|\theta)p(\theta)d\theta}. \end{aligned}$$

That is,

$$p(\theta|y) \propto f(y|\theta)p(\theta), \quad (4.5)$$

where  $f(y|\theta)$  in equation 4.5 is the likelihood function and  $p(\theta)$  is the prior distribution. The parameter vector of the model MS GJR-M (1,1), for  $i = 1, 2$ , specified in 4.2 is given by,

$$\begin{aligned} \Theta_i &= \{\beta_i, \theta_i, \pi_i, \mathbf{S}\}, \\ \theta_i &= (\alpha_{i0}, \alpha_{i1}, \alpha_{i2}, \alpha_{i3}), \\ \pi_i &= (\pi_{i1}, \pi_{i2}), \\ \mathbf{S} &= (S_1, S_2, \dots, S_T). \end{aligned} \quad (4.6)$$

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<sup>4</sup>For example, Perez-Quiros and Timmermann (2001) demonstrate that the regime-switching model with a time-varying transition probability is not applicable for large firms. Chang (2009) finds that the regime-switching model with the time-varying transition probability performs worse in out-of-sample forecasting than the model with fixed transition probability.

To obtain the Bayesian estimators,  $\hat{\Theta}$ , we compute the mean from the sample of the stationary distribution of the simulated  $\Theta_i$ . We need the following conditional posterior distributions:  $f(\beta|\mathbf{R}, \mathbf{S}, \mathbf{H}, \theta_1, \theta_2)$ ,  $f(\theta_i|\mathbf{R}, \mathbf{S}, \mathbf{H}, \theta_{j \neq i})$ ,  $p(\mathbf{S}|\mathbf{H}, \mathbf{R}, \theta_1, \theta_2)$ ,  $f(e_i|\mathbf{S})$ ,  $i = 1, 2$ , where  $\mathbf{R}$  is the observed returns and  $\mathbf{H}$  is the conditional volatility vector and can be computed recursively. Following Tsay (2005), we use conjugate prior distributions to draw  $\beta_i$  and  $e_i$  (see DeGroot, 1990, for a proof).

### Sampling $\beta_i$

Assume  $\beta_i \sim \mathbb{N}(\beta_{i0}, \sigma_{i0}^2)$ , the posterior distribution of  $\beta_i$  depends only on State  $i$ . Define,

$$r_{it} = \begin{cases} \frac{r_t}{\sqrt{(h_t)}} & \text{if } s_t = i; \\ 0 & \text{otherwise.} \end{cases}$$

Then we have,

$$r_{it} = \beta_i + \epsilon_i, \text{ for } s_t = i. \quad (4.7)$$

Let  $\bar{r}_i = (\sum_{s_t=i} r_{it})/n_i$  where  $n_i$  is the total number of data points in state  $i$ , and  $r_{it} \sim \mathbb{N}(\beta_i, \sigma^2)$ . Then the conditional posterior distribution of  $\beta_i$  is normal with mean  $\beta_i^*$  and variance  $\sigma_i^{2*}$ :

$$\beta_i^* = \frac{\sigma^2 \beta_{i0} + n_i \sigma_{i0}^2 \bar{r}_i}{\sigma^2 + n_i \sigma_{i0}^2} \quad \text{and} \quad \sigma_i^{2*} = \frac{\sigma^2 \sigma_{i0}^2}{\sigma^2 + n_i \sigma_{i0}^2}. \quad (4.8)$$

### Sampling $e_i$

The conditional posterior distribution of  $e_i$  only involves  $\mathbf{S}$ . Assume  $e_i \sim \text{Beta}(\varphi_{i1}, \varphi_{i2})$  and let  $\sum_{t=1}^n l_{1t}$  be the number of switches from State 1 to State 2,  $\sum_{t=1}^n l_{2t}$  be the number of switches from State 2 to State 1, and  $n_i$  be the number of the data observations in state  $i$ .  $l_{it}$  are Bernoulli distributed with parameter  $e_i$ ; then the posterior distribution of  $e_i$  is beta as,

$$e_i \sim \text{Beta} \left( \varphi_{i1} + \sum_{t=1}^n l_{it}, \varphi_{i2} + n_i - \sum_{t=1}^n l_{it} \right). \quad (4.9)$$

### Sampling $\alpha_{ij}$

We draw  $\alpha_{ij}$  with a modified Griddy Gibbs sampler. The Griddy Gibbs was first introduced by Tanner (1996). This method is widely applicable when the conditional posterior distribution is univariate. The main idea is to form a simple approximation to the inverse cumulative distribution function (CDF) based on the evaluation of the conditional posterior distribution on a grid of points. In our model, the conditional posterior distribution function of  $\alpha_{ij}$  does not correspond to a well-known distribution; however, as  $h_t$  contains

$\alpha_{ij}$ , it can be evaluated easily:

$$\begin{aligned} g(\alpha_{ij}|\cdot) &\propto \sum_{t=1}^n \left\{ -\frac{1}{2} \left[ \ln(h_t) + \frac{(r_t - \beta_i \sqrt{h_t})^2}{h_t} \right] \right\}, \quad \text{if } s_t = i, \\ f(\alpha_{ij}|\cdot) &\propto \exp(g(\alpha_{ij})). \end{aligned} \quad (4.10)$$

In order to avoid the problem of the fast convergence of the exponential distribution, we modify the Griddy Gibbs by adding a scale factor  $u = \max(g(\alpha_{ij}))$  to the evaluated function:

$$f(\alpha_{ij}|\cdot) \propto \exp(g(\alpha_{ij}) - u). \quad (4.11)$$

The Griddy Gibbs proceeds in the following steps:

1. Evaluate  $f(\alpha_{ij}|\cdot)$  at a grid of points from a properly selected interval of  $\alpha_{ij}$ —for example,  $0 \leq \alpha_{i1} < 1 - \alpha_{i2} - \alpha_{i3}$ —to obtain  $\omega_k = f(\alpha_{ij}^k|\cdot)$  for  $k = 1, \dots, m$ . We choose  $m = 200$ .
2. Use  $\{\omega_{k=1}^m\} = \omega_1, \omega_2, \dots, \omega_k$  to obtain an approximation to the inverse CDF of  $f(\alpha_{ij}|\cdot)$ , which is a discrete distribution for  $\{\alpha_{ij}^k\}_{k=1}^m$  with probability  $p(\alpha_{ij}) = \omega_k / \sum_{v=1}^m \omega_v$ .
3. Draw a uniform  $(0, 1)$  random number and transform the observation via the approximate inverse CDF to obtain a random draw for  $\alpha_{ij}$ .

## Sampling S

Following Henneke et al. (2006), we draw the states  $S_t$  by the “Single Move” procedure. At each step, we sample from the full conditional posterior density of  $S_t$  given by,

$$P(S_t = i | R, \boldsymbol{\theta}_{-s}, S_{-t}), \quad (4.12)$$

where  $\boldsymbol{\theta}_{-s}$  is the parameter vector in equation 4.6 excluding  $S$  and  $S_{-t}$  is the regime path excluding the regime at time  $t$ . In order to save space, we omit the notation of the explicit condition on  $\boldsymbol{\theta}$ . Applying the rules of conditional probability to 4.12, we get,

$$P(S_{t=i} | R, S_{-t}) = \frac{P(S_i, R | S_{-t})}{P(R | S_{-t})} = \frac{P(R | S_{t=i}, S_{-t}) \cdot P(S_{t=i} | S_{-t})}{P(R | S_{-t})}. \quad (4.13)$$

The first term in the numerator,  $P(R | S_{t=i}, S_{-t})$ , is simply the model’s likelihood  $L(S_t = i)$  evaluated at a given regime path, in which  $S_t = i$ , and

$$\begin{aligned} L(S_t = i) &= \prod_{t=j}^n f(\epsilon_t | H) \propto \exp(f_{ji}), \\ f_{ji} &= \sum_{t=1}^n \left\{ -\frac{1}{2} \left[ \ln(h_t) + \frac{(r_t - \beta_i \sqrt{h_t})^2}{h_t} \right] \right\} \quad \text{for } i = 1, 2 \text{ and } t \geq j. \end{aligned} \quad (4.14)$$

Given  $t \geq j$ , one can compute  $h_t$  recursively. The denominator,  $P(R|S_{-t})$ , is the sum of the two probability-weighted conditional distributions,

$$P(R|S_{-t}) = \sum_{i=1}^{s=2} P(R|S_t = i, S_{-t}) \cdot P(S_{t=i}|S_{-t}), \quad (4.15)$$

due to the Markov property of the chain.  $P(S_t = i|S_{-t})$  is only dependent on  $S_{t-1}$  and  $S_{t+1}$ ,

$$P(S_{t=i}|S_{-t}) = P(S_t = i|S_{t-1}, S_{t+1}) = \frac{\pi_{l,i} \cdot \pi_{i,k}}{\sum_{i=1}^{s=2} \pi_{l,i} \cdot \pi_{i,k}} \quad (4.16)$$

Let  $S_{t-1} = l$ ,  $S_{t+1} = k$  and  $\pi_{ij}$  be the respective transition probabilities from the transition probability matrix. Finally, substitute equation 4.15 and equation 4.16 into equation 4.13; we compute the conditional posterior probability as

$$P(S_{t=i}|R, S_{-t}) = \frac{L(S_t = i) \cdot \pi_{l,i} \pi_{i,k}}{\sum_{j=1}^{s=2} L(S_t = j) \cdot \pi_{l,j} \pi_{j,k}}. \quad (4.17)$$

The state  $S_t$  can be drawn using a uniform distribution in the interval  $[0, 1]$ .

### 4.3.2 Monte Carlo Simulation

In order to show that our algorithm works well, we perform a Monte Carlo simulation experiment. We simulate 10 data sets of 1,000 points from Model 4.2 with the same true parameter values for each data set and 5,000 iterations, of which the first 400 of the sample are discarded as burn in. In Table 4.1, we present the estimation results from the randomly chosen 1000 simulated data points. We find that the means of our estimated parameters are quite close to the true parameters and the square roots of the mean squared errors are quite small. Figure 4.1 shows the plots of the true and estimated volatility process, as well as the plot of the true and estimated probability of Regime 1. The estimated probability of Regime 1 is very close to the true probability. Therefore, we can be confident that our algorithm performs very well and is reliable.

## 4.4 Data and Empirical Results

In this section, we present the data used in this paper, perform the empirical study and report the results.

### 4.4.1 Data

The data used in this study consist of the weekly stock index closing price of ten countries that joined the EMU's third stage on January 1, 1999.<sup>5</sup> Specifically, they include Germany's DAX, France's CAC40, Italy's FTSEMIB, Spain's IBEX35, Finland's HEX25, the Netherlands' AEX, Ireland's ISEQ, Austria's ATX, Belgium's BEL20 and Portugal's PSI20. Furthermore, the one-month Euro Interbank Offered Rate (EURIBOR) is the benchmark money market rate for the Euro area. Interest rates with shorter maturities are neglected, and EURIBORs with maturities longer than one month may not be sensitive enough to represent short-term interest rates (see, e.g., Kleimeier and Sander, 2006 Bohl et al., 2008).

The sample period is from January 1, 1999, to March 12, 2010. That is, it begins when the European Central Bank (ECB) replaced the national central banks of EMU members and assumed responsibility for the conduct of unified monetary policy. The data is further divided into in-sample and out-of-sample periods. The in-sample period starts on January 1, 1999, and ends on July 17, 2009, and the out-of-sample period is from July 24, 2009, to March 12, 2010. The total sample size is 589. All the data are obtained from Thomson Financial Datastream.

We calculate the weekly returns as  $\log(y_t/y_{t-1})$  and then annualize them by multiplying by the square root of 52. Table 4.2 presents the statistical description of the EMU stock market indexes' returns. It can be seen from this table that the means of these returns are around zero. The standard deviations range from 0.1815% (Portugal) to 0.256% (Finland). The kurtosis statistics are far greater than the 3 associated with a normal distribution. The negative skewness coefficients are also significantly less than the value (zero) expected for a symmetric normal distribution. The  $p$  values of the Jarque-Bera test show that the null hypothesis of normality is clearly rejected for every series. However, the test statistics from the Augmented Dicky-Fuller test are much less than the critical value, therefore, the null hypothesis of a unit root is rejected at the 5% significance level for all the return series. The  $p$  values of the 10-lag Ljung-Box  $Q$ -test indicate that there are no serial correlations in the series.

### 4.4.2 Empirical results

#### Validation of model estimations

Before the analysis, we examine the validity of the MS GJR-M model in different ways. First, a 20-lag Ljung-Box  $Q$ -test is carried out to check the serial correlation in standardized residuals. The  $p$ -values of the tests presented in the last column in Table 4.3 suggest that the null hypothesis of no serial correlation cannot be rejected. Therefore, the MS GJR-M model fits the data properly.

We then benchmark the proposed MS GJR-M with a standard GARCH model, a GJR model and a single switching GJR-M model described in equation 4.2. We use  $\sqrt{(r_t)^2}$

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<sup>5</sup>The Luxembourg stock market is the only EMU market which is not considered in this study due to a lack of the stock index price data.

as a proxy for true volatility. The mean squared errors (MSE), the mean absolute errors (MAE) and Akaike's information criterion (AIC) are used as adequate model-selection criteria. The MSE, the MAE and the AIC are calculated according to the following formulas.

$$\begin{aligned} MSE &= \frac{1}{N} \sum_{t=1}^N (\hat{\sigma}_t - \sigma_t)^2, \\ MAE &= \frac{1}{N} \sum_{t=1}^N |\hat{\sigma}_t - \sigma_t|, \\ AIC &= 2K + N \left[ \log \left( \frac{2\pi \cdot RSS}{N} \right) + 1 \right], \end{aligned} \quad (4.18)$$

where  $\hat{\sigma}_t$  is the estimated volatility,  $\sigma_t$  is the proxy of true volatility,  $RSS = \sum_{t=1}^N (\hat{\sigma}_t - \sigma_t)^2$ , where  $N$  is the total sample size and  $K$  is the total number of parameters in the model.

The results of the model-selection criteria are shown in the first four columns of Table 4.4, where we present the goodness of fit from various models. The results of the MSE, MAE and AIC all indicate that the MS GJR-M model performs the best compared with the GARCH, GJR and single-regime GJR-M models. We notice that by allowing the conditional variance to enter into the conditional mean equation, the standard GJR-M model improves the conditional variance in most of the EMU countries. For example, in Germany, the MSE is reduced from 0.028 (GARCH model) and 0.027 (GJR model) to 0.025 by the GJR-M mode; the MAE declines from 0.125 to 0.117 from the GARCH model to the GJR-M model; and the AIC is also reduced by roughly 3.2% from the GARCH model to the GJR-M model. By allowing for a Markov-switching effect, the MS GJR-M model further significantly improves the estimated volatility. This is particularly true in the medium and large countries. For example in the Italian market (FTSEMIB), the MSE, MAE and AIC from the MS GJR-M model are 8%, 3% and 2% lower than the ones from the single-regime GJR-M model. This confirms that the GARCH in mean and the Markov switching are all necessary to characterize the return-variance dynamics. Hence, the MS GJR-M model provides a better characterization of the EMU stock returns and the volatility compared with other GARCH family models, e.g., a GJR model or a single regime GJR-M model.

### The time varying relationship between risk and return

Table 4.3 presents the estimated parameters of all indices from the MS GJR-M model described in equation (4.2). The first two columns of Table 4.3 are the estimated parameters  $\beta_1$  and  $\beta_2$ , which are the GARCH in mean coefficients in the conditional mean in Regimes 1 and 2, respectively. The  $\beta_1$  parameters are negative in all countries and the  $\beta_2$  parameters are positive in all countries. A negative/positive beta shows that the mean of returns has a negative/positive correlation with the conditional variance. It is



obvious that in Regime 1, returns are negatively correlated with the volatility, while in Regime 2, returns are positively correlated with the volatility. This means that in Regime 1, a higher risk usually leads to a higher loss in the investment, but in Regime 2, an increased volatility often leads to a higher profitability. Many empirical studies examine the relationship between the conditional mean and the conditional variance. However, the finding of the relationship between risk and return is still controversially.<sup>6</sup> We find a time-varying relationship between risk and return that is in line with such studies as Harvey (1989, 2001), Kandel and Stambaugh (1990) and Whitelaw (1994). In particular, Harvey (2001) argues that the specification of the conditional variance influences the relation between the conditional mean and the conditional variance and provides empirical evidence suggesting that there may be some time variation in the relationship between risk and return. Whitelaw (1994) reports also that the contemporaneous correlation between the first two movements of the return varies from large positive to large negative values. The negative relationship between the conditional mean and the conditional variance in the bear market is intuitive. In the bear market, investors are more risk averse. When investors are scared, they look for safety. They adjust their portfolios to include more safe assets and fewer risky assets. This kind of “flight to quality” leads investors to stay away from risky assets (stocks) which causes stock prices to decline (Barsky, 1989).

### **Bull and bear markets in the EMU stock markets**

By looking at  $\alpha_{10}$  and  $\alpha_{20}$  in Table 4.3, the intercept of the volatility equation in Regimes 1 and 2, respectively, we can see that the values of  $\alpha_{10}$  vary from 0.01 to 0.077, while the values of  $\alpha_{20}$  are all almost zero. This implies that the annualized volatility increases if the market switches from Regime 2 to Regime 1 and vice versa. These distinct characteristics of the two regimes are typical representations of the high-returns stable and the low-returns volatile states in stock returns, which are conventionally labeled bull markets and bear markets in Maheu and McCurdy (2000) and Perez-Quiros and Timmermann (2000). Obviously, the EMU stock markets have well-identified bear (Regime 1) and bull markets (Regime 2). This is similar to Chen’s (2007) finding in the S&P 500 index and Henry’s (2009) in the UK equity market.

The volatility persistence parameters,  $\alpha_{11}$  and  $\alpha_{21}$ , are quite significant in nearly all of the EMU stock markets. Interestingly, in most countries,  $\alpha_{21} > \alpha_{11}$ . This implies that the volatility is less persistent during the bear period. This result is similar to reports from Friedman and Laibson (1989) and Daal et al. (2007). Friedman and Laibson (1989) apply a modified ARCH and a GARCH model that allow for jumps and divide their sample into ordinary- and unusual-returns periods. They find that the volatility of ordinary returns displays persistence, but the volatility of the unusual price movements are less persistent. Daal et al. (2007) find the same pattern with a GARCH model allowing for jumps and asymmetry.

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<sup>6</sup>Some papers (e.g., French et al., 1987 Campbell and Hentschel, 1992 Li, 2003 Guo and Neely, 2006) report a positive relationship and others (e.g., Glosten et al., 1993 Pagan and Hong, 1991 Li et al., 2005 Guedhami and Sy, 2005) indicate a negative relationship, while others (e.g., Bodurtha and Mark, 1992 Baillie and DeGennaro, 1990 Shin, 2005) find no significant relationship at all.

Furthermore, we notice that coefficients  $\alpha_{12}$  are insignificant in all of the EMU countries, and  $\alpha_{22}$  are insignificant at the 5% significance level in the majority of the EMU countries. However, this does not mean that the one-week lagged error term has no effect on current volatility at all. On the contrary, it influences volatility through the channel of leverage effect: When bad news arrives (when the residual is negative), the market displays a remarkably different response to news. Parameters  $\alpha_{13}$  and  $\alpha_{23}$  show this additional sharp response of volatility to bad news in most of the EMU countries. This is generally consistent with the well-documented predicative asymmetrical effect in stock markets (see, e.g., Campbell and Hentschel, 1992 Engle and Ng, 1993). Further, in all EMU countries,  $\alpha_{13} > \alpha_{23}$ , implying that the asymmetry of the volatility response to bad news during volatile periods is greater than during stable periods. For example, the volatility asymmetry coefficient of DAX is 0.3858 in the bear market, which is about 2.1 times that of the bull market. This can be explained by noting that during the bear market, the confidence of investors is greatly damaged and market practitioners become more speculatively oriented and more sensitive to any market news, especially to bad news.

In Figure 4.2, we present the smoothed probability of all of the indices of Regime 1 (the bear period). The solid line is the probability of the bear regime, and the dot is the return. We can see that nearly all of the countries entered into the bear period during 2000 and 2001, during the half burst of the dot-com bubble. Among them, the central European countries most resisted the switch to the bear period, for example, Germany, the Netherlands and Belgium started their bear period in the beginning of 2001. The Irish stock market behaved remarkably differently and remained in the bull period until late 2001. This was due to its outstanding economic performance during that period. From year 1995 to 2000, Ireland's GDP growth was around 10%, while that of most other EMU countries were merely around 3%. A review from the IMF in August 2000 attributed such performance to the roles played by "sound and consistent macroeconomic policies, a generally flexible labor market, a favorable tax regime and the long standing outward orientation of Ireland's trade and industrial policies", and regarded the Irish economy as "well placed to continue to perform strongly in the future". The Irish stock market remained in the bull period until late in 2001, when its GDP growth rate dropped by half.

By the end of 2002 and the beginning of 2003, when key central banks desperately dropped their target rate to a historically low level with the ECB offering a deposit rate of merely 1.5%, most of the EMU stock markets started to see the light at the end of the tunnel and started to reenter the bull period, though the economy of most countries was still sluggish. The exception here is the Austria market. Being the gate from Western to Eastern Europe, Austria enjoyed strong growth in exports and inward investment from 2000 to 2005, which made it the first EMU country to leave the bear period as early as the beginning of 2002.

In most of the EMU countries, the bull period lasted for about 4 years, until the beginning of 2007, when the housing bubble burst and the subprime crisis sparked. The EMU countries then dove into the bear market at the same time again with the exception

of Finland, Germany and Portugal, which delayed a few months. The reason could be that at the beginning of the subprime crisis, the market underestimated its damage, believing that some European countries—which had better economic performance, better risk control and less speculation in the subprime mortgage market—could avoid the crisis. Germany was a typical example.

Finally, the difference between all parameters in both regimes and their respective standard deviations are shown in Table 4.5. Besides the parameters representing the response of the market to market news, the differences between parameters are all statistically significant at the 1% significance level. This confirms that the bear and the bull markets exist in the EMU stock markets. The estimated persistence for the regime  $i$  is  $1/\epsilon_i$  for  $i = 1, 2$ . Regime 1 has a averaged persistence of 22 weeks, while Regime 2 has a averaged persistence of 33 weeks. This is consistent with findings from Napolitano (2006) and Chen (2007) which report that both bull markets and bear markets display persistence but the bear market is less persistent.

### The impact of short-term interest rates on the EMU stock markets

We examine the impact of short-term interest rates by estimating the EMS GJR-M model as specified in equation (4.4). We are particularly interested in studying if an increase in interest rates has an additional effect on stock returns and their volatility and whether the effect varies in the bull and bear markets.

The full results of the interest-rate impact on the EMU stock markets are presented in Table 4.6. The estimated parameters from the EMS GJR-M model are not very different from the ones estimated from the MS GJR-M model, and the characteristics of both regimes are maintained. We find that the relationship between returns and the volatility remains largely unchanged in the EMU countries. The negative and significant parameter  $\beta_1$  in most of the EMU stock markets implies that returns are negatively correlated with volatility in Regime 1. The coefficient  $\beta_2$  is positive and significant in most of the EMU countries, implying a positive relationship between returns and volatility in Regime 2. The intercept of the volatility equation in Regimes 1 and 2 ( $\alpha_{10}$  and  $\alpha_{20}$ ) indicates that the volatility is higher in Regime 1 than in Regime 2. Therefore, the results provide strong evidence in favor of two states in the EMU stock markets, a high-mean low-volatility state (bull market) and a low-mean high-volatility state (bear market). The coefficients  $\alpha_{11}$  and  $\alpha_{21}$  indicate that the volatility is more persistent in the bull market than in the bear market. However, the innovation parameter in both regimes ( $\alpha_{12}$  and  $\alpha_{22}$ ) is insignificantly different from zero. This does not mean that market news has no effect on current volatility. If we look at the parameters  $\alpha_{13}$  and  $\alpha_{23}$ , we can find that market news influences the volatility through the leverage effect. The coefficient  $\alpha_{23}$  is significant in most of the EMU countries (besides Finland, Spain and Austria). The parameter  $\alpha_{13}$  is significant in half of the EMU countries. Moreover,  $\alpha_{13} > \alpha_{23}$  implies that the leverage effect of the bad news is much stronger in bear markets than in bull markets. For example, in Belgium, this additional effect is about 8 times larger in Regime 1 than in Regime 2.

Holding the transition probability constant, the interest-rate fluctuations affect the

equity returns via changes in the volatility. The parameters  $\alpha_{i4}$  and  $\alpha_{i5}$ , for  $i = 1, 2$ , indicate the interest rates' impact on the EMU stock market volatility in bull and bear markets, respectively. If the parameters  $\alpha_{i4}$  are significantly different from zero, then changes in EURIBOR rates affect the conditional variance. Meanwhile, if the parameters  $\alpha_{i5}$  are significantly different from zero, then an increase in interest rates causes an additional effect on the volatility by an amount of  $\alpha_{i5}\chi_{t-1}^2$ . It can be seen from Table 4.6 that the parameter  $\alpha_{24}$  is small in value and is insignificant at the 5% level in most of the EMU countries. The parameter  $\alpha_{14}$  is significant in most of the EMU countries (besides Belgium). This indicates that changes in interest rates have a much stronger effect on volatility in the bear market (the low-mean, high-volatility state) than in the bull market (the high-mean, low-variance state). We find also that  $\alpha_{15}$  is significant at the 5% level in all countries (in Germany, Italy, Spain and Netherlands, it is even significant at the 1% level) and that  $\alpha_{25}$  is only weakly significant in three countries (Finland, Belgium and Portugal). This indicates that an increase in interest rates has an additional effect on current volatility and this effect is also much stronger in the bear market than in the bull market in most of the EMU stock markets. This result is in contrast to the finding from Domian et al. (1996) that drops in interest rates are followed by large positive stock returns while increases in interest rates have little effects. Our finding is generally consistent with the results from Perez-Quiros and Timmermann (2000, 2001), Basistha and Kurov (2008), Chen (2007) and Henry (2009). For example, Perez-Quiros and Timmermann (2000) find that the interest rate can affect the conditional variance only in the low-mean high-volatility regime for large firms. Henry (2009) also reports that the relationship between short-term interest-rate changes and equity volatility in the UK stock market is regime dependent, the effect of interest rates is higher in bear markets than in bull markets. Basistha and Kurov (2008) show that the stock returns' response to monetary shocks is more than twice as large in recessions and tight credit conditions as in good economic times. The reason of this phenomenon may be that during the bull period the market confidence is high and more investors believe in the market itself rather than the information, especially the information from other markets. This makes the market reluctant to respond to changes in short-term interest rates. During the bear period, the market becomes nervous and more volatile, and the volatility becomes more sensitive to information from both the stock market and other markets, and therefore the stock market responds to changes in interest rates. Theoretically, according to recent models with agency costs of financial intermediation (finance constraint), people show that when there is information asymmetry in financial markets, agents may behave as if they are constrained financially. Moreover, the financial constraint is more likely to bind in bear markets (see, e.g., Gertler, 1988 Bernanke and Gertler, 1989 Kiyotaki and Moore, 1997 Garcia and Schaller, 2002). Therefore, a change in short-term interest rates may have greater effect in bear markets than in bull markets.

Further, in an influential study, Gerlach and Smets (1995) conclude that the effects of monetary policy shocks are somewhat larger in Germany than in France or Italy. Clements et al. (2001) have also argued that output in Germany and France is more affected by monetary shocks than in either Spain or Italy. Contrary to results from these

studies, the result from our study suggests that monetary policy is equally transmitted across the EMU stock markets. This may stem from the launching of Euro, which has made the EMU stock markets more integrated than ever.

Finally, we check the goodness of fit of the EMS GJR-M model. As can be seen in the last column of Table 4.4, the goodness-of-fit indicators (MSE, MAE and AIC) suggest that obtaining the interest-rate impact information improves the EMS GJR-M model performance and the fundamental results of the MS GJR-M model in most EMU stock markets.

### Asymmetric effects of bad news and rate increases: The news impact surface

In this section, we investigate the asymmetric news effects (returns residuals) and the asymmetric effect of changes in short-term interest rates on volatility by extending the NIC, introduced by Pagan and Schwert (1990) and christened by Engle and Ng (1993), which shows the implied relationship between the lagged shock from returns and the volatility. We extend the NIC into the news impact surface, in which the conditional variance is evaluated at the level of unconditional variance of stock returns, the shock from conditional returns, and the change in interest rates. The news impact surface of the EMS GJR-M model illustrates the asymmetric effect of stock market news and changes in interest rates on the volatility process:

$$\begin{aligned}
 h_t &= A + \alpha_{i2}\epsilon_{t-1}^2 + \alpha_{i4}\chi_{t-1}^2, & \text{for } \epsilon_{t-1} > 0 \text{ and } \chi_{t-1} < 0, \\
 h_t &= A + (\alpha_{i2} + \alpha_{i3})\epsilon_{t-1}^2 + (\alpha_{i4} + \alpha_{i5})\chi_{t-1}^2, & \text{for } \epsilon_{t-1} < 0 \text{ and } \chi_{t-1} > 0, \\
 h_t &= A + \alpha_{i2}\epsilon_{t-1}^2 + (\alpha_{i4} + \alpha_{i5})\chi_{t-1}^2, & \text{for } \epsilon_{t-1} > 0 \text{ and } \chi_{t-1} > 0, \\
 h_t &= A + (\alpha_{i2} + \alpha_{i3})\epsilon_{t-1}^2 + \alpha_{i4}\chi_{t-1}^2, & \text{for } \epsilon_{t-1} < 0 \text{ and } \chi_{t-1} < 0. \quad (4.19)
 \end{aligned}$$

where  $A = \alpha_{i0} + \alpha_{i1}\sigma^2$ ,  $\sigma^2$  is the unconditional return variance,  $\alpha_{ij}$  ( $i = 1, 2, j = 1, 2, \dots, 5$ ) is the parameter from the estimated EMS GJR-M model,  $\epsilon_{t-1}$  is the unpredictable return at time  $t - 1$ , and  $\chi_{t-1}$  is the change in interest rates. The original NIC of the GJR model from Engle and Ng (1993) does not demonstrate shocks from interest rates and does not distinguish shocks in the bull and the bear markets.

Figure 4.3 plots the news impact surface of the German stock market. Values on the  $X$  axis indicate changes in interest rates, values on the  $Y$  axis indicate shocks from conditional returns, and values on the  $Z$  axis indicate the level of the volatility. The left plot is the news impact surface of German stock market in the bear market, and the right one plots the news impact surface of German stock market in the bull market. If we hold the value on the  $X$  axis constant, then the change in values on the  $Z$  axis with respect to the change in values on the  $Y$  axis shows how the conditional volatility changes with respect to changes in market news. We find that the volatility increases as the value on the  $Y$  axis becomes more negative, and this is more obvious in the left plot than in the right one. This is consistent with our result in the previous section that negative news has an asymmetric effect on volatility in both bear markets and bull markets; however, this effect is greater in bear markets than in bull markets in the German stock market.

If we hold values on the  $Y$  axis constant, then the change in values on the  $Z$  axis with respect to changes in values on the  $X$  axis shows how the conditional volatility changes with respect to interest rates. We find that volatility increases as the value on the  $X$  axis becomes more positive, and this situation is only evident in the left plot (the bear market). This is consistent with our result that a rise in interest rates increases the volatility more than a fall in interest rates. The effect is much stronger in bear markets than in bull markets in the German stock market.

We show the asymmetric effect of shocks from unexpected returns and from changes in interest rates on the volatility of all EMU stock markets in Figure 4.4, where we contour plot the news impact surface of each EMU stock market. Values on the  $X$  axis indicate changes in interest rates and values on the  $Y$  axis indicate shocks from conditional returns. The color indicates the level of the volatility, the higher the volatility, the brighter its color. The first and third columns plot the NIC contours in the bear market, while the second and fourth columns are contour plots of the news impact surface in the bull market. By looking at the  $Y$  axis in the bear market in each EMU country, we find that the slope of the negative side (the left bottom corner) is much sharper and the color is much brighter than that of the positive side (the left top corner). However, in the bull market, the slope of the negative side of the  $Y$  axis (the left bottom corner) is only slightly sharper than that of the positive side (the left top corner). This is consistent with our result that the effect of bad news on the volatility is larger than that of good news in most of the EMU stock markets, and such an impact is also larger in the bear market than in the bull market. On the other hand, by looking at the  $X$  axis in the bear market in each EMU stock market, we can see that the news impact surface captures the asymmetrical effect of changes in interest rates on the volatility because it has a steeper slope and brighter color at the positive side (the right bottom corner where the interest rate moves upward) than the negative side (the left bottom corner where the interest rate follows downward market movements). However, we can only observe this situation in the bear market because in the bull market, the volatility is symmetrically centered at zero on the  $X$  axis in nearly all of the EMU stock markets (except Portugal). This also confirms our result that an increase in short-term interest rates has a considerably larger impact on stock volatility than a decrease in short-term interest rates, and the impact is much stronger during bear periods than during bull periods in most of the EMU markets.

### **Implications of interest-rate impacts on stock markets**

To explain why the interest rate can affect the equity market, we resort to the discounted cash flow (DCF) model pioneered by Williams (1938). The DCF model views the intrinsic value of common stock as the present value of its expected future cash flow. The expected future cash flow is often represented by the “expected dividend”, which is known as a DDM model (dividend distribution model). When interest rates change, first, the expected return must be discounted at a different rate; second, the firms’ future costs to conduct business are changed. These will ultimately affect the firms’ expected profitability and adjust market expectations of the firms’ abilities to pay a dividend. Furthermore, by changing the value of expected future cash flows, interest-rate movements change the

level of real activity in the economy in the medium and long term. Campbell and Ammer (1993) decompose the variance of unexpected excess returns implied by the DDM into three factors, news about future dividends, news about future interest rates, and news about future excess returns, and predict that fluctuations in interest rates should cause equity prices to move and may also result in changes in the variance of equity returns. However, the result from Henry (2009) suggests that events in the money market have no direct influence on the conditional mean of returns in the UK stock market. Our results suggest that the interest-rate market's influence on the conditional mean of stock returns is via the conditional variance because the conditional return and the volatility are negatively related in the bear market and positively related in the bull market. Therefore, the findings of interest rates' impacts from the proposed EMS GJR-M model in our paper support the conclusion that interest rates significantly affect stock returns and volatility and confirm the implications of the DCF model.

The empirical results from our paper have important implications for portfolio selection, asset pricing and risk management. For instance, as implied by the news impact surface, there are significant asymmetric effects of the news and changes in interest rates on the EMU stock market, after a major impact from the money market, the predictable market volatilities given by the EMS GJR-M model and other models such as a standard GJR model or a GJR-M model are very different, this may lead to a significant difference in current option price, portfolio selection, and dynamic hedging strategies.

To further demonstrate the importance of the interest rates' impact when modeling the volatility dynamics, we apply various models to a portfolio choice problem under two scenarios: portfolio choices without and with short-selling constraints.<sup>7</sup> We assume that an investor holds a portfolio consisting of two stocks of German DAX and France CAC40 (risky assets) and that the investor tries to maximize the expected utility function within the mean-variance framework from Best and Grauer (1990),

$$\max \left\{ \lambda \mathbf{w}'\boldsymbol{\mu} - \frac{1}{2} \mathbf{w}'V\mathbf{w} \mid \mathbf{w}'I = 1 \right\}, \quad (4.20)$$

where  $\mathbf{w}$  is the vector of weights invested in risky assets,  $V$  is the variance-covariance matrix of the asset returns,  $\boldsymbol{\mu}$  is the vector of the asset returns, and  $\lambda$  is the risk tolerance coefficient. The purpose is to find the optimum weights of the assets in the portfolio that maximize the utility function. It has been confirmed that investment weights are very sensitive to the first two conditional movements of the risky-asset returns (see, e.g., Best and Grauer, 1990, 1991 Fleming et al., 2001). So the model that can better forecast the conditional mean and variance can provide better performance. Further, as the risk-tolerance coefficient also affects the weight of risky assets, we examine the portfolio performance with different risk-tolerance coefficients. The robustness of the empirical findings in the investment performance can be confirmed if similar results can be obtained under different risk tolerance coefficients. Finally, we compute the optimum weights based on the out-of-sample forecasted conditional mean and variance of the German DAX and

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<sup>7</sup>In the case that the short-selling strategy is not allowed, the investment weight is between 0 and 1

the French CAC40. The average returns of the portfolio and the Sharpe Ratio will be also calculated according to different risk-tolerance coefficients and are used to measure the forecasted portfolio performance.

Table 4.7 presents results of the portfolio performance. Panel 1 shows those from the unrestricted strategy, and Panel 2 shows those from the restricted strategy. Clearly, among the models, the EMS GJR-M model provides the best investment performance in terms of the averaged returns and sharp ratios. This is not surprising because the EMS GJR-M model yields a more accurate volatility forecast than other models in the out-of-sample forecast. This is clear in Figure 4.5, which plots the true volatility proxy and the out-of-sample forecasted volatility of various models in the German DAX and the French CAC40. The solid lines are the estimated volatility from various models, and the dashed lines are the true volatility which is proxied by the absolute values of the returns. We can observe that the volatility estimated from the EMS GJR-M model is closer to the true volatility proxy and can better describe the dynamics of the DAX and the CAC40 return variance compared with the MS GJR-M, the GJR-M and the GJR models.

On the other hand, it is worth noting that the sharp ratio of the non-regime-switching models declines considerably compared to the regime-switching models. Among the non-regime-switching models, the GJR-M model does not perform better than the GJR model in the unrestricted scenario. This may be because of the potential statistical problem with the GARCH-M specification. As pointed out by Christensen et al. (2010) that without the regime switching, the long memory property of the conditional variance may not balance well when entering the short memory property of the conditional mean regression. As shown in many studies (see Diebold and Inoue, 2001), the long memory (high persistence) will disappear after incorporating the structure break in the volatility, e.g., a regime switching specification. These results provide credible evidences that the short-term interest-rate effect, the regime switching play important roles in modeling the dynamics of the EMU stock markets' returns and variance. Only models incorporating these effects can offer more accurate results of the conditional mean and variance. We can observe that the portfolio volatility of the GJR-M and the GJR models are much lower due to ignoring short-term interest rates and regime switching, and consequently result in poor out-of-sample predictive portfolio performance. The poorly forecasted portfolio performance from such models will definitely affect the investor's portfolio choice and risk-management strategy.

## 4.5 Conclusion

The DCF model provides the theoretical background for the possible impact of interest-rate changes on equity prices. With the increased use of short-term interest rates rather than measures of money supply as intermediate targets for monetary policy, many studies have examined the impact of the interest-rate market on the stock market. Unfortunately, most of the studies examine interest rates' impact on the U.S. stock market and heavily consider the effect of changes in interest rates on stock prices and returns. This paper



investigates the spillover effect of interest-rate movements on stock markets in the Euro area, which has received surprisingly little attention. Departing from most previous works examining the effect of interest rates only on stock returns, we analyze the potential impact of short-term interest-rate surprises on both stock returns and the volatility of stock returns. We pay particularly more attention to the asymmetric effect of an increase in interest rates on the EMU stock markets in different market regimes, bull and bear markets. The empirical study is carried out by estimating the EMS GJR-M and MS GJR-M models with a MCMC method, which enjoys several advantages compared with the traditional maximum likelihood method.

Empirical results suggest that two significant regimes exist in the EMU stock markets, a high-mean low-variance regime (bull market) and a low-mean high-variance regime (bear market). The relationship between the conditional mean and variance is time varying. They are positively correlated during bull periods and negatively correlated during bear periods. Furthermore, the negative shock (bad news) from the stock market has a larger effect than the positive shock (good news). Short-term interest rates affect the stock returns and volatility in the EMU countries; this effect is considerably stronger in the bear market than in the bull market in most of the EMU countries, and an increase in interest rates has a larger effect on the EMU stock returns and volatility than a similar drop. It is also confirmed in the out-of-sample forecasted portfolio performance that the EMS GJR-M model can better describe volatility dynamics and provide more powerful portfolio performance prediction than the models without interest rates' impact and regime switching. Our results are of importance not only to the policymaker anticipating the market response to announced and implemented policies, but also to financial-market participants making effective investment decisions and formulating appropriate risk-management strategies.

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# Tables

**Table 4.1:** Estimated Parameters from the Monte Carlo Simulation

	$\beta_1$	$\beta_2$	$\alpha_{10}$	$\alpha_{20}$	$\alpha_{11}$	$\alpha_{21}$	$\alpha_{12}$	$\alpha_{22}$	$\alpha_{13}$	$\alpha_{23}$	$e_1$	$e_2$
True	0.2000	0.6000	0.1000	0.2000	0.4000	0.5000	0.2000	0.2500	0.1000	0.1500	0.0200	0.0100
Mean	0.2106	0.5678	0.1103	0.1859	0.4104	0.5139	0.2124	0.2445	0.0941	0.1531	0.0149	0.0119
RMSE	0.0169	0.0173	0.0132	0.0122	0.0130	0.0302	0.0338	0.0115	0.0092	0.0030	0.0038	0.0047

*Notes:* The RMSE is the square root of the mean squared errors between the true and estimated parameters from all data sets.



**Table 4.2:** Descriptive Statistics for Weekly Returns in the EMU Stock Markets from January 1, 1999, to July 17, 2009, (Weekly Observations)

	Mean	STD	Skewness	Kurtosis	ADF test	JB test
DAX	-0.0044	0.2522	-0.6426	8.0147	-24.0798	0.0010
CAC40	-0.0035	0.2131	-0.8192	8.0416	-24.2817	0.0010
FTSEMIB	-0.0085	0.2310	-1.1698	12.9241	-23.5707	0.0010
IBEX35	-0.0003	0.2177	-1.1080	10.6583	-25.5623	0.0010
HEX25	0.0005	0.2556	-0.7829	6.3504	-23.2265	0.0010
ISEQ	-0.0086	0.2417	-1.9024	18.4048	-24.6172	0.0010
AEX	-0.0093	0.2432	-1.2709	12.6130	-23.4344	0.0010
ATX	0.0079	0.2394	-2.2525	24.8953	-23.5760	0.0010
BEL20	-0.0067	0.2163	-1.4305	13.9584	-23.0917	0.0010
PSI20	-0.0063	0.1815	-1.3557	12.2722	-22.6216	0.0010

*Notes:* This table reports summary statistics for the index return of the EMU countries. The ADF test is the augmented Dicky Fuller test and the test statistics are reported. The JB test is the normality Jarque-Bera test and the  $p$ -values are reported. Weekly returns are calculated as the first difference of the natural logarithm of prices and then annualized with a square root of 52.

**Table 4.3:** Estimated Parameters from the MS GJR-M Model

Index	Volatility equation											LBTest Q(20)	
	Return equation		Intercept		Persistence		Response to news		Additional response to bad news		Transition probability		
DAX	$\beta_1$ -0.2746 (0.1289) **	$\beta_2$ 0.0461 (0.0226) **	$\alpha_{10}$ 0.0769 (0.0153) ***	$\alpha_{20}$ 0.0042 (0.0018) **	$\alpha_{11}$ 0.2630 (0.1573) *	$\alpha_{21}$ 0.7150 (0.0645) ***	$\alpha_{12}$ 0.1260 (0.1287) *	$\alpha_{22}$ 0.0686 (0.0384) *	$\alpha_{13}$ 0.3858 (0.0582) ***	$\alpha_{23}$ 0.1829 (0.0652) ***	$e_1$ 0.0358 (0.0094) ***	$e_2$ 0.0106 (0.0053) **	0.5613
CAC40	-0.0789 (0.0533) **	0.2632 (0.0834) **	0.0121 (0.0010) ***	0.0006 (0.0004) **	0.5676 (0.0364) **	0.7838 (0.0641) ***	0.0326 (0.0317) **	0.0832 (0.0569) **	0.3409 (0.0581) **	0.0917 (0.0571) **	0.0122 (0.0056) **	0.0199 (0.0062) **	0.9045
FTSEMIB	-0.1396 (0.0577) **	0.1193 (0.0616) **	0.0236 (0.0047) ***	0.0006 (0.0002) **	0.1842 (0.0961) **	0.7984 (0.0390) ***	0.1064 (0.0804) **	0.1221 (0.0369) **	0.6018 (0.1313) **	0.0612 (0.0326) **	0.0132 (0.0070) **	0.0110 (0.0063) **	0.6084
IBEX35	-0.1735 (0.0687) **	0.2439 (0.0781) **	0.0133 (0.0035) ***	0.0005 (0.0002) **	0.7177 (0.0653) **	0.8429 (0.0538) ***	0.0265 (0.0238) **	0.0470 (0.0377) **	0.1413 (0.0613) **	0.0768 (0.0391) **	0.0432 (0.0199) **	0.0400 (0.0224) **	0.2582
HEX25	-0.2312 (0.0906) **	0.2089 (0.0672) **	0.0536 (0.0232) **	0.0017 (0.0009) **	0.3843 (0.1771) **	0.7839 (0.0488) **	0.0587 (0.0578) **	0.1437 (0.0399) **	0.3002 (0.1532) **	0.0265 (0.0240) **	0.0442 (0.0181) **	0.0238 (0.0085) **	0.3058
AEX	-0.1129 (0.0606) **	0.0648 (0.0644) **	0.0078 (0.0033) **	0.0028 (0.0017) **	0.7060 (0.0900) **	0.6847 (0.1056) **	0.0350 (0.0347) **	0.1042 (0.0834) **	0.2231 (0.0833) **	0.1938 (0.0931) **	0.0349 (0.0088) **	0.0344 (0.0089) **	0.7190
ISEQ	-0.2346 (0.0970) **	0.1328 (0.0575) **	0.0175 (0.0044) ***	0.0005 (0.0001) **	0.4750 (0.0639) **	0.8903 (0.0332) ***	0.0608 (0.0654) **	0.0521 (0.0321) **	0.3724 (0.1106) **	0.0365 (0.0233) **	0.0613 (0.0184) **	0.0353 (0.0149) **	0.7050
ATX	-0.1497 (0.0902) **	0.2552 (0.0767) **	0.0101 (0.0047) **	0.0017 (0.0008) **	0.5150 (0.1137) **	0.7361 (0.0710) **	0.0739 (0.0616) **	0.2151 (0.0838) **	0.3604 (0.1265) **	0.0272 (0.0301) **	0.0423 (0.0068) **	0.0368 (0.0079) **	0.8360
BEL20	-0.1777 (0.0652) **	0.2108 (0.0701) **	0.0151 (0.0070) **	0.0004 (0.0001) **	0.4611 (0.1559) **	0.8551 (0.0417) **	0.0421 (0.0385) **	0.0765 (0.0415) **	0.4434 (0.1335) **	0.0343 (0.0277) **	0.0345 (0.0086) **	0.0333 (0.0087) **	0.7282
PSI20	-0.3229 (0.0891) **	0.2492 (0.0749) **	0.0155 (0.0035) ***	0.0004 (0.0001) **	0.1858 (0.1326) **	0.8522 (0.0316) **	0.0929 (0.0853) **	0.1132 (0.0322) **	0.6177 (0.1516) **	0.0151 (0.0144) **	0.0372 (0.0085) **	0.0310 (0.0093) **	0.1463

Notes: This table shows the estimated parameters of the MS GJR-M model, without the interest-rate impact, and specified in equation (4.2). Values in parentheses under the estimates indicate standard errors. \*\*\*, \*\*, and \* denote significance at 1%, 5%, and 10% levels, respectively. The sample period is from January 1, 1999, to July 17, 2009, (557 weekly observations). Q(20) is the Ljung-Box test statistic of the standard residuals of order 20 (*p*-values are reported)

**Table 4.4:** The Goodness of Fit of Various Models

MSE					
Index	GARCH	GJR	GJR-M	MSGJR-M	MSGJR-M with interest impact
DAX	0.0279	0.0266	0.0247	0.0241	0.0237
CAC40	0.0240	0.02261	0.0226	0.0212	0.0207
FTSEMIB	0.0275	0.0261	0.0252	0.0232	0.0216
IBEX35	0.0228	0.0226	0.0216	0.0195	0.0191
HEX25	0.0310	0.0311	0.0307	0.0293	0.0277
AEX	0.0297	0.0320	0.0320	0.0263	0.0261
ISEQ	0.0319	0.0295	0.0274	0.0290	0.0295
ATX	0.0332	0.0300	0.0338	0.0315	0.0300
BEL20	0.0269	0.2671	0.0241	0.0224	0.0230
PSI20	0.0190	0.0187	0.0179	0.0160	0.0161

MAE					
Index	GARCH	GJR	GJR-M	MSGJR-M	MSGJR-M with interest impact
DAX	0.1248	0.1206	0.1171	0.1144	0.1157
CAC40	0.1177	0.1126	0.1128	0.1091	0.1077
FTSEMIB	0.1158	0.1134	0.1096	0.1060	0.1035
IBEX35	0.1126	0.1110	0.1097	0.1030	0.1032
HEX25	0.1328	0.1327	0.1298	0.1275	0.1268
AEX	0.1200	0.1229	0.1206	0.1101	0.1103
ISEQ	0.1240	0.1151	0.1112	0.1177	0.1184
ATX	0.1211	0.1194	0.1195	0.1143	0.1106
BEL20	0.1173	0.1141	0.1076	0.1060	0.1096
PSI20	0.0975	0.1016	0.0982	0.0933	0.0930

AIC					
Index	GARCH	GJR	GJR-M	MSGJR-M	MSGJR-M with interest impact
DAX	-1978.8	-2005.5	-2043.1	-2046.7	-2050.2
CAC40	-2062.5	-2095.1	-2092.5	-2119.0	-2124.8
FTSEMIB	-1986.7	-2016.03	-2033.0	-2068.5	-2101.5
IBEX35	-2091.0	-2095.1	-2119.5	-2166.3	-2171.5
HEX25	-1919.5	-1917.0	-1923.8	-1939.4	-1965.1
AEX	-1943.2	-1902.4	-1900.3	-1998.5	-2002.5
ISEQ	-1903.5	-1948.23	-1985.6	-1945.4	-1936.2
ATX	-1881.4	-1854.5	-1870.4	-1898.2	-1919.5
BEL20	-1997.7	-2002.49	-2056.8	-2088.1	-2067.7
PSI20	-2191.1	-2200.34	-2229.0	-2273.4	-2265.5

*Notes:* This table reports the three goodness-of-fit measures in terms of the MSE, the MAE, and the AIC for various models in the EMU countries. These measures are calculated according to equation (4.18). The models are the GARCH, GJR, GJR in Mean, MS GJR in Mean, and the MS GJR in Mean with the interest-rate impact.

**Table 4.5:** Parameter Differences Between Bull and Bear Markets

Index	Return	Intercept	Persistence	Response to news	Additional response to bad news
	$\beta$	$\alpha_0$	$\alpha_1$	$\alpha_2$	$\alpha_3$
DAX	-0.3208 (0.1374) **	0.0727 (0.0154) ***	-0.4520 (0.1700) ***	0.0575 (0.1343)	0.2029 (0.0874) **
CAC40	-0.3421 (0.0990) ***	0.0114 (0.0011) ***	-0.2162 (0.0737) ***	-0.0506 (0.0651)	0.2492 (0.0815) ***
FTSEMIB	-0.2588 (0.0844) ***	0.0230 (0.0047) ***	-0.6141 (0.1037) ***	-0.0157 (0.0885)	0.5405 (0.1353) ***
IBEX35	-0.4173 (0.1040) ***	0.0127 (0.0035) ***	-0.1253 (0.0846)	-0.0205 (0.0446)	0.0646 (0.0727)
HEX25	-0.4402 (0.1128) ***	0.0519 (0.0232) **	-0.3996 (0.1837) **	-0.0850 (0.0703)	0.2737 (0.1551) *
AEX	-0.1777 (0.0884) **	0.0050 (0.0037)	0.0213 (0.1387)	-0.0692 (0.0903)	0.0293 (0.1249)
ISEQ	-0.3675 (0.1127) ***	0.0170 (0.0044) ***	-0.4153 (0.0720) ***	0.0087 (0.0729)	0.3360 (0.1130) ***
ATX	0.4048 (0.1184) ***	0.0084 (0.0048) *	0.2210 (0.1340) *	0.1412 (0.1040)	0.3333 (0.1301) **
BEL20	0.3885 (0.0957) ***	0.0147 (0.0070) **	0.3940 (0.1614) **	0.0344 (0.0566)	0.4092 (0.1364) ***
PSI20	0.5721 (0.1164) ***	0.0151 (0.0035) ***	0.6664 (0.1364) ***	0.0203 (0.0912)	0.6026 (0.1523) ***

*Notes:* This table shows the parameter differences (the parameter value in bear markets minus the parameter value in bull markets). All parameters are estimated from the MS GJR-M model. Values in parentheses under the estimates indicate standard errors. \*\*\*, \*\*, and \* denote significance at the level of 1%, 5%, and 10%, respectively.

**Table 4.6:** Estimated Parameters from the MS GJR-M Model with the Interest Rate Impact

Index	Volatility equation															
	Return equation		Intercept		Persistence		Response to News		Additional response to bad news		Response to interest rates		Additional response to increased interest rates		Transition probability	
	$\beta_1$	$\beta_2$	$\alpha_{10}$	$\alpha_{20}$	$\alpha_{11}$	$\alpha_{21}$	$\alpha_{12}$	$\alpha_{22}$	$\alpha_{13}$	$\alpha_{23}$	$\alpha_{14}$	$\alpha_{24}$	$\alpha_{15}$	$\alpha_{25}$	$\epsilon_1$	$\epsilon_2$
DAX	-0.1573 (0.0793) **	0.0995 (0.0821)	0.0309 (0.0134) **	0.0041 (0.0021) *	0.3387 (0.1908) *	0.6449 (0.1076) ***	0.0710 (0.0757) *	0.0540 (0.0486)	0.4021 (0.1860) **	0.2360 (0.0839) ***	0.1747 (0.0804) **	0.0331 (0.0224)	0.2085 (0.0698) ***	0.0346 (0.0258)	0.0379 (0.0082) ***	0.0288 (0.0091) ***
CAC40	-0.2080 (0.0751) ***	0.2254 (0.0775) ***	0.0247 (0.0076) ***	0.0007 (0.0005) *	0.2288 (0.1658) ***	0.7398 (0.0625) ***	0.0425 (0.0413) *	0.0706 (0.0566)	0.4727 (0.1678) ***	0.1450 (0.0686) **	0.1470 (0.0792) *	0.0290 (0.0216)	0.1854 (0.0825) **	0.0388 (0.0241)	0.0424 (0.0063) ***	0.0389 (0.0078) ***
FTSEMIB	-0.1074 (0.0516) **	0.0746 (0.0489)	0.0246 (0.0088) ***	0.0005 (0.0004)	0.2463 (0.1591) ***	0.8343 (0.0524) ***	0.1145 (0.1115) *	0.0569 (0.0394)	0.4565 (0.1896) **	0.0835 (0.0519)	0.1361 (0.0485) ***	0.0220 (0.0129) *	0.1387 (0.0484) ***	0.0183 (0.0128)	0.0623 (0.0184) ***	0.0312 (0.0111) ***
IBEX35	-0.2165 (0.0855) **	0.1838 (0.0621) ***	0.0207 (0.0090) **	0.0008 (0.0007) **	0.4118 (0.1811) **	0.8059 (0.0690) ***	0.0623 (0.0592) *	0.0522 (0.0357)	0.2186 (0.1378) **	0.1125 (0.0704)	0.2082 (0.0684) ***	0.0174 (0.0146)	0.2297 (0.0605) ***	0.0279 (0.0224)	0.0419 (0.0064) ***	0.0317 (0.0089) **
HEX25	-0.2642 (0.0956) ***	0.1589 (0.0597) ***	0.0580 (0.0377) **	0.0026 (0.0014) *	0.4295 (0.2823) ***	0.7456 (0.0711) ***	0.0538 (0.0568) **	0.1071 (0.0496)	0.1472 (0.1388) **	0.0700 (0.0520)	0.1758 (0.0826) **	0.0367 (0.0250) **	0.1871 (0.0810) **	0.0501 (0.0280) *	0.0401 (0.0070) ***	0.0219 (0.0080) ***
AEX	-0.1149 (0.0613) *	0.1306 (0.0611) **	0.0154 (0.0079) *	0.0012 (0.0007) **	0.5605 (0.1393) ***	0.7656 (0.0619) ***	0.0791 (0.0748) **	0.0350 (0.0295)	0.2777 (0.1306) **	0.1556 (0.0624) **	0.1045 (0.0569) *	0.0221 (0.0125) *	0.1432 (0.0455) ***	0.0192 (0.0136)	0.0545 (0.0202) ***	0.0446 (0.0152) ***
ISEQ	-0.2543 (0.1039) **	0.1041 (0.0703) **	0.0298 (0.0148) **	0.0015 (0.0012) **	0.4203 (0.1584) ***	0.7666 (0.0733) ***	0.0794 (0.0740) **	0.0418 (0.0352)	0.3359 (0.1515) **	0.1496 (0.0651) **	0.1750 (0.0810) **	0.0236 (0.0155) **	0.2208 (0.0660) ***	0.0201 (0.0172)	0.0413 (0.0068) ***	0.0289 (0.0092) ***
ATX	-0.1899 (0.1031) *	0.2072 (0.0660) ***	0.0247 (0.0194) **	0.0061 (0.0030) **	0.3674 (0.1529) **	0.5517 (0.1569) ***	0.1792 (0.1377) **	0.0695 (0.0592)	0.2909 (0.1935) **	0.1832 (0.0954) *	0.1806 (0.0638) ***	0.0209 (0.0191)	0.1818 (0.0653) ***	0.0190 (0.0209)	0.0403 (0.0063) ***	0.0252 (0.0086) ***
BEL20	-0.1075 (0.0673)	0.1446 (0.0662) **	0.0209 (0.0091) **	0.0005 (0.0003) **	0.3488 (0.2004) *	0.8220 (0.0528) ***	0.0486 (0.0514) **	0.0651 (0.0418)	0.4825 (0.1870) ***	0.0571 (0.0427) ***	0.0810 (0.0574) **	0.0203 (0.0131)	0.1108 (0.0560) **	0.0235 (0.0138) *	0.0675 (0.0174) ***	0.0447 (0.0135) ***
PSI20	-0.3200 (0.0942) ***	0.2053 (0.0609) ***	0.0161 (0.0033) ***	0.0003 (0.0000) ***	0.1629 (0.1034) **	0.8978 (0.0287) ***	0.0765 (0.0672) **	0.0347 (0.0316)	0.5371 (0.1722) ***	0.0148 (0.0138) **	0.1712 (0.0863) **	0.0103 (0.0085) **	0.1915 (0.0763) **	0.0516 (0.0207) **	0.0332 (0.0094) ***	0.0239 (0.0085) ***

*Notes:* This table shows the estimated parameters from the MS GJR-M model with the short-term interest-rate impact specified in equation 4.4 in the EMU area. \*\*\*, \*\*, and \* denote significance at 1%, 5%, and 10% levels, respectively. Values in parentheses under the estimates indicate standard errors.

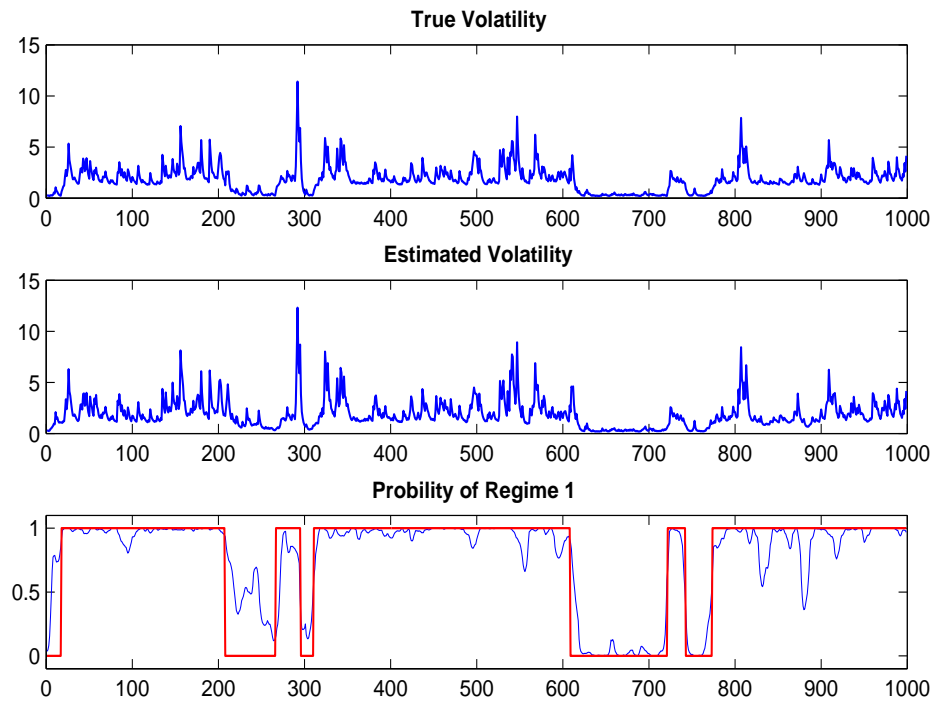
**Table 4.7:** Asset Allocation Performance Results

	$\lambda = 20$	$\lambda = 10$	$\lambda = 5$	$\lambda = 1$
Panel 1: Unrestricted strategy				
(A) Mean				
MS GJR-M with interest	0.7661	0.4036	0.2224	0.0774
MS GJR-M	0.0792	0.0602	0.0508	0.0432
GJR-M	-0.0086	0.0134	0.0244	0.0332
GJR	-0.0123	0.0091	0.0197	0.0283
(B) Volatility				
MS GJR-M with interest	4.9519	2.5311	1.3307	0.4113
MS GJR-M	5.2041	2.6536	1.3889	0.4184
GJR-M	0.5074	0.3210	0.2520	0.2258
GJR	0.2412	0.2272	0.2225	0.2204
(C) Sharp ratio				
MS GJR-M with interest	0.0391	0.0395	0.0397	0.0356
MS GJR-M	0.0118	0.0157	0.0210	0.0307
GJR-M	-0.0119	0.0012	0.0100	0.0181
GJR	-0.0011	0.0056	0.0106	0.0157
Panel 2: Restricted strategy				
(A) Mean				
MS GJR-M with interest	0.0562	0.0558	0.0549	0.0565
MS GJR-M	0.0419	0.0419	0.0419	0.0419
GJR-M	0.0229	0.0278	0.0341	0.0353
GJR	0.0194	0.0218	0.0234	0.0305
(B) Volatility				
MS GJR-M with interest	0.2401	0.2405	0.2410	0.2410
MS GJR-M	0.2289	0.2289	0.2289	0.2289
GJR-M	0.2152	0.2144	0.2151	0.2205
GJR	0.2509	0.2504	0.2486	0.2322
(C) Sharp ratio				
MS GJR-M with interest	0.0318	0.0315	0.0306	0.321
MS GJR-M	0.0261	0.261	0.0261	0.0261
GJR-M	0.0130	0.0156	0.0200	0.0205
GJR	0.0082	0.0105	0.0124	0.0169

*Notes:* This table shows the mean of the portfolio returns, the mean of the portfolio variances, and the mean of the Sharp Ratio of Portfolio over the out-of-sample forecast periods for various models and with respect to different risk tolerance coefficients. Panel 1 is the results for the unrestricted strategy, short selling is allowed. The panel 2 reports the asset allocation results for the restricted results, short selling is not allowed.  $\lambda$  is the risk tolerance coefficient.

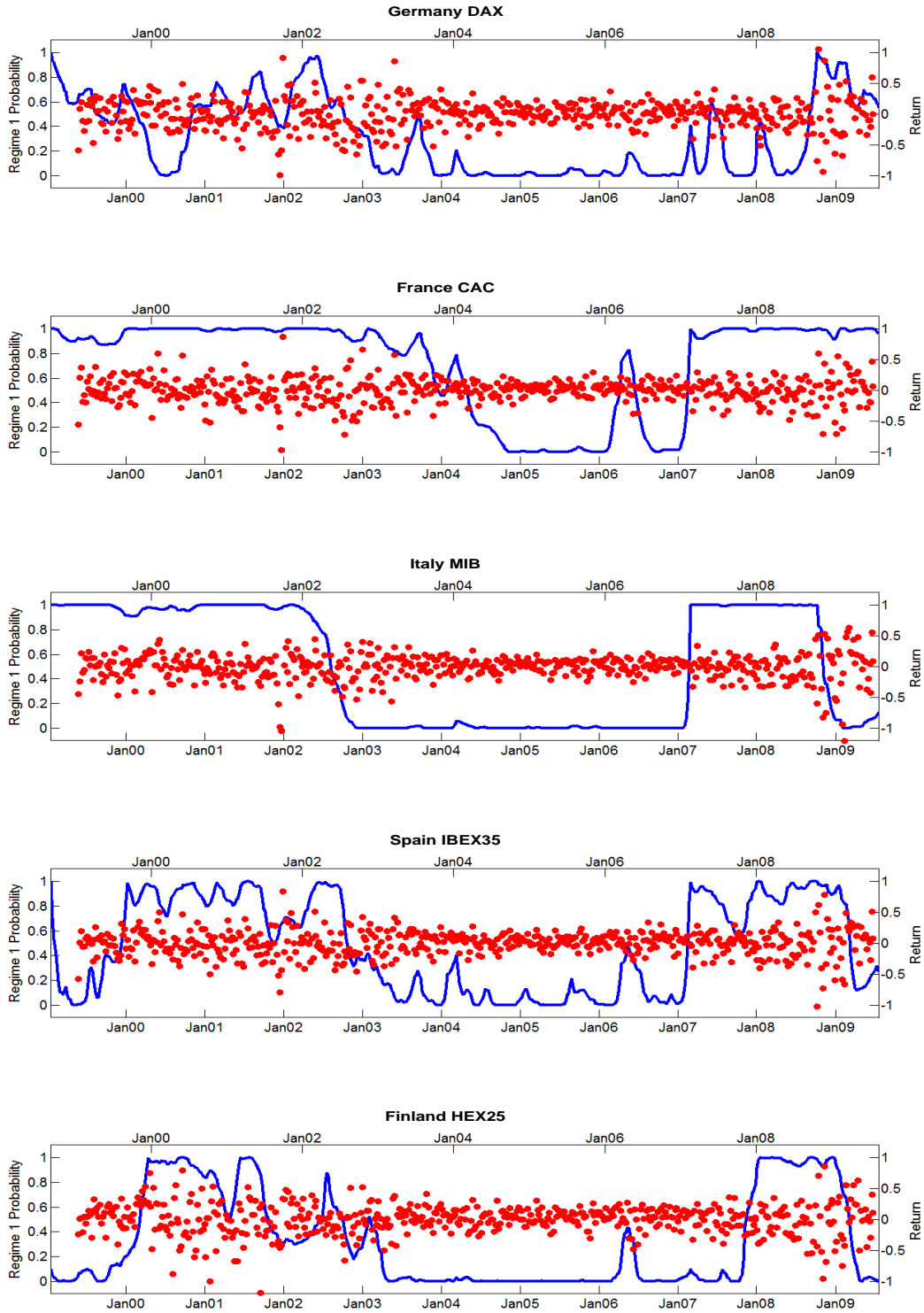
# Figures

**Figure 4.1:** Estimated results for simulated data

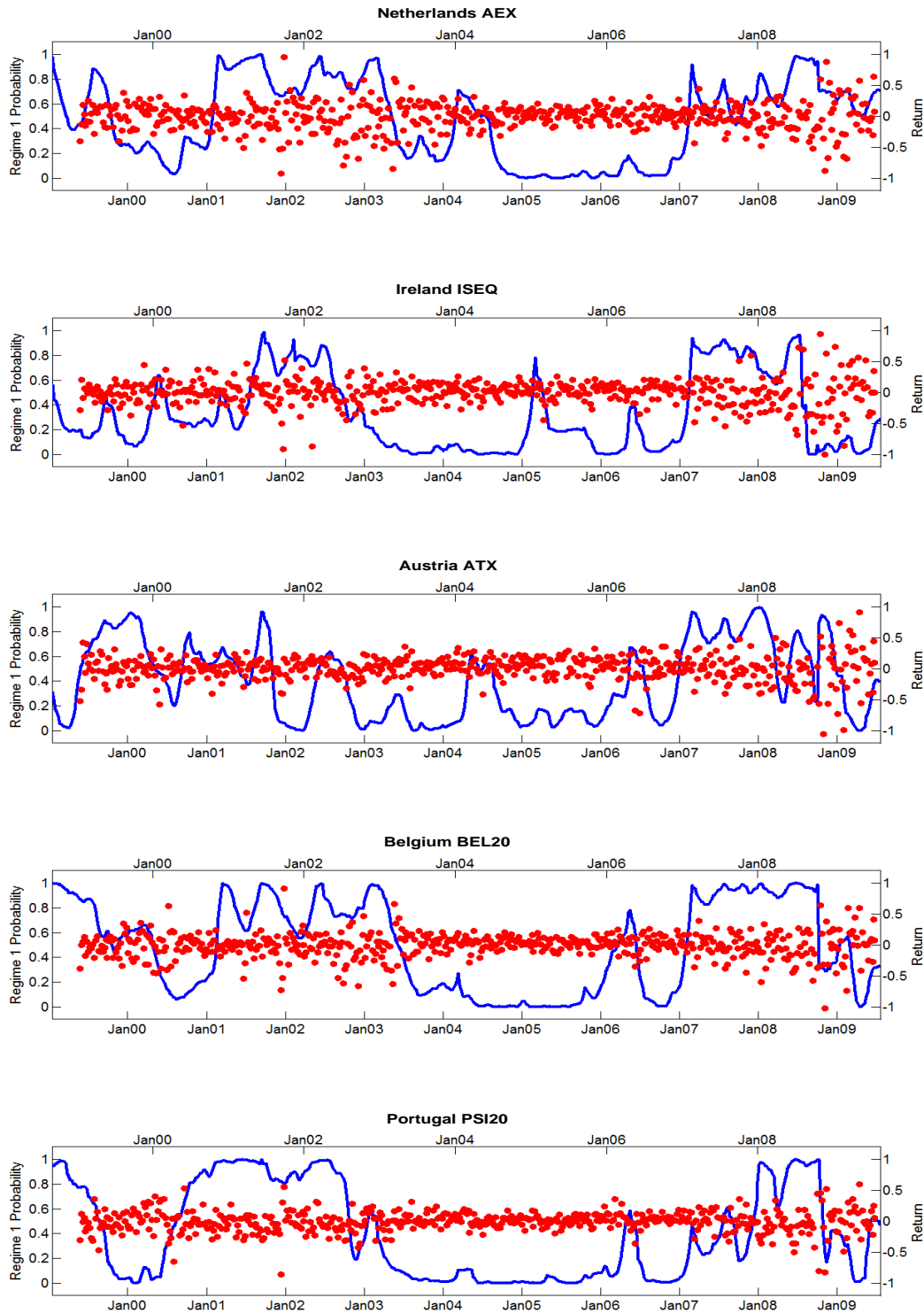


*Note:* This figure plots the estimation results of the randomly chosen simulated 1,000 data points. The first and second plots are the true and estimated volatility, and the last plot is the true and estimated probability of regime 1.

Figure 4.2: Bear regime probability

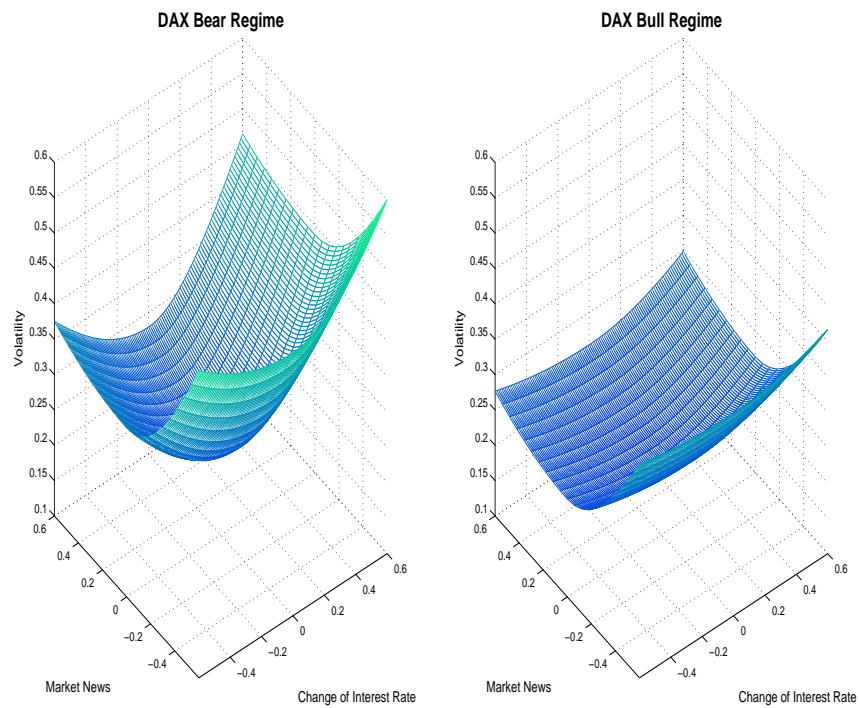






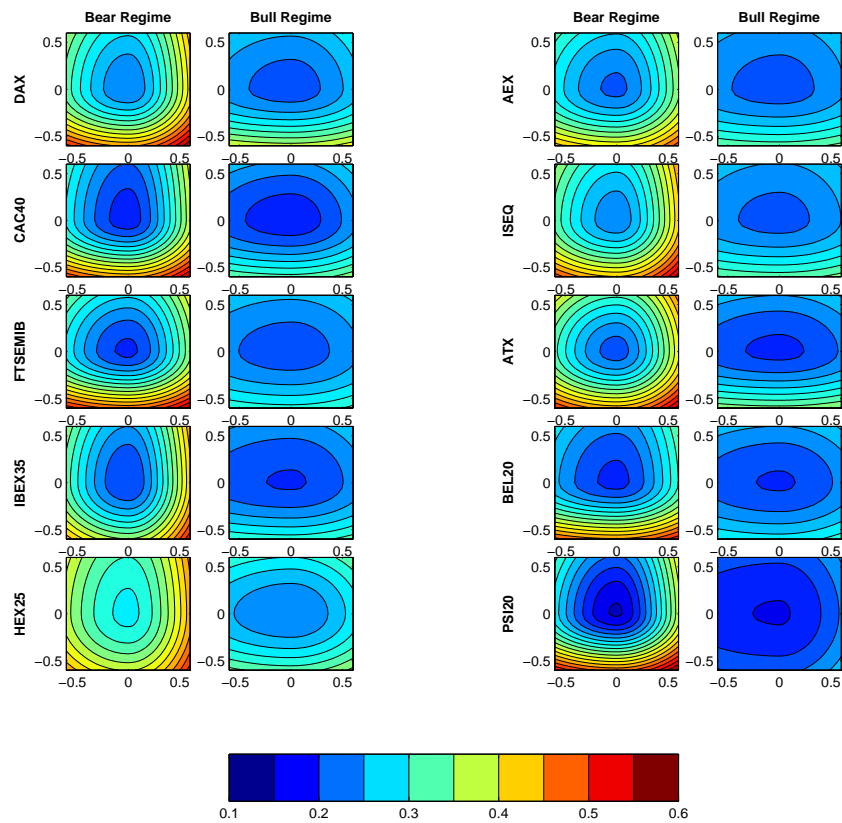
*Note:* This figure plots the estimated probability of the bear regime of the EMU equity markets. The solid line is the estimated probability, and the red dots are the returns. The scale of the return can be found on the y-axis on the right hand side.

Figure 4.3: DAX news impact surface—3D Plot



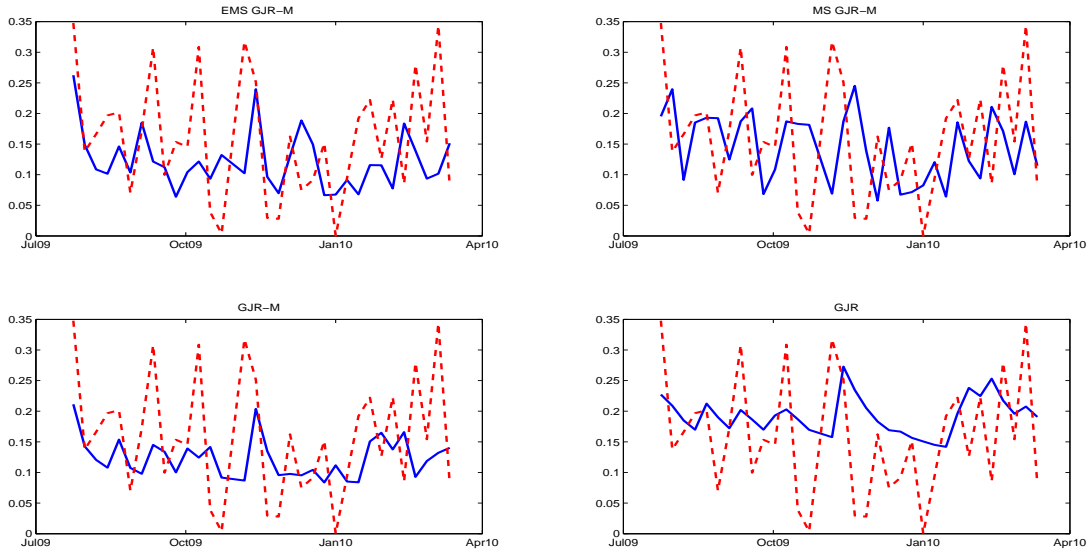
*Note:* This figure plots the news impact surface of the Germany’s DAX index. The  $X$  axis plots the values of the short-term interest rates. The  $Y$  axis plots the values of the unexpected market news.

Figure 4.4: News impact surface contour plots

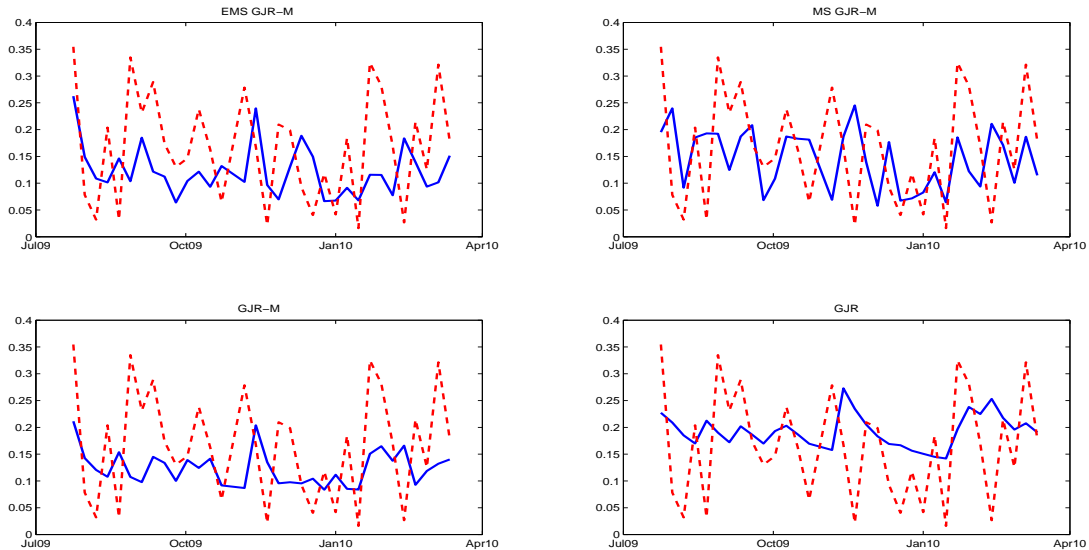


*Note:* This figure plots the news impact surface contour plots. The first and third columns are the contour plots of the news impact surface of each EMU equity market in the bear regime. The second and fourth columns are the contour plots in the bull regime. The X axis represents levels of interest rates, the Y axis represents market news. The color indicates the level of the volatility. The higher the volatility, the brighter its color.

Figure 4.5: Plots of the out-of-sample forecasted and true volatility



(a) Germany DAX



(b) France CAC40

*Note:* This figure plots the true and the out-of-sample forecasted volatility from the GJR, GJR-in-Mean, MS GJR-in-mean, and MS GJR-in-mean with the interest-rate impact models. The upper plot is the volatility for Germany's DAX index. The lower plot is the volatility of France's CAC40 Index. The solid line represents the volatility estimate and the dashed line is the true volatility, where the absolute value of the returns is the true volatility proxy.

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