New regions of stability

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3-B NEW REGIONS OF STABILITY

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§1 INTRODUCTION

The problem of making theoretical predictions about the location of a possible new region of stable elements in some way resembles that of predicting an island outside of a continent from whatever knowledge is available, geographical and geological, about the continent itself.

Several attempts have been made to predict such an island of stability, or maybe a tiny archipelago of islands,outside of the peninsula of stable elements (see Fig. 1). (For a recent review see G.T. Seaborg[1]).

The gross limits of extension of the peninsula of stability are determined by the terms of the liquid-drop model that exhibit a smooth variation in A and Z. The terms embodied in the liquid-drop model provide no prospects for any vast new continents outside of the presently known region.

This is so unless one imagines very radical variations in nuclear shape or the inclusion of additional terms in the liquid-drop formula. One of the former types of variations is, e.g., the change from the solid chunk of nuclear matter to a bubble shape, as considered by Swiatecki[2]: this shape, if ever formed, might be semistable. Furthermore, a piece of nuclear matter so large that the gravitational interaction is strong enough to overcome the probable minor negative binding of pure neutron matter is now thought to exist in the universe and constitute an end product in a super-nova explosion (pulsars).

A principle so far neglected in our discussion of the "geology" of the stability-peninsula is that of shell structure. Shell structure is responsible for mountain ridges on the element peninsula, whose geographical height at each point may be taken as a measure of the nuclear half-lives of the corresponding element. Shell structure is in the same fashion responsible for the existence of islands off the coastline of the peninsula. We shall here mostly limit our

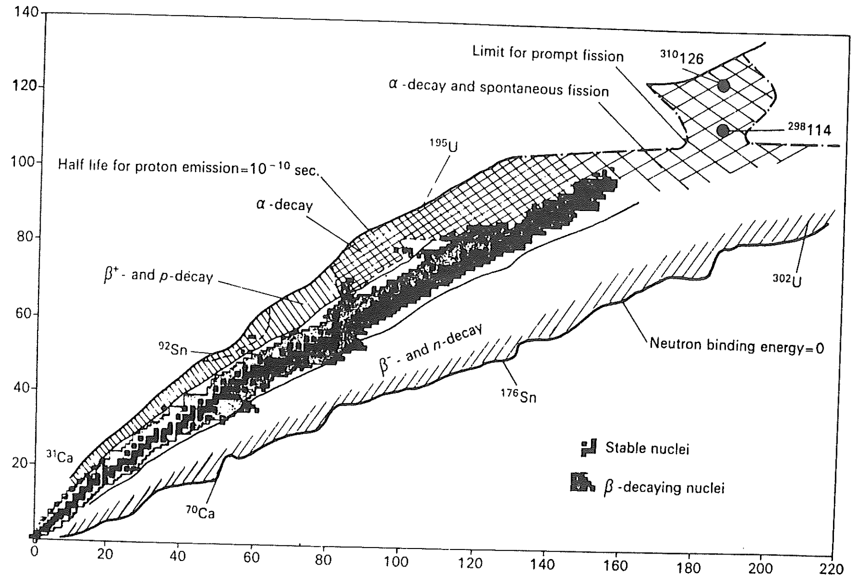

Fig. 1. The peninsula of nuclear stability (as drawn by Prof. G.N. Flerov). For further explanations, see text in figure.

humble considerations to such possible islands formed in connection with proton numbers Z = 114 (and possibly Z = 124 or 126) and neutron numbers N = 184 (and possibly N = 196)[3,4,5]. There is finally the more remote possibility of a minor "reef" connected with Z = 164 and a corresponding semi-magic neutron number (280 - 290).

The most critical test of survival for the super-heavy elements will involve not only spontaneous fission but also alpha and beta-decay. These aspects have been considered by Strutinsky and by Muzichka & Pashkevich[6,7,8,9] and by Nilsson, Nix, Tsang, Thompson, Szymanski and coworkers[10,11,12,13], which latter also have computed detailed fission half-lives. One should at this point also refer to the calculations carried out by Mosel & Greiner[14] treating the shell structure effects entirely in terms of vibrations. This appears an acceptable procedure only in the vicinity of the equilibrium distortion where the vibrational parameters are determined from single-particle structure. Actually Myers & Swiatecki[15,16] were the first to discuss the possibility of a super-heavy island of stability, specifically one around Z = 126, N = 184. These authors also computed a fission barrier based on the liquid-drop formula containing superimposed corrections for shell structure. A half-life estimate for this barrier was made by Wong[17] in terms of the irrotational-flow value of the moment of inertia, a procedure that grossly underestimates the half-lives.

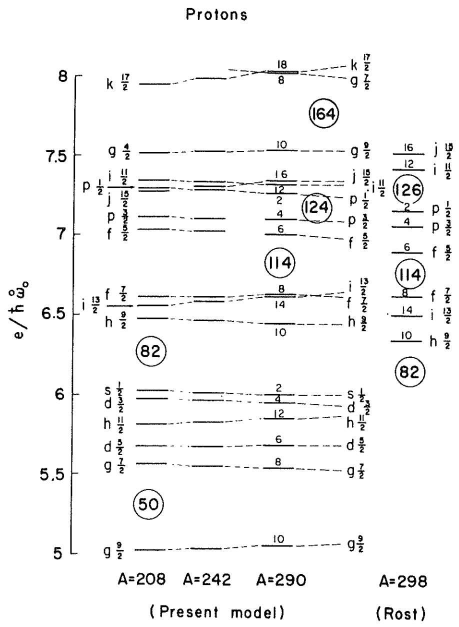

Fig. 2a. Single-proton level diagram for a spherical nuclear shape. The potential parameters κ and μ are fitted so as to reproduce the observed deformed single-particle levels near A=165 and A=242. A linear A-dependence of these parameters is then assumed.

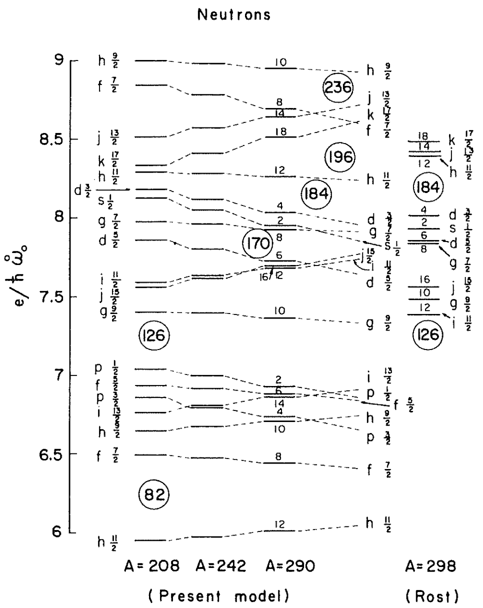

Fig. 2b. Analogous to Fig. 2a, but valid for neutrons.

§2 EXTRAPOLATIONS OF THE NUCLEAR SINGLE-PARTICLE POTENTIAL

The assumption underlying the investigations mentioned is that the nuclear potential remains a useful concept which is easily amenable to extrapolations. Both a deformed oscillator and a Woods-Saxon potential have been employed in these calculations. A useful comparison of the results obtained has been made by Muszichka[9]; he finds surprisingly good agreement between the predictions based on four alternative potentials employed. A relatively important problem here is how one should make the extrapolation of the potential to a different region in A and Z (see Figs. 2a, 2b).

One of the problems is that of the isospin dependence of the potential. This is incorporated into the modified oscillator potential by the use of different oscillator frequencies for neutrons and protons as proportional to $(1 - \frac{1}{3}\frac{N-Z}{A})$ and $(1 + \frac{1}{3}\frac{N-Z}{A})$, respectively. This assumption, for nuclei along the stability line, makes the neutron and proton radii become approximately equal.

To be more specific we give below the modified oscillator potential used in the calculations by our group

$$V = \frac{1}{2}\hbar\omega_{\square}(\varepsilon, \varepsilon_4 \ldots)\,\rho^2\left[1 - \frac{2}{3}\varepsilon P_2 + 2\varepsilon_4 P_4 + 2\varepsilon_6 P_6 + 2\varepsilon_3 P_3\right]$$

$$- \kappa\hbar\overset{o}{\omega}_{\square}\left[\vec{\ell}_t \cdot \vec{s} + \mu(\vec{\ell}_t^{\,2} - \langle\vec{\ell}_t^{\,2}\rangle_N)\right]$$

where

$$\rho^2 \sim \omega_{\perp}(x^2 + y^2) + \omega_z z^2$$

This represents a straight-forward generalisation of the deformed-oscillator potential previously used,with the purpose of taking more general nuclear shapes into account. This is natural as we want to predict half-lives for the fission process. It is found that, for the description of fission configurations, hexadecapole (or P_4) distortions become of increasing importance in addition to the quadrupole ones [12]. The relation between the actual shapes and two alternative sets of distortion parameters is illustrated in Fig. 3, where also the path to fission from the liquid-drop model is indicated.

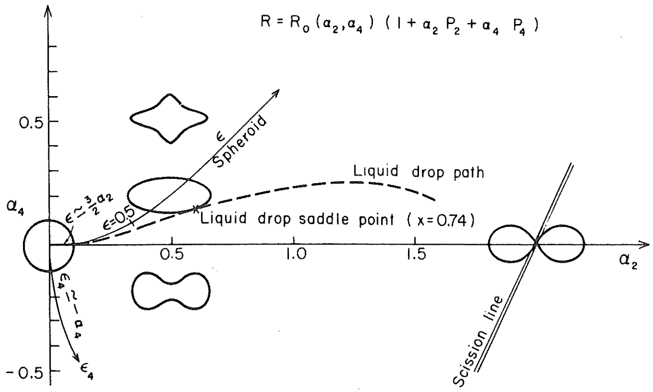

Fig. 3. Relation between distortion parameters (α_2,α_4) and $(\varepsilon,\varepsilon_4)$. Dashed lines shows "path to fission" according to liquid-drop model.

§3 THE BES-SZYMANSKI METHOD FOR THE DETERMINATION OF THE POTENTIAL-ENERGY SURFACE

The earliest equilibrium calculations by Mottelson and Nilsson [18], involving only the quadrupole distortions, followed the recipe of a simple addition of occupied single-particle levels as a function of distortion. The restoring force was provided there by the volume conservation condition. Provided the empirical order of filling was assumed, quadrupole moments in excellent agreement with experiments was obtained. Bés and Szymanski [19] added pairing and Coulomb energies:

$$E(\varepsilon) = \sum e_\nu 2v_\nu^2 - G\,(\sum u_\nu v_\nu)^2 + E_{Coul}$$

Fairly good results were obtained by these authors but it was found necessary to make ad hoc adjustments of higher- and lower-lying shells. (Beyond a certain large distortion also an instability was encountered.) Such unfounded modifications were, however, later made unnecessary by the introduction of the $\langle \vec{\ell}^2 \rangle_N$ term in the deformed oscillator potential [5]. By the introduction of the Strutinsky method [20-23], which will be discussed at some length below, the reliability of equilibrium calculations is further improved . The equilibrium distortions in ε and ε_4 for the modified oscillator potential in the latter method with P_4 couplings between shells up to $N \to N+2$ included are exhibited in Fig. 4. It is found that nuclei at the end of a shell show a trend towards the development of a waist-line relative to the spheroid shape. At the beginning of the shell the tendency is the opposite.

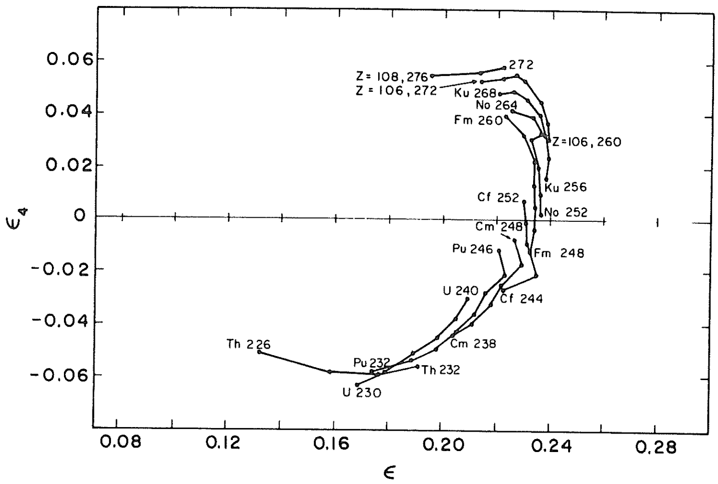

Fig. 4. Equilibrium values of ε and ε_4 in the actinide region, obtained on the basis of the Strutinsky modification of the Bés-Szymanski method.

§4 APPLICATION OF EQUILIBRIUM CALCULATIONS TO NEW REGIONS OF NUCLEI

In our slow progress towards the region of superheavy elements it is useful here to stop for a few brief remarks on deformed regions which, though a part of the main peninsula of stability, were only recently opened to experimental study, namely the very neutron rich and very neutron deficient regions of medium heavy elements. (For experimental studies see, e.g. Johansson [24] and refs 25, 26), the latter contributions to these proceedings.) The calculations [27,28] by the Dubna group covering the region $28\leq Z\leq 50$, $50\leq N\leq 82$ are of particular interest to us for two reasons. First, this region is treated in the calculations exactly in analogy with the superheavy region, i.e. by extrapolation from a better studied region. Secondly the predictions of the Bés-Szymanski method are at some variance (by $\lesssim$ 0.5 MeV) with those of the Strutinsky method [29]. The latter method is more resistive to oblate shapes. The calculations of the Dubna group predict oblate shapes in the lower right 4/5 of the region $28<Z<50$, $50<N<82$ (see Fig. 5), a prediction that should be amenable to experimental tests (oblate s.p. orbitals). Calculations performed in Lund based on the Strutinsky method (Fig. 6) usually favour prolate shapes in the $40\leq Z\leq 50$ part of the region in question [28]. As for the magnitude of distortion, ε = 0.30-0.35 is seldom exceeded in these regions in either calculations. This result is in contradiction to the experimental results of ref. 25 which are taken to imply distortions $|\varepsilon|>0.5$ (Fig. 7). One might also note the failure

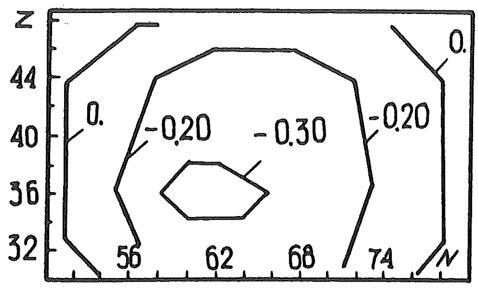

Fig. 5. Equilibrium values of *oblate* ε-distortions in the $28 \leq Z \leq 50$, $50 \leq N \leq 82$ region, from Arseniev et al, ref. 27. The values are obtained on the basis of the Bes-Szymanski method.

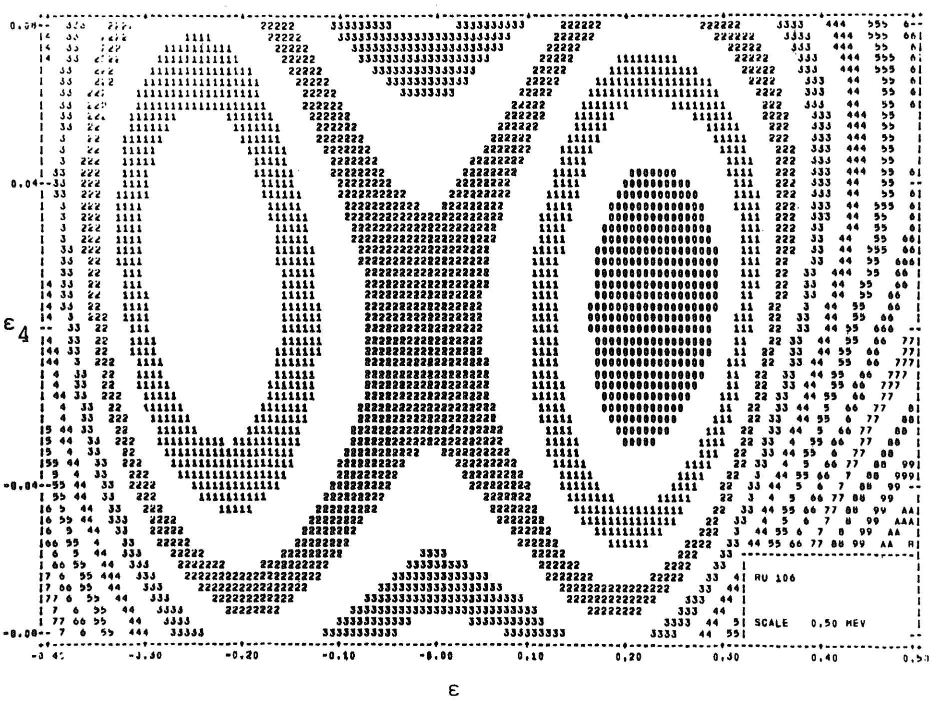

Fig. 6. The potential energy surface in the $(\varepsilon,\varepsilon_4)$ plane for the nucleus ${}^{106}_{44}Ru$ obtained by the Strutinsky normalisation method. The *prolate* minimum is here slightly deeper than the oblate one. For both minima the equilibrium $|\varepsilon|$-value is <0.3. The energy spacing between the middle of the energy regions marked "1" and "2" is 1 MeV.

reported in ref. 26 in an attempt to identify definite deformed orbitals in the ^{133}La level scheme. No detailed comparison with experimental data has as yet been undertaken.

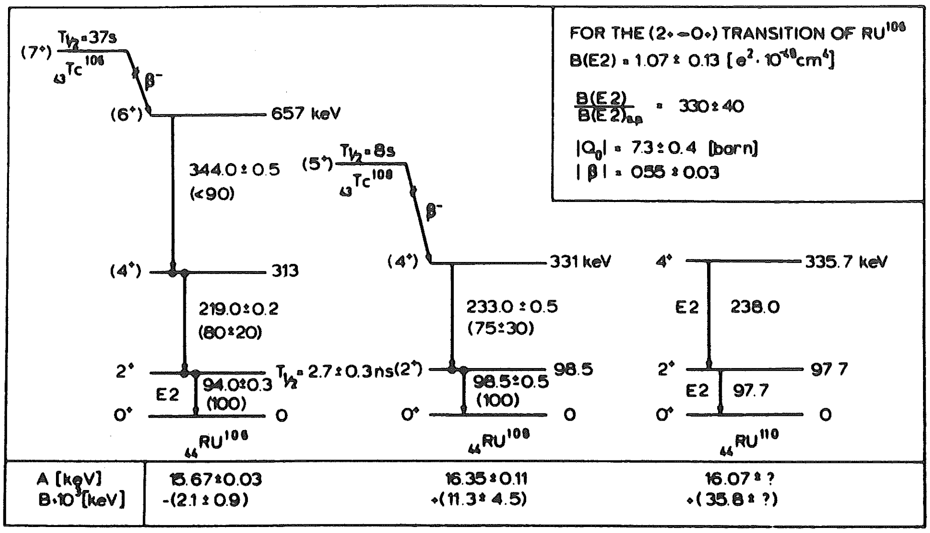

Fig. 7. Level schemes of some even-even Ru isotopes as given by Zicha et al., ref. 26. Note the large quadrupole moment derived for ^{106}Ru suggesting equilibrium distortions in excess of $|\varepsilon|\approx 0.5$.

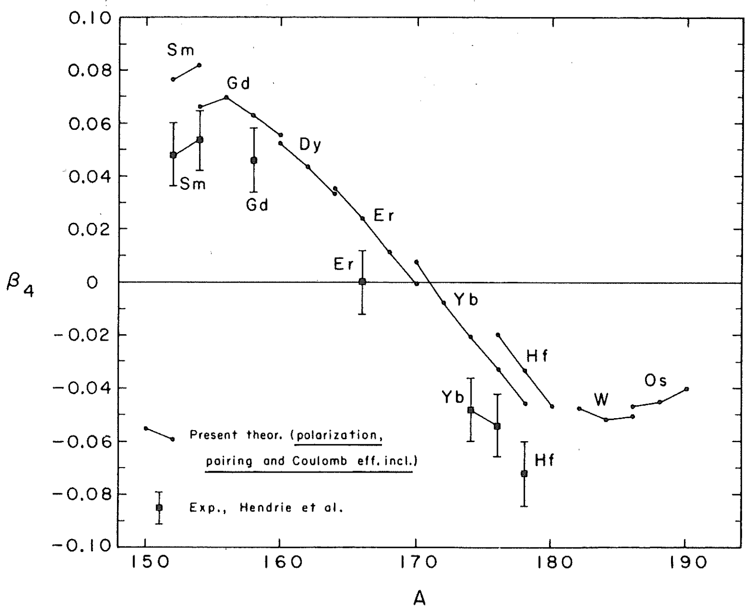

Fig. 8. Equilibrium β_4-values in the rare-earth region. The calculated β_4-values marked by solid lines were obtained by the Bes-Szymanski method with the inclusion of coupling terms in $\rho^2 P_4$ up to N=2. Solid squares with marked errors give experimental β_4-values as obtained from an analysis of inelastic alpha-scattering data.

The calculations by the Dubna group also include the γ degree of freedom. As is the case in the Os-region, treated by Kumar and Baranger [31], the ground state minimum usually moves directly from $\gamma=0°$ to $\gamma=60°$ i.e. from a pure prolate to a pure oblate. A very important theoretical finding is that the low oblate minimum usually communicates directly with the low-prolate minimum via the γ-degree of freedom. One should thus not expect in the even-even case to find both a low-lying prolate and a low-lying oblate minimum. However, for odd-A nuclei, as shown by the Dubna group, some particular oblate configurations, represented by certain spins, may coexist with other odd-A prolate configurations.

§5 CALCULATION OF EQUILIBRIUM P_4 DISTORTIONS

The effect of couplings between different N-shells by the $\rho^2 P_2$ term is taken care of, to all orders, by the introduction of the stretched coordinates. The inclusion of solely diagonal P_4 matrix elements, as studied [32] by Hendrie et al, is found to give essentially the same results as those obtained by the B.S. method with inclusion of couplings between shells with $\Delta N \leq 2$ (see Fig. 8). The results of both recipes are found to agree well with the results of inelastic alpha scattering experiments [32]. The inclusion [33] of higher couplings ($\Delta N > 2$) gives much poorer agreement with experiments (see Fig.8). Good agreement is again restored by the use of the Strutinsky method (see Fig. 9 and ref. 33).

§6 THE STRUTINSKY METHOD

At fission barrier distortions the coupling between very distant shells becomes of increasing importance. The neglect of the $\vec{\ell}^2$ and $\vec{\ell}.\vec{s}$ term in the formulation of the volume conservation condition probably becomes of increasing importance for the height of the fission barrier. Presently the only satisfactory treatment at large distortions appears to be the Strutinsky method of normalisation.

The basic contention of Strutinsky is that *on the average* the potential energy behaviour should be that of the liquid-drop model. The basic idea of a normalisation of the shell structure energies to the liquid-drop model average behaviour appears implicit in the 1959 calculations of Johansson[34], who studied the influence of realistic single-particle levels on fission half-lives in the actinide region. The idea of using a simulated shell structure

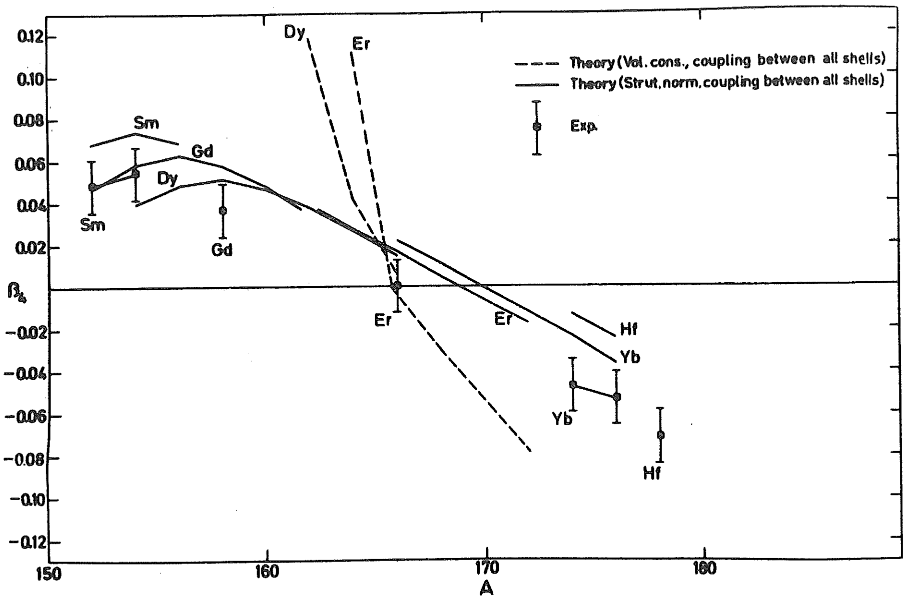

Fig. 9. Equilibrium calculations by the B.S. and the Strutinsky methods. Dashed lines mark equilibrium β_4-values as obtained from the B.S. method, when hexadecapole coupling terms between *all* shells were included. Solid lines reproduce the results of calculations based on the Strutinsky method. The latter show a better fit to the data points.

correction to the liquid-drop barrier is first taken up in a work by Geleilikman [35] in 1960. A similar method of using *ad hoc* shell structure corrections is applied in the calculations by Myers and Swiatecki [15]. Strutinsky proceeds [20-23] to construct such an average by the introduction of a folding function that smears a discrete distribution of single-particle states

$$G(e) = \sum \delta(e-e_\nu)$$

into an average distribution g(e)

$$g(e) = \int \exp\left[-\left(\frac{e-e'}{\gamma}\right)^2\right] \cdot f_{corr} \cdot G(e')\, de'$$

where f_{corr} is a correction function discussed by Strutinsky [23] and also somewhat more completely in ref. [12] which corrects for undesirable folding errors.

On the basis of the smeared level distribution g(e) a new averaged total energy E(g) is calculated for each distortion. The averaged total energy is then subtracted out of the energy sum obtained from the actual level distribution. In the place of the former a surface energy term and a Coulomb energy term are added, with parameters given in the Myers-Swiatecki version of the liquid-drop model [16]. (In this version some rough shell corrections are already introduced.) We have thus

$$E = E(G) - E(g) + E_{pair} + E_{surf} + E_{Coul}$$

The formulation of the volume conservation condition is made less critical by the Strutinsky procedure but is still needed to define the surface, e.g. for the calculation of the surface energy[10].

A few more comments should be added about the pairing energy term, E_{pair}. The basic pairing matrix elements are given an isospin dependence so as to reproduce the empirical gap variation over nuclear isotopes. The empirical average behaviour of the gap at equilibrium distortions is reproduced[12] by

$$\Delta_n \simeq \Delta_p \simeq \frac{12}{\sqrt{A}} \text{ MeV.}$$

At saddle distortions there are experimental indications [36-38] of a large gap that can be reproduced only if the pairing matrix element G is assumed to grow with the surface area S. We have assumed G to be proportional to S.

The separate liquid-drop contribution and the "shell contribution", $E(G) - E(g) + E_{pair}$, to the energy surface can be studied in Figs. 10 and 11 for the ^{252}Fm case.

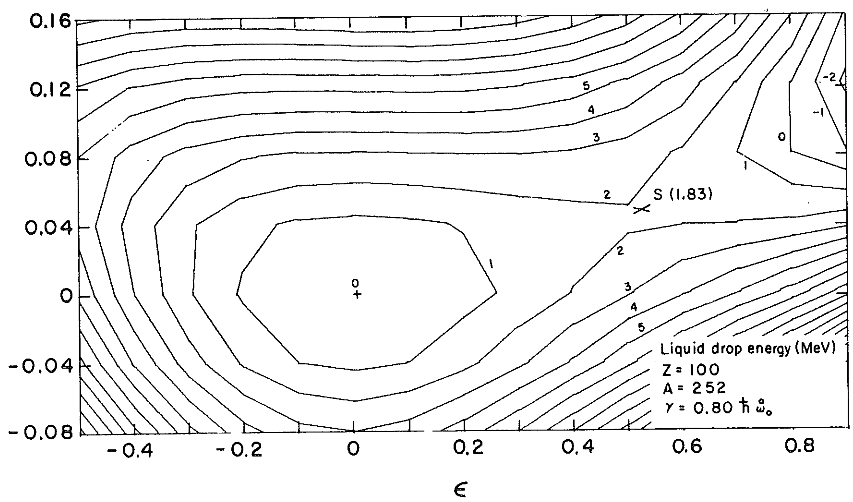

Fig. 10. Contour map of liquid-drop energy surface of ^{252}Fm in $(\varepsilon, \varepsilon_4)$-plane (from ref. 12).

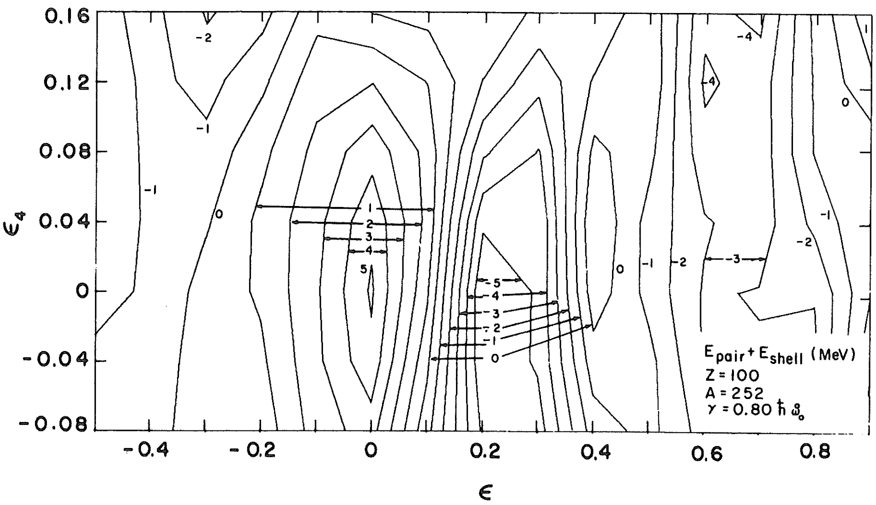

Fig. 11. Contour map of shell and pairing contributions to the nuclear potential energy for ^{252}Fm (ref. 12).

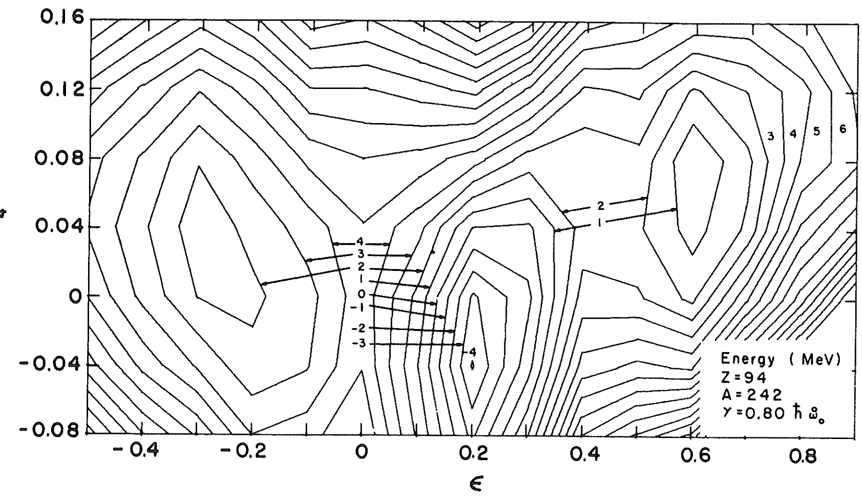

Fig. 12. Total potential-energy surface for ^{242}Pu as obtained by the Strutinsky normalisation method (ref. 12). In these early calculations a G, independent of surface area, was employed.

§7 THE TWO-PEAK BARRIER IN THE ACTINIDE REGION

The resulting energy surfaces are reproduced in Figs. 12 (^{242}Pu), 13 (^{244}Cm), 14 (^{248}Cf), 15 (^{252}Fm) and 16 (^{298}114).

One should note first the occurence of a double-peak barrier both in the actinide and the superheavy region. The ridges and

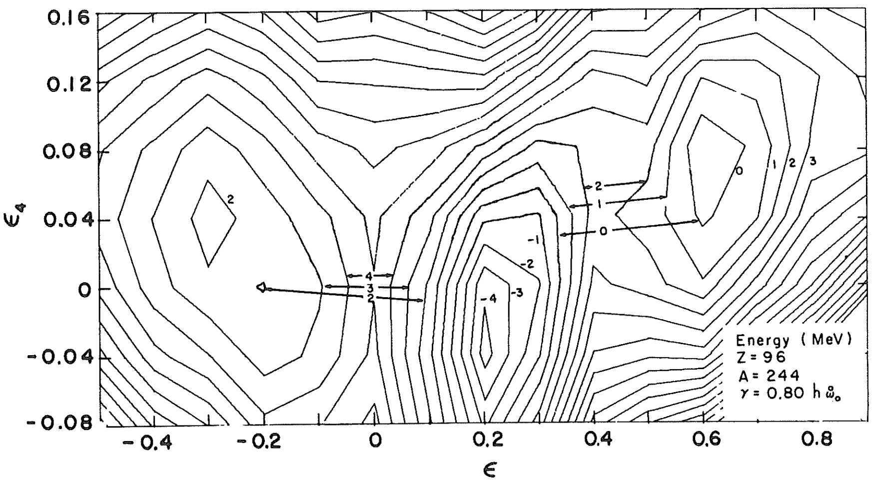

Fig. 13. Same as Fig. 12 for ^{244}Cm.

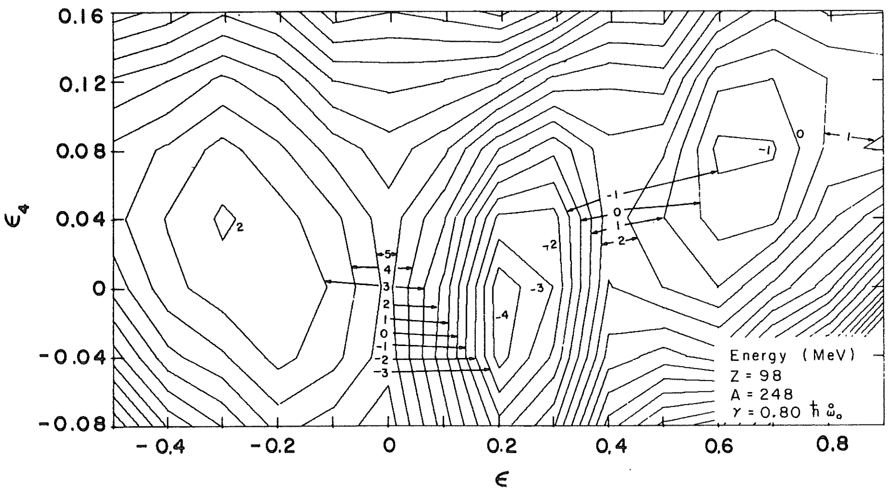

Fig. 14. Same as Fig. 12 for ^{248}Cf.

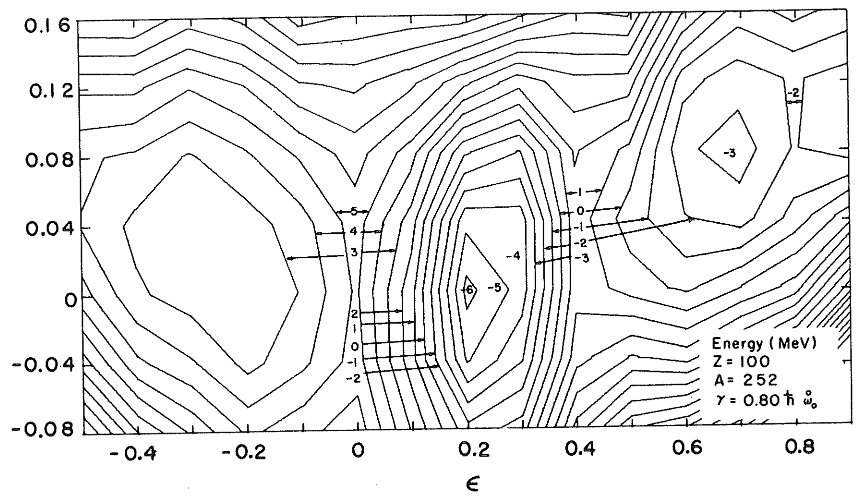

Fig. 15. Same as Fig. 12 for ^{252}Fm.

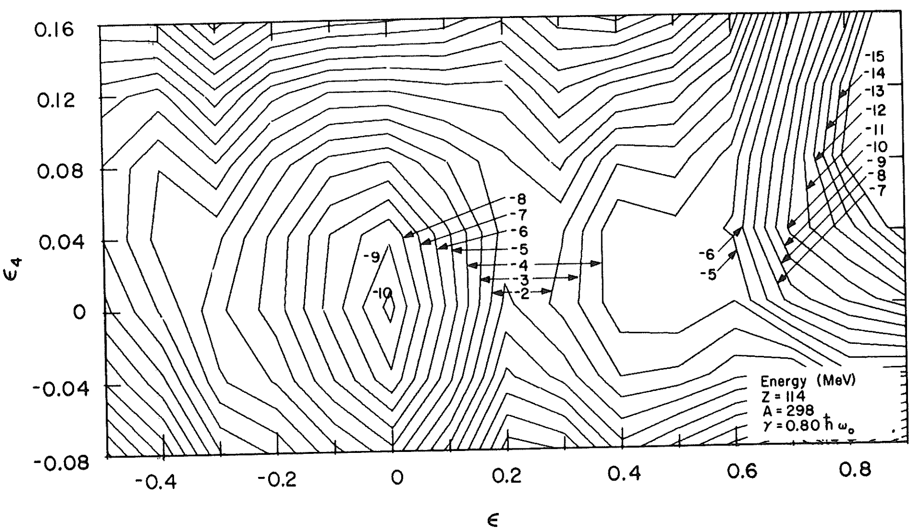

Fig. 16. Same as Fig. 12 for ^{298}114.

valleys relative to the smooth liquid-drop surface should be associated with shell crossings. In the actinide region, e.g., the first tendency for the potential-energy surface to acquire a downward trend is associated with essentially the $\Delta N=1$ crossings. (The mixing of parities within shells due to the spin-orbit term gives rise to some $\Delta N=0$ and 2 crossings in addition.) Volume conservation then provides a restoring force* resulting in the first ground-state minimum. The next downsloping trend is due largely to $\Delta N=2$ crossings and the final downward trends occuring beyond $\varepsilon = 0.85$ is due to $\Delta N=3$ crossings [12].

The fission isomeric states, first discovered by Polikanov, Flerov et al, [38, 39] are now well established experimentally - their energies measured by various threshold experiments etc. The conspicuous centering around the 235-245 region is well borne out by Fig. 17. Their occurence is according to Strutinsky [23] associated with the neutron numbers near N=146.

Recently an entirely new region of fission isomers may have been discovered by F. Ruddy and J. Alexander [40] associated with very neutron deficient nuclei in the rare earth and Pb regions. Some of these elements studied have Z^2/A values in the same range as the actinide elements. Detailed calculations of the potential-energy surface in this region are presently in progress.

Returning to the actinide region we find that the important trend, as apparent from previous figures, is the fact that the fission barrier ends at ever smaller distortions (and our parametrisation consequently becomes more and more satisfactory) with increasing values of Z^2/A. Thus a superheavy region of relative stability with respect to fission, which in turn implies a thick enough fission barrier, more or less seems to require a spherical equilibrium ground-state shape. (See Fig. 18.) This in turn requires the availability of a shell closure gap for both neutrons and protons.

§8 BARRIER PENETRATION PARAMETERS

The multidimensional barrier penetration problem involving several degrees of freedom is a fairly complex problem which is now being studied by various groups. In the calculations of refs 11a-13, we have employed the following simplified procedure. We have assumed the nucleus to exploit the ε_4-degree of freedom completely without

In the Strutinsky method of renormalization the role of volume conservation is replaced by the restoring effect of the surface energy term.

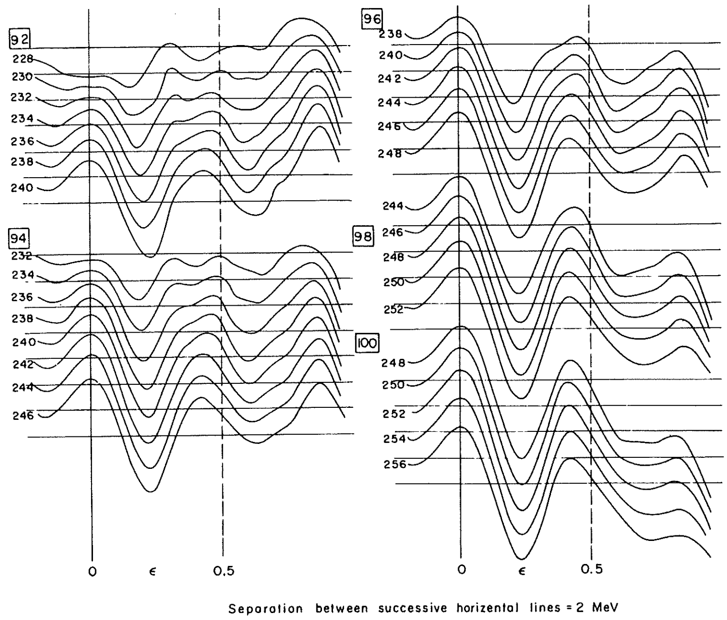

Fig. 17. The two-peak barrier for sequences of isotopes of Z=92-100 (see ref. 12).

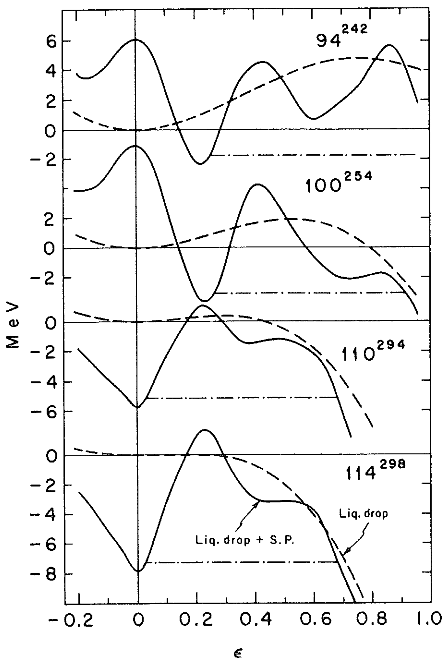

Fig. 18. The potential energy minimized with respect to ε_4 for each ε-value. Dashed lines mark the liquid-drop fission barrier. Solid lines are obtained after the inclusion of shell and pairing terms. Note the deterioration of the liquid-drop barrier with increasing Z-value.

the limitations of dynamics. Thus for each ε-value we assume a barrier height equal to the potential energy minimum with respect to ε_4. Thus barrier peaks are consequently circumvented without regard for dynamics. The path constructed in this arbitrary way is then projected on the ε axis. We then consider the one-dimensional penetration through this projected barrier. For this latter problem the WKB method gives the approximate expression for the half-life in years [12)]

$$\tau_{1/2} = 10^{-28.0} \cdot \exp\left[+\frac{2}{\hbar} \int \sqrt{2B(W(\varepsilon) - E)}\, d\varepsilon \right]$$

In this formula the first factor conceals the reciprocal of the number of barrier assaults per year (assumed to be given by the beta-vibrational frequency). Furthermore $W(\varepsilon)$ represents the potential-energy barrier, while B is the corresponding inertial mass value. The latter has been calculated on the basis of the microscopic model [13)] and found to exceed the lower limit given by the irrotational-flow model by a factor of roughly ten.

Through these calculations the half-lives of the heavier actinides were reasonably well reproduced [12)]. The corresponding overestimates of the half-lives of the lighter actinides appear connected with the fact that there the second barrier is overestimated by a few MeV at least partly because of an insufficient parametrisation. It is found that our estimates of the inertial parameters are in gross agreement with those derived semiempirically by Moretto and Swiatecki [41)] for the actinide region employing the Myer-Swiatecki [16)] modified-liquid-drop barriers and the empirical fission half-lives.

§9 CALCULATION OF NUCLEAR GROUND-STATE MASSES

As first found by Strutinsky [21, 22)], estimates of the nuclear masses can be conveniently and successfully computed from this method of calculation. One may compare the extensive set of masses obtained by our group in ref. 12 with the extensive set of masses obtained by Seeger and Perisho [42)]. In our computation we do not vary the liquid-drop parameters but employ those given by Myers and Swiatecki while the latter authors allow for a free variation of these parameters. In general the fit to empirical masses is of about

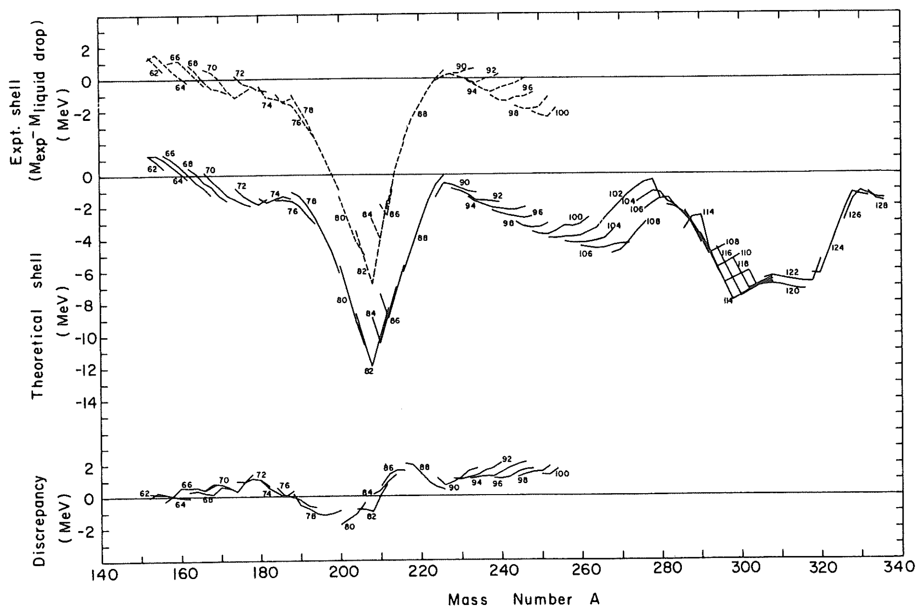

Fig. 19. Experimental and theoretical mass values for elements with A>150 expressed relative to the spherical liquid-drop values of ref. 16.

the same quality. The local improvement of the masses relative to those of Seeger and Perisho in the A≈220 region is clearly due to the inclusion of the P_4 degree of freedom which in the ground state for several nuclei lowers the minimum energy by 2 MeV and more. The mass determination has been extended[12,43] to the (114, 184) and (114, 196) regions as seen in Fig. 19. The gap effect is found to be about 1/2 to 2/3 of the Pb gap effect on the masses.

§10 TOTAL HALF-LIFE DETERMINATIONS FOR THE SUPERHEAVY REGION

We are now prepared for the brave extrapolation to the superheavy region in search for detailed half-life estimates. Given the masses, the alpha half-lives can be easily determined[11]. For the fission half-lives in Figs. 20 and 21 we have used microscopic B-values which, although ε-dependent, are averaged for one ε-value near the center of the barrier (this is so as not to weight unduly the probably slightly overestimated second peak of the fission barrier.) The inaccuracies in the half-life estimates are of course considerable. Thus a 30% error in the value of the inertial parameter gives rise to an error factor of, say, 10^6 in the half-lives; a 30% error in the barrier energy scale gives rise to a similar factor. The barrier height and width depend actually most critically on the predicted

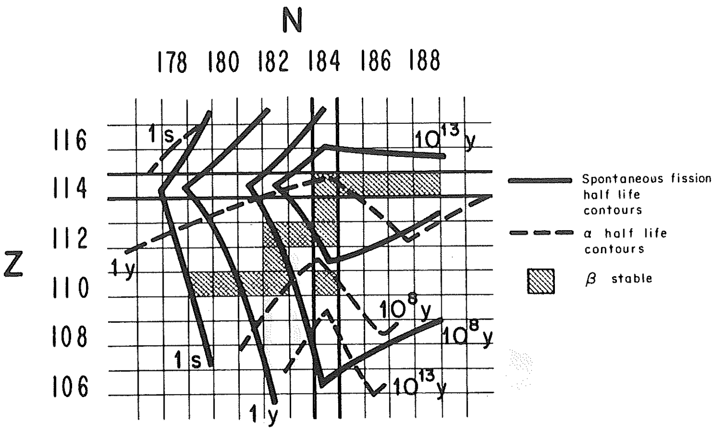

Fig. 20. Contour lines of theoretical half-lives in the mass region around Z=114, N=184. Thick solid lines mark spontaneous-fission half-lives. Dashed lines are contours of alpha decay half-lives. Beta stable elements appear as cross-hatched squares, (ref. 12).

shell structure gap around Z = 114, N = 184. The maps represented by Figs. 20 and 21 should be thought of more as a guide in the search for superheavy elements rather than as a table of half-lives, each subject to experimental verification. From this map the nuclide Z=110, N=184 is calculated to be associated with the best chances of survival. It is found to be beta stable and to have a fission half-life as well as an alpha half-life in excess of 10^8 years [12]. It should be pointed out, however, that a slight difference in the symmetry energy coefficient might shift the position of the stability line either in a direction to make $^{294}110$ beta-unstable or alternatively in a direction to make $^{292}108$ beta stable. Then the alpha half-life in the latter case should be correspondingly changed (probably diminished), $^{292}108$ might be considered a possible alternative candidate for long-range survival. As it is favourable to stay on the high N-side of the stability line to ensure long alpha half-lives*, the corresponding odd-A elements in this region are rarely

*The interesting recent work by Muzychka (Joint Institute of Nuclear Research, Dubna, Preprint R7-4435, 1969) employing three alternative nuclear potentials, those of E. Rost (Phys. Letters 26B (1968) 184) and V.A. Chepurnov (J. Nucl. Phys. USSR 6 (1967) 955) in addition to that used by us, compares the different alpha-decay half-lives obtained. Although the discrepancy factor is sometimes as large as 10^8, in the Z = 110 - 114 region, the total discrepancy is about 10^4 - 10^5. In general the inaccuracy is of the order of magnitude suggested in ref. 12). Finally one may note that the results based on our potential tend to fall between the predictions of the two Woods-Saxon potentials employed. This latter fact makes the current popular predictions concerning the unsuitability of the modified-oscillator potential relative to other potentials as a basis for extrapolations appear poorly founded.

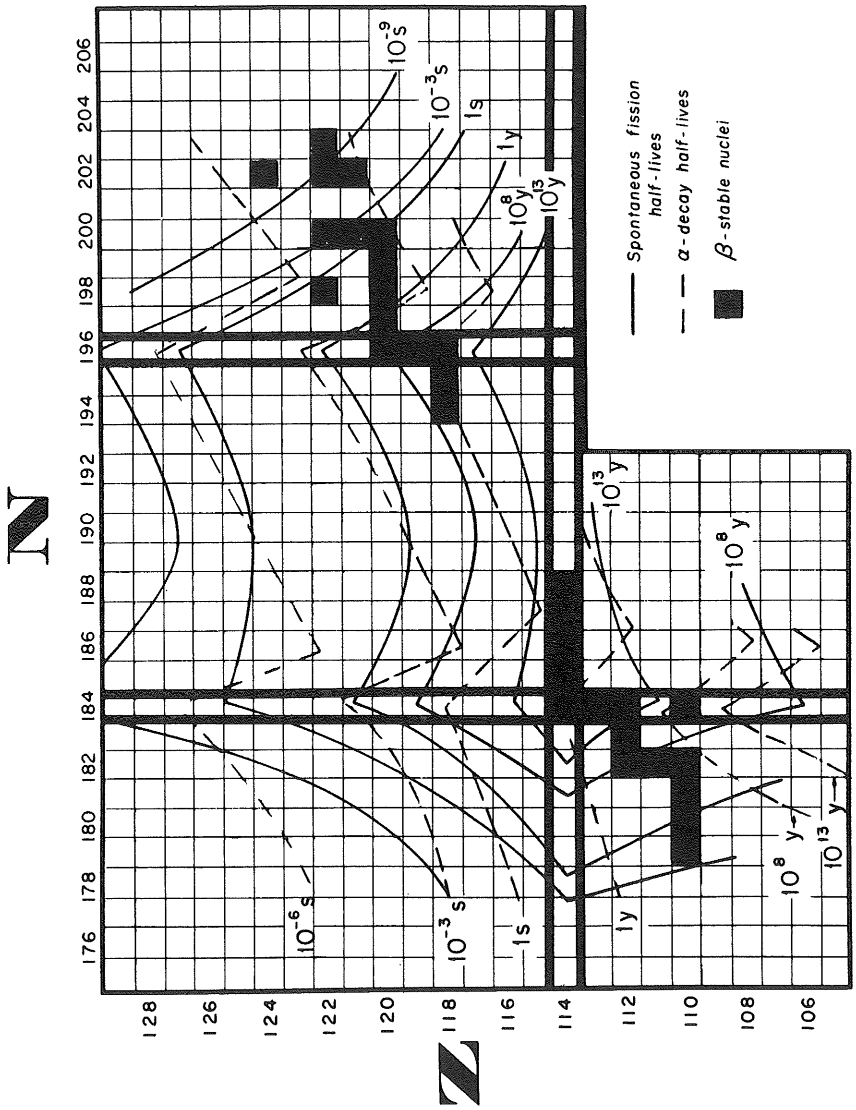

Fig. 21. Half-life contour map enlarged relative to Fig. 20. Beta-stable nuclei are marked as solid squares (ref. 43).

beta-stable. Thus elements with Z=110 or 108 or 112, all with N=184, should be considered the best targets for a search in the ores or in the cosmic radiation. Concerning how such a search is to be carried out, we will be better informed in the next lecture [44], so I will refrain from comments here.

§11 OTHER DEGREES OF FREEDOM

As the Z^2/A value for nuclei in the (114, 184) region is much larger than along the stability line in the actinide region, the set of shapes assumed through the fission barrier deviate much less from the spherical shape than is the case in the latter region. It thus takes rather small quadrupole distortions to bring us through the fission barrier in this region. As by then just two or three half-filled shells are available, it appears unlikely that correlations corresponding to very high multipoles may develop.

The possible importance of the P_3 degree of freedom in the fission process was discussed at an early stage by several authors. Johansson [45] discussed the possible development of a stable P_3 deformation at some point of the fission barrier on the basis of a relatively detailed perturbation calculation of single-particle orbitals in an octupole distorted field. He obtained P_3 instabilities around what was then considered the fission saddle point.

With the present methods which are successful in the P_2 and P_4 degrees of freedom, we have investigated instabilities with respect to P_3 distortions in the actinide region. It is found (see Fig. 22a, b) that the stiffness to such distortions is very much weakened at the second barrier peak which, in the actinide region, occurs near $\varepsilon \simeq 0.85$. Still, no instability is encountered*. In the (114, 184) region at this distortion one is already through the double-peak barrier. The superheavy case is presently being investigated but it appears unlikely that the half-lives are much affected. The P_4 distortion is also of somewhat less importance in the superheavy region than in the actinide region. Still it should be pointed out that the highest peak is lowered by 1-2 MeV in the superheavy region due to the inclusion of this latter degree of freedom. The influence of the P_6 degree of freedom in the actinide region has been investigated by P. Möller and B. Nilsson [46] who find an influence never in excess of half an MeV, usually such as to lower both barrier

It should be pointed out that in these calculations no center-of-mass correction has been applied. Such a correction can, of course, in no case change the sign of the stiffness parameter but its inclusion may affect its magnitude.

Fig. 21. Half-life contour map enlarged relative to Fig. 20. Beta-stable nuclei are marked as solid squares (ref. 43).

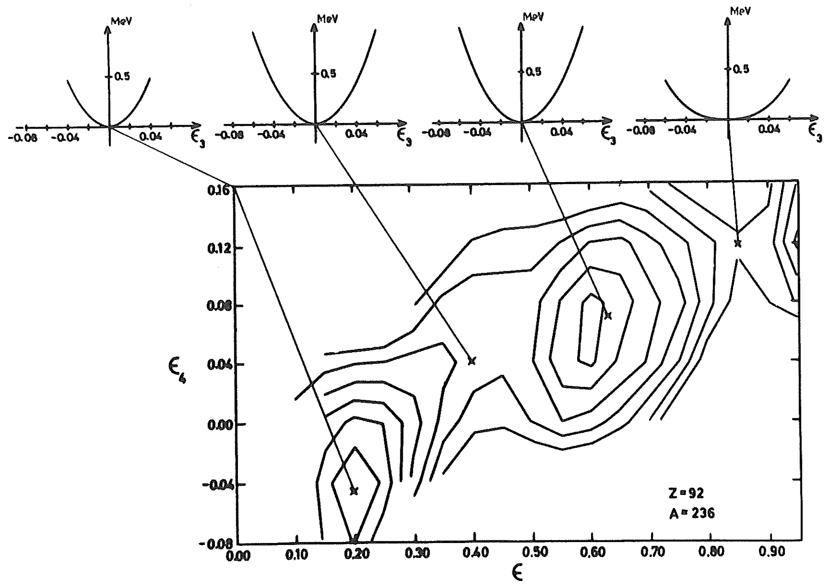

Fig. 22a. Dependence of potential energy surface on ε_3, the octupole degree of freedom, at a few important points along the "path to fission". Note the weakening in stiffness against octupole distortions at the second peak of the fission barrier. Concerns specifically ^{232}U.

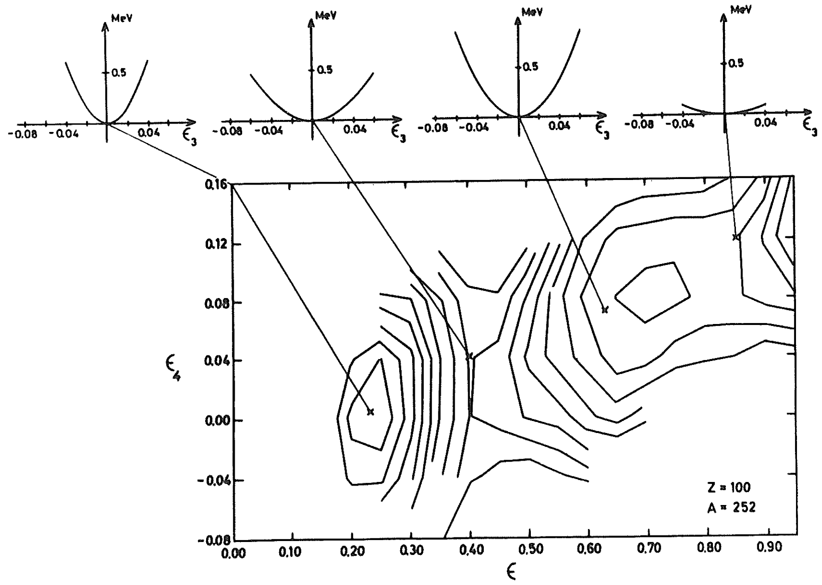

Fig. 22b. Same as Fig. 22a for ^{252}Fm.

peaks very slightly relative to the minima. (Sometimes even the ground-state minimum is lowered more than the barrier peaks.)

There is also the possible influence of a deviation from rotational symmetry of the nuclear shape. Pashkevich [47)] finds the modification of the fission barrier by the inclusion of the gamma degree of freedom to be generally less than 0.5 MeV in the actinide region. The main effect is also found to occur at the very large quadrupole distortion connected with the second barrier peak. Hopefully the effect may be neglected in the superheavy case.

As a conclusion one might say that the parametrisation used in the deformed oscillator calculations is probably satisfactory to determine fission half-lives in the heavy-actinide and superheavy-element region. For a treatment of the fission process all the way out to the scission point the shape parameters employed here appear inadequate. Better prospects may be associated with the Bolsterli, Fiset and Nix [48)] method of shape parametrisation of the "folded Yukawa" potential. Another promising line of approach is that taken by Holzer, Mosel and Greiner [49)] based on a two-center harmonic oscillator potential, as yet without a spin-orbit term. This investigation resembles that in progress by T. Johansson [50)] who employs a modified one center oscillator with an added adjustable central wine bottle bump along the z-axis.

Ultimately the treatment in terms of a potential extrapolated to another mass region may be replaced by full-fledged Hartree-Fock calculations. Up to now the simple modified oscillator potential is found to give surprisingly reliable results, whenever an experimental test is possible.

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RESUME

Le modèle de l'oscillateur déformé en P_2 et P_4 est utilisé pour obtenir des corrections de modèle en couche, au modèle hydrodynamique de la fission. Cette méthode due à Strutinsky prédit l'existence d'isotopes superlourd dans la région $Z = 114$.

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DISCUSSION

Greiner (Frankfurt): My first question concerns zero-point energies. Should one use $(1/2)\hbar\omega$ as you have done or $(5/2)\hbar\omega$ as would be more appropriate for a 5-dimensional oscillator? The amount of zero point energy is also not the same at the top and at the bottom of the barrier. These effects might change the barrier height and therefore the half-life by as much as 10^{10} in some cases.

Nilsson (Lund): I agree that the question of zero-point energy corrections is a difficult one. We have given it a lot of attention. We feel that only the β degree of freedom is relevant so we have not considered the other ones. I am not in favor of adding zero-point corrections for too many degrees of freedom like, say, octupole, hexadecapole etc. I also agree that we could be in error by as much as 10^{10} but I believe the dominant source of error comes from the second barrier which may be improperly parametrised. I think our results for the first barrier are all right. Bjornholm has obtained similar results as ours for the first barrier.

Greiner: There are calculations of the various barriers where the oblate barrier is considerably lower than the prolate barrier for several cases of the superheavy elements. In the case of oblate barriers your estimate is again too large by a very large factor.

Nilsson: You are referring to calculations done without the liquid drop normalization. However the oblate barrier is predicted to be high and quite impenetrable by the Strutinski procedure.

Greiner: My last question is of general interest. Could one improve on the present method of summing up single-particle energies by doing a Hartree-Fock or even a more complex calculation?

Nilsson: I can only say that I stand on the fifth amendment. This is a very delicate question. The Wilets calculations indicate that you don't seem to do very well with any of these recipes but you seem to do a little better with the sum than with anything else. Perhaps Prof. Wilets or Prof. Baranger would like to comment on this.

Baranger (MIT): It is not true that the Strutinski prescription is equivalent to summing the single-particle energies. It seems to me that his prescription does things just about correctly. I don't know whether Wilets would agree, but it does not seem open to the objections that Wilets has made to the other prescriptions.

Nilsson: On the average you have done away with this problem with the Strutinski prescription because you have a liquid drop background. But when it comes to the shell correction, which is a number of the order of ± 5 MeV, one still rests on the summation of single-particle levels.

Baranger: The shell corrections are something that happen near the Fermi surface and there is a theorem which states that near the Fermi surface Hartree-Fock is equivalent to adding or subtracting single-particle energies.

Karnaukhov (Dubna): The fission barrier for magic superheavy nuclei is caused mainly by the shell effect. What can you say about the dependence of the shell effect and consequently the fission barrier on the excitation energy of nucleons? This is important point for the predictions of the reaction cross-sections for the production of superheavy nuclei.

Nilsson: When you start exciting several nucleons does the shell effect remain? Do the magic numbers remain or do these effects deteriorate due to the excitation of as few as two particles? I don't have very much of an answer for this. I take the view that there are many particles contributing to the shell structure, $\sim$ 400 particles in this region. The excitation of a few of them into other orbitals should not radically change the potential. Perhaps there are people who would like to comment on that.