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# A low-complex peak-to-average power reduction scheme for OFDM based massive MIMO systems

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**Abstract**—An Orthogonal Frequency-Division Multiplexing (OFDM) based multi-user massive Multiple-Input Multiple-Output (MIMO) system is considered. The problem of high Peak-to-Average Ratio (PAR) in OFDM based systems is well known and the large number of antennas (RF-chains) at the Base Station (BS) in massive MIMO systems aggravates this further, since large numbers of these Power Amplifiers (PAs) are used. High PAR necessitates linear PAs, which have a high hardware cost and are typically power inefficient. In this paper we propose a low-complex approach to tackle the issue. The idea is to deliberately clip signals sent to one set of antennas, while compensating for this by transmitting correction signals on a set of reserved antennas (antenna-reservation). A reduction of 4 dB in PAR is achieved by reserving 25% of antennas, with only a 15% complexity overhead.

## I. INTRODUCTION

Massive Multiple-Input Multiple-Output (MIMO) is a promising technology to meet the ever increasing demands for high throughput and quality-of-service for next-generation wireless communication systems. Massive MIMO, also coined as "very-large" or "large-scale" MIMO, are systems wherein the Base Stations (BSs) are equipped with a very large number of antennas compared to previously considered systems, serving a relatively low number of users simultaneously in the same frequency and time resource. The advantages of massive numbers of antennas at the BS is well studied in literature [1], [2] and also backed by measurement campaigns [3].

Although the theoretical aspects are very promising there are still a lot of practical implementation challenges. In terms of signal processing, an advantage is that even though there is a quite large number of antennas to be handled at the BS, most of the processing can be performed using simple linear methods. For example, low-complex pre-coding can be deployed which reduces the overall digital (signal processing) hardware cost [4]. It is also important to look at the RF chains where, considering the large number of instances, it is very important to reduce both hardware cost and power consumption.

One of the critical components of the transmitting RF-chain, both in terms of hardware cost and power consumption, are the Power Amplifiers (PAs). Ideally it would be desirable to have a low Peak-to-Average Ratio (PAR), which would both reduce the hardware cost of the PA and, more importantly, improve its power efficiency. Adhering to these characteristics of the PA, an extreme approach is multi-user pre-coding schemes with constant envelope [5], where the resulting PAR is 0 dB. This

strict restriction on PAR may not be optimal in practice<sup>1</sup> and cannot co-exist with well established Orthogonal Frequency-Division Multiplexing (OFDM) techniques for handling frequency selective channels.

OFDM, on the other hand, is known to suffer from high PAR. Hence it requires a linear PA with high dynamic range to avoid in-band distortion and out-of-band components due to non-linearity and signal clipping. Linear PAs are much more expensive and typically have a lower power efficiency than their non-linear counterparts.

Many techniques for handling high PAR in OFDM systems are described in literature [6]. A well known and low-complex approach is the tone-reservation technique [7]. It relies on reserving bandwidth (around 20% for 10 dB reduction in PAR), which, in-turn, reduces the spectral-efficiency significantly since it has a linear (pre-log-factor) relation to capacity. In massive MIMO there is inherently a large degree-of-freedom (due to the large number of antennas at the BS), that can be utilized to reduce the PAR. In [8], a PAR reduction scheme is proposed for massive MIMO systems based on solving a convex optimization problem. Although this approach results in a dramatic improvement of PAR (around 10dB reduction), the complexity of the optimization is a severe hardware challenge. In contrast, we propose an approach which does not deliver the same amount of PAR reduction, but is of much lower complexity and hardware cost. The idea is to reserve antennas ("antenna-reservation" analogy to "tone-reservation"), which will be used to compensate for a (deliberate) clipping of the signals on the remaining antennas.

## II. SYSTEM DESCRIPTION

The system model depicted in Fig. 1 is an OFDM based multi-user massive MIMO system. The BS consists of  $M$  antennas serving  $K (\ll M)$  single antenna users. The  $K$  user symbols on tone  $n$ , of a total of  $N$  tones, is represented by the  $K \times 1$  vector  $\mathbf{s}_n$ . The user symbol normally has an allocation pattern, consisting of guard-bands (unused tones at the edges), pilots, and user data. In this paper we consider pilots as data, and use  $\psi$  as the set of all data tones. The complementary set  $\psi^c$  hence contains the guard band tones, such that  $\mathbf{s}_n = \mathbf{0}_{K \times 1}$  for  $n \in \psi^c$ .

The user symbols on the  $n$ -th tone are pre-coded as

$$\mathbf{p}_n = \mathbf{W}_n \mathbf{s}_n, \quad (1)$$

<sup>1</sup>Although a low cost and highly efficient non-linear PA can be deployed, the overall system performance in-terms of total power-efficiency may not be optimal.

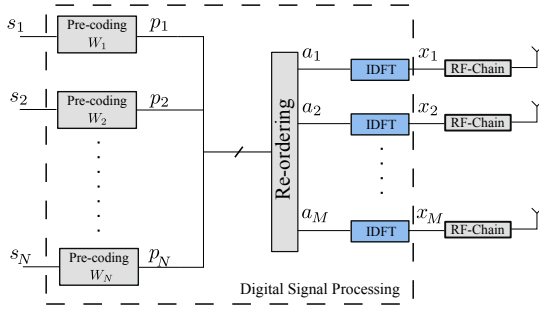


Fig. 1. Multi-User Massive MIMO system model for downlink with per-tone pre-coding. The PAR problem occurs after the IDFT which is related to the pre-coding matrix and the input symbols.

prior to transmission in order to cancel inter-user interference, where  $\mathbf{p}_n$  is the pre-coded vector of size  $M \times 1$  and  $\mathbf{W}_n$  the corresponding pre-coding matrix for the  $n$ -th OFDM tone. In this study we use a standard Zero-Forcing (ZF) linear pre-coder

$$\mathbf{W}_n \propto \mathbf{H}_n^H (\mathbf{H}_n \mathbf{H}_n^H)^{-1}, \quad (2)$$

where  $\mathbf{H}_n$  is the  $n$ -th tone channel matrix of size  $K \times M$ . To satisfy an average power constraint  $\mathbb{E}\{\|\mathbf{p}_n\|_2^2\} = 1$  on the pre-coded vector, a normalization factor is applied.

The overall input-output relation in the frequency-domain, after removal of the cyclic prefix, is described as

$$\mathbf{y}_n = \sqrt{P_n} \mathbf{H}_n \mathbf{p}_n + \mathbf{w}_n, \quad (3)$$

where  $P_n$  is the transmit power on tone  $n$  (constrained by total transmit power  $P_T = \sum P_n$ ) and  $\mathbf{w}_n$  contains unit-variance zero-mean complex Gaussian white noise elements.

To describe the transmitted signals in the time-domain, the pre-coded vectors  $\mathbf{p}_n$  are reshaped as

$$[\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_M] = [\mathbf{p}_1, \mathbf{p}_2, \dots, \mathbf{p}_N]^T, \quad (4)$$

where  $\mathbf{a}_m$  is an  $N \times 1$  vector containing the frequency-domain signal transmitted from antenna  $m$  and  $(\cdot)^T$  denotes the transpose operator. An Inverse Discrete Fourier Transform (IDFT) is performed on the transposed pre-coded signal as

$$\mathbf{x}_m = \mathbf{F} \mathbf{a}_m, \quad (5)$$

where  $\mathbf{F}$  is the  $N \times N$  unitary IDFT matrix,  $\mathbf{x}_m$  is the  $N \times 1$  time-domain OFDM symbol transmitted on antenna  $m$ .

With the addressed system specified, we move on to describing different approaches to handle PAR, before introducing the new low-complex approach.

### III. PAR AWARE PRE-CODING

OFDM transmission causes the signal  $\mathbf{x}_m$  in (5) to have a high dynamic range, due to the very nature of the transformation. This would, in-turn, require expensive linear PAs made to operate at a certain power back-off. Increasing the transmit power implies an increased probability of saturating the PAs and thus generation of both in-band distortion and out-of-band emissions. To avoid this, PAR reduction techniques are essential. The PAR for the OFDM symbol on antenna  $m$  is defined as

$$\text{PAR}_m = \frac{\|\mathbf{x}_m\|_\infty^2}{\|\mathbf{x}_m\|_2^2/N}, \quad (6)$$

and the global PAR as

$$\text{PAR}_{\text{glb}} = \frac{\|\bar{\mathbf{x}}\|_\infty^2}{\|\bar{\mathbf{x}}\|_2^2/(NM)}, \quad (7)$$

where  $\bar{\mathbf{x}}$  is an  $NM \times 1$  vector containing all time-domain OFDM symbols transmitted on all antennas.

#### A. Convex Optimization based PAR Pre-coding

In massive MIMO, the high degree-of-freedom can be used to select a frequency-domain transmission signal  $\mathbf{a}_m$  with very strict requirements on the dynamic range of the corresponding time-domain signal  $\mathbf{x}_m$ , while at the same cancelling the multi-user interference. An example of this is the joint precoding, modulation, and PAR reduction (PMP) scheme presented in [8], where an optimization problem is formulated along the lines of

$$\underset{\bar{\mathbf{x}}}{\text{minimize}} \|\bar{\mathbf{x}}\|_\infty \text{ subject to } \bar{\mathbf{s}} = \bar{\mathbf{G}} \bar{\mathbf{x}}. \quad (8)$$

where all time-domain symbols are stacked in  $\bar{\mathbf{x}}$ , all user data symbols, including nulls on guard-band frequencies, are stacked in  $\bar{\mathbf{s}}$ , and the structure of  $\bar{\mathbf{G}}$  implements the constraints on no user interference and no energy in the guard bands.<sup>2</sup>

Various methods to solve this optimization problem exist [9], including the method described in [8]. This approach gives an optimal solution with very good control on the PAR levels. However, the complexity and the number of dimensions involved in such large scale optimization poses an implementation challenge in hardware. Moreover, the optimization problem is non-linear and depends on the user data, thus requires solving of (8) frequently, which implies a high computational latency.

#### B. Peak Cancellation

An approach that is contrary to the optimization problem (8) is clipping the peak signals. Either the clipping is performed in the analog domain (*i.e.*, by the PA), which introduces both out-of-band components and in-band distortion, or it can be performed in the digital domain. The benefit of clipping in the digital domain is that it does not cause any out-of-band components, only in-band distortion. The idea is to compute a signal  $\mathbf{r}$ , to be added to  $\mathbf{x}$ , that reduces the PAR,

$$\text{PAR}(\mathbf{x} + \mathbf{r}) < \text{PAR}(\mathbf{x}), \quad (9)$$

in-line with the tone-reservation technique [7]. When the signal  $\mathbf{r}$  is mapped to all the tones it is known as the peak-cancellation technique. When  $\mathbf{r}$  is mapped only to the data tones ( $\psi$ ), no out-of-band power exists. Let us now introduce the proposed low-complex approach to reduce PAR in massive MIMO systems.

### IV. LOW-COMPLEX PAR AWARE PRE-CODING

Clipping the transmit signals is very simple but suffers from a certain amount of distortion. An approach to compensate this distortion is to dedicate a subset of the antennas which transmits signals used to mitigate the resulting distortion. Hence, the overall system for the PAR reduction would be a set of antennas ( $\chi$ ) transmitting user data with the clipping technique, and the remaining antennas in the set  $\chi^c$  are used to compensate this distortion as shown in Fig. 2.

<sup>2</sup>For more details, see [8].

### A. Peak-Cancellation

The peak-cancellation functional block of Fig.2 basically imposes a saturation logic (clipping), based on the desired PAR level by applying

$$x' = \begin{cases} x, & \text{if } |x| < T \\ Te^{j\angle x}, & \text{otherwise} \end{cases} \quad (10)$$

element-wise to the signals, where  $T$  is the clipping amplitude threshold and  $x'$  is resulting distorted signal value. The clipping signal on the  $m$ -th antenna, with notation analogous to (10),

$$c_m = x_m - x'_m, \quad (11)$$

is the residual after peak-cancellation. Reducing the desired PAR level would mean more energy in the clipping signal which, in-turn, needs to be compensated by the antennas in the set  $\chi^c$ .

### B. Distortion Prediction

The clipping signal due to peak-cancellation ( $c_m$ ) is transformed back to the frequency domain as

$$b_m = F^H c_m. \quad (12)$$

After which it is reshaped (transposed) similar to (4) as

$$[e_1, e_2, \dots, e_N] = [b_1, b_2, \dots, b_M]^T, \quad (13)$$

and used to compute the distortion

$$d_n = H_n^{(\chi)} e_n, \quad (14)$$

on tone  $n$  at the user end, for all  $K$  users, where  $H_n^{(\chi)}$  is the channel matrix from the antenna set  $\chi$  to the users. The distortion, can now be compensated by transmitting  $-d_n$  through the  $\chi^c$  antenna set.

### C. Complexity Analysis

There are two sets of pre-coding which needs to be performed, for user symbols  $s_n$  and distortion cancellation  $-d_n$ . If we consider the set  $\chi$  to contain  $M_1$  antennas, then the compensation set  $\chi^c$  contains  $M - M_1$  antennas. The total complexity in terms of multiplications for computing the two pre-coding matrices per tone is  $\mathcal{O}(M_1 K^2 + K^3) + \mathcal{O}((M - M_1)K^2 + K^3) = \mathcal{O}(MK^2 + 2K^3)$ , which is almost the same as performing pre-coding on all  $M$  antennas (except for one additional  $K \times K$  matrix inversion).

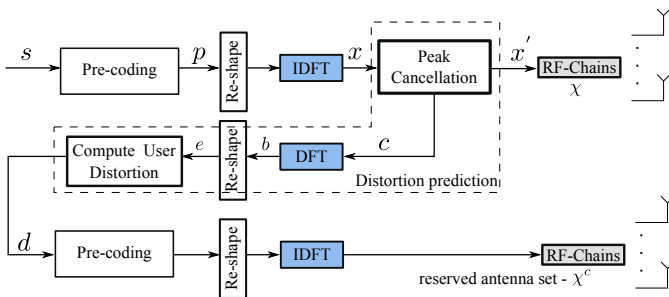


Fig. 2. Top level data-flow of the proposed low complexity PAR reduction approach, where the dedicated set of compensation antennas  $\chi^c$  counteracts the distortion.

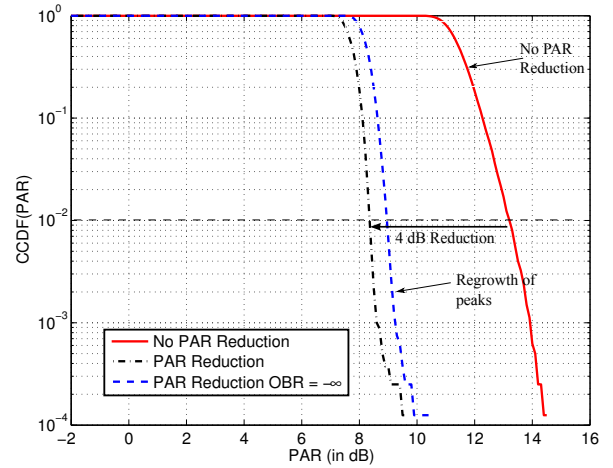


Fig. 3. CCDF of  $\text{PAR}_{\text{gbl}}$  for a MIMO system with  $M = 100$ ,  $K = 10$ ,  $N = 512$ , and using 64-QAM modulation and the number of reserved antennas for compensation in  $\chi^c$  is  $M_2 = 25$ .

The main overhead, in terms of complexity, is from the distortion prediction module. A DFT is performed efficiently using fast Fourier transform, hence requiring  $\mathcal{O}(N \log_2 N)$  multiplications. The matrix-vector multiplication with  $H_n$  requires  $\mathcal{O}(M_1 K)$  multiplications per tone, resulting in a total complexity overhead of  $\mathcal{O}(M_1 N \log_2(N) + N M_1 K + N K^3)$  to perform the proposed PAR reduction as compared to performing only zero-forcing pre-coding on all antennas. Consider a system with  $M = 100$  antennas serving  $K = 10$  users simultaneously, using OFDM modulation with  $N = 512$  tones. If we reserve  $M_2 = 25$  antennas for distortion compensation resulting from the peak cancellation on the remaining 75 antennas, the overhead is only around 15%. This is a minor overhead and what remains is to investigate the performance of this method.

## V. PERFORMANCE ANALYSIS

To measure the PAR characteristics of the proposed method, we use the Complementary Cumulative Distribution Function (CCDF) of the PAR, defined as

$$\text{CCDF}(X) = \text{Prob}(\text{PAR} < X) = 1 - \text{CDF}(X).$$

A tolerance level of 99% is used as benchmark for all the antennas, *i.e.*,  $\text{CCDF}(\text{PAR})=1\%$ .

Fig. 3 demonstrates the  $\text{PAR}_{\text{gbl}}$  characteristics of the proposed low complexity scheme. There is a reduction of around 4 dB when using 25% of the antennas for distortion compensation. Reserving antennas has a negative effect on the spectral efficiency, but it is much lower than when using techniques like tone-reservation to achieve similar PAR reductions. This is because the bandwidth is linearly proportional to capacity, whereas array gain is logarithmically proportional. For the antenna set  $\chi$ , the  $\text{PAR}_m$  is controlled by the amplitude threshold applied in the peak cancellation on these. For the antennas in the compensation set  $\chi^c$  the  $\text{PAR}_m$  gets worse by around 3-5 dB, which may appear a bit concerning. However, it should be noted that the average power of these antennas is considerably lower (around 8 dB lower for  $M_2 = 25$ ) than on the other antennas, resulting in an absolute peak power on a comparable level.

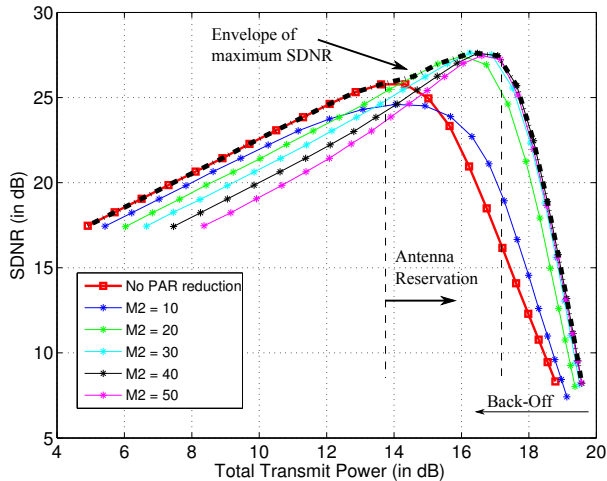


Fig. 4. Antenna-reservation impact on SDNR for varying transmit power. The simulation was performed for a MIMO system with  $M = 100$ ,  $K = 10$ ,  $N = 512$ , noise variance of 1 and a hard-saturation model (maximum amplitude of 1) for PA.

To analyze the impact of antenna-reservation ( $\chi^c$ ) on the system performance, we look at the Signal-to-distortion-plus-noise ratio (SDNR) at the user terminals

$$\text{SDNR} = 10 \log_{10} \left( \frac{P_s}{P_d + \sigma_w^2} \right), \quad (15)$$

where  $P_s$  is the received signal power,  $P_d$  is the power of the distortion introduced by clipping and  $\sigma_w^2$  is the receiver noise variance. In Fig. 4, it can be seen that by reserving antennas there is a drop in SDNR at low transmit powers, because of the associated loss in the array gain. As the transmit power is increased, beyond some point, the PAs saturate, and a higher SDNR is achievable by deliberate clipping and using reserve antennas to mitigate the clipping distortion. This results in a linear extension of SDNR by around 4 dB as shown in Fig. 4. However, further increasing the transmit power results in more power in the reserved antennas, which also start to saturate, hence lowering the SDNR.

Another performance measure of the PAR reduction technique is the resulting Out-of-Band (ratio) Power (OBR), which is the ratio of power on out-of-band tones (guard band) to the power on data tones. For the proposed PAR reduction technique the OBR is around -20 dB for 10% guard band tones ( $|\psi^c| = N/10$ ) as shown in Fig. 5. An OBR =  $-\infty$  can also be achieved by mapping the clipped signals only to the data tones, which would result in a slight re-growth of the peaks, as shown in Fig. 3.

A more intuitive and generalized measure is the Output-Back-off (OBO), the difference between operating power and the 1 dB compression point of the PA, required to guarantee enough linearity. The proposed PAR reduction schemes allows for around 4 dB lower back-off. This in-turn would result in the PAs operating in a more efficient region. Based on the requirement, different configurations of the proposed PAR reduction scheme can be employed.

## VI. CONCLUSION

While the theoretical aspects of massive MIMO are well researched, there are still a lot of challenges when it comes to

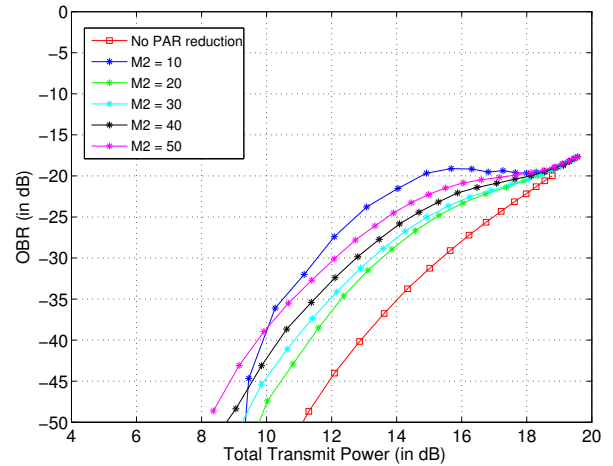


Fig. 5. OBR for the proposed PAR reduction technique.

practical deployment. In this paper we have introduced a low-complexity PAR reduction technique for OFDM based massive MIMO systems. The proposed antenna-reservation method has a low complexity overhead and can be implemented with existing hardware blocks, namely a DFT and a matrix inversion (pre-coder). The PAR reduction can be varied during peak-cancellation to a targeted level. We demonstrated around 4 dB improvement in PAR at only 15% pre-coding complexity increase for a 100-antenna system. Although the proposed technique has a lot of potential, there are many interesting theoretical problems, *e.g.*, how many and which antennas should be reserved and what is the trade-off between complexity and performance.

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