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# A Harmonic Transfer Function Model for a Diode Converter Train

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*Abstract*

A method for analysis of electric networks with nonlinear and switching components is presented. The method is based on linearization around the nominal AC voltage, which results in linear time periodic (LTP) models. For nonlinear and switching components, there is coupling between different frequencies, which may cause stability and resonance problems. The models capture this coupling and can thus be used for small signal stability and robustness analysis. A short introduction to transfer functions for LTP systems is given.

To illustrate the method, an LTP model for the Adtranz locomotive Re 4/4 is derived. The system consists of an AC-side with a transformer, and a DC-side with a DC-motor and a smoothing choke. The AC-side and the DC-side are connected by a diode bridge rectifier. The model clearly shows the coupling between frequencies.

*Keywords:* Power networks, load modeling, nonlinear analysis, periodic systems.

## I. INTRODUCTION

Modern trains use power electronic converters to shape the supplying AC-voltage. These switching converters introduce harmonics in the supplying network. Under unfortunate operating conditions, the introduced harmonics may interact with other trains. This may trigger resonances and cause instability. Known incidents have occurred in:

- Italy: Electrical line disturbances in 1993–95.
- Denmark: Several protective shutdowns of the net in 1994.
- Great Britain: Problem with the signaling system in 1994–95.
- Switzerland: Several modern converter locomotives shut down due to network resonance in 1995.
- Germany: S-bahn in Berlin exceeded the limits for harmonical perturbations in 1995.

This is not a problem for train networks alone, but for all power networks with components that modulate the frequency, for instance HVDC systems [6].

When analyzing electric networks, one is often restricted to time domain simulation. Very accurate and thoroughly validated models have been developed for use with, for instance, EMTP and EMTDC. However, no matter how accurate the models are, there is no way that simulations alone can guarantee that all critical parameter values and operating conditions are found so that new incidents can be avoided in the future. Simulations can only give a yes or no answer to stability, and do not say anything about robustness to a set of uncertainties. Thus, no uncertainty in model parameters is allowed, no unmodelled dynamics, and all possible operating conditions must be analyzed.

In control design, robustness has been a main concern for a long time. There now exist powerful tools for robustness analysis such as $\mu$-analysis, $H_\infty$-design and also nice methods for model aggregation, which makes modularized modeling and analysis easier. Most of these methods are only available for linear systems. The use of power electronics, however, implies that traditional linear analysis does not apply. The switching introduces coupling between different frequencies. For proper analysis, the models have to consider this coupling.

The supplied AC-voltage leads to a periodic excitation of the system. A natural approach is to linearize around the nominal voltage. This results in a linear model, however not time invariant but time periodic. These linear time periodic (LTP) models capture the coupling between frequencies and can thus be used for analysis of networks including nonlinear and switching components.

For periodic signals, an LTP model gives a linear relation between the Fourier coefficients of the inputs and the outputs. This is the reason why frequency domain methods are popular for steady state analysis of power networks. LTP models for steady state analysis have been developed for numerous electric components, for instance transformers with nonlinear saturation curves [11, 12, 1], HVDC converters [13, 2, 16] and static var compensators [17].

When the harmonic balance solution for a network is obtained via Newton iterations [8], and [5], the Jacobians are LTP models that improve the convergence of the solution. Newton's method of harmonic balance has been used for analysis of power networks under various names, *Harmonic Power Flow Study* in [15], it is called *Unified Solution of Newton Type* in [1], and *Harmonic Domain Algorithm* in [3]. Harmonic balance with relaxation is called *Iterative Harmonic Analysis* in [4], and Newton's method with a diagonal Jacobian is called *A Multiphase Harmonic Load Flow Solution Technique*

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in [17].

In [14] and [7] a transfer function for LTP systems is derived and used to analyze vibrations in helicopter rotors. Via this transfer function many stability and robustness results for linear time invariant systems can be generalized to hold also for LTP systems. These references also give a nice historical background and relates the method to Floquet theory, Liapunov exponents and to so called lifting methods.

In this paper, a transfer function for a diode converter locomotive is derived, and it is shown how the Nyquist criterion can be used to guarantee stability when the loco is connected to the power system. For related work see also [10] where the harmonic transfer function method is used to study harmonic interaction for a four-quadrant converter locomotive.

## II. ANALYSIS OF LTP SYSTEMS

For linear time invariant systems, many stability and robustness results are based on the transfer function operator. To generalize these results to LTP systems, we need a corresponding transfer function.

Let the input, $u(t)$, be an exponentially modulated periodic (EMP) signal with period $T$

$$u(t) = e^{st}\sum_m U_m e^{jm\omega_0 t} = \sum_m U_m e^{(s+jm\omega_0)t}, \qquad (1)$$

where $\omega_0 T = 2\pi$. In Appendix A it is shown that an LTP system maps an EMP input to an EMP output, that is, the output too is an EMP signal

$$y(t) = e^{st}\sum_n Y_n e^{jn\omega_0 t} = \sum_n Y_n e^{(s+jn\omega_0)t}.$$

If the EMP input signal $U$ and output signal $Y$ are written on vector form

$$U(s) = [\ldots \quad U_{-1} \quad U_0 \quad U_1 \quad \ldots]^T e^{st},$$
$$Y(s) = [\ldots \quad Y_{-1} \quad Y_0 \quad Y_1 \quad \ldots]^T e^{st},$$

then their relation is described by a linear equation

$$Y(s) = H(s)U(s).$$

The transfer function matrix $H(s)$ defines the coupling between different frequencies and is called the harmonic transfer function (HTF) and can, formally, be represented as a doubly-infinite matrix, see Appendix A.

$$H(s) = \begin{bmatrix} \ddots & \cdots & \cdots & \cdots & \cdots \\ \vdots & H_{-1,-1}(s) & H_{-1,0}(s) & H_{-1,1}(s) & \cdots \\ \vdots & H_{0,-1}(s) & H_{0,0}(s) & H_{0,1}(s) & \cdots \\ \vdots & H_{1,-1}(s) & H_{1,0}(s) & H_{1,1}(s) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}.$$

## III. A DIODE CONVERTER TRAIN

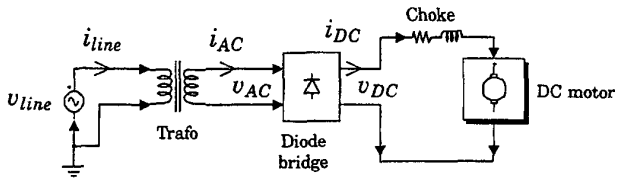


**Fig. 1** A simulink model for the diode converter loco. The harmonic transfer function derived in Section III describes the harmonic interaction between the variable $i_{AC}, v_{AC}, i_{DC}$ and $v_{DC}$, see Fig. 7

An LTP model for the Adtranz locomotive Re 4/4 is derived. A Simulink model for the locomotive, which consists of a transformer, a diode bridge rectifier, a smoothing choke and a DC motor, is shown in Fig. 1.

### A *A Diode Bridge Rectifier Model*

The diode bridge rectifier ensures that the AC-side and the DC-side are related by a time varying modulation

$$\begin{aligned} v_{DC}(t) &= B(t)v_{AC}(t), \\ i_{AC}(t) &= C(t)i_{DC}(t). \end{aligned} \qquad (2)$$

The current and voltage on both sides of the rectifier are shown in Fig. 2.

The diodes in the diode bridge are not ideal, which means that it takes some time for the AC-current to change sign. During this period current flows through all diodes. This is called commutation. The result is that the commutation functions, $B(t)$ and $C(t)$, are not square waves. Typical shapes are shown in Fig. 3. To avoid detailed modeling of the converter, these modulation functions can be obtained via simulation or measurement. We have used data from time domain simulation using Simulink's Power System Blockset toolbox. For a diode bridge rectifier, the switching instants are determined by the zero-crossings of the AC-voltage. A voltage distortion will hence affect the switching instants and thus the periodicity.

Linearizing (2) around the periodic functions, $B_0(t)$ and $C_0(t)$ and the nominal signals, $v^0_{AC}(t)$ and $i^0_{DC}(t)$, gives

$$\Delta v_{DC}(t) = B_0(t)\Delta v_{AC}(t) + \Delta B(t)v^0_{AC}(t), \qquad (3)$$
$$\Delta i_{AC}(t) = C_0(t)\Delta i_{DC}(t) + \Delta C(t)i^0_{DC}(t). \qquad (4)$$

The deviations from the periodicity, $\Delta B(t)$ and $\Delta C(t)$, are due to distortion of $v_{AC}$. The effect of $\Delta B(t)$ is neglectable since $v^0_{AC}(t)$ is small around the zero crossing. The effect of $\Delta C(t)$ is analyzed in the next section, see (9). The analysis will show that an HTF for the diode bridge has the following structure

$$\Delta V_{DC}(s) = \qquad B_0(s)\Delta V_{AC}(s), \qquad (5)$$
$$\Delta I_{AC}(s) = C_0(s)\Delta I_{DC}(s) \quad + D(s)\Delta V_{AC}(s), \qquad (6)$$

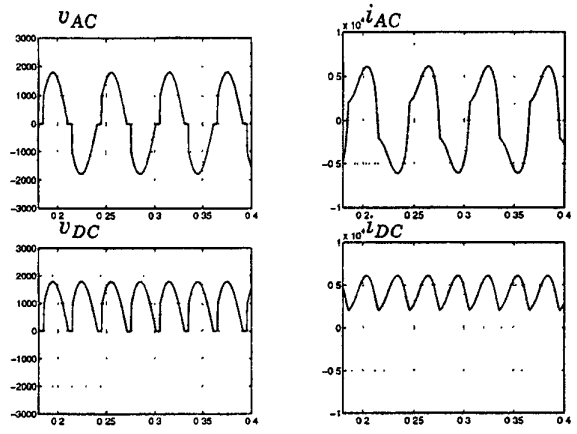


**Fig. 2** Simulation of AC- and DC-voltages and currents for the diode converter loco, see also (2). Small-signal linearization around these nominal trajectories leads to the model in Fig. 4.

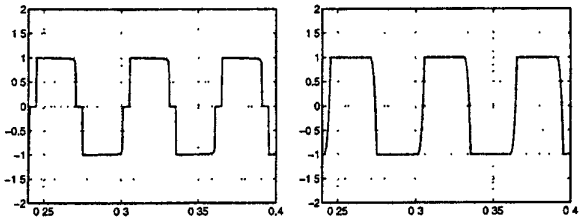

**Fig. 3** Modulation functions, $B_0(t)$ and $C_0(t)$. Changes in $v_{AC}$ change the switch instances but not the form of the modulation functions.

where $D(s)$ is due to non-periodic switching. The rectifier can hence be described by the block diagram in Fig. 4.

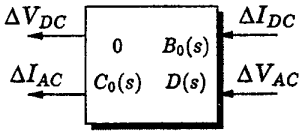


**Fig. 4** A block diagram of the linearized diode rectifier. The HTF models $B_0(s)$, $C_0(s)$ and $D(s)$ are derived in the text and have been verified by time domain simulations. The diode rectifier model is connected with the DC dynamics in Fig. 5.

## B *Deriving the HTF for the Rectifier*

The periodic functions $B_0(t)$ and $C_0(t)$ can be expressed by their Fourier series $B_0(t) = \sum_{k=-\infty}^{\infty} B_k e^{jk\omega_0 t}$. With $\Delta v_{AC}(t)$ being an EMP signal we get

$$\Delta v_{DC}(t) = B_0(t)\Delta v_{AC}(t) = e^{st}\sum_m\sum_k B_k V_{AC,m} e^{j(m+k)\omega_0 t}$$
$$= e^{st}\sum_n\sum_m B_{n-m} V_{AC,m} e^{jn\omega_0 t}.$$

Thus, the HTF is a static Toeplitz matrix

$$B_0(s) = \begin{bmatrix} \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ \ddots & B_0 & B_{-1} & B_{-2} & \ddots & \ddots \\ \ddots & B_1 & B_0 & B_{-1} & B_{-2} & \ddots \\ \ddots & B_2 & B_1 & B_0 & B_{-1} & \ddots \\ \ddots & \ddots & B_2 & B_1 & B_0 & \ddots \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \end{bmatrix},$$

and similarly for $C_0(s)$.

We will now analyze the part of $i_{AC}$ that is due to changes in switching instants, $\Delta C(t) i^0_{DC}(t)$. A good approximation is that a change in switching instant does not affect the shape of the modulation function, but only shifts it in time.

$$C(t) = C_0(t - \Delta t) \approx C_0(t) - \frac{dC_0(t)}{dt}\Delta t,$$

The change in switching at time $t_k$ only affects the current until the next switch occurs around $t_{k+1} = t_k + T/2$. A switch change $\Delta t_k$ at time $t_k$ gives

$$\Delta C(t) = -\Pi(t - t_k)\frac{dC_0(t)}{dt}\Delta t_k = \begin{cases} -\dfrac{dC_0(t)}{dt}\Delta t_k & t \in [t_k, t_{k+1}), \\ 0 & \text{otherwise}, \end{cases} \quad (7)$$

where $\Pi(t)$ is a unit pulse with width $T/2$.

We must now relate the zero crossing change, $\Delta t$, with the voltage distortion. Let the nominal voltage be

$$v^0_{AC}(t) = V_0 \sin \omega_0 t,$$

with zero-crossings at $t_k = kT/2 = k\pi/\omega_0$. A distortion $\Delta v_{AC}(t)$ gives a change in switching time, $\Delta t$. This change is approximately given by the voltage distortion at the nominal switching time, $t_k$

$$\Delta v_{AC}(t_k) \approx -\frac{dv^0_{AC}(t_k)}{dt}\Delta t_k = -(-1)^k V_0 \omega_0 \quad (8)$$

Using (7) and (8) and assuming a constant $i^0_{DC}(t) = i^0_{DC}(t_0)$ now gives

$$\Delta C(t) i^0_{DC}(t) = -\Pi(t - t_k)\frac{dC_0(t)}{dt}\Delta t_k i^0_{DC}(t_0)$$
$$= (-1)^k \frac{i^0_{dc}(t_0)}{V_0\omega_0}\frac{dC_0(t)}{dt}\Pi(t - t_k)\Delta v_{AC}(t_k)$$
$$= \int h(t,\tau)\Delta v_{AC}(\tau)d\tau, \quad (9)$$

where $h(t,\tau)$ is the impulse response. If all zero-crossings are considered, the impulse response is hence

given by

$$h(t,\tau) = \frac{i_{dc}^0(t_0)}{V_0\omega_0}\frac{dC_0(t)}{dt}\sum_k(-1)^k\Pi(t-t_k)\delta(\tau-t_k). \tag{10}$$

The time periodic transfer function becomes

$$\begin{aligned} H(s,t) &= e^{-st}\int h(t,\tau)e^{s\tau}d\tau \\ &= e^{-st}\frac{i_{dc}^0(t_0)}{V_0\omega_0}\frac{dC_0(t)}{dt}\sum_k(-1)^k\Pi(t-t_k)e^{st_k}, \end{aligned}$$

which gives

$$\begin{aligned} \hat{H}_k(s) &= \frac{1}{T}\int_0^T H(s,t)e^{-jk\omega_0 t}dt \\ &= \frac{i_{dc}^0(t_0)}{V_0 2\pi}\left(\int_0^{T/2}\frac{dC_0(t)}{dt}e^{-(s+jk\omega_0)t}dt \right. \\ &\qquad \left. -\int_{T/2}^T\frac{dC_0(t)}{dt}e^{sT/2}e^{-(s+jk\omega_0)t}dt\right) \\ &= \frac{i_{dc}^0(t)}{V_0 2\pi}\int_0^{T/2}\sum_l(1-e^{j(l-k)\omega_0 T/2})jl\omega_0 c_l e^{-(s+j(k-l)\omega_0)t}dt \\ &= -\frac{i_{dc}^0(t)}{V_0 2\pi}\sum_l(1-(-1)^{k-l})jl\omega_0 c_l\frac{e^{-(s+j(k-l)\omega_0)T/2}-1}{s+j(k-l)\omega_0}. \end{aligned}$$

The HTF $D(s)$ is then obtained as in Appendix A.

C *The DC-Side*

A DC motor consists of two windings, the rotating armature winding and field winding. Due to the rotation an electro-magnetic force, $e_a$, is induced in the armature winding, $e_a = k_1\Phi_s(i_s)\omega$, the flux $\Phi_s$ is a function of the stator current and $\omega$ is the rotor speed.

For a series excited DC-motor $i_s = i_a$. Assuming the speed of the train to be constant, a linearized model for the DC motor can be seen as a resistor, $\Delta e_a = R_{ind}\Delta i_a$, where $R_{ind}$ depends on $i_s^0$ and $\omega^0$. The transfer function from DC-voltage to DC-current is hence given by

$$\Delta I_{DC}(s) = \frac{1}{R+sL}\Delta V_{DC}(s) = G(s)\Delta V_{DC}(s),$$

where, $L = L_c + L_a + L_e$ is the sum of the choke inductance and the inductances in the armature and field windings, and similarly $R = R_c + R_a + R_e + R_{ind}$.

D *Assemblying the Loco*

The model for the diode converter including the DC side dynamics is shown in Fig. 5. The HTF, $H_{db}(s)$, for the diode bridge rectifier and the DC side dynamics is hence given by

$$\begin{aligned} \Delta I_{AC}(s) &= H_{db}(s)\Delta V_{AC}(s) \\ &= (C_0(s)\tilde{G}(s)B_0(s) + D(s))\Delta V_{AC}(s), \end{aligned}$$

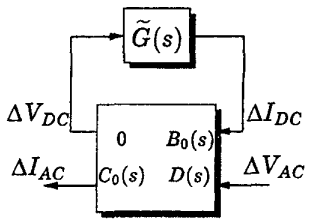


**Fig. 5** The diode converter with the DC dynamics $\tilde{G}(s)$ connected. The resulting HTF, $H_{db}(s)$, is then connected with a model of the transformer $\tilde{Z}_{traf}(s)$, see Fig. 6.

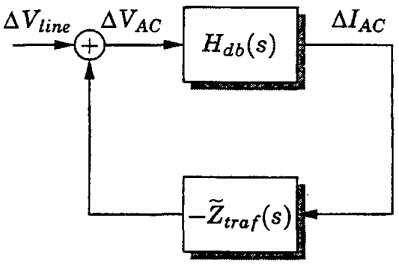


**Fig. 6** The electrical network described by a feedback connection. The relation between harmonic disturbances on the line and in the AC-current is given by $H_{loco}(s)$, see x

where

$$\tilde{G}(s) = \mathrm{diag}(\ldots \quad G(s-j\omega_0) \quad G(s) \quad G(s+j\omega_0) \quad \ldots). \tag{11}$$

The transformer is modeled as an ideal transformer plus an equivalent impedance on the low voltage side. The effect of the impedance is shown in Fig. 6 and gives $\Delta I_{AC}(s) = H_{loco}(s)\Delta V_{line}(s)$, where

$$H_{loco}(s) = (I + H_{db}(s)\tilde{Z}_{traf}(s))^{-1}H_{db}(s). \tag{12}$$

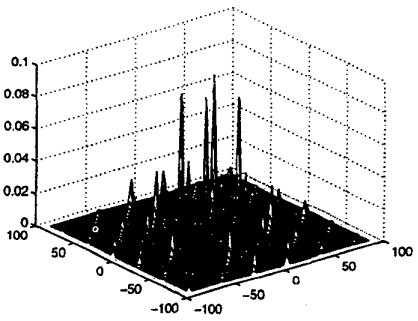


**Fig. 7** The amplitude plot of the HTF $H_{loco}$. Notice the large out-diagonal bands, illustrating the nonlinear coupling between different frequencies in $V_{line}$ and $I_{AC}$.

In Fig. 7, the amplitude of $H_{loco}$ is plotted. The diagonal structure shows that there is only coupling between frequencies separated by $33\frac{1}{3}$ Hz.

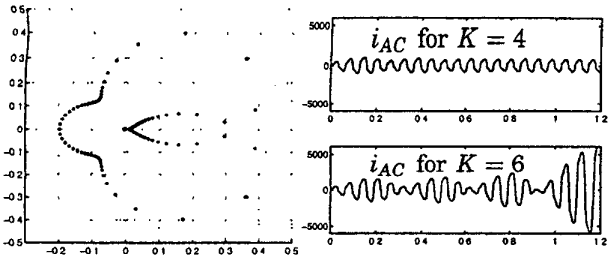


**Fig. 8** The Harmonic Nyquist plot, i.e. the eigenvalues of $\tilde{Z}_{grid} H_{loco}(j\omega)$ for $-\omega_0/2 \leq \omega \leq \omega/2$, for the loco attached to the line. Notice that the curve indicates a harmonic amplitude margin of $A_m = 5$. This corresponds well with the results from time domain simulations.

## IV. A NYQUIST CRITERION FOR LTP SYSTEMS

On matrix form it is clear that an LTP system formally can be treated as a LTI system with infinitely many inputs and outputs. Transmission zeros and poles can thus be derived from theory for multi-input multi-output (MIMO) systems, see for instance [9]. These poles determine the stability of the system.

A Nyquist criterion for LTP systems based on HTFs was presented in [14]. It is based on the generalized Nyquist criterion for MIMO systems. It states that stability of the closed loop system can be determined by plotting the eigenvalue curves of the open loop HTF, $H(j\omega)$ for $-\omega_0/2 < \omega < \omega_0/2$. If the open loop system is stable, and the Nyquist curve does not enclose the point $-1$, then the closed loop system is stable.

Consider the locomotive connected to a fictive (non-passive) line environment, modeled as an impedance given by $Z_{grid}(s) = 5000K/(s^2 + 5s + 5000)$. The open loop system is given by $\tilde{Z}_{grid}(s)H_{loco}(s)$ where $\tilde{Z}_{grid}(s)$ is defined as in (11).

In Fig. 8, the Nyquist plot for the HTF of the locomotive and the grid is plotted for $K = 1$. The curve crosses the negative real axis at $s \approx -0.2$. The Nyquist criterion states that the system is stable for $K < A_m = 1/0.2 = 5$. A time domain simulation shows that the system is stable for $K = 4$ but not for $K = 6$. This indicates that the Nyquist criterion does a good job in predicting harmonic stability of the full nonlinear locomotive. The derived model can now be used for analysis, such as harmonic interaction with other trains on the same line.

## V. CONCLUSIONS

The Harmonic transfer function method of modeling linear time periodic systems has been described. A HTF model has been derived for a diode converter locomotive. The model has been verified with time domain simulations and is a good starting point for further analysis of resonance risks and harmonic interaction.

## VI. ACKNOWLEDGEMENT

The model of the diode converter loco was kindly provided by Markus Meyer at Adtranz, Zürich. The work was financially supported by Elforsk AB under Elektra project 3320.

## APPENDIX A. ANALYSIS OF LTP SYSTEMS

The transfer function plays a central role in stability and robustness analysis as well as control design. For LTI-systems, the transfer function is a linear operator on the class of exponentially modulated sinusoids

$$u(t) = U_0 e^{st}, \qquad y(t) = H_0(s)U_0 e^{st} = Y_0 e^{st}.$$

To get a transfer function for LTP systems, we need a corresponding class of test signals. For an LTP system on state space form, Floquet decomposition reveals that a suitable test signal is the class of exponentially modulated periodic (EMP) signals, see [14]

$$u(t) = e^{st} \sum_m U_m e^{jm\omega_0 t} = \sum_m U_m e^{(s+jm\omega_0)t}. \tag{13}$$

Not all systems have a state space representation. A general LTP system can be defined by its impulse response, $h(t,\tau)$. The periodicity of the system implies that

$$h(t+T, \tau+T) = h(t,\tau), \tag{14}$$

where $T$ is the period time. Assume the input, $U(s)$, is given in Laplace domain. The corresponding time domain signal is

$$u(t) = \mathcal{L}^{-1}(U)(t) = \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} e^{st} U(s) ds.$$

This gives the following output

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} h(t,\tau)u(\tau)d\tau \\ &= \int_{-\infty}^{\infty} h(t,\tau) \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} e^{s\tau} U(s) ds d\tau \\ &= \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} e^{st} H(s,t) U(s) ds. \end{aligned}$$

Here, $H(s,t)$ is a time periodic transfer function

$$H(s,t) = e^{-st} \int_{-\infty}^{\infty} e^{s\tau} h(t,\tau) d\tau$$

which is periodic in $t$. The periodicity implies that $H(s,t)$ can be written as a Fourier series with the fundamental frequency $\omega_0 = 2\pi/T$

$$H(s,t) = \sum_k \hat{H}_k(s) e^{jk\omega_0 t},$$

$$\hat{H}_k(s) = \frac{1}{T} \int_0^T H(s,t) e^{-jk\omega_0 t} dt.$$

The output can now be written

$$y(t) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty}\sum_k \hat{H}_k(s)e^{(s+jk\omega_0)t}U(s)ds$$
$$= \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty}\sum_k \hat{H}_k(s-jk\omega_0)e^{st}U(s-jk\omega_0)ds.$$

Here, we recognize the definition of the inverse Laplace transform. In Laplace domain, the output is hence

$$Y(s) = \sum_k \hat{H}_k(s-jk\omega_0)U(s-jk\omega_0). \qquad (15)$$

From this we conclude that for LTP-systems there is coupling between frequencies that are separated by a multiple of the fundamental frequency of the system, $\omega_0$. Laplace transformation of the EMP signal defined by (13) gives

$$U = 2\pi\sum_m U_m\delta_{s+jm\omega_0}$$

Equation (15) gives that the output too is an EMP signal

$$Y = 2\pi\sum_n Y_n\delta_{s+jn\omega_0},$$

where

$$Y_n = \sum_m \hat{H}_{n-m}(s_0+jm\omega_0)U_m.$$

The doubly-infinite matrix in Section II is hence given by

$$H_{n,m}(s) = \hat{H}_{n-m}(s+jm\omega_0). \qquad (16)$$

## VII. References


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