M-ary Coded Modulation by Butterworth Filtering

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Abstract — **Earlier we have studied the bandwidth and distance of binary codes created by Butterworth lowpass filtering of a pulse stream. Here the work is extended to 4- and 8-ary inputs (16- and 64-ary QAM). Performance reaches into a very narrowband part of the distance–bandwidth plane. An M-algorithm decoder is tested.**

The CPM, TCM and partial response signaling (PRS) classes of coded modulation are well known. In the last class are intersymbol interference removal and codes based on filtering and real-number discrete-time convolution. These classes operate in different parts of the energy–bandwidth plane and PRS coding in particular is characterized by narrow bandwidth and high energy per bit. Here we synthesize PRS-type codes by ordinary but severe filtering of a simple pulse modulation. In earlier work we applied binary modulation to a Butterworth filter, and here we extend that to 4- and 8-ary modulation (16- and 64-ary in QAM application). We calculate the bandwidth and minimum distance $d_{\min}$ of the new schemes. We also test a practical M-algorithm decoder.

Codewords are patterned real-time signals $s(t)$ made up as follows. A pulse train $\sum a_n v(t-nT)$ is formed, where a_n are equi-spaced zero-mean m-ary IID data, $m = 2, 4, 8$, and $v(t)$ is a T-orthogonal pulse. The encoded baseband signal is the convolution of this train with a Butterworth lowpass filter impulse response $g(t)$. We consider 4- and 8-pole filters. The form of $v(t)$ makes little difference at narrow bandwidths; we take a 30% excess bandwidth root-RC pulse. By changing the filter cutoff frequency, codes with different bandwidths and minimum distances are created. Because the modulation is linear, the bandwidth of the signals per data bit is that of the filter, divided by $\log_2 m$.

The normalized Euclidean distance between two of these signals can be calculated via several methods [1], and it depends only on the difference between the symbol sequences, $\Delta\boldsymbol{a}$. We have obtained good refinements to the basic error estimate $KQ(\sqrt{d^2_{\min}E_b/N_o})$ by counting nearest neighbors; K is almost always less than 1.

I. Butterworth Bandwidth and Distance

As the filter cutoff frequency drops, $d^2_{\min}$ remains constant until a critical cutoff is reached. This value is $d^2_{\min}$ for unfiltered m-ary orthogonal-pulse modulation, 4/5 and 2/7 for $m = 4$ and 8. The critical cutoff frequencies are $.31/T$ and $.35/T$ Hz for 4 and 8 poles (and both alphabets). This leads to 99% power bandwidths $.41/T$ and $.40/T$ Hz. A careful study shows that some of the distance between two signals escapes to wide bandwidths, outside the 99% bandwidth. By calculating the distance in the spectral domain, $d_{\min,.99}$ inside the 99% bandwidth can be precisely measured. At very narrow bandwidths, a large fraction of the minimum distance can be so lost.

Figure 1 compares $d^2_{\min}$ versus normalized 99% bandwidth for 2-, 4- and 8-ary input alphabets. Each alphabet has a range of bandwidth over which it works best; as bandwidth falls, the best distance comes with larger alphabet size.

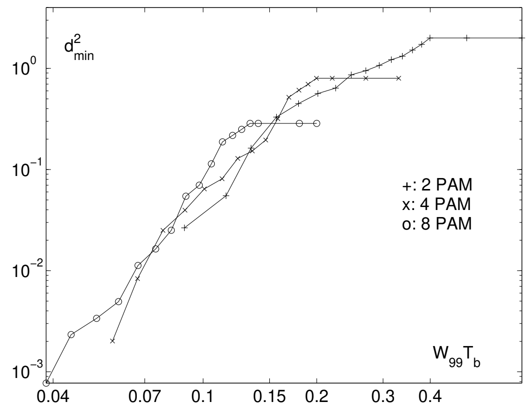

Figure 1: Comparison of $d^2_{\min}$ versus bit normalized 99% bandwidth, for 2-, 4- and 8-ary 8-pole Butterworth codes. Log-log plot.

II. Decoding

An effective decoder for these codes is the M-algorithm. An infinite impulse response code has an infinite trellis if states are based in the usual way on data symbols; the Butterworth response is 10–30 symbols in practice. Thus a Viterbi decoder is impractical. A practical and relatively simple way to decode this is to base an M-algorithm on the 4 or 8 circuit state variables of the Butterworth filter[1]; the decoder identifies all signal paths by their present state variables, updates these variables, and keeps only the M best paths. Distances are computed directly from the analog signals. We find that $M \approx 70$ suffices for 8-ary transmission at cutoff $.4/T$ Hz, and 200 suffices for cutoff $.1/T$; 4-ary transmission requires about 1/4 these paths. The 4-ary $.2/T$ 8-pole case yields a scheme with bandwidth $\approx .11$ Hz-s/data bit and error rate close to $KQ(\sqrt{d^2_{\min}E_b/N_o})$, with $d^2_{\min} = .0814$ and $K = .66$. The decoder needs $M \approx 40$ and $d^2_{\min,.99} = .067$.

References

[1] J.B. Anderson and A. Svensson, *Coded Modulation Systems*, Plenum/Kluwer, New York, 2003.

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