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# Spatial/Spectral Filters for Microphone Arrays

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## ABSTRACT

The spatial and spectral filtering characteristics of a microphone array can be greatly enhanced through the use of an appropriately designed spatial/spectral filter. In this paper, we consider the design of optimum, in the minimax sense, spatial/spectral filters. The optimum minimax design problem is solved using the new efficient computational technique of Jiang *et al.* Numerical results demonstrating the effectiveness of the method are given.

## 1 Introduction

Recently, the study of broadband receiving antenna arrays operating in the nearfield has received much attention in the literature, particularly in the area of microphone array applications such as video conferencing [1, 2], hands-free mobile telephony [3, 4], hearing aids [5-7] and computer applications [8]. In these applications, an array of microphones, together with a spatial/spectral filter, are deployed to exploit the spatial and temporal characteristics of the propagating sound waves to enhance the detection and/or localisation of speakers in a noisy and reverberant environment. The function of the spatial/spectral filter is to take the outputs of the microphones and process them in such a way so that the overall microphone system exhibits a certain frequency response to signal sources located in a specified region in space, and at the same time, is able to suppress all other signal sources located outside this spatial acceptance region.

In this paper, we focus on the design of optimum spatial/spectral filters. By “optimum”, we mean filters whose parameters are derived by solving an optimisation problem. The optimisation problem is formulated from the point of view of requiring the overall system to exhibit a response that is as close as possible, in some sense, to a desired response. The design approach we have adopted here is the minimax approach; and in contrast to the traditional methods which discretize the decision space into a number of grid points, we shall follow the approach of [9] and [10] which performs the design over all points in the decision space. Indeed, the methods of [9] and [10] have many similarities. The minimax problem is first transformed to an equivalent semi-infinite programming problem. Constraint transcription is then applied to the problem. Finally, the solution is obtained through solving a sequence of minimisation problems. The main difference is that the method of [10] requires two parameters to be continuously varied in the solution sequence which may cause the algorithm to fail to converge, while the method of [9] uses an auxiliary cost function to define the original problem and the behaviour of this function is such that the optimal solution to the original problem occurs at its first root.

In Section 2, we formulate the optimal filter design problem. The solution method of [9] is then described in Section 3. An illustrative design example is given in Section 4. Section 5 gives the conclusions.

## 2 Problem Statement

Consider the  $L$ -element microphone array shown in Fig. 1 where  $\mathbf{r}_l$ ,  $l = 1, \dots, L$  are the position vectors of the microphones. The outputs of the microphones are first delayed by the optional delay elements  $\Delta_l$ ,  $l = 1, \dots, L$ . They are then sampled and digitised at a rate of  $1/T$  samples per second before being fed to the spatial/spectral filter. This filter consists of  $L$   $N$ -tap finite impulse response (FIR) filters. Fig. 2 shows the structure of an  $N$ -tap FIR filter. The minimax optimum filter design problem can be posed as follows.

$$\min_{\mathbf{w} \in \mathbb{R}^{NL}} \max_{(\mathbf{r}, f) \in \Omega} v(\mathbf{r}, f) \left| \mathbf{w}^T \mathbf{d}(\mathbf{r}, f) - G_d(\mathbf{r}, f) \right| \quad (\text{P1})$$

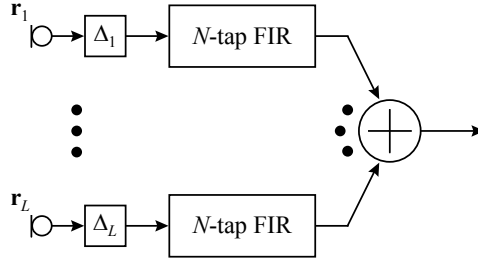
where

$$\mathbf{d}(\mathbf{r}, f) = \left[ A_1(\mathbf{r}, f) e^{-j2\pi f \Delta_1} \mathbf{d}_o^T(f) \quad \dots \quad A_L(\mathbf{r}, f) e^{-j2\pi f \Delta_L} \mathbf{d}_o^T(f) \right]^T, \quad (1)$$

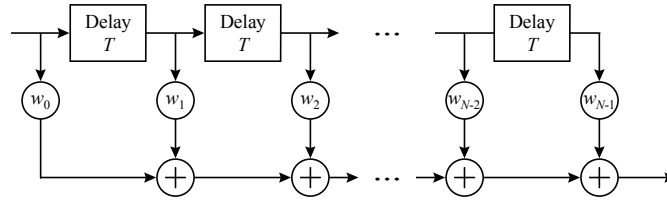
$$\mathbf{d}_o = \left[ 1 \quad e^{-j2\pi f T} \quad \dots \quad e^{-j2\pi f (N-1)T} \right]^T, \quad (2)$$

$$A_l(\mathbf{r}, f) = \frac{1}{\|\mathbf{r} - \mathbf{r}_l\|} \exp\left(-j2\pi f \frac{\|\mathbf{r} - \mathbf{r}_l\|}{c}\right), \quad l = 1, \dots, L, \quad (3)$$

and where  $j = \sqrt{-1}$ ,  $G_d(\mathbf{r}, f)$  is the desired response of the microphone array system,  $\mathbf{w}^T \mathbf{d}(\mathbf{r}, f)$  is the actual response,  $v(\mathbf{r}, f)$  is a positive weighting function, and  $c$  is the speed of sound in air.  $A_l(\mathbf{r}, f)$  is the transfer function from a spatial point with position vector  $\mathbf{r}$  to the  $l$ th microphone located at  $\mathbf{r}_l$ .



**Fig. 1:** Microphone Array with Spatial/Spectral Filter



**Fig. 2:**  $N$ -tap FIR Filter

### 3 Solution Method

#### 3.1 Background Theory

We first state the main results of [9] as applied to problem (P1). Note that (P1) is unconstrained while [9] considers the more general case of constrained minimax problems.

Denote the cost function of (P1) by  $E(\mathbf{w}, \mathbf{r}, f)$ , i.e.

$$E(\mathbf{w}, \mathbf{r}, f) = v(\mathbf{r}, f) \left| \mathbf{w}^T \mathbf{d}(\mathbf{r}, f) - G_d(\mathbf{r}, f) \right|. \quad (4)$$

Suppose  $\Omega$  is a compact set of  $\mathcal{R}^4$  and  $E(\mathbf{w}, \mathbf{r}, f)$  is continuously differentiable within  $\Omega$ . The problem (P1) can be rewritten as a semi-infinite programming problem as follows.

$$\begin{aligned} & \min_{z \in \mathcal{R}} z \\ & \text{subject to} \quad E(\mathbf{w}, \mathbf{r}, f) \leq z, \quad \forall \mathbf{w} \in \mathcal{R}^{NL}, \forall (\mathbf{r}, f) \in \Omega. \end{aligned} \quad (\text{P2})$$

The key idea of [9] is to apply constraint transcription to the constraint of (P2) to yield the following auxiliary function.

$$J_0(\alpha) = \min_{\mathbf{w} \in \mathcal{R}^{NL}} \iint_{\Omega} p(E(\mathbf{w}, \mathbf{r}, f) - \alpha) d\mathbf{r} df \quad (5)$$

$$\text{where} \quad p(t) = \begin{cases} 0, & t < 0 \\ t, & t \geq 0 \end{cases}. \quad (6)$$

The solution of (P1) is given by the first root of  $J_0(\alpha)$ .

Clearly,  $p(t)$  is non-smooth. The optimisation problem in eq. (4) is thus numerically difficult since the gradient of  $p(t)$  is undefined at  $t=0$ . In [9],  $p(t)$  is approximated by the following smooth function where  $\varepsilon$  is the size of the smoothing interval:

$$g_{\varepsilon}(t) = \begin{cases} 0, & t \leq -\varepsilon \\ \frac{(t + \varepsilon)^2}{4\varepsilon}, & -\varepsilon < t \leq \varepsilon \\ t, & t > \varepsilon \end{cases}. \quad (7)$$

The approximate problem to eq. (5) is thus given by

$$J(\varepsilon, \alpha) = \min_{\mathbf{w} \in \mathcal{R}^{NL}} \Phi(\mathbf{w}, \varepsilon, \alpha) \quad (8)$$

$$\text{where} \quad \Phi(\mathbf{w}, \varepsilon, \alpha) = \iint_{\Omega} g_{\varepsilon}(E(\mathbf{w}, \mathbf{r}, f) - \alpha) d\mathbf{r} df. \quad (9)$$

The following results establish the relationship between the solution of the approximate problem and the solution of the original minimax problem.

Firstly, from the definitions of  $g_\varepsilon(t)$  and  $\Phi(\mathbf{w}, \varepsilon, \alpha)$ , it can be shown that:

Lemma 1

- (a) For any  $\varepsilon > 0$ ,  $\Phi(\mathbf{w}, \varepsilon, \alpha)$  is continuously differentiable in  $\mathbf{w}$  and  $\alpha$ .
- (b) For any  $\alpha \in \mathcal{R}$  and  $0 \leq \varepsilon_1 \leq \varepsilon_2$ ,  $J(\varepsilon_1, \alpha) \leq J(\varepsilon_2, \alpha)$ .
- (c) For any  $\varepsilon > 0$  and  $\alpha_1 \leq \alpha_2$ ,  $J(\varepsilon, \alpha_1) \geq J(\varepsilon, \alpha_2)$ .
- (d)  $J(\varepsilon, \alpha)$  is non-negative and is continuous in  $\varepsilon$  and  $\alpha$ .
- (e) For any  $\varepsilon > 0$ , let  $\alpha_\varepsilon^* = \min\{\alpha \in \mathcal{R}: J(\varepsilon, \alpha) = 0\}$ . Then,  $\alpha_\varepsilon^*$  is a quadratic zero of  $J(\varepsilon, \alpha)$  as a function of  $\alpha$ .

Next, suppose all partial derivatives of  $E(\mathbf{w}, \mathbf{r}, f)$  are bounded, i.e.  $\forall(\mathbf{w}, \mathbf{r}, f) \in \mathcal{R}^{NL} \times \Omega$

$$\left| \frac{\partial E(\mathbf{w}, \mathbf{r}, f)}{\partial w_i} \right| \leq M_1, \text{ for } i = 1, \dots, NL \quad (10)$$

$$\left| \frac{\partial E(\mathbf{w}, \mathbf{r}, f)}{\partial r_i} \right| \leq M_2, \text{ for } i = 1, 2, 3 \quad (11)$$

and

$$\left| \frac{\partial E(\mathbf{w}, \mathbf{r}, f)}{\partial f} \right| \leq M_2. \quad (12)$$

It then follows that:

Lemma 2

For any  $\varepsilon > 0$  and  $\alpha \in \mathcal{R}$ , if  $\mathbf{w}_\varepsilon$  satisfies

$$\Phi(\mathbf{w}_\varepsilon, \varepsilon, \alpha) = J(\varepsilon, \alpha) < \frac{\varepsilon^2}{16M_1}, \quad (13)$$

then

$$E(\mathbf{w}_\varepsilon, \mathbf{r}, f) < \alpha, \quad \forall(\mathbf{r}, f) \in \Omega. \quad (14)$$

Theorem 1

For any  $\varepsilon > 0$ , suppose  $\alpha$  satisfies

$$0 < J(\varepsilon, \alpha) < \frac{\varepsilon^2}{16M_1}, \quad (15)$$

then

$$\min_{\mathbf{w} \in \mathcal{R}^{NL}} \max_{(\mathbf{r}, f) \in \Omega} E(\mathbf{w}, \mathbf{r}, f) \leq \alpha \leq \min_{\mathbf{w} \in \mathcal{R}^{NL}} \max_{(\mathbf{r}, f) \in \Omega} E(\mathbf{w}, \mathbf{r}, f) + \varepsilon \quad (16)$$

### Theorem 2

For any  $\varepsilon > 0$ , let  $\mathbf{w}_\varepsilon \in \mathcal{R}^{NL}$  and  $\alpha \in \mathcal{R}$  be such that  $\Phi(\mathbf{w}_\varepsilon, \varepsilon, \alpha) = J(\varepsilon, \alpha)$ , then if the following condition is satisfied

$$J(\varepsilon, \alpha) \geq \frac{\varepsilon}{4} |\Omega|, \quad (17)$$

where  $|\Omega|$  is the Lebesgue measure of the set  $\Omega$ , then

$$\alpha \leq \alpha^* = \min_{\mathbf{w} \in \mathcal{R}^{NL}} \max_{(\mathbf{r}, f) \in \Omega} E(\mathbf{w}, \mathbf{r}, f). \quad (18)$$

### **3.2 Computational Procedure**

Based on the above theoretical results, the following computational procedure is described in [9] to obtain an approximate solution to the minimax optimisation problem (P1).

Step 1: Select the parameters  $\varepsilon, \delta$ .

Step 2: Calculate the initial values  $\alpha_1, \alpha_2$  by choosing 2 points  $(\mathbf{r}_1, f_1), (\mathbf{r}_2, f_2) \in \Omega$  and solving

$$\alpha_1 = \min_{\mathbf{w} \in \mathcal{R}^{NL}} E(\mathbf{w}, \mathbf{r}_1, f_1) \quad \text{and} \quad \alpha_2 = \min_{\mathbf{w} \in \mathcal{R}^{NL}} E(\mathbf{w}, \mathbf{r}_2, f_2).$$

Set  $\alpha_1 < \alpha_2$ .

Step 3: Calculate  $J(\varepsilon, \alpha_1)$ .

Step 4: If  $J(\varepsilon, \alpha_1) = 0$ , decrease  $\varepsilon$  and go to Step 3.

Step 5: Calculate  $J(\varepsilon, \alpha_2)$ .

Step 6: If  $J(\varepsilon, \alpha_2) = 0$ , let  $\alpha_3 = \alpha_1 + 0.618 \times (\alpha_2 - \alpha_1)$ . Replace the value of  $\alpha_2$  by the value of  $\alpha_3$ . Go to Step 5.

If  $J(\varepsilon, \alpha_2) \geq \delta$ , let  $\alpha_3 = \alpha_2 + \frac{\alpha_2 - \alpha_1}{\sqrt{\frac{J(\varepsilon, \alpha_1)}{J(\varepsilon, \alpha_2)} - 1}}$ . Replace the values of  $\alpha_2$  by  $\alpha_3$  and

$\alpha_1$  by  $\alpha_2$ . Go to Step 5.

Step 7:  $J(\varepsilon, \alpha_2) < \delta$ . Check whether the solution satisfies  $E(\mathbf{w}, \mathbf{r}, f) \leq \alpha_2$ . If yes, then stop. Otherwise, reduce  $\delta$  and go to Step 5.

Note that the algorithm ensures the optimum solution  $\alpha^*$  is obtained to an accuracy determined by the user. In [9], it is shown that

### Theorem 3

- (a) The computational procedure terminates in a finite number of steps.
- (b) Denote the successive  $\alpha_1$ 's obtained by  $\mu_i, i = 1, 2, \dots, (M-1)$ . Denote the last  $\alpha_2$  obtained by  $\mu_M$ . Then the optimal solution to the auxiliary function satisfies  $\alpha^* \in (\mu_{i0}, \mu_M]$  where  $\mu_{i0}$  and  $\mu_M$  (lower and upper bound respectively) satisfy the following inequalities.

$$J(\varepsilon, \mu_{i0}) \geq \frac{\varepsilon}{4}|\Omega|, \quad (19)$$

$$J(\varepsilon, \mu_{i0+1}) < \frac{\varepsilon}{4}|\Omega|, \quad (20)$$

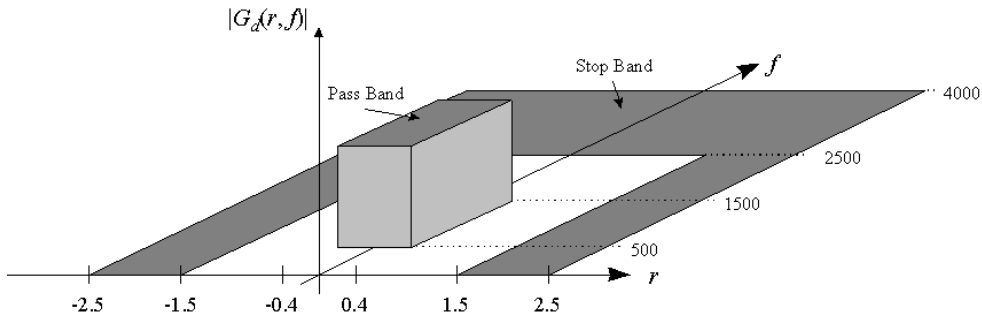
$$\text{and} \quad 0 < J(\varepsilon, \mu_M) \leq \frac{\varepsilon^2}{16M_1}. \quad (21)$$

Note that the algorithm does not guarantee the global solution is obtained. Therefore, the performance depends on the behaviour of the local solutions of the objective function considered. Different starting points may be used to as an attempt to find better solutions.

#### 4 Design Example

In hands-free mobile telephony, an array of microphones is typically positioned horizontally in front of the car driver, usually along the dashboard. The array is typically linear with the microphones equally spaced apart. Because of the car seat, only sideways motion of the driver's head is possible. Also, because of the directional characteristics of the microphones, the response of the overall system will intrinsically be relatively independent of the height of the driver. One can assume that the distance of the driver's mouth from the dashboard is fixed at about 1 m.

In the design example, we assume an array of 5 microphones and that the origin of the positional coordinate system is located at the position of the centre microphone. The orientation of the coordinate system is such that the microphones are arranged along the  $x$ -axis. The position vector of the microphones are thus given by  $(-0.1, 0, 0)$ ,  $(-0.05, 0, 0)$ ,  $(0, 0, 0)$ ,  $(+0.05, 0, 0)$  and  $(+0.1, 0, 0)$ . The desired response of the microphone system is shown in Fig. 3 where  $f$  is the frequency axis, and  $r$  is the horizontal displacement of the driver's mouth from the central position, i.e. the position vector of the driver's mouth is given by  $(r, 1, 0)$ . The spatial acceptance region extends, therefore, from  $r = -0.4$  to  $r = 0.4$  and in the acceptance region, the desired frequency response is from 500 Hz to 1,500 Hz. The region between the pass band and stop band regions, as shown in Fig. 3, is a "don't care" transitional region. Finally, we assume that each of the FIR filters in the spatial/spectral filter has 7 taps.



**Fig. 3:** Desired Response of Spatial/Spectral Filter

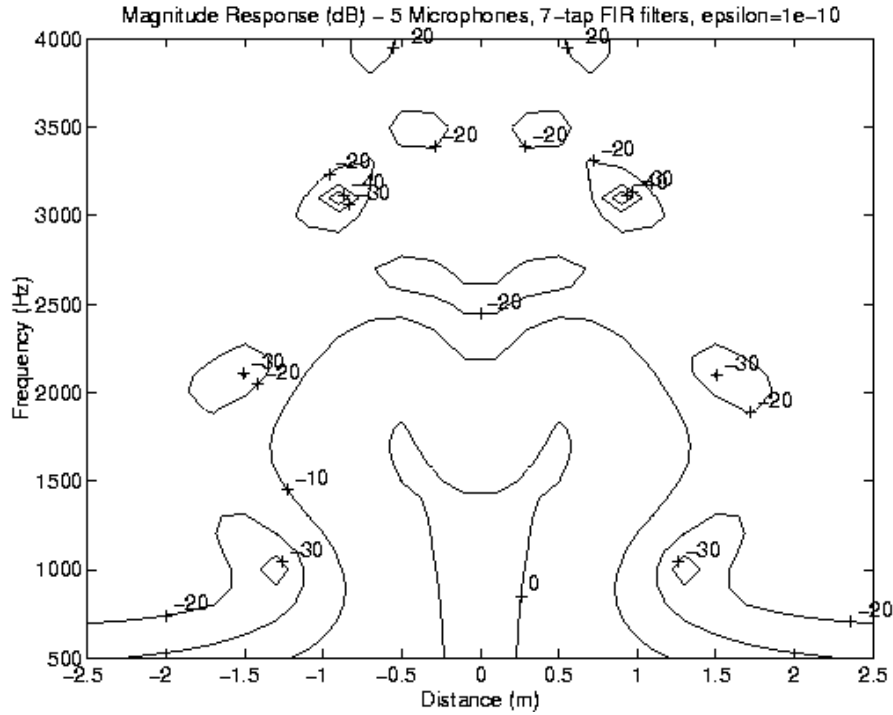
In the solution of the optimal filter design problem, we exploited the symmetry of the array structure to reduce the number of decision variables. In particular, it can be shown that, as

a consequence of the array symmetry, the filter coefficients of the two microphones located at either side of the centre microphone and at the same distance from the centre microphone, are identical. The results obtained from the computer program for different values of  $\varepsilon$  are summarised in Table 1. The magnitude response for  $\varepsilon = 10^{-10}$  is displayed in Fig. 4. Note that the response is plotted in decibels (dB).

As can be seen from Fig. 4, the microphone array is able to provide about 20 dB or more suppression in the stop band. Further suppression can be obtained by either increasing the number of microphone elements, or the number of taps in the FIR filters, or both.

| $\varepsilon$ | $10^{-7}$               | $10^{-8}$               | $10^{-9}$               | $10^{-10}$               |
|---------------|-------------------------|-------------------------|-------------------------|--------------------------|
| $\delta$      | $1.9073 \times 10^{-8}$ | $3.7253 \times 10^{-8}$ | $2.3842 \times 10^{-9}$ | $1.4901 \times 10^{-10}$ |
| $\mu_{i0}$    | 0.18514                 | 0.20656                 | 0.21588                 | 0.21595                  |
| $\mu_M$       | 0.21858                 | 0.21860                 | 0.21856                 | 0.21864                  |

**Table 1:** Computation Results of Design Example



**Fig. 4:** Response of Optimal Spatial/Spectral Filter.

## 5 Conclusion

In this paper, we have successfully applied a new and rather elegant optimisation technique to solve a minimax spatial/spectral filter design problem. Unlike the conventional techniques

which discretise the  $(\mathbf{r}, f)$ -space, the new technique performs the design over all points in the space. The new technique is also computationally efficient.

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