Frequency Domain Adaptive Control

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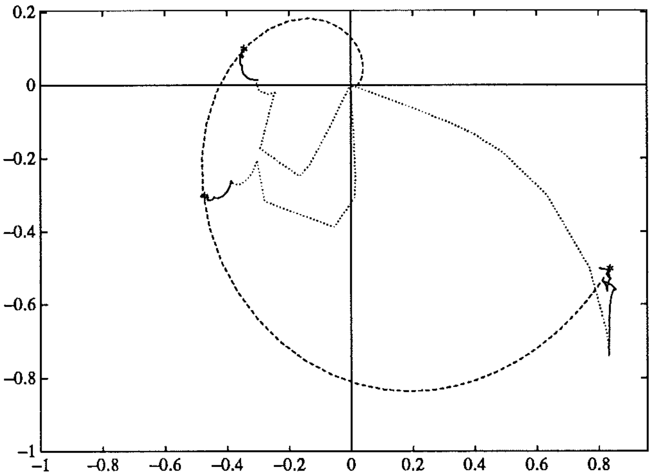

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Abstract

Adaptive controllers are normally based on the assumption that the process can be described by a linear model of known order. Also they rely on the certainty equivalence principle i.e. the model uncertainty is not taken into account in the controller design. In practice, process modeling is always approximate. This raises the question of how robust adaptive controllers are in view of uncertainties and undermodeling. In the literature it has been argued that both the estimation and the controller design must be made robust to uncertainties and undermodeling in order to obtain a robust adaptive controller. This thesis is a step in that direction, not by relying on uncertainty measures but instead by not basing the scheme on a rational transfer function model of the process. Instead both the process modeling and the controller design is formulated in the frequency domain. The process is in the scheme represented by a set of points on the Nyquist curve of the process. The actual process order is then irrelevant from a modeling point of view. The controller design is formulated as an approximation problem in the frequency domain. In this the closed loop response to command signals is fitted to a desired response supplied by the user. The explicit solution to a least squares problem gives the controller parameters. The necessary process knowledge is obtained from a bank of low order estimators. By using a bank of band pass filters the process estimation is decoupled in the frequency domain. Low order controllers can in the scheme be used to control high order plants.

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1

Introduction

1.1 Background

Adaptive controllers are traditionally based on the principle of separation of estimation and control. The design methods used are often quite simplistic. Since the models obtained from the parameter estimation are uncertain it would be desirable to use robust design methods in the adaptive controllers. This thesis is a step in this direction.

There are many approaches to robust control: H_∞ (Francis (1987), Doyle *et al.* (1989), Stoorvogel (1990)), parametric uncertainty (Doyle *et al.* (1982), Weinmann (1991)) and quantitative feedback theory (Horowitz (1963), D'Azzo and Houpis (1988)). Unfortunately none of these methods are well matched to the uncertainty measures. In this thesis an adaptive controller based on a design method developed by Lilja (1989) is investigated. This method which originates from Levy (1959) is based on a frequency domain approach. A number of points on the Nyquist curve of the system are estimated and used in the controller design.

1.2 Estimation

A number of different methods can be used for estimating the frequency response of a system. Non-parametric spectral methods, Zhu (1990), Ljung (1987), Wellstead (1981), directly gives a frequency response estimate. In closed loop these methods can, however, not be used in the same way as for open loop identification. The reason for this is twofold. First correlation between the input and noise acting on the output, as in the feedback case, give biased estimates. Secondly the spectral methods disregard causality which in the worst case results in identifying the inverse of the feedback controller Gustavsson *et al.* (1977). In order to use spectral methods for process estimation in a closed loop configuration, spectral measures must be computed with respect to exogenous signals not correlated with the system noise, Unbehauen *et al.* (1987), and so rather large and continuous external excitation is required.

Based on parametric models the frequency response is trivially obtained by evaluating the frequency response of the models. In the parametric case different strategies are possible. To prevent from undermodeling high order models may be estimated and applied to model reduction schemes Wahlberg (1987). The requirements on excitation should be kept low in an adaptive controller and so overparameterized models are not well suited for the adaptive case. If the process order is known, a full order model can be estimated using any appropriate estimation scheme, Ljung (1987), Goodwin *et al.* (1984). Another possibility, that will be pursued in the thesis, is that of estimating a low order model on band pass filtered data. For high order plants the band pass filters must be rather narrow in order to obtain a good model fit in the pass band of the filter. By estimating one low order parametric model for each of the required points on the Nyquist curve an estimation scheme that is independent of the true process order is obtained. Also it will not suffer from the drawbacks of spectral methods encountered in closed loop estimation. Further, only disturbances inside the pass bands of the filters will influence the different estimated models and so disturbance rejection is automatically obtained. Using the frequency point estimates a design method based on Lilja (1989) is used to obtain a two-degree of freedom controller. The frequency response estimation can be regarded as an extension of the work by Hägglund and Åström (1991), who use a method to track one point on the Nyquist curve.

1.3 Proposed Scheme

The scheme proposed in this work has a structure similar to other adaptive controller schemes, see Figure 1.1. The contents of the building blocks are, however, different from that of many other adaptive controllers. The main building blocks are

- Time variable controller
- Process estimator
- Controller design block
- Filtering block

The building blocks that differ from the normal are described briefly below.

Frequency Domain Least Squares Controller Design

The controller used has a two degree of freedom structure. The parameters of the controller are obtained by solving a least squares problem in the frequency domain. The process is described by a discrete set of points on the Nyquist curve of the process. The fitting is made between a desired command signal response and the response of the closed loop system at a discrete set of frequency points.

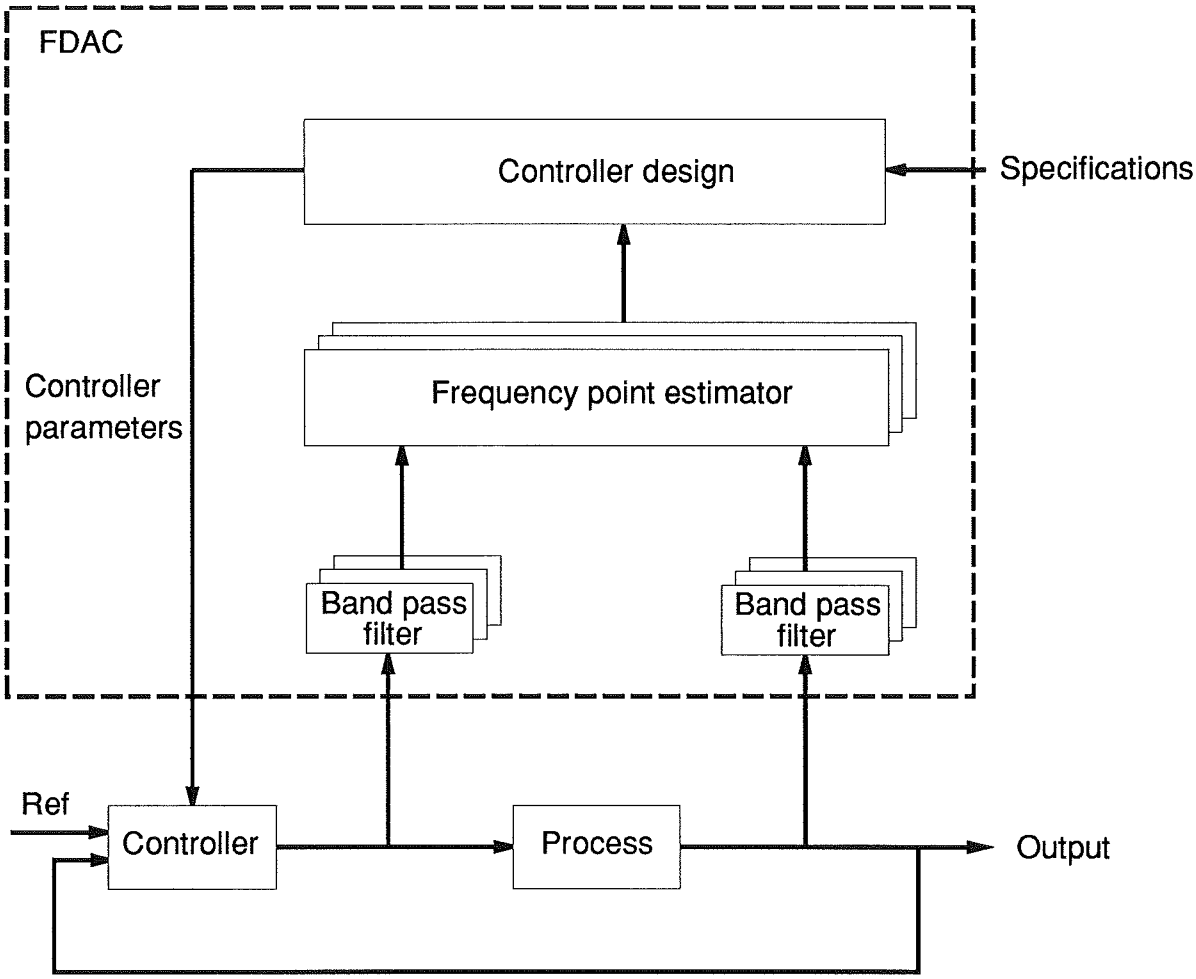

Figure 1.1 Block diagram of the adaptive controller

Frequency Domain Process Model Estimator

In the controller design the process is represented only by a discrete set of points on the Nyquist curve of the process. Therefore it is not necessary to estimate a full order parameterized process model. In this work the process estimation is split up into a bank of frequency point estimators. The output of each frequency point estimator is an estimate of a point on the Nyquist curve of the process. Each point is obtained by estimating a low order parametric process model. The data entering each frequency point estimator is band pass filtered process inputs and outputs. The frequency point is obtained by evaluating the low order process model at the corresponding frequency. By using a bank of frequency point estimators the estimation is decoupled at different frequency points.

Filtering

The data entering each frequency point estimator is band pass filtered. By doing so good modeling is possible for a low order model in the pass band of the filter. This is the case even for processes of high order. The width of the pass band is governed by two contradictory demands. First the filter should be sufficiently narrow in order for the process model to be good at the center frequency. This is important since the low order model is only evaluated at the center frequency of the filter. Secondly the filter should be sufficiently wide to have a response time that is not too long. With a long filter response time the filtered data will be influenced by old data that may no longer be valid due to more recent changes in the process characteristics. Also it will take time before new incoming data show up in the filtered data.

1.4 Summary

An adaptive scheme is proposed in this work that combines frequency domain estimation with frequency domain controller design. The obtained scheme is more robust to bad signal excitation and process assumptions than many other schemes. The content is organized as follows. Chapter 2 gives the problem setup. In Chapter 3 the frequency domain controller design is examined. Chapter 4 discusses the frequency point estimator. The complete adaptive scheme is given in Chapter 5. The filtering problem is handled in Chapter 6. In Chapter 7 design considerations are discussed. The adaptive scheme is evaluated through simulation examples in Chapter 8. Finally conclusions are given in Chapter 9.

The examples in Chapter 8 show that the overall adaptive scheme behaves well. After approximately 30 samples the frequency point estimates have negligible bias. By following the guidelines in Chapter 7 for the choice of desired response a good controller design is obtained. Therefore the closed loop system has a response closed to the desired after a few transients of excitation.

2

Problem Setup

2.1 Preliminaries

A Frequency Domain Adaptive Controller (FDAC) is discussed in this work. In this chapter the problem setup and some aspects related to the FDAC will be presented. Details are given in the chapters that follow.

2.2 Process and Process Representation

Only stable single input single output (SISO) systems are considered. The process is assumed to be time invariant and unknown or slowly time varying. By this assumption the adaptive controller has a reasonable chance of achieving parameter following. It is assumed that there is a decoupling in time scales between the process state dynamics and the rate of change of the process parameters. Variables that vary fast should in this context be considered as process states. It is assumed that the process at each time instant can be modeled by a pulse transfer function

$$Y(z) = G(z)U(z) + V(z) = \frac{B(z)}{A(z)}U(z) + V(z) \tag{2.1}$$

where U and Y are the Z-transform of the process input and output. Unmodeled dynamics as well as disturbances acting on the system are collected in V. V will not be studied further in the following since it is not used in the controller design. The simulations in chapter 8, however, indicate that the influence of V on the frequency response estimates is small.

Define a frequency set

$$\mathbf{\Omega} = \{\omega_1, \ldots, \omega_{\mathrm{M}}\} \tag{2.2}$$

To use the FDAC this set will be chosen by the user. Both the controller design and the process estimation will be made with respect to the chosen $\mathbf{\Omega}$. The frequencies ω_i can be chosen freely but has to be lower than the Nyquist frequency. Indications on how to choose these frequencies will, however, be given by the underlying design. The number of frequencies M can be chosen freely as long as M is larger than a lower limit. The lower limit is determined by the number of parameters in the controller.

Based on the frequency set above define the set

$$\mathbf{Z} = \{z_1, \ldots, z_{\mathrm{M}}\} = \{e^{i\omega_1 h}, \ldots, e^{i\omega_{\mathrm{M}} h}\} \quad \omega_i \in \mathbf{\Omega} \tag{2.3}$$

in the z-plane where h is the sampling interval. This set of points lies on the upper half of the unit circle in the z-plane when all ω_i are lower than the Nyquist frequency.

In the controller design the process will be represented by a set of points on the process Nyquist curve. The corresponding frequencies are given by the set $\mathbf{\Omega}$. For the discrete set of points on the Nyquist curve define

$$\mathbf{G} = \{G(z_1), \ldots, G(z_M)\} \quad z_i \in \mathbf{Z} \tag{2.4}$$

Because $\mathbf{G}$ is used as the process representation it is not necessary to restrict to the rational transfer function description (2.3). Since both the controller and the desired response will be linear, this restriction is, however, natural.

2.3 Controller Structure

The controller, called an RST-controller, has a two degree of freedom structure

$$R(z,t)U(z) = -S(z,t)Y(z) + T(z,t)U_c(z) \tag{2.5}$$

where U and U_c are the control and command signals. Further

$$\begin{aligned} R(z,t) &= z^{n_R} + r_1(t)z^{n_R-1} + \ldots + r_{n_R}(t) \\ S(z,t) &= s_0(t)z^{n_S} + s_1(t)z^{n_S-1} + \ldots + s_{n_S}(t) \\ T(z,t) &= t_0(t)z^{n_T} + t_1(t)z^{n_T-1} + \ldots + t_{n_T}(t) \end{aligned} \tag{2.6}$$

For convenience of notation and readability the time argument is dropped from this point.

2.4 Controller Design

A common design technique that uses the RST-controller structure is the poles placement design. In this the closed loop is made equal to a desired rational transfer function. This design involves the solution of a Diophantine equation for the controller parameters, see Åström and Wittenmark (1990). The controller design considered in this work is related to the pole placement design. The controller parameters are, however, obtained in a quite different way.

The controller design uses frequency domain fitting that is presented in Lilja (1989). In this the frequency response of the closed loop system is fitted to a desired response. The fitting is made at the set $\mathbf{Z}$ in the z-plane. The fitting is performed by solving a least squares problem. Below the different parts are discussed.

Process

The process is in the design represented by (2.4). This represents a limited amount of process information. Therefore the frequencies $\mathbf{\Omega}$ must be chosen properly to achieve a good design.

Controller

The controller is given by (2.5). The structure of the controller is chosen by the user. There are no restrictions on the controller structure as for the pole placement design, i.e. with respect to the orders of the polynomials of the controller. The chosen controller structure must, however, be reasonable. This is further discussed in Chapter 7. The method is well suited for design of low order controllers.

Closed Loop System

By applying the controller (2.5), the closed loop response to command signals becomes

$$G_{cl}(z) = \frac{G(z)T(z)}{R(z) + G(z)S(z)} \tag{2.7}$$

This is in the design evaluated at $\mathbf{Z}$.

Desired Response

The desired response of the closed loop system is given either as a set

$$\mathbf{G_m} = \{G_m(z_1), \ldots, G_m(z_{\mathrm{M}})\} \quad z_i \in \mathbf{Z} \tag{2.8}$$

or as a desired pulse transfer function

$$G_m(z) = \frac{B_m(z)}{A_m(z)} \tag{2.9}$$

When the desired response is given by (2.9) it is in the design only evaluated at **Z** to obtain (2.8).

Loss Function

The frequency domain fitting is obtained by solving a least squares problem in the frequency domain. With no weighting the used loss function is given by

$$J(\vartheta) = \sum_{k=1}^{2M} |\varepsilon_r(z_k)|^2 \quad z_k \in \bar{\mathbf{Z}} = \{\mathbf{Z}, \mathbf{Z}^*\} \tag{2.10}$$

where $\mathbf{Z}^*$ denotes the complex conjugate set of **Z**. ε_r is the relative closed loop model error defined by

$$\varepsilon_r(z) = \frac{G_{cl}(z) - G_m(z)}{G_{cl}(z)} \tag{2.11}$$

and ϑ represents a parametrization of the controller parameters. The controller parameters are obtained by minimizing the loss function (2.10). The set **G** will be estimated in a frequency domain estimation scheme. By doing so and applying the estimates of **G** in the controller design the FDAC is obtained. The parametrization is chosen such that a least squares problem with an explicit solution is obtained. It should be noted here that since the loss function has a small constant number of terms, the design solution can not be obtained by using *recursive* least squares. The obtained least squares problem must therefore be solved for the controller parameters at each sampling instant.

2.5 Process Estimator

To solve the resulting frequency domain least squares problem an estimate of the process frequency response (2.4) has to be known. To achieve this a bank of frequency point estimators is used. There is one frequency point estimator for each element in $\mathbf{\Omega}$. Each frequency point estimator gives an estimate of the process frequency response $G(e^{i\omega_j h})$ at the corresponding frequency $\omega_j \in \mathbf{\Omega}$.

Each frequency point estimator includes

- Band pass filters
- A low order parametric estimator
- Frequency response evaluation

These are described briefly below.

Band Pass Filter

A low order parametric process model is estimated for each point on the Nyquist curve. To ensure that the model is fitted to the relevant frequency range, data entering each estimator is band pass filtered with a rather narrow band pass filter centered at the corresponding frequency in $\mathbf{\Omega}$.

Low Order Estimator

Since the estimator will use band pass filtered data, a low order parametric process model can be used to model the process in the corresponding filter pass band. Depending on properties of the process, different model structures can be used. To estimate the parameters of the low order model, any recursive estimation algorithm can in principle be used. Of the different algorithms that have been considered the normal recursive least squares method (RLS) has shown the best properties.

Frequency Response Evaluation

The estimate of the frequency response of the process is simply obtained as

$$\hat{\mathbf{G}} = \{\hat{G}_1(z_1)\ldots\hat{G}_{\mathrm{M}}(z_{\mathrm{M}})\} \quad z_j \in \mathbf{Z}$$

where $\hat{G}_j(z)$ is the current estimate of the low order process model obtained from the band pass filtered data with center frequency $\omega_j \in \mathbf{\Omega}$.

3

Frequency Domain Controller Design

3.1 Introduction

Most design methods rely on a process model on transfer function or state space form. In this work the controller is instead obtained by fitting transfer functions in the frequency domain. The process is in the design represented by a discrete set of points on the Nyquist curve of the process. At the same set of frequencies the closed loop is determined for the chosen controller structure. By solving a least squares problem in the frequency domain a fit is done to a desired response. The controller parameters are obtained as the solution to this least squares problem. Frequency domain fitting of this type originates from Levy (1959). The formulation used here is based on the work by Lilja (1989) where the technique is used for off-line controller design.

3.2 Preliminaries

Process

It's assumed that the process is represented by **G**, a discrete set of points on the process Nyquist curve defined by (2.4) or more shortly

$$\mathbf{G} = \{G(z_j)\}_{j=1}^{M} \quad z_j = e^{i\omega_j h} \in \mathbf{Z} \ , \ \omega_j \in \mathbf{\Omega}$$

where $\mathbf{\Omega}$ and $\mathbf{Z}$ are defined by (2.2) and (2.3), respectively.

Controller

The RST-controller that is used in the design is given by (2.5)–(2.6). There is, however, one new restriction imposed on the controller. $T(z)$ will be given by

$$T(z) = t_0 A_o(z) \tag{3.1}$$

The polynomial A_o, which can be interpreted as an observer polynomial, is specified by the user. Therefore T will be a fixed known polynomial except for the gain t_0. The number of controller parameters to be determined is

$$n_R + n_S + 2 \tag{3.2}$$

Introduce the notation

$$\mathbf{n_c} = (n_R, n_S, n_T) = (n_R, n_S, n_{A_o}) \tag{3.3}$$

for the polynomial degrees of the controller. When there are prespecified factors in the controller the notation has to be slightly modified. In this case

$$\begin{aligned} R(z) &= R_p(z) \cdot \bar{R}(z) \\ S(z) &= S_p(z) \cdot \bar{S}(z) \end{aligned} \tag{3.4}$$

where R_p and S_p are the prespecified parts of the controller. Including the factor $(z-1)^l$ in R_p introduces integrators in the controller.

The number of unknown controller parameters to be determined is now

$$n_{\bar{R}} + n_{\bar{S}} + 2 \tag{3.5}$$

For this setup the following notation is used

$$\mathbf{n}_{\bar{\mathbf{c}}} = (n_{\bar{R}}, n_{\bar{S}}, n_T) \tag{3.6}$$

Desired Response of the Closed Loop System

When applying the controller (2.5) to the process model (2.1) the closed loop response from command signal to process output is given by (2.7). Our primary design objective is to make this response to reference signals behave in a specified way. In the design the desired response is represented by (2.8).

Controller Orders

Notice the similarity of this setup to that of the normal pole placement design where

$$\frac{Y}{U_c} = \frac{GT}{R + GS} = \frac{BT}{AR + BS}$$

This closed loop transfer function is by polynomial identities made identically equal to the desired response

$$G_m = \frac{B_m}{A_m}$$

The pole placement design requires that

$$\deg R \geq \deg A + \deg R_p - 1 \tag{3.7}$$

From (3.7) it is seen that this controller has to be at least of the same order as the process provided the controller has integral action. The method of frequency domain fitting encountered here has no such restrictions since it only makes an approximate fit. A low order controller can be used for controlling a high order plant. To get a good approximation for a given $\mathbf{n_c}$ the design objectives must be realistic, otherwise the design will not result in a closed loop system close to the desired. For unrealistic demands the controller design may result in an unstable closed loop system. Design considerations are further discussed in Chapter 7.

3.3 Controller Design

In many designs the controller parameters are obtained by solving Diophantine or Riccati equations. Here a different methodology is used. The controller design is formulated as an approximation problem. A loss function formulated in the frequency domain is minimized. The controller parameters are obtained as the solution to a least squares problem which has an explicit solution.

Loss Function

The loss function used in the controller design should be chosen so that it reflects the goodness of the fit to the desired response. Since the frequency response of the process is only known at the frequencies in $\mathbf{\Omega}$ the loss function will be based on the fit only at those frequencies.

Using the relative closed loop model error $\varepsilon_r(z)$ defined by (2.11) a natural choice of loss function is given by

$$J(\vartheta) = \sum_{k=1}^{N} |\varepsilon_r(z_k)|^2 \quad z_k \in \bar{\mathbf{Z}} \tag{3.8}$$

The set of approximation points $\bar{\mathbf{Z}}$ should be chosen as to guarantee a real valued set of controller parameters. Since $\varepsilon_r(z)$ is a complex quantity this is not necessarily true. A real ϑ is obtained by choosing $\bar{\mathbf{Z}}$ as a self conjugated set of approximation points. A normal complex least squares problem with an explicit solution is then obtained.

By definition ε_r is a relative error. How does this reflect the resulting fit? To contribute equally to the loss function a larger deviation is allowed for frequency response points far from the origin than for frequency response points near the origin. This is easily seen by looking closer at (2.11). The numerator of (2.11) represents the distance between the actual and the desired response. This is divided by G_{cl}, giving a larger contribution of $\varepsilon_r(z_j)$ for small values of G_{cl} than for large values for a given deviation $|G_{cl} - G_m|$. In practice the fitting will not be made at frequencies where G_{cl} is very small. Also it is usually desired that G_{cl} has unit amplitude at low frequencies. Therefore the loss function (3.8) can in most cases be used. In specific cases where weighting is preferred, the loss function to minimize is chosen as

$$J_v(\vartheta) = \sum_{k=1}^{N} |v_k \varepsilon_r(z_k)|^2 \quad z_k \in \bar{\mathbf{Z}} \tag{3.9}$$

where v_k is the weighting of $\varepsilon_r(z_k)$.

Regression Model

Rewriting the expression (2.11) for ε_r gives

$$\varepsilon_r(z) = 1 - G_m(z)G_{cl}(z)^{-1} = 1 - F_R(z)\frac{\bar{R}(z)}{t_0} - F_S(z)\frac{\bar{S}(z)}{t_0} \tag{3.10}$$

where

$$\begin{aligned} F_R(z) &= \frac{G_m(z)R_p(z)}{G(z)A_o(z)} \\ F_S(z) &= \frac{G_m(z)S_p(z)}{A_o(z)} \end{aligned} \tag{3.11}$$

The transfer functions F_R and F_S will never be used to filter signals. They will only be evaluated at $\bar{\mathbf{Z}}$ in the design to obtain a set of complex values. Therefore causality and stability of F_R and F_S is not an issue here. Notice also that F_R and F_S are known transfer functions when the process response G is known.

Define the controller parameter vector ϑ by

$$\vartheta = \begin{pmatrix} \frac{1}{t_0} & \frac{r_1}{t_0} & \cdots & \frac{r_{n_{\bar{R}}}}{t_0} & \frac{s_0}{t_0} & \cdots & \frac{s_{n_{\bar{S}}}}{t_0} \end{pmatrix}^T \tag{3.12}$$

Also define the regression vector by

$$\phi(z) = \begin{pmatrix} \phi_R(z) & \phi_S(z) \end{pmatrix} \tag{3.13}$$

where

$$\phi_R(z) = F_R(z) \begin{pmatrix} z^{n_{\bar{R}}} & z^{n_{\bar{R}}-1} & \ldots & 1 \end{pmatrix} \tag{3.14}$$

$$\phi_S(z) = F_S(z) \begin{pmatrix} z^{n_{\bar{S}}} & z^{n_{\bar{S}}-1} & \ldots & 1 \end{pmatrix} \tag{3.15}$$

The expression for ε_r then takes the form

$$\varepsilon_r(z) = 1 - \phi(z)\vartheta \tag{3.16}$$

Notice that ε_r is linear, or rather affine, in the controller parametrization ϑ.

The parameter t_0 acts as a proportional gain in the closed loop response (2.7). Therefore the nominal value of this parameter will never be zero. Problems of dividing by zero in (3.12) should therefore never occur. A simple test for this is otherwise easy to incorporate into the controller design.

The loss function (3.8) can be rewritten by using the linear in the error regression model (3.16). Let Φ^H denote the Hermitian conjugate of Φ i.e. the complex conjugate transpose of Φ. By making the fit at the approximation points given by $\bar{\mathbf{Z}}$ the loss function can be written as

$$J(\vartheta) = \mathbf{E}_c^H \mathbf{E}_c = (\psi_c - \Phi_c\vartheta)^H(\psi_c - \Phi_c\vartheta) \tag{3.17}$$

where

$$\Phi_c = \begin{pmatrix} \phi(z_1) \\ \vdots \\ \phi(z_N) \end{pmatrix}, \quad \psi_c = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}, \quad \mathbf{E}_c = \begin{pmatrix} \varepsilon_r(z_1) \\ \vdots \\ \varepsilon_r(z_N) \end{pmatrix}, \quad z_j \in \bar{\mathbf{Z}} \tag{3.18}$$

This can be minimized by completing the squares. The explicit solution is given by

$$\hat{\vartheta} = (\Phi_c^H \Phi_c)^{-1} \Phi_c^H \psi_c \tag{3.19}$$

With the loss function (3.9) the explicit solution is instead given by

$$\hat{\vartheta} = (\Phi_c^H V \Phi_c)^{-1} \Phi_c^H V \psi_c \tag{3.20}$$

where $V = \text{diag}(|v_1|^2, \ldots, |v_N|^2)$. It remains to specify the set $\bar{\mathbf{Z}}$ that ensures a real valued solution.

Complex LS Problem

To get a least squares problem that is not underdetermined, the equation system must have at least $n_{\bar{R}} + n_{\bar{S}} + 2$ rows. Using only the set $\mathbf{Z}$ as approximation points will not guarantee a real valued solution. However, by choosing a self conjugated set of approximation points on the unit circle the parameters to be determined, ϑ, are guaranteed to be real valued. Let the approximation set be given by

$$\bar{\mathbf{Z}} = \{\mathbf{Z}, \mathbf{Z}^*\} \tag{3.21}$$

where $*$ denotes pure complex conjugation and $\mathbf{Z}$ is the set (2.3) specified by the user. Since $\bar{\mathbf{Z}}$ is a self conjugate set, the matrices in (3.18) can be partitioned as follows

$$\Phi_c = \begin{pmatrix} \Phi_M \\ \Phi_M^* \end{pmatrix}, \quad \psi_c = \begin{pmatrix} \psi_M \\ \psi_M^* \end{pmatrix}, \quad \mathbf{E}_c = \begin{pmatrix} \mathbf{E}_M \\ \mathbf{E}_M^* \end{pmatrix} \tag{3.22}$$

where

$$\Phi_{\mathrm{M}} = \begin{pmatrix} \phi(z_1) \\ \vdots \\ \phi(z_{\mathrm{M}}) \end{pmatrix} \quad , \quad \psi_{\mathrm{M}} = \psi_{\mathrm{M}}^* = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} \quad , \quad \mathbf{E}_{\mathrm{M}} = \begin{pmatrix} \varepsilon_r(z_1) \\ \vdots \\ \varepsilon_r(z_{\mathrm{M}}) \end{pmatrix} \tag{3.23}$$

By using the approximation set (3.21) the matrices involved in the explicit solution satisfy

$$\begin{aligned} \Phi_c^H \Phi_c &= 2\mathrm{Re}\,(\Phi_{\mathrm{M}}^H \Phi_{\mathrm{M}}) \\ \Phi_c^H \psi_c &= 2\mathrm{Re}\,(\Phi_{\mathrm{M}}^H \psi_{\mathrm{M}}) \end{aligned}$$

From this we have

$$\hat{\vartheta}^* = ((\Phi_c^H \Phi_c)^*)^{-1} (\Phi_c \psi_c)^* = (\Phi_c^H \Phi_c)^{-1} \Phi_c \psi_c = \hat{\vartheta}$$

The solution is guaranteed to be real.

If the software used to implement the adaptive controller is able to handle complex arithmetics, the formulation above can be used in the controller design. Since this is normally not the case a real valued formulation is called for.

Real Valued Formulation

In order to obtain a real least squares problem formulation notice that premultiplying with a unitary matrix Q, $(Q^{-1} = Q^H)$, does not change the 2-norm of a matrix. This is seen from

$$\begin{aligned} \|QM\|_2 &= \bar{\sigma}(QM) = \left(\lambda_{max}\{(QM)^H QM\}\right)^{1/2} = \\ &= \left(\lambda_{max}\{M^H Q^H QM\}\right)^{1/2} = \left(\lambda_{max}\{M^H M\}\right)^{1/2} = \bar{\sigma}(M) = \|M\|_2 \end{aligned}$$

where $\bar{\sigma}$ and λ_{max} denotes respectively the maximum singular value and maximum eigenvalue of a matrix. Therefore with a unitary matrix Q

$$\min_{\vartheta} \|\mathbf{E}_c\|_2 = \min_{\vartheta} \|Q\mathbf{E}_c\|_2$$

Also

$$\hat{\vartheta} = \arg\min_{\vartheta} \|Q\mathbf{E}_c\|_2 = \arg\min_{\vartheta} \|kQ\mathbf{E}_c\|_2$$

for any non zero constant k so the solution is independent of k. Using those two properties the complex least squares problem is transformed into a real least squares problem. The obtained system of equations is given by

$$\Phi\vartheta = \psi - \mathbf{E} \tag{3.24}$$

In this all elements are real. This is obtained by chosing

$$Q = \frac{1}{\sqrt{2}}\begin{pmatrix} I_{\mathrm{M}} & I_{\mathrm{M}} \\ I_{\mathrm{M}} & -I_{\mathrm{M}} \end{pmatrix} \quad \text{and} \quad k = \frac{1}{\sqrt{2}}$$

The matrices in (3.24) are then given by

$$\Phi = kQ\Phi_c = \begin{pmatrix} \mathrm{Re}\,\Phi_{\mathrm{M}} \\ \mathrm{Im}\,\Phi_{\mathrm{M}} \end{pmatrix} =$$

$$= \begin{pmatrix} \mathrm{Re}\phi(z_1) \\ \vdots \\ \mathrm{Re}\phi(z_{\mathrm{M}}) \\ \mathrm{Im}\phi(z_1) \\ \vdots \\ \mathrm{Im}\phi(z_{\mathrm{M}}) \end{pmatrix} = \begin{pmatrix} \mathrm{Re}\phi_R(z_1) & \mathrm{Re}\phi_S(z_1) \\ \vdots & \vdots \\ \mathrm{Re}\phi_R(z_{\mathrm{M}}) & \mathrm{Re}\phi_S(z_{\mathrm{M}}) \\ \mathrm{Im}\phi_R(z_1) & \mathrm{Im}\phi_S(z_1) \\ \vdots & \vdots \\ \mathrm{Im}\phi_R(z_{\mathrm{M}}) & \mathrm{Im}\phi_S(z_{\mathrm{M}}) \end{pmatrix} =$$

$$
= \begin{pmatrix}
\operatorname{Re} F_R(z_1) \begin{pmatrix} z_1^{n_{\bar{R}}} & \dots & 1 \end{pmatrix} & \operatorname{Re} F_S(z_1) \begin{pmatrix} z_1^{n_{\bar{S}}} & \dots & 1 \end{pmatrix} \\
\vdots & \vdots \\
\operatorname{Re} F_R(z_{\mathrm{M}}) \begin{pmatrix} z_{\mathrm{M}}^{n_{\bar{R}}} & \dots & 1 \end{pmatrix} & \operatorname{Re} F_S(z_{\mathrm{M}}) \begin{pmatrix} z_{\mathrm{M}}^{n_{\bar{S}}} & \dots & 1 \end{pmatrix} \\
\operatorname{Im} F_R(z_1) \begin{pmatrix} z_1^{n_{\bar{R}}} & \dots & 1 \end{pmatrix} & \operatorname{Im} F_S(z_1) \begin{pmatrix} z_1^{n_{\bar{S}}} & \dots & 1 \end{pmatrix} \\
\vdots & \vdots \\
\operatorname{Im} F_R(z_{\mathrm{M}}) \begin{pmatrix} z_{\mathrm{M}}^{n_{\bar{R}}} & \dots & 1 \end{pmatrix} & \operatorname{Im} F_S(z_{\mathrm{M}}) \begin{pmatrix} z_{\mathrm{M}}^{n_{\bar{S}}} & \dots & 1 \end{pmatrix}
\end{pmatrix} \tag{3.25}
$$

$$
\psi = kQ\psi_c = \begin{pmatrix} \operatorname{Re} \psi_{\mathrm{M}} \\ \operatorname{Im} \psi_{\mathrm{M}} \end{pmatrix} = \begin{pmatrix} \psi_{\mathrm{M}} \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{3.26}
$$

Notice that only approximation points belonging to **Z** appears in the matrices above. Therefore it is in the controller design enough to specify the set **Z**. The controller parameter vector is given by (3.19) or (3.20) with Φ_c and ψ_c replaced by Φ and ψ. It is easily seen that the number of equations are 2M in both the complex and the real least squares formulation. A necessary condition for solvability is therefore that the number of fitting frequencies satisfy

$$
2\mathrm{M} \geq n_{\bar{R}} + n_{\bar{S}} + 2 \tag{3.27}
$$

Interpolating Solution

When Φ is a square and nonsingular matrix we get a zero residual solution. This is equivalent to an interpolating solution i.e. the fit is exact at $\mathbf{Z}$. One way to see this is by computing the residuals

$$
\begin{aligned}
E_{res} &= \mathbf{E}(\hat{\vartheta}, G, \mathbf{Z}) = \psi - \Phi\hat{\vartheta} = \psi - \Phi(\Phi^T\Phi)^{-1}\Phi^T\psi = \\
&= \psi - \Phi\Phi^{-1}\Phi^{-T}\Phi^T\psi = \psi - \psi = 0
\end{aligned}
$$

When Φ is non square i.e. the number of frequencies, $\textsc{m}$, is larger than necessary, the fitting error will normally be different from zero for all frequencies. This does not imply that the fit to the desired frequency response curve is worse than for the interpolating solution since it is the fit for all frequencies that determines the behavior of the closed loop. Simulations has, however, indicated that the frequency region in which $\mathbf{\Omega}$ lies is far more important for the result then is the number of frequency points $\textsc{m}$.

3.4 Example

Above it is stated that the closed loop specifications must be reasonable. What is meant by this is that both process characteristics and controller structure give restrictions on the obtainable performance. To exemplify this consider a process described by

$$
G^c(s) = \frac{1}{(s+1)^5}
$$

The sampled process has -180 degrees phase lag at $\omega \approx 0.63$ rad/s. If a critical frequency larger than this is wanted in the loop gain, the controller must give phase lead around this frequency. Let the controller be specified by

$$
\begin{aligned}
\mathbf{n}_{\bar{c}} &= (1, 2, 2) \\
R_p(z) &= z - 1
\end{aligned}
$$

i.e. a second order controller with integral action. Let the desired response be based on the dominant pole design described in Chapter 8. One choice of desired response is then given by

$$G_m^c(s) = \frac{\omega_m^2}{s^2 + 1.73\omega_m s + \omega_m^2} \cdot \frac{(2\omega_m)^3}{s^3 + 2.43(2\omega_m)s^2 + 2.47(2\omega_m)^2 s + (2\omega_m)^3}$$

The observer polynomial also has to be specified. Since it affects the response to disturbances it should not be chosen too slow. If it is chosen fast, the obtainable performance from reference value will be reduced. The Bessel filter pole pattern is used also for the observer polynomial. The observer is of same order as the T-polynomial giving

$$A_o^c(s) = s^2 + 1.73(0.5\omega_m)s + (0.5\omega_m)^2$$

where the pole radius has been chosen to half that of the dominating poles of the desired response. The second order observer dynamics will then have approximately the same time constant as the desired response which is of order 5. Normally the sampling interval should be chosen with respect to the desired response. Since the desired response will not vary to much, a fixed value $h = 1$ is used.

Let the desired bandwidth vary and study the closed loop response when applying the designed controller. The frequencies of fitting should in general be chosen with respect to the desired response. The frequency set should, however, not include too high frequencies. If this is the case an unstable controller often results. The reason for this is that a large amount of phase lead is required by the controller to fit at high frequencies. Here a fixed set is chosen, namely

$$\Omega = \{\, 0.1 \quad 0.3 \quad 0.5 \,\}$$

In Figure 3.1 the step response and control signal of the obtained closed loop systems are shown when chosing the pole radius ω_m

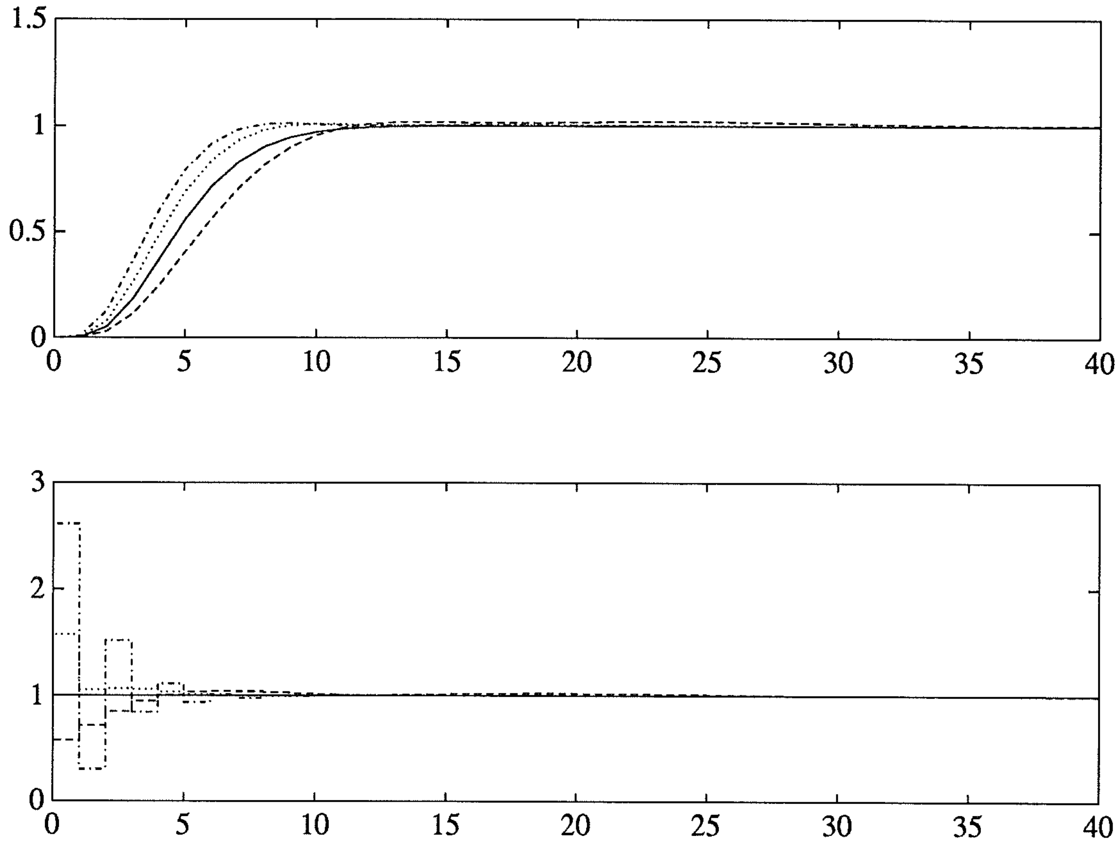

Figure 3.1 Step responses and control signals, of the process (solid), of the closed loop when $\omega_m = 0.5$ (dashed), $\omega_m = 0.7$ (dotted) and $\omega_m = 0.8$ (dash dotted)

to {0.5 0.7 0.8}. The behavior for $\omega_m = 0.5$ and 0.7 is good. For $\omega_m = 0.8$ there is ringing in the control signal because the controller contains a pole on the negative real axis in the unit circle. If ω_m is chosen larger than 0.85 the controller as well as the closed loop system is unstable. By specifying the controller to have integral action it is hard to get the required phase lead. Because of this the desired response can not be specified much faster than the open loop response for a second order controller with integral action.

3.5 Summary

A controller design formulated as an approximation problem in the frequency domain has been derived in this Chapter. The controller parameters are obtained as the explicit solution to a least squares problem. To perform the design the following quantities have to be specified.

- The desired response G_m.
- The sampling interval h.
- The approximation set $\mathbf{\Omega}$.
- The controller structure $\mathbf{n_c}$.
- The observer polynomial A_o.

In the design there are no restrictions in the choice of controller orders. If, however, large phase advance is required for a high closed loop bandwidth the obtainable closed loop bandwidth is limited when using a low order controller. Since this is a fundamental property it is not due to the particular design method considered here.

4

Frequency Point Estimation

4.1 Introduction

The frequency domain controller design is based on knowledge of a set of points on the process frequency response curve. Therefore it is not necessary to estimate a full order parametric process model. There are different possible ways of obtaining the process estimate. The perhaps most obvious is to perform spectral analysis using the Fourier transform. This method does, however, in the normal case give biased estimates in closed loop estimation. Also it requires long data sets in order to get good estimates. In the adaptive case we want to get a good model of the process in a fairly short period of time. Also the used method should not require open loop estimation. If the process order is known, another way would be to estimate a full order parametric process model. This could then be evaluated at the set of frequencies Ω to get the desired process representation $\mathbf{G}$. If the process is of high order, the corresponding model of high order will require substantial excitation in order for convergence. A good property of the estimator would be to require excitation only in frequency regions of interest. This is not the case if a full order process model is used on a high order plant. Taking these considerations into account the following scheme naturally emerge. Let the estimation be split up into a bank of estimators. Each of these estimators model the process in a narrow frequency band around the frequencies in Ω. Therefore the number of estimators is determined by the frequency set Ω used in the controller design. A low order parametric process model is used in each estimator. The data entering each low order process model estimator is filtered by a band pass filter centered at the corresponding frequency. The widths of the band pass filters must be narrow enough in order for a low order process model to be fitted and wide enough for the filter to have reasonably short response time. A narrow filter has both a pulse response with long duration and poles near the stability boundary. Stability problems will easily occur because of numerical sensitivity if an insensitive filter implementation is not used. An insensitive filter implementation is obtained by for example splitting up the filter into low order parts and implement it on cascade or parallel form. The low order process models are estimated using recursive least squares since this algorithm has shown good properties for the estimation task.

4.2 Process Estimator

In the controller design the process is represented by (2.4). This is just a discrete set of complex values that are points lying on the process Nyquist curve. The process has to be well modeled only in neighborhoods of the frequencies in Ω. This property is used to choose the estimator structure. The estimator is split up into a bank of frequency point estimators. Each of these models the process in a narrow frequency band around one of the frequencies in $\mathbf{\Omega}$. The estimation of each element in $\mathbf{G}$ is therefore normally separated from the estimation of the other elements. To make this feasible, the data entering each estimator is band pass filtered. Each of the frequency point estimators will therefore use

- Band pass filtered data
- A low order parametric process model
- The recursive least squares algorithm for the parameter estimation
- Frequency response evaluation to obtain $\hat{G}(z_j)$

Band Pass Filtering

The filtering problem is treated in Chapter 6. Here only some remarks are made. The data entering each frequency point estimator is band pass filtered according to

$$\begin{aligned} Y_f(z) &= H_f(z)Y(z) \\ U_f(z) &= H_f(z)U(z) \end{aligned}$$

using some filter H_f. The same filter is used for filtering the process input and output thereby keeping the process transfer function properties in the filtered signals. At frequencies outside the filter pass band there will be nearly no information in the filtered signals. It is then clear that the filtered data is well suited for obtaining a process model in the pass band of the filter H_f.

Low Order Models

To obtain an estimate of the frequency response at a particular frequency $\omega_j \in \mathbf{\Omega}$ a low order parametric process model is estimated around this frequency using the band pass filtered process inputs and outputs. This low order model should be given a structure that is depending on the characteristics of the process around ω_j. Different structures can therefore be used for the same process at different frequencies. Some examples of possible structures are

$$\frac{b_1 z + b_2}{z^2} \;,\quad \frac{b}{z+a} \;,\quad z^{-d}\frac{b_1 z}{z^2 + a_1 z + a_2}$$

Since the process in many cases is unknown it may seem unrealistic to require the model structure to depend on the characteristics of the process. In many cases it is, however, enough with quite few parameters. To decide on the necessary model complexity different aspects should be considered. The width of the band pass filter is important since it dictates the frequency region to which the model is fitted. More parameters are normally needed for wider pass bands. At high frequencies the process has sometimes large phase lag. This might be handled by introducing a number of delays in the model at high frequencies. The choice of model structure is further discussed in Chapter 7.

Low Order Estimator

To be able to estimate the parameters of the low order models each one is written on regression form in the usual way, see for instance Ljung (1987).

$$y_f(t) = \varphi^T(t)\theta$$

where for instance the model structure

$$\frac{b}{z+a}$$

give

$$\varphi = \begin{pmatrix} -y_f(t-1) \\ u_f(t-1) \end{pmatrix} \;,\quad \theta = \begin{pmatrix} a \\ b \end{pmatrix}$$

To estimate the parameters of this regression model the RLS-algorithm is used. The outcome of each estimator is a transfer function

$$\hat{G}_j(z) = \frac{\hat{B}_j(z)}{\hat{A}_j(z)}$$

An estimate of the frequency response of the process at ω_j is from this model easily obtained as

$$\hat{G}(z_j) = \hat{G}(e^{i\omega_j h}) = \hat{G}_j(z_j) \quad z_j \in \mathbf{Z}$$

Here $\hat{G}_j$ is the low order parametric model estimated around the frequency $\omega_j \in \mathbf{\Omega}$.

Estimation Schemes

To estimate the parameters of the low order models in principle any recursive identification method can be used. A main concern is to obtain a robust adaptive controller partly by choosing a robust estimation scheme. For this, different schemes have been evaluated during the work. Especially directional forgetting schemes like those by Hägglund (1983) and Kulhávý and Karný (1984) seemed promising. Directional forgetting schemes are intuitively interesting since they discount old information only in directions of new incoming information. In this way old information is not lost in directions that are seldom spanned by the regressors. In directions that are frequently spanned by the regressors the new incoming information replaces the old and the stored process information is basically the most recent in all directions. Unfortunately both the scheme by Karný and Hägglund have undesirable properties for estimation on narrow banded signals. The scheme by Karný have a tendency of giving biased estimates with noticeable variance even for rather good excitation. In the scheme by Hägglund the P-matrix converges to a diagonal matrix $a \cdot I$. The limiting case corresponds to a gradient estimation scheme which has approximately the same properties as the scheme by Karný in the estimation application considered. Further, all of the eigenvalues of the P-matrix have to approach the value a before the convergence to $a \cdot I$ occurs. For narrow banded signals only certain directions will be spanned frequently by the regressors. Some eigenvalues will then quickly approach zero and so identifiability is nearly lost in those directions until all eigenvalues approach a. Since the eigenvalues corresponding to seldomly spanned regressor directions decrease slowly, this will in many cases take considerable time.

The normal recursive least squares scheme has, however, shown good properties when the number of estimated parameters is small. Therefore this scheme is used in the low order estimators. When the data is not enough exciting covariance windup occurs for this scheme. This should be taken care of by monitoring the amount of excitation and switching of the estimation when the excitation is poor.

4.3 Weighted Least Squares with Exponential Forgetting

The process is assumed to satisfy

$$y(t) = \varphi^T(t)\theta^0(t) + e_n(t) \tag{4.1}$$

i.e. it can be modeled by a time varying regression model corrupted by noise. The noise is assumed to be a sequence of independent random variables.

In the weighted least squares problem the following loss function is minimized with respect to θ.

$$J(\theta) = \sum_{i=1}^{t} \frac{1}{\omega(t,i)}(y(i) - \varphi^T(i)\theta)^2 = (Y - \Phi\theta)^T V_\omega^{-1}(Y - \Phi\theta) \tag{4.2}$$

where

$$Y(t) = \begin{pmatrix} y(1) \\ \vdots \\ y(t) \end{pmatrix}, \Phi(t) = \begin{pmatrix} \varphi^T(1) \\ \vdots \\ \varphi^T(t) \end{pmatrix} \tag{4.3}$$

and the measurement variance matrix

$$V_\omega(t) = \begin{pmatrix} \omega(t,1) & 0 & \dots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \omega(t,t) \end{pmatrix} \tag{4.4}$$

Here $\omega(t,i)$ should equal the measurement variance at time i given the current time t. For a time invariant plant $\omega(t,i)$ is independent of t. The explicit solution is given by

$$\hat{\theta} = \left(\Phi^T V_\omega^{-1} \Phi \right)^{-1} \Phi^T V_\omega^{-1} Y \tag{4.5}$$

The formulation above covers different types of least squares schemes. By the specific choice

$$\omega(t,i) = \left(\frac{1}{\lambda} \right)^{t-i} v(i) \tag{4.6}$$

where $v(i) = \omega(i,i)$, we get the least squares problem with exponential forgetting and measurement noise weighting. The ordinary LS problem and LS with exponential forgetting are special cases given by

$v(t) = 1, \lambda = 1$: ordinary LS

$v(t) = 1$: LS with exponential forgetting

Let the P-matrix be defined by

$$P(t) = \left(\Phi^T(t) V_\omega(t)^{-1} \Phi(t) \right)^{-1} \tag{4.7}$$

For the choice (4.6) the recursive updating formula for the inverse P-matrix become

$$P(t)^{-1} = \begin{pmatrix} \Phi^T(t-1) & \varphi(t) \end{pmatrix} \begin{pmatrix} \lambda V_\omega(t-1)^{-1} & 0 \\ 0 & v(t)^{-1} \end{pmatrix} \cdot$$

$$\cdot \begin{pmatrix} \Phi(t-1) \\ \varphi^T(t) \end{pmatrix} = P(t-1)^{-1} - (1-\lambda)P(t-1)^{-1} + \frac{1}{v(t)} \varphi(t)\varphi^T(t) \tag{4.8}$$

In this the first term represents old information, the second term discounted information and the third term new incoming information. It is seen that information is forgotten in all directions. λ should therefore not be chosen too small. For a constant parameter process $\lambda = 1$ should be used since all incoming information describes the process properly. In the low order estimators the ordinary recursive least squares with exponential forgetting will be used. The familiar algorithm is given here for easy reference

ALGORITHM 4.1—Recursive Least Squares

$$\hat{\theta}(t) = \hat{\theta}(t-1) + K(t)\left(y(t) - \varphi^T(t)\hat{\theta}(t-1)\right)$$

$$K(t) = P(t)\varphi(t) = \frac{P(t-1)\varphi(t)}{\lambda + \varphi^T(t)P(t-1)\varphi(t)}$$

$$P(t) = \left(I - K(t)\varphi^T(t)\right)P(t-1)/\lambda =$$

$$= \left(P(t-1) - \frac{P(t-1)\varphi(t)\varphi^T(t)P(t-1)}{\lambda + \varphi^T(t)P(t-1)\varphi(t)}\right)\frac{1}{\lambda}$$

4.4 Example

To obtain a well working adaptive system it is necessary that the frequency point estimates are good. If not, the controller design may result in a poor controller. The estimate of **G** will be good if the structure of the low order models is chosen properly. Simulations have indicated that a two parameter model is seldom sufficient since the low order model has to give a good fit in the whole filter pass band. With too many parameters the narrow band signals are not

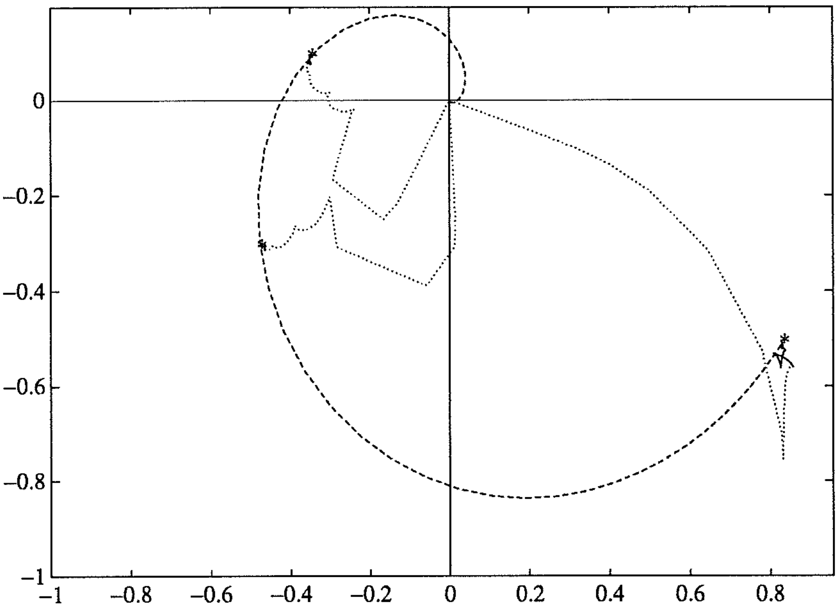

Figure 4.1 Time evolution of the frequency response estimates for the three parameter case. $0 < t \leq 30$ (dotted) , $30 < t < 500$ (solid)

sufficiently exciting. Three parameter models have, however, shown good properties. To exemplify this consider the process

$$G = \frac{1}{(s+1)^5}$$

Let the frequency points corresponding to $\mathbf{\Omega} = \{0.1 \quad 0.5 \quad 0.7\}$ rad/s be estimated in closed loop using a constant gain controller and the sampling interval $h = 1$ sec. The band pass filters proposed in Chapter 6 are used to filter the data. The parameters of the filters are the same as for the examples in Chapter 8 i.e. fourth order filters with pulse response duration of approximately 50 samples are used. Using the same three parameter model structure

$$G_j(z) = z^{-2} \frac{b_{1j} z}{z^2 + a_{1j} z + a_{2j}}$$

for all frequencies, a 500 sec simulation were carried out using a PRBS command signal. The time evolution of the frequency point

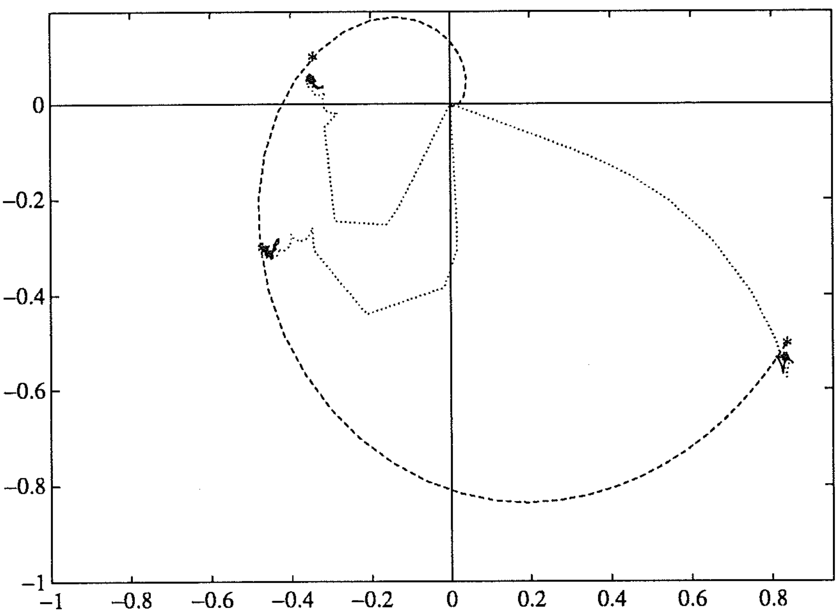

Figure 4.2 Time evolution of the frequency response estimates for the two parameter case. $0 < t \leq 30$ (dotted) , $30 < t < 500$ (solid)

estimates are shown in Figure 4.1 where the estimation has been performed in closed loop using a constant gain controller. The behavior for the first 30 samples are shown with a dotted line and the following with a solid. After 30 samples, corresponding to a single command signal step in the simulation, the estimates are good with nearly no bias. Notice that there are nearly no bias or variance in the estimates for larger t. By instead using the two parameter model structure

$$G_j(z) = z^{-2} \frac{b_{1j}}{z + a_{1j}}$$

the behavior in Figure 4.2 is obtained. The estimates have noticeable bias and variance. The models are not able to capture the process behavior accurately in the filter pass bands. If the controller design does not require very accurate process estimates, i.e. the controller design is robust to estimation errors, a two parameter model may be used. This typically corresponds to low closed loop demands. For high closed loop demands accurate estimates are often required. In this case models with three parameters or more should be used.

4.5 Summary

By using band pass filtered data low order parametric models can be fitted around the frequencies in $\mathbf{\Omega}$. An estimate of $\mathbf{G}$ is obtained by evaluating the frequency response of the models at the corresponding frequencies. To use the estimators the following quantities have to be specified.

- The structure of the low order models $G_j(z)$
- The initial parameter values $\hat{\theta}_j(0)$
- The initial P-matrices $P_j(0)$
- The forgetting factor λ

How to choose these will be discussed in Chapter 7.

5

The Complete Adaptive Scheme

5.1 Introduction

Many adaptive schemes are based on design methods originally intended for off-line controller design, see for instance Åström and Wittenmark (1989). This is true also for the frequency domain controller design encountered in Chapter 3. In Lilja (1989), the method is used in different ways to do controller design off-line. The focus is on obtaining low order controllers for higher order processes. A process model then has to be known prior to the design. To obtain an adaptive scheme based on this design method, the frequency point estimator discussed in Chapter 4 is used. From this a process model is obtained on-line. Based on the process model the frequency domain controller design can be applied on-line.

5.2 The Complete System

A picture of the complete adaptive scheme is shown in Figure 1.1. The building blocks are

- Band pass filter bank
- Frequency point estimator
- Frequency domain controller design
- Time varying controller

The scheme is similar in structure to many other adaptive schemes. A controller design method intended for off-line design is applied. To do this the necessary information about the process is supplied by an estimator. New here is the focusing on frequency domain description. Since the estimated frequency points are obtained from low order parametric process models some time domain concepts are used also in this scheme. The reason for not using spectral or other similar frequency domain methods for the process estimation is the demand on obtaining a good process model in a fairly short period of time.

Band Pass Filters

All process data are band pass filtered before entering the estimator. For each frequency in Ω a specific band pass filter is used. Each filter should have the pass band center frequency equivalent to one of the frequencies in Ω, see Chapter 6 for details. By using band pass filtered signals low order models can be used in the frequency point estimation.

Frequency Point Estimator

The frequency point estimation is split up into a number of estimators. Each of these estimates a low order parametric model using band pass filtered data. The frequency point estimates are obtained by evaluating the frequency response of the low order models at the corresponding frequency, see Chapter 4.

Frequency Domain Controller Design

The controller design is formulated as an approximation problem in the frequency domain. A fit between the closed loop frequency response and a desired response is obtained by formulating the design as a least squares problem, see Chapter 3. The controller parameters are obtained as the explicit solution to this least squares problem. In order for the resulting controller to be good the frequency point estimates must be good. Also the desired response must be chosen properly, see Chapters 7 and 8.

Controller

The controller will normally be of low order since the controller design is primarily intended for low order controllers. It should also have integral action since in a practical situation this is required to obtain zero steady state error in the presence of low frequency disturbances. The obtainable closed loop bandwidth will then be reduced because of the extra phase lag introduced in the loop transfer due to the controller integrator.

Excitation Monitoring

The data entering the estimators are band pass filtered. Disturbances outside the filter pass band will be filtered out in the estimator data. This suggests that the signal to noise ratio for the estimator data is good if the the command signal give enough excitation. If, however, the only system excitation comes from noise, this is not the case. To avoid such situations a safety net that monitors the amount of excitation should be used to get better performance. What measures can then be used to model the amount of excitation or rather the lack of excitation? It seems obvious that if the command signal is constant over a period of time it does not give much excitation. Since the response to an earlier command change may still be in progress, enough excitation for improving the estimates may still be present in the filtered data. Therefore the command signal alone is not easily used to monitor the amount of excitation.

If, however, the process output is close to the command signal for a period of time there is very little excitation. This is because the process output then has frequency content only at very low frequencies. A good detection measure for lack of excitation would therefore be to have near zero control error. Since disturbances give rise to non zero control error the steady state output variance should be taken into account when using such a measure. Also it will only detect near total lack of excitation. There should exist some method that detects lack of excitation in different frequency regions. This implies either computing the spectrum of the interesting signals or using band pass filtering to get the frequency components of the interesting signals.

From the construction of the frequency point estimator we have already access to band pass filtered data. Since this data represents both the frequency content in an interesting frequency band and is also the data used in the estimator it is particularly good for monitoring the current amount of excitation as seen from the estimator around a specific frequency. Also a good property with using this is that the excitation can be checked separately in the different frequency point estimators.

5.3 Summary

In the previous chapters the frequency domain controller design and the frequency point estimators have been discussed. The filtering problem and guidelines for choosing the different parts of the adaptive controller is discussed in Chapters 6 and 7. In Chapter 8 the complete system is evaluated through a number of examples.

6

Filtering

6.1 Introduction

Using the pure form of many adaptive schemes without taking the filtering question into account will result in an overall behavior that is very much dependent on the nature of the exogenous signals acting on the system. However, with carefully selected filtering, excitational robustness can be achieved both with respect to command signals, disturbances, and also to some degree to unmodeled process dynamics. For the process estimator this is understood from the fact that a model will be fitted primarily to the frequency regions that are excited. Therefore data entering the estimator should contain energy only in frequency bands where a good model description is important and possible to achieve.

Disturbances acting on the system should be taken care of separately in all of the blocks of the adaptive controller. In the estimator this is done by filtering the data entering the estimator to suppress disturbances and focus on the frequency interval in which a good model of the process is desired. In the controller, which is not using the estimator filtered data but instead the sampled process output and command signal, disturbances are taken care of by using the internal model principle when determining the controller structure.

Unmodeled dynamics, which in many cases is primarily concentrated to higher frequencies, is handled in the estimator in the same way as disturbances i.e. by filtering. One of the tasks of the controller is to give a closed loop system that is insensitive to process parameter uncertainties. Unmodeled dynamics is therefore in the controller design taken care of by using a robust design, meaning that both design method and design objective are carefully chosen.

6.2 Filter Design Methods

Below some of the general filter design techniques encountered in the literature is shortly summerized, see for instance Oppenheim and Schafer (1989). It's assumed that the filter specifications are given in discrete time as an amplitude frequency response or amplitude tolerance scheme with level restrictions for pass bands and stop bands that has to be met. Two well known filter types are recognized. The finite impulse response (FIR) filter defined by

$$H_f(z^{-1}) = \sum_{j=0}^{m} b_j z^{-j}$$

and the infinite impulse response (IIR) filter which has the form of a rational pulse transfer function

$$H_f(z) = \frac{B_f(z)}{A_f(z)}$$

The FIR filters have advantages like general linear phase solutions and ability to fit to rather arbitrary frequency responses. However, a large number of parameters is required when fitting to sharp filters with narrow transition bands between stop bands and pass bands. Also closed form solution do not in general exist. This is not a large disadvantage because of the existing computational power of today.

The IIR filter offers in some cases closed form discrete time solutions and in other, which may also be closed form, relies on continuous time IIR filter design in which much theory is available. Algorithmic solutions obtained by the use of computer programs are also in some cases required for the IIR filter designs. For standard frequency selective filters like low pass, high pass and band pass filters this is, however, not necessary. For the IIR filter design the procedure below can be used.

A prototype continuous time (low pass, lp) filter is chosen first. The discrete time specifications are transformed, depending on the method, into specifications on the continuous time filter, which is then determined through the use of continuous time filter design. From this point two somewhat different approaches can be used

I Perform algebraic frequency transformations on the designed (lp) filter in order to obtain a continuous time frequency selective filter of the desired type

$$G_f(s) = G_{lp}(s')\Big|_{s'=\gamma(s)}$$

The discrete time filter is obtained using the impulse invariance method or bilinear transformation described below, or some other method.

II Transform the continuous time (lp) filter into a discrete time (lp) filter by the use of the methods described below giving

$$H_{lp}(z)$$

Obtain the desired frequency selective filter by the use of discrete time algebraic frequency transformations giving

$$H_f(z) = H_{lp}(z')\Big|_{z'=\gamma(z)}$$

Two of the methods for transforming between continuous and discrete time filters are described next.

Impulse Invariance Design

Given a continuous time filter $G_{cf}(s)$ meeting the specifications, make a partial fraction expansion

$$G_{cf}(s) = \sum_{j=1}^{n} \frac{\alpha_j}{s - s_j}$$

The discrete time filter is now simply obtained as

$$H_f(z) = h \sum_{j=1}^{n} \frac{\alpha_j}{z - e^{s_j h}}$$

where h is the sampling interval. An advantage of the method is its simplicity and that the filter is given directly on parallel form, which is a good filter implementation form. Also the frequency scale is transformed linearly making the starting point conversion from discrete time to continuous time response easy. For complex poles each pole pair gives a second order transfer function with real coefficients to implement. A large disadvantage with the method is the aliasing effect introduced in the transformation. This makes the method suitable only for band limited continuous time filters, which has a negligible gain above the Nyquist frequency.

The term impulse invariance steams from the fact that the impulse response of G_{cf} is

$$h_{cf}(t) = \sum_{j=1}^{n} \alpha_j e^{s_j t}$$

while the pulse response of H_f is

$$h_f(kh) = h \sum_{j=1}^{n} \alpha_j \left(e^{s_j h}\right)^k \quad k > 0$$

The impulse response of the continuous time filter equals, at the sampling instants, the pulse response of the impulse invariant filter except for the gain h which is due to the gain $1/h$ of the sampling process.

Bilinear Transformation Design

The bilinear transformation is performed, if the precautions given below are taken into account, simply by substituting the Laplace s in the transfer function of the designed continuous time filter with

$$s' = \frac{2}{h}\frac{z-1}{z+1}$$

The bilinear transformation has the disadvantage of distorting the continuous time frequency scale in the transformation to discrete time. Therefore the important frequency points of the discrete time frequency specifications must be prewarped according to

$$\omega_c = \frac{2}{h}\tan\frac{\omega_d h}{2}$$

prior to designing the continuous time filter. These transformed frequency points for pass bands, stop bands etc., are to be used in the filter design, thereby ensuring that when applying the bilinear transformation to obtain the final filter, the frequency points of interest will be warped into desired places on the frequency scale. In this method the s-plane imaginary axis is mapped onto the unit circle in the z-plane and thereby ensuring that a stable discrete time filter is obtained from a stable continuous time filter.

6.3 Filtering and Robustness

To work well in practice all controller designs should be robust to disturbances, noise, and unmodeled dynamics. For nonadaptive designs only unmodeled dynamics can destroy the stability, disturbances will only be more or less attenuated. In the adaptive case also disturbances, noise, and lack of command signal excitation can create instability if precautions are not taken to prevent it. Since many factors can destroy the behavior of an adaptive controller, it should be designed so that each separate block of the controller is designed to cope with those problems, see also Wittenmark (1988). Therefore a robust estimator as well as a robust controller design should be used. One important part of this is filtering which is mostly associated with the estimator. The controller design is made robust by proper choice of design method, controller structure, and closed loop specifications. It is important to note that specifications that are hard to meet easily leads to a sensitive solution even if a good design method is used. An implication of this is that adaptive controllers need to be fed with some amount of proper information in order to work well. They can not be based on a pure black box approach if the behavior is expected to be satisfactory. It would of course be possible for the controller to make experiments on the process and based on the results decide on a proper desired response, compare for example with the relay feedback autotuner in Åström and Hägglund (1984). This would, however, be more of a startup procedure than a part of the adaptive scheme following it, see Lundh (1991).

In the adaptive controller the estimator must deliver a process model that is accurate in some frequency band determined implicitly by the desired response and explicitly by the corresponding loop transfer function. This loop transfer function is in the two-degree of freedom controller case possible to get exactly only by solving the design equation, which requires a process model. Therefore frequency regions of interest is in many cases determined from the desired response even if the loop transfer function would be more appropriate to use. Physical insight of the process and disturbances acting on the system is used to select proper design method, closed loop specifications and controller structure. Because the process model is often simplified, the frequency region over which we want a good process model must be specified. The effect of filtering in adaptive controllers is discussed for instance in Wittenmark (1988).

Disturbances

Assume that the process can be described by

$$y(t) = G(q)u(t) + v(t) + d(t)$$

where the disturbances acting on the system are separated into one stochastic term $v(t)$ and one piecewise deterministic term $d(t)$. The stochastic term can often be modeled by

$$v(t) = H(z)e(t)$$

where $e(t)$ is a sequence of uncorrelated random variables with zero mean.

Piecewise Deterministic Disturbances

Piecewise deterministic disturbances can be generated by

$$d(t) = H_d(q)\delta_s(t) = \frac{B_d(q)}{A_d(q)}\delta_s(t) = q^{-d_d}\frac{B_d^*(q^{-1})}{A_d^*(q^{-1})}\delta_s(t)$$

where the reciprocal polynomials are defined as

$$A^*(q^{-1}) = q^n A(q^{-1}) \ , \quad n = \deg A(q)$$

and δ_s is a sequence of pulses with unknown amplitude and distribution in time. Some examples of generating filters are

$$\text{Step disturbance} \quad : \quad H_d(q) = \frac{q}{q-1}$$

$$\text{Ramp disturbance} \quad : \quad H_d(q) = \frac{hq}{(q-1)^2}$$

$$\text{Sinusoidal disturbance} \quad : \quad H_d(q) = \frac{\sin(\omega h)q}{q^2 - 2\cos(\omega h)q + 1}$$

To reduce the effect of $d(t)$ to a finite length effect in the data entering the estimator, data are filtered by

$$H_f(q) = A_d^*(q^{-1}) \tag{6.1}$$

To see that the desired property is obtained note that filtering $d(t)$ gives

$$H_f(q)d(t) = q^{-d_d} B_d^*(q^{-1})\delta_s(t)$$

which has a finite impulse response with length given by d_d and the order of $B_d(q)$ to each occurring disturbance pulse. As is seen by the examples this response time is in most cases fairly short when this type of filter is used. For low frequency piecewise deterministic disturbances the filter $A_d(q)$ will, however, have high pass character, accentuating high frequency disturbances in the data which can for instance originate from the stochastic noise term $v(t)$. To prevent this, the filter should be modified in order to filter out high frequency disturbances. This can be done by using the filter

$$H_f(q) = \frac{A_d(q)}{A_f(q)}$$

where a $A_f(q)$ gives low pass filtering. For disturbance pulses to have a limited time effect on the filtered data, $A_f(q)$ should have fast dynamics since the finite impulse response of (6.1) will then pass through a fast dynamic system resulting in short response time. Perhaps of more importance for the choice of upper cut off frequency

is the fact that choosing it too high will result in the estimator trying to make a model fit at too high frequencies. Taking this into account, A_f should include factors that breaks down the frequency response at a lower frequency. Since steplike load disturbances are often present in control systems, $A_d(q)$ should in general also include the factor $(q - 1)$ giving high pass character to the filter H_f for low frequencies. All taken together the filter H_f will in most cases be of ordinary band pass filter type possibly including a notch at known disturbance frequencies.
Based on these considerations a rule of thumb is to choose the pass band lower frequency ω_{fl} at least one decade below and the pass band upper frequency ω_{fh} 2-10 times the desired bandwidth of the closed loop system.

Stochastic Disturbances

The standard least squares method will give unbiased estimates only if the disturbance acting on the system correspond to an uncorrelated equation error sequence when the process is described on regression form. This is satisfied if the process can be described by

$$y = \frac{B}{A}u + \frac{1}{A}e$$

where e is an uncorrelated noise sequence. In the case of colored noise the extended least squares, recursive maximum likelihood or instrumental variable methods may be used to get an unbiased estimate of the process parameters when a parameterized process model is used.

Anti Aliasing Filters

In digital control systems high frequency signals are folded into low frequency signals due to the aliasing effect, if this is not prevented by the use of anti-aliasing filters. The resulting low frequency signals can be considered as low frequency disturbances which will give undesirable control actions due to those virtual disturbances. In order to prevent this the analog signals are filtered by an anti-aliasing filter before sampling. The anti-aliasing filter can in most cases be approximated by a time delay which if it is significant must be taken into account both in the estimator by the choice of process model, and in the design, by the choice of specifications which should be based on this process model. As a rule of thumb the anti-aliasing filter can be neglected in the design if the desired closed loop bandwidth lies at least one decade below the bandwidth of the anti-aliasing filter, see for instance Wittenmark and Gustafsson (1991).

Anti Reset Windup

Physical limitations on sensors and actuators makes the closed loop system become nonlinear whenever the signals goes beyond those limitations. When the controller has integral action a frequently occurring situation is that of reset windup in which the integral part is permitted to grow despite the fact that the actuator signal is limited. This results in unwanted overshoot in the response. To overcome this problem anti reset windup which is a kind of control signal filtering should be used, see Åström and Wittenmark (1990) or Rundqwist (1991). Consider the standard form RST-controller

$$Ru = -Sy + Tu_c \tag{6.2}$$

Now let the control signal limitation be described by

$$u_l = f(u)$$

where u_l is the limited control signal entering the system and f is for example described by the saturation function

$$f(u) = \min(u_{\max}, \max(u_{min}, u))$$

Anti reset windup is now obtained by using the controller scheme

$$\begin{aligned} A_{ar}u &= (A_{ar} - R)u_l - Sy + Tu_c \\ u_l &= f(u) \end{aligned} \tag{6.3}$$

where u_l is the control signal used and A_{ar} is a stable polynomial which can be seen as an observer polynomial for the controller. It should be noted that this scheme is only intended to cope with nonlinearities such as saturation. If the function f is nonlinear over the whole working space, local feedback around the actuator should be considered either by the use of a good model for the nonlinearity or if possible by feedback from the actual actuator signal entering the system. In this case and when the actuator includes dynamics of its own it should be included in the process model prior to controller design. Another possibility in the case of static nonlinearity is to include an inverse of the nonlinearity in the signal path in order to linearize the overall loop transfer. It is easily seen that the scheme (6.3) equals (6.2) as long as the control signal is not saturated when the only type of actuator nonlinearity is of saturation type.

6.4 Narrow Band Pass Filters

In the adaptive scheme considered here the process data is to be filtered by a set of rather narrow band pass filters. When using the filter design methods discussed previously in this chapter the obtained filters will satisfy the amplitude response demands. These methods will, however, lead to filters having poles clustered closely together near the point $e^{i\omega_0 h}$ on the unit circle where ω_0 is the center frequency of the pass band. As the pass band get narrower the poles gets closer to the stability boundary. There are two specific problems associated with this type of pole configurations. Firstly the poles lie close to the stability boundary giving small safety margin for stability. Due to numerical sensitivity, filter instability may occur if the accuracy of the filter coefficients is not high enough. In the design methods above it is difficult to setup filter specifications achieving filter poles lying some guaranteed distance from the stability boundary which would be desirable from a stability point of view. Often some of the poles will lie very close to the stability boundary while others are placed on a less critical distance from the unit circle. Secondly the clustering of the poles gives filter properties that are similar to those of a filter with a high order multiple pole. The characteristic polynomial of a high order multiple pole configuration is very sensitive to coefficient errors. To see this take for instance

$$p(s) = (s - s_1)^n = s^n + a_1 s^{n-1} + \ldots + a_n$$

By disturbing the last coefficient a_n with ε the disturbed poles satisfy

$$(s - s_1)^n = \varepsilon$$

The poles are therefore given by

$$s_k = s_1 + |\varepsilon|^{1/n} e^{i\frac{\arg(\varepsilon) + 2k\pi}{n}}, \quad k = 0, \ldots, n-1$$

All poles are evenly spread on a circle with the radius $|\varepsilon|^{1/n}$ centered at s_1. With for instance $\varepsilon = 10^{-10}$ and $n = 10$ all poles are moved a distance $(10^{-10})^{1/10} = 0.1$ from s_1. The large sensitivity to coefficient errors is obvious. Nearly the same behavior is therefore expected for the coefficients of narrow band pass filters resulting in filter realization problems. This sensitivity is especially dangerous with respect to stability since the poles lie so close to the stability boundary. Using the design methods above it is therefore essential that an insensitive filter realization is used in order to avoid filter instability since the pole configuration can in general not be monitored directly. To achieve a less sensitive pole configuration using those methods the easiest but not always accurate way is to specify a less narrow pass band. To prevent from these problems and also to get a simple filter design formula a different filter design technique is presented. The main concern is to get a simple discrete time band pass filter with explicit knowledge of pole configuration in order to get a filter that is not sensitive with respect to realization. To do this let the each pass band filter have the following structure

$$H_{f_j}(z) = H^1_{f_j}(z) \cdot H^2_{f_j}(z) \cdot \ldots \cdot H^{m_j}_{f_j}(z) \tag{6.4}$$

where

$$H^i_{f_j}(z) = \frac{(1 - r_j)(z+1)(z-1)}{z^2 - 2r_j \cos \varphi_{ji} z + r_j^2} \tag{6.5}$$

Here j is the filter index of the filter with pass band center frequency ω_j. The filter which is of order $2m_j$ consists of a number of second order prototype band pass filters (6.5). The parameters of the prototype filters are determined as follows. Choose for the band pass filter with center frequency ω_j

$$r_o < r_j < 1 \tag{6.6}$$

$$\varphi_{ji} = \Phi_j + (\frac{1}{2} - \frac{i-1}{m_j - 1})\Delta\Phi_j \ , \quad i = 1, \ldots, m_j \tag{6.7}$$

where

$$\begin{aligned} r_0 &\approx 1 \ \ (\geq 0.8) \\ \Phi_j &= \omega_j h \end{aligned} \tag{6.8}$$

$\Delta\Phi_j$ is the maximum phase difference between the filter poles in the z-plane. By instead specifying the opening angle between the filter poles $\Delta\varphi_j$ defined by

$$\Delta\varphi_j = \frac{\Delta\Phi_j}{m_j - 1} \tag{6.9}$$

we get instead

$$\varphi_{ji} = \Phi_j + (\frac{m_j + 1}{2} - i)\Delta\varphi_j \ , \quad i = 1, \ldots, m_j \tag{6.10}$$

This choice of parameters makes the filter poles lie evenly spread with equal distance to the nearest pole and with equal distance from the stability boundary given by $1 - r_j$, i.e. on a circle sector, see

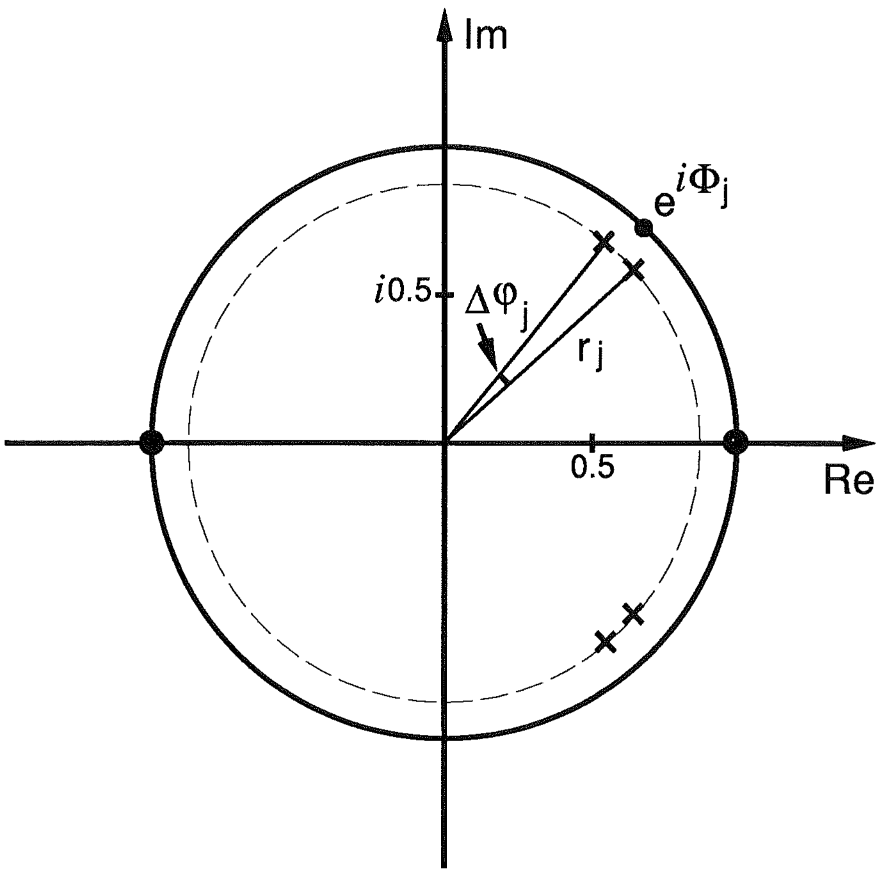

Figure 6.1 Pole-zero configuration of a fourth order band pass filter of the proposed type.

Figure 6.1. The reason for the structure of the proposed filter is to overcome the problems associated with band pass filters as discussed above. The obtained band pass filters should therefore be quite insensitive to coefficient roundoff errors. The risk of filter instability is also reduced since nominally all poles lie on equal distance from the stability border. What is paid for this insensitivity and design simplicity is an irregular amplitude response if the filter is chosen too wide. Also the pass band of the filter is not explicitly given by the design. The resulting pass band of this type of filter is depending on the steepness of the filter i.e. the number of prototype filters used, m_j, as well as of r and $\Delta\Phi$. Even though the filter has been designed to be insensitive, the filter realization should also have that property in order to get a well working system. An insensitive

realization of the prototype filters is given by

$$x(t+h) = \begin{pmatrix} r_j \cos\varphi_{ji} & -r_j \sin\varphi_{ji} \\ r_j \sin\varphi_{ji} & r_j \cos\varphi_{ji} \end{pmatrix} x(t) + \begin{pmatrix} \frac{(1-r_j)}{r_j \sin\varphi_{ji}} \\ 0 \end{pmatrix} u(t)$$

$$y_{ji}(t) = \begin{pmatrix} r^2 \sin 2\varphi_{ji} & (r^2 \cos 2\varphi_{ji} - 1) \end{pmatrix} x(t) + \begin{pmatrix} (1-r_j) \end{pmatrix} u_{ji}$$

The proposed filters are only suitable for rather narrow pass bands. A wider pass band would be achieved by chosing $(\varphi_i)_{max} - (\varphi_i)_{min}$ larger. This will result in an amplitude response of the pass band with local minimas and maximas. For wider pass bands the normal design methods have good properties and should be used instead. In the examples in Chapter 8 fourth order filters with

$$\begin{aligned} r_j &= 0.9 \\ \Delta\varphi_j &= 2 \cdot \frac{\pi}{180} \text{ rad} \end{aligned} \tag{6.11}$$

will be used. The resulting filters have pulse responses with approximately 50 samples duration for all frequencies in $\mathbf{\Omega}$.

Information Delay

A general property of dynamical systems is that the energy of the input signal is delayed in the output signal. For narrow band pass filters the situation is very similar to that of passing narrow band signals through an arbitrary transfer function. The reason for this is that all frequencies except those inside the pass band are filtered out. The behavior of the filter can therefore be studied by using an input signal that includes only pass band frequencies.

In the context of double sided amplitude modulated signals, see Oppenheim and Schafer (1989) and Parks and Burrus (1987)

$$u(t) = s(t) \cdot \cos(\omega_o t)$$

the information signal $s(t)$ satisfy

$$|S(\omega)| = 0 \ , \ |\omega - \omega_o| > \Delta\omega$$

When passing $u(t)$ through an arbitrary transfer function H with $\varphi(\omega) = \arg H(\omega)$ the output signal approximately satisfy

$$y(t) \approx |H(\omega_o)| \cdot s(t - \tau_g(\omega_o)) \cdot \cos(\omega_o(t - \tau_p(\omega_o))) \tag{6.12}$$

where

$$\tau_g(\omega_o) = -\frac{d\varphi(\omega_o)}{d\omega} \quad \text{and} \quad \tau_p(\omega_o) = -\frac{\varphi(\omega_o)}{\omega_o}$$

τ_g and τ_p are denoted the group delay time and phase delay time respectively. The information signal $s(t)$ or equivalently the envelope of the total signal is though approximately delayed a time τ_g in the output signal. This holds also for the pass band frequencies of an arbitrary input signal to a band pass filter. The maximum pulse response of a band pass filter therefore occurs at approximately $t = \tau_g$. To verify this look at the pulse response of $H_1 = H_f$ and $H_2 = H_f^2$. Then

$$y_1(t) = H_f(q)\delta(t)$$
$$y_2(t) = H_f(q)y_1(t)$$

where $\delta(t)$ is a pulse at $t = 0$. Since $y_1(t)$ is a narrow band signal (6.12) apply directly to y_2 when y_1 is considered as the input signal. Therefore the envelope of y_2 should be delayed τ_g relative to that of y_1. This can be verified in Figure 6.2 where a tenth order filter with center frequency $\omega_o = 1$ rad/s and $\tau_g = 40$ sec has been used. The maximum response of y_1 is delayed less than τ_g relative to the input pulse but the result apply approximately also to the pulse response of H_f.

The conclusion is that input energy to the band pass filters will be delayed approximately τ_g. By increasing the filter order τ_g is increased since the phase roll-off is then steeper at the center frequency ω_o. The data entering the estimators should not be delayed

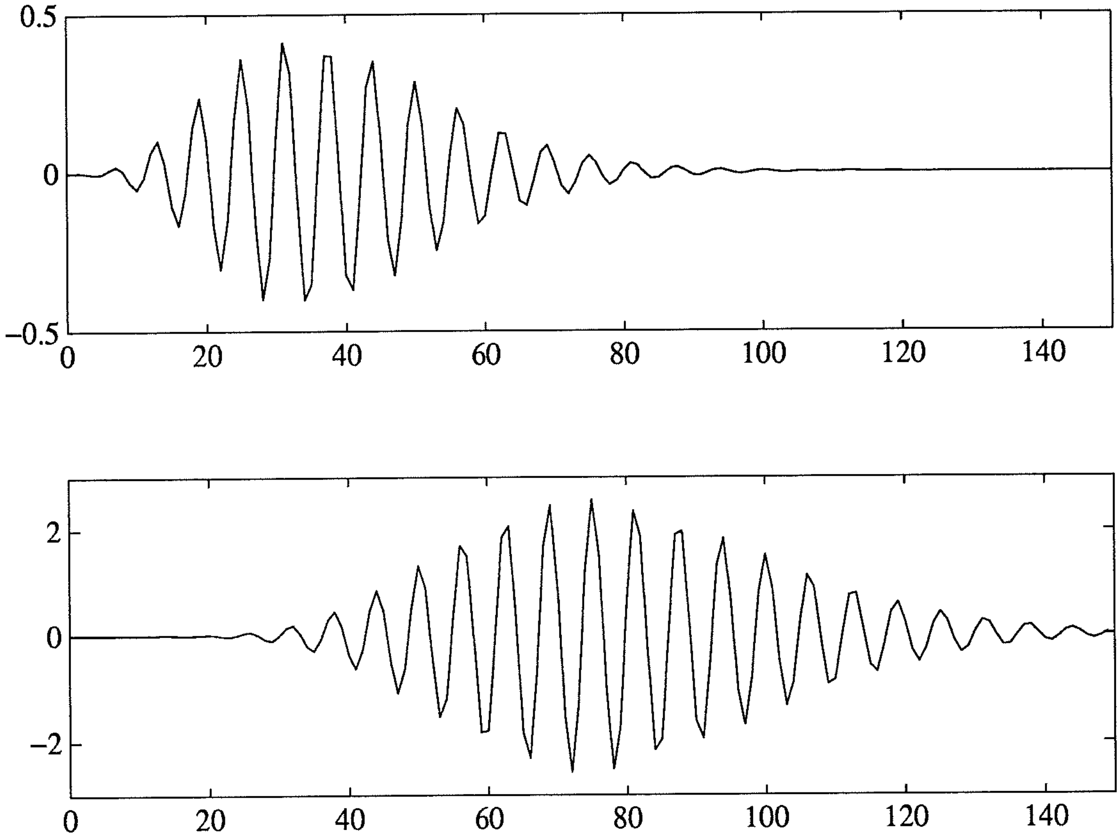

Figure 6.2 Top: y_1, impulse response of H_f. Bottom : (y_2), y_1 filtered with H_f or equivalently impulse response of H_f^2.

too long since it will then take longer time to get good estimates. The filter order should therefore not be too high. In most cases a fourth order filter is adequate. For the parameter choice of (6.11) $\tau_g \approx 20$ sec.

Another important filter property is the response time or the memory length τ_r of the filter which can for instance be defined as

$$|H_f(q)\delta(t)| < \varepsilon \;\; , \;\; t > \tau_r$$

This is closely related to the solution time of the filter step response. The memory length of the filters should not be too long since the process data will then affect the regressor variables of the estimator for an unreasonably long period of time. τ_r is a function of the filter order and the filter parameter r. This is seen as follows. Each factor (6.5) gives rise to a pulse response term of the form

$$Ar^k \sin(kh + \varphi)$$

Since no multiple poles exist the pulse response decay as r^k for any filter order. The memory length can therefore be monitored directly by the choice of r. By not choosing r too close to 1 the filter poles are placed the distance $(1 - r)$ from the stability boundary and the pulse response is guaranteed to decay as r^k. For a higher order filter the gains and coupling of the different pulse response terms will give rise to the information delay τ_g discussed above. Because of this a longer response time τ_r is obtained for a higher order filter despite the fact that all terms in the pulse response decay as r^k. By choosing $\Delta\varphi \approx 0$ the filter poles lie very close to a multiple pole configuration. The multiple pole configuration gives rise to pulse response terms

$$c_o p^k + c_1 k p^k + \ldots + c_{m_j-1} k^{m_j-1} p^k$$

which decay slower than p^k. This behavior is captured by τ_g since the multiple pole configuration has the steepest phase slope and so the largest τ_g is obtained for the multiple poles case.

6.5 Conclusions

In this chapter the necessity of using filtering in adaptive control has been discussed. Normally it is sufficient to use a band pass filter that covers the frequency range where a good process model is needed. In the FDAC rather narrow band pass filters are used in order to allow estimation of a number of low order parametric models. For this a new type of band pass filters have been suggested. The new band pass filter design has been constructed to give easily calculated filters that are robust to coefficient errors and have a reduced risk of filter instability.

7

Design Considerations

7.1 Introduction

As many other adaptive schemes the FDAC naturally separates into two parts, estimation and controller design. In the estimator low order models, fitting frequencies Ω, band pass filters and certain estimator parameters have to be chosen. In the controller design the controller structure n_c, the observer polynomial A_o and the desired response G_m has to be determined. The following sections discuss how to choose the various design parameters.

7.2 Frequency Point Estimator

Low Order Model Structure $G_k(z)$

The number of parameters in the low order models should be small. At least two parameters is, however, required to estimate a complex frequency response point. With too many parameters, the band pass filtered data is not sufficiently exciting for the estimation purpose. This is to give good estimates of the process frequency response at the center frequency of the band pass filters. Also the required amount of computation is significantly increased as the number of parameters increases.

Since the band pass filters should not be chosen too narrow, the filtered data contains frequency information in some frequency band. The low order models must be able to make a proper fit in this frequency band. This means that both levels and slopes of the amplitude and phase curve is to be appropriately modeled. A conclusion made from simulations is that two parameters are seldom enough. Often biased estimates of $\hat{G}$ with a noticeable variance are obtained with two parameter models. The reason is under-modeling even though the modeled frequency band is narrow. The estimates will drift according to the most recent data entering the estimators. However, simulations have shown that three parameters are normally enough. A model structure that has shown good properties is

$$G_k(z) = z^{-d_k} \frac{b_{1k} z}{z^2 + a_{1k} z + a_{2k}}$$

A proper choice of the sampling delay d_k can be obtained from the process step response. Since d_k represents pure time delays it can be taken as the approximate time delay of the process, i.e. the number of samples at which the process step response is nearly zero. In most cases the same model structure can be used for all the fitting frequencies, i.e. all d_k are chosen equal. By chosing d_k in this manner the slope of the process phase curve is to some extent captured by the factor z^{-d_k} in the model. It is then easier for the estimated parameters to model the remaining amplitude and phase characteristics in the desired frequency band. A proper choice of d_k is important predominately at higher frequencies where the process may have large phase lag.

Number of Fitting Frequencies M

A necessary condition for solvability in the controller design is that the number of fitting frequencies M satisfies

$$\text{M} \geq \frac{n_{\bar{R}} + n_{\bar{S}} + 2}{2}$$

When this lower bound is an integer and M is chosen minimal, an interpolating design is obtained. The fitting between the desired response and the closed loop response is then exact at the fitting frequencies provided the true process model $\mathbf{G}$ is used in the controller design. With a large M the fitting will in general not be exact at any frequency in $\mathbf{\Omega}$, instead the fitting errors will be evenly spread. Normally it is sufficient to choose M close to minimal. A definitely more important design choice is the positioning of the fitting frequencies. As long as $\mathbf{\Omega}$ is chosen in a proper frequency region the choice of M is of minor importance. If also the desired response is reasonable it is enough with quite few fitting frequencies in order to obtain a good design. An important reason for not choosing M large is also the implied computational increase in the frequency domain estimator. Since little is gained by chosing M large it is not a disadvantage to choose M small.

Choice of Fitting Frequencies $\mathbf{\Omega}$

If $G_m(z)$ is chosen so that a good fit is possible over a large frequency band, the choice of $\mathbf{\Omega}$ is not crucial. Since it is often hard to obtain a good fit over a large frequency band and especially at high frequencies, the choice of $\mathbf{\Omega}$ is not arbitrary. Especially too high frequencies in Ω may result in a poor design. This is because at high frequencies the process may have considerably larger phase lag than the desired loop gain. The feedback must then give large phase advance implying a high order or unstable controller. Using a frequency dependent weighting in the loss function may reduce this problem. However, if higher frequencies can only be allowed by nearly neglecting them in the loss function, they can as well be totally disregarded.

To be good from all viewpoints the choice of Ω should take desired response, loop transfer, process dynamics, sampling interval, and controller structure into account. Normally it is, however, enough to make a choice with respect to the process characteristics. The highest frequency can often be chosen as the frequency at which the process has −180 degrees phase lag. The remaining frequencies can be logarithmically evenly spread over approximately one decade. If the behavior is not good with this choice it can be wise to decrease the highest frequency.

From a stability point of view it is important to have good knowledge of the loop transfer around its critical frequency ω_o. If a pure unit feedback configuration is used the loop transfer is easily obtained as $G_o(z) = G_{cl}(z)(1 - G_{cl}(z))^{-1}$. In this case the critical frequency of G_o equals the frequency at which G_{cl} has −180 degrees phase lag. Therefore if G_{cl} is close to G_m, the loop gain critical frequency ω_o will be close to the frequency where G_m has −180 degrees phase lag.

By using a two degree of freedom controller this relation is in general no longer true. In this case

$$G_o(z) = \frac{G_{cl}(z)}{\frac{T(z)}{S(z)} - G_{cl}(z)} \tag{7.1}$$

Since $S(z)$ is not known in advance, an exact value of the loop transfer critical frequency can not be obtained without performing a controller design to obtain $S(z)$. When the controller design gives a good fit between G_m and G_{cl}, a good approximation is

$$\hat{G}_o(z) = \frac{G_m(z)}{\frac{T(z)}{S(z)} - G_m(z)} \tag{7.2}$$

Assuming that T/S is close to 1 at ω_o, a simple estimate of ω_o is obtained from above as

$$\arg(G_m(e^{i\hat{\omega}_o h})) = -180^\circ$$

Too get a better off-line estimate of ω_o a controller design can be made based on a priori knowledge or the process frequency response. This can be obtained from a nominal model or by performing a frequency response test. S is then obtained from the controller design and (7.2) can be evaluated to obtain an estimate of ω_o. Since it involves off-line model building this approach is not very attractive.

An on-line refinement of $\hat{\omega}_o$ can be obtained during the use of the adaptive controller by computing $\hat{\omega}_o$ from (7.2). If $\mathbf{\Omega}$ does not contain elements close to $\hat{\omega}_o$, the estimate of ω_o may be included in $\mathbf{\Omega}$. Since the band pass filters have considerable response time it is, however, not wise to use a frequently changing estimate of ω_o in $\mathbf{\Omega}$. New frequency estimates should be included in the controller design only after the estimates have become good. How long this takes depend on the exciting signals and on the filter response time.

Band Pass Filters H_f

The choice of pass band width is a tradeoff between response time and model fitting. A narrow filter gives signals that are nearly sinusoidal, representing the frequency response at the interesting frequency. The pulse response of the filter will then have long duration. New incoming data will therefore influence the filtered data for a long period of time. Also it will take time before new incoming

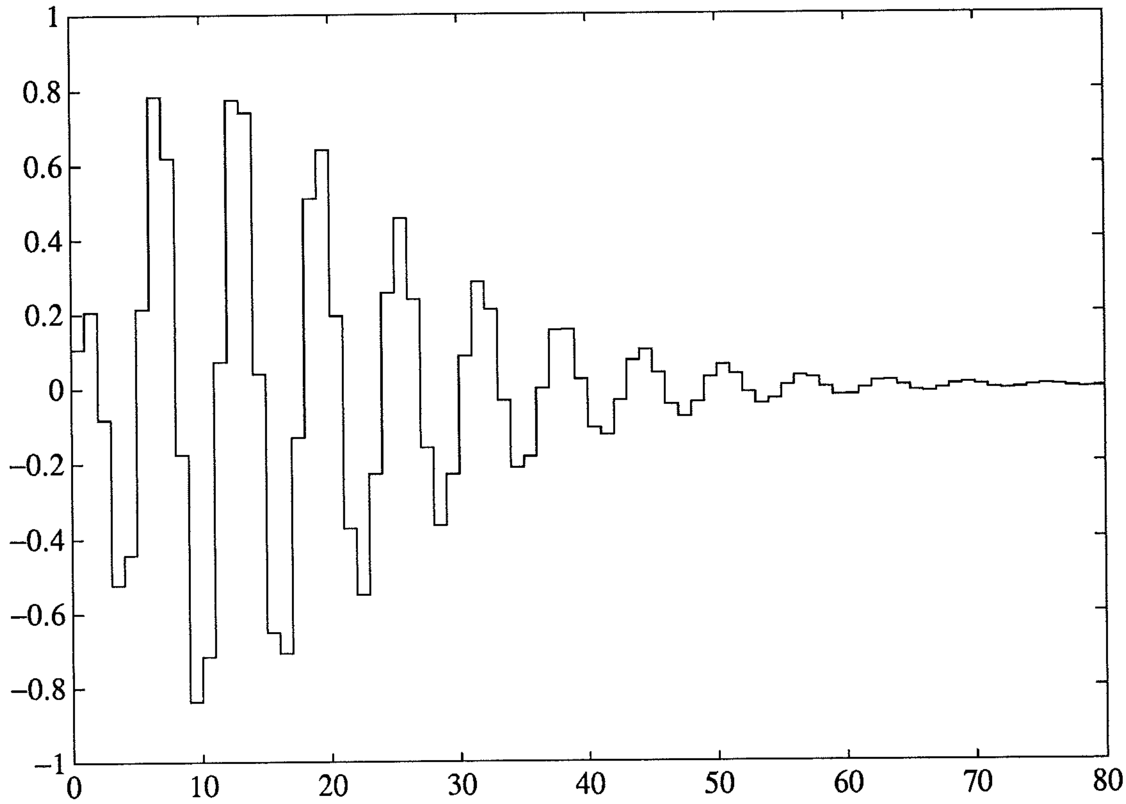

Figure 7.1 Typical band pass filter pulse response when h=1 sec and the center frequency 1 rad/s.

data shows up in the filtered data. These properties apply also to disturbance corrupted data. The requirement of not letting disturbances influence the filtered data for too long is therefore achieved by not chosing the filter too narrow. This gives a lower bound on the width of the pass band.

An upper bound is obtained by requiring a proper model fit in the whole filter pass band. With a filter that is too wide the low order model will not be able to fit properly to the data. In the examples in Chapter 8 fourth order filters with a response time of approximately 50 samples are used. Figure 7.1 shows a typical pulse response of such a filter. The sampling interval is 1 sec and the band pass center frequency 1 rad/s . For a fourth order filter this is typically achieved by chosing the filter parameter $r = 0.9$ in (6.5).

Signal Conditioning

A square wave signal has Fourier coefficients that are inversely proportional to the frequency

$$c_k(\omega_k) \sim \omega_k^{-1}$$

Assume that the command signal is a square wave or PRBS signal. Further assume that the band pass filters have constant pass band width and unit gain at the center frequency. Since the identification is done in closed loop neither the process input nor the process output will equal the command signal. However, by looking at the filtered data it is seen that the average amplitude is almost inversely proportional to the corresponding filter center frequency. The filtered data power will therefore decrease with increased frequency $\omega_j \in \boldsymbol{\Omega}$. To get the same convergence rates for all frequency response estimates it is necessary to use equivalent initial conditions for the different estimators. This is done either by using data scaling or by choosing different initial values for the P-matrices. The following lemma give some insight

LEMMA 7.1
Let a parametric model be estimated using the recursive least squares scheme with exponential forgetting. Using scaled data

$$\bar{y}(t) = ky(t) \ , \ \ \bar{\varphi}(t) = k\varphi(t)$$

in the estimation and

$$P(0) = P_0$$

as the initial value of the P-matrix is equivalent to using the unscaled data

$$y(t) \ , \ \ \varphi(t)$$

and

$$P(0) = k^2 P_0$$

for the initial value of the P-matrix

Proof: Let the P-matrix be defined by

$$P(t,\Phi,P(0)) = (\lambda^t P(0)^{-1} + \Phi^T D\Phi)^{-1} \ , \ D = diag(\lambda^{t-1},\ldots,1)$$

Then the following holds

$$P(t,k\Phi,P_0) = k^{-2}P(t,\Phi,k^2P_0)$$

When scaling the data with a factor k the parameters are updated according to

$$\Delta\hat{\theta}(t) = \hat{\theta}(t) - \hat{\theta}(t-1) = P(t,k\Phi,P_0)\bar{\varphi}(t)(\bar{y}(t) - \bar{\varphi}^T(t)\hat{\theta}(t-1)) =$$
$$= P(t,\Phi,k^2P_0)\varphi(t)(y(t) - \varphi^T(t)\hat{\theta}(t-1))$$

which, if the same initial value $\hat{\theta}(0)$ is used in the two setups, is exactly the updating that would be obtained by using the unscaled data and the initial value $P(0) = k^2P_0$. The two different estimates will then be identical for all times. □

From the lemma it is easily concluded that when estimating θ in the regression model

$$\bar{y}_f = \bar{\varphi}_f^T\theta \ , \ \bar{y}_f = \bar{H}_f\bar{y} \ , \ \bar{\varphi}_f = \bar{H}_f\bar{\varphi} \ , \ \bar{P}(0) = \bar{P}_0$$

the estimate $\hat{\theta}(t,\bar{y},\bar{\varphi},\bar{H}_f,\bar{P}_0)$ satisfies

$$\hat{\theta}(t,ky,k\varphi,H_f,P_0) = \hat{\theta}(t,y,\varphi,kH_f,P_0) = \hat{\theta}(t,y,\varphi,H_f,k^2P_0)$$

The first two alternatives can be used to obtain the same average amplitude in all the filtered data and so the same initial P-matrix can be used for all frequencies. However, the most straightforward alternative is the last one that only involves scaling of the initial

P-matrix. If the scaling factor is chosen proportional to the square of the filter center frequency i.e.

$$P_j(0) = \omega_j^2 P_0 \ , \quad \omega_j \in \mathbf{\Omega}$$

for some P_0, similar convergence speeds will be obtained in the different frequency point estimators. Another solution to the problem would be to use signal normalization. This will, however, cause start up problems giving large weight to initial, low amplitude filtered data. This could be solved by not using signal normalization until the filtered data has reached a certain amplitude.

Forgetting Factor λ

Ideally only information accurately describing the process should be used in the estimator. For a time varying process therefore old information should be forgotten since it may no longer be valid. The value of λ determines how fast old regressor information is forgotten. If data is regarded as forgotten when its contribution to the loss function is reduced by a factor of 10 or more, a rule of thumb for the memory window N is

$$N \approx \frac{2}{1-\lambda}$$

For a disturbance free process with constant parameters the optimal choice is $\lambda = 1$ since all regressor information is accurately describing the process dynamics. When the process is time varying or when data are corrupted by for instance load disturbances λ should be less than 1. Temporary bad data are then forgotten after a period of time.

7.3 Controller Design

Process Characteristics

Since only points on the process frequency response curve are estimated, high order processes are allowed. The obtainable closed loop speed will, however, be limited if large phase advance is required for a high closed loop bandwidth. This will be reflected in the choice of G_m.

The process is assumed to be properly modeled by (2.1). Since the estimator can not follow abrupt changes in the process model the process dynamics should be constant or slowly time varying. It is then possible to estimate $\mathbf{G}$ properly. This is vital since a poor estimate of $\mathbf{G}$ may result in a bad controller. Since closed loop estimation is used, only processes allowing for proper modeling in this situation should be considered. A poorly damped process may be hard to estimate in closed loop if the oscillatory modes are suppressed by the controller. This will show up as slow convergence in the process estimates. In order to achieve good performance in this case the estimation should be performed in near open loop for a period of time to be able to capture the resonant characteristics.

Sampling Interval

Process dynamics, desired command signal response and desired disturbance rejection properties are factors that give restrictions on the choice of sampling interval. The restriction requiring the smallest sampling interval should then be used. Normally G_m will require the smallest sampling interval, see Åström and Wittenmark (1990) for details.

Controller

The frequency domain controller design is intended for low order controller design. The order of the controller $\mathbf{n_c}$ should in general be chosen between 2 and 4. There is, however, no restrictions in the choice of $\mathbf{n_c}$.

In a practical situation the controller should always have integral action to achieve zero steady state error in the presence of low frequency disturbances. Therefore R_p (3.4) should include the factor $(z-1)$. In order not to introduce unnecessary delays in the controller S and T should normally have the same degree as R. It is also important not to reduce the degree of S in order to achieve a good fit in the controller design. The reason is that S is used to give phase advance in the required frequency regions. With deg$S$ < deg$R$ the obtainable phase advance will be reduced.

Response to Disturbances

Assume that the process can be described by a rational pulse transfer function. Using a two degree of freedom RST-controller the general closed loop response is given by

$$\begin{pmatrix} Y \\ U \end{pmatrix} = \frac{1}{AR+BS}\left(\begin{pmatrix} BT \\ AT \end{pmatrix} U_c + \begin{pmatrix} BR \\ -BS \end{pmatrix} V_l + \begin{pmatrix} AR \\ -AS \end{pmatrix} V_o + \begin{pmatrix} -BS \\ -AS \end{pmatrix} V_m\right) \tag{7.3}$$

where V_l, V_o, V_m are load disturbance, output disturbance, and measurement noise respectively acting on the system. From this it is seen that constant load and output disturbances are removed by using a controller with integral action. If a rational process model is known (7.3) can be used to evaluate the design with respect to different types of disturbances. Since a rational process model is normally not known (7.3) can only be evaluated at $\mathbf{\Omega}$.

Desired Response $G_m(z)$

A natural requirement on $G_m(z)$ is that it has a well behaved continuous time counterpart. The desired response is therefore naturally specified as a continuous time transfer function $G_m^c(s)$. $G_m(z)$ is then obtained by sampling $G_m^c(s)$. To be able to specify the desired response it is normally instructive to look at the step response of the open loop process. From this one can conclude the existence of for instance real or fictitious time delays, non-minimum phase properties, resonant modes, slow modes etc. Since the controller will normally be of low order and include an integrator, it can only give limited phase advance. Therefore the critical frequency of the process is of importance in the design. A desired loop gain with phase considerably larger than -180 degrees at this frequency can not be obtained. The critical frequency point can for instance be obtained by a relay feedback experiment, see Åström and Hägglund (1984) or for a thorough analysis Holmberg (1991).

Closed Loop Model Structure To get a good fit between G_m and G_{cl} (2.7) in the controller design it is preferable if the structure of G_m is close to that of G_{cl}. For this, a priori knowledge of the process order can be used to choose the structure of G_m.

To see this for a process

$$G = \frac{B}{A}$$

look at the relative closed loop model error (2.11) used in the controller design

$$\varepsilon_r = \frac{G_{cl} - G_m}{G_{cl}} = \frac{G_{cl}/A_o - G_m/A_o}{G_{cl}/A_o}$$

The frequency domain fitting can according to the last equality be alternatively formulated as fitting

$$\frac{G_{cl}}{A_o} = \frac{Bt_o}{AR + BS} \quad \text{to} \quad \frac{G_m}{A_o} = \frac{B_m}{A_o A_m} \tag{7.4}$$

In general $\deg B = \deg A - 1$ for a process without time delay. Assuming no time delay in $G_m^c(s)$, $\deg B_m = \deg A_m - 1$. With the normal choice $\deg A_o = \deg R$, the transfer functions in (7.4) are given equal orders by choosing

$$\deg A_m = \deg A$$

Assuming two dominant poles, this can be used in a dominant pole design, see for instance Åström and Hägglund (1988), to select the number of fast poles as $\deg A^c(s) - 2$. For a higher order process of unknown order, the number of fast poles in a dominant pole design can normally be chosen to 2 or 3. The dominant pole design is further discussed below.

Sampling Delays Using a causal controller it is not possible to achieve a closed loop system with a shorter time delay than that of the process. True time delays in the process should therefore be kept in the desired response $G_m^c(s)$. Assuming a process with a known time delay τ_d, the desired response is naturally chosen as

$$G_m^c(s) = G_m^{c'}(\frac{s}{k})e^{-s\tau_d}$$

where $G_m^{c'}(s)$ is the desired response that would be chosen if the time delay of the process had been zero. k gives decreased bandwidth if chosen less than unity. This may be necessary since a harder design problem is obtained when time delays are introduced. The parameter k can be used as a tuning parameter.

If the process does not have a pure time delay but large relative degree or non minimum phase zeros, its step response resembles that of a process with a time delay $\tilde{\tau}_d$. By using a controller of sufficiently high order the closed loop may be given an arbitrary response. When using a controller that is, compared to the process, of low order, the situation is different. $\tilde{\tau}_d$ then acts as a limit on the obtainable closed loop response speed. The desired response $G_m^c(s)$ should then either be of high relative degree or include a time delay τ_m.

The choice of high relative degree in $G_m^c(s)$ corresponds to a desired step response with near zero response for small t. The desired response will then resemble that of the process. A difficulty with this approach is to choose the desired high order dynamics. The dominant pole configuration may help in this task.

A more straight forward and easier approach is to include a time delay in $G_m^c(s)$. An approximate time delay $\tilde{\tau}_d$ obtained from the process step response can be used to choose this time delay τ_m. With no direct term or time delay in $G(s)$, $G(z)$ will in general be of relative degree 1. The first sample of the step response will then always be zero. This applies also to $G_m(z)$. Therefore a reasonable choice of τ_m is

$$\tau_m = \tilde{\tau}_d - h$$

where h is the sampling interval and $\tilde{\tau}_d$ is the time with near zero response in the process step response. If the discrete time step response of the process is used, τ_m can be chosen as

$$\tau_m = (\tilde{n}_d - 1)h$$

where $\tilde{n}_d$ is the number of samples with near zero response. A combination of the two approaches above may of course also be used in which $\tau_m < \tilde{\tau}_d - h$ is chosen and the relative degree of $G_m^c(s)$ becomes a tuning parameter to achieve good fitting in the controller design.

Pole Configurations A number of different pole configurations will be used in the following. To get simple expressions for the desired closed loop pole configurations, some notations are needed. Introduce

$$Butt(n, \omega_m, \alpha) \tag{7.5}$$

for the modified Butterworth polynomial of order n with roots lying on the distance ω_m from the origin. The roots are evenly spread on a circular arc with half the opening angle α, see Figure 7.2. The

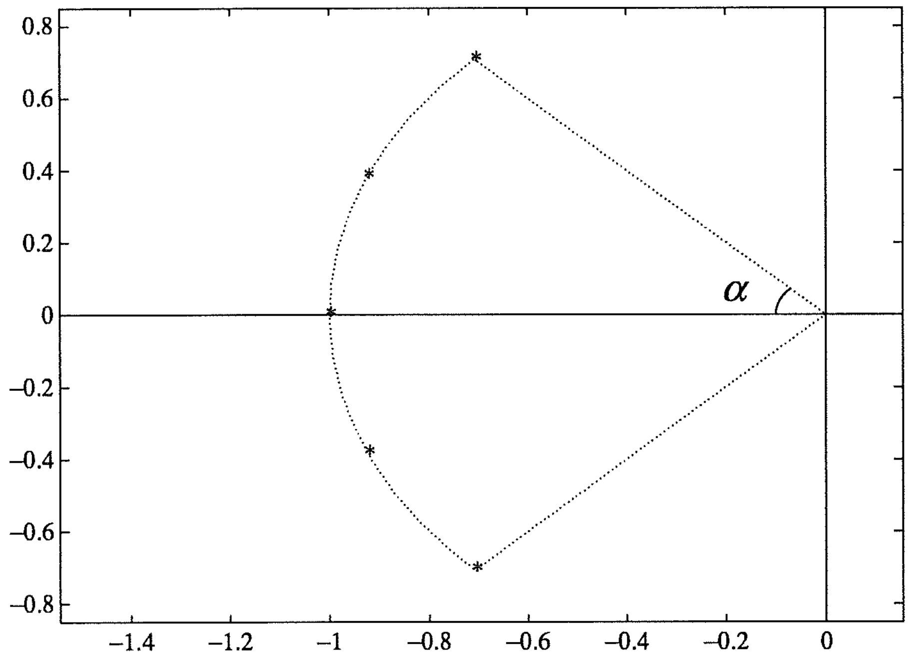

Figure 7.2 The pole pattern of the modified Butterworth polynomial

reason for not using the original Butterworth polynomial is that for high orders it includes poorly damped roots. Also introduce

$$Bess(n, \omega_m) \tag{7.6}$$

for the Bessel polynomial of order n with roots on the approximate radius ω_m from the origin. More precisely, the product of the roots equals $\pm\omega_m^n$. Approximately the roots lie on a circular arc with the circle origin in the right half plane. The Bessel polynomials have linear phase for low frequencies. They also equal the denominator of order n in a Pade-approximation of a time delay. A transfer function

$$G^c(s) = \frac{K}{Bess(n, \omega_m)}$$

has a smooth step response with negligible overshoot, see Figure 7.3. If a well damped step response is desired the Bessel polynomial can be used for the pole locations.

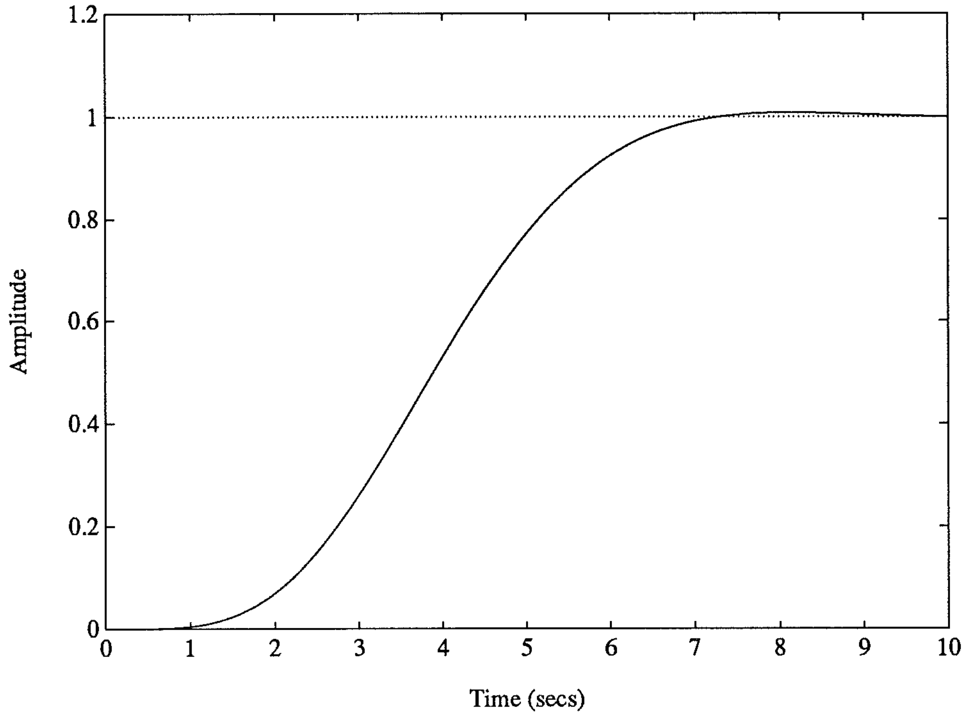

Figure 7.3 Step response for A(s)=*Bess*(5,1)

Also introduce the notation

$$\varpi = (\omega_{-90}, \omega_{-180}, \omega_{-270}) \tag{7.7}$$

for the frequencies at which the sampled process has −90, −180 and −270 degrees phase lag respectively.

Dominant Pole Design Satisfying $G_m(z)$ should not require too large controller gains. Normally this is achieved by not speeding up or slowing down the open loop process poles too much in the closed loop system. It is, however, also wise to choose the poles of $G_m(z)$ not too far away from the pole pattern obtained by pure proportional feedback since the controller will then not have to drastically change the loop gain to satisfy the demands. This pole pattern will be called the natural pole configuration since it is achieved by the simplest of controllers. The natural pole configuration for different feedback gain is simply obtained from the root locus. Assuming a process

$$G(s) = \frac{Q(s)}{P(s)} = \frac{b_o s^m + \ldots + b_m}{s^n + a_1 s^{n-1} + \ldots + a_n}$$

The root locus is given by

$$P(s) + KQ(s) = 0$$

m poles will go to the zeros of $Q(s)$ as $K \to \infty$. The remaining $n-m$ poles will go to infinity in directions given by

$$\arg(s - s_o) = \frac{\pi}{n-m} + k \cdot \frac{2\pi}{n-m} \quad , \quad k = 1, \ldots, n-m$$

where s_o is the center of the pole asymptotes. It is interesting to note that some of the poles will move far into the left half plane while the dominating dynamics will be determined by two or three poles that move toward the stability boundary. The natural pole configuration therefore normally consists of a complex conjugated pole pair and the remaining poles in a Butterworth like pole pattern further into the left half plane. To exemplify this, consider the root locus obtained by proportional feedback of

$$G(s) = \frac{1}{(s+1)^5}$$

This is shown in Figure 7.4 for $k \in [0, 5]$. The root locus clearly follows the suggested paths. By keeping this structure in the pole pattern of $G_m^c(s)$ the controller will be focused on placing the dominating pole pair as desired. In many cases a natural choice of characteristic polynomial is for instance

$$Butt(2, \omega_m, \alpha) \cdot Butt(n-2, k\omega_m, \alpha)$$

Here $k > 1$, α and ω_m are half the opening angle and the pole radius of the desired dominating pole pair. Åström and Hägglund

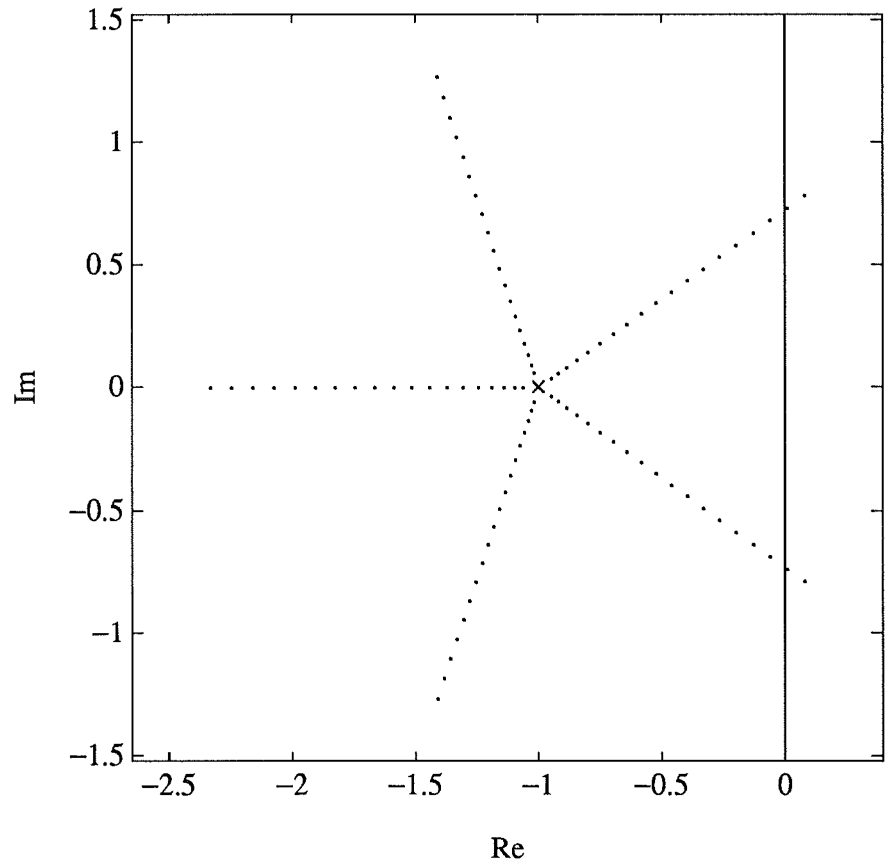

Figure 7.4 Natural pole configuration given by the root locus.

(1988) treats the dominant pole design using PID controllers. In Persson (1992) the dominant pole design is used to optimize the closed loop disturbance rejection using low order controllers. Here only the structure of the dominant pole design is used to choose a proper structure of $G_m^c(s)$.

Bandwidth In the controller design, desired responses of different orders may be considered to achieve a certain closed loop bandwidth ω_b. With a desired response

$$G_m^c(s) = \frac{K}{A_m^c(s)}$$

the bandwidth is determined primarily by the slowest dynamics. When all poles have approximately the same magnitude i.e. the same distance to the origin in the s-plane, the bandwidth is in general depending on both the pole magnitudes and the order of $A_m^c(s)$. When changing the order of $A_m^c(s)$ the pole magnitudes should be changed accordingly not to alter the bandwidth. For specific cases, like the Butterworth filter, the bandwidth is independent of the filter order. It is determined solely by the filter break frequency or equivalently by the pole magnitudes.

Consider for example a multiple real pole configuration

$$G_m^c(s) = \frac{a^n}{(s+a)^n}$$

for which the bandwidth is a good measure of the step response speed. The bandwidth is given by

$$\omega_b = a\sqrt{2^{1/n} - 1} \tag{7.8}$$

For a specific bandwidth, a is depending on the order n. To evaluate the controller design for different orders (7.8) can be used for this particular choice of G_m^c. When expressions like (7.8) can not be obtained, the bandwidth ω_b is achieved by choosing

$$G_m^c(s) = G^c(s\frac{\omega_1}{\omega_b}) \ , \ |G^c(i\omega_1)| = \frac{1}{\sqrt{2}}$$

where $G^c(s)$ has the correct order and pole pattern but not the correct bandwidth.

For systems without time delays, the bandwidth is a good measure of the response speed. Including a time delay does not affect the frequency response amplitude of a system. The bandwidth is therefore not affected by time delays. The response speed is, however, very much depending on time delays. Therefore, for systems with a time delay, the bandwidth is no longer a good measure of the response speed. It is instead a measure of the speed of the transient part of the response.

Phase Properties To obtain a closed loop system with a behavior close to the desired it is not sufficient that the fitting is good at $\mathbf{\Omega}$. Instead the fitting between G_{cl} and G_m must be good over a wider frequency range. To achieve this the structure of G_m should resemble that of G_{cl}. If not, good fitting requires $\mathbf{\Omega}$ to contain frequencies covering the whole of this frequency range. This contradicts the rules for choosing $\mathbf{\Omega}$. Since $\mathbf{\Omega}$ will not contain high frequencies, good fitting at high frequencies can be achieved by taking the structure of G_{cl} into account.

Consider a unit feedback system

$$G_{cl}(z) = \frac{G_o(z)}{1+G_o(z)}$$

where $G_o(z)$ is the loop transfer function. At high frequencies where G_o is small

$$G_{cl}(e^{i\omega h}) \approx G_o(e^{i\omega h}) \tag{7.9}$$

For a two degree of freedom controller

$$G_{cl}(z) = \frac{T(z)}{S(z)} \frac{G_o(z)}{1+G_o(z)}$$

Normally $\deg T = \deg S$. To give phase advance near the critical frequency of the loop transfer, the roots of S will normally lie not far from the the point 1 in the right half of the unit circle. Since also the roots of T (A_o) normally lie in this region, T/S will approach a constant near the Nyquist frequency. Therefore (7.9) holds approximately also for the two degree of freedom case at high frequencies.

To get a good fit at high frequencies the phase characteristics of $G_m^c(s)$ should according to (7.9) be close to that of the process since the low order feedback can only give limited phase advance. With a process of high relative degree G_o will have large phase lag at high frequencies. To give a good fit at high frequencies $G_m^c(s)$ should have similar phase characteristics. This is in accordance with the discussion on closed loop model structure.

Observer Dynamics $A_o(z)$

To get a good fit in the controller design A_o can not be chosen arbitrarily. In fact, A_o and A_m play a very similar role in the frequency domain fitting. This is seen in (7.4) where the transfer function to be fitted is given as

$$\frac{B_m}{A_o A_m}$$

Since A_m has to be chosen properly this obviously apply also to A_o. The interesting observation here is that for high demands a tradeoff has to be made between desired dynamics A_m and observer dynamics A_o. The demands are regarded as high when they, for the controller structure in question, require near the maximum phase lead obtainable with a stable controller. This normally corresponds to chosing the dynamics $A_o A_m$ as fast as possible.

A rule of thumb in the pole placement design is that the observer dynamics is chosen at least as fast as the dynamics of A_m. The reason is that the disturbance rejection, which is influenced by the observer dynamics, should not be too slow. In the frequency domain fitting the observer dynamics will not be totally cancelled in the command signal response, as is the case in the pole placement design. It will therefore also have at least a small influence on the command signal response. The observer dynamics should therefore not be chosen too slow. A rule of thumb is to choose the dynamics of A_o approximately as fast as that of A_m. To do this it should be kept in mind that the observer dynamics is normally of lower order than the desired dynamics. Assuming a process of higher order than the controller a normal choice is

$$\deg A_o = n_R < \deg A_m = n_m \approx \deg A = n$$

If, as is customary, the same pole pattern is used for both A_o and A_m normally smaller pole magnitudes should be used in A_o. To see this take for instance $A_o^c = (s + a_o)^{n_R}$ and $A_m^c = (s + a_m)^n$. A number of cascaded first order systems

$$\frac{1}{(1+sT)^n}$$

has a time time constant that is approximately $n \cdot T$. To get approximately the same time constant for the dynamics of A_o and A_m we get

$$a_0 \approx \frac{n_R}{n_m} \cdot a_m < a_m$$

Since a tradeoff between A_o and A_m only occurs for high demands, A_o can be chosen faster when the desired response is not so demanding. For high demands the desired response can be chosen faster by slowing down the observer dynamics.

Too High Demands

For too high demands still resulting in a stable closed loop system, the closed loop system will typically have at least one pole near the point -1 in the z-plane. The controller will then have corresponding poles at approximately the same positions. Controller poles of this type causes a "ringing" control signal and should therefore be avoided. A different formulation of this effect is that closed loop poles are not moved far from open loop poles lying in this region. The reason for this is that in this region, the process has low gain. To explain this, notice that for any process pulse transfer function

$$G(z) = K\frac{\prod_{j=1}^{m}(z-z_j)}{\prod_{j=1}^{n}(z-p_j)}$$

we have

$$\frac{dG}{dz}(z) = \left(\sum_{j=1}^{m}\frac{1}{z-z_j} - \sum_{j=1}^{n}\frac{1}{p-p_j}\right) G(z)$$

Looking at a point $z = z_R$ where $G(z)$ is small and where no z_j or p_j are close to z_R we have

$$\left|\frac{dG}{dz}(z_R)\right| \leq (m+n)\cdot \max\left(\frac{1}{|z_R - z_j|}, \frac{1}{|z_R - p_j|}\right) |G(z_R)|$$

Therefore $G(z)$ is almost constant in a neighborhood around z_R since $|G(z_R)|$ is small.

Now let the controller have a pole at z_R at which the process has low gain. Also let no other poles or zeros of the controller or the process lie close to z_R. We then have

$$G_P(z) \approx \varepsilon_p e^{i\varphi_P} \quad , \quad |z - z_R| < \varepsilon$$
$$G_R(z) \approx \frac{G'_R(z_R)}{(z - z_R)} \approx \frac{a'_R e^{i\varphi'_R}}{(z - z_R)} \quad , \quad |z - z_R| < \varepsilon$$

Now find the point

$$z_c = z_R + \varepsilon e^{i\varphi}$$

that is a pole of the closed loop system. For the controller we have

$$G_R(z_c) \approx \frac{G'_R(z_R)}{\varepsilon e^{i\varphi}} = \frac{a'_R}{\varepsilon} e^{i(\varphi'_R - \varphi)}$$

We see that $G_R(z_c)$ can be given any phase by proper choice of φ. Also the gain will decrease with increasing ε. The closed loop will have a pole at z_c when

$$G_P(z_c)G_R(z_c) = -1$$

This is satisfied when

$$1 = |G_P(z_c)| \cdot |G_R(z_c)| \approx \varepsilon_p \cdot \frac{a'_R}{\varepsilon}$$
$$\pi + 2k\pi = \arg G_P(z_c) + \arg G_R(z_c) \approx \varphi_P + \varphi'_R - \varphi$$

The closed loop pole is therefore approximately given by $z_c = z_R + \varepsilon e^{i\varphi}$ where

$$\varepsilon = a'_R \cdot \varepsilon_p$$
$$\varphi = \varphi_p + \varphi'_R + \pi + 2k\pi$$

The interesting to note here is that ε will in fact be quite small as long as a'_R is not too large. The conclusion is that when, due to increased demands on the closed loop response, controller poles are moved out of the unit circle at a point where the process has low gain, the closed loop will become unstable or lie very close to the stability boundary. This effect can be expected to be most noticeable for processes with large pole excess that is sampled fast since the gain is then very small around the point -1.

7.4 Conclusions

A number of quantities has to be specified in order to use the FDAC. Below a list that can be followed step by step summerizes the choice of design parameters.

1) Process characteristics

- Determine ω_{-180} from a relay feedback experiment or from a priori knowledge of the plant.
- Determine a true or fictive time delay $\tilde{\tau}_d$ from the step response of the process.

2) Controller structure n_c

- Use a controller of order 2 to 4 with integral action. If the closed loop demands can not be fulfilled a controller of higher order may be necessary in some cases.
- Normally choose $\deg S = \deg T = \deg R$ not to introduce extra time delays in the controller.

3) Approximation frequencies Ω

- In most cases M can be chosen close to minimal. The number of fitting frequencies must satisfy

$$\mathrm{M} \geq \frac{n_{\bar{R}} + n_{\bar{S}} + 2}{2}$$

- Often the highest fitting frequency can be chosen as the frequency where the process has −180 degrees phase lag. The remaining frequencies in Ω can be logarithmically evenly spread over approximately one decade.

4) Desired response $G_m(z)$

- The desired response is naturally specified as a continuous time transfer function $G_m^c(s)$. For a process without time delays choose a bandwidth not far above ω_{-180} where the process has −180 degrees phase lag. If delays are introduce in $G_m^c(s)$ the bandwidth may be increased.
- If the process order is known a priori a natural choice to achieve good fit in the controller design is

$$\deg A_m^c = \deg A^c$$

- If the desired response is chosen according to the dominant pole design the number of fast poles can be chosen as $\deg A^c(s) - 2$. It the process order is not known 2 or 3 fast poles can in most cases be used for a higher order process.
- True time delays in the process should be kept in the desired response $G_m^c(s)$. Also for processes with high relative degree a desired response with a time delay can be used. In this case the fictitious time delay can be obtained from the process step response.
- To obtain good fitting at higher frequencies choose a desired response with phase properties close to that of the process.
- High demands will give rise to controller poles close to the stability boundary. This will in many cases give rise to large control signal variance. The desired closed loop bandwidth should be chosen so that this is avoided.

5) Observer polynomial $A_o(z)$

- Normally choose the observer polynomial A_o such that the corresponding time constant of A_o and A_m are approximately the same. By slowing down the observer dynamics a faster response to command signals can be obtained.

6) Sampling interval

- Normally choose the sampling interval h with respect to the desired closed loop response. One rule of thumb for the choice of h is

$$\omega h \approx 0.2 - 0.6$$

where ω in most cases can be taken as the desired closed loop bandwidth. In some cases good disturbance rejection requires shorter sampling interval.

7) Low order model structure $G_j(z)$

- Use low order process models with at least three parameters. A good model structure is given by

$$G_j(z) = z^{-d_j} \frac{b_{1j}z}{z^2 + a_{1j}z + a_{2j}}$$

d_j can be chosen as the number of samples with near zero response in the process step response

$$d_j = \text{int}(\tilde{\tau}_d/h)$$

8) Filtering

- Choose band pass filters with appropriate response time to obtain a good model fit. Fourth order filters with $r = 0.9$ and $\Delta\varphi_j = 2 \cdot \frac{\pi}{180}$ rad can in most cases be used. This gives a response time of approximately 50 samples i.e. the pulse response of the filters are small after 50 samples.

9) Estimator

- With a linear time invariant process use $\lambda = 1$ in the estimators. For time varying processes use a smaller λ in the interval [0.98,1].
- Let the initial values of the P-matrices be proportional to the squares of the fitting frequencies i.e.

$$P_j(0) \sim \omega_j^2 \ , \quad \omega_j \in \mathbf{\Omega}$$

  to obtain signal conditioning. An alternative is to let the peak response of the band pass filters be proportional to their corresponding center frequencies.

8

Examples

Introduction

The frequency domain controller design is intended for controllers of relatively low order. To be practically usable the controller should have integral action. By including integral action the amount of phase lead achievable by the controller is limited. Therefore in order to obtain a good design the desired response of the closed loop system must be chosen with some care. For a process of high order the closed loop response can not be made much faster than that of the open loop. Therefore knowledge of the open loop step response is important for deciding how to choose the desired closed loop response. The fitting frequencies Ω should also be chosen with respect to the process behavior. The critical frequency at which the process has -180 degrees phase lag can in many cases be taken as the highest frequency in Ω. This frequency can for instance be obtained by a relay feedback experiment on the process, see Åström and Hägglund (1984). Below a number of examples are used to examine the properties of the FDAC.

In all the examples below the process dynamics are chosen such that $h = 1$ is a reasonable choice of sampling interval. Therefore the choice of sampling interval will not be further discussed below. It should only be commented here that in general h should be chosen with respect to both the process dynamics and the desired closed loop dynamics. Since the desired closed loop dynamics is normally faster it will in most cases determine the the sampling interval. Also the ability to control out disturbances give restrictions on the choice of sampling interval. See Åström and Wittenmark (1990) for a more detailed discussion on the choice of sampling interval.

Example 1 - Fifth Order System

Consider the fifth order process described by

$$G(s) = \frac{1}{(s+1)^5}$$

A look at the process step response reveals that the output is nearly zero at the first two samples. The structure of the low order models used in the frequency point estimators can then be chosen as

$$G_k(z) = z^{-2}\frac{b_{1k}z}{z^2 + a_{1k}z + a_{2k}}$$

The phase cross over frequencies (7.7) are $\varpi \approx (0.29, 0.63, 1.1)$. The controller will be of low order with integral action. By requiring a stable controller the obtainable phase advance is limited. Therefore the highest frequency in $\boldsymbol{\Omega}$ is chosen only slightly above ω_{-180}. For a second order controller $\boldsymbol{\Omega} = \{0.1 \quad 0.3 \quad 0.7\}$ is used while for a third order controller the choice is $\boldsymbol{\Omega} = \{0.1 \quad 0.3 \quad 0.5 \quad 0.7\}$. From a stability point of view it would be desirable to choose $\boldsymbol{\Omega}$ with respect to the loop transfer. For a pure unit feedback configuration this is easy. However, for a two degree of freedom controller, knowledge of the controller parameters is required. Since those are not known a priori the choice of $\mathbf{\Omega}$ is based on the desired closed loop response, the controller structure and the a priori knowledge of the process. The chosen $G_m^c(s)$ should not require too much of the controller since it is of low order. For a high order closed loop system it can be expected harder to get a good fit if the order of $G_m^c(s)$ is chosen too low. By taking this into account and by using the dominant pole configuration in this example, the desired pole polynomial is chosen as

$$A_m^c(s) = Bess(2, \omega_m) \cdot Bess(3, 3\omega_m)$$

$B_m^c(s)$ is chosen as a constant giving unit steady state gain. The observer polynomial is chosen as $A_o^c(s) = Bess(n_R, \omega_o)$ with $\omega_o = \omega_m$ if not specified differently. As discussed earlier $\deg A_o(z) = \deg R(z)$ in order not to introduce extra phase lag in $G_{cl}(z)$. In some figures below the nominal design is evaluated. This is obtained by using the true value of $\mathbf{G}$ in the controller design. By using this constant gain nominal controller the nominal closed loop system is obtained.

Second Order Controller - Base Line Case

First a second order controller with integral action is considered using

$$\mathbf{n_c} = (2, 2, 2)$$
$$R_p = (z - 1) \ , \ S_p = 1$$

i.e. the controller has the form

$$(z-1)(z+r_1)u(t) = -(s_0 z^2 + s_1 z + s_2)y(t) + t_0(z^2 + a_{o1}z + a_{o2})u_c(t)$$

The set of feasible values of the desired closed loop speed, ω_m, can be obtained by evaluating the nominal design for different values of ω_m. In this setup $\omega_m \in [0.3, 0.6]$ give reasonable designs. For higher values of ω_m the obtained controller will be unstable in order to give enough phase advance. As explained earlier this will result

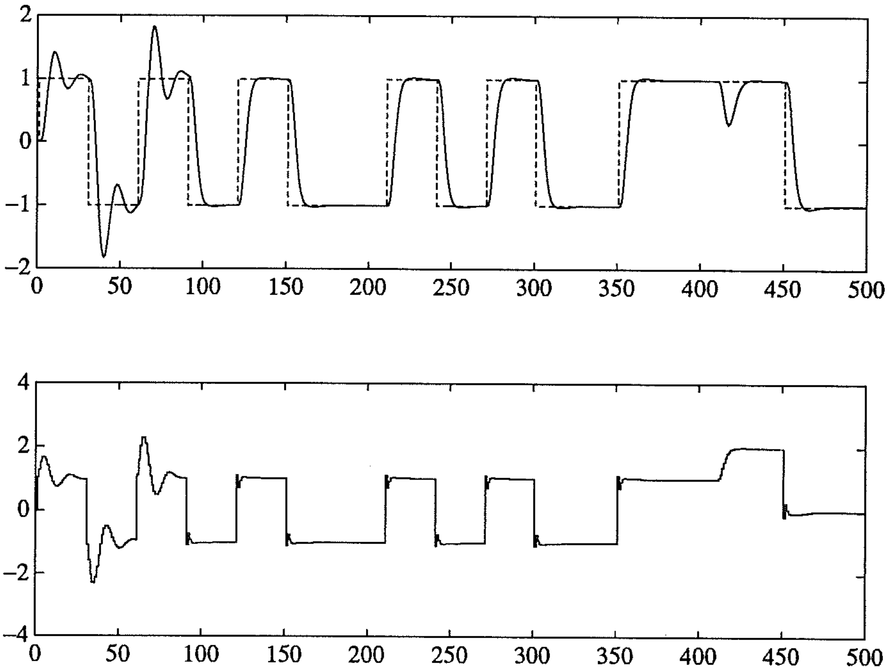

Figure 8.1 Behavior of the adaptive system using a second order controller and $\omega_m = 0.5$. Command signal and process output (top) and control signal (bottom) for the base line case in Ex 1.

in an unstable closed loop system if the process gain is very small at the unstable controller poles. Therefore unstable controllers can normally not be allowed.

The behavior of the adaptive system for $\omega_m = 0.5$ is shown in Figure 8.1. The command signal is a pseudo random binary sequence, (PRBS). All parameters of the low order models have initial values equal to zero. To get a good estimate of G before closing the adaptive loop, a constant gain controller is used up to $t = 80$. At $t = 80$ the adaptive loop is closed. At $t = 400$ a unit load disturbance is applied at the process input. The behavior is closed to the desired. The time evolution of the frequency point estimates is shown in Figure 8.2. For $t > 80$ the estimates are very good so the adaptive system should perform in accordance with the nominal design. To show the initial behavior of the estimators, the first 15 samples of $\hat{G}$ are dotted. In Figure 8.3 the time evolution of the controller parameters

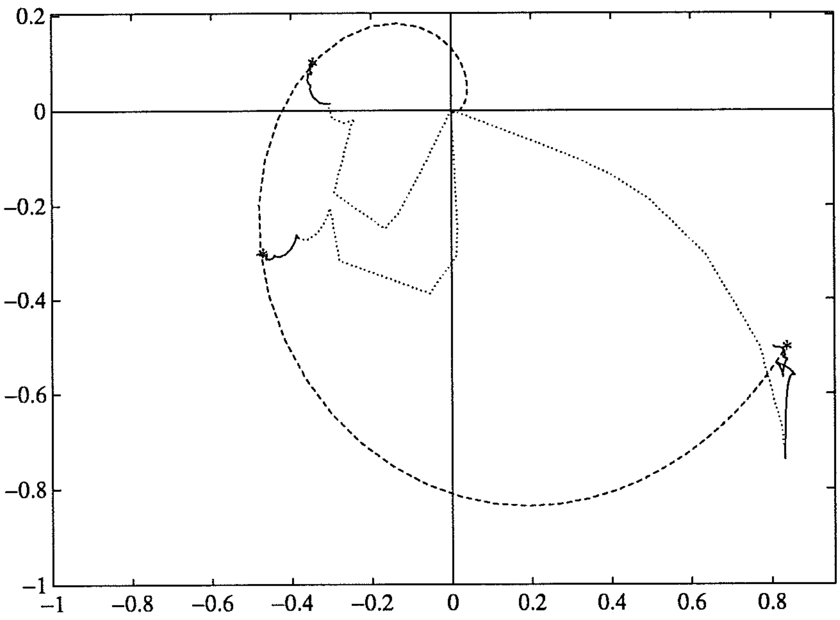

Figure 8.2 Time evolution of the frequency point estimates in Ex 1. The dashed curve is the true Nyquist curve of the process. The estimates are dotted for $t \leq 15$.

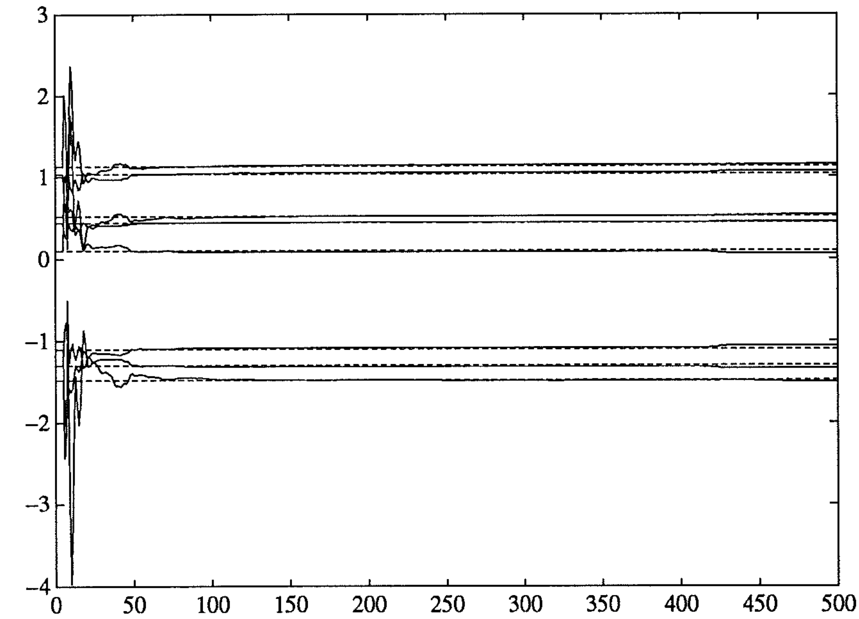

Figure 8.3 Controller parameter evolution in Ex 1. The dashed lines show the nominal values of the controller parameters.

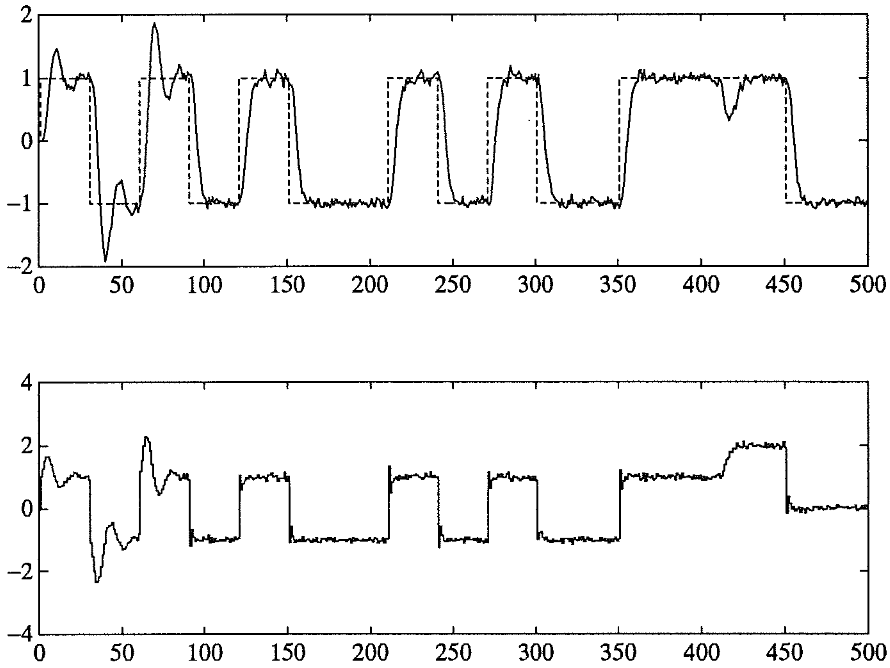

Figure 8.4 Behavior in a noisy situation (Ex 1).

are shown together with their nominal values. The violent behavior for $t < 20$ is due to the zero initial conditions of the low order model parameters. Even though small in this case the consequence of the load disturbance is seen as a temporary biasing of the controller parameters.

The behavior of the adaptive system in a noisy situation is depicted in Figure 8.4. A normal distributed zero mean noise with standard deviation $\sigma = 0.05$ is acting on the system output. As can be expected the frequency point estimates are slightly disturbed by the noise. However, the behavior of the overall system is good.

Slower Observer Dynamics

In Section 7.3 the close connection between $A_o(z)$ and $A_m(z)$ in the controller design was discussed. By using slower observer dynamics the command following can be made faster i.e. ω_m can be increased further before obtaining an unstable controller. To show this the

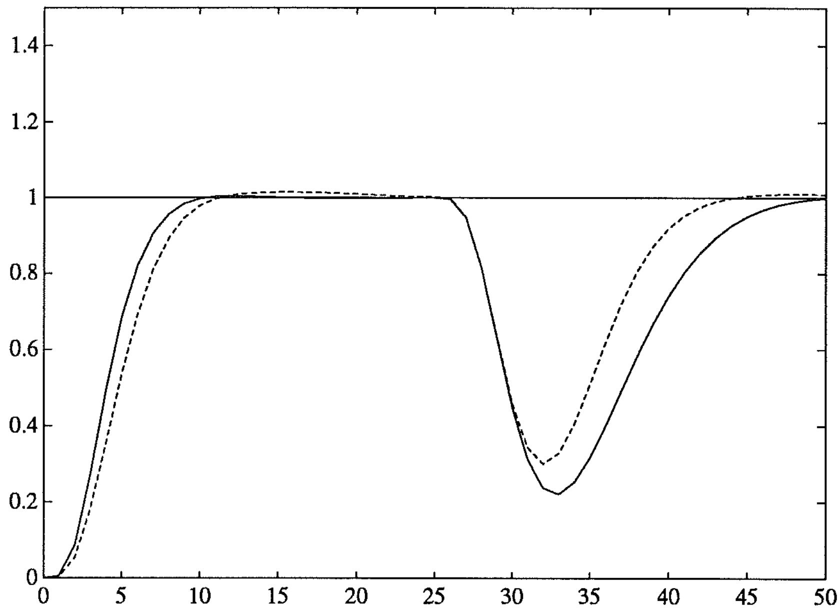

Figure 8.5 Nominal behavior with $\omega_m = 0.6$ and slower observer dynamics $\omega_o = 0.3$ (solid) and base line design (dashed) where $\omega_m = \omega_o = 0.5$ in Ex 1.

observer dynamics is now chosen as

$$Bess(n_R, 0.5 \cdot \omega_m)$$

Notice that the speed of both this and the previous observer dynamics is always proportional to the command following speed (ω_m). Since ω_m can be increased, the observer dynamics does not have to be slowed down by as much as a factor of 2 compared to the previous case. However, ω_m can not be increased enough to compensate for the relative speed reduction of the observer polynomial. In this case a reasonable design is obtained for $\omega_m \in [0.4, 0.7]$. The choice $\omega_m = 0.6$ gives the nominal design shown in Figure 8.5. Notice that the obtained system is faster but the load disturbance rejection slower. The behavior of the adaptive system is shown in Figure 8.6 where the adaptive loop is closed at $t = 80$.

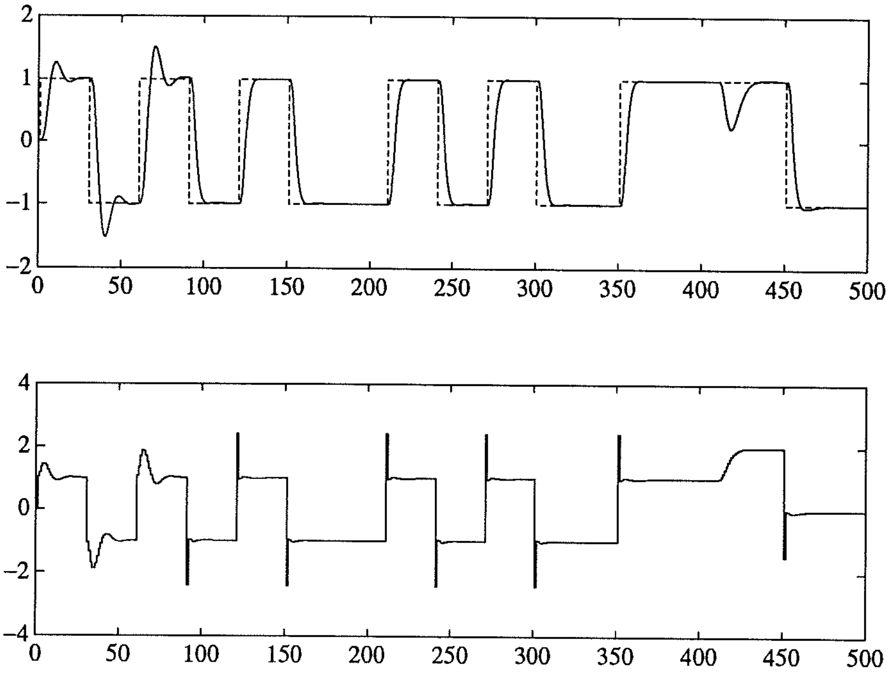

Figure 8.6 Behavior with slower observer dynamics when $\omega_m = 0.6$ in Ex 1.

Variable Process Gain

To study the behavior for a time varying process the process gain is increased linearly with time by a factor of two in the time interval [200, 250]. Figure 8.7 shows the behavior when $\lambda = 0.98$ and $\omega_m = 0.5$ is used. Since the estimators can not follow too fast changes in the process dynamics the behavior deteriorates slightly during and for a while after the change in the process dynamics. However, the behavior approaches the desired after some time. Figure 8.8 shows that the frequency response estimates follows the change in process dynamics.

Third Order Controller

By increasing the controller order to three the obtainable speed is increased. For this the slower observer dynamics is used and

$$\mathbf{n_c} = (3, 3, 3)$$
$$R_p = (z - 1) \;,\; S_p = 1$$

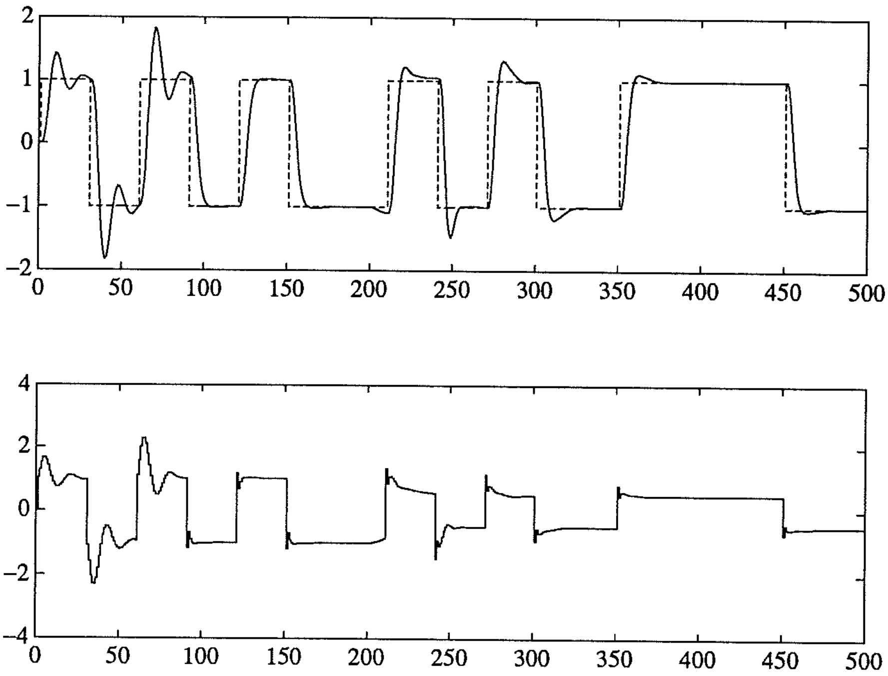

Figure 8.7 Behavior with gain increase by a factor of two in the time interval [200, 250] when $\omega_m = 0.5$

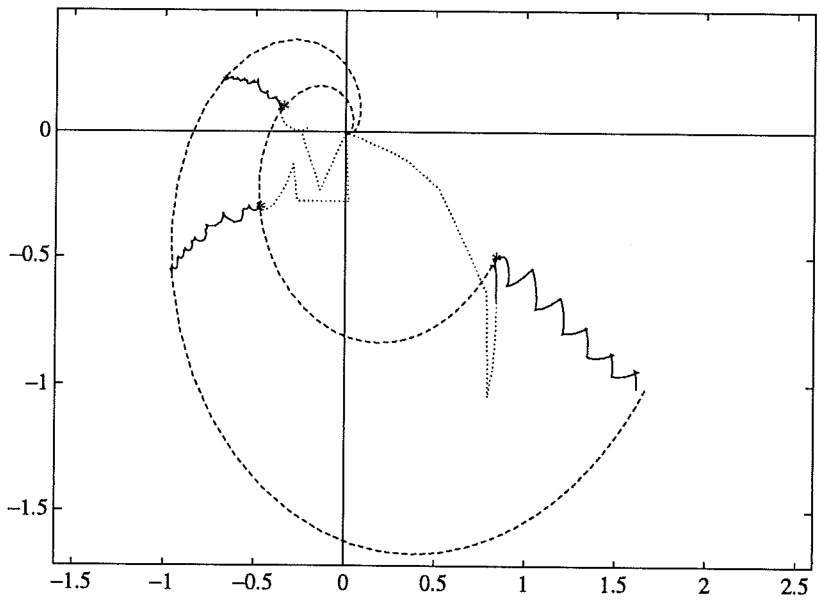

Figure 8.8 Time evolution of the frequency point estimates in Ex 1. The dashed curves are the true Nyquist curves before and after the change in process dynamics. The estimates are dotted for $t \leq 15$.

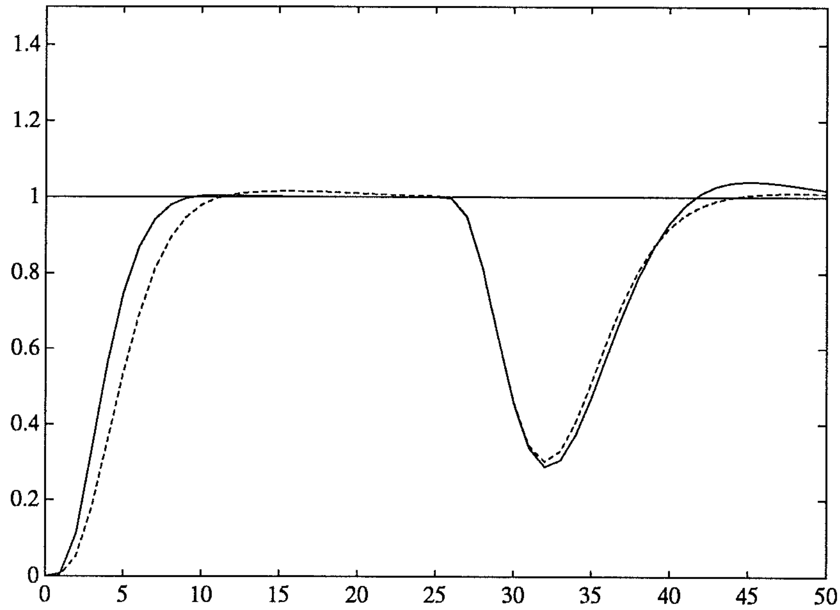

Figure 8.9 Nominal behavior with $\omega_m = 0.65$ and a third order controller (solid) and the base line design (dashed) where $\omega_m = 0.5$ in Ex 1.

The command following speed can now be chosen in the region $\omega_m \in [0.4, 0.8]$. The behavior of the nominal design with $\omega_m = 0.65$ is shown in Figure 8.9 where it is compared with the base line design. Notice that the command following is faster but the disturbance rejection of the same order as for the base line case. This is because the slower observer dynamics is used in the design. The behavior of the adaptive system is shown in Figure 8.10. Rather large control signals are applied at the command signal steps to speed up the closed loop system. Finally the behavior of the adaptive system is shown for moderate demands on the closed loop system in Figure 8.11 when $\omega_m = 0.4$. In this case $A_o^c(s) = Bess(n_R, 5\omega_m)$ is used. As can be expected the control signal is much smoother.

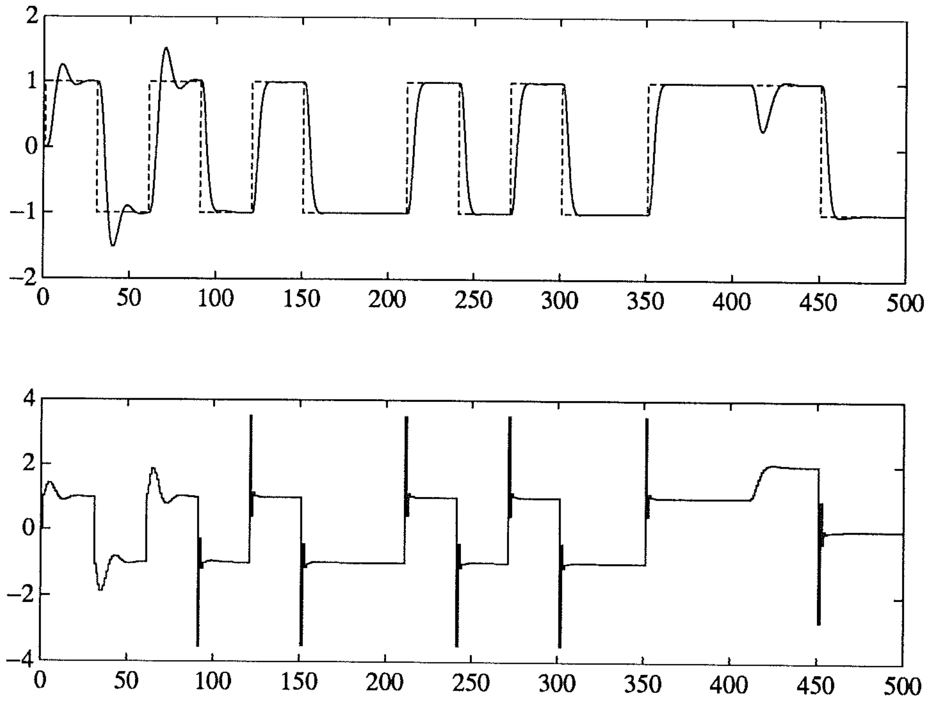

Figure 8.10 Behavior with a third order controller when $\omega_m = 0.65$ in Ex 1.

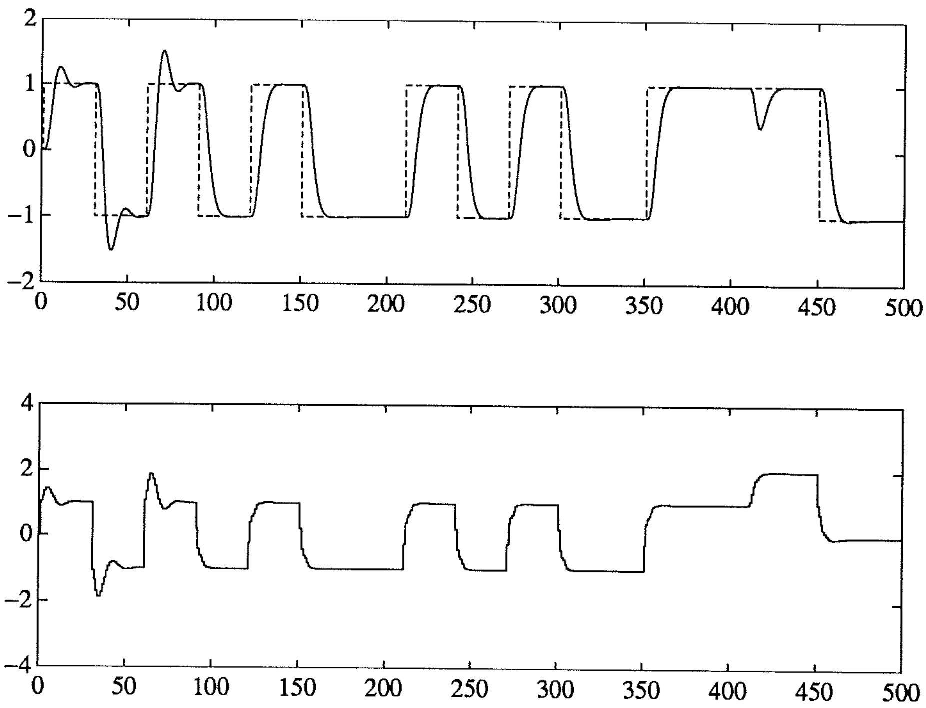

Figure 8.11 Behavior with a third order controller when $\omega_m = 0.4$ in Ex 1.

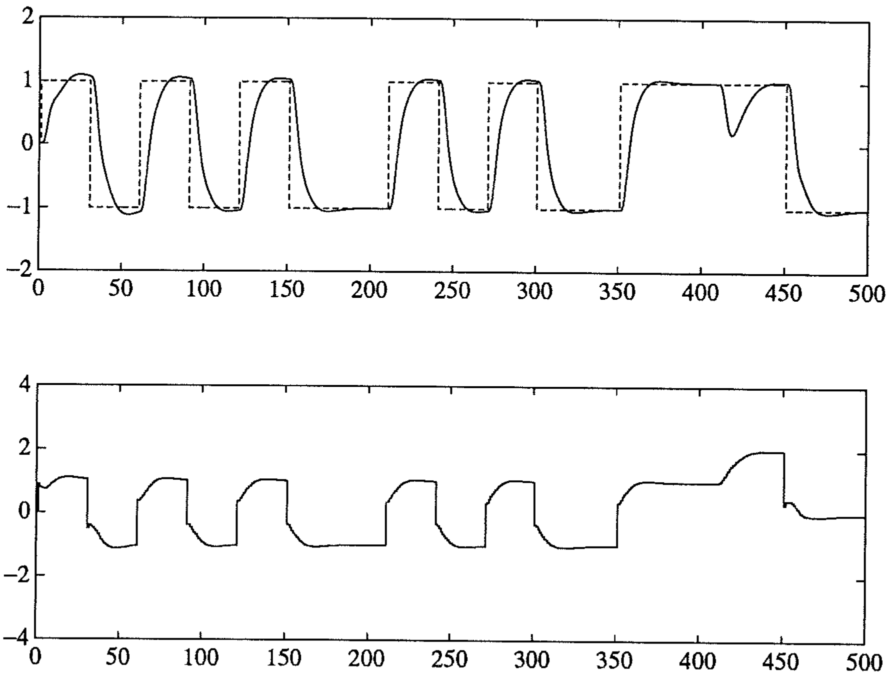

Figure 8.12 Behavior of the STR when $\omega_m = 0.3$.

Comparison with a Self Tuning Regulator

In practical situations parametric models give only approximate descriptions of the plants. What is a good model depends on the intended use of the model. From a control point of view it's the modeling of the process around the loop transfer critical frequency that is most important. This implies that the model order selection in a practical situation may be considered a design parameter and not a fixed quantity. By applying this point of view to an indirect STR scheme a comparison with the FDAC on equal terms can be made. Specifying a second order controller with integral action implies that a second order process model is used in the STR. For a fair comparison the estimator data is filtered with a fourth order band pass filter with pass band [0.05 1] rad/s. Further

$$A_m^c(s) = A_o^c(s) = Bess(2, \omega_m)$$

In Figures 8.12 and 8.13 the behavior of the STR scheme is shown for $\omega_m = 0.3$ and 0.5 respectively. In the simulations the initial

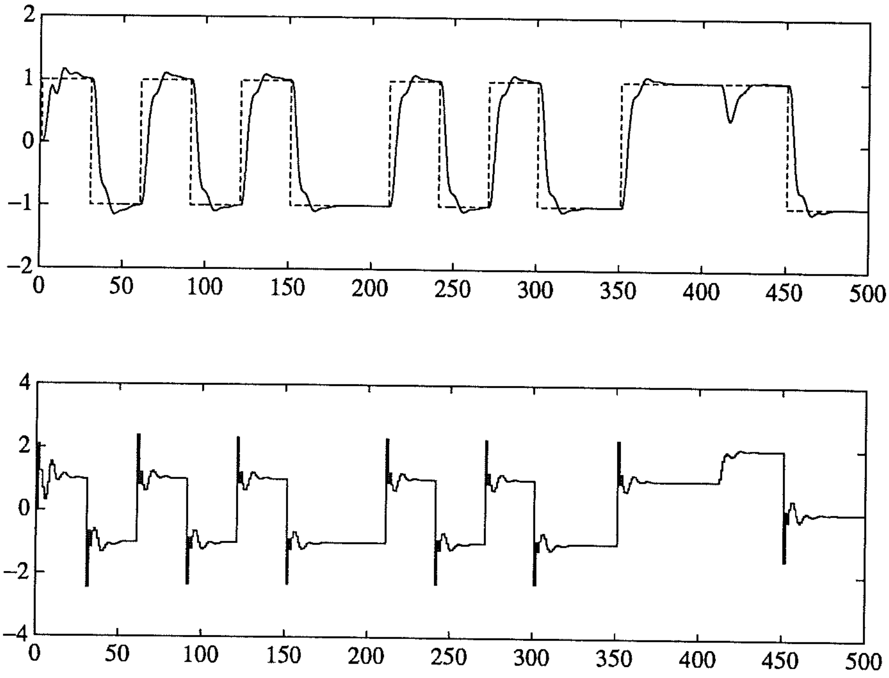

Figure 8.13 Behavior of the STR when $\omega_m = 0.5$. Compare Figure 8.1.

parameter values are chosen close to the final and therefore only minor transients occurs initially. The figures show that the STR scheme is less robust to choice of controller order. Consequently, the closed loop can be made faster when using the FDAC without obtaining a deteriorated behavior, compare Figure 8.1.

Example 2 - Non-Minimum Phase Process

Consider the process

$$G(s) = \frac{1 - 2s}{(s+1)^5}$$

Introducing non-minimum phase zeros gives increased high frequency gain and at the same time increased phase lag. A harder design problem is then obtained since demands on amplitude and phase margins are more difficult to meet without decreasing the desired closed loop bandwidth. The non-minimum phase properties will also show up in the closed loop system since it is not possible to cancel

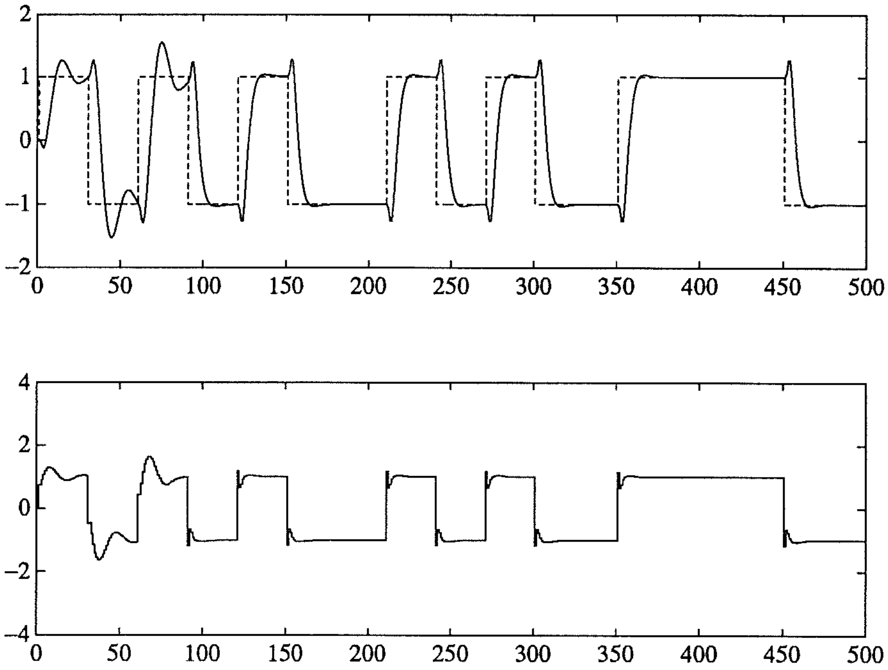

Figure 8.14 Behavior of the adaptive system when $\omega_m = 0.6$ for the non-minimum phase process in Ex 2.

non-minimum phase zeros in the closed loop response.

The non-minimum phase zero is not known in the design. In the desired response G_m^c it is taken care of by including an approximate time delay $\tau_m = 3$ sec obtained from the process step response. The high frequency amplitude roll-off of the process is decreased by introducing a non-minimum phase zero. Compared to a process without the non-minimum phase zero it is natural to decrease the relative degree of G_m^c to obtain a good fit in the controller design at high frequencies. By again using the dominant pole structure the desired closed loop poles are chosen to

$$A_m^c(s) = Bess(2, \omega_m) \cdot Bess(2, 3\omega_m)$$

With a second order controller, $\omega_m = 0.6$ gives a good design when $A_o^c(s) = Bess(2, 0.5\omega_m)$ is chosen. This corresponds to a closed loop system with approximately the same response speed as the open loop process. The behavior of the adaptive system is shown in Figure

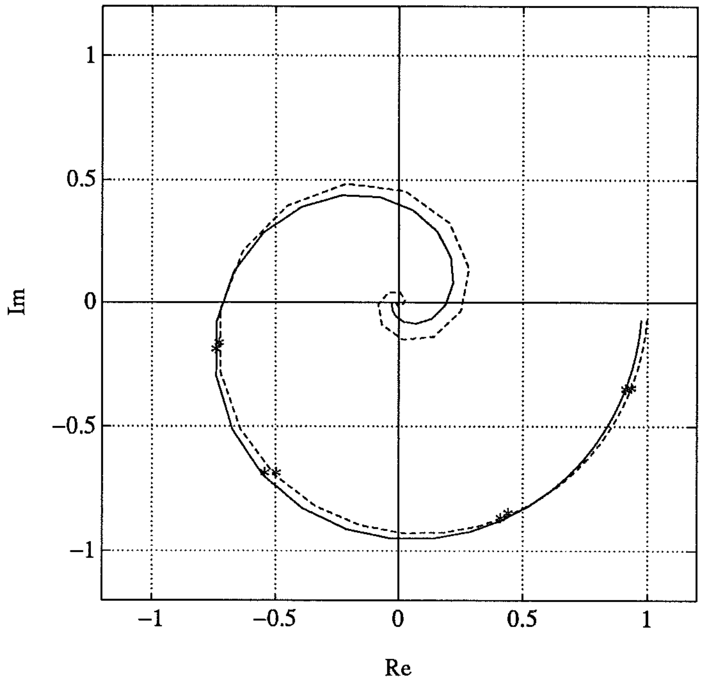

Figure 8.15 Nyquist curve of the closed loop at t=500 (solid) and of the desired response (dashed) in Ex 2.

8.14. In Figure 8.15 the desired and the obtained closed loop Nyquist curves at $t = 500$ is shown. The fitting is good considering that the non-minimum phase zero is not known in the design and that the used T polynomial differs slightly from that obtained from the controller design. The used T polynomial is given by

$$T_{used}(z) = \frac{S(1)}{T(1)} T(z)$$

where S and T are obtained from the controller design. This gives exact unit steady state gain when a controller with integral action is used despite the fact the closed loop fit at $\omega = 0$ is not exact. The low order models are chosen as

$$G_k(z) = z^{-5} \frac{b_{1k} z}{z^2 + a_{1k} z + a_{2k}}$$

where the non-minimum phase properties are approximated with a time delay obtained from the process step response. In the design

$\Omega = \{0.05 \ \ 0.15 \ \ 0.3 \ \ 0.4\}$ is used. An inspection shows that the frequency point estimates have a bias of approximately 1 percent which is quite small. The closed loop system therefore behaves as can be predicted from the nominal design.

Example 3 - Process with a Time Delay

The process

$$G(s) = \frac{1}{(s+1)^2}e^{-5s}$$

has a time delay that is considerable compared to the other dynamics of the process. This time delay is kept in the desired response. For this process a good fit is obtained by choosing real closed loop poles. Since the process, excluding the time delay, is of second order

$$A_m^c(s) = (s+\omega_m)^2$$

is chosen. With $A_o^c = Bess(n_R, 0.5\omega_m)$, $\omega_m = 0.65$ give a good design for a second order controller. With a third order controller $\omega_m = 1$ can be chosen. The behavior of the adaptive system for the third order controller is shown in Figure 8.16. The response is close to that of the open loop system. From the process step response it is seen that the low order models can be chosen as

$$G_k(z) = z^{-6}\frac{b_{1k}z}{z^2 + a_{1k}z + a_{2k}}$$

This is used in the frequency point estimators with $\Omega = \{0.05 \ 0.15 \ 0.3 \ 0.45\}$. The obtained frequency point estimates have very little bias.

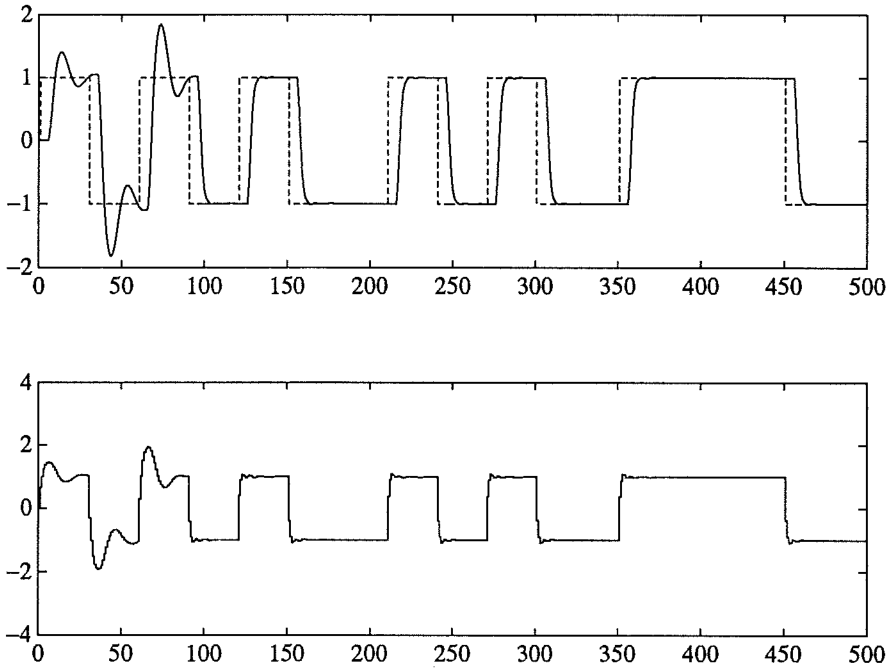

Figure 8.16 Behavior of the adaptive system when $\omega_m = 1$ for the process with time delay in Ex 3.

Example 4 - Process with Integrator and Time Delay

Let the process be described by

$$G(s) = \frac{1}{s(s^2 + s + 1)} e^{-2s}$$

Since both the process and the controller have integrators the loop transfer will have −180 degrees phase at low frequencies. To obtain a fast closed loop system the feedback must give phase advance around the crossover frequency of the loop transfer. Since the process has a time delay this is included in the desired response. The desired characteristic polynomial is chosen as

$$A_m^c = Bess(2, \omega_m) \cdot Bess(3, 4\omega_m)$$

B_m^c is chosen as a constant giving unit steady state gain. The observer polynomial is chosen as $A_o^c = Bess(n_R, \omega_m)$. By taking the

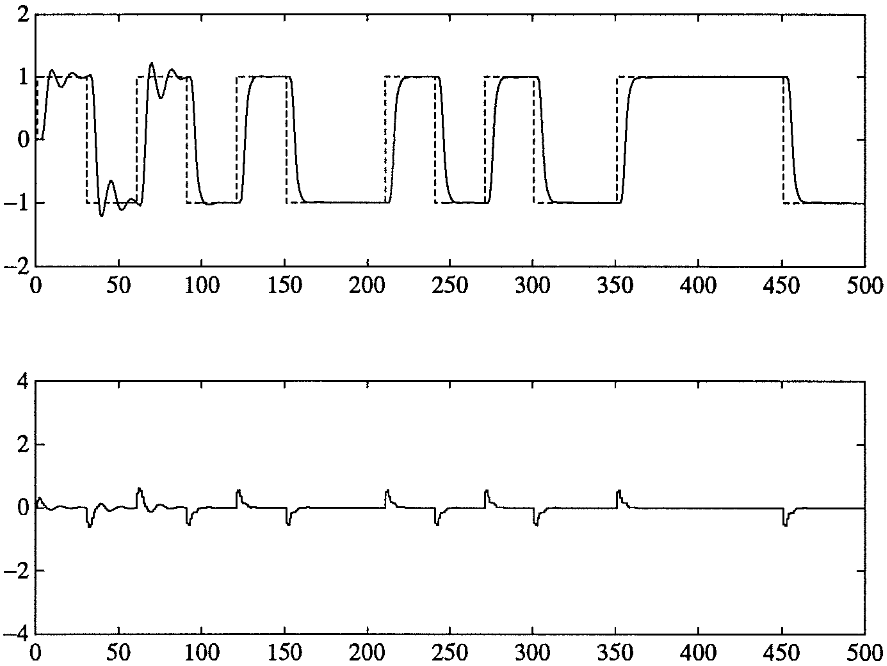

Figure 8.17 Behavior of the adaptive system for a third order controller when $\omega_m = 0.7$ in Ex 4.

time delay into account the low order models are chosen as

$$G_k = z^{-3} \frac{b_{1k} z}{z^2 + a_{1k} z + a_{2k}}$$

where the fitting frequencies are chosen as $\boldsymbol{\Omega} = \{0.2 \ 0.3 \ 0.5 \ 0.7\}$. For a second order controller $\omega_m = 0.4$ give a good design while for a third order controller $\omega_m = 0.7$ can be used. In Figure 8.17 the behavior of the adaptive system is shown when $\omega_m = 0.7$ for a third order controller. The fitting to the desired response is good as can be seen in Figure 8.18 where the desired and the obtained closed loop system at $t = 500$ is shown.

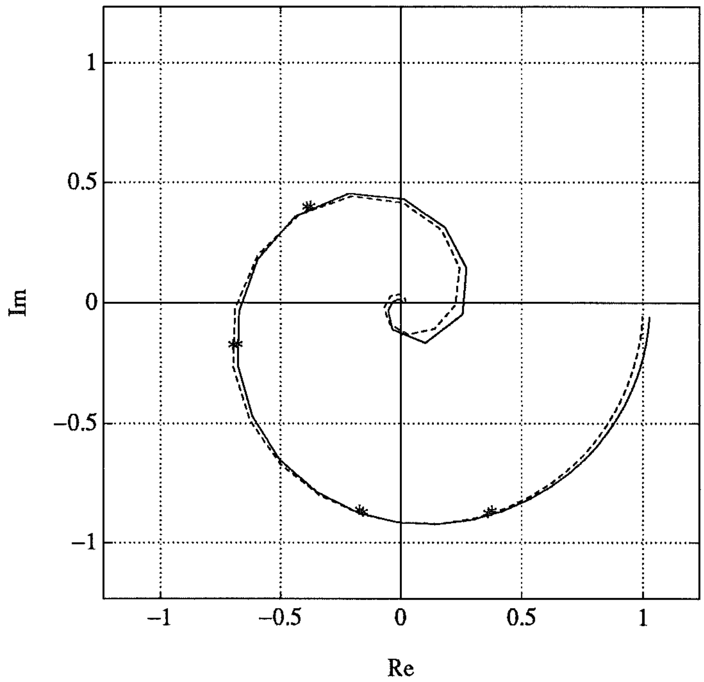

Figure 8.18 Nyquist curves of the desired (dashed) and obtained closed loop system at t=500 (solid) when ω_m = 0.7 in Ex 4.

9

Conclusions

Most adaptive controllers are based on time domain concepts. In this thesis a different approach has been used. The goal has been to derive a frequency domain based adaptive control scheme. In this both the process estimation and the controller design are formulated in the frequency domain. Low order parametric process models are used to estimate points on the Nyquist curve of the process. The controller design is formulated as an approximation problem in the frequency domain and the controller parameters are obtained as the explicit solution to a least squares problem. By using this approach the adaptive scheme can be applied to processes of arbitrary order and without knowledge of the actual process order. Furthermore, low order controllers can be used to control high order processes. The main advantages with the scheme are

- The estimation is concentrated to the frequencies used in the design. Disturbances with frequency content differing from those frequencies will therefore to a large degree be filtered out.
- Demands on excitation is not increased with process order since only points on the process Nyquist curve are estimated.

- Low order controllers can be used to control high order processes.
- The process order does not have to be known in the design.

The examples in Chapter 8 show that the adaptive system behaves well for various types of processes when proper choices of closed loop specifications are used. Further, a comparison with the STR example shows that the FDAC is more robust with respect to choice of controller order.

Comparison

In the FDAC the quantities that has to be supplied by the user is similar to those of an indirect STR scheme. A comparison gives

FDAC	STR
n_R , n_S	n_A , n_B
G_m	G_m
A_o	A_o
h	h
H_{f_j}	H_f
Ω	

Here the model orders n_A and n_B of the STR are considered as design parameters that do not have to equal the unknown true process order. The design variables are similar except for Ω in the FDAC. However, Ω is related to H_f in the STR scheme since both determine the frequency region of interest. The comparison between STR and FDAC in Example 1 shows that the FDAC is at least in some cases more robust than the STR scheme. The reason for this is that the FDAC does not rely on a specific process model order.

Future Research

In order to obtain a well working adaptive system a number of quantities have to be specified. Since this may be time consuming it would be preferable if this task, that is currently left to the user, could be automated. A future research topic is to develop methods that automatically chooses proper parameters for the adaptive controller. This can, for instance, be performed in an initial phase where the controller extracts information about the process and from this decides on proper choices of parameters, see Lundh (1991). With a time varying process a continuous updating of the design parameters should be considered. The desired response will then depend on the process characteristics.

In the controller design it is import to choose the desired closed loop bandwidth properly in order to obtain a stable controller. This is a property common to many design methods. Since, if possible, stable controllers should be used, an important research topic is to investigate conditions for obtaining stable controllers. The allowable closed loop bandwidth could in this context be regarded as an output parameter to be determined instead of input parameter as is the case in many design methods. The desired controller properties could then be regarded as new input parameters.

To obtain estimators that are more robust to disturbances and lack of excitation, further research can be devoted to safety nets for the estimation and if necessary, to derive modifications to the estimation scheme.

10

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