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Serial and Parallel Concatenations Based on Faster Than Nyquist Signaling

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***Abstract*— We investigate the performance of concatenated coding schemes based on Faster Than Nyquist (FTN) signaling over the AWGN channel. We test both serial and parallel concatenations. In serial concatenation the FTN signaling is considered as the inner encoder and the outer code is a rate b/c convolutional code. In parallel schemes we use two parallel Gaussian channels and transmit FTN pulse trains in both; here a precoding device turns out to be crucial. The convergence behaviour is analysed using EXIT charts. The overall spectral density of the schemes varies but is roughly 1–2 bit/s/Hz. The results, in terms of needed E_b/N_0 for reliable communication versus spectral density, are very good.**

Key Words: *Coded modulation, Faster Than Nyquist, Concatenated coding, iterative decoding, EXIT charts*

I. Introduction

Since the landmark paper [1] was published in 1993 concatenated coding schemes combined with iterative decoding have been very popular. The so called turbo codes have seen major resaerch effort and the original parallel structure was soon extended to serial concatenation [2]. These systems offer near capacity operation but still at a medium decoding complexity. However, they are usually limited to spectral density 1 bit/s/Hz, due to the binary structure. To achieve a higher throughput a higher order modulation is employed, such as 8 PSK or 16 QAM; examples of these systems are Turbo TCM [3] and serially concatenated TCM [4] to mention a few. When decoded with iterative decoding these systems can also operate close to capacity.

The convergence behaviour of turbo like schemes is today usually analysed by EXIT charts. Although they have not been mathematically proven to be perfectly valid, the mutual information upon which they are based seems to give the most accurate prediction of convergence behaviour [5].

An older, but less investigated, coded modulation is Faster Than Nyquist (FTN) signaling. If a PAM system is based on T-orthogonal pulses, the pulses can be packed closer than the Nyquist rate 1/T without suffering any distance loss. In a bandpass system a QAM signal constellation is used. The rate where the square distance falls below 2 for the first time is called the Mazo limit. Mazo showed that for ideal sinc pulses the rate can be increased by as much as 25% without sacrifying the minimum squared Euclidean distance [6]. Mazo limits for the more practical root raised cosine pulse are found in [7], and a generalization of FTN is given in [8]. FTN achieves spectral efficiency not by expanding the signaling alphabet but simply by sending pulses more often; this is major difference in strategy. For systems operating close to capacity the minimum distance plays little role.

In this paper we will investigate the performance of several concatenated schemes where FTN is used as modulation. When serial concatenation is done the iterative decoding goes under the name turbo equalization [9]. It is generally assumed that the intersymbol interference (ISI) channel is unintentional, but here we add it intentionally. The contribution of the paper can be described by posing the following question: Given an arbitrary convolutional code $\mathcal{C}$, an interleaver and bit rate R_b, what is the smallest necessary bandwidth to maintain the performance of $\mathcal{C}$?

To answer the above question we will suggest, using EXIT charts, the theoretical limit for our type of system with a selection of convolutional codes. The limits will then be verified by simulations. The outcome will be that a rate 1/2 convolutional code can maintain its performance at roughly one third of its normal bandwidth.

In our work the parallel concatenated schemes appear to be better than serial concatenations. In parallel concatenations a rate 1 precoding device is used. The parallel system can be seen as an instance of bit interleaved coded modulation (BICM) with a rate $1/2$ repetition code as an outer code and a rate 1 convolutional code as inner code and modulation via FTN; in that case the structure is not parallel but serial. However, due to its resemblance to the original turbo structure of [1] we still refer to it as parallel concatenation.

The paper proceeds as follows. In section 2 we give the system models and some basics of decoding. In section 3 and 4 results on serial and parallel concatenations are given.

II. System models

Consider a baseband QAM system based on a time continuous pulse $\psi(t)$. The signal transmitted over the channel is given by

$$s_{\mathbf{a}}(t) = \sum_{n=-\infty}^{\infty} a[n]\psi(t - nT_\Delta), \tag{1}$$

where $a[n] \in \{\pm1\pm\jmath\}$ are the data symbols and T_Δ is the symbol time. We shall refer to this as the symbol time separation. Let $\mathbf{a}$ denote the sequence $\{\ldots, a[-1], a[0], a[1], \ldots\}$. Furthermore we assume $\psi(t)$ to be unit energy, i.e. $\int_{-\infty}^{\infty} |\psi(t)|^2 dt = 1$. It is well known that if the data symbols are uncorrelated the

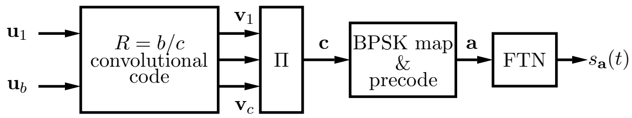

Fig. 1. System model for serial concatenation. The convolutional code has 2^b branches out of each trellis node and c bits on each branch.

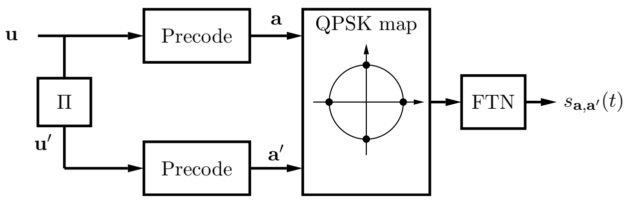

Fig. 2. System model for parallel concatenation.

power spectral density of the transmission equals $|\Psi(f)|^2$. The one sided baseband bandwidth of the modulation is denoted by W, and $R = 1/T_\Delta$ is the symbol rate. For the pulse $\psi(t)$ we use root raised cosine pulses with excess bandwidth α. When $\alpha = 0$ we get an ideal sinc pulse. Throughout this paper we use $\alpha = .3$, a common value in practice, although not strongly bandwidth efficient. All pulses within this family are orthogonal if packed at the Nyquist rate $1/T$, where $T = (1+\alpha)/2W$. The spectral density can be increased by decreasing T_Δ below T. Mazo showed [6] that for ideal sinc pulses the signaling rate can be increased almost 25% without any Euclidean distance loss, i.e., $T_\Delta = .802T$.

FTN will be used as modulation for two concatenated schemes. The system models for the serial and parallel concatenations are shown in figure 1 and 2. The interleaver used is an S-random interleaver of block length 10000 symbols and $S = 25$.

In the serial concatenation scheme an outer rate b/c convolutional code first encodes the data which belongs to the alphabet $\{0, 1\}$, and this is followed by interleaving. The resulting bit sequence is then mapped to $\{-1, 1\}$ and possibly precoded; the resulting symbol sequence is then transmitted using a FTN/BPSK scheme having time compression ratio

$$\tau = \frac{T_\Delta}{T}. \tag{2}$$

In the parallel case the symbol sequence $\mathbf{u}$ is assumed to take values in $\{-1, 1\}$; then interleaving $\mathbf{u}$ yields $\mathbf{u}'$, and the two data streams are precoded with a rate 1 precoder. A mapping follows where $\mathbf{a}$ is mapped onto the I component and $\mathbf{a}'$ onto the Q component of a QPSK constellation. Finally the transmitted signal is formed by FTN modulation.

The resulting spectral densities for the two cases are

$$R_b/W = \begin{cases} (2b/c)/(1+\alpha)\tau & \text{serial} \\ 1/(1+\alpha)\tau & \text{parallel} \end{cases} \tag{3}$$

Assuming that the precoder generates an uncorrelated sequence the average energy per bit equals

$$E_b = \begin{cases} c/b & \text{serial} \\ 2 & \text{parallel,} \end{cases} \tag{4}$$

where we have used that $\int_{-\infty}^{\infty} |\psi(t)|^2 dt = 1$.

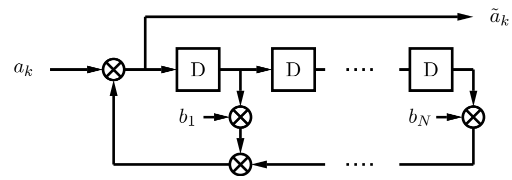

Fig. 3. System model for the precoder.

The system model of the precoding device is shown in figure 3, where $a_k \in \{-1, +1\}$ and $b_k \in \{0, 1\}$. If the length N of the precoder is less than the length of the ISI response, no extra complexity is introduced by the precoder.

Decoding is done by standard iterative decoding for turbo like systems, see for example [10]. The only non standard thing is that we do not assume the Forney model [11] in the decoder; instead the decoder works directly on samples from a fliter matched to $\psi(t)$; this is the so called Ungerboeck model [12]. A problem arises since the resulting model is not causal and the noise samples are not white; therefore the standard BCJR algorithm cannot be applied. In a recent paper [13], a BCJR like algorithm was derived for the Ungerboeck model. It was also shown that this algorithm gives identical results compared to a BJCR on the corresponding Forney model.

This decoder encounters the sequence

$$y_k = \sum_{l=-\infty}^{\infty} g_l a_{k-l} + n_k, \tag{5}$$

where g_l is

$$g_l = \psi(t) \star \psi^*(-t)|_{t=lT_\Delta} \tag{6}$$

The noise samples $\{n_k\}$ have autocorrelation $R_n(m) = 2N_0 g_m$ where N_0 is the one sided noise power spectral density. Since $\psi(t)$ is theoretically infinite and practically very long, the ISI sequence $\{g_l\}$ is also very long and the number of states in the decoder is astronomical. This we resolve by truncating the impulse response used by the decoder to finite length, say L on each side, and treat the rest as Gaussian noise; this is refered to as Truncated Viterbi (TV) in the literature. It will of course decrease the performance, but if L is large enough the performance drop is not significant. As was demonstrated in [7] a linear post processing device can be applied to improve the quality of the soft decisions, but this has not been used here. Throughout we have used $L = 5$ which results in 32 states in the decoder. For time compressions τ close to 1, L could probably be decreased further. For small τ, $L = 5$ is probably too small. The actual length of $\psi(t)$ was $80T$ for all simulations.

To analyze the behaviour of iterative decoding EXIT charts are used. EXIT charts provides quick insight into the convergence mechanism although they do not give a hard indication of convergence. Typically they offer good guidelines when the block size is reasonable. Due to lack of space we will not explain EXIT charts here; instead we refer to [14].

III. Serial concatenations

We will briefly review the performance of these types of systems; a full treatment is found in [15]. We start with no precoding. Consider a low weight error word $\mathbf{v} = \{\ldots, 0, 0, e_1, e_2, \ldots, e_\nu, 0, \ldots\}$; ideally the interleaver will break this event up into a signal error word $\mathbf{c}$ where the nonzero entries are randomly spread over $\mathbf{c}$. The generated Euclidean distance will now become $d^2 = 2\nu b/c$. This implies that the asymptotic BER is limited by the minimum distance of the outer code.

When applying recursive precoding the story is different. Low weight error codewords are mapped into high weight signal error words $\mathbf{e}$. Only for some specific signal words $\mathbf{a}$ will this signal error word $\mathbf{e}$ generate an error event with small Euclidean distance. This implies that a precoder weight gain of $1/2^{|\mathbf{e}|}$ is achieved, with $|\mathbf{e}|$ the number of nonzero components in $\mathbf{e}$. Although the Euclidean distance might be smaller than for the no precoder case this weight gain is significant and leads to smaller BERs.

By assuming that EXIT charts do provide full insight into the convergence behaviour, the problem of finding inner and outer codes becomes equivalent to finding curves with an open convergence tunnel. Assume that the inner and outer codes have EXIT curves $T_i(x)$ and $T_o(x)$ respectively. The system will converge to a point where $T_i(x) = T_o^{-1}(x)$. However, since

$$T_i(1) < 1, \tag{7}$$

this point is not $(I_A, I_E) = (1, 1)$ for non precoded systems. It can be shown that $T_i(1)$ will be equal to $T_{orth}(1)$, where $T_{orth}(1)$ denotes the EXIT curve of an orthogonal system corresponding to $\tau = 1$. The reason is that if the decoder has full knowledge about interfering bits, i.e. $I(\mathbf{a}, LLR(\mathbf{a}|\{y_k\})) = 1$, then all the interfering bits can be subtracted from (5) and the ISI channel becomes a memoryless channel with colored noise. Furthermore, $T_{orth}(x)$ is a horizontal line, i.e. $T_{orth}(x) = \lambda, \quad 0 \le x \le 1$. Given a target BER, assume that the outer convolutional code is powerful enough to achieve this target BER when $T_{orth}(x) = \lambda_o$. Then, via iterative decoding, the convolutional code can also decode a scheme where $\tau < 1$ if $T_i(\lambda_i) = \lambda_o$, where (λ_i, λ_o) is the intersection point between $T_i(x)$ and $T_o(x)$, conditioned that $T_i(x) > T_o^{-1}(x), \quad x < \lambda_i$.

For powerful outer codes, having very steep EXIT curves for inputs close to 1, we have $\lambda_i \approx 1$, which means that nothing is lost in terms of E_b/N_0; the gain is a pure rate gain, given by $1/\tau$, traded off against decoding complexity. Furthermore, it is interesting to observe that the curves $T_i(x)$ are very well approximated by straight lines. Assuming a target BER of 10^{-5}, table I lists necessary E_b/N_0 for some optimum distance profile codes (ODP) [16], with memory lengths 2–4, together with the minimum τ such that there is an open convergence tunnel. If the number of iterations is limited, e.g., to only two, one can search for the smallest time separation T_Δ that fulfills

$$T_i(T_o(T_i(0))) = \lambda_o. \tag{8}$$

generator	E_b/N_0	τ
(7,5)	5.9	.32
(74,54)	5.5	.33
(62,56)	5.0	.34

TABLE I

CONVOLUTIONAL CODES AND NECESSARY E_b/N_0 TO ACHIEVE BER 10^{-5} AND SMALLEST τ FOR OPEN CONVERGENCE TUNNELS.

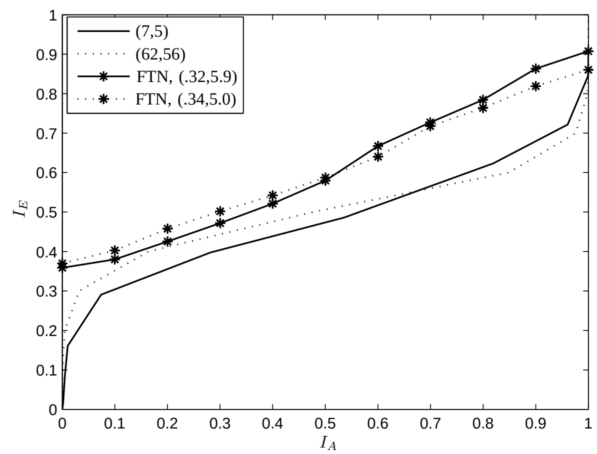

Fig. 4. EXIT charts for serial concatenations. The legend gives the generator polynomial and $(\tau, E_b/N_0)$ for FTN

The EXIT charts for two of the convolutional codes and corresponding FTN schemes given in table I are shown in figure 4. From these we observe that weak codes have more bandwidth reduction. The bandwidth reductions are significant: the performance of a normal rate $1/2$ code can be maintained but at roughly one third of its normal bandwidth. Simulations are shown in figure 5; it is seen that for BER 10^{-5} the time compressed system suffers only very small losses compared to the orthogonal case. By increasing the interleaver length this loss probably gets even smaller. The number of iterations for the (74,54) code at high E_b/N_0 was 5 for $\tau = .4$ and 7 for $\tau = .36$. For the (62,56) code the number of iterations was 7. A simulation where the underlying code is the (634,564) convolutional code is shown in figure 6. The number of iterations is 2 and $\tau = .57$; the resulting spectral density is 1.35 bit/s/Hz. Also shown is a simulation of the systematic rate 2/3 convolutional code (4 0 5;0 4 7) combined with an FTN scheme with $\tau = .42$; the resulting spectral density is 1.83 bit/s/Hz. The number of iterations was 7.

A non precoded serial system is always limited to the error performance of the underlying outer code, but precoding the FTN modulation by a rate 1 recursive convolutional code opens up the possibility of lowering the SNR. By regarding

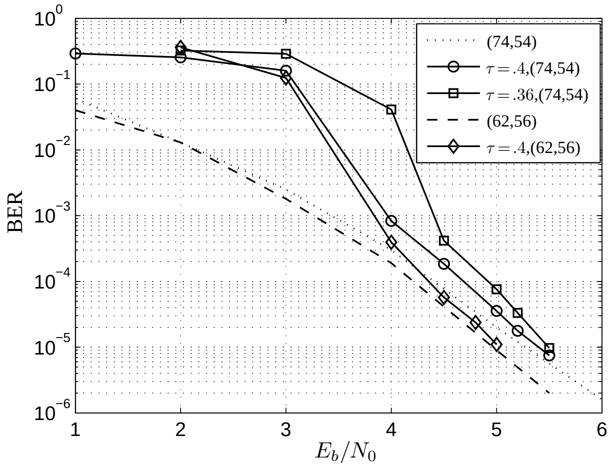

Fig. 5. Simulations for non precoded serial concatenations. The dashed and dotted curves show the performance of the underlying convolutional codes over an AWGN channel.

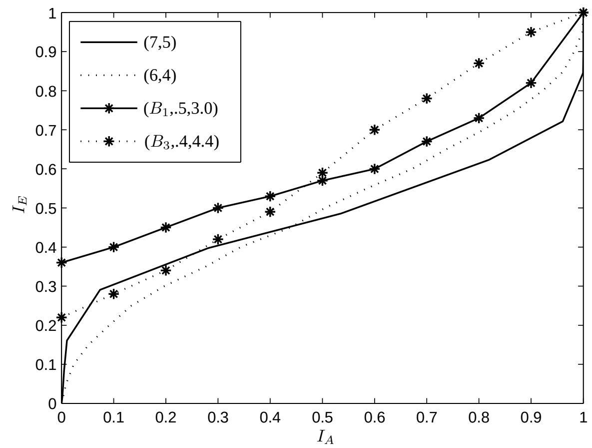

Fig. 7. EXIT charts for serial concatenations. The legend gives the generator polynomial and (precoder,τ, E_b/N_0) for FTN

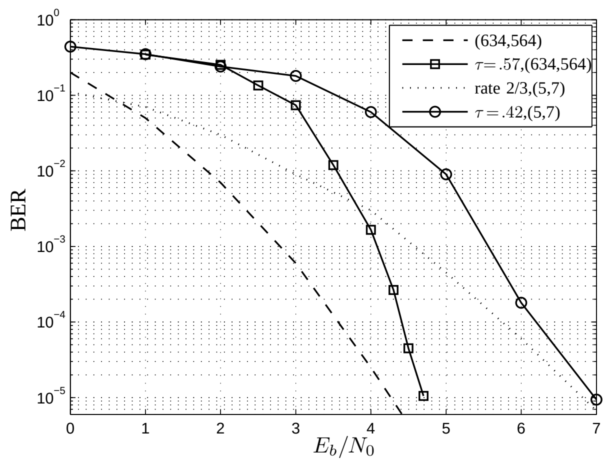

Fig. 6. Simulations of a scheme with a systematic rate 2/3 outer code and a scheme designed for two iterations.

the precoder as a part of the inner code it can be seen that $T_i(1) = 1$ for precoded systems which means that the performance is not limited to the outer code any more. In this paper we consider three different precoders, namely $B_1 = [11]$, $B_2 = [111]$ and $B_3 = [11111]$. However, for precoded systems the EXIT charts get worse for lower input mutual informations, which leads to less bandwidth reduction.

Two examples of systems are shown in figure 7. The outer codes are the (6,4) and (7,5) convolutional codes and the inner precoders are B_1 and B_3. The parameters for the FTN modulation are $\tau = .5$ and $E_b/N_0 = 3.0$ dB for the solid asterix marked curve and $\tau = .4$ and $E_b/N_0 = 4.4$ dB for the dotted asterix curve. As seen, the precoded system is more power efficient but has lower spectral density since the time compression ratio is higher. Moreover, the convergence tunnels are still quite open and could be further closed, but this would require an enormous blocklength. These two precoded systems are just examples of good systems, and there exist many more. The resulting spectral densities are 1.923 bit/s/Hz and 1.539 bit/s/Hz, respectively.

IV. Parallel concatenations

Since $T_i(1) < 1$ for non precoded systems it is clear that precoding must be applied for parallel concatenation. In fact, consider a single input bit error for a non precoded parallel system. This single error will cause distance $D(e) = 4$ in each signal dimension. After normalizing with $2E_b$ the normalized distance will be $d(e) = 2$, so the performance of such a system will never be better than an uncoded system, although bandwidth is lower. By investigation we have found that this bandwidth reduction is not much better than the saving for normal FTN [6]; therefore we did not investigate non precoded parallel concatenations further.

We now investigate the case where $\tau = .5$. This case is particularly important because competing systems, working with an outer rate 2/3 code and using 8 PSK as modulation, will end up at exactly the same spectral density if the same pulse shape $\psi(t)$ is used. Therefore we can directly compare them. The 2/3–8PSK system has Shannon limit at $E_b/N_0 = 2.8$ dB, and we will here demonstrate a system based on parallel FTN that operates below 2.8 dB. However, if a multicarrier modulation system is employed the orthogonal schemes can avoid excess bandwidth (asymptotically) and must therefore be compared to FTN schemes where τ is smaller than .5; therefore our results are strongest and most suited for single carrier modulation.

By replacing the outer code in serial concatenation by a rate $1/2$ repetition code, a system very similar to the parallel concatenation system is obtained, but the two systems are different. This can be seen by considering a single information bit error event. In parallel concatenation this error will generate two error events that last for the rest of the block, and therefore generate high distance; for serial systems the generated error

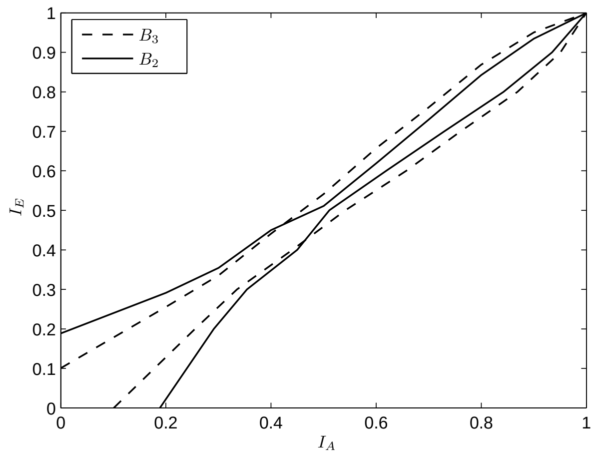

Fig. 8. EXIT charts for parallel concatenations. $\tau = .5$ for both schemes, the solid lines refers to B_2 and the dotted lines to B_3, $E_b/N_0 = 2.2$ dB for both schemes.

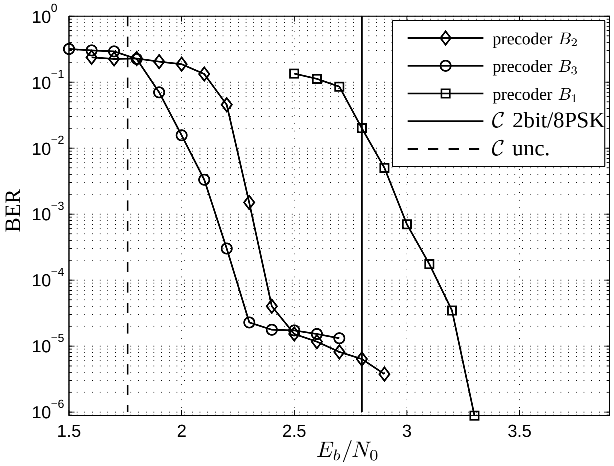

Fig. 9. Simulations for parallel concatenations. The vertical dashed line shows the Shannon limit for an unconstrained input alphabet (Gaussian) with equal throughput for same basic pulse transmitted at the Nyquist rate.

event after precoding can still be short (although larger than S for S-random interleavers). From EXIT charts it can be seen that the two systems will converge at exactly the same E_b/N_0 but the convergence tunnel is exactly twice as large for parallel concatenations.

EXIT charts for the parallel concatenations are shown in figure 8 for two different precoders. Even for very small E_b/N_0 the convergence tunnel is still open. Simulations are given in figure 9. The number of iterations was 35. It is seen that the most powerful system is the system precoded with B_3, and the error floor is just above BER 10^{-5}. Furthermore, the error floor increases with the weight of the precoder but the convergence threshold occurs for smaller values of E_b/N_0. This is opposite to the serial concatenations in [15]. Note that for B_2 and B_3 the system operates below 2.8 dB which is the Shannon limit for standard 2/3-8PSK systems based on orthogonal pulses; this limit appears in the figure. Also shown in the picture is the Shannon limit (1.76 dB) for an unconstrained input alphabet having the same throughput and being transmitted at the Nyquist rate with the same basic pulse. It is possible to have different precoders in the two rails, but we have not explored this.

A careful BER analysis for a given interleaver is possible. If the analysis is done for an uniform interleaver, a particular interleaver can perform much better than the analysis predicts. An S-random interleaver almost always performs much better. Due to lack of space this analysis is not presented here; it is essentially an adjustment of the analysis in [15] to the case of parallel concatenation.

V. Conclusion

We have demonstrated that by means of FTN and iterative decoding a convolutional code can maintain its performance at roughly one third of its normal bandwidth. By precoding the FTN modulations even stronger systems can be found. We tried both serial and parallel concatenations and it seems that parallel is stronger. The best precoders are multiweight precoders. They give performance that is out of reach for standard methods based on orthogonal root raised cosine pulses, such as turbo TCM, BICM, multilevel coding and serially concatenated TCM. When searching for these new coding schemes EXIT charts play a critical role.

VI. Acknowledgments

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