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# On the Distribution of Relevant Radio Channel Figures in Different Propagation Environments for Performance Evaluation of WCDMA Systems

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## Abstract

*A statistical distribution for the number of CDMA RAKE fingers  $N_T$  to be assigned in order to track a given percent of the total received power has been derived. The obtained distribution reveals a strong dependence of  $N_T$  on the system bandwidth, the median rms delay spread, its standard deviation and other environment specific parameters. Also, a statistical distribution for the fading width  $fw$ , is presented. Some numerical results for both  $N_T$  and  $fw$  are presented as a function of the system bandwidth at different probability levels.*

## 1 Introduction

As is well known, the propagation environment set constraints on the performance of a mobile radio communications system. In general, most radio channels show a multipath nature. The radio signals, that are electromagnetic waves, propagate through different pathways and arrive with different delays at the receiver. One parameter that may have a major impact when designing a mobile radio system is the rms delay spread, defined as the second moment with respect to the measured average power delay profile. In the sequel we will refer to the rms delay spread as simply the delay spread.

In real world, the numerous propagation scenarios are characterized by corresponding values of delay spread. From that universal set of scenarios, some representative propagation environments, such as the urban, suburban, rural and mountainous (or hilly terrain) environments may be distinguished. Each of them may then be characterized by a statistical distribution for the delay spread with environment specific parameters, [1].

In CDMA systems the RAKE receiver technique is used to overcome the impairments set by the propagation channel. Two relevant issues for the implementation of RAKE

receiver algorithms are the RAKE finger assignment and the resulting diversity gain. A thorough analysis of those problems depends on many system and channel parameters and therefore may become too complex. Still, both experimental results and computer simulations have shown that reliable conclusions may be drawn by means of rather simple models.

In the following sections, the statistical distribution for the number  $N_T$  of CDMA RAKE fingers to be assigned in different environments in order to track a given percent of the total received power is presented. The distribution for the fading width  $fw$ , is also presented without derivation due to lack of space. Both distributions were found using the model distribution for the delay spread, given by Greenstein et.al., [1]. A brief review of this model follows in section 2.

## 2 Review of the delay spread distribution model.

The delay spread model presented in [1] is based on the three following assumptions:

- The delay spread ( $\tau_{rms}$ ) is lognormal distributed at every base-to-mobile distance  $d$
- The median delay spread ( $\tau_{med}$ ) increases as a power law of the distance ( $d^c$ )
- The delay spread tends to increase with the shadow fading

These assumptions are according to Greenstein et.al, supported by evidence presented in literature. In the present paper we shall pay attention only to points a) and b). The implications of assumption c) are quite obvious due to the direct proportionality between the delay spread and the shadow fading.

In [1] the probability density function (*pdf*) of  $\tau_{rms}$ ,  $p_{\tau}(\tau_{rms})$ , after the integration of the conditional *pdf* over all the mobile locations  $d$  within a given propagation area (or

cell area), is given by the following expressions:

$$p_{\tau}(\tau_{rms}) = \frac{1}{\varepsilon \tau_{rms}} e^{\left( \frac{2y}{\varepsilon} + \frac{2\sigma_y^2}{\varepsilon^2} \right)} \operatorname{erfc} \left( \frac{y}{\sqrt{2}\sigma_y} + \frac{\sqrt{2}\sigma_y}{\varepsilon} \right) \quad (2.1)$$

where,

$$y = \ln \left( \frac{\tau_{rms}}{T_1 d_{max}^{\varepsilon}} \right)$$

The power coefficient  $\varepsilon$  takes values between 0.5 and 1,  $T_1$  is defined as the median delay spread at 1 km,  $\sigma_y$  is the standard deviation in nepers and is related to the standard deviation in decibels as  $\sigma_y = (10/(\ln(10)))\sigma_d$ . The mobile locations  $d$  are distributed over an area of radius  $d_{max}$  as follows:  $p_d(d) = 2(d/d_{max}^2)$ .

Area	#	$d_{max}$ (km)	$\tau_{med}$ ( $\mu$ s)	$\sigma$ (dB)	$T_1$ ( $\mu$ s)	$\sigma_y$ (dB)	$\varepsilon$
Urban	1	7.0	1.9	3.7	0.94	3.6	0.5
	2	3.5	1.1	2.7	0.77	2.6	
	3	1.0	0.7	2.0	0.92	1.9	
	4	5.0	0.7	2.0	0.41	1.9	
	5	0.6	0.2	2.4	0.34	2.3	
Suburban	6	2.0	0.3	4.9	0.28	4.7	0.5
	7	2.2	0.3	2.1	0.27	2.0	
Rural	8	5.0	0.13	5.5	0.076	5.3	0.5
	9	2.2	0.08	4.2	0.071	4.0	
Mountains	10	12.9	3.8	3.3	0.45	3.2	1.0
	11	6.0	1.8	2.5	0.45	2.4	

**Table 1.**  $d_{max}$ :radius of the propagation area,  $\tau_{med}$ :median delay spread over the propagation area,  $\sigma$ : standard deviation over the propagation area,  $T_1$ : median delay spread at 1Km,  $\sigma_y$ : standard deviation at a distance  $d$ ,  $\varepsilon$ : power law coefficient ( $\tau_{med}(d)=T_1 d^{\varepsilon}$ ). The enumeration corresponds to the following propagation environments, (1)4 US Cities, (2)Manhattan, (3)Toronto, (4)Denver, (5)Red Bank (m-cells), (6)Toronto, (7)Hague, (8)Colorado, (9)The Netherlands, (10)Vevey, (11)Korfu.

From the corresponding cumulative probability functions (cpf) (2.1) expressions for the median delay spread are derived in [1] yielding the following result for  $\varepsilon=0.5$   $\tau_{med}(area) = 0.76 T_1 d_{max}^{0.5}$ . The corresponding result for  $\varepsilon=1$  is  $\tau_{med}(area) = 0.66 T_1 d_{max}$ . The standard deviation for all  $\varepsilon$  is computed as  $\sigma(area) = 1.04 \sigma_y$ .

Observe that the values of the median delay spread and the standard deviation are given for an area of radius  $d_{max}$ , not for a circle of constant radius.

Table 1 contains data (locations 1 to 11) presented in [1] with both published measurements in other works and calculations results given by Greenstein et.al.

### 3 Distribution function for the number of RAKE fingers

In order to evaluate the multipath diversity performance of CDMA RAKE receivers in various propagation environments, corresponding simulations were performed. Firstly [2&3], a simplified channel model was assumed, where the power delay profile (pdp) of the channel was assumed to consist of only one exponentially shaped cluster and the fading of each tap was simulated as uncorrelated Rayleigh processes. A further development of the model was done in [4]. There, a more realistic channel was considered. The pdp was now modelled as a three-cluster channel, each of them exponentially shaped with random decay constants, covering in this manner a wider range of probable profiles and delay spread. The RAKE receiver fingers were assumed to be able to track the tap power instantly, and the search for taps occurred in an infinite delay window. Making these assumptions, it is shown in [2&4] that the number of CDMA RAKE receiver fingers ( $N_T$ ) to be assigned in order to track a given percent of the total power measured at the receiver location is a function of the product of the system bandwidth ( $W$ ) and the delay spread ( $\tau_{rms}$ ):

$$N_T = C(W\tau_{rms})^D \quad (3.1)$$

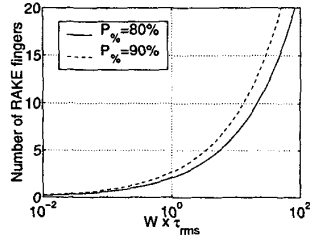
The constants  $C$  and  $D$  are different for different percents of tracked power,  $P\%$ , that is defined as the ratio of the power tracked by a number of RAKE finger and the available power at the receiver. The values of  $C$  and  $D$  corresponding to 80% and 90% tracked power are given in Table. 2 (see [4]). Equation (3.1) is plotted in Fig.1.

$P\%$	$C$	$D$
90%	2.7	0.51
80%	2.1	0.51

**Table 2.** Model parameters (Eq. 3.1)

In [5] the proportionality of the number of RAKE fingers to the system bandwidth was established, but the given formulas did not provide a general connection to the delay spread. In [6], a similar dependence for the number of RAKE fingers needed to track a given percent of the available power is also given, where the unity is added to equation (3.1). Those results were obtained experimentally. Similar

studies regarding the multipath richness impact on CDMA systems performance are presented in [10 & 11].



**Figure 1. Number of RAKE fingers as a function of the system bandwidth times delay spread**

Further, the derivation of the distribution function for the number of CDMA RAKE fingers is straightforward. Using the fundamental theorem for finding the distribution function of a random variable [7], we obtain after some elementary algebraic manipulations the following pdf:

$$P_N(N_T) = \frac{1}{\varepsilon D N_T} e^{\left( \frac{2y}{\varepsilon} + \frac{2\delta^2 y^2}{\varepsilon^2} \right)} \operatorname{erfc} \left( \frac{y}{\sqrt{2}\delta y} + \frac{\sqrt{2}\delta y}{\varepsilon} \right) \quad (3.2)$$

where,

$$y = \frac{1}{D} \ln \left( \frac{N_T}{C(\kappa W \tau_{med})^D} \right)$$

All the parameters remain the same as described in the section above. The cumulative distribution function (cdf) is computed as follows:

$$P_N(N_T) = 1 - \frac{1}{2} \operatorname{erfc} \left( \frac{y}{\sqrt{2}\delta y} \right) + \frac{1}{2} e^{\left( \frac{2y}{\varepsilon} + \frac{2\delta^2 y^2}{\varepsilon^2} \right)} \operatorname{erfc} \left( \frac{y}{\sqrt{2}\delta y} + \frac{\sqrt{2}\delta y}{\varepsilon} \right) \quad (3.3)$$

The above function was obtained by integration by parts where the argument of the complementary error function was chosen as the new variable of integration.

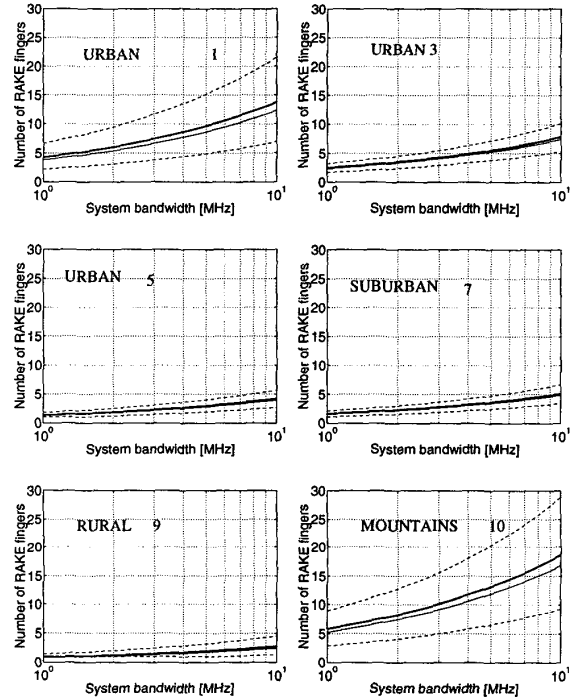
The average number of RAKE fingers related to a specific kind of propagation environment is easily obtained and is given by:

$$E\{N_T\} = \frac{2C(\kappa W \tau_{med})^D}{\varepsilon D + 2} e^{\left( \frac{D^2 \delta^2 y^2}{2} \right)} \quad (3.4)$$

where  $\kappa$  is the inverse of coefficients 0.76 and 0.66 in the expression for the mean delay calculated over the entire propagation area and varies from 1.3158 (=1/0.76) to 1.5152 (=1/0.66) as  $\varepsilon$  varies from 0.5 to 1, [1]. As it is clear from (3.4), the estimation of the multipath diversity can be done very easy for a wide range of parameters that charac-

terizes different kind of time dispersive propagation environments. Equation (3.4) is the 1-st order moment. To obtain higher order moments of the distribution (3.2), let say  $m$ -th order moments, the parameters  $C$  and  $D$ , must be substituted by  $C^m$  and  $mD$ , respectively in equation (3.4).

The average number of needed CDMA RAKE receiver fingers depends upon the median delay spread computed over the propagation area, the standard deviation of the measured delay spread and other coefficients ( $C$ ,  $D$ ,  $\varepsilon$ ) that are environment specific. Observe that the dependence on the radius of the propagation area is included in the median delay spread.



**Figure 2. Number of CDMA RAKE fingers needed to track the 90% of the total available power as a function of the system bandwidth. Results are shown for different areas according to Table 1 and equation (3.3). The dashed line corresponds to the 10% and 90% probability level according to eq. (3.3). The thin continuous line corresponds to the 50% and the thicker line is the average (3.4)**

The number of RAKE fingers required to track the 90% of the total available power versus the system bandwidth is shown in Fig.2. Results for the 10%, 50% and 90% probability levels are given as well as for the mean according to equation (3.4). From these plots conclusions may be drawn about the number of RAKE fingers required the system to perform well.

It is worthwhile to notice that the figures for the number of RAKE fingers obtained here are a result of the model

used for the average PDP profile. The delay spread is, in general, sensitive to the shape of the power delay profile. So for different PDP shapes different values of constants C and D might be obtained, that in turn affects the final number of the needed RAKE receiver fingers. Though, the results presented here may be regarded as generally applicable due to the way the simulations were performed.

Finally, if we compare the multipath diversity of two systems we see, as expected, that the multipath diversity works better for the system with larger bandwidth. The improvement factor is given by the ratio:

$$\frac{N_T(W_2)}{N_T(W_1)} = \left(\frac{W_2}{W_1}\right)^D \approx \sqrt{\frac{W_2}{W_1}}$$

where  $W_1$  and  $W_2$  are the bandwidths of the systems to be compared.  $D$  is the constant given in Table 2, which in general is a function of the shape of the power delay profile.

#### 4 Distribution function for the fading width

As mentioned in the introduction, the fading variation of a wideband channel, is, together with the delay spread one of the most significant figures to be taken into account when designing a digital mobile radio system. There exist several publications that treat this problem both theoretically, experimentally and by means of simulations, among them [8], [2&3] and [9&10]. A parameter that is used to characterize the amount of fading is the fading width ( $fw$ ), which is also called fading depth, and directly relates to the diversity gain. The fading width, in general is defined as the difference in decibels of the power amplitudes corresponding to two different cdf levels. In [8] it was found that the fading width was a function of the product of the system bandwidth and the delay spread. In [2&3] simulation and measurement results are given for the fading width.

Further, we will make use of an empirical formula presented in [2&3], which shows a very good agreement with measurements. The equation is given by:

$$fw = A(1 - \gamma \tanh(\alpha \ln(\beta W \tau_{rms}))) \quad (4.1)$$

where  $fw$  is the fading width defined for the 1-99% levels in the cdf curve,  $A=15.54$  dB,  $\alpha=0.4$ ,  $\beta=1.49$  and  $\gamma=0.83$ .

Equation (4.1) represents the fading variation of the signal output of a RAKE receiver where all the available taps have been combined assuming a Maximum Ratio Combining (MRC) scheme for simulated Rayleigh fading channels. The above equation also fits very well to measurements results obtained from experiments given in [3], and is in good agreement with other measurement results [8].

Applying the same ideas as in the previous sections we may conclude that the fading width may also be regarded as a statistically distributed variable. Further, applying the fun-

damental theorem for finding the distribution function of a random variable [7], we obtain after some algebraical manipulations the following pdf for the fading width:

$$p_{fw}(fw) = \frac{p_\tau \left( \frac{1}{\beta W} \left( \frac{1 + (1 - fw/A)/\gamma}{1 - (1 - fw/A)/\gamma} \right)^{\frac{1}{2\alpha}} \right) / (\alpha \beta \gamma A W)}{(1 - (1 - fw/A)/\gamma)^{1+1/2\alpha} (1 + (1 - fw/A)/\gamma)^{1-1/2\alpha}} \quad (4.2)$$

The parameters in equation (4.2) are defined above and  $p_\tau(\tau_{rms})$  is given by (2.1). The corresponding cdf is similar to equation (3.3) and is trivial to find. On the contrary, to obtain the average fading width and higher order moments seems not to be trivial. Below in Fig.3, results for the fading width at different cdf levels are given as function of the system bandwidth for different areas according to Table.1 and equation (4.2)

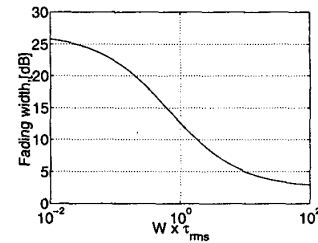


Figure 3. Fading width as a function of the system bandwidth times delay spread (Equation (4.1)) and references [2&3]

As may be seen from Fig.4 the fading width will decay with the system bandwidth. In lower dispersive media the increment of the system bandwidth needed to overcome the fading is greater than in higher dispersive media where the fading width is already small and an additional increment of the system bandwidth would not yield a considerable improvement.

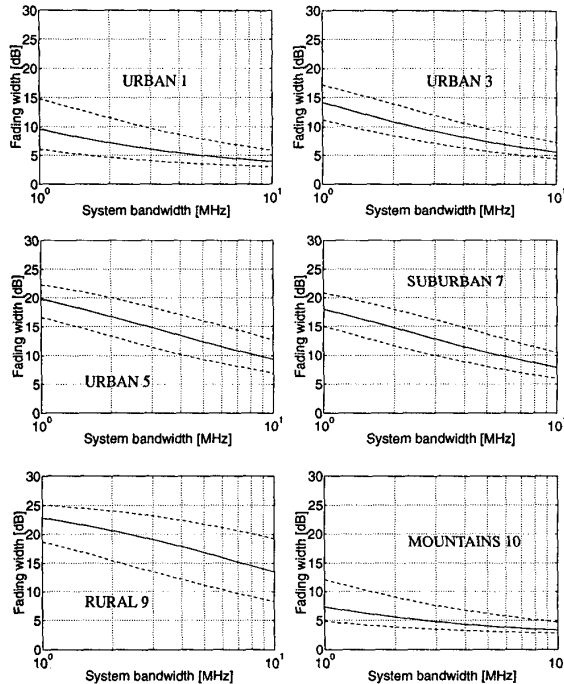
The diversity gain may be directly assessed from the plots in Fig.4 by comparing corresponding values of the fading width at the system bandwidths of interest. The range of variation may also be obtained for different probability levels.

Finally, the approach presented above could easily be extended to obtain the distribution function for the channel capacity. The effects of antenna and polarization diversity may also be included likewise.

#### 5 Summary

The probability density function for both the number of CDMA RAKE fingers required in order to track a given percent of the total available power at the receiver and the fading width of the signal output of a RAKE receiver where all the available taps have been combined assuming a Maximum Ratio Combining (MRC) scheme have been presented.

The moments of the obtained distributions are tightly related to the time dispersive characteristics of different mobile radio propagation environments due to the connection to the rms delay spread.



**Figure 4.** Fading width as a function of the system bandwidth. Results are shown for different areas according to Table. 1 and equation (4.2). The dashed line corresponds to the 10% and 90% probability level according to eq. (4.2). The continuous line corresponds to the 50% level.

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