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Multiple-Antenna Reception and Reduced-State Viterbi Detection for Block Transmission Systems

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Abstract— This paper presents a receiver structure employing multiple receiving antennas and reduced-state Viterbi detection. The presentation is couched in terms of matrices representing a discrete-time, symbol-sampled system with frequency-selective, slow fading and additive Gaussian noise. The receiver design involves determining two matrices: one matrix defining the metric of the reduced-state Viterbi detector and one representing a pre-processor operating on the output from a maximal-ratio combiner. The presented receiver is a generalization of those used earlier in single-antenna, combined linear-Viterbi equalizers [3], [5], [7], [11], [13], [16], [17] for continuous transmission systems. Performance comparisons are made with the minimum mean-square error (MMSE) linear equalizer and the MMSE decision-feedback equalizer for block transmission systems [10], indicating that the presented class of receivers offers superior performance.

I. INTRODUCTION

The use of multiple receiving antennas in wireless communication to take advantage of spatial diversity has been widely studied in the literature, see e.g., [1], [4], [8], [9] and the references therein. For flat fading channels it is shown in [8] that the use of multiple receiving antennas can improve the average signal-to-noise ratio (SNR) and lower the probability of a deep fade. In a frequency-selective fading environment the channel will be time-dispersive, which may in turn cause intersymbol interference. Several methods of multiple-channel equalization have been put forward to combat this. These are based on, for example, the minimum mean-square error criterion [11], decision-feedback techniques [1], or maximum likelihood sequence detection [6].

In this paper we propose one other method of performing multiple-channel equalization; we examine a receiver structure that uses multiple receiving antennas and a reduced-state Viterbi Algorithm (VA). A system with a maximal-ratio combiner (MRC) [8], [9] is modelled as a single-antenna, white-noise system by using a data transformation matrix preserving the sufficient statistics. Linear pre-filtering and a reduced, or fixed, state VA are then applied, producing a system with an MRC, a linear operation and a fixed complexity VA.

II. MODEL

Consider a system using \( M \) receiving antennas and assume that there is one linear, frequency-selective, slow fading channel for each antenna. Each channel is modelled as a discrete-time, symbol-sampled, additive Gaussian channel (DTGC) with intersymbol interference (ISI). Let \( b \) be an \( N \times 1 \) random vector containing the \( N \) independent symbols to be transmitted. For each block the received signal at each antenna can then be described in terms of the matrix representation

\[
x_i = H_i b + n_i
\]

where \( H_i \) is an \((N + L_i - 1) \times N \) stochastic, complex valued matrix representing the ISI, \( n_i \) is a complex, jointly Gaussian, zero mean random vector with a \( N(0, R_i) \) distribution and where \( x_i \) is a vector of observable channel outputs.

In a mobile situation, where a moving transmitter or receiver creates a non-stationary environment, the channels will vary in between the time instants two blocks are transmitted. Throughout this paper we assume that, for each block to be transmitted, the outcomes of all the channel matrices \( H_i \) are known to the receiver.

III. MAXIMAL-RATIO COMBINER

The Maximal-Ratio Combiner (MRC) [8], [9] is an optimal method to combine the received signals \( x_i \) in the sense that it gives a minimal sufficient statistic for the detection of \( b \). In this section we re-derive the MRC by generalizing a result for single-antenna block transmission systems by Barbosa [2] to the multiple-antenna case.

With the following block matrices:

\[
x \triangleq \begin{bmatrix} x_1 \\ \vdots \\ x_M \end{bmatrix}, \quad n \triangleq \begin{bmatrix} n_1 \\ \vdots \\ n_M \end{bmatrix} \quad \text{and} \quad H \triangleq \begin{bmatrix} H_1 \\ \vdots \\ H_M \end{bmatrix},
\]

a multiple-antenna model can be expressed as

\[
x = Hb + n.
\]

Define the \( N \times N \) matrix \( M \triangleq H^H R^{-1} H \), where \( R = \mathbb{E}\{nn^H\} \) and is assumed to be invertible and where the superscript \( H \) denotes Hermitian transpose. Assume that \( H \) has a column space of rank \( N \), meaning there is information in \( x \) about every symbol in \( b \). Then \( M \) is invertible and

\[
\hat{b}_a = M^{-1} H^H R^{-1} x = b + M^{-1} H^H R^{-1} n
\]
is the output of the simultaneous zero-forcing equalizer, cf. the result by Barbosa in [2] for the single-antenna case. From [2] we know that \( \hat{b}_n \) is a minimal sufficient statistic [14] for \( b \) given \( x \). Because \( M \) is an invertible \( N \times N \) matrix, it follows from (4) that

\[
y \triangleq H^{H} R^{-1} x
\]

is also a minimal sufficient statistic. Hence the vectors \( x \) and \( y \) both hold the same information for the purpose of detecting \( b \), in spite of \( y \) being only of dimension \( N \times 1 \). Equation (5) is the basic appearance of the MRC. In the case that the noise at each particular antenna is correlated with the noise at all other antennas, the outputs of the whitened matched filters are summed to form the output from the MRC, see figure 2 and, e.g., [9].

A. An equivalent single-antenna model

Before beginning with designing receivers, we would like to transform the model (3) into an "equivalent" single-antenna, white noise model. Given \( H \), the filtered additive noise in (4), \( M^{-1} H^H R^{-1} n \), is a Gaussian zero mean noise process with a correlation matrix equal to \( M^{-1} \). To whiten this noise we use the Cholesky factorization of \( M \), producing an upper triangular matrix \( V \) such that \( VM^{-1} V^H = I \), where \( I \) is the identity matrix. Let

\[
r \triangleq V \hat{b}_n = VM^{-1} y = Vb + n_v.
\]

Then \( n_v = VM^{-1} H^H R^{-1} n \sim N(0, I) \) given \( H \). Equation (6) can be seen as a single-antenna model that is equivalent to (3) in the sense that an optimal receiver will perform equally well operating on either \( x \) or \( r \), since both are sufficient statistics for \( b \), cf. the derivations by Kaleh in [10] for the single-antenna case. The main difference between (6) and (3) lies in the structures of the model matrices \( H \) and \( V \). The multiple-antenna model matrix \( H \) is an \((M(N-I)+L_1+\ldots+L_M) \times N\) matrix while \( V \) is an \( N \times N \) matrix that can be seen as an anti-causal, time-variant filter.

Note that with this model, variations in the signal-to-noise ratio take the form of varying signal energy in the matrix \( V \) while the noise \( n_v \) always has unit variance. We define the block signal-to-noise ratio (SNR) as

\[
\gamma \triangleq \frac{\text{tr} \{ V^H V \}}{N}.
\]

Note that the block-SNR is a stochastic variable (with one outcome for each transmitted block) since the channel matrix \( H \), and thereby \( V \), is stochastic.

B. The Effects of Using Multiple Receiving Antennas and Maximal-Ratio Combining

Let us look at how the number of antennas and the lengths of the impulse response at each antenna affect the characteristics of the system model (6). A simple measure of the amount of ISI is the normalized energy in the elements outside the main diagonal of \( V \), i.e., with \( B \) as a matrix containing the off-diagonal elements of \( V \), as

\[
\rho \triangleq \frac{\text{tr} \{ B^H B \}}{\text{tr} \{ V^H V \}}.
\]

Consider an example where each channel is frequency-selective, Rayleigh-faded and independent of all other channels. For each block to be transmitted let the impulse response of each channel be the outcome of a stochastic vector of length \( L \) with independent, identically distributed (i.i.d.), complex, zero mean Gaussian random variables. Furthermore let the block length \( N = 20 \) and the noise correlation \( R = \sigma^2 I \). In figure 1 we have plotted the estimated mean value of \( \rho \) versus the number of antennas, \( M \), and the length of the impulse response of each channel, \( L \). (Note that origo is in the lower, right part of the figure.) The mean value of \( \rho \) was estimated by averaging (8) over 3000 independent outcomes of \( H \).

From figure 1 we see that if we add more independent diversity paths by means of more antennas, the ISI in the equivalent model (6) tends to decrease in the sense that \( \rho \) of definition (8) tends to decrease. Thus, the ISI increases with the length of the channel responses but decreases with the number of antennas. Alternative characterizations of the ISI in block transmission systems, are the colourization of the noise in equation (4) [2] and the eigenvalue spread of the matrix \( M [11] \).

In the example above the block-SNR \( \gamma \) of (7) is the sum of the squared absolute value of \( 2LM \) i.i.d. real valued Gaussian variables giving the block-SNR a chi-squared distribution with \( 2LM \) degrees of freedom. Hence, in this example the number of antennas and the length of the impulse responses have an identical effect on the probability distribution of the block-SNR of definition (7): an increase of either factor would decrease the probability of a deep fade and increase the mean of the block-SNR \( \gamma \).

Although we assume the diversity paths to be independent in this example, these characteristics may well apply to a real situation with, at least, partly uncorrelated antennas.
IV. CLVEs for Block Transmission Systems

Consider a receiver where the output of the MRC, $y$, is filtered by a linear system represented by a matrix $G$ and an estimate of the transmitted sequence $b$ is obtained as

$$
\hat{b} = \arg\min_b \|Gy - Qb\|_2^2,
$$

where $\|x\|_A \triangleq x^HAx$. A receiver structure implementing this, consisting of a linear prefilter and a VA, is depicted in figure 2. We refer to this class of receivers as “combined linear-Viterbi equalizers for block transmission systems” (B-CLVE), cf. [5], [7], [13], [16], [17].

![Fig. 2. The proposed receiver structure for the case when the noise at each particular antenna is uncorrelated with the noise at all other antennas.](image)

For the VA to be a meaningful tool for solving (9), the matrix $Q$ should be a band matrix [10], i.e.,

$$
Q = \begin{bmatrix}
0 & \cdots & 0 \\
0 & \ddots & \vdots \\
\vdots & \ddots & 0 \\
0 & \cdots & 0
\end{bmatrix},
$$

where $l$ is the width of the band of non-zero elements. Because a wider band of non-zero elements in $Q$ would require more states in the VA trellis, it is the width of this band that determines the complexity of the VA. By choosing a given value of $l$, the computational complexity of B-CLVEs can be controlled; conversely, for a given VA complexity (determined, perhaps, by hardware), a prescribed value for $l$ may be indicated.

If we apply the model (6) to the structure displayed in figure 2, we get a model suitable for the design of B-CLVEs, see the system described in figure 3. The linear distortion of the received signal is described by the matrix $V$. The matrix $P$ is the prefilter to be designed together with the matrix $Q$ which is given to the VA as a channel model. The function of the prefilter $P$ is to concentrate the energy of the system $PV$ to a band of width $l$ matching the non-zero band of the matrix $Q$, thus matching the complexity constraint of the VA. Comparing figures 2 and 3 and using the model (6) we see that $G = PV^{-1}$.

![Fig. 3. The structure of CLVEs for block transmission systems.](image)

We display two special cases of the receiver structure in figure 3, both replacing the VA with a hard limiter, or equivalently setting $Q = I$. If we choose the matrix

$$
P = R_b V^H (V R_b V^H + I)^{-1},
$$

with $R_b$ as $E\{bb^H\}$, then the mean-square error $E\{\|b - b\|^2\}$ is minimized [14, p. 303] and the receiver of figure 2 turns into the minimum mean-square error linear equalizer in [10]. On the other hand, if $P = V^{-1}$, then the receiver is equivalent to the zero-forcing equalizer of (4) followed by a hard limiter.

The design of the B-CLVE that minimizes the probability of choosing the wrong sequence $b$, is given by $P = I$ and $Q = V$, which is a description of the maximum likelihood sequence decoder [6]. The complexity of the VA is then determined by the structure of $V$, or rather, by the width of the band of non-zero elements in $V$. This, in turn, is equal to the length of the largest ISI time spread of the channels, cf. the Cholesky factorization in [15, p. 55].

V. Design Methods for Block-CLVE’s

In this section we will discuss the design of the matrices $P$ and $Q$ with respect to what has been done earlier for continuous transmission systems. We will also, as an example, generalize one of these earlier methods to our block transmission system environment.

Hereinafter we discuss only the case when both $P$ and $Q$ are of dimension $N \times N$. In order to restrict $Q$ to a band-matrix, let $q_i = \tilde{q}_i D_i$ for all $i$, where $\tilde{q}_i$ is a $1 \times l$ vector with the model of the system’s impulse response given to the VA. The matrix $D_i$ positions the vector $\tilde{q}_i$ in the $i$th row of the matrix $Q$, and is given in Appendix A.

In [16] it is recognized by Sundstrom et al. that in block transmission systems “edge effects” will appear that could be incorporated in B-CLVE design. It is also noted that the CLVEs for continuous transmission systems that are presented in [5], [7], [16], [17] all are related to the same criterion: the variance of a signal formed as the difference between the actual signal given to the VA and the model of the same signal.

Let us here consider the error vector

$$
\epsilon = Pr - Qb = PVb + Pn_b - Qb,
$$

i.e., the difference between the filtered received signal and the model signal, as displayed in figure 4.

The expected energy in the error vector $\epsilon$ is given by

$$
J(P, Q) = E\{\epsilon^H \epsilon\} = \sum_{i=1}^{N} j(p_i, q_i),
$$

where each term

$$
J(p_i, q_i) = \|p_i V - q_i\|^2_{R_b} + \|p_i\|^2_{I}
$$

and where $p_i$ and $q_i$ are the $i$th rows of $P$ and $Q$, respectively.
Some earlier design methods can be generalized to block transmission systems by applying their respective foundational ideas to the criterion (13). The method presented by Falconer and Magee in [5] is, for B-CLVE, to minimize (14) with respect to \( q_i \) and \( p_i \) for all \( i \), under the constraint \( q_i q_i^H = 1 \). The following section describes one way of applying the weighted least squares (WLS) method of [17] to B-CLVE design. The concept of spectral matching in CLVE design, presented by Fredricsson [7], later investigated by Beare [3], seems, however, to be more difficult to apply directly to B-CLVE.

**A. The WLS-design for Block CLVEs**

In [17] a promising method for CLVE design is presented based on the WLS criteria in [12]. We will generalize the WLS-CLVE to block transmission systems by applying the method to (14), i.e., to every term in (13). This generalized method, in principle, recalculates a new FIR-filter \( p_i \) for each transmitted symbol, a complex process currently too impractical for most applications. We present it as one demonstration of block CLVE design, not as a practical design method.

Consider

\[
J'(p_i, q_i) = \| (p_i V - \delta_i) W_i^H + p_i^H \|_F^2, \tag{15}
\]

where \( W_i \) is a weighting matrix and \( \delta_i \) is a \( 1 \times N \) vector with a one in position \( i \) and zeros in all other positions. Completing the square in (15) gives

\[
J'(p_i, q_i) = \| (p_i - p_i,0) W_i^H \|_{A_i}^2 + \| \delta_i^H \|_{B_i}^2, \tag{16}
\]

where

\[
A_i = V W_i R_b W_i^H V_H + I,
B_i = W_i (R_b - R_b W_i^H V_H A_i^{-1} V W_i R_b) W_i^H,
p_i,0 = \delta_i W_i R_b W_i^H V_H A_i^{-1}.
\]

Following [17], let the weighting matrix be a diagonal matrix, as \( W_i = \text{diag} \{ 1_{1 \times N} - 1_i \}, \) where \( 1_{1 \times N} \) is a \( 1 \times N \) vector containing ones, and \( D_i \) is given in Appendix A.

In the presence of noise, the matrices \( A_i \) and \( B_i \) are positive definite, so the minimum of (16), with respect to \( p_i \), is obtained if \( p_i = p_i,0 \), with a residual error of \( \| \delta_i^H \|_{B_i} \). Let \( P_{WLS} \) and \( Q_{WLS} \) denote the pre-filter and the model obtained by the WLS-design, respectively. Each row in the matrix \( P_{WLS} \), \( p_{i,WLS} \), is chosen as the vector \( p_i \) that minimizes (15), that is

\[
p_{i,WLS} = \delta_i W_i R_b W_i^H V_H A_i^{-1} \quad \forall \ i \in [1 \ldots N]. \tag{17}
\]

Each row in \( Q_{WLS}, q_{i,WLS} \), is then assigned an exact copy of the positions of the vector \( p_{i,WLS} V \) corresponding the non-zero band on \( Q \) and zero otherwise as

\[
q_{i,WLS} = p_{i,WLS} V D_i, \tag{18}
\]

giving the VA a correct channel model for those positions.

![Fig. 5. An example where the WLS method has been used to design P and Q from V. Here \( R_b = I \) and the block-SNR is equal to 10dB. The absolute value of the entries in the matrices are displayed.](image)

**VI. SIMULATIONS**

Using simulation, we compare the performance of the B-CLVE presented in section V-A with the performance of the minimum mean-square error linear equalizer (MMSE-LE) and the minimum mean-square error decision-feedback equalizer (MMSE-DFE) for block transmission systems, both described by Kaleh in [10]. The MMSE-LE and the MMSE-DFE are applied directly to \( y \), the output of the MRC. Also using simulation, we illustrate the necessity of prefiltering when the actual impulse response is longer than \( l \), the permitted length of the model. We plot the performance of a receiver that assigns a “truncated” version of the channel \( V \) as the channel model \( Q \) by setting \( P = I \) and the rows of \( Q \) as in (18). We refer to this receiver as the receiver without pre-filtering (WPF).

In the simulations each block consisted of \( N=20 \) antipodally modulated, i.i.d. bits, thus \( b \in \{ -1, +1 \} \). Only one antenna was used. The impulse response of the channel was modelled as an i.i.d. complex, jointly Gaussian, zero mean random vector of length 10, thus, the number of states in the channel was \( 2^{10} = 1024 \). For each block to be transmitted, the channel was assigned a new impulse response independent of the other channels. The noise was white with
covariance matrix variance $R = \sigma^2 I$. The number of states in the VA was 32 ($= 2^5$), as compared with the channel's 1024. Figure 6 shows the simulated Bit-Error Rate (BER) for the four receivers versus average block-SNR.

![Fig. 6. The estimated BER for the B-CLVE, MMSE-DFE and the MMSE-LE against average block-SNR.](image)

Figure 7 shows the simulated BER for the three receivers versus fixed block-SNR. Here the energy in $H$ has been normalized to one for each outcome of $H$ and the noise variance $\sigma^2$ has then been varied.

![Fig. 7. The estimated BER for the B-CLVE, MMSE-DFE and the MMSE-LE against fixed block-SNR.](image)

Looking at the simulations in figures 6 and 7 we see that the performance of the B-CLVE based on the WLS-design is better than the performance of the MMSE-DFE, indicating a potential for the presented type of receivers. We also see that the performance of the VA without pre-filtering (WPF) is the worst among the simulated receivers.

**VII. SUMMARY**

This paper presents ideas about the use of reduced-state Viterbi detection in block transmission systems employing multiple receiving antennas and maximal-ratio combining. The effect of using a maximal-ratio combiner in terms of sufficient statistics, ISI, and the mean and variance of the block-SNR over a series of transmitted blocks is briefly discussed. The presented receiver structure, referred to as B-CLVE, is a generalization of the earlier presented combined linear-Viterbi equalizers for single-antenna, continuous transmission systems. Simulations using a version of the WLS-design method in [12], [16], [17] adopted for B-CLVE design indicate a potential for this class of receivers.

**APPENDIX**

**I. APPENDIX: THE POSITIONING MATRIX $D_i$**

Let $0_{lxl}$ denote an $l \times l$ matrix containing zeros and $I_{lxl}$ denote an $l \times l$ identity matrix. If $0 < i \leq N - l + 1$ let

$$D_i = \begin{bmatrix} 0_{lx(i-1)} & I_{xl} & 0_{lx(N-i-l+1)} \end{bmatrix},$$

(19)

and if $N - l + 1 < i \leq N$

$$D_i = \begin{bmatrix} 0_{lx(i-1)} & 0_{lx(N-i-l+1)} & I_{lx(N-i-l+1)} & 0_{lx(N-i-l+1)} \end{bmatrix}.$$  

(20)

**REFERENCES**


