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# New Results on Estimations of the Burst and Bit Error Probabilities for Fixed Convolutional Codes

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**Abstract** — Improved upper bounds on both burst and bit error probabilities for maximum-likelihood decoding of fixed binary convolutional codes on the binary symmetric channel are derived. The bounds are evaluated for rate  $R = 1/2$  encoders and comparisons are made with simulations and with the bounds of Viterbi and Van de Meeberg. The new bounds are significantly better than Van de Meeberg's bounds for rates above the computational cut-off rate  $R_0$ .

## I. BACKGROUND AND IDEAS

When using a maximum-likelihood (ML) decoder of convolutional codes a typical error event consists of bursts of erroneously decoded information digits. If we use very long frames the bit error probability is a natural quality measure. We call the error probability that an error burst starts at a given node the *burst error probability*. It is sometimes called the *first-event error probability*. In general, it is easier to obtain good bounds for the burst error probability than for the bit error probability. The burst error probability is *not* the same for all nodes along the correct path. Since the burst error probability at depth  $i, i > 0$ , is not greater than that at the root we consider only upper bounding of the burst error probability at the root.

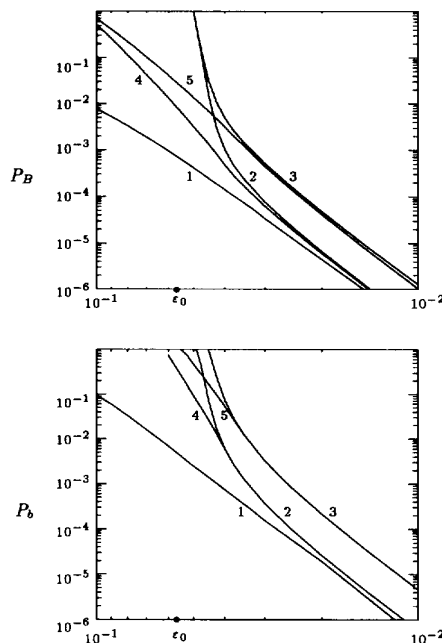
By using a random walk technique to separate the error events into two disjoint events corresponding to 'few' and 'many' errors, respectively, Cedervall, Johannesson, and Zigangirov [1] derived upper bounds on the burst error probability that are significantly better than Viterbi's [2] and van de Meeberg's [3] bounds for rates above the computational cut-off rate  $R_0$ .

In this paper, we extend this approach by using the fact that all good convolutional codes of rate  $R = 1/c$  have the property that the code symbols on one branch stemming from a node is the complement of the code symbols on the other branch stemming from the same node. Thus, we obtain an even tighter bound on the burst error probability for fixed convolutional codes.

More importantly, we make a non-obvious extension of the technique and obtain an upper bound on the bit error probability that is significantly tighter than both Viterbi's and van de Meeberg's bounds for rates above  $R_0$ . As before, we use the union bound technique to handle the 'few' error case. To upper bound the 'many' error case we again use a random-walk argument. This latter bound is the product of the expectation of the last crossing time and the probability of crossing a certain barrier.

## II. NUMERICAL RESULTS

For the memory  $m = 6$ , rate  $R = 1/2$  encoder  $G = (554, 744)$  the numerical results show that the regions of crossover probabilities for which the bounds are non-trivial, i.e.,  $< 1$ , are essentially increased. Van de Meeberg's bounds are non-trivial



$P_B$  and  $P_b$  versus  $\epsilon$  for the encoder  $G = (554, 744)$ .

- 1 - Simulation.
- 2,3 - van de Meeberg's and Viterbi's bounds.
- 4,5 - van de Meeberg- and Viterbi-type bounds.

if  $\epsilon < 0.05$ , while our bounds on the burst and bit error probability are non-trivial for  $\epsilon < 0.115$  and  $\epsilon < 0.062$ , respectively. Furthermore, at crossover probability  $\epsilon_0 \approx 0.045$ , corresponding to  $R_0 = R = 1/2$ , both our bounds are an order of magnitude tighter (Fig.1 and 2). For encoders with larger memories the improvements will be even greater.

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