



# LUND UNIVERSITY

## An Improved Stochastic Send-on-Delta Scheme for Event-Based State Estimation

Thelander Andrén, Marcus; Cervin, Anton

2016

*Document Version:*

Publisher's PDF, also known as Version of record

[Link to publication](#)

*Citation for published version (APA):*

Thelander Andrén, M., & Cervin, A. (2016). *An Improved Stochastic Send-on-Delta Scheme for Event-Based State Estimation*. Poster session presented at Reglermöte 2016, Göteborg, Sweden.

*Total number of authors:*

2

**General rights**

Unless other specific re-use rights are stated the following general rights apply:

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

Read more about Creative commons licenses: <https://creativecommons.org/licenses/>

**Take down policy**

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

LUND UNIVERSITY

PO Box 117  
221 00 Lund  
+46 46-222 00 00





# An Improved Stochastic Send-on-Delta Scheme for Event-Based State Estimation

Marcus Thelander Andrén Anton Cervin

Department of Automatic Control, Lund University  
marcus.thelander\_andren@control.lth.se anton@control.lth.se

LUND UNIVERSITY



LUND UNIVERSITY

## Introduction

Event-based sensing and communication holds the promise of lower resource utilization and/or better performance for remote state estimation applications in e.g networked control systems (NCS).

However, the problem of designing an optimal event-based state estimator often becomes untractable due to nonlinear measurements. This complexity is avoided with stochastic event-triggering.

In this work [1], we extend the work on stochastic triggering in [2] by proposing a simple predictor in the sensor to further improve the estimation performance.

## The MMSE Estimator

Bayes' theorem gives case dependent Kalman filter:

### Time Update:

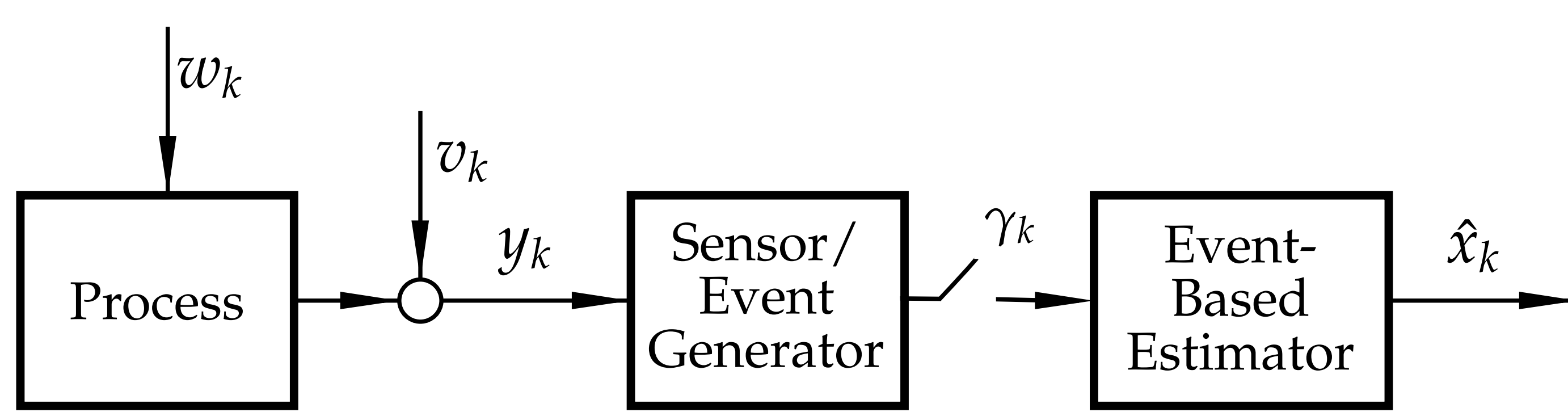
$$\begin{aligned}\hat{x}_k^- &= A\hat{x}_{k-1} \\ \hat{y}_k^- &= C\hat{x}_k^- \\ P_k^- &= AP_{k-1}A^T + Q\end{aligned}$$

### Measurement Update:

$$\begin{aligned}\hat{x}_k &= \hat{x}_k^- + K_k[\gamma_k y_k + (1 - \gamma_k)S_l y_{k-l} - \hat{y}_k^-] \\ P_k &= (I - K_k C)P_k^- \\ K_k &= P_k^- C^T [C P_k^- C^T + R + (1 - \gamma_k)Y^{-1}]^{-1}\end{aligned}$$

## The Remote Estimation Problem

Compute optimal estimates both with and without transmission:



### Process:

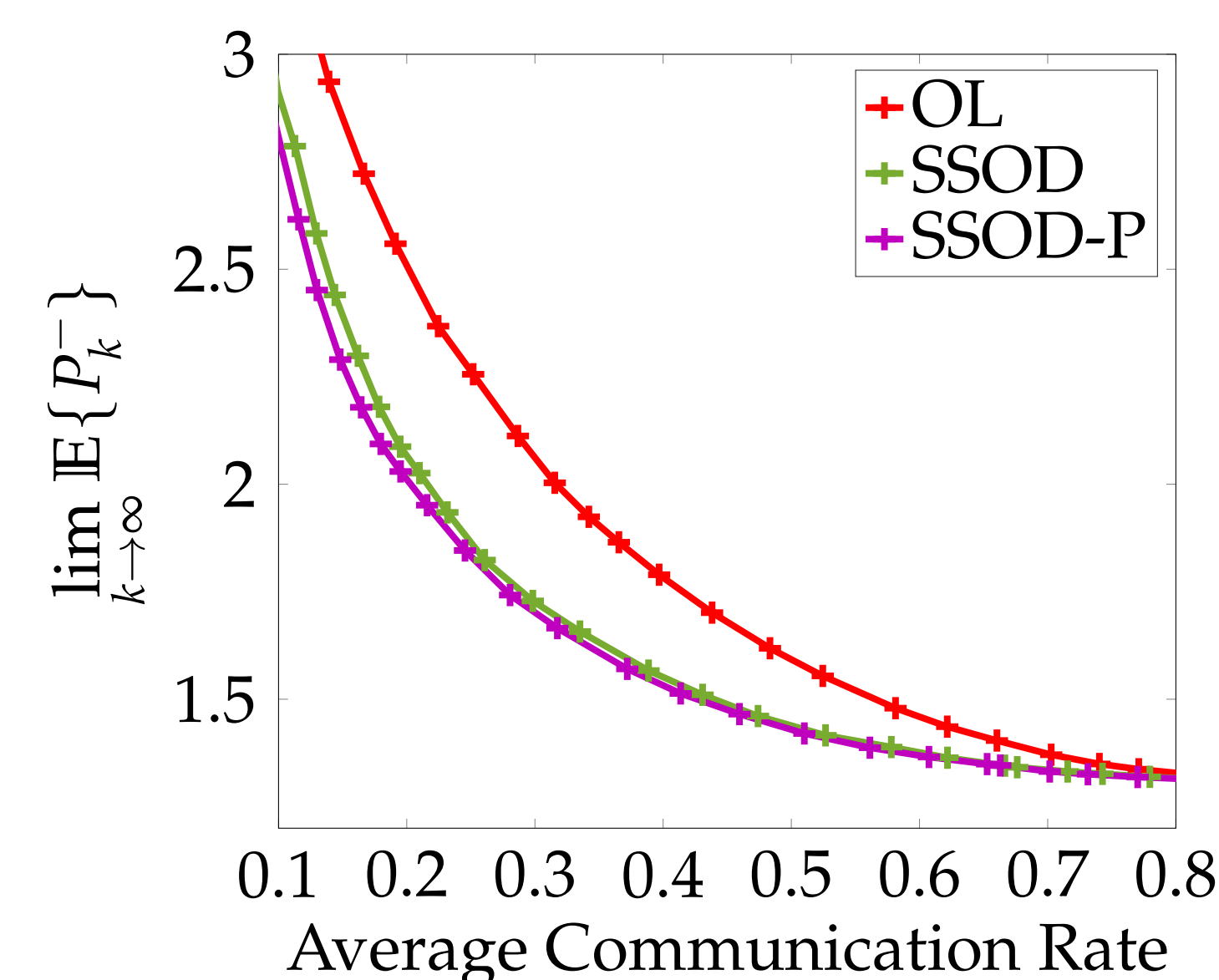
$$\begin{aligned}x_{k+1} &= Ax_k + w_k \\ y_k &= Cx_k + v_k \\ w_k &\sim \mathcal{N}(0, Q) \\ v_k &\sim \mathcal{N}(0, R)\end{aligned}$$

### Two Cases:

$$\gamma_k = \begin{cases} 1 & \Rightarrow \text{Transmission} \\ 0 & \Rightarrow \text{No transmission} \end{cases}$$

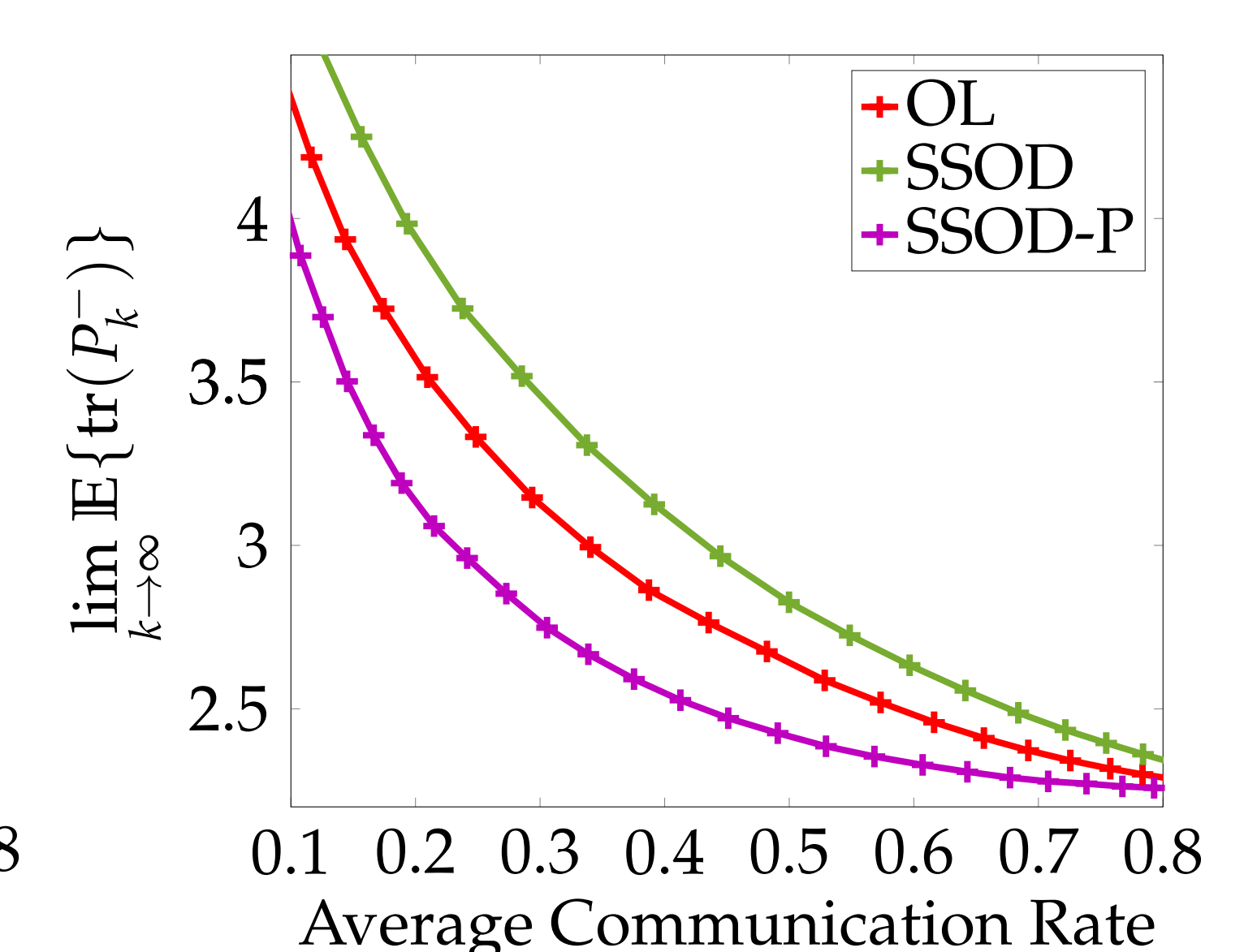
## Numerical Performance Comparison

Performance of SSOD and OL depends on process, while SSOD-P takes the process configuration into account:



### Slow 1<sup>st</sup>-order process:

$$\begin{aligned}A &= 0.95, C = 1 \\ Q &= 0.8, R = 1\end{aligned}$$



### Highly oscillatory 2<sup>nd</sup>-order process

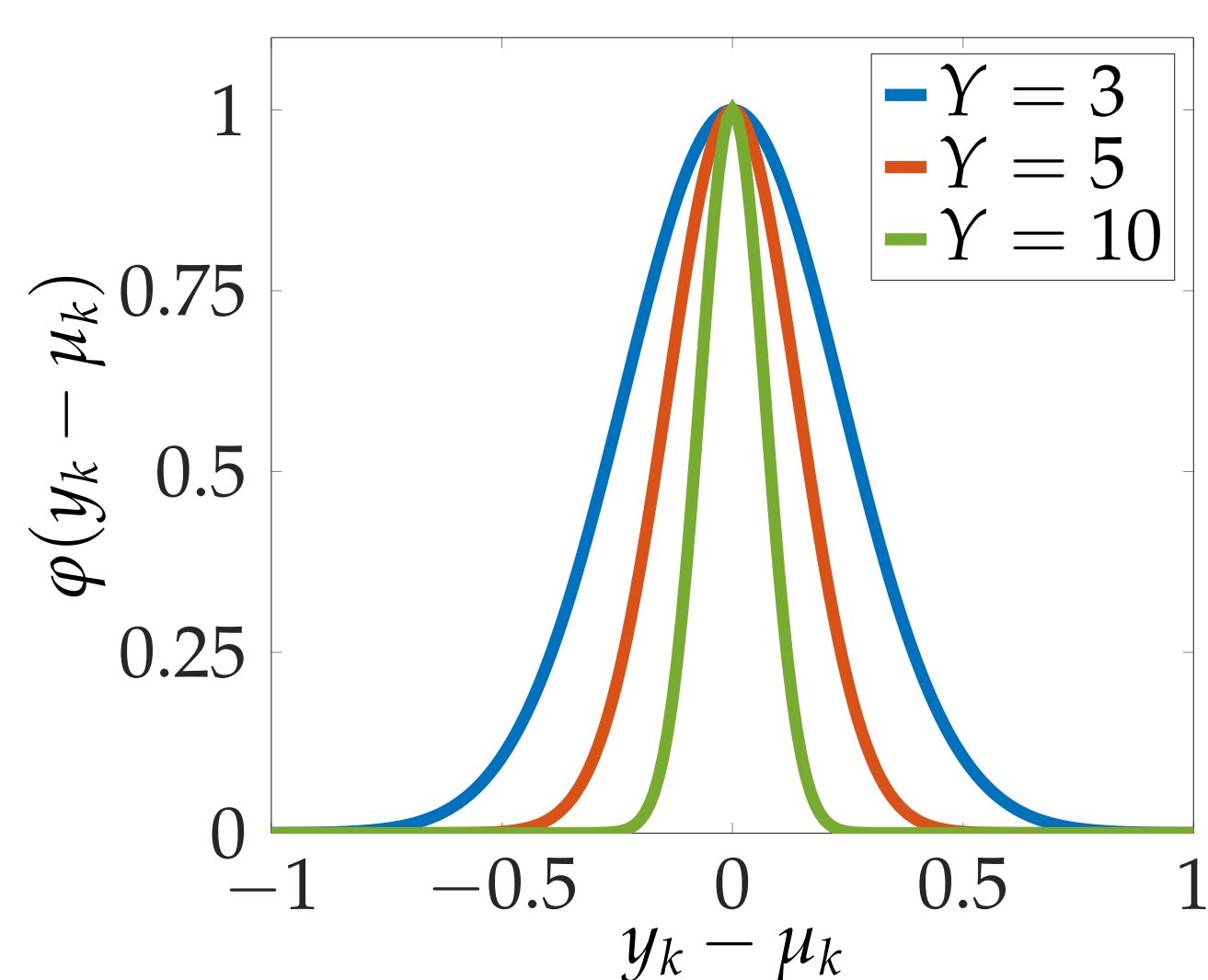
$$\begin{aligned}A &= \begin{bmatrix} -0.85 & -0.35 \\ 0.35 & -0.85 \end{bmatrix}, C = [1 \ 0] \\ Q &= \text{diag}(10^{-3}, 1), R = 0.1\end{aligned}$$

## Stochastic Event-Triggering

Trigger transmission with certain probability:

### Decision Function:

$$\varphi(y_k - \mu_k) = e^{-\frac{1}{2}(y_k - \mu_k)^T Y (y_k - \mu_k)}$$



### Event-Generator:

$$\zeta_k \sim \mathcal{U}(0, 1)$$

$$\gamma_k = \begin{cases} 1, & \text{if } \zeta_k > \varphi(y_k - \mu_k) \\ 0, & \text{else} \end{cases}$$

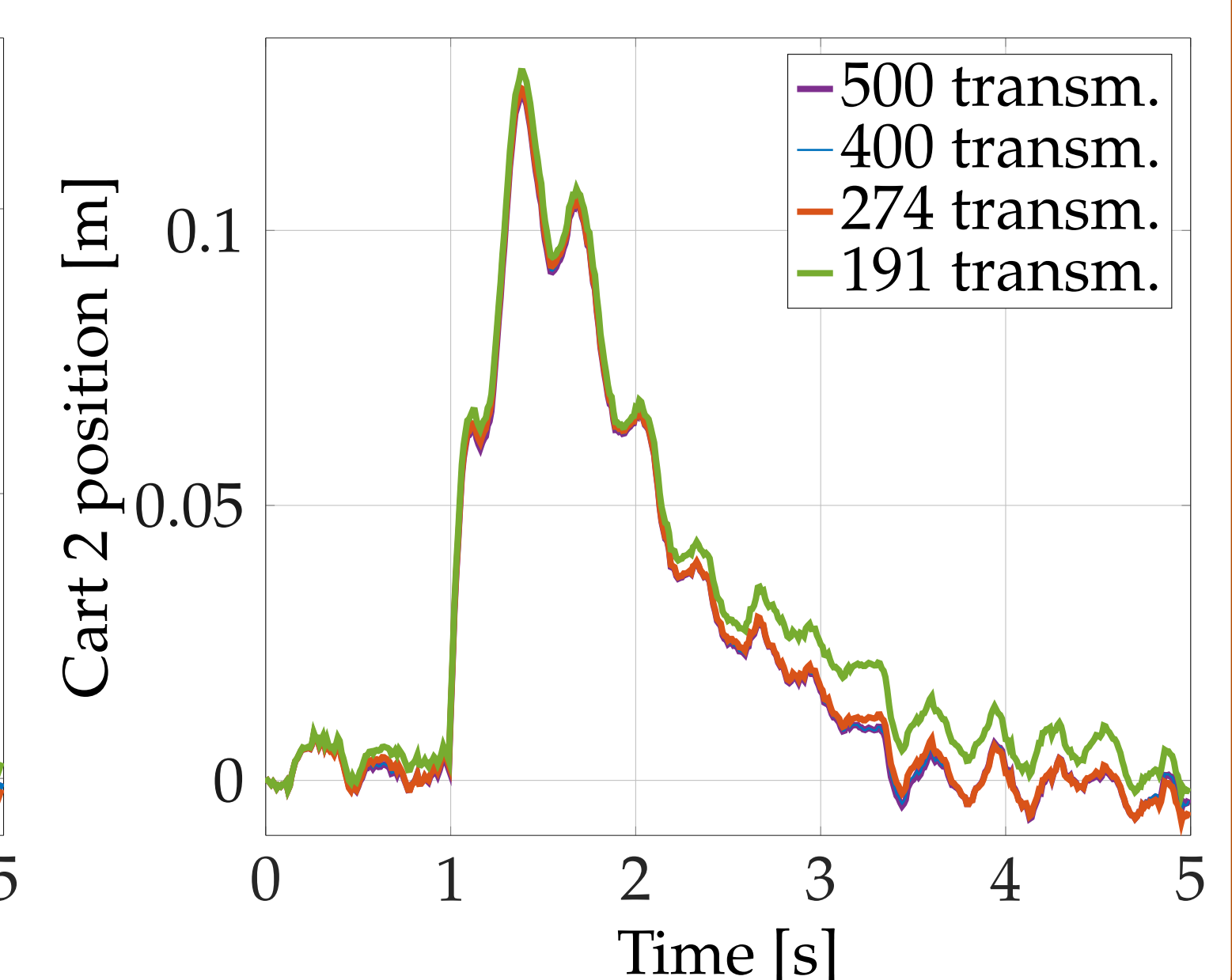
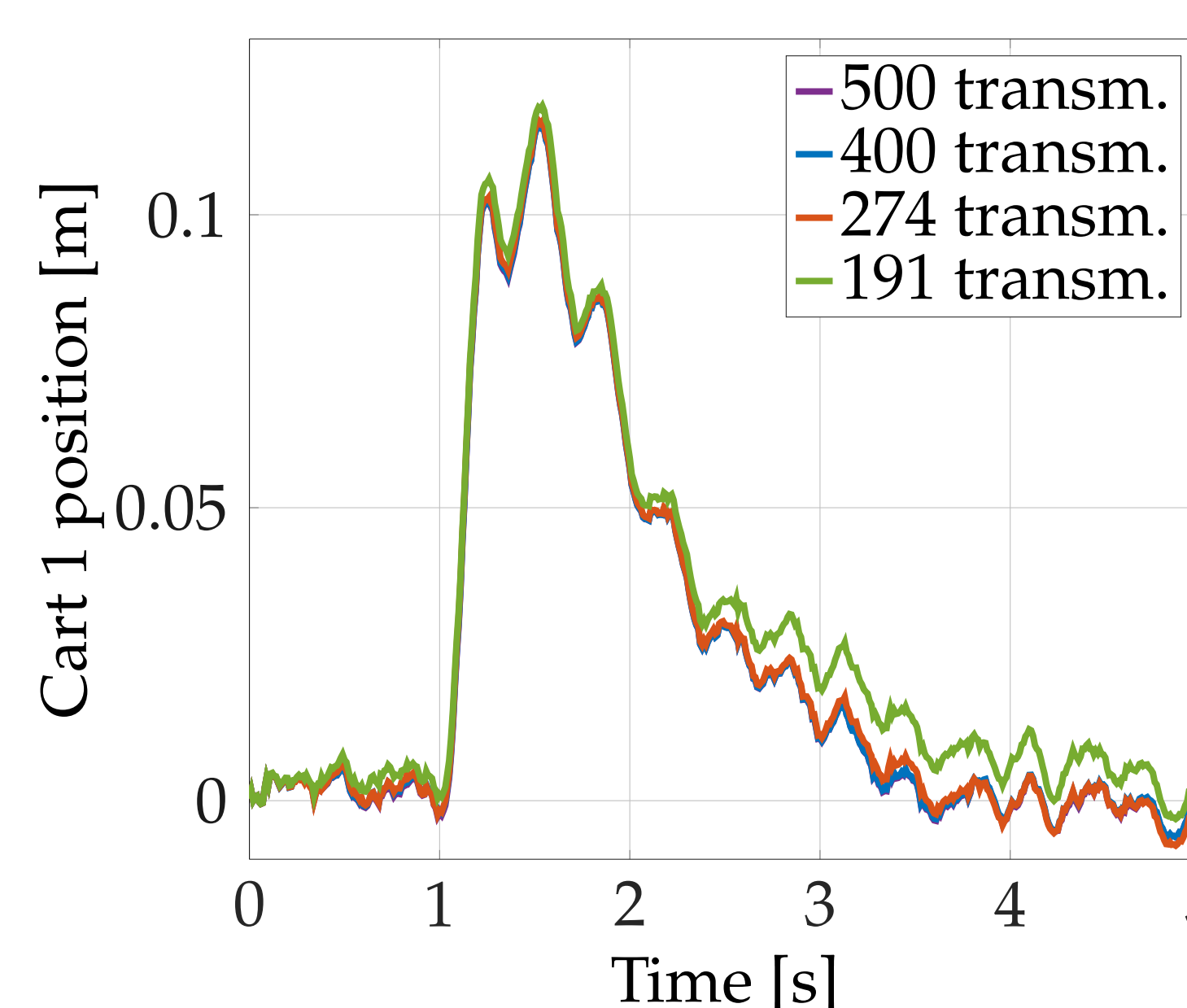
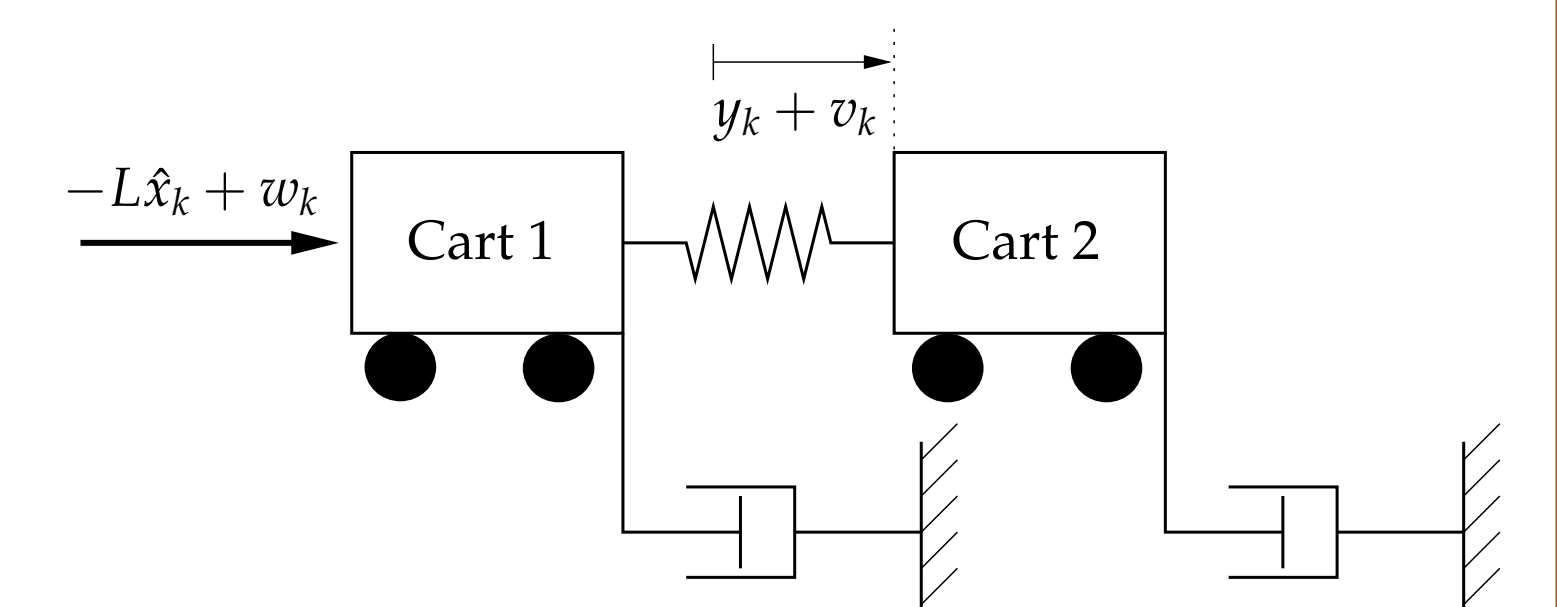
### Property of Scheme:

$$\Pr(\gamma_k = 0) = \varphi(y_k - \mu_k)$$

## Simulation Study

Position control of two carts with state-feedback and SSOD-P:

- System discretized with time step 0.01 s
- Impulse in cart 1 velocity at time 1 s
- Degradation in control performance small even at low communication rates



## A Simple Prediction

Proposed  $\mu_k$  in [2] with no estimator-to-sensor feedback are:

### Open-Loop (OL):

$$\mu_k = 0$$

### Stoch. Send-on-Delta (SSOD):

$$\mu_k = y_{k-l} \text{ (Transmit } l \text{ steps ago)}$$

Based on stationarity, we instead propose:

### Stoch. Send-on-Delta with Simple Prediction (SSOD-P):

$$\mu_k = \mathbb{E}\{y_k | y_{k-l}\} = S_l y_{k-l}$$

$$S_l = CA^l \Sigma C^T [C \Sigma C^T + R]^{-1}, \quad \Sigma = \text{Cov}(x_k) \text{ in stationarity}$$

## Conclusions

- **Stochastic Triggering** enables simple remote estimator design
- We propose a **simple sensor prediction** for improved performance
- Prediction implies a **scaling of last transmitted value**
- Scaling factors can be **pre-computed offline**
- Proposed scheme **compares well** in numerical examples

## Acknowledgments

This work has been supported by the Swedish Research Council. The authors are members of the LCCC Linnaeus Center and the ELLIIT Excellence Center at Lund University.

## References

- [1] M. Thelander Andrén and A. Cervin *Event-Based State Estimation Using an Improved Stochastic Send-on-Delta Sampling Scheme* In 2nd Int. Conf. on Event-Based Control, Communication and Signal Processing (EBCCSP) (Accepted), Krakow, Poland, June, 2016.
- [2] Shi, D., Shi, L. and Tongwen, C. *Event-Based State Estimation – A Stochastic Perspective* Springer, 2016.