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The Lambda Method for Tuning PI Controllers

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Abstract This report describes the Lambda method as a way to design PI controllers. The method is attractive to an industrial user as only one tuning parameter is needed. In the report the Lambda method is compared to a new design for PI controllers, which also requires just one tuning parameter. Also a new idea is presented which gives a new interpretation to the tuning parameter of the Lambda method.

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1. Introduction

The PID controller is by far the most commonly used algorithm for process control. In spite of all research there is still a need for good tuning methods for a variety of purposes. Lambda tuning is one method which is suggested as an alternative to empirical tuning rules. In the report this method is analyzed. Based on this analysis some suggestions are also given for the choice of design parameters.

Lambda tuning is an approximative pole placement method. The method was first suggested by Dahlin (1968) and Higham (1968), as a method for tuning digital controllers for systems with time delays and was widely used in the early period of digital computer control. The method was incorporated as a standard block in several early systems for direct digital control. The name "lambda-tuning" derives from the fact that Dahlin specified that the closed loop system should have one closed loop pole at $s = -\lambda$. Dahlin treated λ as a tuning parameter. The Dahlin-Higham method is closely related to the Smith Predictor, see Smith (1957). In the development of internal model control it was shown by Rivera and Morari, see Rivera *et al.* (1986), that the method could be viewed as a special case of Internal Model Control. This viewpoint is elaborated in Chien and Fruehauf (1990). Bialkowski has strongly advocated the method as a standard technique for tuning industrial controllers, see Sell (1995). It is also the basis of the Swedish SSG initiative. SSG is the Swedish short name for Pulp and Paper Industries' Engineering Co which handles all the documentation and standardization of the Swedish pulp and paper industry. In later developments the parameter λ has often changed both meaning and dimension. For example in Bialkowskis work λ is a time constant.

The tuning method has the potential to be a simple straight forward tuning method that can be used routinely, but first a few unclear points have to be straightened out. The following issues need to be dealt with:

- Good guidelines for choice of closed loop poles.
- The consequences of cancellation of the process pole.
- There are different tuning methods for systems with and without integration. It would be highly desirable to be able to merge these methods.
- As the design method is based on a simple three-parameter process model, therefore approximations are used to derive the method. The consequences of the approximation and the modeling errors should be investigated.

2. The Process Model

It is assumed that the process dynamics can be approximated by simple models. Two cases are considered. Process without integration are described by a three-parameter model

$$G(s) = K_p \frac{e^{-sL}}{1+sT} \tag{1}$$

and processes with integration by a two-parameter model

$$G(s) = K_v \frac{e^{-sL}}{s}. \tag{2}$$

The three-parameter model is characterized by three parameters: the static gain K_p, the time constant T and the dead time L. The two-parameter model is characterized by two parameters: the velocity gain K_v and the dead time L. A three-parameter model with a very long time constant can be approximated with a two-parameter model with $K_v = K_p/T$.

The parameters are usually determined graphically by a step response experiment on the process, which is thoroughly discussed in Åström and Hägglund (1995). Note that these models are good approximations for low frequencies which is often sufficient for PID controller tuning.

3. The Lambda Method

In this section the expressions of the controller parameters will be given for a PI controller designed with the Lambda method. Also, a new idea is presented where the consequences of cancelling process poles are considered.

The theory of the Lambda method is based on two assumptions. The first one where the process is modeled as a first order process with dead time. The second one where the closed loop transfer function is specified as

$$G_{cl}(s) = \frac{e^{-sL}}{1+sT_{cl}},$$

where T_{cl} is the time constant of the closed loop.

A simple straight forward tuning method is obtained if we separate the cases between processes with and without integral action.

3.1 Design with Cancellation for Stable Processes

We begin by considering the case of stable processes which have been modeled by the first order process model described by (1). The PI controller is given by the transfer function

$$G_c(s) = K_c \frac{1+sT_i}{sT_i}. \tag{3}$$

where K_c is the controller gain and T_i is the integral time. Consequently, the closed loop transfer function is

$$G_{cl} = \frac{GG_c}{1+GG_c} = \frac{K_c K_p (1+sT_i)e^{-sL}}{(1+sT)sT_i + K_p K_c (1+sT_i)e^{-sL}}.$$

By approximating e^{-sL} with $1-sL$, G_{cl} can be written as

$$G_{cl} \approx \frac{K_c K_p (1+sT_i)e^{-sL}}{(1+sT)sT_i + K_p K_c (1+sT_i)(1-sL)}. \tag{4}$$

By choosing $T_i = T$ in Equation (4) we get

$$G_{cl}(s) \approx K_p K_c \frac{e^{-sL}}{K_p K_c + s(T - K_p K_c L)}, \tag{5}$$

i.e. we have cancelled the process pole at $1/T$ with the controller zero. Now, using pole placement we will obtain the desired controller parameters.

The characteristic equation of system (5) is,

$$s(\frac{T}{K_p K_c} - L) + 1 = 0.$$

Comparing this with the desired characteristic equation

$$sT_{cl} + 1 = 0,$$

where T_{cl} is the desired closed loop time constant, gives the controller parameters which are given by

$$\begin{aligned} K_c &= \frac{1}{K_p}\frac{T}{L+T_{cl}}, \\ T_i &= T. \end{aligned} \tag{6}$$

Note that the expressions of the controller parameters in formula (6) are just the ones which are used in the process industry when tuning PI controllers with the Lambda method, see Sell (1995).

3.2 Design without Cancellation for Stable Processes

Now a new idea is presented where the cancellation of the process pole is taken under consideration. The controller parameters in (6) were obtained by first positioning two poles: one at $1/T$ and the other in $1/T_{cl}$. Then the process pole, $1/T$, is cancelled by the controller zero. The consequence of cancelling the process pole may give a system with poor rejection of load disturbances in those cases when $T > T_{cl}$. A thorough discussion of the effects of cancelling process poles may be found in Åström and Hägglund (1995).

When $T > T_{cl}$, one can omit the cancellation and calculate the characteristic equation of (4), given by

$$s^2(\frac{T_iT}{K_pK_c} - T_iL) + s(\frac{T_i}{K_pK_c} + T_i - L) + 1 = 0.$$

Comparing this with the desired characteristic equation obtained by placing both poles at $1/T_{cl}$

$$T_{cl}^2 s^2 + 2T_{cl}s + 1 = 0,$$

gives the controller parameters

$$\begin{aligned} K_c &= \frac{1}{K_p}\frac{TL + 2T_{cl}T - T_{cl}^2}{(L+T_{cl})^2}, \\ T_i &= \frac{TL + 2T_{cl}T - T_{cl}^2}{(T+L)}. \end{aligned} \tag{7}$$

3.3 Design without Cancellation for Unstable Processes

Finally we will consider unstable processes which have been modeled by the first order process model described by (2). For this case the same approach with no cancellation of the process pole has to be taken. Consider the unstable process (2) for which the closed loop transfer function is given by

$$G_{cl} = \frac{GG_c}{1+GG_c} = \frac{K_vK_c(1+sT_i)e^{-sL}}{s^2T_i + K_vK_c(1+sT_i)e^{-sL}}.$$

By approximating e^{-sL} with $1 - sL$, G_{cl} can be written as

$$G_{cl} \approx \frac{K_vK_c(1+sT_i)e^{-sL}}{s^2T_i + K_vK_c(1+sT_i)(1-sL)}. \tag{8}$$

Consequently, the characteristic equation of (8) is given by

$$s^2(\frac{T_i T}{K_v K_c} - T_i L) + s(T_i - L) + 1 = 0.$$

Comparing this with the desired characteristic equation

$$T_{cl}^2 s^2 + 2T_{cl} s + 1 = 0,$$

where T_{cl} is the desired time constant of the closed loop system, gives the controller parameters,

$$\begin{aligned} K_c &= \frac{1}{K_v} \frac{L + 2T_{cl}}{(L + T_{cl})^2} = \frac{1}{K_v} \left(\frac{1}{L + T_{cl}} + \frac{T_{cl}}{(L + T_{cl})^2} \right), \\ T_i &= L + 2T_{cl}. \end{aligned} \tag{9}$$

Note that when $T \to \infty$ the expressions of the controller parameters of (9) and (7) coincide if setting $K_v = K_p/T$. Also, the expressions of the controller parameters in formula (7) are just the ones which are used in the process industry when tuning PI controllers with the Lambda method, see Sell (1995).

Summarizing, we find that a sensible way to design the controller in the case of processes without integral action is to use formula (6) when T_{cl} is larger than T and Equation (7) when T_{cl} is smaller than T. This means that tuning a PI controller with our method gives an extra benefit by separating the cases of fast and slow process poles. For the case of processes with integral action formula (9) should be used.

Furthermore, the Lambda method requires only one tuning parameter, that is the desired closed loop time constant, T_{cl}. The choice of T_{cl} is a key decision.

Note that in all the following verifications only formula (6) and (9) has been used to compute the controller parameters. The motivation is that those PI controllers in the process industry which has been tuned with the Lambda method uses only the formulas of (6) and (9).

4. Design Choices

Formally we can choose any value of the closed loop time constant T_{cl}. An arbitrary choice may, in practice, lead to poor performance or even instability. Because the simple process models are only valid in certain frequency regions. An indication that the model is valid for a particular choice is the fact that the closed loop time constant agrees with its specified value. Therefore, the closed loop time constant should be related to the process dynamics. The model (1) has a time constant T and a time delay L. The model (2) has only a time delay L. Therefore it is natural to relate T_{cl} to T or L for the model (1) and to L for model (2). To obtain a fast response with good rejection of disturbances it is desirable to have a small value of T_{cl}. However, a large value of T_{cl} gives a system that is more insensitive to parameter variations.

4.1 How to Select T_{cl}

The most common way to determine T_{cl} is by multiplying the time constant of the process, T, with a factor λ, i.e. $T_{cl} = \lambda T$. This is not a good choice when $L > T$. In these cases the tuning parameter T_{cl} is better determined as $T_{cl} = \lambda L$.

If we have limited knowledge of the process dynamics it would be appropriate to use a relatively high value of T_{cl}, i.e. a large value of λ. Conversely, if we were very confident in the process dynamics, and there was a small amount of dead time, a relatively small T_{cl} would be appropriate, i.e. a small value of λ. In the process industry the λ factor is set normally to values between $0.5 - 3.0$, the interpretation of T_{cl} is the time it takes for the process to reach the new set point.

Note that in the case of integrating processes, the value of T_{cl} may be determined in only one way by multiplying the dead time of the process L with a factor λ, i.e. $T_{cl} = \lambda L$. Also, in the process industry the tuning parameter T_{cl} is interpreted as the arrest time, i.e. the time between the occurrence of the load and the peak of the disturbance.

4.2 The Possibility of Obtaining the Specified T_{cl} for Non-Integrating Processes

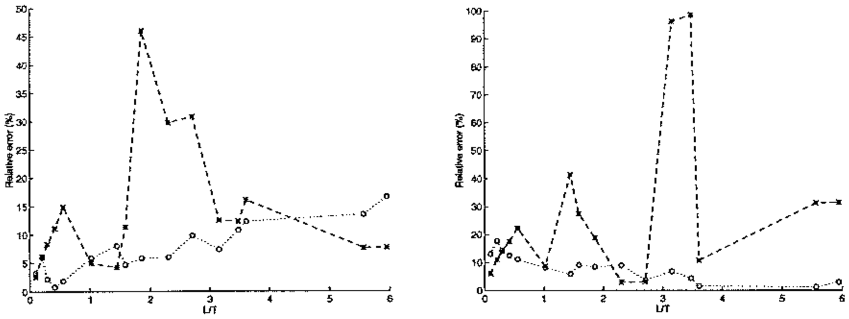
In the Lambda method for non-integrating processes only one design parameter needs to be chosen, T_{cl}, which is the desired speed of response of the closed loop. Now the reader may wonder if, in practice, it is possible to obtain this specified value and which of the two proposed ways to set it, i.e. $T_{cl} = \lambda T$ or $T_{cl} = \lambda L$, gives the best result depending on the characteristic of the process?

The answer to these questions are given by verifying the following process: $G(s) = e^{-sL}/(s+1)$ where $L \in [0.1, 6.0]$. To make the comparison between the two suggested ways of determining T_{cl} the relative errors are computed, i.e.,

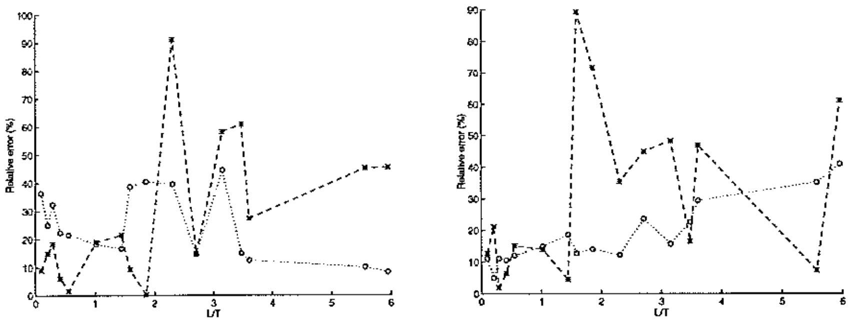
$$T_{rel} = |T_{\lambda cl} - T_{cl}|/T_{cl},$$
$$\tilde{T}_{rel} = |\tilde{T}_{\lambda cl} - \tilde{T}_{cl}|/\tilde{T}_{cl},$$

where T_{cl}, $\tilde{T}_{cl}$ are the closed loop time constants in practice and $T_{\lambda cl} = \lambda T$ and $\tilde{T}_{\lambda cl} = \lambda L$. The results are shown in Figure 1. The following conclusions may be drawn on the possibility to obtain, in practice, the specified value of T_{cl} from Figure 1,

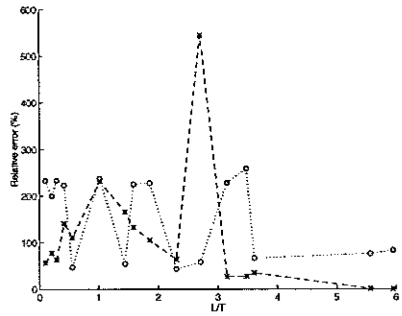
- The first thing to note: in practice it is not possible to obtain the specified value of T_{cl} as the relative error is different from zero.
- The second thing to note: it is generally possible to determine T_{cl} systematically from process knowledge. If $L/T < 1$ then the least relative error is given by setting $T_{cl} = \lambda T$ and if $L/T > 1$ then the least relative error is given by setting $T_{cl} = \lambda L$. Note that Figure 1 confirms the statement of a general rule as it will not be valid for all cases.
- The third thing to note: as the factor λ decreases the relative error of T_{cl} increases.



Case when $\lambda = 3.0$ (left) and $\lambda = 2.0$ (right).



Case when $\lambda = 1.5$ (left) and $\lambda = 1.0$ (right).



Case when $\lambda = 0.5$.

Figure 1 Plots of the relative error of T_{cl} as a function of L/T, for the process $G(s) = e^{-sL}/(s+1)$, $L \in [0.1, 6.0]$. Comparison has been done for determining T_{cl} as $T_{cl} = \lambda T$ ($*$) and $T_{cl} = \lambda L$ ($\circ$), where $\lambda = 3.0,\ 2.0,\ 1.5,\ 1.0,\ 0.5$.

4.3 The Possibility of Obtaining the Specified T_{cl} in Practice for Integrating Processes

The Lambda method for integrating processes requires only one design parameter to be chosen, namely T_{cl} which is the time it takes to arrest a load disturbance. The reader may wonder if this interpretation really holds in practice.

The answer to this question is given by verifying the following process $G(s) = e^{-sL}/s$ where $L \in [0.1, 6.0]$. To be able to draw any conclusions out of this the relative error of T_{cl} is computed, i.e.,

$$T_{rel} = |T_{\lambda cl} - T_{cl}|/T_{cl},$$

where T_{cl} is the time to arrest a load disturbance in practice and $T_{\lambda cl} = \lambda L$.

The results are shown in Figure 2. The following conclusion may be drawn from Figure 2,

- The first thing to note: it is not correct that the arrest time is equal to the specified value of T_{cl}, as the relative error is different from zero.
- The second thing to note: for each value of the λ factor the relative error takes on a constant value independently of the dead time L. This implies that there exists a linear relation between T_{cl} and L, that is $T_{cl} = cL$ where the constant rate of change c depends on the value of the λ factor.
- The third thing to note: as the factor λ decreases the relative error of T_{cl} increases. The ratio of change of the relative error for one unit increase of the λ factor is approximately a decrease of 20%.

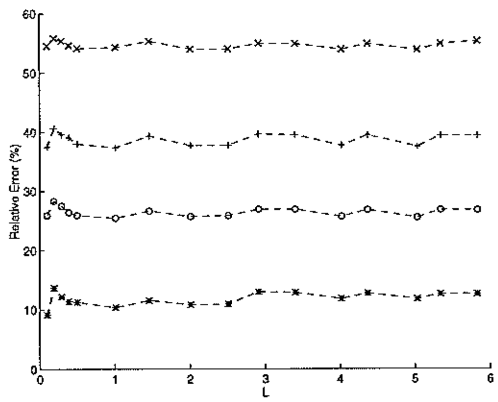


Figure 2 Lambda method on $G(s) = e^{-sL}/s$ where $L \in [0.1, 6.0]$ for $\lambda = 3.0(*)$, $2.0(o)$, $1.5(+)$, $1.0(x)$. The relative error for the time of the maximum load disturbance is computed.

5. A Systematic Approach to Determine T_{cl}

In Section 4.1 it was shown how the tuning parameter T_{cl} should be chosen so that the knowledge of the process was included. Also, we examined the possibility of the Lambda method to obtain, in practice, the specified value of T_{cl}.

In this section we will present an idea which gives us a systematic way to determine the tuning parameter T_{cl} from user specifications on the robustness of the system. That is, the user specifies a desired robustness of the closed loop system, i.e. the the maximum of the sensitivity function, denoted M_s. Recall that the sensitivity function is given by

$$S(s) = \frac{1}{1 + G(s)G_c(s)}. \tag{10}$$

Thus, $M_s = \max |S(i\omega)|$. Given the value M_s, our idea makes it possible to compute the corresponding value of T_{cl}, which in turn makes it possible for the Lambda method to fulfill the robustness requirement. Note that this idea has only been applied for systems without integral action.

We begin by computing the loop transfer function obtained by designing a PI controller for the process model (1) with the lambda method. It is given by

$$GG_c(s) = \frac{T}{L + T_{cl}} \frac{e^{-sL}}{sT} = \frac{1}{1 + T_{cl}/L} \frac{e^{-sL}}{sL}. \tag{11}$$

It follows from (11) that the shape of the Nyquist curve is uniquely determined by the quantity T_{cl}/L. Thus the sensitivity function is a unique function of T_{cl}/L. Remark that the sensitivity constraint indicates that T_{cl} should be proportional to L. From Equation (10) we find that

$$\frac{1}{S(i\tilde{\omega})} = 1 + \frac{\kappa}{i\tilde{\omega}}(\cos\tilde{\omega} - i\sin\tilde{\omega}),$$

where

$$\begin{aligned} \kappa &= 1/(1 + T_{cl}/L), \\ \tilde{\omega} &= \omega L. \end{aligned} \tag{12}$$

Hence

$$|\frac{1}{S(i\tilde{\omega})}|^2 = 1 - 2\frac{\kappa}{\tilde{\omega}}\sin\tilde{\omega} + (\frac{\kappa}{\tilde{\omega}})^2.$$

This expression has a minimum when

$$\kappa - \tilde{\omega}\sin\tilde{\omega} + \tilde{\omega}^2\cos\tilde{\omega} = 0. \tag{13}$$

Equation (13) can easily be solved by Newton-Raphson which gives the following iteration

$$\tilde{\omega}_{n+1} = \tilde{\omega}_n + \frac{\kappa - \tilde{\omega}_n sin\tilde{\omega}_n + \tilde{\omega}_n^2\cos\tilde{\omega}_n}{(1+\tilde{\omega}_n^2)\sin\tilde{\omega}_n - \tilde{\omega}_n\cos\tilde{\omega}_n}. \tag{14}$$

The solution of Equation (14) will show what value the maximum of the sensitivity function, M_s, obtains for a given value of κ, i.e. T_{cl}/L. Some interesting values of M_s and the corresponding values of κ and T_{cl}/L are shown in Table 1.

Depending on how we determine the closed loop speed of response T_{cl} we will obtain different expressions on λ when solving Equation (12). We get

$$\begin{aligned} T_{cl} = \lambda T: &\quad \lambda = \frac{1-\kappa}{\kappa}\cdot\frac{L}{T}, \\ T_{cl} = \lambda L: &\quad \lambda = \frac{1-\kappa}{\kappa}. \end{aligned} \tag{15}$$

It follows from Equation (15), that in the first equation, the λ factor is a function of κ, T, L, and in the second equation, the λ factor is only a function of κ. Thus to obtain a specified M_s value with the first equation in

M_s	κ	T_{cl}/L
1.0	0.01	99
1.2	0.21	3.76
1.4	0.38	1.63
1.6	0.51	0.96
2.0	0.71	0.41

Table 1 Using the Lambda method to obtain a specified maximum sensitivity value, M_s.

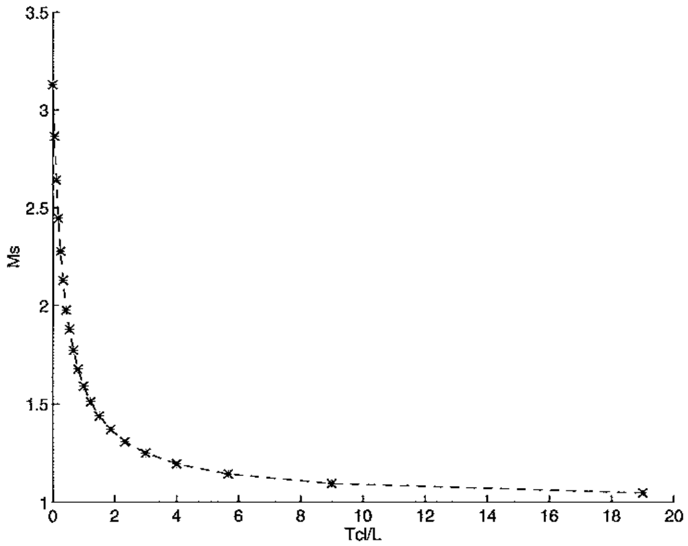


Figure 3 Maximum of the sensitivity function, M_s, as a function of T_{cl}/L.

(15) it will be possible with different values of the λ factor as it depends on the process considered. But, in the second equation only one value of the λ factor will give the specified M_s value as it is independent of what process is considered. Note that for both equations in (15) the tuning parameter T_{cl} takes on the same expression, i.e. $T_{cl} = (1-\kappa)/\kappa \cdot L$.

In Figure 3 we show the maximum of the sensitivity function, M_s, as a function of T_{cl}/L. Note that for the interesting cases $M_s = 1.4,\ 2.0$, then $T_{cl}/L < 1$, i.e. the time delay dominates the behavior of the closed loop system. This result indicates that it seems better to base the choice of the closed loop time constant on L rather than T.

5.1 The Possibility of Obtaining the Specified M_s Value

Will it in practice be possible to obtain the desired value of the maximum sensitivity function, M_s, by specifying λ according to the theory in the previous Subsection? Too answer this question we tried our idea on the process $G(s) = e^{-sL}/(s+1)$ for $L \in [0.1, 6.0]$. We calculated the relative error of the M_s value for the cases $M_s = 1.2,\ 1.4$ and 2.0. As was mentioned in the previous Subsection, the choice of how to set the tuning parameter T_{cl} is indifferent. We end up with the same expression, i.e. $T_{cl} = (1-\kappa)/\kappa \cdot L$ in either case.

The results are presented in Figure 4. The following conclusions may be drawn from Figure 4 on the possibility of obtaining a specified M_s value with the Lambda method:

- The first thing to note: it is possible to obtain the specified M_s value with the Lambda method with a relative error less then 10%.
- The second thing to note: the relative error of the M_s value decreases for increasing values of T_{cl}/L.
- The third thing to note: the relative error of the M_s value decreases for decreasing values of M_s.

It is possible to explain these observations. Let us start with the second observation. According to Figure 3 a desired robustness of the system, i.e. to specify a desired M_s-value, corresponds to a specific value of T_{cl}/L. Consequently, the value of the tuning parameter needed for the Lambda method can be calculated. As we have a one to one relation between T_{cl}/L and M_s in Figure 3, this implies that if the the value of T_{cl} obtained in practice with the Lambda method differs from the specified one we will get a different M_s value from the desired one. As it was shown in Section 4.2, the assertiveness to obtain T_{cl} in practice with the Lambda method decreases for decreasing values of the specified T_{cl}. Thus, the possibility to obtain the specified M_s value decreases for decreasing values of T_{cl}/L.

The third observation can also be explained from Figure 3. According to Figure 3 the value T_{cl}/L increases as M_s decreases. This means that the possibility to obtain the specified M_s value increases for decreasing values of M_s according to the second observation.

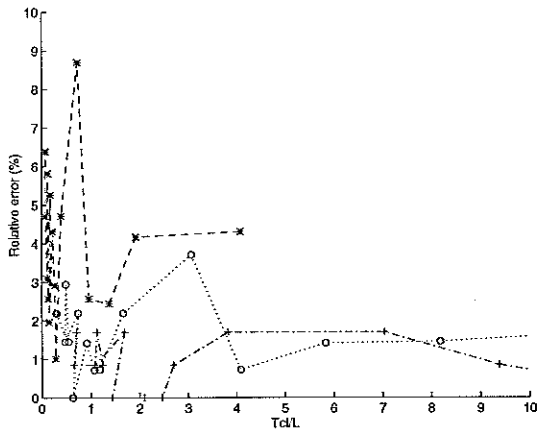


Figure 4 Plot of the relative error of the M_s value as a function of T_{cl}/L, for the process $G(s) = e^{-sL}/(s+1)$ where $L \in [0.1, 6.0]$. The cases $M_s = 2.0$ ($*$), 1.4 ($\circ$), 1.2 ($+$) have been examined.

6. The Connection Between Integral Action and T_{cl}

It is always useful to have a simple way to judge if the integral action of the controller is too weak. For a good closed loop performance it is necessary to have sufficient integral action, see Åström and Hägglund (1995). In this Section we will present an idea where the tuning parameter T_{cl} of the

Lambda method gives a rough estimate of the amount of integral action the controller will generate. The analysis is only performed for stable processes.

Let us consider stable processes which are modeled by the first order process model in (1). Compute the loop transfer function which is given by

$$GG_c = \frac{1}{1+T_{cl}/L}\frac{e^{-sL}}{sL}. \tag{16}$$

Make the approximation e^{-sL} to $1 - sL$ for (16), then

$$GG_c \approx -\frac{1}{1+T_{cl}/L} + \frac{1}{sL(1+T_{cl}/L)}. \tag{17}$$

This implies that the imaginary part of the of the Nyquist curve starts off at $-\infty$ and tends towards 0 for increasing values of the frequency ω, with a constant negative real part for small values of ω.

To have a good closed loop performance in view of enough integral action, it is required that the real part of the loop transfer function should be less than -0.5 for small s, that is,

$$\frac{1}{1+T_{cl}/L} > 0.5$$

which gives the following requirement

$$T_{cl} < L. \tag{18}$$

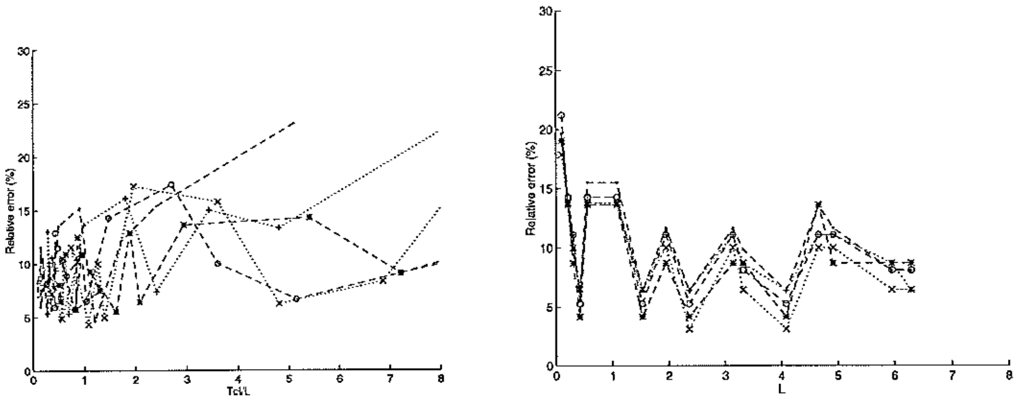


Figure 5 Plot of the relative error of the real part of the loop transfer function versus T_{cl}/L and L when using the Lambda method on $G(s) = e^{-sL}/(s+1)$ where $L \in [0.1, 6.0]$ for $\lambda = 3.0$ (*), 2.0(x), 1.5(o), 1.0(+), 0.5(.). In the left figure $T_{cl} = \lambda T$ and in the right figure $T_{cl} = \lambda L$.

6.1 The Relation Between Integral Action and T_{cl}

Will in practice the Nyquist curve take on a constant real part for small ω? The answer to this question has been obtained by computing the relative error of the real part of the loop transfer function for the process $G(s) = e^{-sL}/(s+1)$, where $L \in [0.1, 6.0]$. To compute the relative error we use the approximation (17) and the true value of the real part of the loop transfer function for low frequencies. Both ways has been examined to set the tuning parameter T_{cl} of the Lambda method, i.e. $T_{cl} = \lambda T$ and $T_{cl} = \lambda L$.

The results are shown in Figure 5. Note that in the right figure the relative error is plotted versus L just to give a more descriptive result. The following conclusions may be drawn from Figure 5,

- The first thing to note: with an average value of 10% of the relative error it is possible to obtain the approximate value of the real part of the loop transfer function in (17) with the Lambda method.
- The second thing to note: for the setting $T_{cl} = \lambda T$ the average value of the relative error is about 7%, for $T_{cl}/L < 1$ and for $T_{cl}/L > 1$ it is in general larger than 7%. Also, for increasing values of the λ factor the relative error decreases.
- The third thing to note: for the setting $T_{cl} = \lambda L$ the average value of the relative error is about 7%, for $L > 1$ and for for $L < 1$ it is in general larger than 7%. Also, for increasing values of the λ factor the relative error decreases.

To sum up: we find that the approximation in (17) makes sense for the cases $T_{cl} = \lambda T$ when $T_{cl}/L < 1$ and $T_{cl} = \lambda L$ when $L > 1$. Consequently, in these cases the requirement (18) will be fulfilled so that enough integral action will be provided to the closed loop system.

Also in the two lower figures of Figure 6, we really see that when applying the Lambda method to stable processes the loop transfer function will have a constant real part for low frequencies.

7. Typical Process Control Problems

In this section the Lambda method is compared to another design method, see Åström *et al.* (1997), which we denote as the M_s-method. We will give a number of examples illustrating the properties of the controllers obtained from the two design methods.

It is interesting to compare the two design methods, because only one tuning parameter needs to be chosen. For the Lambda method the tuning parameter, T_{cl}, is directly related to the speed of response of the closed loop system, and for the M_s-method the tuning parameter is directly related to the maximum of the sensitivity function.

The following transfer function describes the structure of the PI controller to be designed

$$u(t) = K_c(by_{sp}(t) - y(t)) + \frac{K_c}{T_i}\int (y_{sp}(\tau) - y(\tau))d\tau,$$

where u is the controller output, y is the process output and y_{sp} is the set point. The controller parameters are: K_c the controller gain, T_i the integral time and b the set point weighting. In the M_s-method the b parameter is included in the design which it is not in the case of the Lambda method. So in this case the b parameter is set to 1 to make a fair comparison between the two methods.

When using the Lambda method the tuning parameter T_{cl} is set to either $T_{cl} = \lambda T$ when $L/T < 1$ or $T_{cl} = \lambda L$ when $L/T > 1$. The comparison of the Lambda method with the M_s-method is done for the values $\lambda = 1.0,\ 3.0$ and $M_s = 2.0,\ 1.4$. These choices are based on the fact that with $\lambda = 1.0$ and $M_s = 2.0$ a more tuff control is obtained on the contrary to the choices $\lambda = 3.0$ and $M_s = 1.4$ which gives more careful control. Also, these choices of the λ factor and the M_s value are usually recommended to the process industry when choosing suitable design parameters.

We will consider the following transfer functions which are representative systems normally encountered in process control,

$$
\begin{aligned}
G_1(s) &= \frac{1}{(s+1)(1+0.2s)(1+0.04s)(1+0.008s)}, & G_2(s) &= \frac{1}{(s+1)^3},\\
G_3(s) &= \frac{e^{-5s}}{(s+1)^3}, & G_4(s) &= \frac{1}{s(s+1)^2},\\
G_5(s) &= \frac{1-2s}{(s+1)^3}, & G_6(s) &= \frac{9}{(s+1)(s^2+2s+9)}.
\end{aligned}
$$

Systems G_1 and G_2 represent processes that are relatively easy to control. System G_3 has a long dead time, and G_4 models an integrating process. System G_5 has a zero in the right half plane, and system G_6 has complex poles with relative damping 0.33. Systems of type G_5 and G_6 are not common in process control, but they have been included to demonstrate the wide applicability of the two design procedures.

Figure 6 shows the Nyquist curves of the loop transfer functions obtained using the M_s-method and the Lambda method. The responses to step changes in set point and load are shown in Figure 7 and 8, and the details of the design calculations and simulations are summarized in Table 2.

In Table 2 the following information is given: the first column marks the verified process. The second indicate the design method and the third column the given design parameters. The fourth till the twelfth column show characteristic parameters of the closed loop system. They are the following: K_c is the controller gain, T_i is the integration time of the controller, b is the set point weighting parameter, IE and IAE are respectively the integrated error and the integrated absolute error after a load disturbance, IE/IAE is the quotient between the integrated error and the integrated absolute error, ω_0 is the frequency at which the sensitivity function takes on its maximum value, t_s is the settling time after a load disturbance, M_s is the maximum value of the sensitivity function and M_p is the maximum value of the closed loop system.

Although the systems $G_1 - G_6$ represent processes with large variations in process dynamics, Figure 7 and 8 show that the resulting closed loop responses of the M_s-method are quite similar. This is important because it means that this design procedure gives closed loop systems with desired and predictable properties. The fact that it even treats integrating processes in the same way as stable processes is interesting. Whereas for the Lambda method the resulting closed loop responses are not obviously similar compared to the previous method. Also, it requires, like so many other design approaches, that stable and integrating processes have to be treated separately, see Åström and Hägglund (1995).

Process			K_c	T_i	b	IE	IE/IAE	w_0	t_s	M_s	M_p
$G_1(s)$	M_s	1.4	1.93	0.74	0.89	0.39	1.00	3.37	2.49	1.40	1.10
		2.0	4.13	0.59	0.52	0.14	0.92	4.47	1.30	2.00	1.66
	λ	3.0	0.31	1.05	1.00	3.35	1.00	2.26	14.0	1.06	1.00
		1.0	0.83	1.05	1.00	1.26	1.00	2.93	6.50	1.14	1.00
$G_2(s)$	M_s	1.4	0.63	1.95	1.00	3.07	1.00	0.74	10.8	1.40	1.00
		2.0	1.22	1.78	0.50	1.46	0.77	0.86	12.8	2.00	1.45
	λ	3.0	0.28	1.92	1.00	6.90	1.00	0.61	25.8	1.17	1.00
		1.0	0.63	1.92	1.00	3.06	1.00	0.73	10.8	1.40	1.00
$G_3(s)$	M_s	1.4	0.19	2.99	1.00	15.6	1.00	0.22	38.2	1.40	1.00
		2.0	0.31	2.68	0.00	8.60	0.73	0.23	17.9	2.00	1.19
	λ	3.0	0.09	2.23	1.00	25.9	1.00	0.17	72.2	1.23	1.00
		1.0	0.17	2.23	1.00	13.1	0.97	0.20	28.4	1.53	1.00
$G_4(s)$	M_s	1.4	0.17	14.0	0.70	84.0	0.97	0.29	35.3	1.40	1.40
		2.0	0.33	8.00	0.50	24.0	0.96	0.41	15.7	2.00	1.77
	λ	3.0	0.24	11.9	1.00	49.1	1.00	0.37	27.9	1.58	1.42
		1.0	0.38	5.37	1.00	14.0	0.62	0.42	25.9	2.77	2.57
$G_5(s)$	M_s	1.4	0.18	1.78	1.00	9.92	0.90	0.38	28.6	1.40	1.00
		2.0	0.29	1.60	0.00	5.44	0.70	0.42	13.5	2.00	1.20
	λ	3.0	0.09	1.31	1.00	15.5	1.00	0.28	45.7	1.25	1.00
		1.0	0.17	1.31	1.00	7.65	0.87	0.34	18.8	1.60	1.00
$G_6(s)$	M_s	1.4	0.31	0.37	0.88	1.19	0.87	1.98	4.13	1.40	1.04
		2.0	0.48	0.31	0.00	0.65	0.63	2.11	2.56	2.00	1.37
	λ	3.0	0.29	0.83	1.00	2.92	1.00	2.63	9.95	1.20	1.00
		1.0	0.66	0.83	1.00	1.25	1.00	2.69	4.80	1.62	1.00

Table 2 Properties of the controllers obtained when designing with the M_s-method and the Lambda method for the systems $G_j(s)$, $j = 1 \ldots 6$.

There is also a large similarity between the responses obtained with the M_s-method for different values of the tuning parameter M_s. This shows that the M_s-value is a suitable tuning parameter. Responses obtained with M_s=1.4 show little or no overshoot. This is normally desirable in process control. Responses obtained with M_s=2.0 give faster responses. The settling time at load disturbances, t_s, is significantly shorter with this larger value of M_s. On the other hand, these responses are oscillatory with a larger overshoot. This can be seen from the quotient IE/IAE in Table 2.

On the other hand there are also similarities between the responses obtained of the Lambda method for different values of the tuning parameter T_{cl}. Responses obtained when setting the factor λ=3.0, i.e. large T_{cl}, show no overshoot and gives a slower response compared to setting the factor λ=1.0, i.e. small T_{cl}. This shows the fact that λ is directly related to the speed of response of the closed loop system. The settling time at load disturbances, t_s, is in general almost half as short with the smaller value of λ. On the other hand, most will show a little overshoot. This can be seen from the comparison between IE/IAE in Table 2.

According to Table 2 the controller gain K_c varies significantly with the design parameter M_s and λ, i.e. T_{cl}. However, integral time T_i is fairly constant for the M_s-method and completely constant for the Lambda method in the case of stable processes, i.e., all processes except G_4. This means that, for PI control, the different design specifications are mainly obtained by adjusting only the gain. This observation is made earlier, see Åström and Hägglund (1995).

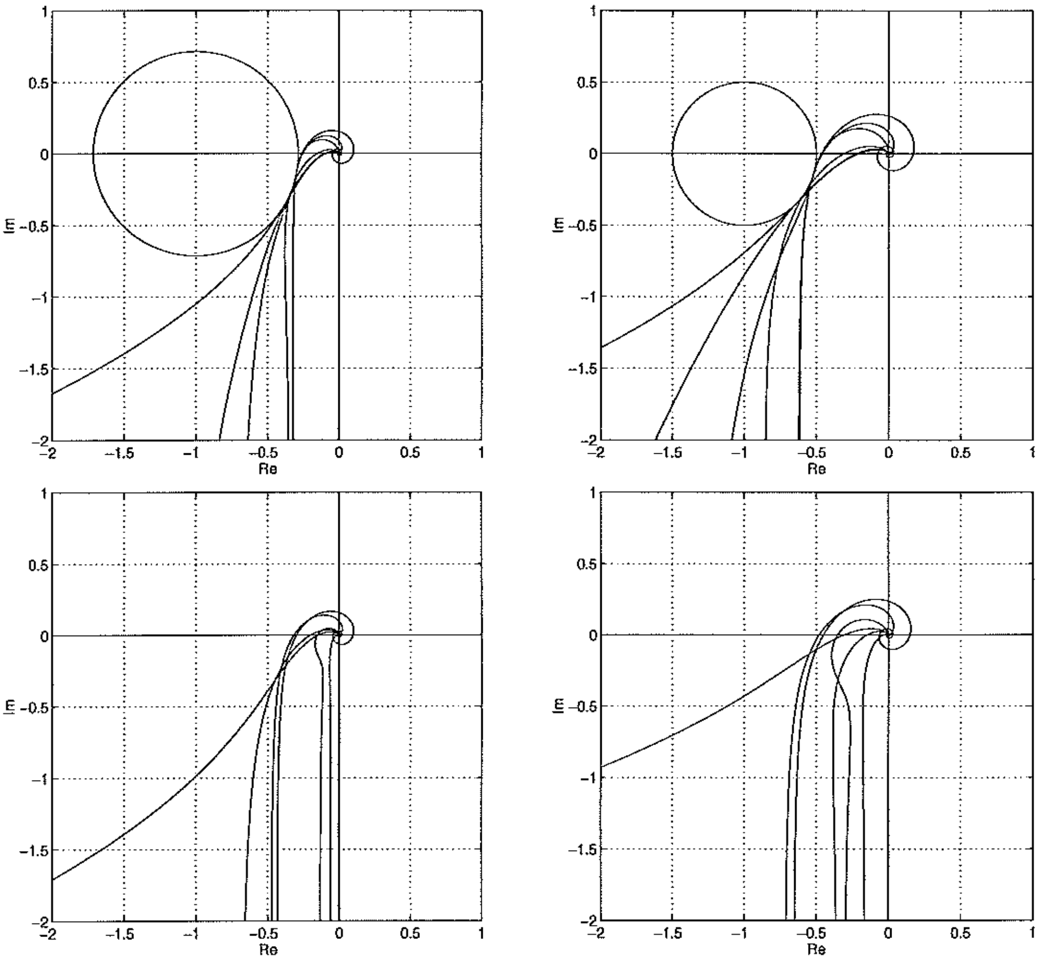
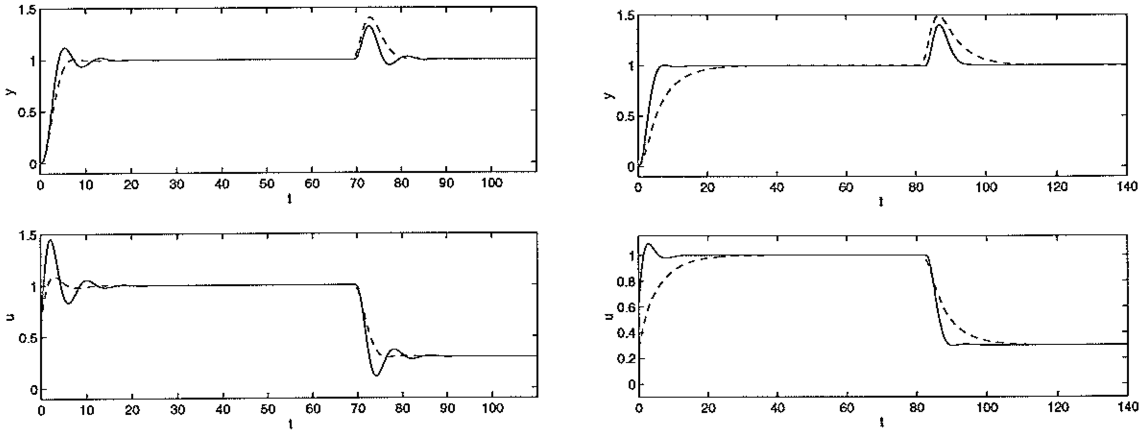


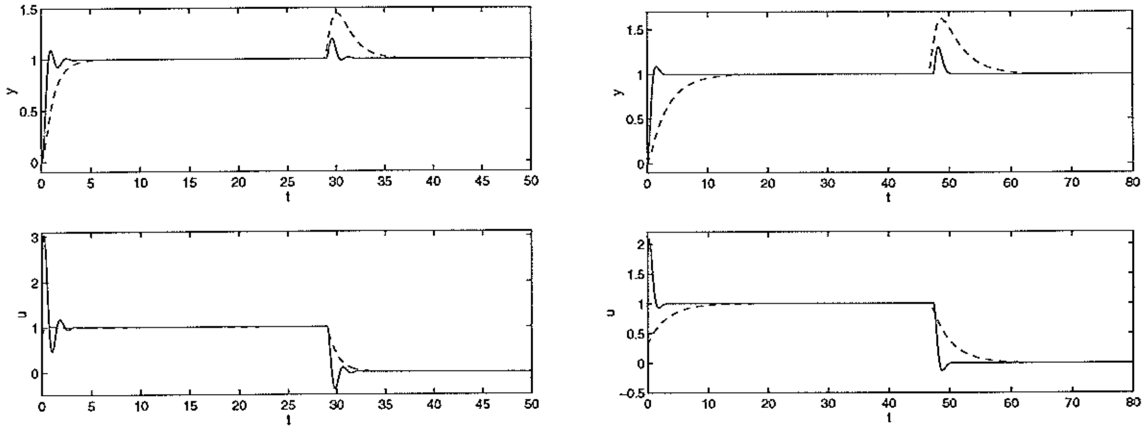
Figure 6 In the two upper figures the Nyquist plots of the open loop frequency response for the systems $G_j(s)$, $j = 1 \dots 6$ when using the M_s-method with $M_s = 1.4$ (left) and $M_s = 2.0$ (right.). In the lower figures the corresponding Nyquist plots of the open loop frequency response when using the Lambda method with $\lambda = 3.0$ (left) and $\lambda = 1.0$ (right).

Except for the integrating process G_4, the M_p values obtained for M_s=1.4 are all close to one. Consequently, parameter b is also close to one. For M_s=2.0, the M_p values are, however, larger. This means that the overshoots would have been significant if the set point weighting were chosen to $b = 1$. However, acceptable set point responses are obtained by using small values of b.

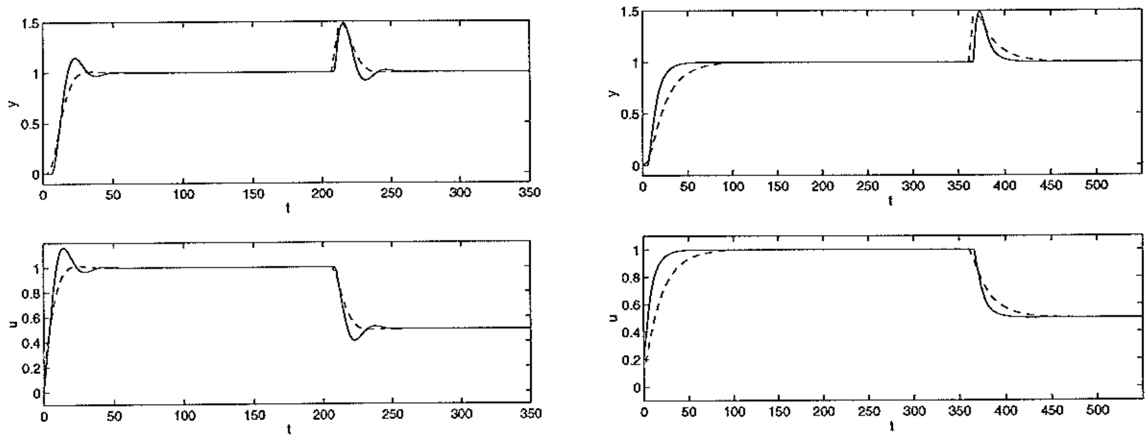
In the case of the Lambda method the M_p values obtained are all equal to one except for the integrating process G_4.



Control of the system G_1 with $M_s = 2.0$ and $\lambda = 1.0$ (left) and with $M_s = 1.4$ and $\lambda = 3.0$ (right).

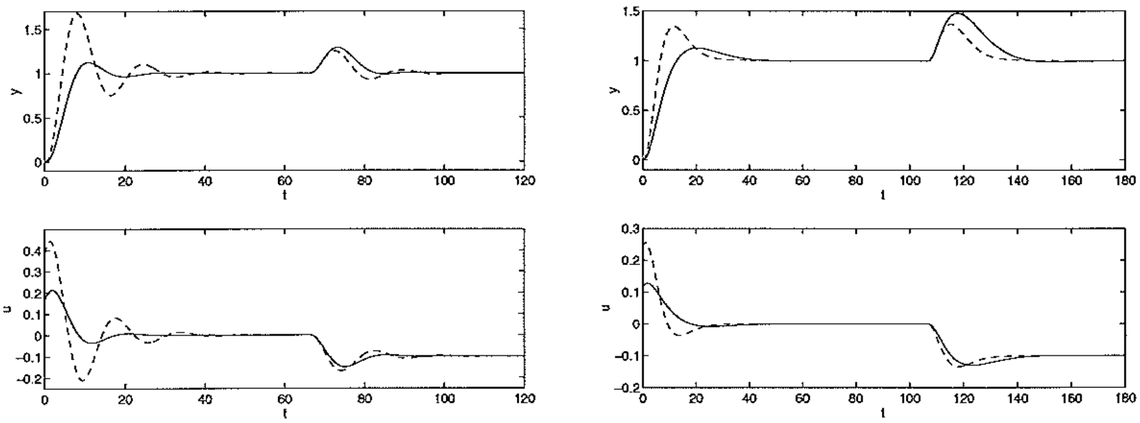


Control of the system G_2 with $M_s = 2.0$ and $\lambda = 1.0$ (left) and with $M_s = 1.4$ and $\lambda = 3.0$ (right).

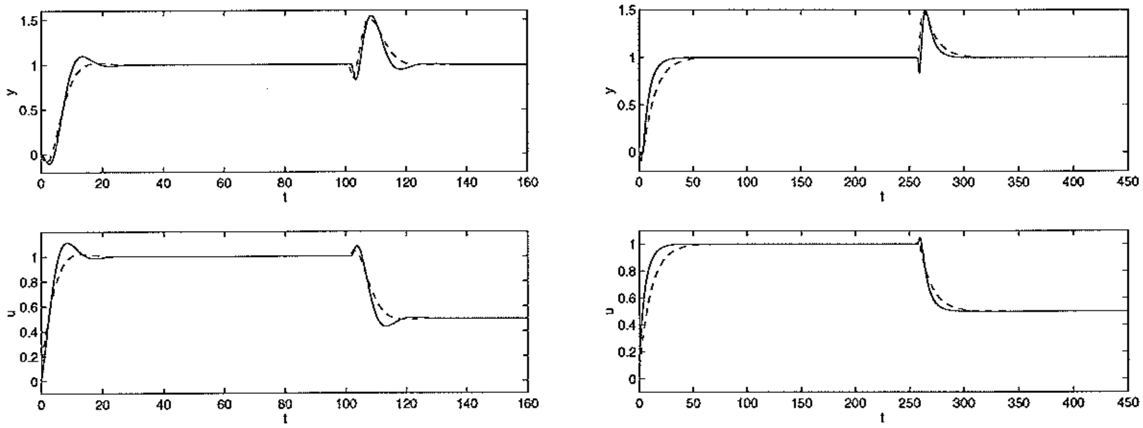


Control of the system G_3 with $M_s = 2.0$ and $\lambda = 1.0$ (left) and with $M_s = 1.4$ and $\lambda = 3.0$ (right).

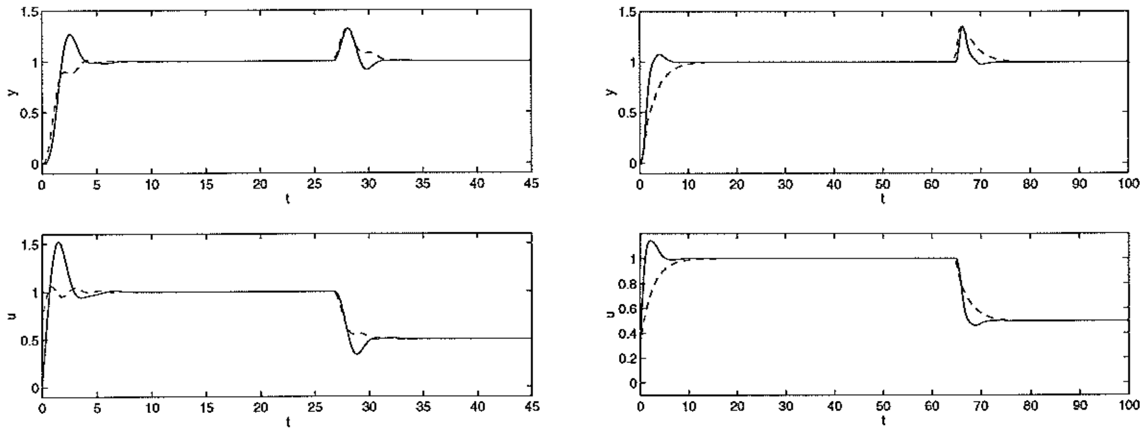
Figure 7 Comparison between the PI controllers obtained for the M_s-method and the Lambda method, where the design has been done for $M_s = 1.4,\ 2.0$ and $\lambda = 1.0,\ 3.0$. The graphs shows a step response followed by a load disturbance where the responses obtained with the M_s-design are drawn with solid lines and with the Lambda method are drawn with dashed lines.



Control of the system G_4 with $M_s = 2.0$ and $\lambda = 1.0$ (left) and with $M_s = 1.4$ and $\lambda = 3.0$ (right).



Control of the system G_5 with $M_s = 2.0$ and $\lambda = 1.0$ (left) and with $M_s = 1.4$ and $\lambda = 3.0$ (right).



Control of the system G_6 with $M_s = 2.0$ and $\lambda = 1.0$ (left) and with $M_s = 1.4$ and $\lambda = 3.0$ (right).

Figure 8 Comparison between the PI controllers obtained for the M_s-method and the Lambda method, where the design has been done for $M_s = 1.4,\ 2.0$ and $\lambda = 1.0,\ 3.0$. The graphs shows a step response followed by a load disturbance where the responses obtained with the M_s-design are drawn with solid lines and with the Lambda method are drawn with dashed lines.

8. Conclusion

In this report the Lambda method for tuning PI controllers has been reviewed, new ideas based on the method have been derived and the validity of already known features of the method has been verified.

The report begins by designing a PI controller with the Lambda method. In earlier presentations, a process pole is cancelled with a controller zero when the process is stable, whereas no cancellation is performed for integrating processes. Here, it is suggested to avoid cancellation also for stable processes when the process pole is small compared to the closed loop pole.

Next the design parameter of the Lambda method was examined. The strength of the method is the fact that only one tuning parameter, T_{cl}, has to be chosen. Also a great advantage is its intuitive interpretation as the time constant of the closed loop system. This report shows that in practice the obtained time constant of the closed loop system will not be equal to T_{cl}. On the other hand the report shows that the discrepancy between the specified value of T_{cl} and the true one can be reduced depending on how T_{cl} is determined, i.e. $T_{cl} = \lambda T$ or $T_{cl} = \lambda L$.

Later the report presents of two new ideas: The first one gives a systematic way to determine the tuning parameter T_{cl} from user specifications on the robustness of the system. The second idea gives an easy way to judge if the integral action of the controller is too weak or not. This judgment is based on knowledge of the design parameter and the process.

In the end the report compares the Lambda method to a new design method, see Åström *et al.* (1997), which also requires just one tuning parameter. A test batch of typical processes found in the process industry has been used for the comparisons.

To sum up: this report contributes with new insights of the Lambda method as a design method for PI controllers, especially, the fact that it is not always possible to obtain the specified closed loop time constant in practice. The report gives also a systematic way to determine the design parameter T_{cl} and an easy way to judge the performance of the closed loop based on knowledge of the tuning parameter T_{cl} and the process.

9. References

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