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Damping Structure in Power Systems[†]

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Abstract. To enhance the basic damping of power systems, a variety of stabiliser configurations can be used for the generators, SVCs and HVDC links. A study is made of how the overall damping matrix is built up from these contributions. This qualitative picture is useful in preliminary sensitivity studies to determine placement of stabilisers and their coordinated tuning.

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1. Introduction

Following problems with low frequency oscillations in the 1960's, most power systems have installed power system stabilisers (PSS) on generators with fast excitation systems. These are generally tuned using local response information for the generator [1]. Intuitively, this can only be expected to succeed in damping the so-called 'local modes' where individual generators oscillate against large parts of the system. However, in many modern systems, there is difficulty with self-excited slow oscillations corresponding to 'system modes' where large groups of generators swing against each other. These problems require a system-wide rather than single generator view. Preliminary studies need to pin-point those PSS which can have significant effect in improving the damping of the troublesome modes. Beginning with DeMello et al. [2], sensitivity studies on simple models have been suggested. These models have so far been generator oriented in that the usual impedance network reduction is employed [3]. This report suggests a model which retains simplicity but gives an opportunity to study the influence of other damping sources such as loads, SVC and HVDC links. The work forms part of a larger programme on analytical aspects of power system dynamics and security using network oriented (or structure preserving) models [4].

The use of generator/network oriented (rather than generator oriented) models is based on the proposition that a complete resolution of the system mode damping problem will come with use of network information in stabilisers. Typical conventional PSS use signals derived from generator power, frequency or speed. Use of tie-line powers which connect the two swinging areas in a mode would appear much more effective [4]. Also SVC and HVDC links are known to have significant damping effect when fitted with stabilisers. Issues such as these are discussed.

From the model presented here a picture of overall damping structure can be built up (with reference to a particular damping matrix). The various damping forces from PSS, SVC and HVDC links are related to the classifications used in mechanics.

2. Basic Multimachine Model

In this section, the basic multimachine model is derived. Later sections consider the inclusion of the various supplementary damping influences. The steps taken in deriving the model are similar to less general exercises carried out in [5,6].

Suppose there are m generators which are interconnected by a network of transmission lines and transformers. The network has a total of n buses. The $n - m$ buses without generation only have power injection from loads. Let the generator terminal buses be numbered as $i = 1, \ldots m$ and the load buses as $i = m + 1, \ldots n$. Let δ_j be the rotor angle of the j:th generator with respect to a synchronously rotating reference frame. Then $\omega_j = \dot{\delta}_j$ is the frequency deviation from the synchronous frequency.

The dynamics of each generator is given by the swing equation

$$M_i \frac{d\omega_i}{dt} + D_i \omega_i = P_{mi} - P_{gi} \tag{2.1}$$

where M_i is the inertia constant, D_i the damping coefficient, Pm_i the mechanical power input and P_{gi} is the electrical power generated. To simplify calculation of P_{gi}, we make a number of assumptions:

A1. The network is assumed to be in sinusoidal steady-state with transmission lines represented by series impedances.

A2. Each generator is modelled as an internal voltage source E'_j behind a transient reactance X'_{dj}.

A3. The phase angle of the internal machine voltage E'_i coincides with the rotor angle δ_i.

A4. The powers P_{mi} are constant.

A5. The transmission lines are assumed to be lossless.

A6. The loads are modelled as real and reactive power demands which are a function of the magnitude of the bus voltage.

Assumptions A1–A5 are standard in simplified models for power system stability analysis [3]. The generator model is often further simplified to a constant voltage $|E'_i|$ behind transient reactance [3]. The present model is motivated by problems which are caused by the use of fast excitation systems. These attempt to regulate the terminal voltages to set values via fast changes in $|E'_i|$.

Assumption A4 requires that the frequency control system occupies a different (lower in practice) bandwidth than the voltage control system. This is not always the case, but seems to be the preferred situation.

It is also common to simplify the model by assuming the loads are impedances. In general the load powers are nonlinear functions of frequency and voltage of the load bus. The frequency dependence of loads is often neglected. This practice will be followed here according to Assumption A6.

From Assumption A1, there are four variables to consider at each network bus, namely, the voltage magnitude $|V_i|$, the voltage phase angle θ_i, the real power injection P_i and the reactive power injection Q_i. Let $\delta =$

$[\delta_1, \delta_2, \ldots, \delta_m]^t$, $|E'| = [|E'_1|, |E'_2|, \ldots, |E'_m|]^t$, $\theta = [\theta_1, \theta_2, \ldots, \theta_n]^t$, and $|V| = [|V_1|, |V_2|, \ldots, |V_n|]^t$. It is useful to write $|V| = [|V_g|^t |V_l|^t]$, where V_g denotes generator terminal voltages and V_l the load bus voltages.

In the formulation of models it is sometimes convenient to regard the internal generator voltages E'_j as corresponding to fictitious network buses in an augmented network [5]. These are then numbered $i = n+1, \ldots, n+m$ and $V_i = E'_{i-n}$, $i = n+1, \ldots, n+m$. We use the notation $|V_a| := (|V|, |E|)$ and $\delta_a := (\delta, \theta)$.

From Assumptions A1, A5 all transmission lines are represented as pure reactances. Thus in the overall augmented network all buses are conneted by reactances. (Assumption A2 gives that each fictitious bus is attached to a generator terminal bus through the transient reactance.) Let the bus admittance matrix for the transmission network and the augmented network by Y and Y_a, respectively. Y_a is obtained from Y in the form

$$Y_a = \begin{pmatrix} Y & 0 \\ 0 & 0 \end{pmatrix} + Y_d$$

where Y_d has every row (and column) containing the terms $\pm j(1/X'_{dj})$ in the pattern of an admittance matrix. Both Y and Y_a are purely imaginary with Y having elements $Y_{ij} = jB_{ij}$, where B_{ij} is the susceptance between buses i and j.

At each bus, real and reactive power is exchanged between some of the generators, loads and/or transmission lines. At an internal generator bus, we have real power balance given by (2.1) with

$$P_{gi}(\delta, \theta, |V|) = \frac{|E'_i|\,|V_i^0|}{X'_{di}} \sin(\delta_i - \theta_i) \tag{2.2}$$

The reactive power balance is given by

$$Q_{gi}(\delta, \theta, |V|) + Q_{ei}(|E'_i|, |V_i|) = 0 \tag{2.3}$$

where Q_{ei} is the reactive power injected at the generator internal bus and Q_{gi} is given by

$$Q_{gi}(\delta, \theta, |V|) = \frac{|E'_i|\,|V_i^0|}{X'_{di}} \cos(\delta_i - \theta_i) - \frac{|V_i^0|^2}{X'_{di}} \tag{2.4}$$

We make a further assumption:

A7. Q_{ei} is a known characteristic of the excitation system.

If we adopt the classical model where the $|E'_i|$ are constant, the internal generator buses become PV buses in the usual load flow sense. In this case, the reactive power equation (2.3) is not needed. The terminal buses are PQ buses. For fast excitation systems, it makes more sense to require the terminal voltages $|V_g|$ to be constant [6]. The terminal buses are then PV buses (with their reactive power injection to be determined).

The remaining network buses have no generation attached. The injected powers are determined by loads and control devices.

At each network bus, the injected powers are balanced by powers entering transmission lines. We use the augmented network view. Let P_{bi} and Q_{bi}

denote the total real and reactive powers leaving the i:th bus via transmission lines. Then

$$\begin{aligned} P_{bi}(\delta_a, |V_a|) &= \sum_{j=1}^{n+m} |V_i|\,|V_j|\,B_{aij}\sin(\delta_i-\delta_j) \\ Q_{bi}(\delta_a, |V_a|) &= -\sum_{j=1}^{n+m} |V_i|\,|V_j|\,B_{aij}\cos(\delta_i-\delta_j) \end{aligned} \tag{2.5}$$

For specific bus types, these compact expressions can be rewritten in terms of δ, θ and so on. For instance, at a generator terminal bus, the real power balance is given by

$$\begin{aligned} P_i(\delta,\theta,|E'|,|V|) &= P_{bi}(\delta_a,|V_a|) \\ &= \frac{|E_i'|\,|V_i^0|}{X_{di}'}\sin(\theta_i-\delta_i) + \sum_{j=1}^{n} |V_i|\,|V_j|\,B_{ij}\sin(\theta_i-\theta_j) \\ & \qquad i = 1,2,\ldots,m \end{aligned} \tag{2.6a}$$

where P_i is the nett power injected into bus i from loads and/or control devices. A similar expression for Q_i can be easily stated. At the remaining load buses, we have

$$\begin{aligned} P_i(\delta,\theta,|E'|,|V|) &= \sum_{j=1}^{n} |V_i|\,|V_j|\,B_{ij}\sin(\theta_i-\theta_j) \\ & \qquad i = m+1,\ldots,n \end{aligned} \tag{2.6b}$$

Combining (2.1), (2.3) and power balançe at terminal and load buses, it is clear that a model can be written in the form

$$\begin{aligned} P_n(\delta,\theta,|E'|,|V|) &= P_1 \\ M_g\omega_g + D_g\omega_g + P_g(\delta,\theta,|E'|,|V|) &= P_2 \end{aligned} \tag{2.7}$$

$$\begin{aligned} Q_n(\delta,\theta,|E'|,|V|) &= Q_1 \\ Q_g(\delta,\theta,|E'|,|V|) &= Q_2 \end{aligned} \tag{2.8}$$

where $M_g = \mathrm{diag}\{M_i\}$, $D_g = \mathrm{diag}\{D_i\}$, $P_1 = [P_1, P_2, \ldots, P_m]^T$, and other terms are defined in the obvious way. P_n, Q_n refer to bus powers for network buses.

Now assume an operating point $(\delta^0, \theta^0, |E^{0\prime}|, |V^0|)$ is known. Suppose small deviations occur under the influence of small disturbances. At such an operating point, we have $\omega_g^0 = 0$ and (2.7), (2.8) reduce to standard load flow equations (for the augmented network). Since these equations have translational symmetry, it is standard to refer the angles to a reference which can be taken here as $\delta_m (= \delta_{an}) = 0$.

Small disturbance stability is studied via linearization of equations (2.7), (2.8) about the operating point. In setting up the linearized equations, we make use of the Jacobian J of the load flow equations. We have

$$\begin{pmatrix} P_{n\delta} & P_{n\theta} & \vdots & P_{ne} & P_{nv} \\ P_{g\delta} & P_{g\theta} & \vdots & P_{ge} & P_{gv} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ Q_{n\delta} & Q_{n\theta} & \vdots & Q_{ne} & Q_{nv} \\ Q_{g\delta} & Q_{g\theta} & \vdots & Q_{ge} & Q_{gv} \end{pmatrix} := \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} \tag{2.9}$$

where

$$P_{n\delta} = \frac{\partial P_n}{\partial \delta}$$

and so on in the obvious way. Then $J^0, P^0_{n\delta}$ etc denote these matrices evaluated at the operating point. The components of the various submatrices in J are easily built up using (2.5) or (2.6). From (2.6), we see that elements of $P_{n\delta}$ are given by

$$J_{ij} = \begin{cases} -\frac{|E'_i|\,|V^0_i|}{X'_{di}} \cos\left(\theta^0_i - \delta^0_i\right) & ; \quad 1 \le i \le m, j = 1 \\ 0 & ; \quad \text{otherwise for } 1 \le i \le n, 1 \le j \le m \end{cases}$$

Now consider the block $P_{n\theta}$. The corresponding elements are given by (2.6) as

$$J_{ij} = \begin{cases} \frac{|E'_i|\,|V^0_i|}{X'_{di}} \cos\left(\theta^0_i - \delta^0_i\right) + \sum_{j=1}^{n} |V^0_i|\,|V^0_j|\, B_{ij} \cos\left(\theta^0_i - \delta^0_j\right) & ; \\ \qquad\qquad 1 \le i \le m, j = 1 & \\ \sum_{j=1}^{n} |V^0_i|\,|V^0_j|\, B_{ij} \cos\left(\theta^0_i - \delta^0_j\right) \quad ; \quad m+1 \le i \le n, j = 1 & \\ |V^0_i|\,|V^0_j|\, B_{ij} \cos\left(\theta^0_i - \delta^0_j\right) \quad ; \text{ otherwise for } 1 \le i \le n, 1 \le j \le n & \end{cases}$$

We should note that the nonzero terms in J are closely related to the network structure. They correspond to a physical connection between buses. The elements of J can clearly be expressed in a more compact way using (2.5). For instance, the submatrix J_{11} which relates real power to angles is given by

$$J_{ij} = \begin{cases} \sum_{j=1}^{n+m} |V^0_i|\,|V^0_j|\, B_{aij} \cos\left(\delta^0_{ai} - \delta^0_{aj}\right) & ; \quad i = j \\ -|V^0_i|\,|V^0_j|\, B_{aij} \cos\left(\delta^0_{ai} - \delta^0_{aj}\right) & ; \quad i \ne j \end{cases}$$

The linearized version of (2.7), (2.8) can now be written as

$$P_{n\delta}\delta + P_{n\theta}\theta + P_{ne}|E'| + P_{nv}|V| = P_1 \qquad (2.10a)$$

$$M_g\dot{\omega}_g + D_g\omega_g + P_{n\delta}\delta + P_{n\theta}\theta + P_{ge}|E'| + P_{gv}|V| = P_2 \qquad (2.10b)$$

$$Q_{n\delta}\delta + Q_{n\theta}\theta + Q_{ne}|E'| + Q_{nv}|V| = Q_1 \qquad (2.11a)$$

$$Q_{g\delta}\delta + Q_{g\theta}\theta + Q_{ge}|E'| + Q_{gv}|V| = Q_2 \qquad (2.11b)$$

where $\delta, \theta, |E'|, |V|, \omega_g$ now refer to perturbations from the operating values. (For convenience of notation, we have not introduced new symbols for the perturbation variables.)

We now include the effect of nonlinear loads at the network buses. From Assumption A6, we can write

$$\begin{aligned} P_1 &= -N_1|V| + \tilde{P}_1 \\ Q_1 &= -N_2|V| \end{aligned} \qquad (2.12)$$

where $\tilde{P}_1$ refers to other real power sources at the buses. For example, suppose the load demand characteristics have the exponential form

$$P_{Li} = a_i|V_i|^{p_i}$$
$$Q_{Li} = b_i|V_i|^{q_i}$$

Then $N_1 = \text{diag}\left\{a_i p_i |V_i|^{p_i-1}\right\}$ and $N_2 = \text{diag}\left\{b_i q_i |V_i|^{q_i-1}\right\}$.

Substituting (2.12) into (2.10a), (2.11a) gives

$$\begin{pmatrix} P_{n\theta} & P_{nv}+N_1 \\ Q_{n\theta} & Q_{nv}+N_2 \end{pmatrix} \begin{pmatrix} \theta \\ |V| \end{pmatrix} = -\begin{pmatrix} P_{n\delta} & P_{ne} \\ Q_{n\delta} & Q_{ne} \end{pmatrix} \begin{pmatrix} \delta \\ |E'| \end{pmatrix} + \begin{pmatrix} \tilde{P}_1 \\ 0 \end{pmatrix} \tag{2.13}$$

We now consider the condition:

C1. The matrix $\begin{pmatrix} P_{n\theta} & P_{nv}+N_1 \\ Q_{n\theta} & Q_{nv}+N_2 \end{pmatrix}$ is nonsingular.

Under C1, (2.13) can be solved to yield $\theta, |V|$. On substitution into (2.10b), it is straightforward to check that this yields the form

$$M_g\dot{\omega}_g + D_g\omega_g + K_1\delta + K_2|E'| = P_2 + L\tilde{P}_1 := P_2^* \tag{2.14}$$

Note that the matrices K_1, K_2, and L can be readily obtained in practice from a linear load flow solution of the network.

In (2.14), we have maintained $|E'|$ as an independent input. Thus, it has the general character of a PV bus and equation (2.11b) is not used. Sometimes, we also need $|V|$ as a controlled variable. Then equation (2.11a) is also redundant. The condition of interest is then:

C2. The matrix $P_{n\theta}$ is nonsingular.

The elements of $P_{n\theta}$ were given above. From equations (2.10), we obtain

$$M_g\dot{\omega}_g + D_g\omega_g + P_{g\delta}^*\delta + P_{ge}^*|E'| + P_{gv}^*|V| = P_2^* \tag{2.15}$$

where

$$\begin{aligned} P_{g\delta}^* &= P_{g\delta} - P_{g\theta}P_{n\theta}^{-1}P_{n\delta} \\ P_{ge}^* &= P_{ge} - P_{g\theta}P_{n\theta}^{-1}P_{ne} \\ P_{gv}^* &= P_{gv} - P_{g\theta}P_{n\theta}^{-1}P_{nv} \\ P_2^* &= P_2 - P_{g\theta}P_{n\theta}^{-1}P_1 \end{aligned} \tag{2.16}$$

The use of the notation K_1, K_2 in (2.14) is consistent with earlier discussion based on reduced network models [2].

There are special cases of interest which simplify the models (2.14) and (2.15). One which we use later is suggested by [7]. Suppose system reduction has been used and each M_i represents several generators in parallel. Then a reasonable approximation is to ignore the X'_{di} relative to other reactances. Further, each bus in the reduced system may have both generation and load attached. Then we have $\delta = \theta$ and $|E'| = |V|$. Model (2.15) can be simplified with

$$P_{g\delta}^* = P_{g\delta} \qquad P_{ge}^* = P_{ge}$$
$$P_{gv}^* = 0 \qquad P_2^* = P_2$$

We henceforth refer to this case as the aggregation system model.

Of course, it may happen that some $|E_i'|$ and some $|V_i|$ are controlled by stabilisers. Then clearly parts of (2.11) and (2.10a) are to be solved. For simplicity, we will only give details here for the cases given above.

A further case of interest is where we allow for load frequency dependence. This has influence on the overall damping matrix. Then (2.12) are written

$$\begin{aligned} P_1 &= -N_1|V| - D_{lp}\omega_l + \tilde{P}_1 \\ Q_1 &= -N_2|V| - D_{lq}\omega_l \end{aligned} \tag{2.17}$$

where D_{lp}, D_{lq} are diagonal matrices corresponding to frequency dependence in real, reactive loads respectively. Each element of these matrices is taken to be nonzero. We suppose that load buses are not voltage controlled for simplicity. Substituting (2.17) into (2.10a), (2.11a) gives

$$\begin{pmatrix} D_{lp} & P_{nv} + N_1 \\ D_{lp} & Q_{nv} + N_2 \end{pmatrix} \begin{pmatrix} \omega_1 \\ |V| \end{pmatrix} = - \begin{pmatrix} P_{n\delta} & P_{n\theta} & P_{ne} \\ Q_{n\delta} & Q_{n\theta} & Q_{ne} \end{pmatrix} \begin{pmatrix} \delta \\ |\theta| \\ |E'| \end{pmatrix} + \begin{pmatrix} \tilde{P}_1 \\ 0 \end{pmatrix} \tag{2.18}$$

The relevant solvability condition is:

C3. The matrix $\begin{pmatrix} D_{lp} & P_{nv} + N_1 \\ D_{lq} & Q_{nv} + N_2 \end{pmatrix}$ is nonsingular.

Under C3, (2.18) can be solved to give $\omega_l, |V|$. We then have from (2.10)

$$\begin{aligned} D_{lp}\dot{\theta} + P_{n\delta}^*\delta + P_{n\theta}^*\theta + P_{ne}^*|E'| &= L_1\tilde{P}_1 := P_1^* \\ M_g\dot{\omega}_g + D_g\omega_g + P_{g\delta}^*\delta + P_{g\theta}^*\theta + P_{ge}^*|E'| &= P_2 + L_2\tilde{P}_1 := P_2^* \end{aligned} \tag{2.19}$$

where $P_{n\delta}^*$, other starred matrices, L_1 and L_2 are all derived from solving (2.18) and substitution. Clearly, θ is now a part of the system state.

If we need $|V|$ as independent variables, then set $N_1 = 0$, $N_2 = 0$ in (2.17); only the frequency dependent component of the loads is relevant. Then we have in place of (2.19)

$$\begin{aligned} D_{lp}\dot{\theta} + P_{n\delta}\delta + P_{n\theta}\theta + P_{ne}|E'| + P_{nv}|V| &= \tilde{P}_1 \\ M_g\dot{\omega}_g + D_g\omega_g + P_{g\delta}\delta + P_{g\theta}\theta + P_{ge}|E'| + P_{g\theta}|V| &= \tilde{P}_2 \end{aligned} \tag{2.20}$$

Typically, these equations would accommodate some network buses as controlled and others uncontrolled (regular load buses).

3. Simple Stabiliser Models

In this section, we look at the inclusion of contributions of damping provided by stabilisers into the models of Section 2.

3.1 AVR Power System Stabiliser

Suppose that a power system stabiliser (PSS) is fitted to each generator AVR. These come in a variety of forms which use different input signals. The main ones are rotor speed, terminal frequency and electrical power. Irrespective of what signal is used, the following assumption is reasonable in simplified studies [2]:

A8. With proper compensation of the excitation/voltage regulation loop, stabiliser action can be approximated by gains

$$g_i = \frac{|E_i'|}{\omega_i} \tag{3.1}$$

The matrix version of (3.1) is

$$|E'| = G\omega_g \tag{3.2}$$

where $G = \text{diag}\{g_i\}$.

Substituting (3.2) into model (2.14) gives

$$M_g\ddot{\delta} + D\ddot{\delta} + K_1\delta = P_2^* \tag{3.3}$$

where

$$\begin{aligned} D &= D_g + D_s \\ D_s &= K_2 G \end{aligned} \tag{3.4}$$

The structure of D_s for the single stabiliser on k_1:th generator is

$$\begin{bmatrix} & D_{1k_1} & \\ & D_{2k_1} & \\ 0 & & 0 \\ & \vdots & \\ & D_{mk_1} & \end{bmatrix}$$

i.e. each stabiliser provides a nonzero column in D.

For stabilisers based on electrical power, a more complete version of (3.2) has been proposed [7] as

$$|E'| = G_1\omega_g + G_2\dot{\omega}_g \tag{3.5}$$

Clearly, this effectively modifies both the damping matrix as in (3.4) and the inertia matrix as

$$\begin{aligned} M &= M_g + M_s \\ M_s &= K_2 G_2 \end{aligned} \tag{3.6}$$

3.2 HVDC Links

Reference [4] presents a simple model for a HVDC link in stability analysis. The effect of this is to provide a controllable influence at $\tilde{P}_1$. Suppose each of the n_c links is given a reference direction for power flow.

We write

$$\tilde{P}_1 = T_1 P_{lc} \tag{3.7}$$

where P_{lc} is an n_c-vector of dc link controller outputs and T_1 is an $n \times n_c$ connectivity matrix whose elements are given by

$$t_{ij} = \begin{cases} -1, & \text{link } j \text{ leaves bus } i \\ 0, & \text{link } j \text{ not incident on bus } i \\ +1, & \text{link } j \text{ enters bus } i \end{cases}$$

There are ± 1 elements in each column; only one nonzero element exists per row in the normal situation where links have no common buses.

For simplified analysis, we make the following assumption about the links.

A9. The link stabiliser action can be approximated as a gain

$$P_{lcj} = g_{ij}(\dot{\theta}_i - \dot{\theta}_j) \tag{3.8}$$

Thus if $\dot{\theta}_i > \dot{\theta}_j$, the link stabiliser transports a power increment towards the lower frequency part of the system. More elaborate models are considered in [8]. From (3.8), it follows that

$$P_{lc} = -G_3 T_1^t \omega_1 \tag{3.9}$$

where G_3 is a diagonal $n_c \times n_c$ matrix of the gains. Combining (3.7) and (3.9) gives

$$\tilde{P}_1 = -T_1 G_3 T_1^t \dot{\theta} \tag{3.10}$$

Assuming load damping satisfying condition C3, we substitute (3.10) into model (2.19). This gives

$$\begin{aligned} \left(D_{lp} + L_1 T_1 G_3 T_1^t\right) \dot{\theta} + P_{n\delta}^* \delta + P_{n\theta}^* + P_{ne}^* |E'| &= 0 \\ L_2 T_1 G_3 T_1^t \dot{\theta} + M_g \dot{\omega}_g + D_g \omega_g + P_{g\delta}^* \delta + P_{g\theta}^* \theta + P_{ge}^* |E'| &= P_2 \end{aligned} \tag{3.11}$$

Again we have a generalized linear system model [9]. If it is nonsingular, it can be transformed to normal form.

The models where θ is not part of the state do not readily accommodate the HVDC model. One exception is the aggregation system model [7] where $\dot{\theta} = \omega_g$. Then from (2.15) and (3.10) it follows that

$$M_g \dot{\omega}_g + \left(D_g + T_1 G_3 T_1^t\right) \omega_g + P_{g\delta} \delta + \left(P_{ge} + N_1\right) |E'| = 0 \tag{3.12}$$

The structure of the contribution to the damping matrix from one stabiliser connecting buses i and j is

$$\begin{array}{cc} & \begin{array}{cc} i & \quad j \end{array} \\ \begin{array}{c} i \\ j \end{array} & \begin{pmatrix} g_{ij} & -g_{ij} \\ -g_{ij} & g_{ij} \end{pmatrix} \end{array}$$

Other elements are zero.

3.3 Static VAR Compensators (SVC)

We consider the SVC in damping mode and make a simplifying assumption. The stabiliser is typically a similar device to that used for a generator PSS. For instance, it may sense the power flow in a line incident on the bus which it supports.

A11. The SVC stabiliser action on bus i can be approximated as a gain

$$|V_i| = g_{ij}(\dot{\theta}_i - \dot{\theta}_j) \tag{3.13}$$

where j denotes some other bus.

Following similar steps to the HVDC damping description, we get

$$|V| = G_4 T_2^t \dot{\theta} \tag{3.14}$$

where G_4 is a diagonal matrix of gains and T_2 a connectivity matrix. We can clearly substitute (3.14) into model (2.20) to obtain the total model for damping studies.

3.4 Tie-line Stabilisers

The conventional generator stabilisers as described in Section 3.1 rely on local frequency information only. In [4], it is suggested that a more effective damping contribution for low frequency system modes would be obtained if frequency difference information was the input signal. This could be derived from measurement of a tie-line power and clearly gives some similarity to SVC stabilisers. Equation (3.14) is replaced by

$$|E'| = G_5 T_3^t \dot{\theta} \tag{3.15}$$

This method of stabilisation needs development at the detailed level, but it would be interesting to explore its potential at the simplified level.

The common facility in the HVDC, SVC and tie-line stabilisers is the ability to sense relative frequency between two separate areas of the system. If these areas are swinging against each other in a slow and system wide mode, we can expect a substantial contribution to damping in a well-tuned stabilizer.

3.5 The Total Damping Matrix

Using similar steps to those in Sections 3.1–3.4, the separate damping source models can be combined with the basic system (2.10), (2.11) to produce a relatively simple model for damping studies. The details are given in Appendix A. A very concise model arises in the special case of the aggregation model.

Using superposition (2.14) gives

$$M\dot{\omega}_g + D\omega_g + K_1\delta = 0 \tag{3.16}$$

where

$$M = M_g + K_2 G_2 \tag{3.17a}$$
$$D = D_g + K_2\left(G_1 + G_4 T_2^t + G_5 T_3^t\right) + T_1 G_3 T_1^t \tag{3.17b}$$

The overall damping matrix consists of three terms: inherent generator damping, voltage control stabiliser damping (PSS, SVC, tie-line) and HVDC stabiliser damping. This model clearly highlights the role of system network structure on mode damping.

One interesting question concerns the degrees of freedom in (3.17b) to introduce damping on all system modes.

4. Stability Analysis

The models derived above are simple enough to explore stability analysis beyond conventional eigenvalue calculations. We aim to state general algebraic conditions which reveal the qualitative role of the various aspects of the system in preserving stability. For simple structure preserving models, such results have been explored in [6,10]. In this section, we merely review the possibilities of applying this analysis to our models.

4.1 Normal Form Models

From Appendix A, we see that the models naturally come out in the form of a generalised linear system

$$E\dot{x} = Ax + Bu \tag{4.1}$$

with $x = (\theta, \delta, \omega_g)$. However, in all but unusual cases, E is nonsingular and the model can be reduced to the usual normal form, i.e. where $E = I$. To illustrate, (3.16) can be written as

$$\dot{x} = Ax \tag{4.2a}$$

where $x = (\delta, \omega_g)$ and

$$A = \begin{pmatrix} 0 & I \\ -M^{-1}K_1 & -M^{-1}D \end{pmatrix} \tag{4.2b}$$

4.2 Connection to Classical Mechanics

Model (3.16) is the subject of much discussion in the mechanics literature [11]. There M is accepted as symmetric while various cases, where D, K_1 may or may not be symmetric, are studied. Here, of course, M is asymmetric if $G_2 \neq 0$.

Consider the symmetric and skew symmetric parts of D, K_1. We note that K_1 is symmetric if the real load powers are assumed voltage independent [6].

In terms of mechanics terminology, we have:

1. voltage dependence in loads contributes circulatory forces;
2. HVDC links contribute only symmetric damping and so pure dissipative forces;
3. all stabilisers on generators and SVCs also contribute anti-symmetric damping and so gyroscopic forces.

4.3 Stability Results

Again we restrict attention to the simple model of the form (3.16). Stability results for asymmetric D, K_1 cases make use of the following concept.

DEFINITION 4.1
A real matrix A is ***symmetrisable*** if it is made symmetric upon multiplication by a real, symmetric, positive definite matrix S. □

Symmetrisable matrices are characterised by real eigenvalues and a full set of eigenvectors. Symmetric matrices are trivially symmetrisable with $S = I$.

We note that translational symmetery in angle variables for the load flow equations makes any equilibrium point of the dynamic models a point in a one-dimensional manifold of equilibria. It only makes sense to study stability of the entire manifold [12] unless a reduced set of state variables is chosen [13].

There are many results available which require the system to be closely related to a symmetric system. For instance, if $M^{-1}D$ and $M^{-1}K_1$ have a common symmetrising matrix, simple results can be stated [10]. Such conditions are unlikely to be satisfied for our more general models.

One result is available which does not require similarity to a symmetric system. Model (3.16) is equivalent to

$$\ddot{\delta} + M^{-1}D\dot{\delta} + M^{-1}K_1\delta = 0 \tag{4.3}$$

Using the symmetrising matrix S for $M^{-1}K_1$ gives

$$S\ddot{\delta} + (C + G)\dot{\delta} + SM^{-1}K_1\delta = 0 \tag{4.4}$$

where C and G are the symmetric and skew-symmetric parts of $SM^{-1}D$ (corresponding to the dissipative and gyroscopic forces of system (4.4) respectively).

Assume that K_1 has positive real eigenvalues except for precisely one zero eigenvalue corresponding to the translational symmetry. Then the following result can be adapted from [10].

THEOREM 4.1
If $C \geq 0$ and the $m^2 \times m$ matrix

$$\begin{pmatrix} C \\ C\left(SM^{-1}K_1\right) \\ \vdots \\ C\left(SM^{-1}K_1\right)^{m-1} \end{pmatrix}$$

has rank m, then the equilibrium manifold is asymptotically stable. □

This condition ensures that the damping is distributed appropriately on all modes.

4.4 Sensitivity Studies

As stated earlier the primary purpose of the simplified models is as a basis for preliminary investigation of stabiliser placement to improve damping of low frequency oscillatory modes. The basic ideas are well established [2,14]: study eigenvalues and eigenvectors for the simplified models as stabiliser gains are changed. A sophisticated way to achieve this is via perturbation formulae for eigenvalues of matrices [15].

Suppose A is a matrix dependent on parameters $\mu \in \mathbb{R}^k$. Then from [16], we have

$$\frac{\partial \lambda_i}{\partial \mu_j} = \frac{v_i^H \frac{\partial A}{\partial \mu_j} u_i}{v_i^H u_i} \tag{4.5}$$

where

- λ_i is the i:th eigenvalue of A
- u_i is the eigenvector of A associated with λ_i
- v_i is the eigenvector of A^t associated with λ_i^*

Superscripts $*$ and H denote conjugate and Hermitian transpose operators respectively.

Once the model is in normal form with stabiliser gains shown explicitly as parameters, (4.5) can be used to calculate sensitivities. We look for stabilisers which increase damping on troublesome modes.

For example, consider model (3.16) expressed as (4.2). When $G_2 = 0$, it is easy to see that A is linear in all gain parameters. For PSS, we can write

$$A = \begin{pmatrix} 0 & I \\ -M_g^{-1}K_1 & -M_g^{-1}D_g \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & -M_g^{-1}K_2 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & G_1 \end{pmatrix}$$

It follows that

$$\frac{\partial A}{\partial g_{ij}} = \begin{pmatrix} 0 & 0 \\ 0 & -M_g^{-1}K_2^j \end{pmatrix} \tag{4.6}$$

where K_2^j is the matrix obtained by nulling all of K_2 except the j:th column. Similarly, we can derive formulae for other stabiliser types. Note that in writing down (4.5) only the frequency half of the eigenvectors are needed.

The formula (4.6) for gains G_2 is a little more complicated, since they enter A through M^{-1}. The details are given in Appendix B.

Note that the sensitivity matrix defined by (4.5) is defined at an operating point. Thus it is only valid for small gain changes. For large changes K_1 and K_2 need updating in principle as the operating point changes. It is observed in practice that one base case is often adequate for gain changes over the range of interest [2].

5. Conclusions

This report has given simplified power system models for study of the effect of all stabiliser types on damping. The use of the models will involve some established ideas in eigenvalue sensitivity analysis. A major study on the NORDEL system will be reported in a subsequent report.

The models presented here probably represent the crudest level of generality which would be used in practice. Without going to complete linear models typically used in commercial software, we may wish to use slightly more sophisticated models. For instance, inclusion of the negative damping contributions from AVRs requires a multimachine Heffron-Phillips model [17,18]. Inclusion of time-constants in stabiliser models is very easy.

6. References

[1] E. V. Larsen and D. A. Swann (1981): "Applying power system stabilisers, Part I: General concepts," *IEEE Trans. Power Apparatus and Systems*, **PAS-100**, No 6, 3017–3024.

[2] F. P. deMello, P. J. Nolan, T. F. Laskowski and J. M. Undrill (1980): "Coordinated application of stabilisers in multimachine power systems," *IEEE Trans. Power Apparatus and Systems*, **PAS-99**, No 3, 892–901.

[3] P. M. Anderson and A. A. Fouad (1977): *Power System Control and Stability*, The Iowa State University Press, Ames, Iowa.

[4] D. J. Hill (1986): "Status report on project Power System Stabilisation TFRT-3183, Department of Automatic Control, Lund Institute of Technology, Lund, Sweden,".

[5] A. R. Bergen and D. J. Hill (1981): "A structure preserving model for power system stability analysis," *IEEE Trans. Power Apparatus and Systems*, **PAS-100**, No 1, 25–35.

[6] F. F. Wu and C-C. Liu (1986): "Characterization of power system small disturbance stability with models incorporating voltage variation," *IEEE Trans. Circuits and Systems*, **CAS-33**, No 4, 406–417.

[7] T. Lysfjord et al. (1982): "Förbättrad dämpning av effektpendlingar i nordelnätet genom optimering av inställbara reglerparametrar för dämptillsatser," Technical report.

[8] K. R. Padiyar, M. A. Pai and C. Radhakrishna (1981): "A versatile system model for the dynamic stability analysis of power systems including HVDC links," *IEEE Trans. Power Apparatus and Systems*, **PAS-100**, No 4, 1871–1879.

[9] G. C. Verghese, B. C. Lévy and T. Kailath (1981): "A generalised state-space for singular systems," *IEEE Trans. Automatic Control*, **AC-26**, No 4, 811–831.

[10] H. G. Kwatny, L. Y. Bahar and A. K. Pasrija (1985): "Energy-like Liapunov functions for power system stability analysis," *IEEE Trans. Circuits and Systems*, **CAS-32**, 1140–1149.

[11] K. Hüseyin (1978): *Vibrations and Stability of Multiple Parameter Systems*, Sijthoff and Noordhoff.

[12] J. P. LaSalle (1976): *The Stability of Dynamical Systems*, Siam.

[13] D. J. Hill, "On the equilibria of power systems with nonlinear loads," to appear, *IEEE Trans. Circuits and Systems*.

[14] H. Rudnick, F. M. Hughes and A. Brameller (1983): "Steady state instability: Simplified studies in multimachine power systems," *IEEE Trans. Power Apparatus and Systems*, **PAS-102**, No 12, 3859–3867.

[15] P. J. Nolan, N. K. Sinha and R. T. H. Alden (1976): "Eigenvalue sensitivities of power systems including network and shaft dynamics," *IEEE Trans. Power Apparatus and Systems*, **PAS-95**, No 4, 1318–1324.

[16] D. K. Fadeev and V. N. Fadeeva (1963): *Computational Methods of Linear Algebra*, W. H. Freeman and Co.

[17] C. D. Vournas and R. J. Fleming (1978): "Generalisation of the Heffron-Phillips Model of a synchronous generator," *IEEE PES Summer Meeting*, Los Angeles.

[18] T. S. Bhatti and D. J. Hill (1987): "A multimachine Heffron-Phillips model for power systems with frequency and voltage dependent loads, Part I: General case," Technical Report EE8739, University of Newcastle, Australia.

Appendix A

For a general model incorporating all damping soruces we start with (2.20)

$$\begin{aligned} D_{lp}\dot{\theta} + P_{n\delta}\delta + P_{n\theta}\theta + P_{ne}|E'| + P_{nv}|V| &= \tilde{P}_1 \\ M_g\dot{\omega}_g + D_g\omega_g + P_{g\delta}\delta + P_{g\theta}\theta + P_{ge}|E'| + P_{gv}|V| &= P_2 \end{aligned} \tag{A.1}$$

where $|V|$ is the vector of controlled bus voltages (* superscripts have been dropped).

We now superimpose the effect of the damping sources. From (3.5) and (3.15) the generator stabilisers give

$$|E'| = G_1\omega_g + G_2\dot{\omega}_g + G_5T_3^t\dot{\theta} \tag{A.2}$$

From (3.10), the HVDC links give

$$\tilde{P}_1 = -T_1G_3T_1^t\dot{\theta} \tag{A.3}$$

From (3.14), the SVCs give

$$|V| = G_4T_2^t\dot{\theta} \tag{A.4}$$

Substituting (A.2) to (A.4) into (A.1) gives

$$\begin{aligned} D_1\dot{\theta} + P_{n\delta}\delta + P_{n\theta}\theta &= 0 \\ M\dot{\omega}_g + D_2\omega_g + D_3\dot{\theta} + P_{g\delta}\delta + P_{g\theta}\theta &= P_2 \end{aligned} \tag{A.5}$$

where

$$\begin{aligned} D_1 &:= D_{lp} + T_1G_3T_1^t + P_{ne}G_5T_3^t + P_{nv}G_4T_2^t \\ M &:= M_g + P_{ge}G_2 \\ D_2 &:= D_g + P_{ge}G_1 \\ D_3 &:= P_{gv}G_4T_2^t \end{aligned} \tag{A.6}$$

With $x = (\theta, \delta, \omega_g)$ and $u = P_2$, (A.5) is of the form

$$E\dot{x} = Ax + Bu \tag{A.7}$$

where

$$E = \begin{pmatrix} D_1 & 0 & 0 \\ D_3 & 0 & M \\ 0 & I & 0 \end{pmatrix} \qquad A = \begin{pmatrix} -P_{n\theta} & -P_{n\delta} & 0 \\ -P_{g\theta} & -P_{g\delta} & D_2 \\ 0 & 0 & I \end{pmatrix}$$

$$B = \begin{pmatrix} 0 \\ I \\ 0 \end{pmatrix}$$

Appendix B

Suppose that matrices X, Y depend on parameter $\alpha \epsilon \mathbb{R}$. Then

$$\frac{\partial}{\partial \alpha}(XY) = X\frac{\partial Y}{\partial \alpha} + \frac{\partial X}{\partial \alpha}Y \tag{B.1}$$

We can write

$$A = \begin{pmatrix} I & 0 \\ 0 & -M^{-1} \end{pmatrix} \begin{pmatrix} 0 & I \\ K_1 & D \end{pmatrix} \tag{B.2}$$

where M, D are given by (3.17). Consider gain parameter g_{2j}. From (B.2)

$$\frac{\partial A}{\partial g_{2j}} = \begin{pmatrix} 0 & 0 \\ 0 & -\frac{\partial M^{-1}}{\partial g_{2j}} \end{pmatrix} \begin{pmatrix} 0 & I \\ K_1 & D \end{pmatrix} \tag{B.3}$$

Now apply (B.1) again to $MM^{-1} = I$ to get

$$\frac{\partial M^{-1}}{\partial g_{2j}} = -M^{-1}\frac{\partial M}{\partial g_{2j}}M^{-1}$$

As in the derivation of (4.6)

$$\frac{\partial M}{\partial g_{2j}} = K_2^j$$

So

$$-\frac{\partial M^{-1}}{\partial g_{2j}} = M^{-1}K_2^jM^{-1}$$

Substituting into (B.3) gives

$$\frac{\partial A}{\partial g_{2j}} = \begin{pmatrix} 0 & 0 \\ M^{-1}K_2^jM^{-1}K_1 & M^{-1}K_2^jM^{-1}D \end{pmatrix}$$