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THE SEA STRATUS EXPERIMENTS, APRIL 1976  
PART I

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THE SEA STRATUS EXPERIMENTS, APRIL 1976

PART I

CLAES KÄLLSTRÖM



## ABSTRACT

Experiments of steady state course keeping and turning on a tanker in ballast condition are presented. An adaptive autopilot, consisting of a Kalman filter, a self-tuning regulator for straight course keeping and a turning regulator, was tested in different speed and weather conditions. Comparative experiments with a conventional autopilot based on a PID-regulator were also performed.

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## 1. INTRODUCTION

The purpose of the experiments was to test an adaptive autopilot, consisting of a Kalman filter, a self-tuning regulator and a yaw regulator, in different weather conditions and at different speeds. Comparative experiments performed with a PID-regulator were also recorded.

The ship, T/T Sea Stratus, is a 355 000 tdw oil tanker built for the Salén Shipping Co in Stockholm by Kockums in Malmö. The length between perpendiculars is 350 m and the beam is 60 m. The draught of the ship is 22.3 m in full load condition. The main engine power is 29.4 MW (40 000 shp), which implies that the speed obtained is 15.5 knots when the tanker is fully loaded. The adaptive autopilot was implemented by use of the programming language FORTRAN as a task of the Integrated Navigation System, Kockums Bridge System type 546, which is a standard software package designed for the process computer Kongsberg SM 406 S. A special paper tape punch was installed to record the experiments.

Extensive computer simulations had preceded the experiments. A mathematical model of a 255 000 tdw tanker was used in Aspernäs and Foisack (1975), Aspernäs and Källström (1975), and Källström (1976a). A model, which describes the steering dynamics of the actual tanker T/T Sea Stratus, was simulated in Källström (1976b) and (1976c). The latter report shows the performance of the same adaptive autopilot, which is used in the experiments described here, for different load and speed conditions.

Preliminary full scale experiments on 255 000 tdw tankers, T/T Sea Scout and T/T Sea Swift, are described in Källström (1974) and (1975), resp. Most of the Sea Swift experiments were performed in full load condition, while the Sea Scout was ballasted during the experiments.

The experiments on the Sea Stratus were performed during a voyage from Lisbon, Portugal, to Cape Town, South Africa, between 1976-04-18 and 1976-05-03. At the departure from Lisbon the ship had a ballast of 134 000 tdw, which implies a mean draught of 10.5 m. The ballast was changed to 160 000 tdw on 1976-04-26 and then the mean draught obtained was 11.9 m. The average speed of the ship varied between 12.1 knots and 15.9 knots. The largest wind velocity measured was 15 m/s, which means moderate gale, but complete calm was also prevailing during some of the experiments.

Thirty-six experiments, A1-A36, of straight course keeping were recorded. The PID-regulator was used during some of the experiments, but most of them were performed using the adaptive autopilot. Three experiments, B1-B3, were specially carried out to record the performance of the Kalman filter by punching data at a high sampling rate. Six turning experiments, C1-C6, and three normal operating experiments, D1-D3, were also recorded as well as four experiments, E1-E4, for off-line identification of the ship steering dynamics. Finally four straight course keeping experiments, F1-F4, were performed to compare the steering quality of the adaptive autopilot to that of the PID-regulator. Notice that no data were punched on paper tape during these experiments. The Kalman filter was running during all the experiments, but the estimates were not always used by the regulators. All the fifty-six experiments are described in Appendix D. The total experimental time was about 33 h, but the testing was going on during almost all the days and nights of the voyage, i.e. during about 2 weeks.

The notations used in this report are explained in Appendix A and the standard parameter values of the autopilot are summarized in Appendix B. The listings of the FORTRAN sub-routines of the autopilot are given in Appendix C.

## 2. MEASUREMENT EQUIPMENT

Several measurement signals are usually available in the computer, and no special equipment, besides the paper tape punch, had to be installed to carry through the experiments. Following measurement signals were recorded:

- o Rudder servo position  $\delta_s$  [deg], scan cycle 1 s.
- o Rudder angle  $\delta$  [deg], scan cycle 1 s, accuracy about 0.2 deg.
- o Propeller effect  $P_s$  [MW] (mean value during 5 min), scan cycle 10 s.
- o Number of propeller revolutions  $n$  [rpm], scan cycle 2 s.
- o Forward velocity  $u$  [knots], scan cycle 1 s, measured by a doppler log, type Atlas, with an accuracy of 0.02 knots.
- o Transversal velocity of bow  $v_1$  [knots], scan cycle 1 s, measured by the same device and with the same accuracy as the forward velocity.
- o Yaw angular velocity or yaw rate  $r$  [deg/s], scan cycle 1 s, measured with an accuracy of about 0.005 deg/s by a rate gyro manufactured by ATEW AB.
- o Heading angle  $\psi$  [deg], scan cycle 0.25 s, measured by a Sperry gyro compass, and transformed by a synchro-digital converter with an accuracy of about 0.02 deg.

During two experiments, A1 and D1, the forward velocity  $u$  was measured by Sperrys log. Sometimes that signal had an incorrect value in the computer. No measurements of the transversal velocity of bow  $v_1$  were recorded during these two experiments. The sign of  $v_1$  was incorrect during experiment C1 and a defect rudder servo was used during experiment A1.

The relative wind direction and the relative wind velocity were manually recorded once at the beginning of each experiment. However, absolute values have been calculated and these are used in this report.

The distance from the origin, which is placed half-way between the perpendiculars, to the forward doppler log ( $l_1$ ) is 164.35 m. The centre of mass is situated 9.3 m in front of the origin in full load condition. The distance is 6.9 m when the ship is ballasted.

In this report the convention of the sign of the rudder angle is chosen in such a way that a positive rudder angle (starboard rudder) gives a positive yaw rate (starboard yaw).



### 3. AUTOPILOT

The structure of the autopilot is shown in Fig. 3.1. To obtain a good performance of all speeds, the autopilot performs a speed scaling using the speed  $V_s$  [m/s].  $V_s$  is computed with sampling interval  $T_k = 1$  s according to

$$V_s = u \cdot \text{CKM} \quad \text{if} \quad \text{IVVC} = 1 \quad (3.1)$$

or

$$V_s = V_c \quad \text{if} \quad \text{IVVC} = 3 \quad (3.2)$$

where  $\text{CKM} = 0.514444$  denotes the conversion factor from knots to m/s and  $V_c$  [m/s] is a constant user-defined speed. The standard value of IVVC is equal to 1.

The Kalman filtering as well as the computation of the transversal velocity of the origin  $v$  [knots] according to

$$v = v_1 - \ell_1 \cdot r \cdot \text{CGR}/\text{CKM} \quad (3.3)$$

is performed with sampling interval  $T_k$ . The lever arm  $\ell_1$  is equal to 164.35 m and the conversion factor from degrees to radians CGR is equal to 0.0174533. Either the estimates from the Kalman filter  $\hat{v}$  [knots],  $\hat{r}$  [deg/s], and  $\hat{\psi}$  [deg] or the measurements  $v$ ,  $r$ , and  $\psi$  are used by the self-tuning regulator, the PID-regulator, and the yaw regulator. The standard configuration is the one when Kalman filter estimates are fed into the different regulators.

The rudder command  $\delta_c$  [deg] is computed with sampling interval  $T_s$ , where  $T_s$  mostly is equal to 10 s. The PID-regulator for straight course keeping is only used for comparison. The reference course  $\psi_{\text{ref}}$  [deg] and the reference yaw rate  $r_{\text{ref}}$  [deg/s] for turning as well as the rudder limit  $\delta_\ell$  [deg] are also used by the autopilot. The complete autopilot is implemented by the FORTRAN subroutines AUTP3 and STUR listed in Appendix C.

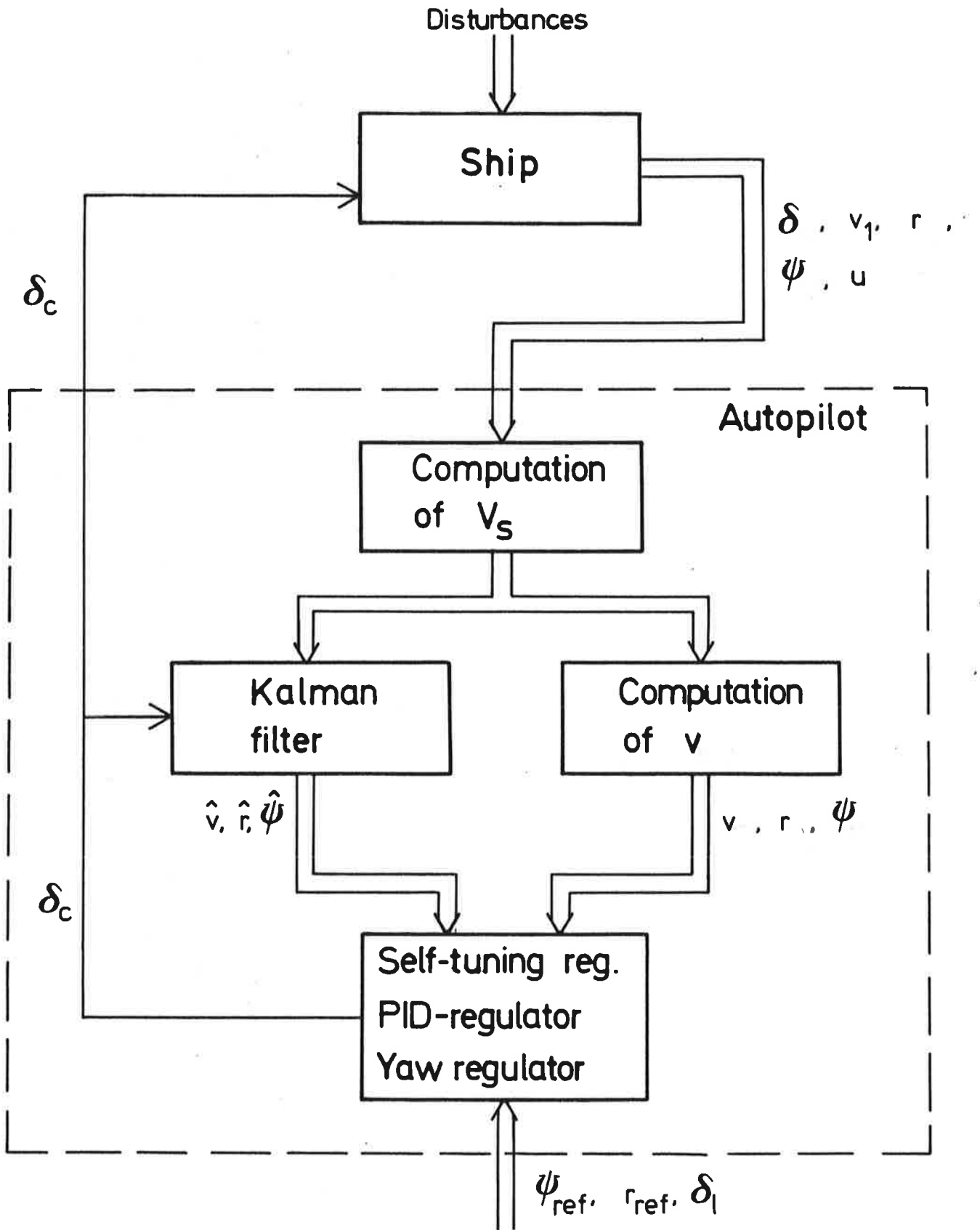


Fig. 3.1 - Structure of the autopilot.

### 3.1 Kalman Filter

The design of the Kalman filter follows in detail the description given in Källström (1976c). A model of a fully loaded tanker is used. Notice, however, that the covariance matrix  $R_1$  for the process noise is changed somewhat:

$$\begin{aligned} \text{diag}(R_1) &= & (3.4) \\ &= \left( 2.5 \cdot 10^{-5} \quad 5 \cdot 10^{-9} \quad 8 \cdot 10^{-8} \quad 8 \cdot 10^{-8} \quad 1.2 \cdot 10^{-8} \quad 4 \cdot 10^{-7} \quad 3 \cdot 10^{-12} \quad 8 \cdot 10^{-11} \right) \end{aligned}$$

The following Kalman filter equations are obtained:

$$\begin{cases} \hat{x}'(t|t-1) &= A \hat{x}'(t-1|t-1) + B u'(t-1) \\ \hat{x}'(t|t) &= \hat{x}'(t|t-1) + K \varepsilon'(t) \\ y'(t) &= C \hat{x}'(t|t-1) + \varepsilon'(t) \end{cases} \quad (3.5)$$

where all variables are normalized using the length of the ship  $L$  ( $L = 350$  m) as unit of length and the time to cover the length  $L$ , i.e.  $L/V_s$ , as unit of time. The notation  $x'(t|t-1)$  stands for the estimate of  $x'$  at the time  $t$  based on measurements up to and including time  $t-1$ . The vectors and matrices of (3.5) are explained by:

$$\begin{aligned} u' &= \delta_c \cdot \text{CGR} \\ (\hat{x}')^T &= \left( \hat{v}' \quad \hat{r}' \quad \hat{\psi}' \quad \hat{\delta}'_d \quad \hat{\delta}'_o \quad \hat{d}'_v \quad \hat{d}'_r \quad \hat{d}'_\delta \right) \\ (y')^T &= \left( \delta \cdot \text{CGR} \quad v_1 \cdot \text{CKM}/V_s \quad r \cdot \text{CGR} \cdot L/V_s \quad \psi \cdot \text{CGR} \right) \\ (\varepsilon')^T &= \left( \varepsilon'_\delta \quad \varepsilon'_v \quad \varepsilon'_r \quad \varepsilon'_\psi \right) \end{aligned}$$

$$A = \begin{pmatrix} a_{11} & a_{12} & 0 & a_{14} & a_{15} & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & a_{24} & a_{25} & 0 & 0 & 0 \\ a_{31} & a_{32} & 1 & a_{34} & a_{35} & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{44} & a_{45} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B^T = \begin{pmatrix} b_{11} & b_{21} & b_{31} & b_{41} & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & \ell_1/L & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$K = \begin{pmatrix} -9.27 \cdot 10^{-5} & 8.82 \cdot 10^{-2} & -2.40 \cdot 10^{-2} & -0.367 \\ 6.30 \cdot 10^{-4} & 0.481 & 9.65 \cdot 10^{-2} & 1.02 \\ 1.69 \cdot 10^{-5} & 7.86 \cdot 10^{-3} & 3.14 \cdot 10^{-3} & 0.326 \\ 1.66 \cdot 10^{-2} & -2.90 \cdot 10^{-3} & 5.65 \cdot 10^{-3} & 6.51 \cdot 10^{-2} \\ 1.03 \cdot 10^{-3} & 2.98 \cdot 10^{-3} & -5.63 \cdot 10^{-3} & -6.49 \cdot 10^{-2} \\ -1.26 \cdot 10^{-4} & 2.00 \cdot 10^{-2} & -6.69 \cdot 10^{-5} & -5.06 \cdot 10^{-3} \\ 1.63 \cdot 10^{-6} & -1.68 \cdot 10^{-3} & 3.02 \cdot 10^{-3} & -5.77 \cdot 10^{-2} \\ 2.54 \cdot 10^{-3} & 7.61 \cdot 10^{-6} & 1.24 \cdot 10^{-5} & 1.41 \cdot 10^{-4} \end{pmatrix}$$

where

$$\begin{array}{ll}
 a_{11} = 0.99163 & a_{32} = 0.0224515 \\
 a_{12} = -0.0100810 & a_{34} = 0.000195418 \\
 a_{14} = -0.00208207 & a_{35} = -0.0000132723 \\
 a_{15} = 0.000212358 & a_{44} = 0.81873 \\
 a_{21} = -0.0759537 & a_{45} = -0.18127 \\
 a_{22} = 0.96485 & b_{11} = -0.000212358 \\
 a_{24} = 0.0164706 & b_{21} = 0.00170991 \\
 a_{25} = -0.00170991 & b_{31} = 0.0000132723 \\
 a_{31} = -0.000874514 & b_{41} = -0.18127
 \end{array}$$

The elements of the state estimate vector  $\hat{\mathbf{x}}'$  are normalized estimates of the following variables: transversal velocity of the origin ( $\hat{v}'$ ), yaw rate ( $\hat{r}'$ ), heading ( $\hat{\psi}'$ ), rudder angle minus rudder bias ( $\hat{\delta}'_d = \hat{\delta}' - \hat{\delta}'_o$ ), rudder bias ( $\hat{\delta}'_o$ ), measurement biases of  $v_1$ ,  $r$ ,  $\delta$ , resp. ( $\hat{d}'_v$ ,  $\hat{d}'_r$ ,  $\hat{d}'_\delta$ ). The initial state estimate vector is equal to

$$(\hat{\mathbf{x}}'(t_0))^T = \left[ v \cdot \text{CKM} / V_s \quad r \cdot \text{CGR} \cdot L / V_s \quad \psi \cdot \text{CGR} \quad \delta \cdot \text{CGR} \quad 0 \quad 0 \quad 0 \quad 0 \right]$$

where  $v$  is obtained from (3.3) and all the measurements are from the time  $t_0$ .

If

$$\begin{array}{l}
 |\varepsilon'_\delta| > t'_\delta, \\
 |\varepsilon'_v| > t'_v, \\
 |\varepsilon'_r| > t'_r, \\
 \text{or} \\
 |\varepsilon'_\psi| > t'_\psi,
 \end{array}$$

where

$$\begin{aligned}
t'_\delta &= 1 \\
t'_v &= 0.06 \\
t'_r &= 0.25 \\
t'_\psi &= 0.015
\end{aligned} \tag{3.6}$$

then the corresponding measurement or measurements are rejected, when the state estimate vector is updated. A measurement signal is definitively rejected when IMX (IMX = 10) consecutive measurements are rejected. However, during the first IKMX = 900 s after the Kalman filter is initialized, no measurements are rejected because the bias states of the Kalman filter must be fairly estimated to avoid incorrect rejectings. The rejecting of a measurement is performed by putting the corresponding column of K equal to zero. Notice that the values of (3.6) are chosen very large to avoid rejectings when the Kalman filter is tested.

The only measurement signal used by the Kalman filter in the experiments A26 and A27 is the heading angle. The first three columns of the filter gain K are cancelled in experiment A27, while the following filter gain, designed correctly for the case of heading measurements only, is used in experiment A26:

$$K^T = \begin{bmatrix} -0.481 & 2.40 & 0.390 & 0.0720 & -0.0716 & 0 & 0 & 0 \end{bmatrix} \tag{3.7}$$

The bias estimates  $\hat{\delta}'_o$ ,  $\hat{d}'_v$ ,  $\hat{d}'_r$ , and  $\hat{d}'_\delta$  were not updated during the turning experiments C3 and C4.

The non-normalized state estimate vector is obtained as:

$$(\hat{x})^T = \begin{bmatrix} \hat{v} & \hat{r} & \hat{\psi} & \hat{\delta}_d & \hat{\delta}_o & \hat{d}_v & \hat{d}_r & \hat{d}_\delta \end{bmatrix}$$

where

$$\begin{aligned}
\hat{v} &= \hat{v}' \cdot V_S / \text{CKM} && [\text{knots}] \\
\hat{r} &= \hat{r}' \cdot V_S / (\text{CGR} \cdot L) && [\text{deg/s}] \\
\hat{\psi} &= \hat{\psi}' / \text{CGR} && [\text{deg}] \\
\hat{\delta}_d &= \hat{\delta}_d' / \text{CGR} && [\text{deg}] \\
\hat{\delta}_o &= \hat{\delta}_o' / \text{CGR} && [\text{deg}] \\
\hat{d}_v &= \hat{d}_v' \cdot V_S / \text{CKM} && [\text{knots}] \\
\hat{d}_r &= \hat{d}_r' \cdot V_S / (\text{CGR} \cdot L) && [\text{deg/s}] \\
\hat{d}_\delta &= \hat{d}_\delta' / \text{CGR} && [\text{deg}]
\end{aligned}$$

Notice the following expressions, too:

$$\begin{aligned}
\hat{v}_1 &= \hat{v} + \ell_1 \cdot \hat{r} \cdot \text{CGR} / \text{CKM} && [\text{knots}] \\
\hat{\delta} &= \hat{\delta}_d + \hat{\delta}_o && [\text{deg}]
\end{aligned}$$

The non-normalized residual vector is obtained as:

$$\epsilon^T = \left( \begin{array}{cccc} \epsilon_\delta & \epsilon_v & \epsilon_r & \epsilon_\psi \end{array} \right)$$

where

$$\begin{aligned}
\epsilon_\delta &= \epsilon_\delta' / \text{CGR} && [\text{deg}] \\
\epsilon_v &= \epsilon_v' \cdot V_S / \text{CKM} && [\text{knots}] \\
\epsilon_r &= \epsilon_r' \cdot V_S / (\text{CGR} \cdot L) && [\text{deg/s}] \\
\epsilon_\psi &= \epsilon_\psi' / \text{CGR} && [\text{deg}]
\end{aligned}$$

### 3.2 Self-tuning Regulator

A simple self-tuning regulator based on least squares identification and minimum variance control is used for straight course keeping. The basic self-tuning regulator is described in Wittenmark (1973).

The following model of the ship is used by the self-tuning regulator:

$$\begin{aligned}
 & (\hat{\psi}(t) - \psi_{\text{ref}}) + a_1(\hat{\psi}(t-k-1) - \psi_{\text{ref}}) + \dots + \\
 & + a_{\text{NA}}(\hat{\psi}(t-k-\text{NA}) - \psi_{\text{ref}}) = \\
 & = (V_s(t-k-1)/V_o)^2 b_o \nabla \delta_c(t-k-1) + \\
 & + (V_s(t-k-2)/V_o)^2 b_o b_1 \nabla \delta_c(t-k-2) + \dots + \\
 & + (V_s(t-k-\text{NB}-1)/V_o)^2 b_o b_{\text{NB}} \nabla \delta_c(t-k-\text{NB}-1) + \\
 & + V_s(t-k-1) c_1 \nabla \hat{v}(t-k-1) + \\
 & + c_2 \nabla \hat{r}(t-k-1) + e(t) \tag{3.8}
 \end{aligned}$$

where the design speed is denoted  $V_o$  [m/s]. The minimum variance control is given by

$$\begin{aligned}
 \nabla \delta_c(t) = & \left( \frac{V_o}{V_s(t)} \right)^2 \frac{1}{b_o} \left[ a_1(\hat{\psi}(t) - \psi_{\text{ref}}) + \dots + \right. \\
 & + a_{\text{NA}}(\hat{\psi}(t-\text{NA}+1) - \psi_{\text{ref}}) - \\
 & - (V_s(t-1)/V_o)^2 b_o b_1 \nabla \delta_c(t-1) - \dots - \\
 & - (V_s(t-\text{NB})/V_o)^2 b_o b_{\text{NB}} \nabla \delta_c(t-\text{NB}) - \\
 & \left. - V_s(t) c_1 \nabla \hat{v}(t) - c_2 \nabla \hat{r}(t) \right] \tag{3.9}
 \end{aligned}$$



where

$$\nabla \delta_c(t) = \delta_c(t) - \delta_c(t-1)$$

$$\nabla \hat{v}(t) = \hat{v}(t) - \hat{v}(t-1)$$

$$\nabla \hat{r}(t) = \hat{r}(t) - \hat{r}(t-1)$$

Notice that the speed scaling of (3.8) and (3.9) is introduced in such a way that the parameters  $a_1, \dots, a_{NA}$ ,  $b_1, \dots, b_{NB}$ ,  $c_1$  and  $c_2$  are approximately independent of the forward speed. This fact will, of course, simplify the mission of the self-tuning regulator.

The following standard values are used:

$$NA = 4$$

$$NB = 2$$

$$k = 7$$

$$T_s = 10 \text{ s}$$

$$\lambda_f = 0.99$$

$$b_o = 1$$

$$V_o = 6 \text{ m/s}$$

where  $T_s$  is the sampling interval and  $\lambda_f$  the exponential forgetting factor. Other values of  $k$ ,  $T_s$ , and  $V_o$  are sometimes used. The following initial values of the parameters and of the covariance matrix  $P$  are used:

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ b_1 \\ b_2 \\ c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} -6.91 \\ 5.95 \\ 3.88 \\ -3.57 \\ 0.48 \\ 0.11 \\ -2.10 \\ 34.73 \end{pmatrix} \quad P = \begin{pmatrix} 1 & & & & & & & \\ & 1 & & & & & & \\ & & 1 & & & & & \\ & & & 1 & & & & \\ & & & & 1 & & & \\ & & & & & 0.01 & & \\ & & & & & & 0.01 & \\ & & & & & & & 1 \\ & & & & & & & & 100 \end{pmatrix} \quad (3.10)$$

Sometimes  $c_1 = 0$  or  $c_2 = 0$ , which is indicated by assigning NC1 resp NC2 the value zero. The corresponding diagonal element of P is then also equal to zero. The standard values are NC1 = 1 and NC2 = 1.

By use of the minimum variance control (3.9) the following criterion is minimized:

$$J_1 = \sum_{n=0}^{\infty} (\hat{\psi}(n+k+1) - \psi_{\text{ref}})^2 \quad (3.11)$$

If the criterion

$$J_2 = \sum_{n=0}^{\infty} \left[ (\hat{\psi}(n+k+1) - \psi_{\text{ref}})^2 + q_2 (\nabla \delta_c(n))^2 \right] \quad (3.12)$$

is minimized instead, a penalty on the rudder motions is introduced by the parameter  $q_2$ . However, a proper solution of this problem requires the solving of a Riccati equation. A self-tuning regulator, which performs this, is used in Källström (1976a).

If the criterion (3.12) is modified to read

$$J_3(n) = (\hat{\psi}(n+k+1) - \psi_{\text{ref}})^2 + q_2 (\nabla \delta_c(n))^2 \quad (3.13)$$

$n = 0, 1, 2, \dots$

and if (3.13) is minimized at every sample event, then a simpler regulator is obtained. By inserting (3.8) into (3.13) and then performing the minimization, the following control is obtained:

$$\overline{\nabla\delta_c(t)} = \frac{(V_s(t)/V_o)^4 b_o^2}{(V_s(t)/V_o)^4 b_o^2 + q_2} \nabla\delta_c(t) \quad (3.14)$$

where  $\nabla\delta_c(t)$  is the minimum variance control given by (3.9). If  $q_2 = 0$ , then minimization of (3.13) gives the same result as minimization of (3.11) and consequently the controls (3.14) and (3.9) are equivalent. Notice that (3.14) only is a very small modification of (3.19) and that the identification part of the self-tuning regulator is unchanged. However, the control (3.14) has the serious disadvantage that no guarantee of closed loop stability is obtained in the general case.

The minimum variance control (3.9) is approximately scaled by  $(V_o/V_s(t))^2$  when the speed changes, i.e. (3.14) may be rewritten

$$\overline{\nabla\delta_c(t)} = \frac{b_o^2}{b_o^2 + \left(\frac{V_o}{V_s(t)}\right)^4 q_2} \left(\frac{V_o}{V_s(t)}\right)^2 [\nabla\delta_c(t)]_{V_o} \quad (3.15)$$

where  $[\nabla\delta_c(t)]_{V_o}$  denotes the minimum variance control when  $V_s = V_o$ . By introducing  $q = (V_o/V_s(t))^4 q_2$  we obtain

$$\begin{aligned} \overline{\nabla\delta_c(t)} &= \frac{b_o^2}{b_o^2 + q} \left(\frac{V_o}{V_s(t)}\right)^2 [\nabla\delta_c(t)]_{V_o} = \\ &= \frac{b_o^2}{b_o^2 + q} \nabla\delta_c(t) \end{aligned} \quad (3.16)$$

which is the actual control used in the autopilot. The standard value of  $q$  is equal to zero.

The estimates from the Kalman filter are used in all formulas of this section. Notice, however, that it is possible to use the non-filtered measurements instead.

### 3.3 PID-regulator

The following discrete PID-regulator for straight course keeping is also implemented for comparison:

$$\delta_c(nT_s) = - \left( \frac{V_o}{V_s(nT_s)} \right)^2 \left[ k_p (\hat{\psi}(nT_s) - \psi_{ref}) + k_D \hat{r}(nT_s) + k_I T_s \sum_{i=0}^{n-1} (\hat{\psi}(iT_s) - \psi_{ref}) \right] \quad (3.17)$$

$$n = 0, 1, 2, \dots$$

The following standard values are used:

$$\begin{aligned} k_p &= 3 \\ k_D &= 75 \text{ s} \\ k_I &= 0.02 \text{ 1/s} \\ T_s &= 10 \text{ s} \\ V_o &= 6 \text{ m/s} \end{aligned} \quad (3.18)$$

Notice that it is possible to use the non-filtered measurements instead of the Kalman filter estimates in (3.17). The special speed scaling used in (3.17) will approximately give the same course keeping performance independent of the forward speed. The rudder deviations, however, are increased proportional to  $(V_o/V_s)^2$  when the speed is decreased.

### 3.4 Yaw Regulator

A yaw performed by the yaw regulator consists of four different phases, viz. the initial phase (phase 1), the phase of constant yaw rate (phase 2), the checking rudder phase (phase 3), and the terminating phase (phase 4). However, if the requested heading change  $\Delta\psi_{\text{ref}}$  is small, one or more of the phases may be skipped. The Kalman filter estimates used by the yaw regulator are the yaw rate  $\hat{r}$  and the heading  $\hat{\psi}$ , and the reference values used are the requested yaw rate  $r_{\text{ref}}$  and the new requested heading  $\psi_{\text{ref}}$ . The phase of straight course keeping is denoted phase 0.

Modified discrete, fixed gain PID-regulators are used in the different phases (note that  $n = 0, 1, 2, \dots$ ):

Phase 1:

$$\delta_c(nT_s) = - \left( \frac{V_o}{V_s(nT_s)} \right)^2 k_4 (\hat{r}(nT_s) - r_o) + \bar{\delta}_c$$

$$| k_4 (\hat{r}(nT_s) - r_o) | \leq | \bar{c}_1 r_o |$$

Phase 2:

$$\delta_c(nT_s) = - \left( \frac{V_o}{V_s(nT_s)} \right)^2 \left[ k_5 (\hat{r}(nT_s) - r_o) + \right.$$

$$\left. + k_6 T_s \sum_{i=0}^{n-1} (\hat{r}(iT_s) - r_o) \right] + \bar{\delta}_c$$

Phase 3:

$$\delta_c(nT_s) = - \left( \frac{V_o}{V_s(nT_s)} \right)^2 \left[ k_7 (\hat{\psi}(nT_s) - \psi_{\text{ref}}) + k_8 \hat{r}(nT_s) \right]$$

$$| k_7 (\hat{\psi}(nT_s) - \psi_{\text{ref}}) + k_8 \hat{r}(nT_s) | \leq | \bar{c}_3 r_o |$$

Phase 4:

$$\delta_c(nT_s) = - \left( \frac{V_o}{V_s(nT_s)} \right)^2 \left[ k_1 (\hat{\psi}(nT_s) - \psi_{ref}) + k_2 \hat{r}(nT_s) + k_3 T_s \sum_{i=0}^{n-1} (\hat{\psi}(iT_s) - \psi_{ref}) \right]$$

The moving average  $\bar{\delta}_c$  of the rudder commands  $\delta_c$  is only updated during phase 0 and phase 4:

$$\bar{\delta}_c((k+1)T_s) = \bar{\delta}_c(kT_s) + \left( \frac{1-\gamma}{k+1} + \gamma \right) (\delta_c(kT_s) - \bar{\delta}_c(kT_s))$$

$$k = 0, 1, 2, \dots$$

$$\bar{\delta}_c(0) = 0$$

The reference yaw rate  $r_o$  including sign is computed once, when the yaw is initiated, as

$$r_o = r_{ref} \quad \text{if} \quad \hat{\psi} - \psi_{ref} \leq 0$$

or as

$$r_o = -r_{ref} \quad \text{if} \quad \hat{\psi} - \psi_{ref} > 0$$

Notice that the value of  $r_{ref}$  always is positive.

The conditions to jump from one phase to another read:

Phase 0 → phase 4:

$$\psi_1 < \Delta\psi_{ref} \leq \psi_2$$

Phase 0 → phase 1:

$$\Delta\psi_{ref} > \psi_2$$

Phase 1 → phase 2:

$$r_o \geq 0 \quad \text{and} \quad \hat{r} - r_o > -\varepsilon_1$$

or

$$r_o < 0 \quad \text{and} \quad \hat{r} - r_o < \varepsilon_1$$

or

$$(\text{time in phase 1}) > T_1$$

Phase 1 or 2 → phase 3:

$$r_o \geq 0 \quad \text{and} \quad -\bar{c}_2 \hat{r} < \hat{\psi} - \psi_{\text{ref}}$$

or

$$r_o < 0 \quad \text{and} \quad -\bar{c}_2 \hat{r} > \hat{\psi} - \psi_{\text{ref}}$$

Phase 3 → phase 4:

$$|\hat{r}| < \varepsilon_2$$

or

$$r_o \geq 0 \quad \text{and} \quad \hat{\psi} - \psi_{\text{ref}} > -\varepsilon_3$$

or

$$r_o < 0 \quad \text{and} \quad \hat{\psi} - \psi_{\text{ref}} < \varepsilon_3$$

or

$$(\text{time in phase 3}) > T_3$$

Phase 4 → phase 0:

$$(\text{time in phase 4}) > T_4$$

The condition to remain in phase 0 is:

$$\Delta\psi_{\text{ref}} \leq \psi_1$$

If the reference yaw rate  $r_{\text{ref}}$  is changed during a yaw, the new value is immediately used, but no other changes. It is also possible to change the reference course  $\psi_{\text{ref}}$  during a yaw, and then a new yaw is initiated by entering phase 1, if

$$|\Delta\psi_{\text{ref}}| > \psi_3$$

and

(3.19)

the actual phase is 3 or 4

or if

$$|\Delta\psi_{\text{ref}}| > \psi_3,$$

the actual phase is 1 or 2,

and one of the two conditions

(3.20)

$$r_o > 0, \quad \hat{\psi} - \psi_{\text{ref}} > 0$$

and

$$r_o \leq 0, \quad \hat{\psi} - \psi_{\text{ref}} < 0$$

is satisfied.

If neither condition (3.19) nor condition (3.20) is fulfilled, the new value of  $\psi_{\text{ref}}$  is used, but no other changes.

The following parameter values of the yaw regulator are used:

$$k_1 = 5$$

$$k_7 = 2$$

$$k_2 = 200 \text{ s}$$

$$k_8 = 200 \text{ s}$$

$$k_3 = 0.005 \text{ 1/s}$$

$$\varepsilon_1 = 0 \text{ deg/s}$$

$$k_4 = 200 \text{ s}$$

$$\varepsilon_2 = 0.02 \text{ deg/s}$$

$$k_5 = 200 \text{ s}$$

$$\varepsilon_3 = 1 \text{ deg}$$

$$k_6 = 8$$



$$\begin{array}{ll}
 \bar{c}_1 = 60 \text{ s} & \psi_1 = 0.35 \text{ deg} \\
 \bar{c}_2 = 50 \text{ s} & \psi_2 = 2.5 \text{ deg} \\
 \bar{c}_3 = 60 \text{ s} & \psi_3 = 2.5 \text{ deg} \\
 T_1 = 30 \text{ s} & V_o = 6 \text{ m/s} \\
 T_3 = 100 \text{ s} & T_s = 10 \text{ s} \\
 T_4 = 300 \text{ s} & \gamma = 0.05
 \end{array}$$

A special indicator  $M_y$  is used to describe the actual yaw phase, i.e.  $M_y = 0, 1, 2, 3, 4$  corresponds to phase 0, 1, 2, 3, 4, resp. Notice that it is possible to use the non-filtered measurements instead of the Kalman filter estimates in the yaw regulator. The special speed scaling used in the yaw regulator will approximately give the same performance of the yaw rate and the heading independent of the forward speed. The rudder deviations, however, are increased proportional to  $(V_o/V_s)^2$  when the speed is decreased.

#### 4. DISCUSSION OF THE EXPERIMENTS

All the experiments are described in Appendix D, where also plots of the recorded variables are shown. A complete summary of the experiments is given in the Tables D.1 - D.6 at the beginning of Appendix D.

To make it possible to compare the steering quality of different autopilot structures, two loss functions are now introduced:

$$V_1 = \frac{1}{\tau} \int_0^{\tau} \left[ (\psi(t) - \psi_{\text{ref}})^2 + \lambda (\delta_c(t) - m_{\delta_c})^2 \right] dt \quad (4.1)$$

and

$$V_2 = \frac{1}{\tau} \int_0^{\tau} \left[ (\psi(t) - \psi_{\text{ref}})^2 + \lambda (\delta(t) - m_{\delta})^2 \right] dt \quad (4.2)$$

where the mean values of  $\delta_c$  and  $\delta$  are denoted  $m_{\delta_c}$  and  $m_{\delta}$ , resp., and the weighting factor  $\lambda$  is equal to 1/12. The duration of the experiment is denoted  $\tau$ . The loss functions are approximated by:

$$V_1 = \frac{1}{N_1} \sum_{n=0}^{N_1-1} \left[ (\psi(nT_s) - \psi_{\text{ref}})^2 + \lambda (\delta_c(nT_s) - m_{\delta_c})^2 \right] \quad (4.3)$$

and

$$V_2 = \frac{1}{N_2} \sum_{n=0}^{N_2-1} \left[ (\psi(nh) - \psi_{\text{ref}})^2 + \lambda (\delta(nh) - m_{\delta})^2 \right] \quad (4.4)$$

where  $N_1 T_s = \tau$  and  $N_2 h = \tau$ , resp. Notice that  $T_s$  is the sampling interval of the regulator, usually equal to 10 s, while  $h$  always is equal to 1 s. The loss function  $V_1$  is used to compare the experiments A1 - A36, while the loss function  $V_2$  is computed during experiments F1 - F4.

#### 4.1 Kalman Filter Testing

The Kalman filter was running during all the experiments. Discussions and comparisons of some of the experiments are given in this section.

Three experiments, B1 - B3, were performed to test the Kalman filter by recording some of the estimates at the same sampling rate as the filter was running at, i.e. with sampling interval 1 s. The wind velocity during the experiments was 6, 8, and 15 m/s, resp., and the forward speed was 13.7, 13.3, and 15.9 knots, resp. The experiments are summarized in Table D.2 of Appendix D. It can be concluded from the plots that the performance of the Kalman filter seems to be very good in the different wind and speed conditions. Notice, however, that the yaw rate measurements are filtered in a rather hard way, although the filtering is not at all unreasonable. Another yaw rate estimate  $\hat{r}_d$  is also computed every second:

$$\hat{r}_d(t) = \frac{\psi(t) - \psi(t-5)}{5} \quad (4.5)$$

It can be concluded that these estimates are not at all as smooth as the Kalman filter estimates, but that they are preferable compared to the measurements. By use of the measurement devices of the heading and the yaw rate, which are installed on the Sea Stratus, it thus can be concluded that a yaw rate estimate computed by the difference approximation formula (4.5) seems to be preferable in front of the noisy measurement signal  $r$ .

A summary of 8 experiments of straight course keeping by the self-tuning regulator using Kalman filter estimates, when the measurement signals used by the filter are varied, is given in Table 4.1. The Kalman filter estimates obtained when  $\delta$ ,  $v_1$  or  $r$  is not used by the filter, experiments A22, A23, and A24, resp., are approximately of the same

Ex- peri- ment	Rel. wind direc- tion	Wind velo- city [m/s]	$\delta_c$		$\psi - \psi_{ref}$		Mean value of $u$ [knots]	$V_1$	Remarks
			Mean value [deg]	Std. dev. [deg]	Mean value [deg]	Std. dev. [deg]			
A 19	1	9	0.45	1.11	-0.005	0.176	13.0	0.134	
A 22	1	13	2.32	0.95	-0.006	0.204	12.8	0.117	$\delta$ not used by filter
A 23	1	13	1.71	0.94	0.015	0.217	12.6	0.121	$v_1$ not used by filter
A 24	1	14	1.34	1.25	0.130	0.317	12.5	0.248	$r$ not used by filter
A 25	1	14	1.25	0.90	-0.034	0.240	12.5	0.126	After 9.5 min, $\psi$ was not used by filter
A 26	1	6	0.24	0.89	0.030	0.230	13.4	0.120	$v_1$ , $r$ and $\delta$ not used by filter. Correct K.
A 27	1	7	0.30	0.92	0.043	0.197	13.2	0.111	$v_1$ , $r$ and $\delta$ not used by filter
A 28	1	4	0.16	0.95	0.006	0.239	13.5	0.132	

Table 4.1 - Experiments of straight course keeping by the self-tuning regulator using Kalman filter estimates, when the measurement signals used by the filter are varied. If nothing else is remarked, all measurement signals  $v_1$ ,  $r$ ,  $\psi$ , and  $\delta$  are used by the Kalman filter. An explanation of the relative wind direction is given in Appendix A. The tuning time before the experiment started was equal to 30 min. The following standard values are used:  $T_s = 10$  s,  $V_0 = 6$  m/s,  $IVVC = 1$ ,  $NC1 = 1$ ,  $NC2 = 1$ ,  $k = 7$ , and  $q = 0$ . The experiments were performed during a period of 3 days.

quality as the estimates obtained in experiments A19 and A28, where all the measurement signals are used. The steering performances obtained are also approximately of the same quality, with one exception; extraordinary disturbances seem to have affected experiment A24, where  $r$  is not used by the filter. Experiment A25 shows that it is possible to obtain an acceptable course keeping during at least 15 min after the heading measurement signal  $\psi$  is rejected by the filter.

The Kalman filter estimates obtained, when the only measurement signal used is the heading  $\psi$ , are quite acceptable, which can be concluded from experiments A26 and A27. It is also concluded that the difference between using a correctly designed filter gain  $K$  (cf. (3.7)) and cancelling the corresponding columns of the original filter gain is very insignificant. The steering performance obtained is approximately of the same quality, when the only measurement signal used by the Kalman filter is the heading  $\psi$  (experiments A26 and A27), compared to the performance obtained when all measurement signals are used (experiments A19 and A28). From the experiments of Table 4.1 it thus can be concluded that it is not necessary to use all the measurement signals to obtain a good performance of the Kalman filter. In fact, acceptable filter estimates are obtained when the only measurement signal used is the heading  $\psi$ .

By comparing experiment A14, where complete calm was prevailing, with experiment A22, where the wind velocity was 13 m/s, it can be concluded that the quality of the Kalman filter estimates is rather insensitive to wind changes. Experiments A34 and A35 show that the performance of the Kalman filter is approximately of the same quality at the forward speed 15.8 knots compared to the forward speed 12.1 knots.

Experiments A1, D2 and D3 show the initial performance of the Kalman filter. The bias estimates of the filter are approximately tuned after about 30 min.

The performance of the Kalman filter during turns is shown in experiments C1-C6 and D1. From experiments C1, C2, and D1 it is immediately concluded that it is very important to skip the updating of the bias estimates  $\hat{\delta}'_o$ ,  $\hat{d}'_v$ ,  $\hat{d}'_r$ , and  $\hat{d}'_\delta$  when turning, because otherwise the bias estimates are disturbed significantly. The updating is skipped during experiments C3 and C4. From experiments C3-C6 it is concluded that the performance of the Kalman filter is approximately of the same good quality during turning as during straight course keeping. Experiments E1-E4 also illustrate the good performance of the Kalman filter.

Considering all the experiments it is obvious that the test values (3.6) of the Kalman filter are chosen too large, at least when the speed of the ship is larger than about 10 knots. The following non-normalized values seem to be suitable when the speed is about 13 knots:

$$\begin{aligned}
 t_\delta &= 10 \text{ deg} \\
 t_v &= 0.15 \text{ knots} \\
 t_r &= 0.15 \text{ deg/s} \\
 t_\psi &= 0.3 \text{ deg}
 \end{aligned}
 \tag{4.6}$$

Then the normalized test values are (cf. (3.6)):

$$\begin{aligned}
 t'_\delta &= 0.17 \\
 t'_v &= 0.012 \\
 t'_r &= 0.14 \\
 t'_\psi &= 0.0052
 \end{aligned}
 \tag{4.7}$$

The test values (4.7) seem to be reasonable when the speed is larger than about 10 knots, while the values (3.6), obtained from simulations in Källström (1976c), are suitable at lower speeds.

#### 4.2 Straight Course Keeping

Experiments of straight course keeping by the self-tuning regulator using Kalman filter estimates, when the parameters  $T_s$ ,  $V_o$ ,  $NC1$ ,  $NC2$ ,  $k$  and  $q$  are varied, are summarized in Table 4.2.

The straight course keeping is mainly affected by the value of the design speed  $V_o$  during the initial phase, when the parameters of the self-tuning regulator not yet are tuned. However, from experiments A2, A5, A6, and A10 it can be concluded that  $V_o = 7$  m/s is to prefer in front of  $V_o = 6$  m/s. Since the parameter  $V_o$  also is used in the yaw regulator, the value 6 m/s is nevertheless used during most of the experiments by reasons explained in Section 4.3.

By comparing experiments A2-A9 it is concluded that  $T_s = 10$  s,  $k = 7$  (experiments A2, A5, A6) in average is the best choice. An acceptable alternative is  $T_s = 15$  s,  $k = 5$  (experiment A9). Experiments A10-A13 show clearly that the best steering performance is obtained when  $q = 0$ .

The importance of the feedforward signals is illustrated by experiments A15-A17. It is concluded that the quality of the steering performance is significantly decreased when both feedforward signals are omitted. The comparative experiment A14 was unfortunately disturbed by a temporarily reduction of the number of propeller revolutions immediately before the experiment started. No comparisons with experiment A14 are thus performed.

Ex- peri- ment	Rel. wind dir.	Wind velo- city [m/s]	$T_s$ [s]	$V_0$ [m/s]	Structure of self- tuning regulator				$\delta_c$		$\psi - \psi_{ref}$		Mean value of $u$ [knots]	$V_1$	Remarks
					NC1	NC2	k	q	Mean value [deg]	Std. dev. [deg]	Mean value [deg]	Std. dev. [deg]			
A 2	5	5	10	7	1	1	7	0	0.41	1.14	0.017	0.182	13.9	0.142	Tuning time: > 120 min.  The speed was reduced. Tuning time: > 120 min.
A 3	5	10	10	7	1	1	5	0	0.74	1.20	0.009	0.239	14.2	0.177	
A 4	5	5	10	7	1	1	6	0	1.39	1.41	-0.009	0.246	14.0	0.226	
A 5	5	4	10	7	1	1	7	0	0.66	1.17	0.014	0.219	14.0	0.162	
A 6	4	4	10	7	1	1	7	0	-0.73	1.32	-0.051	0.290	13.5	0.232	
A 7	3	4	10	7	1	1	8	0	-0.17	1.19	0.034	0.260	14.1	0.187	
A 8	5	4	15	6	1	1	4	0	0.17	1.46	-0.027	0.309	13.9	0.274	
A 9	5	4	15	6	1	1	5	0	0.05	1.24	-0.022	0.301	13.8	0.219	
A 10	5	3	10	6	1	1	7	0	0.08	1.62	0.013	0.344	13.7	0.337	
A 11	5	3	10	6	1	1	6	0.05	-0.06	1.79	-0.033	0.400	13.9	0.428	
A 12	5	3	10	6	1	1	6	0.1	0.05	1.69	0.013	0.444	13.7	0.435	
A 13	6	3	10	6	1	1	6	0.2	0.24	1.84	0.016	0.420	13.6	0.459	
A 14	-	0	10	6	1	1	7	0	-0.61	1.66	-0.017	0.111	13.5	0.242	
A 15	-	0	10	6	1	0	7	0	-0.68	0.82	0.015	0.158	13.3	0.081	
A 16	1	5	10	6	0	1	7	0	-0.43	0.62	-0.066	0.169	12.9	0.065	
A 17	1	5	10	6	0	0	7	0	0.01	0.89	-0.188	0.247	12.9	0.162	

Table 4.2 - Experiments of straight course keeping by the self-tuning regulator using Kalman filter estimates, when the parameters  $T_s$ ,  $V_0$ ,  $V_1$ , NC1, NC2, k and q are varied. All measurement signals  $v_1$ ,  $r$ ,  $\psi$  and  $\delta$  are used by the Kalman filter. An explanation of the relative wind direction is given in Appendix A. The tuning time before the experiment started was equal to 30 min, if nothing else is remarked. The value of IVVC is equal to 1. The experiments were performed during a period of 3 days.



The following standard parameter values are used during most of the remaining experiments:

$$\begin{aligned}
 T_s &= 10 \text{ s} \\
 V_0 &= 6 \text{ m/s} \\
 IVVC &= 1 \\
 NC1 &= 1 \\
 NC2 &= 1 \\
 k &= 7 \\
 q &= 0
 \end{aligned}
 \tag{4.8}$$

The good performance of the self-tuning regulator in different weather conditions is illustrated by e.g. experiments A15 and A22, where the wind velocity was 0 resp. 13 m/s. Experiments A34 and A36 show the good performance of the self-tuning regulator when the forward speed was 15.8 resp. 12.1 knots.

Experiments A1, D2 and D3 show the initial performance of the self-tuning regulator when the autopilot is switched on. It is concluded that the steering quality is quite acceptable from the beginning. Experiment D3 also illustrates the performance of the self-tuning regulator in connection with the Sailmaster and the Course Correction. From experiments C1-C6 and D1 it is concluded that the self-tuning regulator has no initial problems after the yaw regulator has terminated a turn.

Straight course keeping experiments to compare the performance of the self-tuning regulator and the PID-regulator are summarized in Table 4.3. The performance of both the self-tuning regulator and the PID-regulator is improved when Kalman filter estimates are used instead of non-filtered measurements. When Kalman filter estimates are used, the performance of the self-tuning regulator is

Ex-periment	Rel. wind dir.	Wind velocity [m/s]	NC1	NC2	$\delta_c$		$\psi - \psi_{ref}$		Mean value of u [knots]	$V_1$	$V_2$
					Mean value [deg]	Std. dev. [deg]	Mean value [deg]	Std. dev. [deg]			
A 16	1	5	0	1	-0.43	0.62	-0.066	0.169	12.9	0.065	-
A 17	1	5	0	0	0.01	0.89	-0.188	0.247	12.9	0.162	-
A 18	1	5	0	0	0.24	1.01	-0.052	0.245	12.8	0.148	-
A 19	1	9	1	1	0.45	1.11	-0.005	0.176	13.0	0.134	-
A 20	1	9	-	-	0.53	2.48	0.026	0.556	13.1	0.822	-
A 21	1	4	-	-	0.53	0.91	-0.018	0.293	12.9	0.155	-
A 27	1	7	1	1	0.30	0.92	0.043	0.197	13.2	0.111	-
A 28	1	4	1	1	0.16	0.95	0.006	0.239	13.5	0.132	-
A 29	1	3	0	0	-0.01	1.51	0.001	0.324	13.4	0.295	-
A 30	1	5	-	-	-0.10	0.89	-0.026	0.256	13.5	0.132	-
A 31	1	3	-	-	0.22	2.41	-0.103	1.085	13.4	1.672	-
A 35	1	11	-	-	0.01	1.66	0.008	0.295	12.1	0.317	-
A 36	1	11	1	1	-0.05	1.08	-0.058	0.206	12.1	0.143	-
F 1	6	3	1	1	-	-	-0.002	-	14.4	-	0.038
F 2	7	3	-	-	-	-	-0.016	-	14.4	-	0.259
F 3	1	11	1	1	-	-	-0.073	-	12.1	-	0.160
F 4	1	11	-	-	-	-	-0.009	-	12.2	-	0.304

Table 4.3 - Comparable straight course keeping experiments. An explanation of the relative wind direction is given in Appendix A. All measurement signals  $v_1$ ,  $r$ ,  $\psi$  and  $\delta$  are used by the Kalman filter, if nothing else is remarked. The following standard values are used, if nothing else is remarked:  $T_s = 10$  s,  $V_0 = 6$  m/s,  $IVVC = 1$ ,  $k = 7$ ,  $q = 0$ ,  $k_p = 3$ ,  $k_D = 75$  s,  $k_I = 0.02$  1/s. The standard values of the PID-regulator are obtained by simulations, see Källström (1976c). The standard values are used in the experiments A20, A21, and F2, while the values of  $k_p$ ,  $k_D$ , and  $k_I$  were manually tuned before the experiments A30, A31, A35, and F4 started. The groups of experiments A16-A21 and A27-A31 were both performed during a period of 6 h, while the groups A35-A36, F1-F2, and F3-F4 were performed during a period of 2 h each. The experiments within each group are fairly comparable.

Ex- peri- ment	Kalman filter esti- mates used		Non-filtered measurements used		Remarks
	Self- tuning regulator	PID- regulator	Self- tuning regulator	PID- regulator	
A 16	x				
A 17	x				
A 18			x		
A 19	x				
A 20				x	
A 21		x			
A 27	x				$v_1$ , $r$ and $\delta$ not used by filter.
A 28	x				
A 29			x		
A 30		x			$k_D = 120$ s.
A 31				x	$k_P = 2$ , $k_D = 100$ s.
A 35		x			$k_D = 140$ s.
A 36	x				
F 1	x				$\left\{ \begin{array}{l} V_0 = 8 \text{ m/s, IVVC} = 3, V_c = 7 \text{ m/s.} \\ v_1 \text{ not used by filter. Sailmaster} \end{array} \right.$
F 2				x	
F 3	x				$\left\{ \begin{array}{l} V_0 = 8 \text{ m/s, IVVC} = 3, V_c = 7 \text{ m/s.} \\ \text{Sailmaster.} \end{array} \right.$
F 4		x			

Table 4.3 - Continued.

significantly better than the performance of the PID-regulator, particularly when the wind speed is increased. The good steering quality of the self-tuning regulator compared to the PID-regulator is still more emphasized, when non-filtered measurements are used in both regulators. Of course, the self-tuning regulator using Kalman filter estimates is consequently also to prefer in front of the PID-regulator using non-filtered measurements. Finally, it is not possible to conclude anything definitively when the self-tuning regulator using non-filtered measurements is compared to the PID-regulator using Kalman filter estimates.

It is now possible to sum up that, even if the heading is the only measurement signal available, a self-tuning regulator with the standard values (4.8) combined with a Kalman filter is very much advantageous to a well tuned PID-regulator.

### 4.3 Yawing

Experiments C1-C6 and D1 show turns performed by the yaw regulator. The Kalman filter estimates were fed into both the self-tuning regulator and the yaw regulator during all the experiments, which are summarized in Tables D.3 and D.4 of Appendix D.

It is concluded from experimen D1, where  $V_0 = 8$  m/s, that the rudder deviations are too large during phase 4.

A simple way to improve the performance of the yaw regulator in this respect is to decrease the value of  $V_0$ .

Thus  $V_0 = 7$  m/s during experiment C1 and  $V_0 = 6$  m/s during experiments C2-C6. Experiments C1, C2 and D1 show the importance of skipping the updating of the bias estimates

$\hat{\delta}'_0$ ,  $\hat{d}'_v$ ,  $\hat{d}'_r$ , and  $\hat{d}'_\delta$  during large course changes. This is performed during experiments C3 and C4, where the very good performance of the yaw regulator for large turns is illustrated. The same good performance for small course changes is shown in experiments C5 and C6.

#### 4.4 Experiments for Identification

Two different kinds of identification experiments were performed, viz. open loop experiments (E1, E2, E4) and closed loop experiment using additive rudder disturbances (E3). The experiments are summarized in Table D.5 of Appendix D. The sampling interval was equal to 10 s. Notice that experiment E2 is a  $5^\circ/5^\circ$  zig-zag test. The rudder commands  $\delta_c$  during experiments E1 and E4 and the additive rudder disturbances  $\delta_{add}$  during experiment E3 were approximately chosen as a PRBS containing the values  $+\delta_{amp}$ ,  $-\delta_{amp}$ , and zero.

The feedback is explained by Fig. 4.1. The rudder command  $\delta_c$  is obtained as

$$\delta_c = -k_{id}(\psi - \psi_{ref}) + \delta_{add}$$

Obviously an open loop experiment is performed by assigning  $k_{id}$  the value zero.

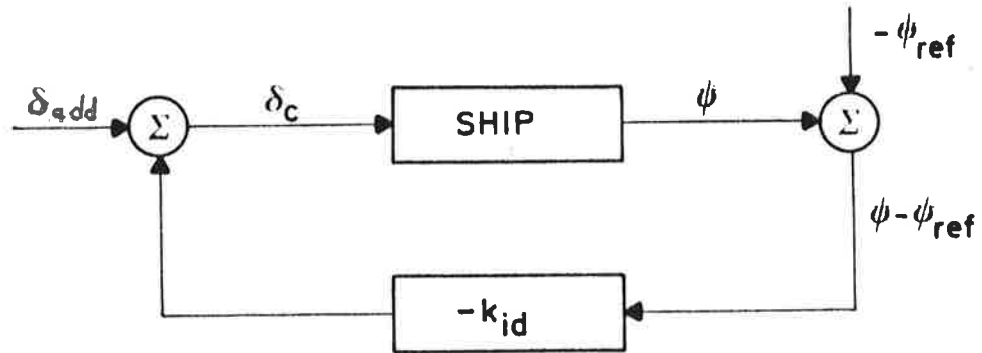


Fig. 4.1 - Proportional feedback from the course error used for identification experiments.

## 5. CONCLUSIONS

An adaptive autopilot, consisting of a Kalman filter, a self-tuning regulator and a yaw regulator, was tested on a ballasted 355 000 tdw oil tanker. Comparative straight course keeping experiments with a well tuned PID-regulator were also performed. The average speed of the ship varied between 12.1 knots and 15.9 knots during the experiments. The range of the wind velocity measured was 0-15 m/s, which corresponds to complete calm and moderate gale. Fifty-six experiments were performed during a total time of 33 h. The testing, however, was in fact going on during almost all the days and nights of the voyage, i.e. during about 2 weeks.

The performance of the Kalman filter was very good in the different speed and wind conditions during straight course keeping as well as during turning. Notice, however, that it is important to skip the updating of the bias estimates during turns. Usually all measurement signals, i.e. the transversal velocity of bow, the yaw rate, the heading and the rudder angle were used by the Kalman filter. However, the performance, when the only measurement signal used was the heading angle, was approximately of the same quality as the performance when all measurement signals were used. One experiment showed that it was possible to obtain an acceptable course keeping during at least 15 min. after the heading measurement signal was rejected by the Kalman filter, when the three other measurement signals were used. The initial performance of the Kalman filter, when the autopilot was switched on, was quite acceptable and the bias estimates were approximately tuned after about 30 min.

The straight course keeping performance of the self-tuning regulator using Kalman filter estimates was very good in the different speed and wind conditions. The use of the estimates of transversal velocity and yaw rate as

feedforward signals was improving the steering quality significantly, even if the only measurement signal used by the Kalman filter was the heading angle. The initial performance of the self-tuning regulator when the autopilot was switched on as well as the performance after the termination of a turn was quite satisfactory.

The performance of both the self-tuning regulator and the PID-regulator was improved when Kalman filter estimates were used instead of non-filtered measurements. When Kalman filter estimates were used, the performance of the self-tuning regulator was significantly better than the performance of the PID-regulator, particularly when the wind speed was increased. The good steering quality of the self-tuning regulator compared to the PID-regulator was still more emphasized when non-filtered measurements were used in both regulators. The self-tuning regulator using Kalman filter estimates was consequently also to prefer in front of the PID-regulator using non-filtered measurements. It was not possible to conclude anything definitively when the self-tuning regulator using non-filtered measurements was compared to the PID-regulator using Kalman filter estimates.

The yaw regulator was tested by performing small course changes as well as large turns. The performance was very good in both cases.

It is now possible to confirm a rather good consistency between the experiments on the Sea Stratus and the simulations reported in Källström (1976c). However, the performance of a well tuned PID-regulator seems to be overestimated in simulations compared to full-scale experiments.



## 6. ACKNOWLEDGEMENTS

I would like to express my gratitude to the Salén Shipping Co for their willingness to allow experiments to be performed with their ship. I am particularly grateful to the captain of the Sea Stratus, Mr B Svensson.

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## APPENDIX A - NOTATIONS

A	system matrix of Kalman filter
$a_i, i = 1, 2, \dots$	parameters of self-tuning regulator
$a_{ij}, i=1, 2, \dots, j=1, 2, \dots$	elements of system matrix A
B	system matrix of Kalman filter
BV	weighting factor
$b_0$	parameter of self-tuning regulator
$b_i, i = 1, 2, \dots$	parameters of self-tuning regulator
$b_{il}, i = 1, 2, 3, 4$	elements of system matrix B
C	system matrix of Kalman filter
CGR	conversion factor from degrees to radians
CKM	conversion factor from knots to m/s
$c_1, c_2$	parameters of self-tuning regulator
$\bar{c}_1, \bar{c}_2, \bar{c}_3$	parameters of yaw regulator
$\hat{d}_v, \hat{d}_r, \hat{d}_\delta$	estimated measurement biases of $v_1$ , $r$ , and $\delta$ , resp.
$\hat{d}'_v, \hat{d}'_r, \hat{d}'_\delta$	normalized estimates corresponding to $\hat{d}_v, \hat{d}_r$ , and $\hat{d}_\delta$ , resp.
e	disturbances of the self-tuning regulator model
h	sampling interval of $V_2$
IDEXP	control parameter of AUTP3
IFLAG	control parameter of printing and punching
IKAL	control parameter of AUTP3
IKMX	number of times the measurement testing is skipped when the Kalman filter is initialized
ILOS	control parameter of AUTP3

IMX	number of consecutive times a measurement signal must be rejected to definitively reject the signal
IPC	control parameter of punching
IPID	control parameter of AOTP3
IPK	control parameter of punching
IPR	sampling interval of punching
IREGY	sampling interval of yaw regulator
IREGYT	sampling interval of yaw testing
IST	control parameter of AOTP3
IVVC	control parameter of computation of $V_s$
IYAW	control parameter of AOTP3
$J_1, J_2, J_3$	criteria of self-tuning regulator
K	gain matrix of Kalman filter
k	number of pure time-delays of self-tuning regulator
$k_P, k_D, k_I$	parameters of PID-regulator
$k_{id}$	parameter of P-regulator used for identification experiments
$k_i, i = 1, 2, \dots, 8$	parameters of yaw regulator
L	length of ship
$\ell_1$	distance from origin to forward doppler log
MEAS	control parameter vector of AOTP3
$M_y$	yaw phase indicator
$m_\delta$	mean value of $\delta$
$m_{\delta_c}$	mean value of $\delta_c$
NA	number of a-parameters of self-tuning regulator
NB	number of b-parameters of self-tuning regulator

NC1	indicator of parameter $c_1$ of self-tuning regulator
NC2	indicator of parameter $c_2$ of self-tuning regulator
$N_1, N_2$	number of sampling events to compute $V_1$ and $V_2$ , resp.
$n$	number of propeller revolutions
$P$	covariance matrix of self-tuning regulator
PI	parameter of AOTP3 containing $\pi$
PI2	parameter of AOTP3 containing $2 \times \pi$
PP0	vector containing initial diagonal values of $P$
PSI0	reference course used for identification experiments
$P_s$	propeller effect
$q$	penalty on rudder deviation when self-tuning regulator is used
$q_2$	weighting factor of $J_2$ and $J_3$
$R_1$	covariance matrix of process noise
$r$	yaw rate
$\hat{r}$	estimated yaw rate
$\hat{r}'$	normalized estimate corresponding to $\hat{r}$
$\hat{r}_d$	estimated yaw rate computed by difference approximation
$r_{ref}$	reference value of yaw rate
$r_0$	reference value of yaw rate including sign
TH0	vector containing initial parameter values of self-tuning regulator
$T_k$	sampling interval of Kalman filter
$T_s$	sampling interval
$T_1, T_3, T_4$	parameters of yaw regulator

$t$	time
$t_0$	initial time
$t_\delta, t_v, t_r, t_\psi$	test values of Kalman filter for $\delta, v_1, r,$ and $\psi,$ resp.
$t'_\delta, t'_v, t'_r, t'_\psi$	normalized test values corresponding to $t_\delta, t_v, t_r,$ and $t_\psi,$ resp.
$u$	forward velocity
$u'$	normalized input signal of Kalman filter
VMAX	parameter of AOTP3
VMIN	parameter of AOTP3
$V_c$	parameter containing a constant forward velocity
$V_s$	scaling speed
$V_0$	design speed
$V_1, V_2$	loss functions
$v$	transversal velocity of origin
$\hat{v}$	estimated transversal velocity of origin
$\hat{v}'$	normalized estimate corresponding to $\hat{v}$
$v_1$	transversal velocity of bow
$\hat{v}_1$	estimated transversal velocity of bow
$\hat{x}$	state estimate vector of Kalman filter
$\hat{x}'$	normalized state estimate vector of Kalman filter
$y'$	Normalized measurement vector of Kalman filter
$\gamma$	parameter of yaw regulator
$\Delta\psi_{ref}$	change of reference value of course
$\delta$	rudder angle
$\hat{\delta}$	estimated rudder angle

$\hat{\delta}$	normalized estimate corresponding to $\hat{\delta}$
$\delta_{\text{add}}$	additive rudder disturbance signal used for identification experiments
$\delta_{\text{amp}}$	rudder amplitude used for identification experiments
$\delta_c$	rudder command
$\bar{\delta}_c$	moving average value of $\delta_c$
$\hat{\delta}_d$	estimated value equal to $\hat{\delta} - \hat{\delta}_0$
$\hat{\delta}'_d$	normalized estimate corresponding to $\hat{\delta}_d$
$\delta_\ell$	rudder limit
$\delta_s$	rudder servo position
$\hat{\delta}_0$	estimated rudder bias
$\hat{\delta}'_0$	normalized estimate corresponding to $\hat{\delta}_0$
$\varepsilon$	residual vector of Kalman filter
$\varepsilon'$	normalized residual vector of Kalman filter
$\varepsilon_\delta, \varepsilon_v, \varepsilon_r, \varepsilon_\psi$	elements of residual vector $\varepsilon$
$\varepsilon'_\delta, \varepsilon'_v, \varepsilon'_r, \varepsilon'_\psi$	normalized values corresponding to $\varepsilon_\delta, \varepsilon_v, \varepsilon_r,$ and $\varepsilon_\psi,$ resp., i.e. elements of normalized residual vector $\varepsilon'$
$\varepsilon_1, \varepsilon_2, \varepsilon_3$	parameters of yaw regulator
$\lambda$	weighting factor of $V_1$ and $V_2$
$\lambda_f$	exponential forgetting factor of self-tuning regulator
$\tau$	duration of experiment
$\psi$	heading angle
$\hat{\psi}$	estimated heading angle
$\hat{\psi}'$	normalized estimate corresponding to $\hat{\psi}$
$\psi_{\text{ref}}$	reference value of course
$\psi_1, \psi_2, \psi_3$	parameters of yaw regulator
$\nabla$	change of value (e.g. $\nabla\delta_c(t) = \delta_c(t) - \delta_c(t-1)$ )

The absolute wind direction related to the ship is explained by Fig. A.1.

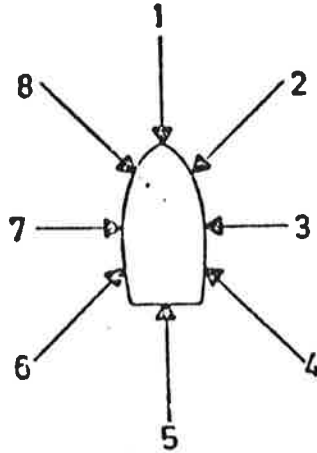


Fig. A.1 - Explanation of the relative wind direction.  
Notice that the absolute wind is used.



## APPENDIX B - STANDARD PARAMETER VALUES

The standard parameter values of the autopilot used during the experiments are summarized in this appendix. When the notation used in this report differs from the variable name used in the computer programs, the latter name is given in brackets.

IDEXP	0
IFLAG	30
MEAS (1)	0
MEAS (2)	0
MEAS (3)	0
MEAS (4)	0
CGR	0.0174533
CKM	0.514444
PI	3.141593
PI2	6.283185
L (AL)	350
$\ell_1$ (AL1)	164.35
$a_{11}$ (A(1))	0.99163
$a_{12}$ (A(2))	-0.0100810
$a_{14}$ (A(3))	-0.00208207
$a_{15}$ (A(4))	0.000212358
$a_{21}$ (A(5))	-0.0759537
$a_{22}$ (A(6))	0.96485
$a_{24}$ (A(7))	0.0164706
$a_{25}$ (A(8))	-0.00170991
$a_{31}$ (A(9))	-0.000874514

$a_{32}$ (A(10))	0.0224515
$a_{34}$ (A(11))	0.000195418
$a_{35}$ (A(12))	-0.0000132723
$a_{44}$ (A(13))	0.81873
$a_{45}$ (A(14))	-0.18127
$b_{11}$ (A(15))	-0.000212358
$b_{21}$ (A(16))	0.00170991
$b_{31}$ (A(17))	0.0000132723
$b_{41}$ (A(18))	0.18127
$K(1,1)$ (AK(1,1))	$-9.27 \cdot 10^{-5}$
$K(2,1)$ (AK(2,1))	$6.30 \cdot 10^{-4}$
$K(3,1)$ (AK(3,1))	$1.69 \cdot 10^{-5}$
$K(4,1)$ (AK(4,1))	$1.66 \cdot 10^{-2}$
$K(5,1)$ (AK(5,1))	$1.03 \cdot 10^{-3}$
$K(6,1)$ (AK(6,1))	$-1.26 \cdot 10^{-4}$
$K(7,1)$ (AK(7,1))	$1.63 \cdot 10^{-6}$
$K(8,1)$ (AK(8,1))	$2.54 \cdot 10^{-3}$
$K(1,2)$ (AK(1,2))	$8.82 \cdot 10^{-2}$
$K(2,2)$ (AK(2,2))	0.481
$K(3,2)$ (AK(3,2))	$7.86 \cdot 10^{-3}$
$K(4,2)$ (AK(4,2))	$-2.90 \cdot 10^{-3}$
$K(5,2)$ (AK(5,2))	$2.98 \cdot 10^{-3}$
$K(6,2)$ (AK(6,2))	$2.00 \cdot 10^{-2}$
$K(7,2)$ (AK(7,2))	$-1.68 \cdot 10^{-3}$
$K(8,2)$ (AK(8,2))	$7.61 \cdot 10^{-6}$
$K(1,3)$ (AK(1,3))	$-2.40 \cdot 10^{-2}$
$K(2,3)$ (AK(2,3))	$9.65 \cdot 10^{-2}$
$K(3,3)$ (AK(3,3))	$3.14 \cdot 10^{-3}$

K(4,3) (AK(4,3))	$5.65 \cdot 10^{-3}$
K(5,3) (AK(5,3))	$-5.63 \cdot 10^{-3}$
K(6,3) (AK(6,3))	$-6.69 \cdot 10^{-5}$
K(7,3) (AK(7,3))	$3.02 \cdot 10^{-3}$
K(8,3) (AK(8,3))	$1.24 \cdot 10^{-5}$
K(1,4) (AK(1,4))	-0.367
K(2,4) (AK(2,4))	1.02
K(3,4) (AK(3,4))	0.326
K(4,4) (AK(4,4))	$6.51 \cdot 10^{-2}$
K(5,4) (AK(5,4))	$-6.49 \cdot 10^{-2}$
K(6,4) (AK(6,4))	$-5.06 \cdot 10^{-3}$
K(7,4) (AK(7,4))	$-5.77 \cdot 10^{-2}$
K(8,4) (AK(8,4))	$1.41 \cdot 10^{-4}$
$t'_\delta$ (TEST(1))	1
$t'_v$ (TEST(2))	0.06
$t'_r$ (TEST(3))	0.25
$t'_\psi$ (TEST(4))	0.015
$V_c$ (VCONST)	8
VMIN	0.2
VMAX	12
$V_0$ (V0)	6
TH0(1) (initial value of $-a_1$ )	6.91
TH0(2) ( " - $-a_2$ )	-5.95
TH0(3) ( " - $-a_3$ )	-3.88
TH0(4) ( " - $-a_4$ )	3.57
TH0(5) ( " - $b_1$ )	0.48
TH0(6) ( " - $b_2$ )	0.11
TH0(7) ( " - $c_1$ )	-2.10
TH0(8) ( " - $c_2$ )	34.73

TH0(9)	0
TH0(10)	0
PP0(1) (initial value of P(1,1))	1
PP0(2) ( " - P(2,2))	1
PP0(3) ( " - P(3,3))	1
PP0(4) ( " - P(4,4))	1
PP0(5) ( " - P(5,5))	0.01
PP0(6) ( " - P(6,6))	0.01
PP0(7) ( " - P(7,7))	1
PP0(8) ( " - P(8,8))	100
PP0(9)	0
PP0(10)	0
$\lambda_f$ (RL)	0.99
$b_0$ (B0)	1
$q$ (Q2)	0
$k_P$ (AK1)	3
$k_D$ (AK2)	75
$k_I$ (AK3)	0.02
$\psi_1$ (Y(1))	0.35
$\psi_2$ (Y(2))	2.5
$\psi_3$ (Y(3))	2.5
$\varepsilon_1$ (Y(4))	0
$\varepsilon_2$ (Y(5))	0.02
$\varepsilon_3$ (Y(6))	1
$\bar{c}_1$ (Y(7))	60
$\bar{c}_2$ (Y(8))	50
$\bar{c}_3$ (Y(9))	60
$k_1$ (Y(10))	5
$k_2$ (Y(11))	200

$k_3$ (Y(12))	0.005
$k_4$ (Y(13))	200
$k_5$ (Y(14))	200
$k_6$ (Y(15))	8
$k_7$ (Y(16))	2
$k_8$ (Y(17))	200
$\gamma$ (BD)	0.05
BV	0.05
$\lambda$ (ALAM)	0.0833333
$\delta_{amp}$ (DELAMP)	3
PSI0	0
$k_{id}$ (AKID)	0
$T_k$ (IREGK)	1
IKAL	1
IKMX	900
IMX	10
IVVC	1
ILOS	1
IREGYT	5
$T_s$ (IREG)	10
IPID	0
IST	2
NA	4
NB	2
NC1	1
NC2	1
$k$ (K)	7
IREGY	10

IYAW	2
T <sub>1</sub> (IT1Y)	30
T <sub>3</sub> (IT3Y)	100
T <sub>4</sub> (IT4Y)	300
IPK	0
IPC	0
IPR	0

```

SUBROUTINE AOTP3
C
C   AUTOPILOT FOR SHIP, INCLUDING KALMAN FILTER,
C   SELF-TUNING REGULATOR AND PID-REGULATOR FOR
C   STRAIGHT COURSE KEEPING, AND YAW REGULATOR.
C
C   AUTHOR, C.KALLSTROM 1976-02-22,
C   REVISED, C.KALLSTROM 1976-04-04.
C
C   SUBROUTINE REQUIRED
C       STUR
C       COSC
C
COMMON /DATA/ ITIME, IDELC, MODYAW, IDEXP, ISTBD, IPORT,
*   IFLAG, IPRINT, INAUT, IKX, MEAS(4),
*   DELCO, DELTA(2), V1(2), R(2), PSI(2), DEL0, DELCOM, DELTAS, U, AN,
*   P, VEST, PSIREF, RREF, DLIM, V(2), DELTA0, D1, D2, D3, TH(10),
*   CGR, CKM, PI, PI2, AL, AL1, A(18),
*   AK(8,4), TEST(4), VCONST, VMIN, VMAX, V0, TH0(10),
*   PP0(10), RL, B0, Q2, AK1, AK2, AK3, Y(17),
*   BD, BV, ALAM, DELAMP, PS10, AK10,
*   IREGK, IKAL, IKMX, IMX, IVVC, ILOS, IREGYT, IREG, IPID,
*   IST, NA, NB, NC1, NC2, K, IREGY, IYAW, IT1Y, IT3Y, IT4Y, IPK, IPC, IPR,
*   EPS(4), VV, PP(55), VLOS(8),
*   CN, EDELTA, EPS12, VLOS2, ENM2,
*   MEASUM(4), IVV1,
*   X(8), VVV(3), V0V, V0V2,
*   PF01, PF02, DELOLD, VOLD, ROLD, DAT(46), SINT, RRF, AINT2,
*   AINT4, STD, STV, SL1, SL2, SL3, SL4, SL5, DUM(10),
*   IRK, IMEAS(4), IRYT, IR, NC, NAB, NP, K1, NDAT, NDAT1,
*   NU1, N1, IRY, ITIM1, ITIM3, ITIM4, IP, I, J, L
C
C   COMPUTE THE SWAY VELOCITY V(1).
C
C   V(1)=V1(1)-AL1*R(1)*CGR/CKM
C
C   IF(INAUT) 80,80,10
C
C   INITIALIZE IF INAUT=1.
C
10   IVV1=IVVC
    VVV(1)=U*CKM
    VVV(2)=AN*0.18*CKM
    VVV(3)=VCONST
12   IF(VVV(IVV1)-VMIN) 16,14,14
14   IF(VVV(IVV1)-VMAX) 18,18,16
16   IVV1=IVV1+1
    GO TO 12
18   VV=VVV(IVV1)
    V0V=V0/VV
    V0V2=V0V*V0V
    VEST=VV/CKM
C
    IRK=IREGK
    IKX=0
    DELCOM=0.
    DELCO=0.
    IF(IKAL) 50,50,20
C
20   IF(MEAS(1)) 22,22,24
22   X(4)=DELTA(1)*CGR
    GO TO 26
24   X(4)=0.

```

```

26   IF(MEAS(3)) 28,28,30
28   X(2)=R(1)*AL*CGR/VV
    GO TO 32
30   X(2)=0.
    GO TO 36
32   IF(MEAS(2)) 34,34,36
34   X(1)=V(1)*CKM/VV
    GO TO 38
36   X(1)=0.
38   IF(MEAS(4)) 40,40,999
40   X(3)=PSI(1)*CGR
    X(5)=0.
    X(6)=0.
    X(7)=0.
    X(8)=0.
    DO 42 I=1,4
    IMEAS(I)=0
    MEASUM(I)=0
42   C
50   IRYT=IREGYT
    PFO1=PSI(1)
    MODYAW=0
    IR=IREG
    NC=NC1+NC2
    NAB=NA+NB
    NP=NAB+NC
    K1=K+1
    NDAT=NAB+(K1+1)*(NC+2)-2
    NDAT1=NDAT+1
    NU1=NA+K+2
    N1=NU1+K
    J=NU1-1
    SL1=PSI(1)-PSIREF
    IF(SL1 .LE. -180.) SL1=SL1+360.
    IF(SL1 .GT. 180.) SL1=SL1-360.
    DO 52 I=1,J
52   DAT(I)=SL1
    J=J+1
    DO 54 I=J,46
54   DAT(I)=0.
    DELOLD=0.
    VOLD=0.
    ROLD=0.
    SINT=0.
    DO 60 I=1,10
    DO 60 J=1,I
    L=I*(I-1)/2+J
    IF(I-J) 58,56,58
56   PP(L)=PP0(I)
    GO TO 60
58   PP(L)=0.
60   CONTINUE
    DO 62 I=1,10
62   TH(I)=TH0(I)
    EDELTA=0.
    STD=1.-BD
    EPSI2=0.
    VLOS2=0.
    ENM2=0.
    STV=1.-BV
    ISTBD=0
    IPORT=0
    IP=IPR
    INAUT=0

```



```

80   IF(IREGK) 200,200,90
90   IF(IRK-IREGK) 180,100,200
C
C   LOOP WITH SAMPLING INTERVAL IREGK.
C
100  IRK=1
     IF(IKAL) 140,140,102
C
C   KALMAN FILTER.
C
102  SL5=CGR*DELCOM
     SL1=A(1)*X(1)+A(2)*X(2)+A(3)*X(4)+A(4)*X(5)+A(15)*SL5
     SL2=A(5)*X(1)+A(6)*X(2)+A(7)*X(4)+A(8)*X(5)+A(16)*SL5
     SL3=A(9)*X(1)+A(10)*X(2)+X(3)+A(11)*X(4)+A(12)*X(5)+A(17)*SL5
     SL4=A(13)*X(4)+A(14)*X(5)+A(18)*SL5
     X(1)=SL1
     X(2)=SL2
     X(3)=SL3
     X(4)=SL4
C
     EPS(1)=DELTA(1)*CGR-X(4)-X(5)-X(8)
     EPS(2)=V1(1)*CKM/VV-X(1)-X(2)*AL1/AL-X(6)
     EPS(3)=R(1)*CGR*AL/VV-X(2)-X(7)
     EPS(4)=PSI(1)*CGR-X(3)
     IF(EPS(4) .LE. -PI) EPS(4)=EPS(4)+PI2
     IF(EPS(4) .GT. PI) EPS(4)=EPS(4)-PI2
C
     IF(IKX-IKMX) 104,108,108
104  IKX=IKX+1
     DO 106 I=1,4
106  IMEAS(I)=0
     GO TO 122
C
108  DO 120 I=1,4
     IF(MEAS(I)) 112,112,118
112  IF(ABS(EPS(I))-TEST(I)) 118,118,114
114  IMEAS(I)=IMEAS(I)+1
     MEASUM(I)=MEASUM(I)+1
     IF(IMEAS(I)-IMX) 120,116,116
116  MEAS(I)=1
118  IMEAS(I)=0
120  CONTINUE
C
122  DO 134 I=1,4
     IF(MEAS(I)+IMEAS(I)) 130,130,134
130  DO 132 J=1,8
     IF((J .EQ. 8) .AND. (MEAS(1) .GT. 0)) GO TO 132
     IF((J .EQ. 6) .AND. (MEAS(2) .GT. 0)) GO TO 132
     IF((J .EQ. 7) .AND. (MEAS(3) .GT. 0)) GO TO 132
     X(J)=X(J)+AK(J,I)*EPS(I)
132  CONTINUE
134  CONTINUE
C
     IF(X(3) .LT. 0.) X(3)=X(3)+PI2
     IF(X(3) .GE. PI2) X(3)=X(3)-PI2
C
     V(2)=VV*X(1)/CKM
     R(2)=VV*X(2)/(AL*CGR)
     PSI(2)=X(3)/CGR
     DELTA(2)=(X(4)+X(5))/CGR
     V1(2)=VV*(X(1)+X(2)*AL1/AL)/CKM
     DELTA0=X(5)/CGR
     D1=VV*X(6)/CKM
     D2=VV*X(7)/(AL*CGR)

```

```

      D3=X(8)/CGR
C
C      COMPUTE THE FORWARD SPEED VV.
C
140  VVV(1)=U*CKM
      VVV(2)=AN*0.18*CKM
      VVV(3)=VCONST
C
148  IF(VVV(1)-VMIN) 152,150,150
150  IF(VVV(1)-VMAX) 154,154,152
152  IVV1=IVV1+1
      GO TO 148
154  VV=VVV(1)
      VOV=V0/VV
      VOV2=VOV*VOV
      VEST=VV/CKM
C
C      COMPUTE THE LOSS FUNCTIONS.
C
160  IF(ILOS) 170,166,162
162  ILOS=0
      CN=0.
      DO 164 I=1,8
164  VLOS(I)=0.
166  CN=CN+1.
      SL1=(CN-1.)/CN
      DO 168 I=1,2
      SL2=PSI(I)-PSIREF
      IF(SL2 .LE. -180.) SL2=SL2+360.
      IF(SL2 .GT. 180.) SL2=SL2-360.
      VLOS(I)=SL1*VLOS(I)+SL2/CN
      L=I+2
      VLOS(L)=SL1*VLOS(L)+DELTA(I)/CN
      L=L+2
      J=L-2
      VLOS(L)=SL1*VLOS(L)+(SL2*SL2+ALAM*(DELTA(I)-VLOS(J))*
* (DELTA(I)-VLOS(J)))/CN
      L=L+2
168  VLOS(L)=SL1*VLOS(L)+P/(CKM*U*COSC(SL2*CGR)*CN)
C
170  IF(IPK .GT. 0) IPRINT=1
      GO TO 200
C
180  IRK=IRK+1
C
200  IF(IDEXP) 201,201,700
C
C      LOOP WITH SAMPLING INTERVAL IREGYT FOR YAW TEST.
C
201  IF(IRYT-IREGYT) 206,202,208
C
202  IRYT=1
      SL1=PSIREF-PF01
      PF01=PSIREF
      IF(MODYAW) 204,204,300
204  PF02=PSIREF-SL1
      IF(SL1 .LE. -180.) SL1=SL1+360.
      IF(SL1 .GT. 180.) SL1=SL1-360.
      IF(ABS(SL1)-Y(1)) 500,500,302
C
206  IRYT=IRYT+1
C
208  IF(MODYAW) 500,500,300
C

```

```

C      LOOP WITH SAMPLING INTERVAL IREGY FOR YAWING.
C
300    IF(IRY-IREGY) 390,302,392
C
302    IRY=1
        SL1=PSIREF-PF02
        PF02=PSIREF
        IF(SL1 .LE. -180.) SL1=SL1+360.
        IF(SL1 .GT. 180.) SL1=SL1-360.
        SL2=PSI(IYAW) - PSIRFF
        IF(SL2 .LE. -180.) SL2=SL2+360.
        IF(SL2 .GT. 180.) SL2=SL2-360.
        IF(MODYAW) 309,309,304
304    IF(ABS(SL1)-Y(3)) 320,320,306
306    IF(MODYAW-2) 314,314,308
308    MODYAW=1
        ITIM1=-IREGY
309    IF(SL2) 310,310,312
310    RRF=RREF
        GO TO 320
312    RRF=-RREF
        GO TO 320
314    IF(RRF) 316,316,318
316    IF(SL2) 308,320,320
318    IF(SL2) 320,320,308
C
320    RRF=RREF*ABS(RRF)/RRF
        SL3=R(IYAW)-RRF
C
        IF(MODYAW) 322,322,324
322    IF(ABS(SL1)-Y(2)) 338,338,326
324    IF(MODYAW-2) 328,332,325
325    IF(MODYAW-4) 336,339,339
C
326    MODYAW=1
        ITIM1=-IREGY
328    ITIM1=ITIM1+IREGY
        IF(RRF .GE. 0. ,AND. SL3 .GT. -Y(4)) GO TO 330
        IF(RRF .LT. 0. ,AND. SL3 .LT. Y(4)) GO TO 330
        IF(ITIM1-IT1Y) 332,332,330
C
330    MODYAW=2
        AINT2=0.
332    IF(RRF .GE. 0. ,AND. -Y(8)*R(IYAW) .LT. SL2) GO TO 334
        IF(RRF .LT. 0. ,AND. -Y(8)*R(IYAW) .GT. SL2) GO TO 334
        IF(MODYAW-1) 340,340,350
C
334    MODYAW=3
        ITIM3=-IREGY
336    ITIM3=ITIM3+IREGY
        IF(ABS(R(IYAW)) .LT. Y(5)) GO TO 338
        IF(RRF .GE. 0. ,AND. SL2 .GT. -Y(6)) GO TO 338
        IF(RRF .LT. 0. ,AND. SL2 .LT. Y(6)) GO TO 338
        IF(ITIM3-IT3Y) 360,360,338
C
338    MODYAW=4
        ITIM4=-IREGY
        AINT4=0.
339    ITIM4=ITIM4+IREGY
        IF(ITIM4-IT4Y) 370,370,400
C
C      YAW PHASE 1.
C
340    SL4=Y(13)*SL3

```

```

SL5=ABS(Y(7)*RRF)
IF(SL4 .GT. SL5) SL4=SL5
IF(SL4 .LT. -SL5) SL4=-SL5
DELCOM=-V0V2*SL4+EDELTA
GO TO 380

C
C   YAW PHASE 2.
C
350  DELCOM=-V0V2*(Y(14)*SL3+Y(15)*AINT2)+EDELTA
      AINT2=AINT2+SL3*FLOAT(IREGY)
      GO TO 380

C
C   YAW PHASE 3.
C
360  SL4=Y(16)*SL2+Y(17)*R(IYAW)
      SL5=ABS(Y(9)*RRF)
      IF(SL4 .GT. SL5) SL4=SL5
      IF(SL4 .LT. -SL5) SL4=-SL5
      DELCOM=-V0V2*SL4
      GO TO 380

C
C   YAW PHASE 4.
C
370  DELCOM=-V0V2*(Y(10)*SL2+Y(11)*R(IYAW)+Y(12)*AINT4)
      AINT4=AINT4+SL2*FLOAT(IREGY)

C
C
380  IF(DELCOM .GT. DLIM) DELCOM=DLIM
      IF(DELCOM .LT. -DLIM) DELCOM=-DLIM
      GO TO 600

C
390  IRY=IRY+1
C
392  GO TO 900

C
C   INITIALIZING OF STRAIGHT COURSE KEEPING.
C
400  J=NU1-1
      SL1=PSI(IST)-PSIREF
      IF(SL1 .LE. -180.) SL1=SL1+360.
      IF(SL1 .GT. 180.) SL1=SL1-360.
      DO 402 I=1,J
402  DAT(I)=SL1
      J=J+1
      DO 404 I=J,46
404  DAT(I)=0.
      DELOLD=EDELTA
      VOLD=V(IST)
      ROLD=R(IST)
      MODYAW=0
      SINT=0.
      GO TO 502

C
C   LOOP WITH SAMPLING INTERVAL IREG FOR STRAIGHT COURSE KEEPING,
C
500  IF(IR-IREG) 540,502,542
C
502  IR=1
      SL1=PSI(IST)-PSIREF
      IF(SL1 .LE. -180.) SL1=SL1+360.
      IF(SL1 .GT. 180.) SL1=SL1-360.
      IF(IPID) 504,504,520

C
504  SL2=V(IST)-VOLD

```

```

SL3=R(IST)-ROLD
VOLD=V(IST)
ROLD=R(IST)
DAT(1)=SL1
IF(NC-1) 514,506,508
506 IF(NC1) 512,512,508
508 J=NAB+2*K+3
DAT(J)=SL2*VV
IF(NC-1) 514,514,510
510 J=NAB+3*K+5
DAT(J)=SL3
GO TO 514
512 J=NAB+2*K+3
DAT(J)=SL3
C
514 CALL STUR(DAT,TH,PP,DUM,RL,NA,NAB,NP,K1,NDAT,NDAT1,NU1,N1)
C
SL2=V0V2*B0/(B0*B0+Q2)
DELCOM=SL2*DAT(NU1)+DELOLD
GO TO 530
C
520 DELCOM=-V0V2*(AK1*SL1+AK2*R(IST)+AK3*SINT)
SINT=SINT+SL1*FLOAT(IREG)
C
530 IF(DELCOM .GT. DLIM) DELCOM=DLIM
IF(DELCOM .LT. -DLIM) DELCOM=-DLIM
IF(IDEXP) 531,531,800
531 IF(IPID) 532,532,534
532 DAT(NU1)=B0*(DELCOM-DELOLD)/V0V2
DELOLD=DELCOM
534 GO TO 600
C
540 IR=IR+1
C
542 GO TO 900
C
C COMPUTE THE MEAN RUDDER COMMAND EDELTA AND THE LOSS FUNCTIONS.
C
600 IF(MODYAW) 604,604,602
602 IF(MODYAW-4) 800,604,604
604 EDELTA=EDELTA+(STD+BD)*(DELCOM-EDELTA)
STD=(1.-BD)*STD/(1.-RD+STD)
IF(MODYAW) 606,606,800
606 EPSI2=EPSI2+(STV+BV)*(SL1-EPSI2)
SL2=DELCOM-EDELTA
SL3=SL1*SL1+ALAM*SL2*SL2
VLOS2=VLOS2+(STV+BV)*(SL3-VLOS2)
SL2=P/(CKM*U*COSC(SL1*CGR))
ENM2=ENM2+(STV+BV)*(SL2-ENM2)
STV=(1.-BV)*STV/(1.-BV+STV)
GO TO 800
C
C IDENTIFICATION EXPERIMENT.
C
700 IF(IR-IREG) 720,702,900
C
702 IR=1
IF(ISTBD+IPORT-1) 712,704,710
704 IF(ISTBD) 708,708,706
706 DEL0=DELAMP
GO TO 712
708 DEL0=-DELAMP
GO TO 712
710 DEL0=0.

```

```
712  ISTBD=0
      IPORT=0
      SL1=PSI(1)-PSI0
      IF(SL1 .LE. -180.) SL1=SL1+360.
      IF(SL1 .GT. 180.) SL1=SL1-360.
      DELCOM=DELO-AKID*SL1
      GO TO 530

C
720  IR=IR+1
      GO TO 900

C
C      INDICATE RUDDER CHANGE.
C
800  DELCO=DELCOM
      IDELC=1

C
      IF(IPC .GT. 0) IPRINT=1

C
900  IF(IPR) 999,999,902
C
902  IF(IP-IPR) 906,904,999
C
904  IP=1
      IPRINT=1
      GO TO 999

C
906  IP=IP+1
C
999  RETURN
      END
```

SUBROUTINE STUR(DAT,TH,P,DUM,RL,NA,NAB,NP,K1,NDAT,NDAT1,NU1,N1)

SELFTUNING REGULATOR BASED ON LEAST SQUARES IDENTIFICATION  
AND MINIMUM VARIANCE CONTROL, ADMITS FEEDFORWARD AND  
EXPLOITS SYMMETRY OF P.

AUTHOR. C.KALLSTROM 1976-02-18.

THE ALGORITHM IS BASED ON THE MODEL

$$Y(T)+A(1)*Y(T-K-1)+\dots+A(NA)*Y(T-K-NA)=$$

$$B0*(U(T-K-1)+B(1)*U(T-K-2)+\dots+B(NB)*U(T-K-NB-1))+$$

$$C(1)*V1(T-K-1)+C(2)*V2(T-K-1)+\dots+C(NC)*VNC(T-K-1)+EPS(T)$$

AT EACH STEP THE LEAST SQUARES ESTIMATES OF THE PARAMETERS  
OF THE MODEL ARE COMPUTED. THE CONTROL VARIABLE U(T) TO  
BE APPLIED AT TIME T IS THEN COMPUTED FROM

$$US(T)= AE(1)*Y(T)+\dots+AE(NA)*Y(T-NA+1)$$

$$-BE(1)*US(T-1)-\dots-BE(NB)*US(T-NB)$$

$$-CE(1)*V1(T)-\dots-CE(NC)*VNC(T)$$

WHERE AE,BE AND CE ARE THE PARAMETER ESTIMATES  
AND US THE SCALED CONTROL SIGNAL I.E.  $US=B0*U$

WHEN USING THE ALGORITHM THE PROCESS OUTPUT Y(T) AND THE  
FEEDFORWARD SIGNALS V(T) ARE READ AT TIME T AND THE CONTROL  
SIGNAL U(T) TO BE APPLIED AT TIME T IS THEN COMPUTED

DAT- VECTOR OF DIMENSION  $NA+NB+(K+2)*(NC+2)-2$  CONTAINING  
PROCESS OUTPUTS Y, SCALED CONTROL VARIABLES U  
AND FEED FORWARD SIGNALS V ORGANIZED AS FOLLOWS

DAT(1)=Y(T)	RETURNED AS Y(T)
DAT(2)=Y(T-1)	RETURNED AS Y(T)
DAT(3)=Y(T-2)	RETURNED AS Y(T-1)
.	
DAT(NA+K+1)=Y(T-K-NA)	RETURNED AS Y(T-K-NA+1)
DAT(NA+K+2)=US(T-1)	RETURNED AS US(T)
DAT(NA+K+3)=US(T-2)	RETURNED AS US(T-1)
.	
DAT(NA+NB+2*K+2)=US(T-K-NB-1)	RETURNED AS US(T-K-NB)
DAT(NA+NB+2*K+3)=V1(T)	RETURNED AS US(T-K-NB-1)
DAT(NA+NB+2*K+4)=V1(T-1)	RETURNED AS V1(T)
.	
DAT(NA+NB+3*K+4)=V1(T-K-1)	RETURNED AS V1(T-K)
.	
DAT(NA+NB+(K+2)*(NC+1)-1)=VNC(T)	RETURNED AS V(NC-1)(T-K-1)
.	
DAT(NA+NB+(K+2)*(NC+2)-2)=VNC(T-K-1)	RETURNED AS VNC(T-K)

TH- VECTOR OF DIMENSION  $NP=NA+NB+NC$  CONTAINING THE PARAMETER  
ESTIMATES ORGANIZED AS FOLLOWS

TH(1)=-AE(1)  
TH(2)=-AE(2)  
.  
TH(NA)=-AE(NA)  
TH(NA+1)=BE(1)  
TH(NA+2)=BE(2)  
.  
TH(NA+NB)=BE(NB)  
TH(NA+NB+1)=CE(1)  
TH(NA+NB+2)=CE(2)

```

C      .
C      TH(NA+NB+NC)=CF(NC)
C
C      P- COVARIANCE MATRIX STORED AS FOLLOWS
C      P(1)=P(1,1)
C      P(2)=P(2,1)
C      P(3)=P(2,2)
C      .
C      P(I*(I-1)/2+J)=P(I,J)
C      .
C      P(NP*(NP+1)/2)=P(NP,NP)
C
C      DUM- DUMMY VECTOR OF DIMENSION NP
C      RL- BASE OF EXPONENTIAL WEIGHTING FACTOR
C      NA- NUMBER OF A-PARAMETERS (MAX 10, MIN 0)
C      NB- NUMBER OF B-PARAMETERS (MAX 10, MIN 0)
C      NC- NUMBER OF C-PARAMETERS (MAX 10, MIN 0)
C      K -NUMBER OF TIME DELAYS IN THE MODEL
C      (MAX ((46-NA-NB+2)/(NC+2))-2 , MIN 0)
C      NAB- NA+NB
C      NP- NA+NB+NC (MAX 10, MIN 1)
C      K1- K+1
C      NDAT- NAB+(K1+1)*(NC+2)-2
C      NDAT1- NDAT+1
C      NU1- NA+K+2
C      N1- NU1+K
C
C      SUBROUTINE REQUIRED
C      NONE
C
C      DIMENSION DAT(46),TH(10),P(55),DUM(10)
C
C      RES=DAT(1)-DAT(N1)
C      DENOM=1.
C      DO 12 I=1,NP
C      R=0.
C      DO 10 J=1,NP
C      L=I*(I-1)/2+J
C      IF (J.GT.I) L=J*(J-1)/2+I
C      M=K1+J
C      IF (J.GT.NA) M=M+K1
C      IF (J.GT.NAB) M=2*K1+(J-NAB)*(K1+1)+NAB
10    R=R+P(L)*DAT(M)
C      DUM(I)=R
C      M=K1+I
C      IF (I.GT.NA) M=M+K1
C      IF (I.GT.NAB) M=2*K1+(I-NAB)*(K1+1)+NAB
12    DENOM=DENOM+R*DAT(M)
C      RES=RES-DAT(M)*TH(I)
C
C      DO 20 I=1,NP
C      R=DUM(I)/DENOM
C      TH(I)=TH(I)+R*RES
C      DO 20 J=1,I
C      L=I*(I-1)/2+J
20    P(L)=(P(L)-R*DUM(J))/RL
C
C      R=0.
C      DO 30 I=1,NP
C      L=I
C      IF (I.GT.NA) L=L+K1
C      IF (I.GT.NAB) L=NAB+K1+(K1+1)*(I-NAB)

```



```
30 R=R-TH(I)*DAT(L)
C
DO 32 I=2,NDAT
L=NDAT1-I
32 DAT(L+1)=DAT(L)
DAT(NU1)=R
C
RETURN
END
```

## APPENDIX D - EXPERIMENTS

Plots from all the experiments, excluding F1 - F3 where no recording of data was performed, are shown in this appendix. Several plots are often shown in the same figure, and the plots are then slided in relation to each other. The corresponding straight line is the level zero. Usually the data were punched on paper tape every time a new rudder command  $\delta_c$  was computed, i.e. with sampling interval  $T_s$ . However, during experiments B1 - B3 the data were punched every second. In experiments B1 - B3 a yaw rate estimate  $\hat{r}_d$  is computed from the heading  $\psi$  and plotted.

The Kalman filter was running during all the experiments, including E1 - E4, but the estimates were not always used by the regulators. If nothing else is remarked, all measurement signals  $v_1$ ,  $r$ ,  $\psi$  and  $\delta$  were used by the Kalman filter. If the value of the rudder limit is not given, the rudder limit has not been active during the experiment. The water below the keel was deep, except during the last 15 min of experiment D1 when the water depth was about 100 m.

Experiments of straight course keeping are shown in Table D.1. A summary of experiments of Kalman filter testing is given in Table D.2, and the yawing experiments are summarized in Table D.3. Normal operating experiments are shown in Table D.4. Note that yaws were performed during experiment D1, while D2 and D3 are straight course keeping experiments. Table D.5 shows the experiments for identification and Table D.6 the experiments of straight course keeping, where no recording of data was performed. A summary of the final parameter values of the self-tuning regulator is given in Table D.7.

Ex-periment	Dura-tion [min]	Draught		Rel. wind dir.	Wind velocity [m/s]	$T_s$ [s]	$V_0$ [m/s]	IVVC	Structure of self-tuning reg.			Parameters of PID-reg.			
		For-ward [m]	Aft [m]						NC1	NC2	k	q	$k_P$	$k_D$ [s]	$k_I$ [1/s]
A1	51	8.5	12.5	3	9	10	8	3	1	1	7	0	-	-	-
A2	26	8.5	12.5	5	5	10	7	1	1	1	7	0	-	-	-
A3	24	8.5	12.5	5	10	10	7	1	1	1	5	0	-	-	-
A4	24	8.5	12.5	5	5	10	7	1	1	1	6	0	-	-	-
A5	24	8.5	12.5	5	4	10	7	1	1	1	7	0	-	-	-
A6	24	8.5	12.5	4	4	10	7	1	1	1	7	0	-	-	-
A7	25	8.5	12.5	3	4	10	7	1	1	1	8	0	-	-	-
A8	24	8.5	12.5	5	4	15	6	1	1	1	4	0	-	-	-
A9	25	8.5	12.5	5	4	15	6	1	1	1	5	0	-	-	-
A10	33	8.5	12.5	5	3	10	6	1	1	1	7	0	-	-	-
A11	24	8.5	12.5	5	3	10	6	1	1	1	6	0.05	-	-	-
A12	26	8.5	12.5	5	3	10	6	1	1	1	6	0.1	-	-	-
A13	24	8.5	12.5	6	3	10	6	1	1	1	6	0.2	-	-	-
A14	24	8.5	12.5	-	0	10	6	1	1	1	7	0	-	-	-
A15	25	8.5	12.5	-	0	10	6	1	1	0	7	0	-	-	-
A16	24	8.5	12.5	1	5	10	6	1	0	1	7	0	-	-	-
A17	25	8.5	12.5	1	5	10	6	1	0	0	7	0	-	-	-
A18	24	8.5	12.5	1	5	10	6	1	0	0	7	0	-	-	-
A19	25	8.5	12.5	1	9	10	6	1	1	1	7	0	-	-	-
A20	25	8.5	12.5	1	9	10	6	1	-	-	-	-	3	75	0.02

Table D.1a - Summary of experiments of straight course keeping. An explanation of the relative wind direction is given in Appendix A. If nothing else is remarked, all measurement signals  $v_1$ ,  $r$ ,  $\psi$  and  $\delta$  were used by the Kalman filter and the tuning time before the experiment started was equal to 30 min. The estimates from the Kalman filter were used by the regulator, if it is not remarked that non-filtered measurements were used.

Ex- peri- ment	$\delta_c$		$\psi - \psi_{ref}$		Mean value			$V_1$	Remarks
	Mean value [deg]	Std. dev. [deg]	Mean value [deg]	Std. dev. [deg]	n	u [knots]	$P_s$ [MW]		
A1	-0.83	2.24	-0.004	0.161	72.3	14.2	16.4	0.444	Defect rudder servo. $v_1$ not used by filter. Tuning time: 0 min. $V_C = 7$ m/s. Tuning time: > 120 min
A2	0.41	1.14	0.017	0.182	69.6	13.9	14.5	0.142	
A3	0.74	1.20	0.009	0.239	69.4	14.2	14.5	0.177	
A4	1.39	1.41	-0.009	0.246	68.5	14.0	14.0	0.226	
A5	0.66	1.17	0.014	0.219	69.7	14.0	14.4	0.162	
A6	-0.73	1.32	-0.051	0.290	69.1	13.5	13.9	0.232	
A7	-0.17	1.19	0.034	0.260	69.5	14.1	14.2	0.187	
A8	0.17	1.46	-0.027	0.309	69.2	13.9	14.1	0.274	
A9	0.05	1.24	-0.022	0.301	69.0	13.8	14.0	0.219	
A10	0.08	1.62	0.013	0.344	69.0	13.7	14.1	0.337	
A11	-0.06	1.79	-0.033	0.400	69.2	13.9	14.2	0.428	The speed was reduced. Tuning time: > 120 min
A12	0.05	1.69	0.013	0.444	69.2	13.7	14.2	0.435	
A13	0.24	1.84	0.016	0.420	66.7	13.6	13.1	0.459	
A14	-0.61	1.66	-0.017	0.111	69.7	13.5	14.6	0.242	
A15	-0.68	0.82	0.015	0.158	69.4	13.3	14.4	0.081	
A16	-0.43	0.62	-0.066	0.169	69.3	12.9	14.4	0.065	Non-filt. meas. used. Non-filt. meas. used. Stand. param. values
A17	0.01	0.89	-0.188	0.247	69.2	12.9	14.4	0.162	
A18	0.24	1.01	-0.052	0.245	69.2	12.8	14.3	0.148	
A19	0.45	1.11	-0.005	0.176	69.3	13.0	14.4	0.134	
A20	0.53	2.48	0.026	0.556	69.5	13.1	14.5	0.822	

Table D.1a - Contd.

Experiment	Duration [min.]	Draught		Rel. wind dir.	Wind velocity [m/s]	$T_s$ [s]	$V_0$ [m/s]	IWC	Structure of self-tuning reg.				Parameters of PID-reg.		
		Forward [m]	Aft [m]						NC1	NC2	k	q	$k_P$	$k_D$ [s]	$k_I$ [1/s]
A21	25	8.5	12.5	1	4	10	6	1	-	-	-	3	75	0.02	
A22	24	8.5	12.5	1	13	10	6	1	1	7	0	-	-	-	
A23	25	8.5	12.5	1	13	10	6	1	1	7	0	-	-	-	
A24	25	8.5	12.5	1	14	10	6	1	1	7	0	-	-	-	
A25	25	8.5	12.5	1	14	10	6	1	1	7	0	-	-	-	
A26	27	8.5	12.5	1	6	10	6	1	1	7	0	-	-	-	
A27	22	8.5	12.5	1	7	10	6	1	1	7	0	-	-	-	
A28	24	10.9	12.9	1	4	10	6	1	1	7	0	-	-	-	
A29	24	10.9	12.9	1	3	10	6	1	0	7	0	-	-	-	
A30	25	10.9	12.9	1	5	10	6	1	-	-	-	3	120	0.02	
A31	24	10.9	12.9	1	3	10	6	1	-	-	-	2	100	0.02	
A32	25	10.9	12.9	1	9	10	6	1	1	7	0	-	-	-	
A33	51	10.9	12.9	1	10	10	6	1	1	7	0	-	-	-	
A34	48	10.9	12.9	1	10	10	6	1	1	7	0	-	-	-	
A35	45	10.9	12.9	1	11	10	6	1	-	-	-	3	140	0.02	
A36	47	10.9	12.9	1	11	10	6	1	1	7	0	-	-	-	

Table D.1b

Ex- peri- ment	$\delta_c$		$\psi - \psi_{ref}$		Mean value				$V_1$	Remarks
	Mean value [deg]	Std. dev. [deg]	Mean value [deg]	Std. dev. [deg]	n	u	$P_s$	$V_1$		
A21	0.53	0.91	-0.018	0.293	69.3	12.9	14.4	0.155	Stand. parameter values. $\delta$ not used by filter. $V_1$ not used by filter. $r$ not used by filter. After 9.5 min, $\psi$ was not used by filter.	
A22	2.32	0.95	-0.006	0.204	68.9	12.8	14.5	0.117		
A23	1.71	0.94	0.015	0.217	69.0	12.6	14.5	0.121		
A24	1.34	1.25	0.130	0.317	68.9	12.5	14.5	0.248		
A25	1.25	0.90	-0.034	0.240	69.6	12.5	14.9	0.126		
A26	0.24	0.89	0.030	0.230	69.7	13.4	14.9	0.120	$V_1$ , $r$ and $\delta$ not used by filter. Correct K.	
A27	0.30	0.92	0.043	0.197	68.9	13.2	14.4	0.111	$V_1$ , $r$ and $\delta$ not used by filter.	
A28	0.16	0.95	0.006	0.239	69.9	13.5	15.0	0.132	Non-filt. meas. used. Tuned param. values.	
A29	-0.01	1.51	0.001	0.324	69.5	13.4	14.6	0.295		
A30	-0.10	0.89	-0.026	0.256	69.6	13.5	14.6	0.132		
A31	0.22	2.41	-0.103	1.085	69.4	13.4	14.5	1.672	Non-filt. meas. used. Tuned param. values. Tuning time: 120 min. Tuning time: > 120 min. Tuning time: > 120 min. Tuned param. values. Tuning time: > 120 min.	
A32	-0.01	0.98	-0.082	0.223	69.9	13.6	14.9	0.136		
A33	-0.24	1.18	0.016	0.181	69.0	13.2	14.4	0.149		
A34	-0.33	0.95	0.001	0.251	87.6	15.8	29.6	0.138		
A35	0.01	1.66	0.008	0.295	68.6	12.1	14.7	0.317		
A36	-0.05	1.08	-0.058	0.206	68.5	12.1	14.7	0.143		

Table D.1b - Contd.

Ex- peri- ment	Dura- tion [min]	Draught		Rel. wind dir.	Wind velo- city [m/s]	Standard dev.			Mean value		
		For- ward [ m ]	Aft [m]			r [deg/s]	$\hat{r}$ [deg/s]	$\hat{r}_d$ [deg/s]	n	u	$P_s$
B1	26	10.9	12.9	1	6	0.022	0.009	0.014	69.2	13.7	14.9
B2	25	10.9	12.9	1	8	0.023	0.007	0.014	70.1	13.3	15.2
B3	26	10.9	12.9	1	15	0.034	0.011	0.021	87.4	15.9	29.5

Table D.2 - Summary of experiments of Kalman filter testing, where the data were punched on paper tape every second. An explanation of the relative wind direction is given in Appendix A. All measurement signals  $v_1$ ,  $r$ ,  $\psi$  and  $\delta$  were used by the Kalman filter.

Ex-periment	Dura-tion [min]	Draught		Rel. wind dir.	Wind velocity [m/s]	T <sub>s</sub> [s]	V <sub>0</sub> [m/s]	IWC	Structure of self-tuning reg.			ψ <sub>ref</sub> [deg]	r <sub>ref</sub> [deg/s]	Mean value			Remarks
		For-ward [m]	Aft [m]						NC1	NC2	k			q	n [rpm]	u [knots]	
C1	41	9.0	12.0	5	9	10	7	1	1	1	7	0	0.05,0.1	69.5	14.2	14.4	The sign of v <sub>1</sub> was incorrect.
C2	33	8.5	12.5	4,5	6	10	6	1	1	1	8	0	0.1	69.3	13.7	14.0	
C3	63	10.9	12.9	1	7	10	6	1	1	1	7	0	0.05,0.1	69.8	13.4	14.9	$\hat{\delta}_0^i, \hat{\delta}_y^i, \hat{\delta}_r^i$ and $\hat{\delta}_\delta^i$ were not updated.
C4	49	10.9	12.9	1	11	10	6	1	1	1	7	0	0.1,0.2	69.1	12.5	14.5	$\hat{\delta}_0^i, \hat{\delta}_y^i, \hat{\delta}_r^i$ and $\hat{\delta}_\delta^i$ were not updated.
C5	27	10.9	12.9	1	8	10	6	1	1	1	7	0	0.1	69.7	13.2	14.7	
C6	72	10.9	12.9	1	12	10	6	1	1	1	7	0	0.1	69.0	13.0	14.4	

Table D.3 - Summary of experiments of yawing. An explanation of the relative wind direction is given in Appendix A. All measurement signals v<sub>1</sub>, r, ψ and δ were used by the Kalman filter. The estimates from the Kalman filter were used by the self-tuning regulator and the yaw regulator.



Ex- peri- ment	Dura- tion [min.]	Draught		Rel. wind dir.	Wind velo- city [m/s]	$T_s$ [s]	$V_0$ [m/s]	IWC	Structure of self-tuning reg.			$\psi_{ref}$ [deg]	$r_{ref}$ [deg/s]	Mean value			Remarks	
		For- ward [m]	Aft [m]						NC1	NC2	k			q	n	u		$P_s$
D1	83	8.5	12.5	7	5	10	8	3	1	1	7	0	180-205	0.1	69.6	14.2	14.7	Water depth was abt 100 m during the last 15 min. Rudder limit: 15 deg. $v_1$ not used by $v_1$ -filter $V_C = 7$ m/s Tuning time: > 120 min.
D2	40	8.5	12.5	5	10	10	7	1	1	1	7	0	180	0.1	69.7	14.2	14.7	Tuning time: 0 min.
D3	68	10.9	12.9	1	9	10	6	1	1	1	7	0	144.3- -147.0	0.1	69.9	13.4	15.4	Sailmaster, Course Corre- tion. Tuning time: 0 min.

Table D.4 - Summary of normal operating experiments. An explanation of the relative wind direction is given in Appendix A. If nothing else is remarked, all measurement signals  $v_1$ ,  $r$ ,  $\psi$  and  $\delta$  were used by the Kalman filter. The estimates from the Kalman filter were used by the self-tuning regulator and the yaw regulator.

Ex-periment	Dura-tion [min]	Draught		Rel. wind dir.	Wind velocity [m/s]	T <sub>s</sub> [s]	IWC	δ <sub>amp</sub> [deg]	k <sub>id</sub>	Mean value			Remarks
		For-ward [m]	Aft [m]							n [rpm]	u [knots]	P <sub>s</sub> [MW]	
E1	89	10.9	12.9	1	11	10	1	3,5	0	69.7	12.5	15.4	5°/5° zig-zag test Rudder limit: 15 deg
E2	19	10.9	12.9	1	14	10	1	5	0	70.5	12.5	15.7	
E3	90	10.9	12.9	2	9	10	1	5	2	70.0	12.9	15.4	
E4	26	10.9	12.9	1	11	10	1	10	0	69.8	12.7	15.3	

Table D.5 - Summary of experiments for identification. An explanation of the relative wind direction is given in Appendix A.

Ex-periment	Dura-tion [min]	Draught		Rel. wind dir.	Wind velocity [m/s]	$T_s$ [s]	$V_0$ [m/s]	IVWC	Structure of self-tuning reg.				Parameters of PID-reg.		
		For-ward [m]	Aft [m]						NC1	NC2	k	q	$k_p$	$k_D$ [s]	$k_I$ [1/s]
F1	60	8.5	12.5	6	3	10	8	3	1	1	7	0	-	-	-
F2	49	8.5	12.5	7	3	10	8	3	-	-	-	-	3	75	0.02
F3	57	10.9	12.9	1	11	10	6	1	1	1	7	0	-	-	-
F4	54	10.9	12.9	1	11	10	6	1	-	-	-	-	3	140	0.02

**Table D.6** - Summary of experiments of straight course keeping, where no data were punched on paper tape. An explanation of the relative wind direction is given in Appendix A. If nothing else is remarked, all measurement signals  $v_1$ ,  $r$ ,  $\psi$  and  $\delta$  were used by the Kalman filter. The estimates from the Kalman filter were used by the regulator, if it is not remarked that non-filtered measurements were used. The rudder limit was 15 deg during the experiments.

Ex- peri- ment	Mean value						V <sub>2</sub>	Remarks
	δ [deg]	ψ-ψ <sub>ref</sub> [deg]	n [rpm]	u [knots]	P <sub>s</sub> [MW]			
F1	1.71	-0.002	69.8	14.4	14.7	0.038	v <sub>1</sub> not used by filter. V <sub>C</sub> = 7 m/s. Sailmaster.	
F2	1.86	-0.016	69.6	14.4	14.7	0.259	Non-filt. meas. used. Stand. param. values. V <sub>C</sub> = 7 m/s. Sailmaster.	
F3	1.67	-0.073	69.1	12.1	15.1	0.160		
F4	1.54	-0.009	69.7	12.2	15.5	0.304	Tuned parameter values.	

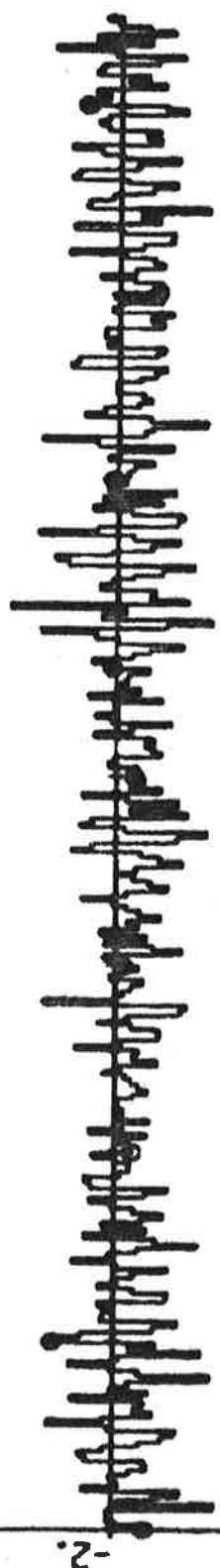
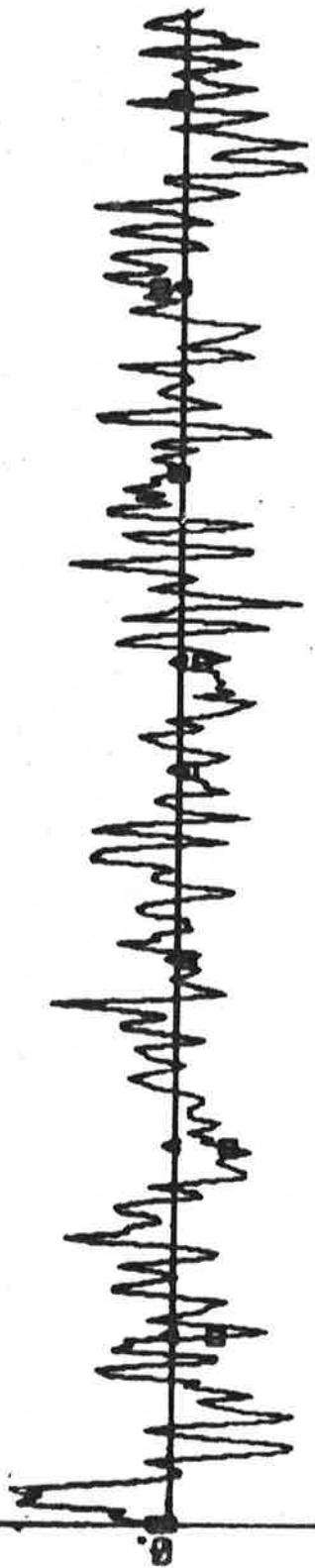
Table D.6 - Contd.

Ex- peri- ment	$a_1$	$a_2$	$a_3$	$a_4$	$b_1$	$b_2$	$c_1$	$c_2$	$\Sigma a_i$
A1	-6.93	6.69	3.50	-3.61	0.51	0.18	-2.02	35.77	-0.35
A2	-15.17	19.33	-2.62	-1.87	0.63	0.52	0.35	113.66	-0.33
A3	-8.60	6.39	4.38	-2.62	0.53	0.26	-2.34	44.87	-0.45
A4	-8.92	6.49	4.88	-2.75	0.48	0.16	-1.79	50.52	-0.30
A5	-8.00	6.67	4.55	-3.59	0.49	0.17	-1.40	49.73	-0.37
A6	-7.23	6.42	4.21	-3.31	0.55	0.16	-0.79	41.84	0.09
A7	-7.00	6.36	4.36	-3.64	0.48	0.09	-1.89	39.41	0.08
A8	-8.29	5.98	4.34	-2.58	0.50	0.18	-2.32	39.92	-0.55
A9	-7.36	5.86	4.03	-2.62	0.51	0.16	-1.97	36.59	-0.09
A10	-9.56	6.81	4.39	-1.97	0.53	0.26	-2.25	40.29	-0.33
A11	-9.16	6.39	4.29	-1.86	0.46	0.22	-2.60	37.18	-0.34
A12	-8.73	7.02	4.66	-3.33	0.29	0.08	-2.89	44.49	-0.38
A13	-9.65	6.81	5.19	-2.67	0.36	0.20	-1.29	44.29	-0.32
A14	-21.78	26.83	-2.30	-3.17	0.50	0.44	-0.54	74.46	-0.42
A15	-7.76	5.70	4.43	-2.61	0.49	0.17	-1.81	-	-0.24
A16	-7.26	6.59	4.46	-4.05	0.42	0.05	-	33.84	-0.26
A17	-7.84	6.75	4.75	-3.89	0.43	0.02	-	-	-0.23
A18	-7.88	6.86	4.67	-3.50	0.45	0.11	-	-	0.15
A19	-8.42	6.56	4.42	-2.96	0.49	0.16	-1.26	45.72	-0.40
A22	-8.04	6.23	4.52	-2.99	0.51	0.16	-1.89	35.94	-0.28
A23	-7.24	6.33	4.12	-3.37	0.50	0.12	-1.82	37.53	-0.16
A24	-8.40	7.15	4.96	-3.79	0.51	0.13	-1.76	40.37	-0.08
A25	-8.06	6.68	4.51	-3.45	0.48	0.13	-1.96	41.91	-0.32
A26	-7.72	6.23	4.27	-2.95	0.49	0.10	-2.20	34.95	-0.17
A27	-7.76	6.63	4.48	-3.73	0.45	0.08	-3.32	41.43	-0.38
A28	-8.13	6.65	4.67	-3.33	0.47	0.10	-1.94	39.40	-0.14
A29	-9.74	8.48	6.22	-5.25	0.29	-0.06	-	-	0.21
A32	-10.51	9.96	4.86	-4.33	0.49	0.12	-0.69	70.44	-0.02
A33	-13.33	16.86	-2.60	-1.14	0.41	0.23	-0.39	100.94	-0.21
A34	-9.25	10.23	-0.26	-0.85	0.48	0.21	-0.03	103.99	-0.13
A36	-11.12	12.52	1.53	-3.00	0.59	0.29	0.45	89.21	-0.07
C1	-7.27	6.10	2.67	-2.07	0.48	0.08	-0.09	95.45	-0.57
C2	-10.20	11.11	2.38	-3.34	0.54	0.26	-0.71	65.62	-0.05
C3	-10.55	12.90	-0.97	-1.57	0.38	0.17	-0.54	89.70	-0.19
C4	-15.46	18.76	-1.80	-2.39	0.40	0.19	0.29	108.85	-0.89
C5	-10.07	8.90	5.69	-4.90	0.41	0.05	-1.54	48.49	-0.38
C6	-11.31	11.45	4.34	-4.95	0.37	0.08	-0.96	61.38	-0.47
D1	-7.32	6.37	3.37	-2.70	0.38	0.02	1.41	112.69	-0.28
D2	-7.57	5.86	4.14	-2.95	0.51	0.12	-1.69	39.59	-0.52
D3	-8.24	6.08	4.67	-2.61	0.52	0.16	-1.30	46.13	-0.10
F1	-7.05	5.88	3.03	-1.67	0.46	0.06	3.88	152.75	0.19
F3	-11.47	13.11	0.48	-2.31	0.58	0.31	-0.07	79.27	-0.19

Table D.7 - Final parameter values of the self-tuning regulator.

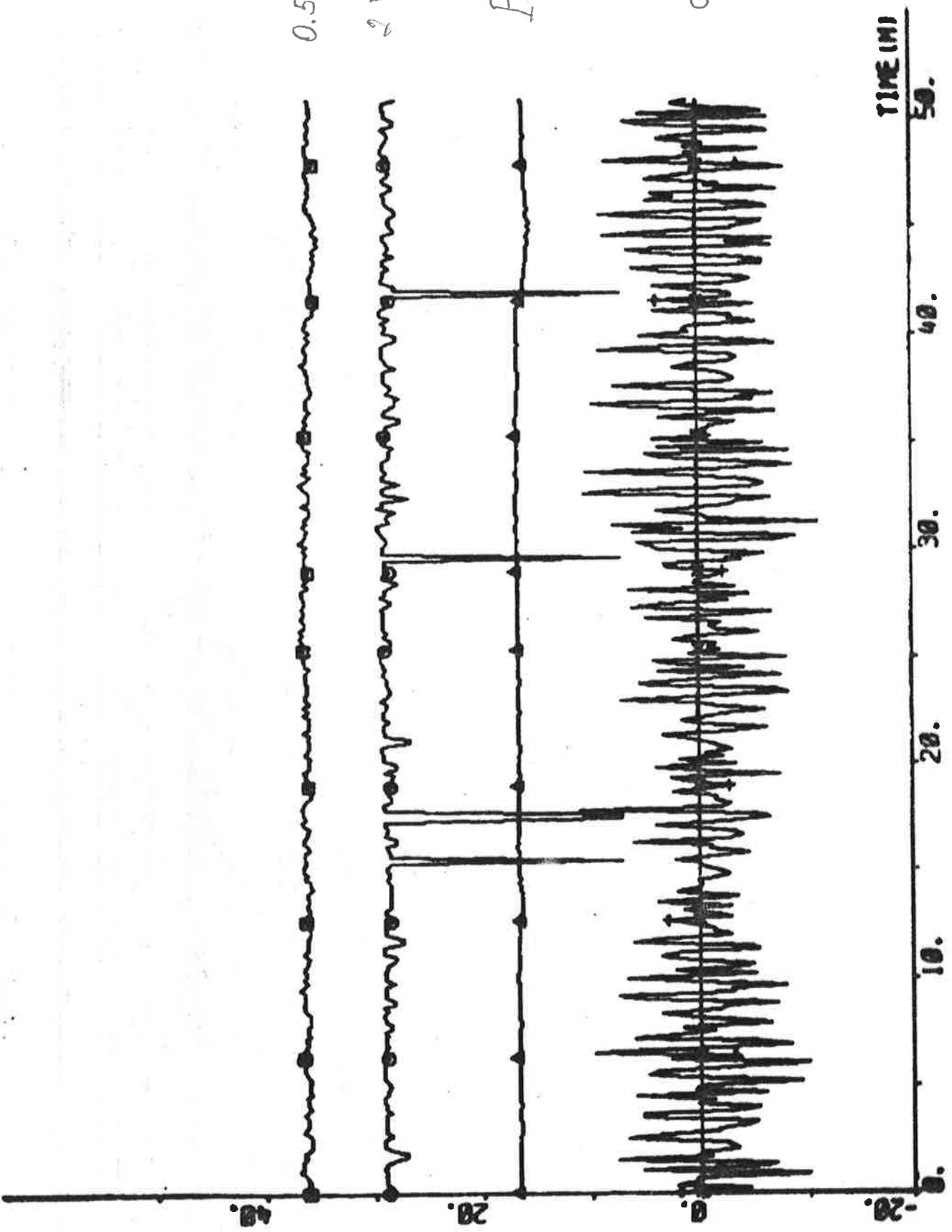


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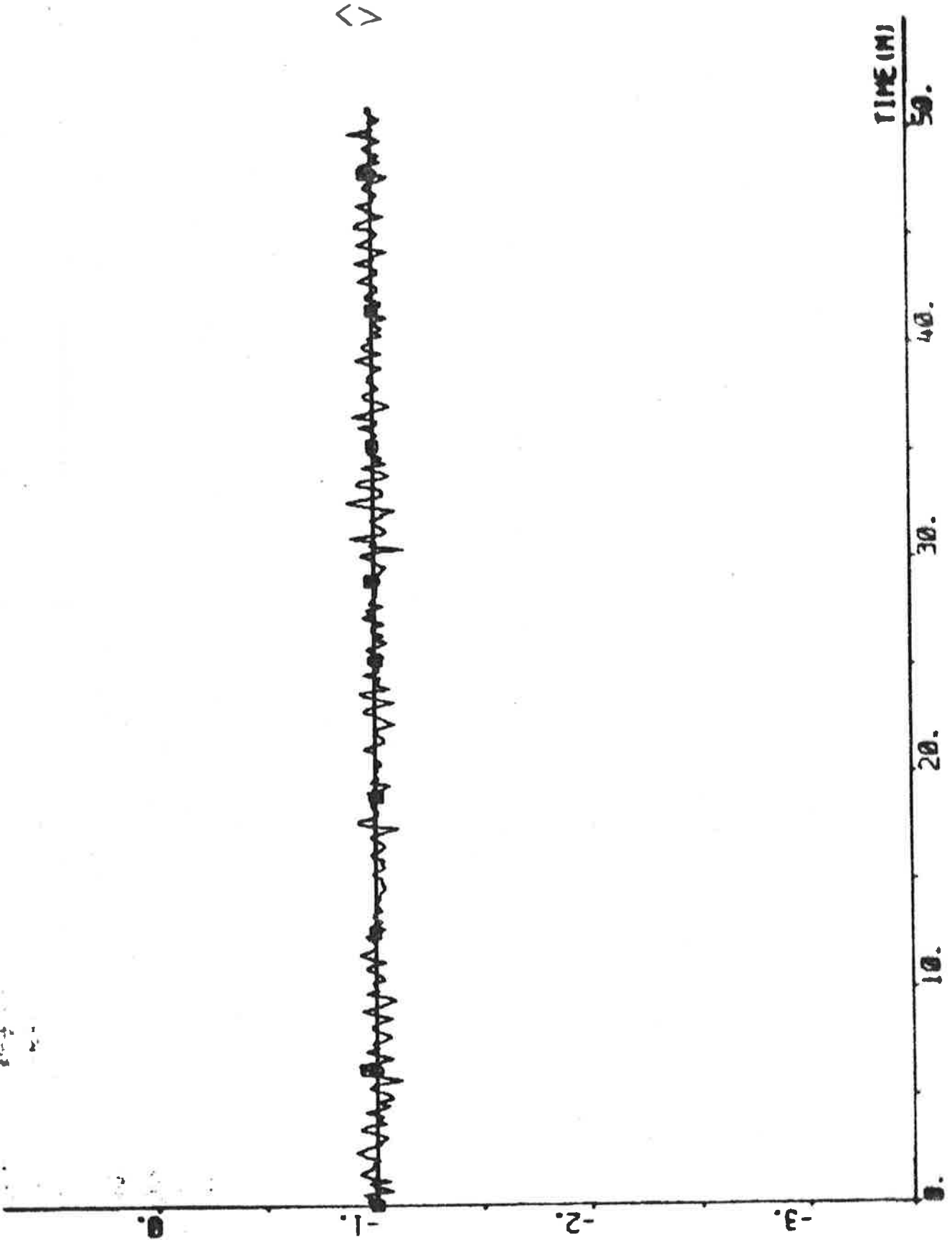
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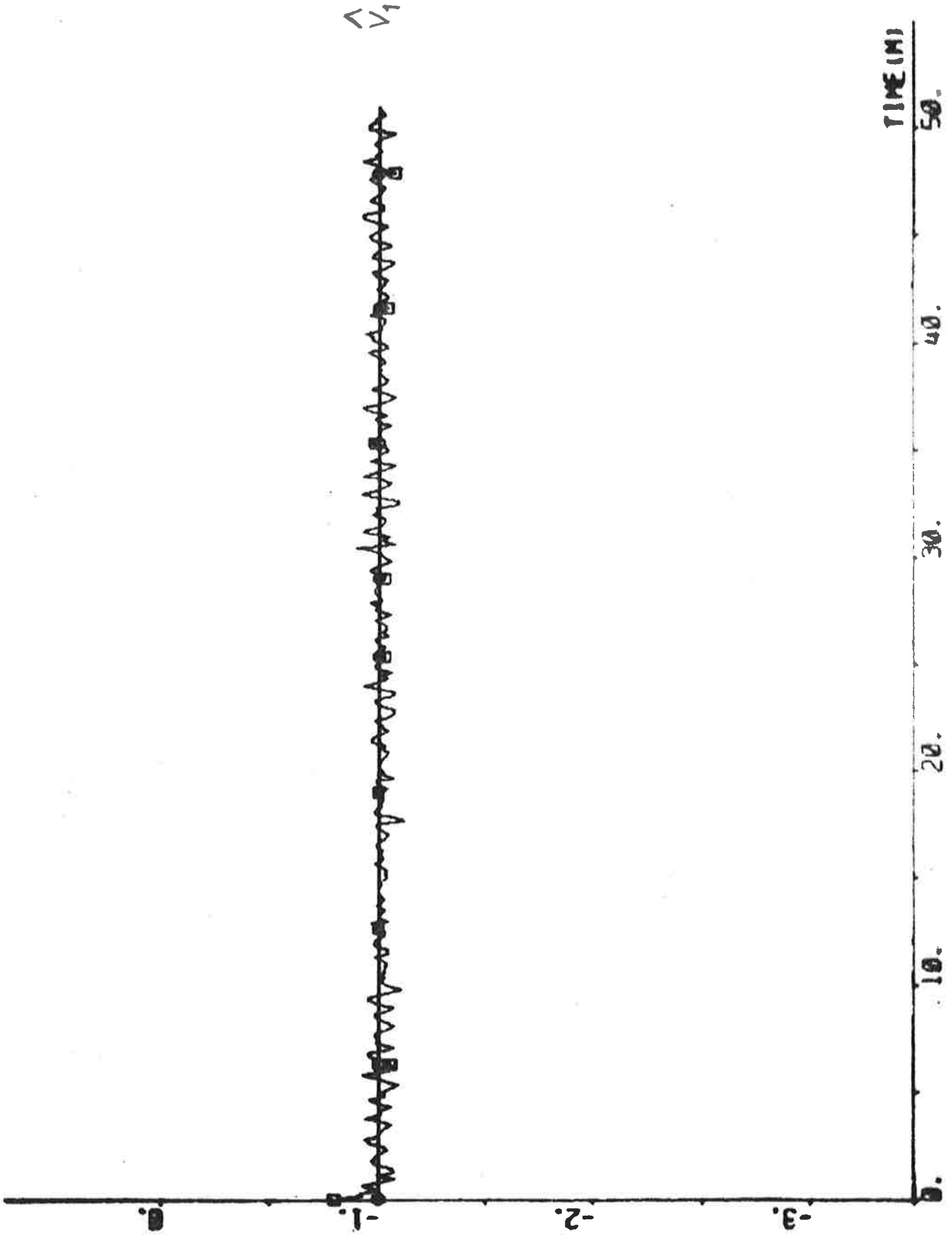




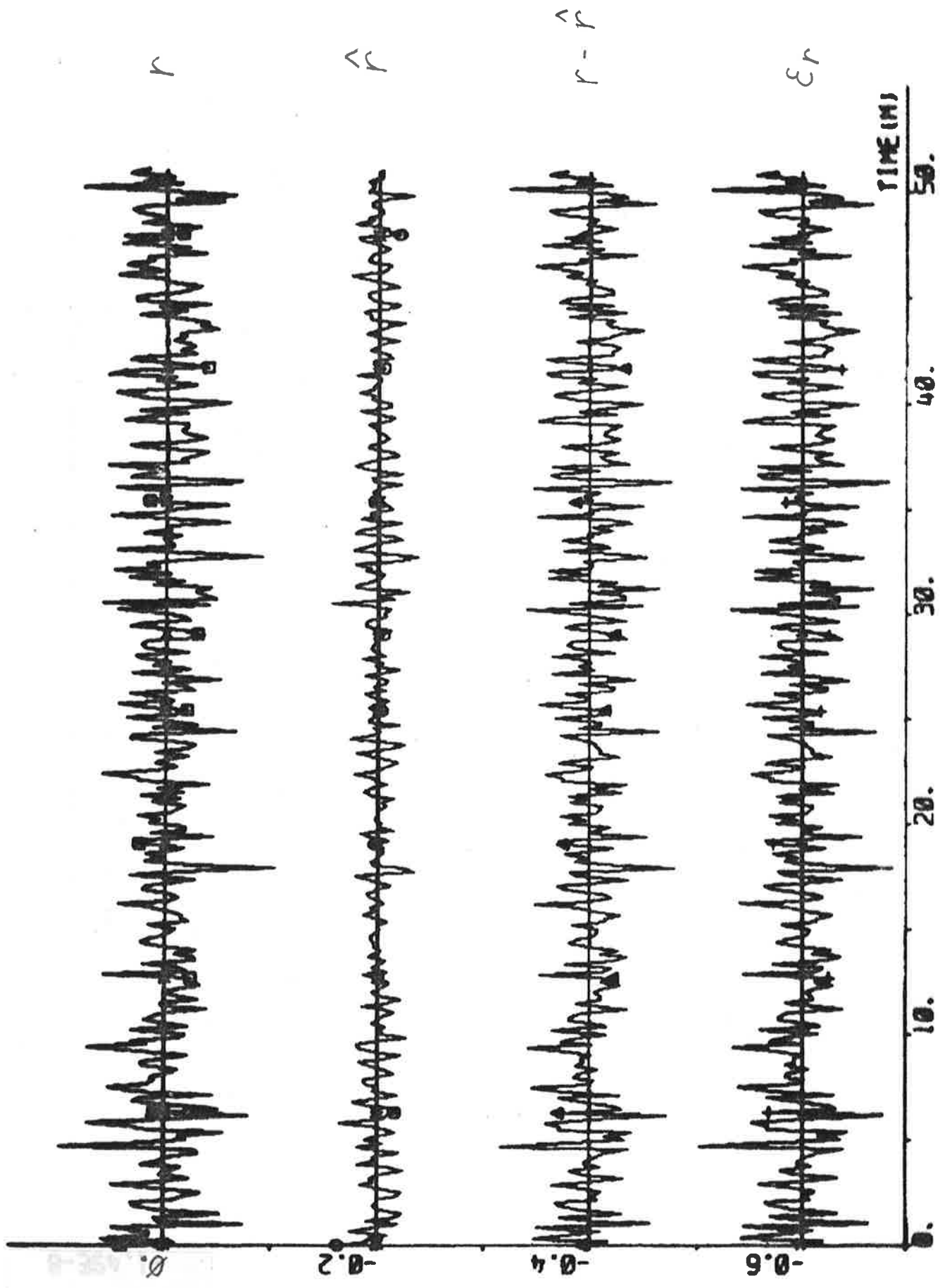
PLOT AIP1(1)-AIP2(2) @1 -3.4 0.0 - KNOTS



PLOT AIP1(1)-AIP1(5) 01 -3.4 0.0 - KNOTS



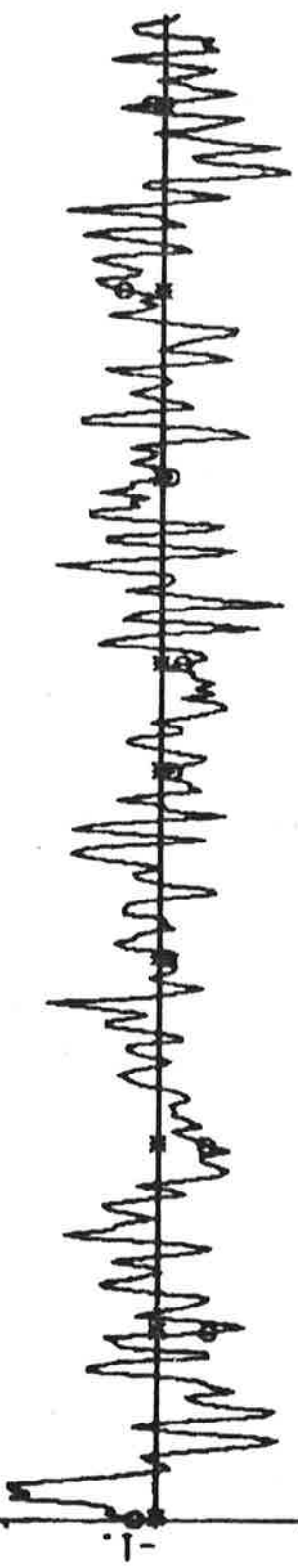
PLOT A1P1(1)-A1P1(6) A1P1(7) ERR3 EPS3 00 002 004 006 -0.7 0. - 0000



PLOT AIP1(1)-AIP1(8) AIP1(8) ERR4 EPS4 00 01 02 03 -3.4 0.0 - DEC



$\gamma - \gamma_{ref}$



$\hat{\gamma} - \gamma_{ref}$



$\gamma - \hat{\gamma}$



$\epsilon_2$

TIME (M) 10. 20. 30. 40. 50.

PLOT AIP1(1)-AIP1(2) AIP1(3) ERR1 EPS1 00 020 040 060 -03 10 - 000



δ



δ



δ-δ



εδ

TIME (MI) 0. 10. 20. 30. 40. 50.

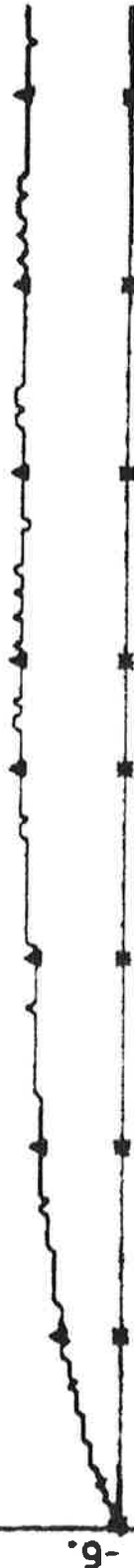
PLOT A1P1(1)-A1P2(3) A1P2(5) A1P2(6) 00 04 08 -8.5 1.5



$0.5 * \hat{\delta}_0$  deg



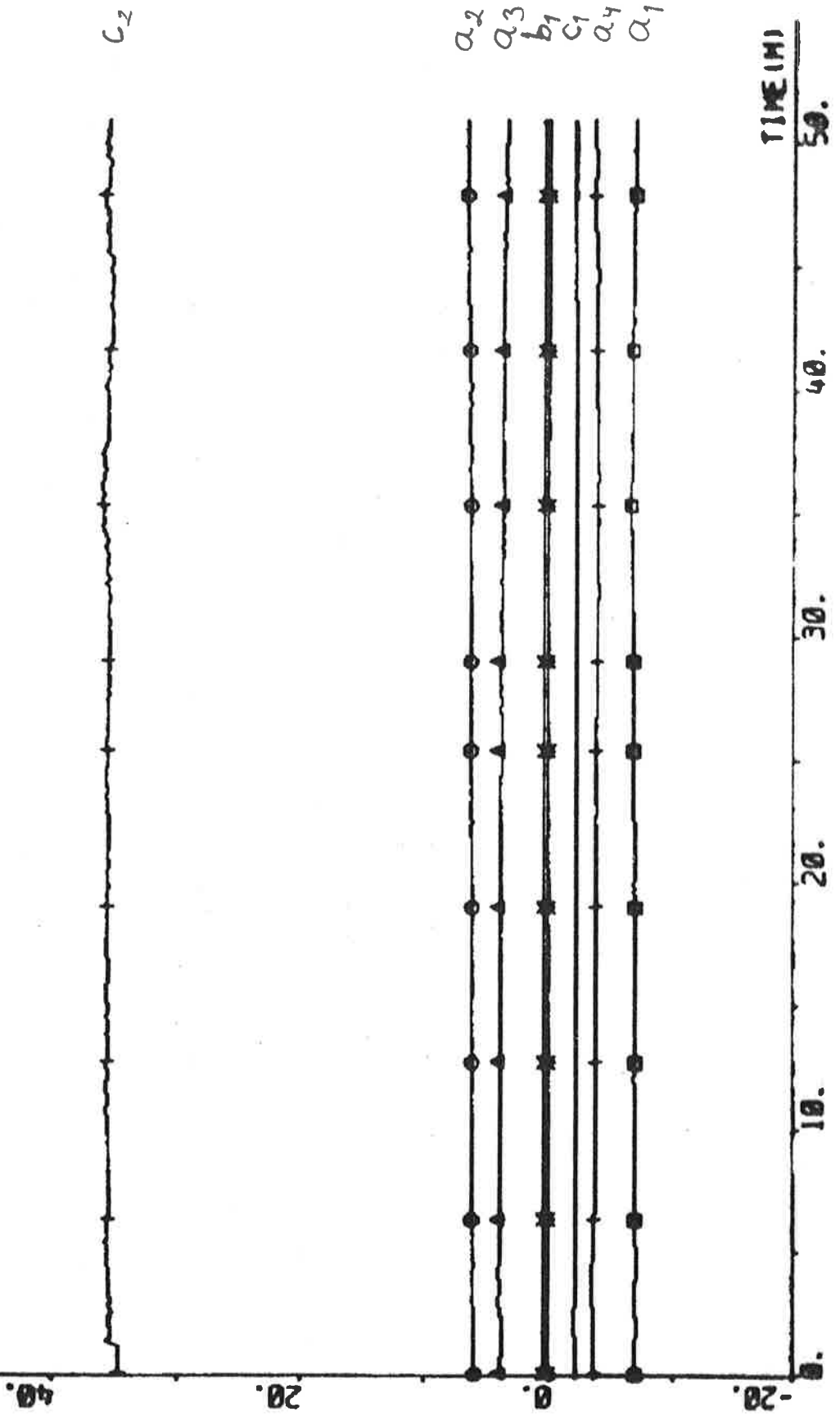
$100 * \hat{d}_r$  deg/s



$0.5 * \hat{d}_s$  deg

TIME (M) 0. 10. 20. 30. 40. 50.

PLOT AIP1(1)-AIP2(7) AIP2(8) AIP2(9) AIP2(10) AIP2(11) AIP2(12) AIP2(13)



## EXPERIMENT A2

Date	1976-04-22	Forward draught	8.5 m
Time	08.19	Aft draught	12.5 m
Duration	26 min	Wind direction	N (5; see App. A)
Position	N 12°39' W 18°29'	Wind velocity	5 m/s (gentle breeze)
$\psi_{ref}$	180 deg	Wave height	Low swell from N

An incorrect measurement of the yaw rate  $r$  was obtained after about 6 min. This measurement was not skipped by the Kalman filter.

Self-tuning regulator using estimates from the Kalman filter.

Tuning time before the experiment started: > 120 min.

$NC1 = 1$	$NC2 = 1$	$k = 7$	$q = 0$
$T_s = 10$ s	$V_0 = 7$ m/s	$IVVC = 1$	

Final values:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ b_1 \\ b_2 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} -15.17 \\ 19.33 \\ -2.62 \\ -1.87 \\ 0.63 \\ 0.52 \\ 0.35 \\ 113.66 \end{bmatrix} \quad P = \begin{bmatrix} 13.56 & & & & & & & & \\ -18.18 & 32.91 & & & & & & & \\ -0.39 & -14.74 & 29.92 & & & & & & \\ 5.72 & -0.62 & -15.50 & 11.07 & & & & & \\ -0.33 & 0.12 & 0.62 & -0.43 & 0.03 & & & & \\ -0.25 & 0.21 & 0.17 & -0.14 & 0.01 & 0.02 & & & \\ -1.85 & 2.54 & 0.28 & -1.04 & 0.03 & 0.00 & 0.86 & & \\ 61.97 & -69.20 & -45.68 & 49.88 & -2.82 & -1.84 & 7.66 & 1171.60 & \end{bmatrix}$$

$$a_1 + a_2 + a_3 + a_4 = -0.33$$

$$\hat{\delta}_0 = 0.4 \text{ deg} \quad \hat{a}_v = 0.20 \text{ knots} \quad \hat{a}_r = -0.001 \text{ deg/s} \quad \hat{a}_\delta = 1.3 \text{ deg}$$

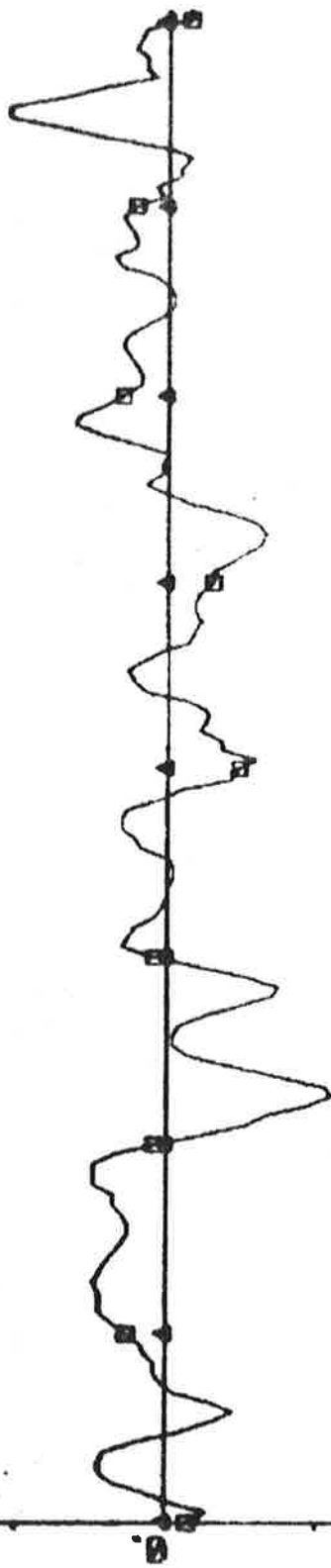
Statistics (mean value and standard deviation)

$\delta_c$	0.41 ± 1.14 deg	$P_s$	14.5 ± 0.1 MW
$\delta$	1.77 ± 1.03 deg	$\epsilon_v$	0.00 ± 0.02 knots
$\psi - \psi_{ref}$	0.017 ± 0.182 deg	$\epsilon_r$	0.00 ± 0.02 deg/s
$n$	69.6 ± 0.5 rpm	$\epsilon_\psi$	0.00 ± 0.02 deg
$u$	13.9 ± 0.1 knots	$\epsilon_\delta$	0.0 ± 0.5 deg

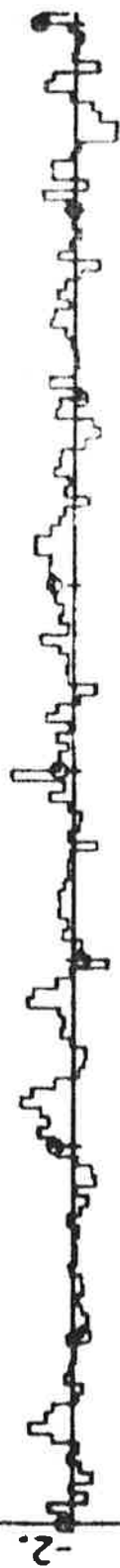
$V_1 = 0.142$



PLOT A2P1(1)-A2P1(8) HP A2P1(10) A2P1(15) 02 -3 1 - DEG



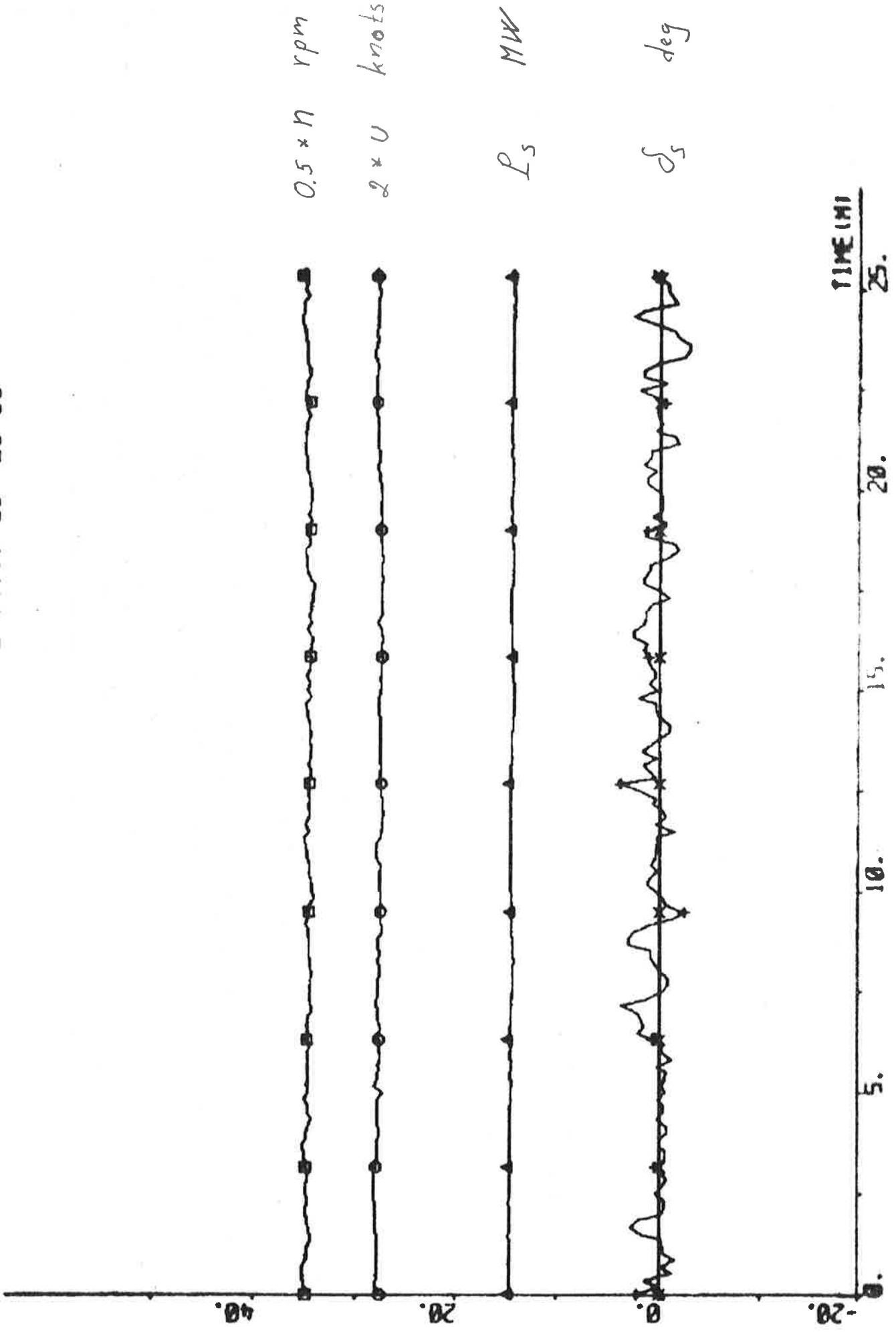
$\gamma - \gamma_{ref}$



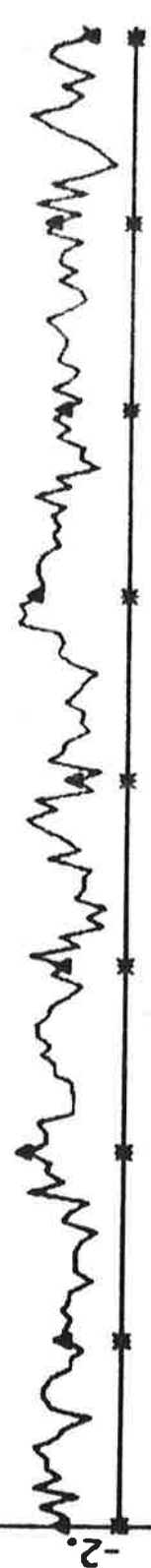
$0.05 \times \delta_c$

TIME (MI) 0 5 10 15 20 25

PLOT A2P1(1)-A2P1(13) A2P1(12) A2P1(14) A2P1(11) 00 -20 00

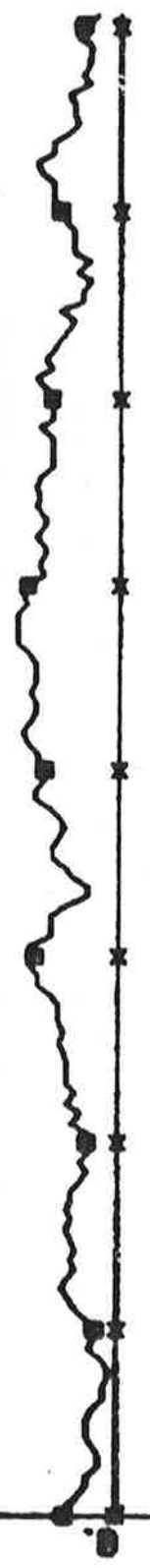


PL0T A2P1(1)-A2P2(1) A2P2(2) ERRS 00 01 02 -3.4 0.0 - KNOTS



TIME (MI)  
25.

PLOT A2P1(1)-A2P1(4) A2P1(5) ERR2 EP32 00 01 02 03 -3.4 0.0 - KNOTS



TIME (H) 25.

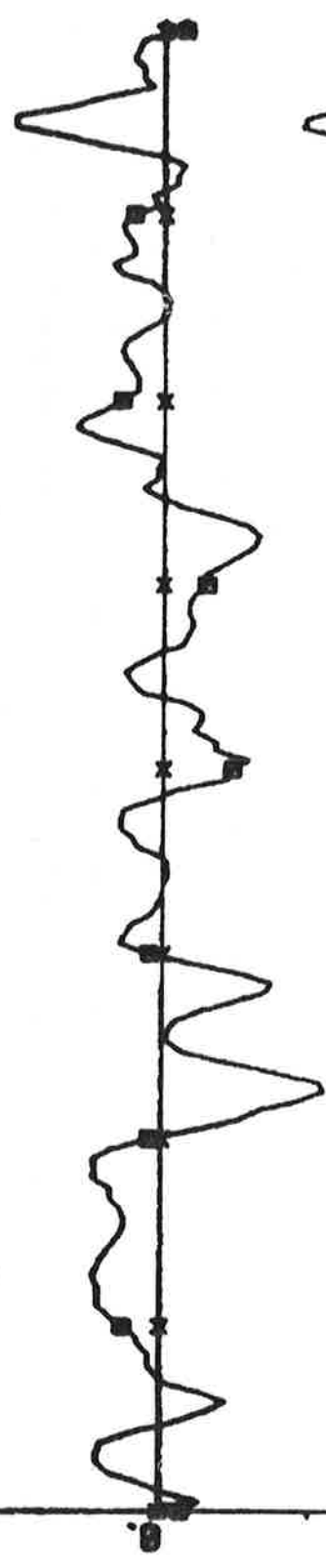
PLOT A2P1(1)-A2P1(6) A2P1(7) ERR3 EPS3 00 002 004 000 -0.7 0. - DECS



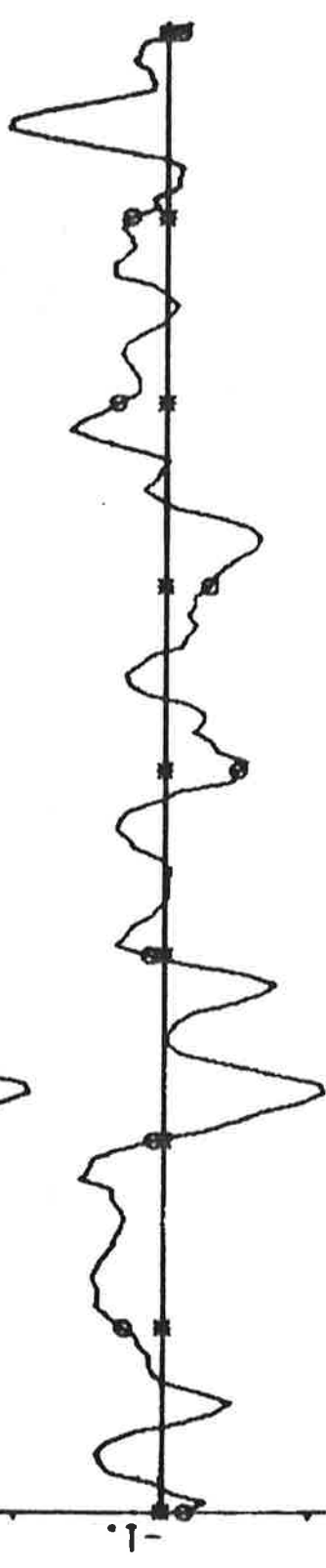
TIME (M)

0. 5. 10. 15. 20. 25.

PLOT R2P1(1)-R2P1(0) R2P1(0) ERR4 EPS4 00 01 02 03 -3.4 0.0 - DEG



$\gamma - \gamma_{ref}$



$\hat{\gamma} - \gamma_{ref}$



$\hat{\gamma} - \hat{\gamma}$



$\epsilon_4$

TIME (H)

0. 5. 10. 15. 20. 25.

PLOT A2P1(1)-A2P1(2) A2P1(3) ERR1 EPS1 00 020 040 060 -03 16 - DEG



$\delta$



$\delta$



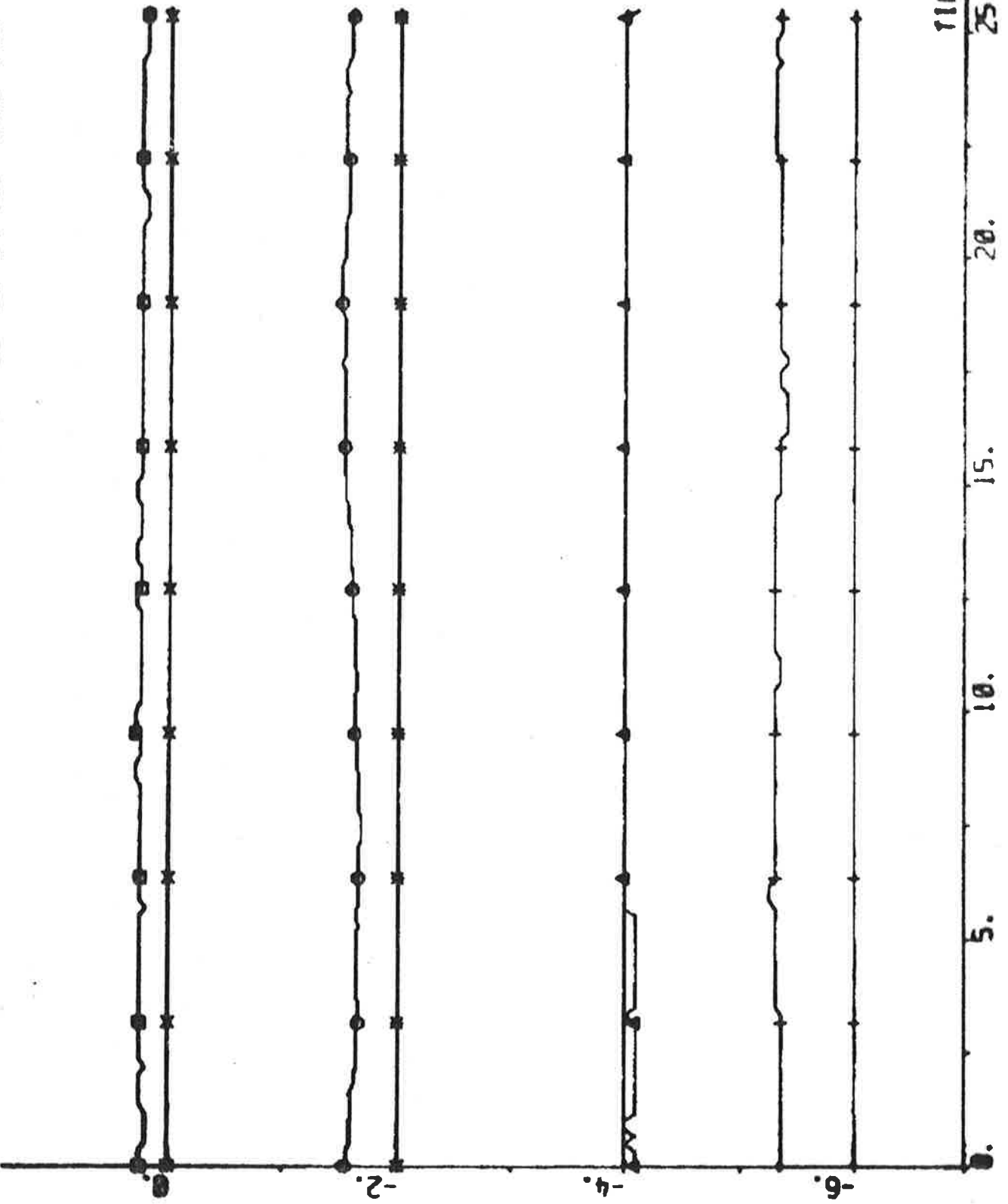
$\delta - \delta$



$\epsilon \delta$

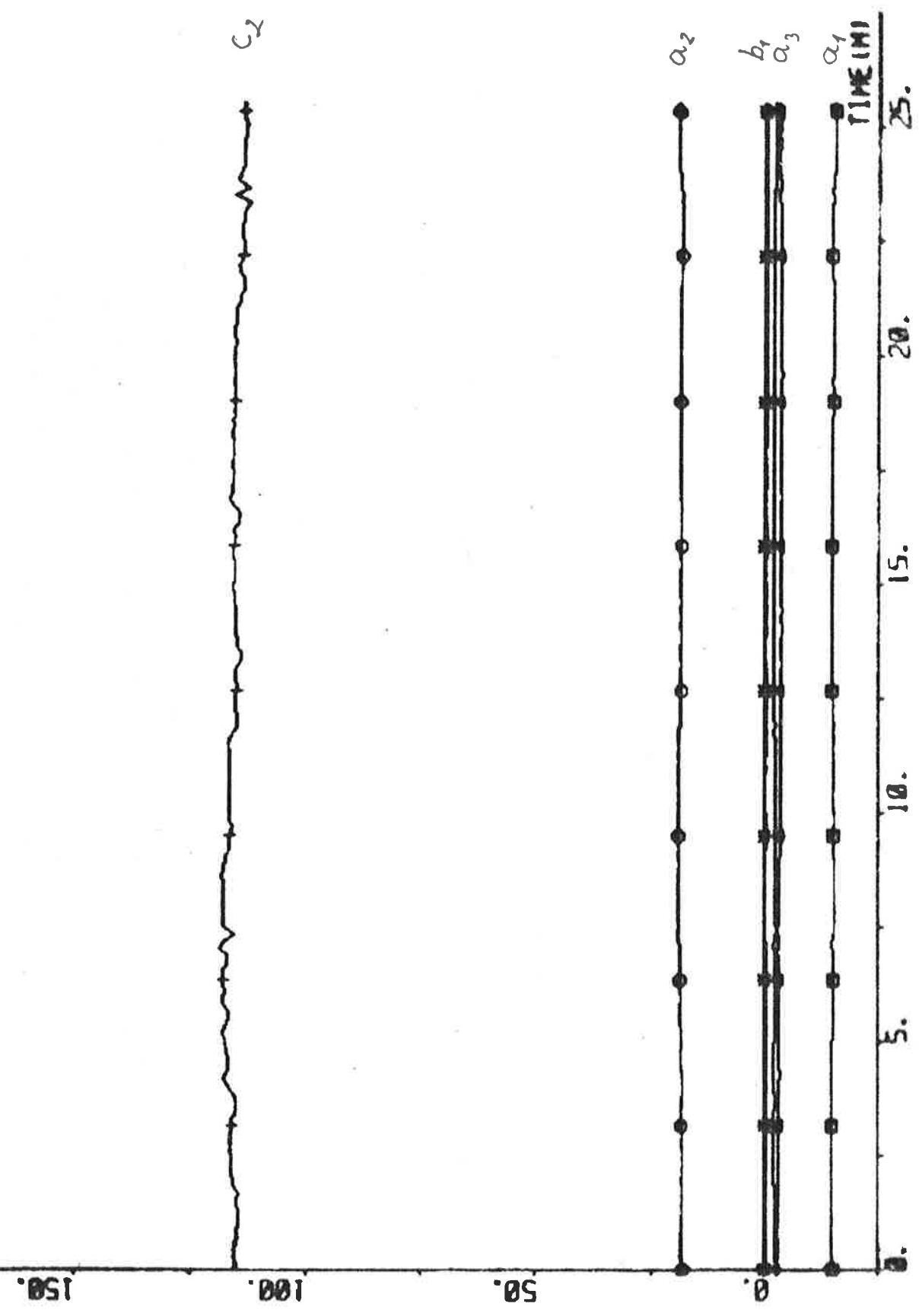
TIME (MI)  
0. 5. 10. 15. 20. 25.

PLOT A2P1(1)-A2P2(3) A2P2(4) A2P2(5) A2P2(6) 00 02 04 06 -0.5 1.5





PLOT A2P1(1)-A2P2(7) A2P2(8) A2P2(9) A2P2(10) A2P2(11) A2P2(12) A2P2(13)

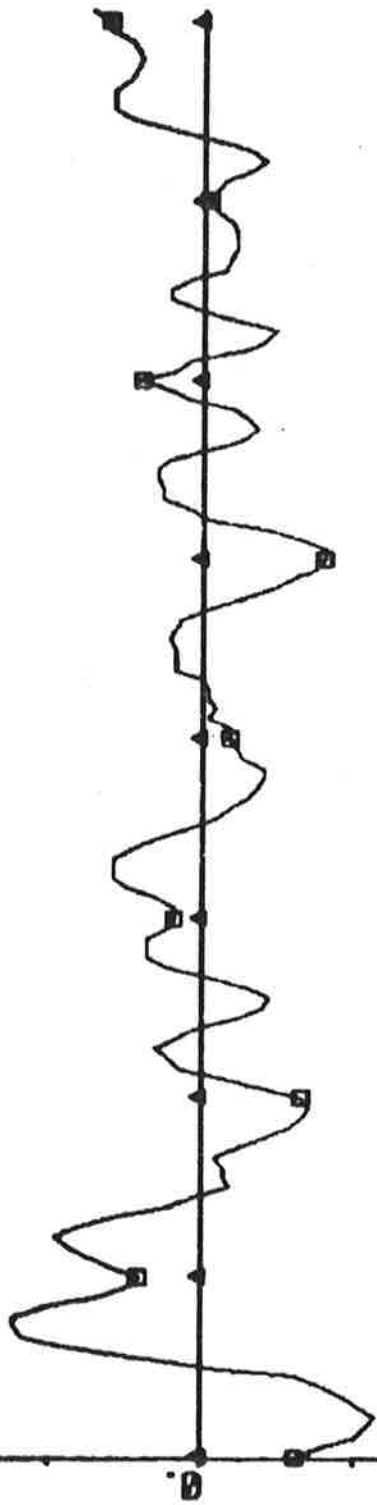


$a_2$   
 $b_1$   $c_1$   
 $a_3$   $a_4$

TIME (MI)  
25.



PLOT ACP1(1)-ACP1(8) HP ACP1(10) ACP1(15) Q2 -3 1 - DEG



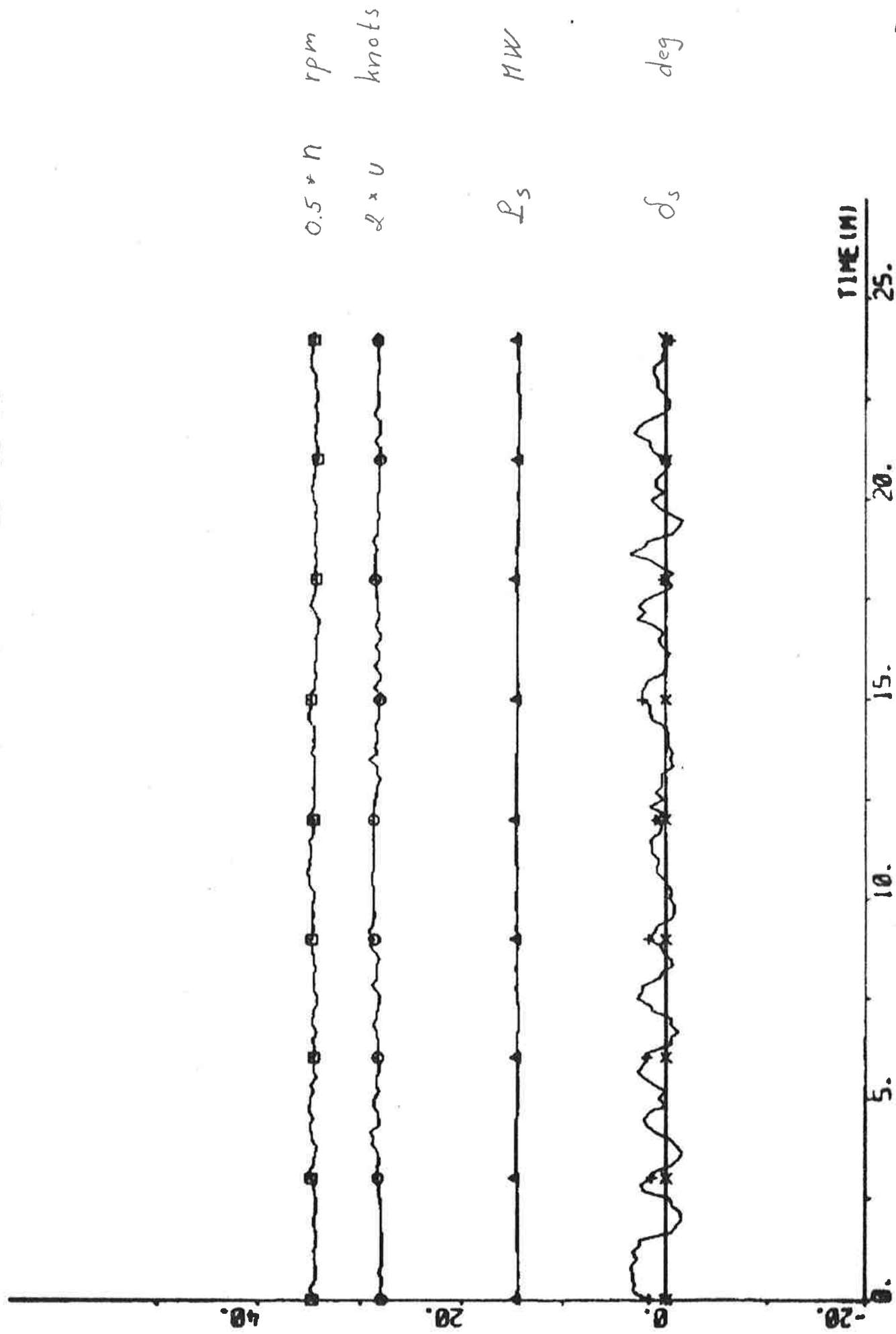
$\gamma - \gamma_{ref}$



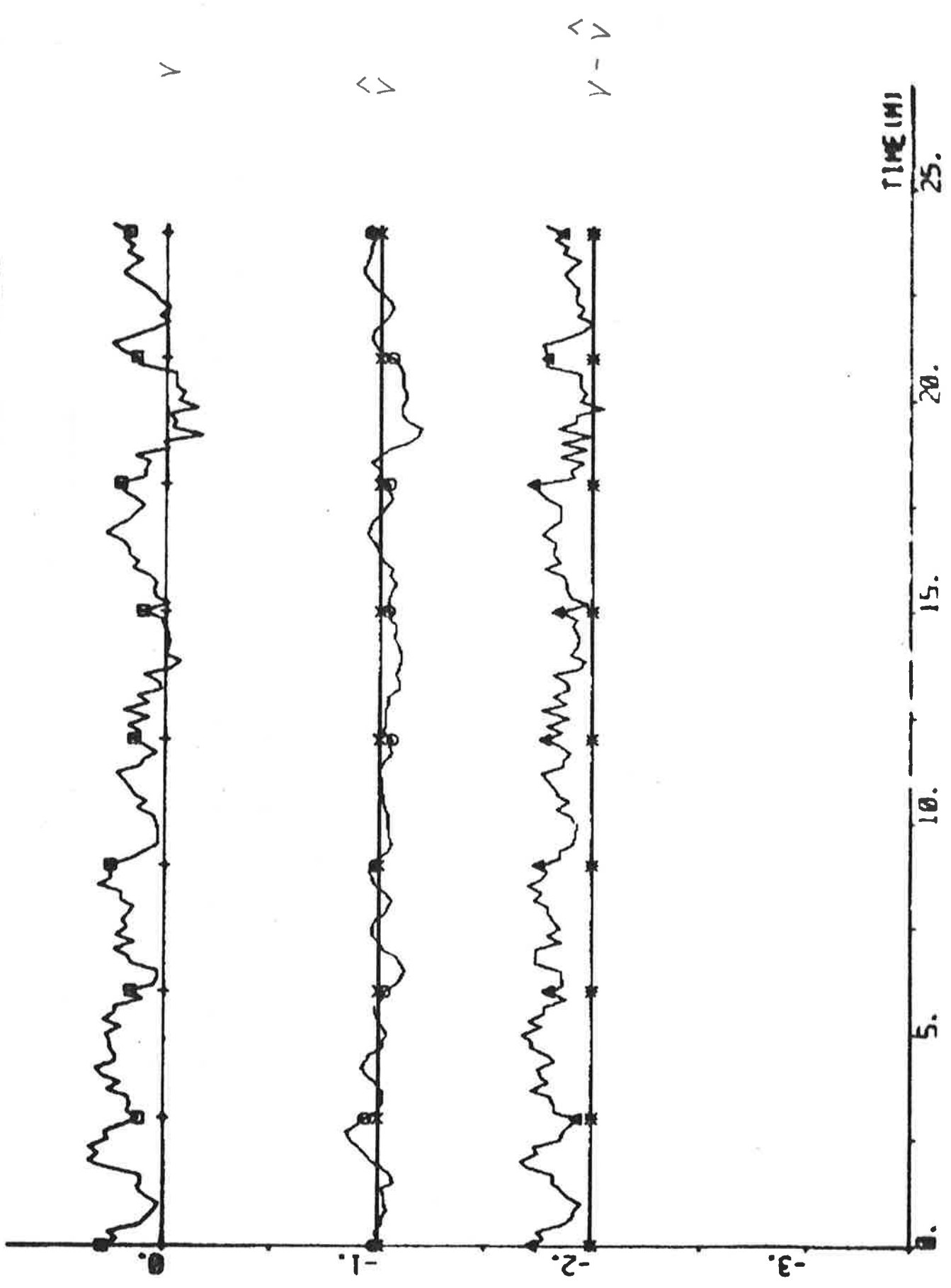
$0.05 * d_c$

TIME (M) 0. 5. 10. 15. 20. 25.

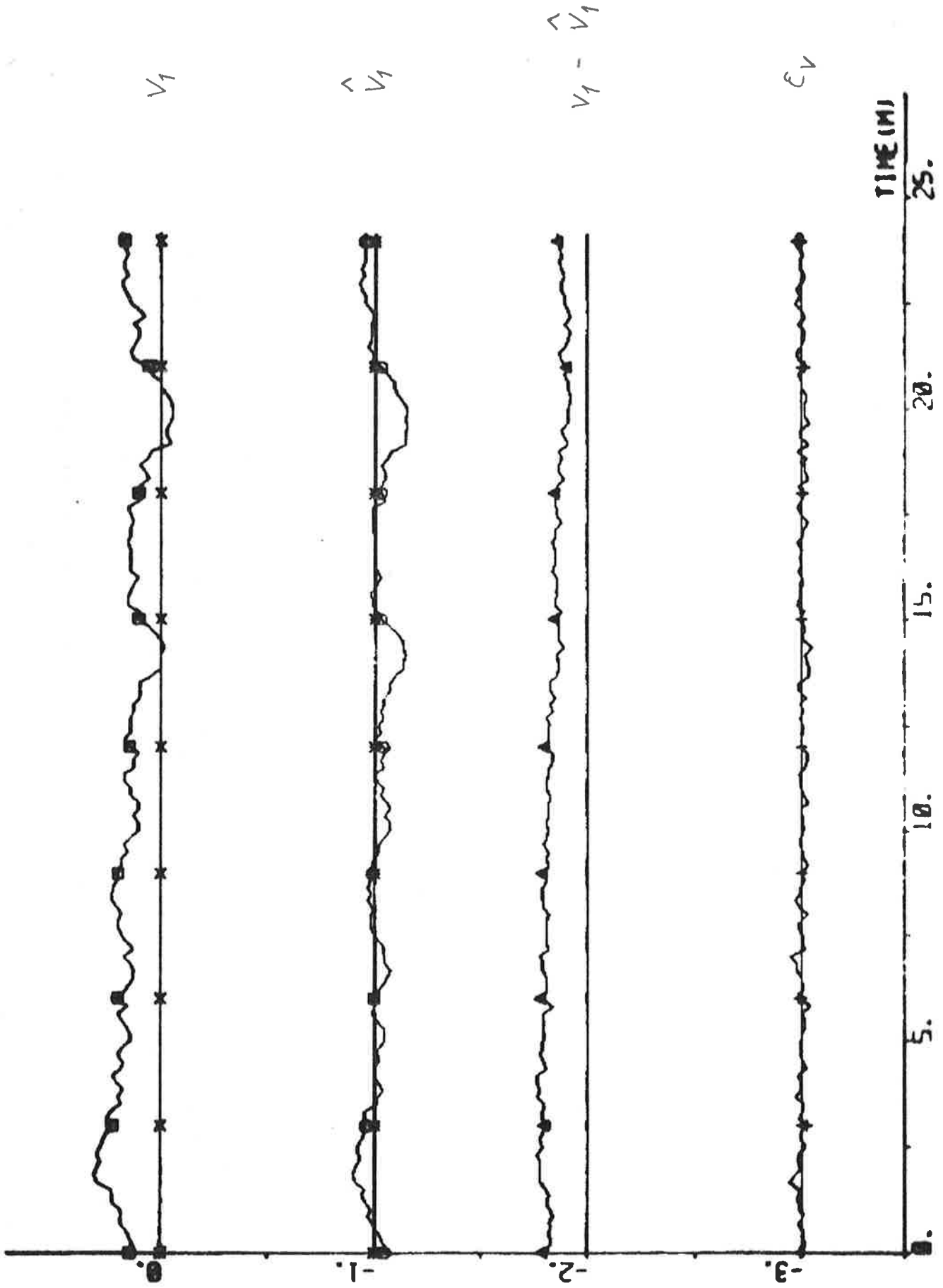
PLOT A3P1(1) A3P1(13) A3P1(12) A3P1(14) A3P1(11) 00 -20 50



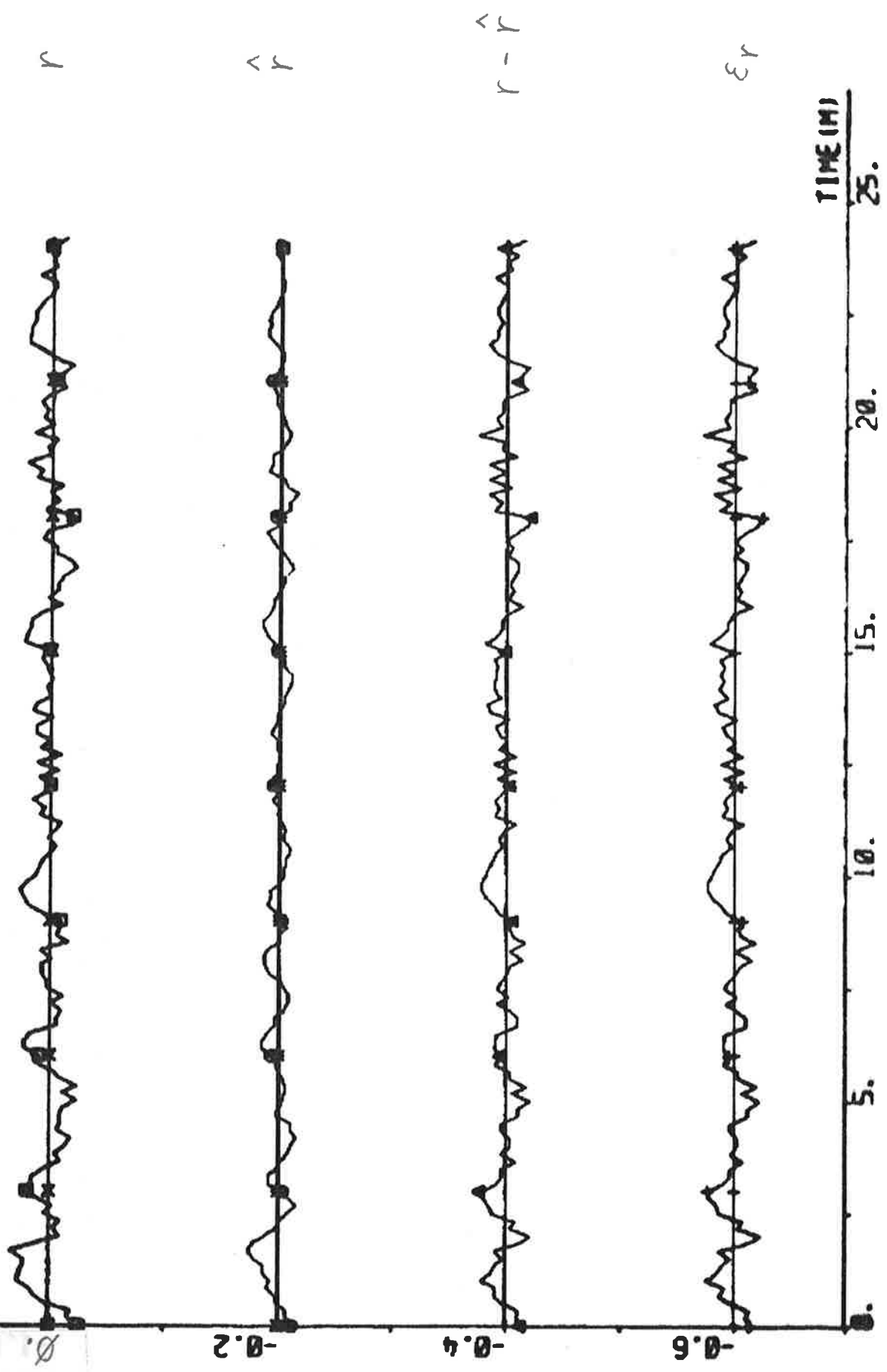
PLOT ACP1(1)-ACP2(1) ACP2(2) ERR6 00 01 02 -3.4 0.0 - KNOTS



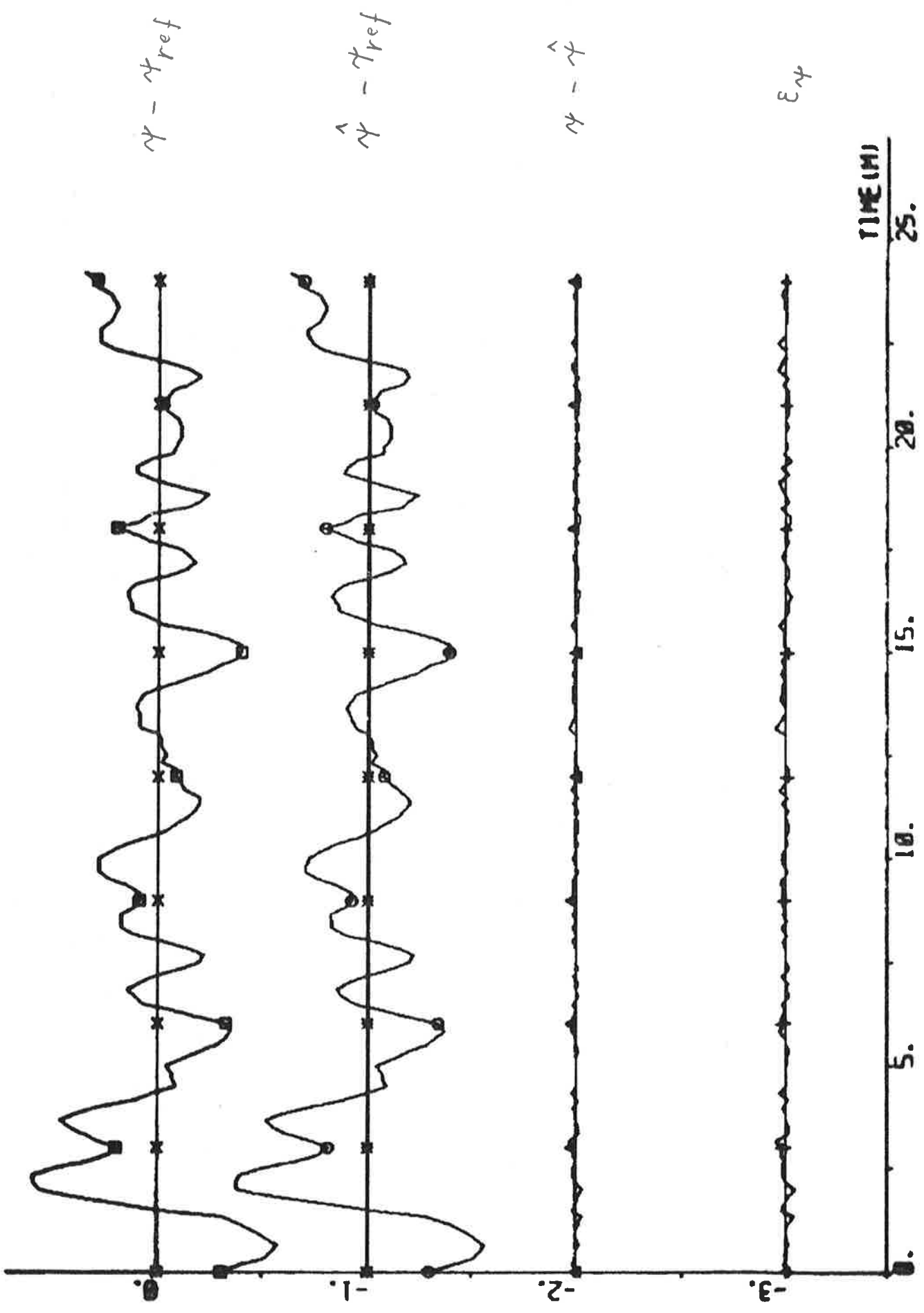
PLOT A3P1(1) A3P1(4) A3P1(5) ERR2 EPS2 00 01 02 03 -3.4 0.0 - KNOTS



PLOT ACP1(1)-ACP1(6) ACP1(7) ERR3 EPS3 00 002 004 006 -0.7 0. - DEC/S

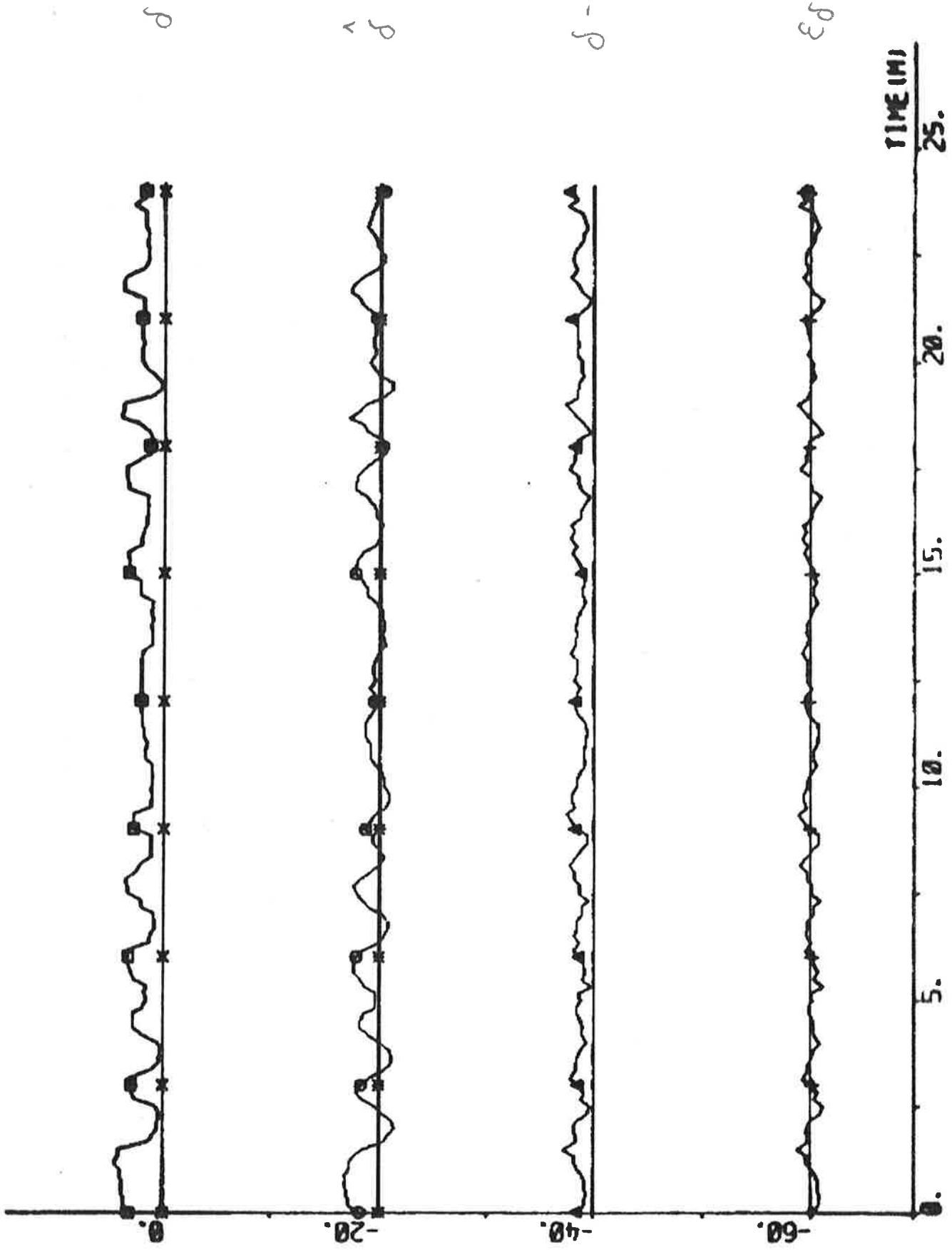


PLOT ACP1(1)-ACP1(9) ACP1(9) ERR4 EPS4 00 01 02 03 -3.4 0.0 DEG

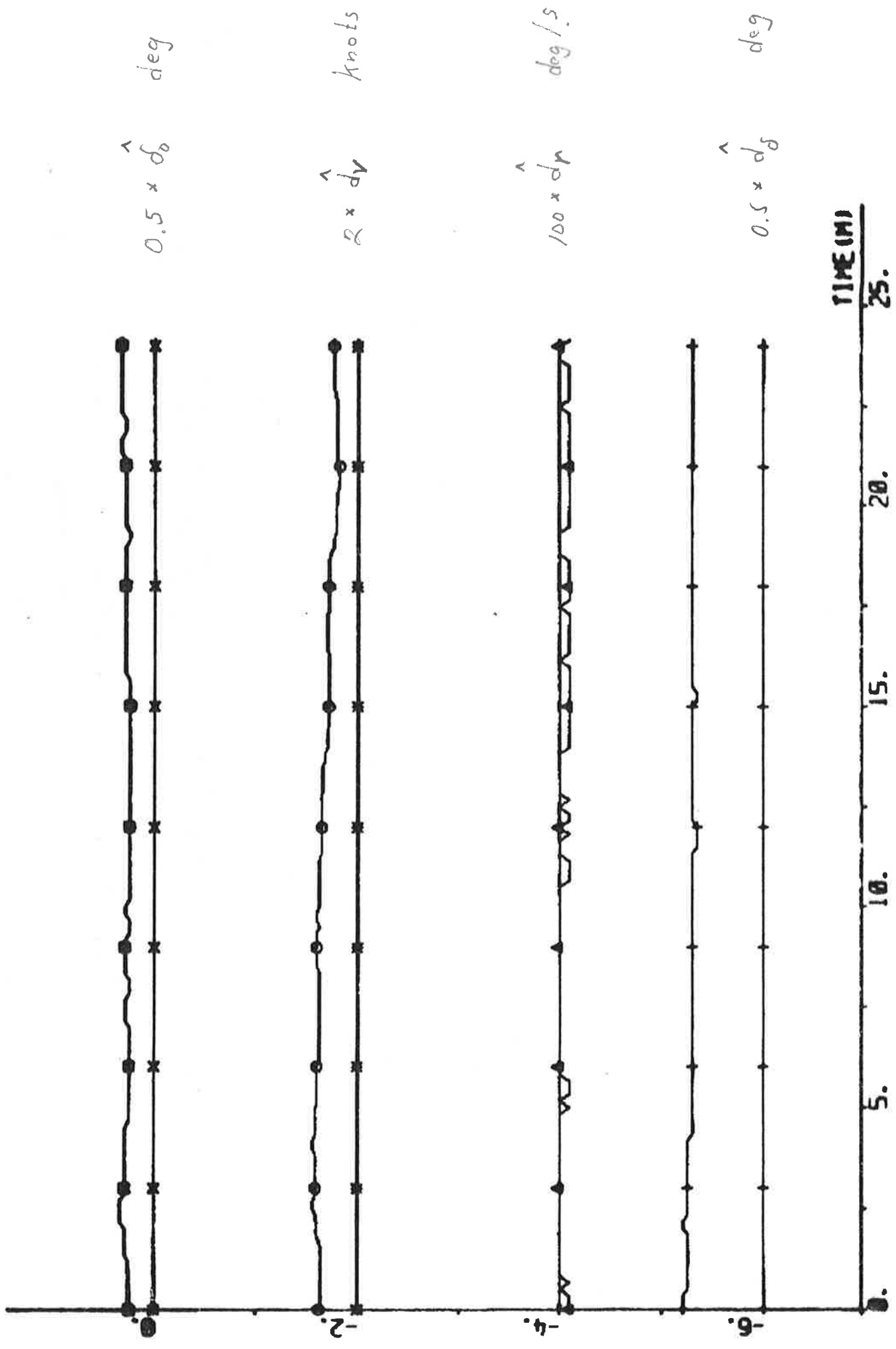




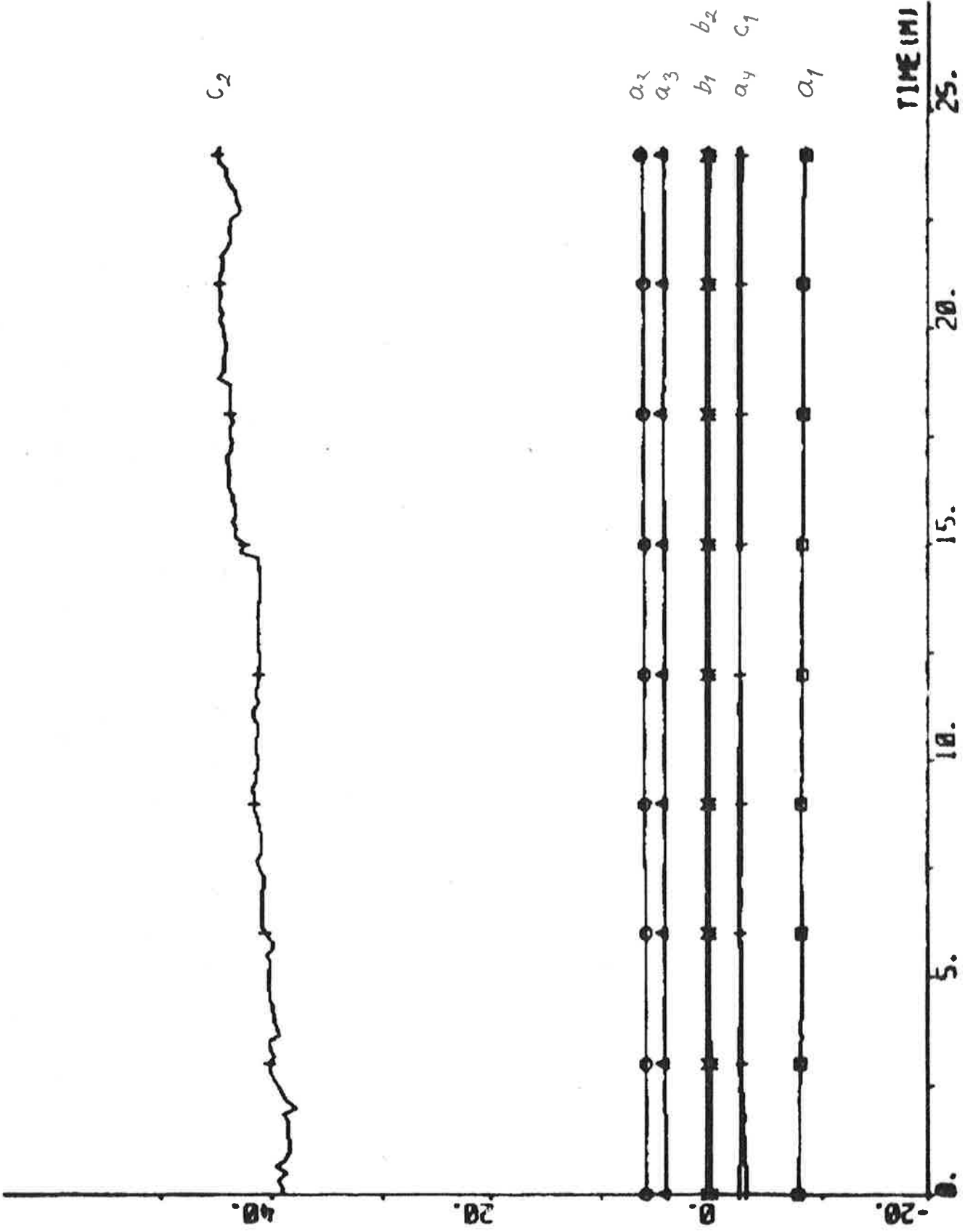
PLOT A3P1(1)-A3P1(2) A3P1(3) ERR1 EPS1 00 020 040 060 -06 16 ° DEG



PLOT R3P1(1) R3P2(3) R3P2(4) R3P2(5) R3P2(6) 00 02 04 06 -0.5 1.5



PLOT R3P1(1)-R3P2(7) R3P2(8) R3P2(9) R3P2(10) R3P2(11) R3P2(12) R3P2(13)



## EXPERIMENT A4

Date	1976-04-22	Forward draught	8.5 m
Time	10.53	Aft draught	12.5 m
Duration	24 min	Wind direction	N (5; see App. A)
Position	N 12°01' W 18°28'	Wind velocity	5 m/s (gentle breeze)
$\psi_{\text{ref}}$	180 deg	Wave height	Low swell from N

Self-tuning regulator using estimates from the Kalman filter.

Tuning time before the experiment started: 30 min.

NC1 = 1	NC2 = 1	k = 6	q = 0
$T_s = 10$ s	$V_0 = 7$ m/s	IVVC = 1	

Final values:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ b_1 \\ b_2 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} -8.92 \\ 6.49 \\ 4.88 \\ -2.75 \\ 0.48 \\ 0.16 \\ -1.79 \\ 50.52 \end{bmatrix} \quad P = \begin{bmatrix} 4.78 & & & & & & & & \\ -5.62 & 11.34 & & & & & & & \\ -1.02 & -5.32 & 12.23 & & & & & & \\ 2.15 & -0.53 & -6.13 & 4.83 & & & & & \\ -0.14 & -0.07 & 0.36 & -0.15 & 0.03 & & & & \\ -0.10 & 0.02 & 0.00 & 0.08 & 0.01 & 0.03 & & & \\ -0.65 & 1.02 & 0.22 & -0.62 & -0.01 & -0.01 & 0.70 & & \\ 17.24 & -25.17 & -10.77 & 16.08 & -0.30 & -0.23 & 9.34 & 881.25 \end{bmatrix}$$

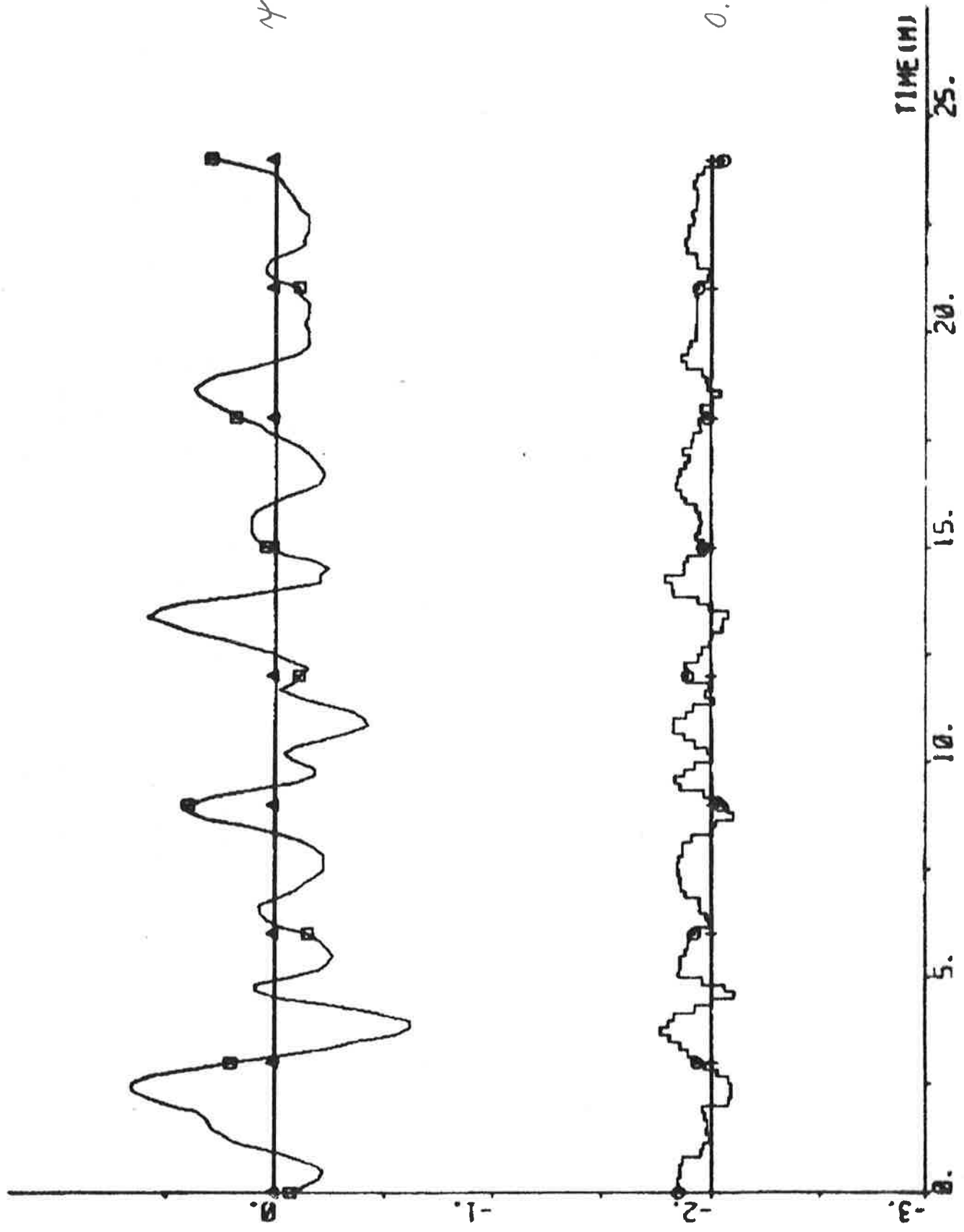
$$a_1 + a_2 + a_3 + a_4 = -0.30$$

$$\hat{\delta}_0 = 1.1 \text{ deg} \quad \hat{d}_v = 0.38 \text{ knots} \quad \hat{d}_r = 0.000 \text{ deg/s} \quad \hat{d}_\delta = 1.5 \text{ deg}$$

Statistics (mean value and standard deviation)

$\delta_c$	1.39	± 1.41	deg	$P_s$	14.0	± 0.2	MW
$\delta$	2.82	± 1.33	deg	$\epsilon_v$	0.01	± 0.02	knots
$\psi - \psi_{\text{ref}}$	-0.009	± 0.246	deg	$\epsilon_r$	0.00	± 0.01	deg/s
n	68.5	± 0.6	rpm	$\epsilon_\psi$	0.00	± 0.02	deg
u	14.0	± 0.2	knots	$\epsilon_\delta$	0.0	± 0.6	deg
$V_1 = 0.226$							

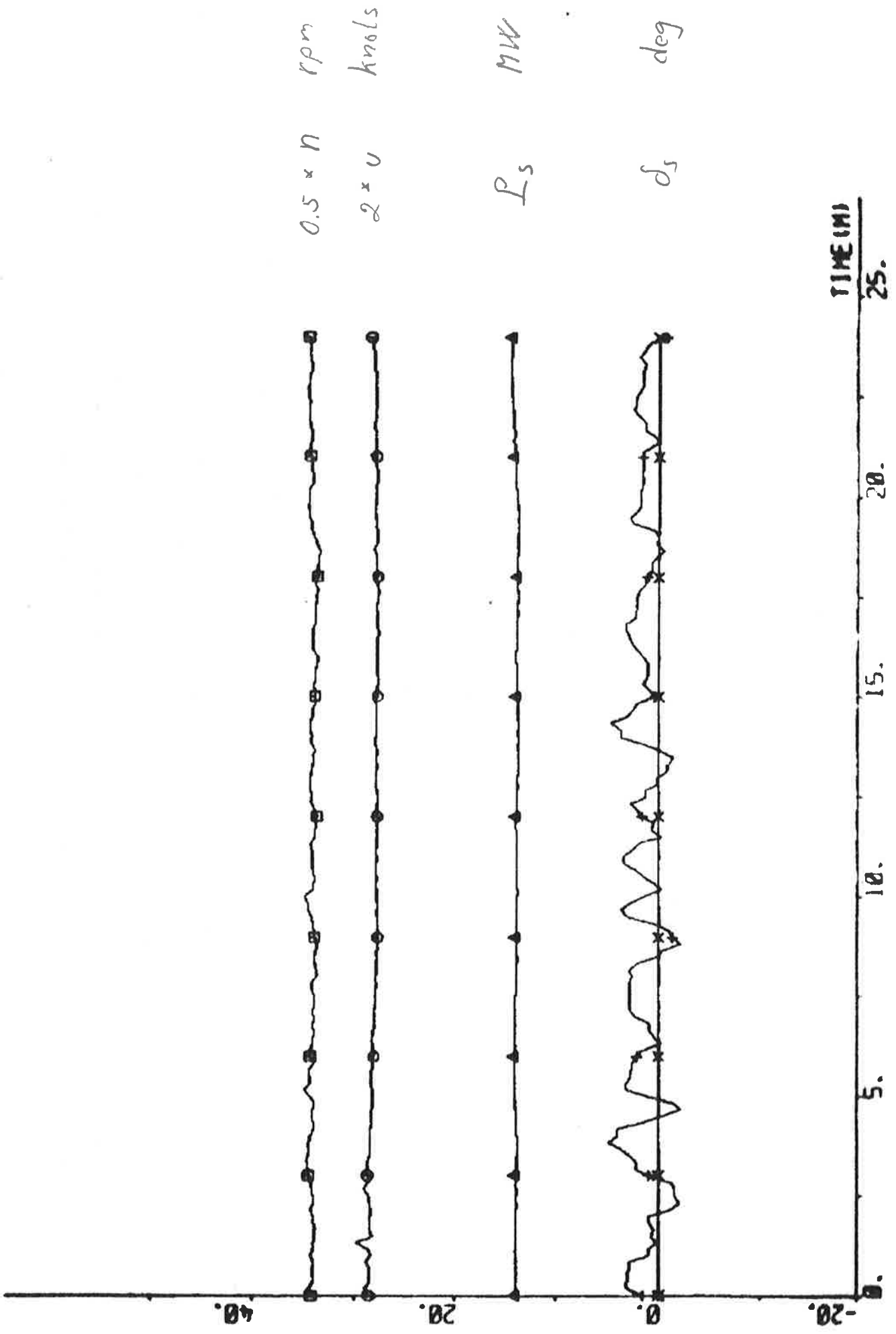
PLOT ANP1(1) ANP1(8) HP ANP1(10) ANP1(15) 02 -3 1 - DEG



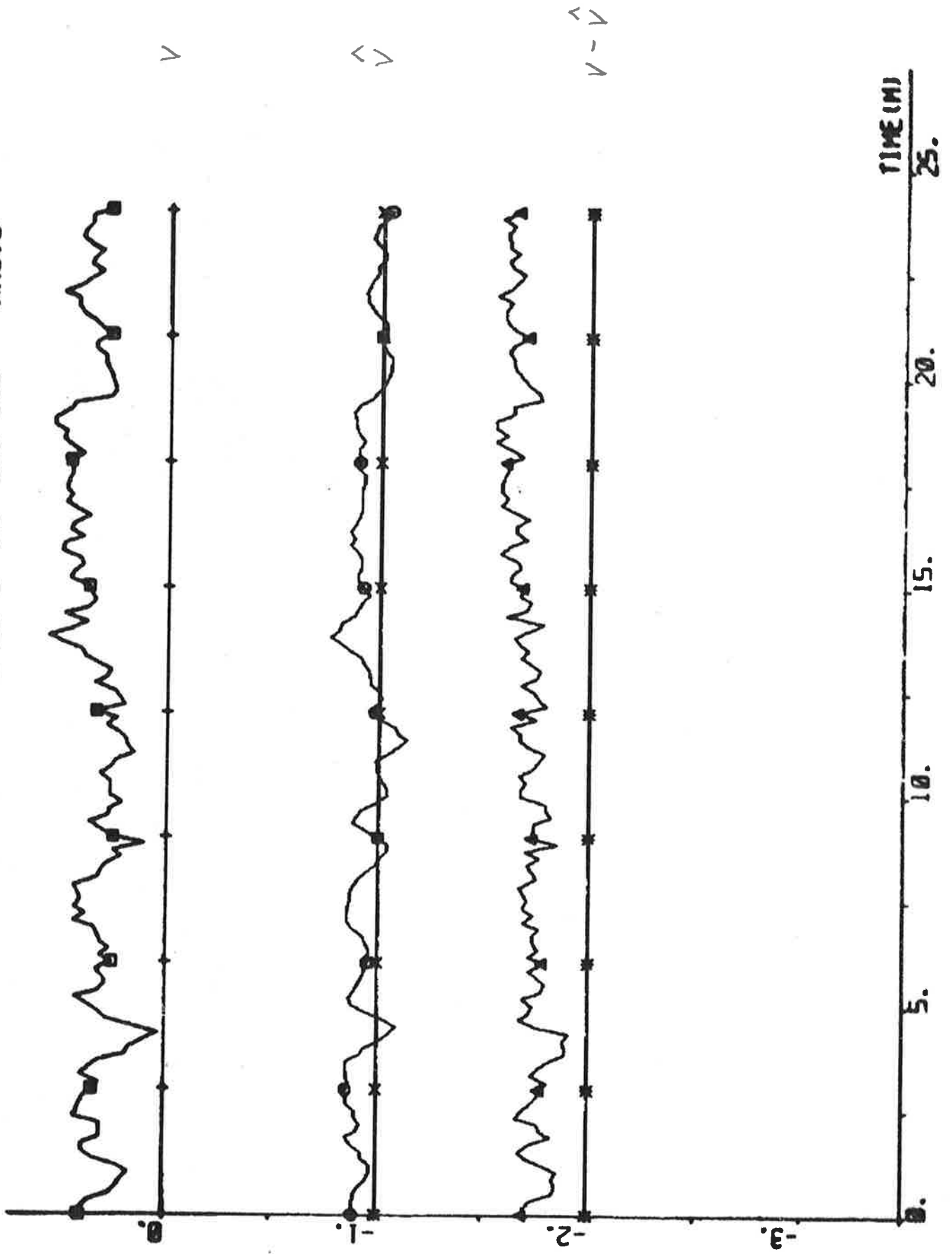
$\psi - \psi_{ref}$

$0.05 * \delta_c$

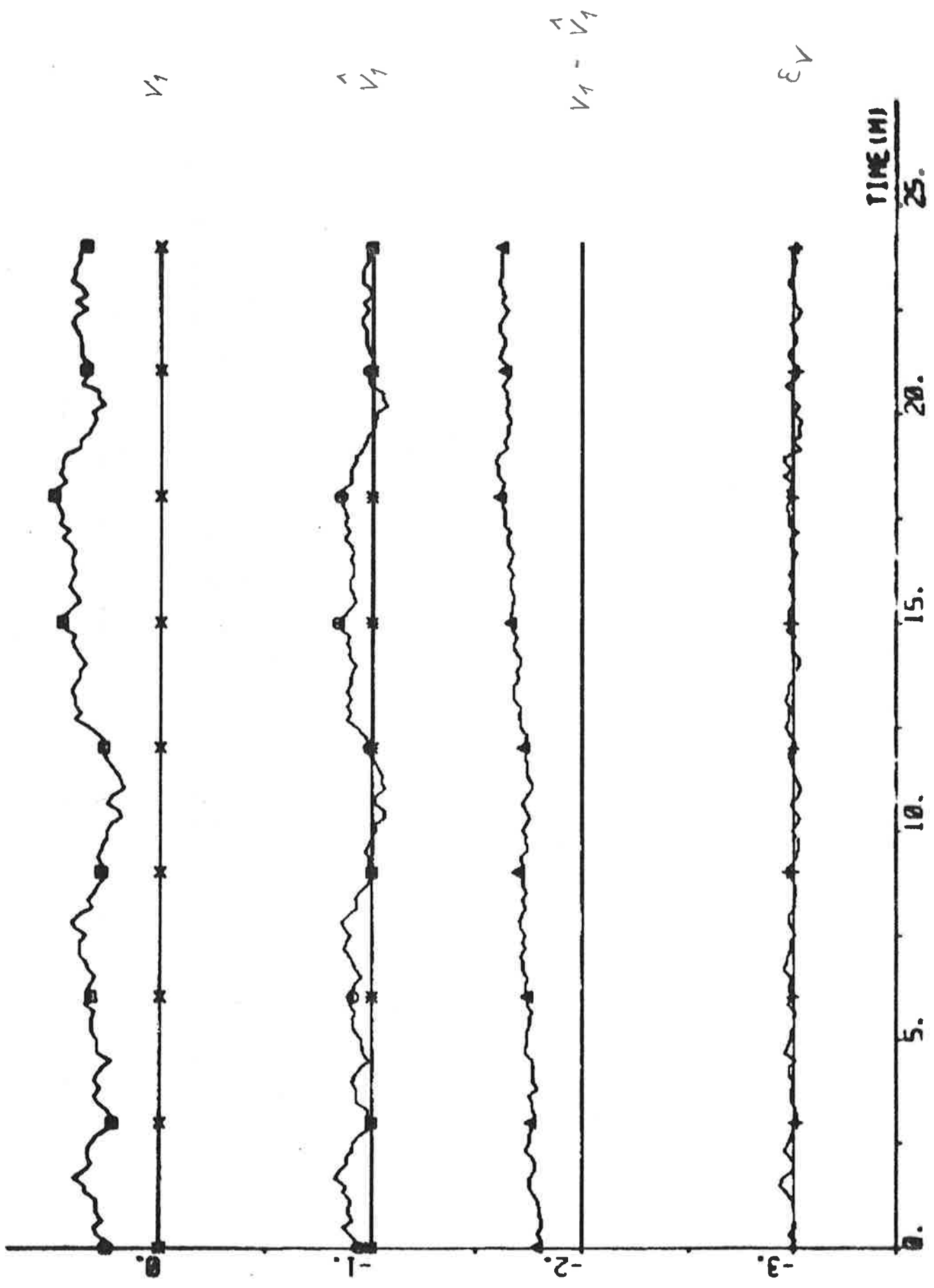
PLOT AWP1(1)-AWP1(13) AWP1(12) AWP1(14) AWP1(11) 00 -20 50



PLOT RHP1(1)-RHP2(1) RHP2(2) ERRS 00 01 02 -3.4 0.0 - KNOTS

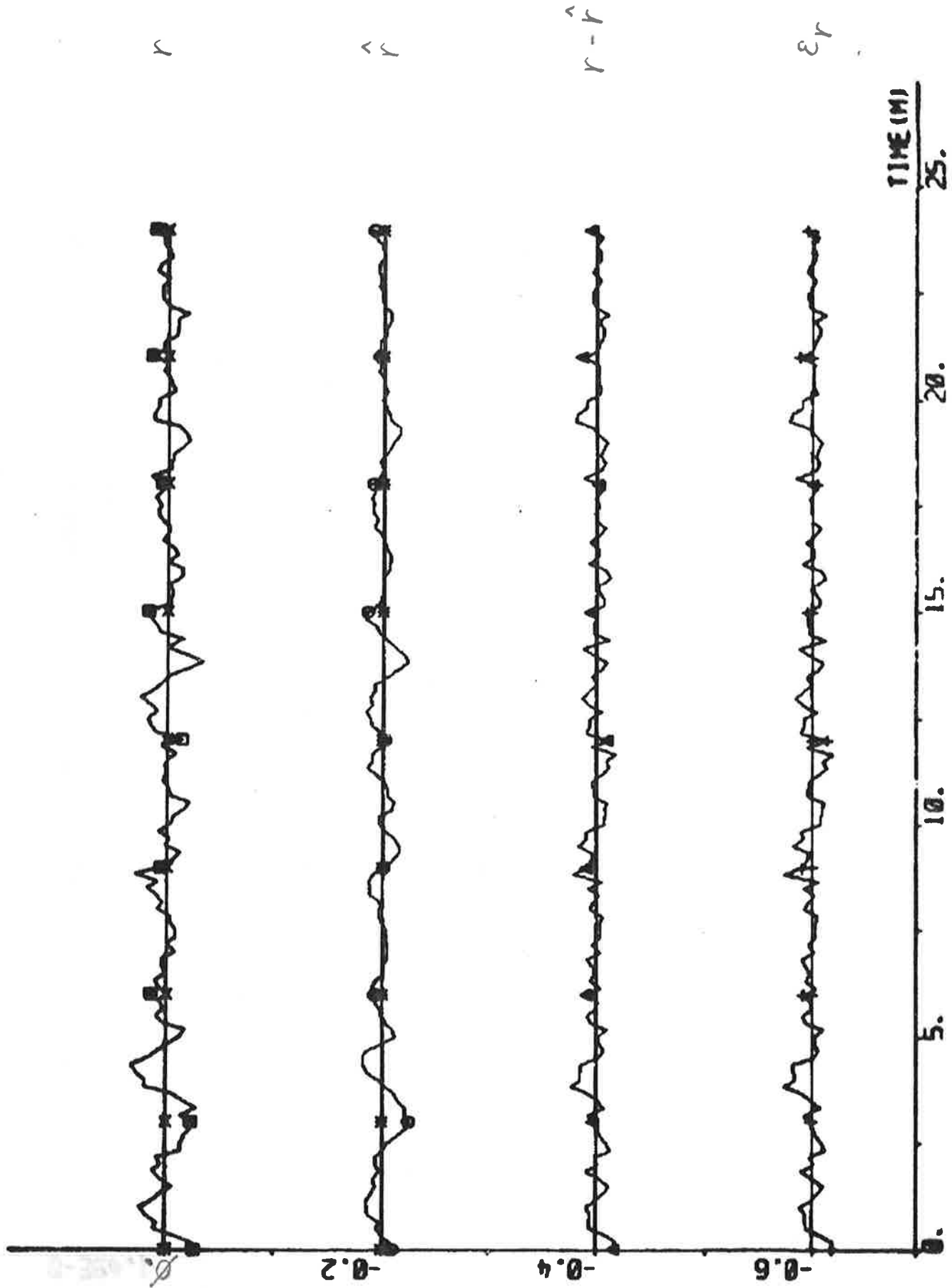


PLOT AWP1(1)-AWP1(4) AWP1(5) ERR2 EPS2 00 01 02 03 -3.4 0.0 - KNOTS

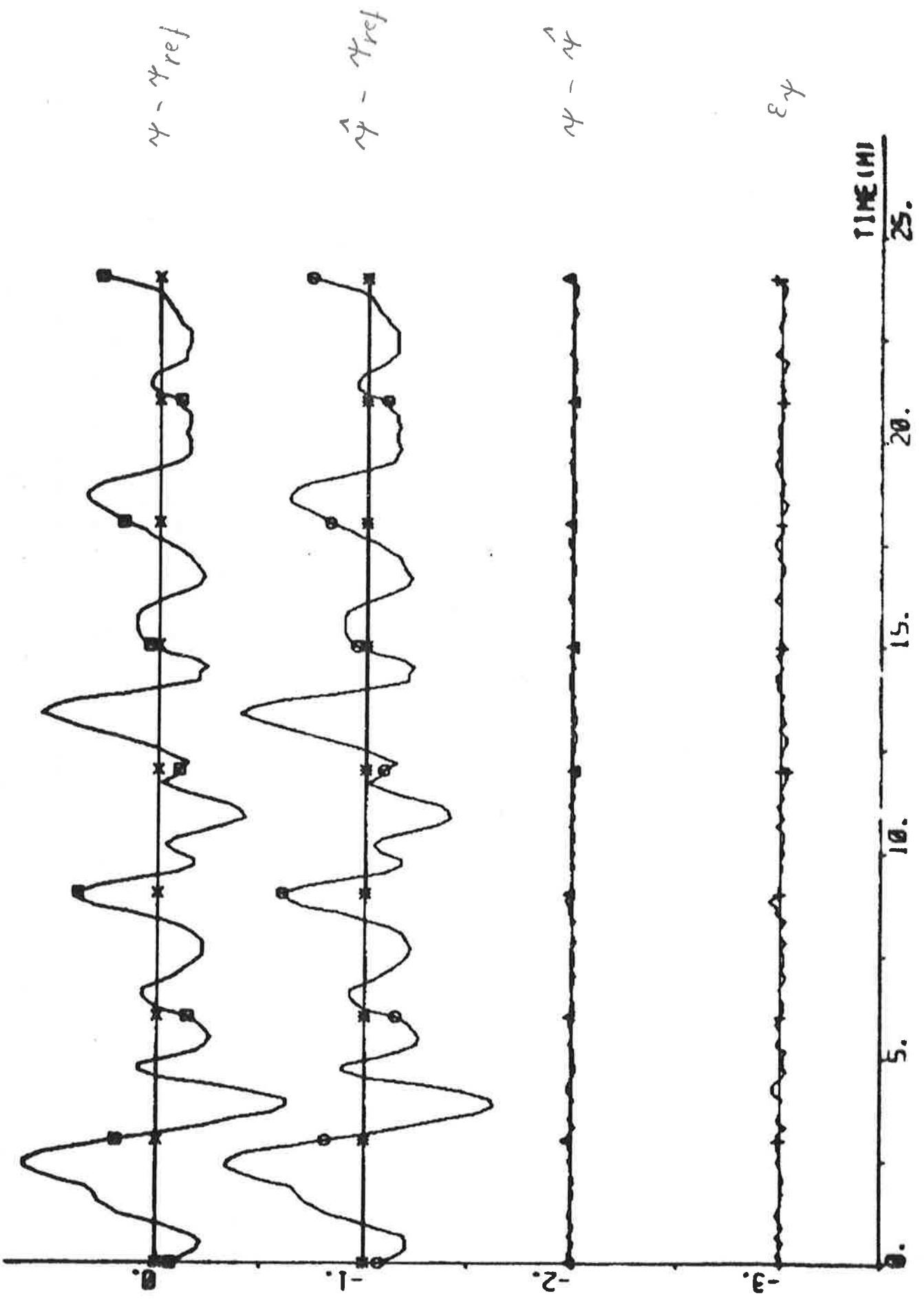




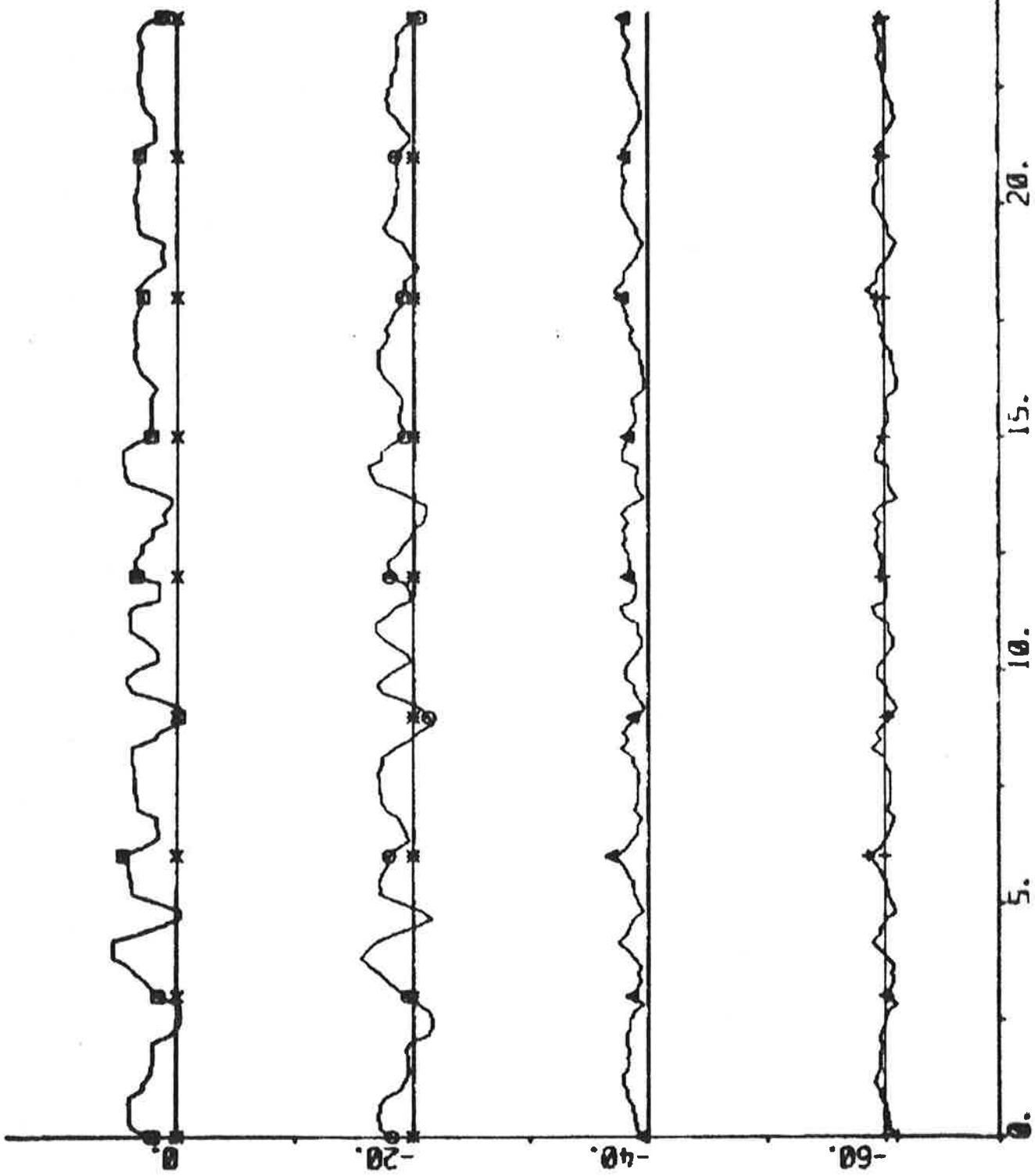
PLOT AMP1(1)-AMP1(6) AMP1(7) ERR3 EPS3 00 002 004 006 -0.7 0. - DECS-VS



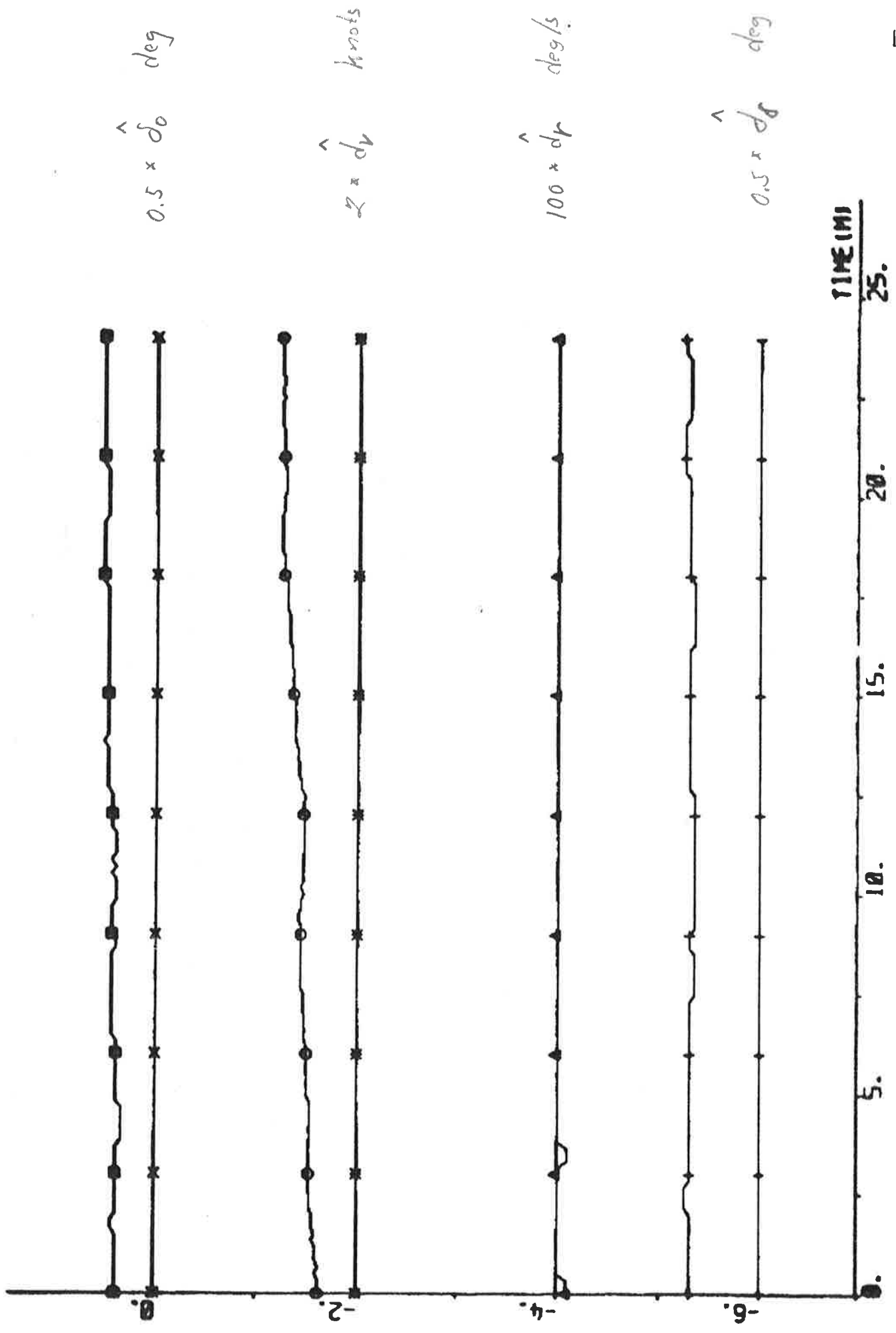
PLOT AMP1(1)-AMP1(8) AMP1(8) ERR4 EPS4 00 01 02 03 -3.4 0.0 - DEG



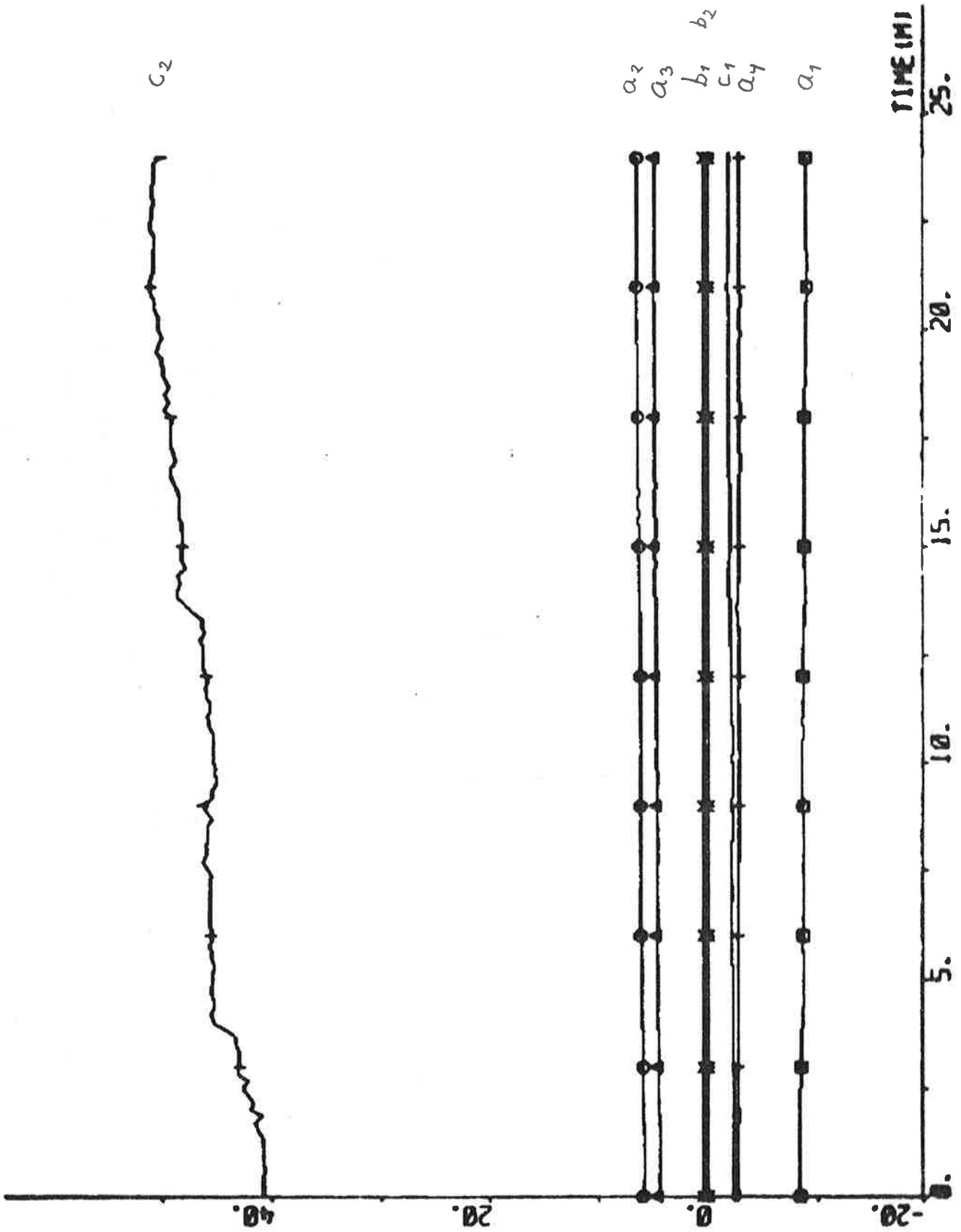
PLOT AWP1(1)-AWP1(2) AWP1(3) ERR1 EPS1 00 020 040 060 .05 15 ° DEG



PLOT AMP1(1)-AMP2(3) AMP2(4) AMP2(5) AMP2(6) 00 02 04 08 -0.5 1.5



PLOT AMP1(1)-AMP2(7) AMP2(8) AMP2(9) AMP2(10) AMP2(11) AMP2(12) AMP2(13)



## EXPERIMENT A5

Date	1976-04-22	Forward draught	8.5 m
Time	11.57	Aft draught	12.5 m
Duration	24 min	Wind direction	N (5; see App. A)
Position	N 11°46' W 18°28'	Wind velocity	4 m/s (gentle breeze)
$\psi_{\text{ref}}$	180 deg	Wave height	Low swell from N

Self-tuning regulator using estimates from the Kalman filter

Tuning time before the experiment started: 30 min.

NC1 = 1	NC2 = 1	k = 7	q = 0
$T_s = 10$ s	$V_0 = 7$ m/s	IVVC = 1	

Final values:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ b_1 \\ b_2 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} -8.00 \\ 6.67 \\ 4.55 \\ -3.59 \\ 0.49 \\ 0.17 \\ -1.40 \\ 49.73 \end{bmatrix}$$

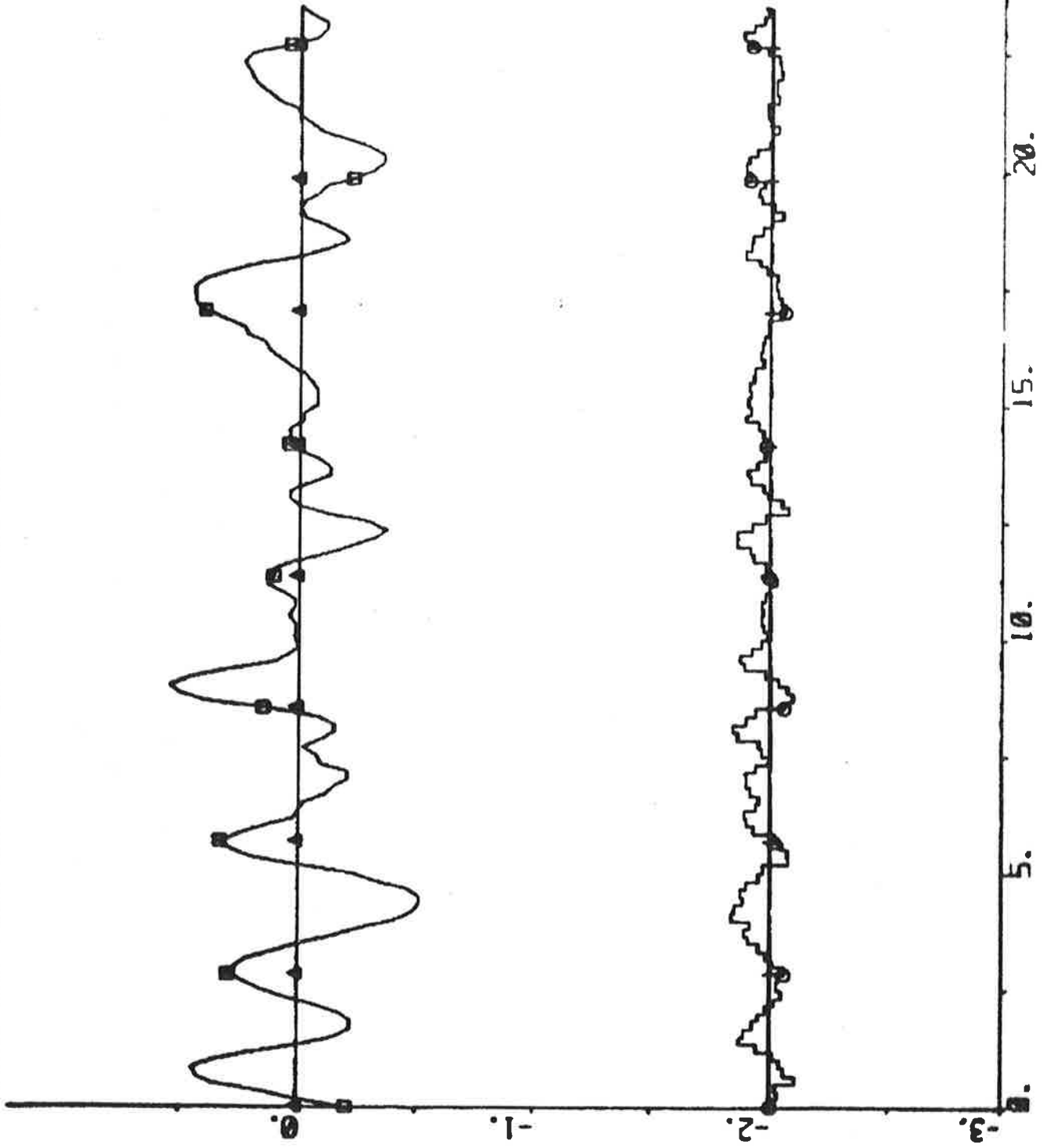
P unknown

$$\begin{aligned} a_1 + a_2 + a_3 + a_4 &= -0.37 \\ \hat{\delta}_0 &= 0.9 \text{ deg} \quad \hat{d}_v = 0.15 \text{ knots} \quad \hat{d}_r = 0.000 \text{ deg/s} \quad \hat{d}_\delta = 1.5 \text{ deg} \end{aligned}$$

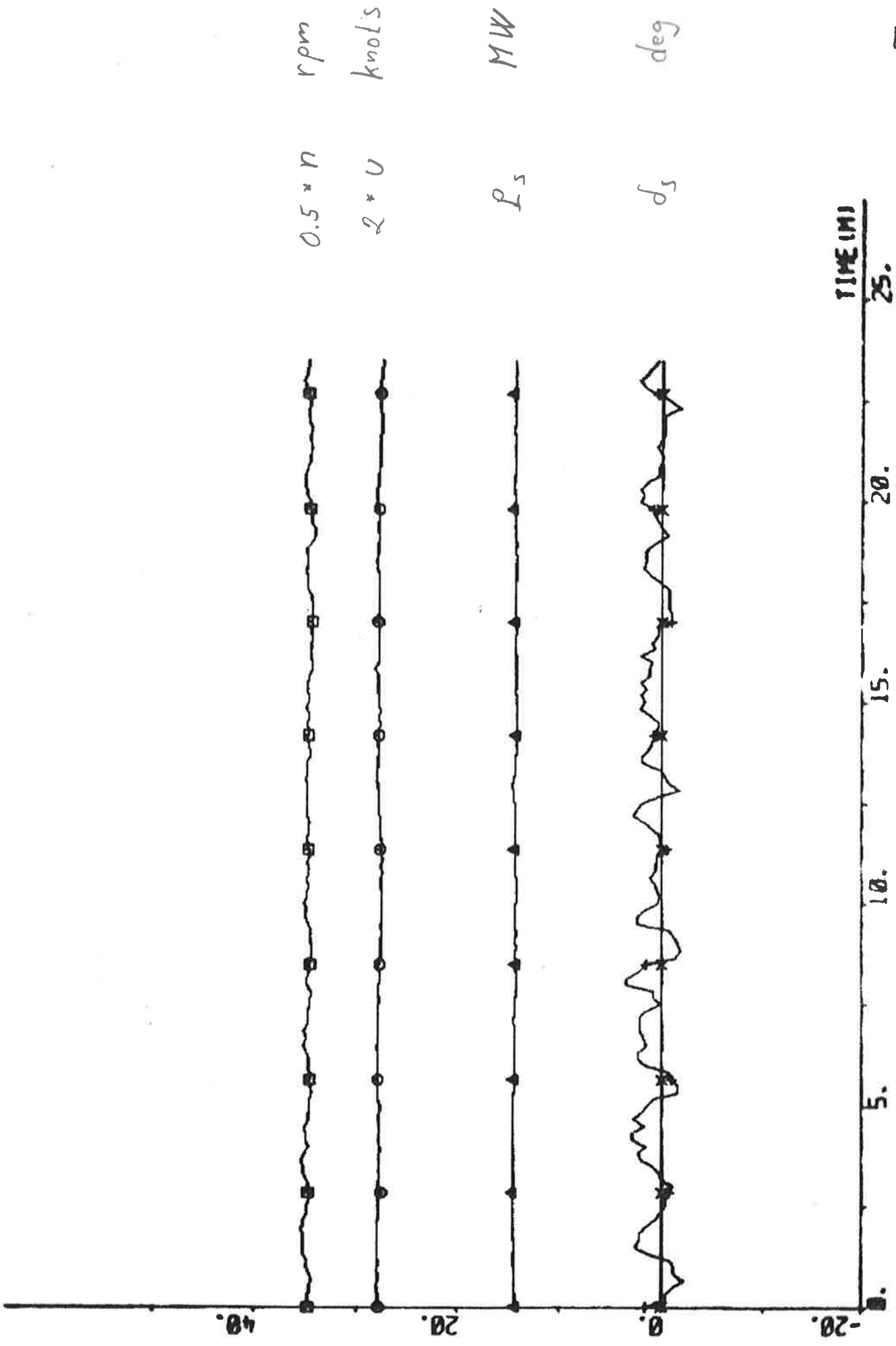
Statistics (mean value and standard deviation)

$\delta_c$	0.66	± 1.17	deg	$P_s$	14.4	± 0.1	MW
$\delta$	2.16	± 1.10	deg	$\epsilon_v$	-0.01	± 0.02	knots
$\psi - \psi_{\text{ref}}$	0.014	± 0.219	deg	$\epsilon_r$	0.00	± 0.01	deg/s
n	69.7	± 0.4	rpm	$\epsilon_\psi$	0.00	± 0.02	deg
u	14.0	± 0.1	knots	$\epsilon_\delta$	0.1	± 0.6	deg
$V_1 = 0.162$							

PLOT ASP1(1) ← ASP1(8) HP ASP1(10) ASP1(15) Q2 -3 1 - DEG

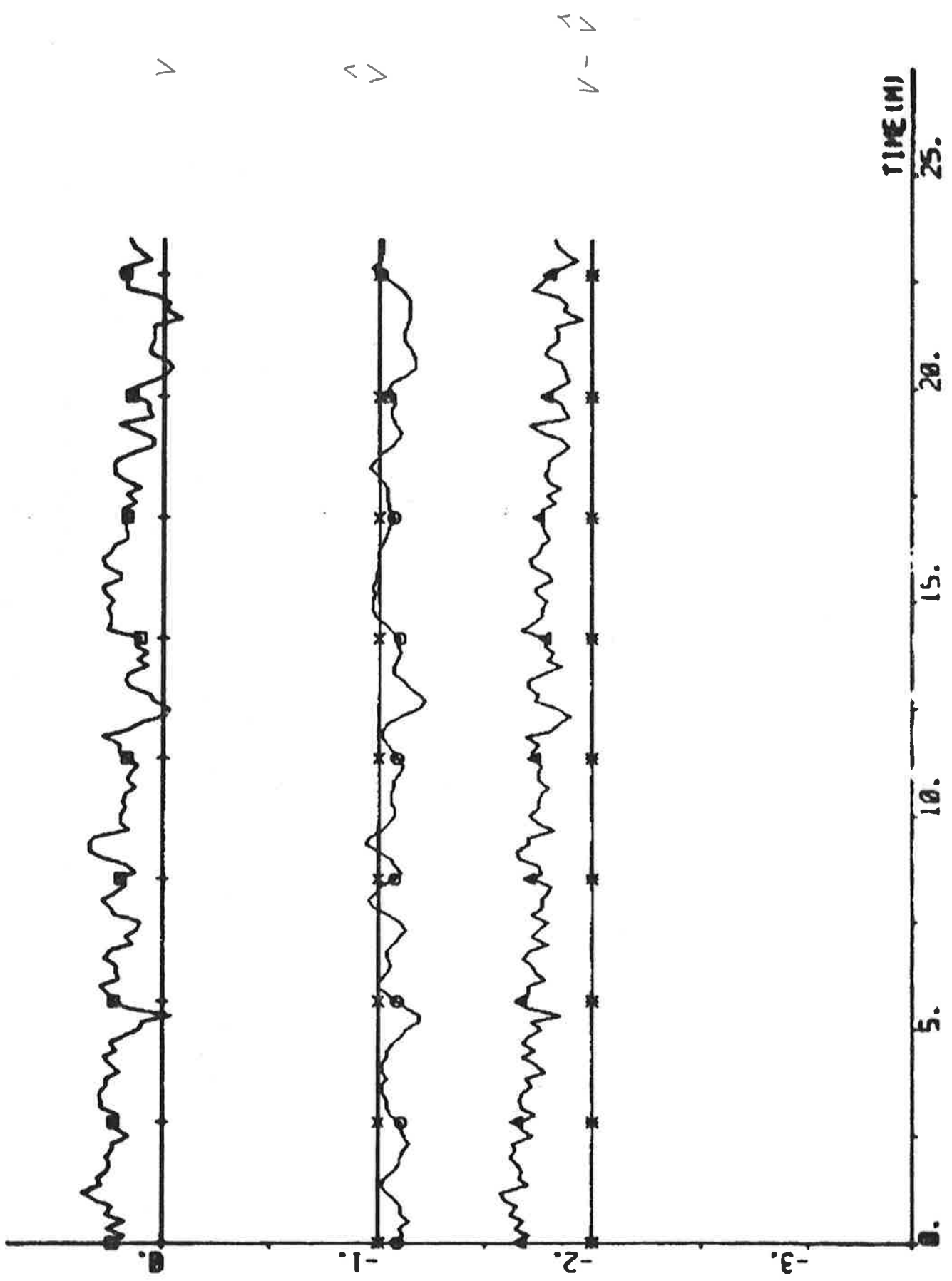


PLOT ASP1(1) ASP1(13) ASP1(12) ASP1(14) ASP1(11) 00 -20 50

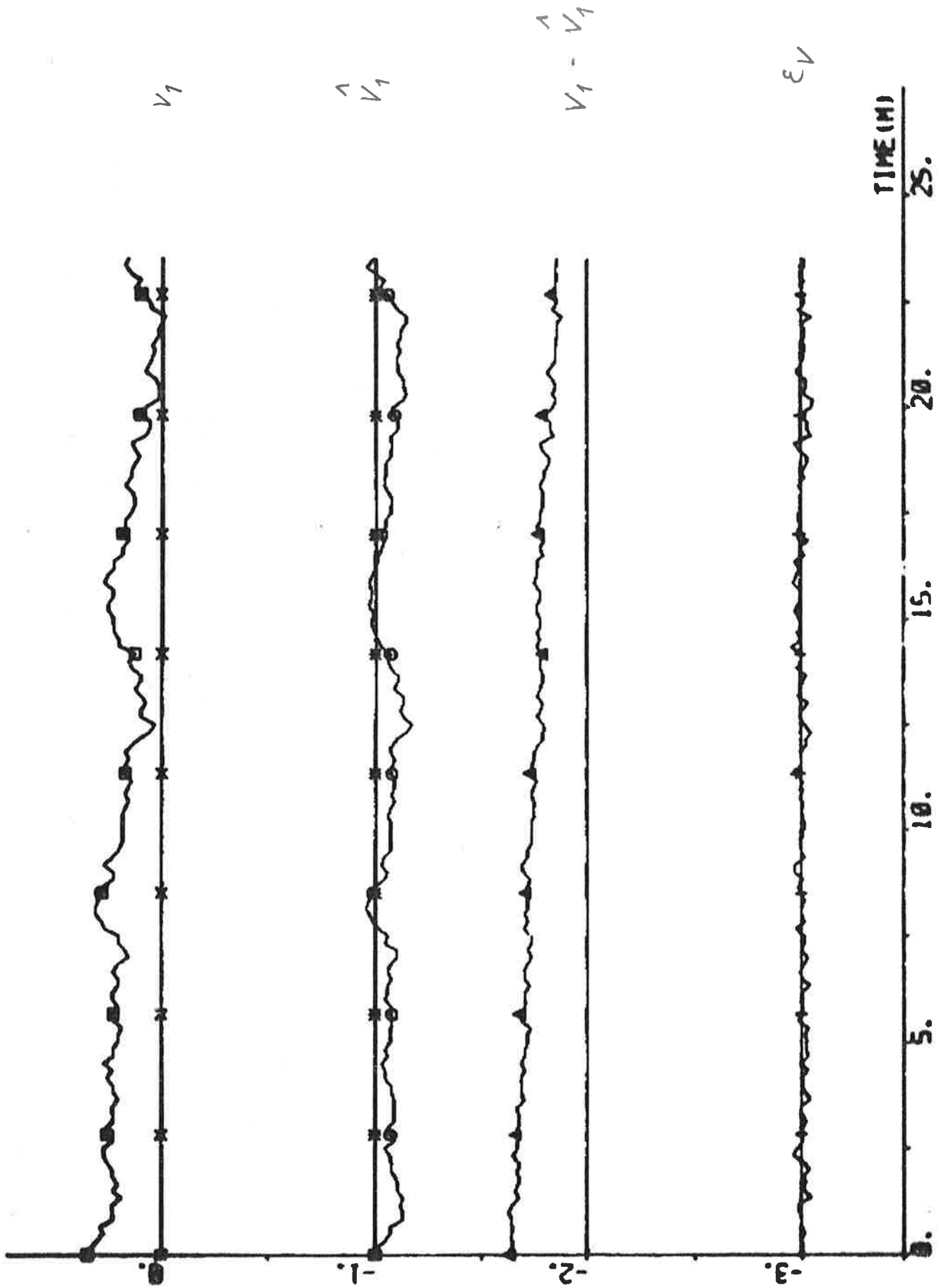




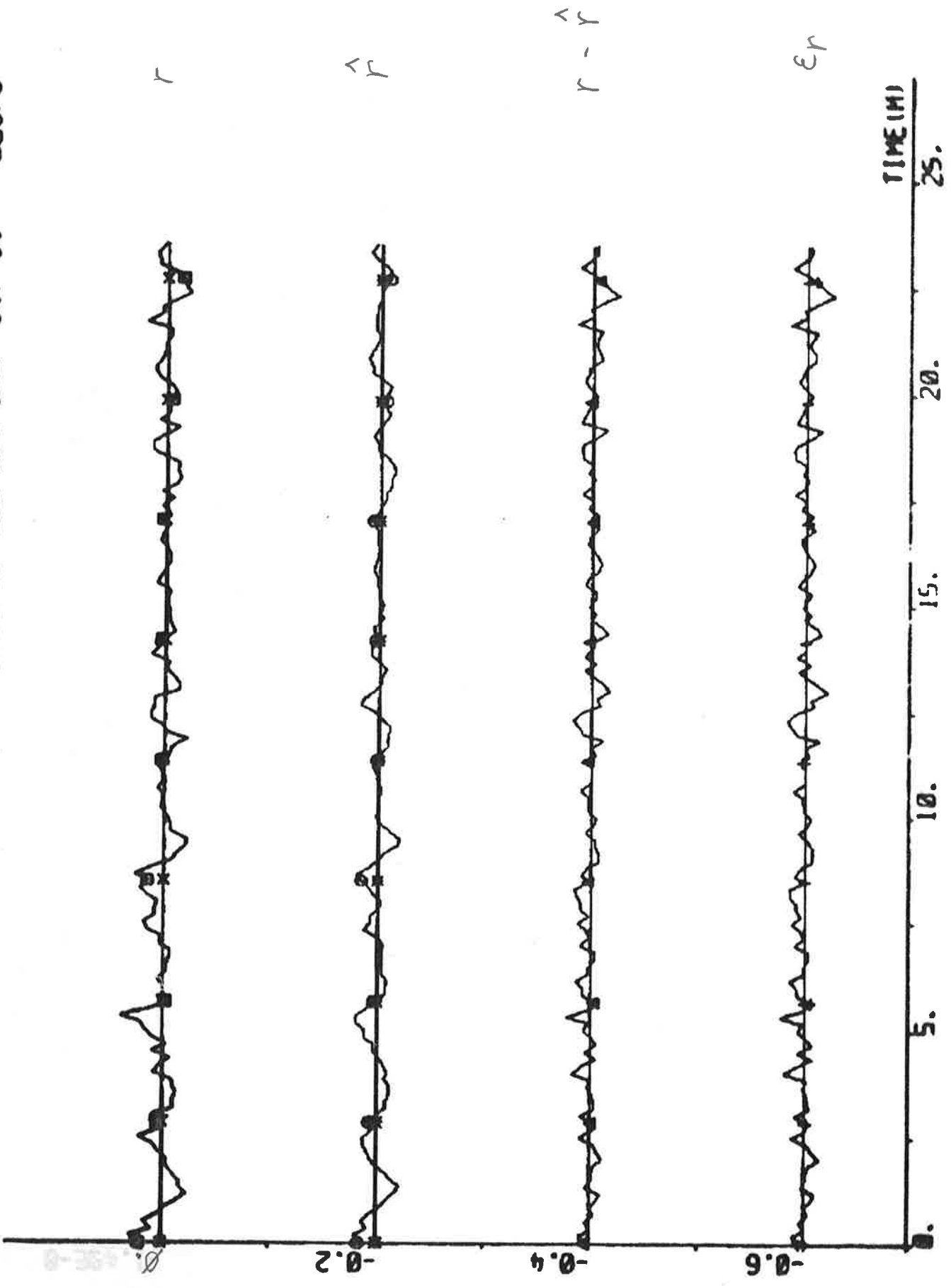
PLOT RSP1(1)-RSP2(1) RSP2(2) ERRS 00 01 02 -3.4 0.0 - KNOTS



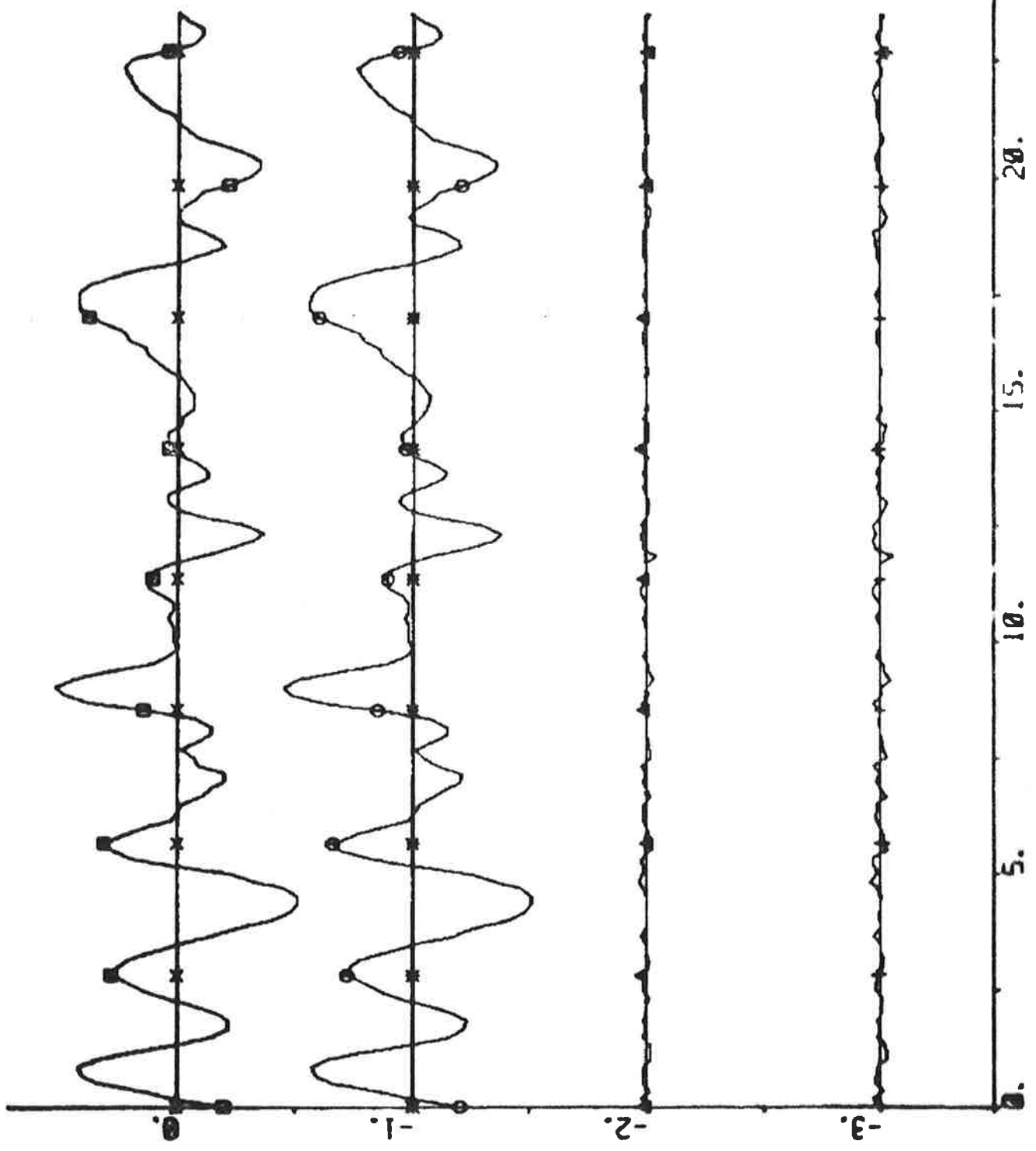
PLOT ASP1(1)←ASP1(4) ASP1(5) ERR2 EPS2 00 01 02 03 -3.4 0.0 • KNOTS



PLOT PEP1(1)-ASP1(8) ASP1(7) ERR3 EPS3 00 002 004 008 -0.7 0. - DEGRS



PLOT ASP1(1)-ASP1(8) ASP1(9) ERR4 EPS4 Q0 Q1 Q2 Q3 -3.4 0.8 - DEG



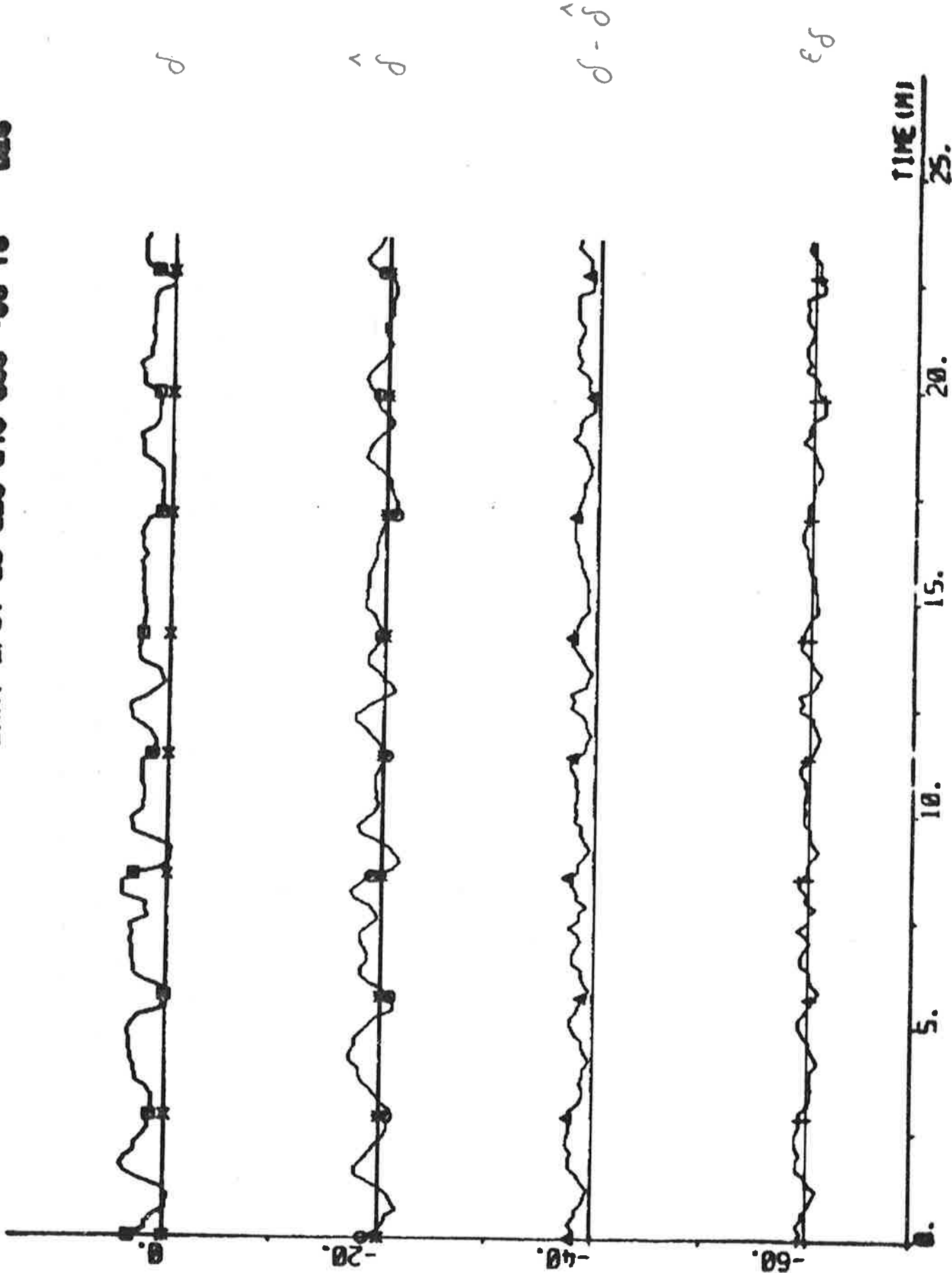
$y - y_{ref}$

$\hat{y} - y_{ref}$

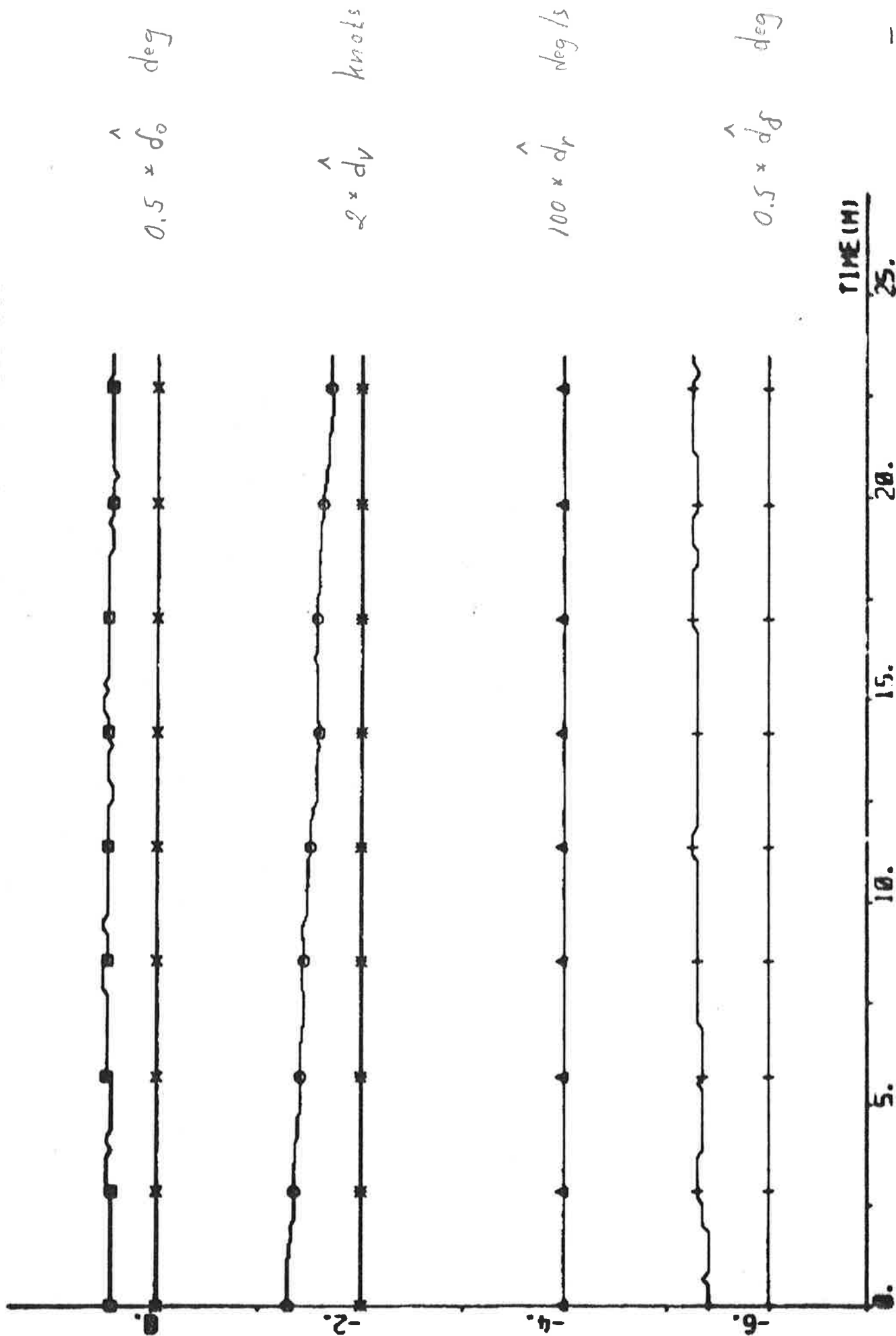
$\hat{y} - y$

$Ey$

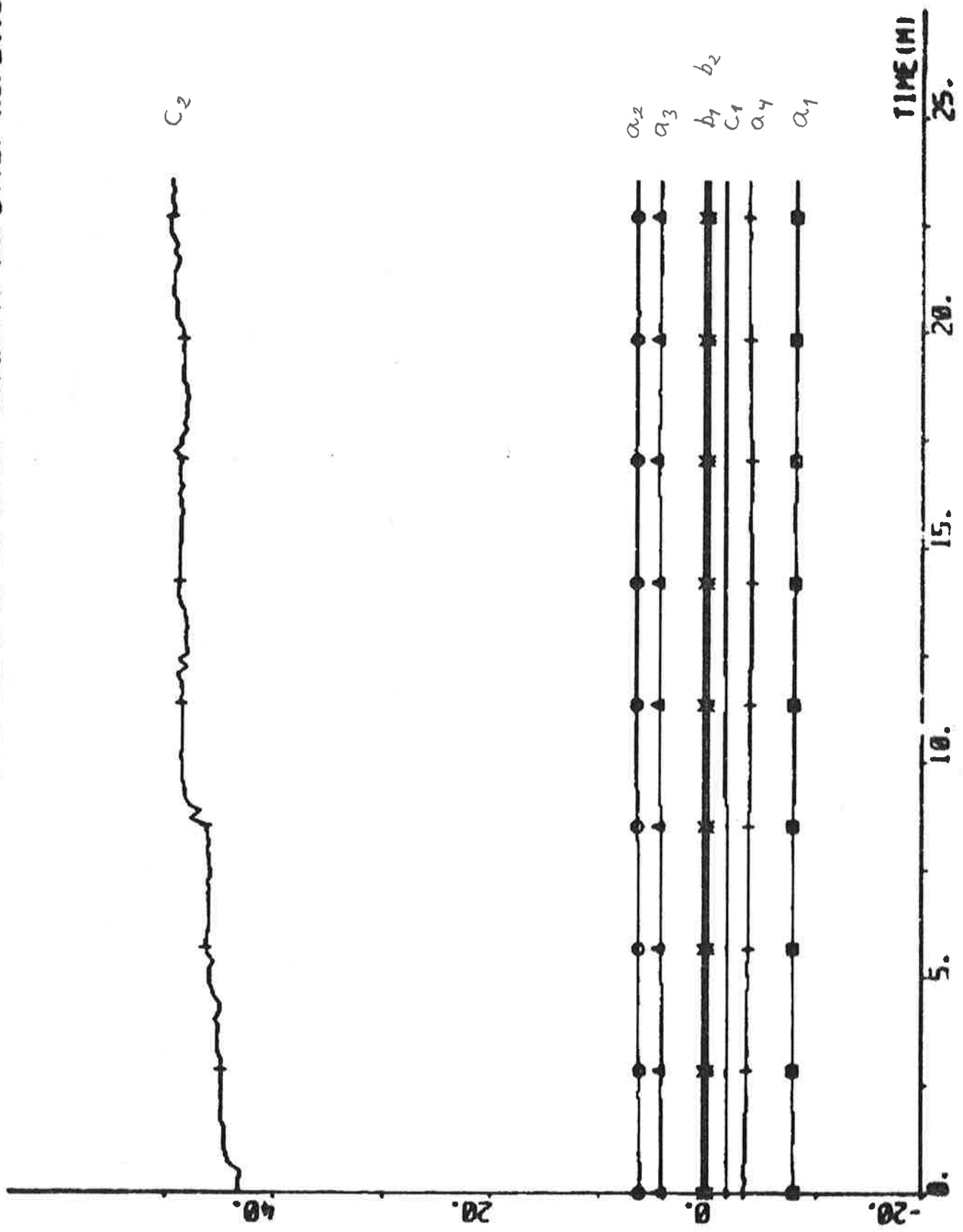
PLOT RESP1(1)-RESP1(2) RESP1(3) ERR1 EPS1 00 020 040 060 -05 15 - 000



PLOT ASP1(1)-ASP2(3) ASP2(4) ASP2(5) ASP2(6) 00 02 04 06 -6.5 1.5



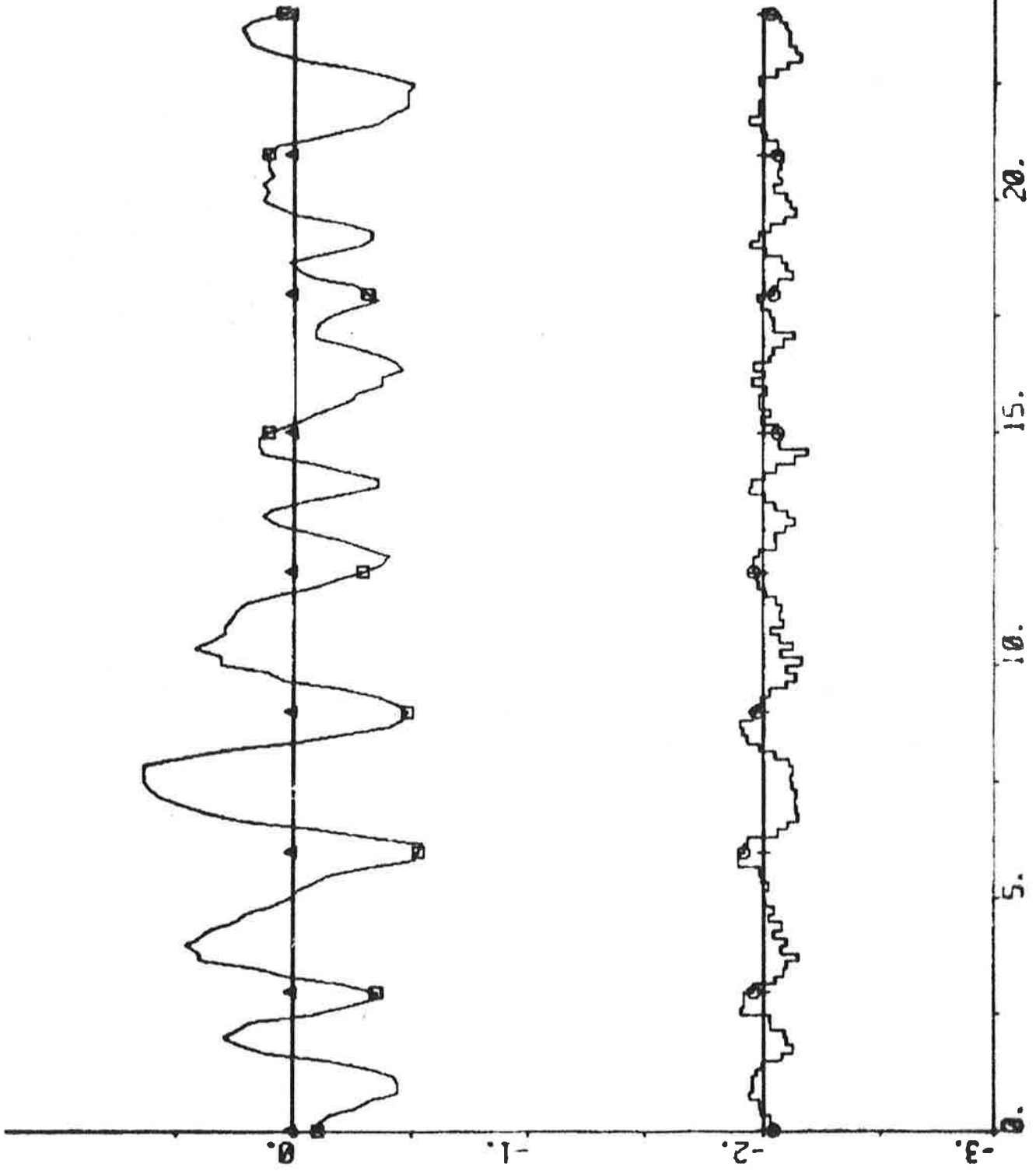
PLOT ASP1(1)-ASP2(7) ASP2(8) ASP2(9) ASP2(10) ASP2(11) ASP2(12) ASP2(13)



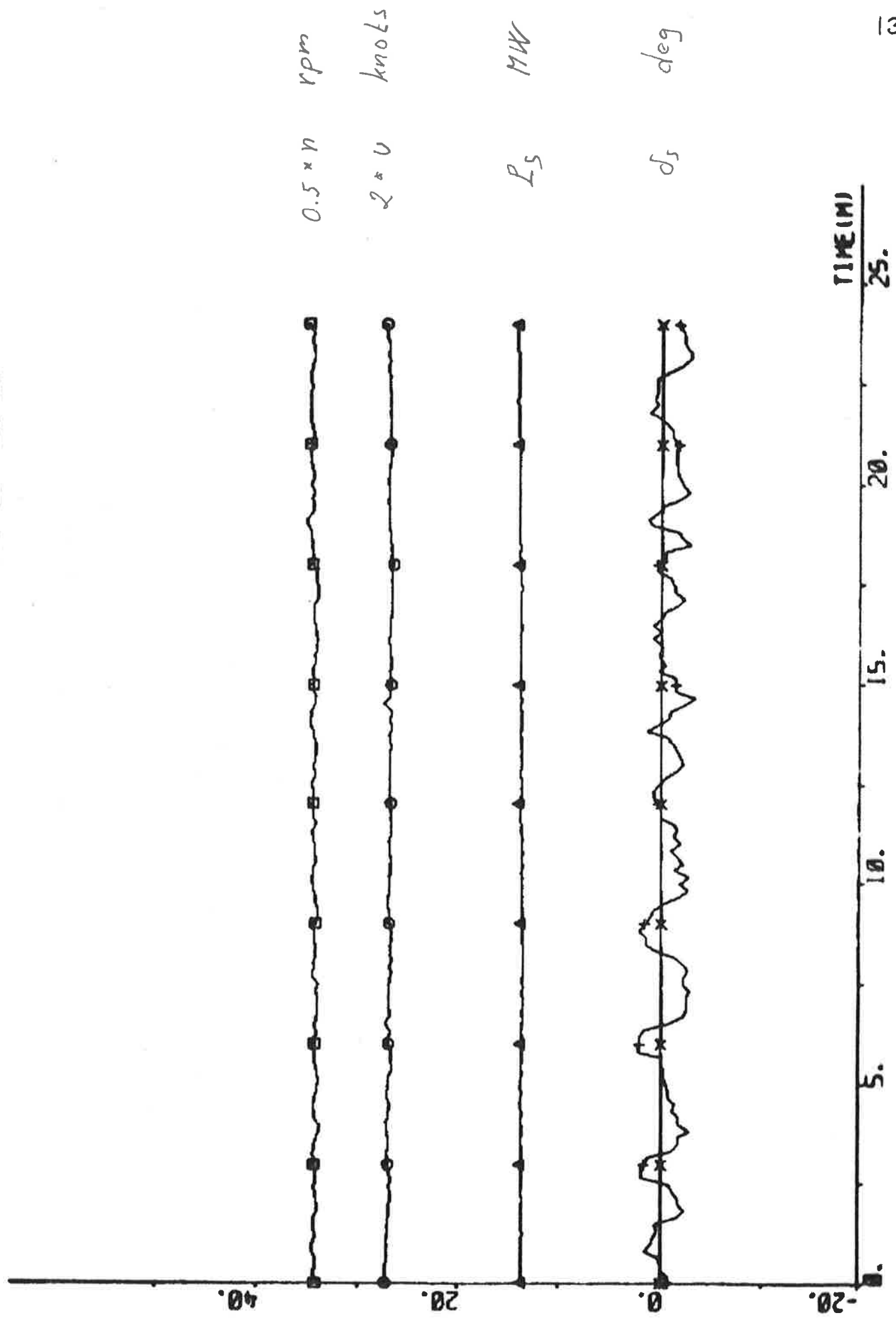




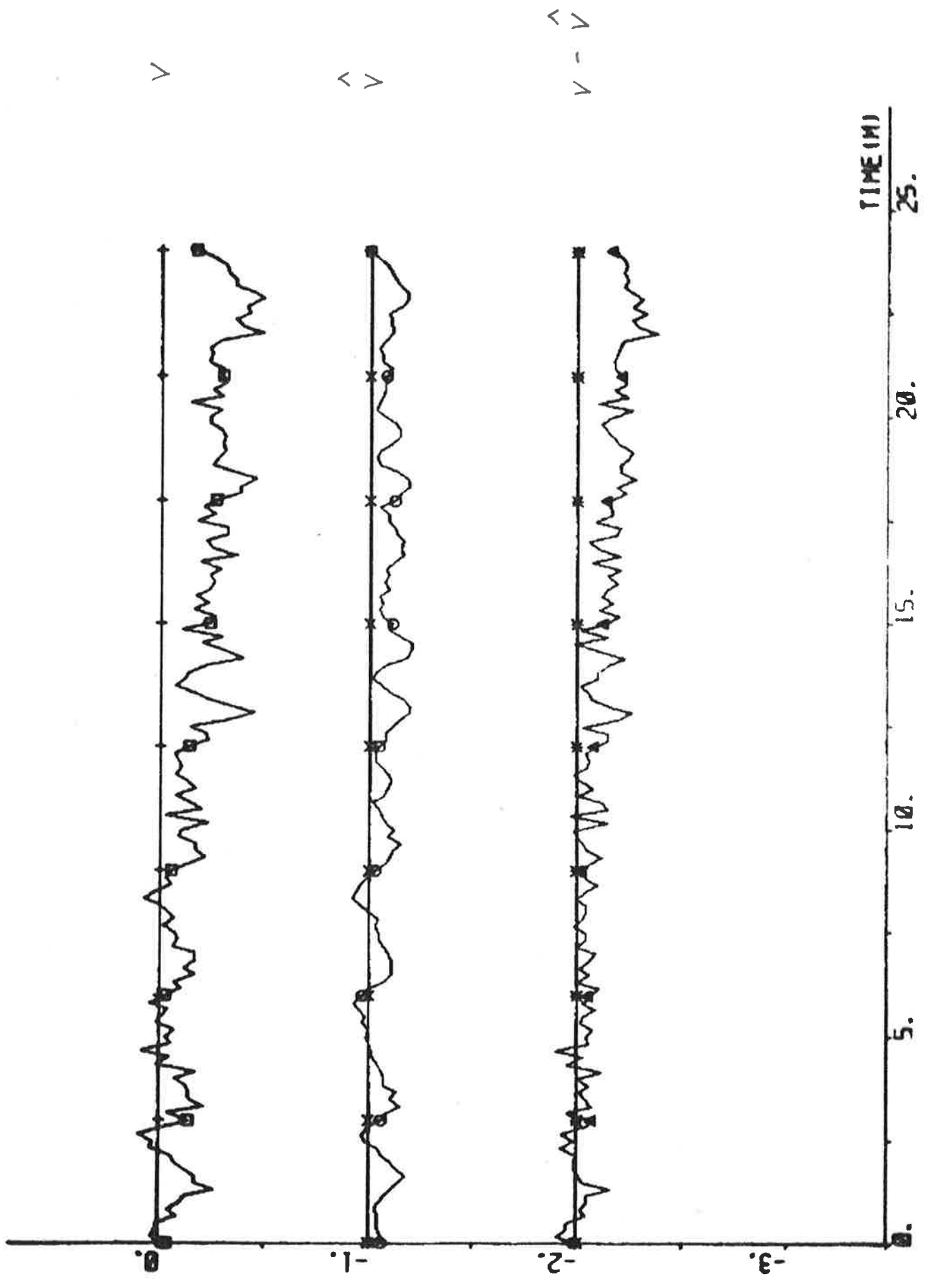
PLOT ASP1(1)-ASP1(8) HP ASP1(10) ASP1(15) 02 -3 1 - DEG



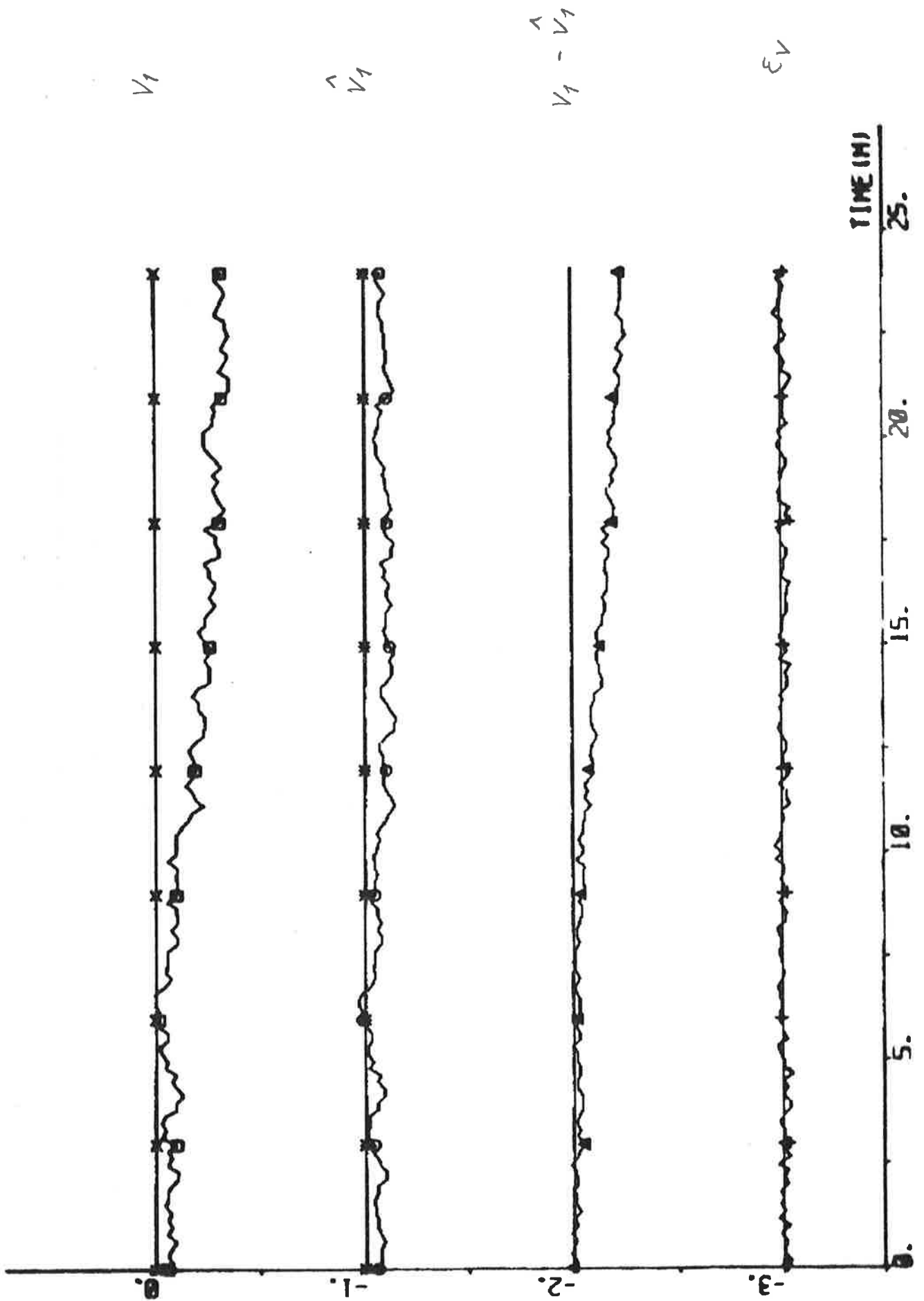
PLOT ASP1(1)-ASP1(13) ASP1(12) ASP1(14) ASP1(11) 00 -20 50



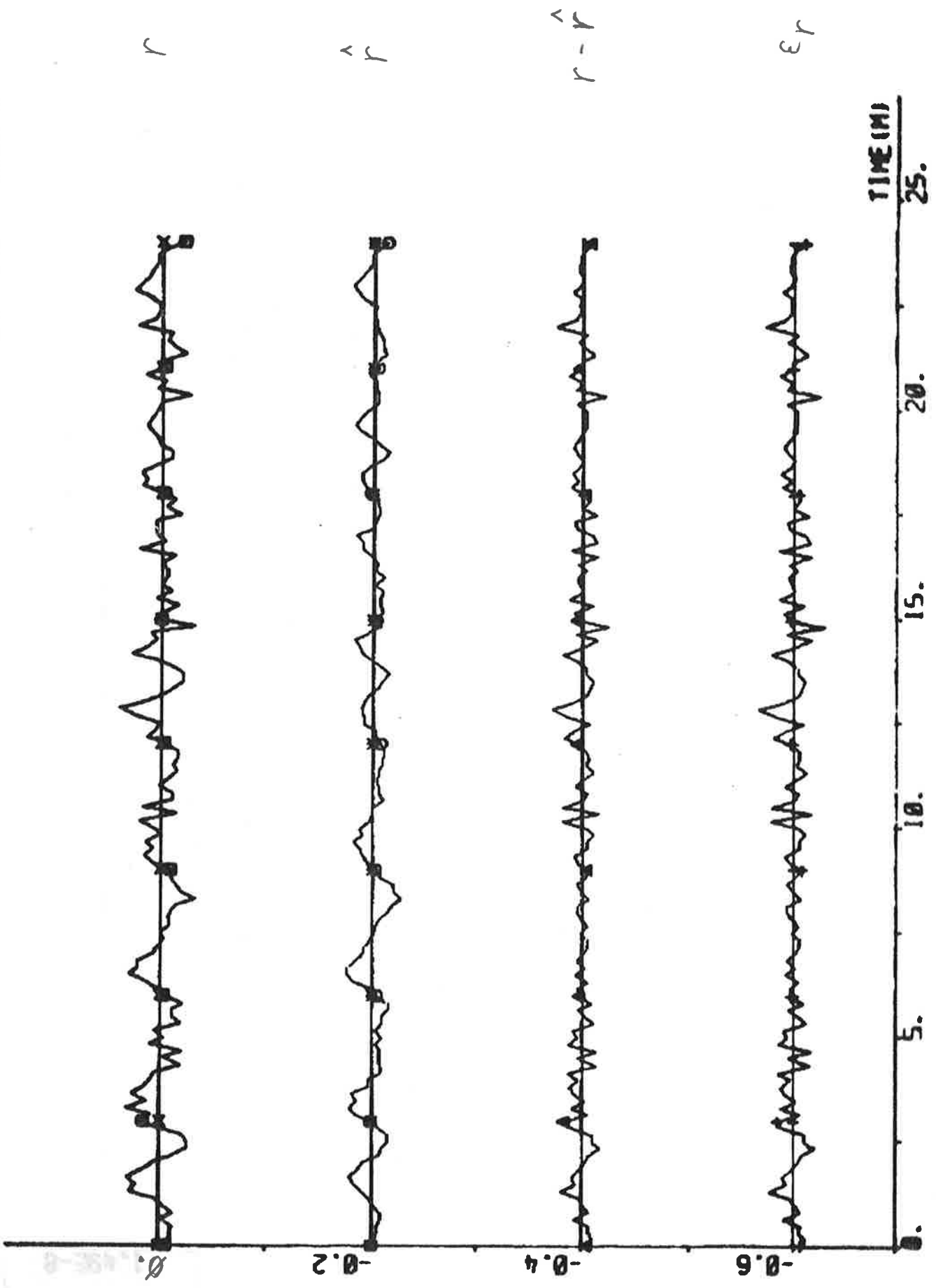
PLOT RSP1(1)-RSP2(1) RSP2(2) ERRS Q0 Q1 Q2 -3.4 0.0 - KNOTS



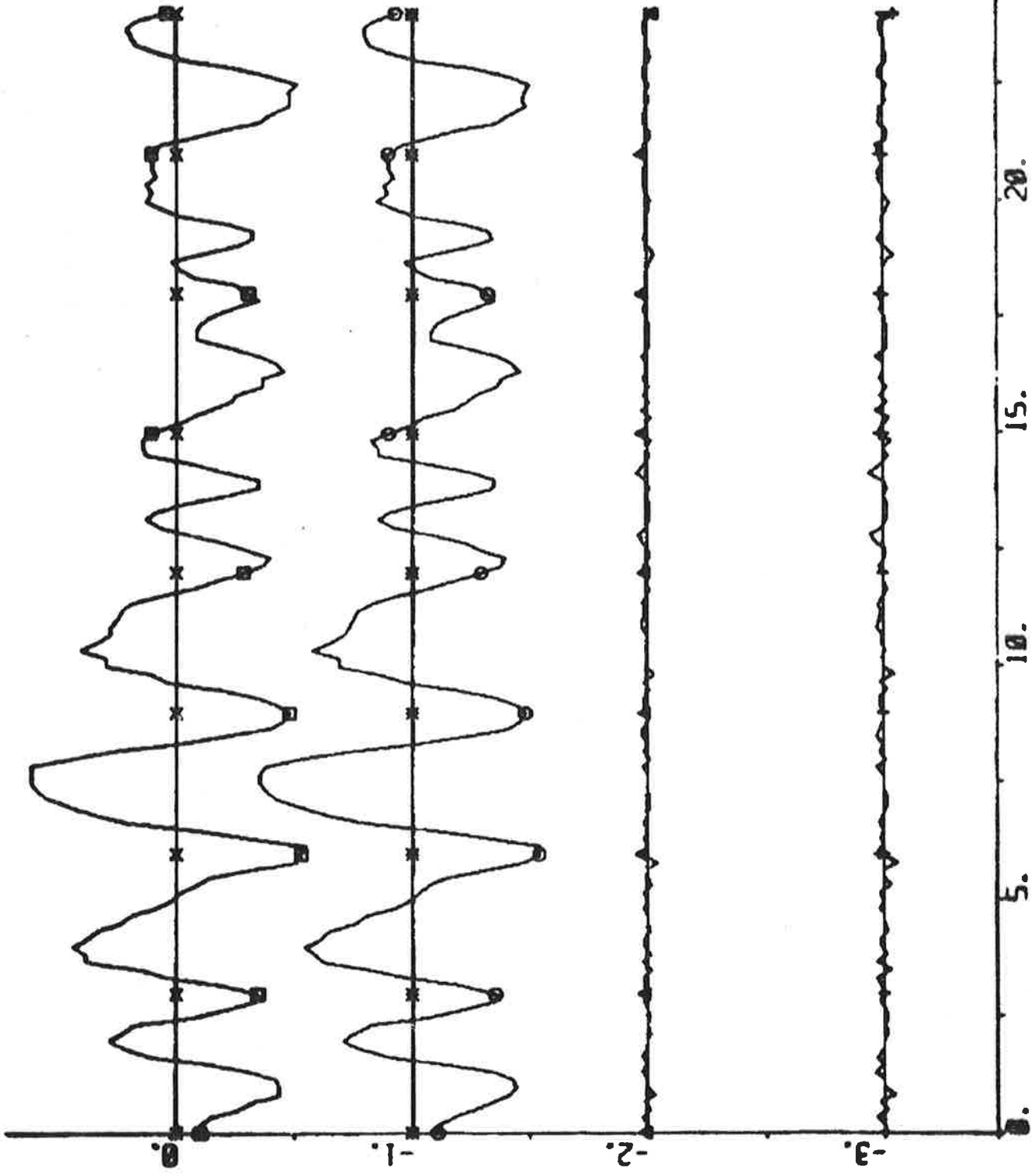
PLOT ASP1(1) ASP1(4) ASP1(5) ERR2 EPS2 00 01 02 03 -3.4 0.0 - KNOTS



PLOT ASP1(1)-ASP1(6) ASP1(7) ERR3 EPS3 00 002 004 008 -0.7 0. " DECS



PLOT ACP1(1) - ACP1(0) ACP1(0) ERR4 EPS4 00 01 02 03 -3.4 0.0 DEG



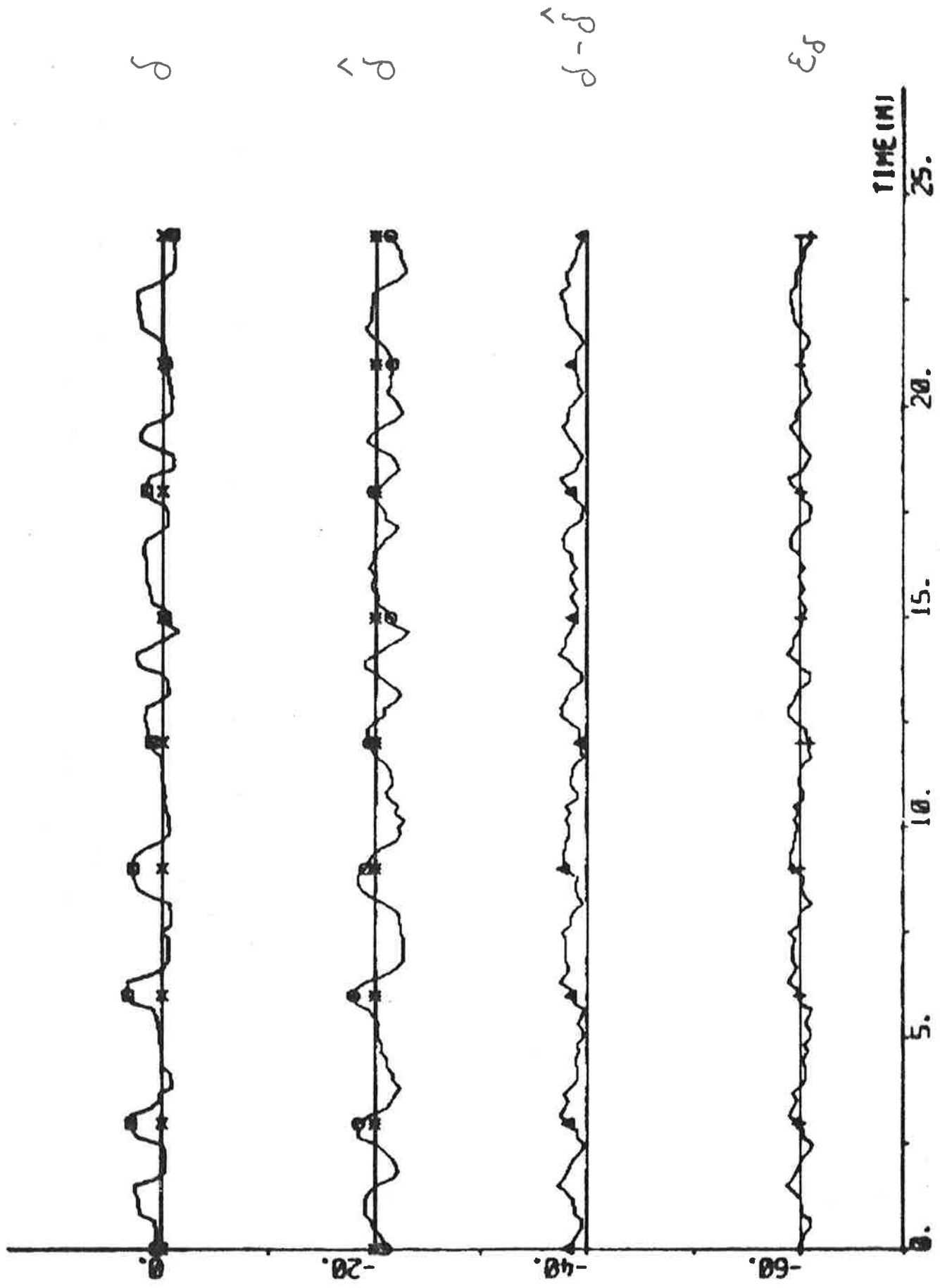
$\hat{y} - y_{ref}$

$\hat{y} - y_{ref}$

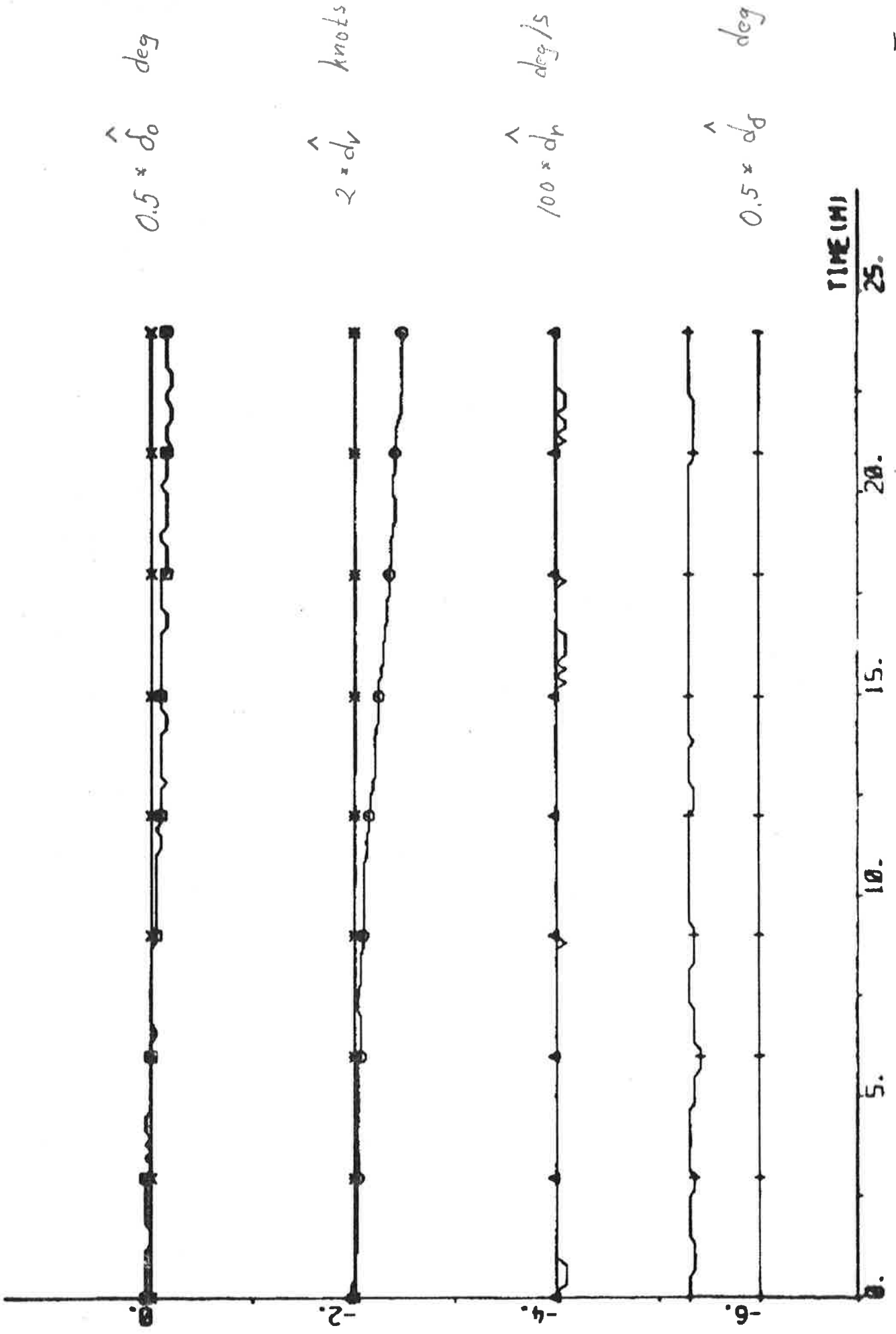
$\hat{y} - y$

$\epsilon_3$

PLOT ASP1(1)-ASP1(2) ASP1(3) ERR1 EPS1 00 020 040 060 -05 16 " DEG

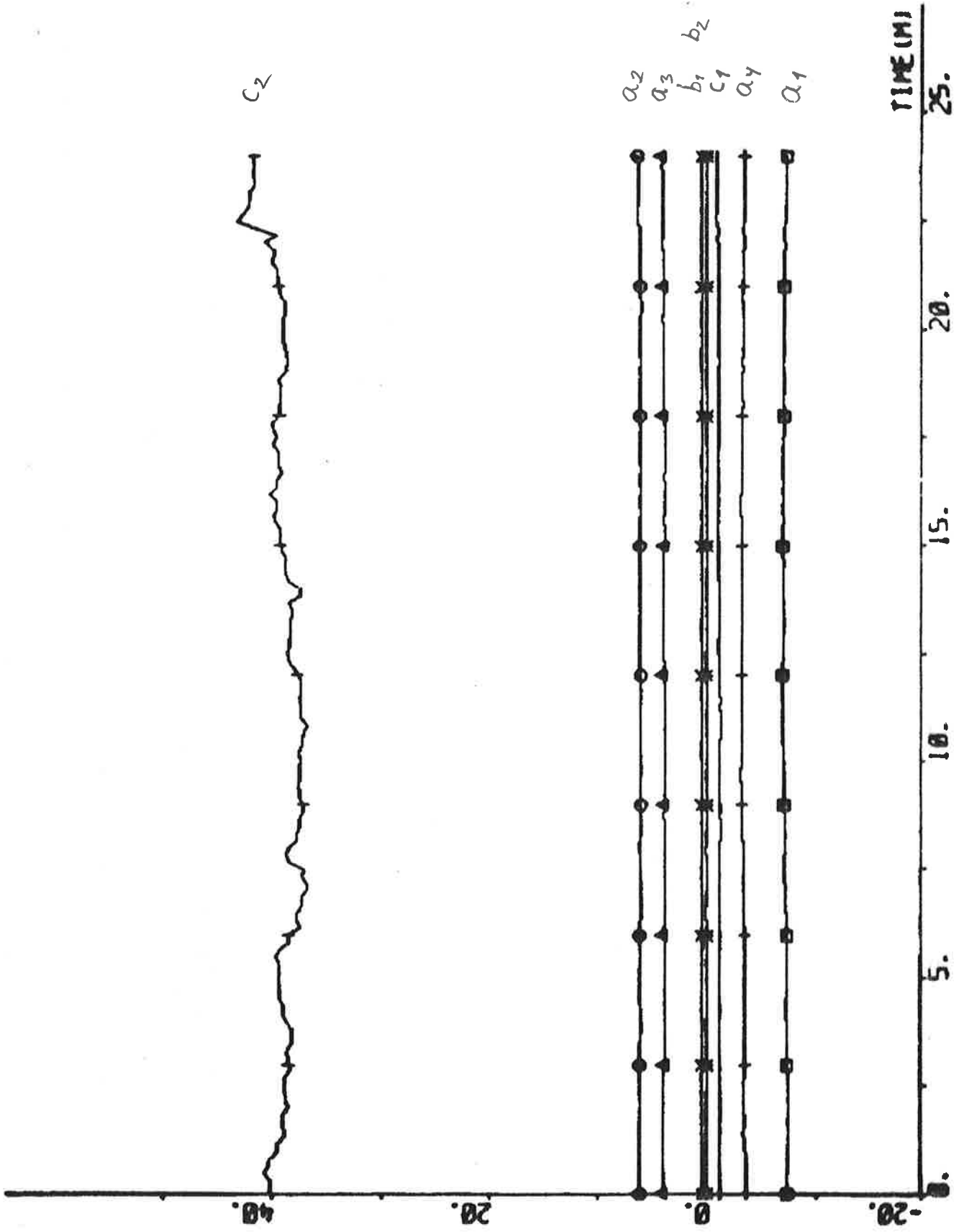


PLOT ASP1(1) ASP2(3) ASP2(4) ASP2(5) ASP2(6) 00 02 04 06 -0.5 1.5



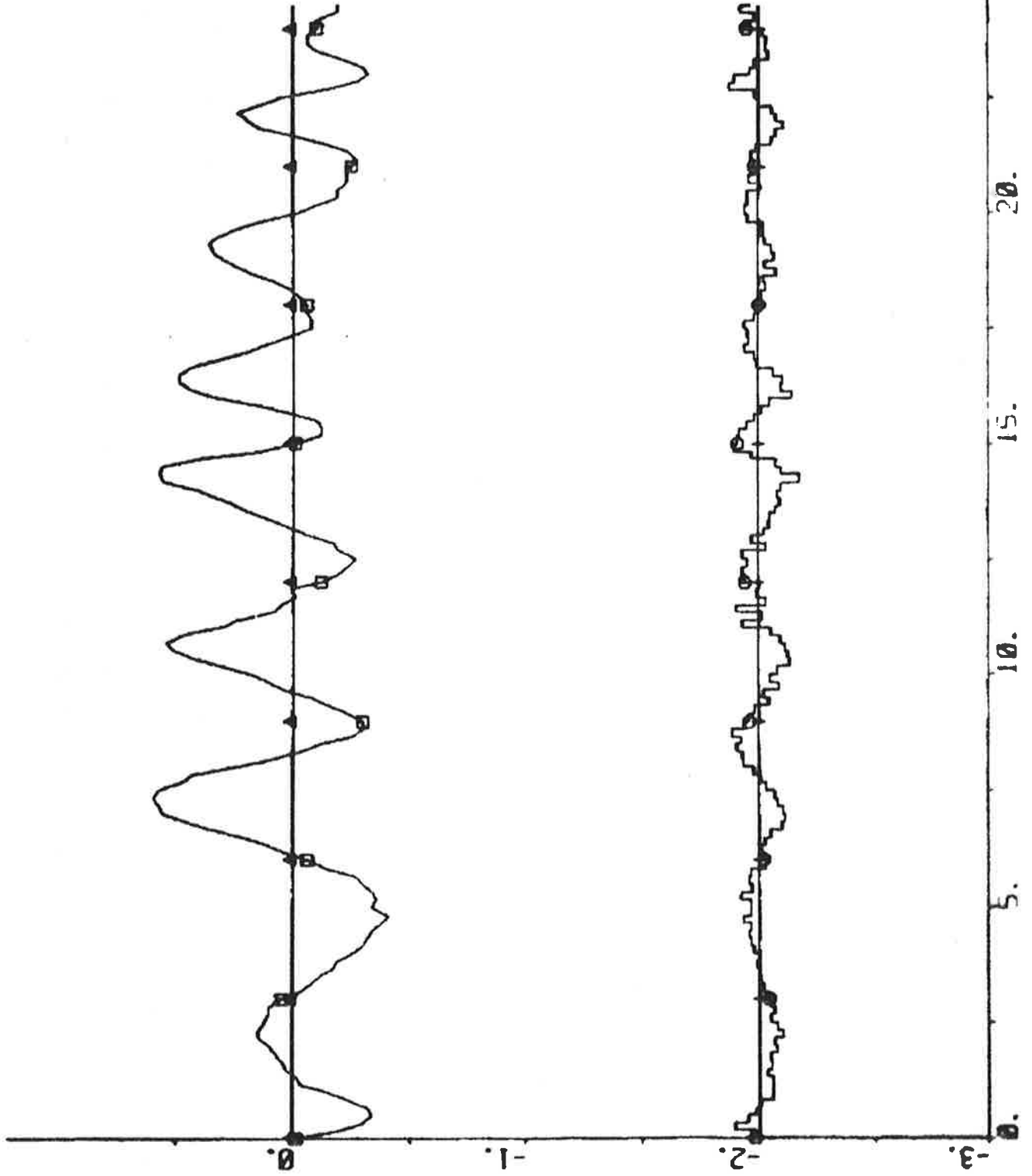


PLOT ASP1(1)-ASP2(17) ASP2(8) ASP2(9) ASP2(10) ASP2(11) ASP2(12) ASP2(13)

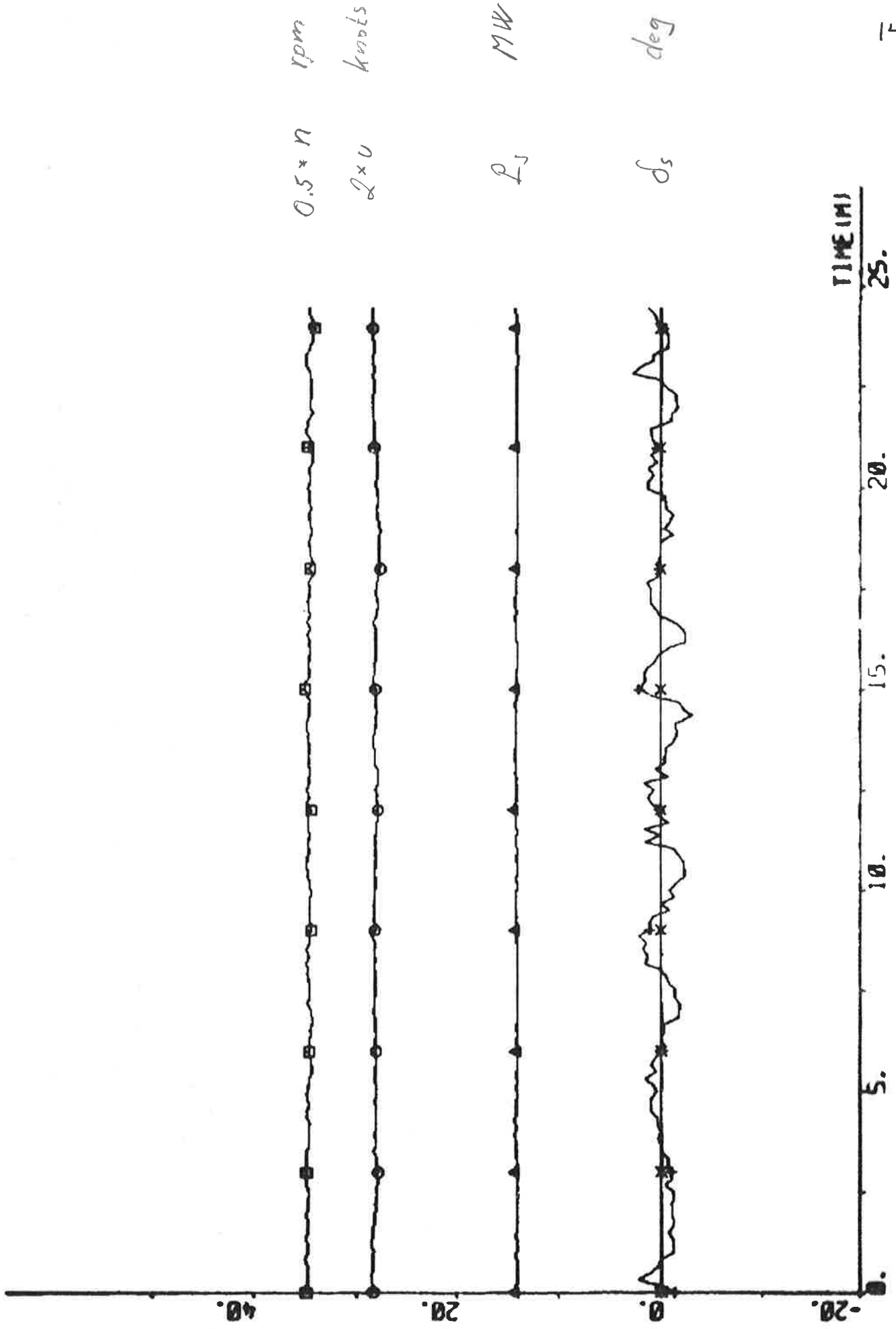




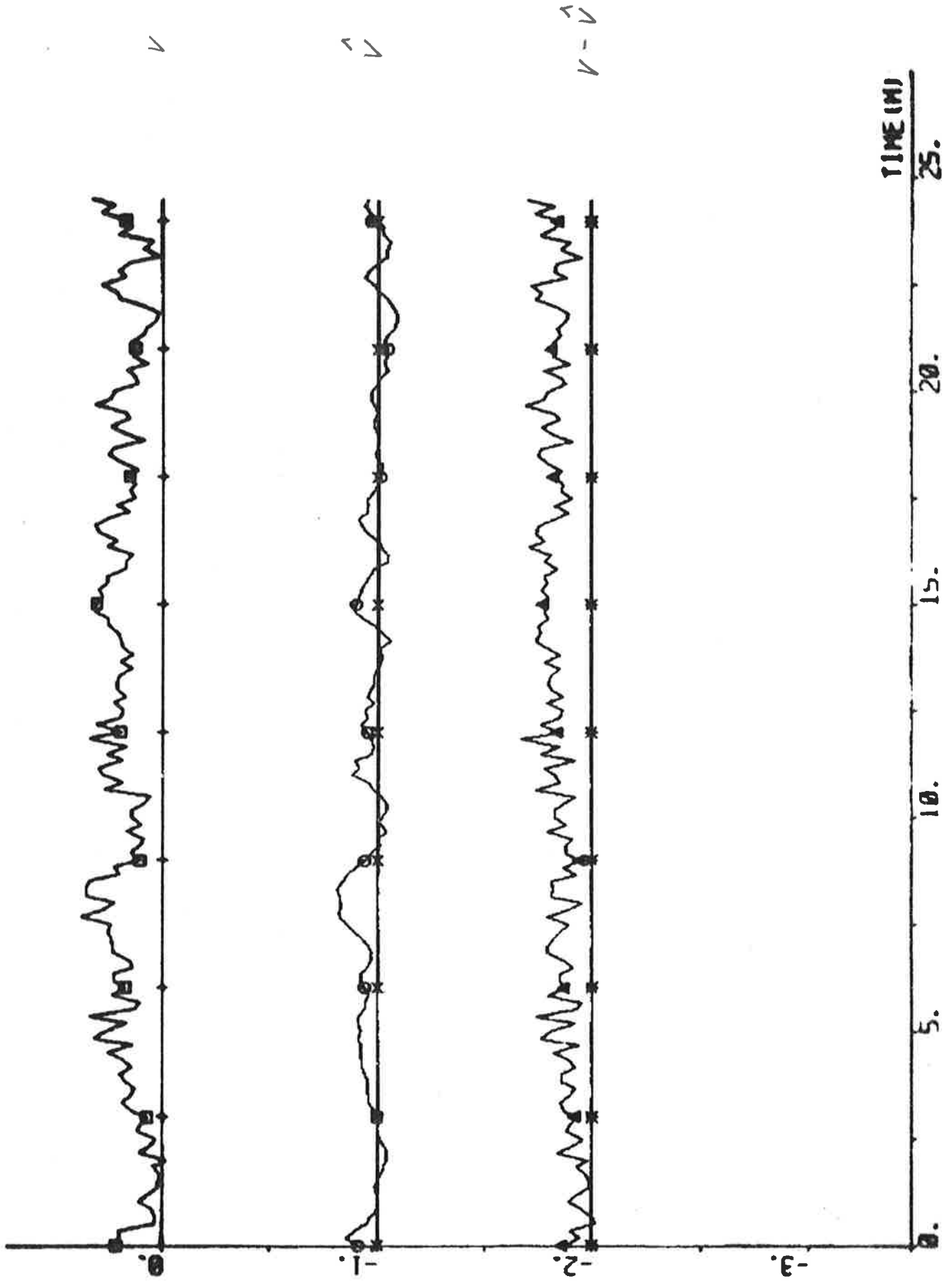
PLOT A7P1(1)-A7P1(8) HP A7P1(10) A7P1(15) Q2 -3 1 - DEG



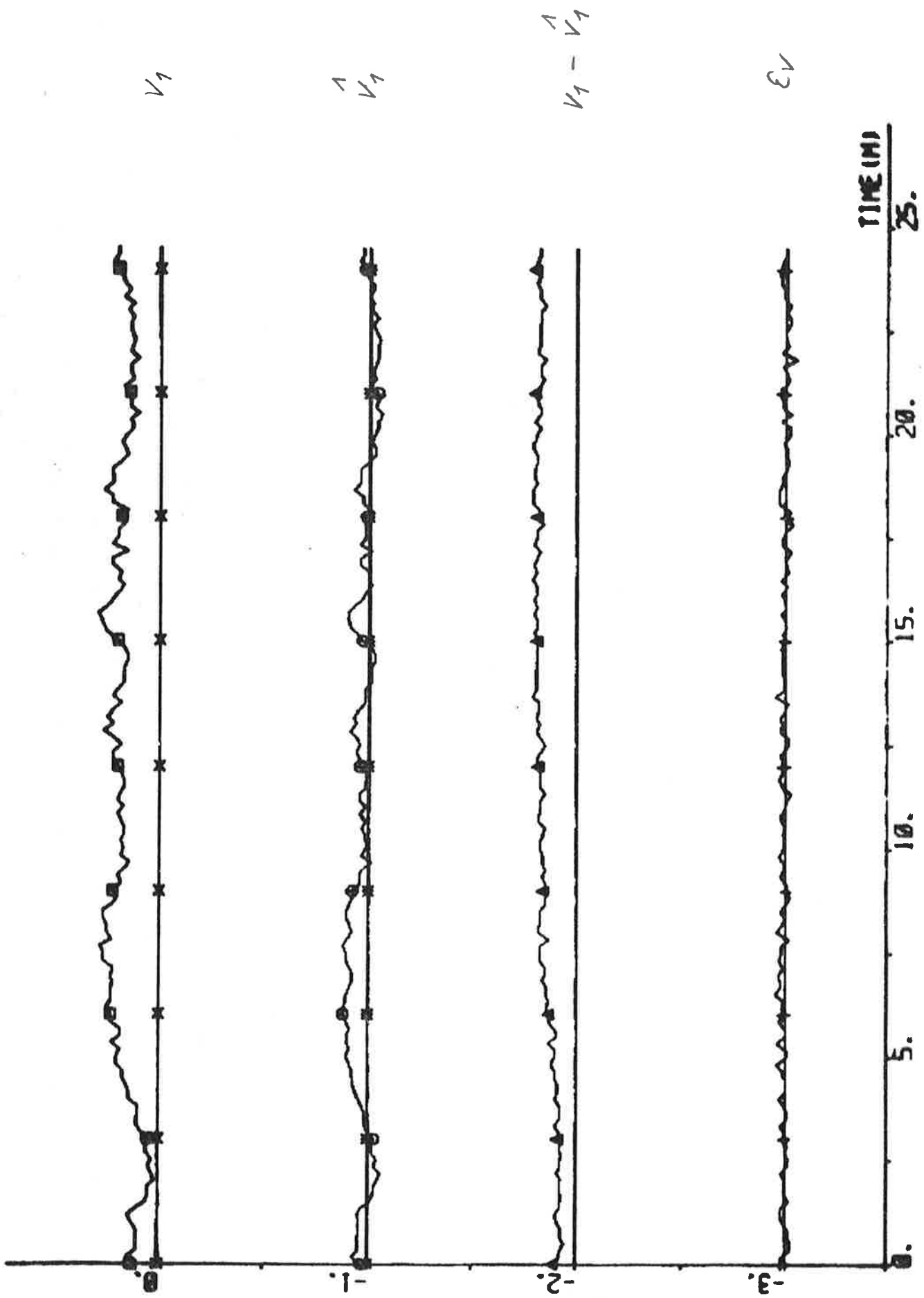
PLOT A7P1(1)-A7P1(13) A7P1(12) A7P1(14) A7P1(11) 00 -20 50



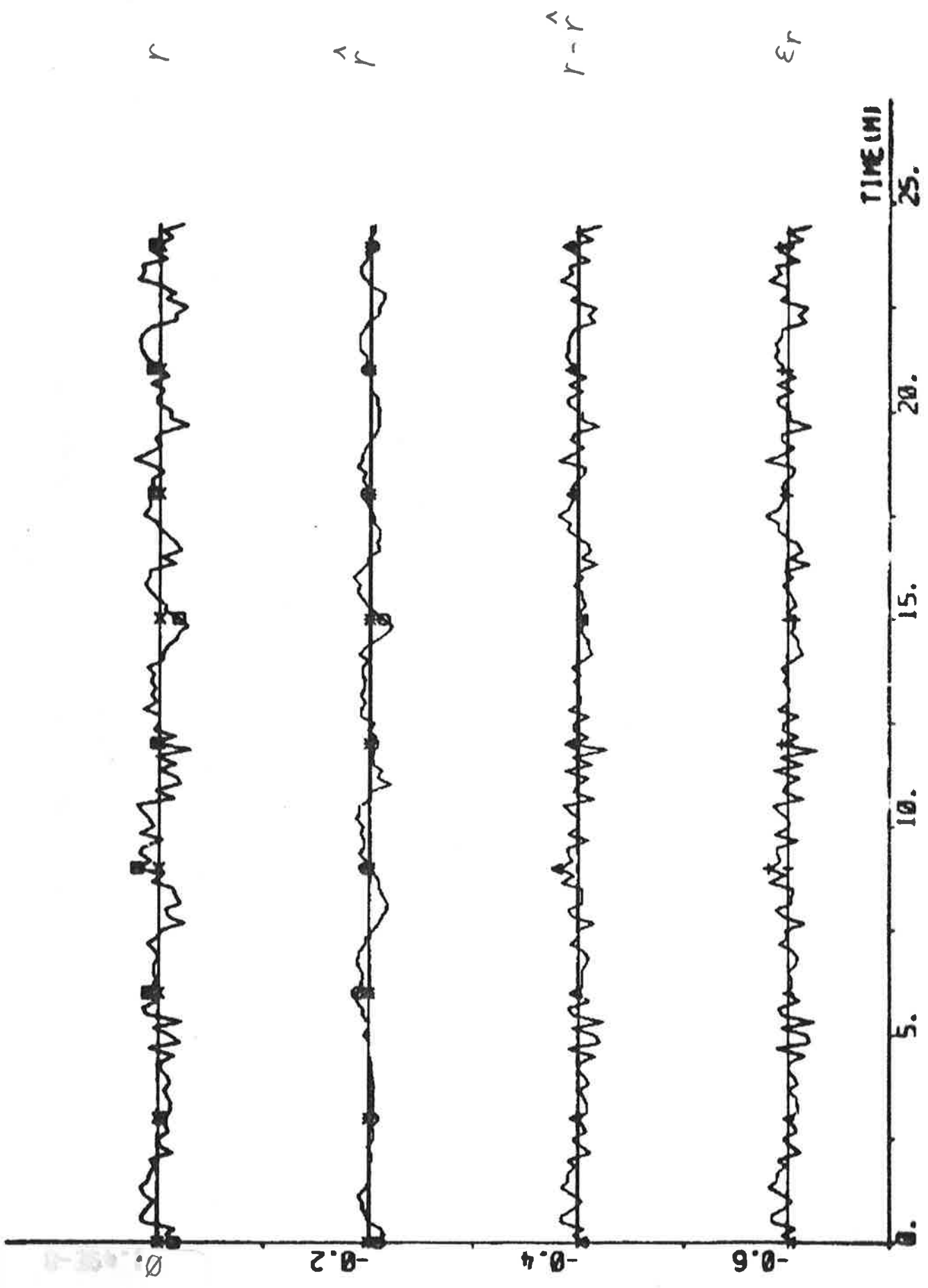
PLOT A7P1(1)-A7P2(1) A7P2(2) ERRS 00 01 02 -3.4 0.0 - KNOTS



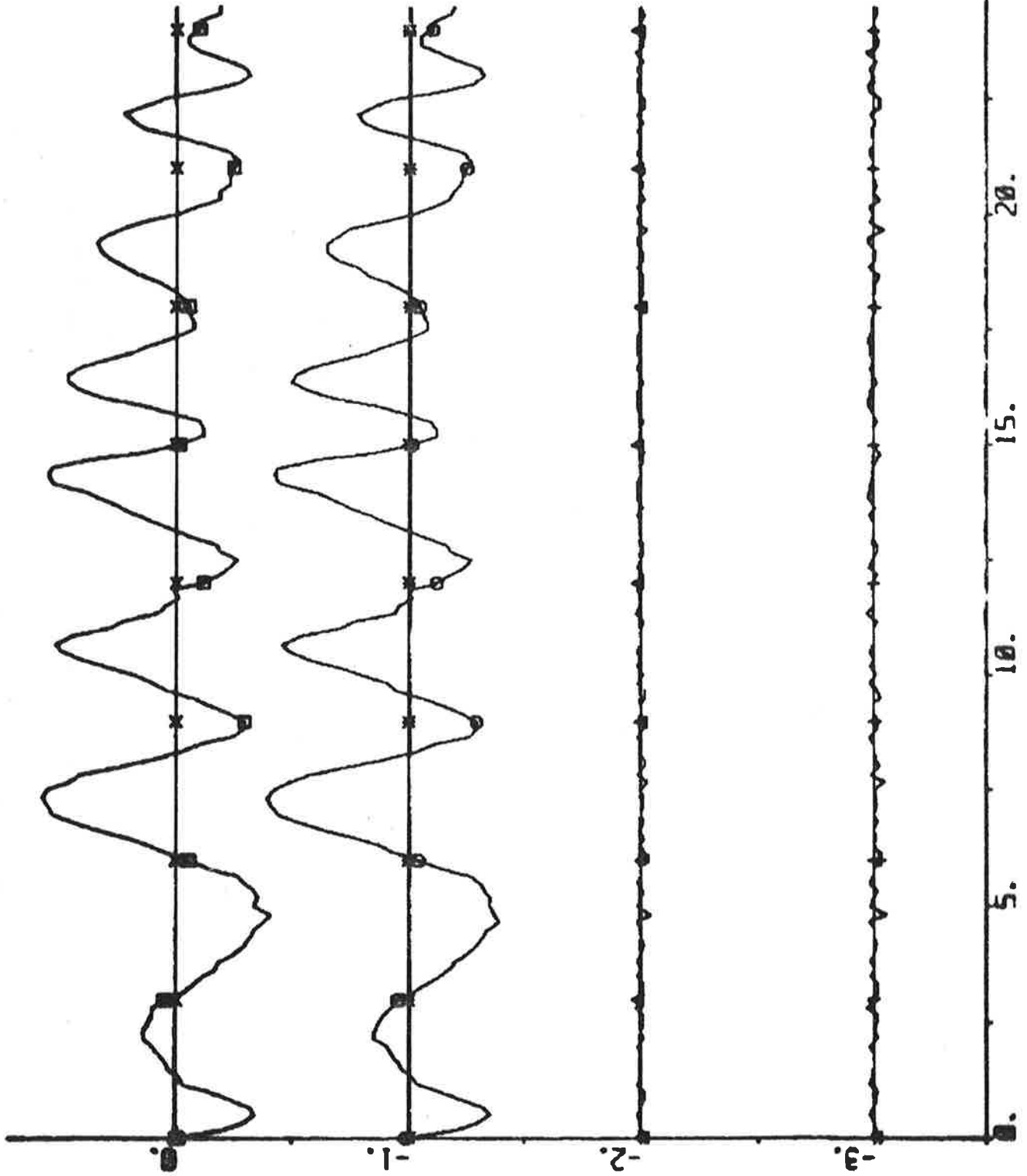
PLOT A7P1(1)-A7P1(4) A7P1(5) ERR2 EPS2 00 01 02 03 -3.4 0.0 - KNOTS



PLOT A7P1(1)-A7P1(8) A7P1(7) ERR3 EPS3 00 002 004 008 -0.7 0. - DEGS

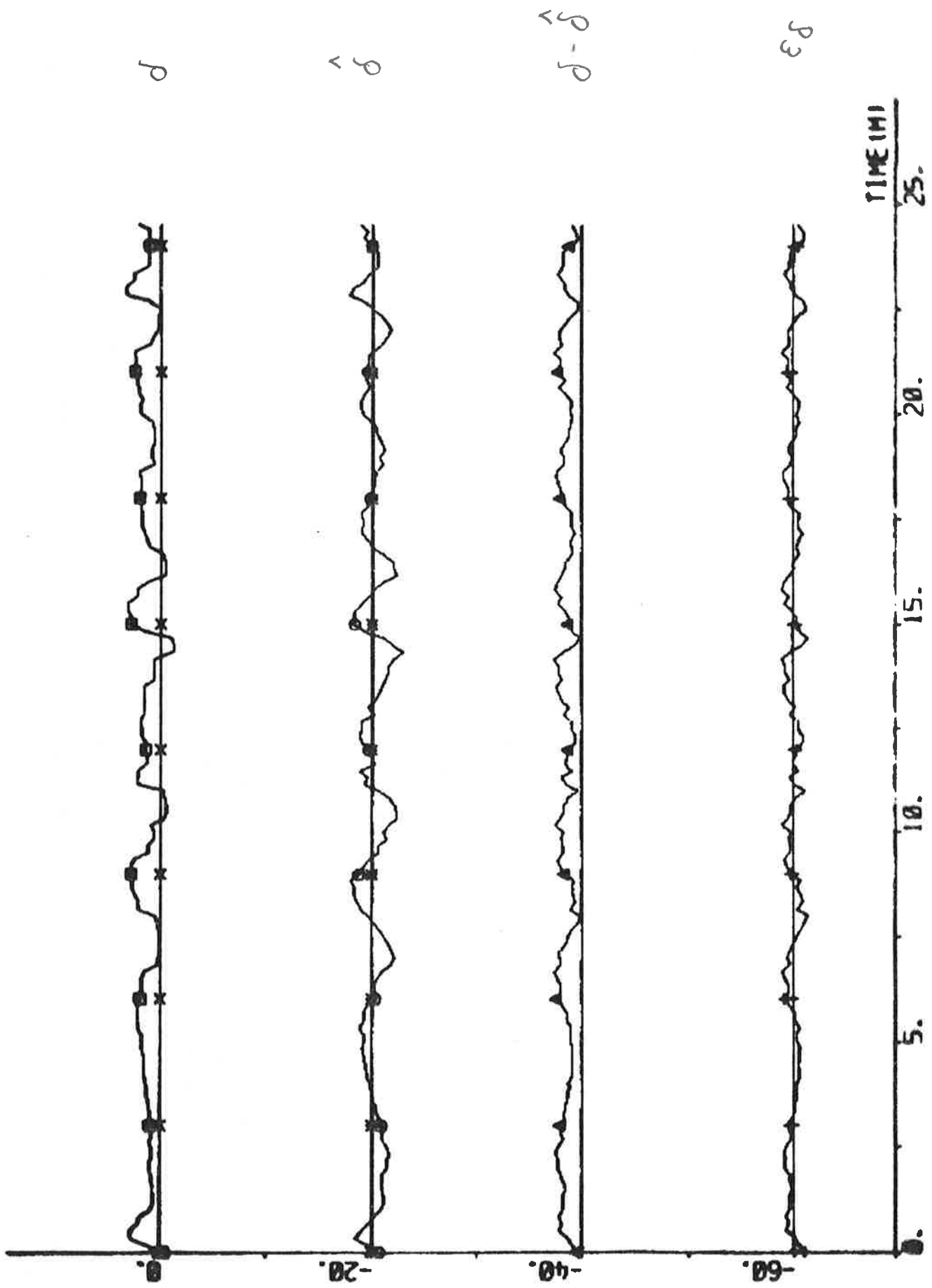


PLOT A7P1(1)-A7P1(8) A7P1(9) ERR4 EPS4 00 01 02 03 -3.4 0.0 - DEG

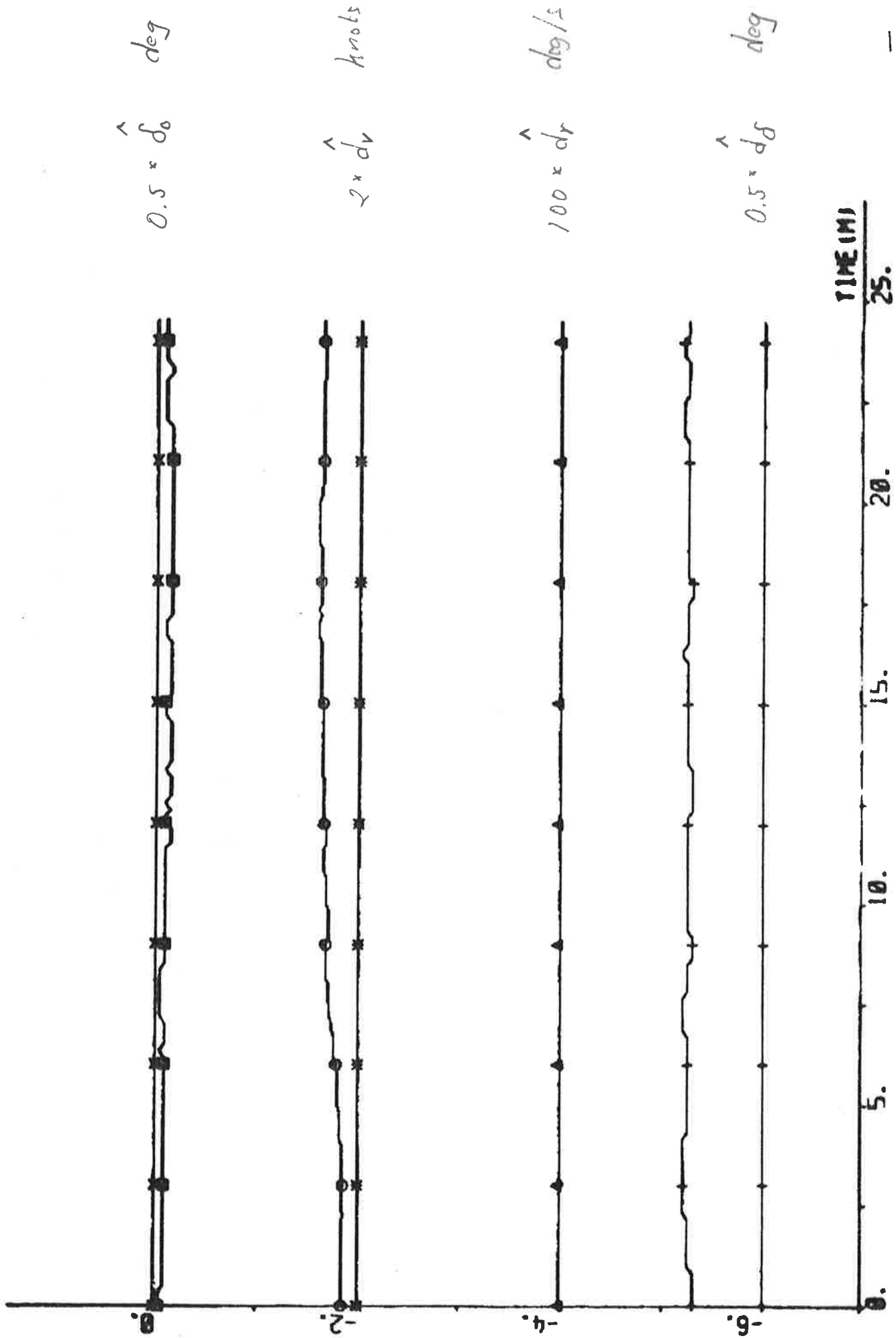




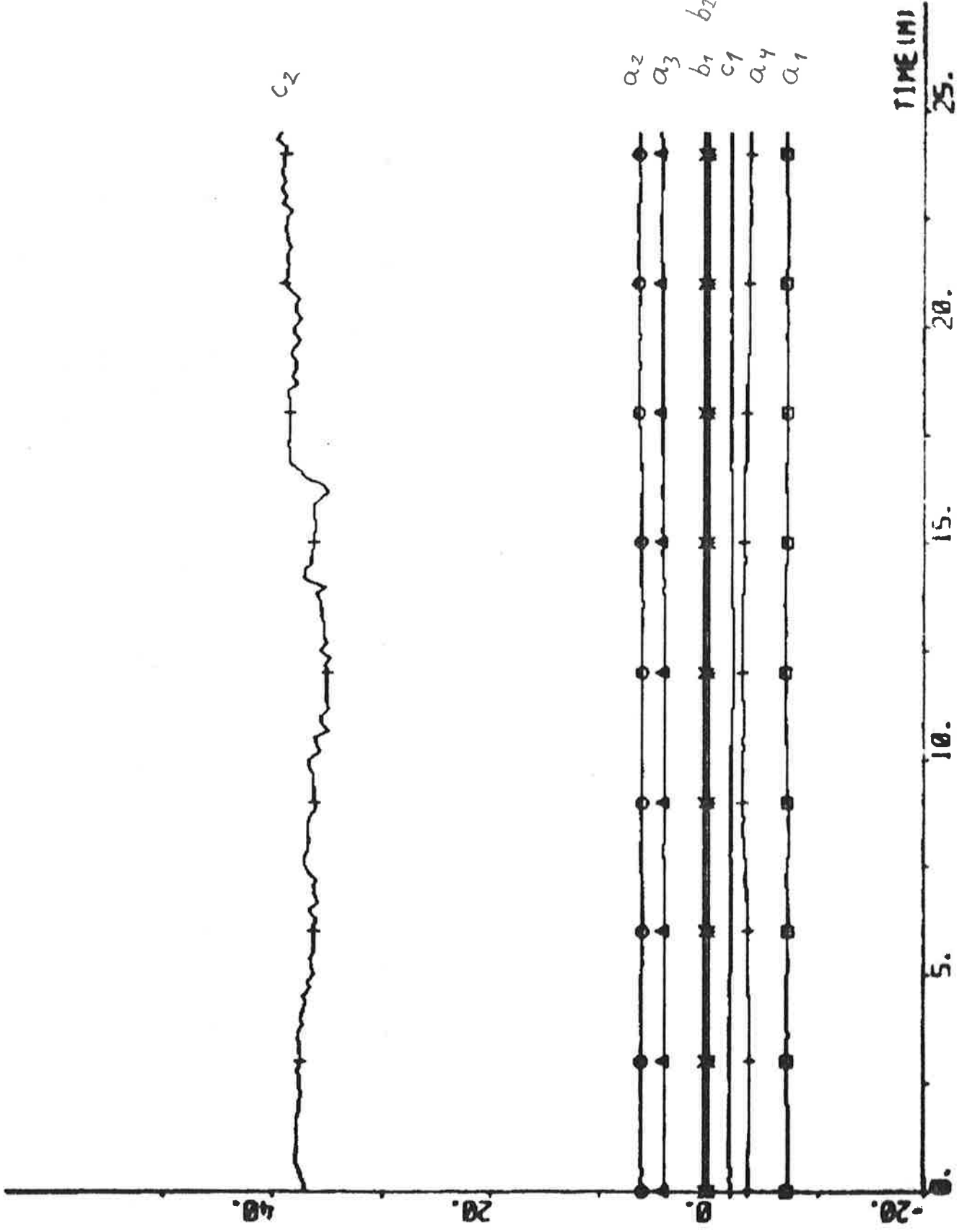
PLOT A7P1(1)-A7P1(2) A7P1(3) ERR1 EPS1 00 020 040 060 -65 15 - DEG



PLOT A7P1(1)-A7P2(3) A7P2(4) A7P2(5) A7P2(6) 00 02 04 06 -0.5 1.5

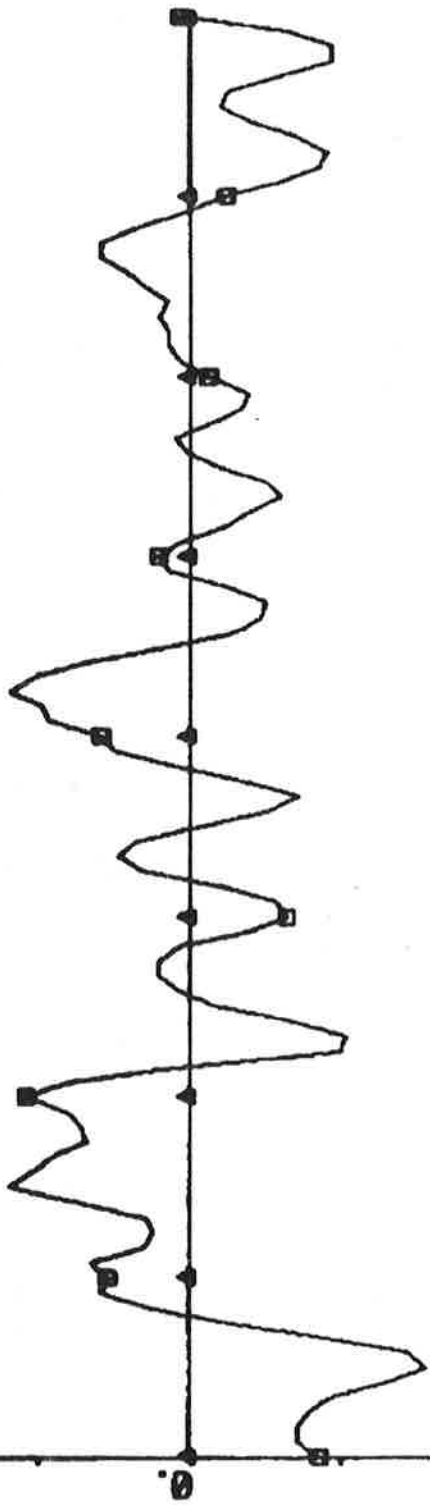


PLOT A7P1(1)-A7P2(7) A7P2(8) A7P2(9) A7P2(10) A7P2(11) A7P2(12) A7P2(13)





PLOT AOP1(1)-AOP1(8) HP AOP1(10) AOP1(15) Q2 -3 1 - DEG



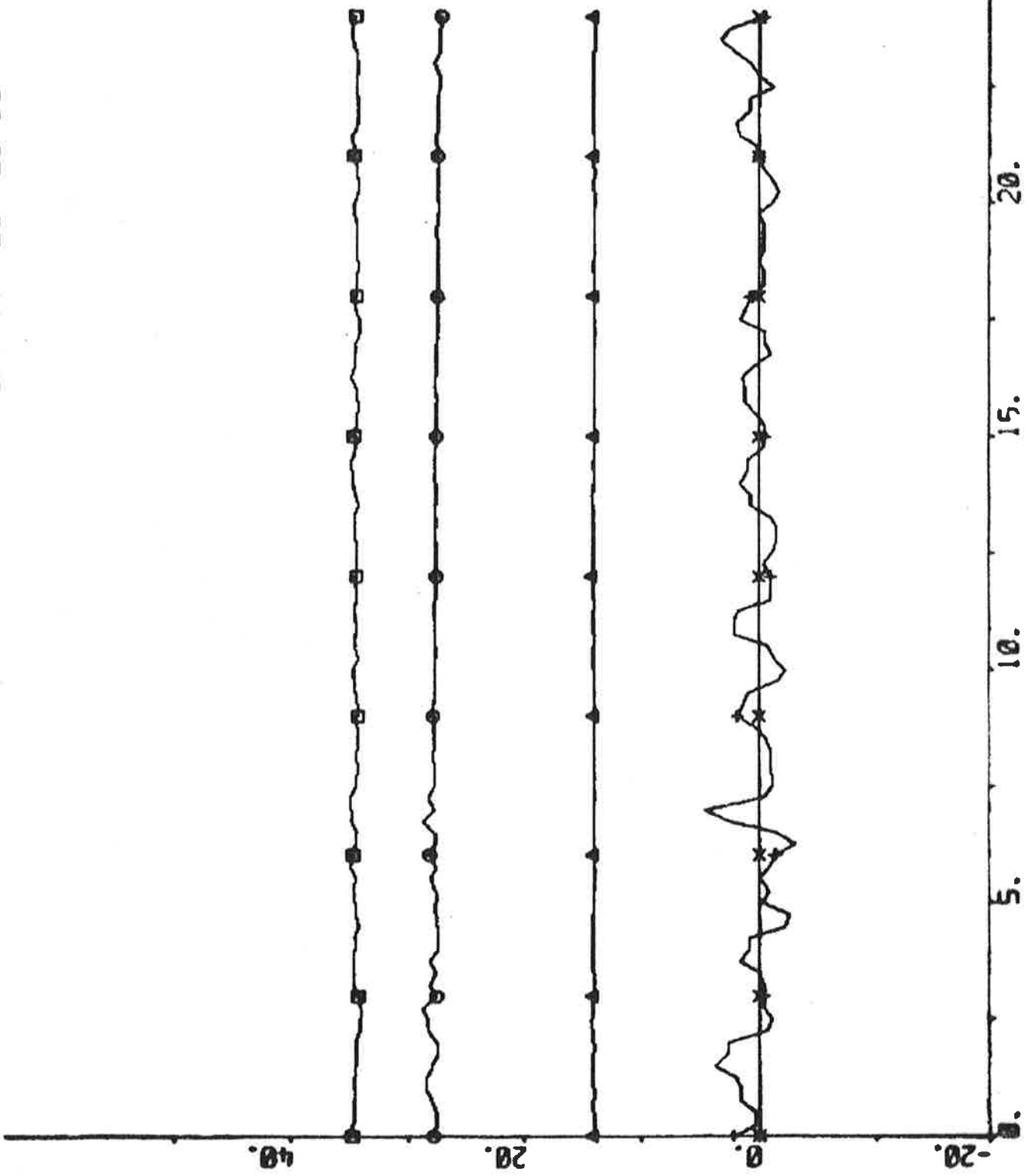
$\gamma - \gamma_{ref}$



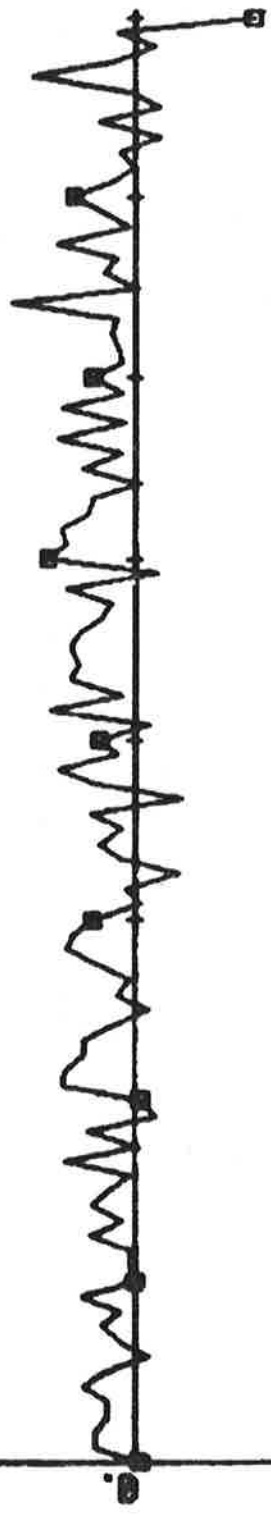
$0.05 * d_c$

TIME(M) 25.

PLOT ACP1(1)-ACP1(13) ACP1(12) ACP1(14) ACP1(11) 00 -20 60



PLOT ROP1(1)-ROP2(1) ROP2(2) ERRS 00 01 02 -3.4 0.8 - KNOTS



TIME (H)

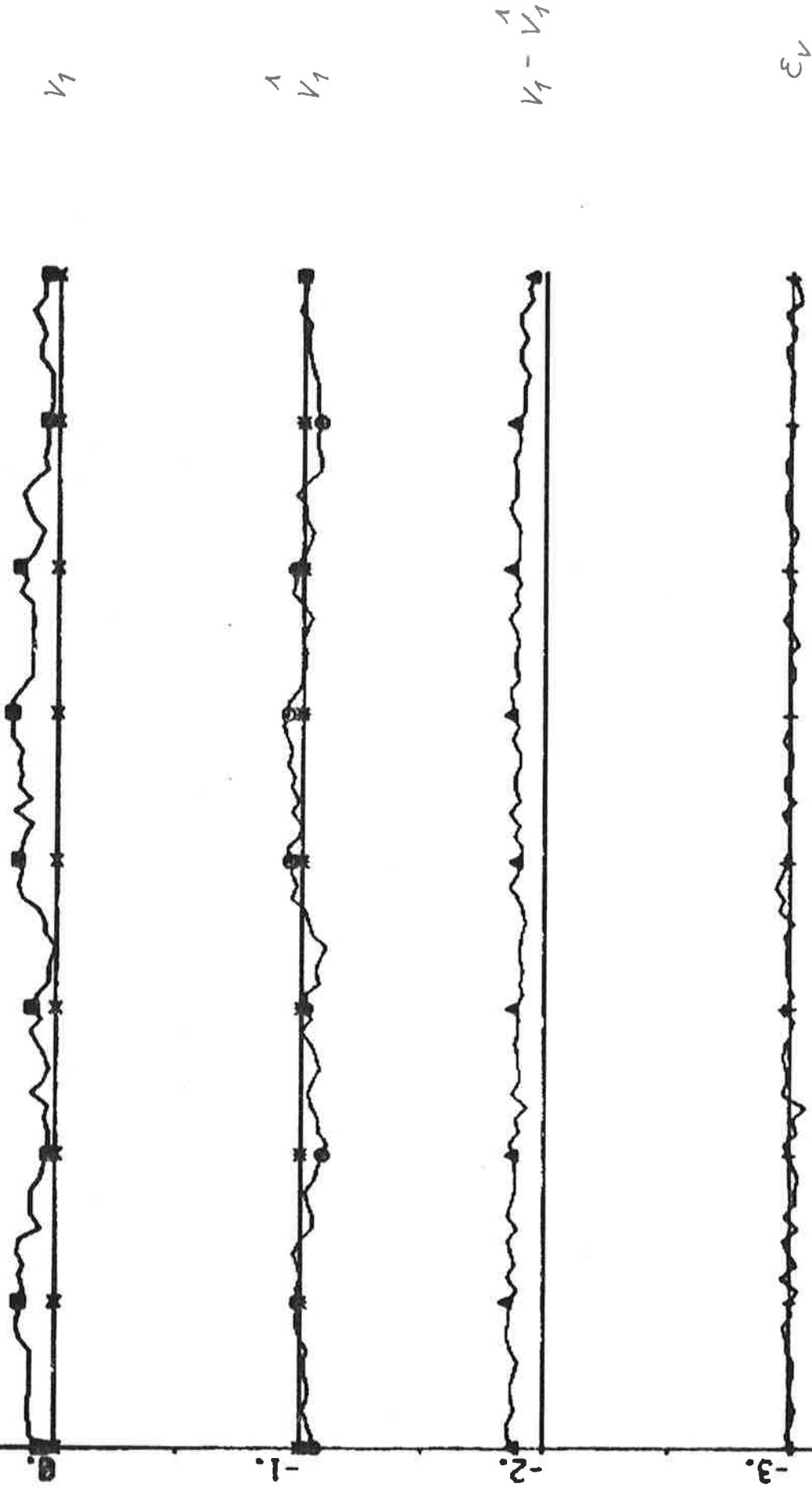
0. 5. 10. 15. 20. 25.

✓

✓

✓ - ✓

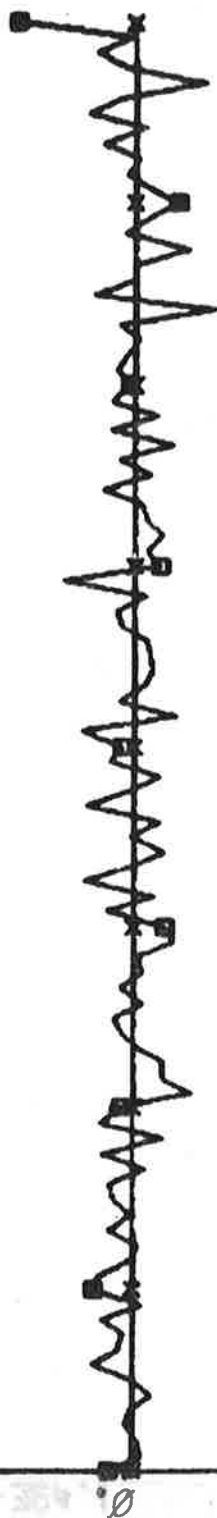
PLOT ACP1(1)-ACP1(4) ACP1(5) ERR2 EPS2 00 01 02 03 -3.4 0.0 - KNOTS



TIME (M)  
25.



PLOT AOP1(1)-AOP1(6) AOP1(7) ERR3 EPS3 00 002 004 006 -0.7 0. - 000/S



r



r



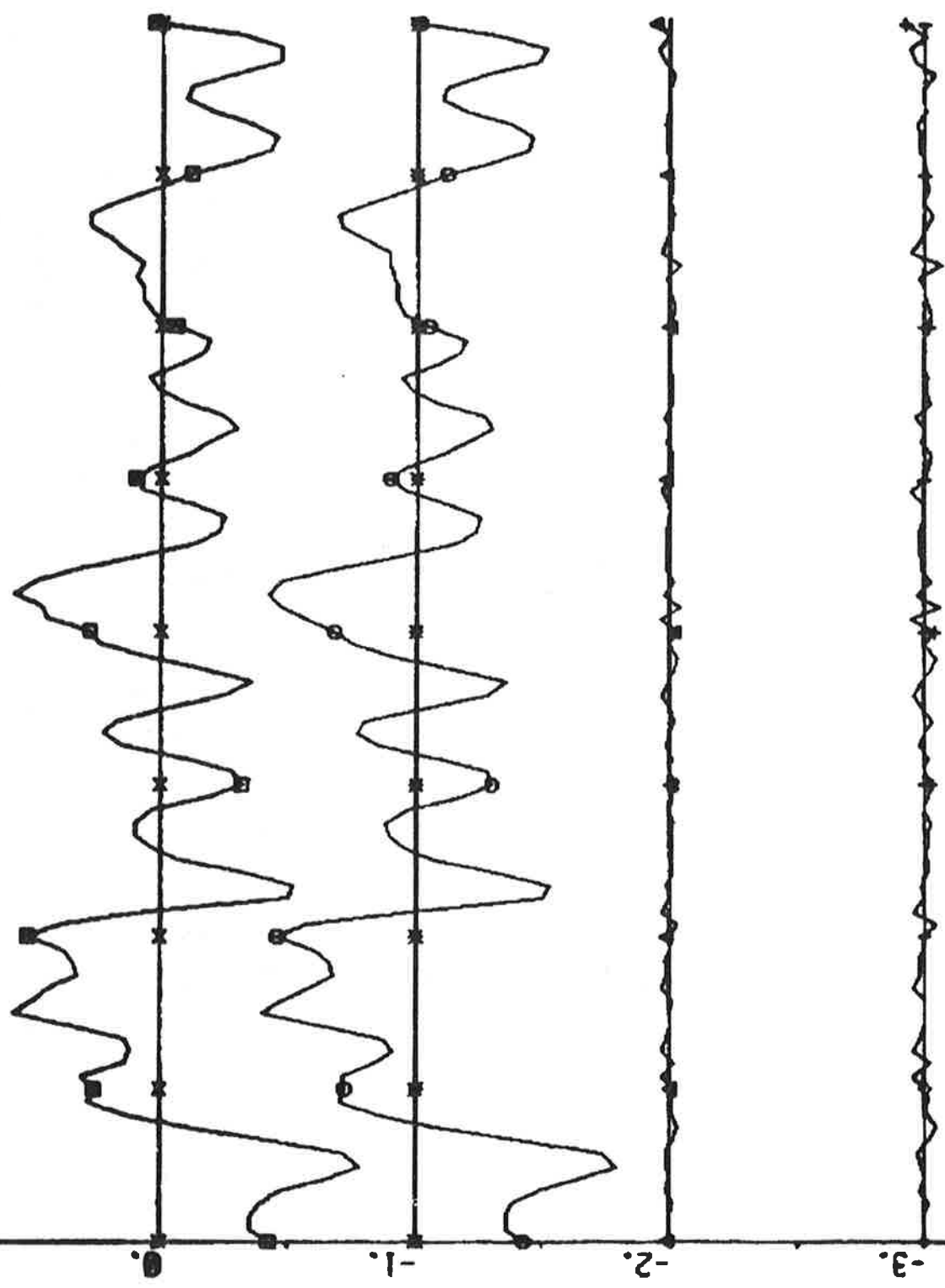
r - r



Er

TIME (M)  
25.

PLOT AOP1(1)-AOP1(8) AOP1(9) ERR14 EPS4 00 01 02 03 -3.4 0.0 - DEG



TIME (M)  
25.

PLOT AOP1(1)-AOP1(2) AOP1(3) ERR1 EPS1 00 020 040 060 -66 15 - DEG



δ



δ



δ-δ



εδ

TIME (M)  
25.

PLOT ROP1(1) ROP2(3) ROP2(4) ROP2(5) ROP2(6) 00 02 04 06 -6.5 1.5

$0.5 \times \hat{\delta}_0$  deg

$2 \times \hat{d}_r$  knots

$100 \times \hat{d}_r$  deg/s

$0.5 \times \hat{d}_\delta$  deg

TIME (M)

25.

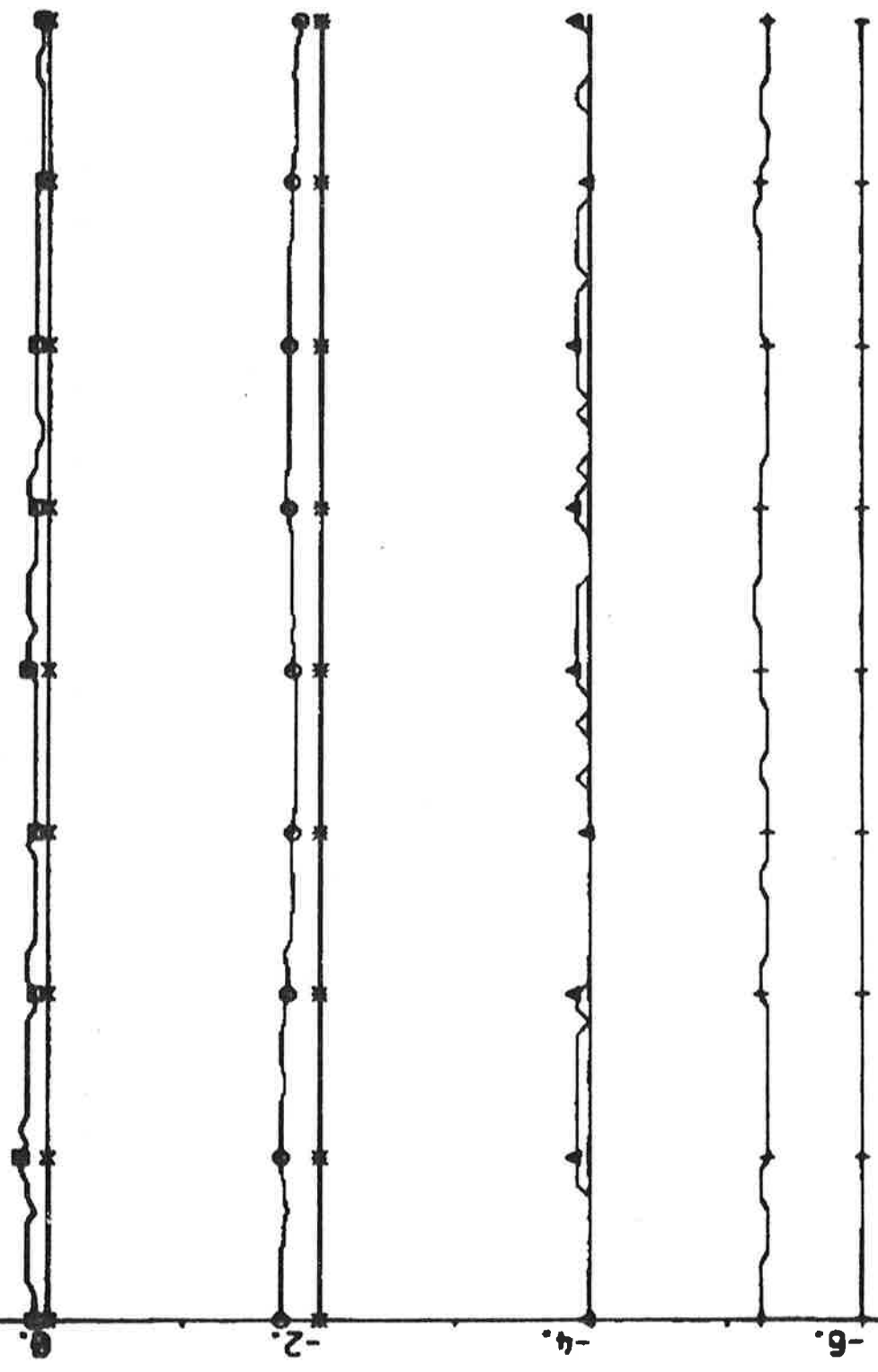
20.

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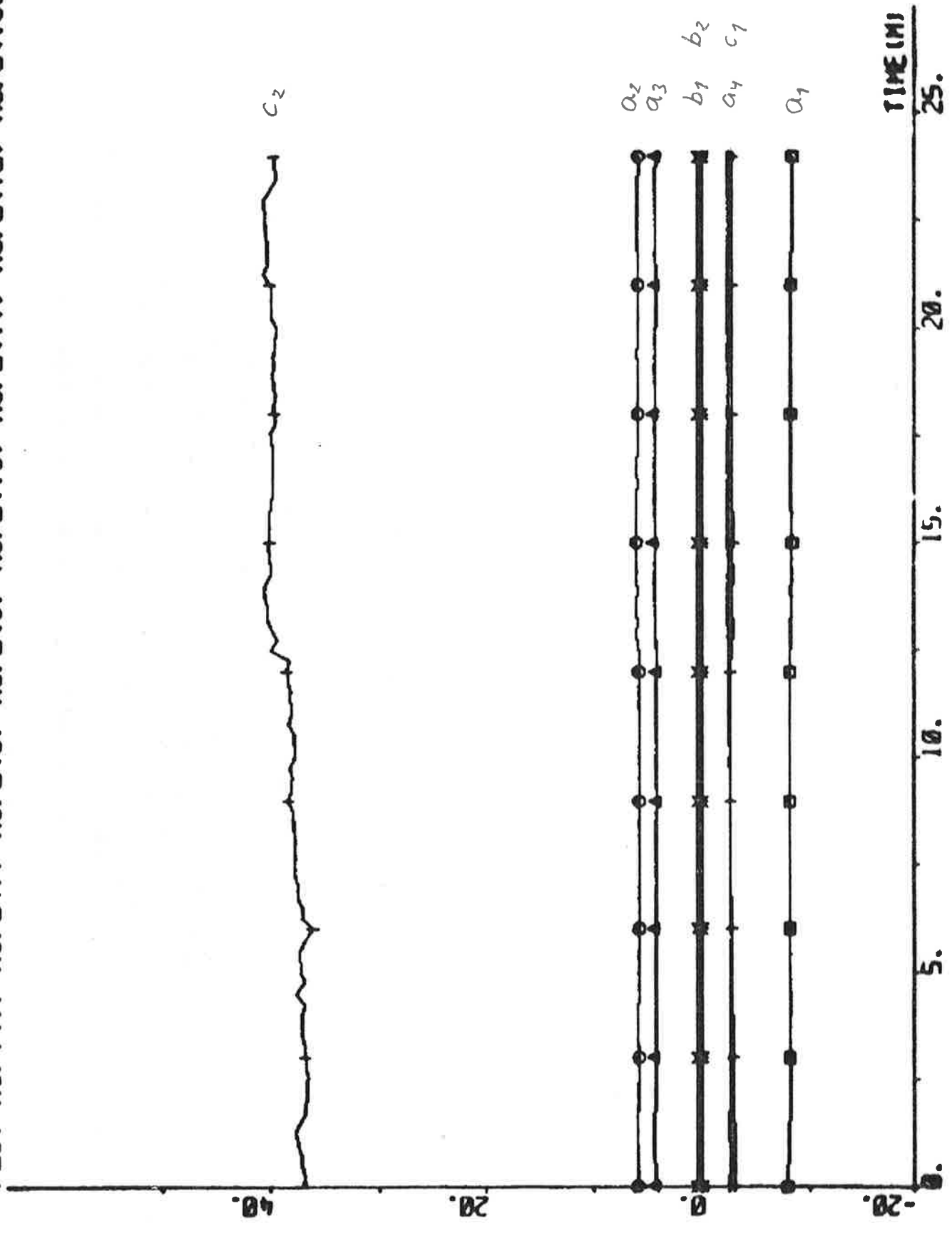
10.

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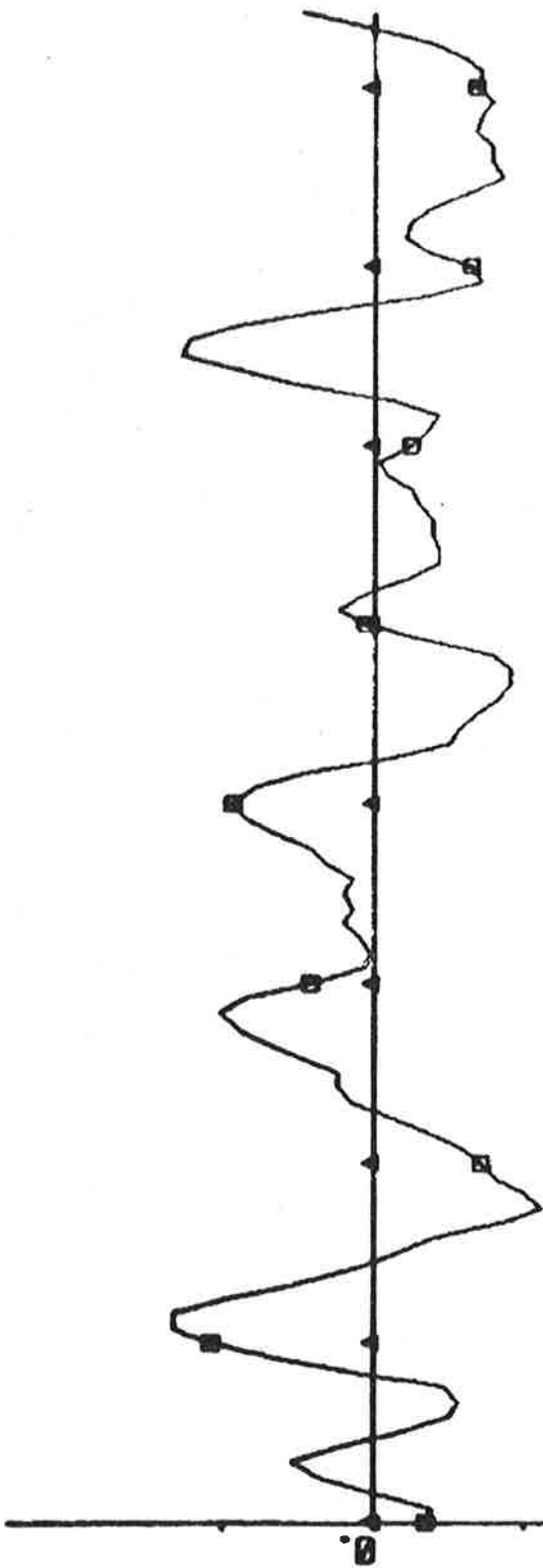


PLOT AOP1(1)-AOP2(7) AOP2(8) AOP2(9) AOP2(10) AOP2(11) AOP2(12) AOP2(13)





PLOT ASP1(1) - ASP1(8) HP ASP1(10) ASP1(15) Q2 -3 1 - DEG



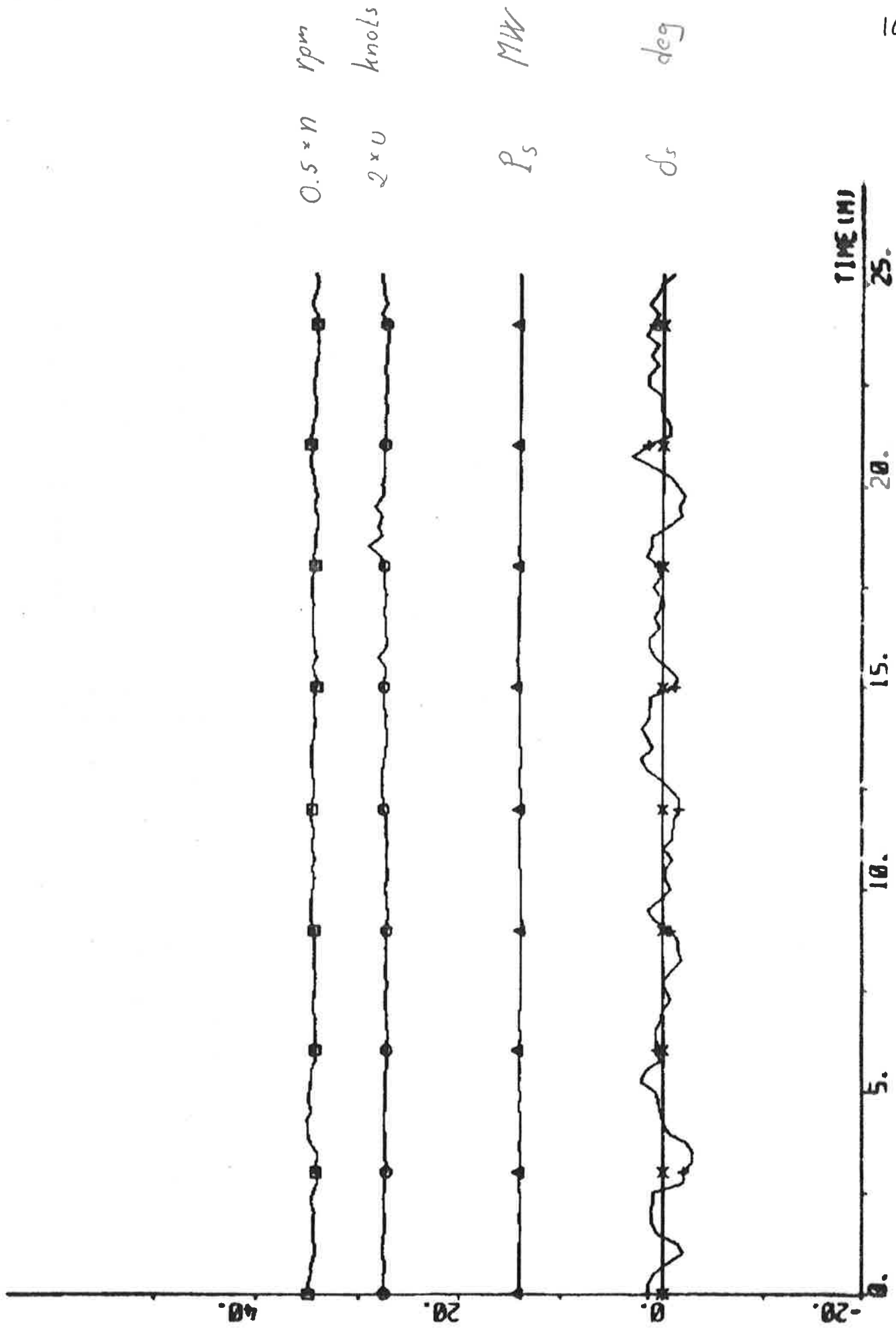
$y - y_{ref}$

$0.05 \times d_c$



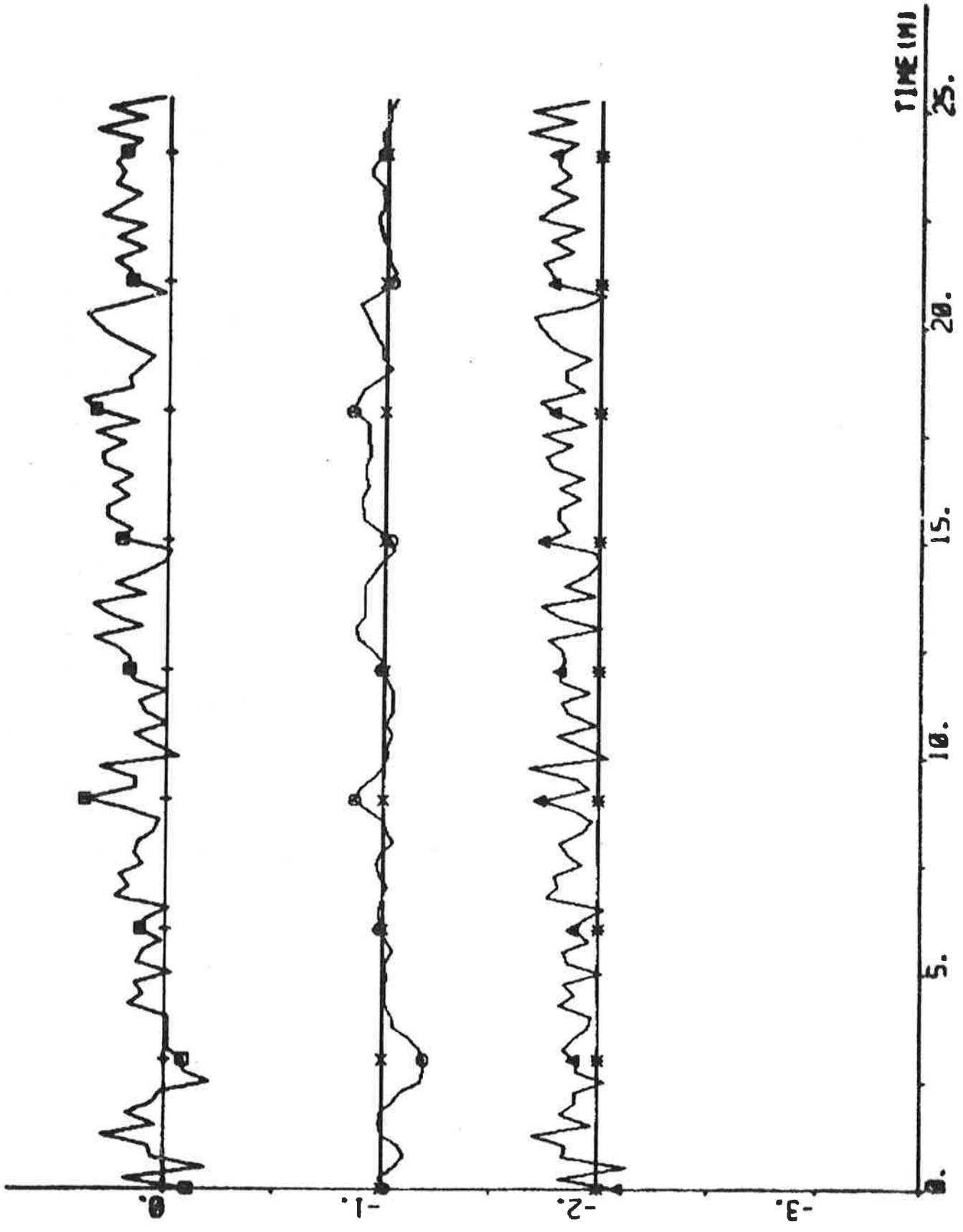
TIME (M)

PLOT ASP1(1)-ASP1(13) ASP1(12) ASP1(14) ASP1(11) 00 -20 50





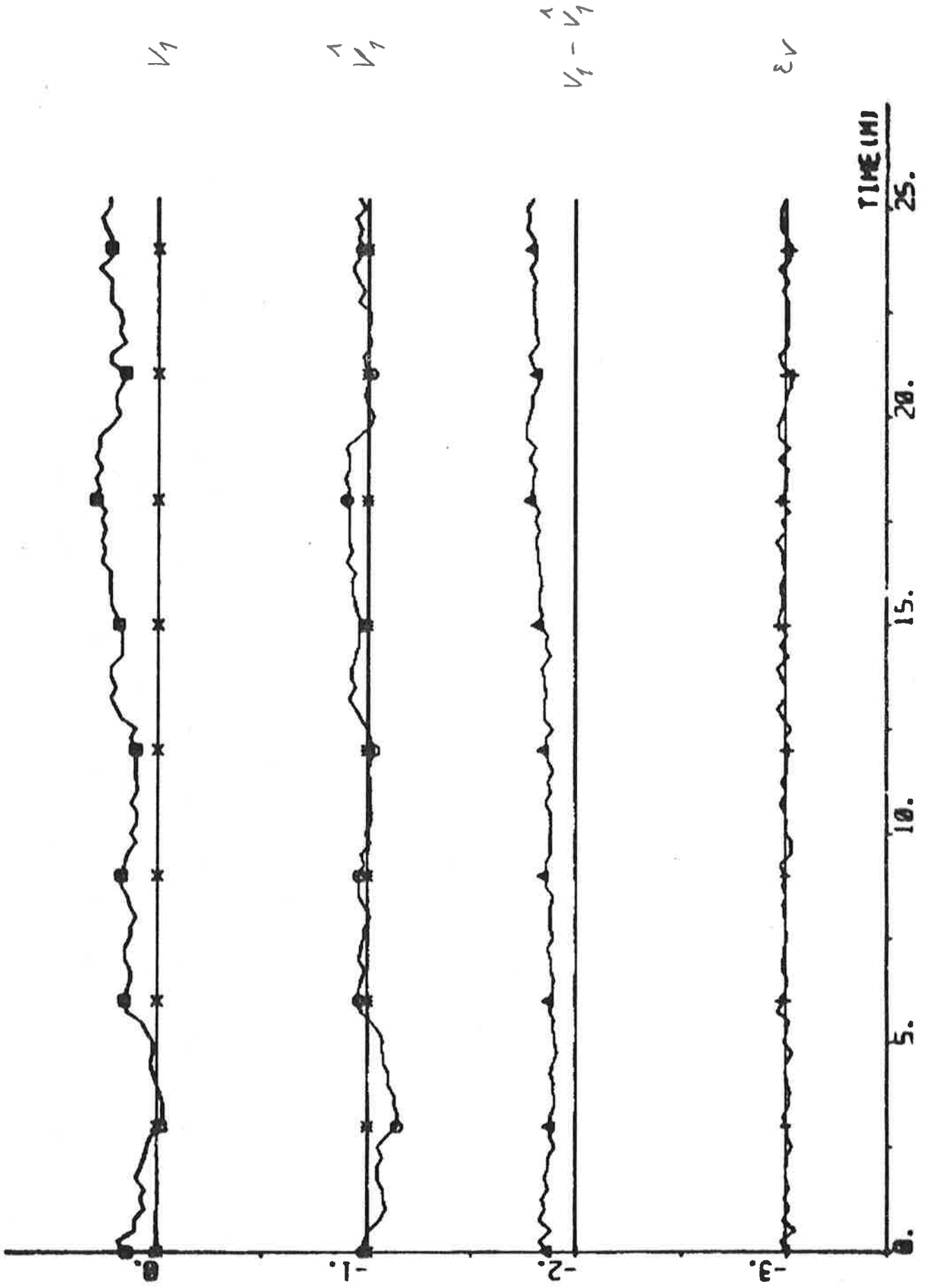
PLOT ASP1(1)-ASP2(1) ASP2(2) ERRS 00 01 02 -3.4 0.0 - KNOTS



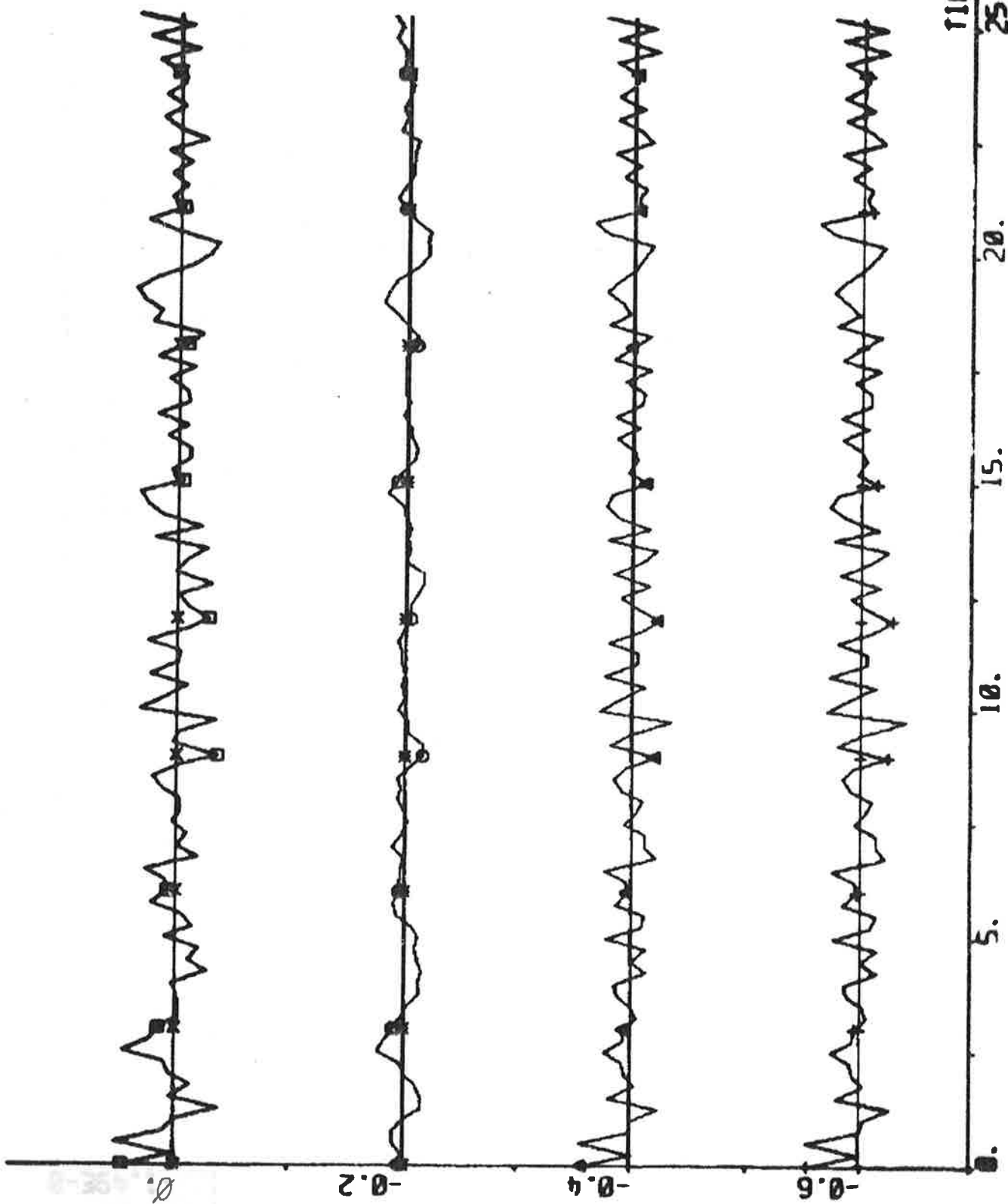
V-U

TIME (M)  
25.

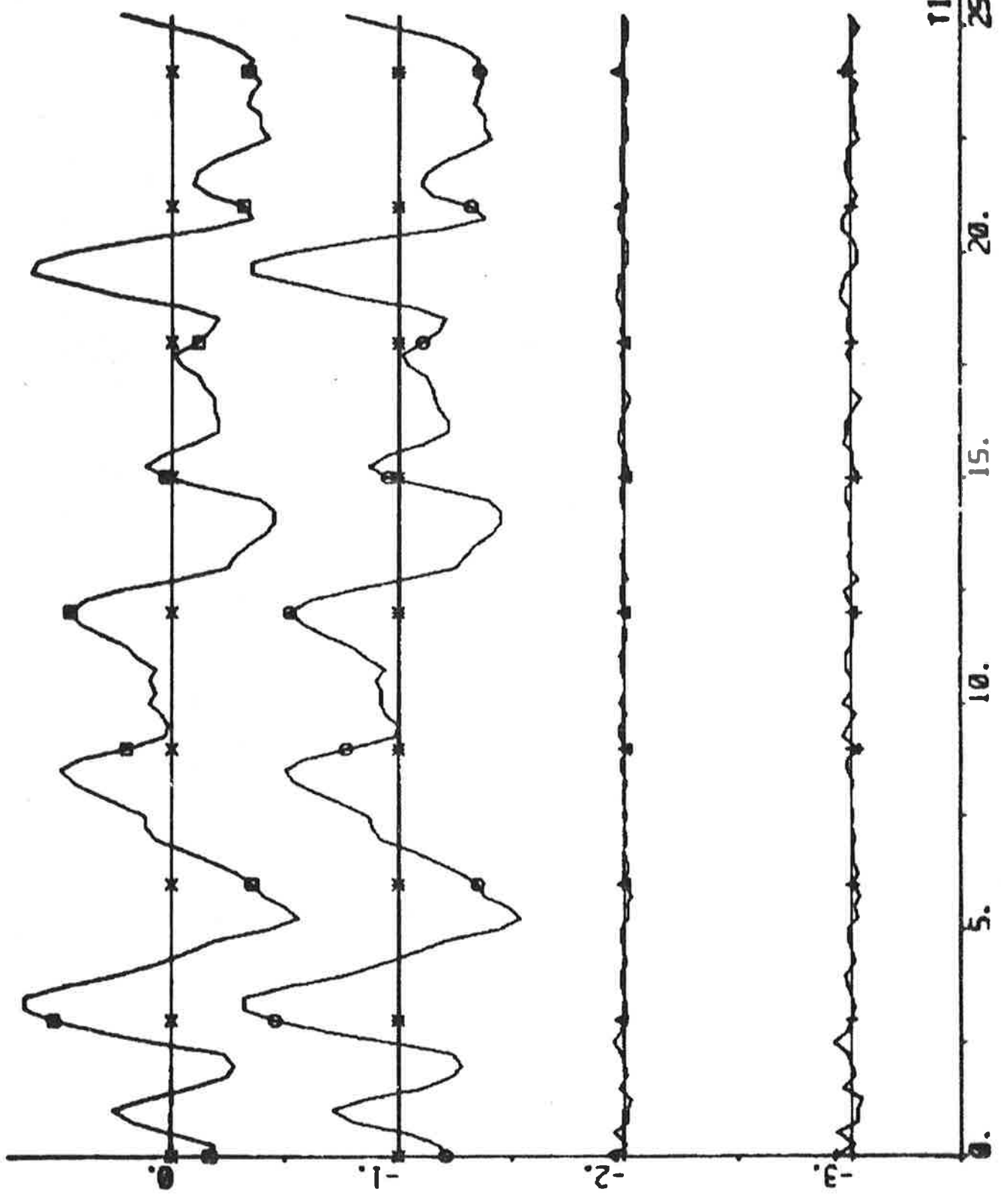
PLOT ASP1(1)-ASP1(4) ASP1(5) ERR2 EPS2 00 01 02 03 -3.4 0.0 - KNOTS



PLOT ASP1(1)-ASP1(6) ASP1(7) ERR3 EPS3 00 002 004 006 -0.7 0. - DECS



PLOT ASP1(1)-ASP1(0) ASP1(0) ERR4 EPS4 00 01 02 03 -3.4 0.0 - DEG



$y - y_{ref}$

$\hat{y} - y_{ref}$

$\hat{y} - y$

$\epsilon_4$

TIME (M) 25.

20.

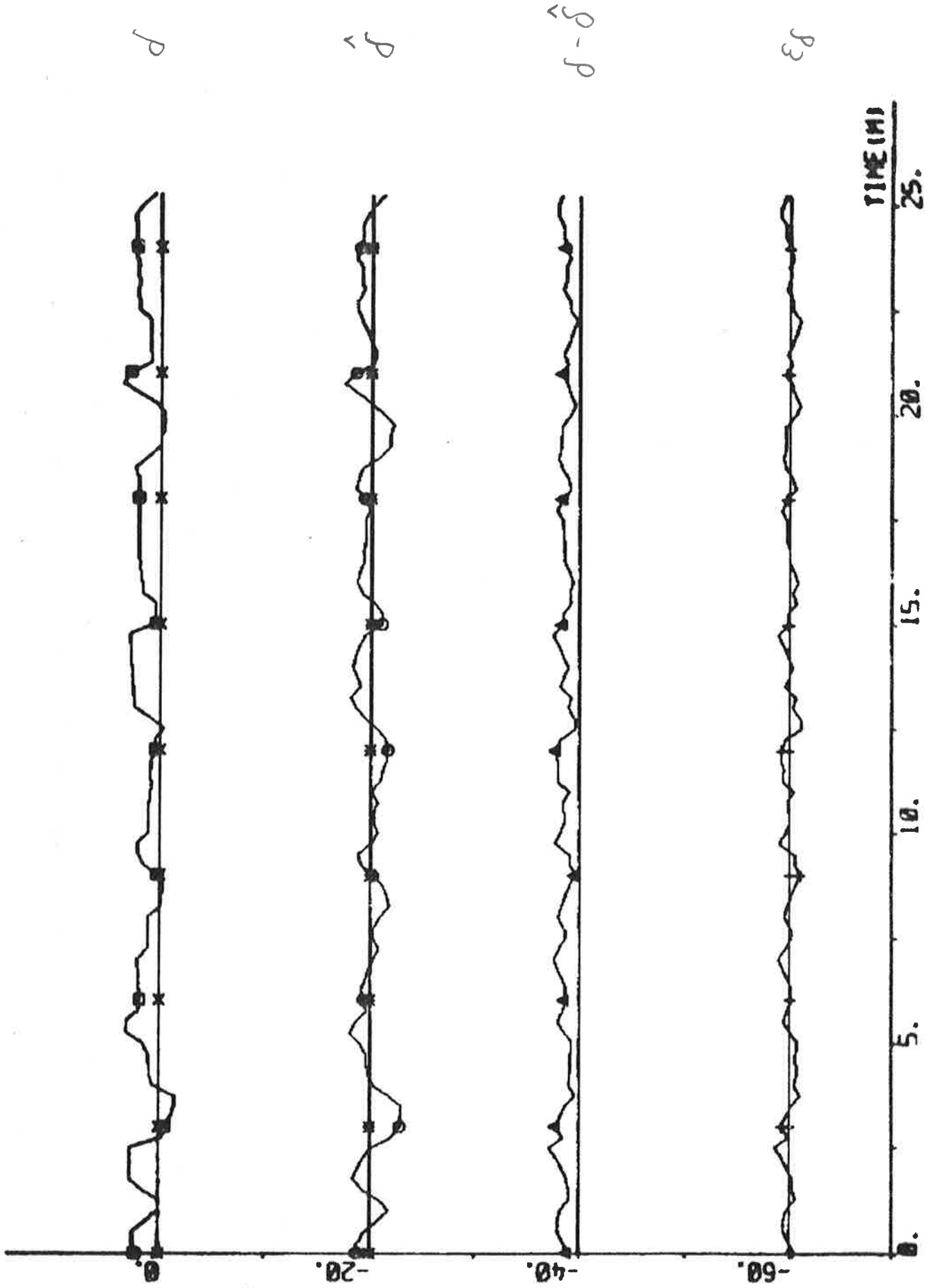
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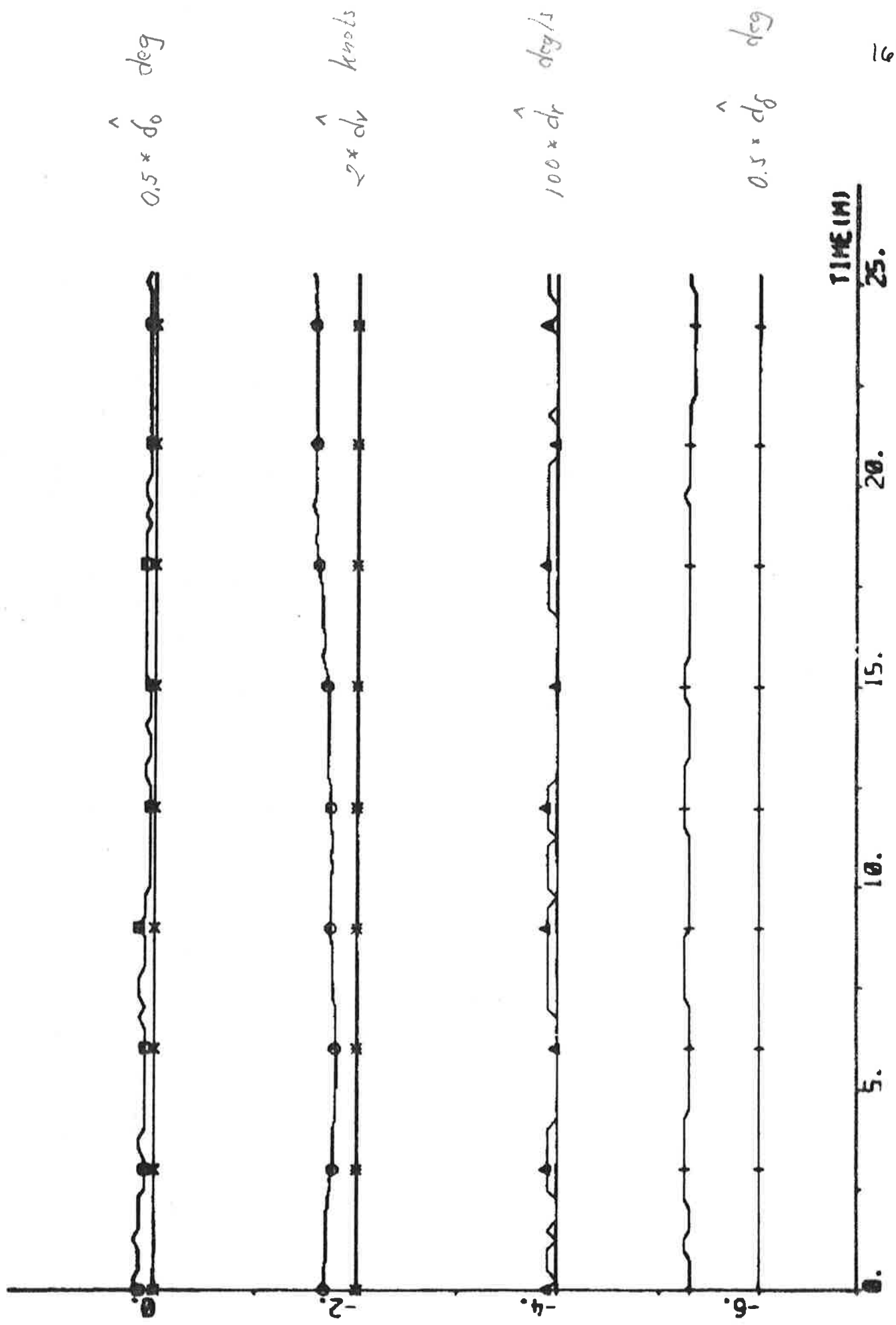
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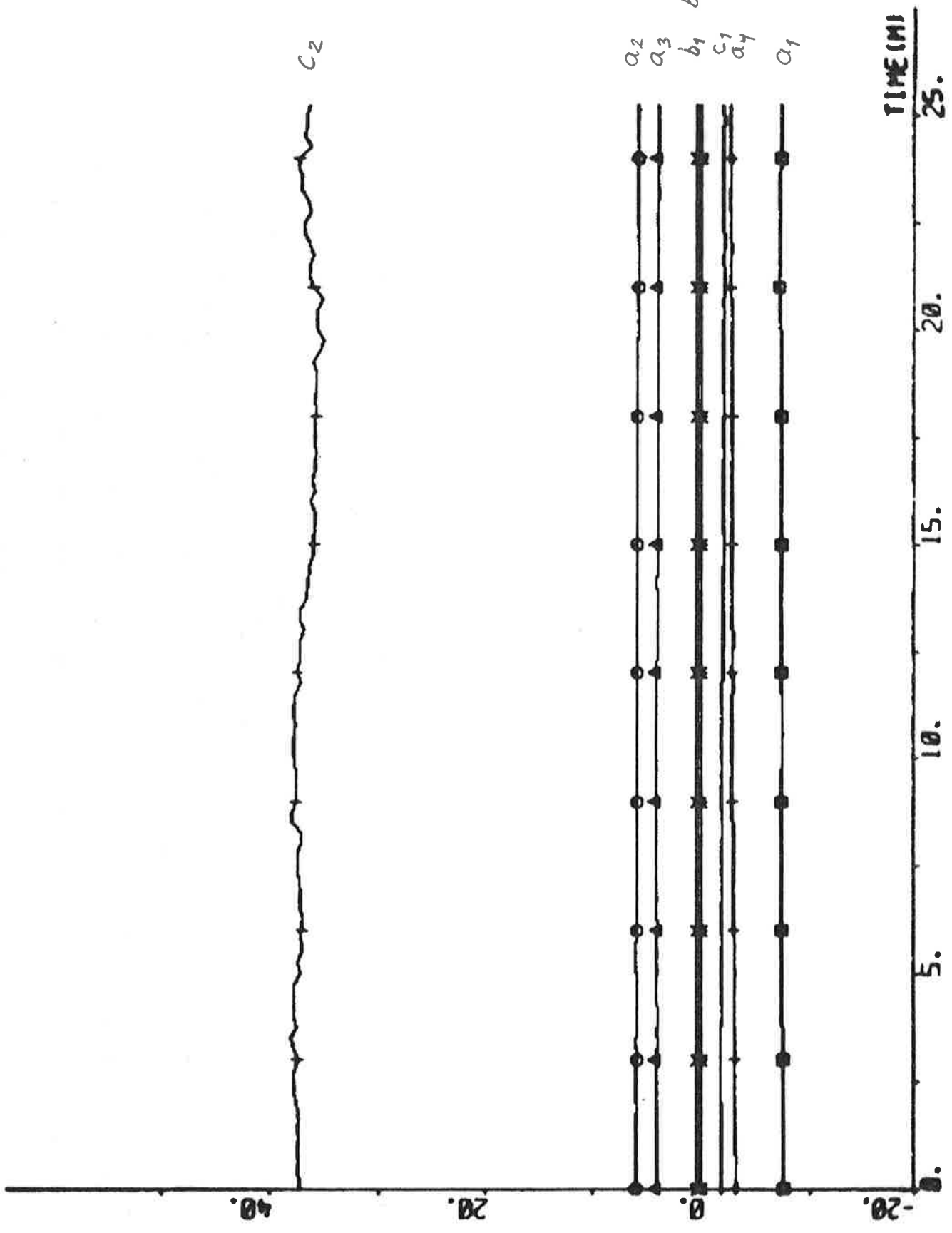
PLOT ASP1(1)-ASP1(2) ASP1(3) ERR1 EPS1 00 020 040 060 -05 15 - DEG



PLOT ASP1(1) ASP2(3) ASP2(4) ASP2(5) ASP2(6) 00 02 04 06 -0.5 1.5



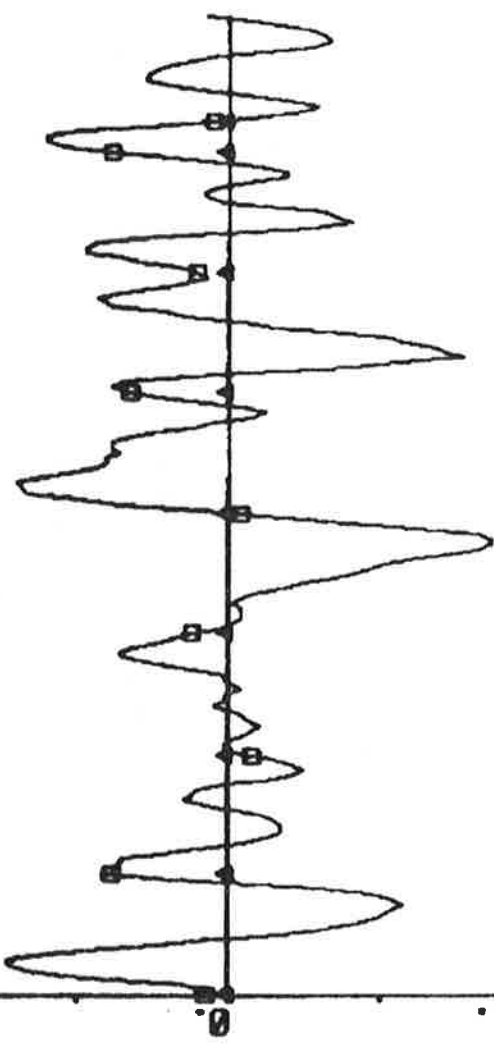
PLOT ASP1(1) ASP2(7) ASP2(8) ASP2(9) ASP2(10) ASP2(11) ASP2(12) ASP2(13)



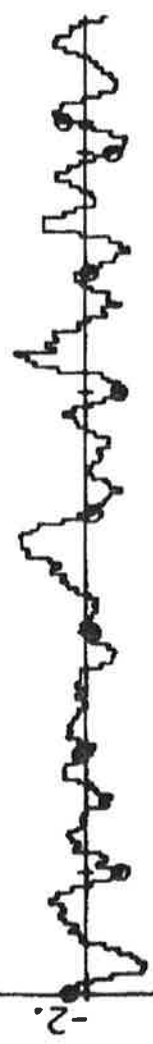




PLOT A1OP1(1)-A1OP1(9) HP A1OP1(10) A1OP1(15) 02 -3 1 - DEG



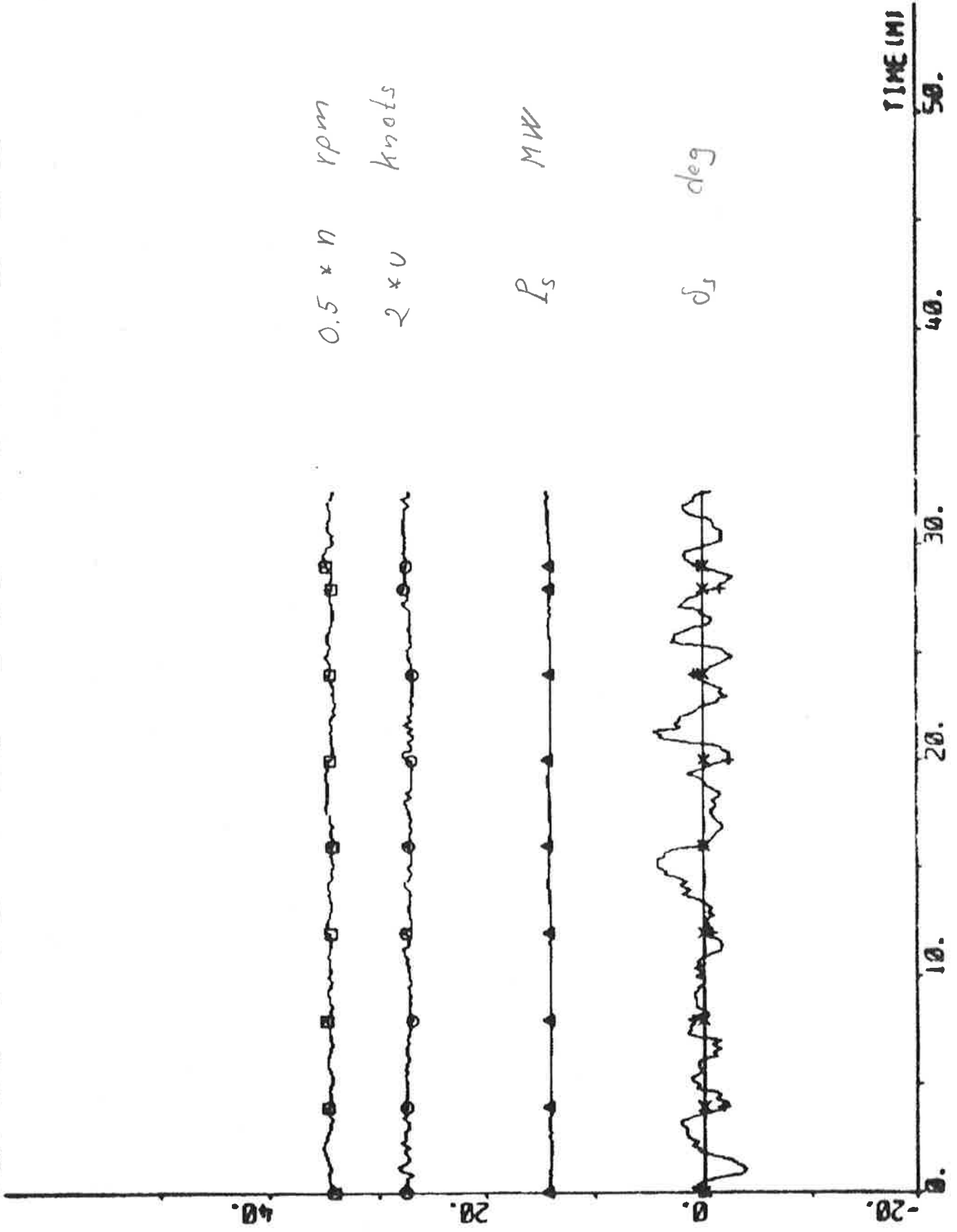
$\gamma - \gamma_{ref}$



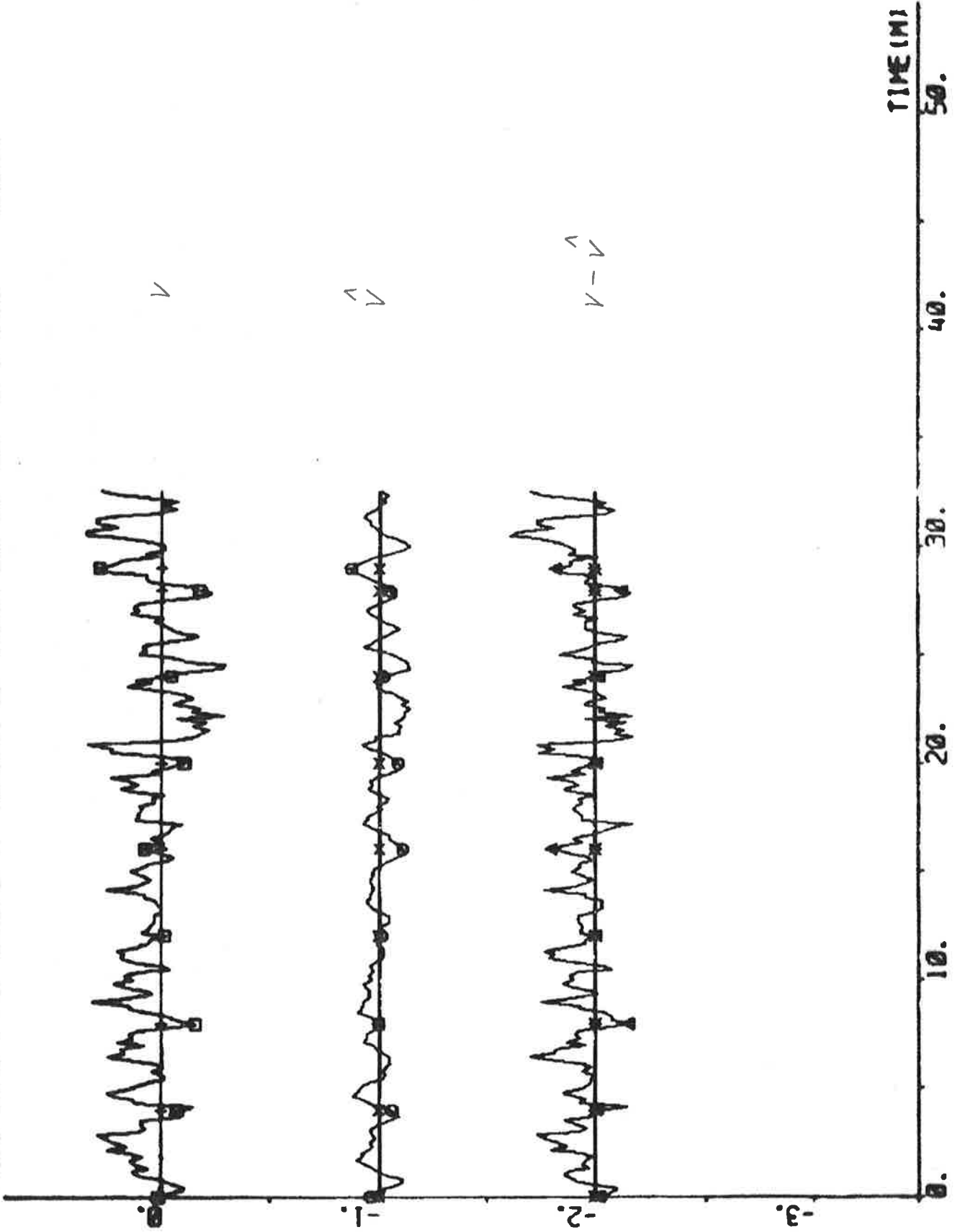
$0.05 * \delta_e$

TIME (M) 0. 10. 20. 30. 40. 50.

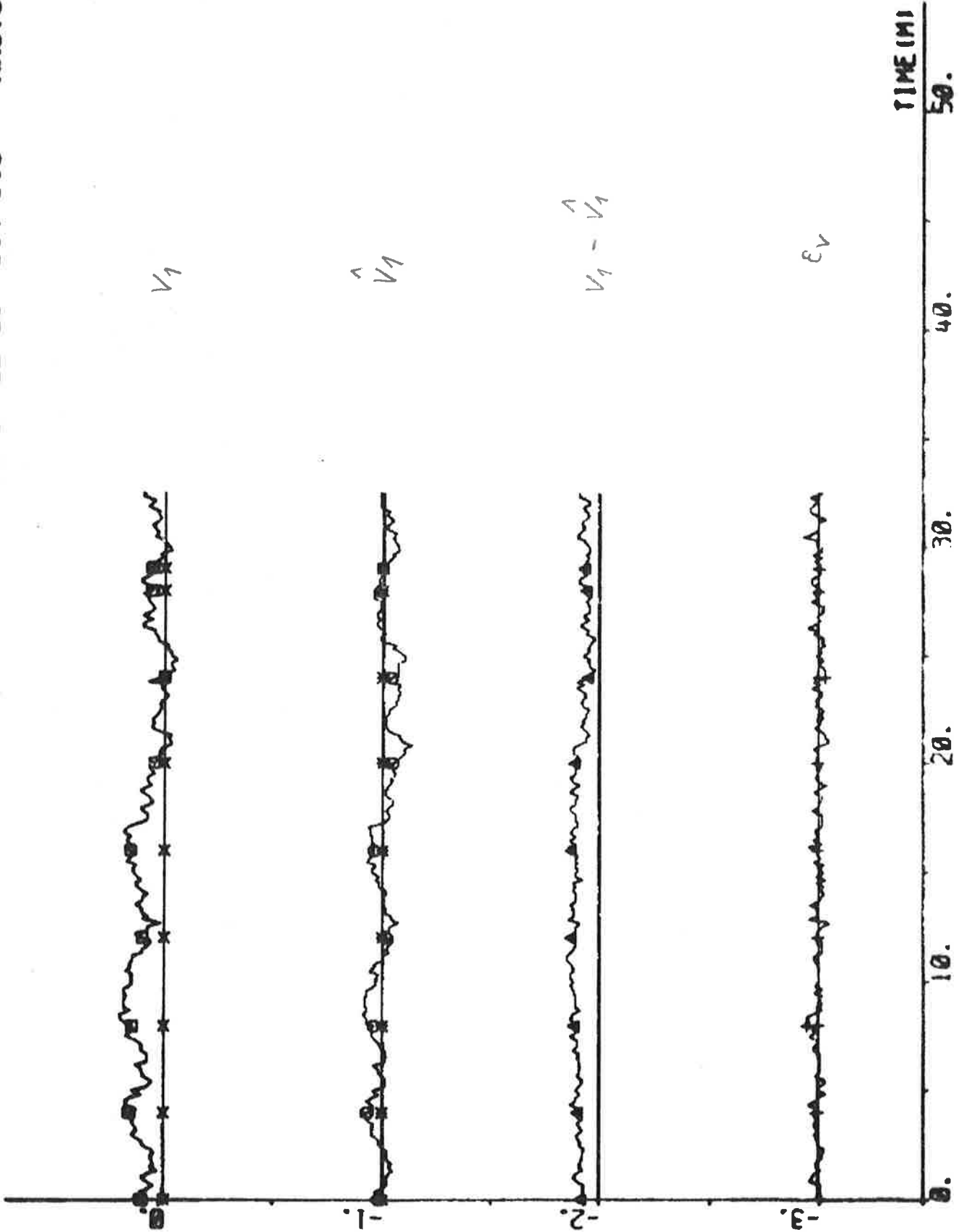
PLOT A10P1(1)-A10P1(13) A10P1(12) A10P1(14) A10P1(11) 00 -20 50



PLOT A10P1(1)-A10P2(1) A10P2(2) ERRS 00 01 02 -3.4 0.8 - KNOTS



PLOT A10P1(1)-A10P1(4) A10P1(5) ERR2 EPS2 00 01 02 03 -3.4 0.0 - KNOTS



PLOT A1OP1(1)-A1OP1(6) A1OP1(7) ERR3 EPS3 00 002 004 006 -0.7 0. - 00003



r



$\hat{r}$



$r - \hat{r}$



$\epsilon_r$

TIME (MI)  
50.

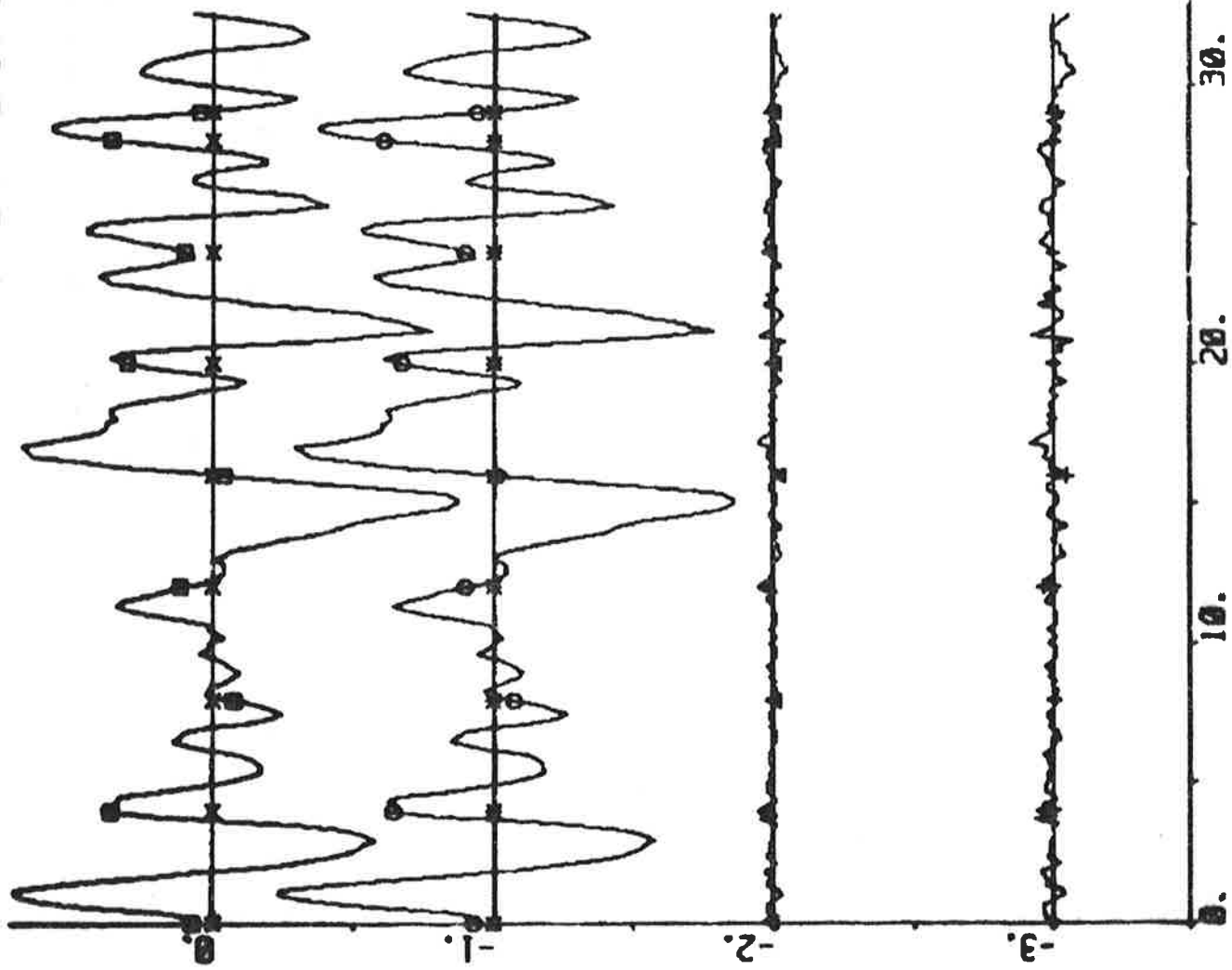
40.

30.

20.

10.

PLOT A1OP1(1)-A1OP1(8) A1OP1(9) ERR4 EPS4 00 01 02 03 -3.4 0.0 - DEG



$N - N_{ref}$

$N - N_{ref}$

$N - N$

$E_N$

TIME (M)

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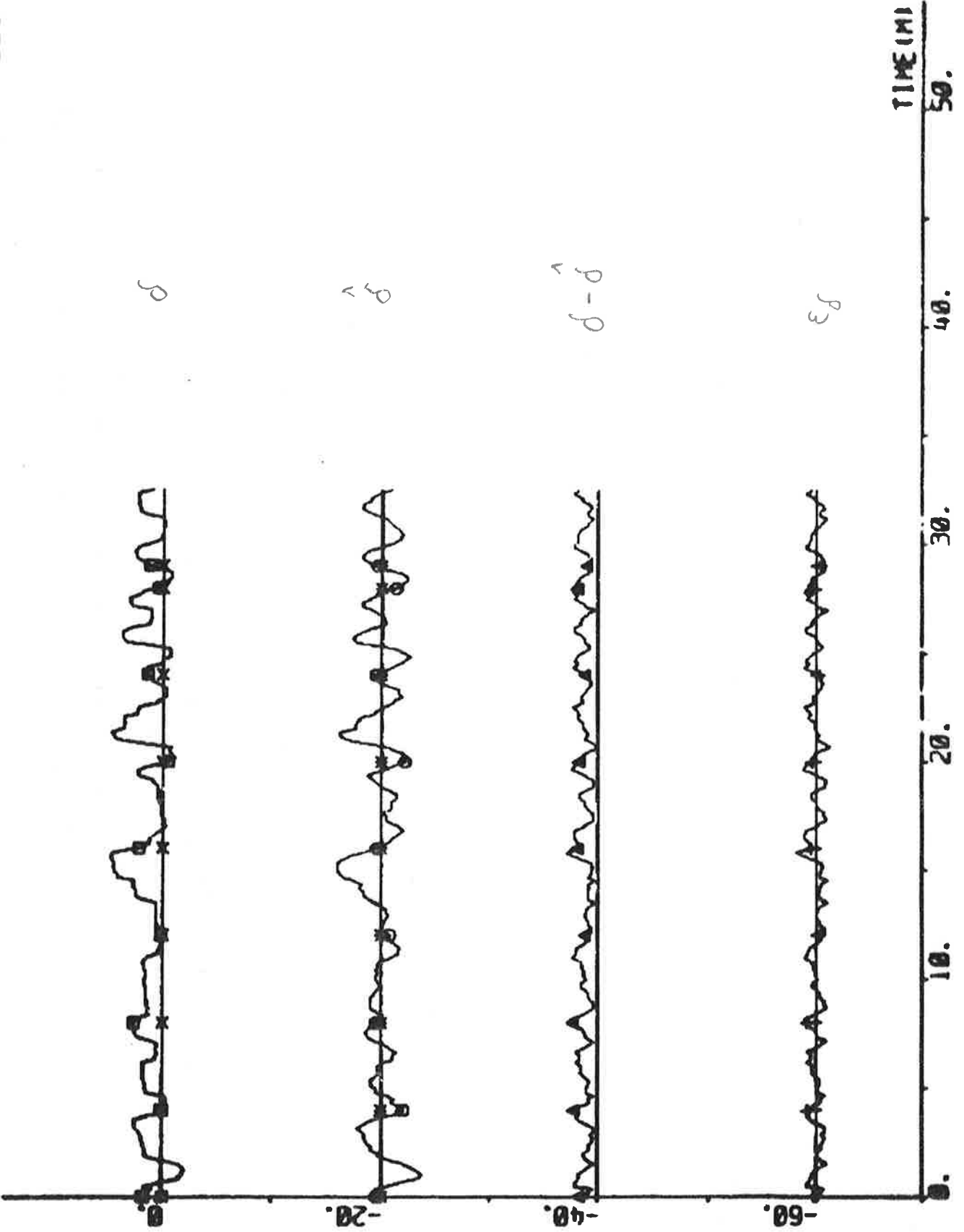
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PLOT A1OP1(1)-A1OP1(2) A1OP1(3) ERR1 EPS1 00 020 040 060 -05 15 - DEG



PLOT A1OP1(1) A1OP2(3) A1OP2(4) A1OP2(5) A1OP2(6) 00 02 04 06 -0.5 1.5

$\hat{d}_0 \times 0.5$  deg

$\hat{d}_v \times 2$  knots

$\hat{d}_r \times 100$  deg/s

$\hat{d}_\delta \times 0.5$  deg

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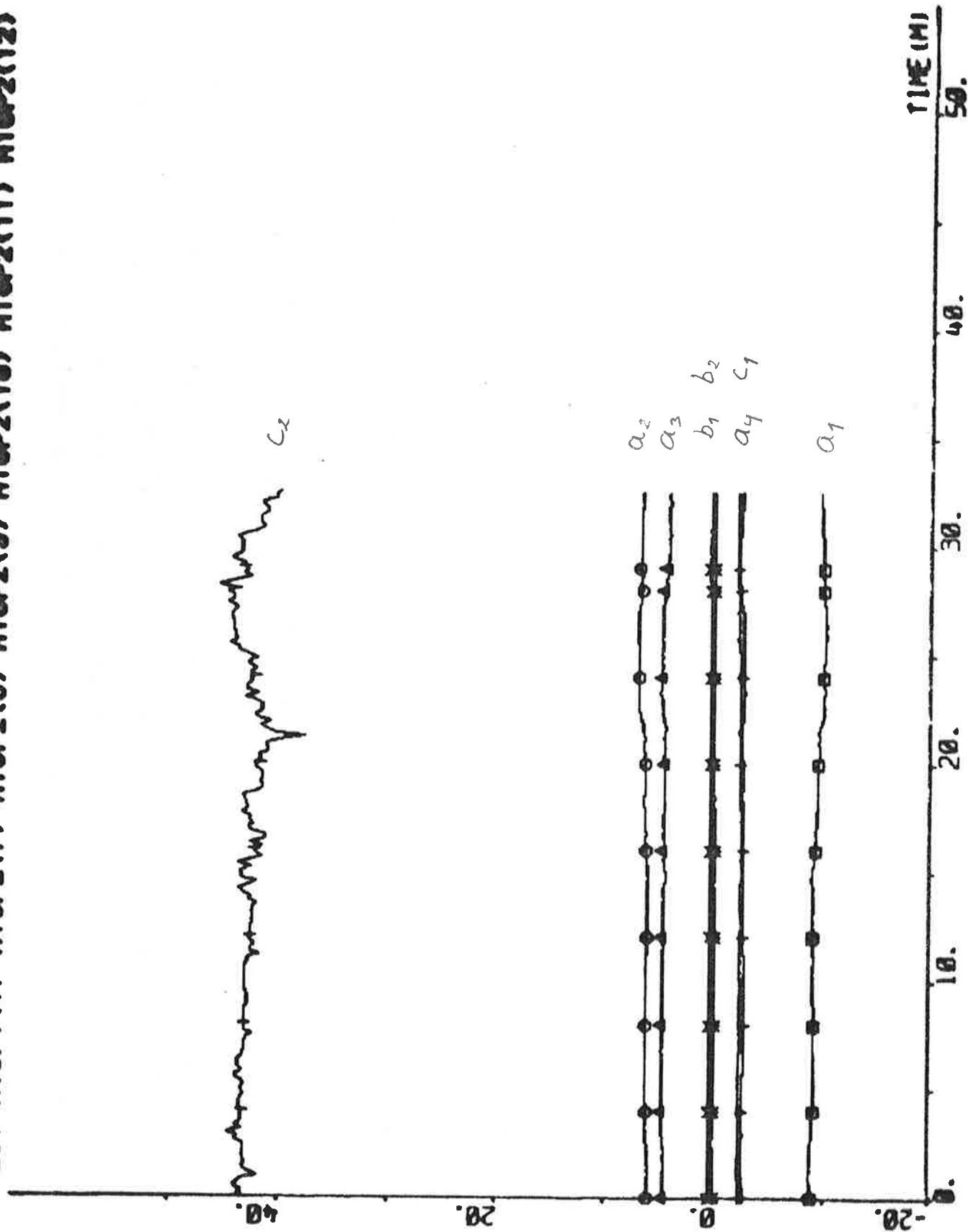
0.

0.

0.



PLOT A1OP1(1)-A1OP2(7) A1OP2(8) A1OP2(9) A1OP2(10) A1OP2(11) A1OP2(12) A1



## EXPERIMENT A11

Date	1976-04-23	Forward draught	8.5 m
Time	13.06	Aft draught	12.5 m
Duration	24 min	Wind direction	NW (5; see App. A)
Position	N 06°48' W 15°57'	Wind velocity	3 m/s (light breeze)
$\psi_{ref}$	145 deg	Wave height	-

Self-tuning regulator using estimates from the Kalman filter.

Tuning time before the experiment started: 30 min.

NC1 = 1	NC2 = 1	k = 6	q = 0.05
$T_s = 10$ s	$V_0 = 6$ m/s	IVVC = 1	

Final values:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ b_1 \\ b_2 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} -9.16 \\ 6.39 \\ 4.29 \\ -1.86 \\ 0.46 \\ 0.22 \\ -2.60 \\ 37.18 \end{bmatrix} \quad P = \begin{bmatrix} 5.17 & & & & & & & & & \\ -6.49 & 13.13 & & & & & & & & \\ -0.84 & -6.88 & 14.72 & & & & & & & \\ 2.41 & 0.01 & -7.29 & 5.08 & & & & & & \\ -0.11 & -0.12 & 0.39 & -0.16 & 0.03 & & & & & \\ -0.07 & 0.04 & -0.04 & 0.08 & 0.00 & 0.02 & & & & \\ -0.95 & 1.32 & 0.35 & -0.75 & 0.00 & -0.02 & 0.83 & & & \\ 15.49 & -16.91 & -15.74 & 15.85 & -0.33 & -0.51 & 12.37 & 860.40 & & \end{bmatrix}$$

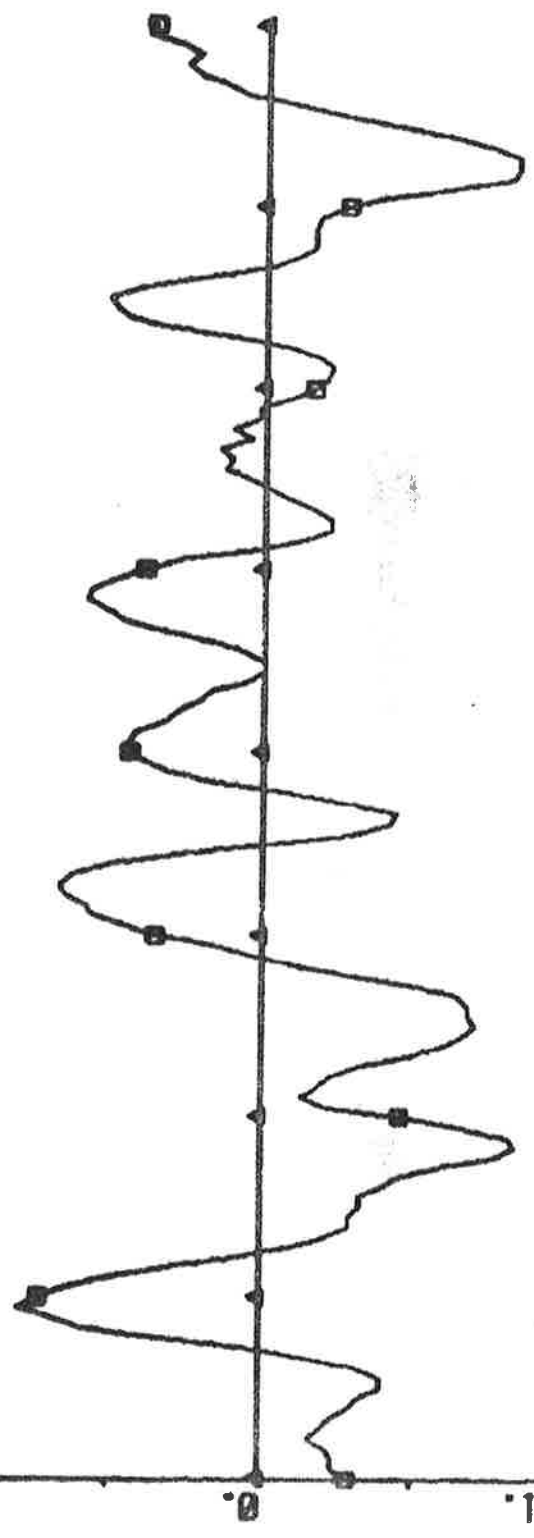
$$a_1 + a_2 + a_3 + a_4 = -0.34$$

$$\hat{\delta}_0 = 0.2 \text{ deg} \quad \hat{d}_V = 0.06 \text{ knots} \quad \hat{d}_r = 0.001 \text{ deg/s} \quad \hat{d}_\delta = 1.5 \text{ deg}$$

Statistics (mean value and standard deviation)

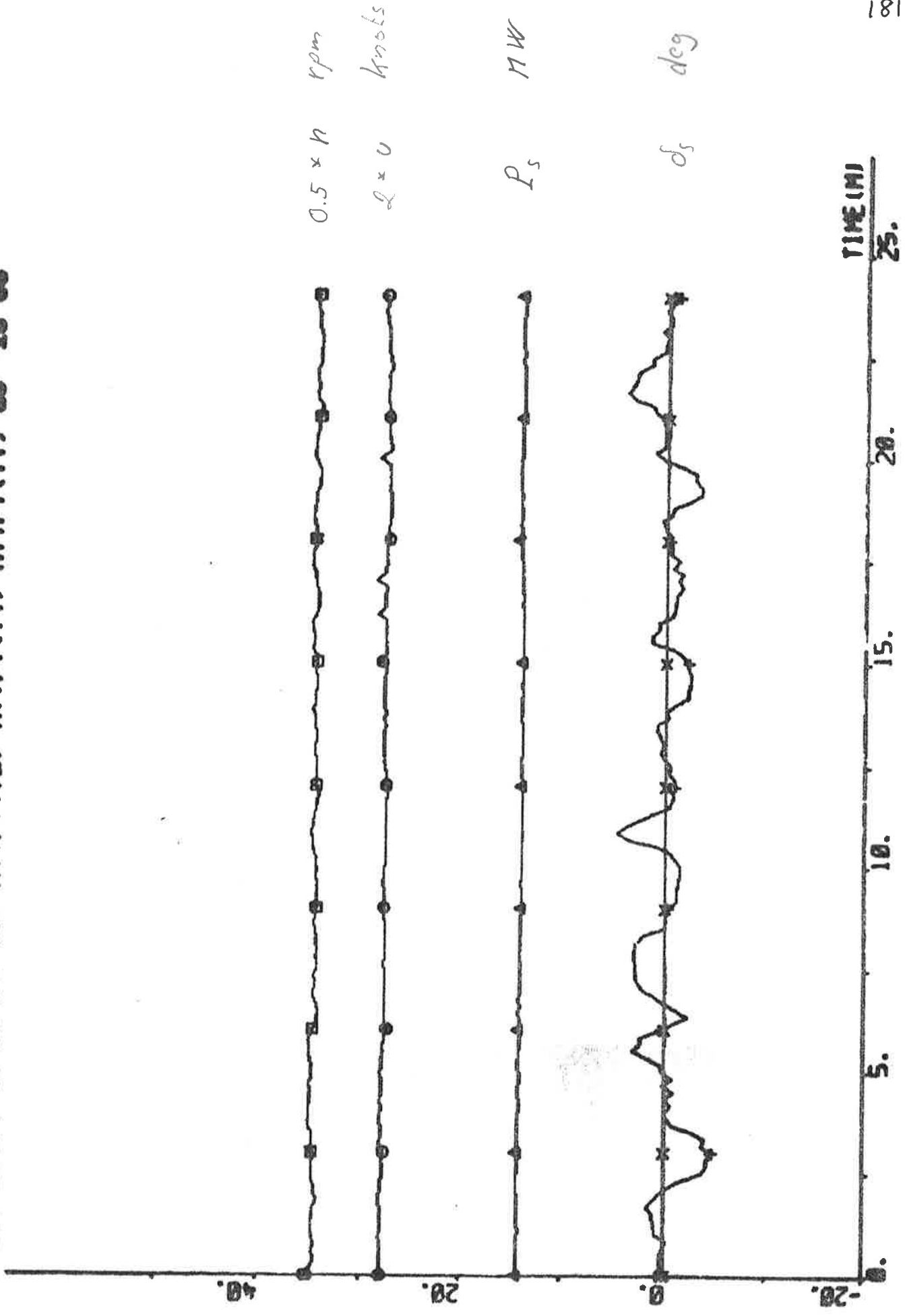
$\delta_c$	-0.06	± 1.79	deg	$P_s$	14.2	± 0.1	MW
$\delta$	1.33	± 1.61	deg	$\epsilon_V$	0.00	± 0.02	knots
$\psi - \psi_{ref}$	-0.033	± 0.400	deg	$\epsilon_r$	0.00	± 0.02	deg/s
n	69.2	± 0.4	rpm	$\epsilon_\psi$	0.00	± 0.03	deg
u	13.9	± 0.1	knots	$\epsilon_\delta$	0.1	± 0.6	deg
$V_1 = 0.428$							

PLOT A11P1(1)-A11P1(0) HP A11P1(10) A11P1(15) 02 -3 1 - DEG

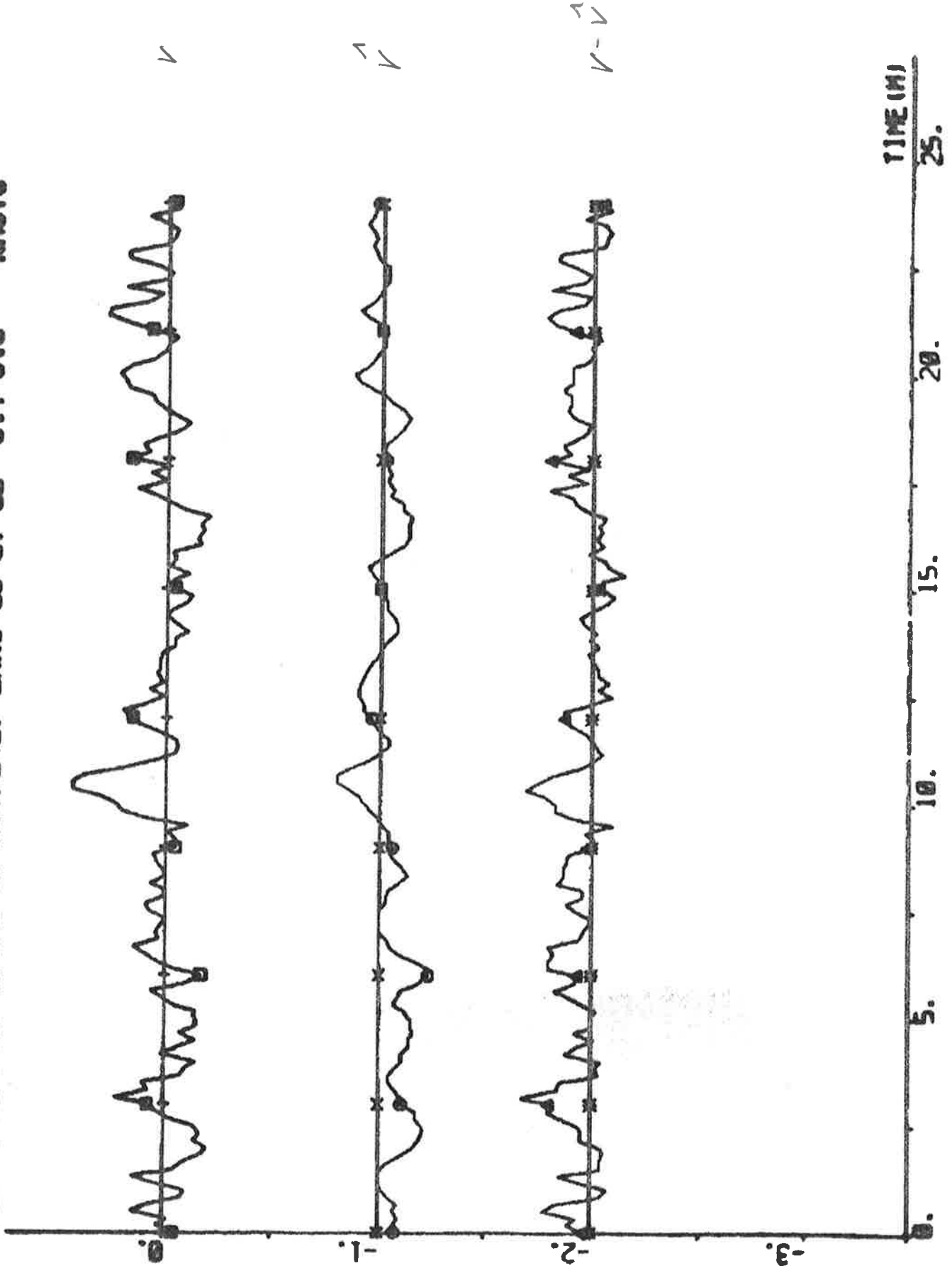


TIME (M) 25.

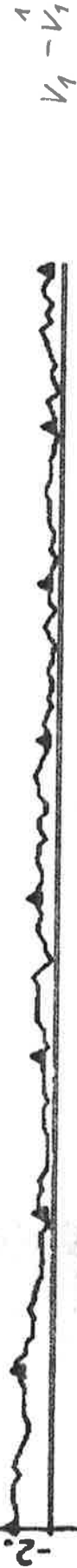
PLOT A11P1(1)-A11P1(13) A11P1(12) A11P1(14) A11P1(11) 00 -20 60



PLOT A11P1(1)-A11P2(1) A11P2(2) ERRS 00 01 02 -3.4 0.0 - KNOTS

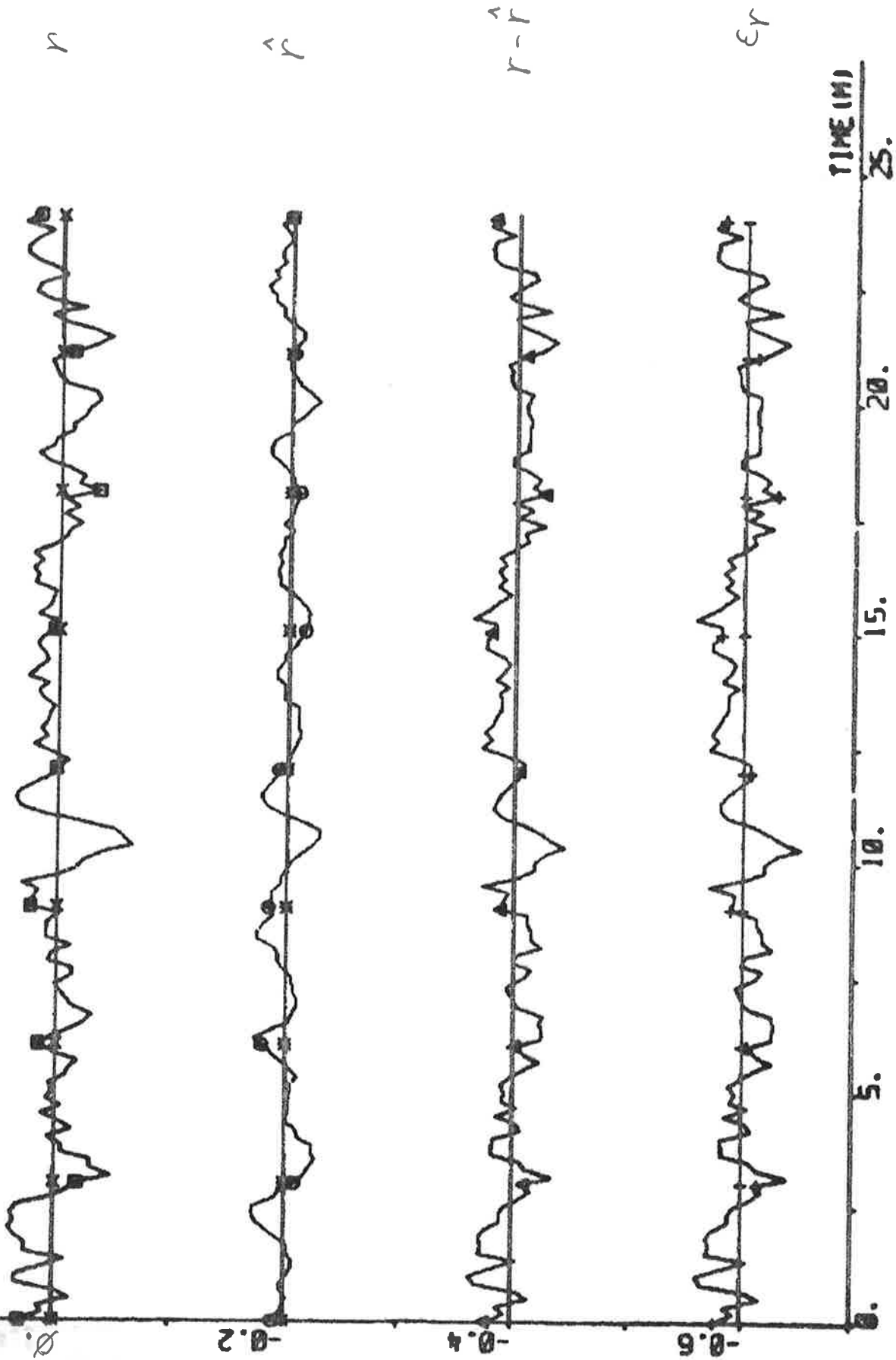


PL0T A11P1(1)-A11P1(4) A11P1(5) ERR2 00 01 02 03 -3.4 0.0 - KNOTS



TIME (M)  
25.

PLOT A11P1(1)-A11P1(6) A11P1(7) ERR3 EPS3 00 002 004 008 -0.7 0. • DECS



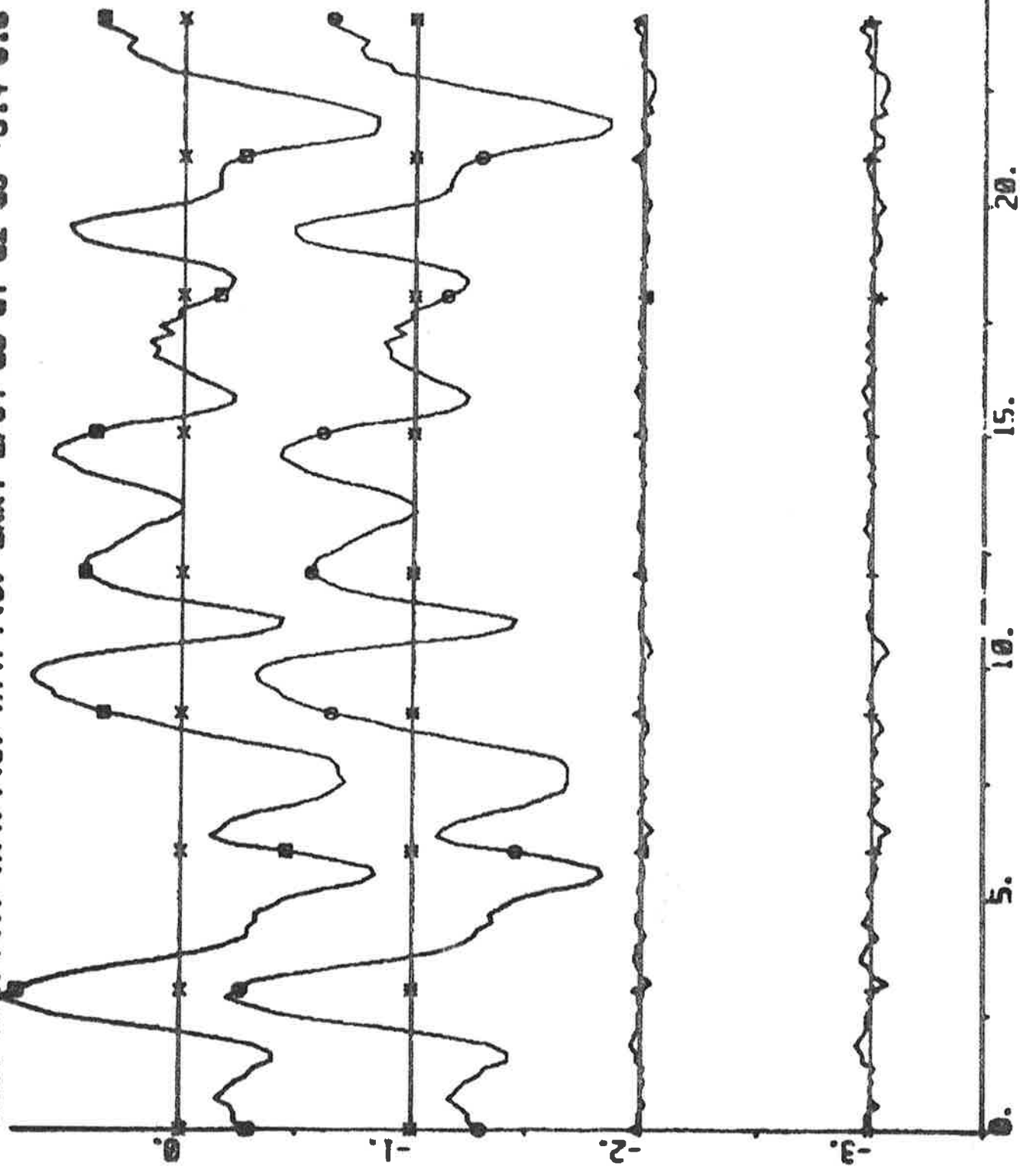
PLOT A11P1(1)-A11P1(0) A11P1(0) ERR4 EPS4 00 01 02 03 -3.4 0.0 - DEG

$\gamma - \gamma_{ref}$

$\hat{\gamma} - \gamma_{ref}$

$\gamma - \gamma$

$\epsilon \gamma$



TIME (M)  
25.



PLOT A11P1(1)-A11P1(2) A11P1(3) ERR1 EPS1 00 020 040 060 -65 16 - DEG



$\delta$



$\delta$



$\delta - \delta$



$\epsilon \delta$

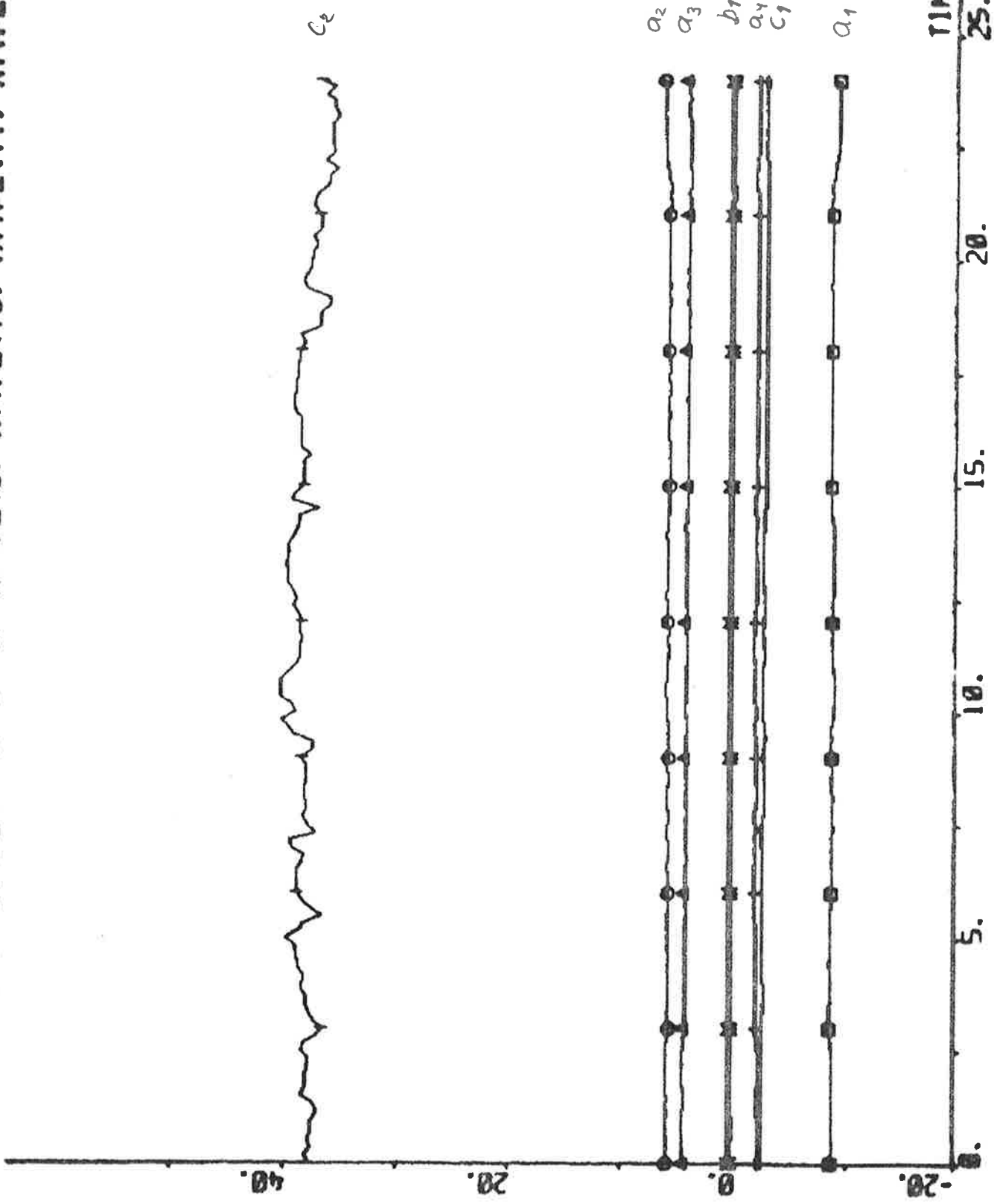
TIME (H) 25.

PLOT A11P1(1)-A11P2(3) A11P2(4) A11P2(5) A11P2(6) 00 02 04 06 -0.5 1.5



TIME (M)  
0. 5. 10. 15. 20. 25.

PLOT A11P1(1)-A11P2(7) A11P2(8) A11P2(9) A11P2(10) A11P2(11) A11P2(12) A1



## EXPERIMENT A12

Date	1976-04-23	Forward draught	8.5 m
Time	14.09	Aft draught	12.5 m
Duration	26 min	Wind direction	N (5; see App. A)
Position	N 06°35' W 15°48'	Wind velocity	3 m/s (light breeze)
$\psi_{\text{ref}}$	145 deg	Wave height	-

Self-tuning regulator using estimates from the Kalman filter.

Tuning time before the experiment started: 30 min.

NC1 = 1	NC2 = 1	k = 6	q = 0.1
$T_s = 10$ s	$V_0 = 6$ m/s	IVVC = 1	

Final values:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ b_1 \\ b_2 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} -8.73 \\ 7.02 \\ 4.66 \\ -3.33 \\ 0.29 \\ 0.08 \\ -2.89 \\ 44.49 \end{bmatrix} \quad P = \begin{bmatrix} 5.36 & & & & & & & & & \\ -6.80 & 13.38 & & & & & & & & \\ -0.84 & -6.55 & 14.33 & & & & & & & \\ 2.47 & -0.08 & -7.15 & 4.95 & & & & & & \\ -0.10 & -0.10 & 0.37 & -0.16 & 0.03 & & & & & \\ -0.10 & 0.09 & -0.05 & 0.06 & 0.00 & 0.03 & & & & \\ -1.01 & 1.34 & 0.26 & -0.58 & -0.01 & 0.01 & 0.92 & & & \\ 18.54 & -24.21 & -13.08 & 17.88 & -0.34 & -0.27 & 12.52 & 915.41 & & \end{bmatrix}$$

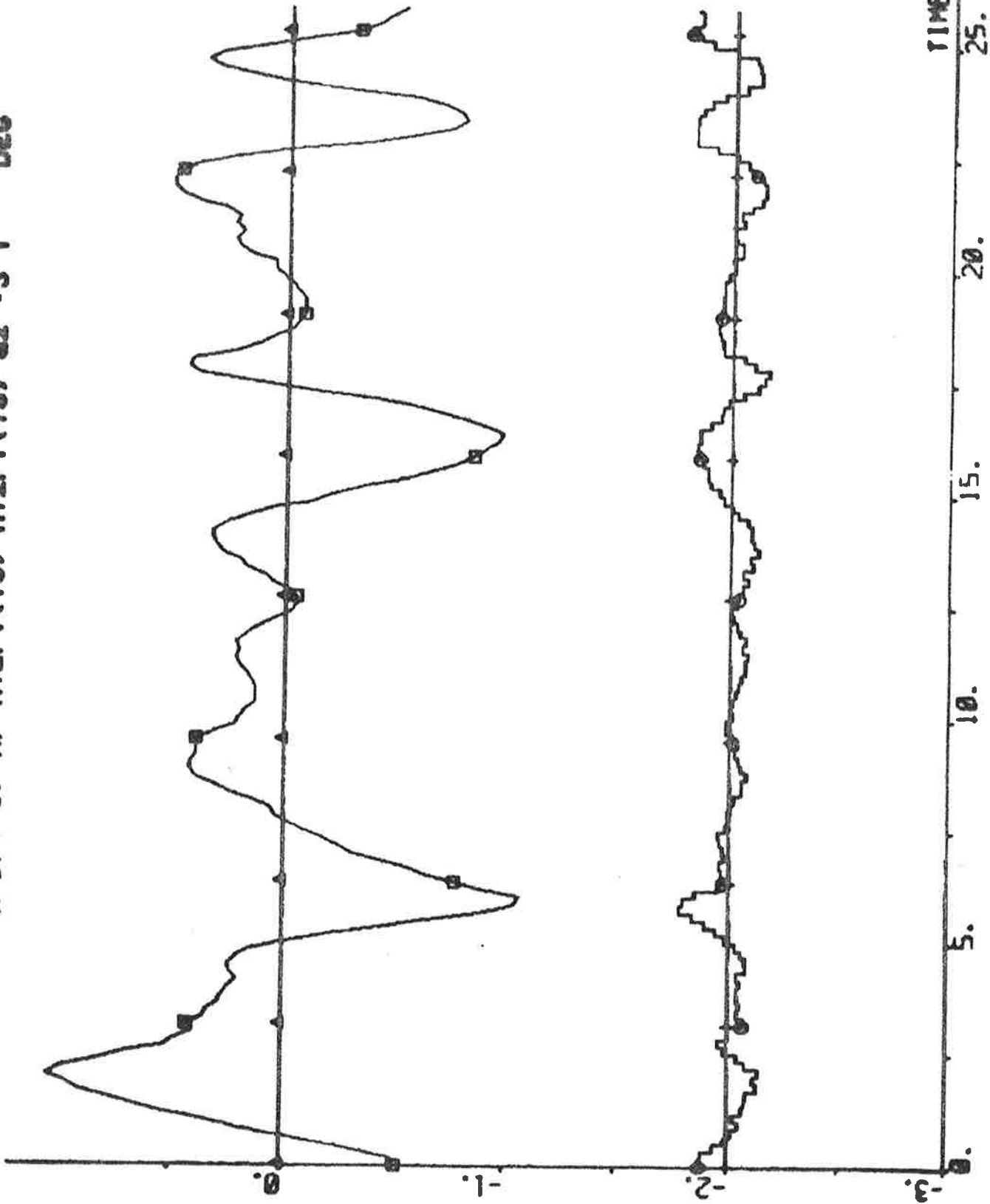
$$a_1 + a_2 + a_3 + a_4 = -0.38$$

$$\hat{\delta}_0 = 0.3 \text{ deg} \quad \hat{d}_v = 0.07 \text{ knots} \quad \hat{d}_r = 0.000 \text{ deg/s} \quad \hat{d}_\delta = 1.3 \text{ deg}$$

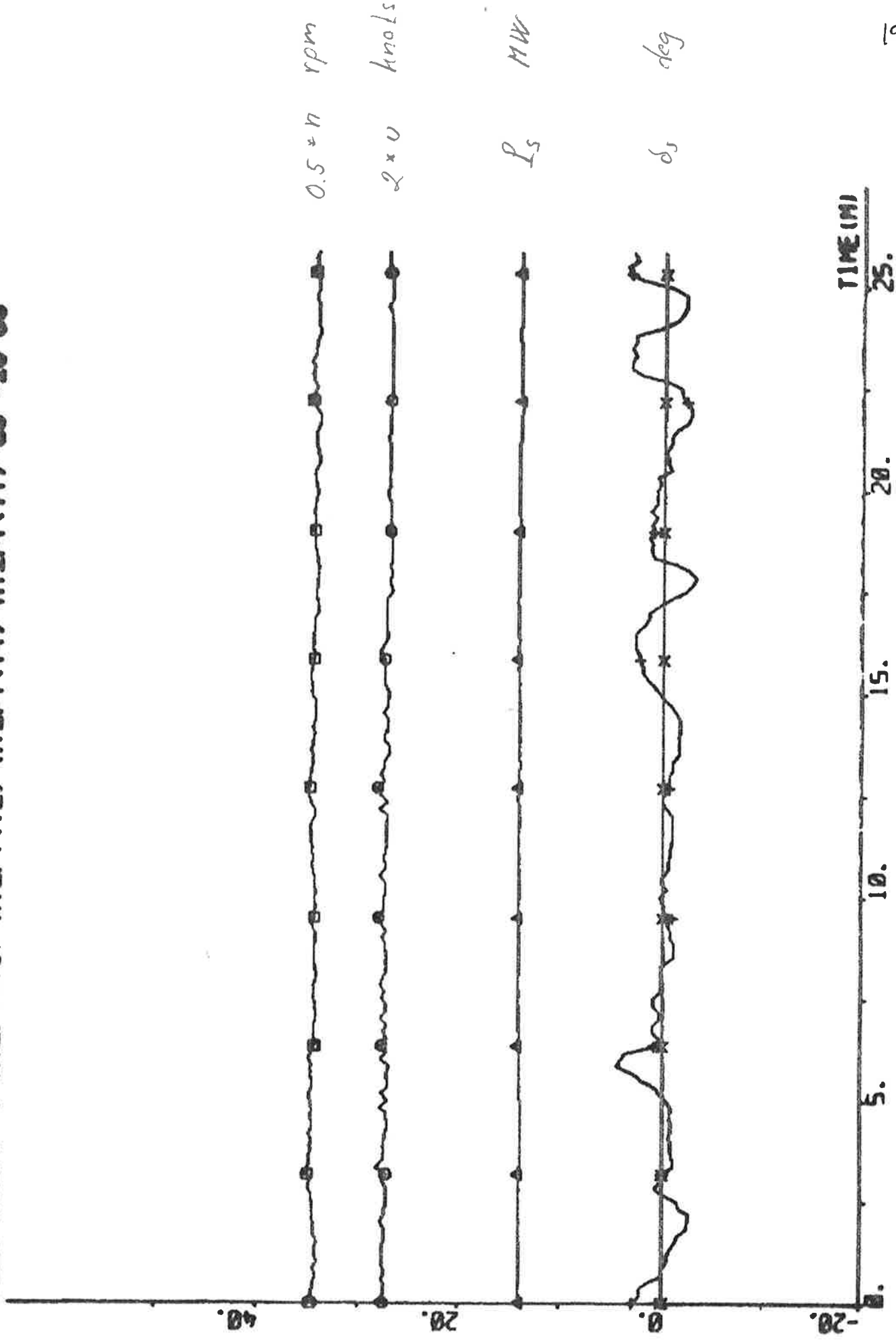
Statistics (mean value and standard deviation)

$\delta_c$	0.05	± 1.69	deg	$P_s$	14.2	± 0.1	MW
$\delta$	1.48	± 1.41	deg	$\epsilon_v$	0.00	± 0.02	knots
$\psi - \psi_{\text{ref}}$	0.013	± 0.444	deg	$\epsilon_r$	0.00	± 0.02	deg/s
n	69.2	± 0.4	rpm	$\epsilon_\psi$	0.00	± 0.02	deg
u	13.7	± 0.1	knots	$\epsilon_\delta$	0.1	± 0.7	deg
$V_1 = 0.435$							

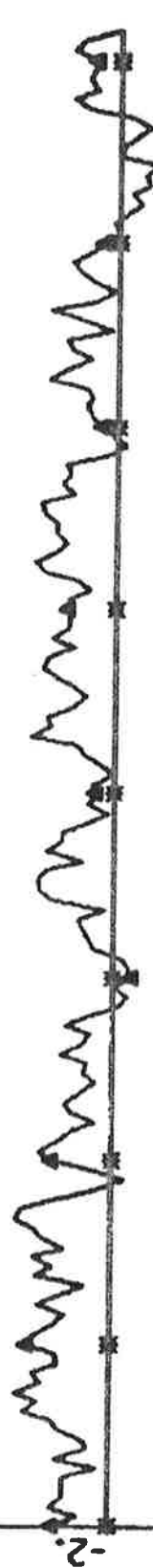
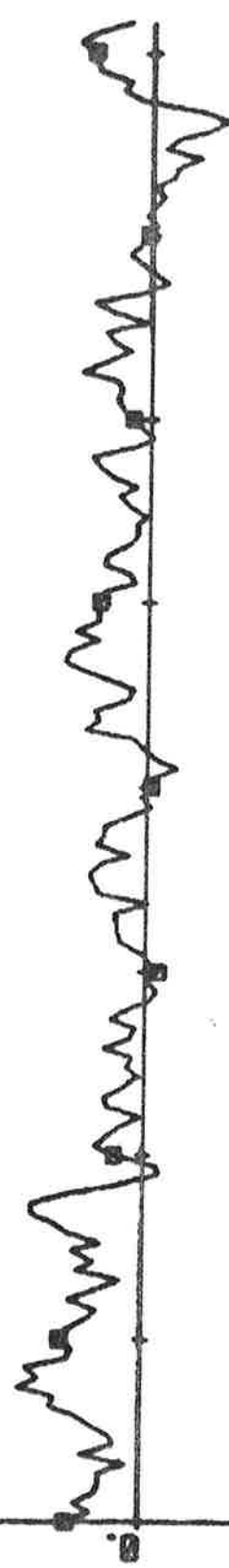
PLOT A12P1(1)~A12P1(8) HP A12P1(10) A12P1(15) Q2 -3 1 - DEG



PLOT A12P1(1)-A12P1(13) A12P1(12) A12P1(14) A12P1(11) 00 -20 50



PLOT A12P1(1)-A12P2(1) A12P2(2) ERRS 00 01 02 -3.4 0.0 - KNOTS



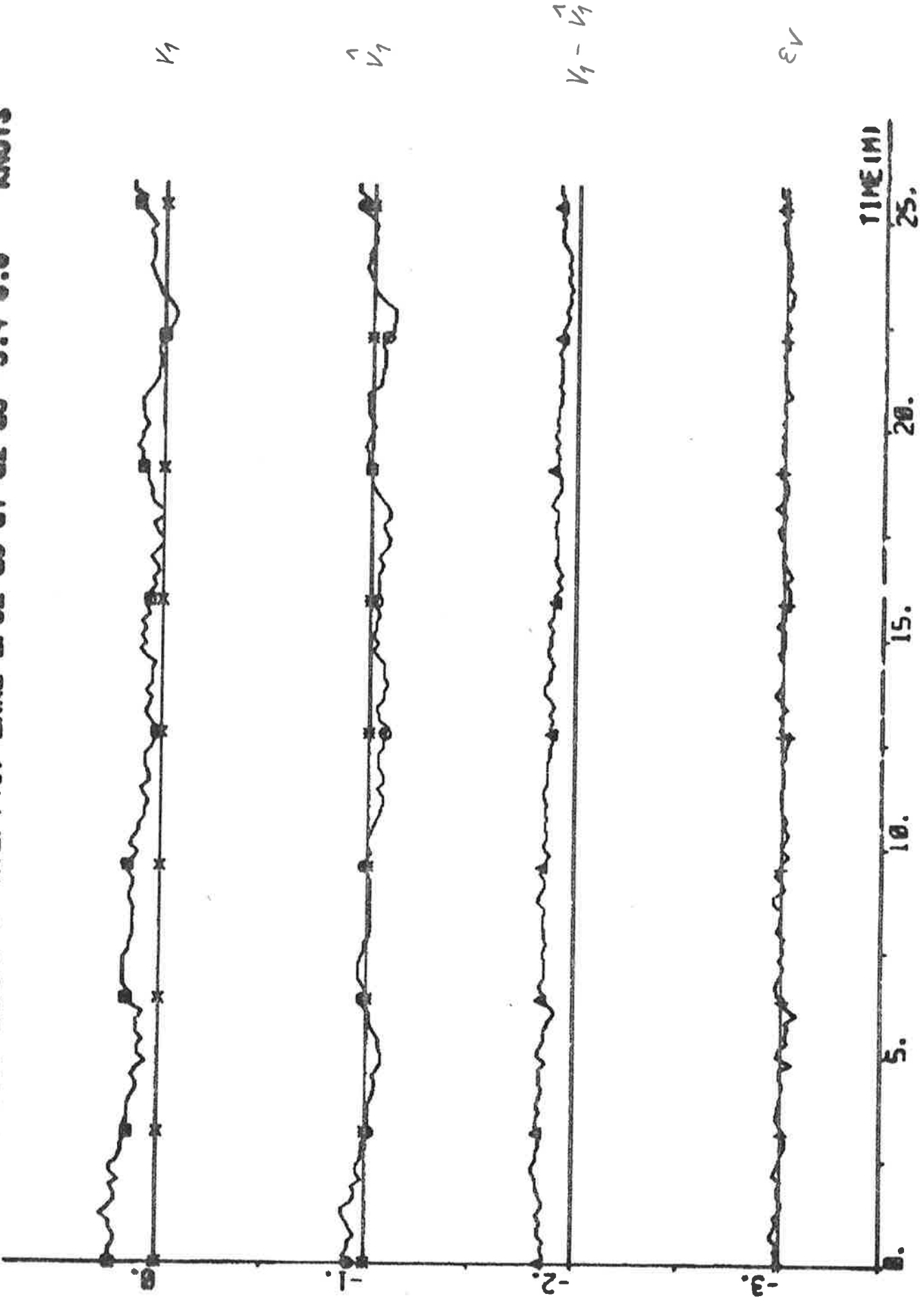
0. 5. 10. 15. 20. 25. TIME (M)

>

>

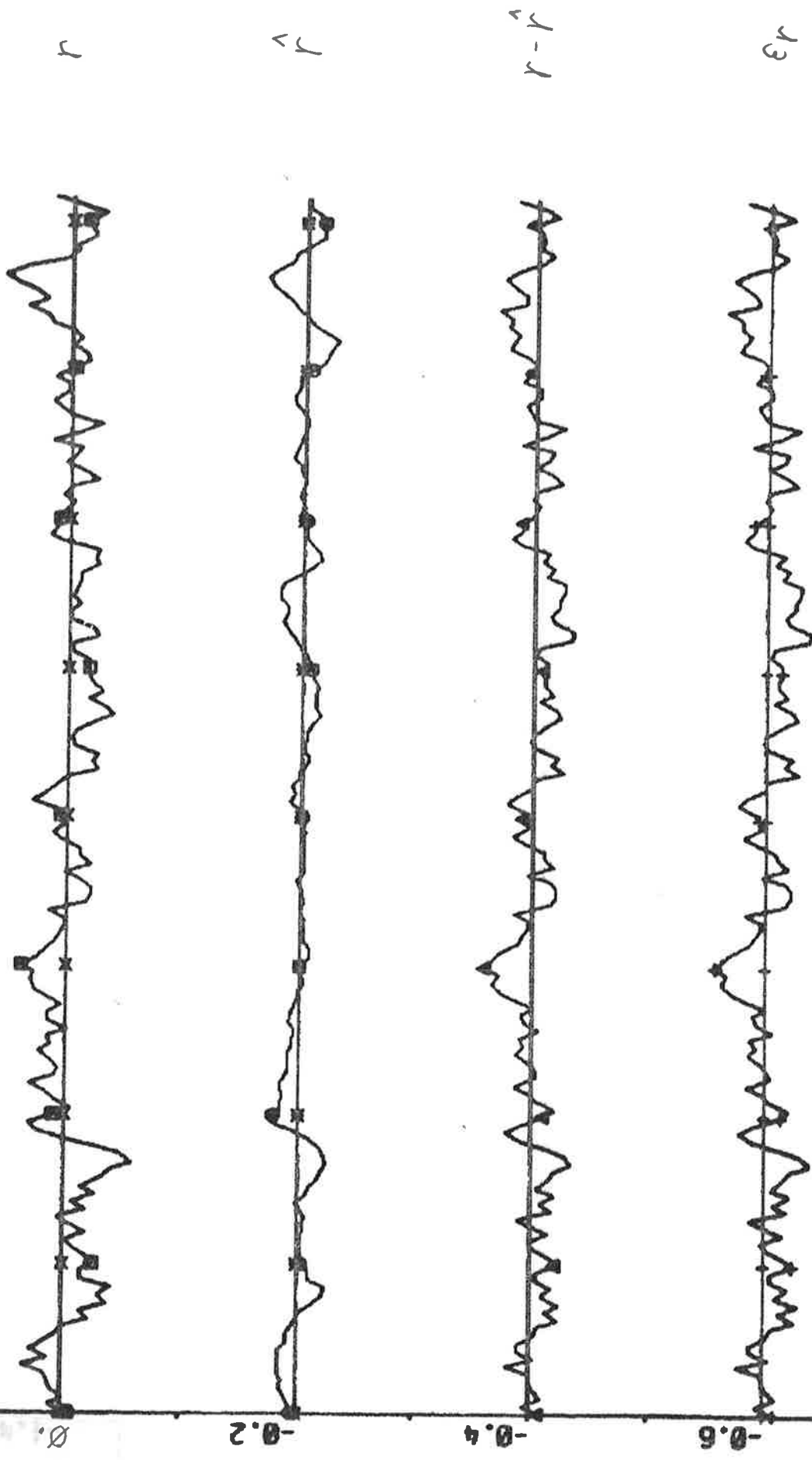
>

PLOT A12P1(1)-A12P1(4) A12P1(5) ERR2 EPS2 00 01 02 03 -3.4 0.0 - KNOTS

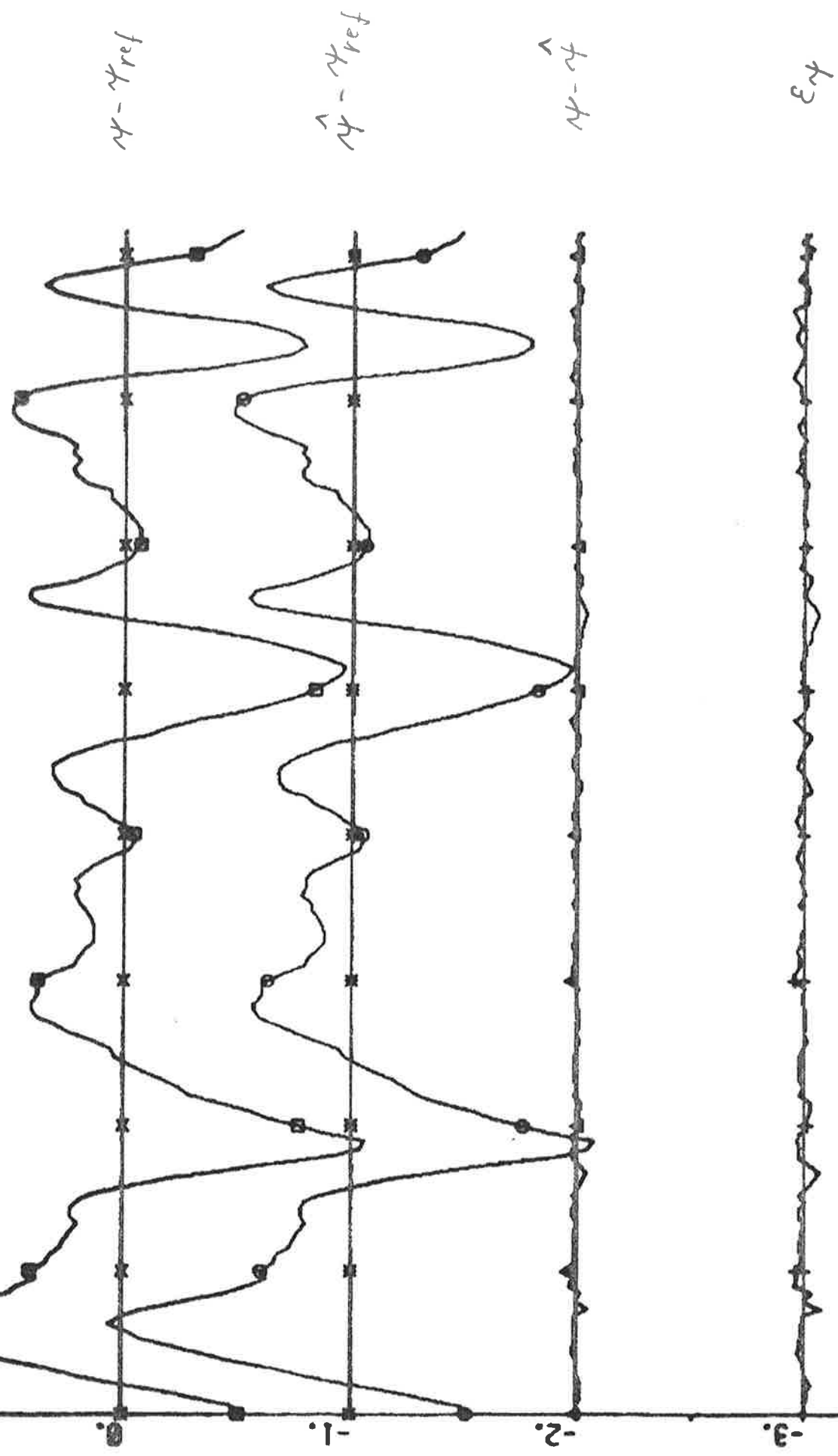




PLOT A12P1(1)←A12P1(6) A12P1(7) ERR3 EPS3 00 002 004 008 -0.7 0. - 005-3



PLOT A12P1(1)-A12P1(8) A12P1(9) ERR4 EPS4 00 01 02 03 -3.4 0.0 - DEG



0  
-1  
-2  
-3  
5. 10. 15. 20. 25.  
TIME (M)

PLOT A12P1(1)-A12P1(2) A12P1(3) ERR1 EPS1 00 020 040 060 -65 15 - DEG



$\rho$



$\hat{\rho}$



$\rho - \hat{\rho}$



$\epsilon \rho$

0. 5. 10. 15. 20. 25. TIME (M)

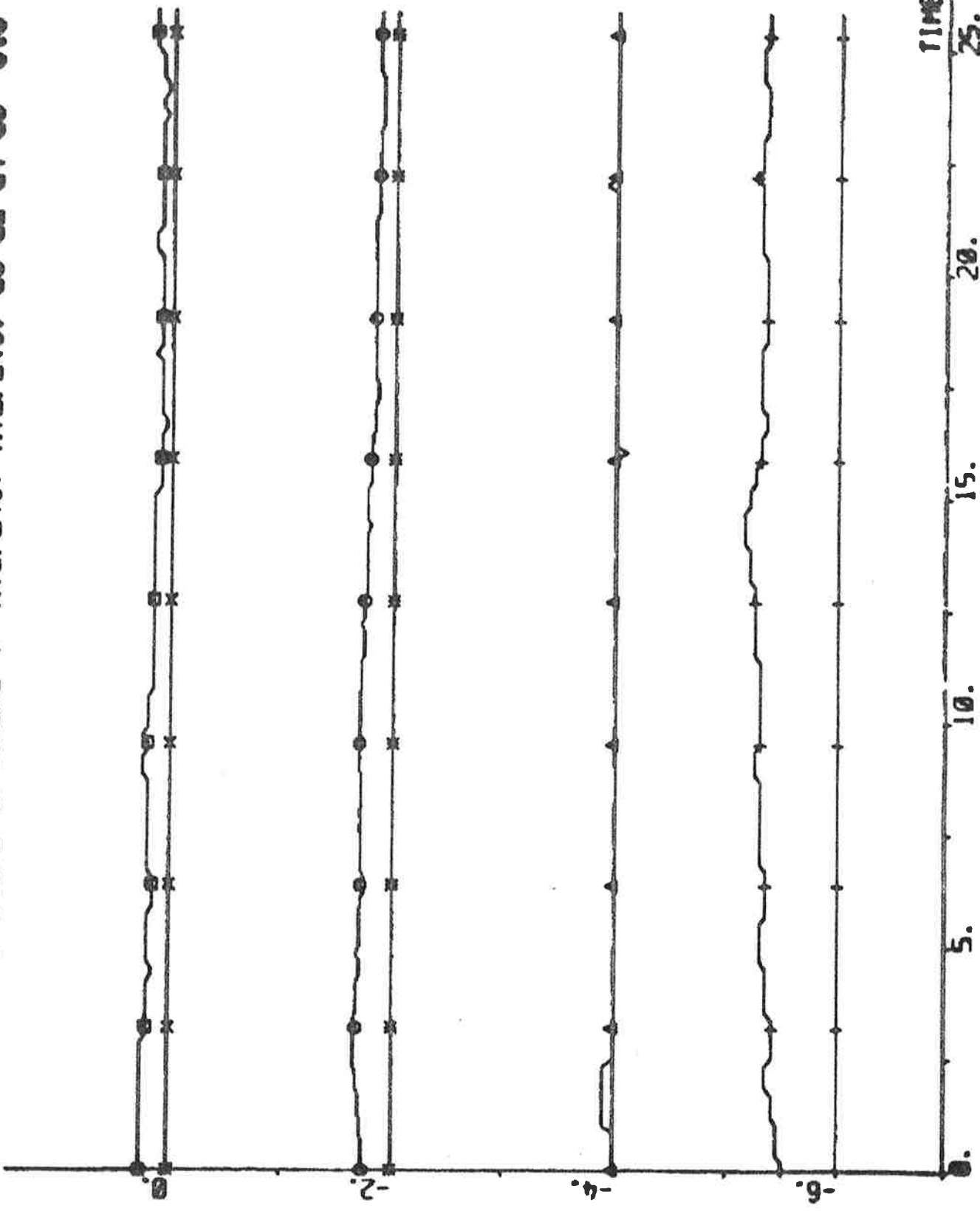
PLOT A12P1(1)-A12P2(3) A12P2(4) A12P2(5) A12P2(6) 00 02 04 06 -6.6 1.6

$0.5 \times \hat{d}_0$  deg

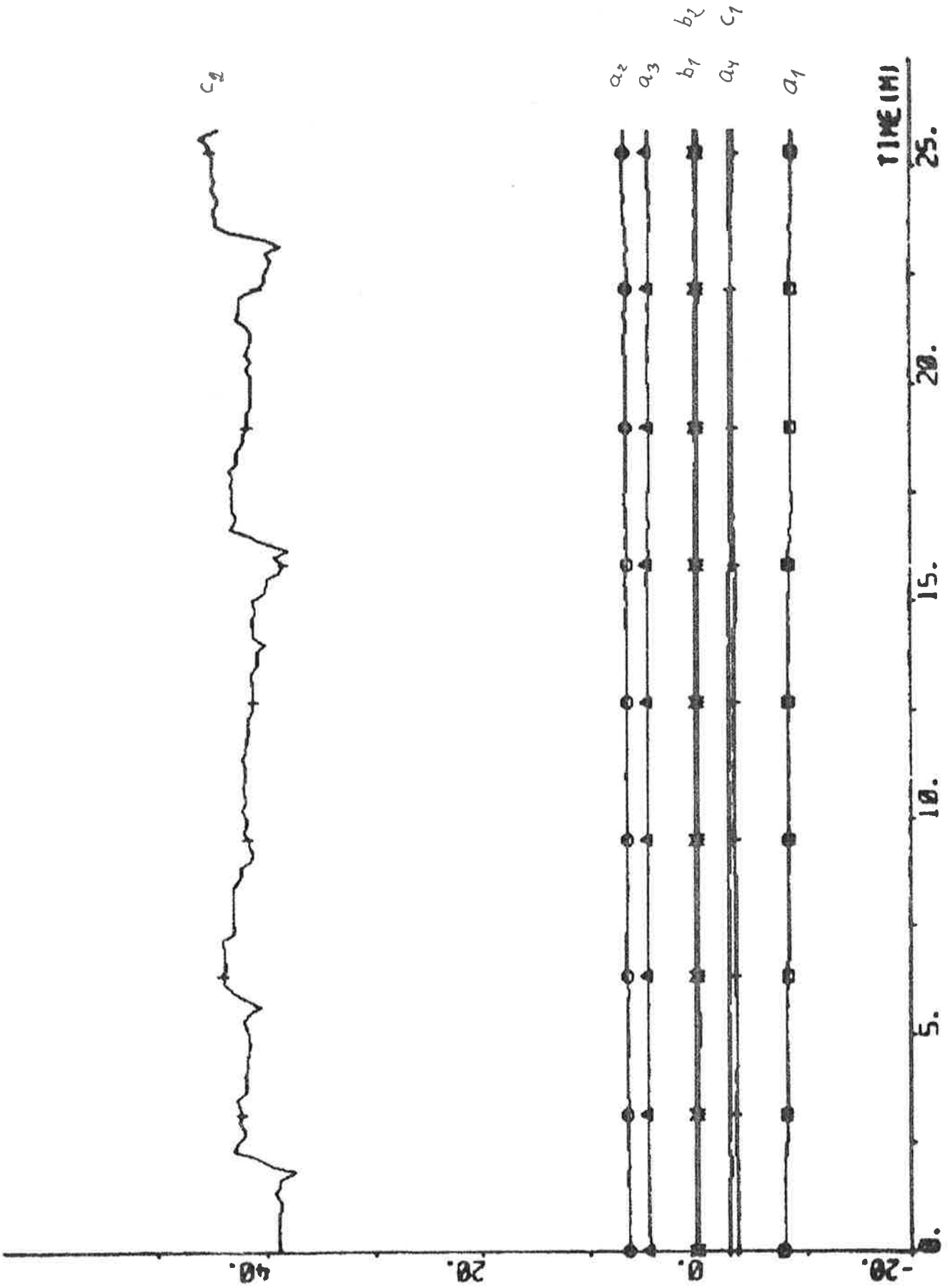
$\hat{z} \times \hat{d}_v$  knots

$100 \times \hat{d}_r$  deg/s

$0.5 \times \hat{d}_f$  deg

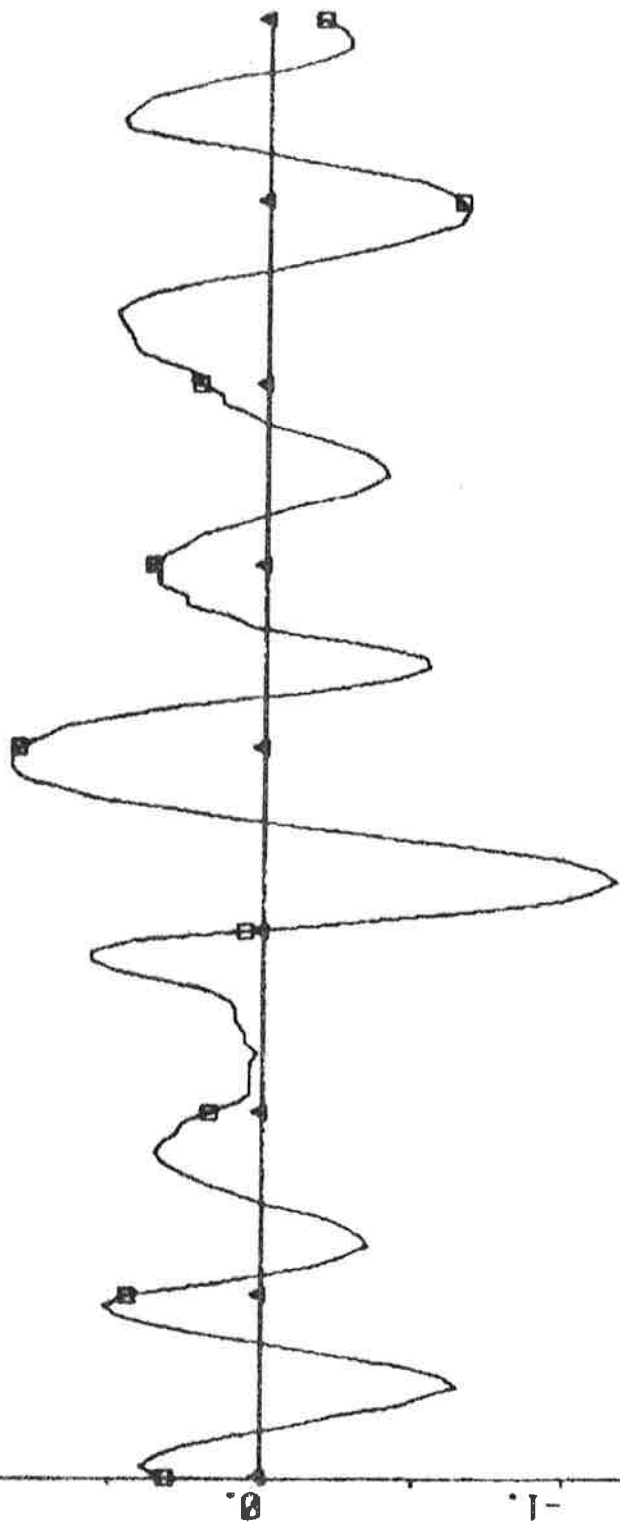


PLOT A12P1(1)-A12P2(7) A12P2(8) A12P2(9) A12P2(10) A12P2(11) A12P2(12) A1

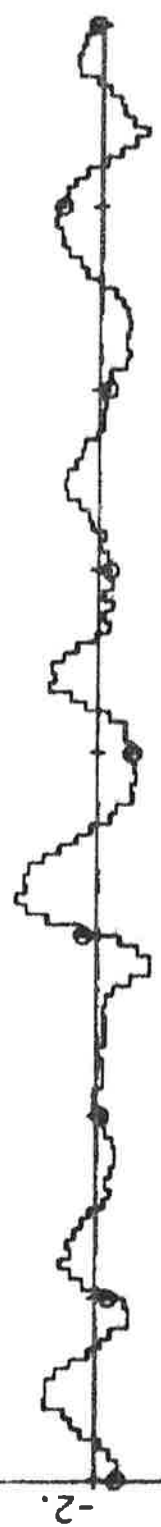




PLOT A13P1(1)-A13P1(8) HP A13P1(10) A13P1(15) 02 -3 1 - DEC



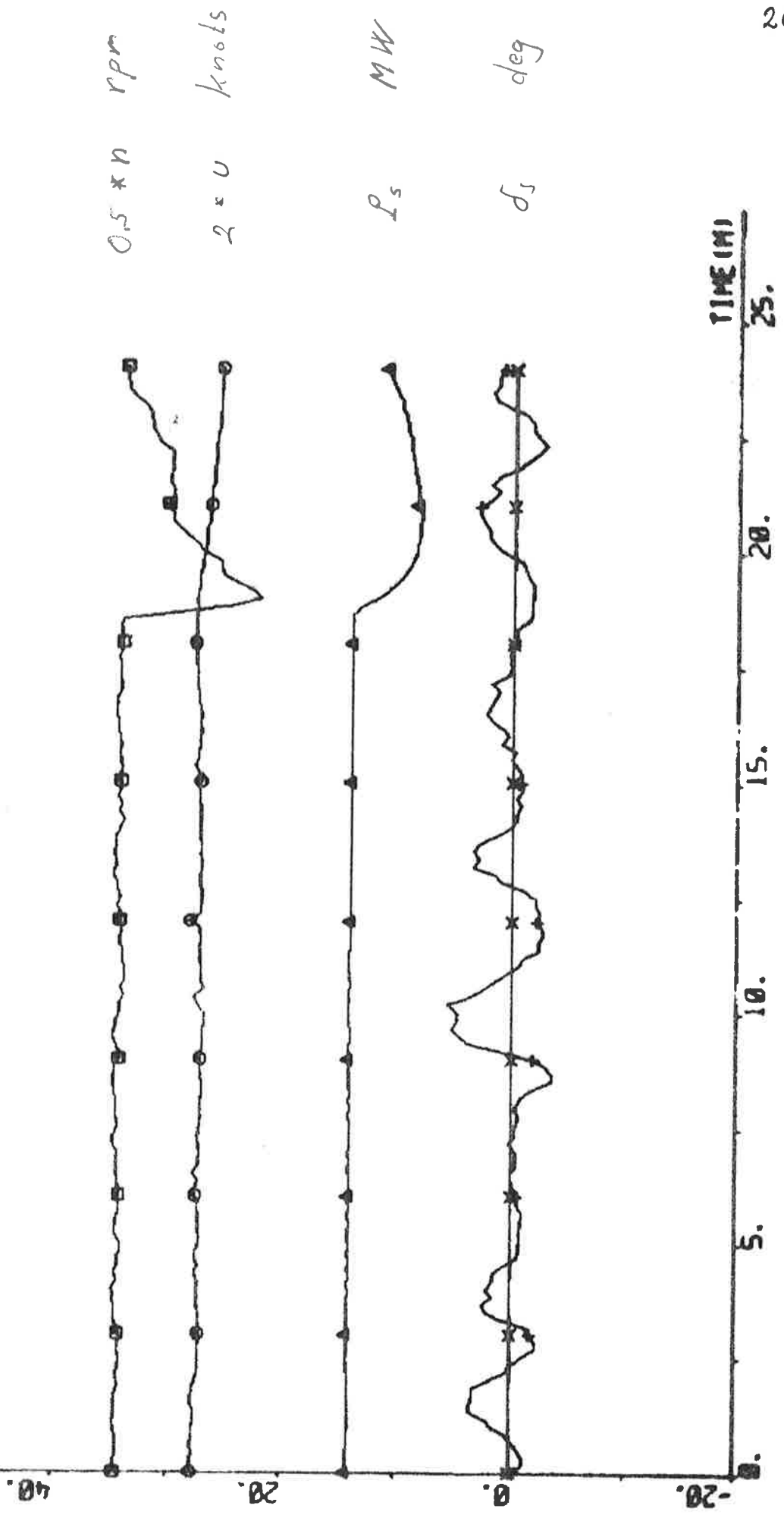
$y = y_{ref}$



$0.05 \times d_c$

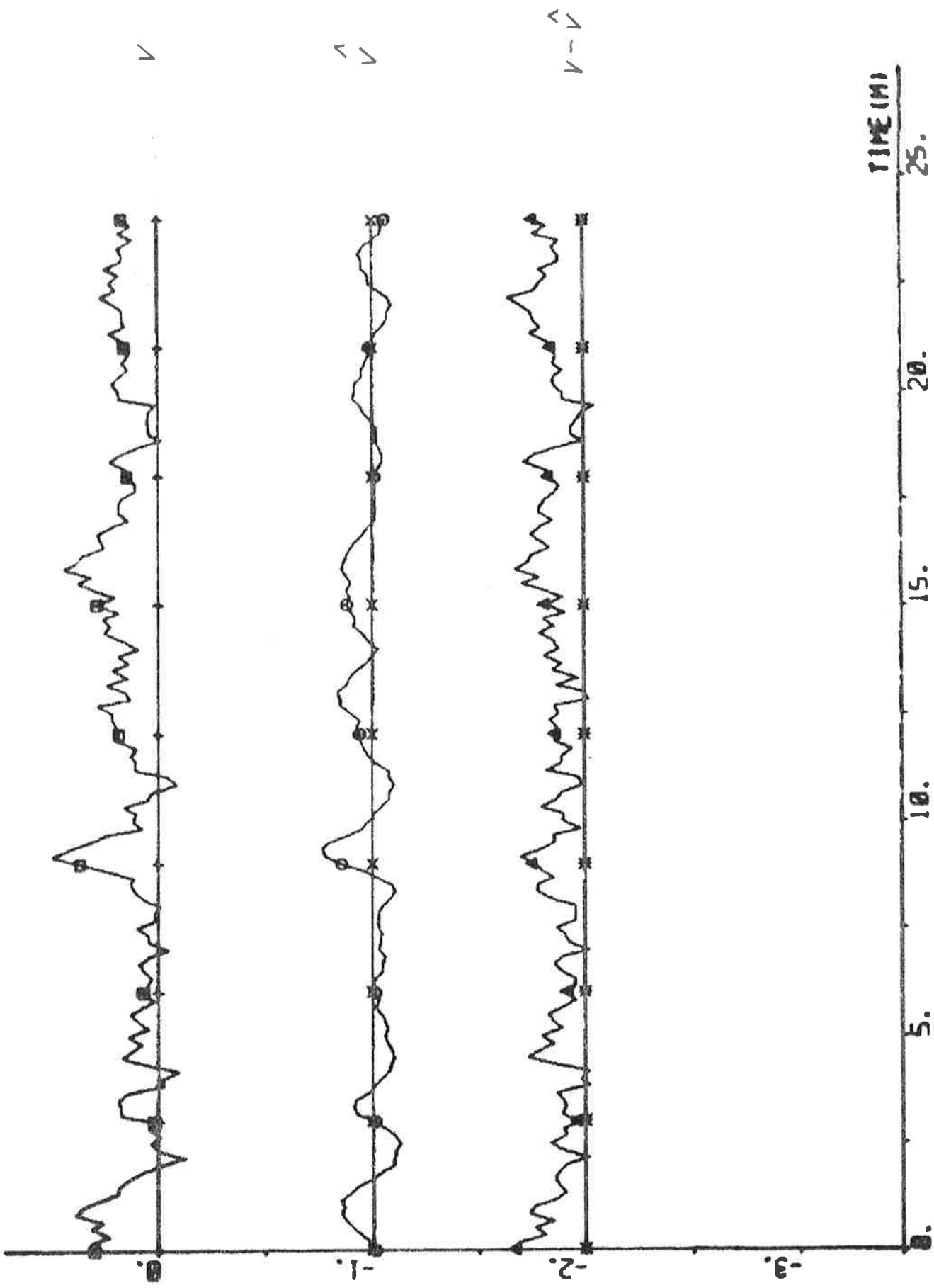
TIME (M) 0. 5. 10. 15. 20. 25.

PLOT A13P1(1)-A13P1(13) A13P1(12) A13P1(14) A13P1(11) 00 -20 50

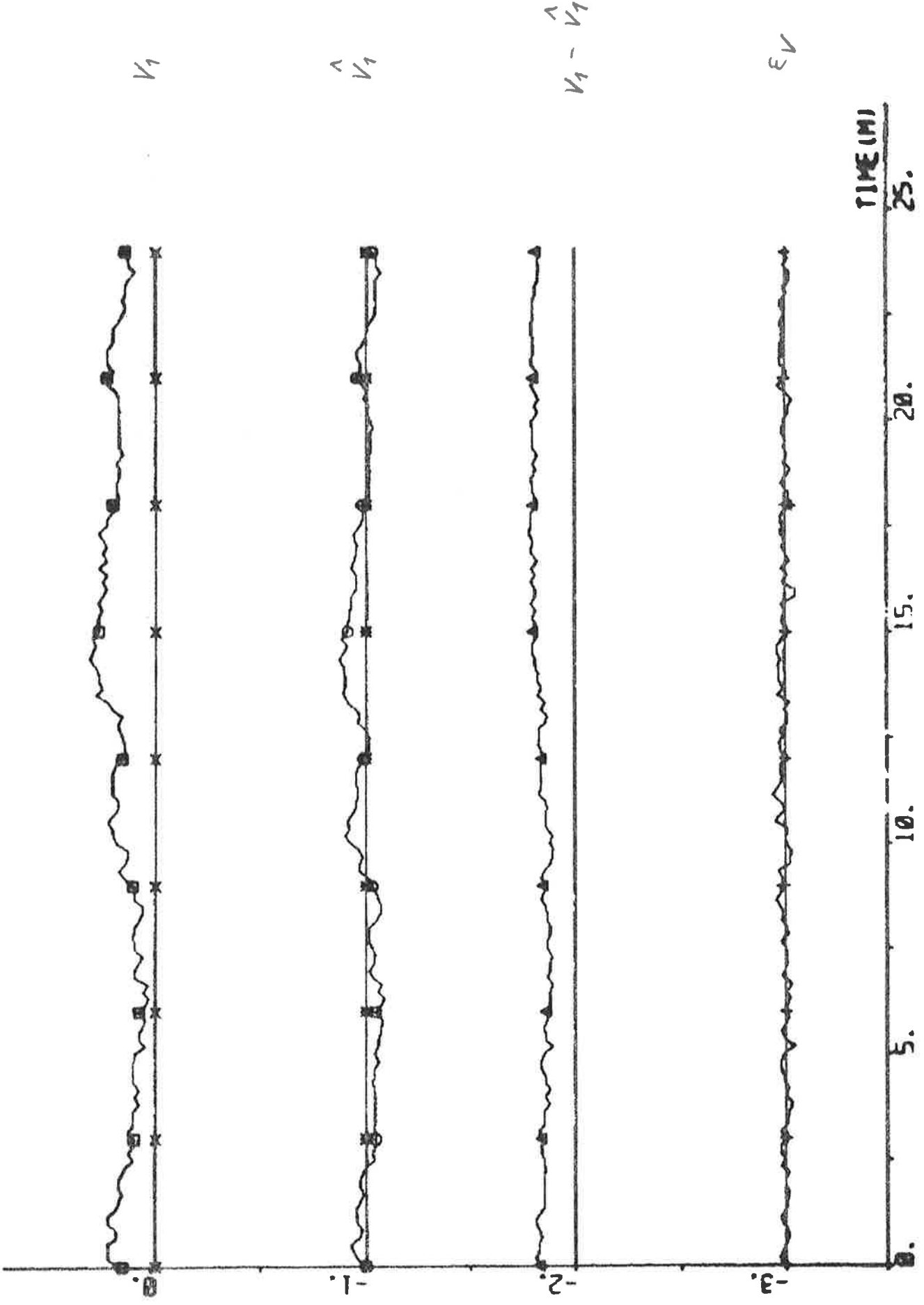




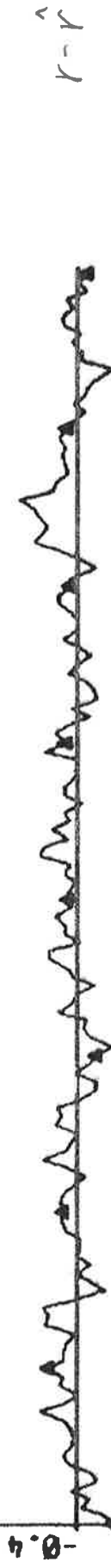
PLOT A13P1(1)-A13P2(1) A13P2(2) ERRS 00 01 02 -3.4 0.0 - KNOTS



PLOT A13P1(1)-A13P1(4) A13P1(5) ERR2 EPS2 00 01 02 03 -3.4 0.0 - KNOTS

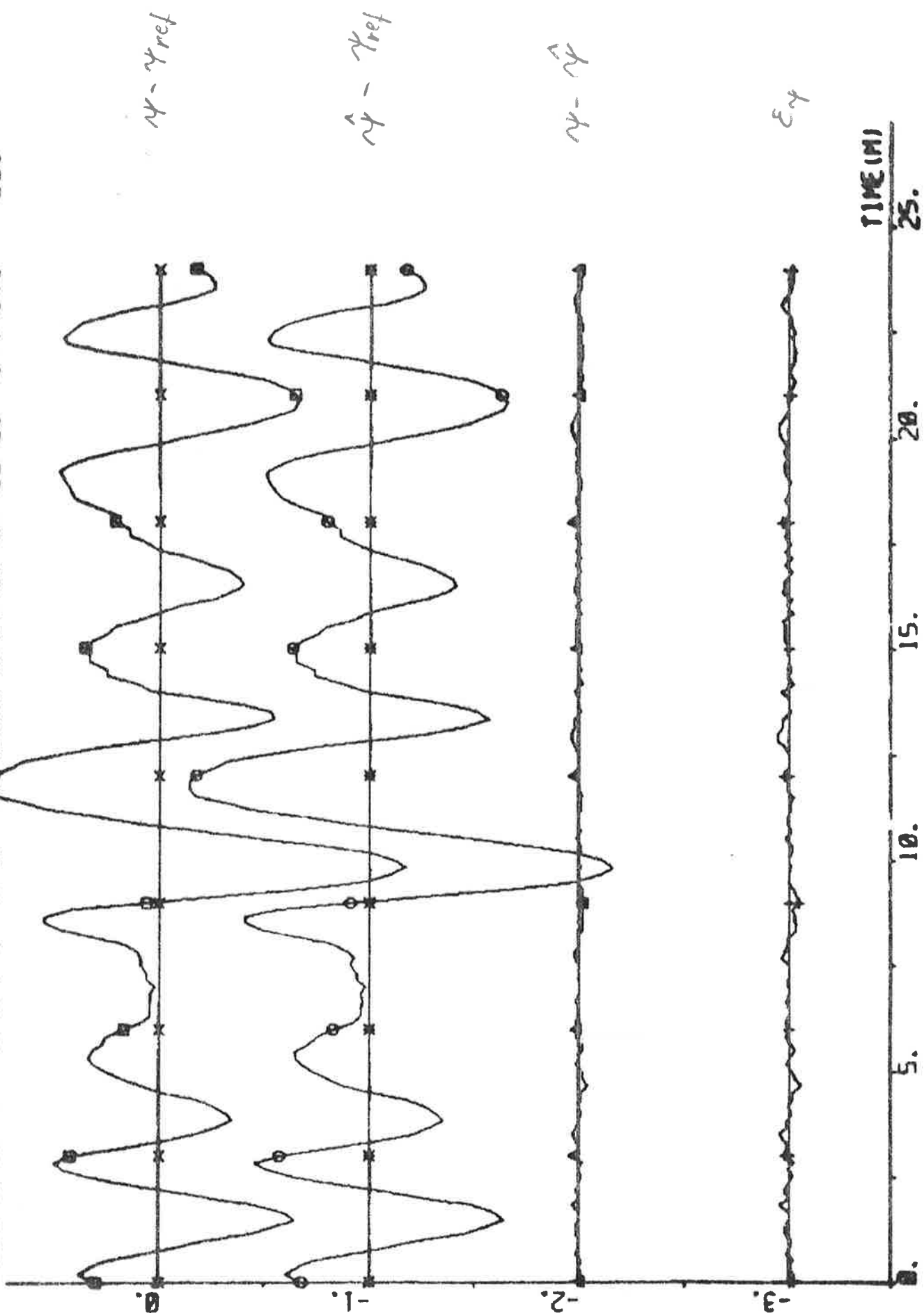


PLOT A13P1(1)-A13P1(6) A13P1(7) ERR3 EPS3 00 002 004 006 -0.7 0. - DEC-73

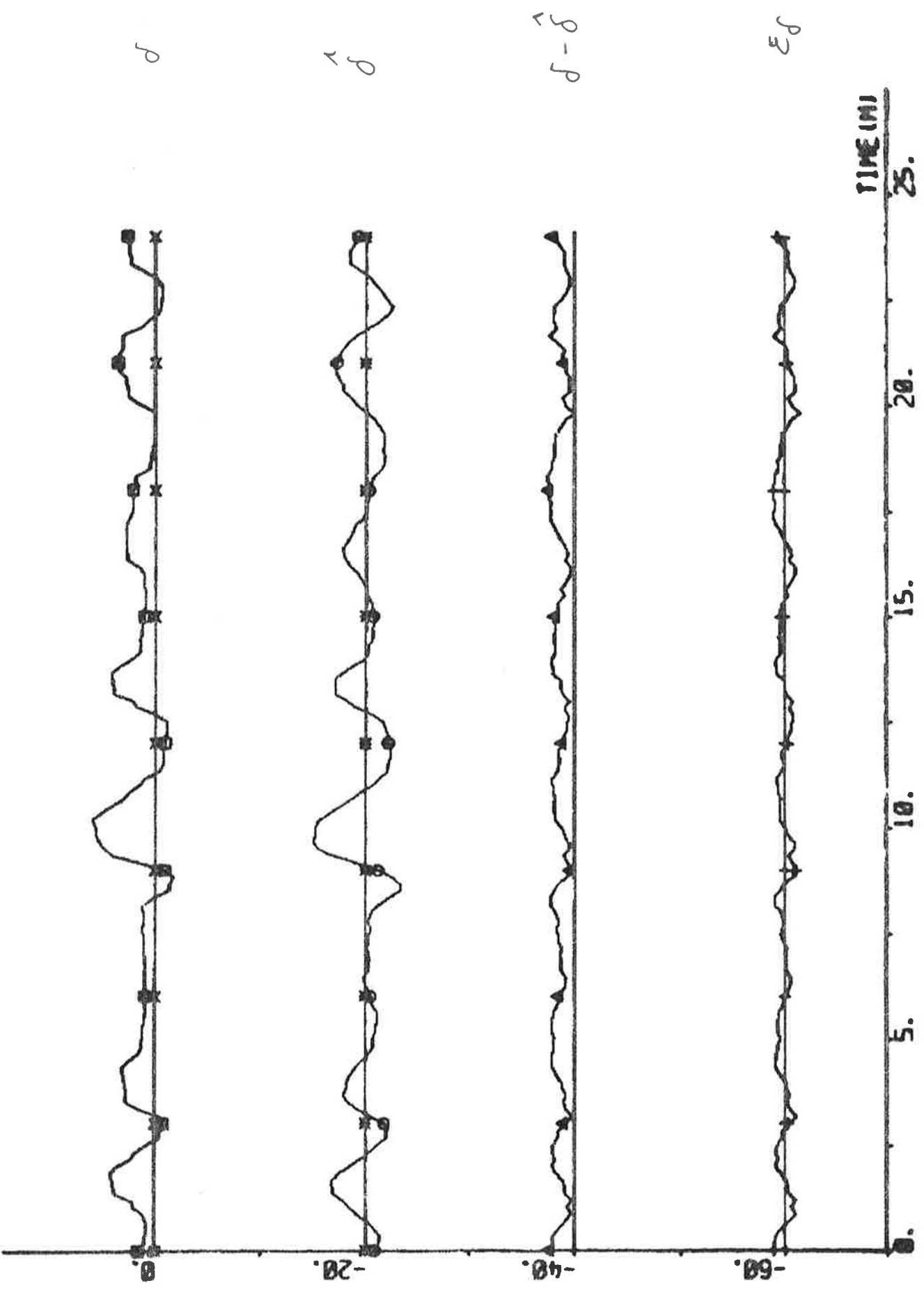


TIME (M)  
25.

PLOT A13P1(1)-A13P1(0) A13P1(9) ERR4 EPS4 00 01 02 03 -3.4 0.0 - DEG



PLOT A13P1(1)-A13P1(2) A13P1(3) ERR1 EPS1 00 020 040 060 -05 15 - DEG



PLOT A13P1(1)-A13P2(3) A13P2(4) A13P2(5) A13P2(6) Q0 Q2 Q4 Q6 -5.5 1.5

$0.5 \times \hat{\delta}_0$  deg



$2 \times \hat{d}_v$  knots



$100 \times \hat{d}_r$  deg/s



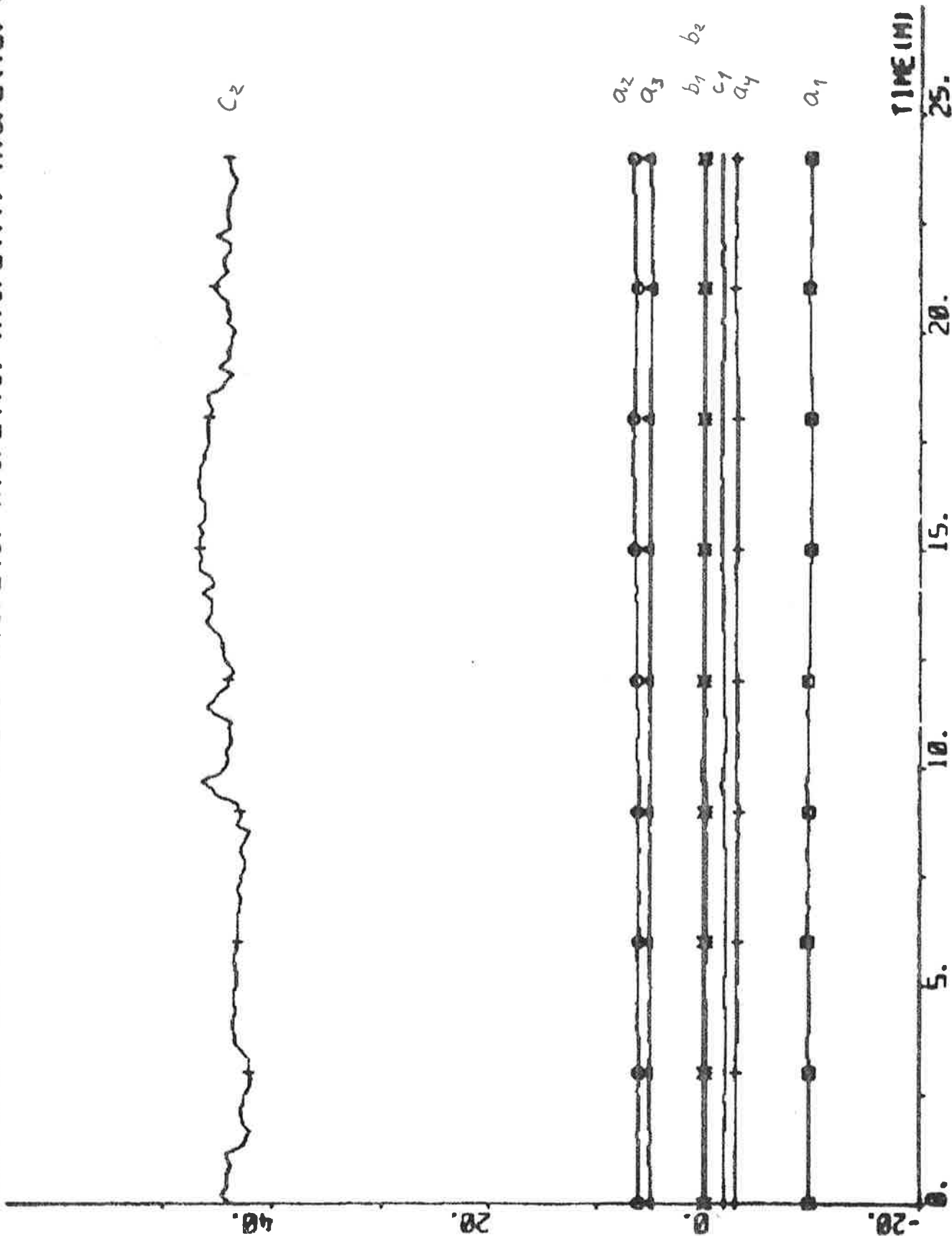
$0.5 \times \hat{d}_\theta$  deg



TIME (M)



PLOT A13P1(1) A13P2(7) A13P2(8) A13P2(9) A13P2(10) A13P2(11) A13P2(12) A1



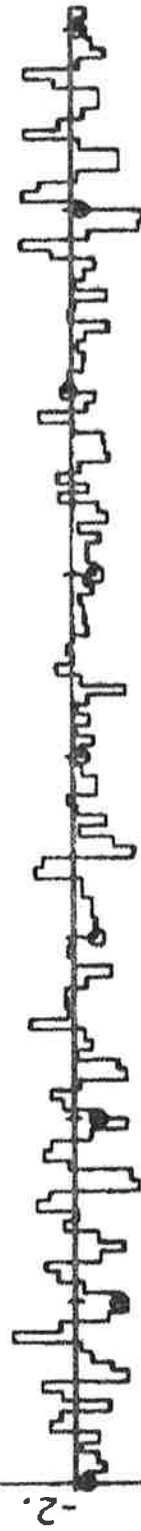




PLOT A14P1(1) - A14P1(6) HP A14P1(10) A14P1(15) Q2 -3 1 - DEC



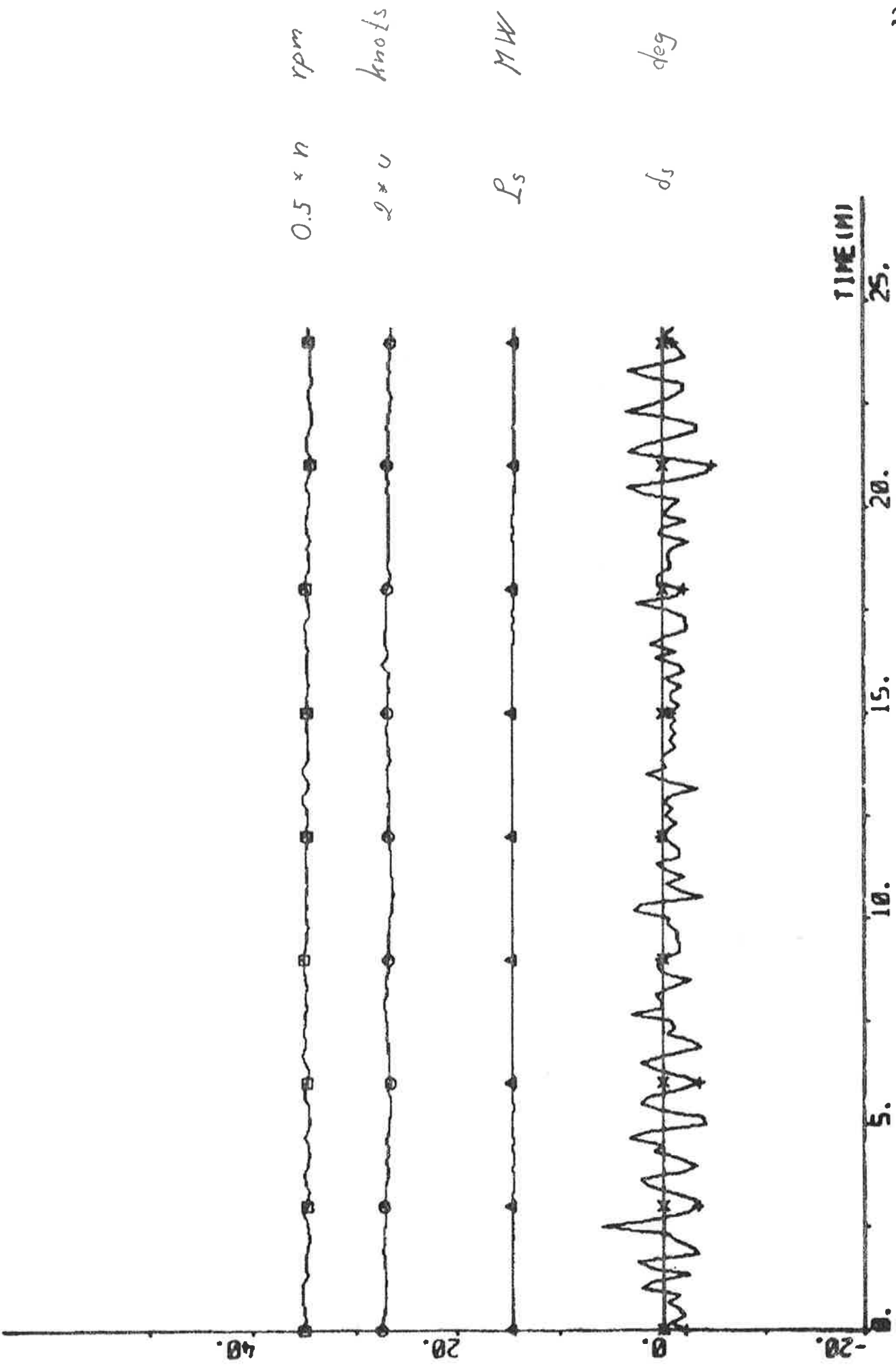
$\gamma - \gamma_{ref}$



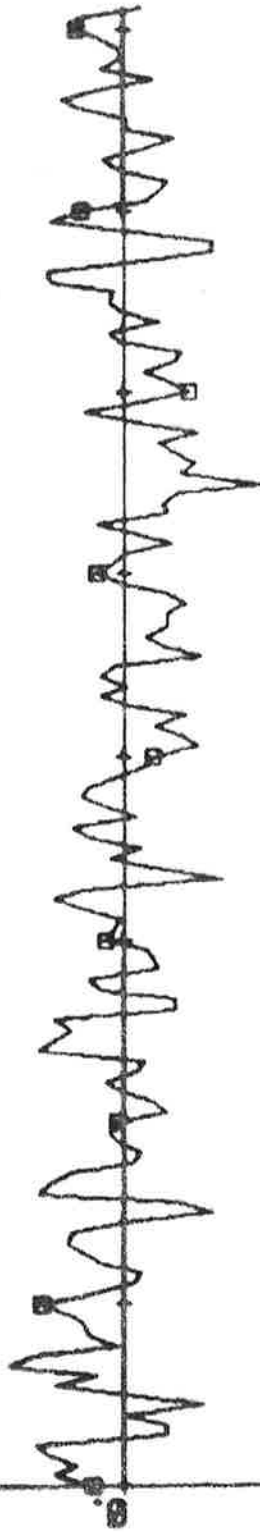
$0.05 \times d_c$

0  
-1  
-2  
TIME (H)  
0. 5. 10. 15. 20. 25.

PLOT A14P1(1)-A14P1(13) A14P1(12) A14P1(14) A14P1(11) 00 -20 50



PLOT A14P1(1)-A14P2(1) A14P2(2) ERRS Q0 Q1 Q2 -3.4 0.0 - KNOTS



TIME (M)  
25.

20.

15.

10.

5.

0.

PLOT A14P1(1)-A14P1(4) A14P1(5) ERR2 EPS2 00 01 02 03 -3.4 0.0 - KNOTS

$V_1$



$\hat{V}_1$



$V_1 - \hat{V}_1$

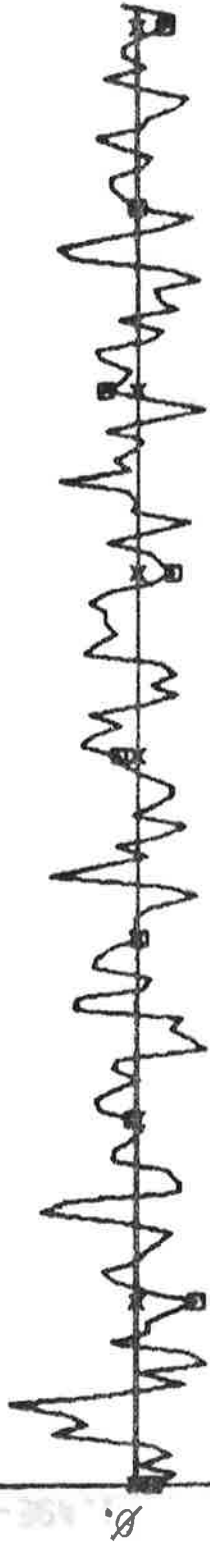


$\epsilon_V$



TIME (M)  
25.

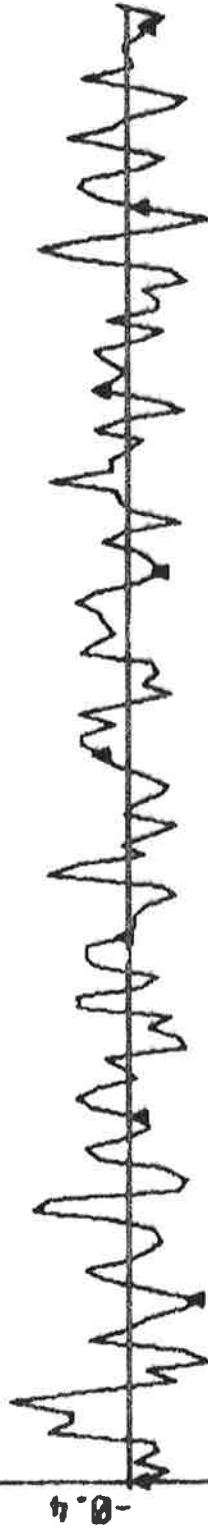
PLOT A14P1(1)-A14P1(6) A14P1(7) ERR3 EPS3 00 002 004 006 -0.7 0. - DEC-73



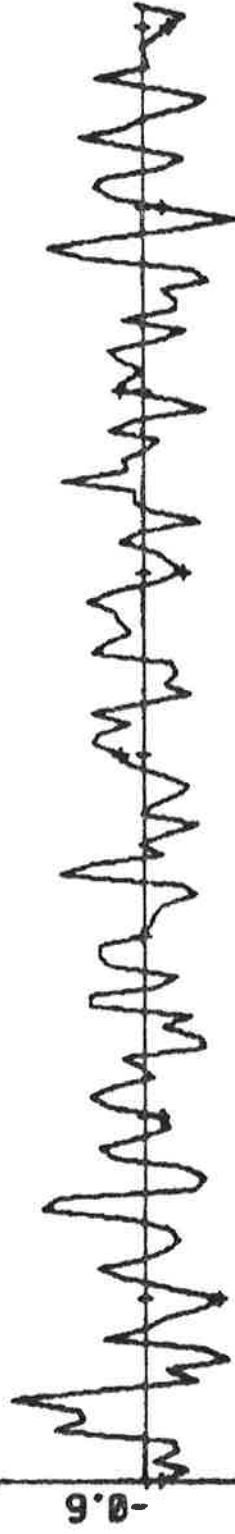
r



r



r-r



r

TIME (M)  
25.

PL0T A14P1(1)-A14P1(8) A14P1(9) ERR4 EPS4 00 01 02 03 -3.4 0.0 - DEC



$\gamma - \gamma_{ref}$



$\hat{\gamma} - \gamma_{ref}$



$\hat{\gamma} - \gamma$



$\epsilon \gamma$

TIME (MI)  
25.

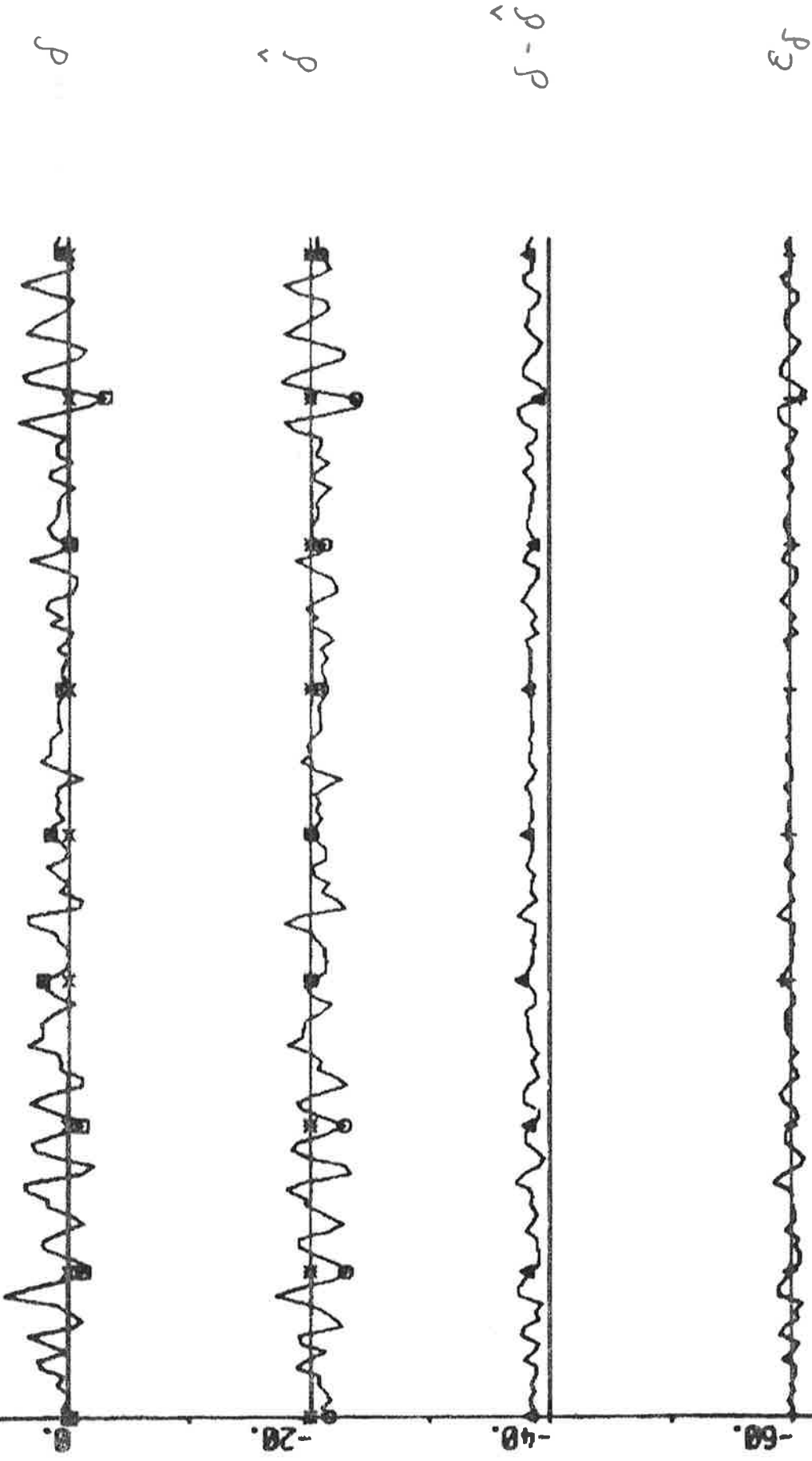
20.

15.

10.

5.

PLOT A14P1(1)-A14P1(2) A14P1(3) ERR1 EPS1 00 020 040 000 -65 15 - DEG



PLOT A14P1(1)-A14P2(3) A14P2(4) A14P2(5) A14P2(6) 00 02 04 06 -0.5 1.5

$0.5 \times \hat{d}_0$  deg

$\hat{v} \times d_r$  knots

$100 \times \hat{d}_r$  deg/s

$0.5 \times \hat{d}_\delta$  deg

TIME (M)

25.

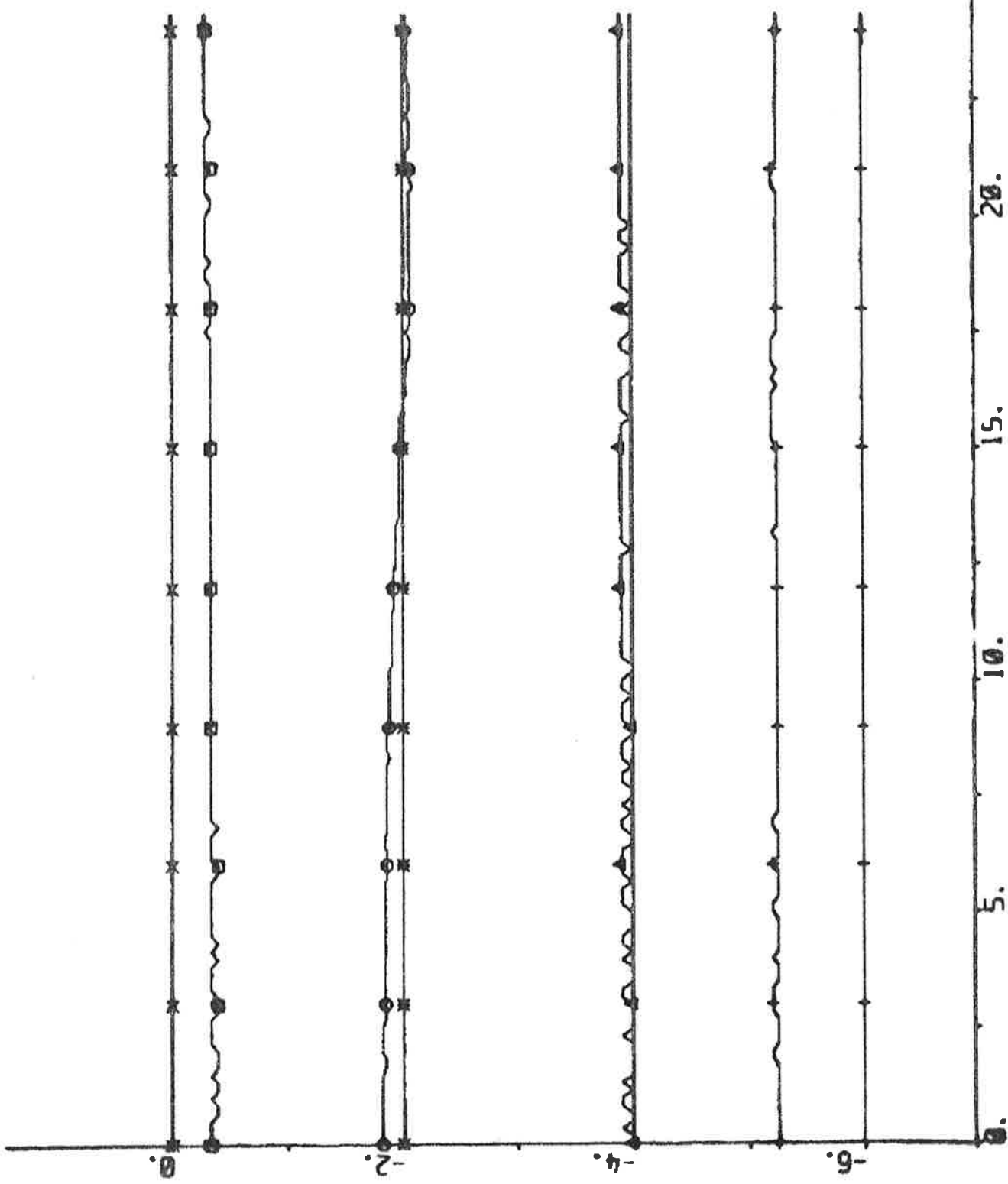
20.

15.

10.

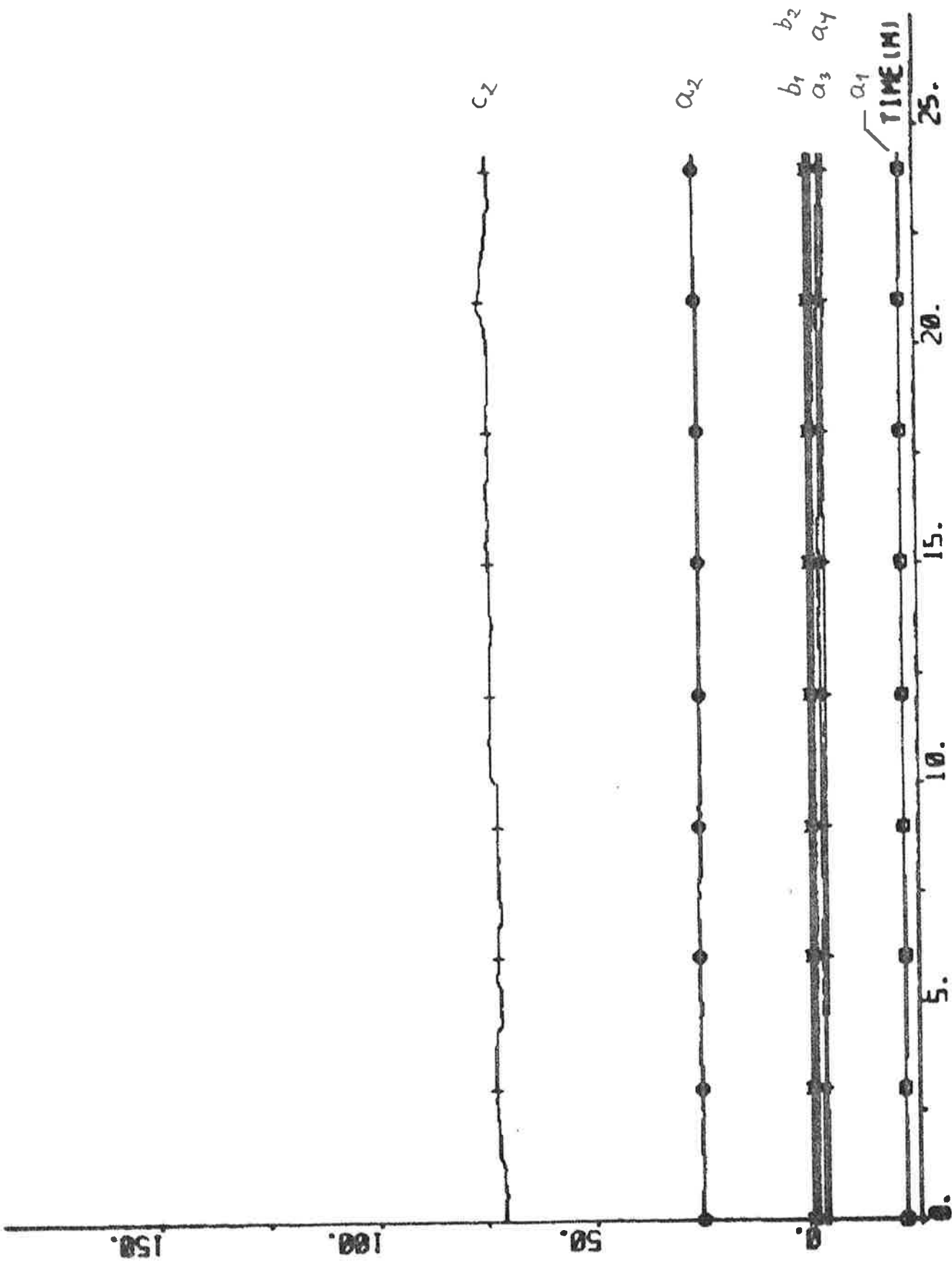
5.

0.





PLOT A14P1(1)-A14P2(7) A14P2(8) A14P2(9) A14P2(10) A14P2(11) A14P2(12) A1



## EXPERIMENT A15

Date	1976-04-24	Forward draught	8.5 m
Time	10.09	Aft draught	12.5 m
Duration	25 min	Wind direction	-
Position	N 02°45' W 12°57'	Wind velocity	0 m/s (calm)
$\psi_{\text{ref}}$	149 deg	Wave height	Swell

Self-tuning regulator using estimates from the Kalman filter.

Tuning time before the experiment started: 30 min.

NC1 = 1	NC2 = 0	k = 7	q = 0
$T_S = 10$ s	$V_0 = 6$ m/s	IVVC = 1	

Final values:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ b_1 \\ b_2 \\ c_1 \end{bmatrix} = \begin{bmatrix} -7.76 \\ 5.70 \\ 4.43 \\ -2.61 \\ 0.49 \\ 0.17 \\ -1.81 \end{bmatrix} \quad P = \begin{bmatrix} 3.74 & & & & & & & \\ -5.01 & 11.78 & & & & & & \\ 0.55 & -8.18 & 14.43 & & & & & \\ 1.11 & 1.15 & -7.13 & 5.49 & & & & \\ 0.08 & -0.39 & 0.41 & -0.09 & 0.03 & & & \\ -0.02 & 0.05 & -0.22 & 0.21 & 0.00 & 0.02 & & \\ -0.86 & 1.03 & 0.34 & -0.53 & -0.02 & 0.01 & 0.45 & \end{bmatrix}$$

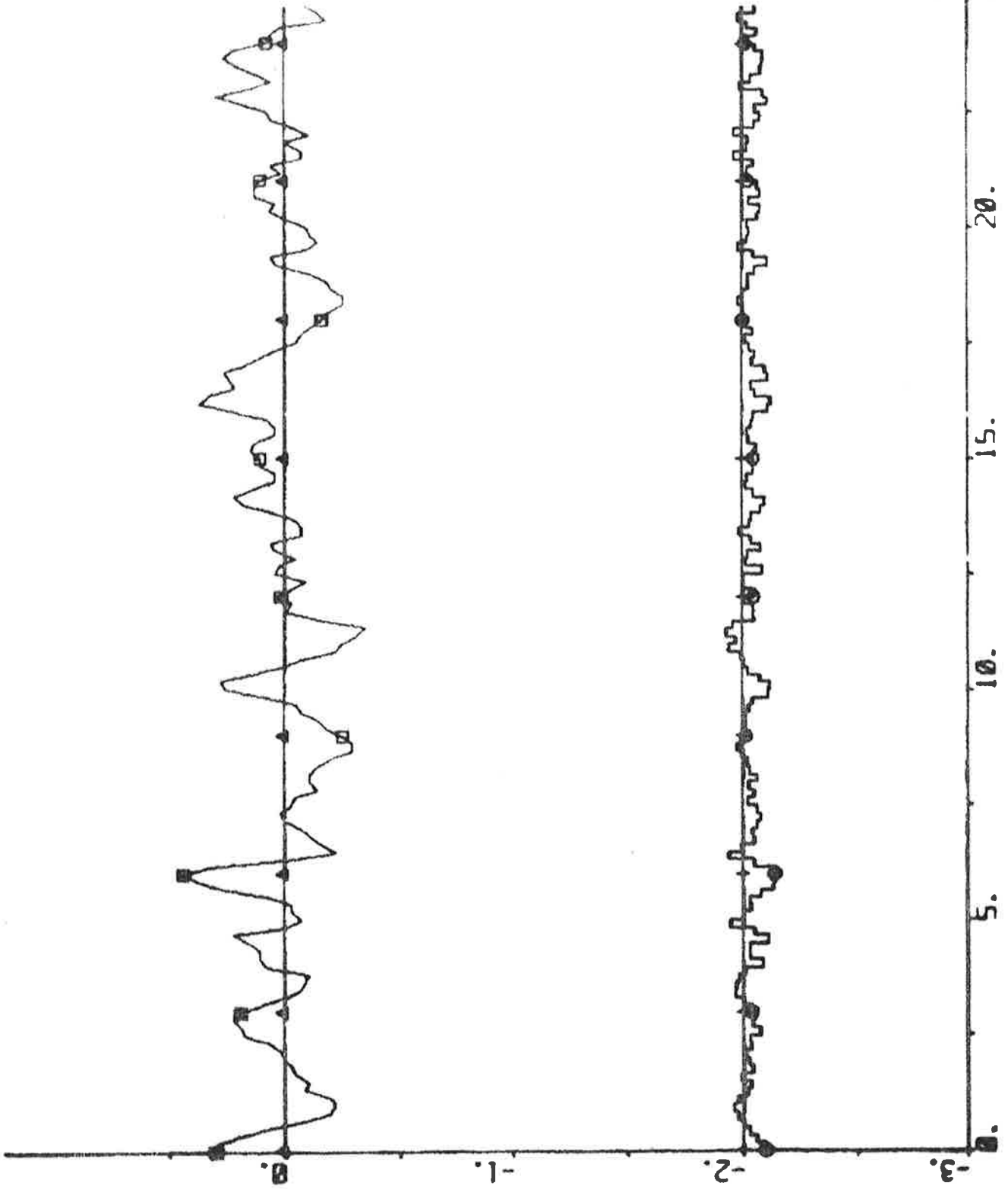
$$a_1 + a_2 + a_3 + a_4 = -0.24$$

$$\hat{\delta}_0 = -0.6 \text{ deg} \quad \hat{d}_V = 0.07 \text{ knots} \quad \hat{d}_r = 0.000 \text{ deg/s} \quad \hat{d}_\delta = 1.6 \text{ deg}$$

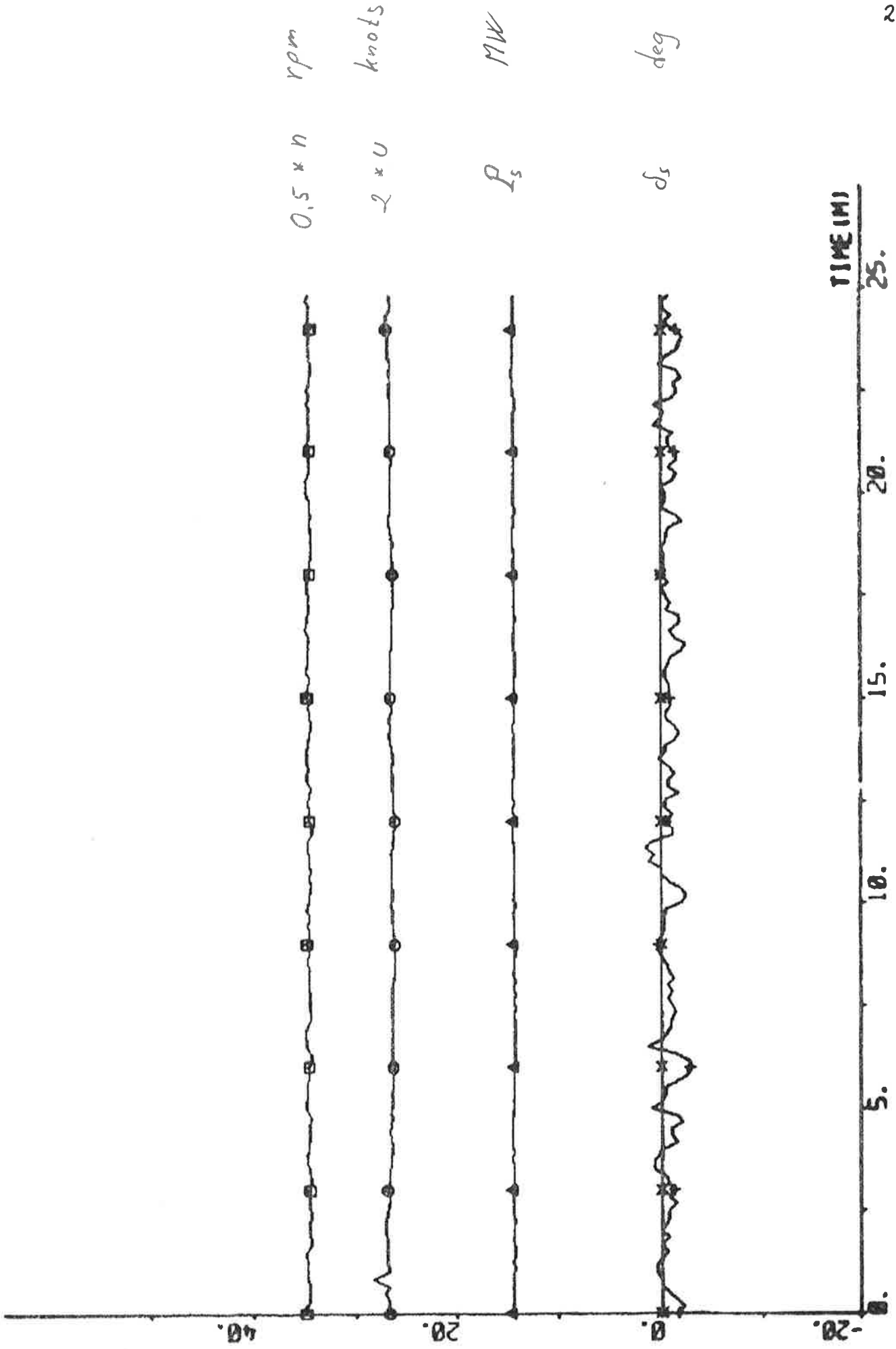
Statistics (mean value and standard deviation)

$\delta_c$	-0.68 ± 0.82 deg	$P_S$	14.4 ± 0.1 MW
$\delta$	0.88 ± 0.70 deg	$\epsilon_V$	0.00 ± 0.02 knots
$\psi - \psi_{\text{ref}}$	0.015 ± 0.158 deg	$\epsilon_r$	0.00 ± 0.03 deg/s
n	69.4 ± 0.3 rpm	$\epsilon_\psi$	0.00 ± 0.04 deg
u	13.3 ± 0.1 knots	$\epsilon_\delta$	0.0 ± 0.4 deg
$V_1 = 0.081$			

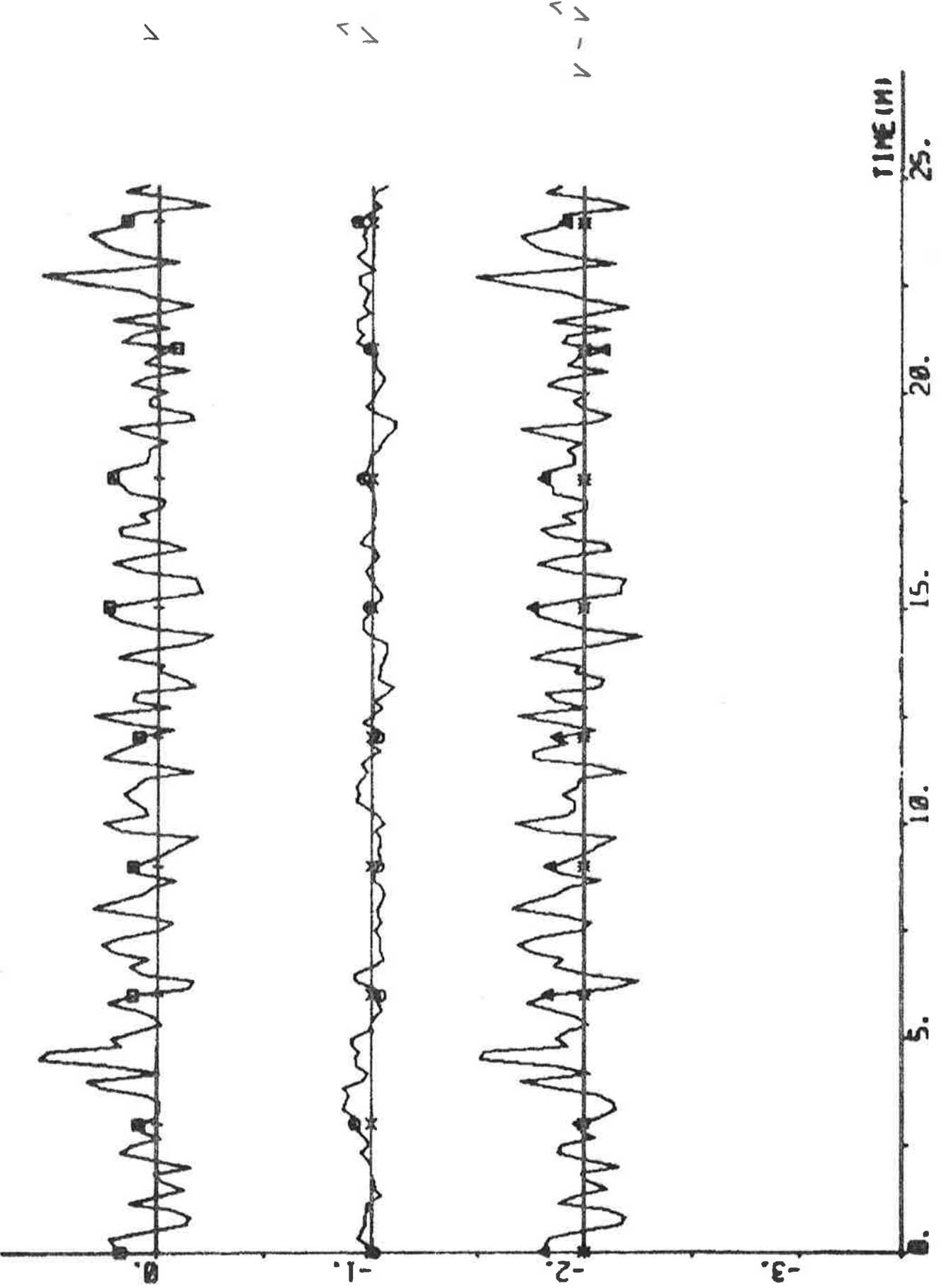
PLOT A15P1(1)-A15P1(8) HP A15P1(10) A15P1(15) Q2 -3 1 - DEG



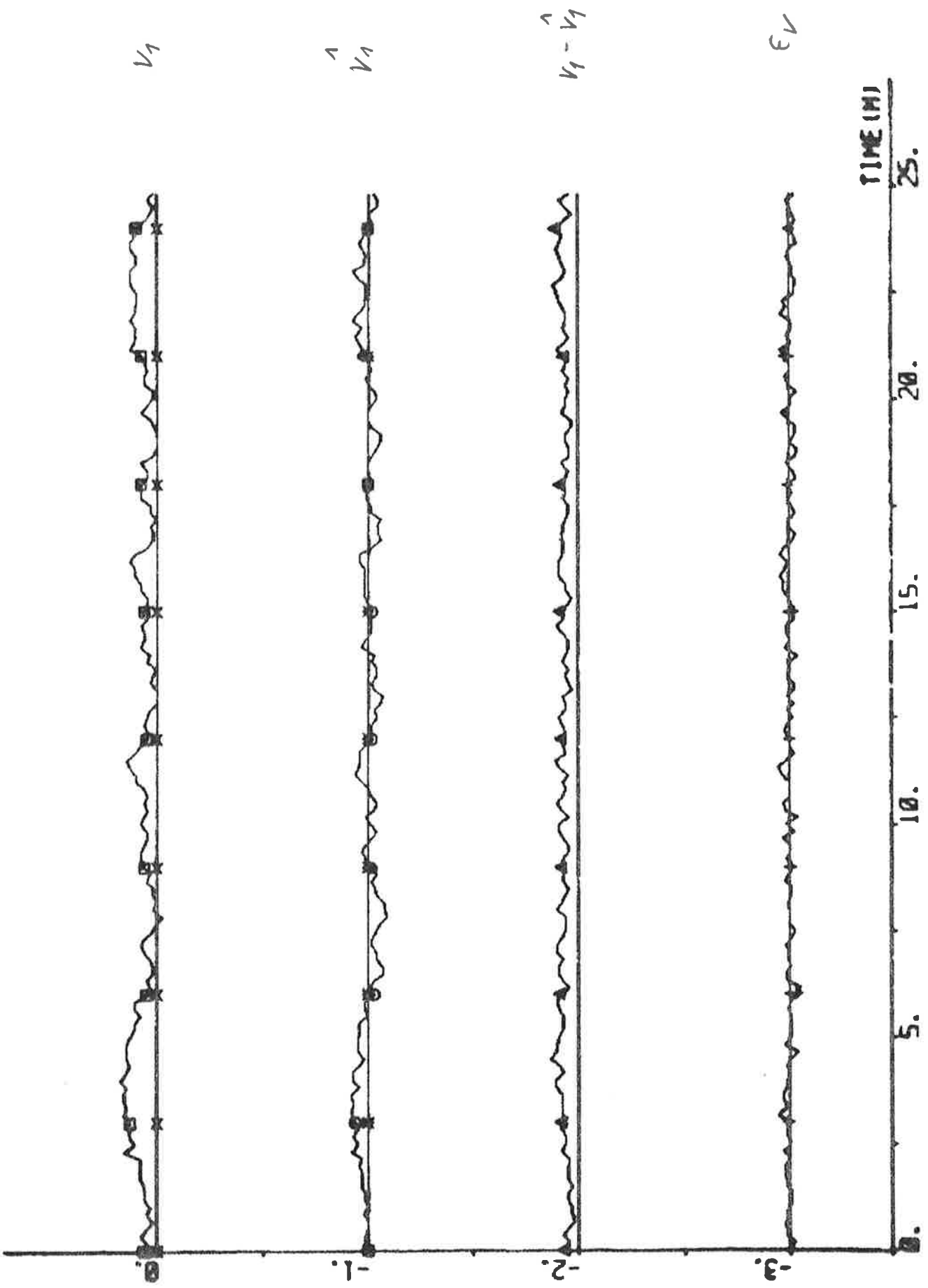
PLOT A16P1(1)-A16P1(13) A16P1(12) A16P1(14) A16P1(11) 00 -20 60



PLOT A1SP1(1)-A1SP2(1) A1SP2(2) ERRS 00 01 02 -3.4 0.0 - KNOTS



PLOT A1SP1(1)-A1SP1(4) A1SP1(5) ERR2 EPS2 00 01 02 03 -3.4 0.0 - KNOTS



THE SEA STRATUS EXPERIMENTS, APRIL 1976  
PART II

CLAES KÄLLSTRÖM

Lund Institute of Technology  
Department of Automatic Control  
September 1976

THE SEA STRATUS EXPERIMENTS, APRIL 1976

PART II

CLAES KÄLLSTRÖM



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## EXPERIMENT A22

Date	1976-04-25	Forward draught	8.5 m
Time	11.14	Aft draught	12.5 m
Duration	24 min	Wind direction	SE (1; see App. A)
Position	S 01°39' W 09°50'	Wind velocity	13 m/s (strong breeze)
$\psi_{ref}$	141 deg	Wave height	-

Self-tuning regulator using estimates from the Kalman filter

The rudder angle  $\delta$  was not used by the Kalman filter.

Tuning time before the experiment started: 30 min.

NC1 = 1            NC2 = 1            k = 7            q = 0

$T_s = 10$  s         $V_0 = 6$  m/s        IVVC = 1

Final values:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ b_1 \\ b_2 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} -8.04 \\ 6.23 \\ 4.52 \\ -2.99 \\ 0.51 \\ 0.16 \\ -1.89 \\ 35.94 \end{bmatrix} \quad p = \begin{bmatrix} 4.01 & & & & & & & & \\ -4.83 & 10.86 & & & & & & & \\ -0.07 & -6.92 & 13.00 & & & & & & \\ 1.10 & 0.75 & -6.22 & 4.74 & & & & & \\ -0.07 & -0.17 & 0.33 & -0.10 & 0.03 & & & & \\ -0.04 & -0.04 & -0.04 & 0.12 & 0.00 & 0.02 & & & \\ -0.51 & 0.67 & 0.17 & -0.32 & -0.01 & -0.01 & 0.81 & & \\ 7.52 & -9.92 & -5.56 & 8.39 & 0.02 & -0.21 & 15.47 & 744.94 & \end{bmatrix}$$

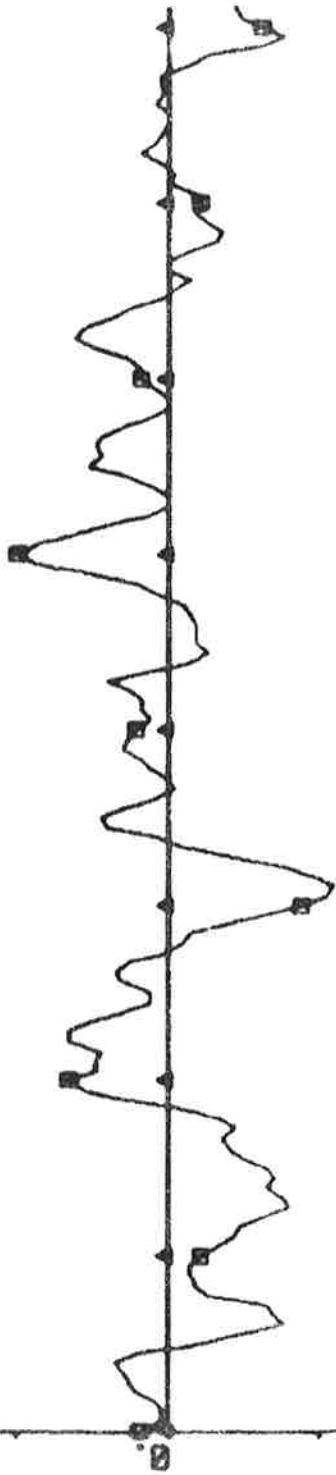
$$a_1 + a_2 + a_3 + a_4 = -0.28$$

$$\hat{\delta}_0 = 1.8 \text{ deg} \quad \hat{d}_v = 0.27 \text{ knots} \quad \hat{d}_r = 0.000 \text{ deg/s} \quad \hat{d}_\delta = -$$

Statistics (mean value and standard deviation)

$\delta_c$	2.32	± 0.95	deg	$P_S$	14.5	± 0.1	MW
$\delta$	3.82	± 0.80	deg	$\varepsilon_v$	0.00	± 0.02	knots
$\psi - \psi_{ref}$	-0.006	± 0.204	deg	$\varepsilon_r$	0.00	± 0.03	deg/s
n	68.9	± 0.3	rpm	$\varepsilon_\psi$	-0.01	± 0.03	deg
u	12.8	± 0.2	knots	$\varepsilon_\delta$	-		
$V_1 = 0.117$							

PLOT A22P1(1)-A22P1(8) HP A22P1(10) A22P1(15) Q2 -3 1 - DEG



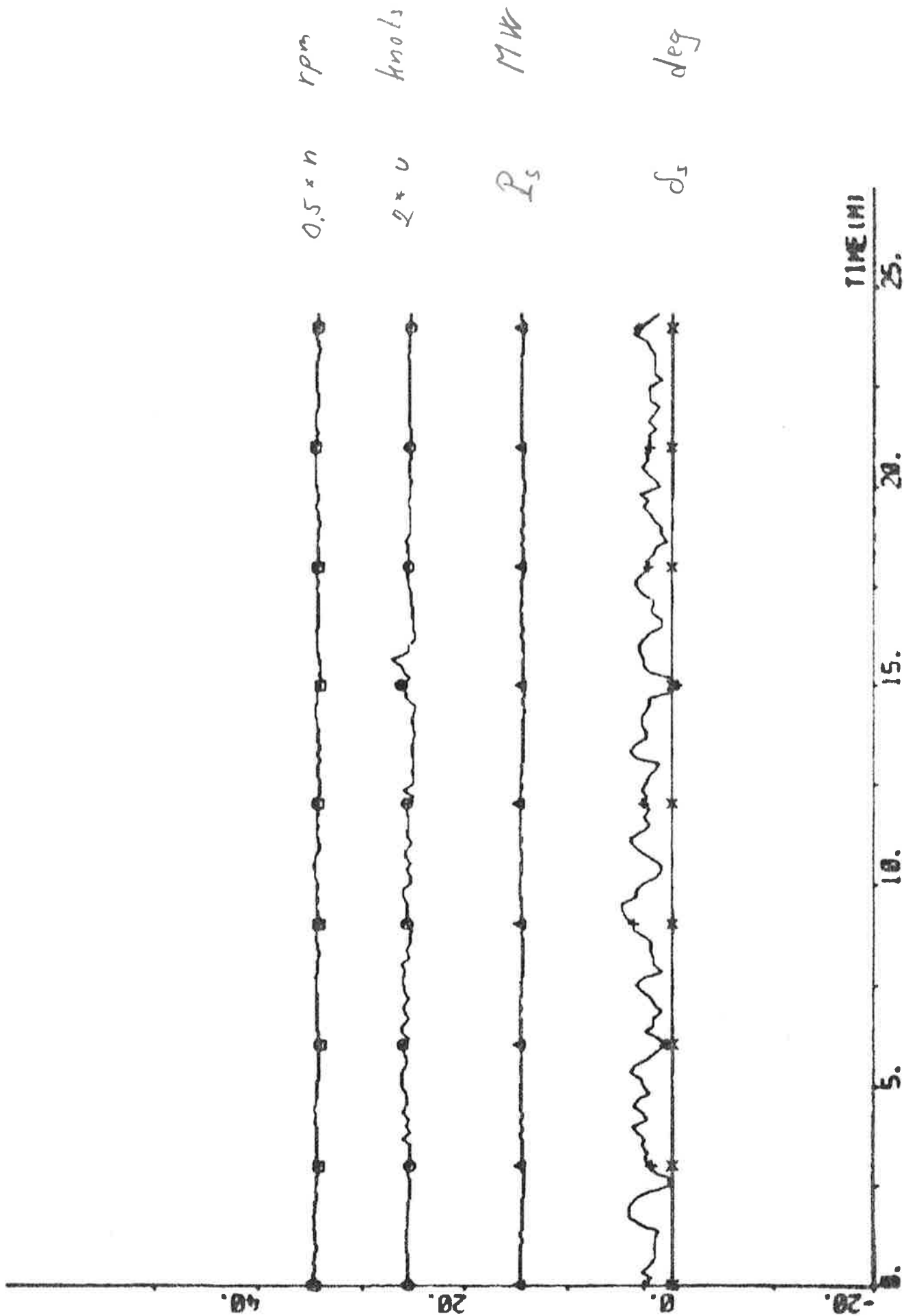
$\gamma - \gamma_{ref}$



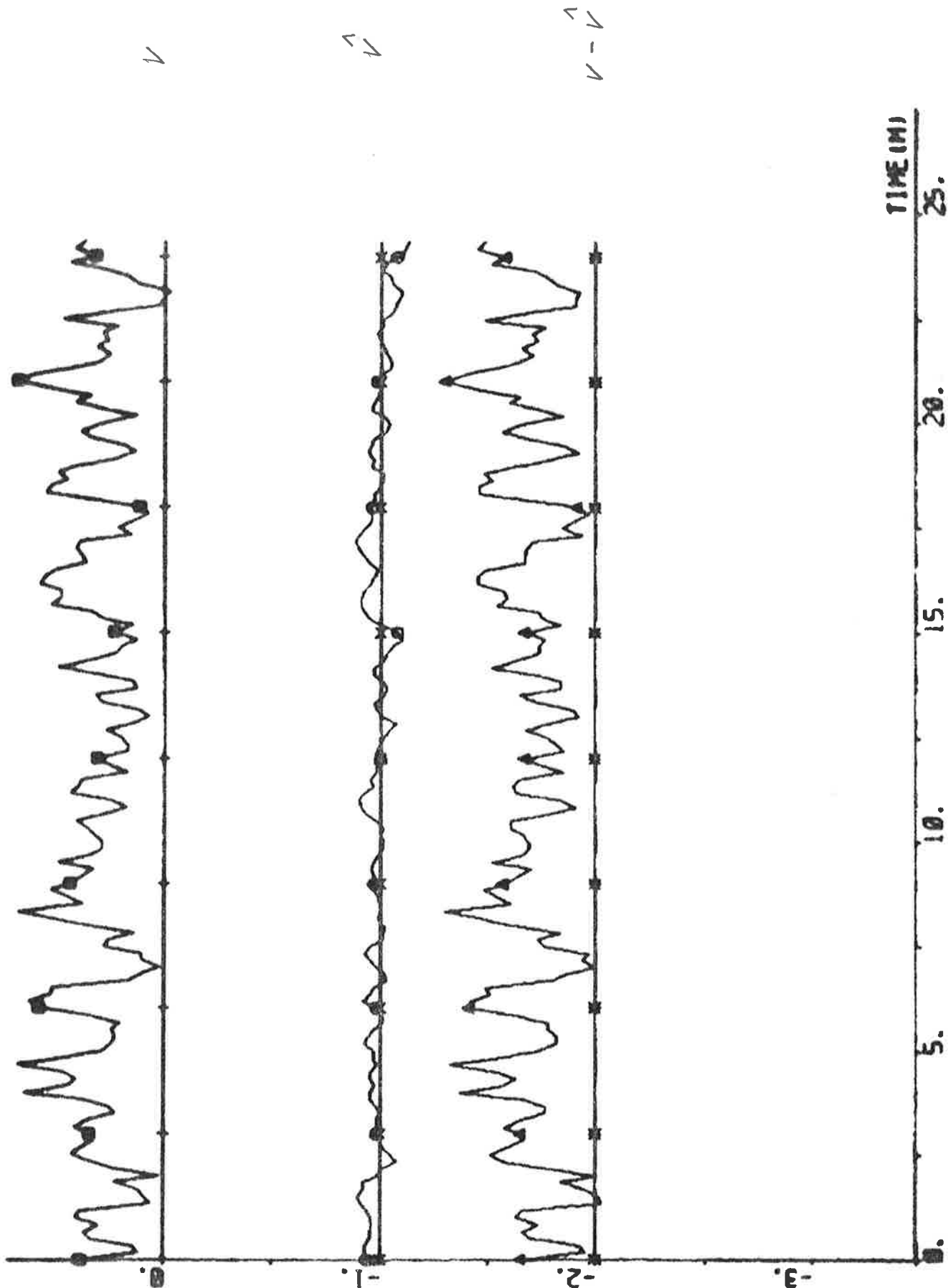
$0.05 \times d_c$

TIME (M) 25. 20. 15. 10. 5.

PLOT A22P1(1)-A22P1(13) A22P1(12) A22P1(14) A22P1(11) 00 -20 50

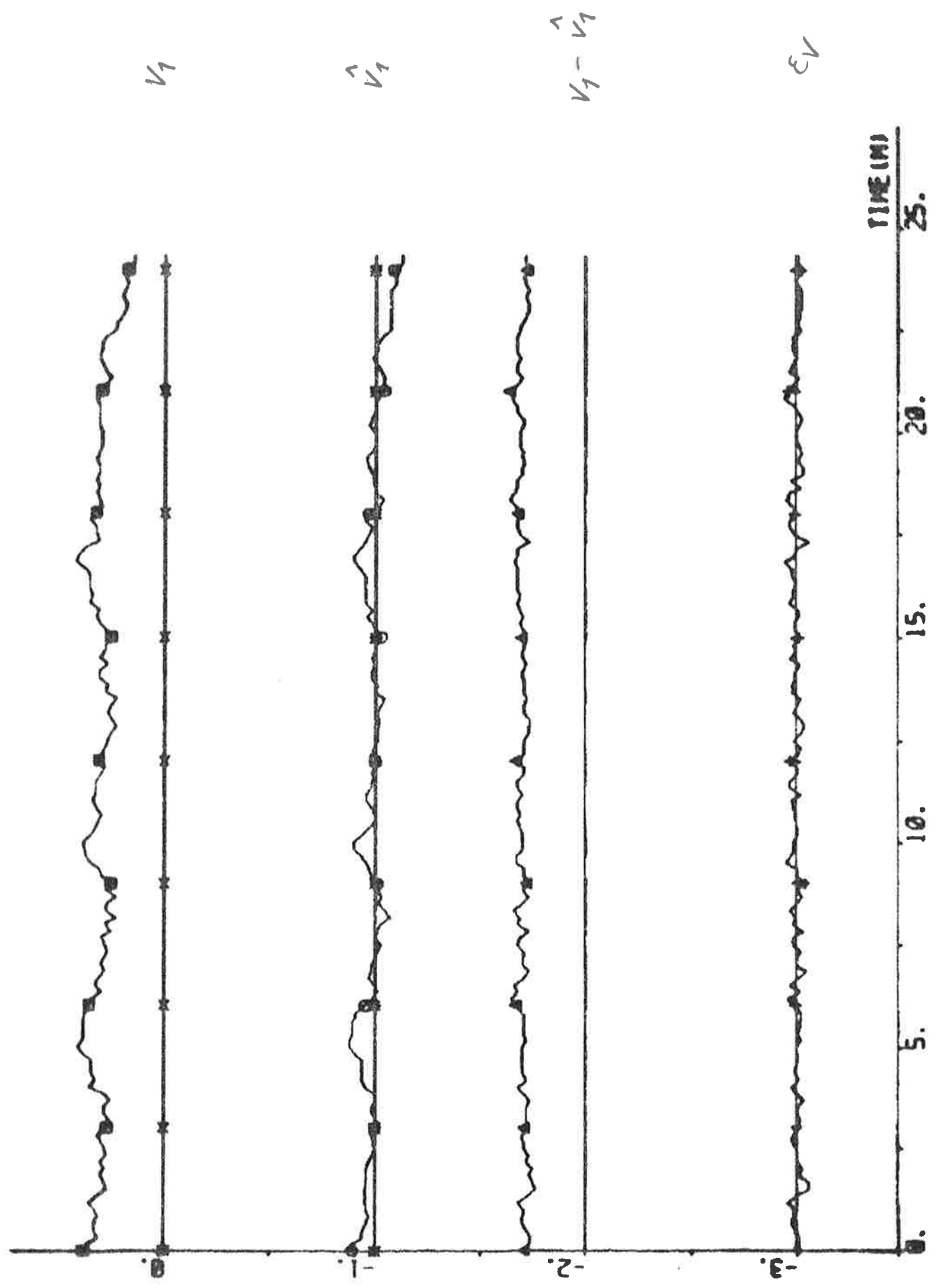


PL0T A22P1(1)-A22P2(1) A22P2(2) ERRS 00 01 02 -3.4 0.0 - KNOTS



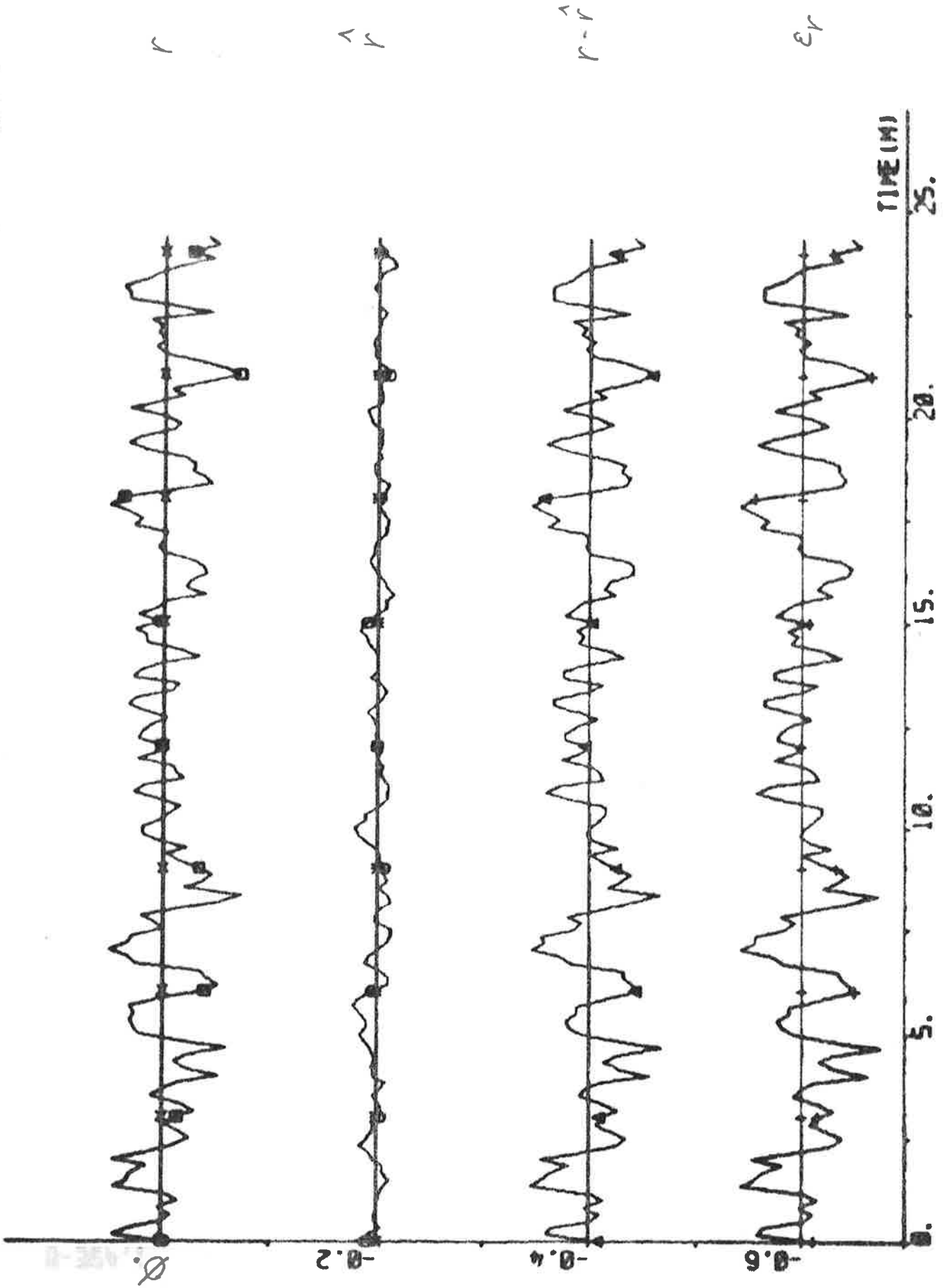
TIME (M)  
25.  
20.  
15.  
10.  
5.

PLOT A22P1(1)-A22P1(4) A22P1(5) ERR2 EPS2 00 01 02 03 -3.4 0.6 - KNOTS

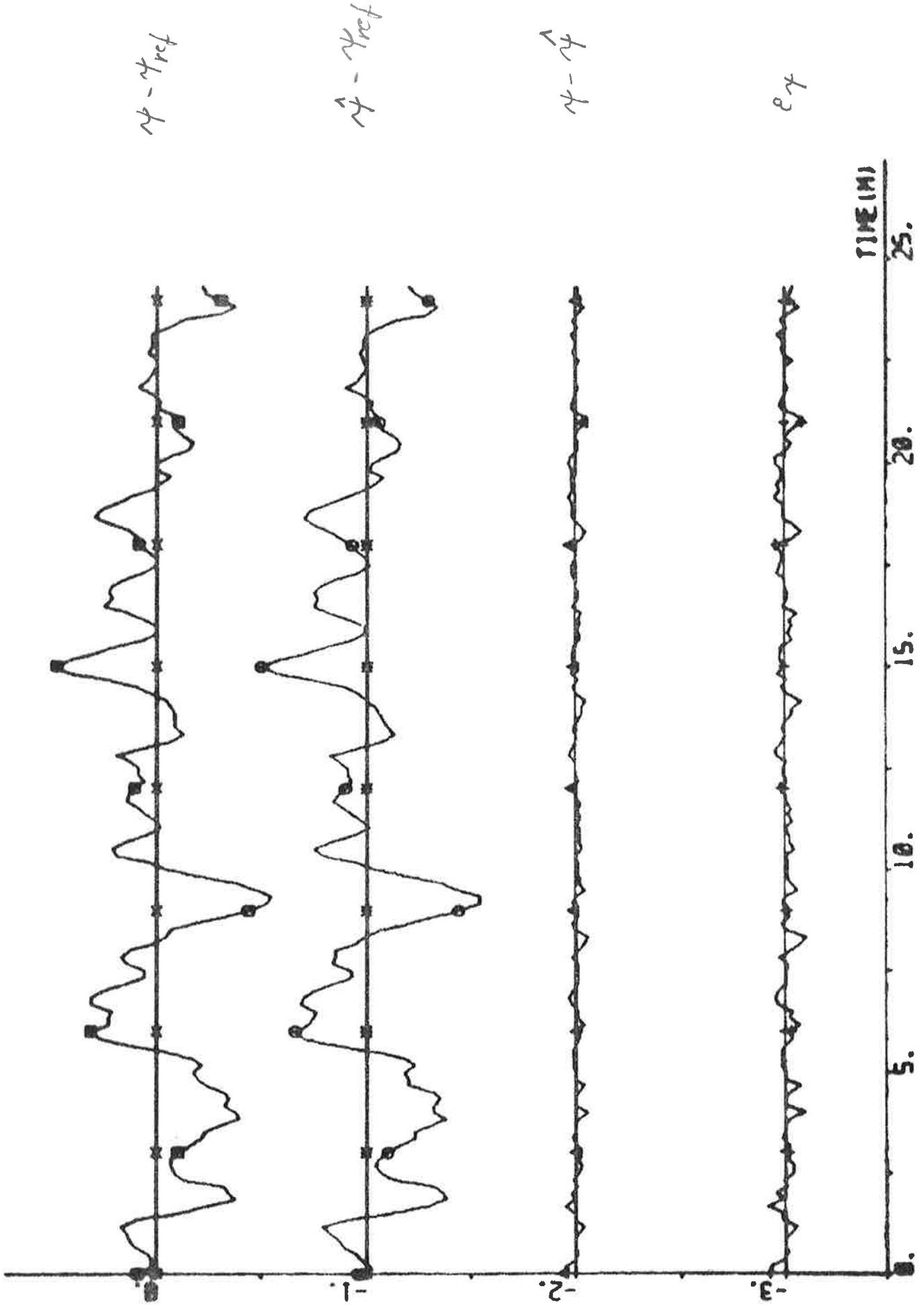




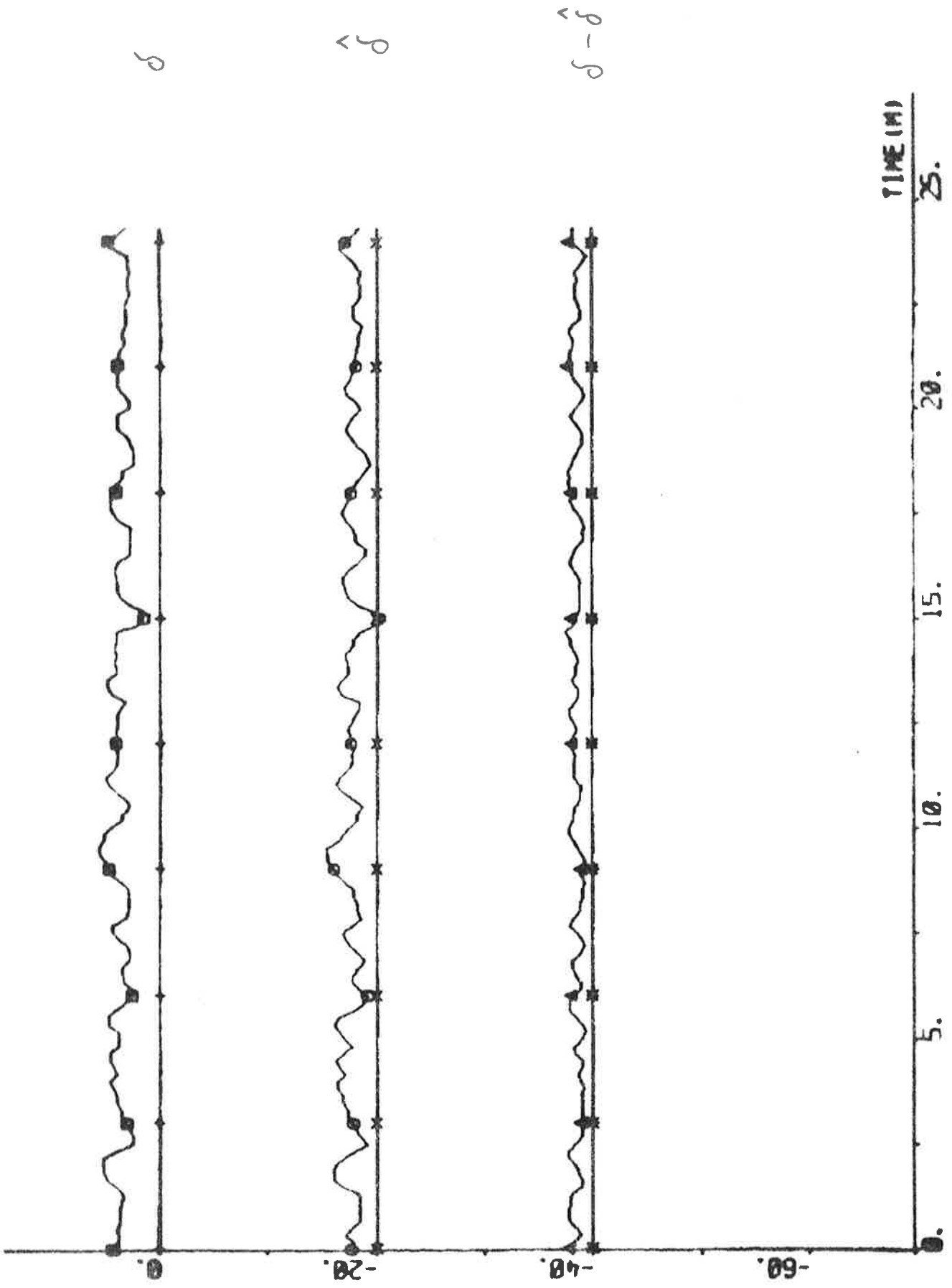
PLOT A22P1(1)-A22P1(6) A22P1(7) ERR3 EPS3 00 002 004 006 -0.7 0. - 000-3



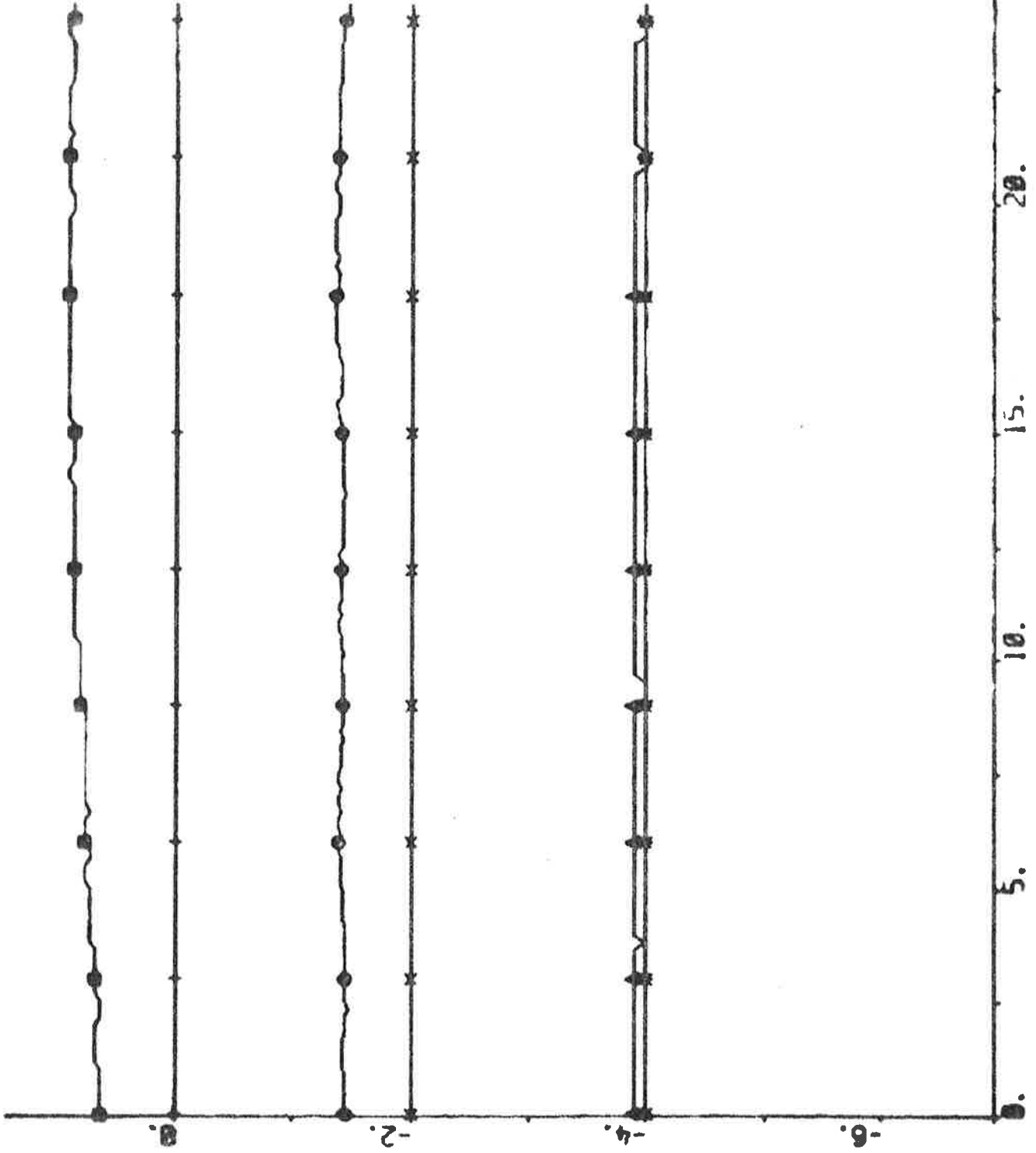
PLOT A22P1(1)-A22P1(8) A22P1(8) ERR4 EPS4 00 01 02 03 -3.4 0.0 - 003



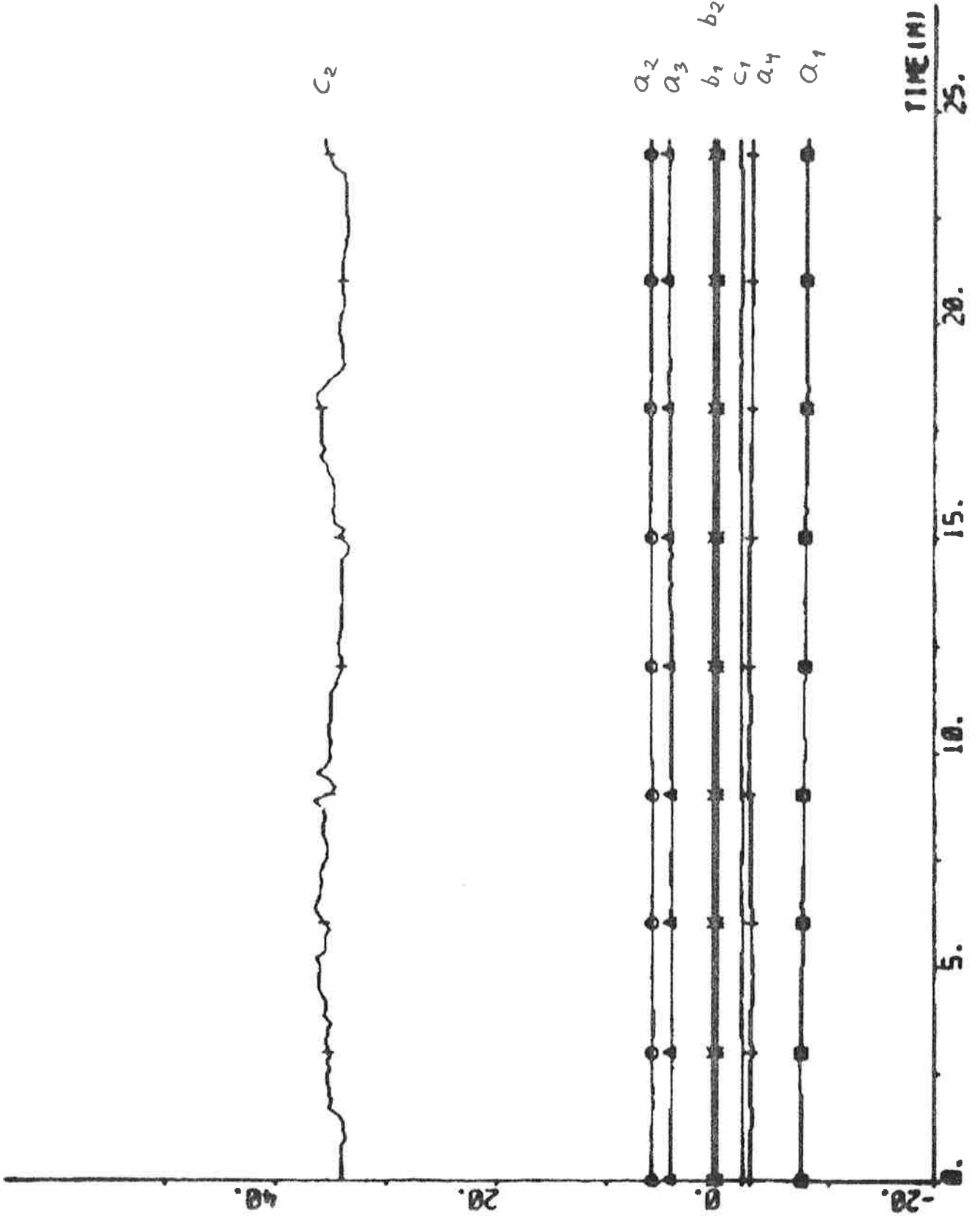
PLOT A22P1(1)-A22P1(2) A22P1(3) ERR1 00 020 040 -66 16 - DEG



PLOT A22P1(1)-A22P2(3) A22P2(4) A22P2(5) 00 02 04 -0.5 1.5

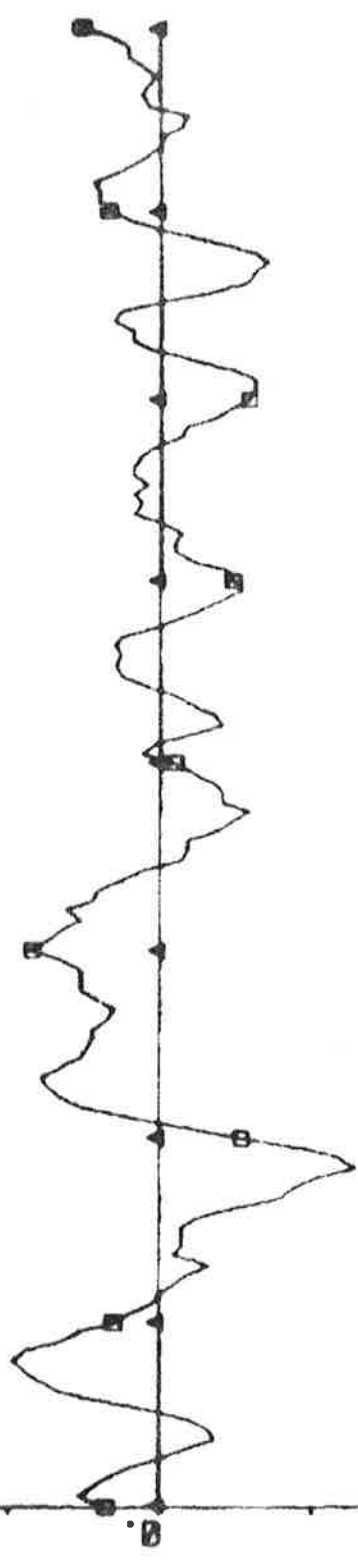


PLOT A22P1(1)-A22P2(7) A22P2(8) A22P2(9) A22P2(10) A22P2(11) A22P2(12) A2





PLOT A23P1(1)-A23P1(8) HP A23P1(10) A23P1(15) 02 -3 1 - DEC



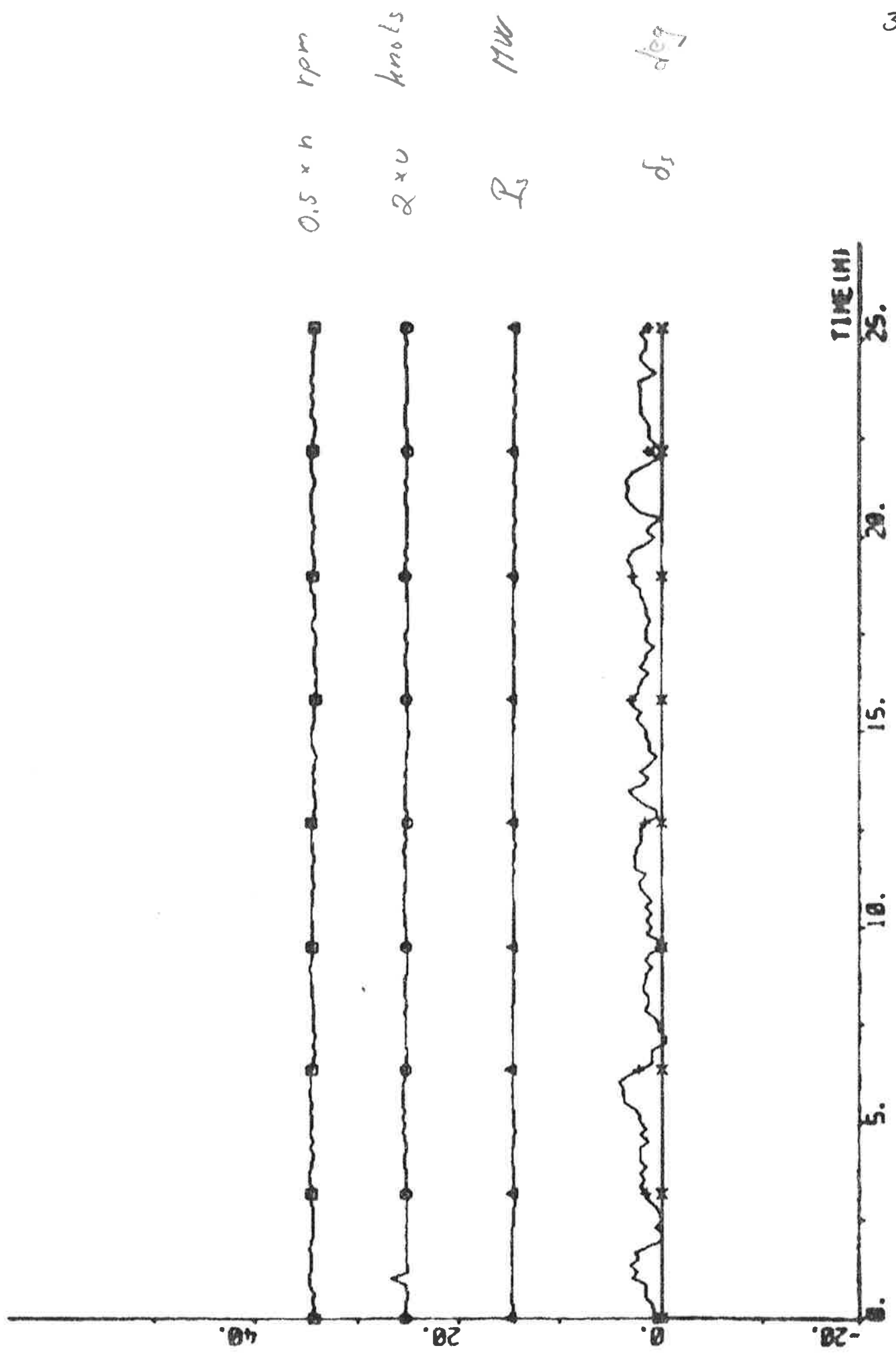
$y - y_{ref}$



$0.05 \times d_c$

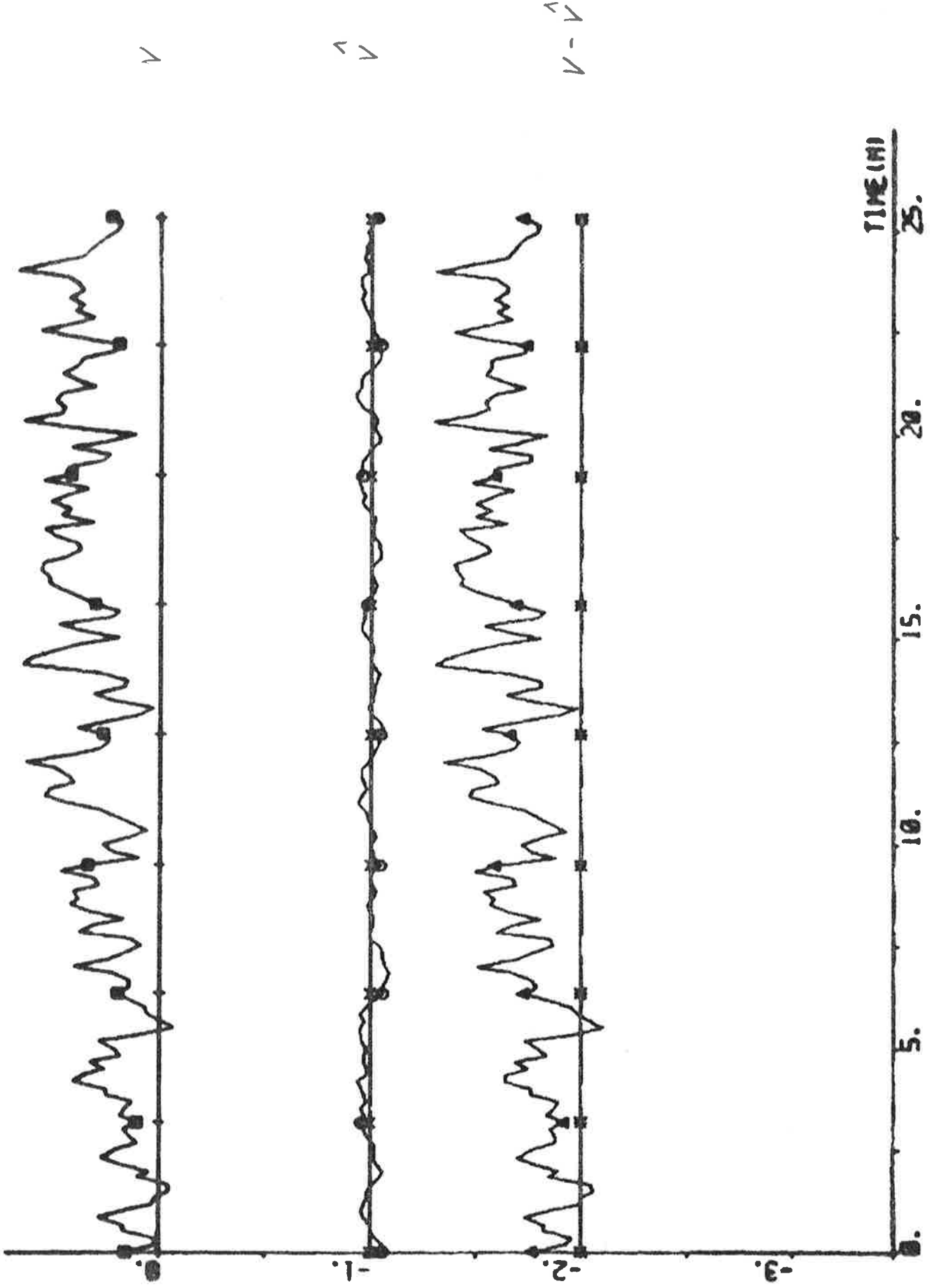
TIME (M)  
5. 10. 15. 20. 25.

PL0T A23P1(1)-A23P1(13) A23P1(12) A23P1(14) A23P1(11) 00 -30 00

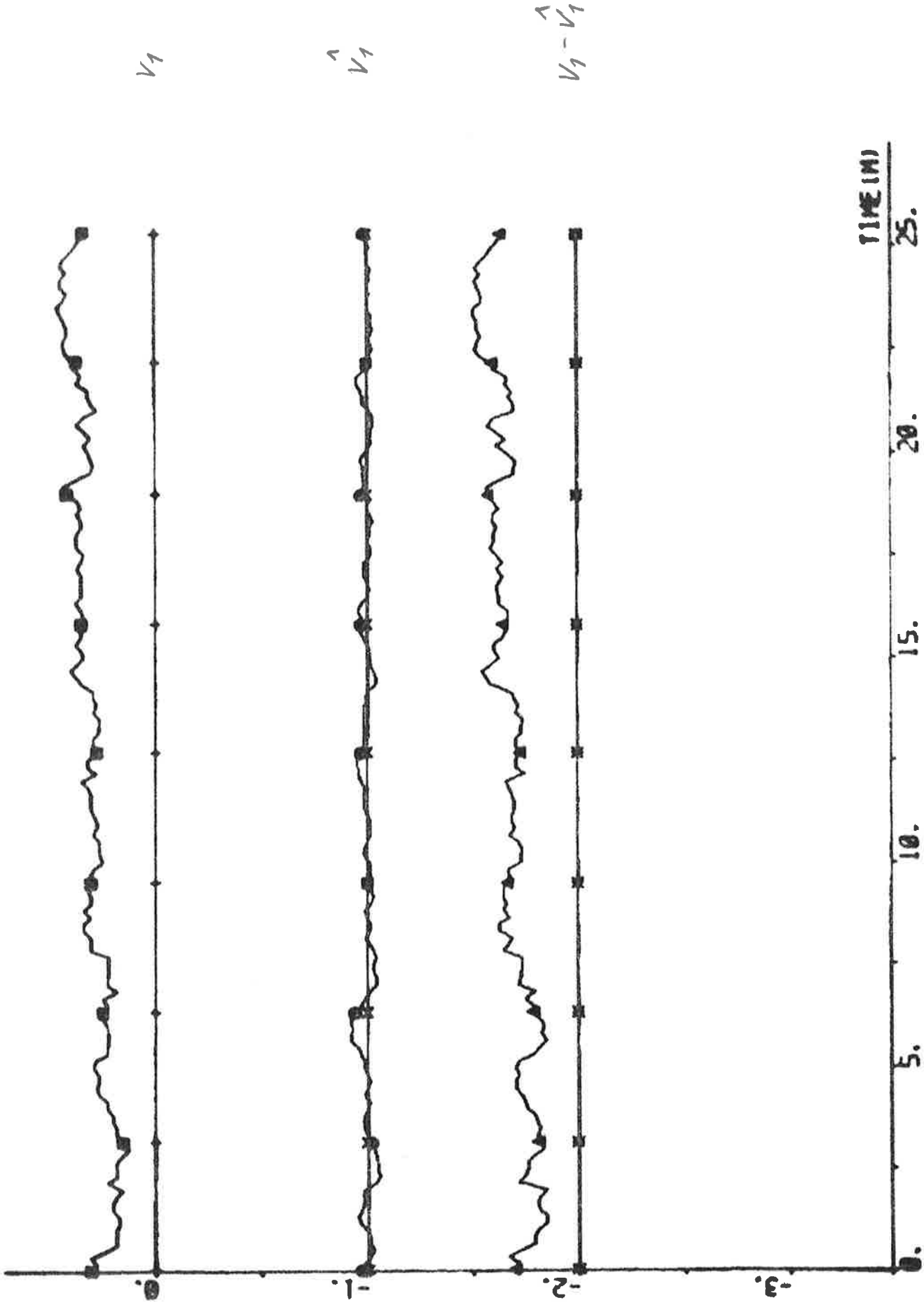




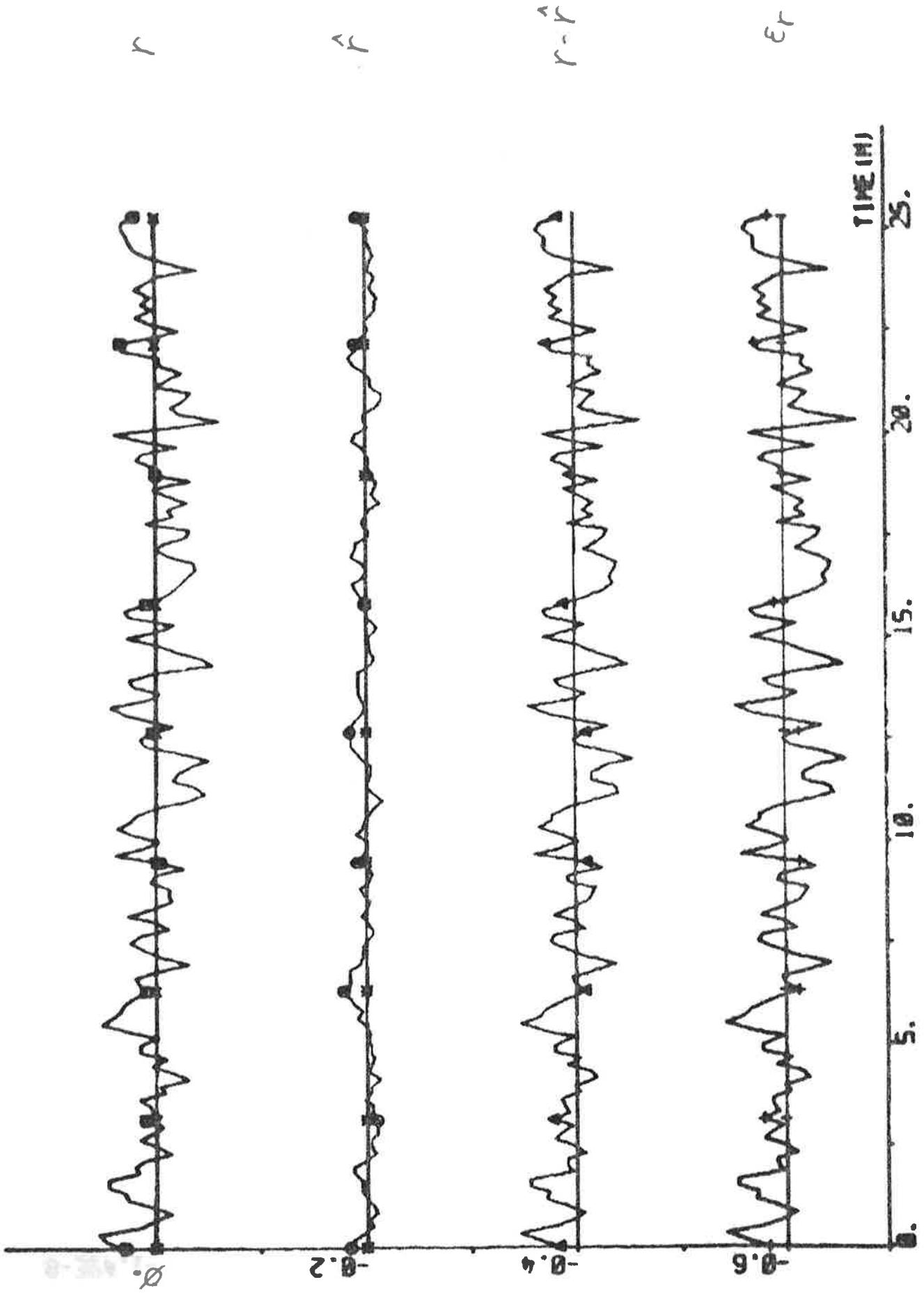
PLOT A23P1(1)-A23P2(1) A23P2(2) ERAS 00 01 02 -3.4 0.0 · KNOTS



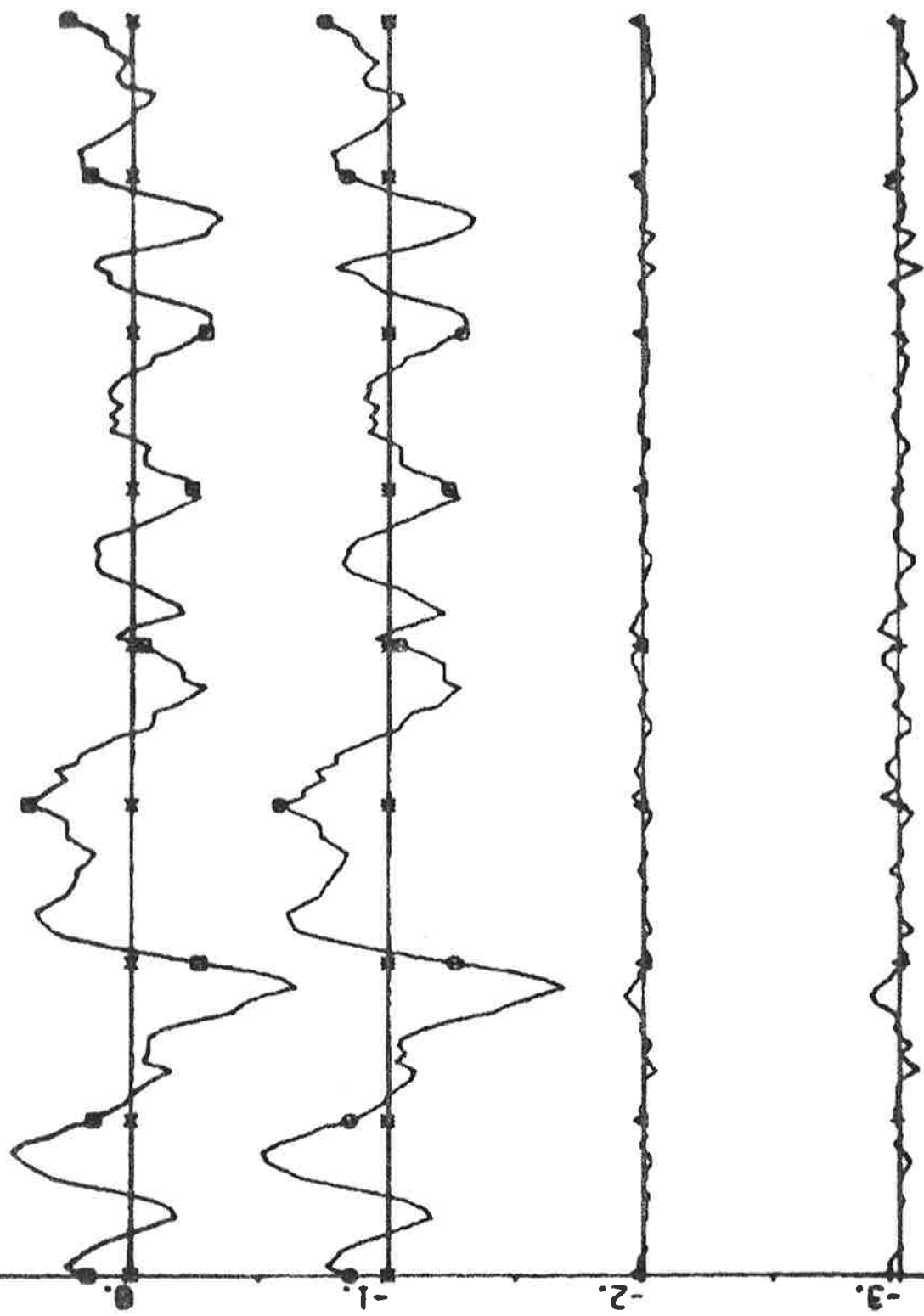
PLOT A23P1(1)-A23P1(4) A23P1(5) ERR2 00 01 02 -3.4 0.0 - KNOTS



PLOT A23P1(1)-A23P1(6) A23P1(7) ERR3 EPS3 00 002 004 008 -0.7 0.0 000000



PLOT A23P1(1)-A23P1(8) A23P1(9) ERR4 EPS4 00 01 02 03 -3.4 0.0 - 003



$\gamma - \gamma_{ref}$

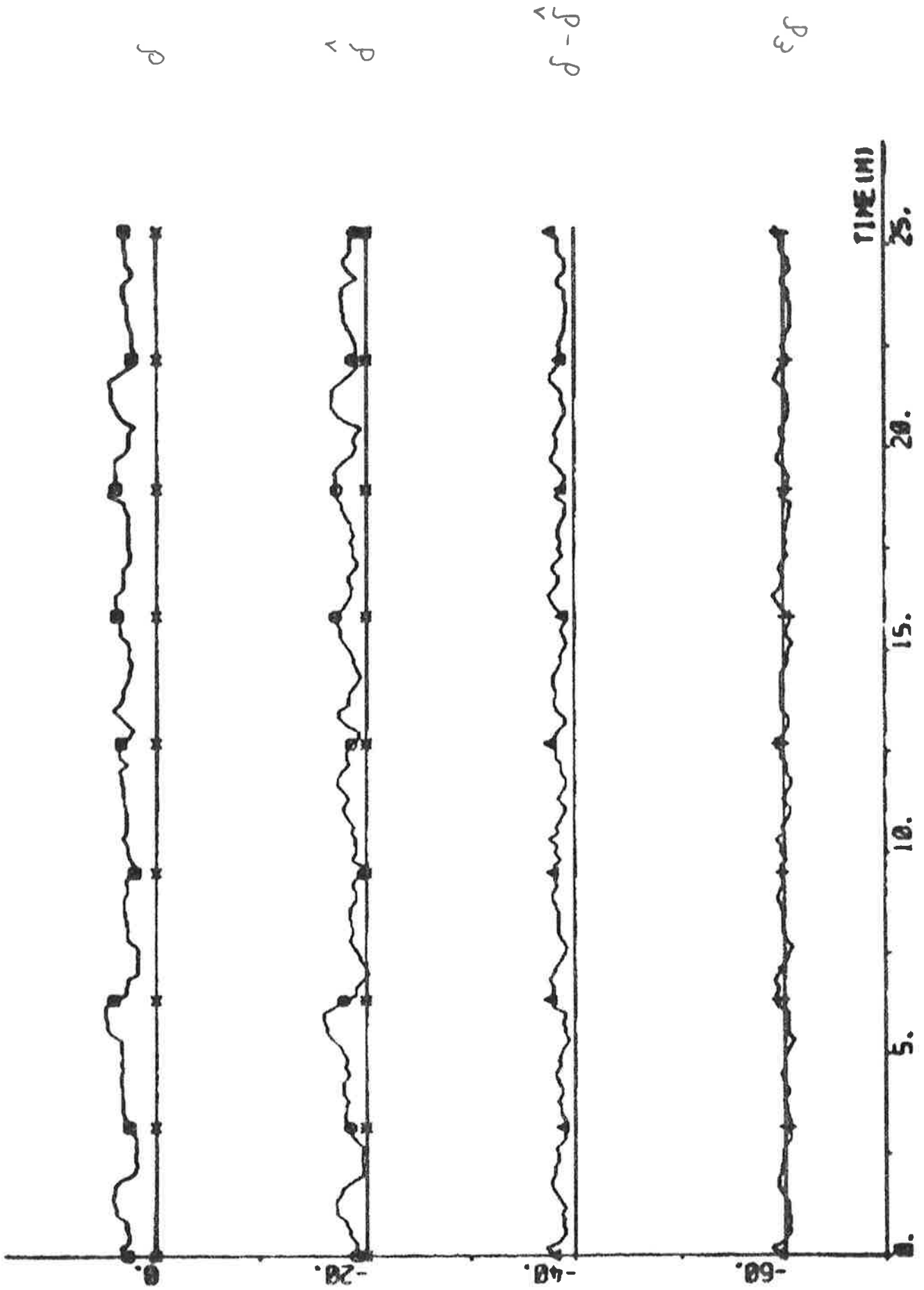
$\gamma - \gamma_{ref}$

$\gamma - \gamma^1$

$E_{\gamma}$

TIME (M)  
25.

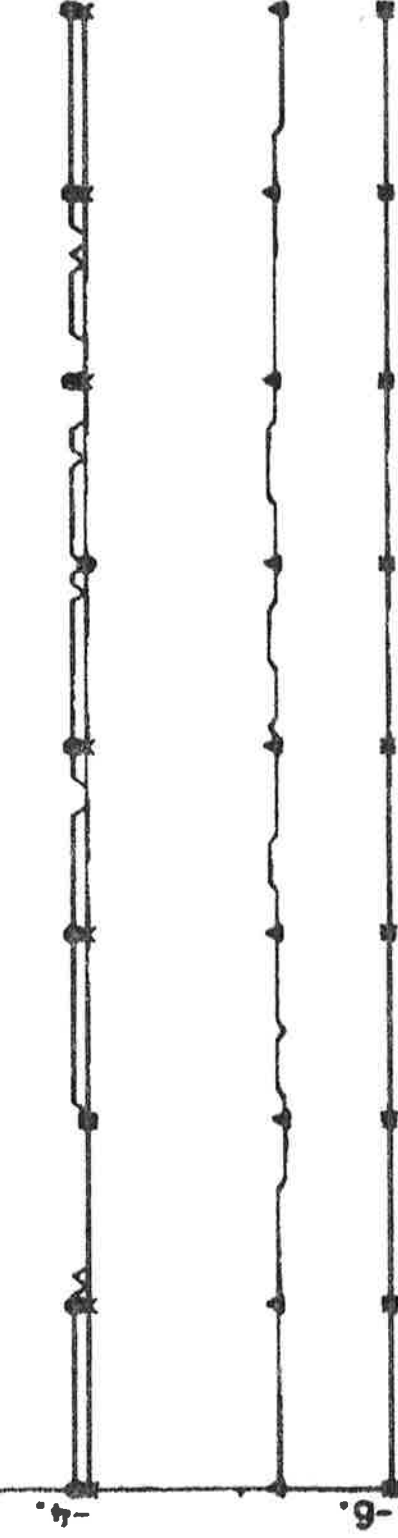
PLOT A23P1(1)-A23P1(2) A23P1(3) ERR1 EPS1 00 020 040 060 -08 10 - 0000



PLOT A23P1(1)-A23P2(3) A23P2(5) A23P2(6) 00 04 08 -6.5 1.5



$0.5 \times \hat{d}_0$  deg

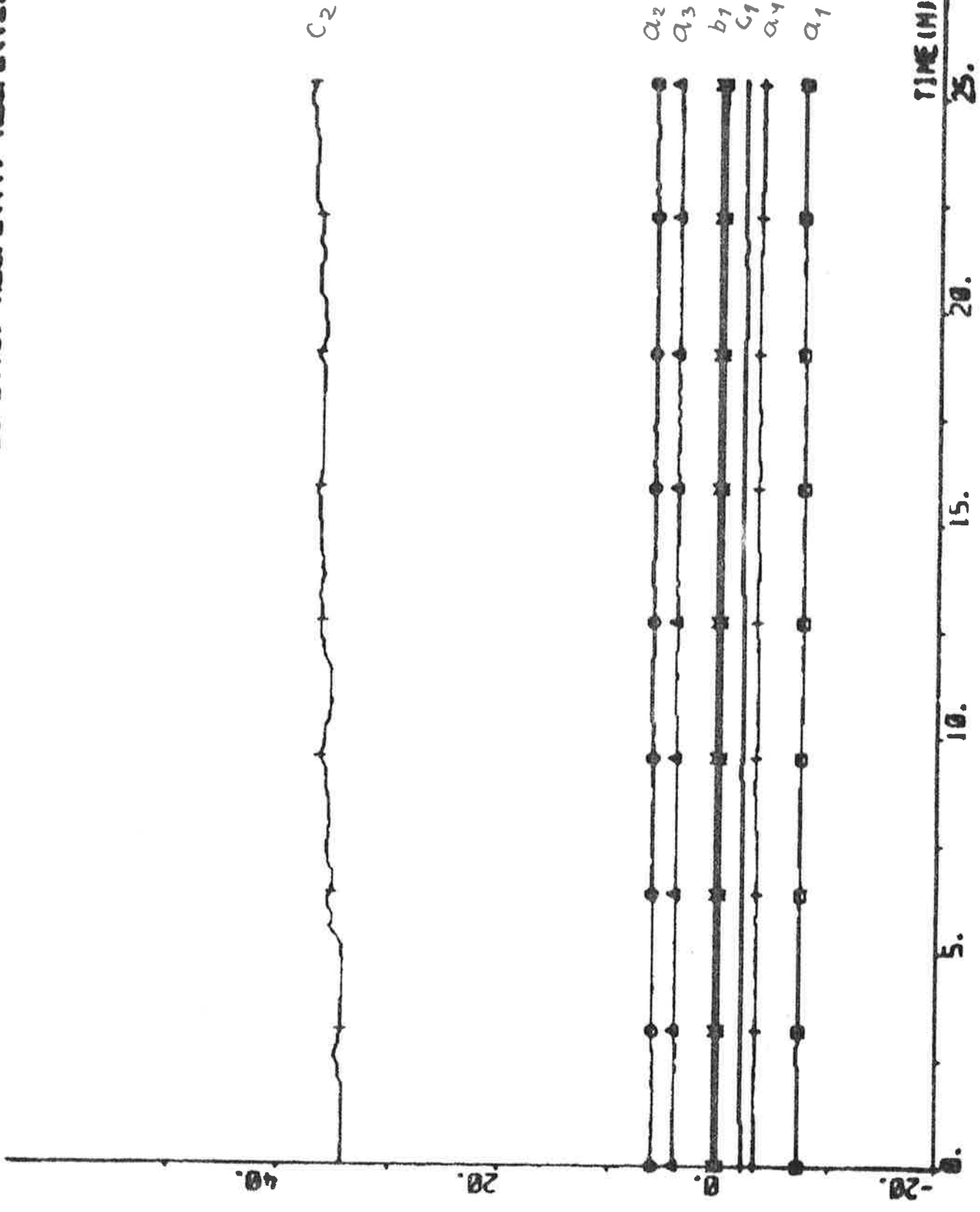


$100 \times \hat{d}_r$  deg/s

$0.5 \times \hat{d}_r$  deg



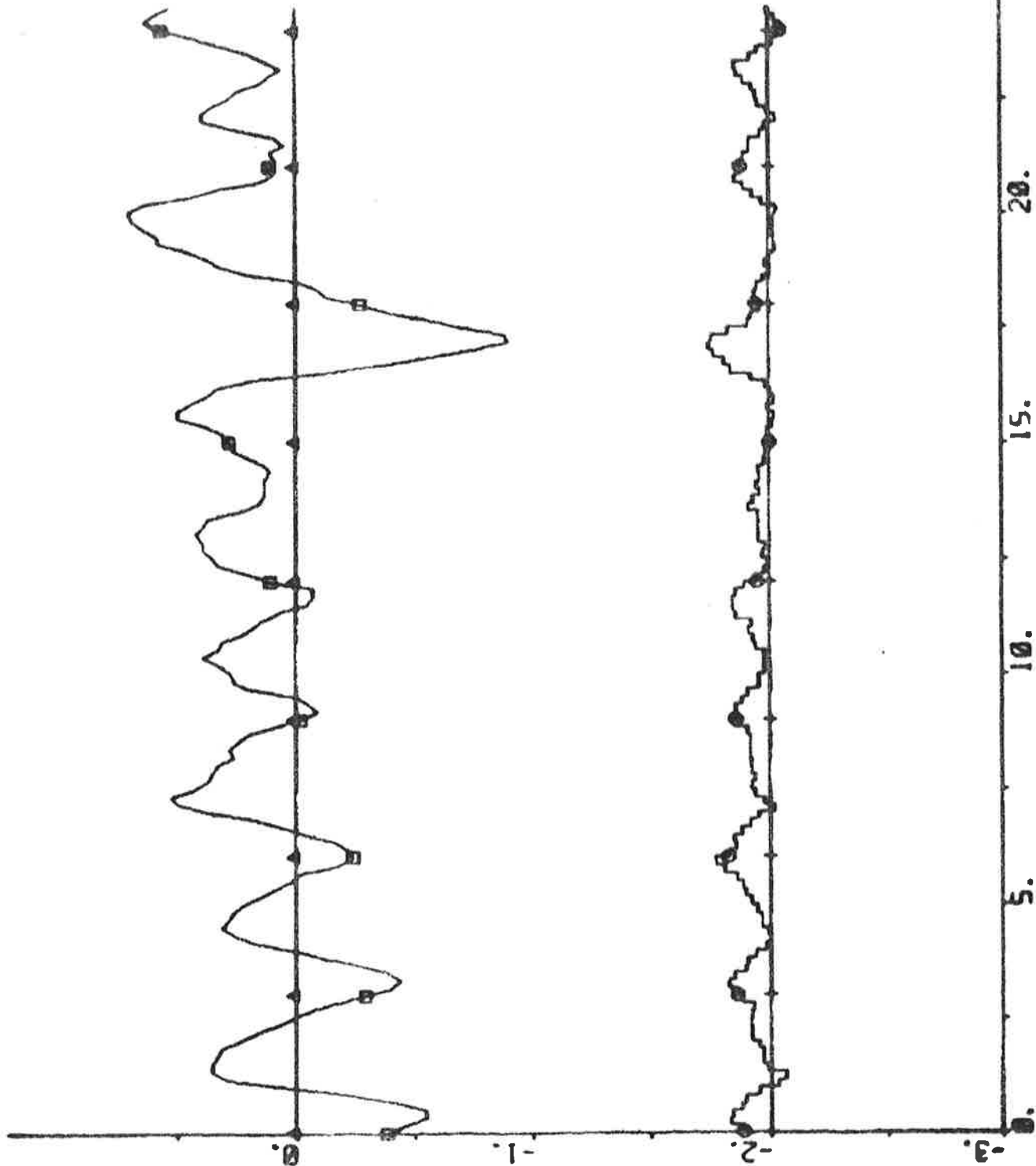
PLOT A23P1(1)-A23P2(7) A23P2(8) A23P2(9) A23P2(10) A23P2(11) A23P2(12) A2



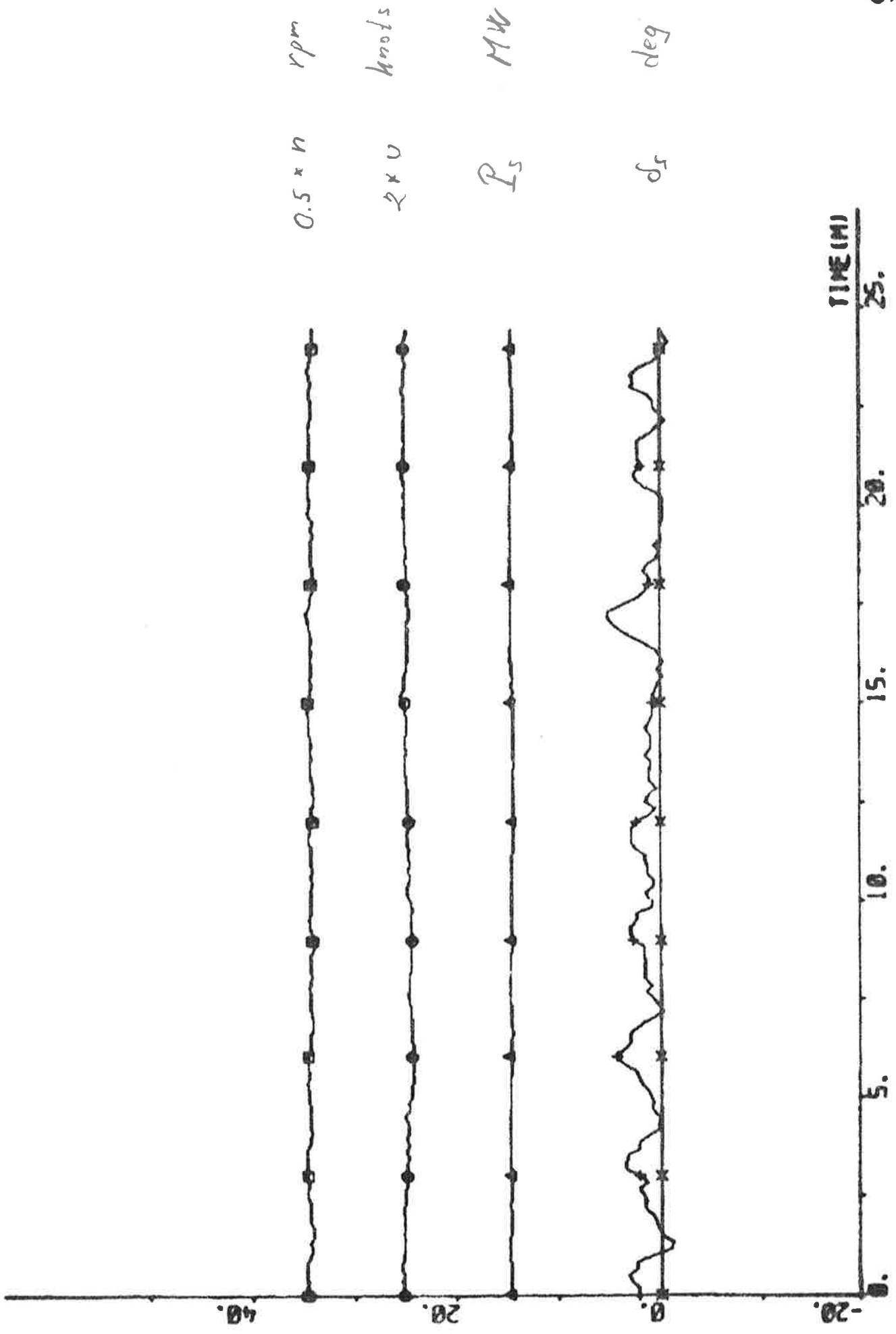




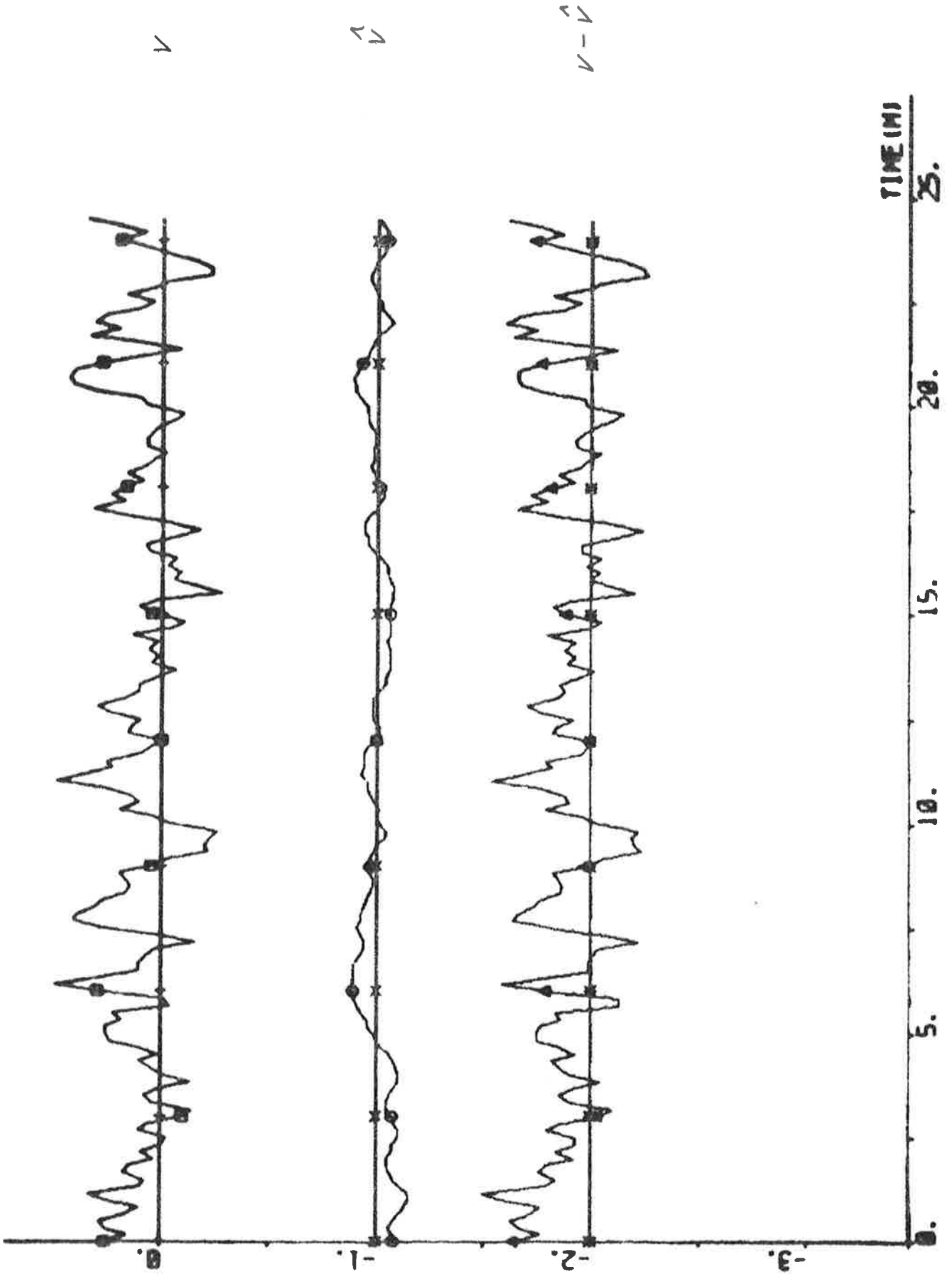
PLOT A24P1(1)-A24P1(8) HP A24P1(10) A24P1(15) Q2 -3 1 - DEC



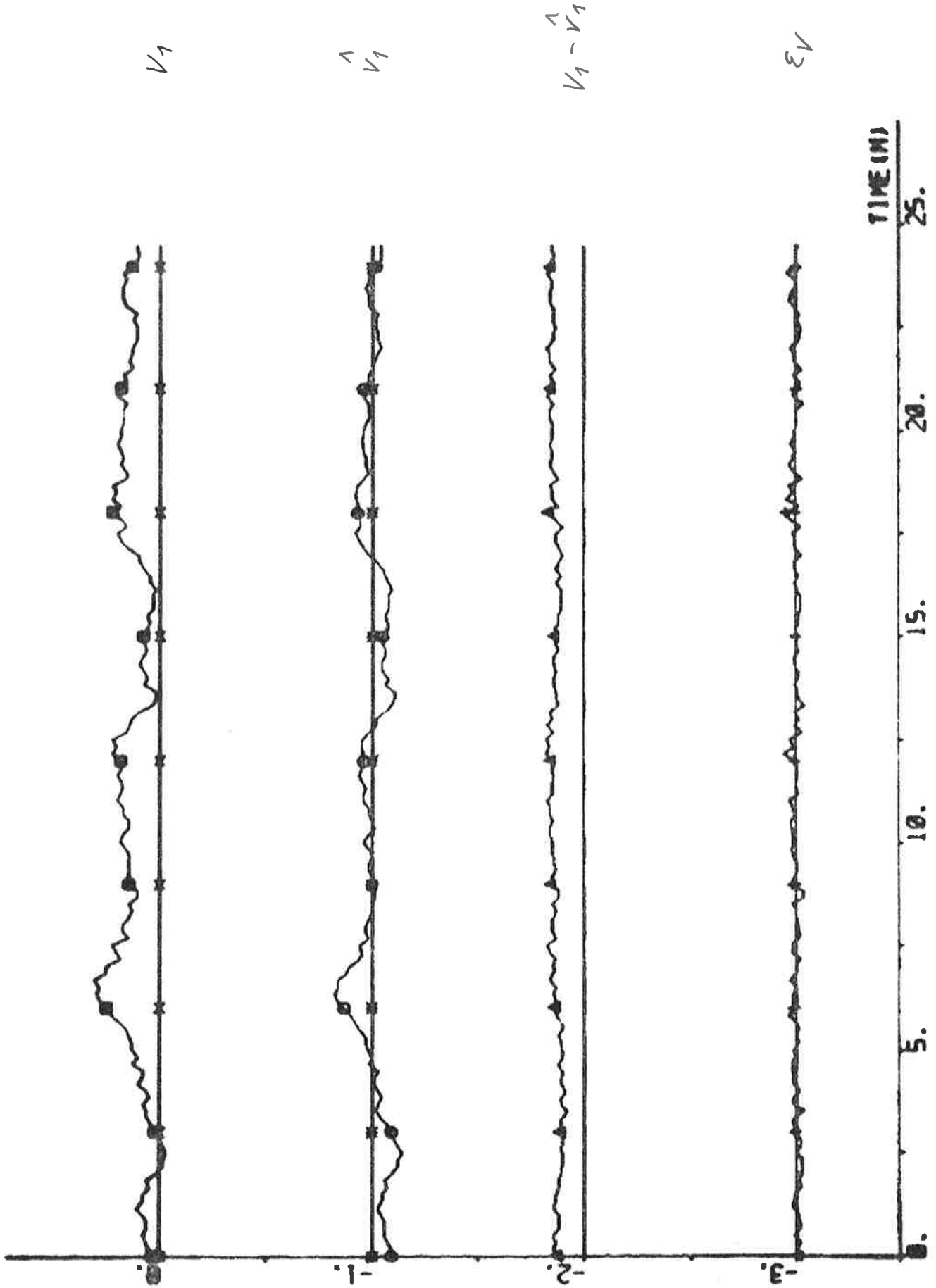
PLOT A24P1(1)-A24P1(13) A24P1(12) A24P1(14) A24P1(11) 00 -20 50



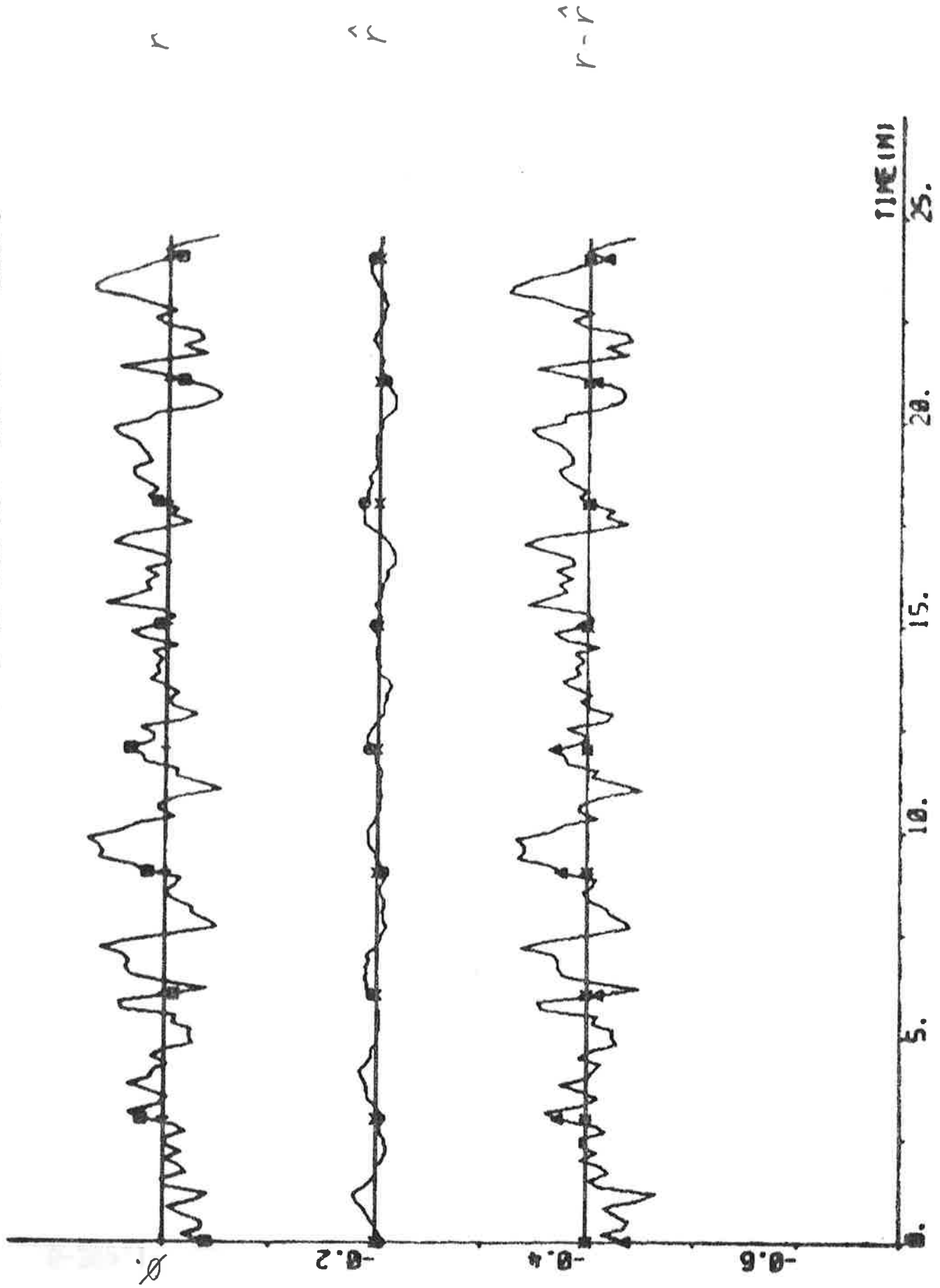
PL0T A24P1(1)-A24P2(1) A24P2(2) ERR6 00 01 02 -3.4 0.0 - KNOTS



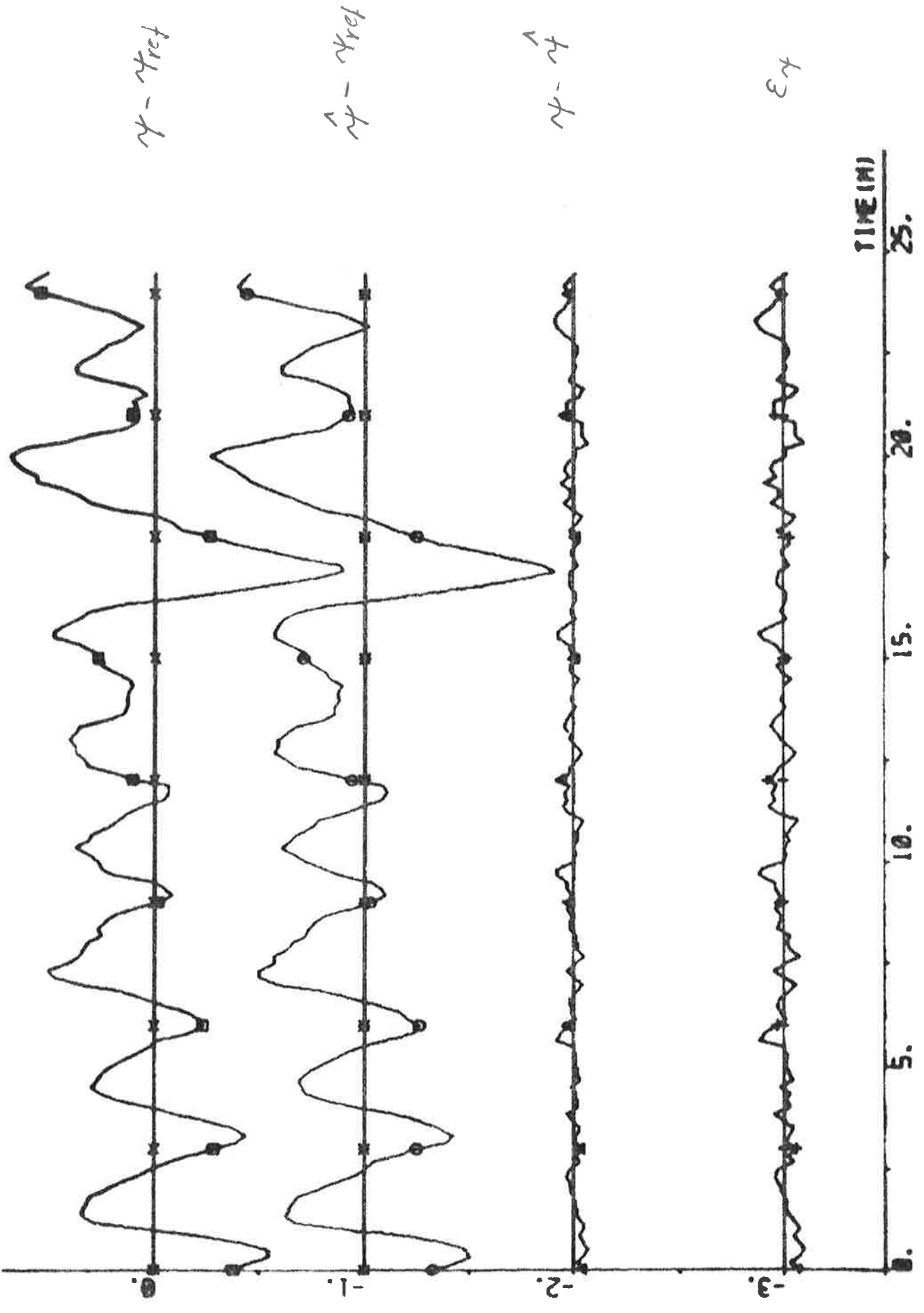
PLOT A24P1(1)-A24P1(4) A24P1(5) ERR2 EPS2 00 01 02 03 -3.4 0.0 - KNOTS



PLOT A24P1(1)-A24P1(8) A24P1(7) ERR3 90 002 004 -0.7 0. - DECS

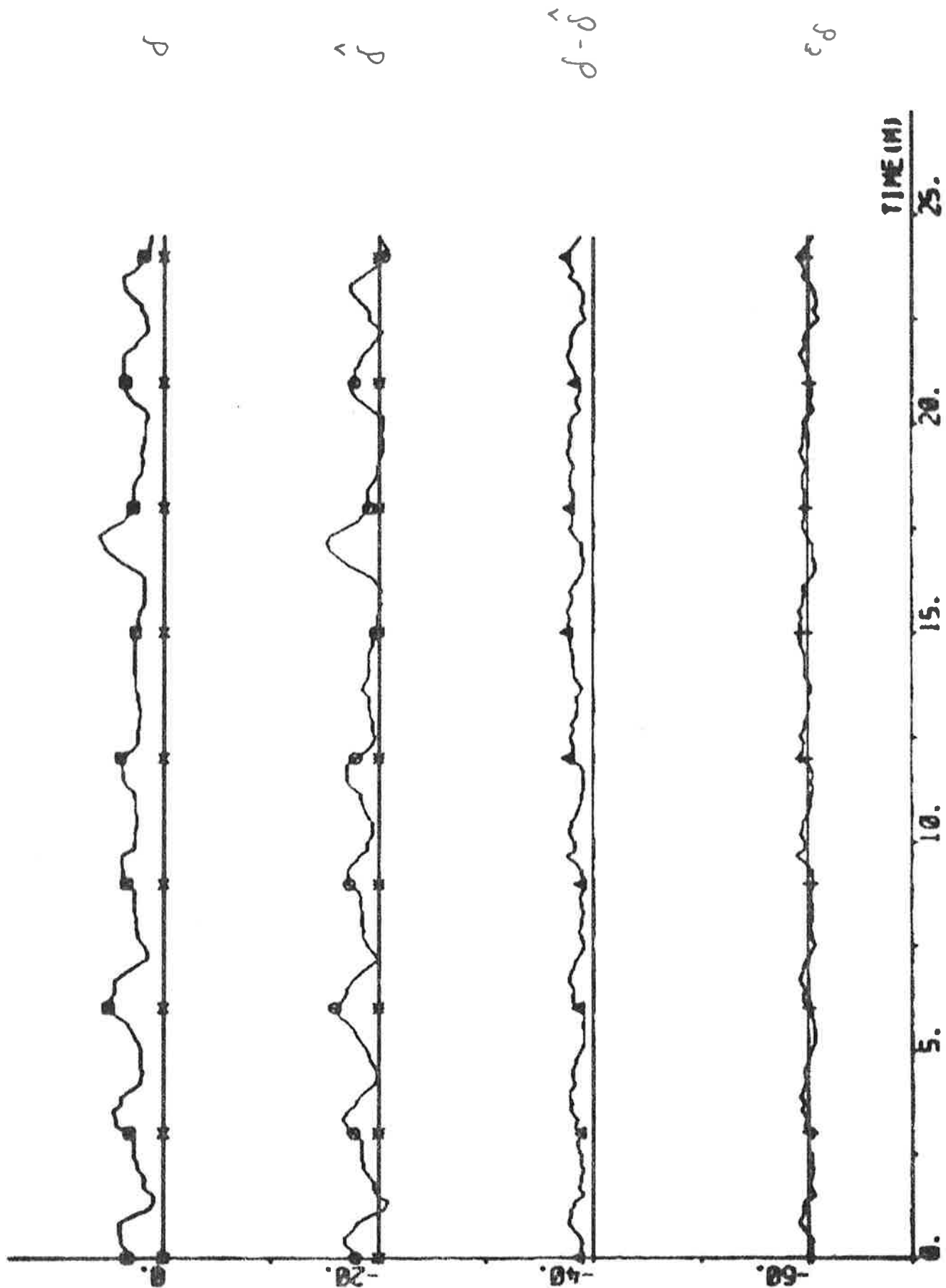


PLOT A24P1(1)-A24P1(8) A24P1(9) ERR4 EPS4 00 01 02 03 -3.4 0.0 - 003

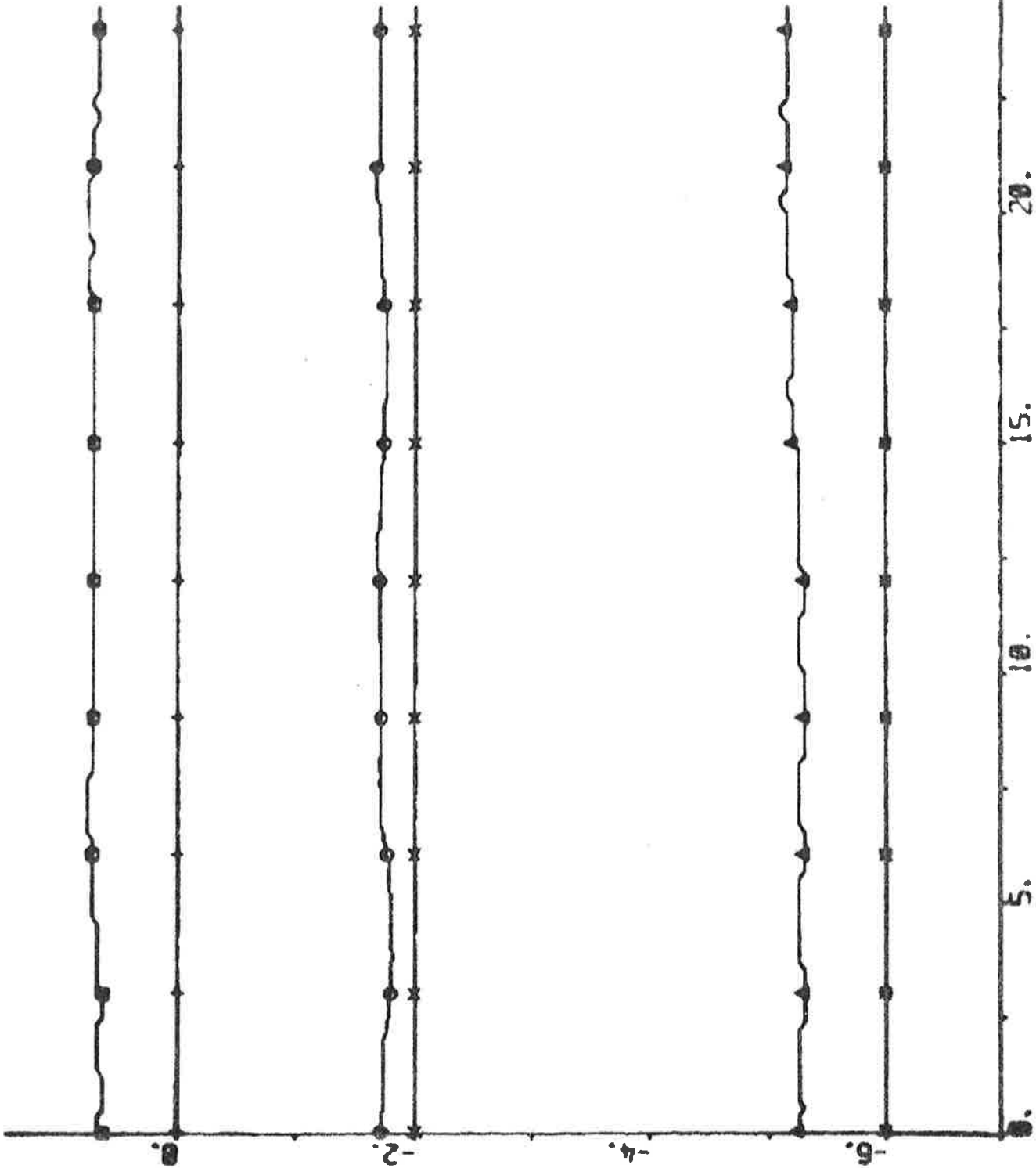


TIME (M)  
25.  
20.  
15.  
10.  
5.

PLOT A24P1(1)-A24P1(2) A24P1(3) ERR1 EPS1 00 020 040 060 -06 16 ° DEG



PLOT A24P1(1)-A24P2(3) A24P2(4) A24P2(6) 69 02 06 -0.5 1.5



$0.5 \times \hat{d}_0$  deg

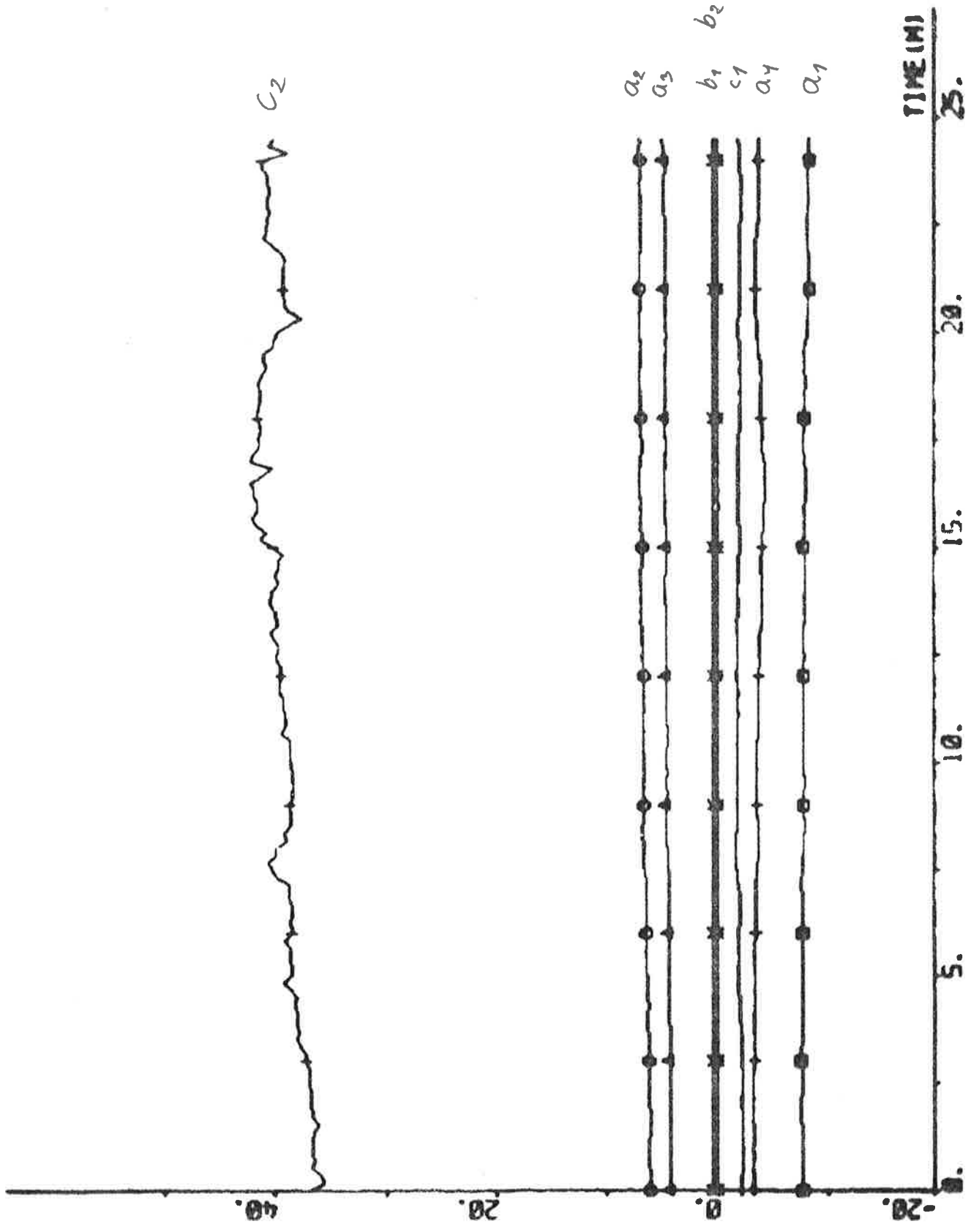
$2 \times \hat{d}_1$  knots

$0.5 \times \hat{d}_8$  deg

TIME (M)

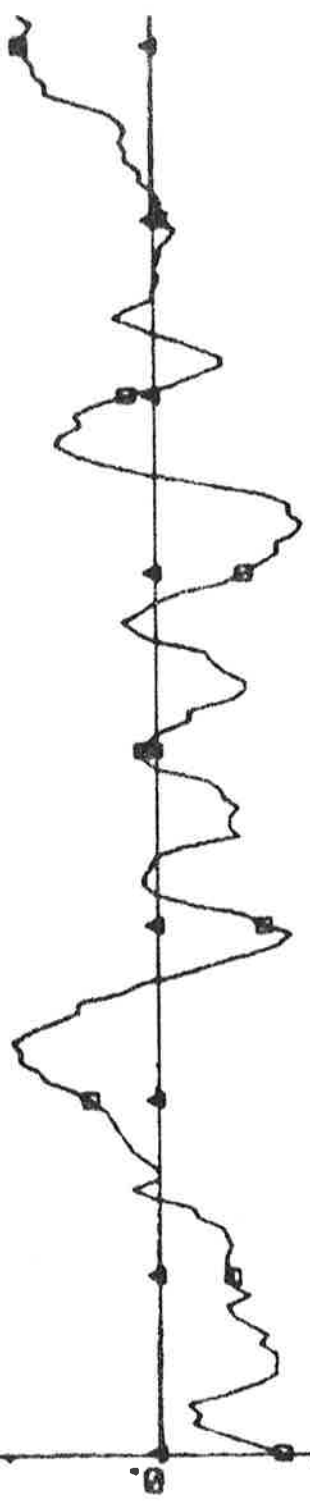


PLOT R24P1(1)-R24P2(7) R24P2(8) R24P2(9) R24P2(10) R24P2(11) R24P2(12) R2





PL0T A25P1(1) - A25P1(8) HP A25P1(10) A25P1(15) Q2 -3 1 - DEG

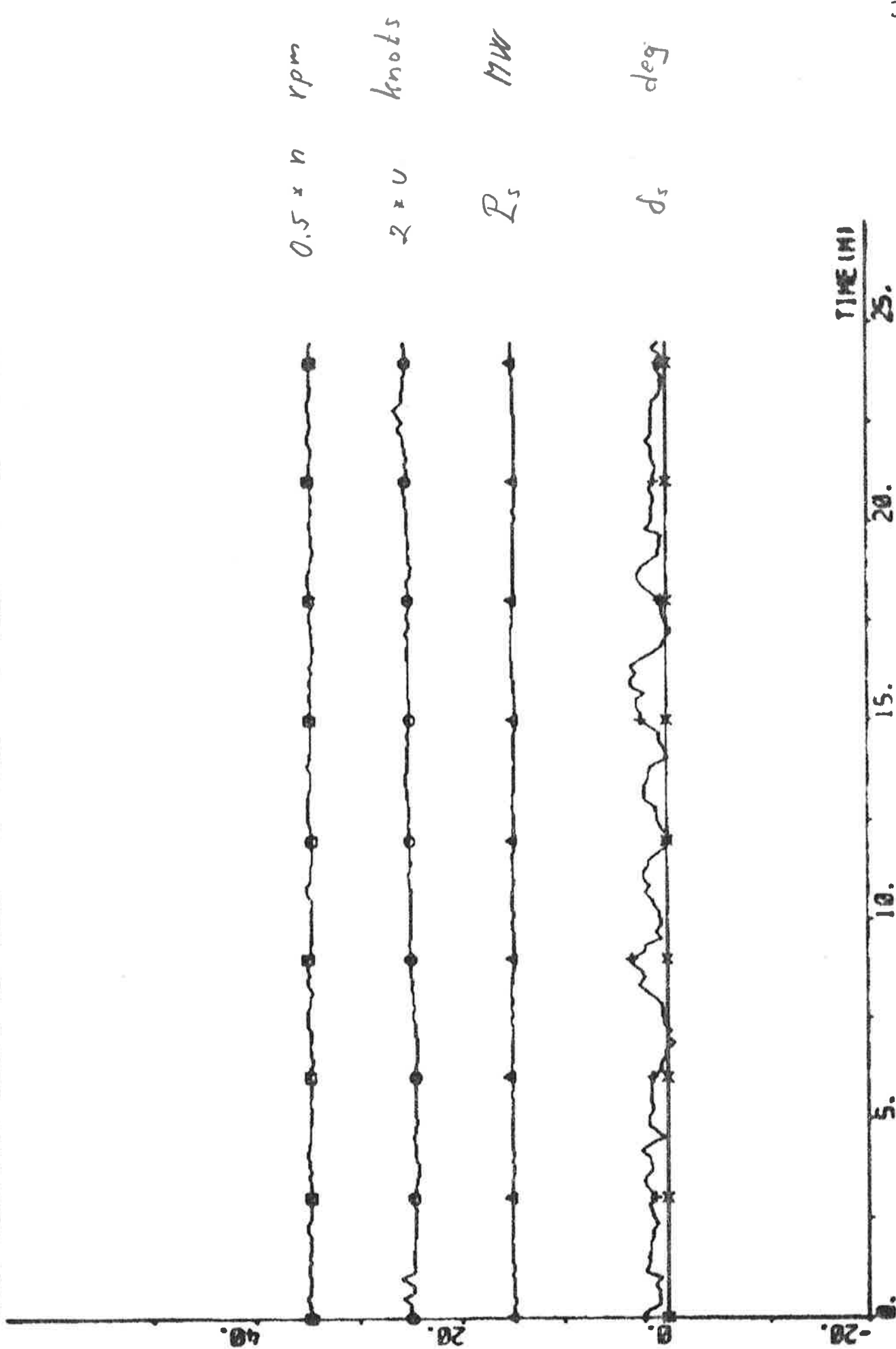


$y - y_{ref}$

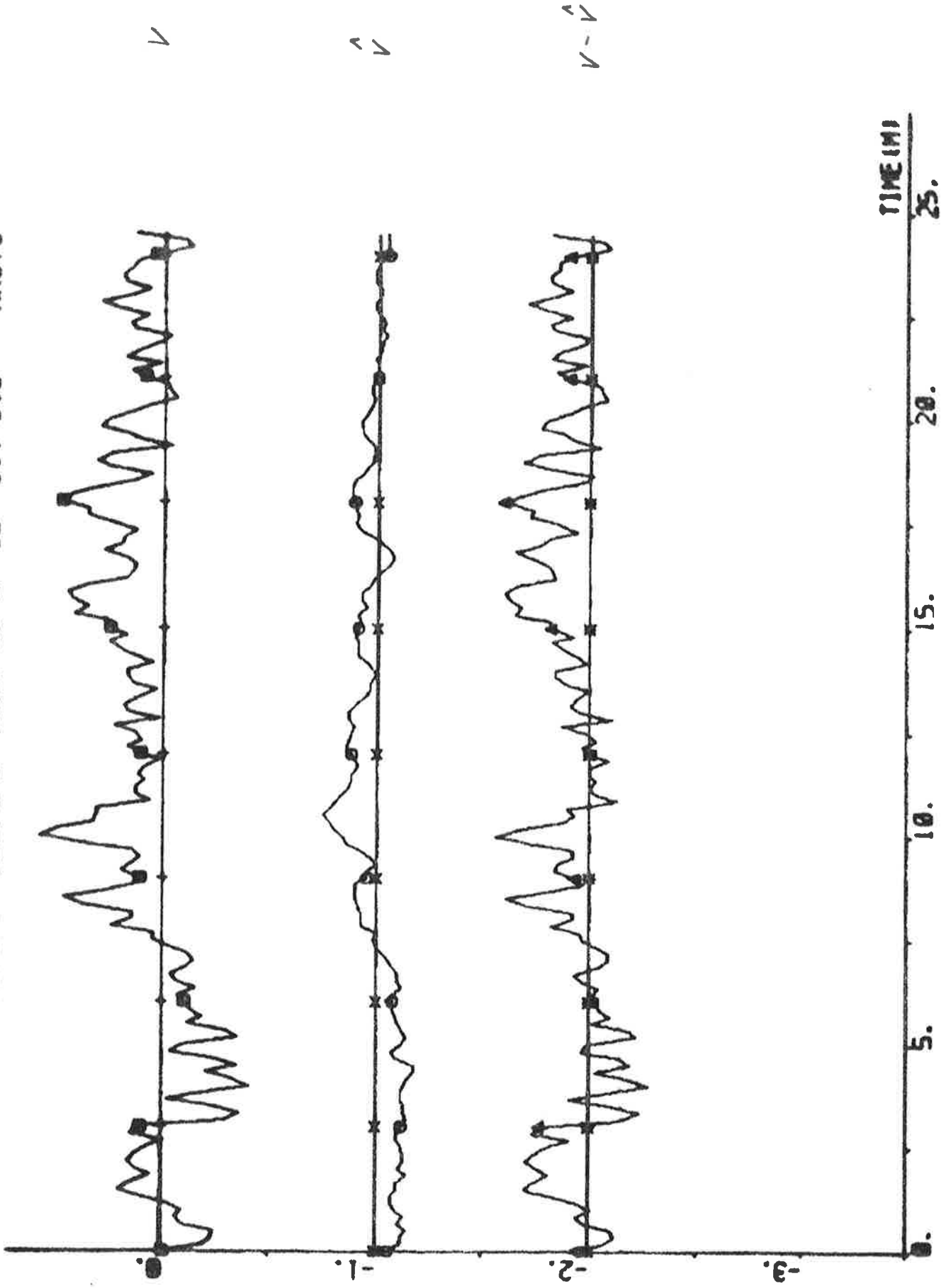
$0.05 \times d_c$



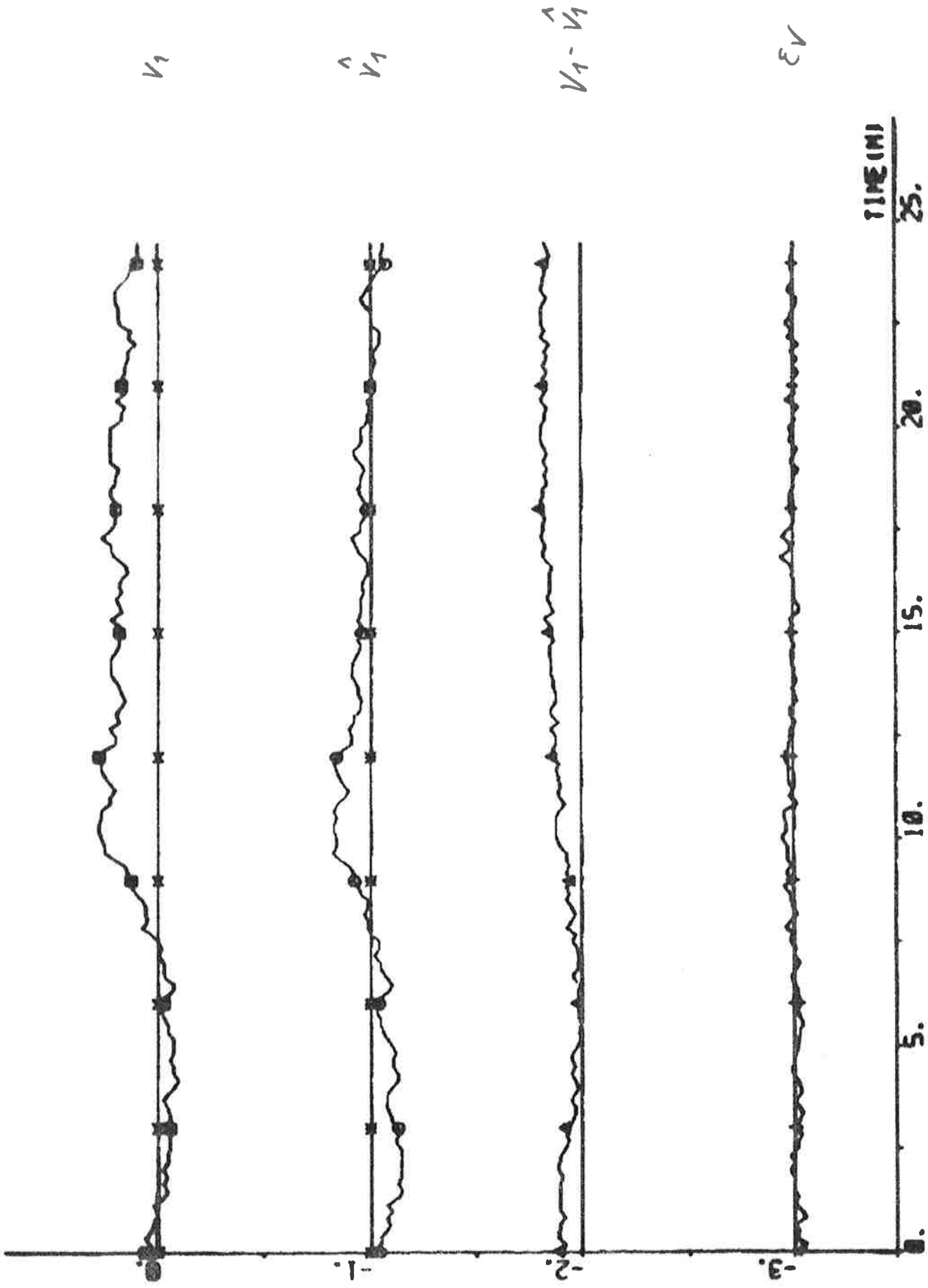
PLOT A25P1(1) A25P1(13) A25P1(12) A25P1(14) A25P1(11) 00 -20 50



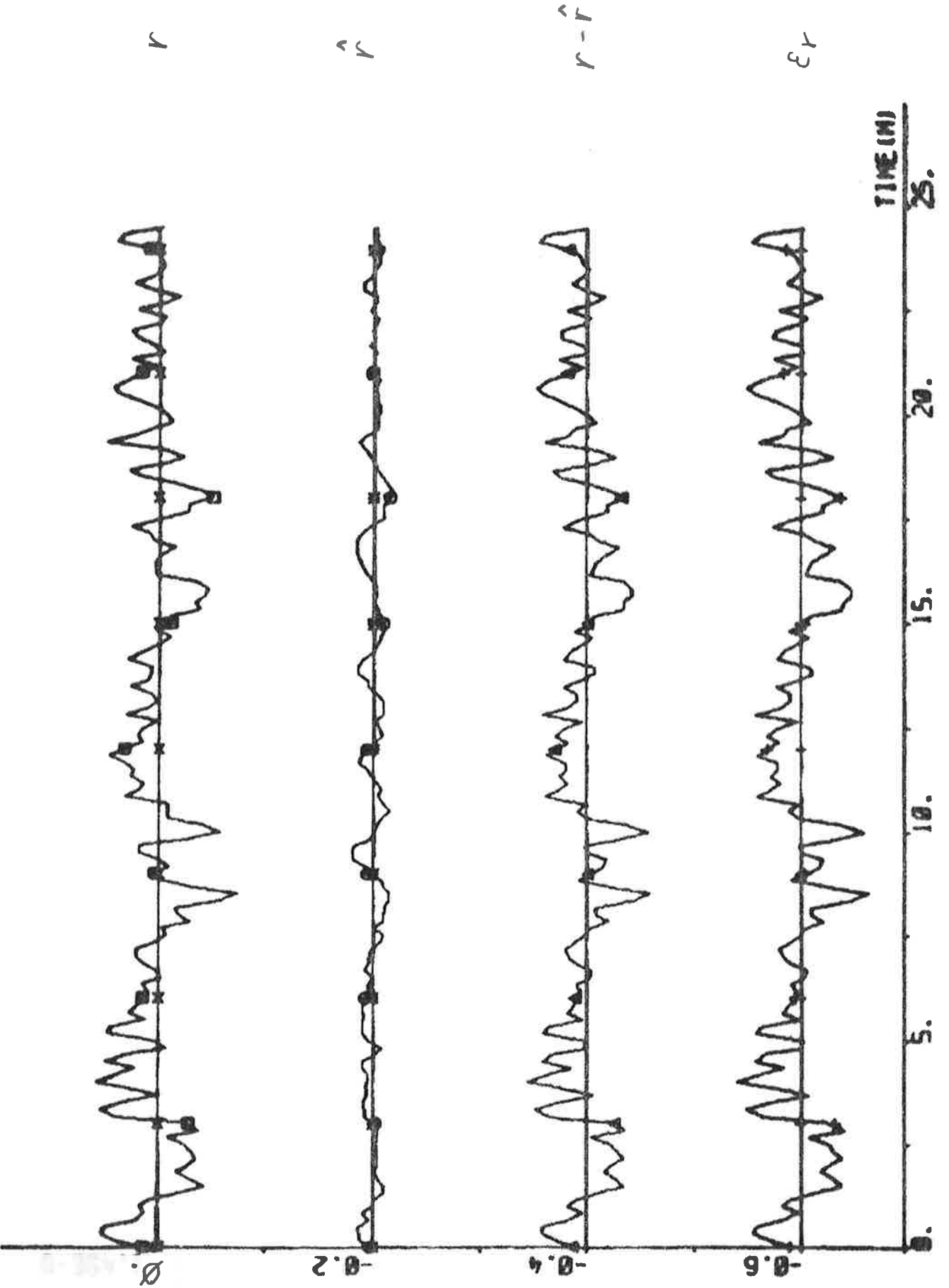
PL0T A2SP1(1) - A2SP2(1) A2SP2(2) ERR6 00 01 02 -3.4 0.0 - 10MOTS



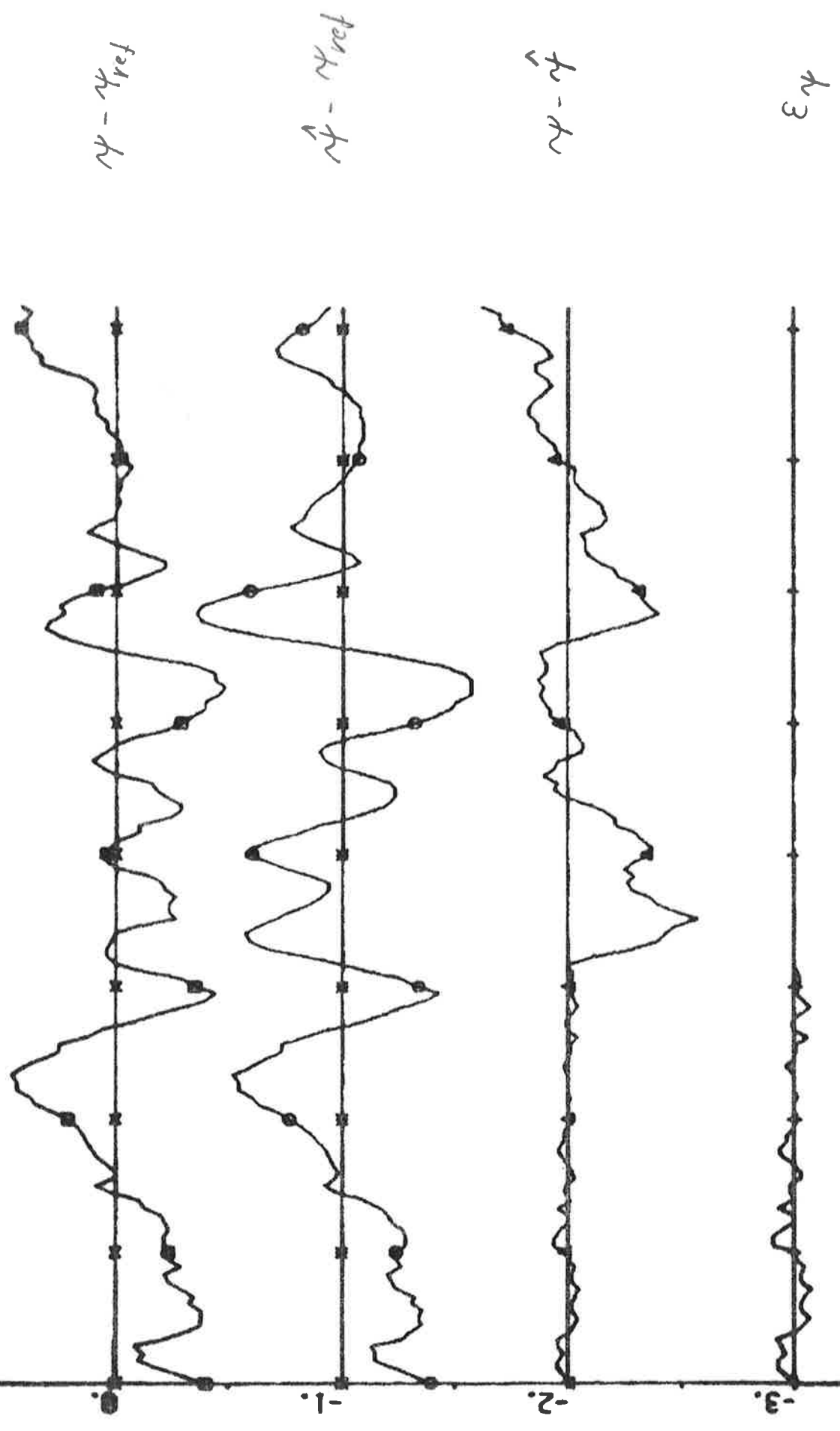
PLOT A2SP1(1) A2SP1(4) A2SP1(6) ERR2 00 01 02 03 -3.4 0.0 - KNOTS



PL0T A2SP1(1)-A2SP1(6) A2SP1(7) ERR03 EP33 00 002 004 006 -0.7 0. • 00000



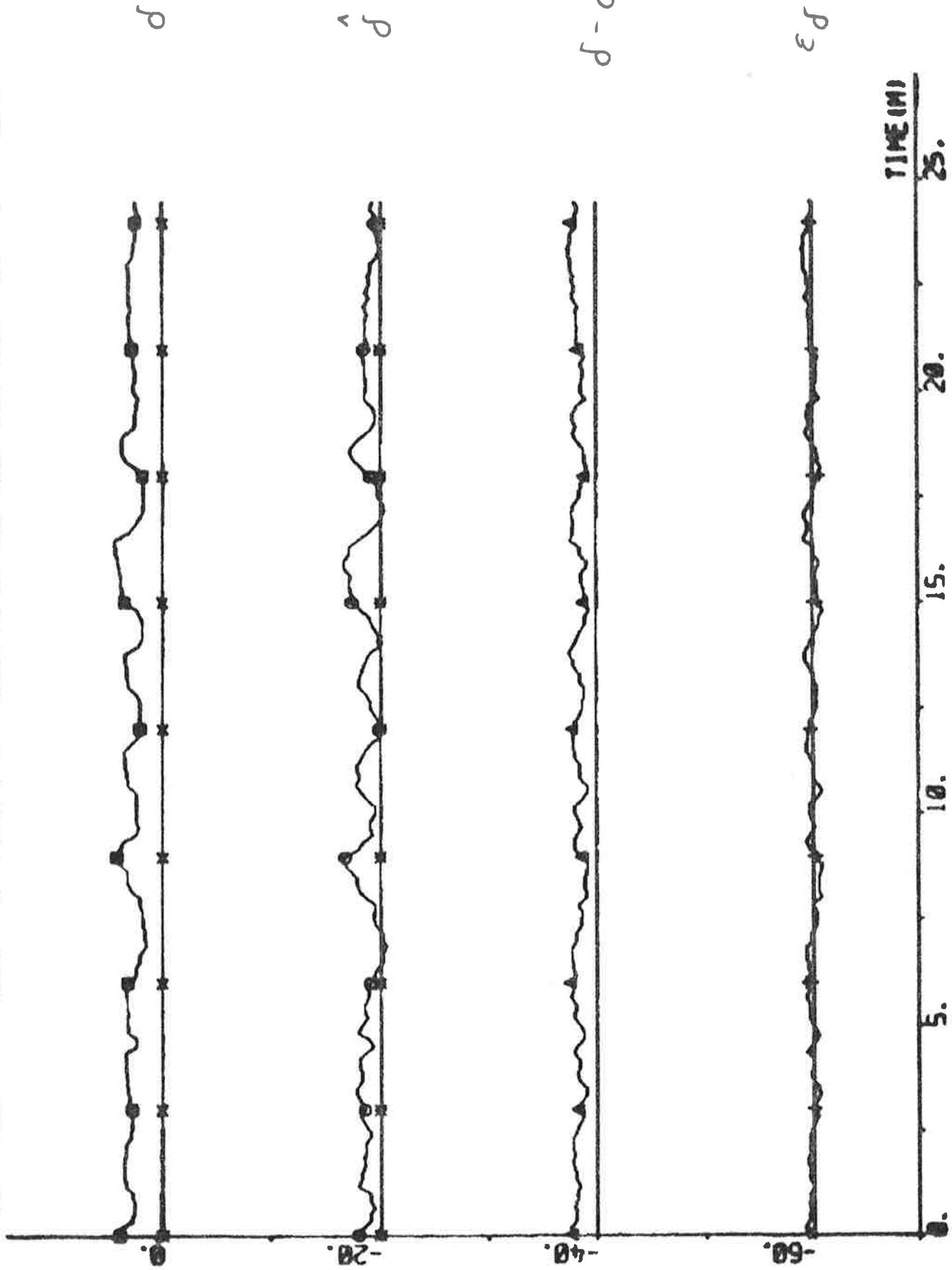
PLOT A25P1(1)-A25P1(8) A25P1(9) ERR4 EPS4 00 01 02 03 -3.4 0.0 DEG



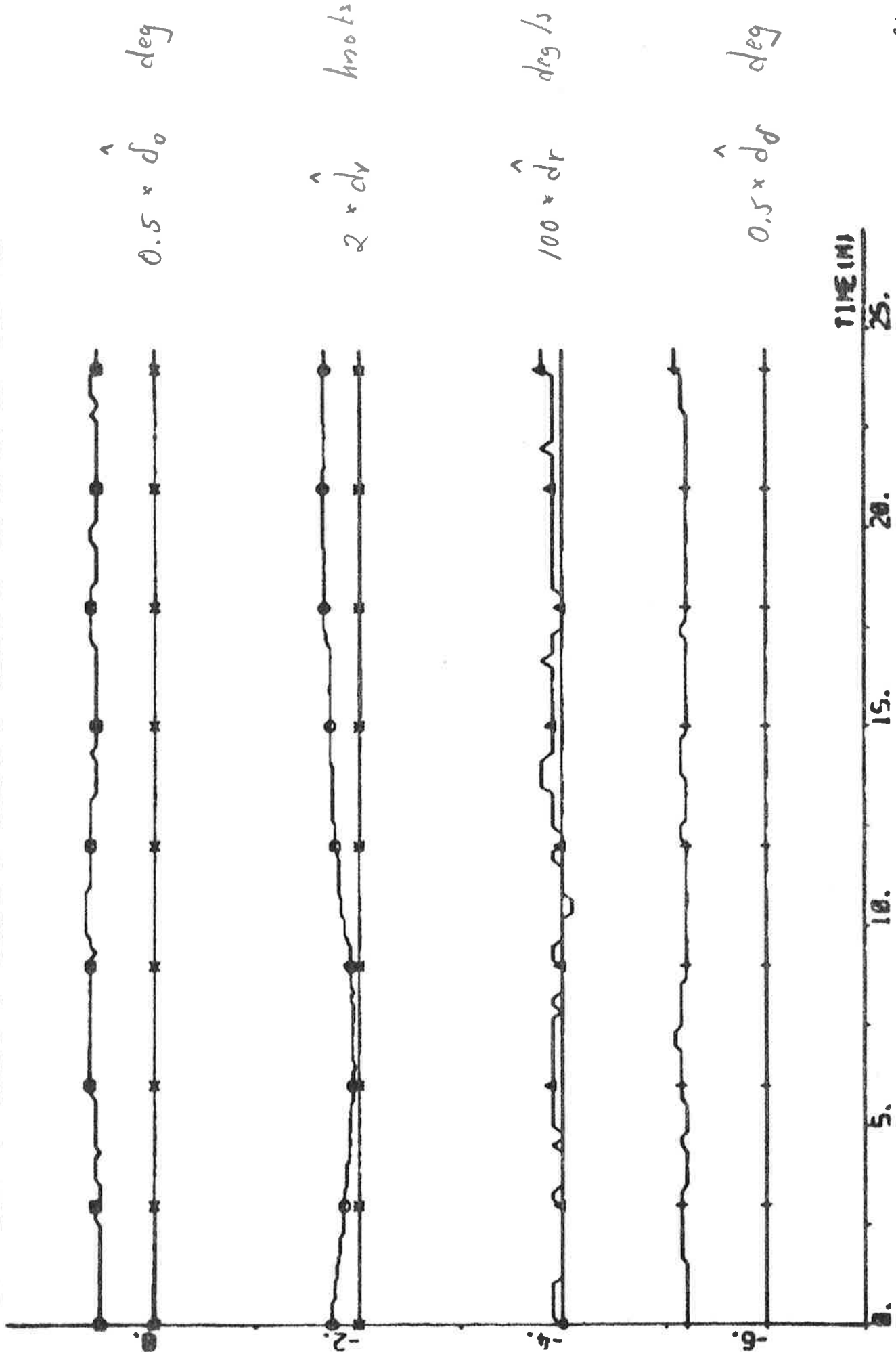
TIME (M)  
25.



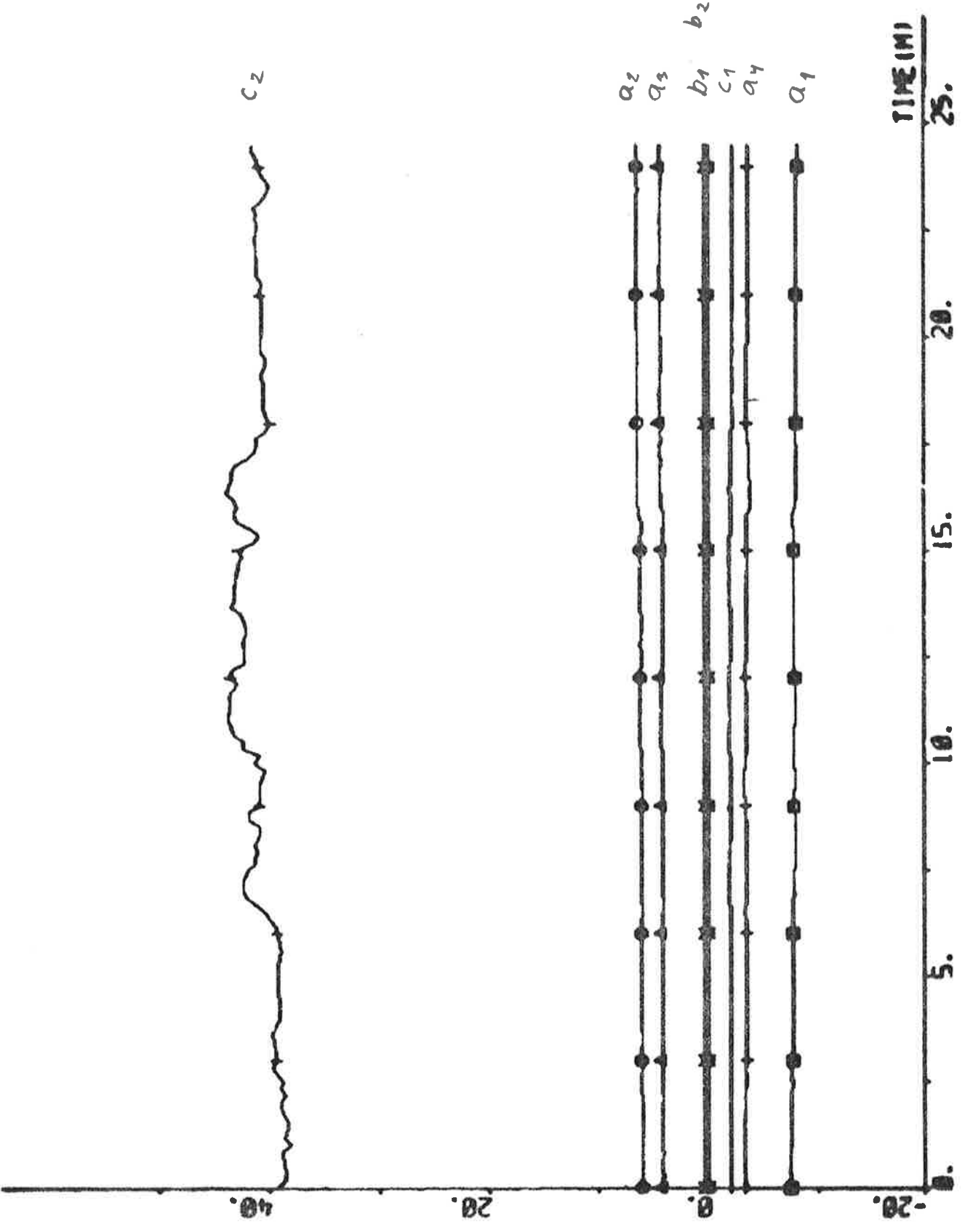
PL0T A2SP1(1)-A2SP1(2) A2SP1(3) ERR1 EPS1 00 020 040 060 -05 15 - 000



PLOT A25P1(1)-A25P2(3) A25P2(4) A25P2(5) A25P2(6) 00 02 04 06 -0.5 1.5

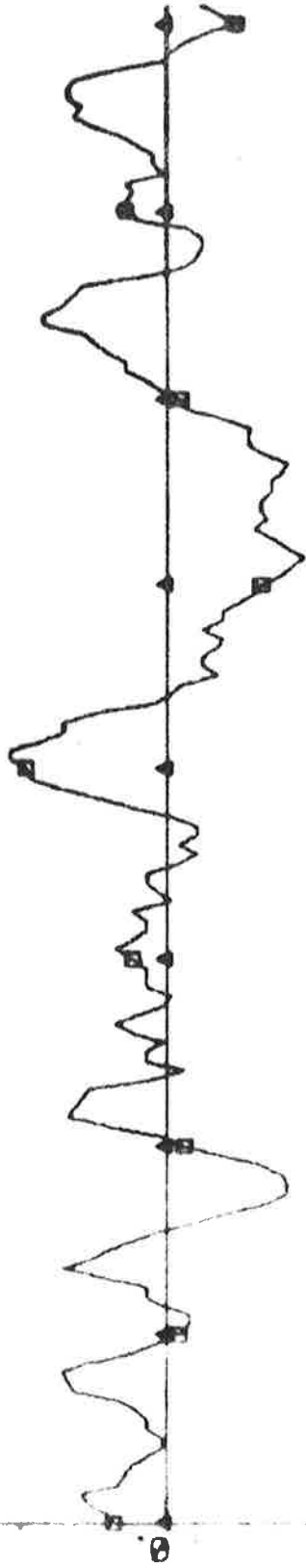


PLOT A2SP1(1)-A2SP2(7) A2SP2(8) A2SP2(9) A2SP2(10) A2SP2(11) A2SP2(12) A3





PLOT A26P1(1)-A26P1(8) MP A26P1(10) A26P1(15) Q2 -3 1 - DEG



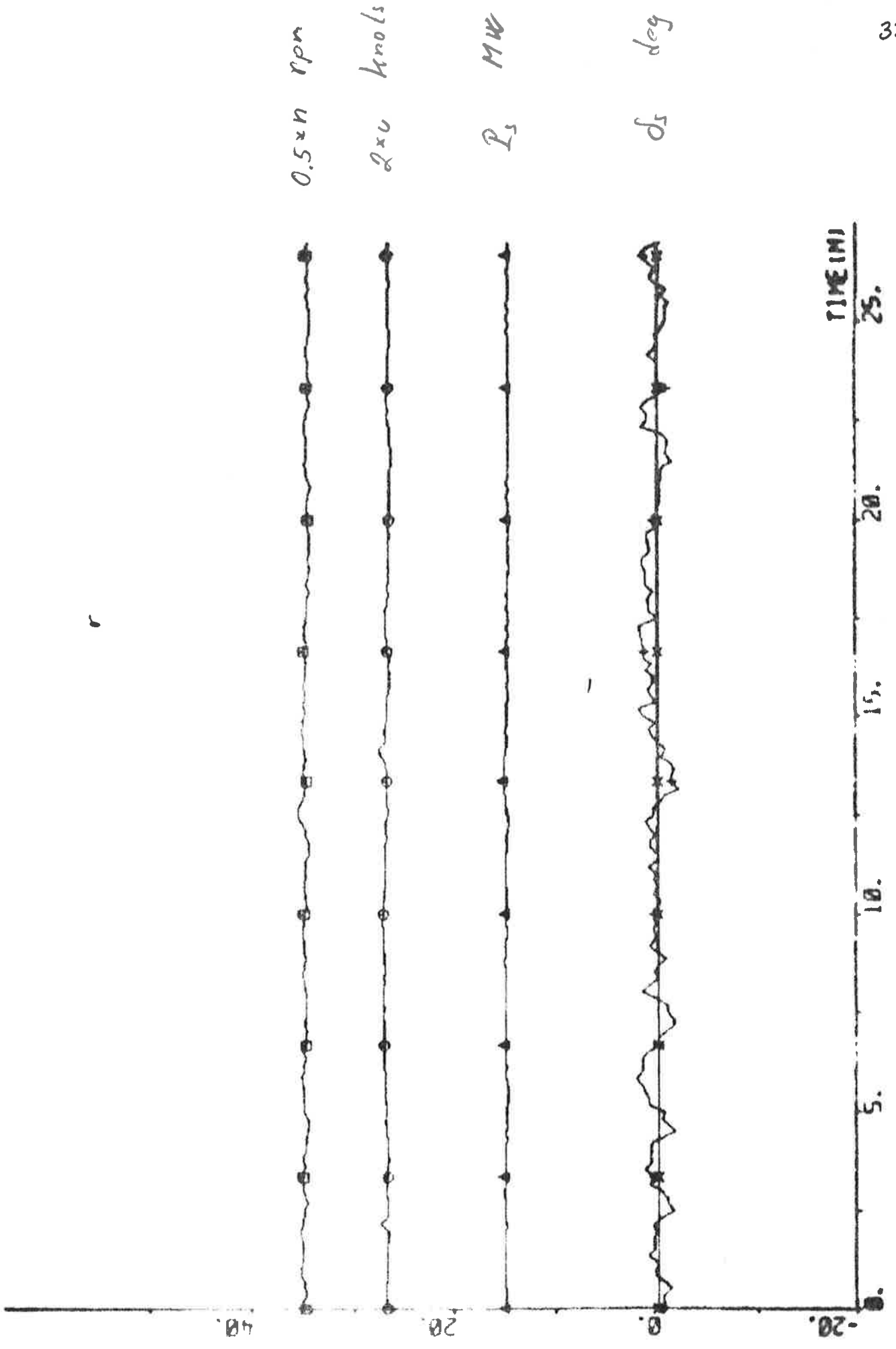
$Y_{ref}$



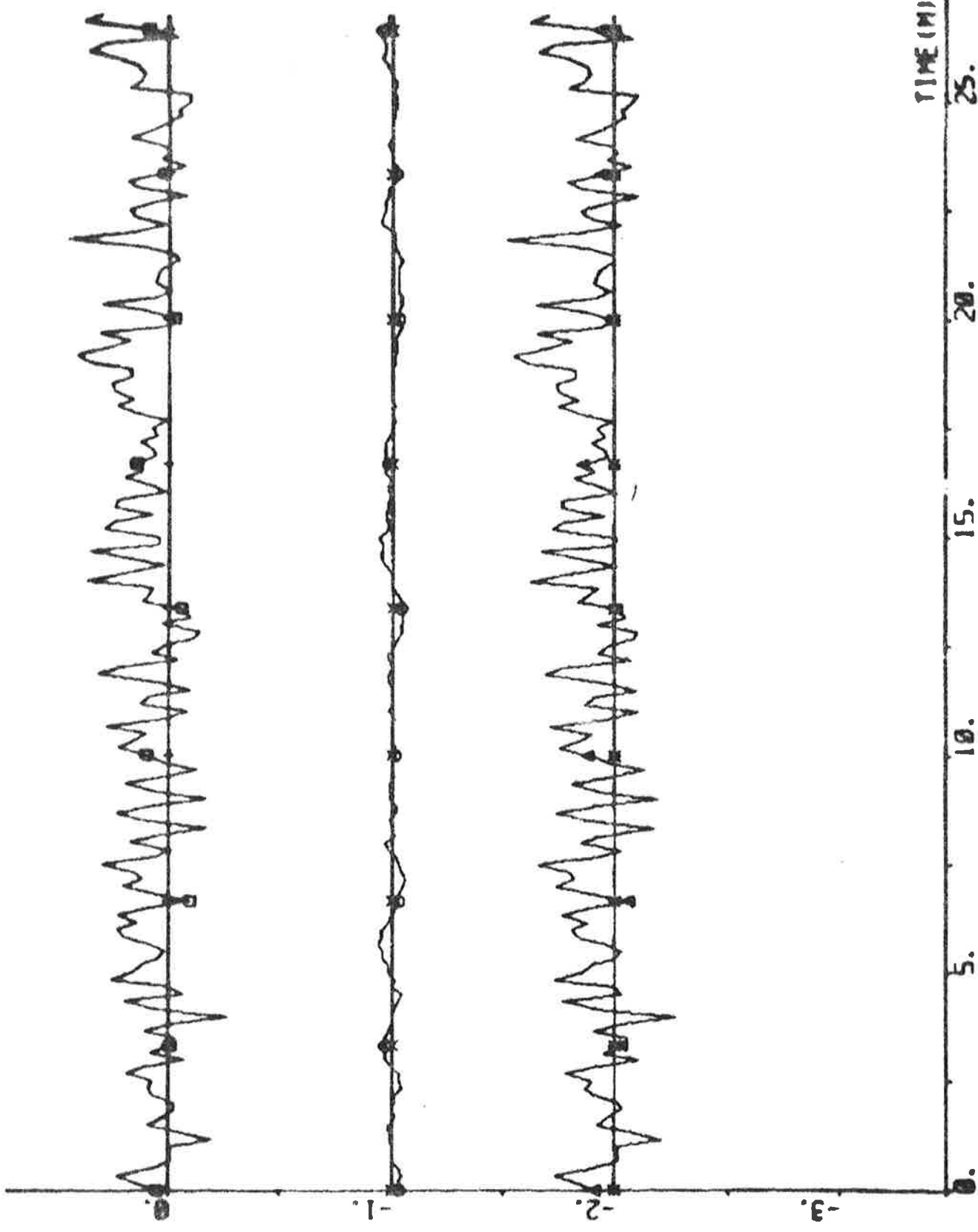
$0.05 * d_c$



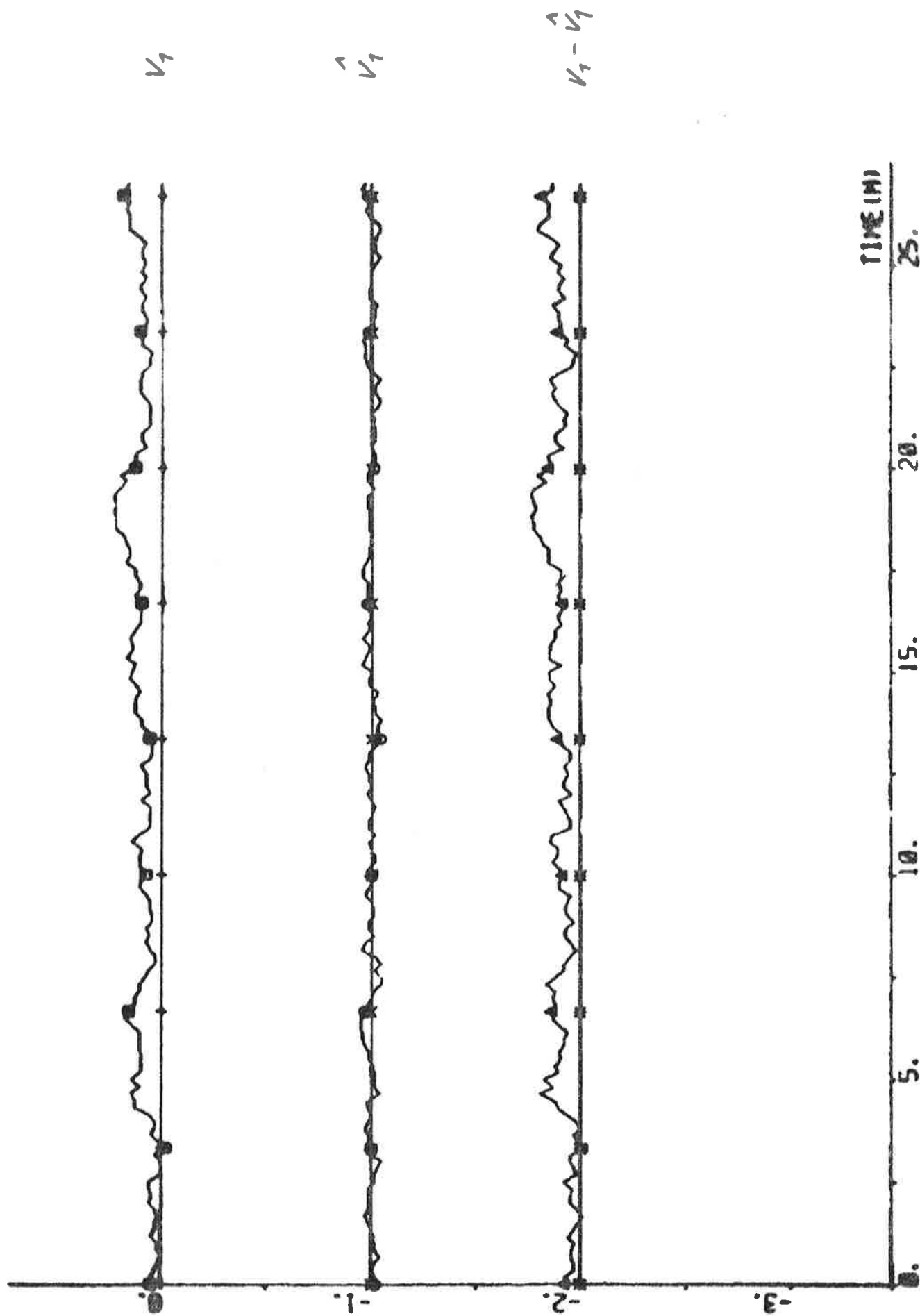
PLOT A26P1(1) A26P1(10) A26P1(12) A26P1(14) A26P1(11) 00 -20 50



PLOT A26P1(1)-A26P2(1) A26P2(2) ERRS 00 01 02 -3.4 0.0 - KNOTS

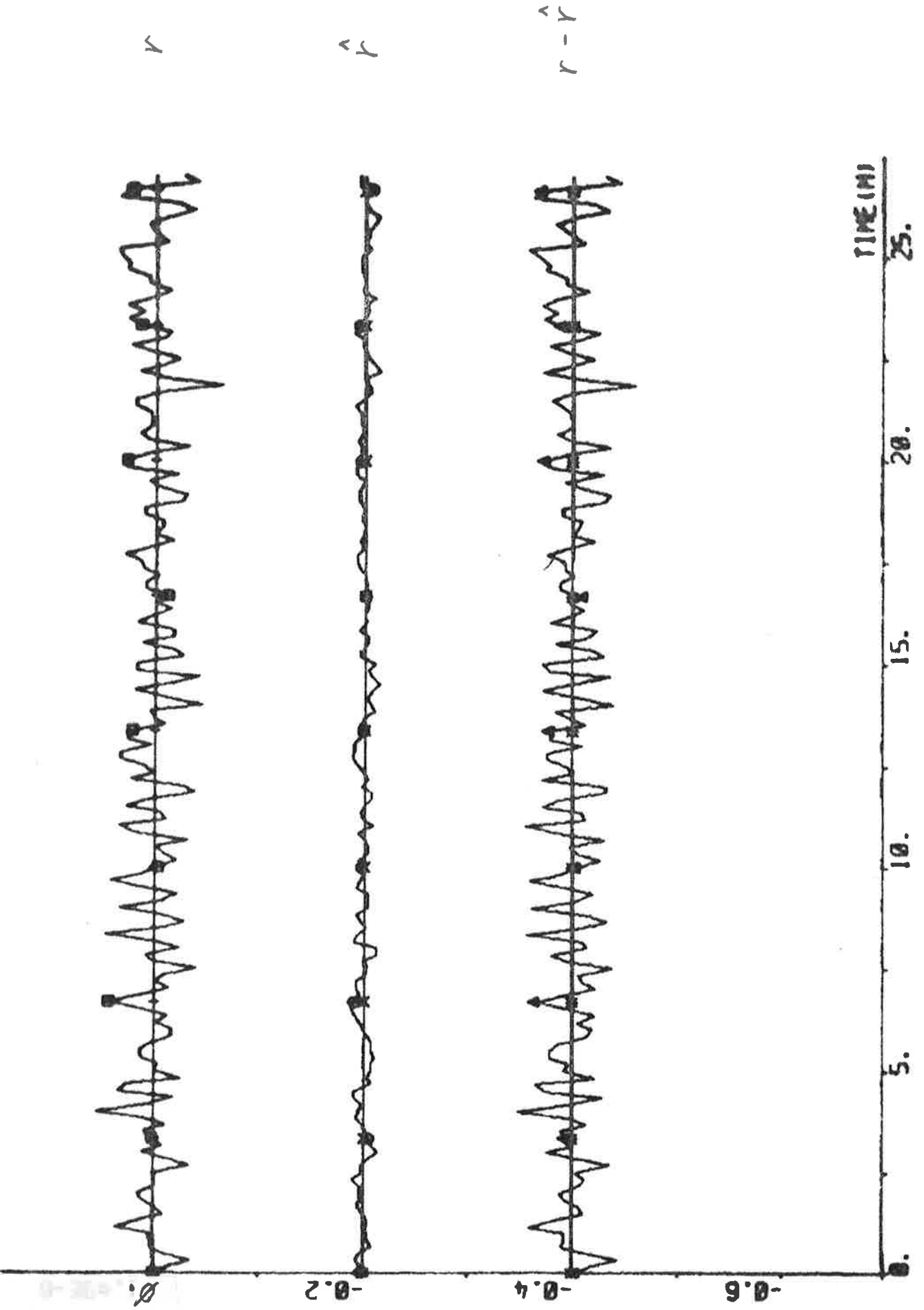


PLOT A2SP1(1)-A2SP1(4) A2SP1(5) ERR2 00 01 02 -3.4 0.0 - KNOTS

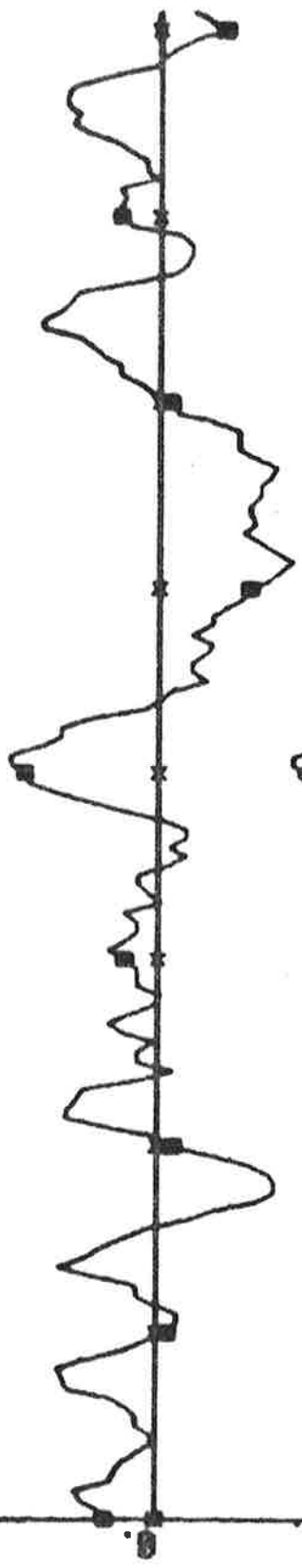




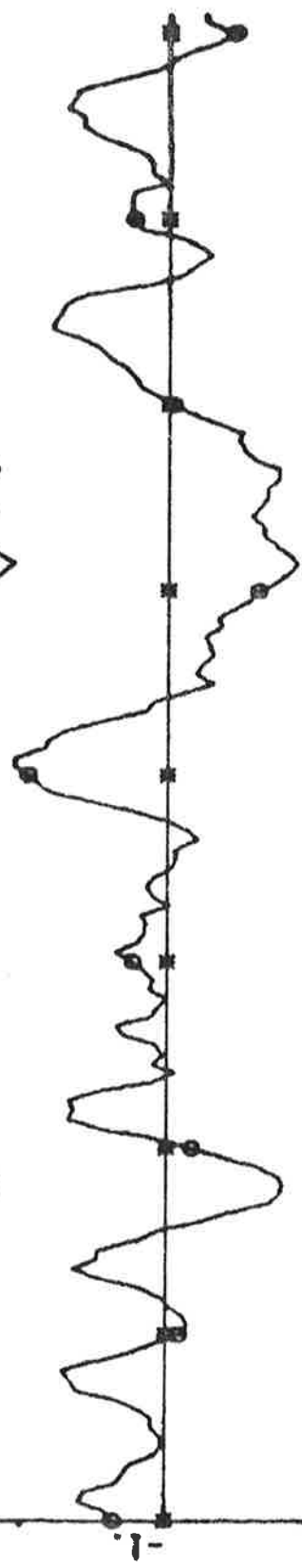
PLOT A26P1(1)-A26P1(6) A26P1(7) ERR3 00 002 004 -0.7 0. - DEC-3



PL0T A2SP1(1)-A2SP1(8) A2SP1(9) ERR4 EPS4 00 01 02 03 -3.4 0.0 DEG



$\hat{y} - y_{ref}$



$\hat{y} - y_{ref}$



$\hat{y} - y$



$\hat{y}$



PLOT A20P1(1)-A20P1(2) A20P1(3) ERR1 00 020 040 -05 15 - DEG



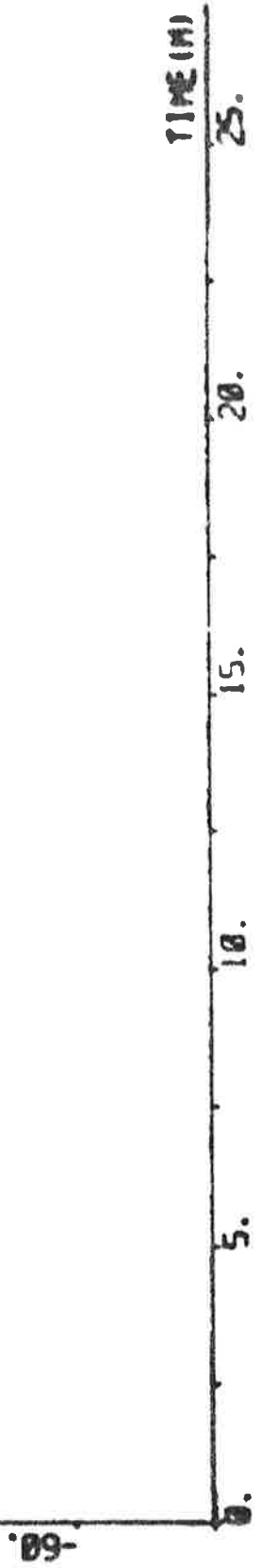
$\delta$



$\hat{\delta}$

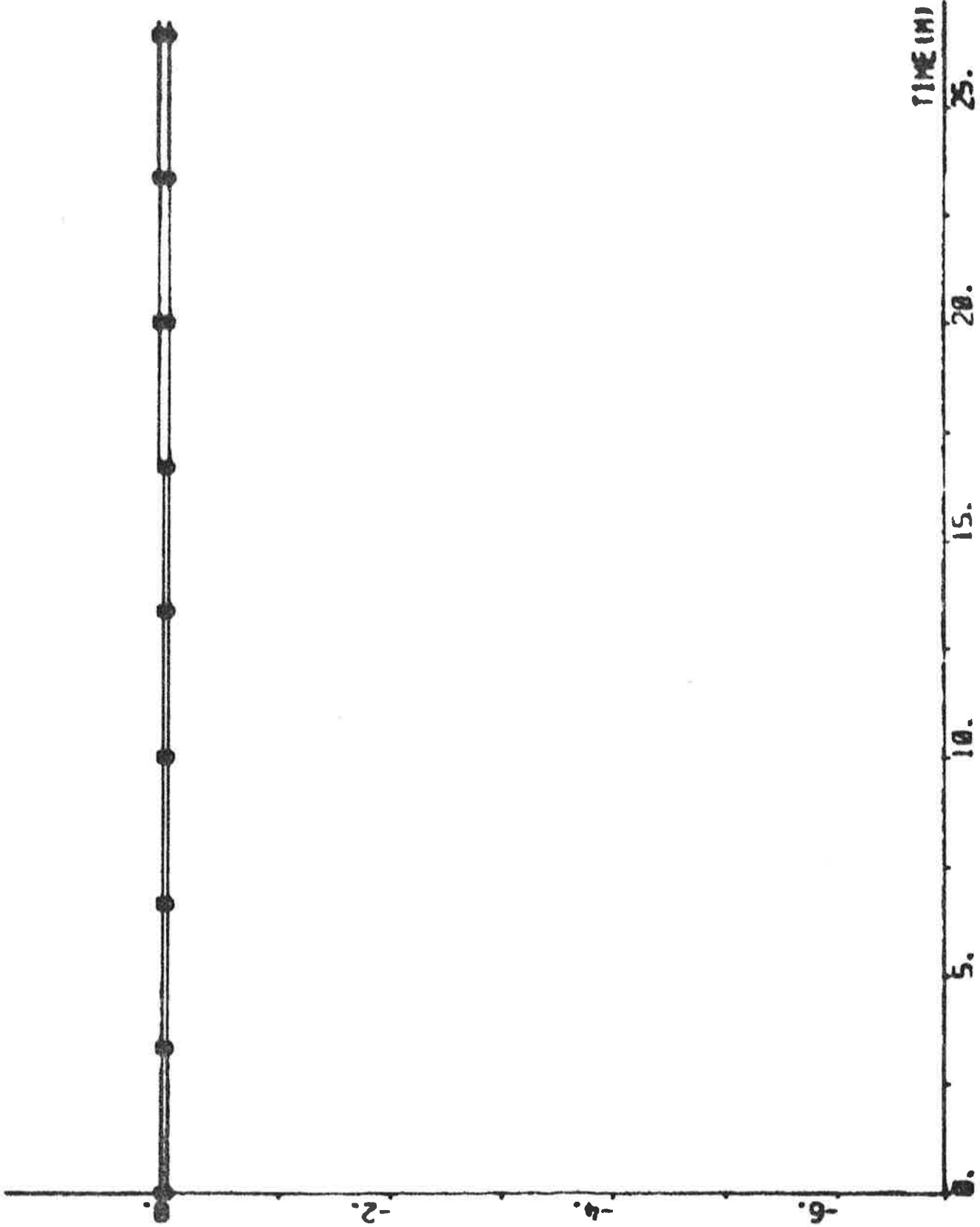


$\delta - \hat{\delta}$

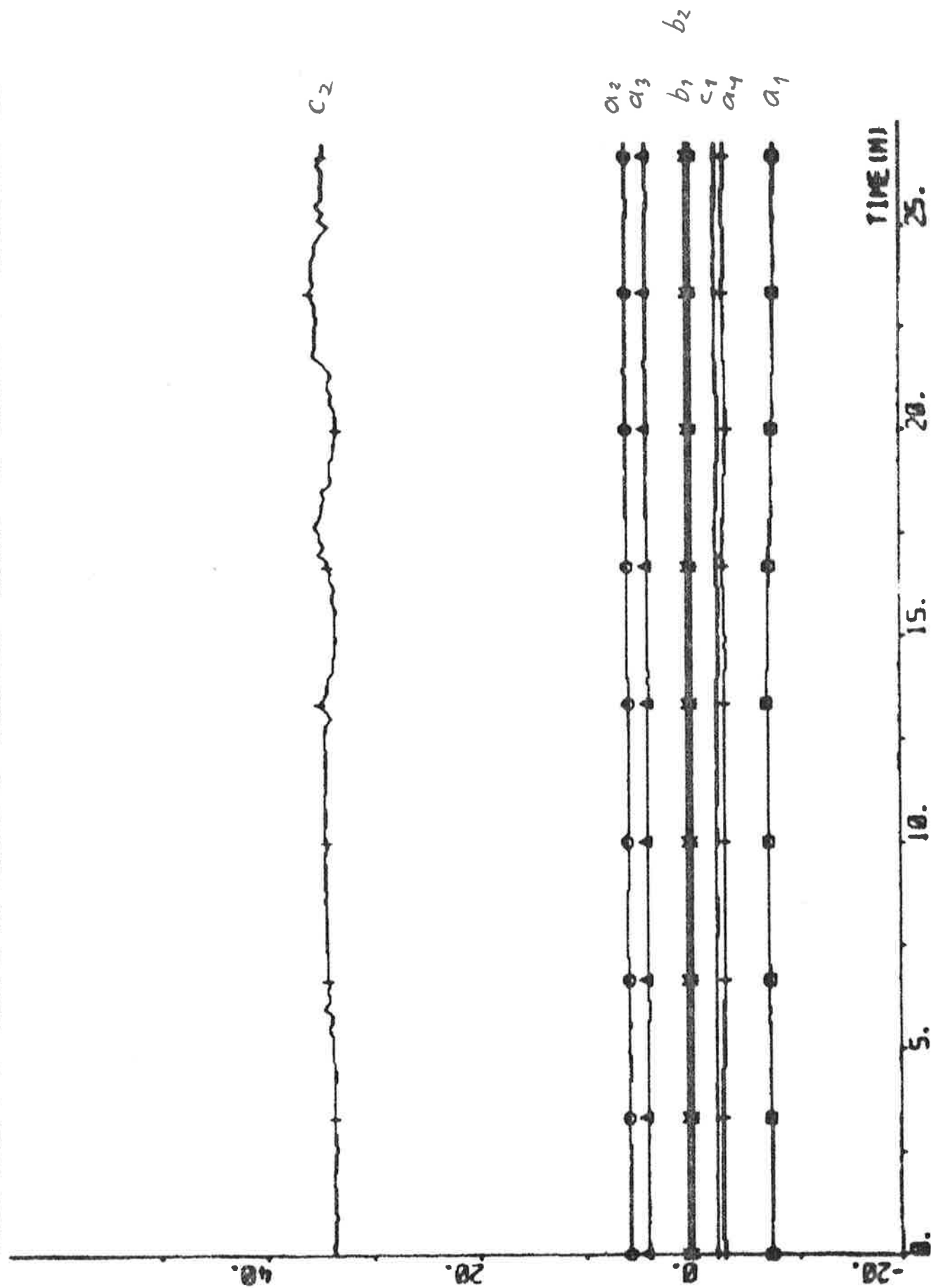


FL0T A20P1(1)-A20P2(3) 00 -0.5 1.5

$0.5 \times d_0$  deg

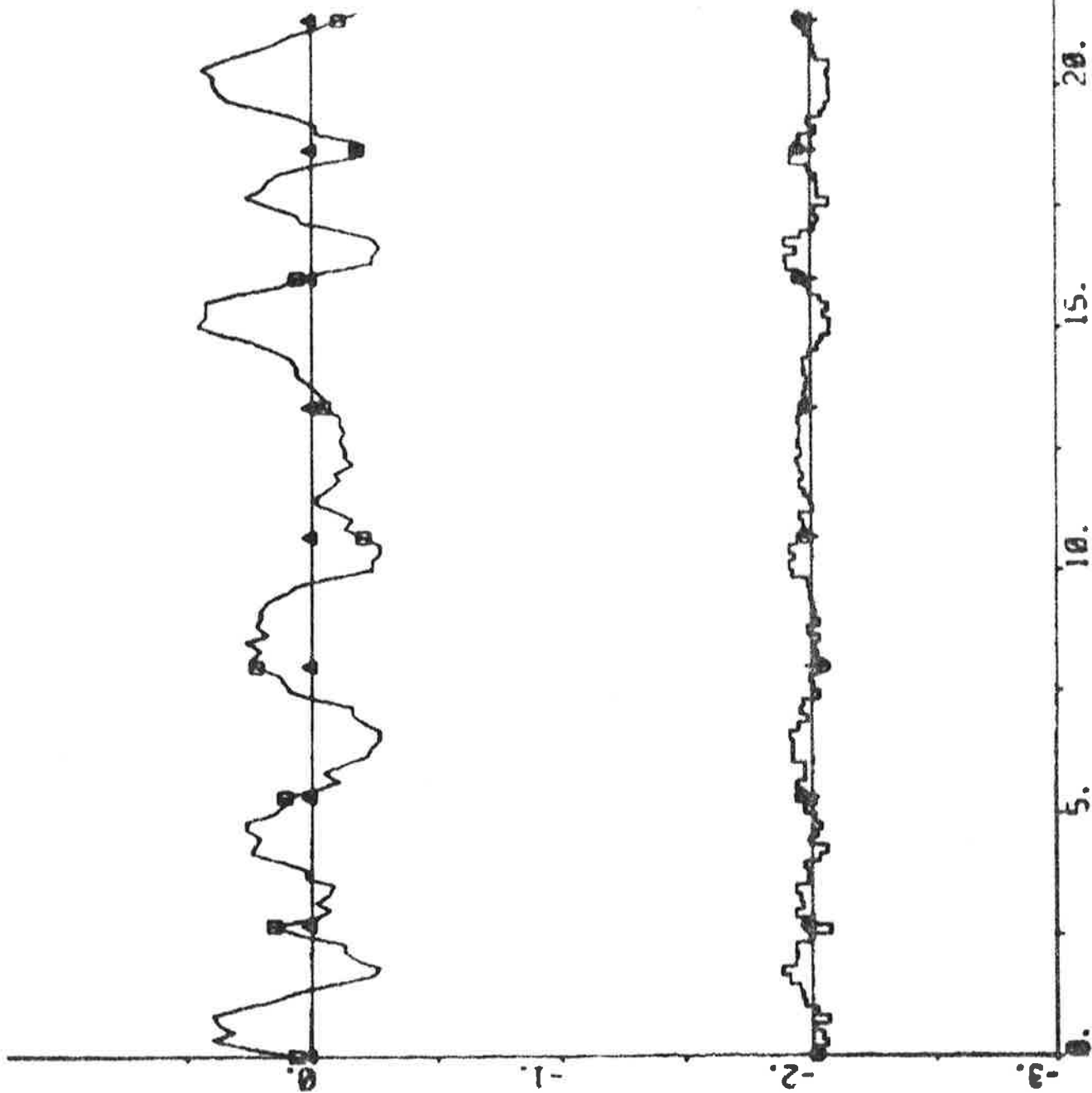


PLOT A20P1(1) A20P2(7) A20P2(8) A20P2(9) A20P2(10) A20P2(11) A20P2(12) A2





PLOT A27P1(1)-A27P1(8) HP A27P1(10) A27P1(15) Q2 -3 1 - DEG



$\psi - \psi_{ref}$

$0.05 * d_c$

TIME (MI)

25.

20.

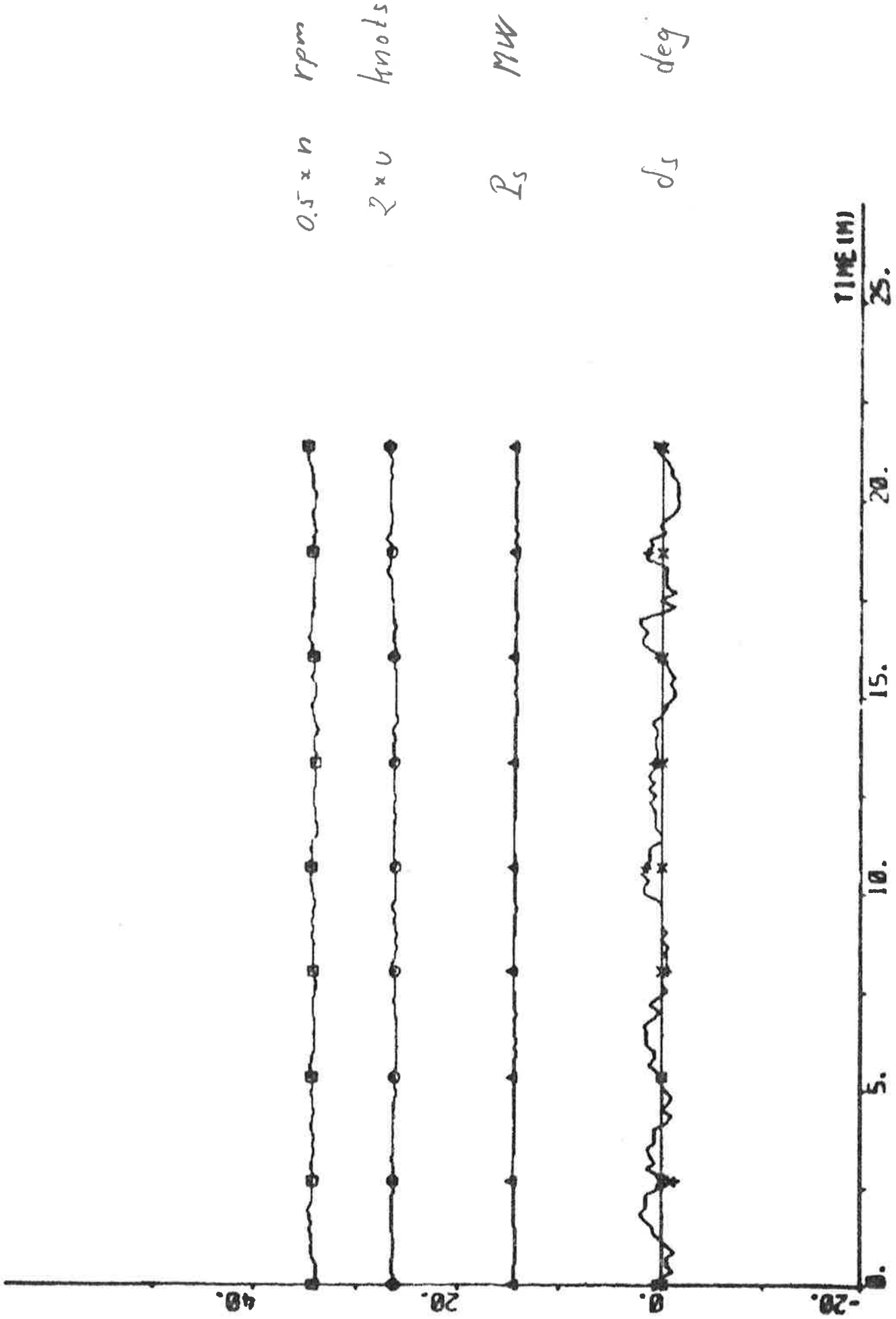
15.

10.

5.

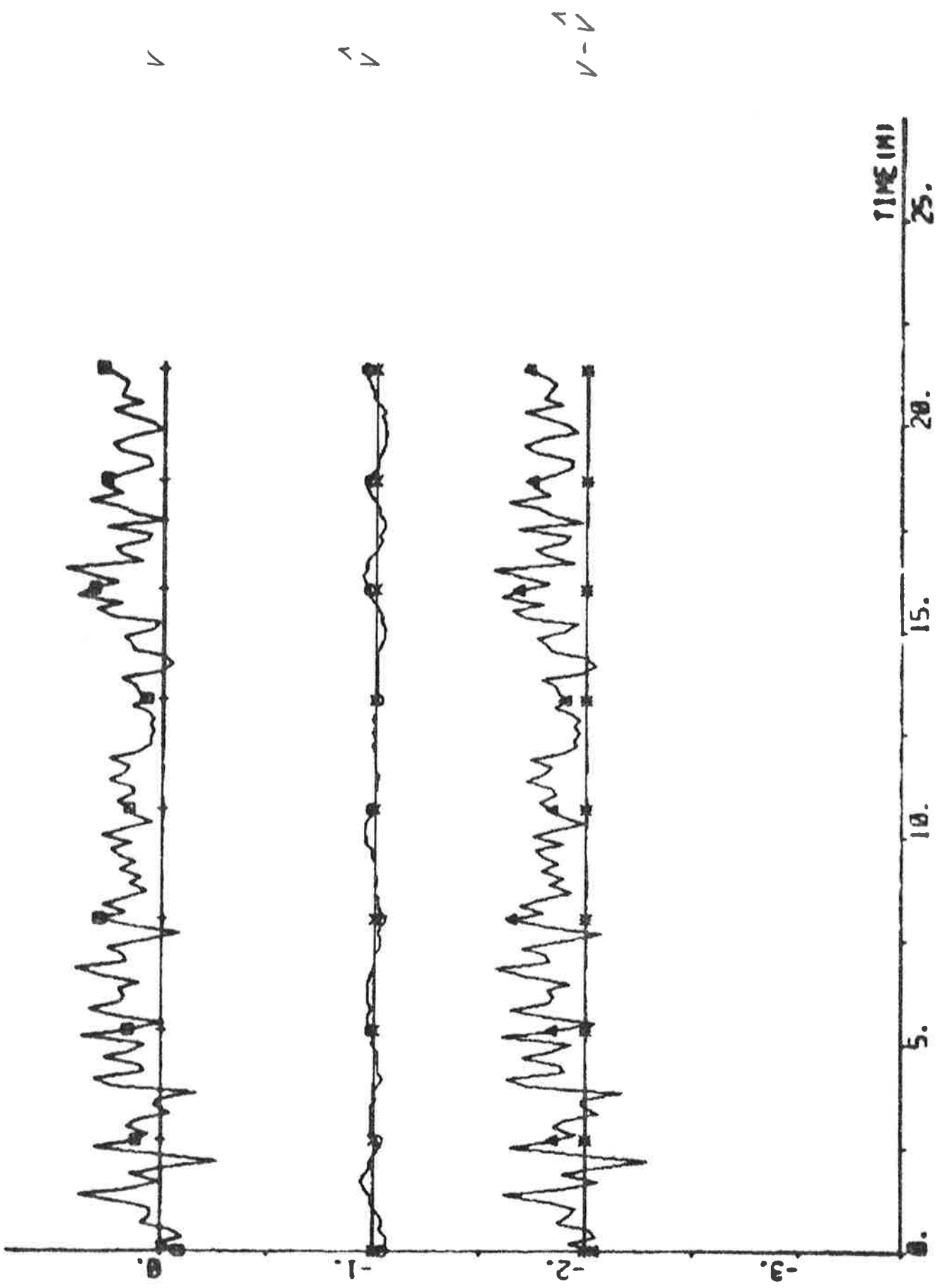
0.

PLOT A27P1(1)-A27P1(13) A27P1(12) A27P1(14) A27P1(11) 00 -20 50

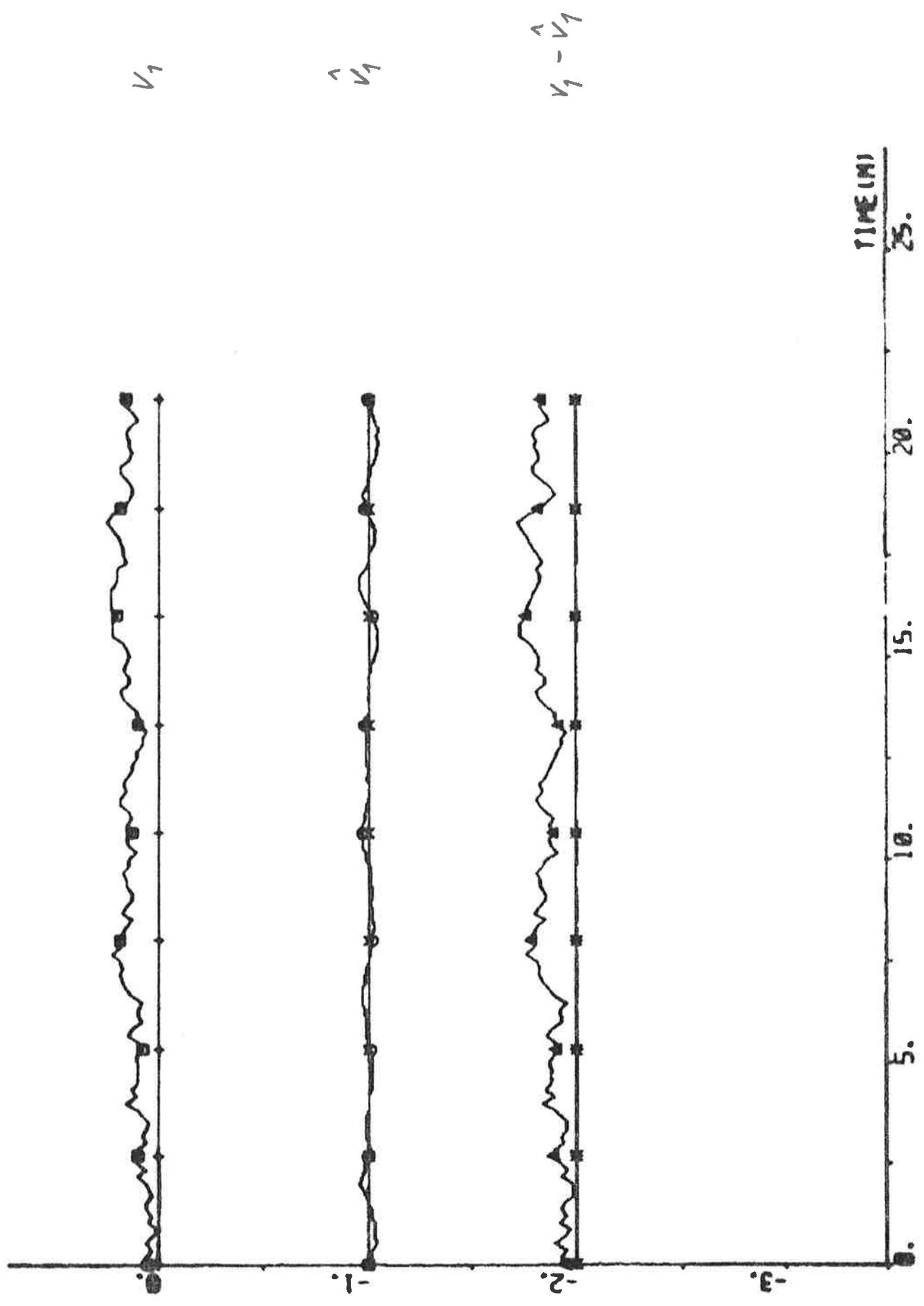




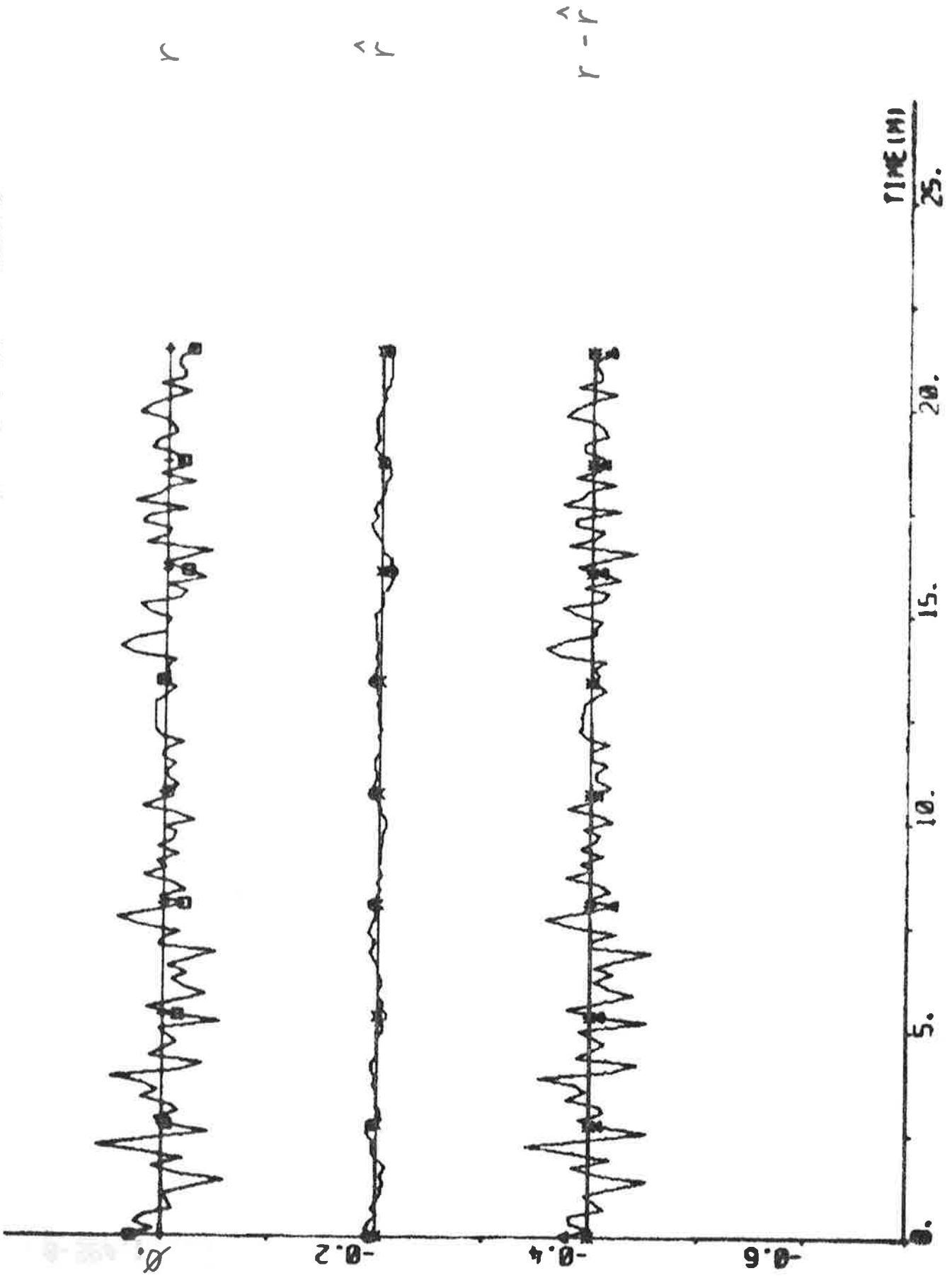
PLOT A27P1(1)-A27P2(1) A27P2(2) ERRS 00 01 02 -3.4 0.0 - KNOTS



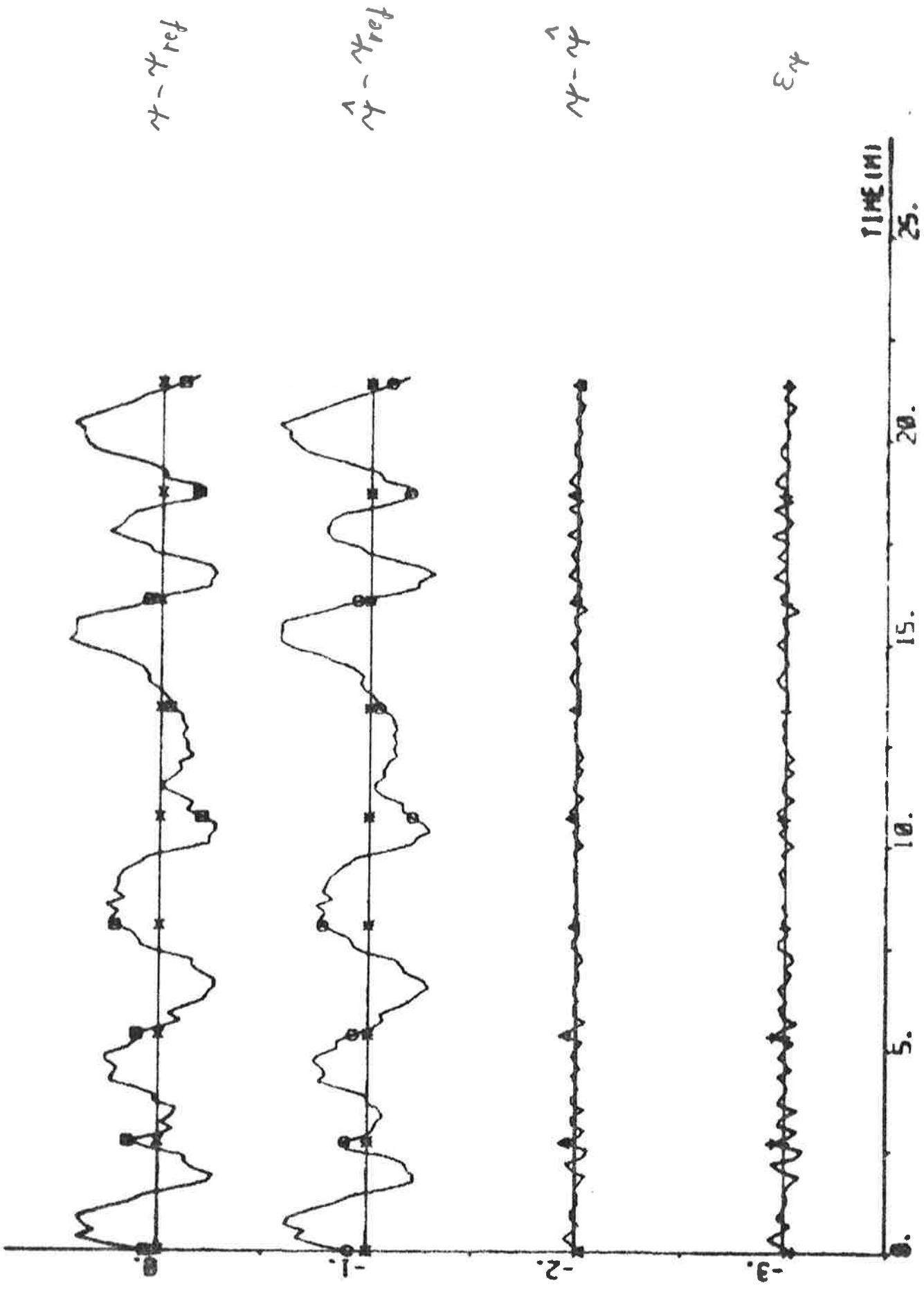
PLOT A27P1(1)-A27P1(4) A27P1(5) ERR2 00 01 02 -3.4 0.0 - KNOTS



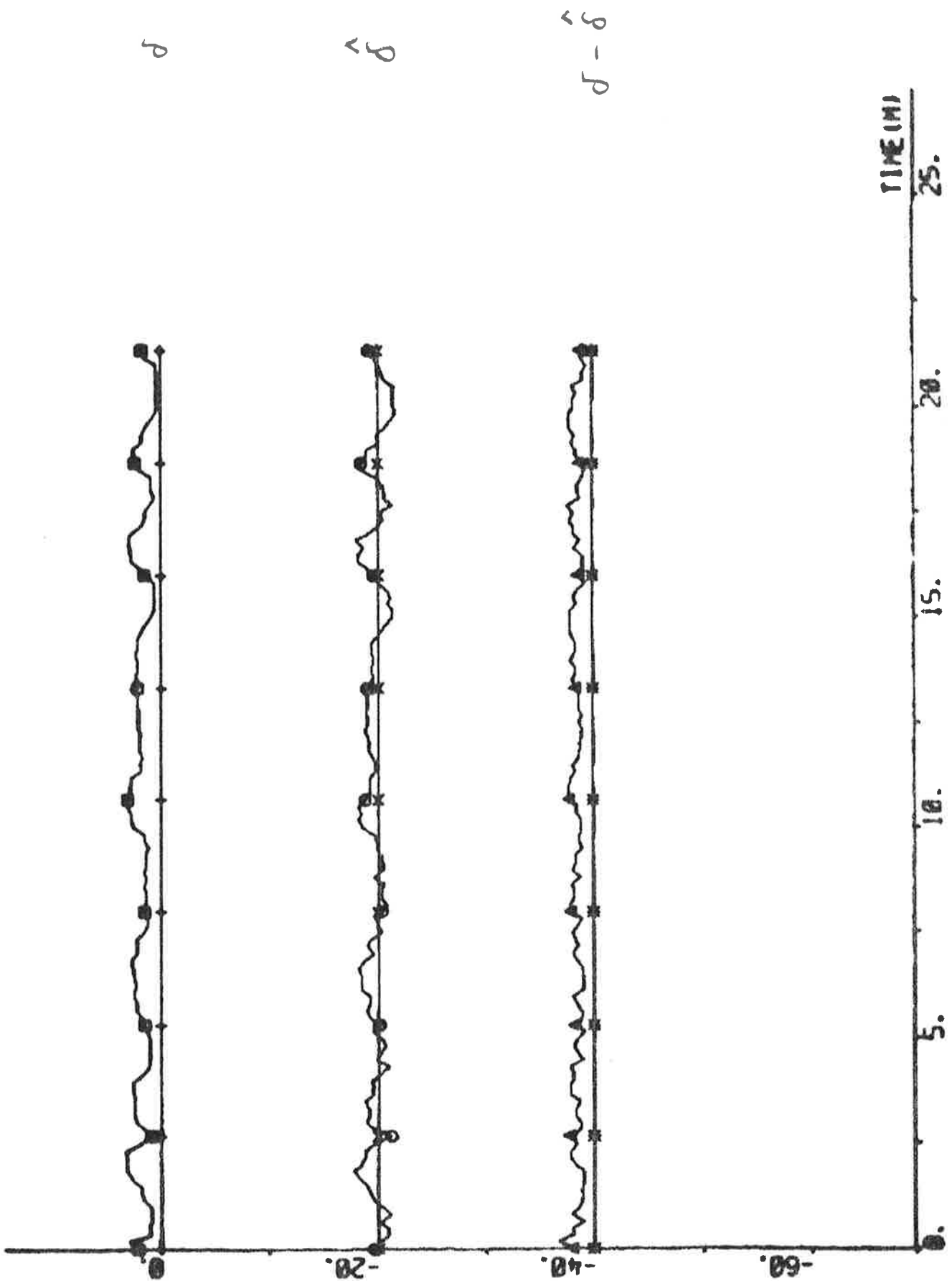
PLOT A27P1(1)-A27P1(8) A27P1(7) ERR3 00 002 004 -0.7 0. - DEG-S



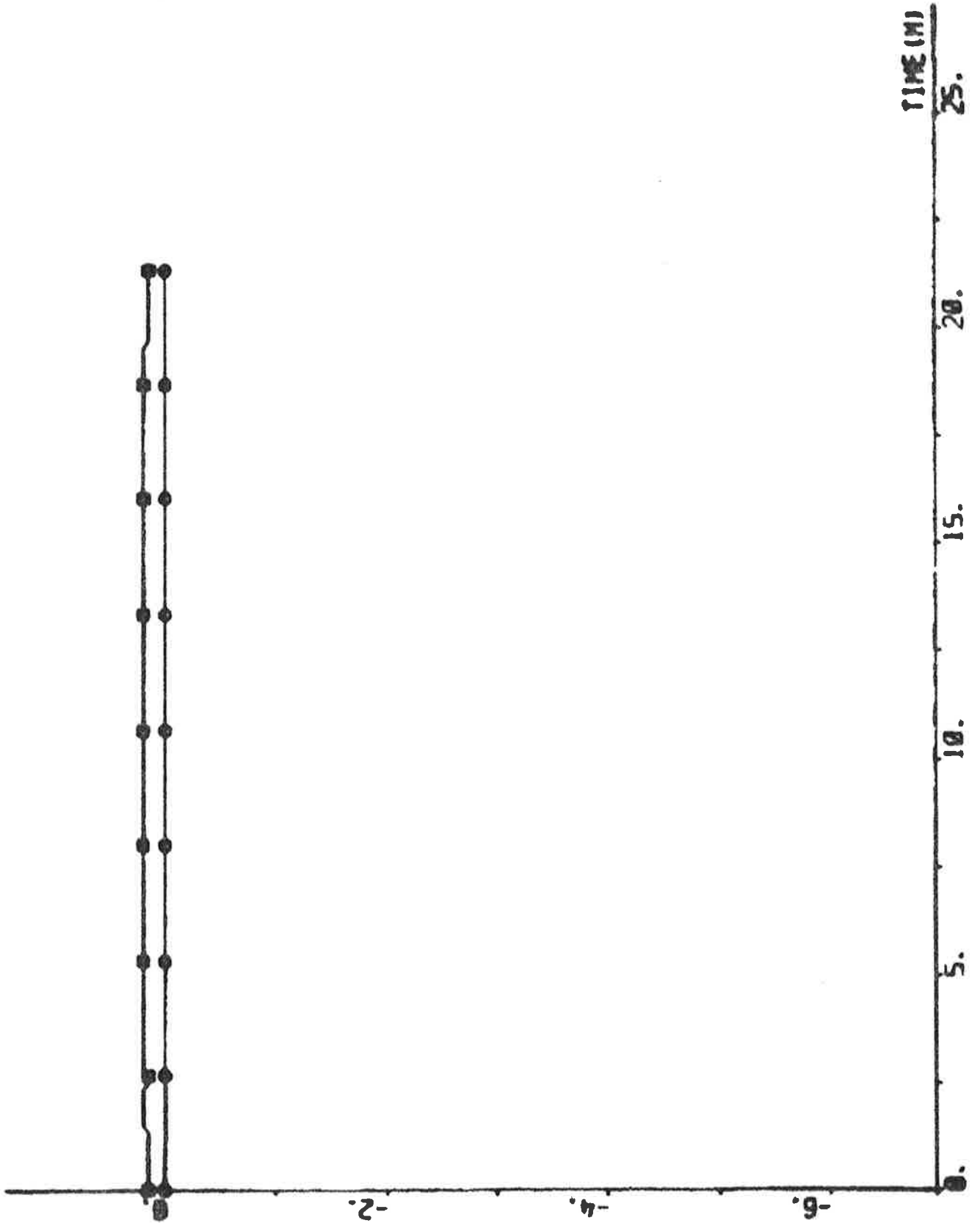
PLOT A27P1(1)-A27P1(8) A27P1(9) ERR4 00 01 02 03 -3.4 0.0 - DEG



PLOT A27P1(1)-A27P1(2) A27P1(3) ERR1 00 020 040 -05 15 - DEG

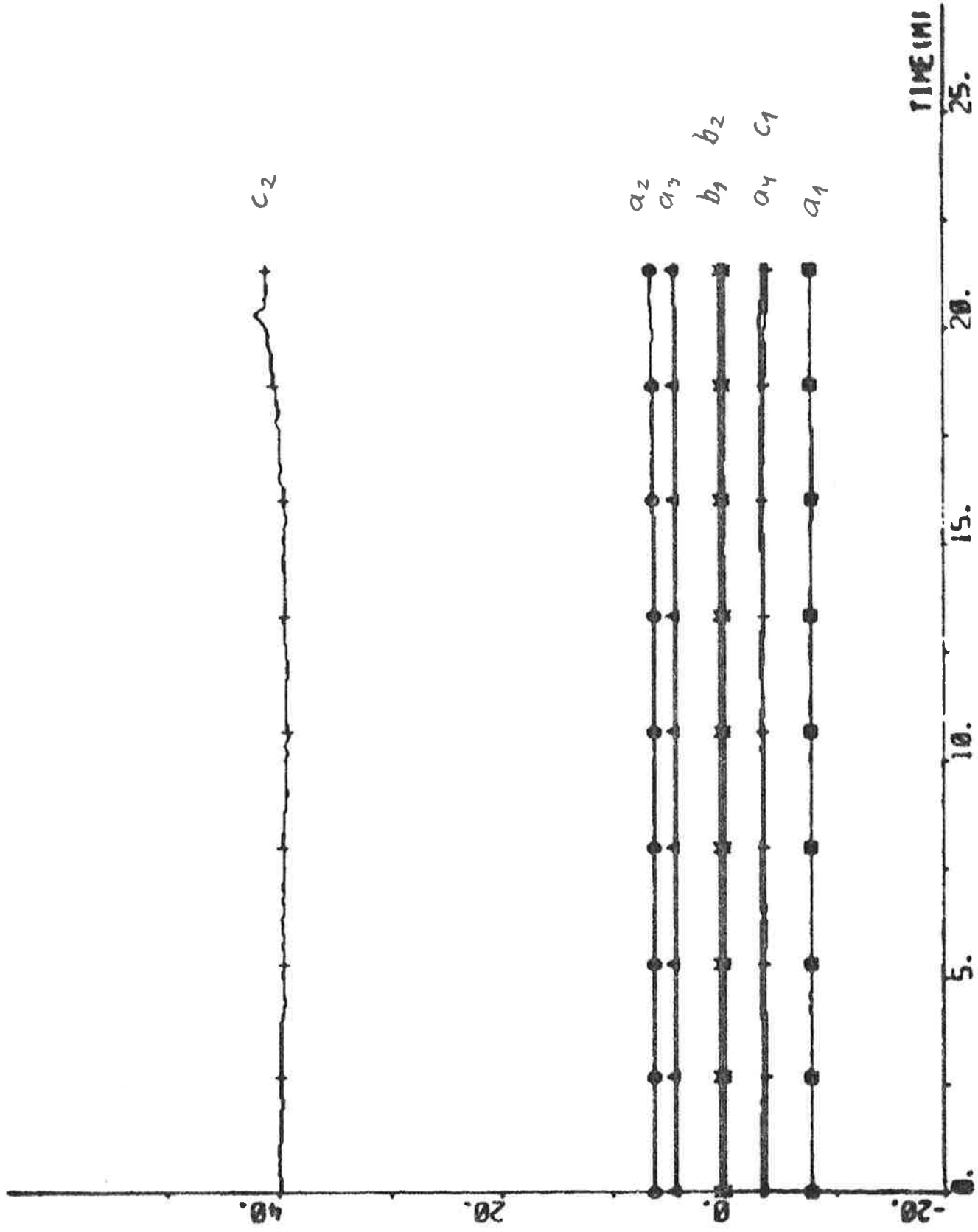


PLOT A27P1(1)-A27P2(3) 00 -0.5 1.5



$0.5 \times d_0$  deg

PLOT A27P1(1)-A27P2(7) A27P2(8) A27P2(9) A27P2(10) A27P2(11) A27P2(12) A2



## EXPERIMENT A28

Date	1976-04-26	Forward draught	10.9 m
Time	12.52	Aft draught	12.9 m
Duration	24 min	Wind direction	SE (1; see App. A)
Position	S 06°06' W 06°32'	Wind velocity	4 m/s (gentle breeze)
$\psi_{\text{ref}}$	143 deg	Wave height	-

Self-tuning regulator using estimates from the Kalman filter

Tuning time before the experiment started: 30 min

NC1 = 1	NC2 = 1	k = 7	q = 0
$T_s = 10$ s	$V_0 = 6$ m/s	IVVC = 1	

Final values:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ b_1 \\ b_2 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} -8.13 \\ 6.65 \\ 4.67 \\ -3.33 \\ 0.47 \\ 0.10 \\ -1.94 \\ 39.40 \end{bmatrix} \quad p = \begin{bmatrix} 3.92 \\ -5.51 & 12.92 \\ 0.54 & -8.61 & 15.22 \\ 1.55 & 0.86 & -7.45 & 5.55 \\ 0.07 & -0.34 & 0.33 & -0.07 & 0.02 \\ 0.02 & 0.02 & -0.17 & 0.16 & 0.00 & 0.02 \\ -0.75 & 1.00 & 0.19 & -0.48 & -0.01 & 0.01 & 0.79 \\ 1.67 & -2.97 & -2.81 & 3.37 & 1.03 & -0.40 & 13.89 & 682.35 \end{bmatrix}$$

$$a_1 + a_2 + a_3 + a_4 = -0.14$$

$$\hat{\delta}_0 = 0.3 \text{ deg} \quad \hat{d}_v = 0.19 \text{ knots} \quad \hat{d}_r = 0.001 \text{ deg/s} \quad \hat{d}_\delta = 1.5 \text{ deg}$$

Statistics (mean value and standard deviation)

$\delta_c$	0.16 ± 0.95 deg	$P_s$	15.0 ± 0.1 MW
$\delta$	1.72 ± 0.87 deg	$\epsilon_v$	0.00 ± 0.02 knots
$\psi - \psi_{\text{ref}}$	0.006 ± 0.239 deg	$\epsilon_r$	0.00 ± 0.02 deg/s
n	69.9 ± 0.4 rpm	$\epsilon_\psi$	0.00 ± 0.03 deg
u	13.5 ± 0.1 knots	$\epsilon_\delta$	0.0 ± 0.5 deg
$V_1 = 0.132$			



PLOT A2BP1(1)-A2BP1(8) HP A2BP1(10) A2BP1(15) Q2 -3 1 - DEG

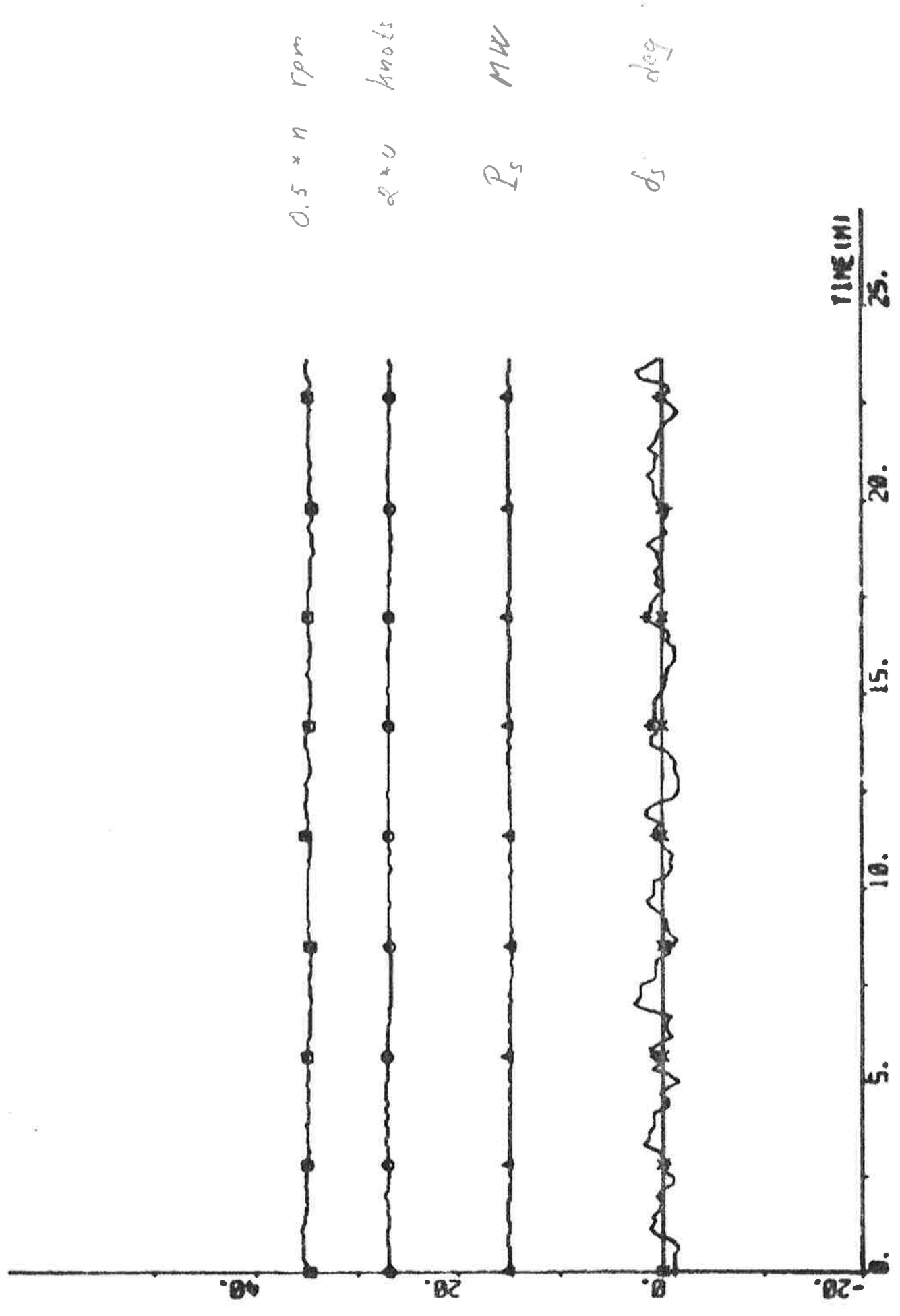


$\gamma - \gamma_{ref}$

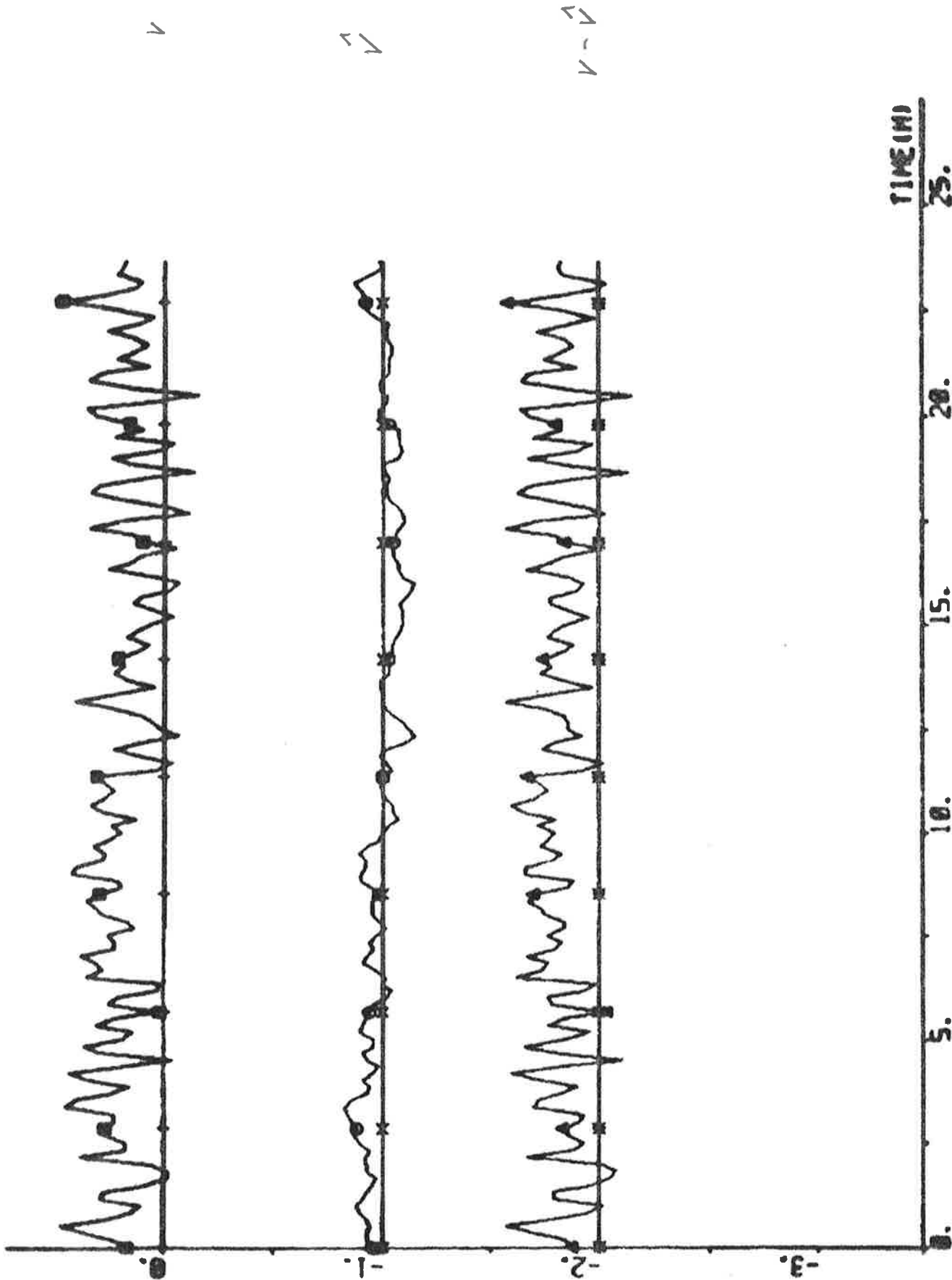
$0.05 \times d_c$



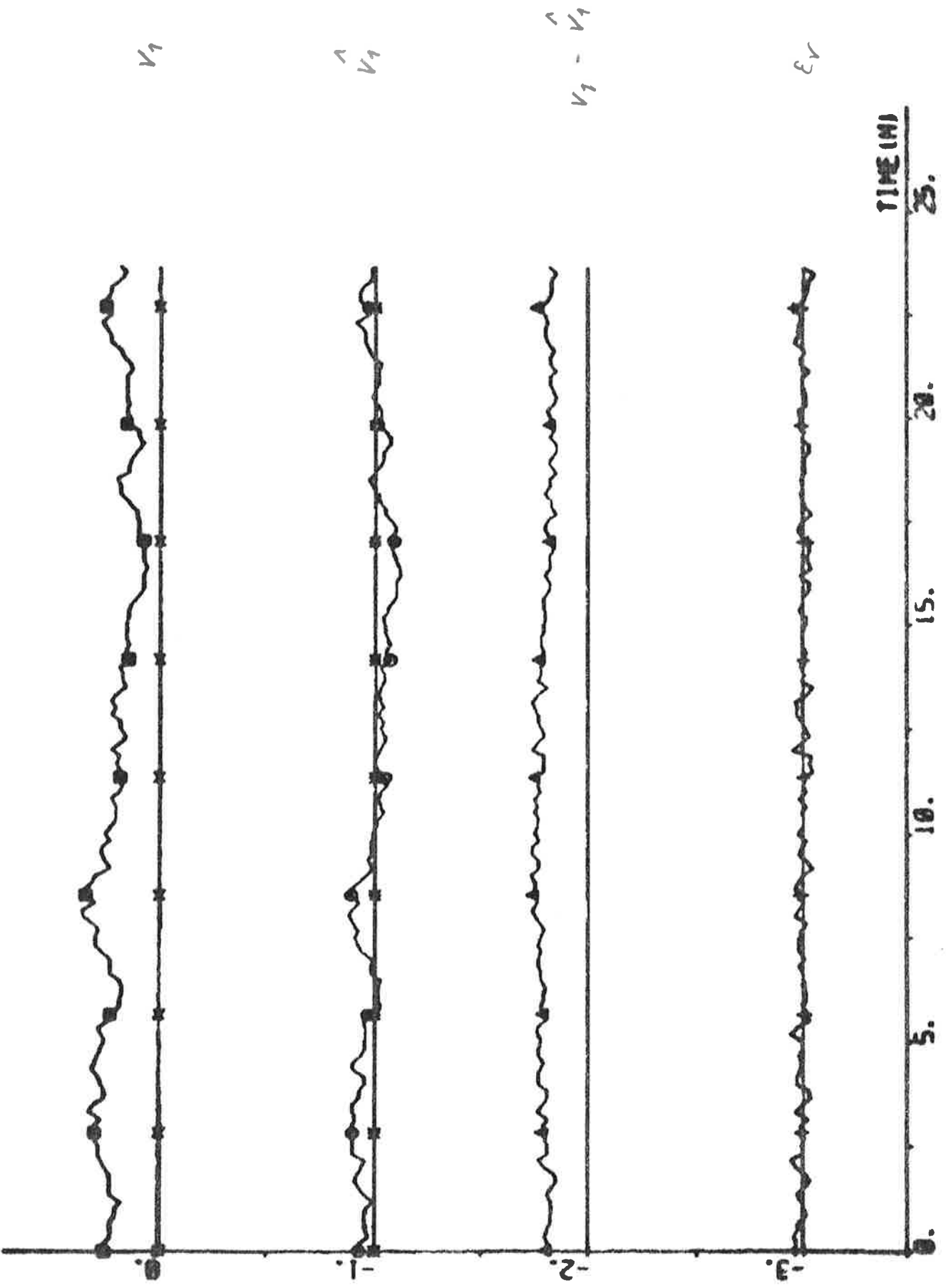
PLOT A20P1(1)-A20P1(13) A20P1(12) A20P1(14) A20P1(11) 00 -20 00



PL0T A2SP1(1)-A2SP2(1) A2SP2(2) ERMS 00 01 02 -3.4 0.0 - KNOTS



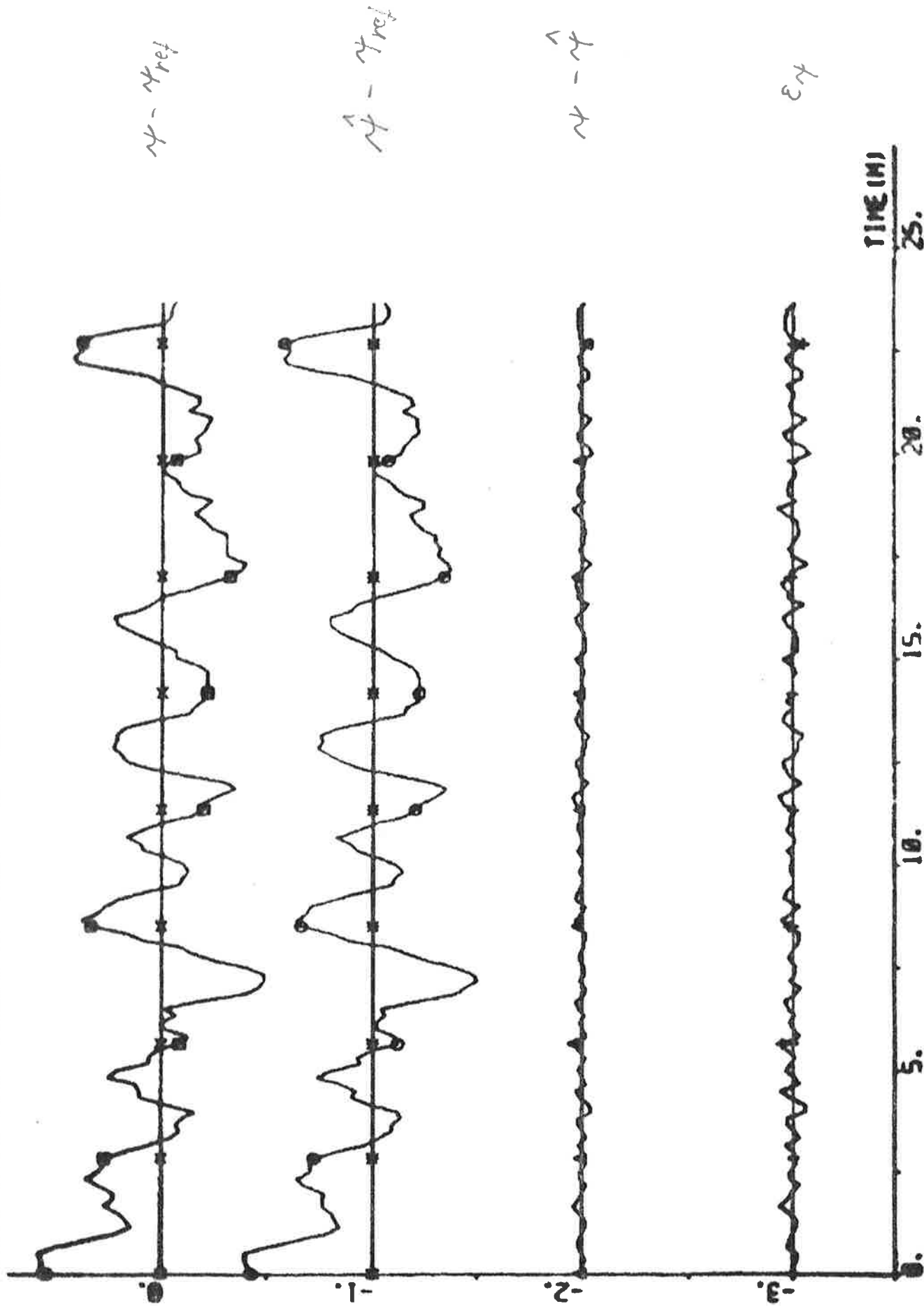
PLOT A20P1(1)-A20P1(4) A20P1(5) ERR2 EPS2 00 01 02 03 -3.4 0.0 - KNOTS



PLST R20P1(11)-R20P1(6) R20P1(7) ERR3 EPS3 00 002 004 006 -0.7 0. - 000-1



PLOT A20P1(1)-A20P1(8) A20P1(9) ERR4 EPS4 G0 01 02 03 -3.4 0.0 - DEG



$y - y_{ref}$

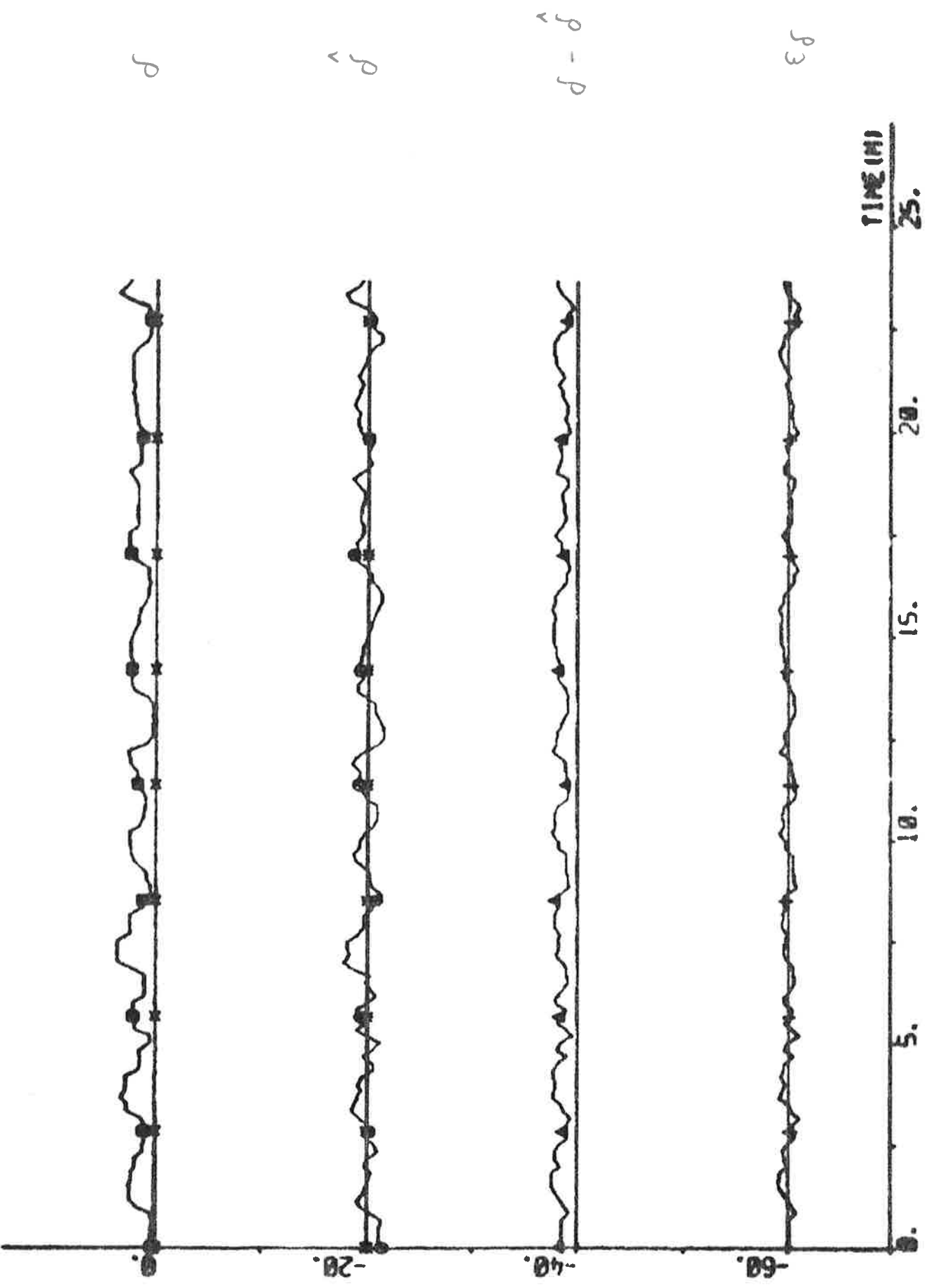
$y - y_{ref}$

$y - y$

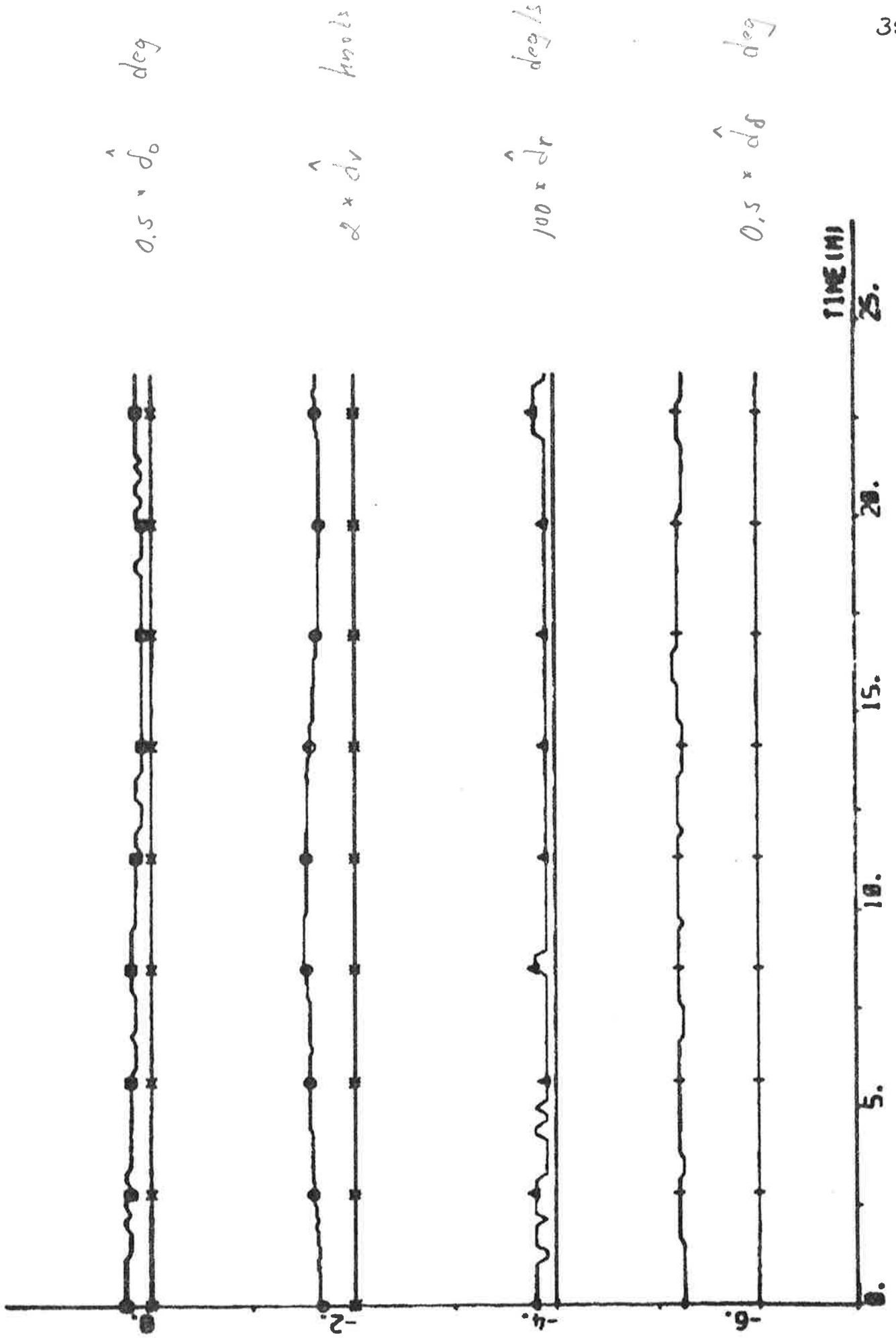
$E4$

TIME (H)  
5. 10. 15. 20. 25.

PLOT A2SP1(1)-A2SP1(2) A2SP1(3) ERR1 EPS1 00 020 040 060 -03 18 - 000

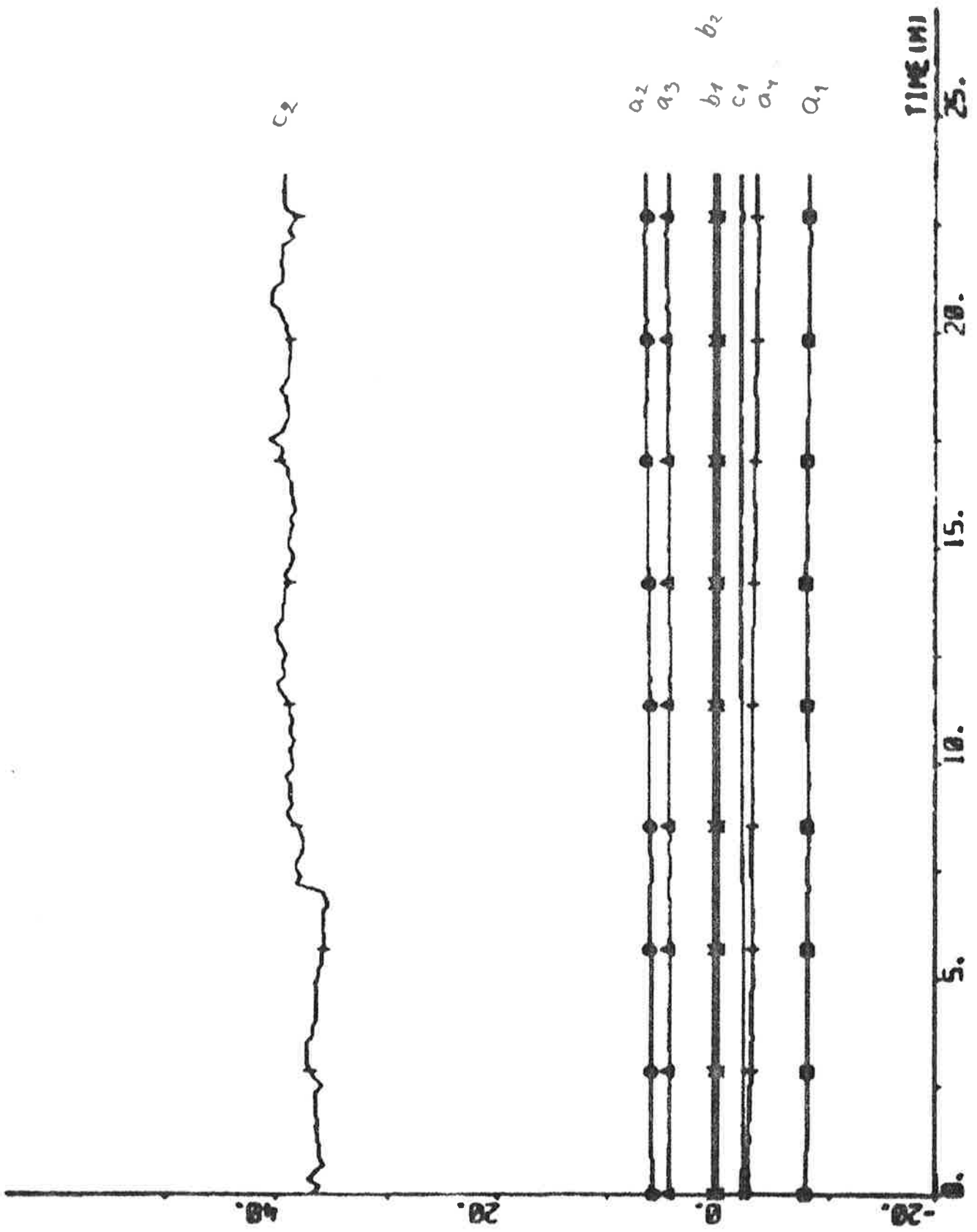


PLOT A2OP1(1)-A2OP2(3) A2OP2(4) A2OP2(5) A2OP2(6) 00 02 04 06 -0.5 1.5





PLOT A20P1(1)-A20P2(7) A20P2(8) A20P2(9) A20P2(10) A20P2(11) A20P2(12) A2



## EXPERIMENT A29

Date	1976-04-26	Forward draught	10.9 m
Time	13.58	Aft draught	12.9 m
Duration	24 min	Wind direction	SE (1; see App. A)
Position	S 06°18' W 06°24'	Wind velocity	3 m/s (light breeze)
$\psi_{\text{ref}}$	143 deg	Wave height	-

Self-tuning regulator using non-filtered measurements

Tuning time before the experiment started: 30 min.

$NC1 = 0$              $NC2 = 0$              $k = 7$              $q = 0$   
 $T_s = 10 \text{ s}$          $V_0 = 6 \text{ m/s}$          $IVVC = 1$

Final values:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} -9.74 \\ 8.98 \\ 6.22 \\ -5.25 \\ 0.29 \\ -0.06 \end{bmatrix} \quad P = \begin{bmatrix} 1.46 & & & & & & \\ -1.98 & 6.56 & & & & & \\ -0.10 & -5.46 & 10.48 & & & & \\ 0.85 & 0.59 & -5.04 & 4.00 & & & \\ -0.01 & -0.34 & 0.47 & -0.11 & 0.05 & & \\ 0.07 & -0.07 & -0.30 & 0.33 & 0.00 & 0.04 & \end{bmatrix}$$

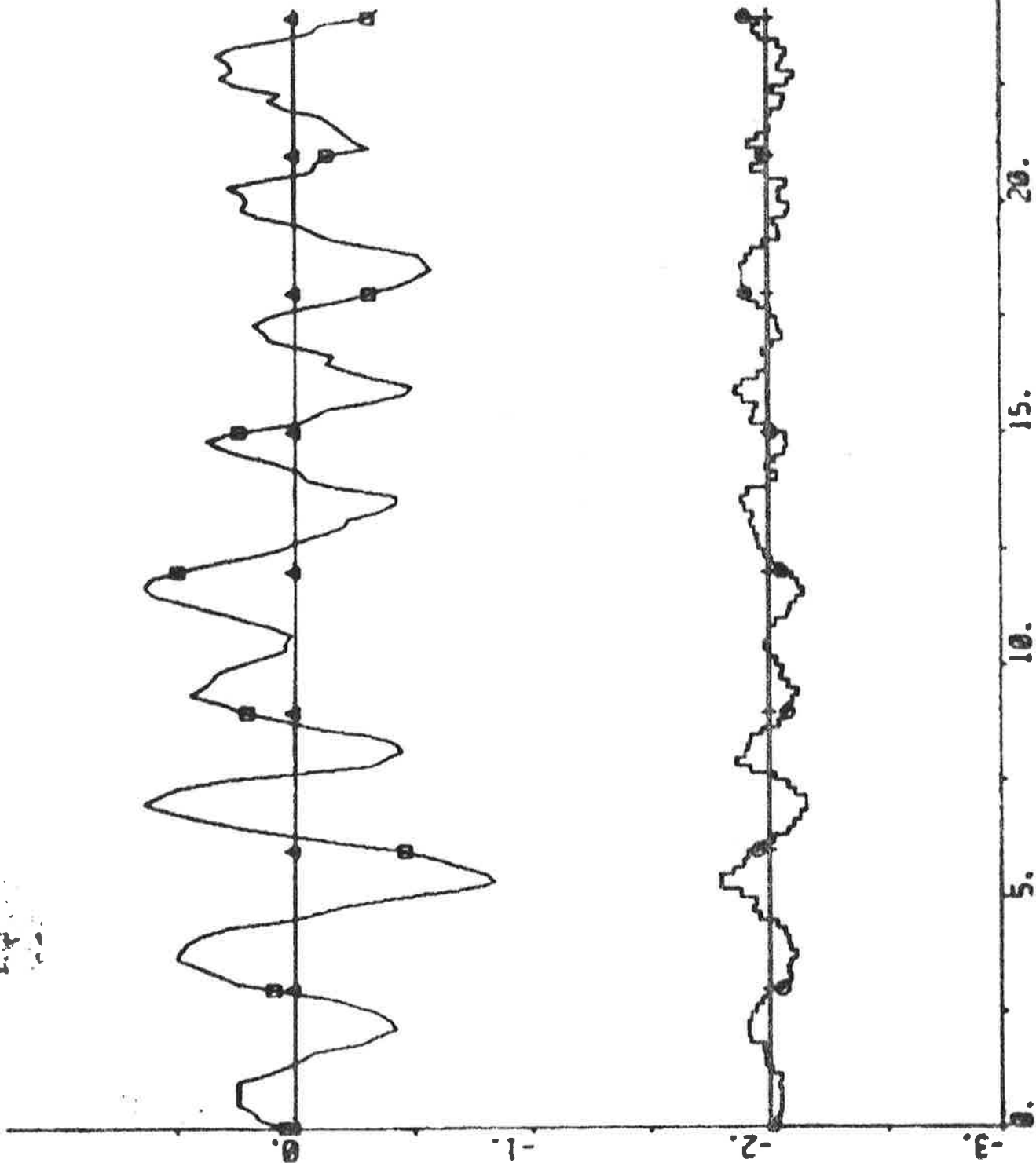
$$a_1 + a_2 + a_3 + a_4 = 0.21$$

$$\hat{\delta}_0 = 0.1 \text{ deg} \quad \hat{d}_V = 0.17 \text{ knots} \quad \hat{d}_r = 0.001 \text{ deg/s} \quad \hat{d}_\delta = 1.6 \text{ deg}$$

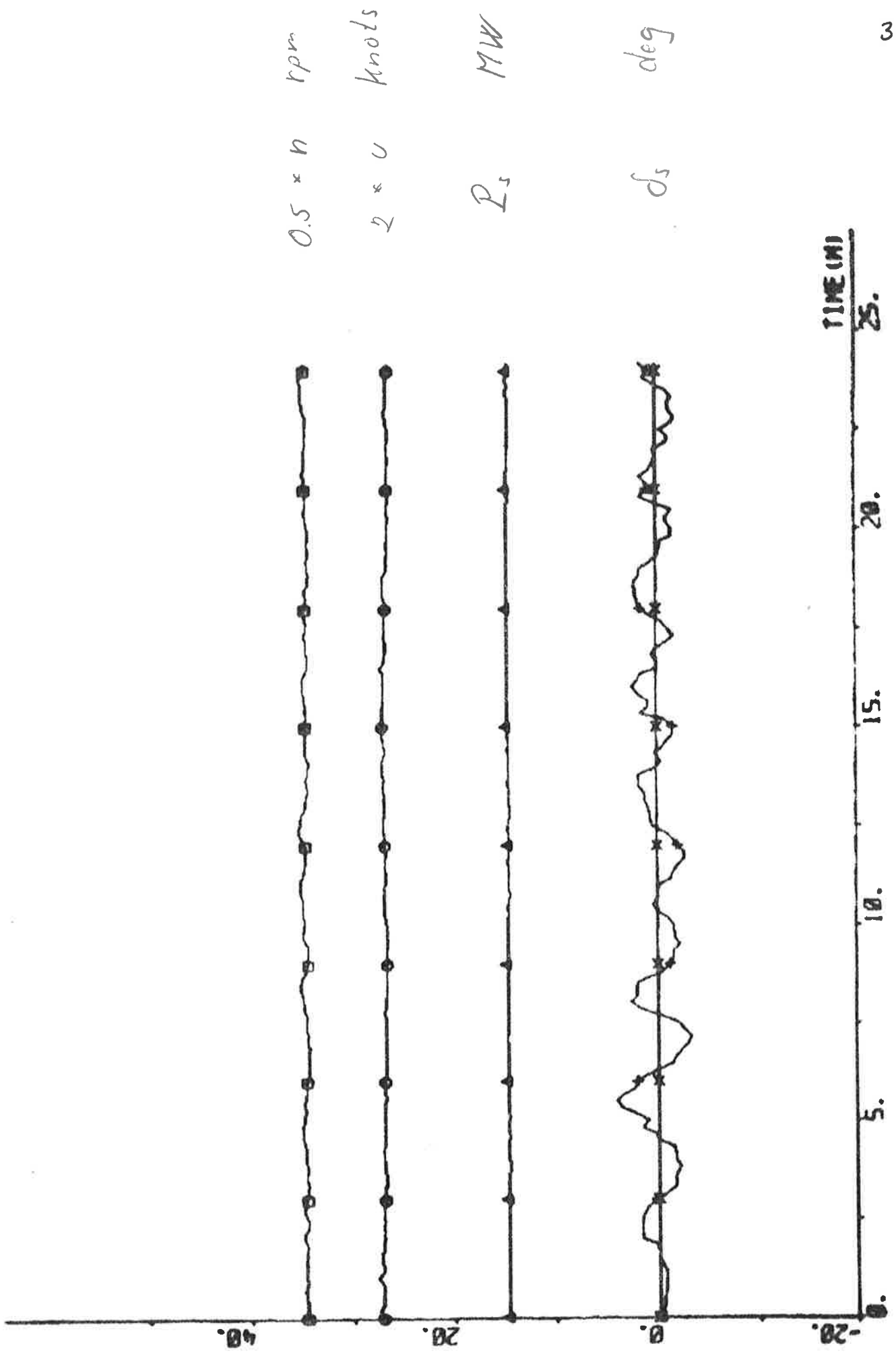
Statistics (mean value and standard deviation)

$\delta_c$	-0.01	± 1.51	deg	$P_s$	14.6	± 0.1	MW
$\delta$	1.53	± 1.27	deg	$\epsilon_V$	0.00	± 0.02	knots
$\psi - \psi_{\text{ref}}$	0.001	± 0.324	deg	$\epsilon_r$	0.00	± 0.02	deg/s
$n$	69.5	± 0.4	rpm	$\epsilon_\psi$	0.00	± 0.03	deg
$u$	13.4	± 0.1	knots	$\epsilon_\delta$	0.0	± 0.5	deg
$V_1 = 0.295$							

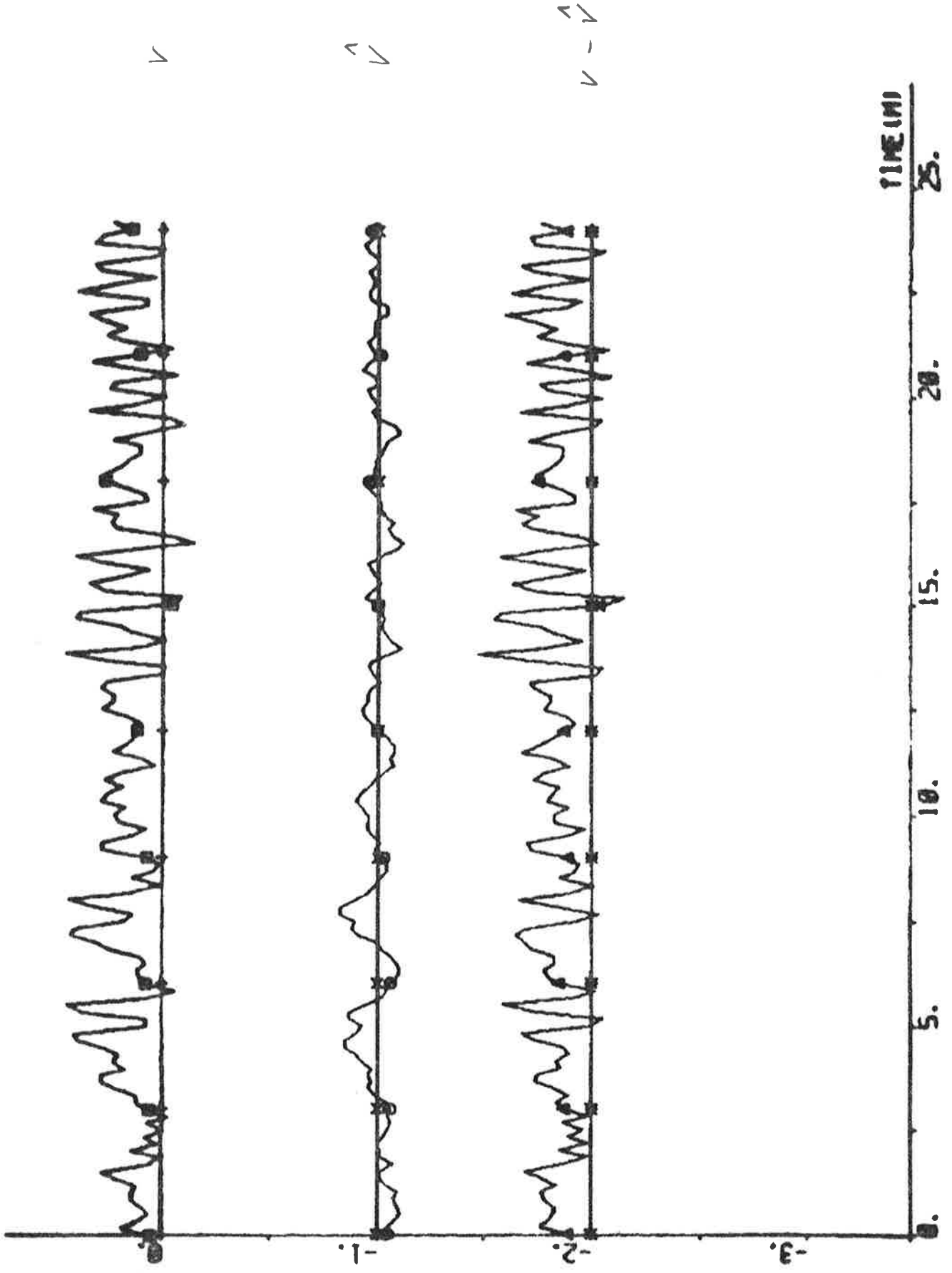
PL0T A23P1(1)-A23P1(9) HP A23P1(10) A23P1(15) 02 -3 1 - DEG



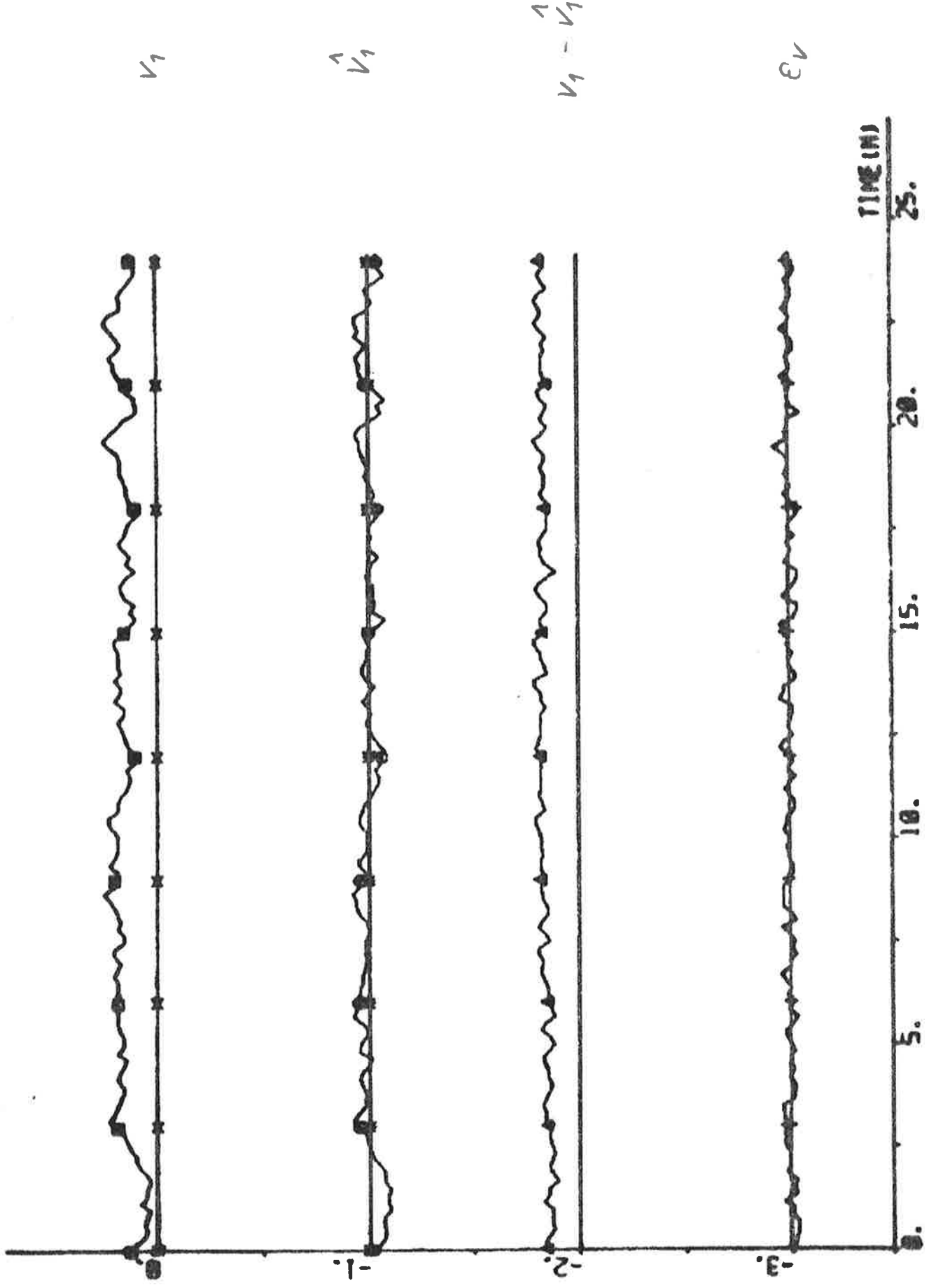
PLOT A29P1(1)-A29P1(13) A29P1(12) A29P1(14) A29P1(11) 00 -20 50



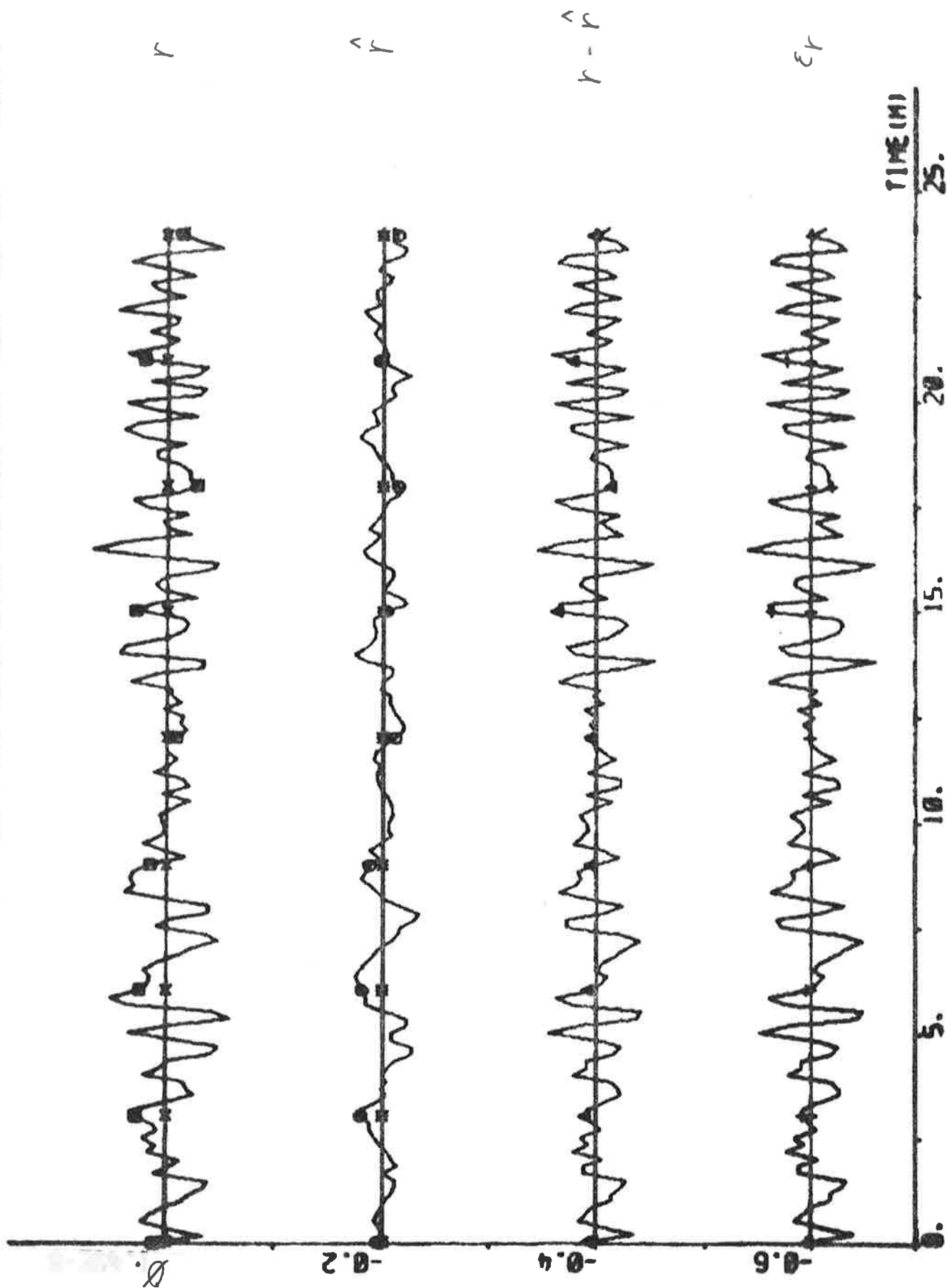
PLOT A2SP1(1)-A2SP2(1) A2SP2(2) ERRS 00 01 02 -3.4 0.0 - 100TS



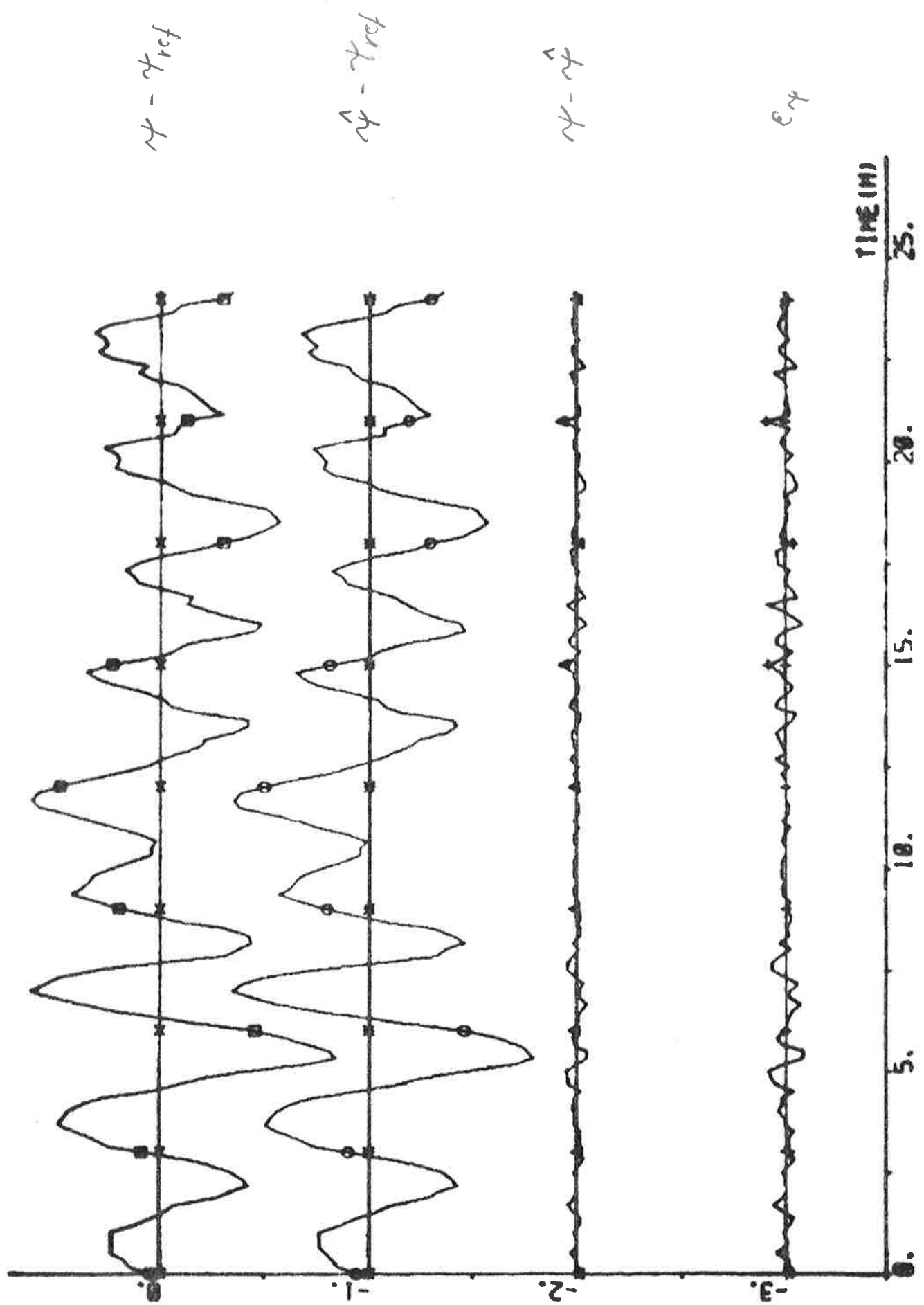
PLOT A20P1(1)-A20P1(4) A20P1(5) ERR2 EPS2 00 01 02 03 -3.4 0.0 - 100013



PLOT A29P1(1)-A29P1(6) A29P1(7) ERRO EPS3 00 002 004 006 -0.7 0. - DECS

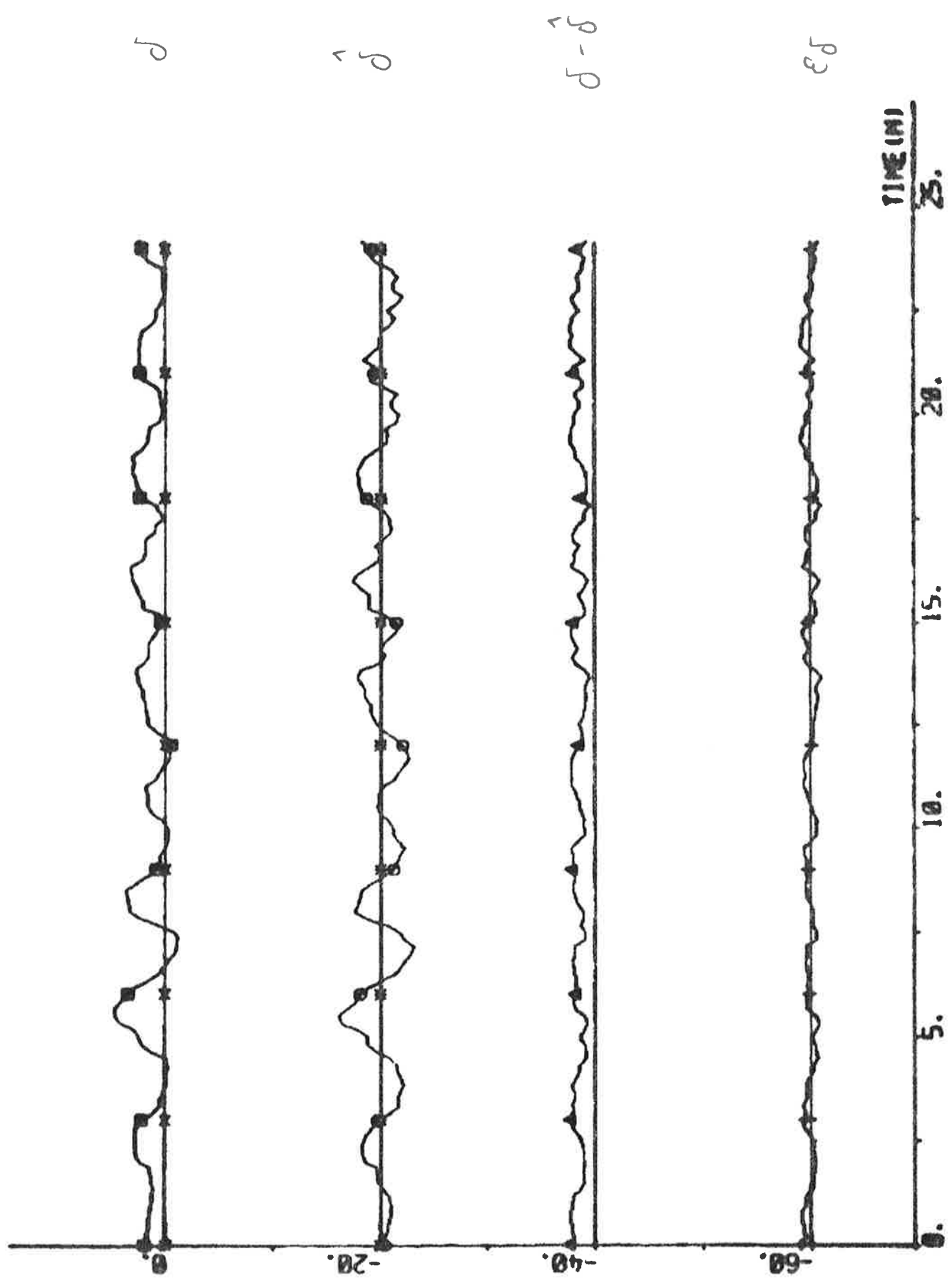


PLOT A29P1(1)-A29P1(8) A29P1(9) ERR4 EPS4 00 01 02 03 -3.4 0.0 - DES

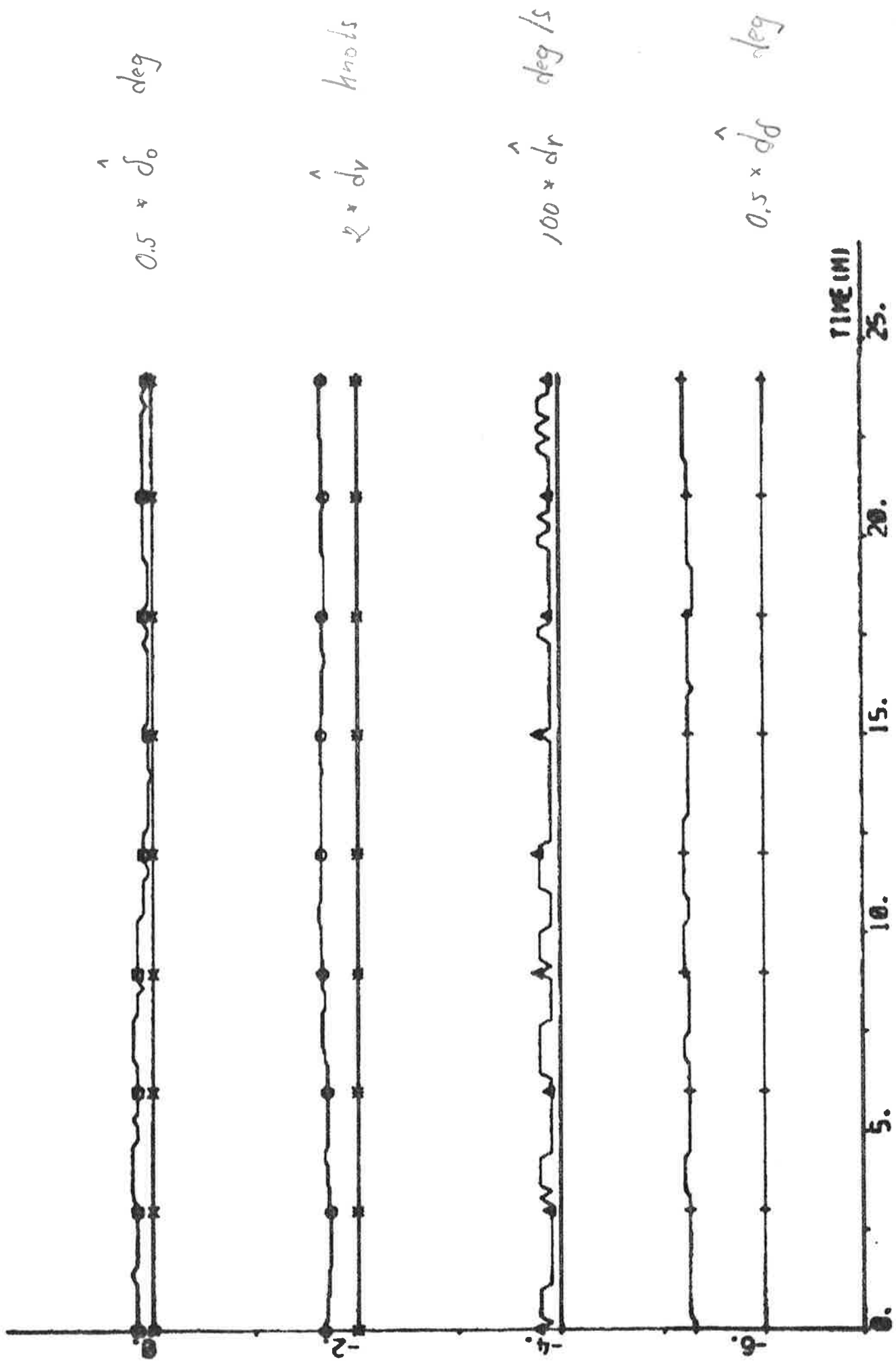




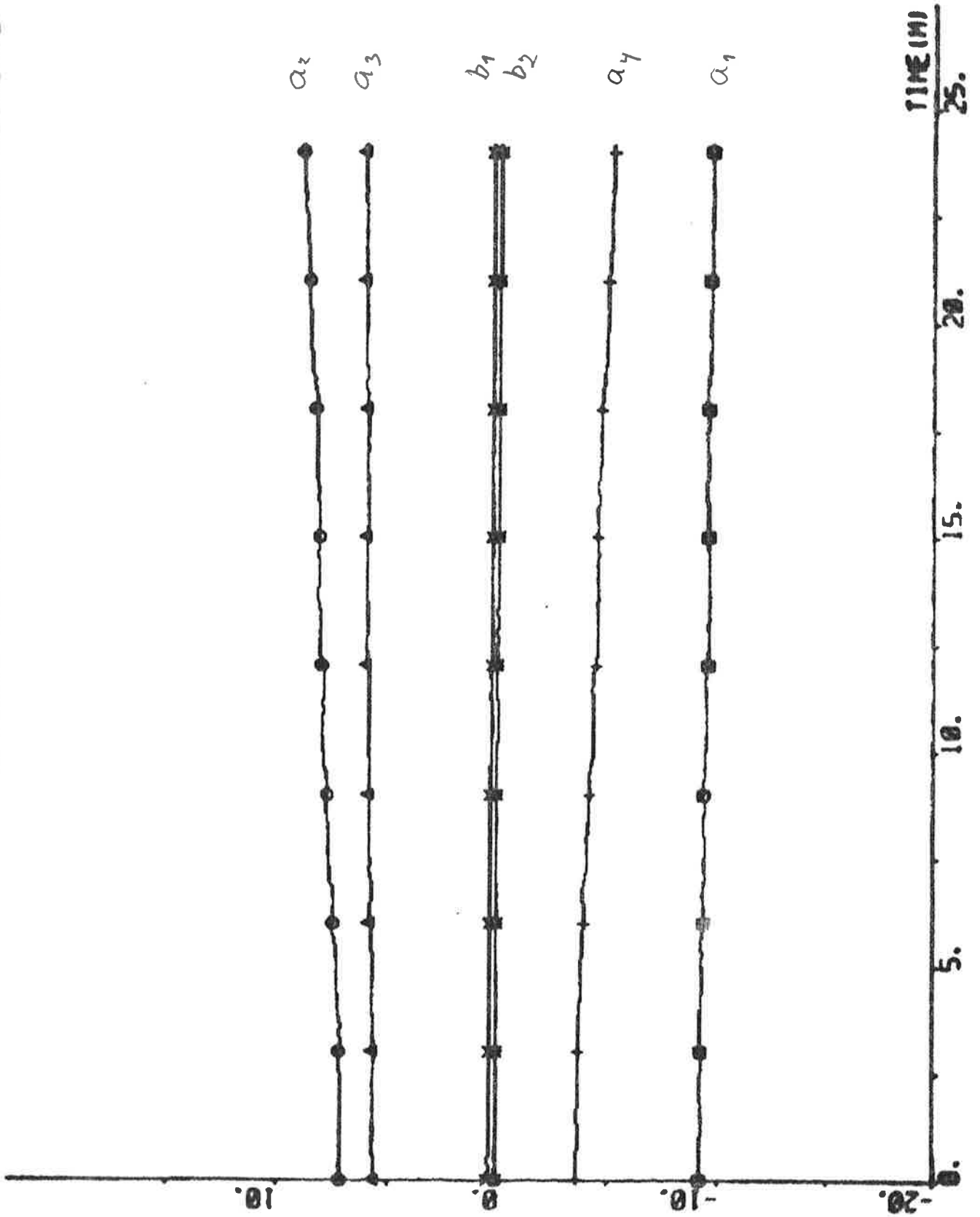
PLOT A2SP1(1)-A2SP1(2) A2SP1(3) ERR1 EPS1 00 020 040 060 -06 16 - 000



PLOT A20P1(1)-A20P2(3) A20P2(4) A20P2(5) A20P2(6) 00 02 04 06 -0.6 1.6



PL0T A20P1(1) A20P2(7) A20P2(8) A20P2(9) A20P2(10) A20P2(11) A20P2(12) -2



## EXPERIMENT A30

Date	1976-04-26	Forward draught	10.9 m
Time	15.12	Aft draught	12.9 m
Duration	25 min	Wind direction	SE (1; see App. A)
Position	S 06°32' W 06°13'	Wind velocity	5 m/s (gentle breeze)
$\psi_{\text{ref}}$	143 deg	Wave height	-

PID-regulator using estimates from the Kalman filter.

$k_P = 3$                        $k_D = 120 \text{ s}$                        $k_I = 0.02 \text{ 1/s}$

$T_s = 10 \text{ s}$                        $V_0 = 6 \text{ m/s}$                        $IVVC = 1$

The parameters were manually tuned before the experiment started.

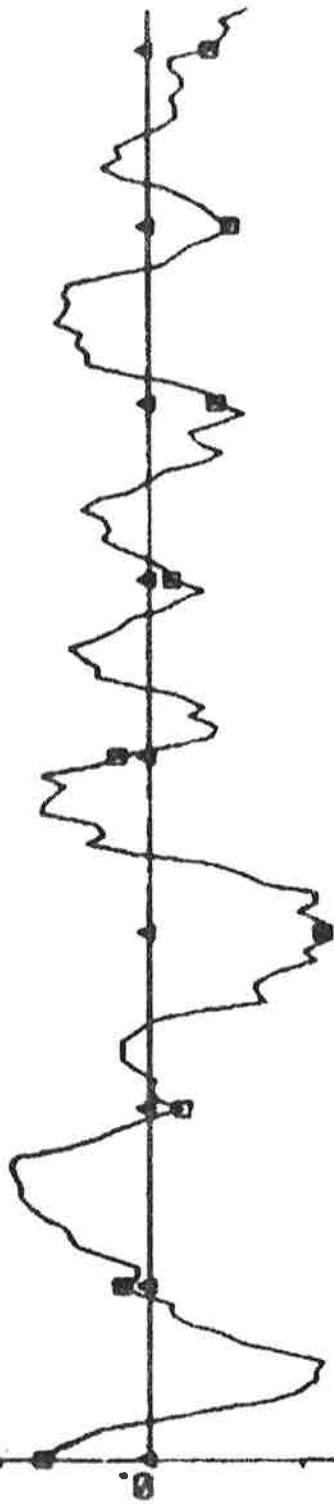
Final values:

$\hat{\delta}_0 = 0.0 \text{ deg}$      $\hat{d}_v = 0.16 \text{ knots}$      $\hat{d}_r = 0.001 \text{ deg/s}$      $\hat{d}_\delta = 1.5 \text{ deg}$

Statistics (mean value and standard deviation)

$\delta_c$	-0.10	± 0.89	deg	$P_S$	14.6	± 0.1	MW
$\delta'$	1.44	± 0.71	deg	$\epsilon_v$	0.00	± 0.02	knots
$\psi - \psi_{\text{ref}}$	-0.026	± 0.256	deg	$\epsilon_r$	0.00	± 0.02	deg/s
$n$	69.6	± 0.4	rpm	$\epsilon_\psi$	0.00	± 0.03	deg
$u$	13.5	± 0.1	knots	$\epsilon_\delta$	0.0	± 0.5	deg
$V_1 = 0.132$							

PLOT ACOP1(1)-ACOP1(8) HP ACOP1(10) ACOP1(15) 02 -3 1 ° DEG



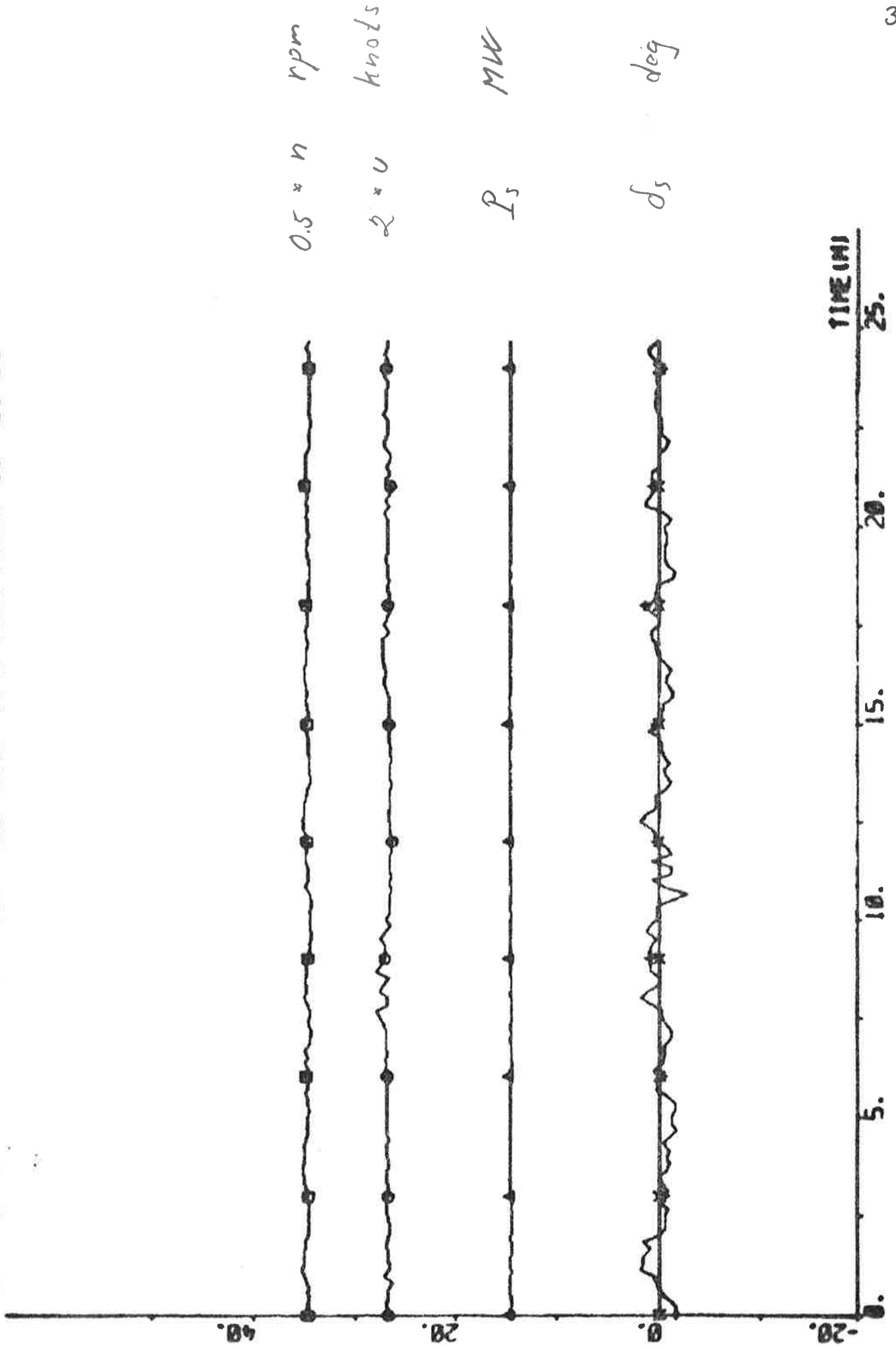
$y - y_{ref}$

$0.05 \times d_c$

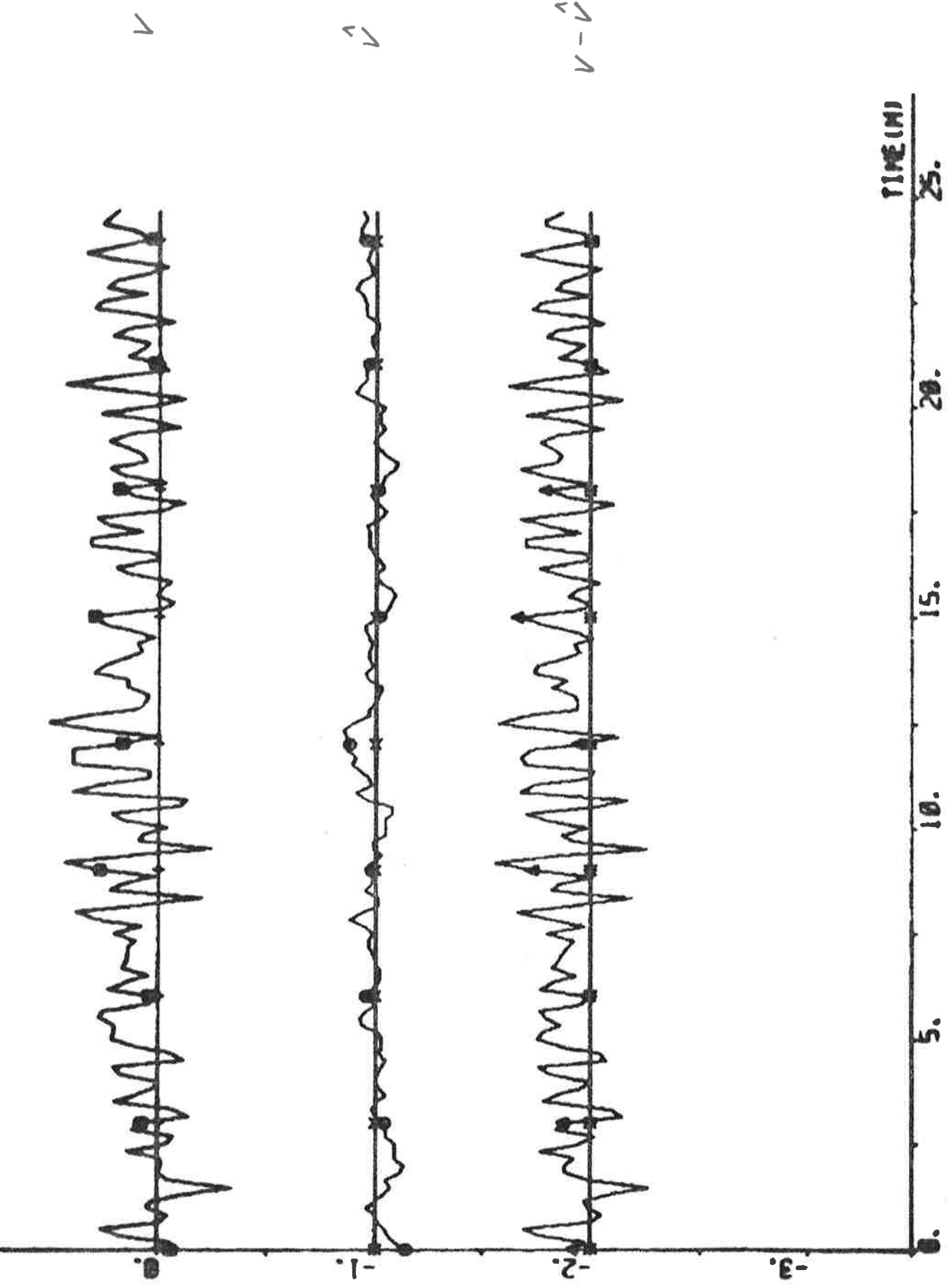


0  
-1  
-2  
0  
5  
10  
15  
20  
25  
TIME (H)

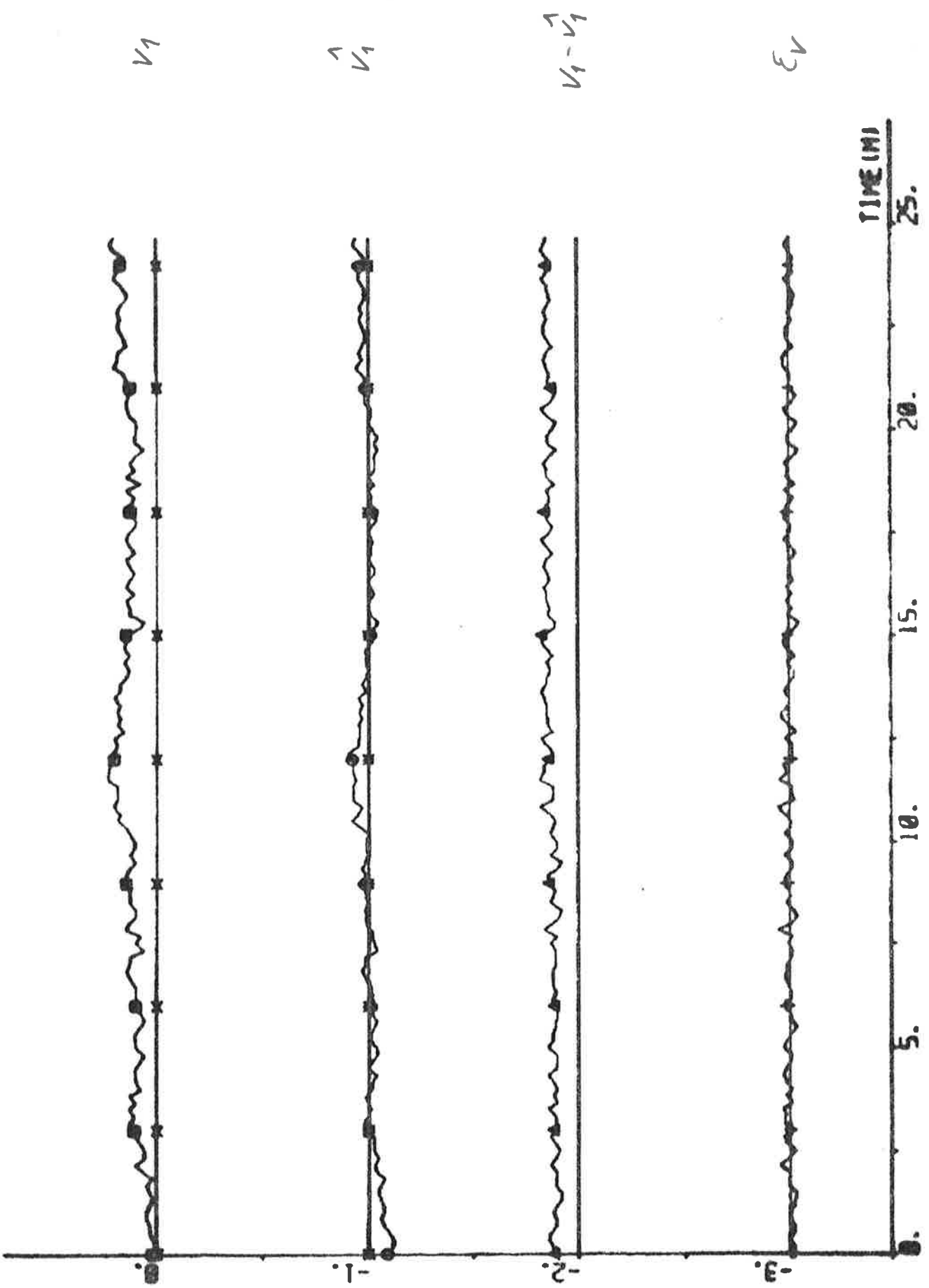
PL0T A00P1(1)-A00P1(13) A00P1(12) A00P1(14) A00P1(11) 00 -20 50



PLOT ACBP1(1)-ACBP2(1) ACBP2(2) ERRS 00 01 02 -3.4 0.0 - KNOTS

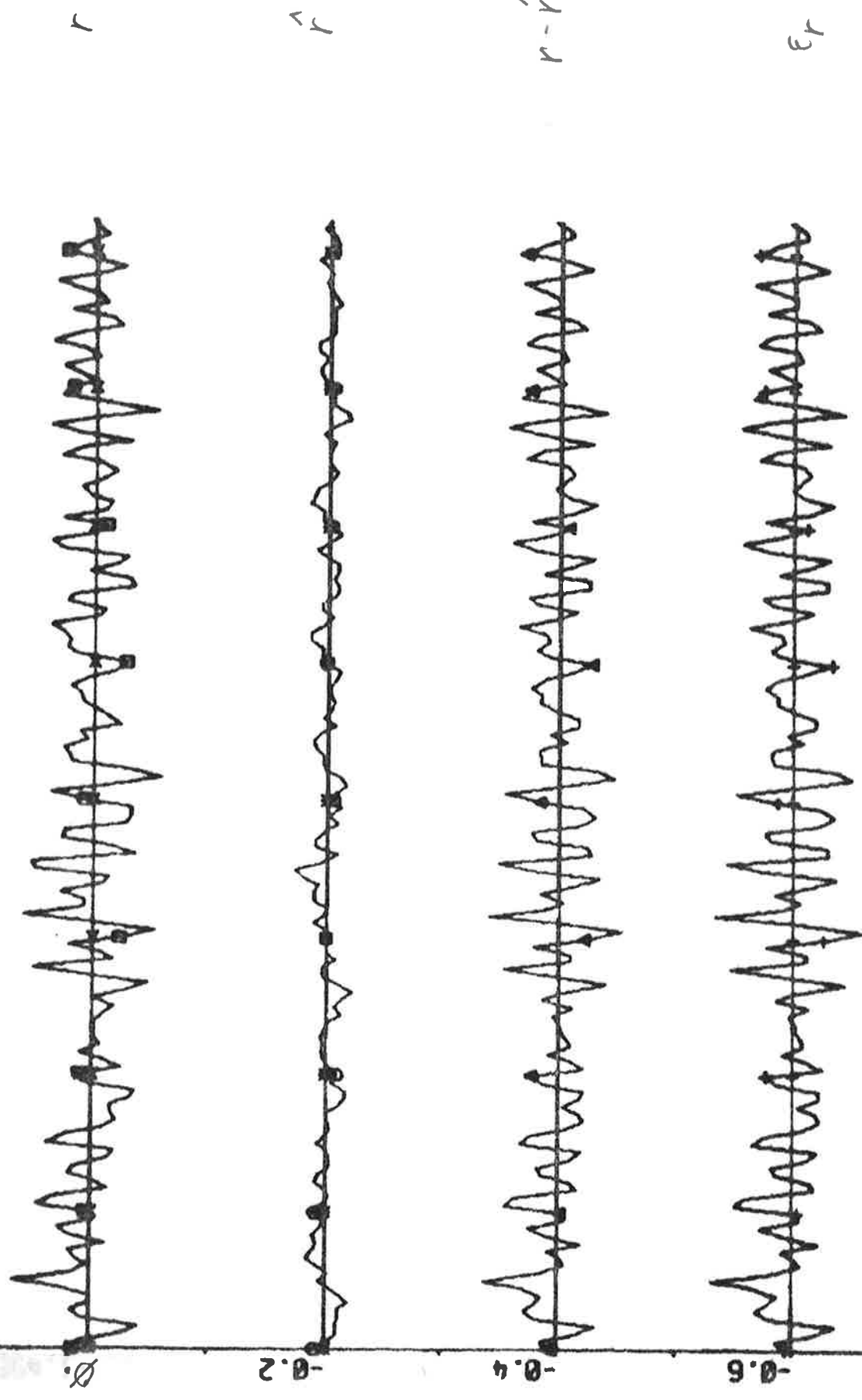


PLOT A3BP1(1)-A3BP1(4) A3BP1(5) ERR2 EPS2 00 01 02 03 -3.4 0.0 - KNOTS

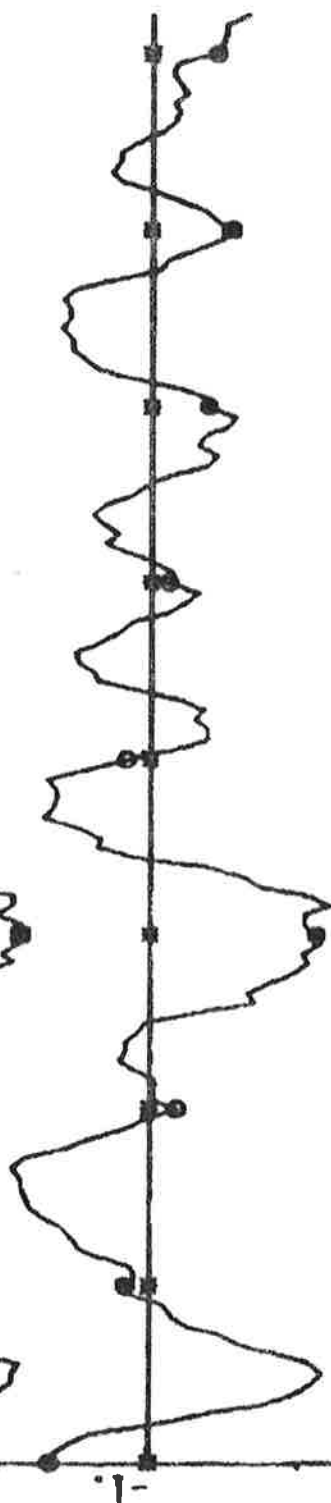
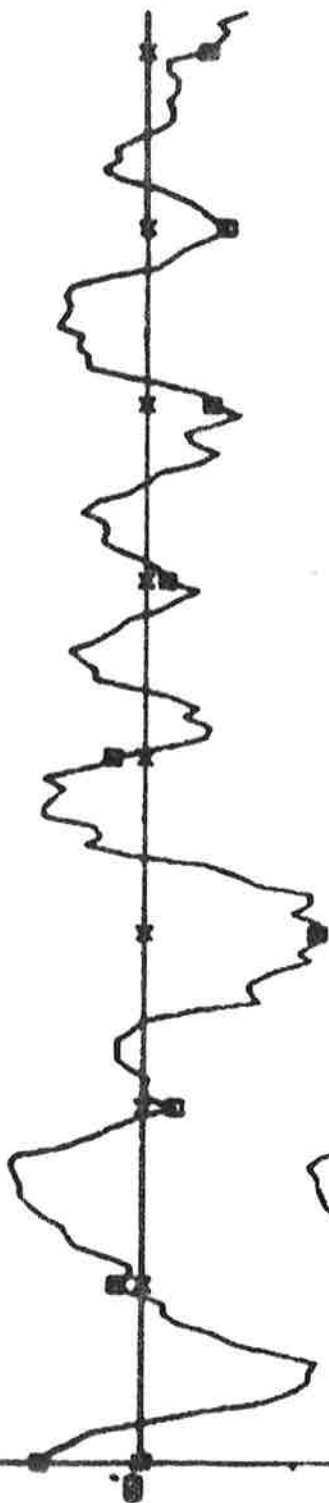




PLOT AOSP1(1)-AOSP1(6) AOSP1(7) ERR3 EPS3 ON 002 004 006 -0.7 0. - 0000



PLOT ACBP1(1)-ACBP1(8) ACBP1(8) ERR4 EPS4 00 01 02 03 -3.4 0.0 - DEG



TIME (M)

25.

20.

15.

10.

5.

PLUT ACP1(1)-ACDP1(2) ACDP1(3) ERR1 EPS1 00 020 040 060 -05 15 - 000



TIME (M)  
5. 10. 15. 20. 25.

PLOT A30P1(1)-A30P2(3) A30P2(4) A30P2(5) A30P2(6) 00 02 04 06 -0.5 1.5

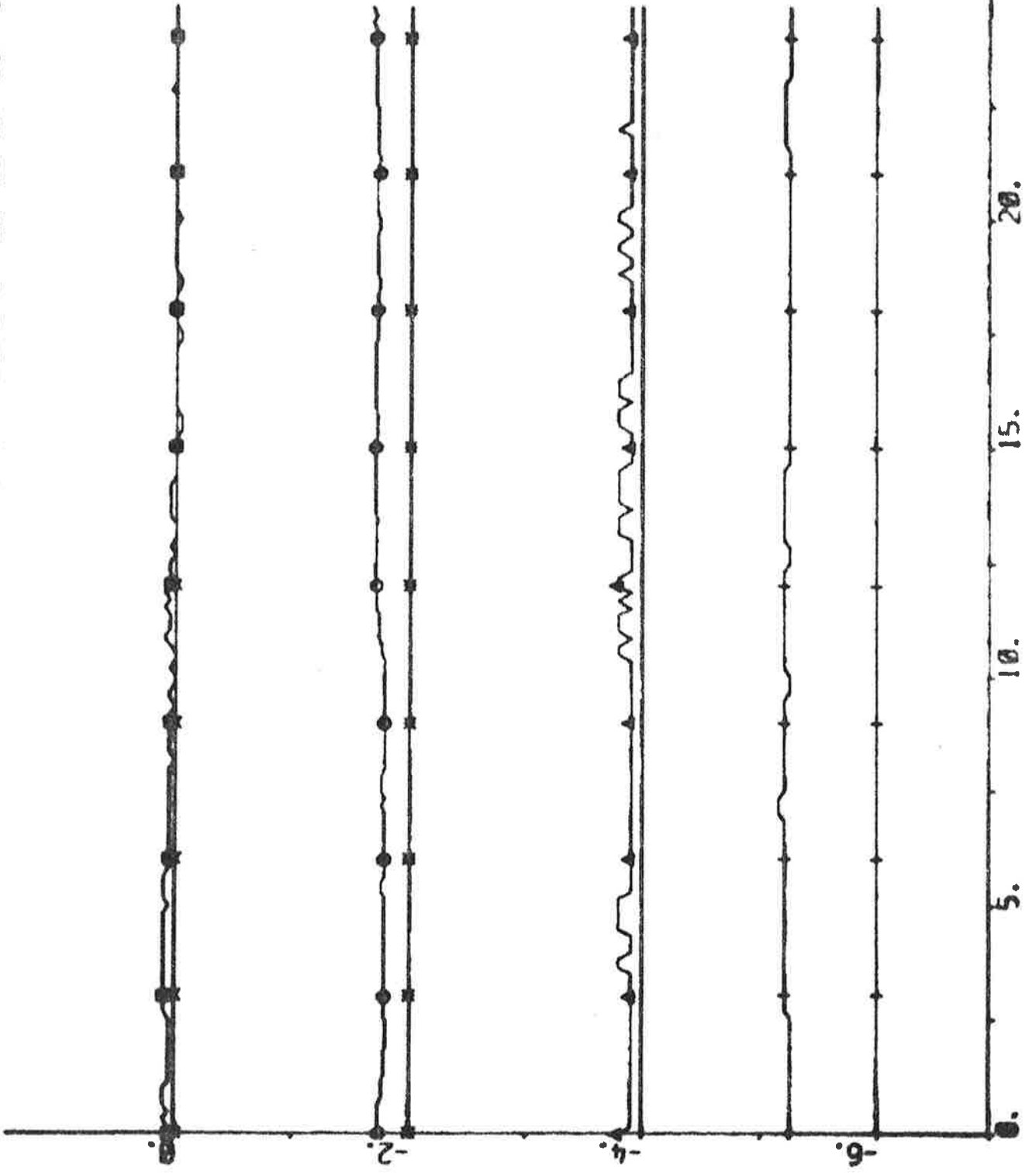
$\hat{d}_0$  deg  
0.5 x  $\hat{d}_0$

$\hat{d}_v$  knots  
2 x  $\hat{d}_v$

$\hat{d}_r$  deg/s  
100 x  $\hat{d}_r$

$\hat{d}_\sigma$  deg  
0.5 x  $\hat{d}_\sigma$

TIME (M)  
25.



## EXPERIMENT A31

Date	1976-04-26	Forward draught	10.9 m
Time	16.05	Aft draught	12.9 m
Duration	24 min	Wind direction	SE (1; see App. A)
Position	S 06°42' W 06°06'	Wind velocity	3 m/s (light breeze)
$\psi_{\text{ref}}$	143 deg	Wave height	-

PID-regulator using non-filtered measurements.

$k_P = 2$	$k_D = 100 \text{ s}$	$k_I = 0.02 \text{ 1/s}$
$T_s = 10 \text{ s}$	$V_0 = 6 \text{ m/s}$	IVVC = 1

The parameters were manually tuned before the experiment started.

Final values:

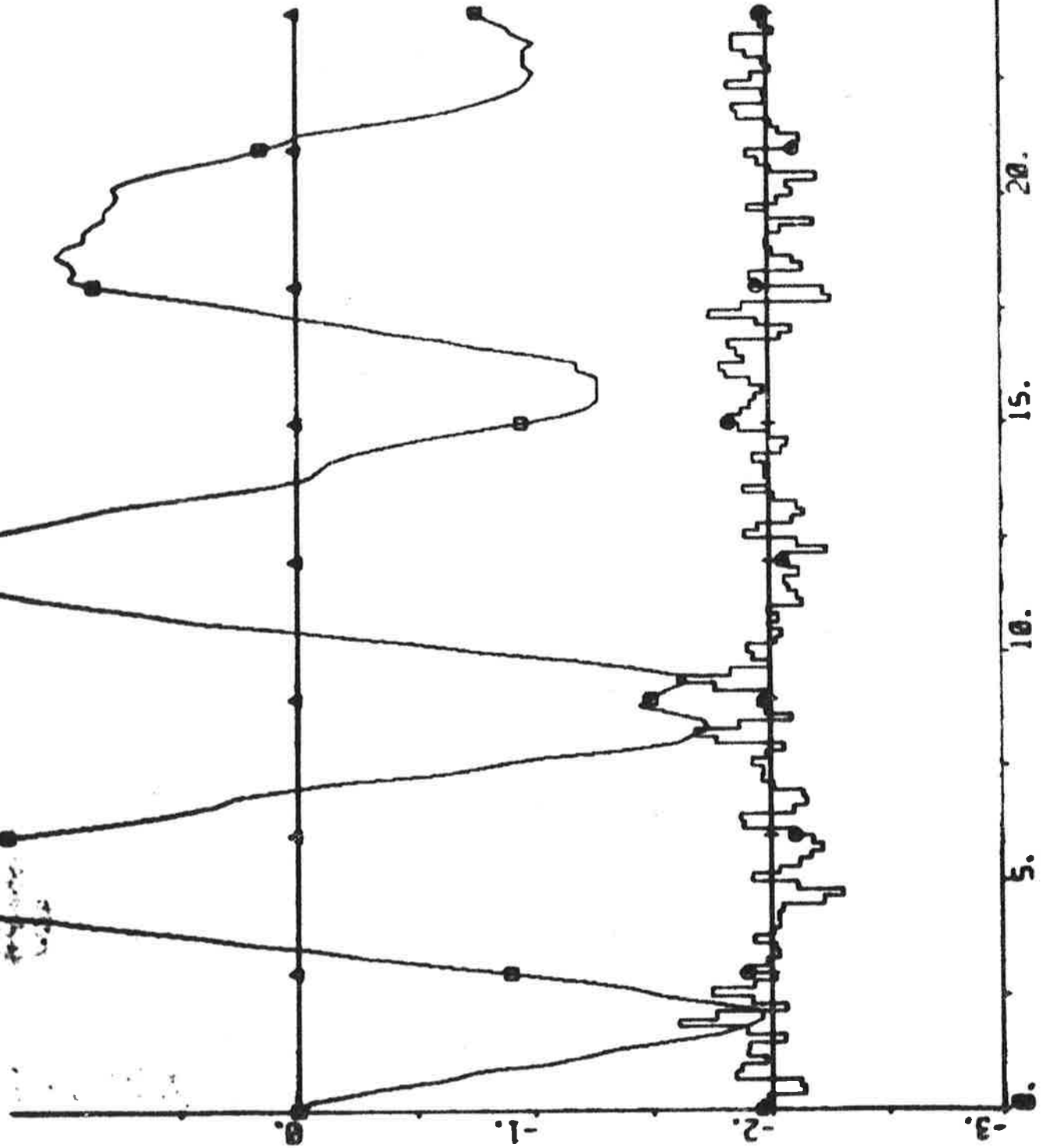
$$\hat{\delta}_0 = 0.0 \text{ deg} \quad \hat{d}_v = 0.11 \text{ knots} \quad \hat{d}_r = 0.001 \text{ deg/s} \quad \hat{d}_\delta = 1.5 \text{ deg}$$
Statistics (mean value and standard deviation)

$\delta_c$	0.22 ± 2.41 deg	$P_S$	14.5 ± 0.1 MW
$\delta$	1.67 ± 2.05 deg	$\epsilon_v$	0.00 ± 0.03 knots
$\psi - \psi_{\text{ref}}$	-0.103 ± 1.085 deg	$\epsilon_r$	0.00 ± 0.02 deg/s
n	69.4 ± 0.6 rpm	$\epsilon_\psi$	0.00 ± 0.03 deg
u	13.4 ± 0.1 knots	$\epsilon_\delta$	0.0 ± 0.6 deg
$V_1 = 1.672$			

PL0T R01R1(1)-R01P1(0) MP R03 P1(10) R01P1(15) 02 -3 1 - DEG

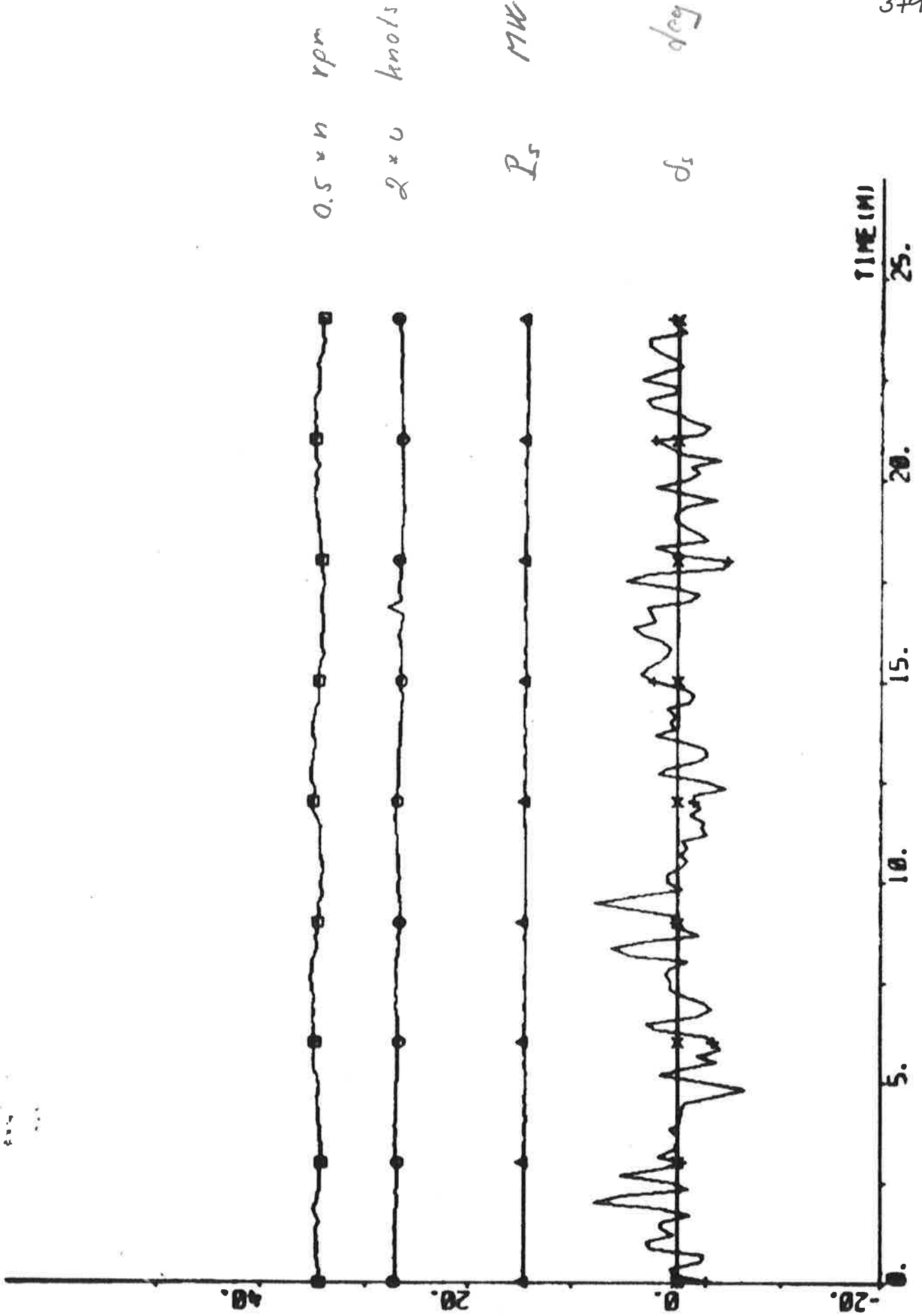
$N - N_{ref}$

$0.05 * d_c$

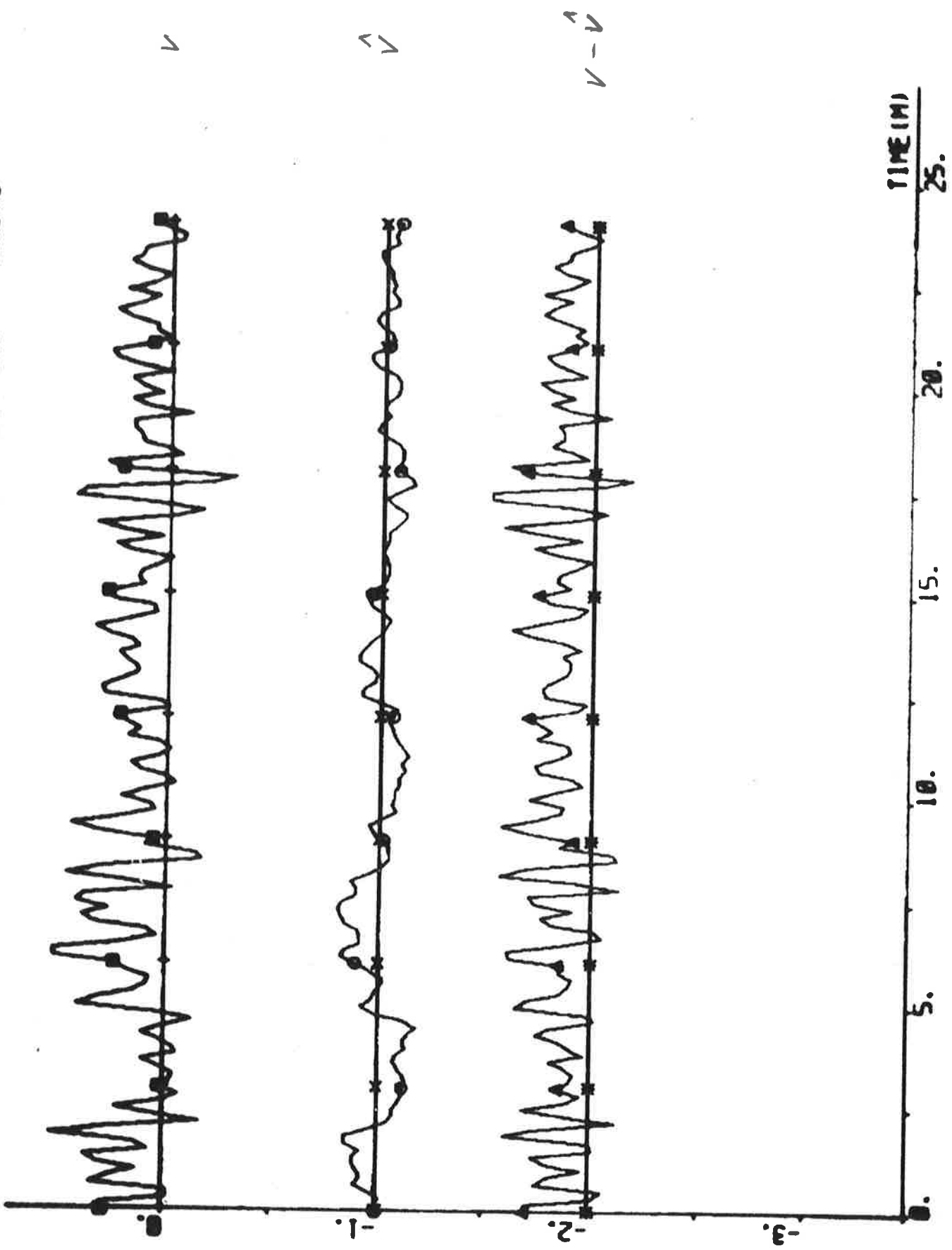


TIME (M)

PL0T R01P1(1)-R01P1(13) R01P1(12) R01P1(14) R01P1(11) 00 -20 00

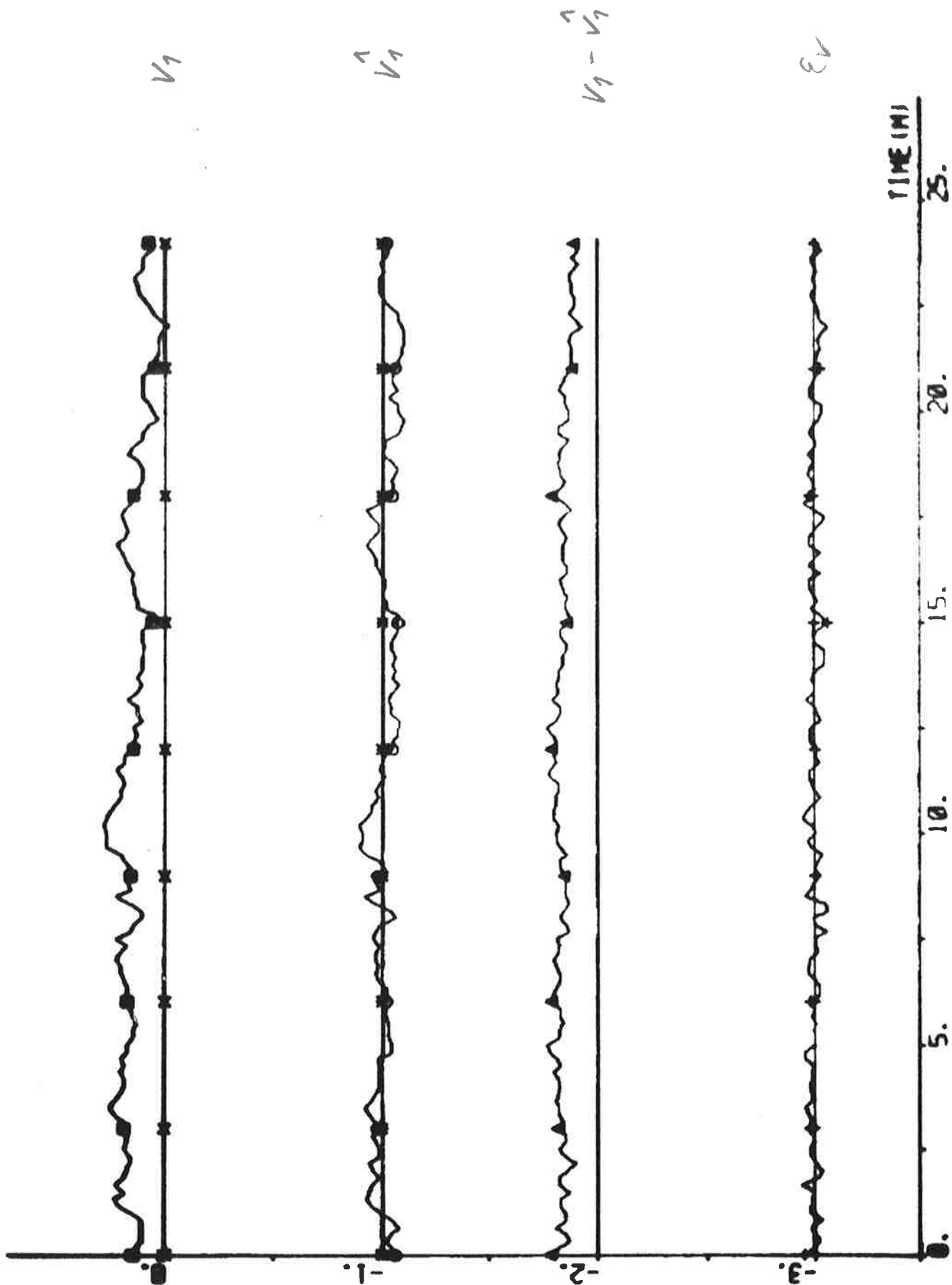


PL0T A31P1(1)-A31P2(1) A31P2(2) ERMS 00 01 02 -3.4 0.0 - 1000TS

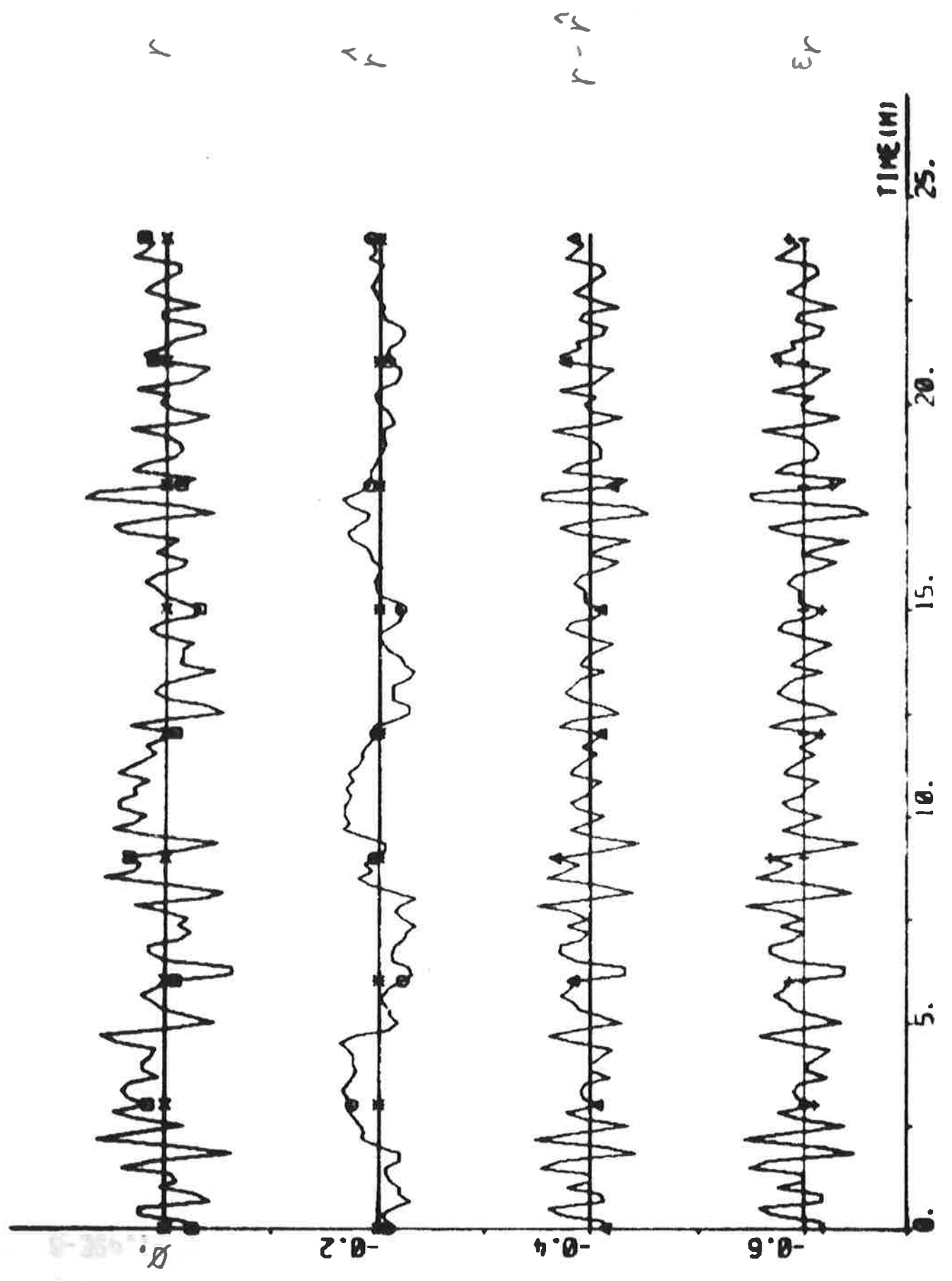




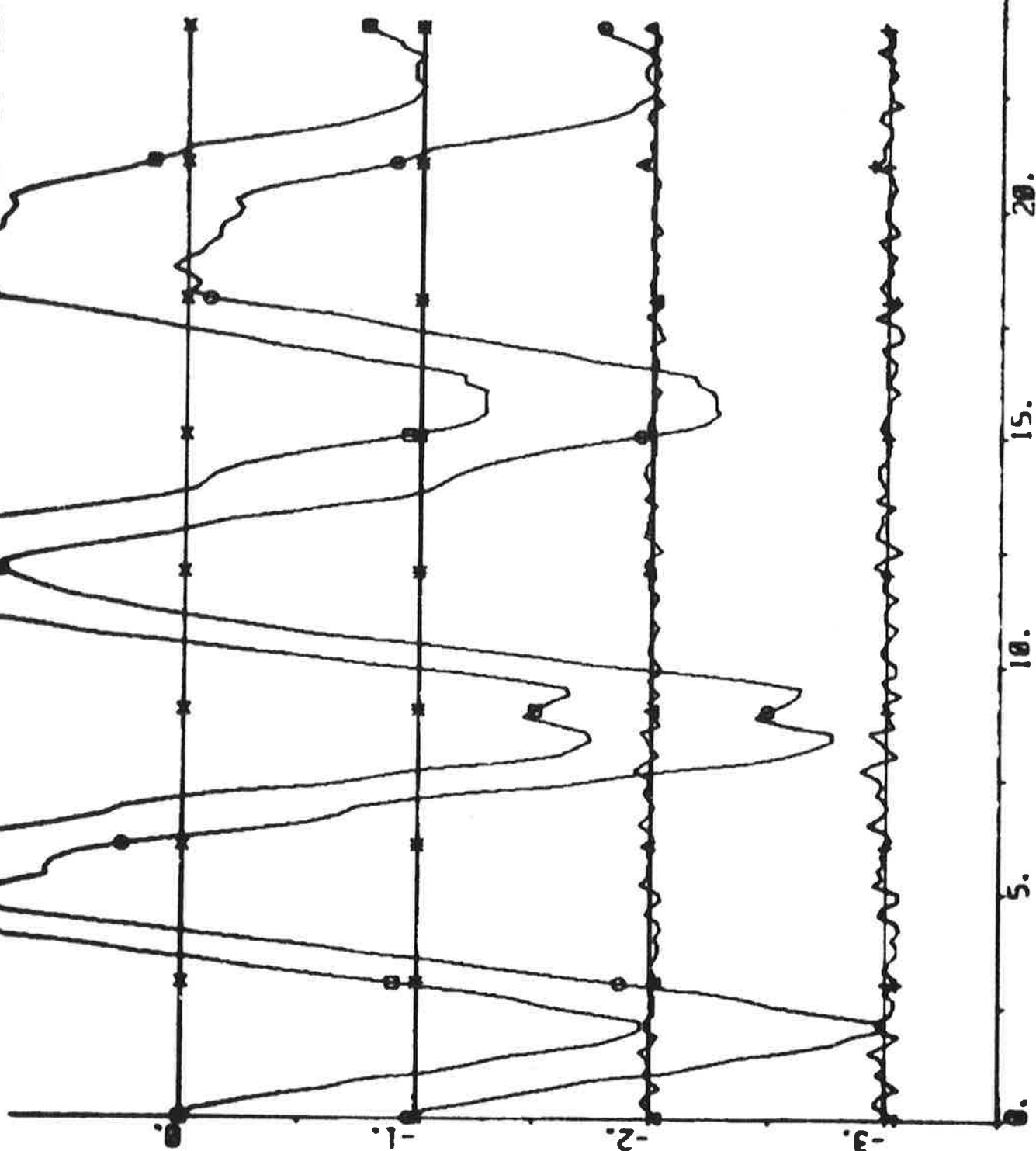
PL0T A01P1(1)-A01P1(4) A01P1(5) ERR2 EPS2 00 01 02 03 -3.4 0.0 - 1000TS



PLOT A31P1(1)-A31P1(6) A31P1(7) ERR3 EPS3 00 002 004 006 -0.7 0. - 00000



PLOT NSIP1(A)-NSIP1(B) NSIP1(C) NSIP1(D) NSIP1(E) NSIP1(F) NSIP1(G) NSIP1(H) NSIP1(I) NSIP1(J) NSIP1(K) NSIP1(L) NSIP1(M) NSIP1(N) NSIP1(O) NSIP1(P) NSIP1(Q) NSIP1(R) NSIP1(S) NSIP1(T) NSIP1(U) NSIP1(V) NSIP1(W) NSIP1(X) NSIP1(Y) NSIP1(Z) NSIP1(1) NSIP1(2) NSIP1(3) NSIP1(4) NSIP1(5) NSIP1(6) NSIP1(7) NSIP1(8) NSIP1(9) NSIP1(0) NSIP1(-) NSIP1(+)



$\gamma - \gamma_{ref}$

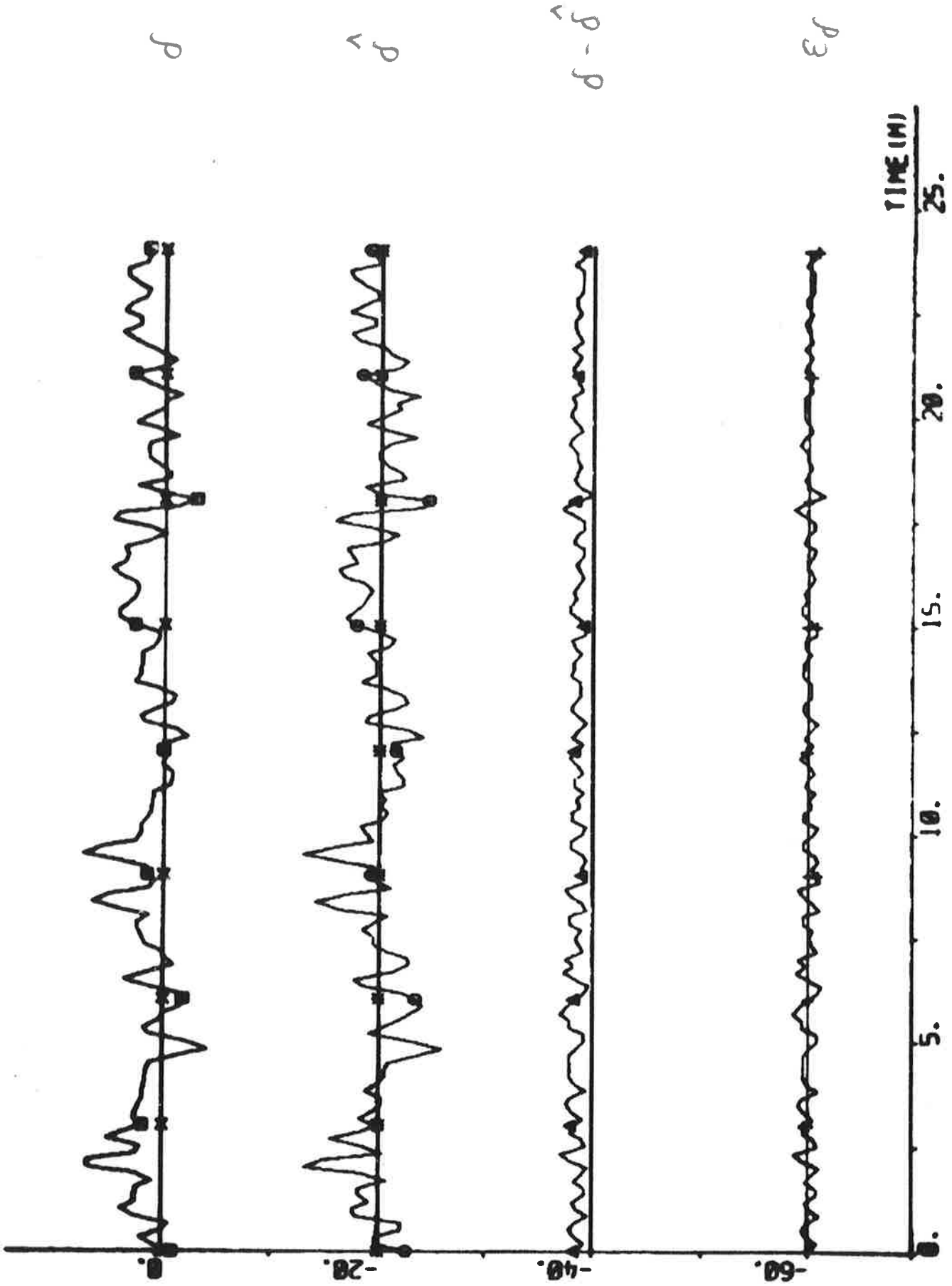
$\gamma - \gamma_{ref}$

$\gamma - \gamma_{ref}$

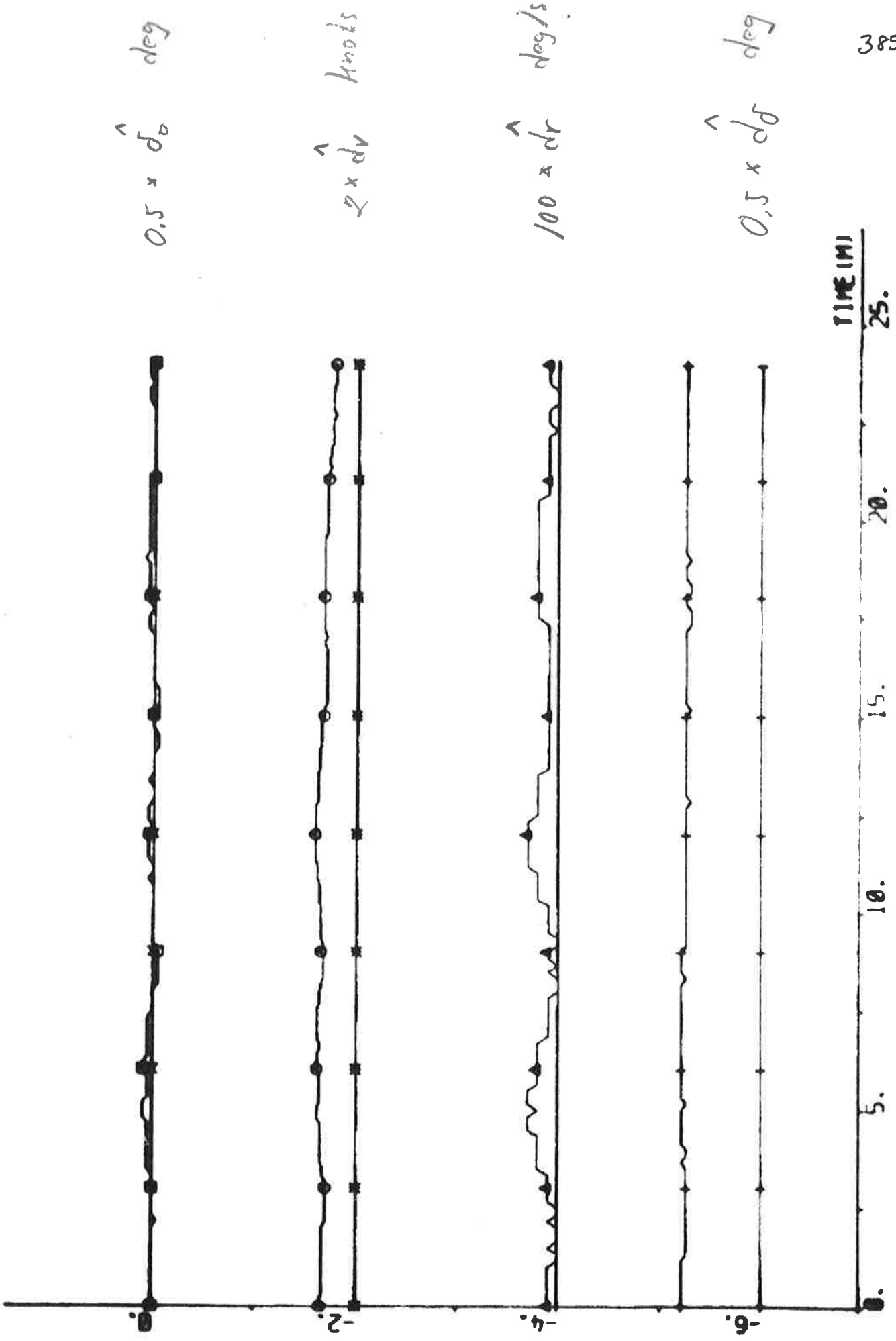
$\epsilon \gamma$

TIME (H)  
25.

PL0T A01P1(1)-A01P1(2) A01P1(3) ERR1 EPS1 00 020 040 060 -05 15 - 000

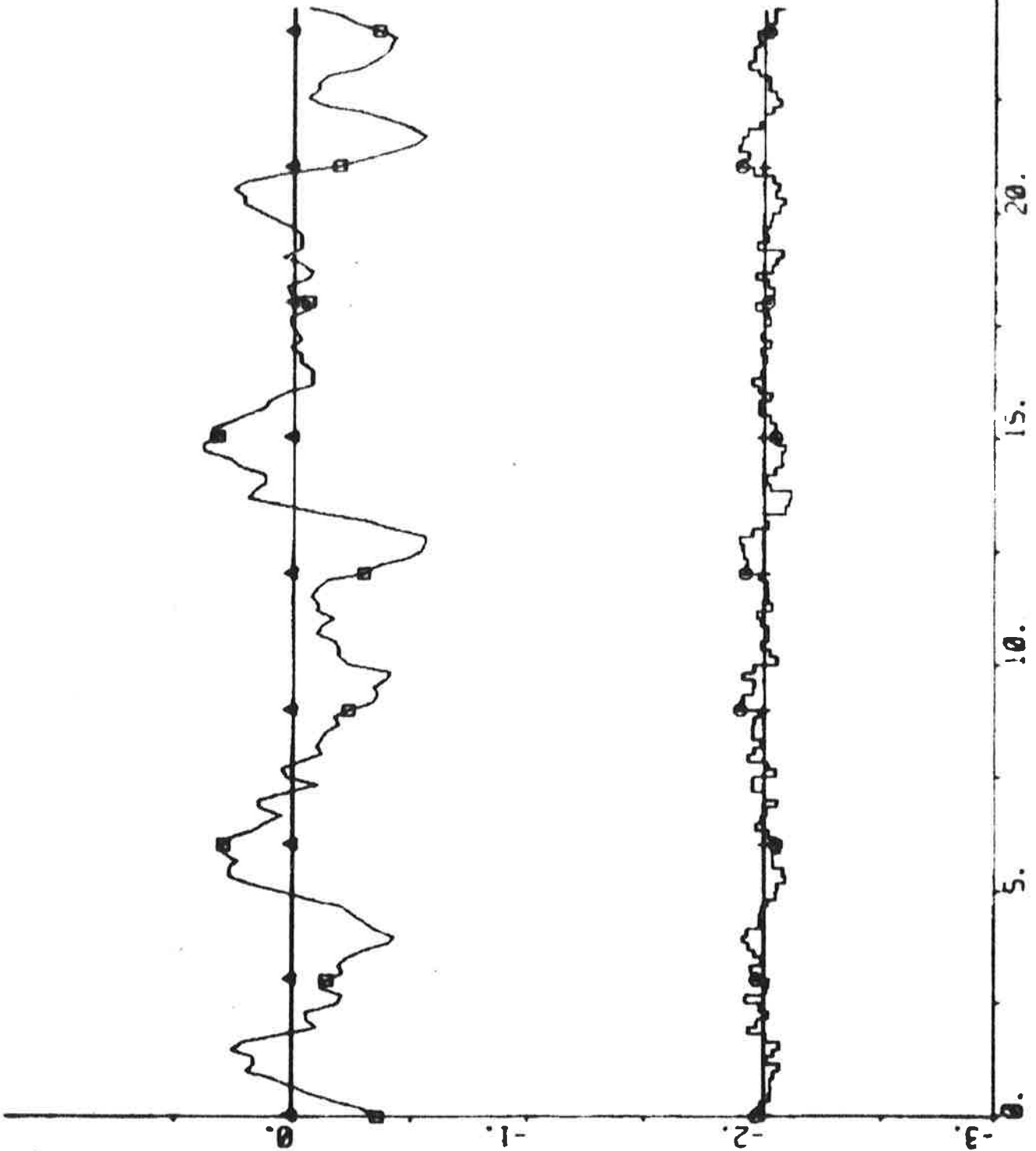


PLOT A31P1(1)-A31P2(3) A31P2(4) A31P2(5) A31P2(6) 00 02 04 06 -0.5 1.5

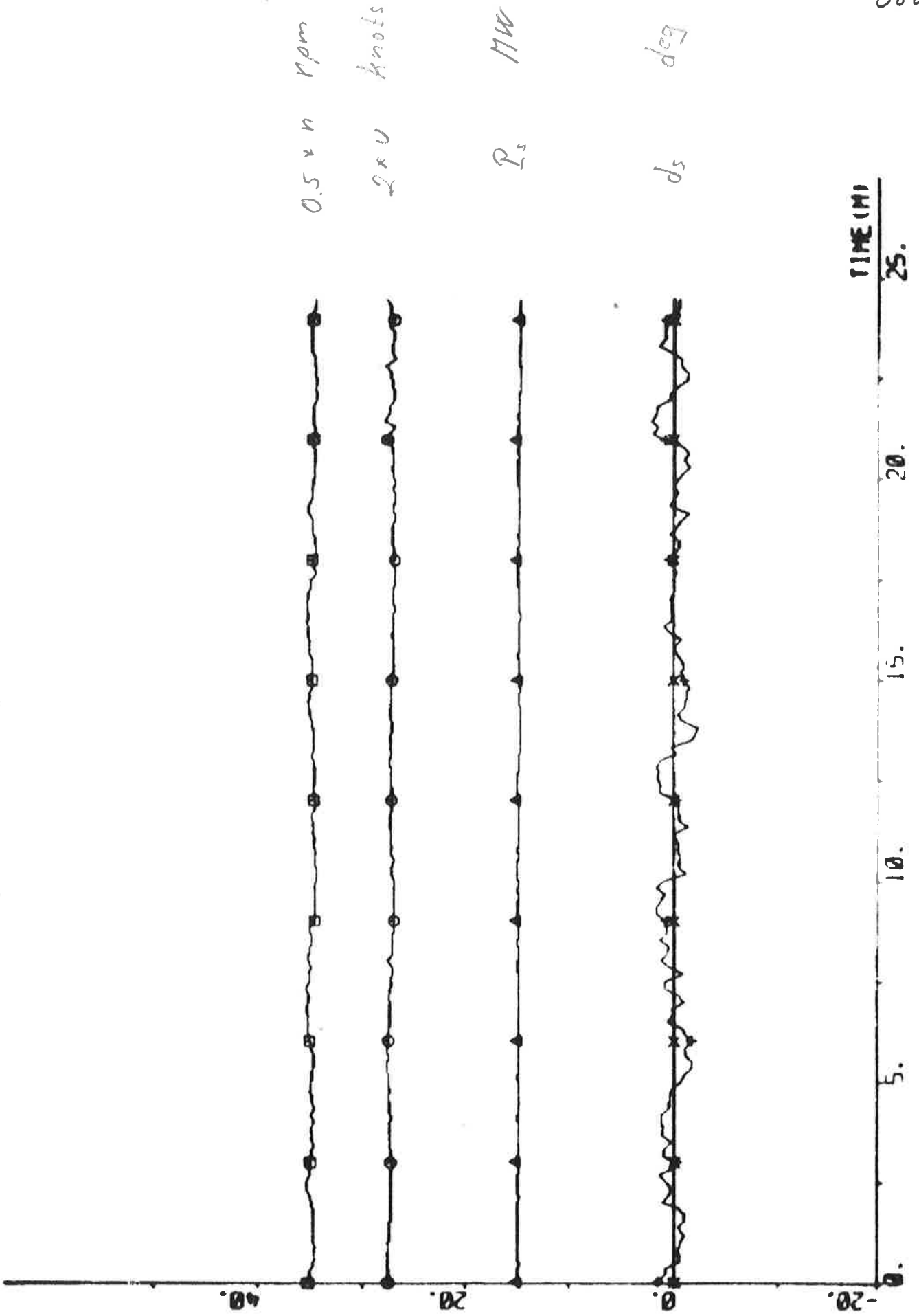




PLOT A32P1(1)-A32P1(8) HP A32P1(10) A32P1(15) 02 -3 1 - DEG

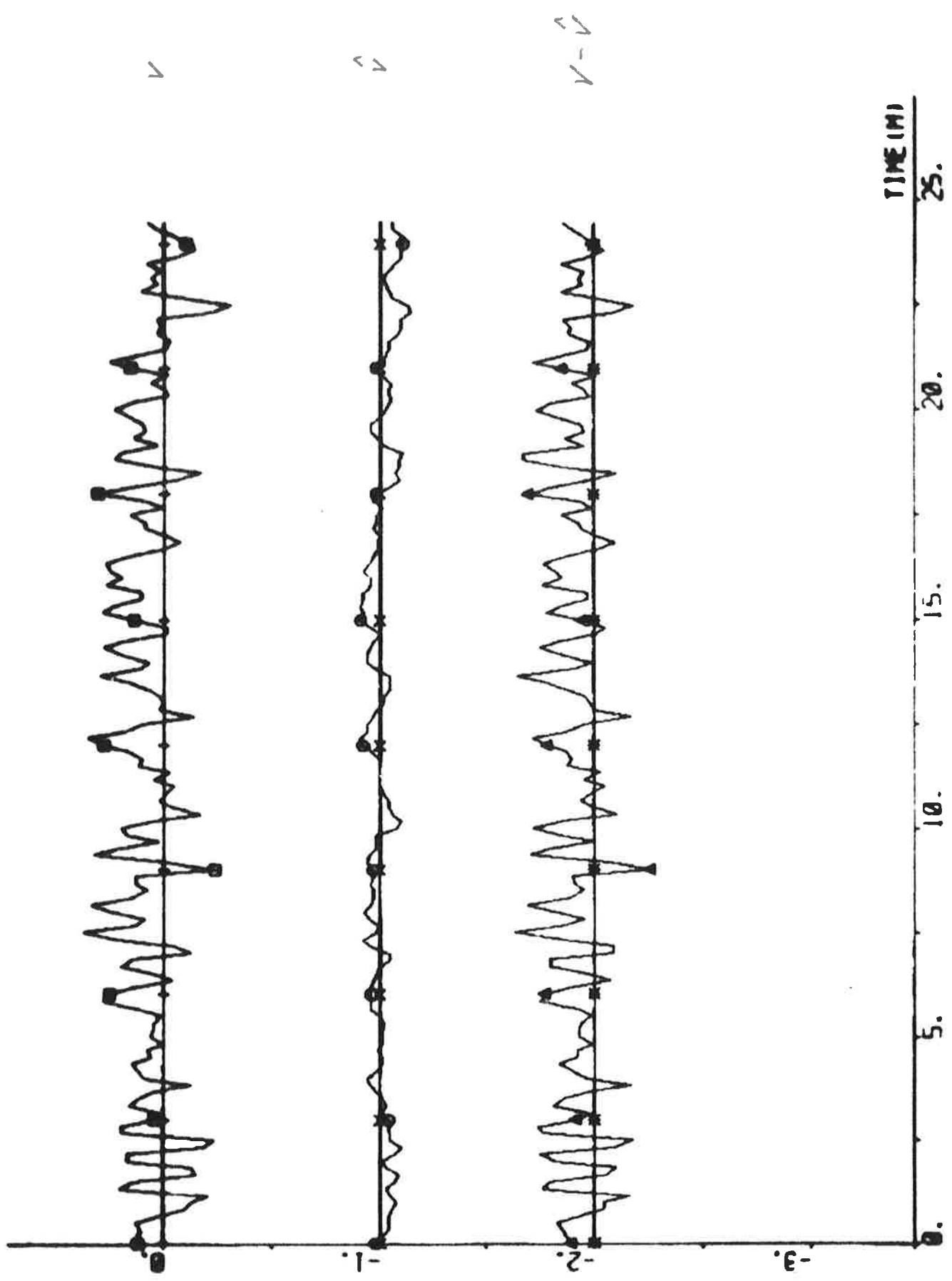


PLOT A32P1(1)-A32P1(13) A32P1(12) A32P1(14) A32P1(11) 00 -20 00

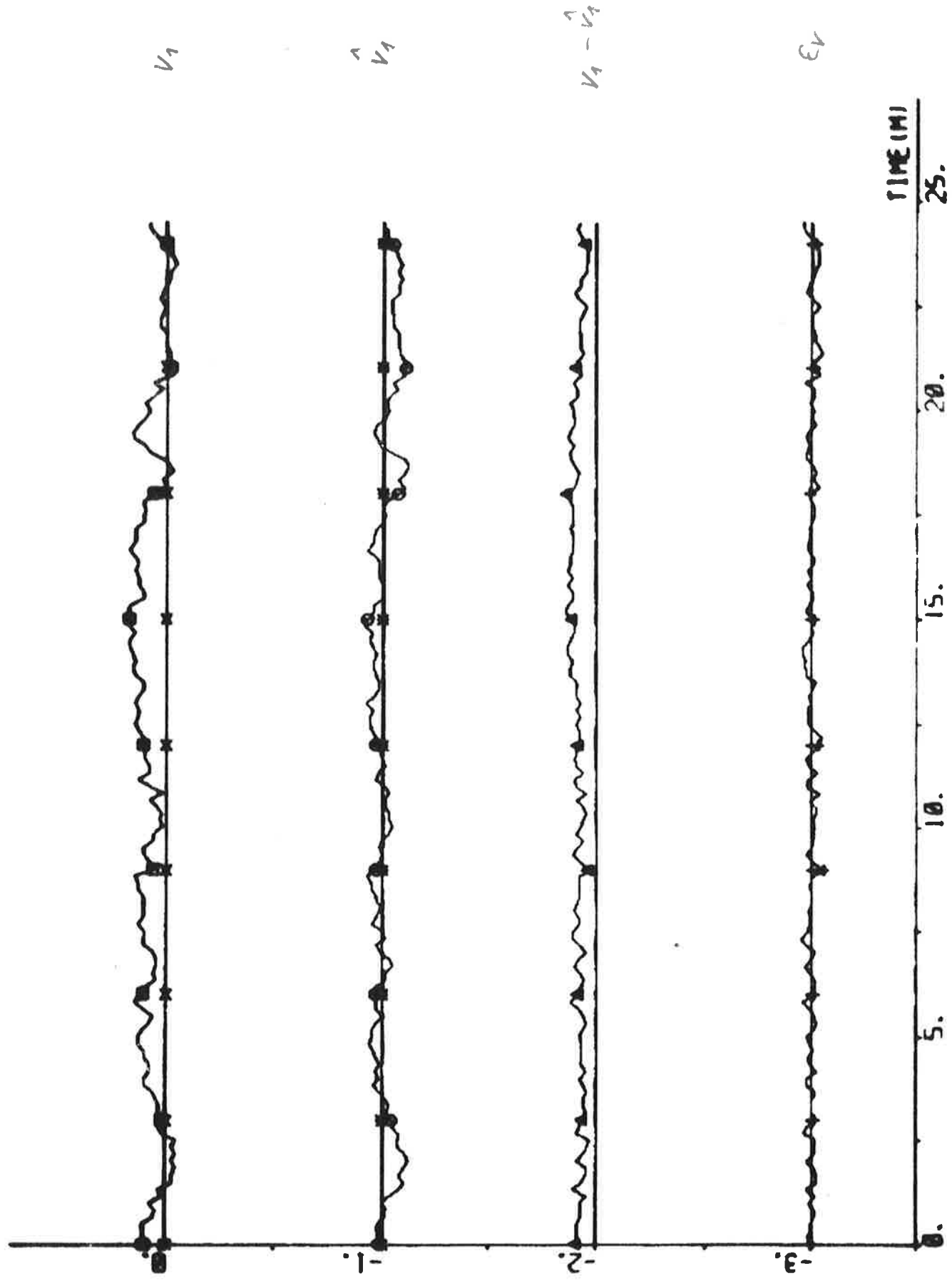




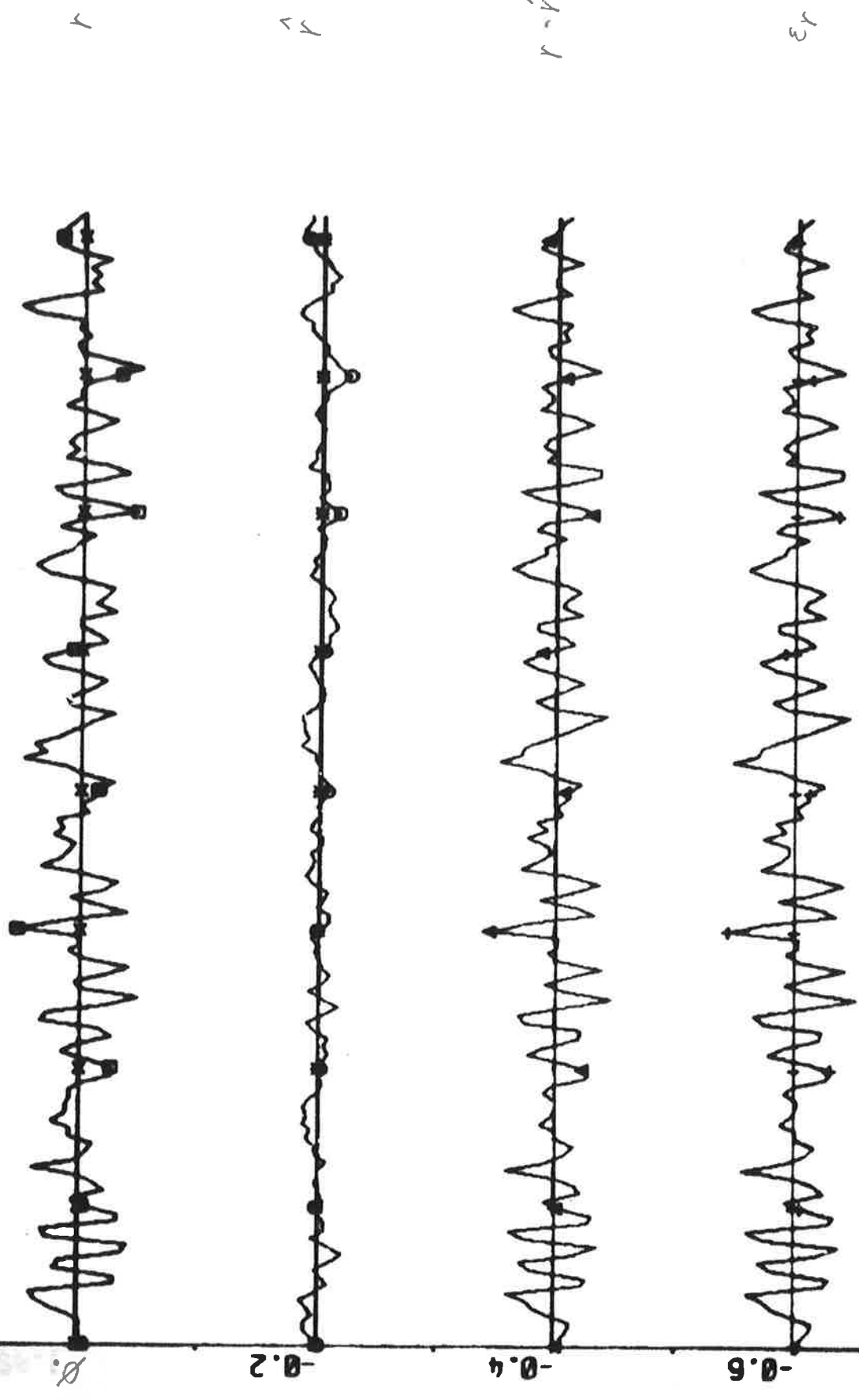
PL0T A32P1(1)-A32P2(1) A32P2(2) ERRS 00 01 02 -3.4 0.0 - KNOTS



PL0T A02P1(1)-A02P1(4) A02P1(5) ERR2 EPS2 00 01 02 03 -3.4 0.0 - 100T3

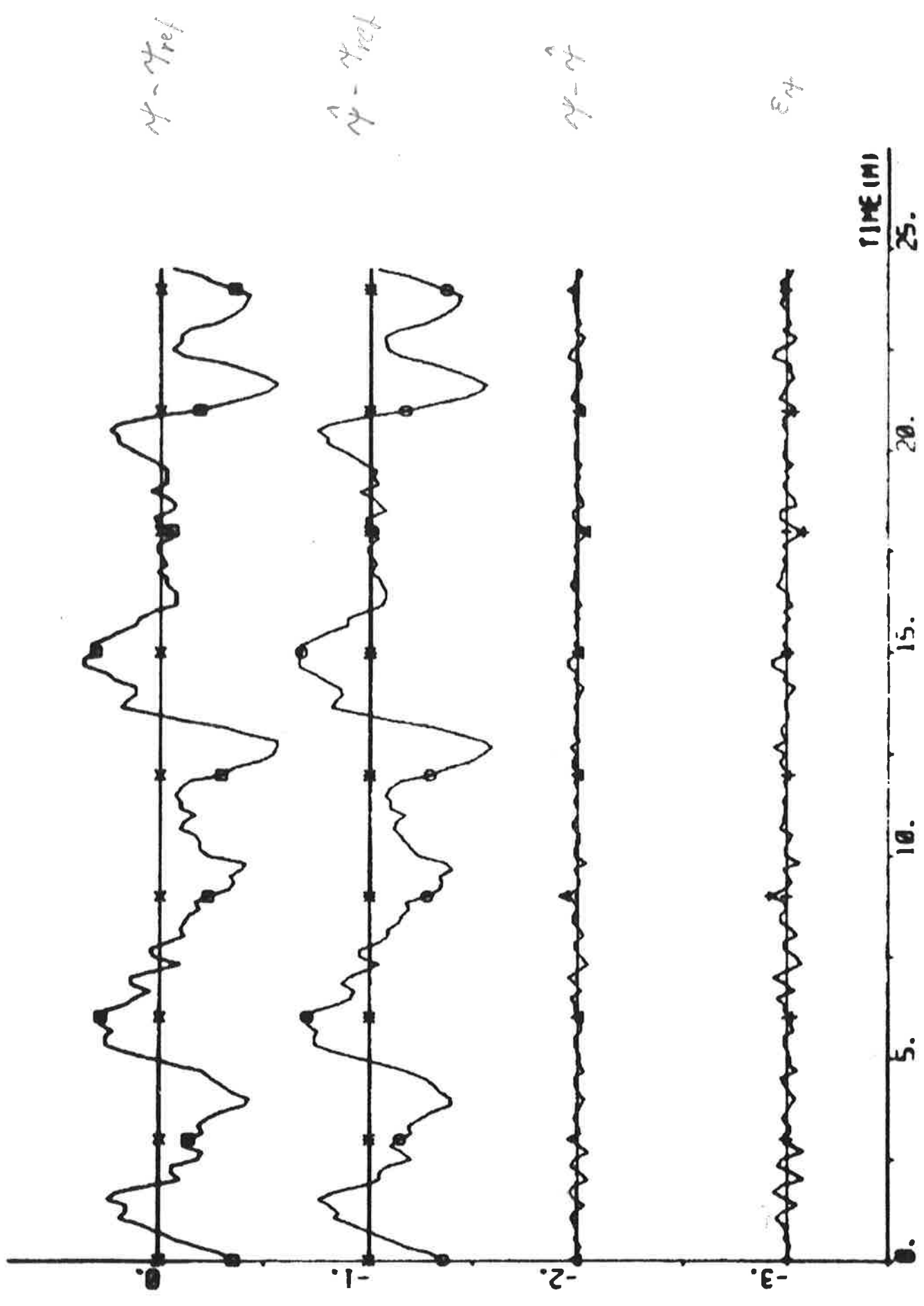


PLOT A32P1(1)-A32P1(6) A32P1(7) ERRO EPS3 00 002 004 006 -0.7 0. - 00000

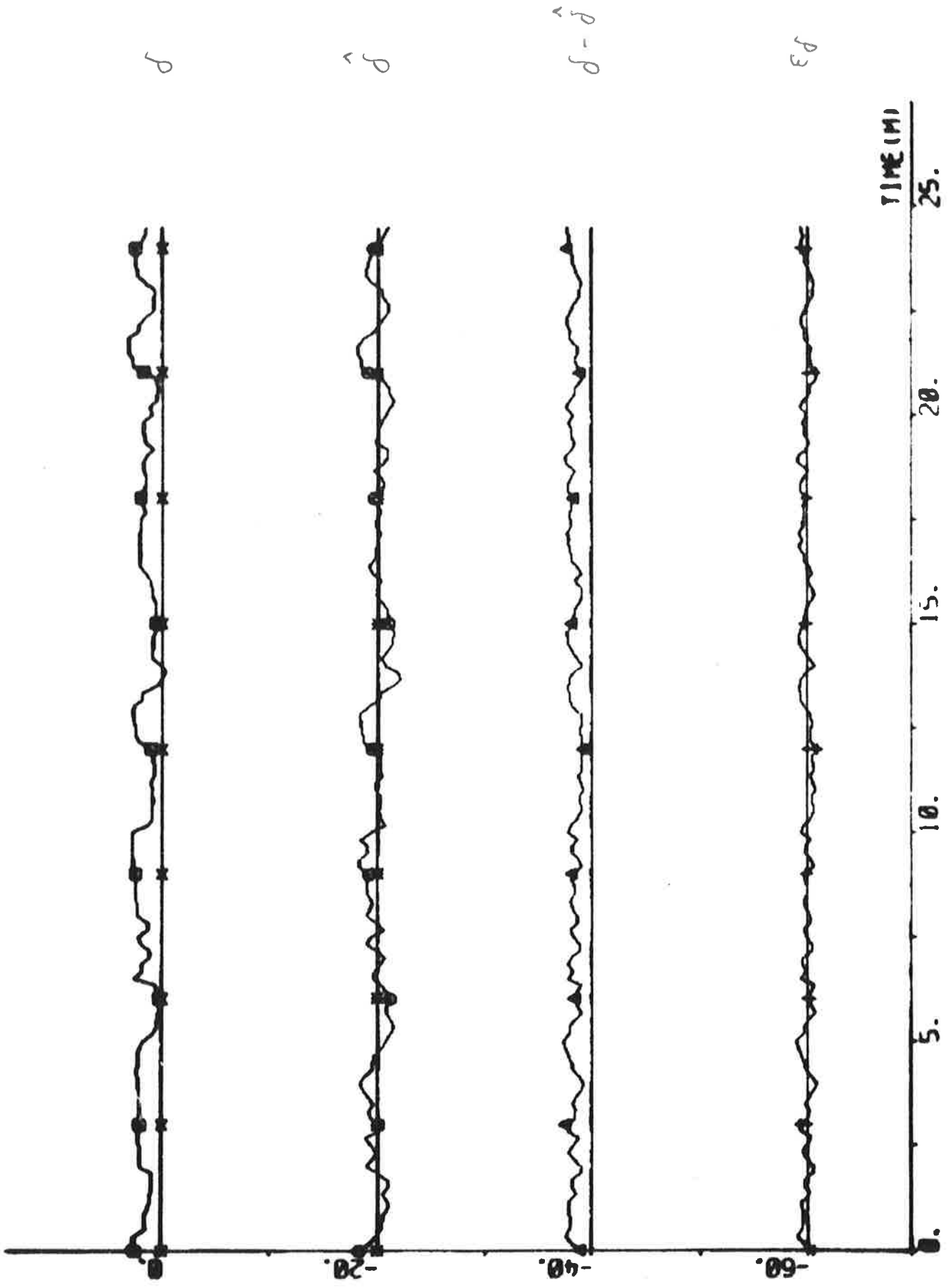


TIME (M)  
25.

PLOT A32P1(1)-A32P1(8) A32P1(9) ERR4 EPS4 00 01 02 03 -3.4 0.0 - 000

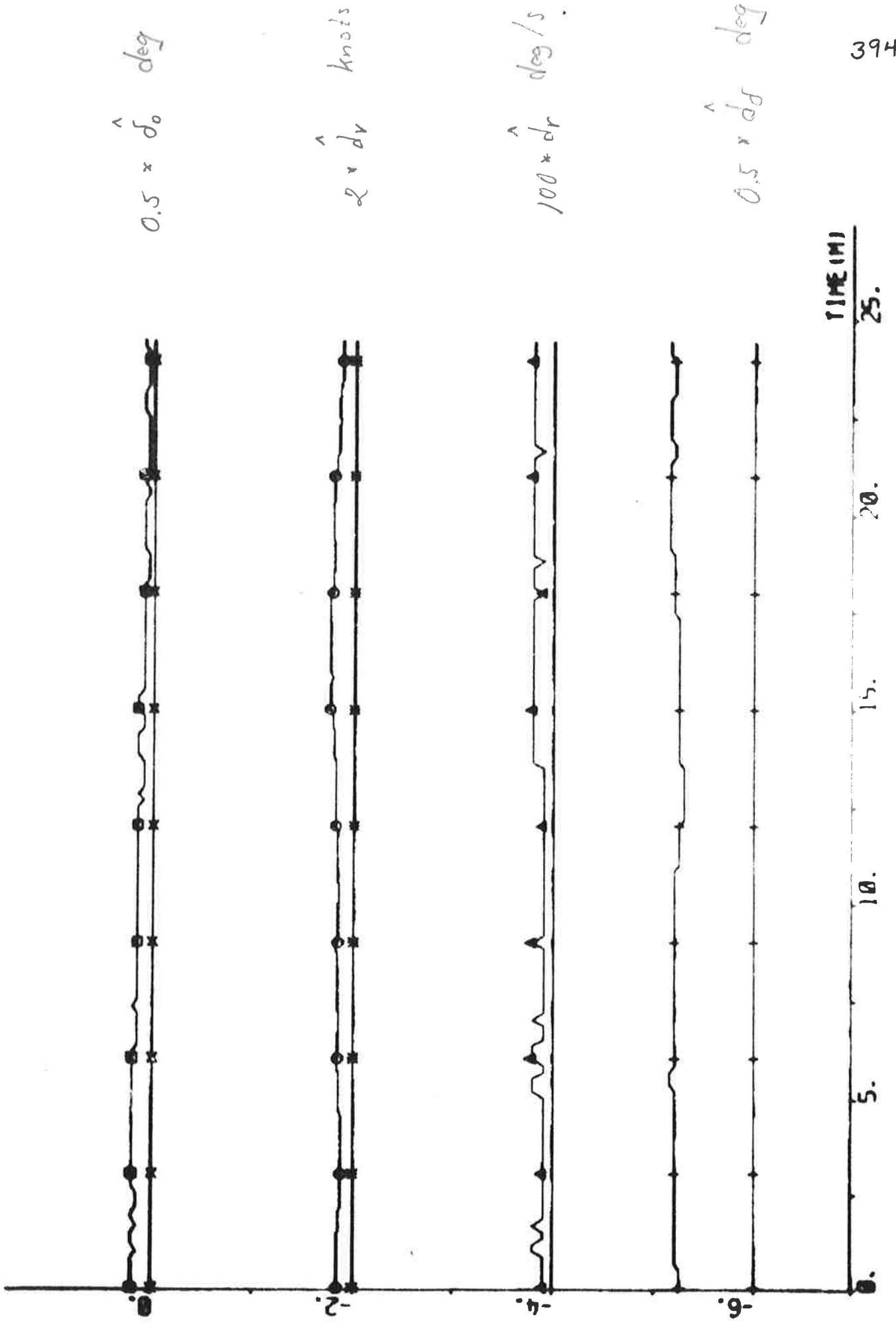


PLOT A32P1(1)-A32P1(2) A32P1(3) ERR1 EPS1 00 020 040 060 .05 15 . 000

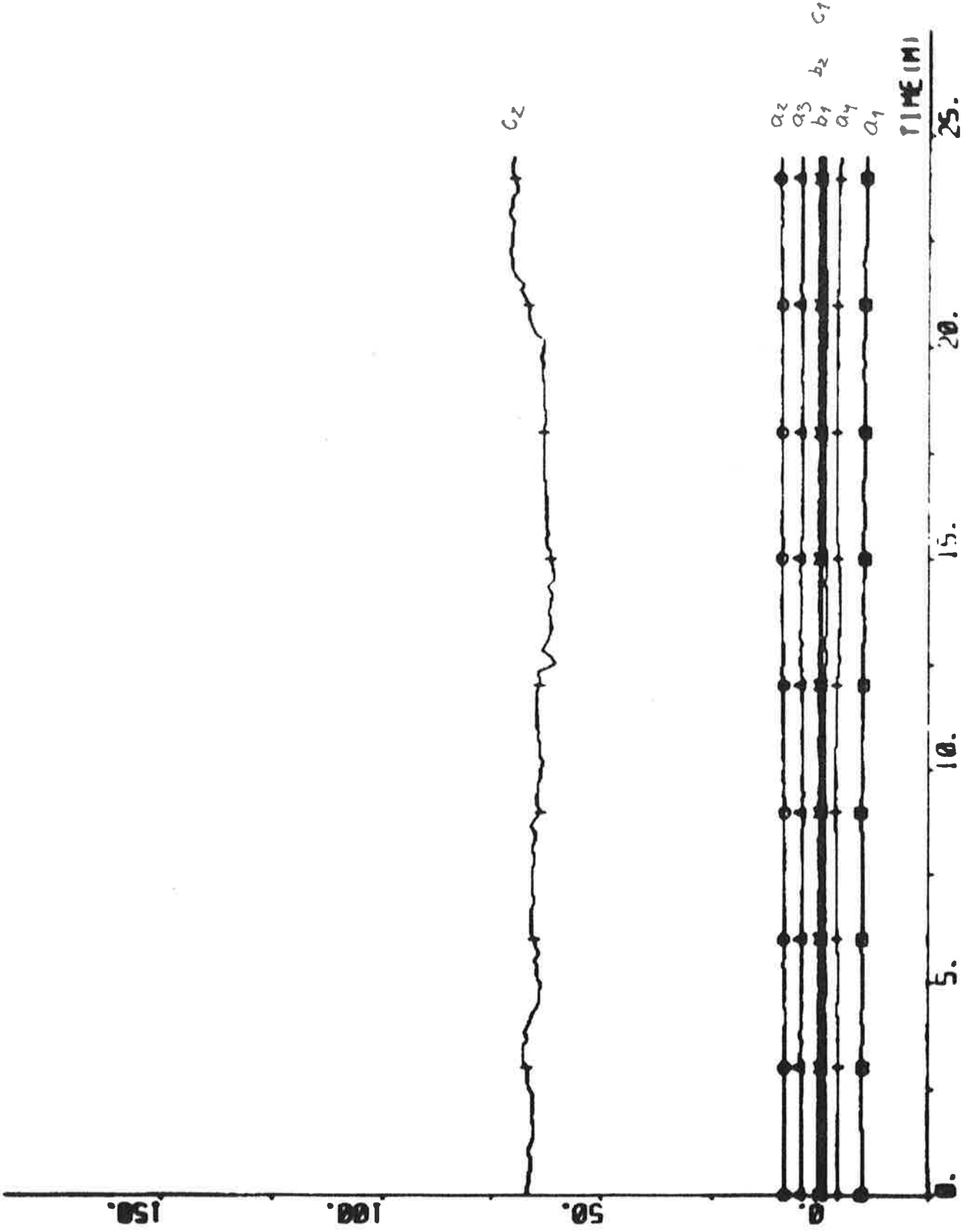


TIME (M)  
25.  
20.  
15.  
10.  
5.

PLOT A32P1(1)-A32P2(3) A32P2(4) A32P2(5) A32P2(6) 00 02 04 06 -0.8 1.8



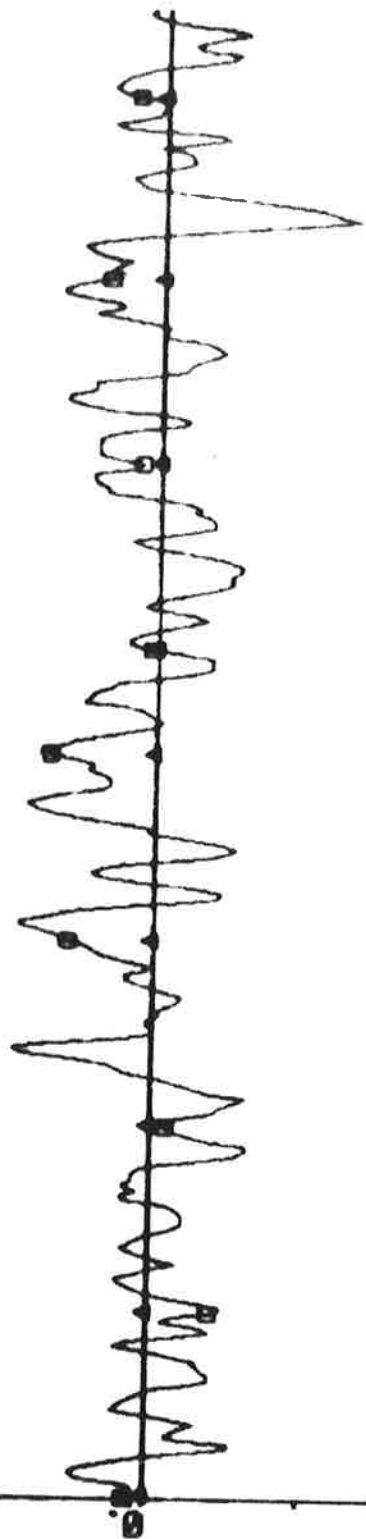
PLOT A32P1(1)-A32P2(7) A32P2(8) A32P2(9) A32P2(10) A32P2(11) A32P2(12) A3







PLOT ACCP1(1)-ACCP1(8) HP ACCP1(10) ACCP1(15) Q2 -3 1 - DEG



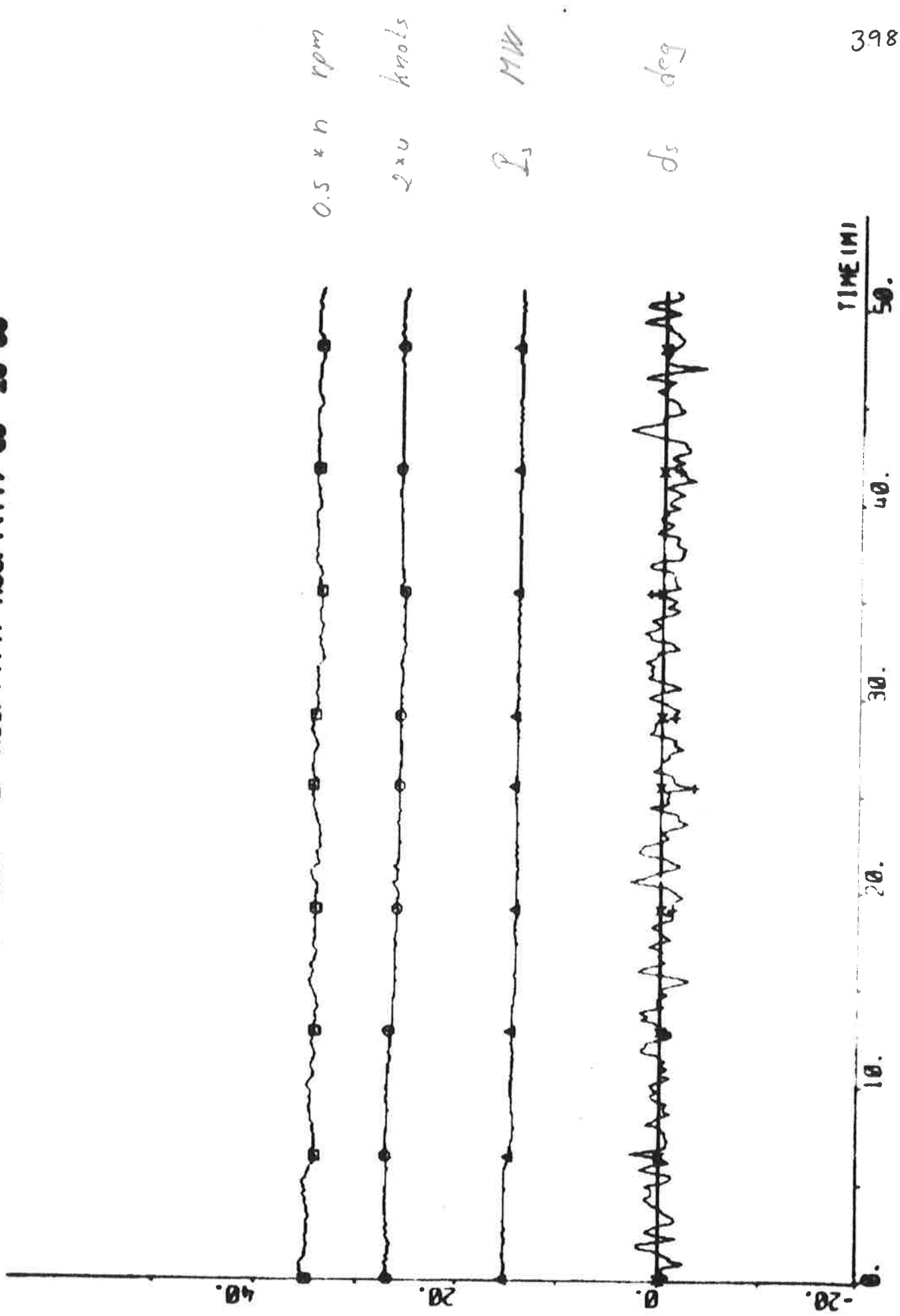
$n - n_{ref}$



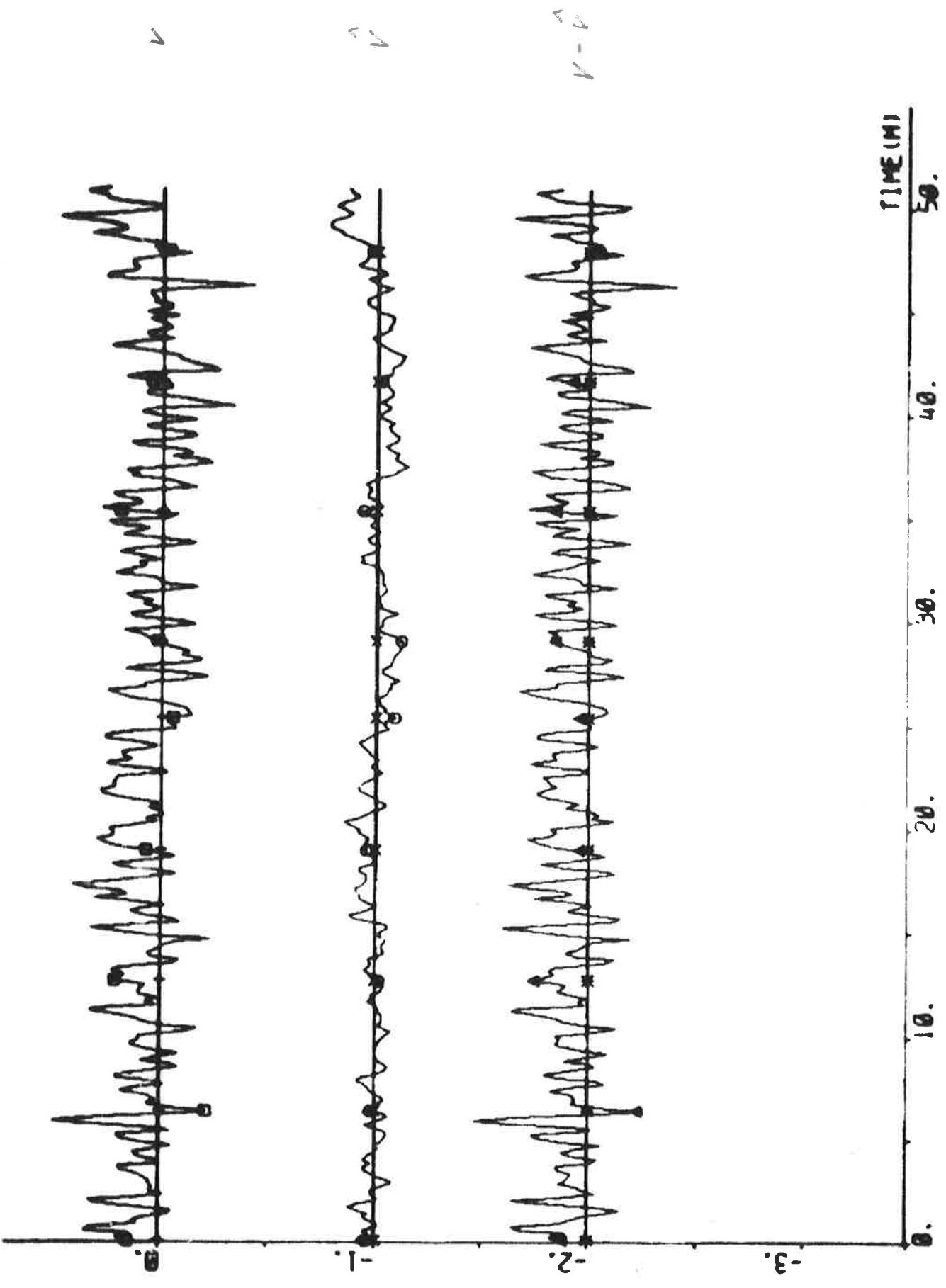
$0.05 \times d_c$

TIME (M)  
50.  
40.  
30.  
20.  
10.

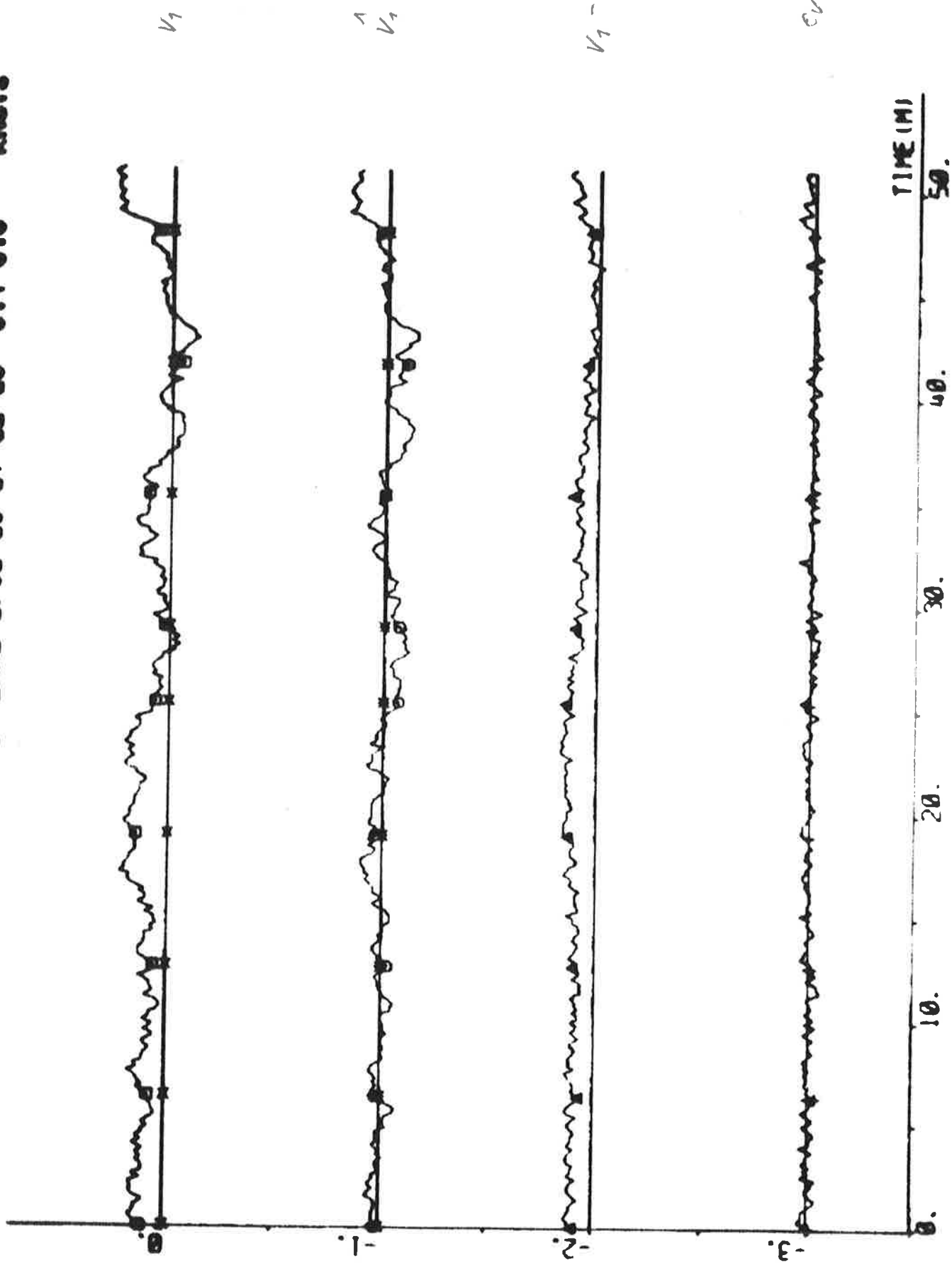
PLOT ACCP1(1)-ACCP1(13) ACCP1(12) ACCP1(14) ACCP1(11) 00 -20 50



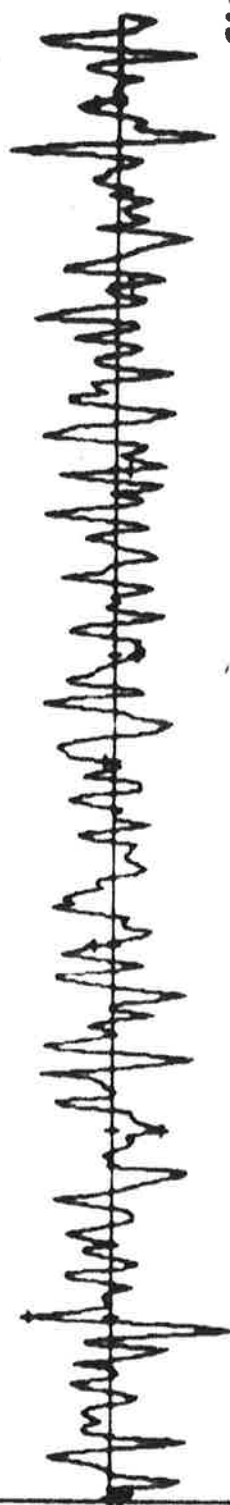
PLOT A33P1(1)-A33P2(1) A33P2(2) ERRS 00 01 02 -3.4 0.0 - KNOTS



PLOT ACCP1(1)-ACCP1(4) ACCP1(5) ERR2 EPS2 00 01 02 03 -3.4 0.0 - KNOTS



PLOT ACCP1(1)-ACCP1(6) ACCP1(7) ERR3 EP33 00 002 004 006 -0.7 0. - 00000



0. 10. 20. 30. 40. 50.  
TIME (M)

PLOT A33P1(1)-A33P1(8) A33P1(9) ERR4 EPS4 00 01 02 03 -3.4 0.0 DEG



$\gamma - \gamma_{ref}$



$\gamma - \gamma_{ref}$



$\gamma - \gamma_{ref}$



$\gamma - \gamma_{ref}$



PLOT ACCP1(1)-ACCP1(2) ACCP1(3) ERR1 EPS1 00 020 040 060 -05 16 - DEG



$\delta$



$\delta$



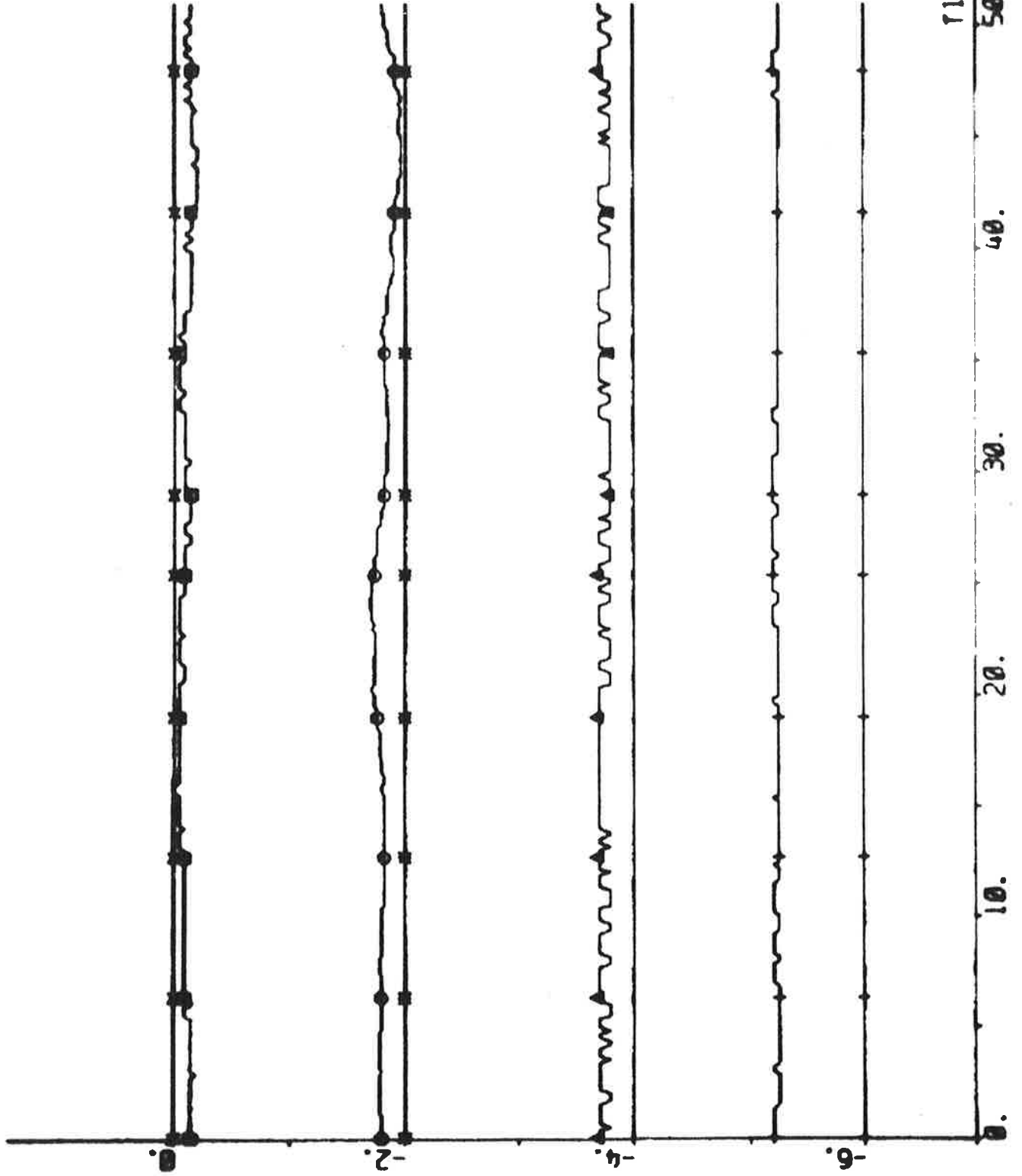
$\delta - \delta$



$\epsilon \delta$

0. 10. 20. 30. 40. 50. TIME (M)

PLOT ACCP1(1)-ACCP2(3) ACCP2(4) ACCP2(5) ACCP2(6) 00 02 04 06 -0.6 1.6



$0.5 \times d_0$  deg

$2 \times d_1$  knots

$100 \times dr$  deg/s

$0.5 \times d_g$  deg

TIME (M)  
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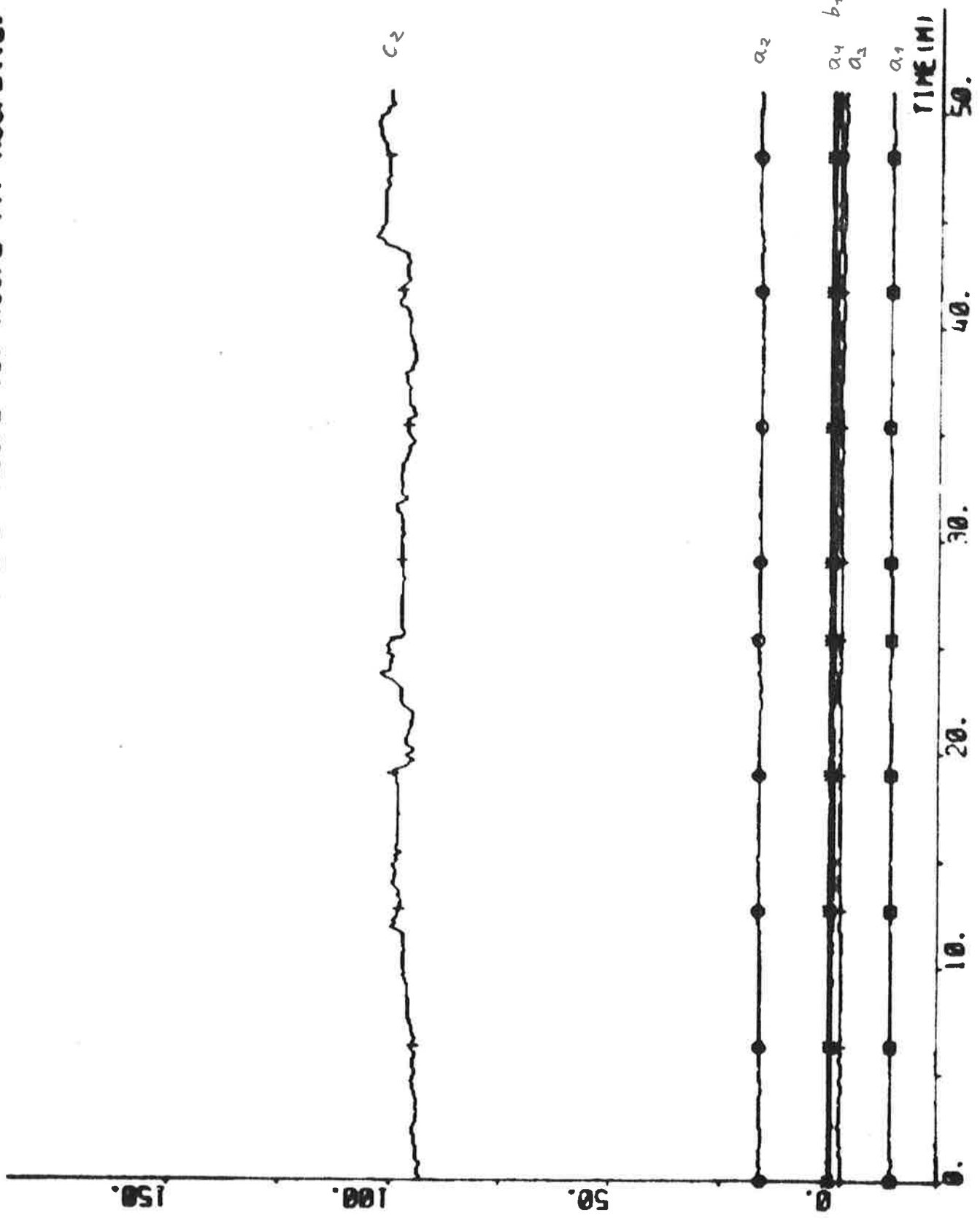
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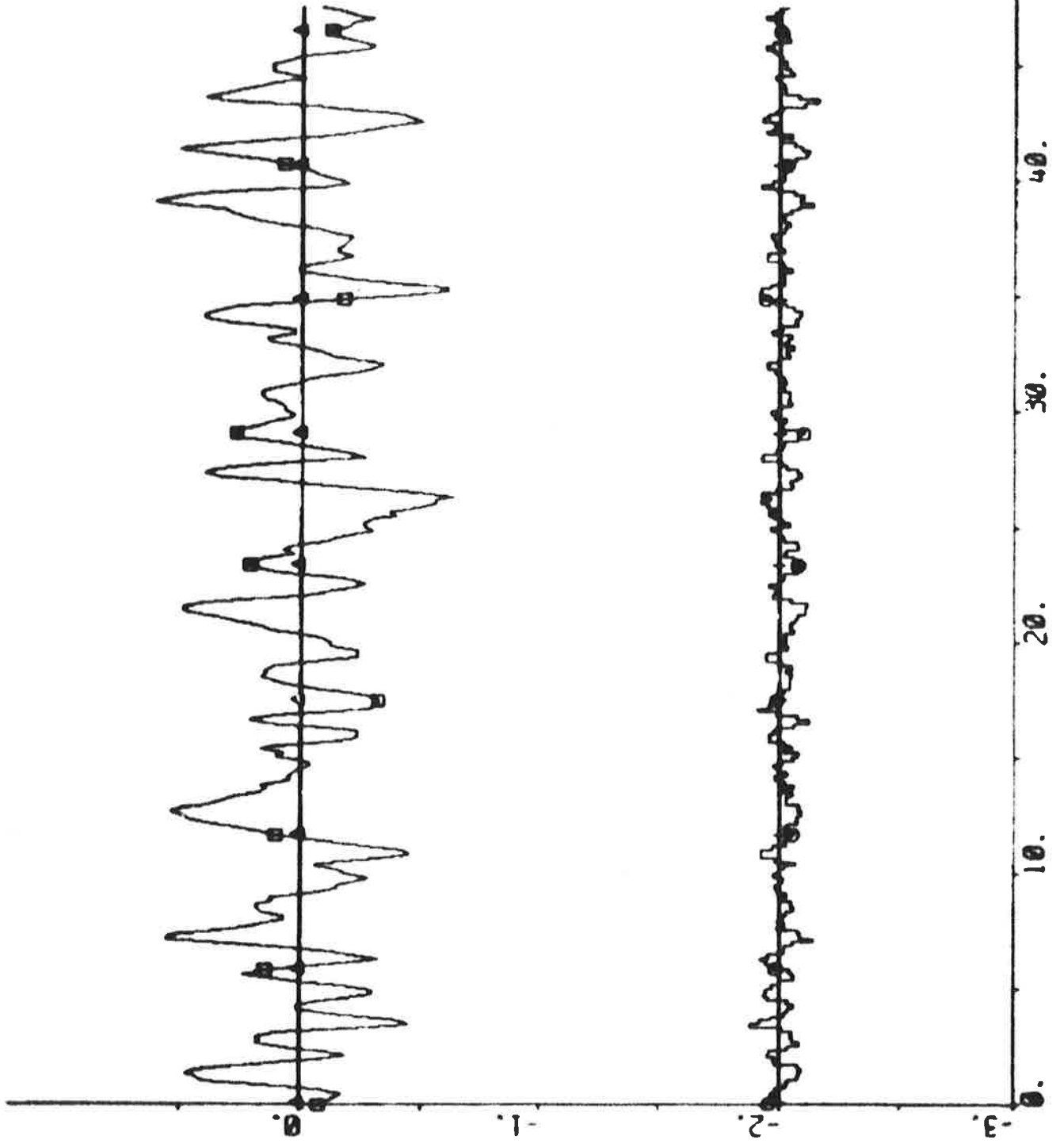


PLOT A33P1(1)-A33P2(7) A33P2(8) A33P2(9) A33P2(10) A33P2(11) A33P2(12) A3

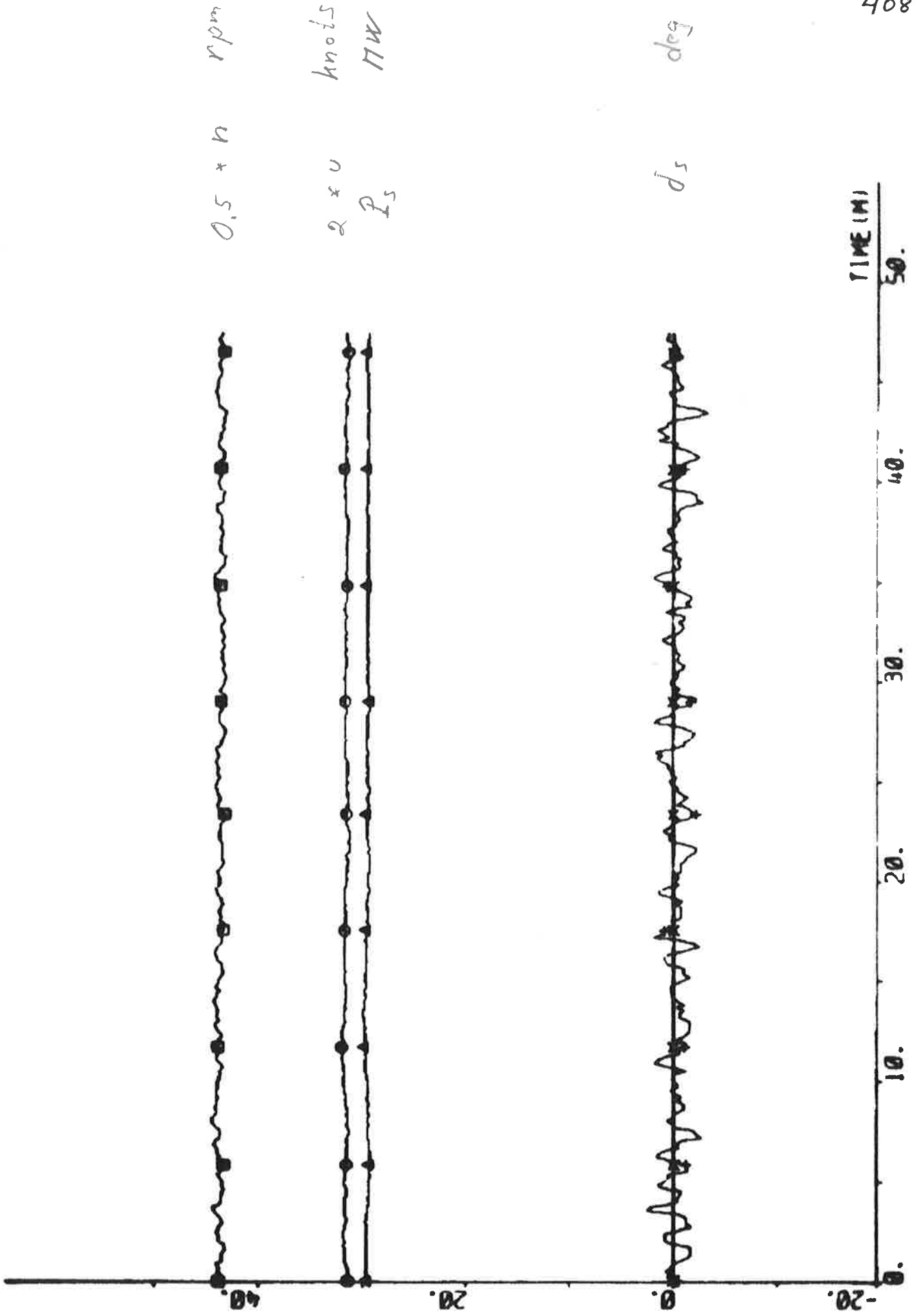




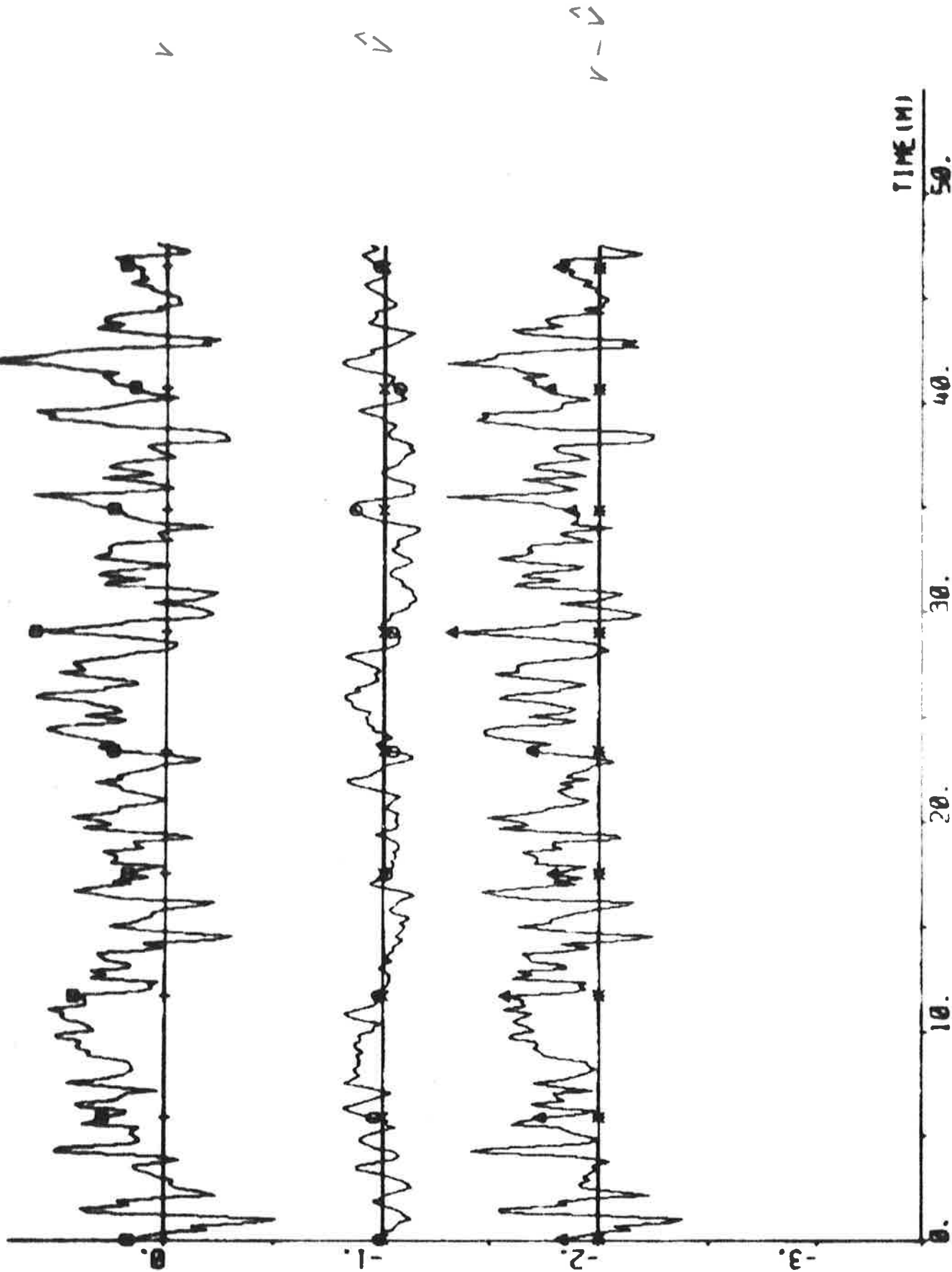
PLOT ACOMP1(1)-ACOMP1(8) HP ACOMP1(10) ACOMP1(15) Q2 -3 1 - DEG



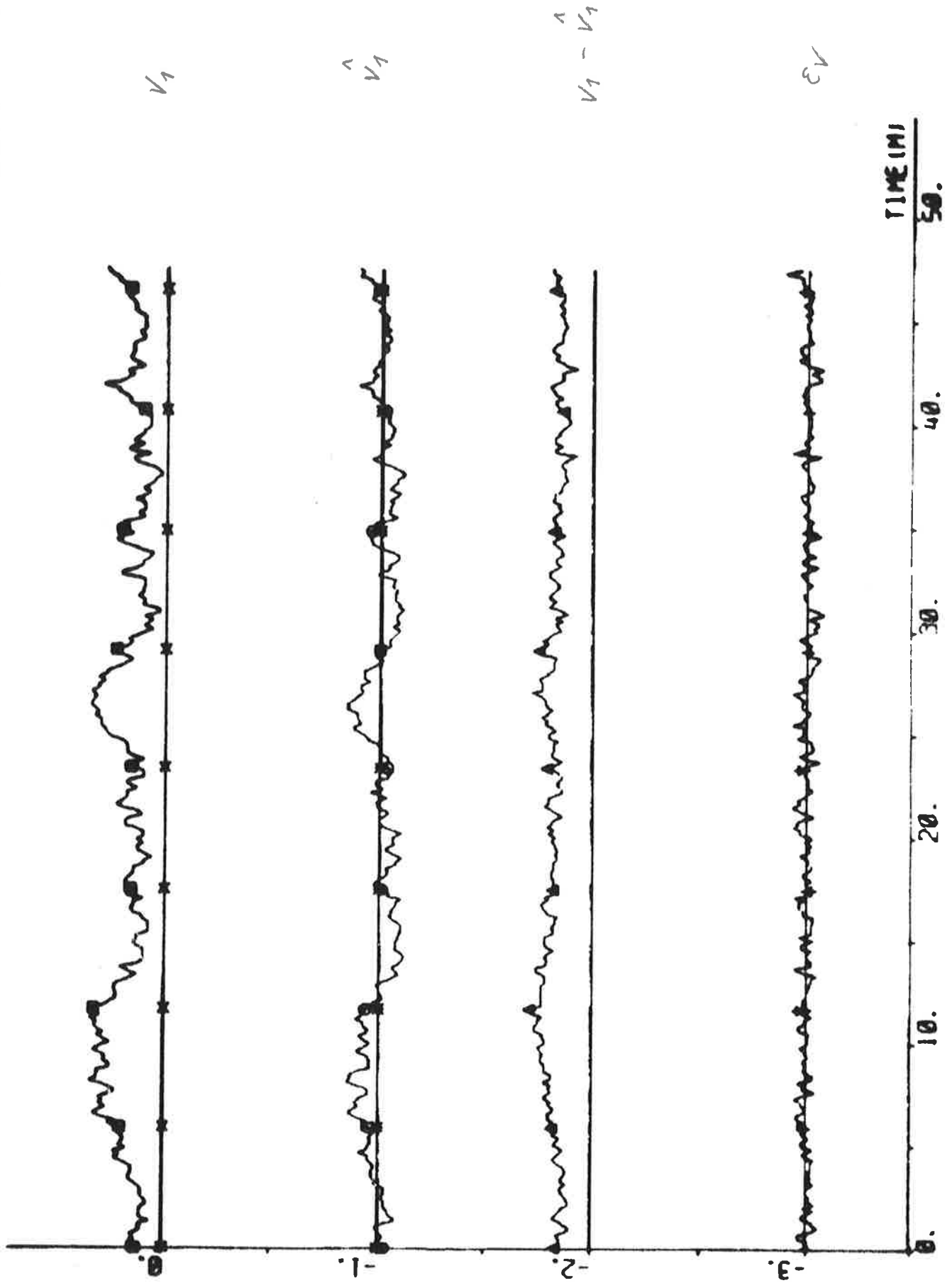
PLOT ACOMP1(1)-ACOMP1(13) ACOMP1(12) ACOMP1(14) ACOMP1(11) 00 -20 00



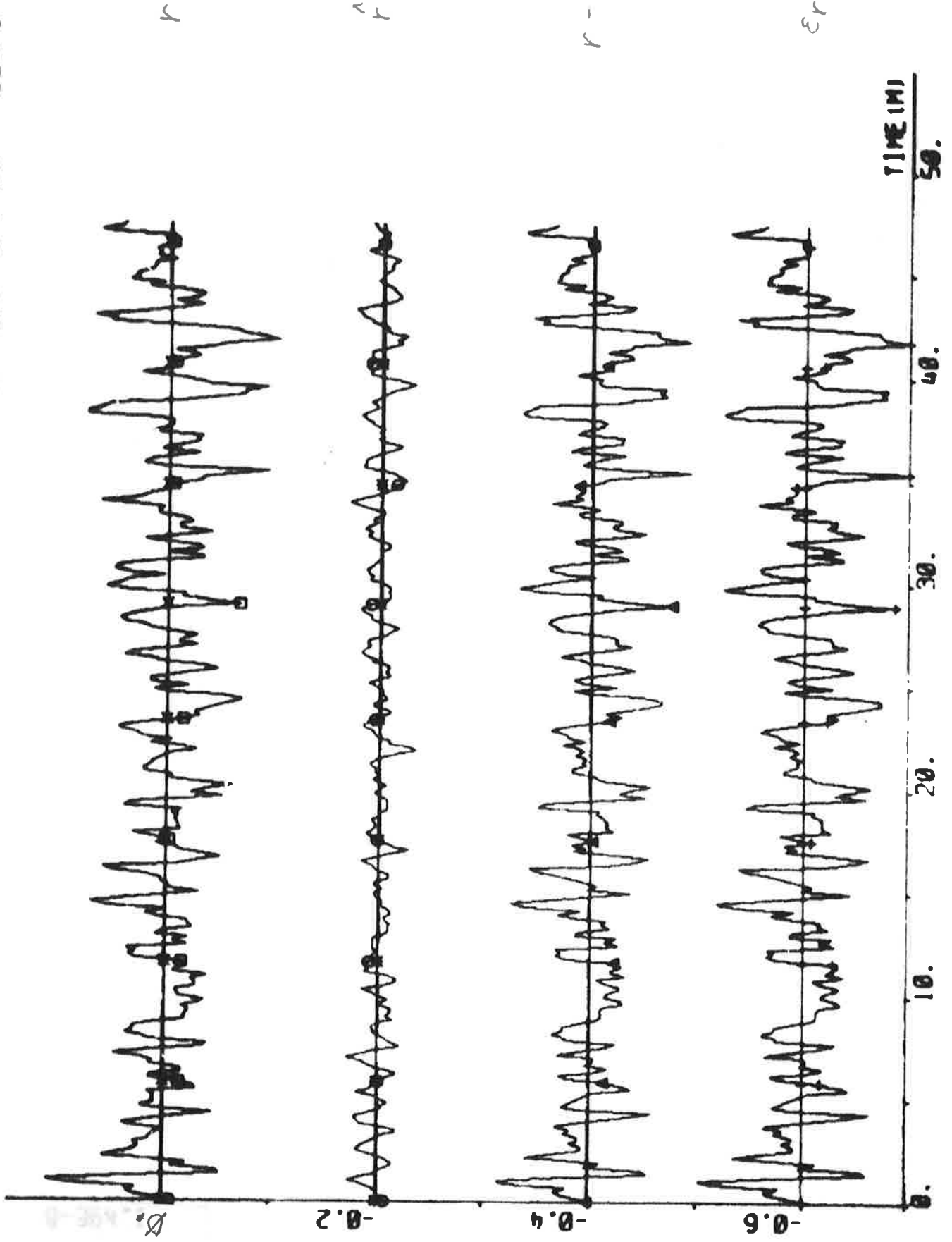
PLOT ACMP1(1)-ACMP2(1) ACMP2(2) ERRS 00 01 02 -3.4 0.0 - KNOTS



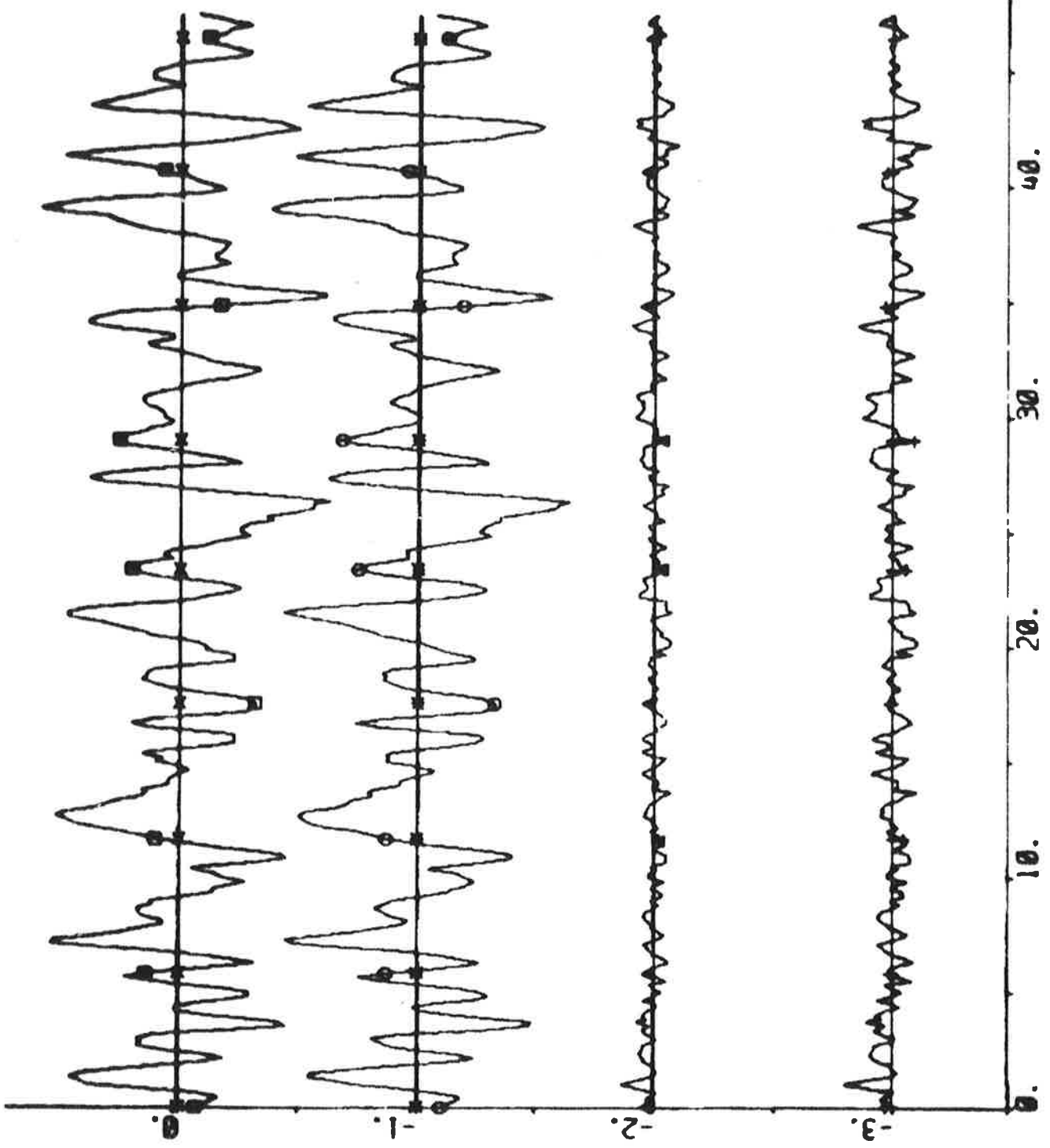
PLOT A34P1(1)-A34P1(4) A34P1(6) ERR2 EPS2 00 01 02 03 -3.4 0.0 - ROOTS



PLOT ACOMP1(1)-ACMP1(6) ACMP1(7) ERR3 EPS3 00 002 004 006 -0.7 0.0 - 00000



PLOT ACOMP1(1)-ACMP1(8) ACOMP1(9) ERR4 EPS4 00 01 02 03 -3.4 0.0 - 000



$y - y_{ref}$

$\hat{y} - y_{ref}$

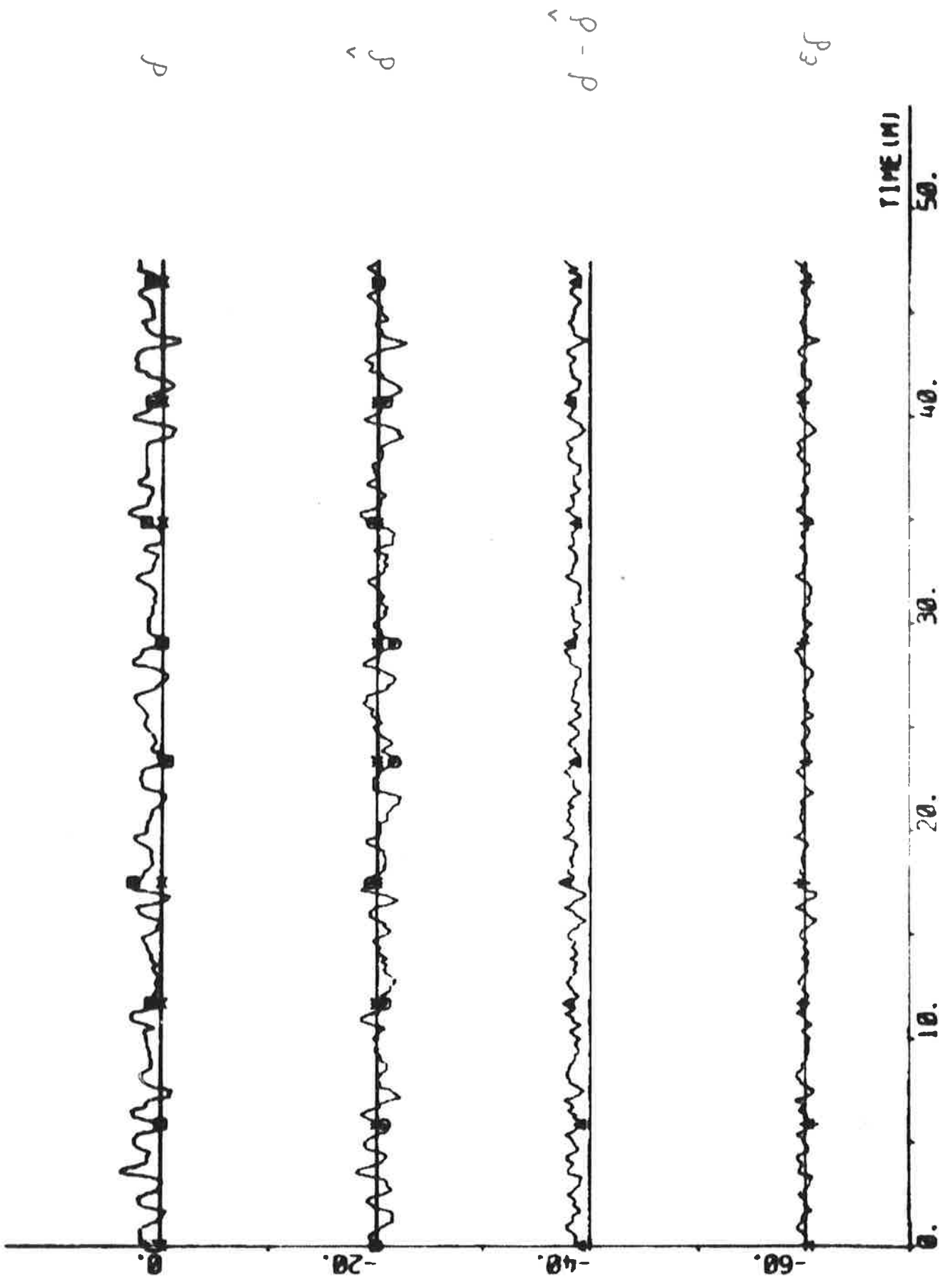
$\hat{y} - y$

$\hat{y}$

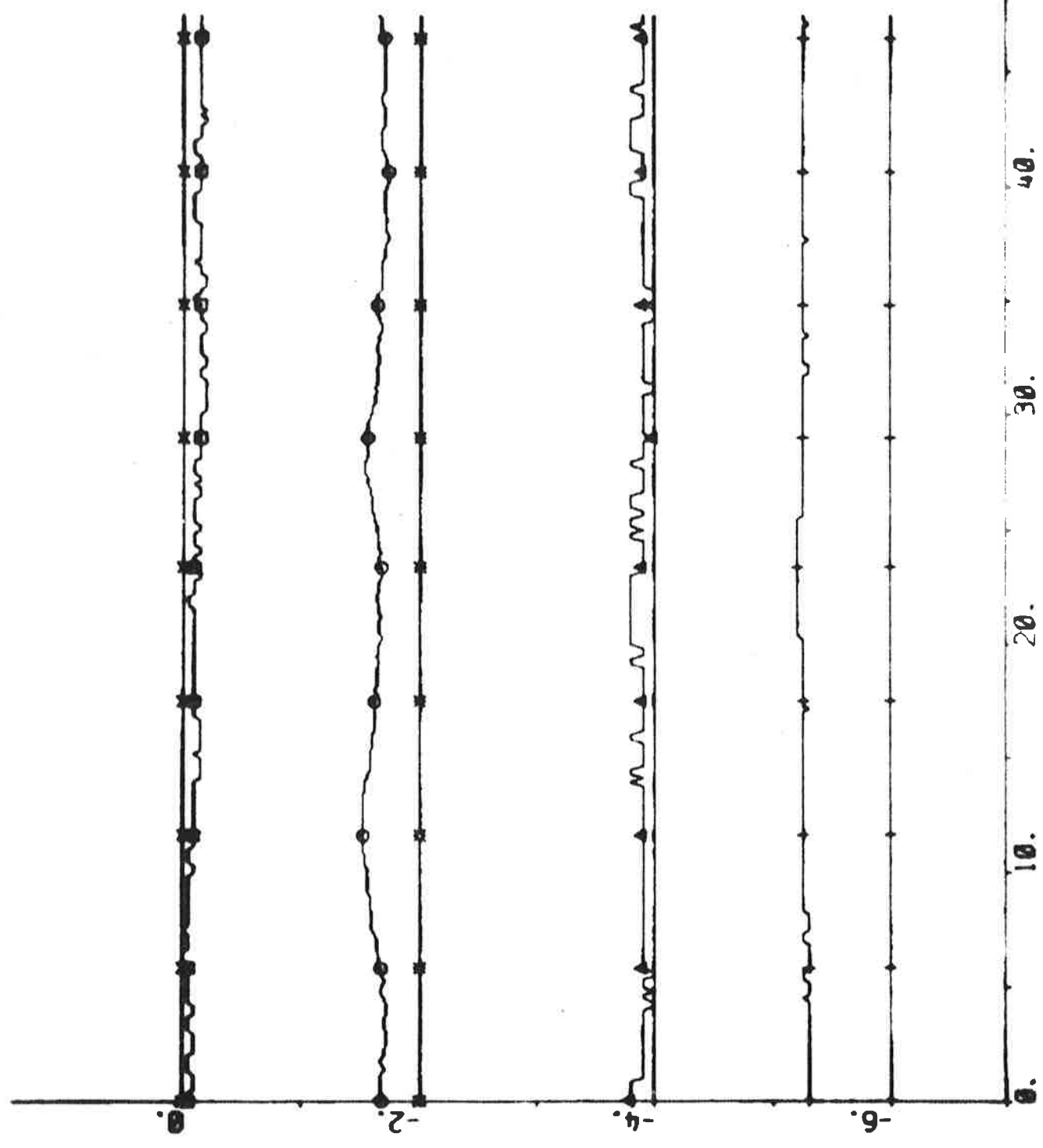
TIME (M)  
50.



PLOT ACOMP1(1)-ACOMP1(2) ACOMP1(3) ERR1 EPS1 00 020 040 060 -05 15 - 0000



PLOT A34P1(1)-A34P2(3) A34P2(4) A34P2(5) A34P2(6) 00 02 04 06 -0.6 1.6



$0.5 \times d_0$  deg

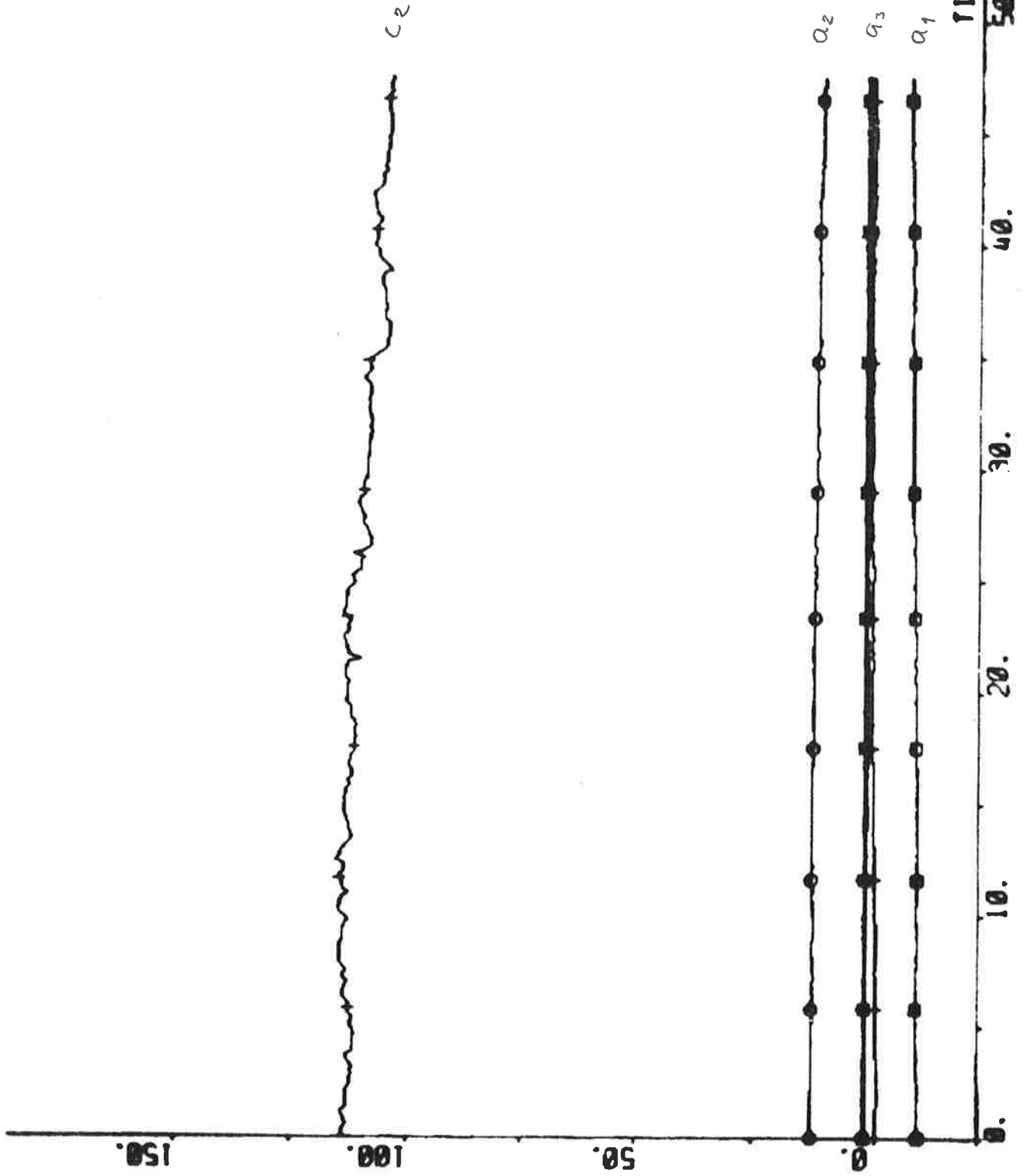
$2 \times dv$  knots

$100 \times dr$  deg/s

$0.5 \times d_\delta$  deg

TIME (M)  
50.

PLOT ACOMP1(1)-ACOMP2(7) ACOMP2(8) ACOMP2(9) ACOMP2(10) ACOMP2(11) ACOMP2(12) NO



## EXPERIMENT A35

Date	1976-04-29	Forward draught	10.9 m
Time	14.45	Aft draught	12.9 m
Duration	45 min	Wind direction	SE (1; see App. A)
Position	S 18°30' E 05°15'	Wind velocity	11 m/s (strong breeze)
$\psi_{\text{ref}}$	143 deg	Wave height	High sea from SE

PID-regulator using estimates from the Kalman filter

$k_P = 3$	$k_D = 140 \text{ s}$	$k_I = 0.02 \text{ 1/s}$
$T_s = 10 \text{ s}$	$V_0 = 6 \text{ m/s}$	IVVC = 1

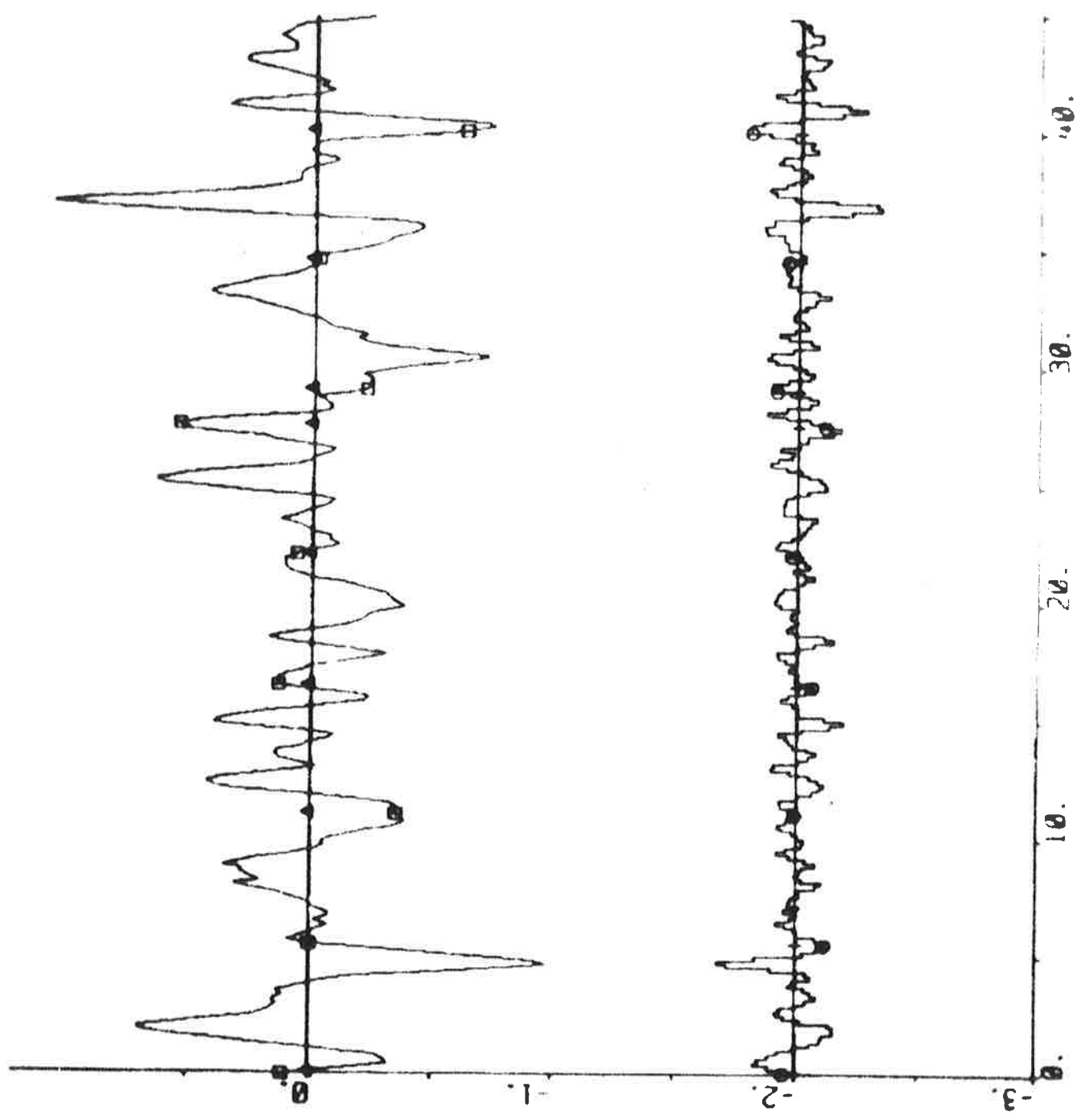
The parameters were manually tuned before the experiment started.

## Final values:

$$\hat{\delta}_0 = 0.0 \text{ deg} \quad \hat{d}_V = 0.14 \text{ knots} \quad \hat{d}_r = 0.001 \text{ deg/s} \quad \hat{d}_\delta = 1.4 \text{ deg}$$
Statistics (mean value and standard deviation)

$\delta_c$	0.01	± 1.66	deg	$P_S$	14.7	± 0.3	MW
$\delta$	1.46	± 1.55	deg	$\varepsilon_V$	0.00	± 0.03	knots
$\psi - \psi_{\text{ref}}$	0.008	± 0.295	deg	$\varepsilon_r$	0.00	± 0.04	deg/s
$n$	68.6	± 0.5	rpm	$\varepsilon_\psi$	0.00	± 0.04	deg
$u$	12.1	± 0.1	knots	$\varepsilon_\delta$	0.0	± 0.4	deg
$V_1 = 0.317$							

PLOT A36P1(1)-A36P1(8) HP A36P1(10) A36P1(15) Q2 -3 1 - DEC

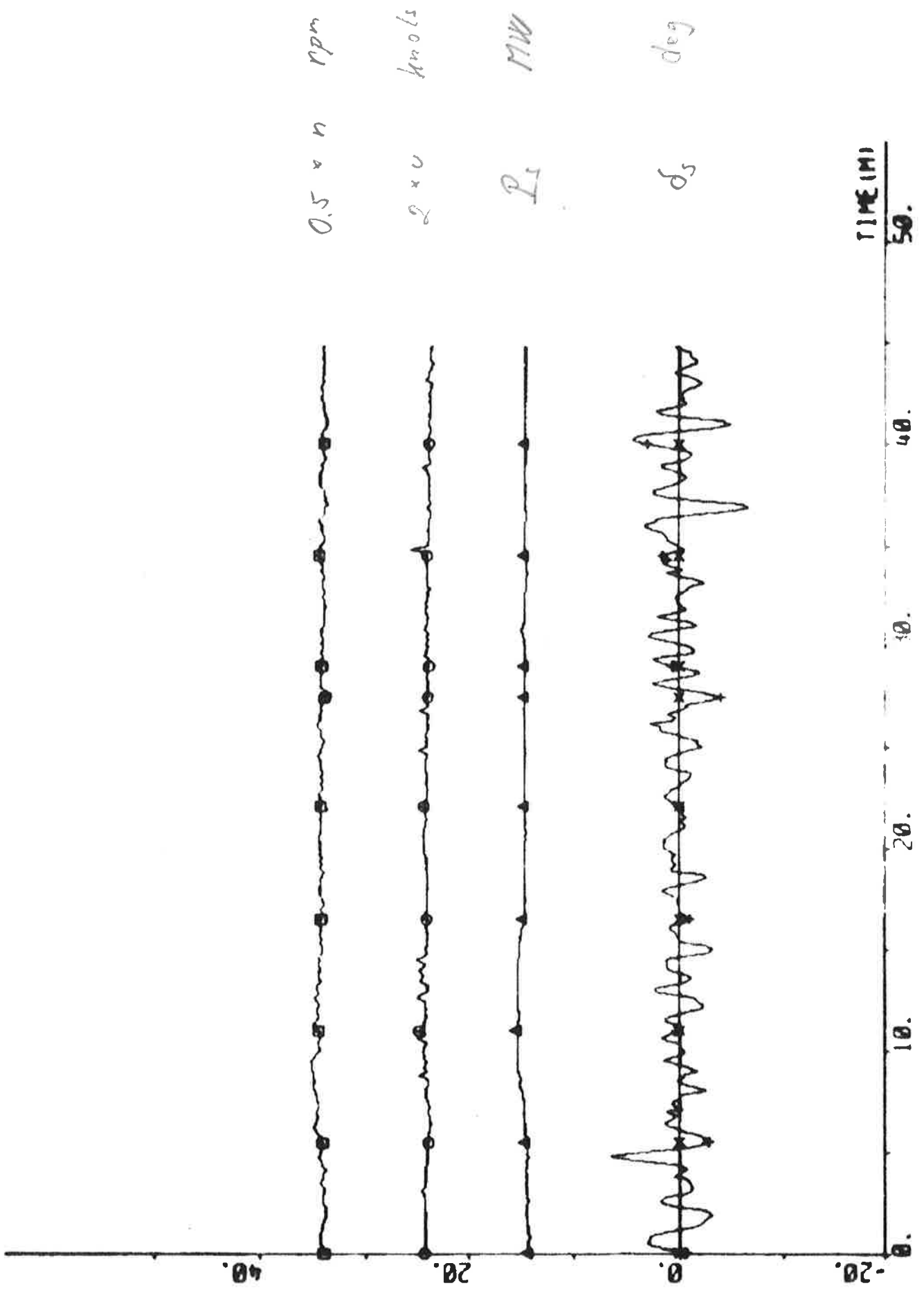


$y - y_{ref}$

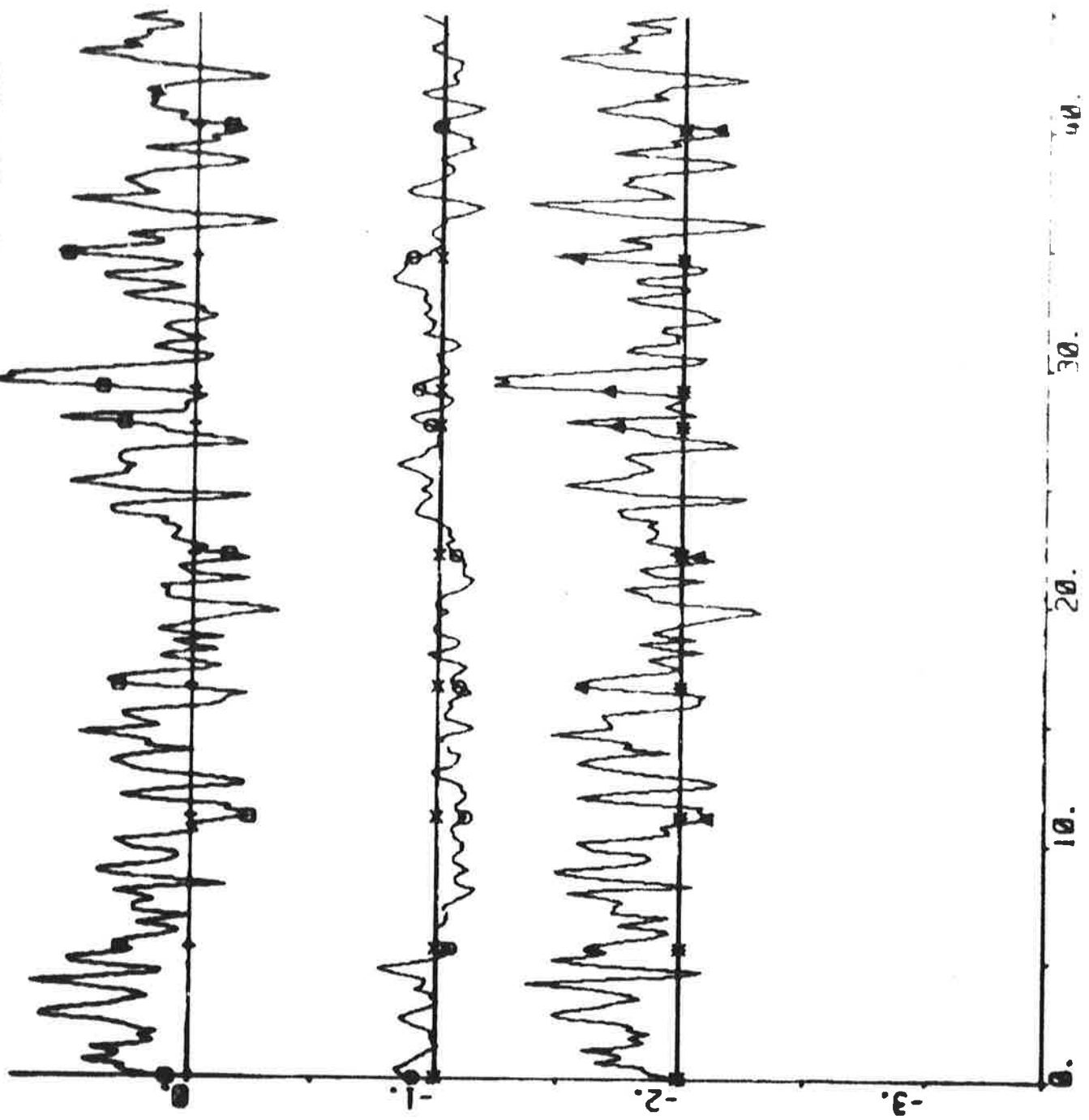
$0.05 \times d_c$

TIME (M)  
50.

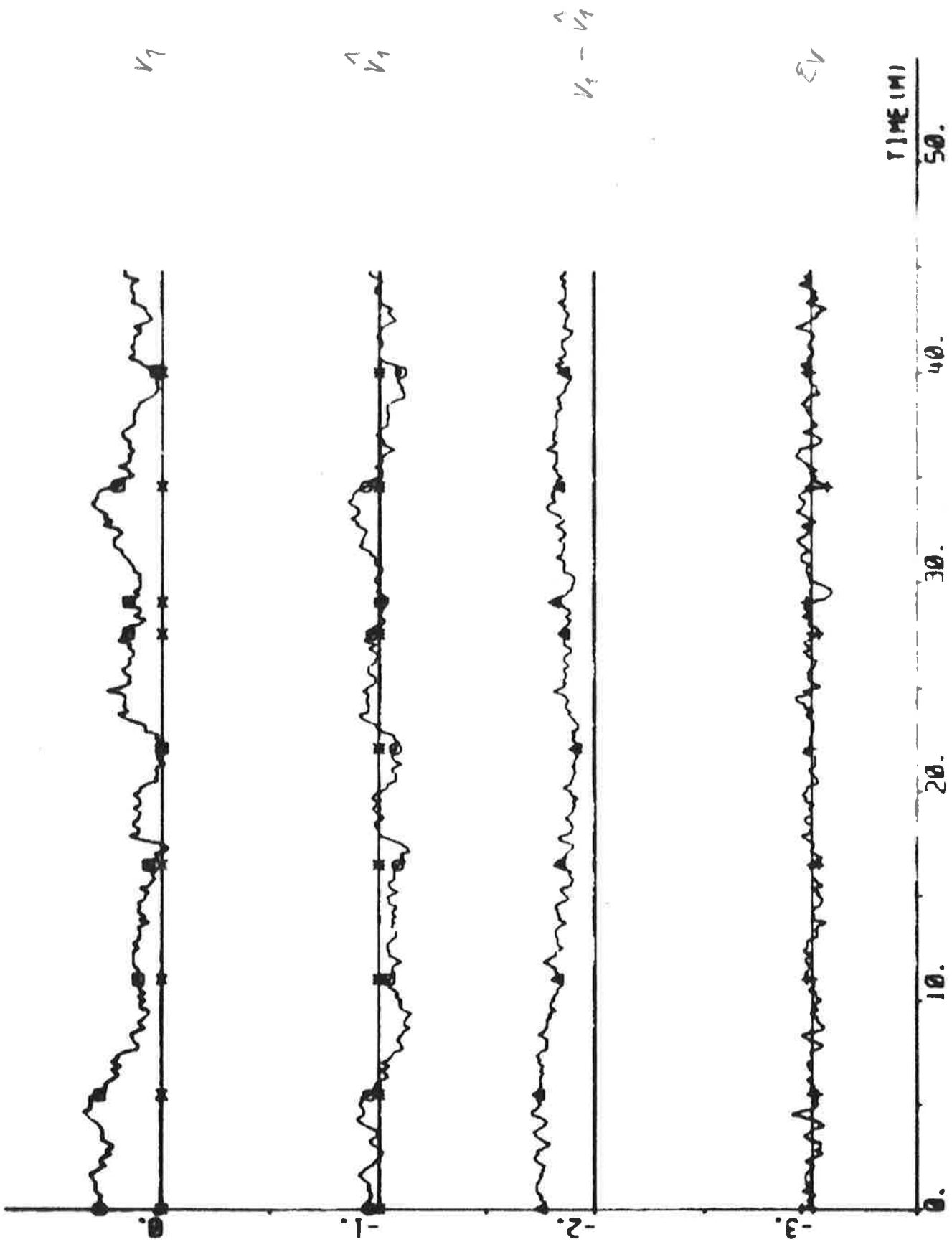
PLOT R36P1(1)-R36P1(13) R36P1(12) R36P1(14) R36P1(11) 00 -20 00



PLOT A36P1(1)-A36P2(1) A36P2(2) ERRS 00 01 02 -3.4 0.0 - KNOTS

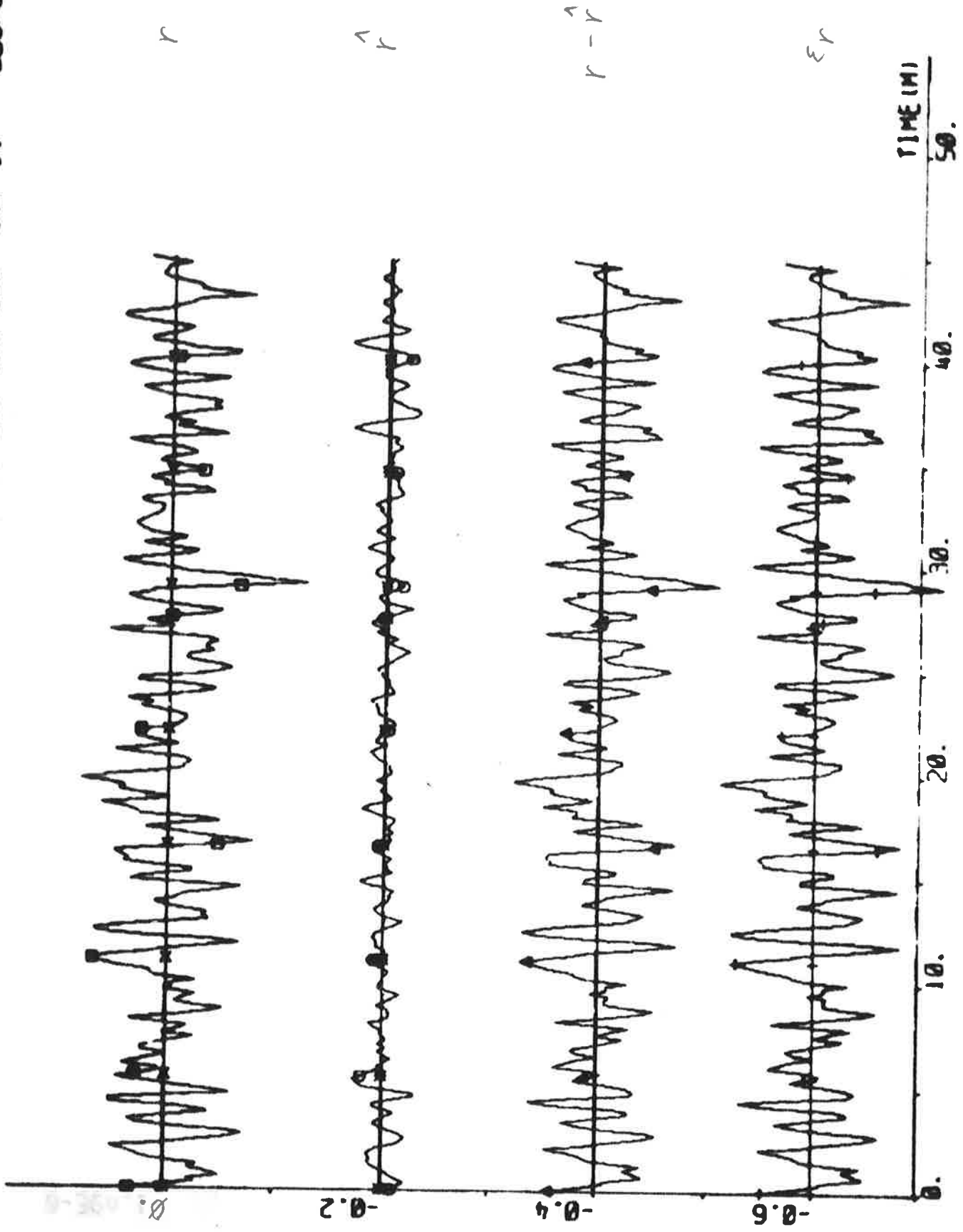


PLOT ASEP1(1)-ASEP1(4) ASEP1(5) ERR2 EPS2 00 01 02 03 -3.4 0.0 - 1000TS

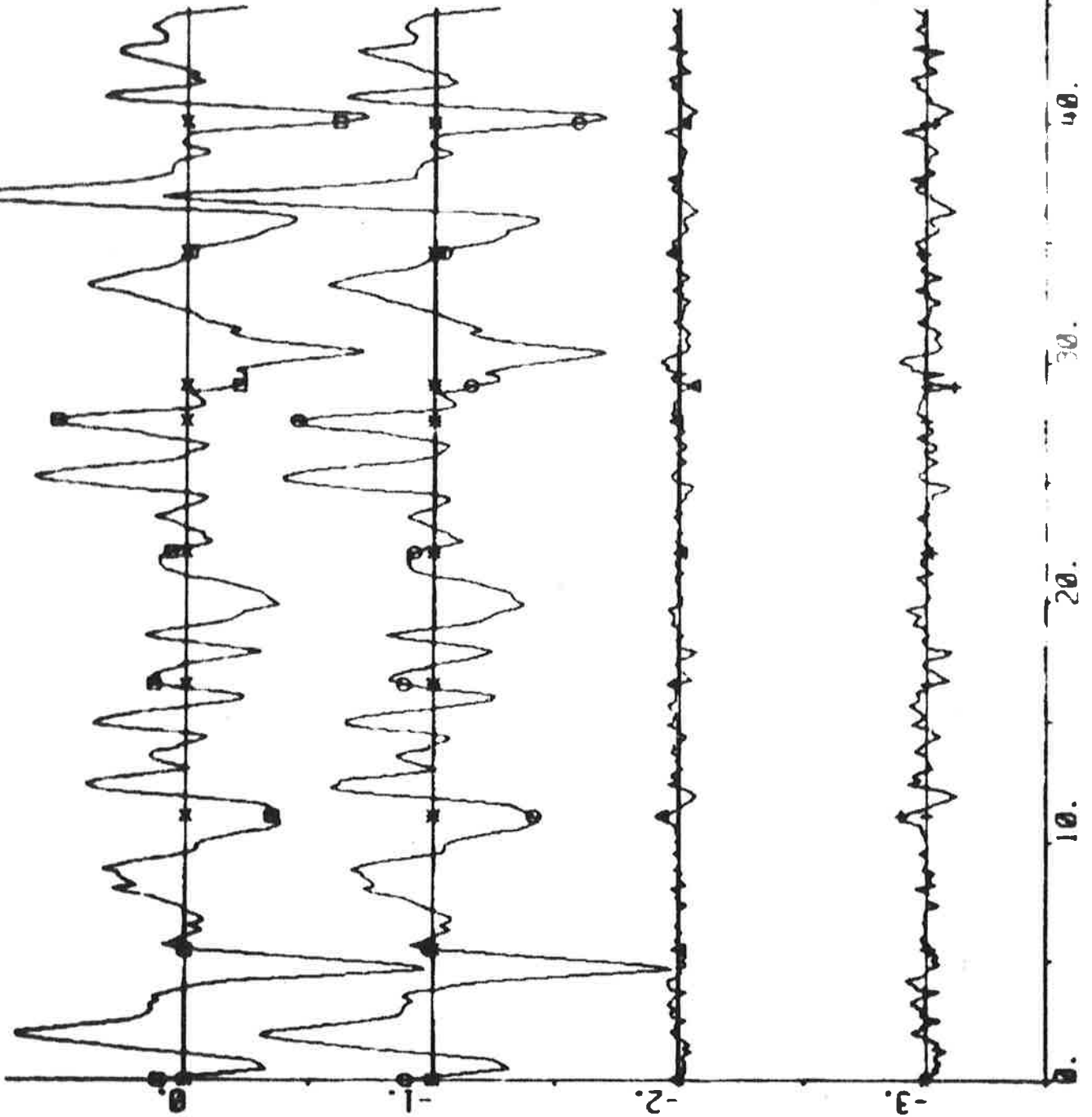




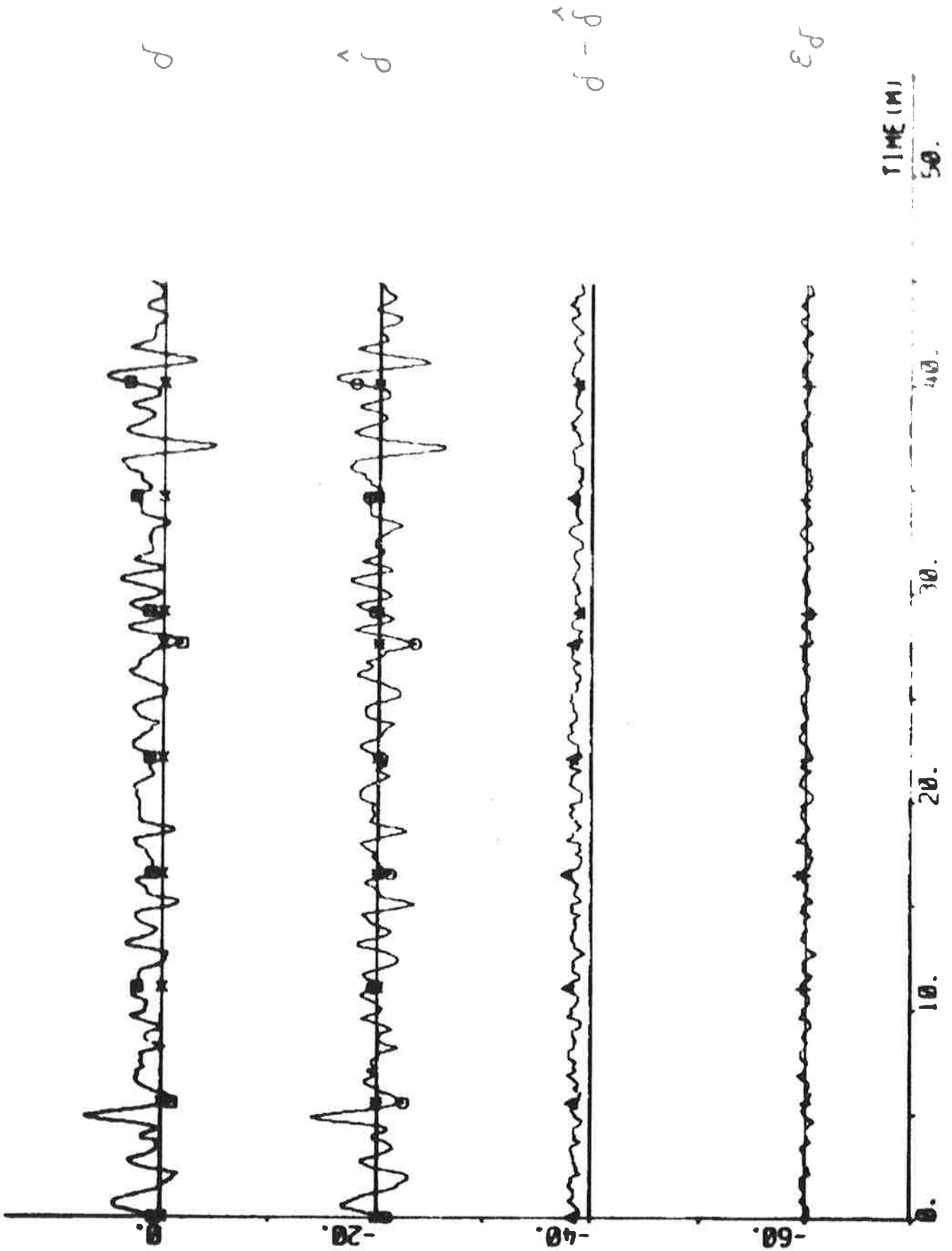
PL0T A3SP1(1)-A3SP1(8) A3SP1(7) ERR3 EPS3 00 002 004 006 -0.7 0. - 000-3



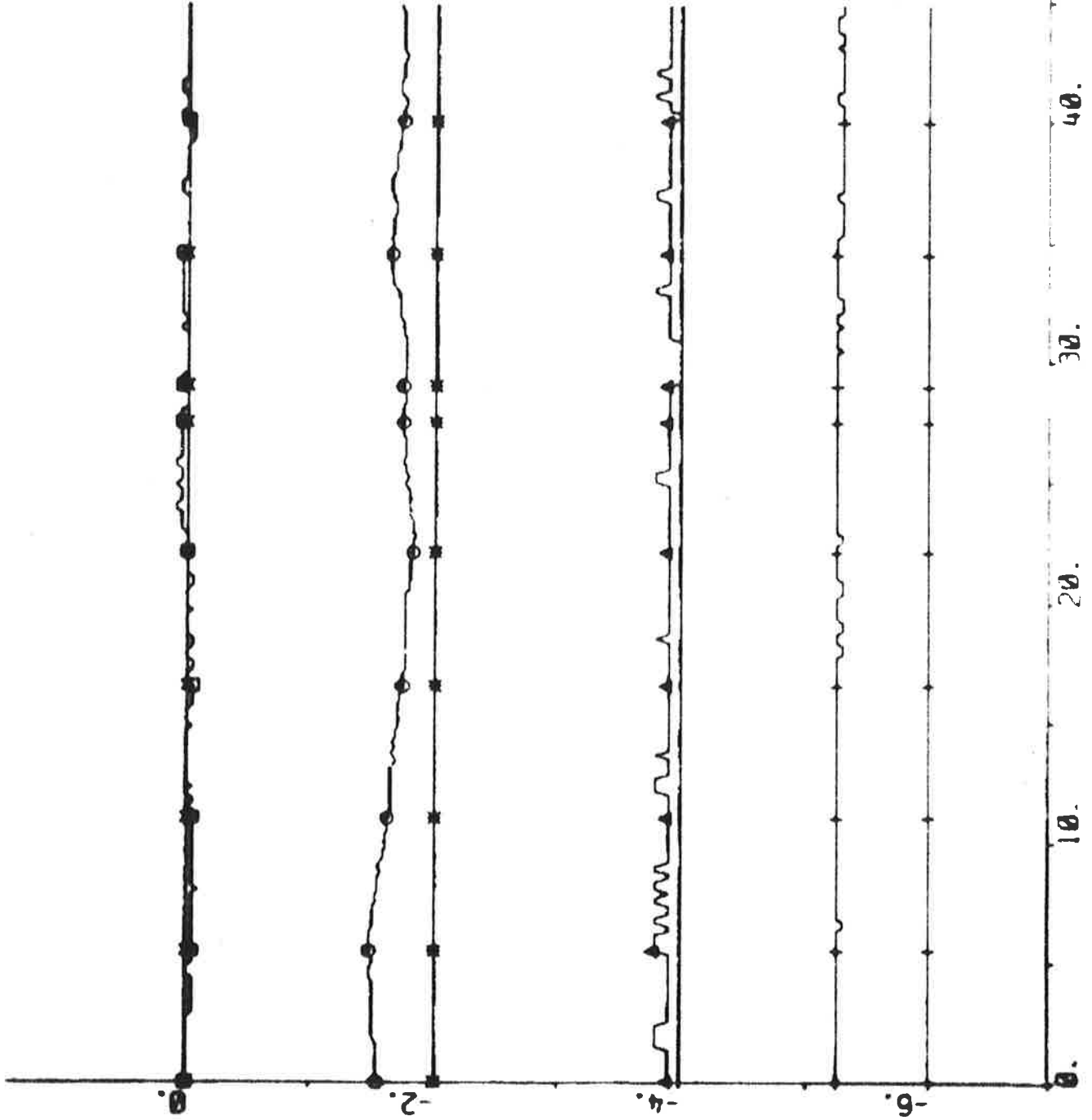
PLOT A36P1(1)-A36P1(8) A36P1(9) ERR4 EPS4 00 01 02 03 -3.4 0.0 - DEC



PLOT A3SP1(1)-A3SP1(2) A3SP1(3) ERR1 EPS1 00 020 040 060 -66 16 - DEG



PLOT A36P1(1)-A36P2(3) A36P2(4) A36P2(5) A36P2(6) 00 02 04 06 -0.6 1.6



$0.5 \times \hat{d}_0$  deg

$2 \times \hat{d}_V$  knots

$100 \times \hat{d}_T$  deg/hr

$0.5 \times \hat{d}_F$  deg

TIME (MI)  
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10.

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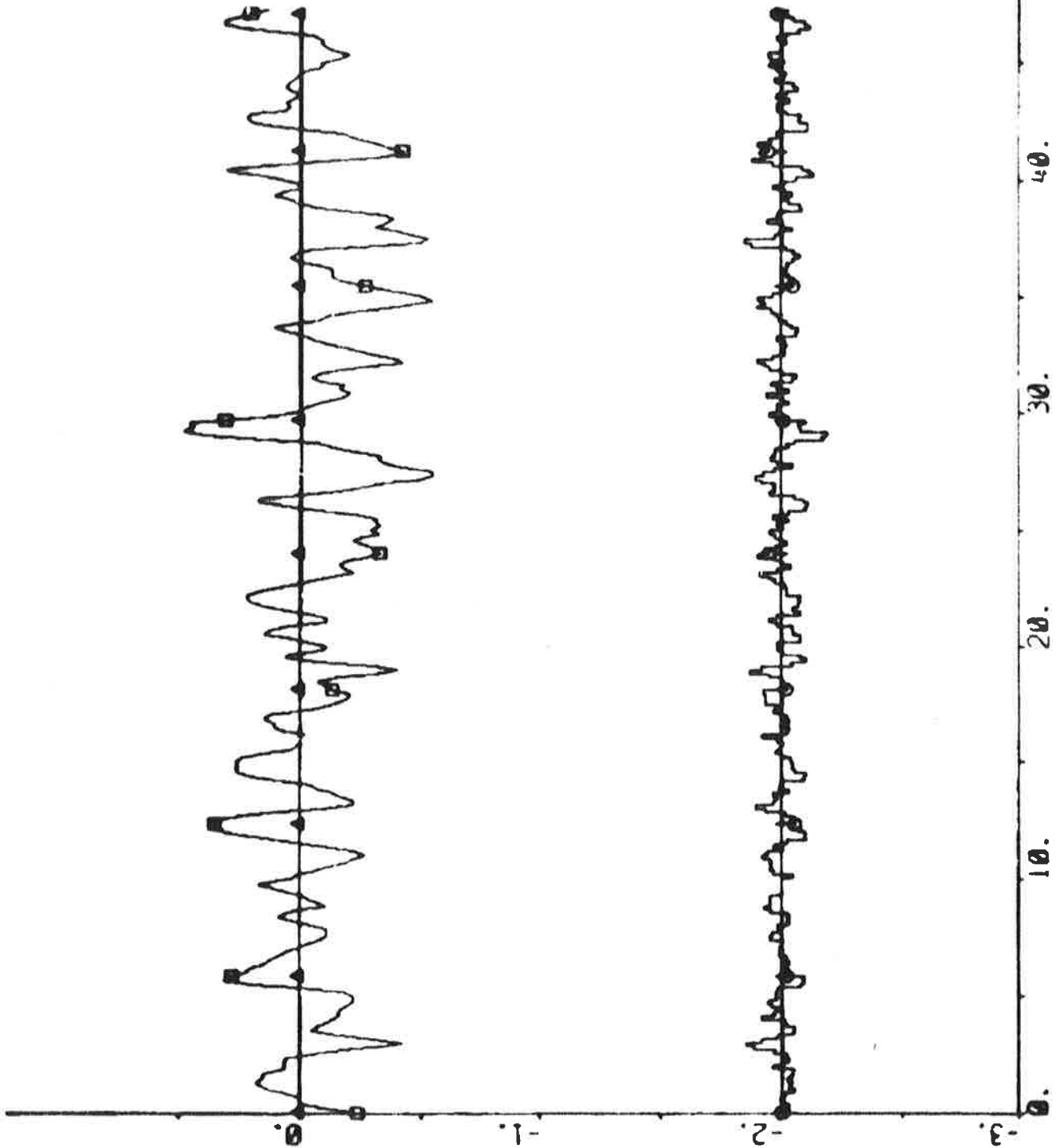
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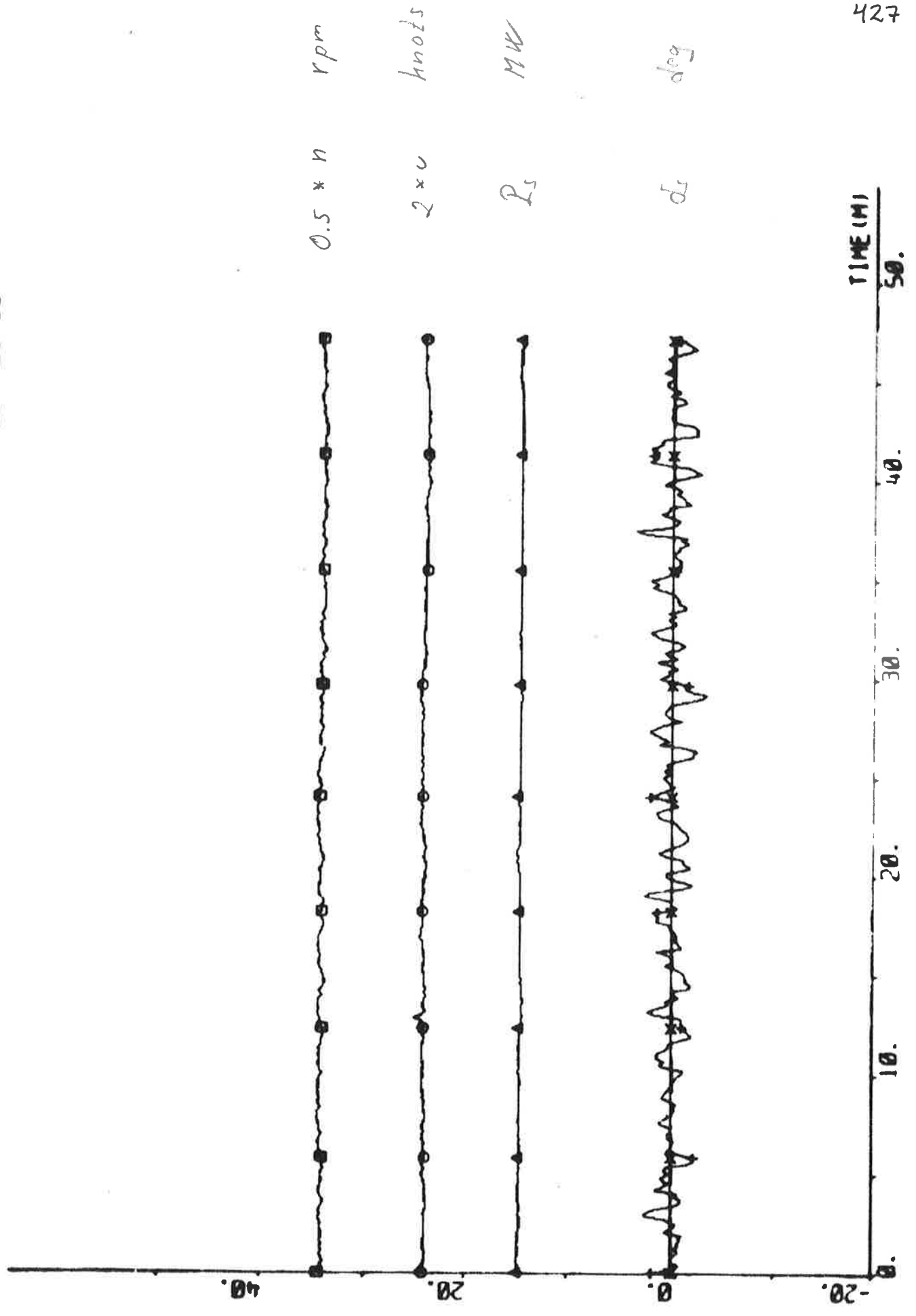
PLOT A3BP1(1)-A3BP1(8) HP A3BP1(10) A3BP1(15) 02 -3 1 - DEG



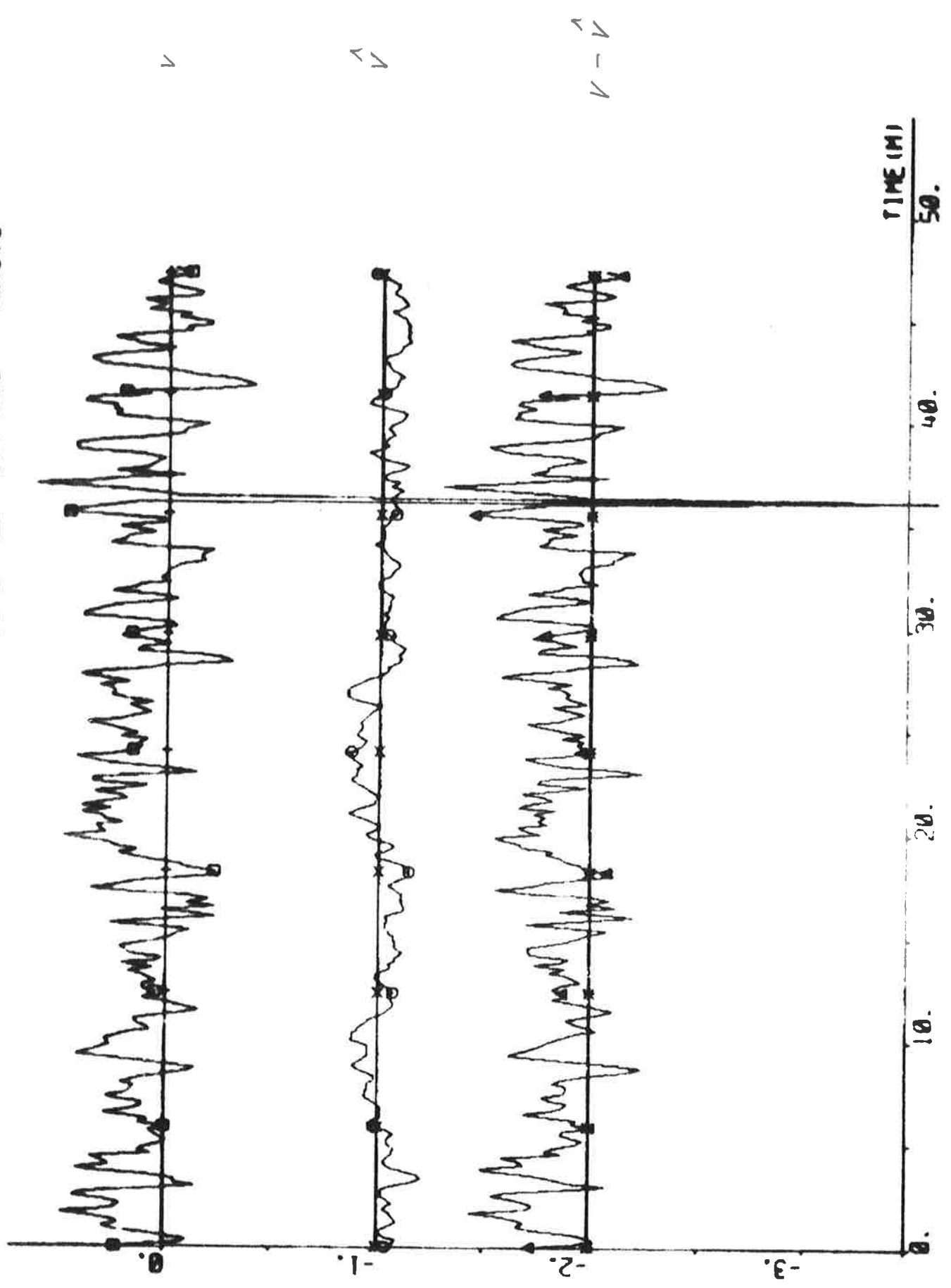
$y - y_{ref}$

$0.05 * d_c$

PLOT A3BP1(1)-A3BP1(13) A3BP1(12) A3BP1(14) A3BP1(11) 00 -20 00

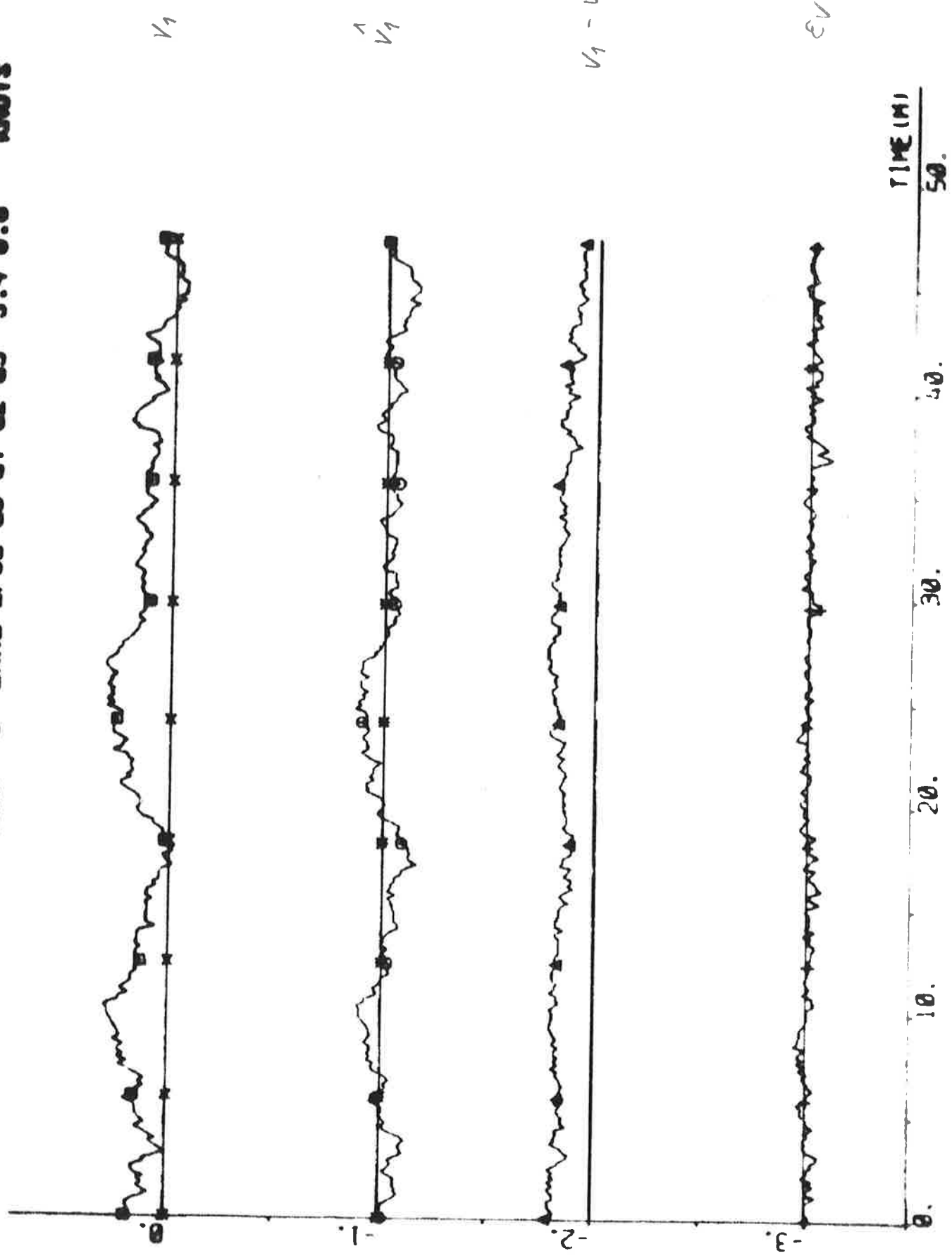


PLOT A36P1(1)-A36P2(1) A36P2(2) ERRS 00 01 02 -3.4 0.0 - KNOTS

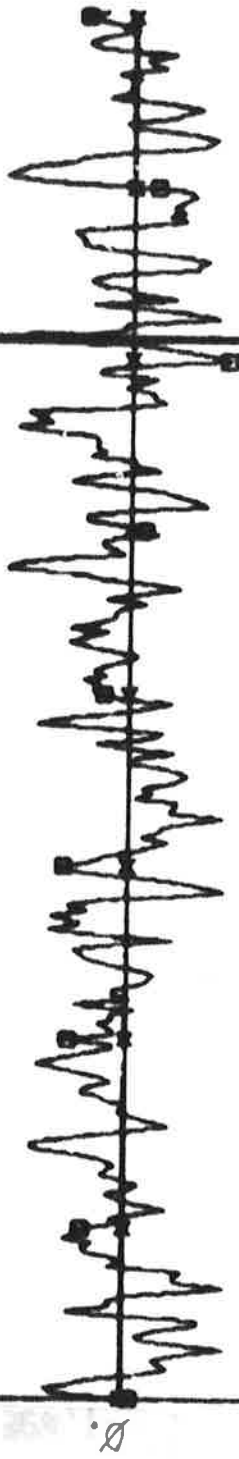




PLOT A36P1(1)-A36P1(4) A36P1(5) ERR2 EPS2 00 01 02 03 -3.4 0.0 - 1000TS



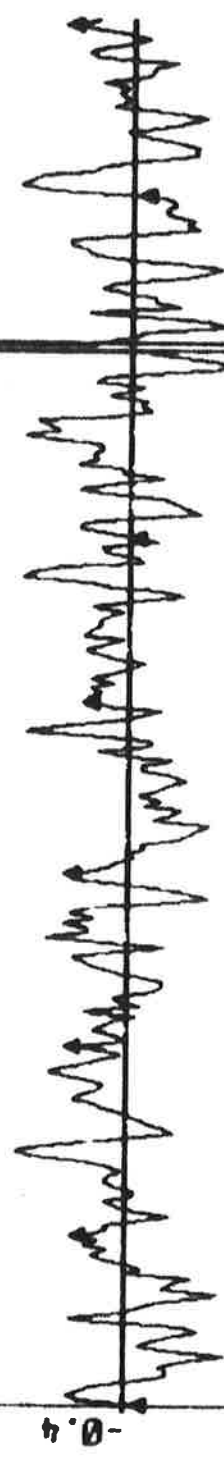
PLOT R3CP1(1)-R3CP1(6) R3CP1(7) ERR3 EPS3 00 002 004 008 -0.7 0.0 - 000000



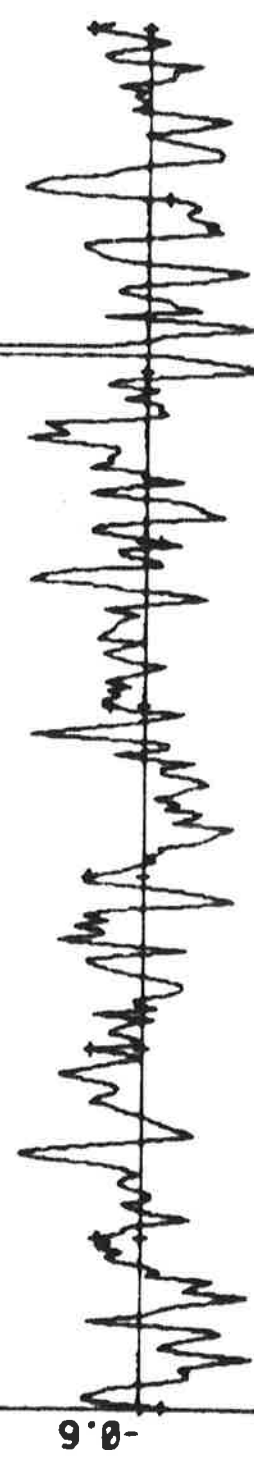
r



r



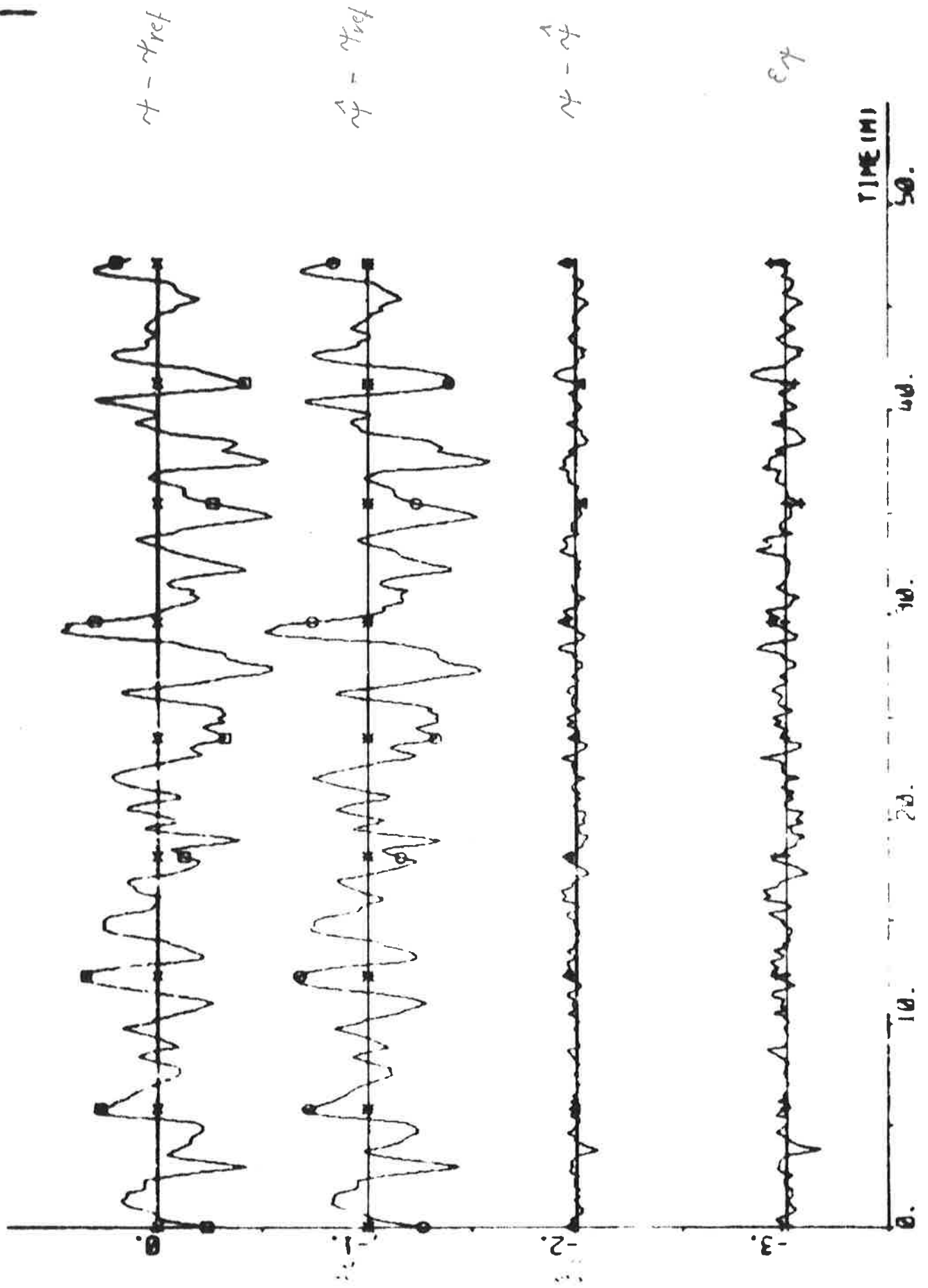
r - r



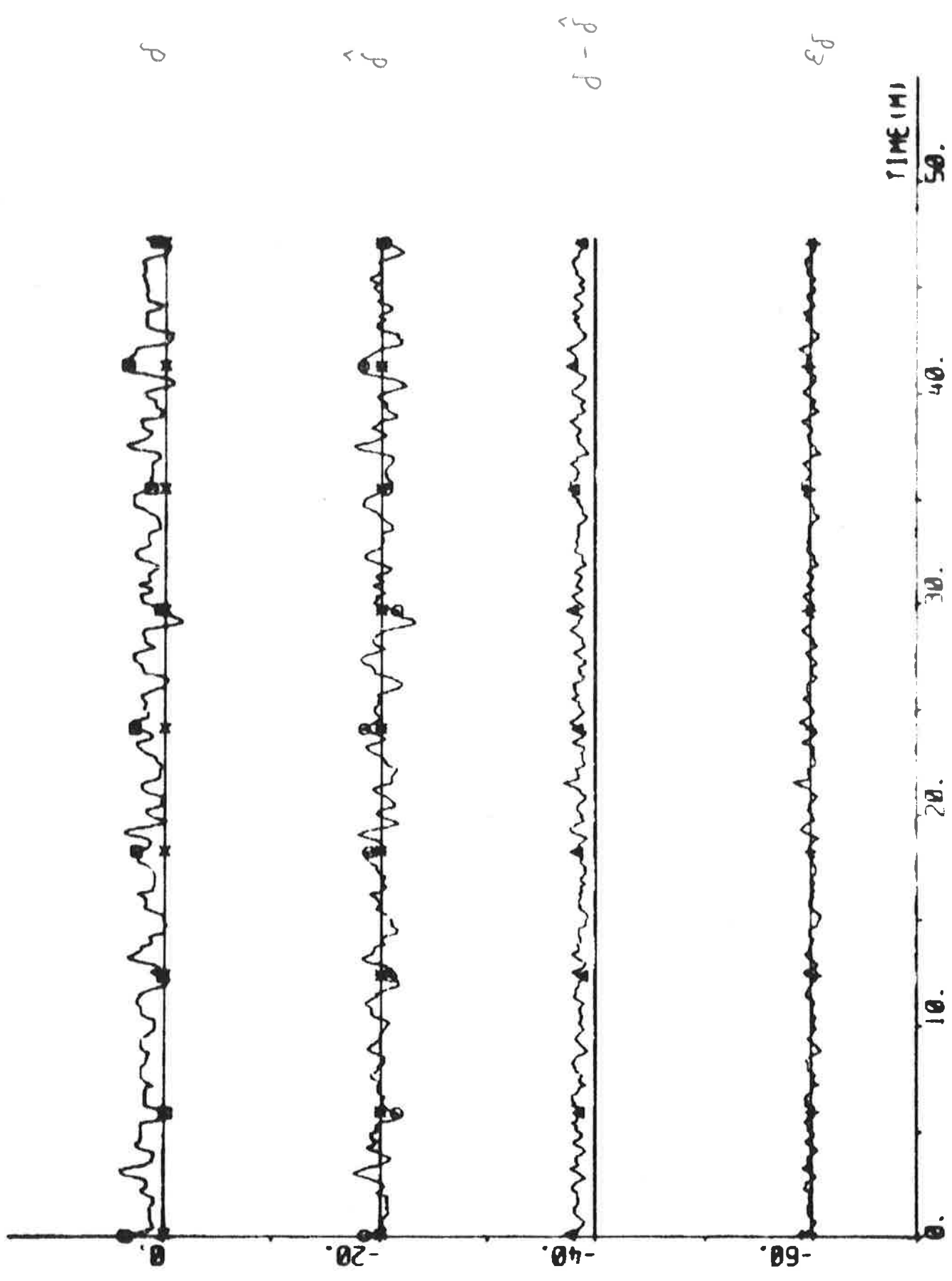
Er

TIME (M) 0. 10. 20. 30. 40. 50.

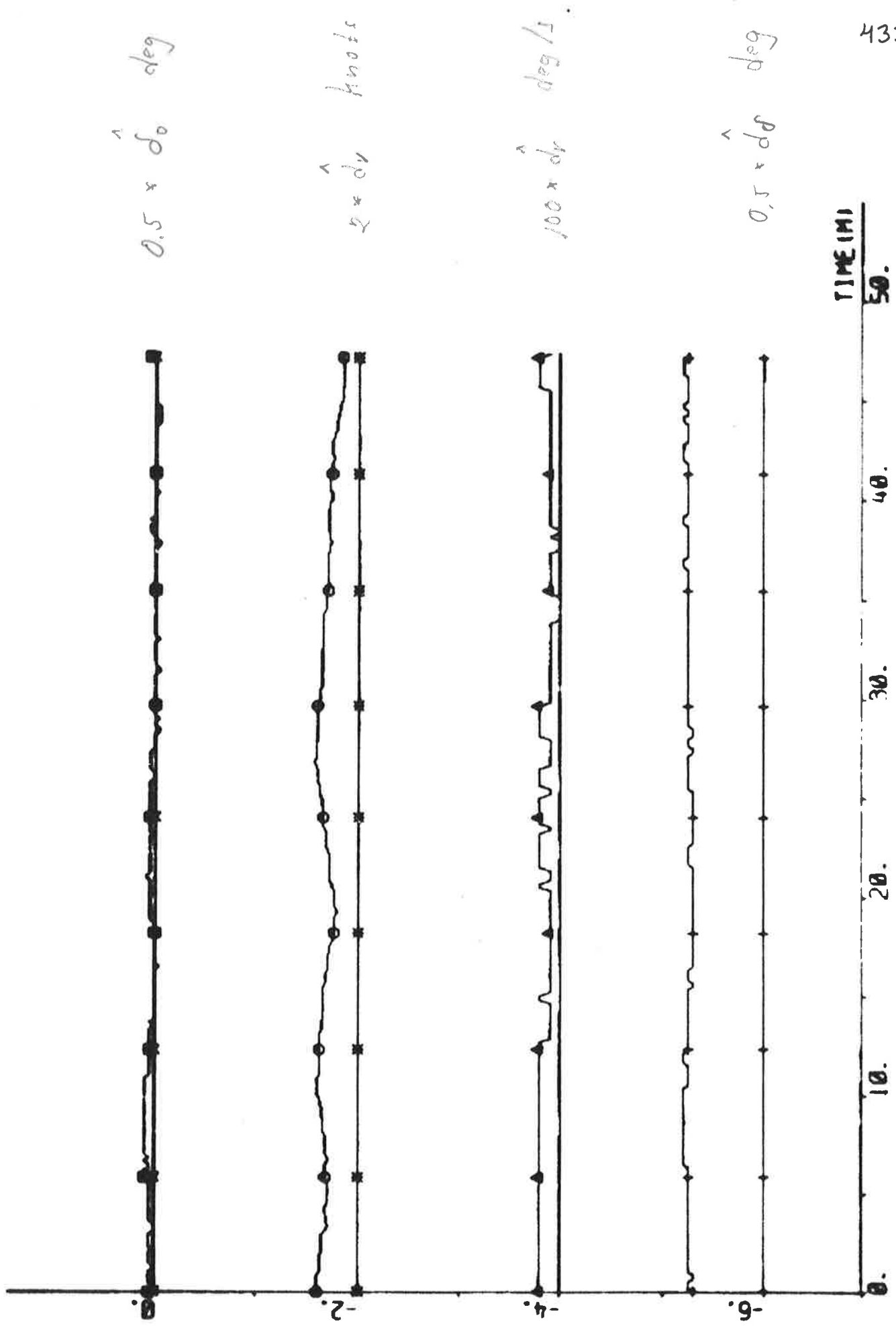
PLOT A36P1(1)-A36P1(8) A36P1(9) ERR4 EPS4 00 01 02 03 -3.4 0.8 - 000



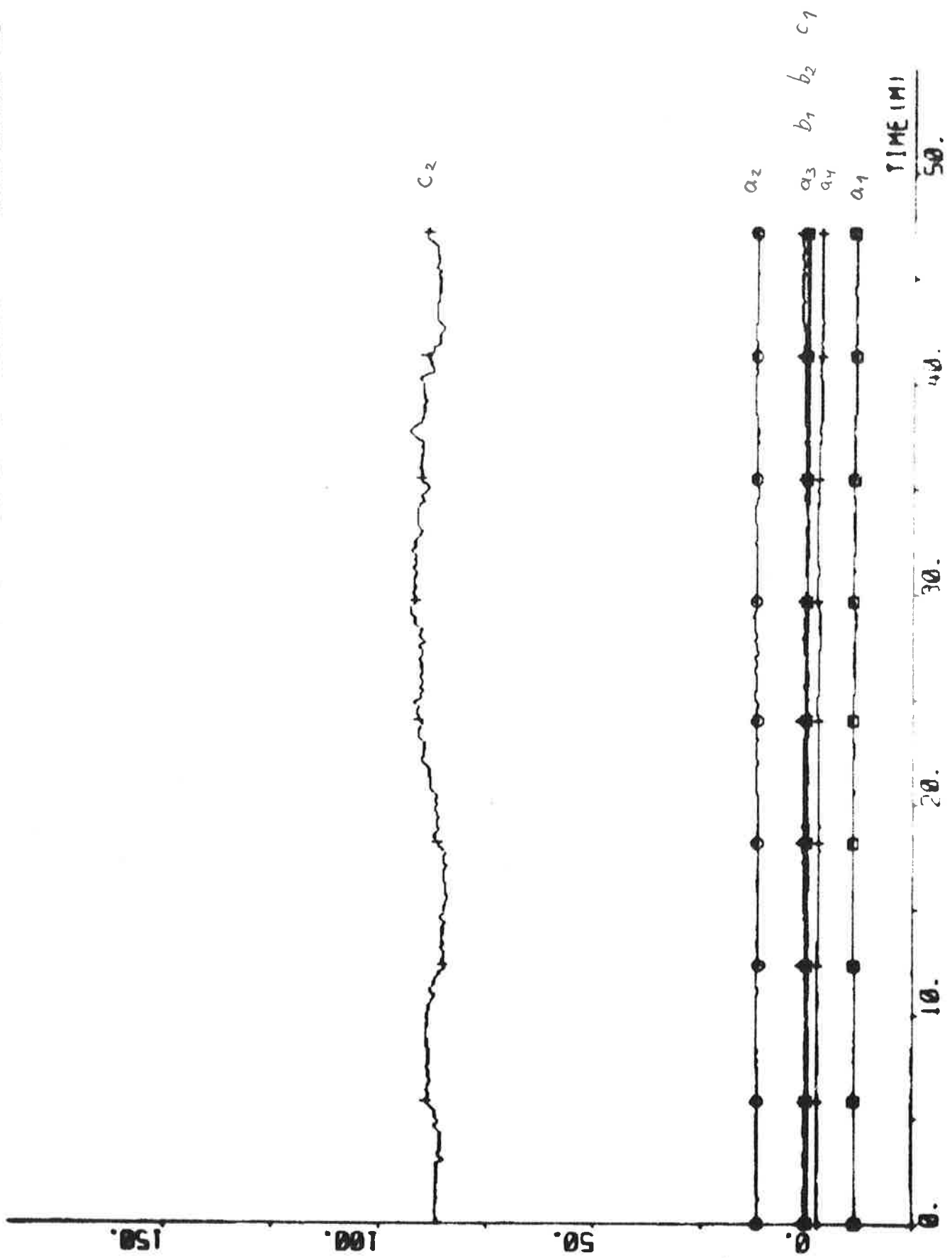
PLOT A3BP1(1)-A3BP1(2) A3BP1(3) ERR1 EPS1 00 020 040 060 -05 15 - 000



PLOT A36P1(1)-A36P2(3) A36P2(4) A36P2(5) A36P2(6) 00 02 04 06 -0.5 1.5



PLOT A36P1(1)-A36P2(7) A36P2(8) A36P2(9) A36P2(10) A36P2(11) A36P2(12) A3



## EXPERIMENT B1

Date	1976-04-26	Forward draught	10.9 m
Time	20.13	Aft draught	12.9 m
Duration	26 min	Wind direction	SE (1; see App. A)
Position	S 07°28' W 05°32'	Wind velocity	6 m/s (moderate breeze)
$\psi_{\text{ref}}$	142, 143, 142 deg	Wave height	-

The data were punched on paper tape every second. A yaw rate estimate is computed every second by the difference approximation

$$\hat{r}_d(t) = \frac{\psi(t) - \psi(t-5)}{5}$$

Final values:

$$\hat{\delta}_0 = -0.1 \text{ deg} \quad \hat{d}_v = 0.12 \text{ knots} \quad \hat{d}_r = 0.002 \text{ deg/s} \quad \hat{d}_\delta = 1.7 \text{ deg}$$

Statistics (mean value and standard deviation)

$r$  0.002  $\pm$  0.022 deg/s

$\hat{r}$  0.000  $\pm$  0.009 deg/s

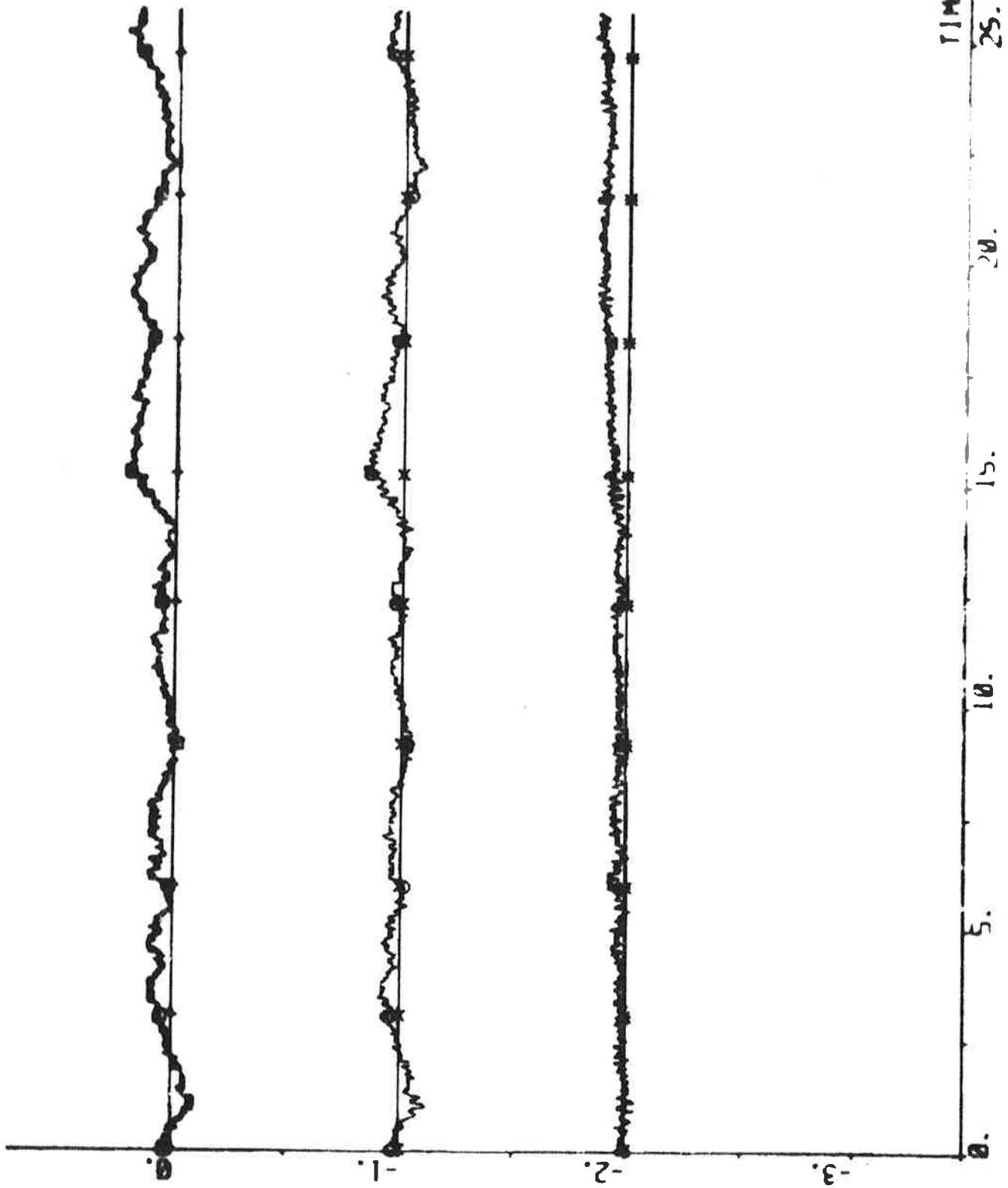
$\hat{r}_d$  0.000  $\pm$  0.014 deg/s

$n \approx 69.2$  rpm

$u \approx 13.7$  knots

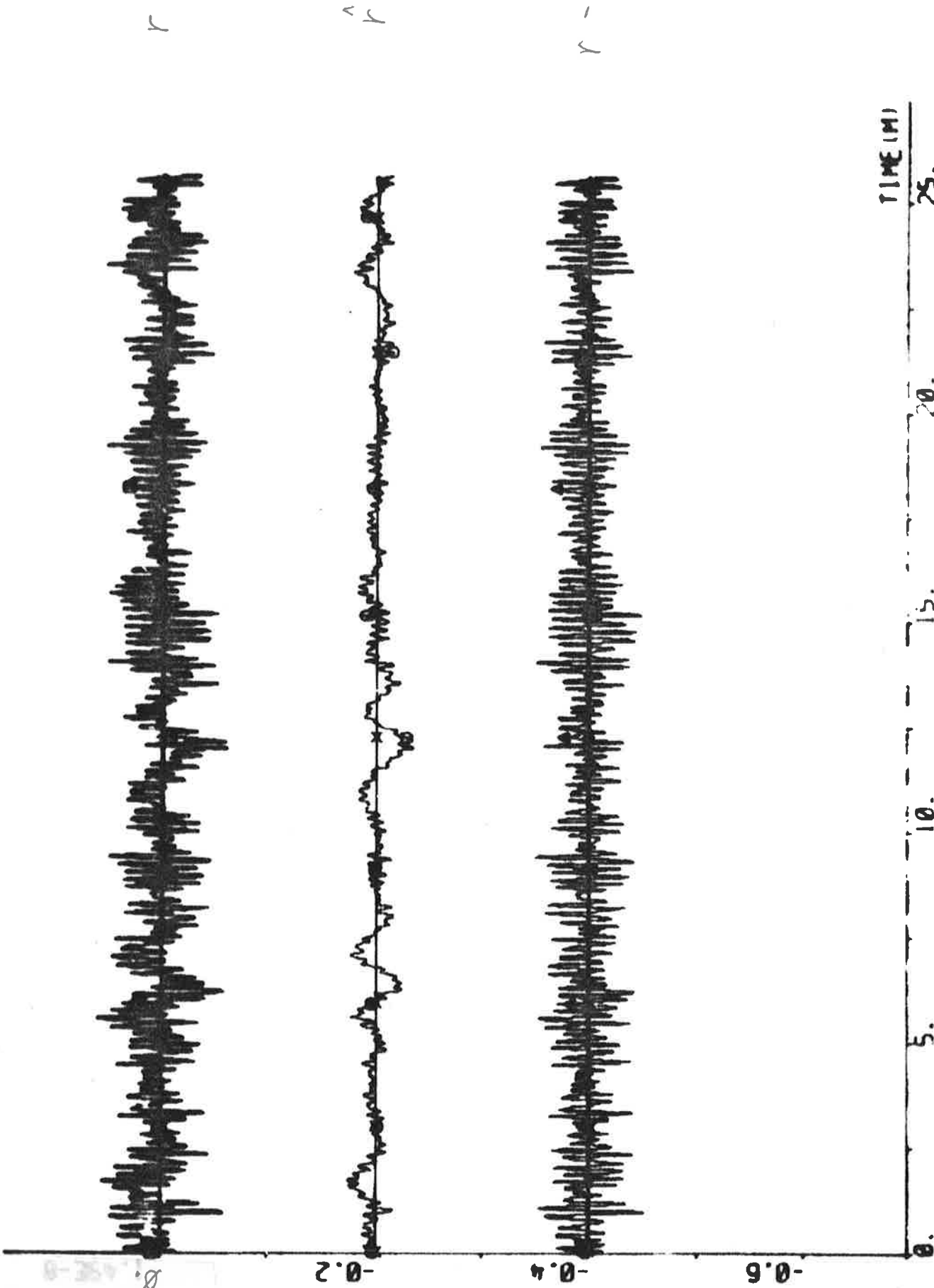
$P_s \approx 14.9$  MW

PLOT B1P1(1)-B1P1(4) B1P1(5) ERR2 00 01 02 -3.4 0.0 - KNOTS





PLOT BIP1(1)-BIP1(6) BIP1(7) ERR3 00 002 004 -0.7 0. - DEC/S



TIME (M)  
25.

20.

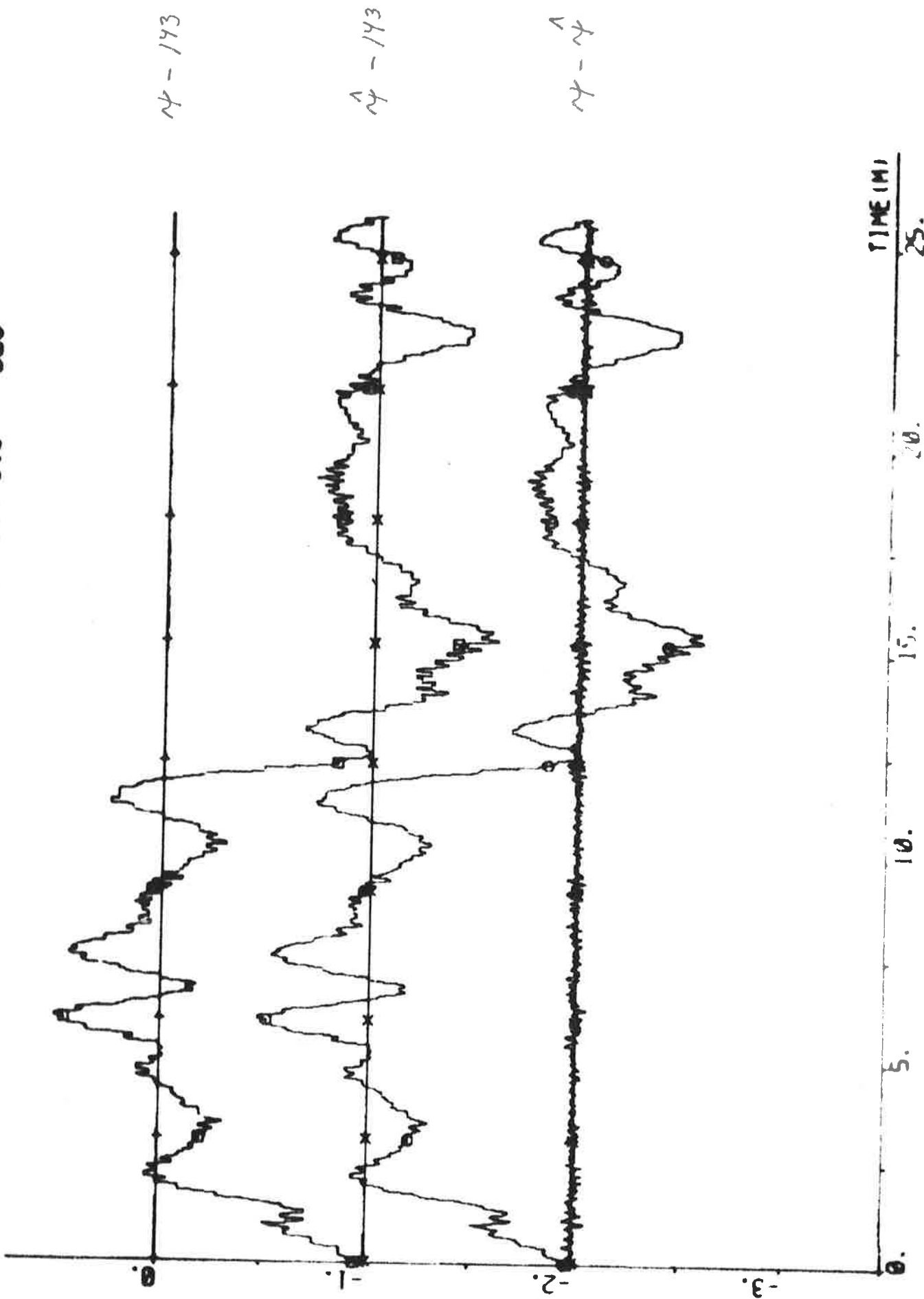
15.

10.

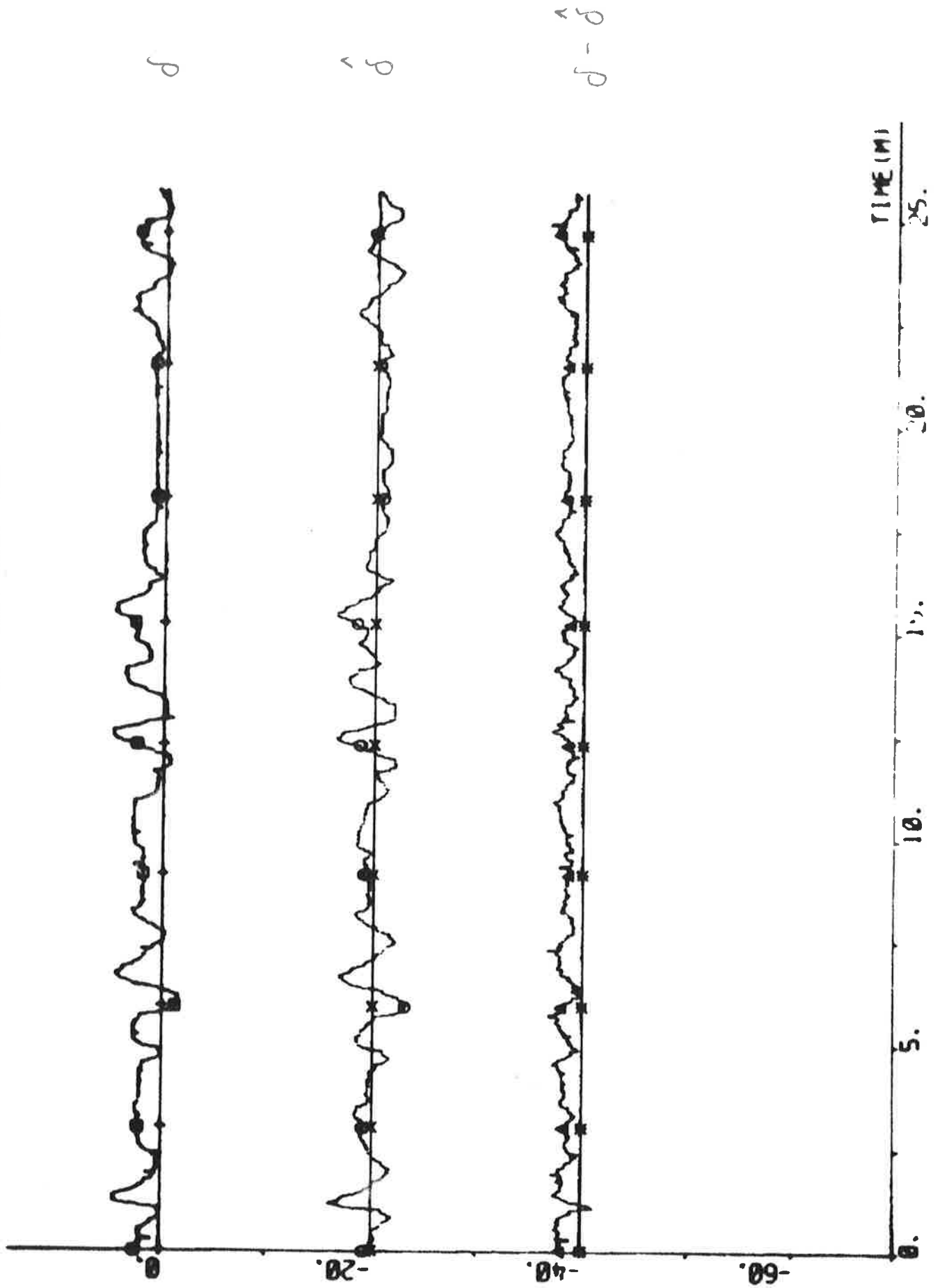
5.

0.

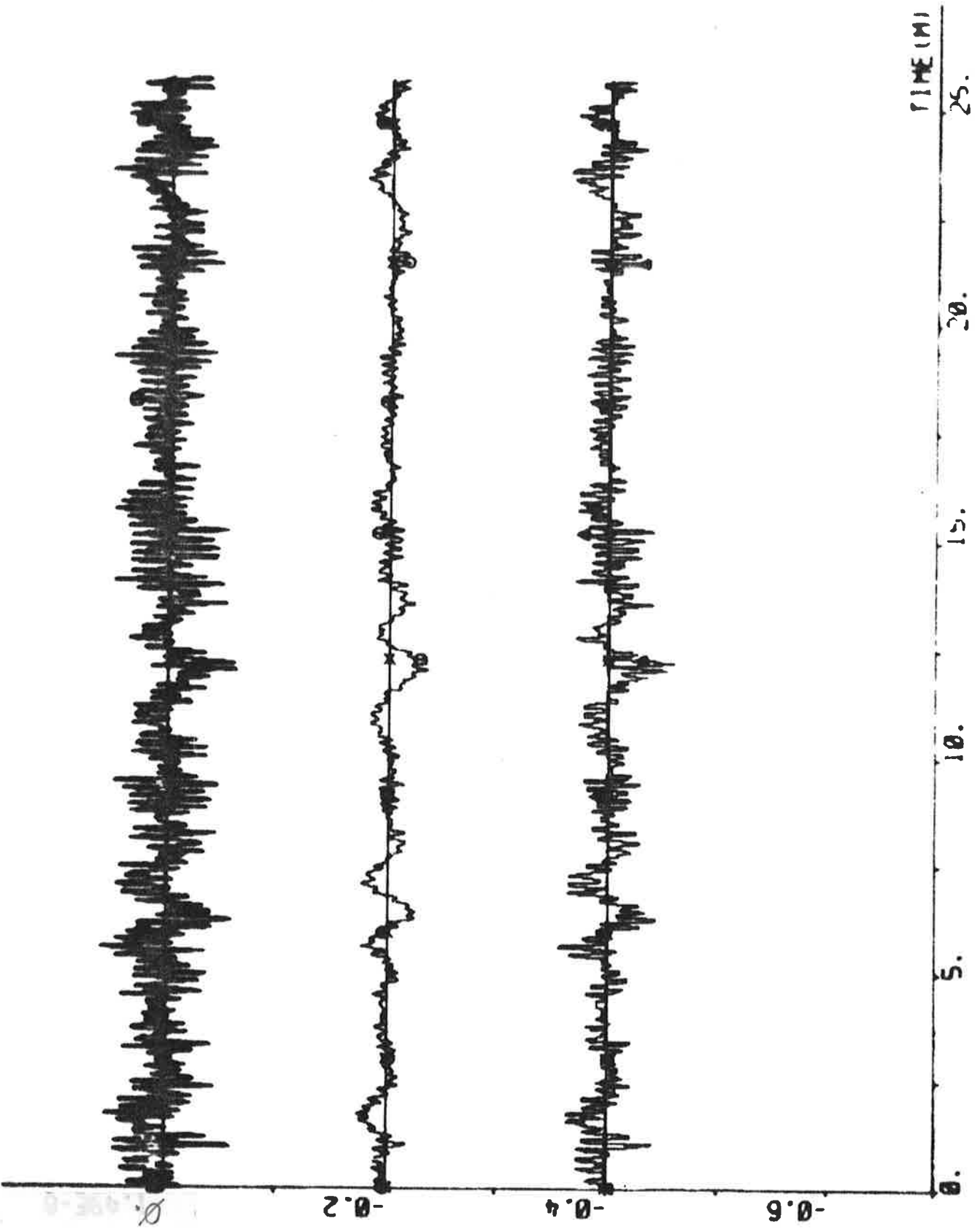
PLOT B1P1(1)-B1P1(8) B1P1(9) ERR4 00 01 02 -3.4 0.0 - DEC



PLOT B1P1(1)-B1P1(2) B1P1(3) ERR1 00 020 040 -65 15 - DEC



PLOT BIP1(1)-BIP1(6) BIP1(7) DPS1 00 002 004 -0.7 0. - DECS



## EXPERIMENT B2

Date	1976-04-27	Forward draught	10.9 m
Time	08.48	Aft draught	12.9 m
Duration	25 min	Wind direction	SE (1; see App. A)
Position	S 09°31' W 03°57'	Wind velocity	8 m/s (moderate breeze)
$\psi_{\text{ref}}$	145 deg	Wave height	-

The data were punched on paper tape every second. A yaw rate estimate is computed every second by the difference approximation

$$\hat{r}_d(t) = \frac{\psi(t) - \psi(t-5)}{5}$$

Final values:

$$\hat{\delta}_0 = -0.4 \text{ deg} \quad \hat{d}_v = 0.06 \text{ knots} \quad \hat{d}_r = 0.002 \text{ deg/s} \quad \hat{d}_\delta = 1.6 \text{ deg}$$

Statistics (mean value and standard deviation)

$r$             0.003 ± 0.023 deg/s

$\hat{r}$             0.000 ± 0.007 deg/s

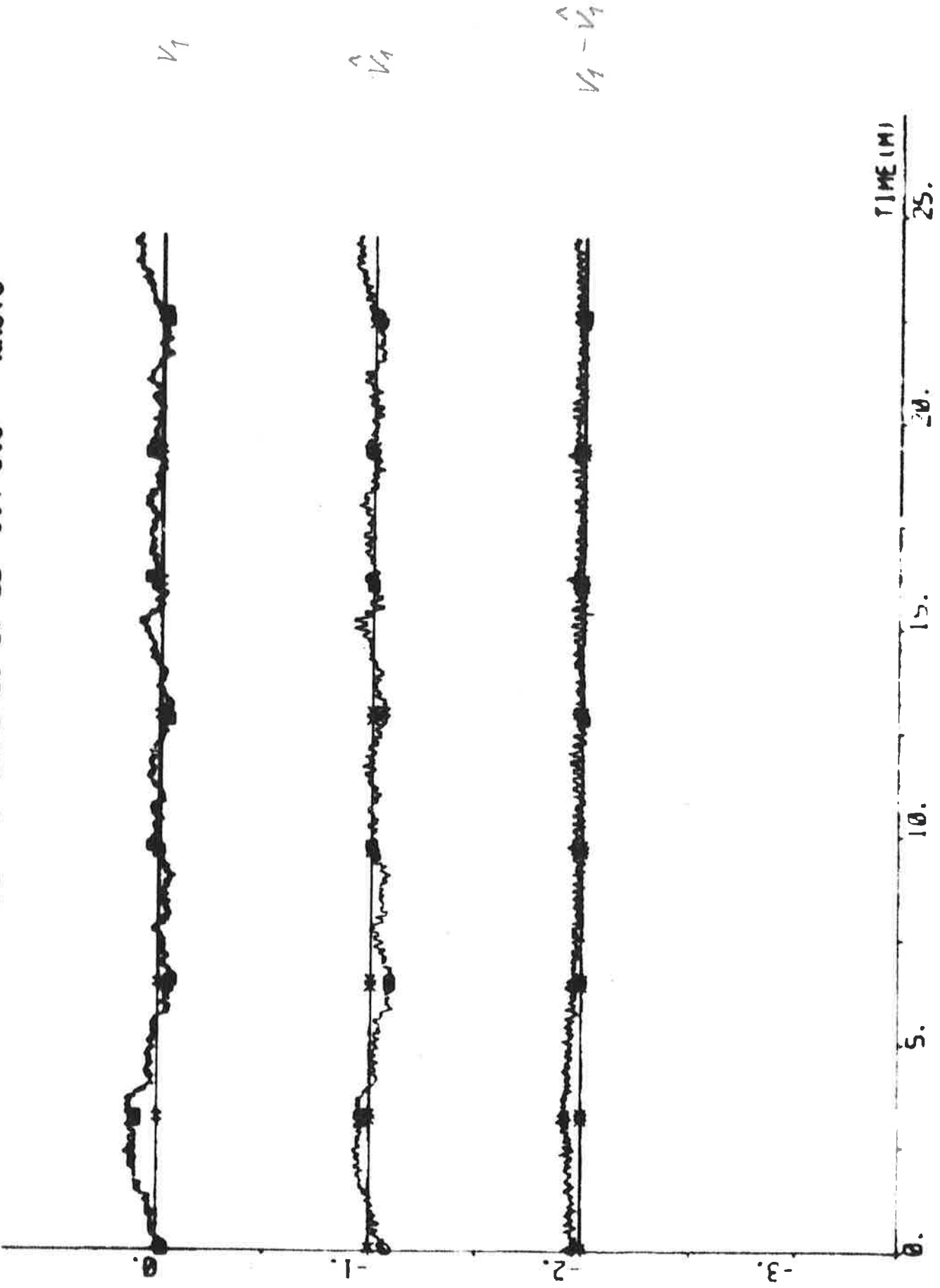
$\hat{r}_d$           0.000 ± 0.014 deg/s

$n \approx 70.1$  rpm

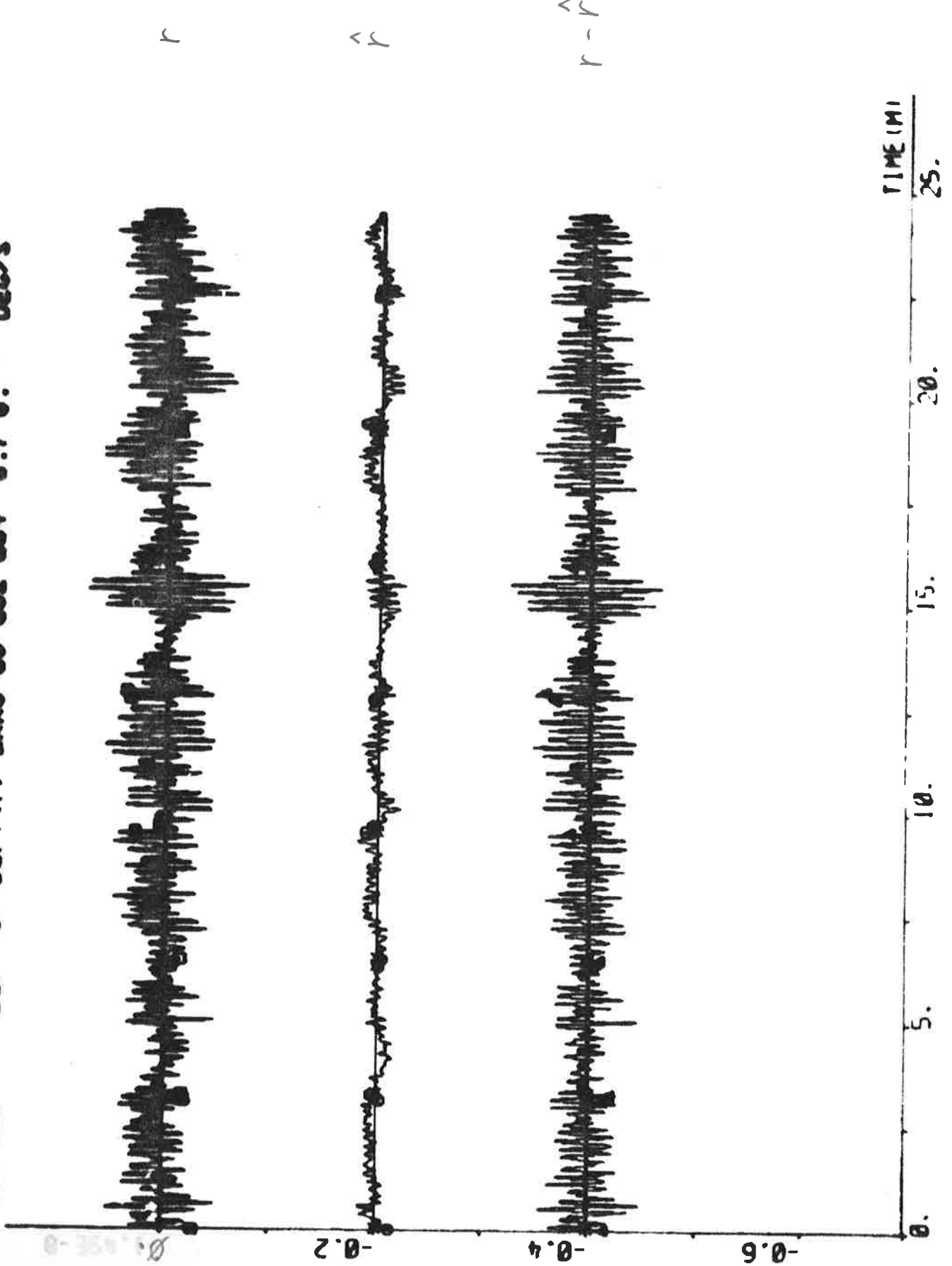
$u \approx 13.3$  knots

$P_s \approx 15.2$  MW

PLOT B2P1(1)-B2P1(4) B2P1(5) ERR2 00 01 02 -3.4 0.0 - KNOTS

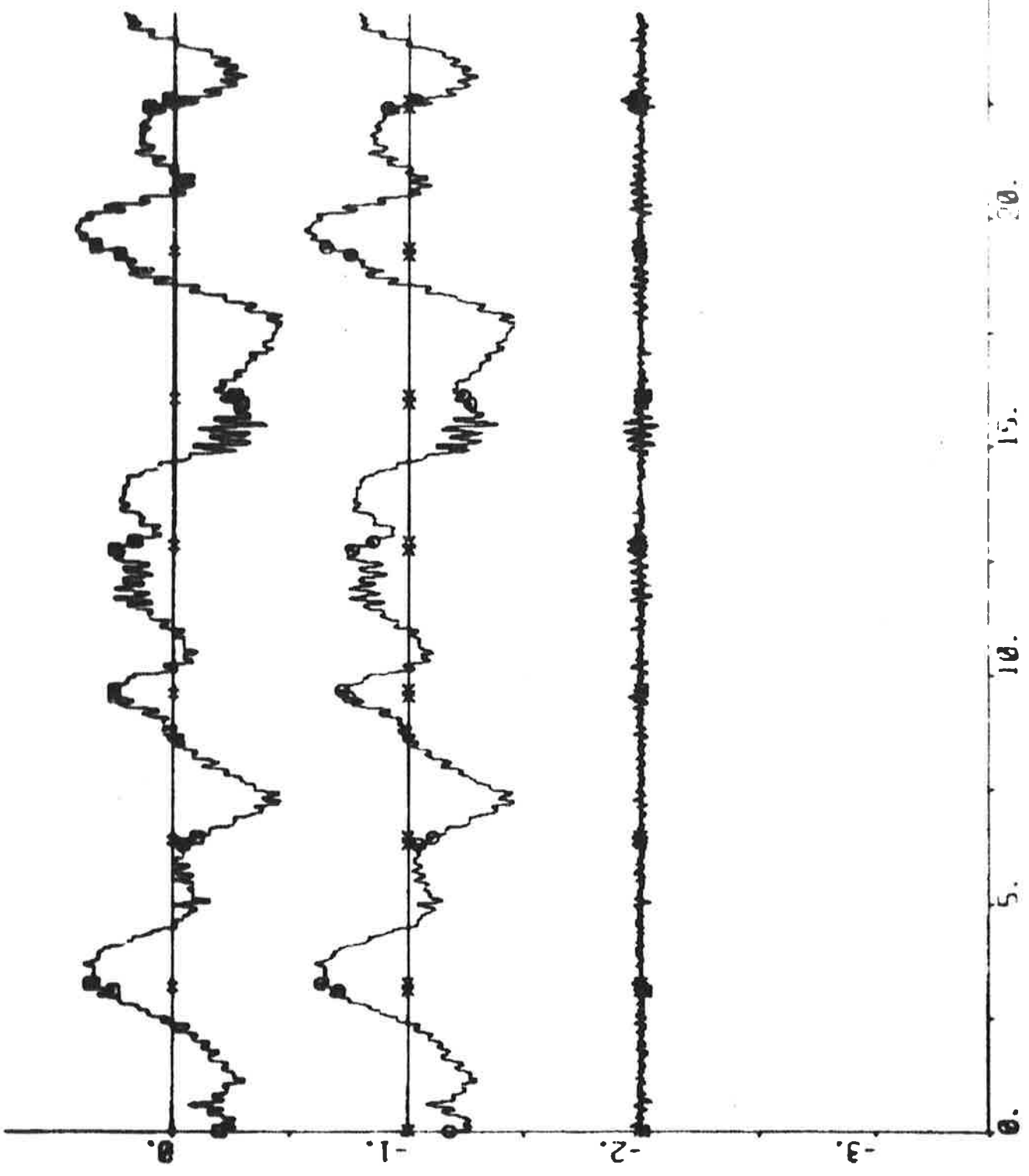


PLOT 02P1(1)-02P1(6) 02P1(7) ERR3 00 002 004 -0.7 0. - DEG/S



TIME (M)  
25.

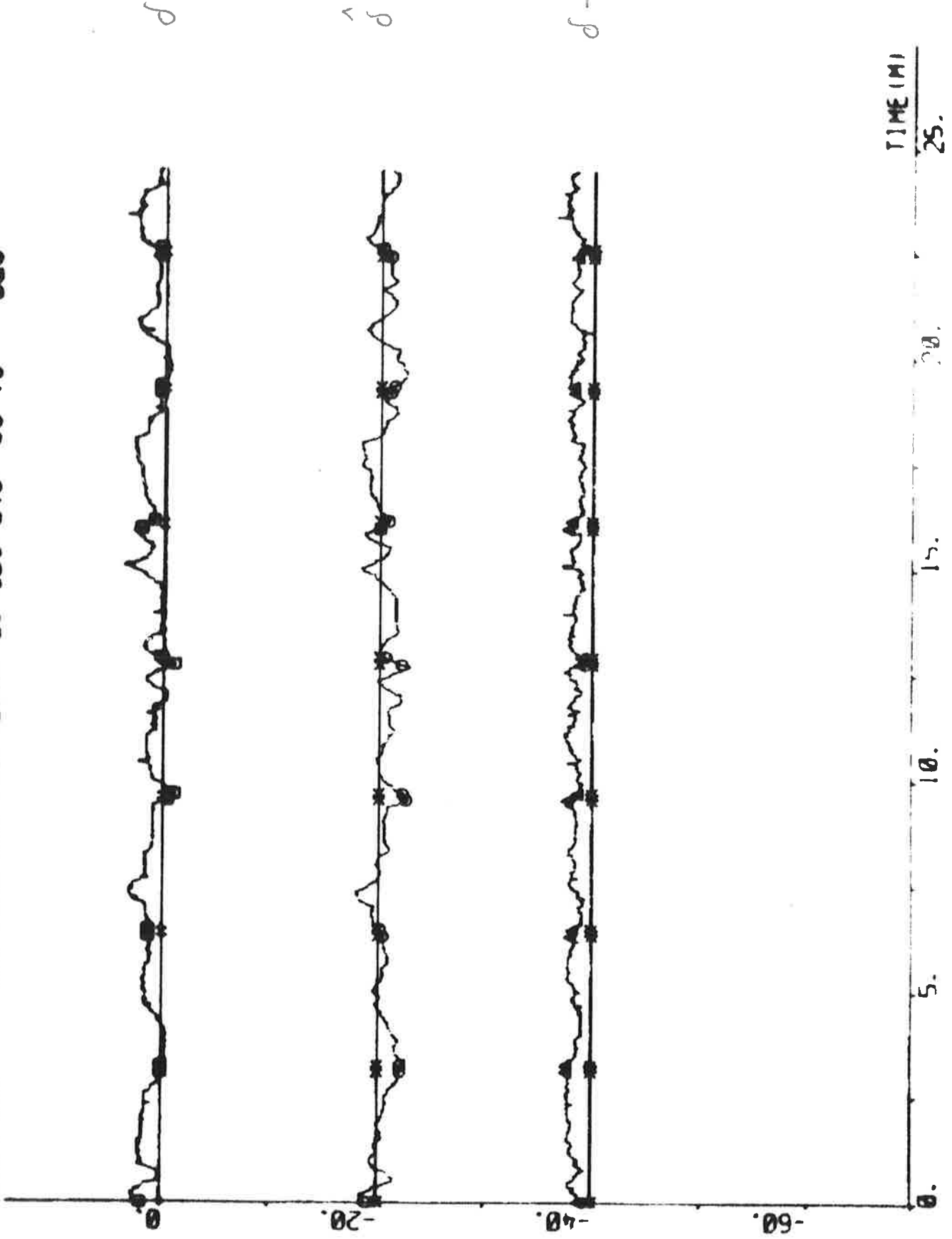
PLOT B2P1(1)-B2P1(8) B2P1(9) ERR4 00 01 02 -3.4 0.0 - DEC



TIME (M)  
25.



PLOT B2P1(1)-B2P1(2) B2P1(3) ERR1 00 020 040 -65 15 - DEG



PLOT B2P1(1)-B2P1(6) B2P1(7) DPS1 00 002 004 -0.7 0. - DEC/S



TIME (M) 25.

20.

15.

10.

5.

0.

## EXPERIMENT B3

Date	1976-04-29	Forward draught	10.9 m
Time	12.59	Aft draught	12.9 m
Duration	26 min	Wind direction	SE (1; see App. A)
Position	S 18°07' E 04°58'	Wind velocity	15 m/s (moderate gale)
$\psi_{\text{ref}}$	145 deg	Wave height	High sea from SE

The data were punched on paper tape every second. A yaw rate estimate is computed every second by the difference approximation

$$\hat{r}_d(t) = \frac{\psi(t) - \psi(t-5)}{5}$$

Final values:

$$\hat{\delta}_0 = -0.3 \text{ deg} \quad \hat{d}_v = 0.15 \text{ knots} \quad \hat{d}_r = 0.000 \text{ deg/s} \quad \hat{d}_\delta = 1.4 \text{ deg}$$

Statistics (mean value and standard deviation)

$$\bar{r} \quad 0.001 \pm 0.034 \text{ deg/s}$$

$$\hat{r} \quad 0.000 \pm 0.011 \text{ deg/s}$$

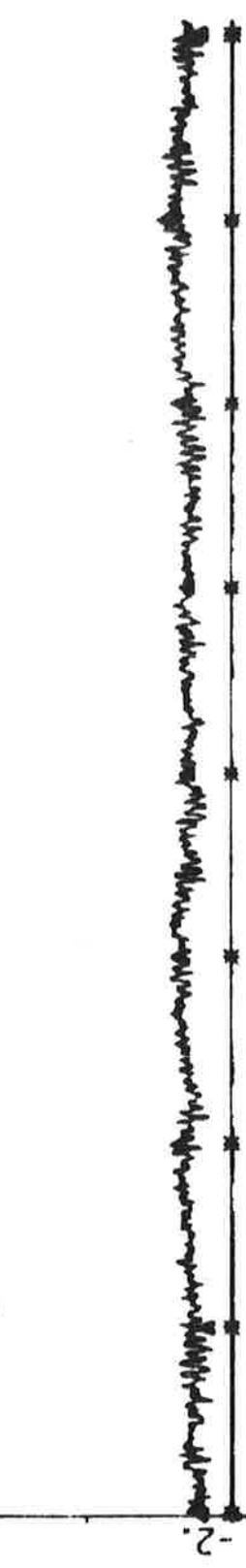
$$\hat{r}_d \quad 0.000 \pm 0.021 \text{ deg/s}$$

$$n \approx 87.4 \text{ rpm}$$

$$u \approx 15.9 \text{ knots}$$

$$P_s \approx 29.5 \text{ MW}$$

PLOT BCP1(1)-BCP1(4) BCP1(5) ERR2 00 01 02 -3.4 0.0 - KNOTS



TIME (MI)  
0. 5. 10. 15. 20. 25.

PLOT B3P1(1)-B3P1(6) B3P1(7) ERR3 00 002 004 -0.7 0. - DEC/S



r



r

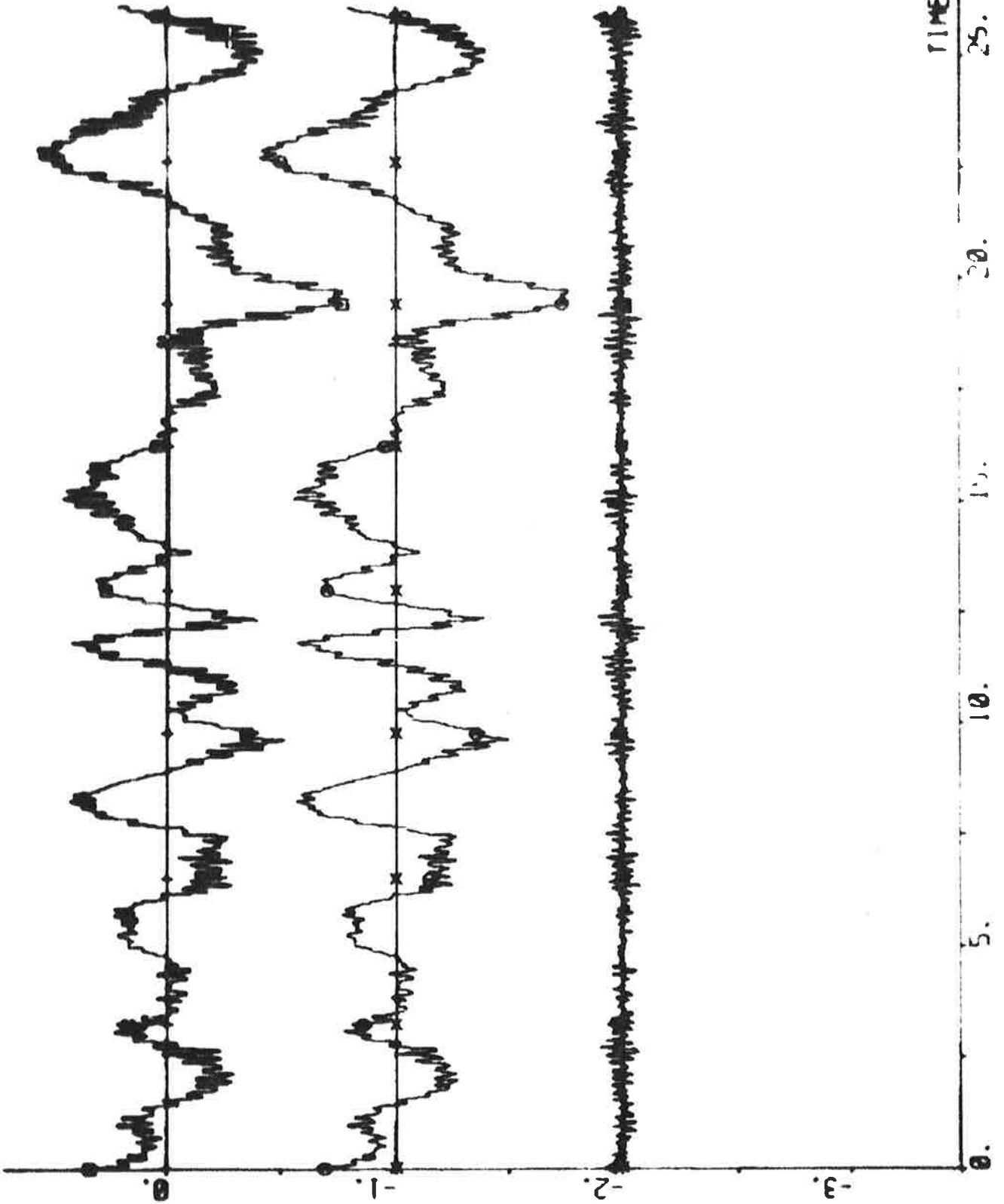


r

0. -0.2 -0.4 -0.6 0.

0. 5. 10. 15. 20. 25. TIME (M)

PLOT SCPI(1)-SCPI(8) SCPI(9) ERR4 00 01 02 -3.4 0.0 - DEG



N - Nref

N - Nref

N - N

TIME (MI)

25.

20.

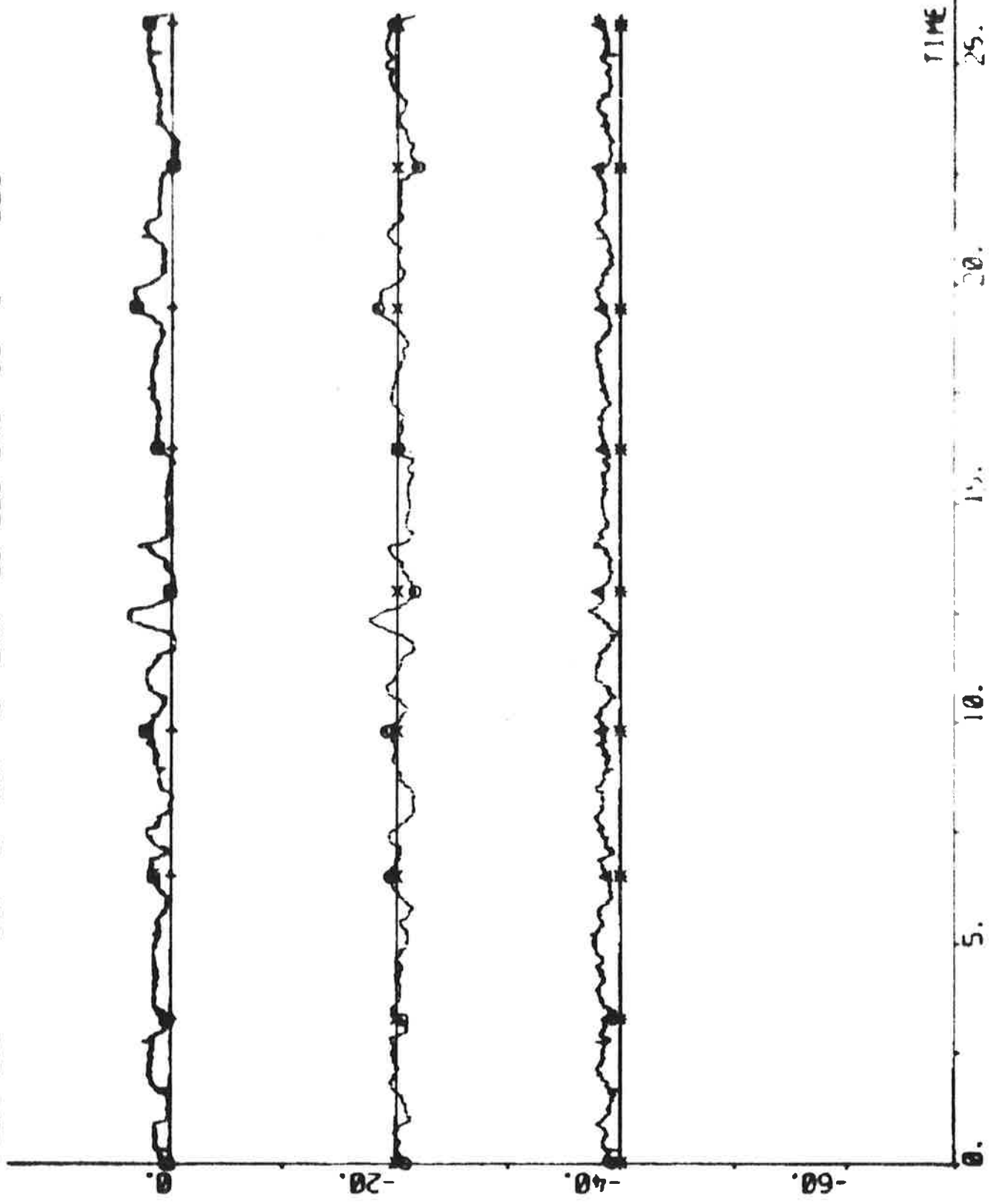
15.

10.

5.

0.

PLOT BCP1(1)-BCP1(2) BCP1(3) ERR1 00 020 040 -65 15 - DEC



PLOT BCP1(1)-BCP1(6) BCP1(7) DPS1 00 002 004 -0.7 0. - DEC/S



^



^



^



0.

-0.2

-0.4

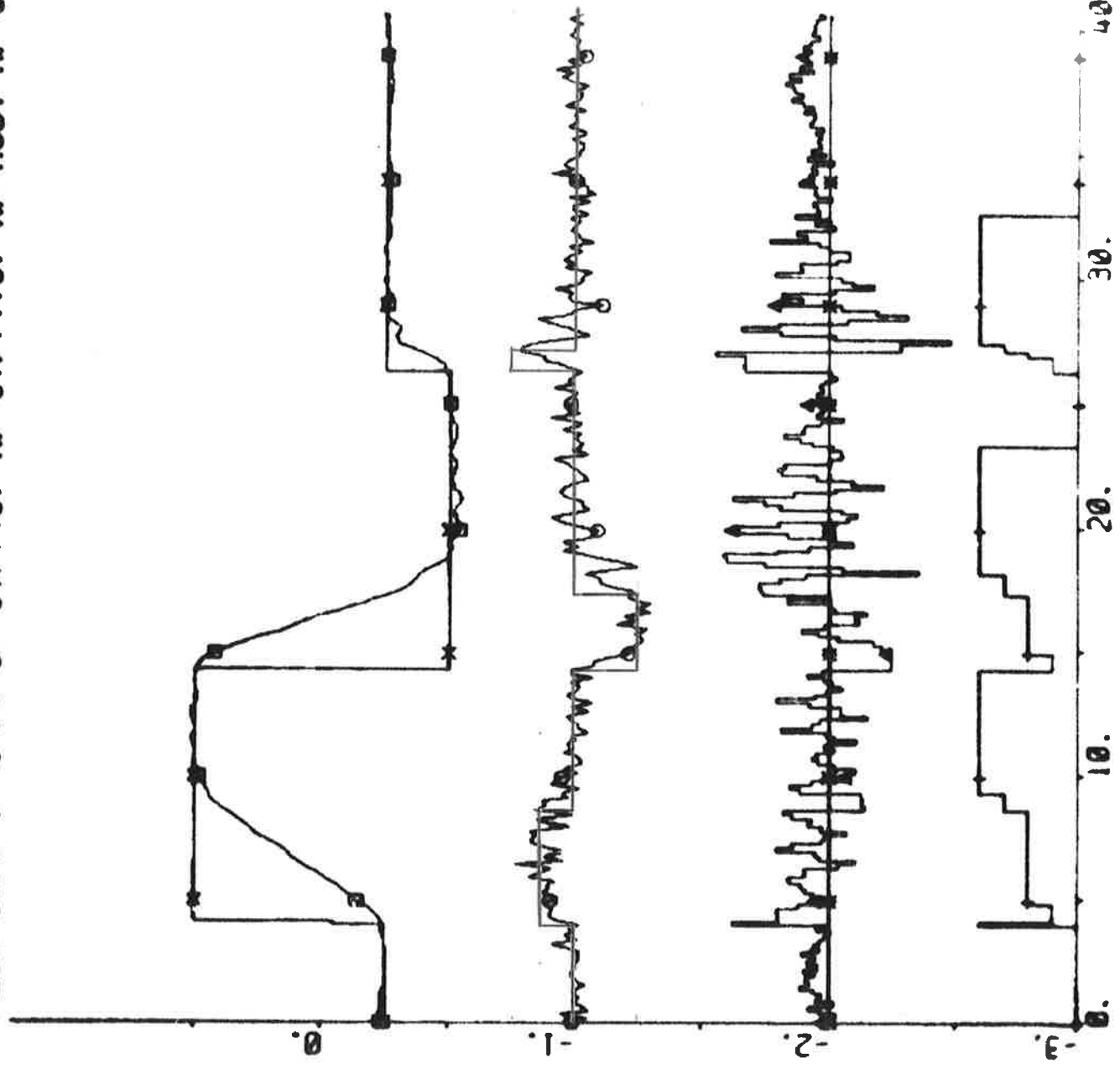
-0.6

0.





PLOT CIP1(1)~CIP1(8) CIP1(9) HP CIP1(10) HP MODY HP CIP1(15) 02 -3 1



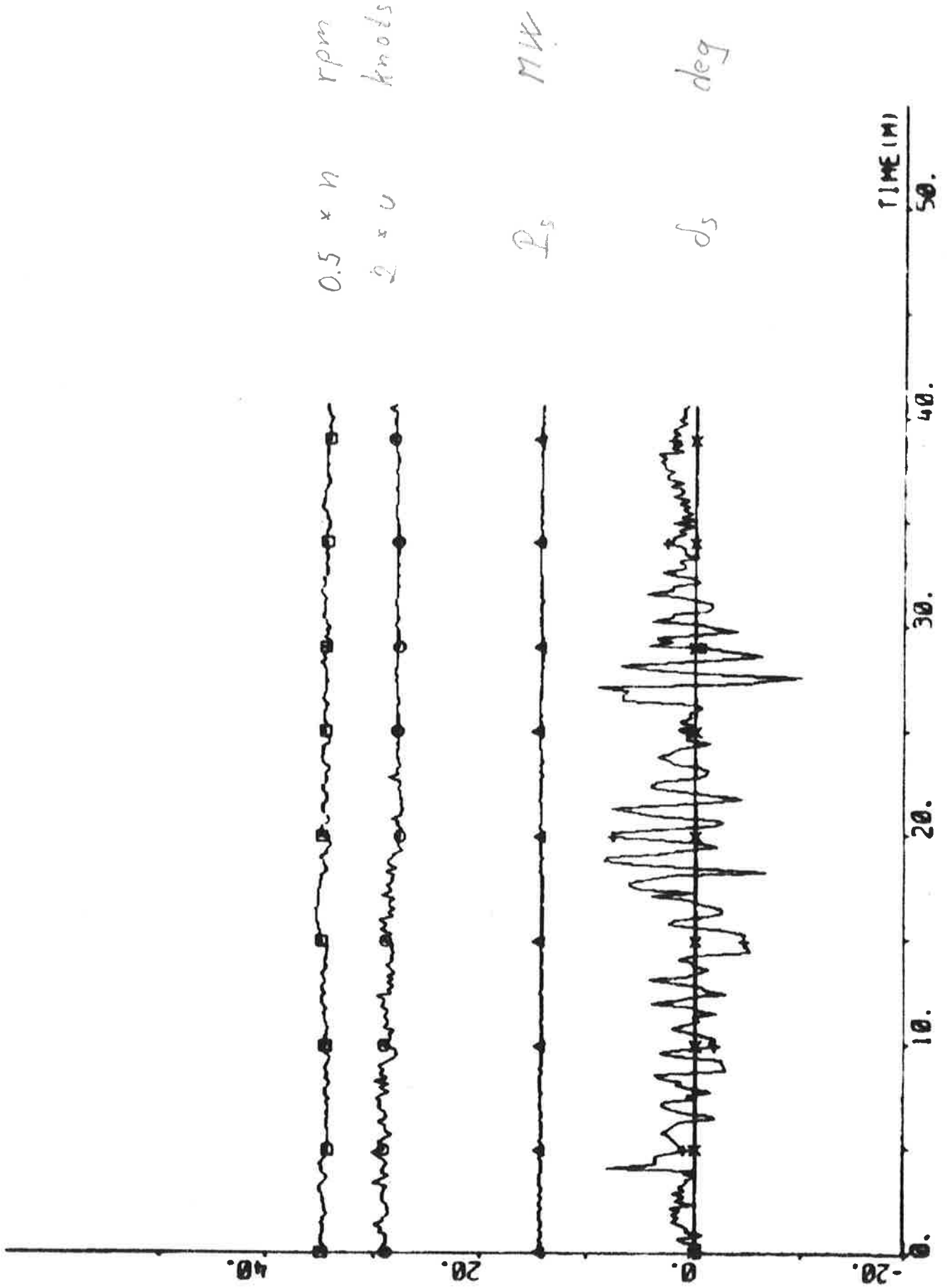
$$0.05 * \left\{ \begin{array}{l} \gamma - 185 \\ \gamma_{ref} - 185 \end{array} \right\} \text{ deg}$$

$$2.5 * \left\{ \begin{array}{l} r \\ r_0 \end{array} \right\} \text{ deg/s}$$

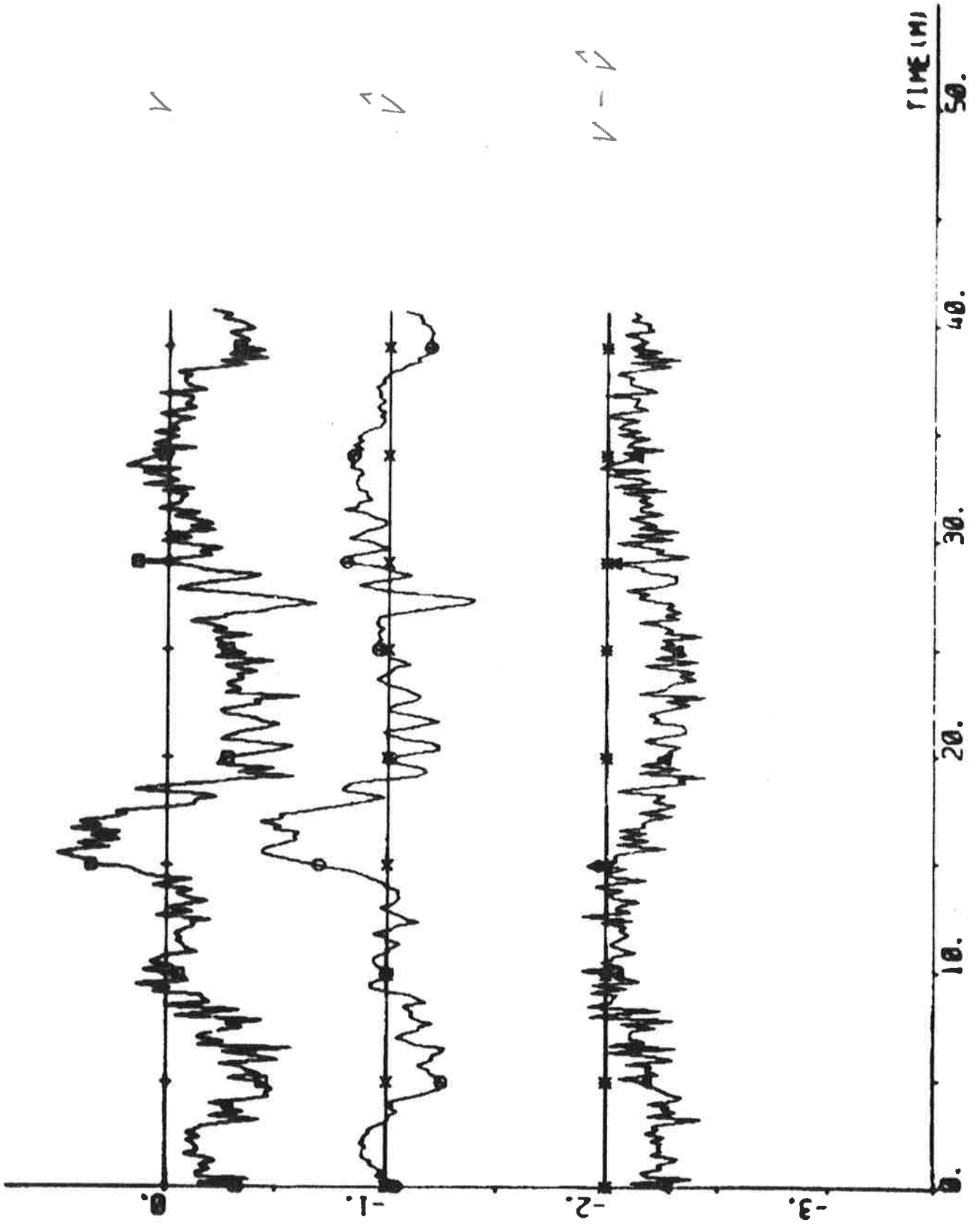
$$0.05 * d_c \text{ deg}$$

$$0.1 * M_y$$

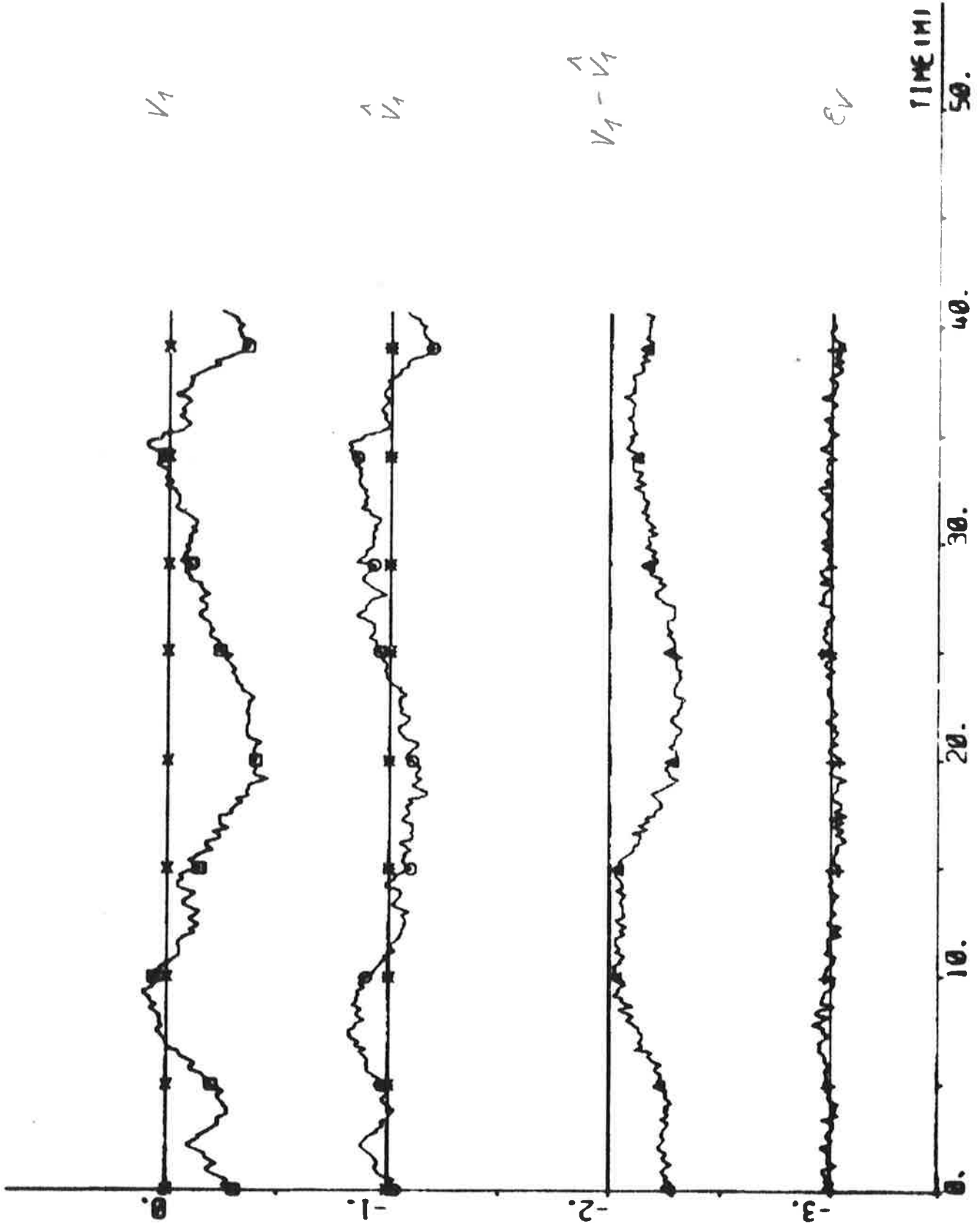
PLOT CIP1(1)-CIP1(13) CIP1(12) CIP1(14) CIP1(11) 00 -20 50



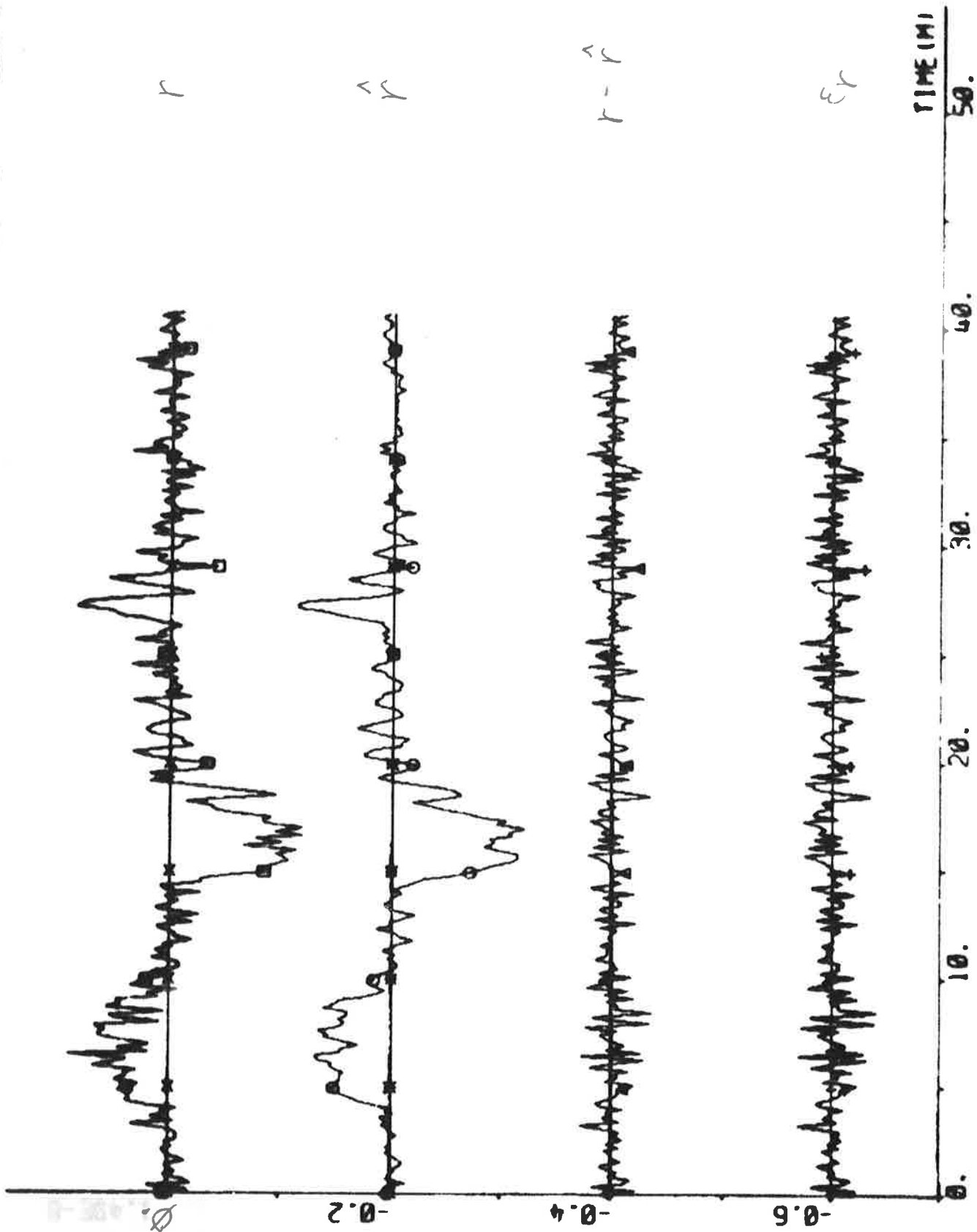
PLOT CIP1(1)-CIP2(1) CIP2(2) ERAS 00 01 02 -3.4 0.0 - KNOTS



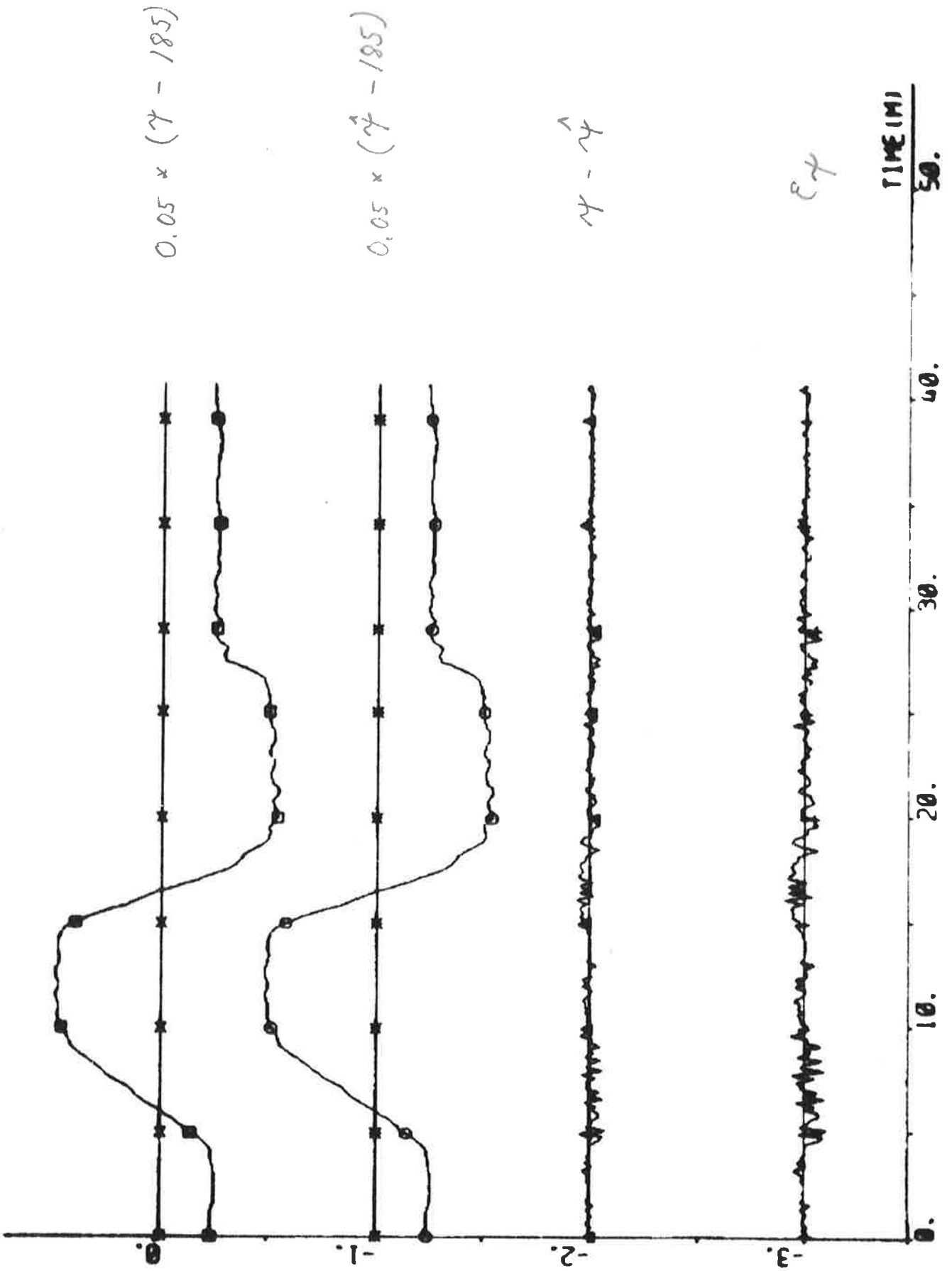
PLOT CIP1(1)-CIP1(4) CIP1(5) ERR2 EPS2 00 01 02 03 -3.4 0.0 - KNOTS



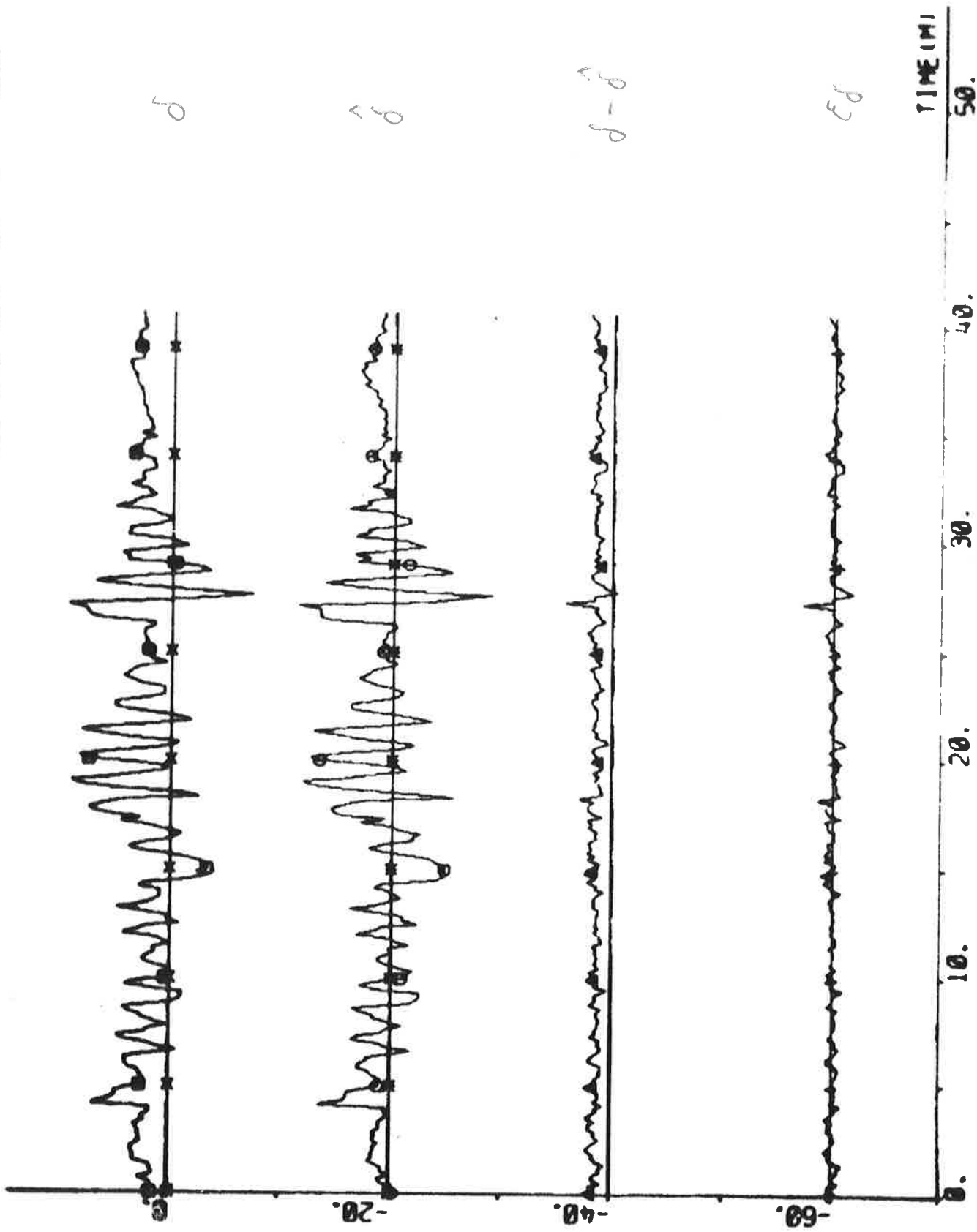
PLOT CIP1(1)-CIP1(6) CIP1(7) ERR3 EPS3 00 002 004 006 -0.7 0. - 00000



PLOT CIP1(1)-CIP1(9) CIP1(9) ERR4 EPS4 00 01 02 03 -3.4 0.0 - 003

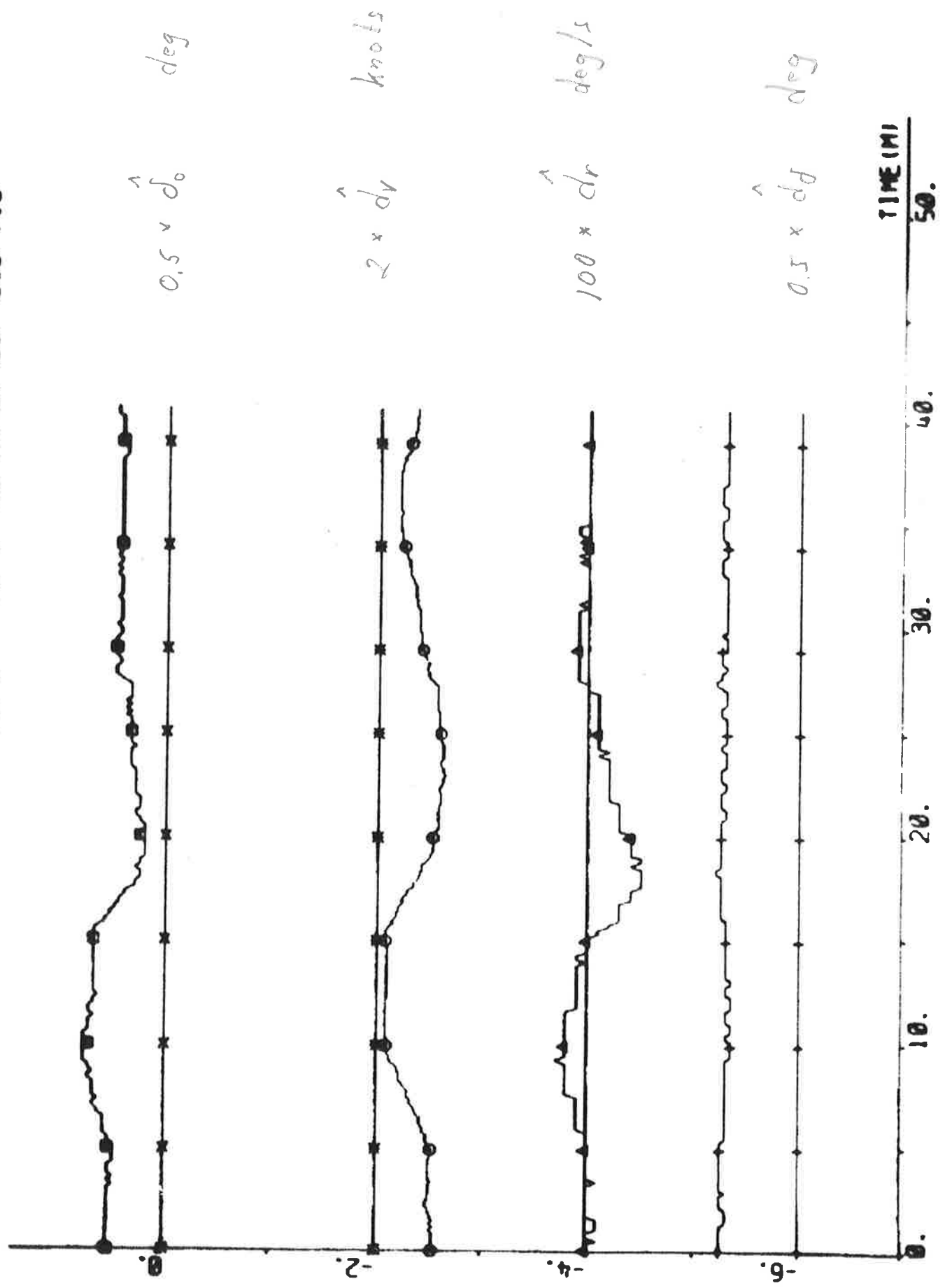


PLOT CIP1(1)-CIP1(2) CIP1(3) ERR1 EPS1 00 020 040 060 -05 15 - DEG

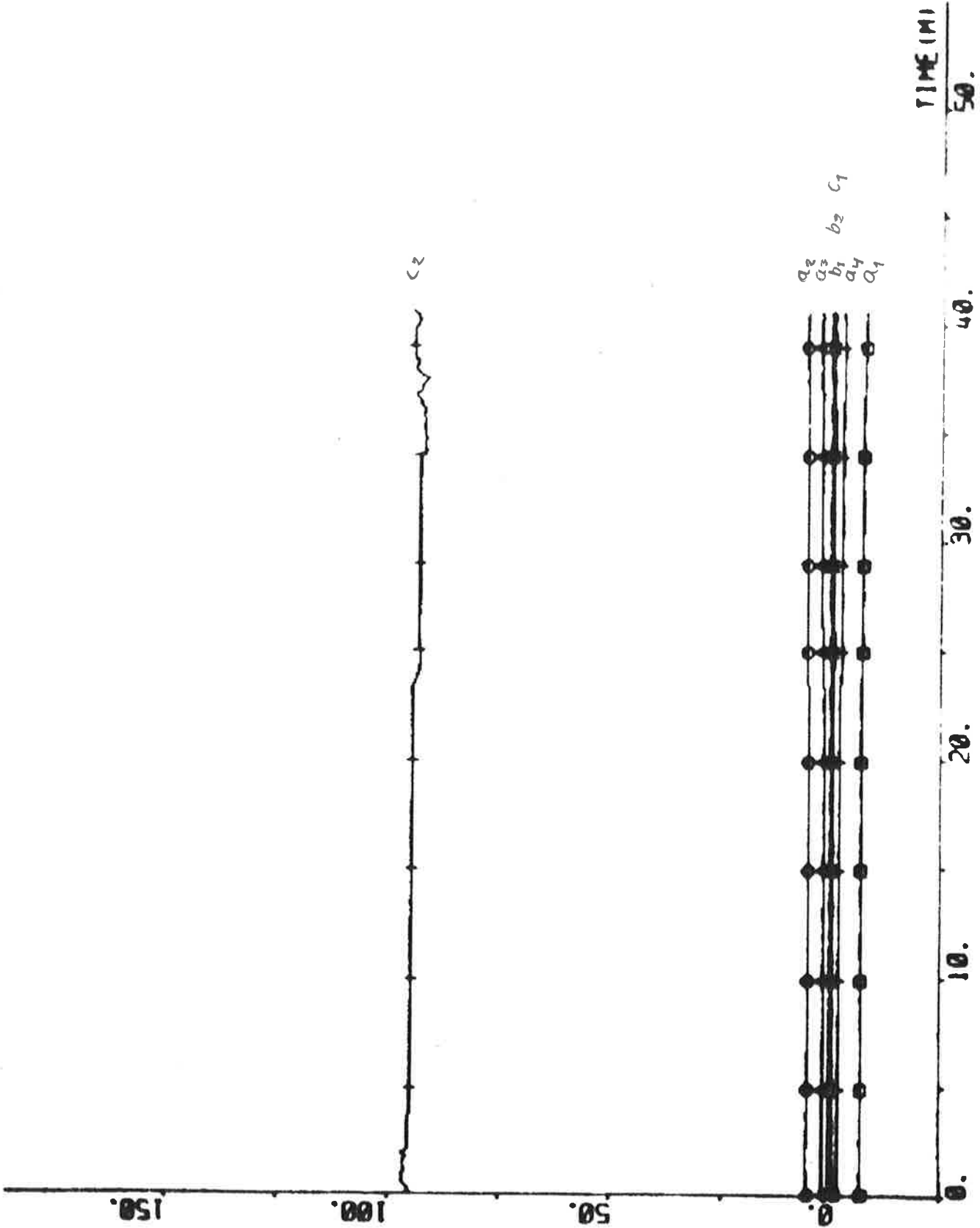




PLOT CIP1(1)-CIP2(3) CIP2(4) CIP2(5) CIP2(6) 00 02 04 06 -0.5 1.5



PLOT CIP1(1)-CIP2(7) CIP2(8) CIP2(9) CIP2(10) CIP2(11) CIP2(12) CIP2(13)





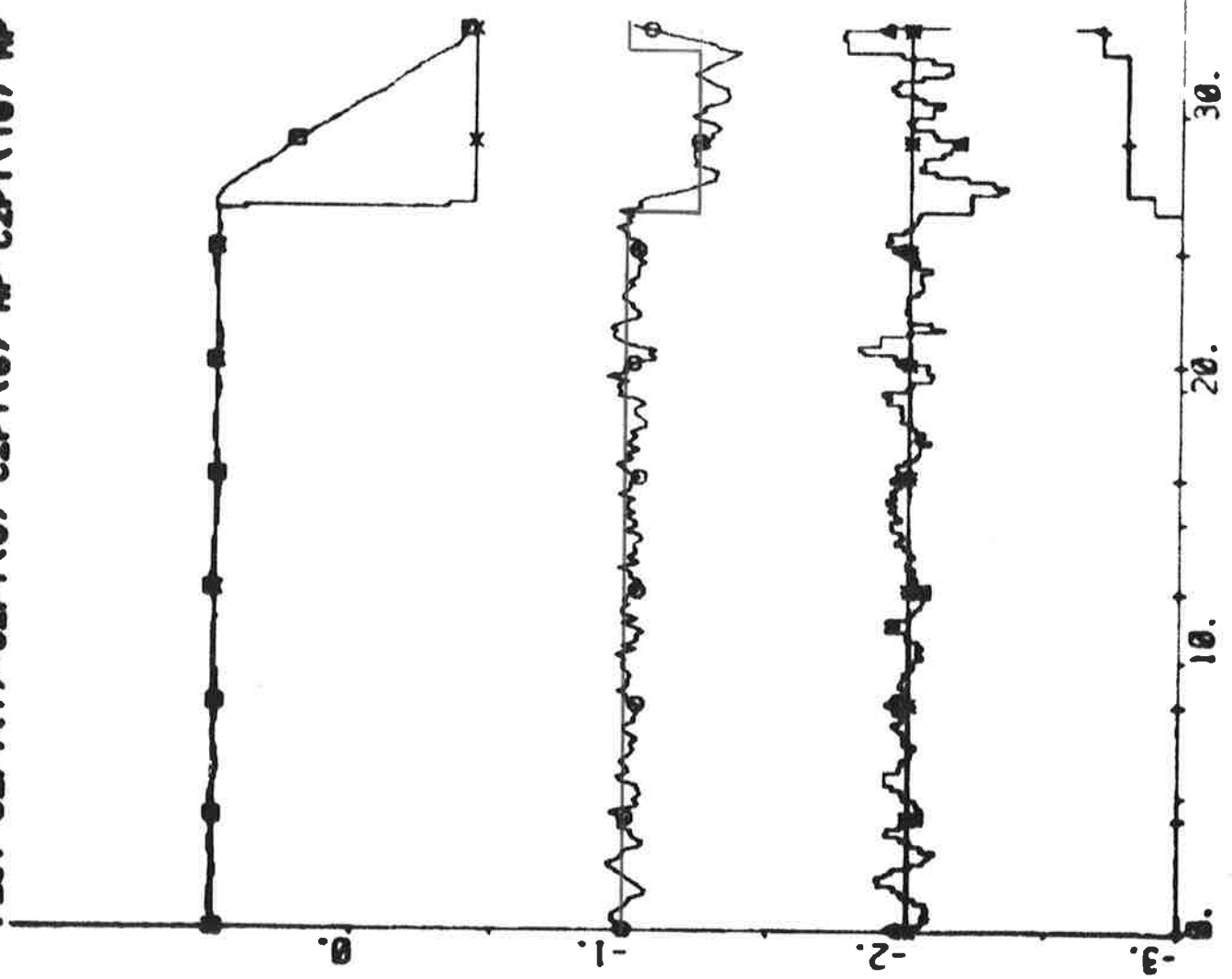
PL0T C2P1(1)-C2P1(8) C2P1(8) HP C2P1(10) HP C2P1(16) 02 -3 1

$$0.025 * \left\{ \begin{array}{l} \gamma - 160 \\ \gamma_{ref} - 160 \end{array} \right\} \text{ deg}$$

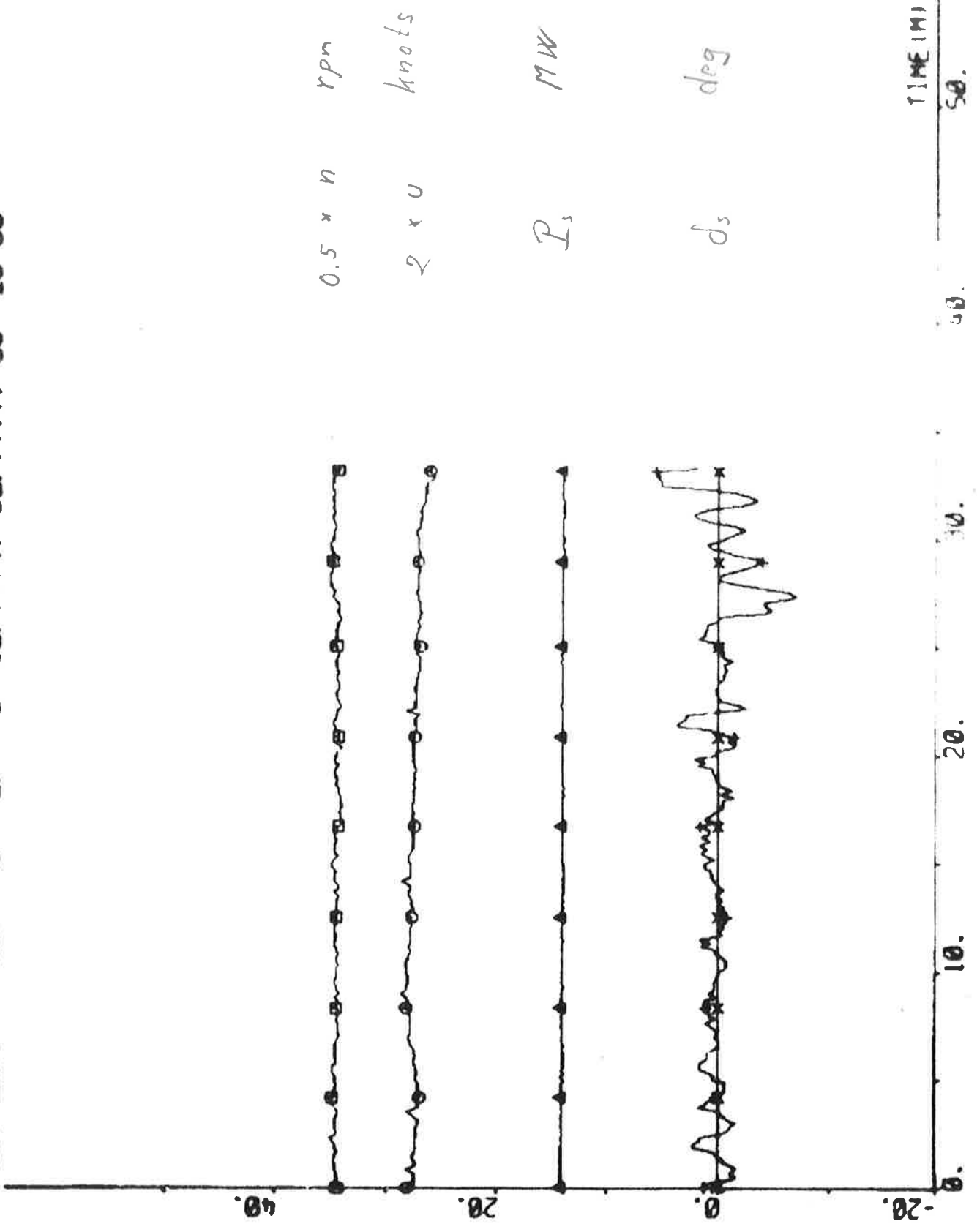
$$2.5 * \left\{ \begin{array}{l} r \\ r_0 \end{array} \right\} \text{ deg/s}$$

$$0.05 * \delta_c \text{ deg}$$

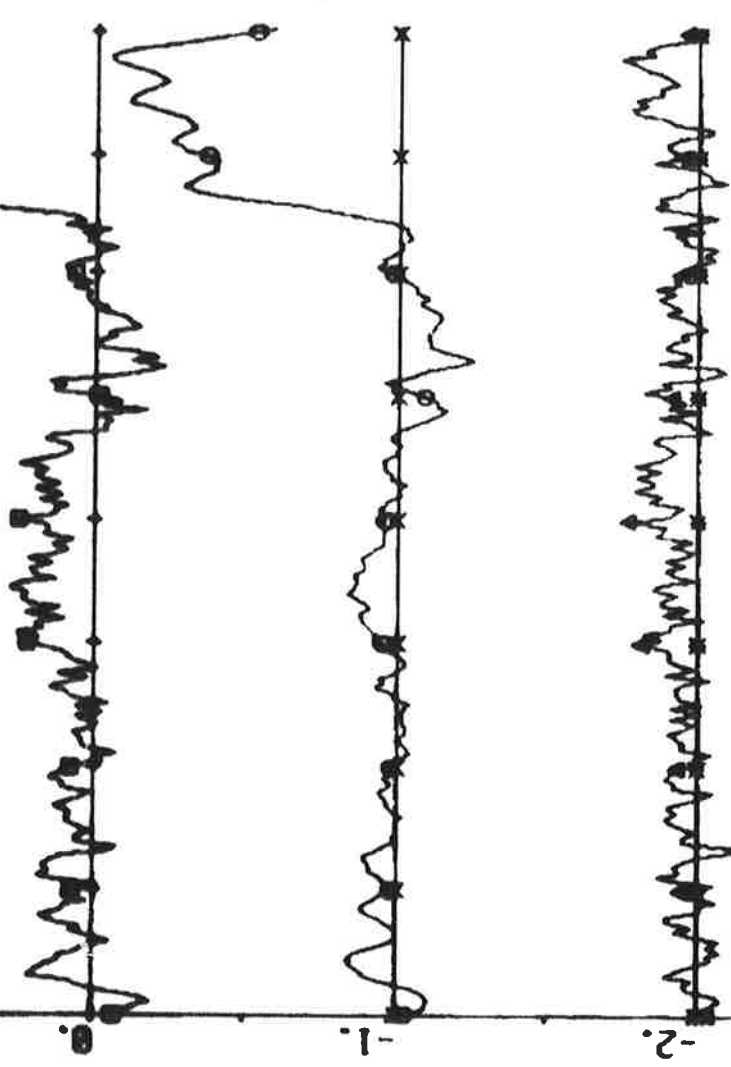
$$0.1 * M_y$$



PLOT C2P1(1)-C2P1(13) C2P1(12) C2P1(14) C2P1(11) 00 -20 50



PLOT C2P1(1)-C2P2(1) C2P2(2) ERRS 00 91 02 -3.4 0.8 - 10NOTS



TIME (M)  
50.

PLOT C2P1(1)-C2P1(4) C2P1(5) ERR2 EPS2 00 01 02 03 -3.4 0.0 - KNOTS



0. 10. 20. 30. 40. 50. TIME (MI)

PL0T C2P1(1)-C2P1(6) C2P1(7) ERR3 EPS3 00 002 004 006 -0.7 0. - 00E-3

r



r-hat



r-hat-tilde



E\_r



TIME (M)  
50.

40.

30.

20.

10.



PL0T C2P1(1)-C2P1(8) C2P1(8) ERR4 EPS4 00 01 02 03 -3.4 0.0 - DEG



0.025 \* (4 - 160)



0.025 \* (7 - 160)



4 - 4



E4



TIME (M)  
50.

PLOT C2P1(1)-C2P1(2) C2P1(3) ERR1 EPS1 00 020 040 060 -05 16 - DEC



$\rho$



$\delta$



$\rho - \delta$



$\beta$

TIME (M) 0. 10. 20. 30. 40. 50.

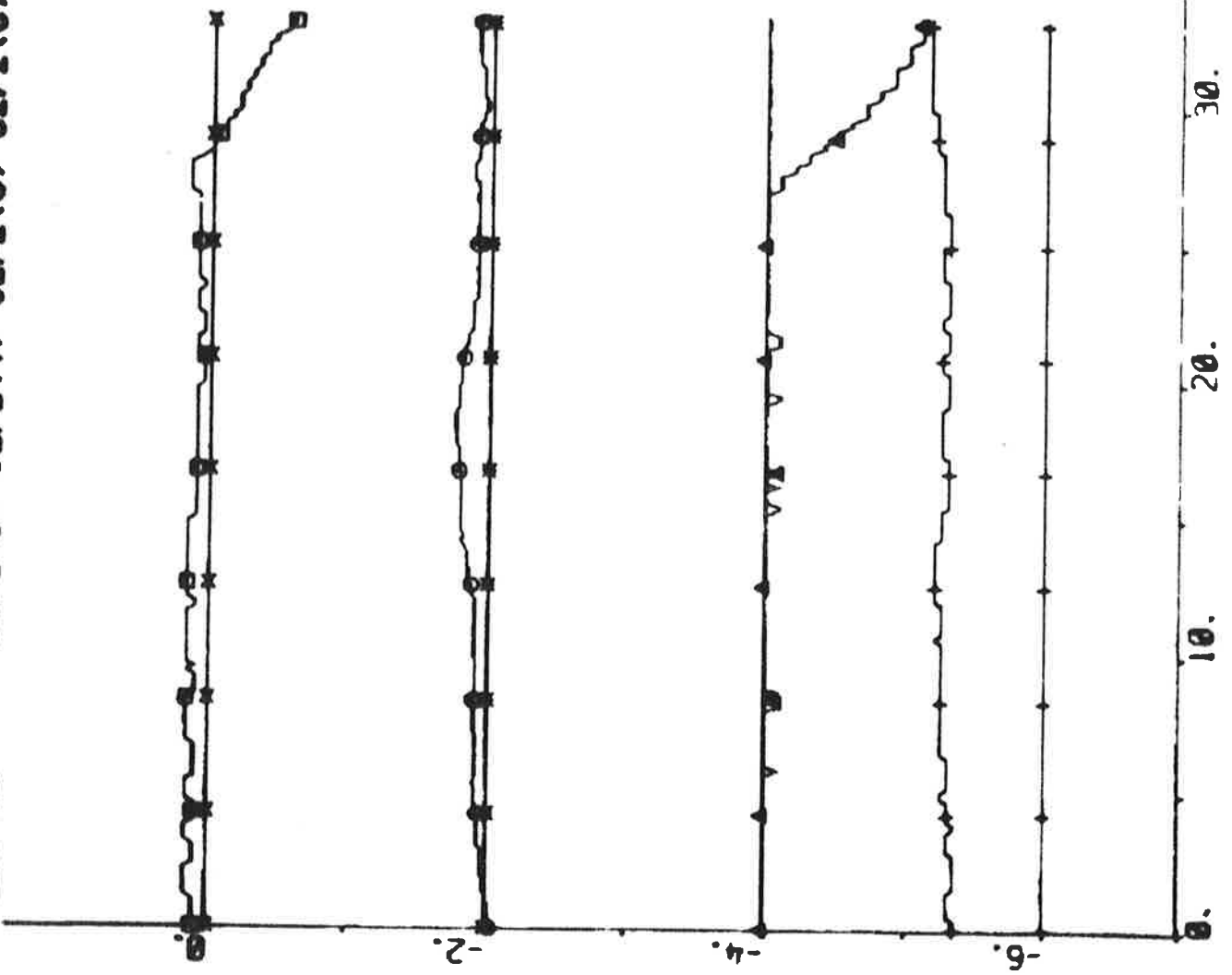
PLOT C2P1(1)-C2P2(3) C2P2(4) C2P2(5) C2P2(6) 00 02 04 06 -6.6 1.6

$0.5 \times \hat{d}_0$  deg

$2 \times \hat{d}_v$  knots

$100 \times \hat{d}_r$  deg/s

$0.5 \times \hat{d}_\delta$  deg



TIME (M)  
50.

PLOT CZP1(1)-CZP2(7) CZP2(8) CZP2(9) CZP2(10) CZP2(11) CZP2(12) CZP2(13)

$c_2$



40.

20.

0.

-20.

$a_2$

$a_3$

$b_2$

$b_1$

$c_1$

$a_4$

$a_1$



10.

20.

30.

40.

50.

TIME (MI)

## EXPERIMENT C3

Date	1976-04-26	Forward draught	10.9 m
Time	21.16	Aft draught	12.9 m
Duration	63 min	Wind direction	SE (1; see App. A)
Position	S 07°39' W 05°24'	Wind velocity	7 m/s (moderate breeze)
$\psi_{\text{ref}}$	142, (148), 162, (159), (131), 122, (134), 142 deg	Wave height	-
$r_{\text{ref}}$	0.05, 0.1 deg/s		

Self-tuning regulator and yaw regulator using estimates from the Kalman filter

The estimates  $\hat{\delta}'_0$ ,  $\hat{d}'_v$ ,  $\hat{d}'_r$  and  $\hat{d}'_\delta$  were not updated during the experiment.

NC1 = 1	NC2 = 1	k = 7	q = 0
$T_s = 10$ s	$V_0 = 6$ m/s	IVVC = 1	

Final values:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ b_1 \\ b_2 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} -10.55 \\ 12.90 \\ -0.97 \\ -1.57 \\ 0.38 \\ 0.17 \\ -0.54 \\ 89.70 \end{bmatrix} \quad P = \begin{bmatrix} 8.37 & & & & & & & & \\ & -15.05 & 45.05 & & & & & & \\ & 6.78 & -44.53 & 67.66 & & & & & \\ & 0.16 & 14.52 & -30.18 & 16.19 & & & & \\ & 0.23 & -1.19 & 1.57 & -0.60 & 0.05 & & & \\ & -0.04 & 0.24 & -0.47 & 0.30 & -0.01 & 0.02 & & \\ & -1.78 & 2.64 & 0.15 & -1.09 & -0.03 & -0.01 & 1.26 & \\ & 1.83 & -4.92 & 5.09 & -1.17 & 0.59 & 0.04 & 22.65 & 961.96 \end{bmatrix}$$

$$a_1 + a_2 + a_3 + a_4 = -0.19$$

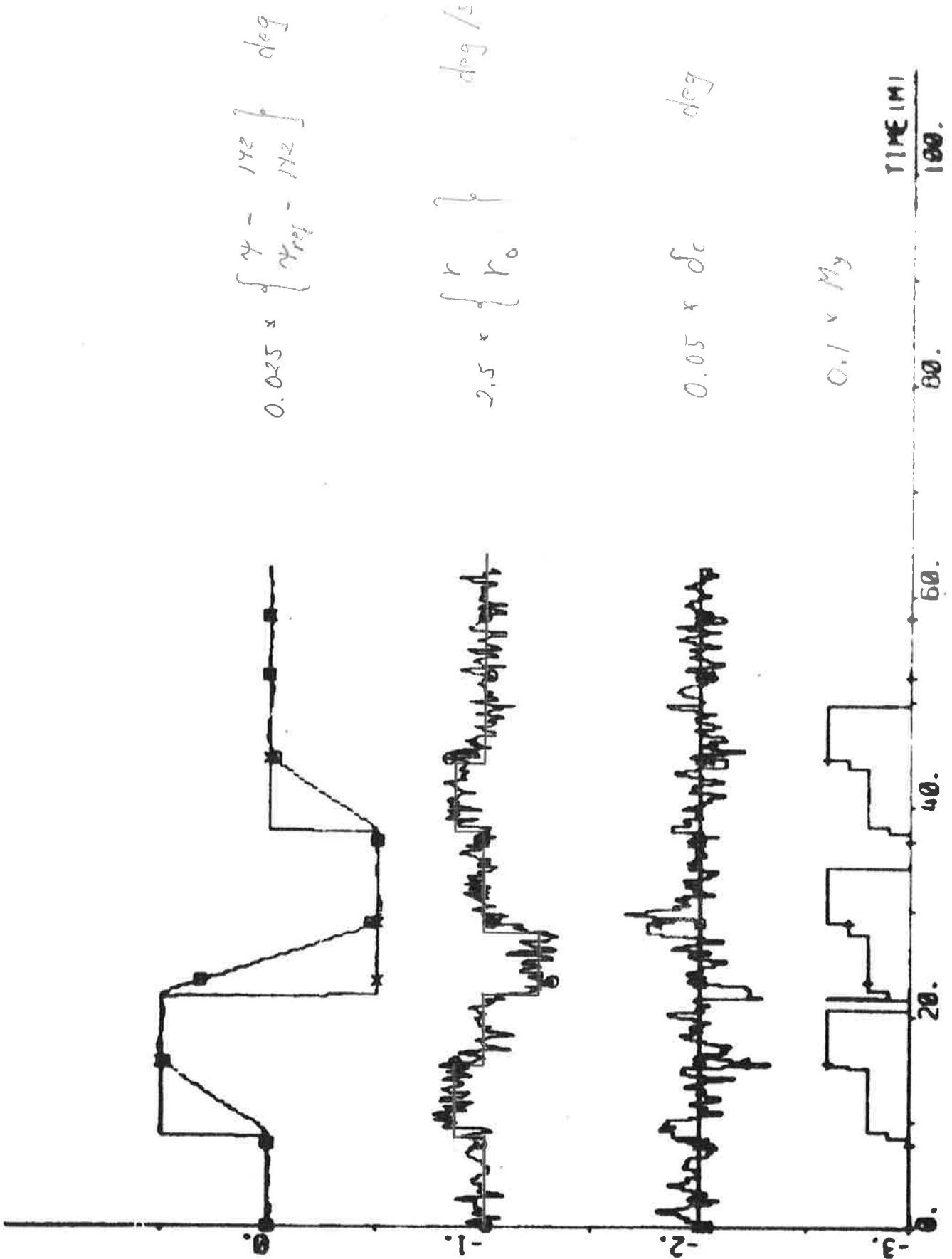
$$\hat{\delta}_0 = 0.0 \text{ deg} \quad \hat{d}_r = 0.21 \text{ knots} \quad \hat{d}_r = 0.002 \text{ deg/s} \quad \hat{d}_\delta = 1.7 \text{ deg}$$

$$\bar{\delta}_c = 0.8 \text{ deg} \quad (\text{Initial value: } 0.0 \text{ deg})$$

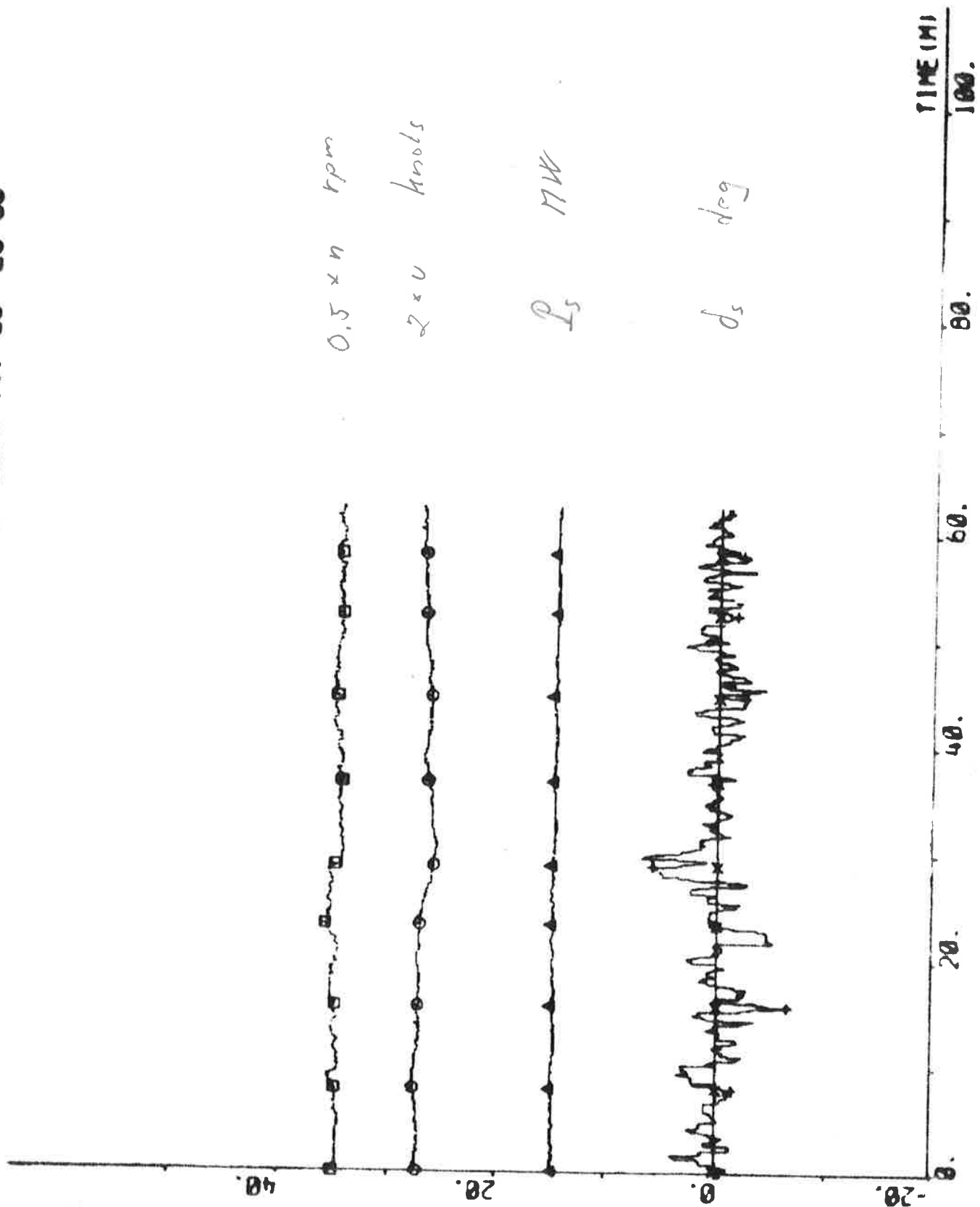
Statistics (mean value and standard deviation)

n	69.8 ± 0.7 rpm	$\epsilon_v$	-0.01 ± 0.02 knots
u	13.4 ± 0.2 knots	$\epsilon_r$	0.00 ± 0.02 deg/s
$P_s$	14.9 ± 0.1 MW	$\epsilon_\psi$	0.01 ± 0.06 deg
		$\epsilon_\delta$	-0.1 ± 0.5 deg

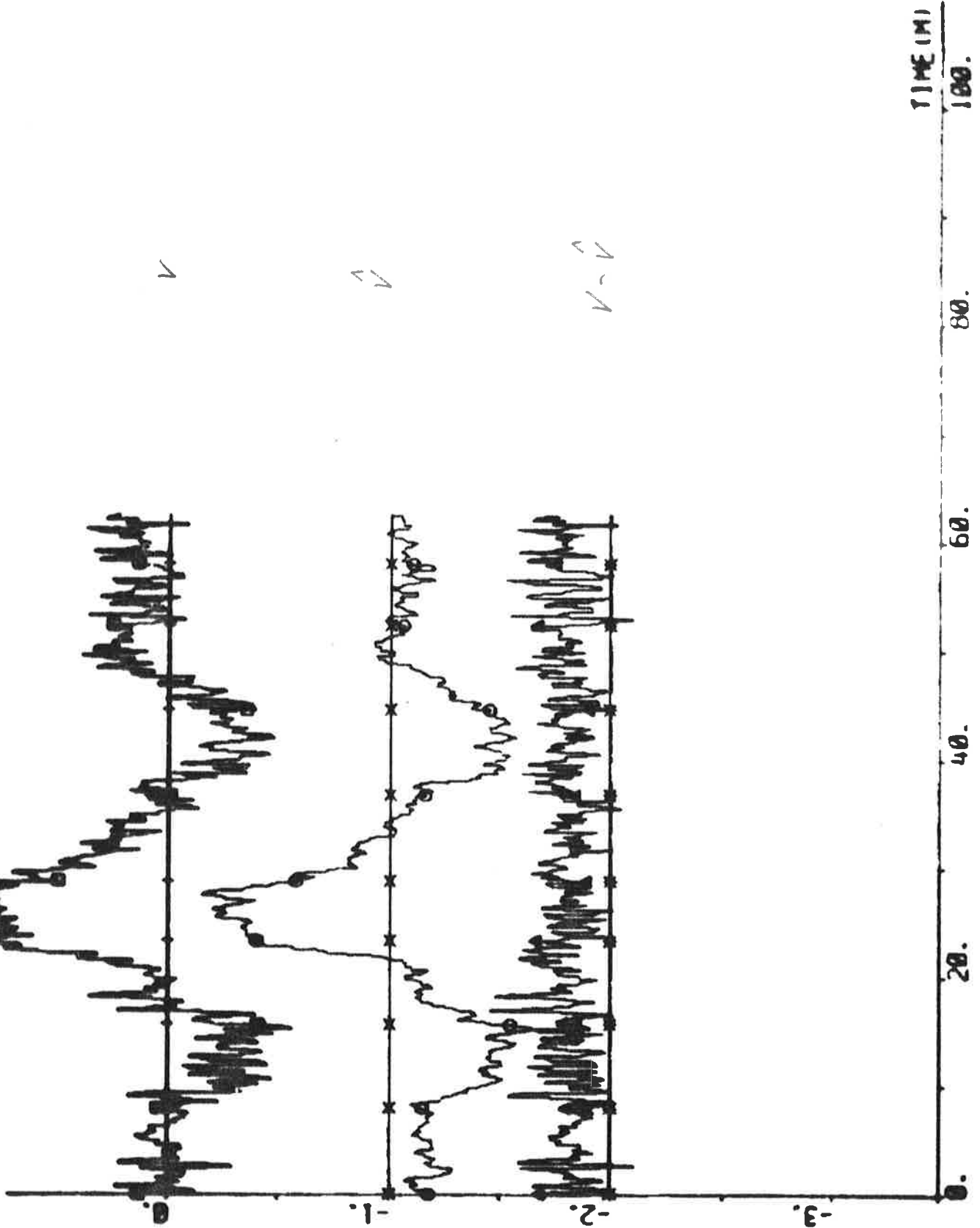
PLOT C3P1(1)-C3P1(8) C3P1(6) HP C3P1(10) HP C3P1(15) 02 -3 1



PLOT C3P1(1)-C3P1(13) C3P1(12) C3P1(14) C3P1(11) 00 -20 50

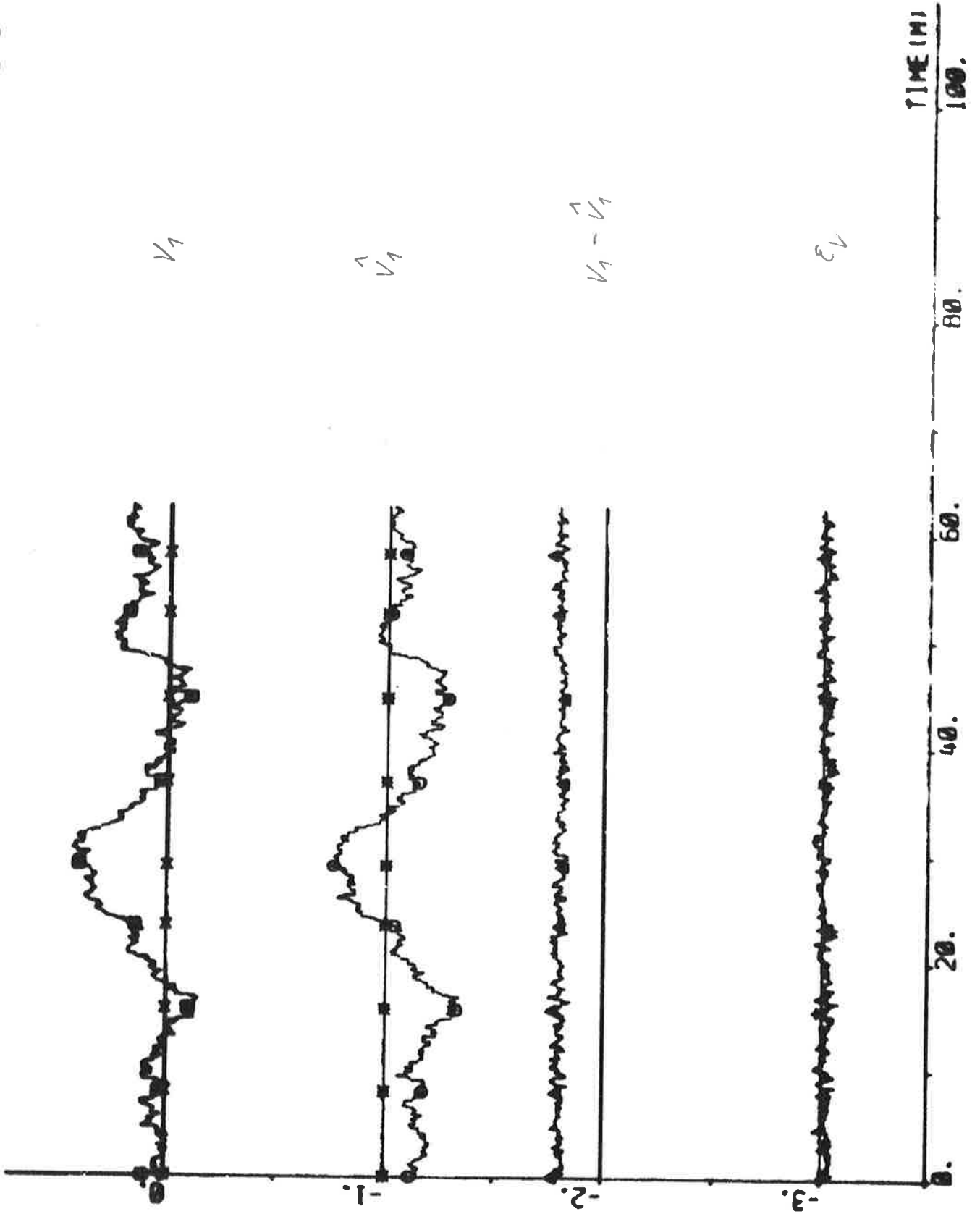


PLOT C3P1(1)-C3P2(1) C3P2(2) ERRS 00 01 02 -3.4 0.8 - KNOTS

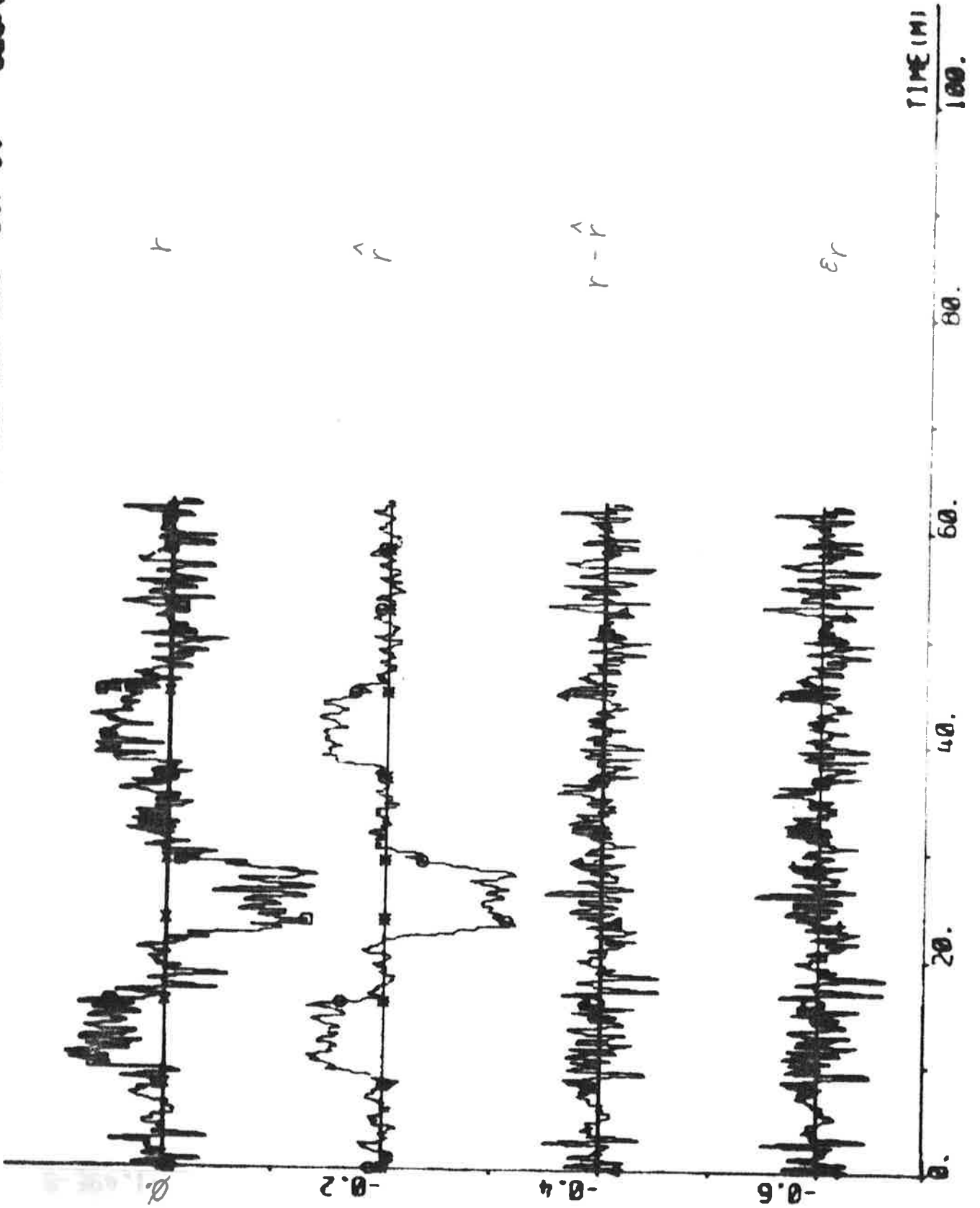




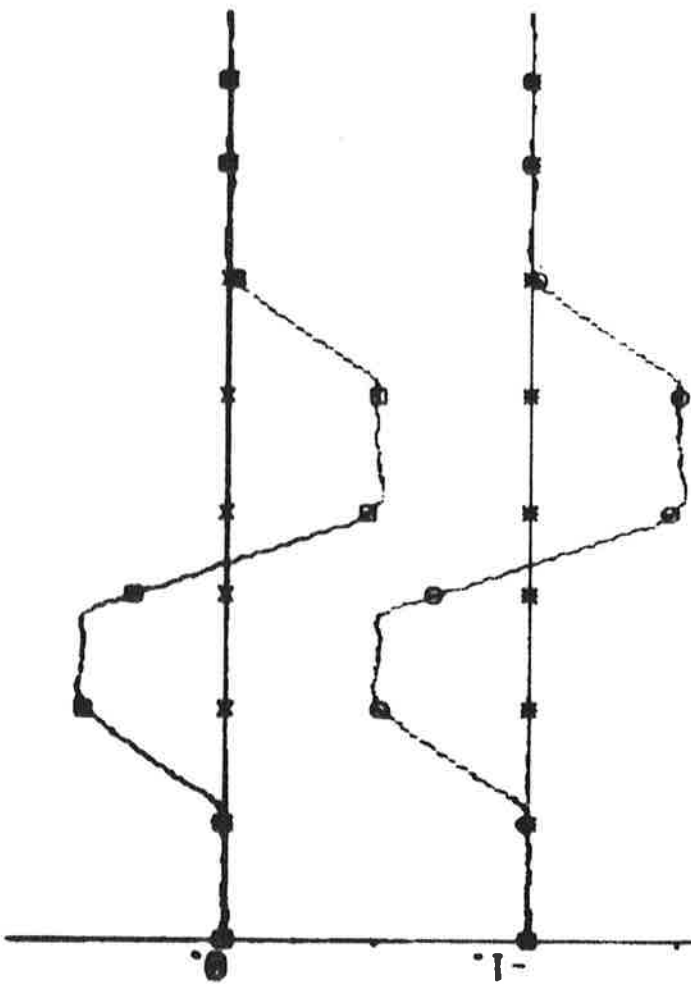
PLOT C3P1(1)-C3P1(4) C3P1(5) ERR2 EP32 00 01 02 03 -3.4 0.0 - KNOTS



PLOT C3P1(1)-C3P1(6) C3P1(7) ERR3 EPS3 00 002 004 006 -0.7 0. - DECS



PLOT C3P1(1)-C3P1(8) C3P1(9) ERR4 EPS4 00 01 02 03 -3.4 0.0 - DEG



$0.025 * (\gamma - 172)$

$0.025 * (\hat{\gamma} - 172)$

$\gamma - \hat{\gamma}$



Err

TIME (M)  
100.

60.

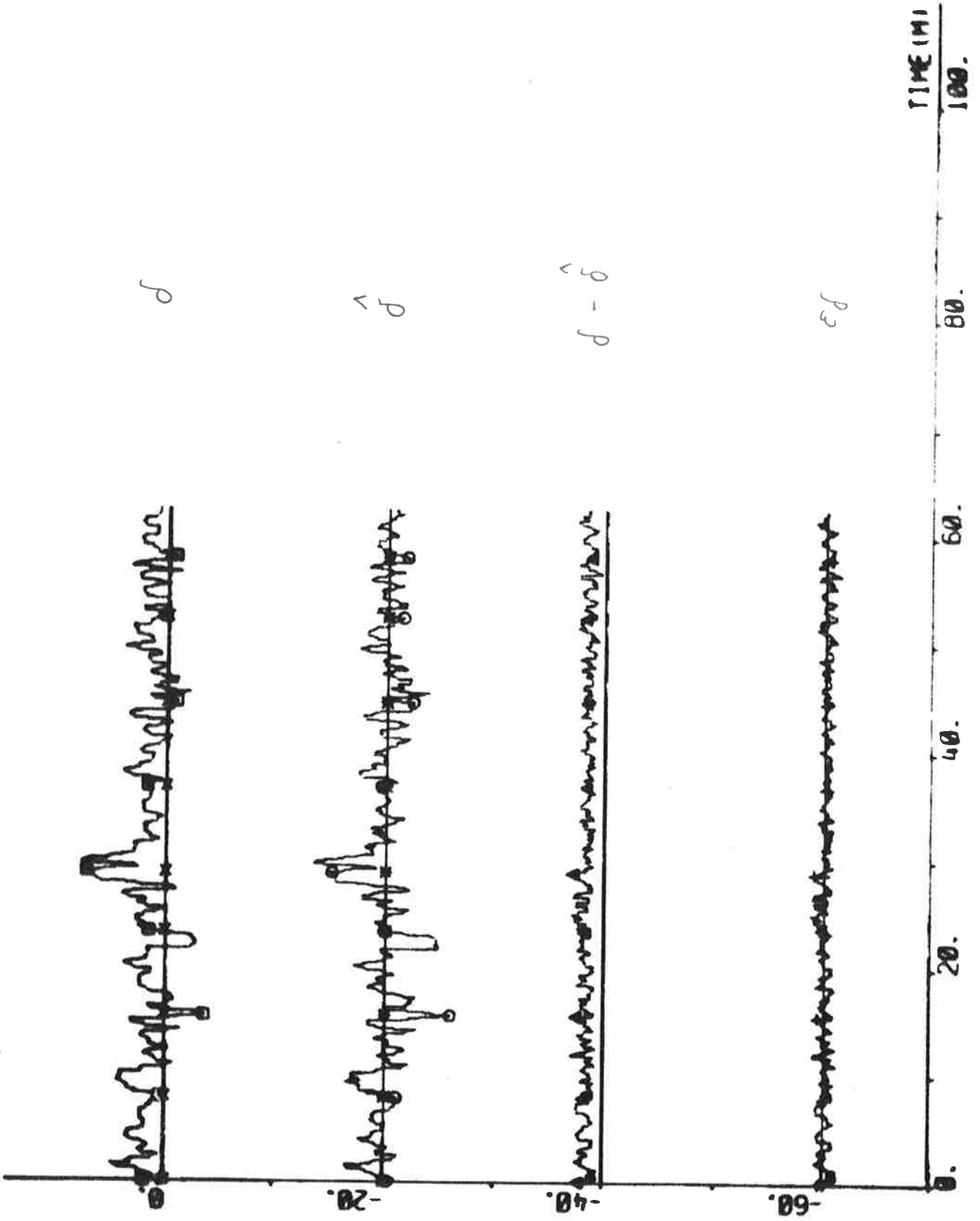
40.

20.

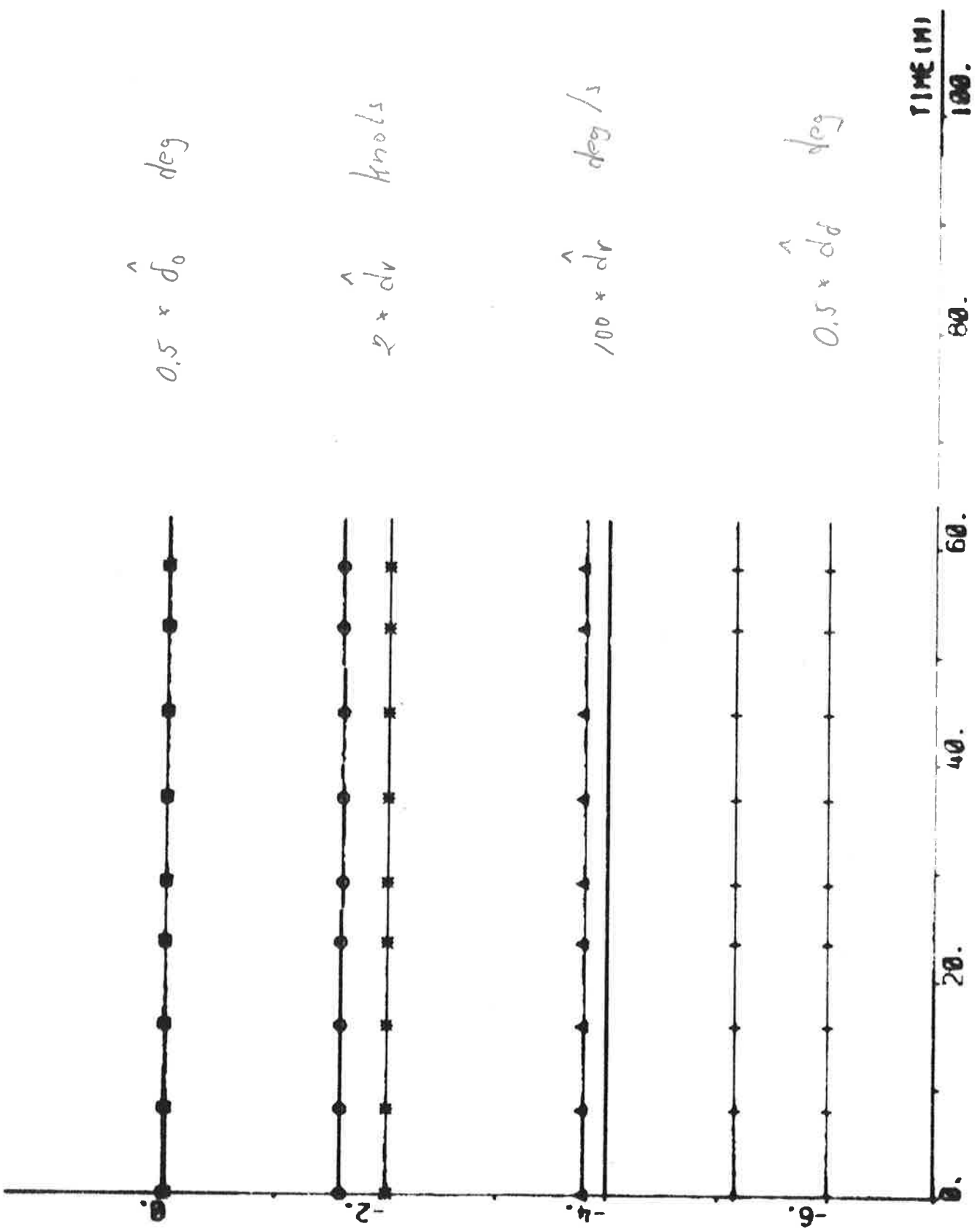
0.

80.

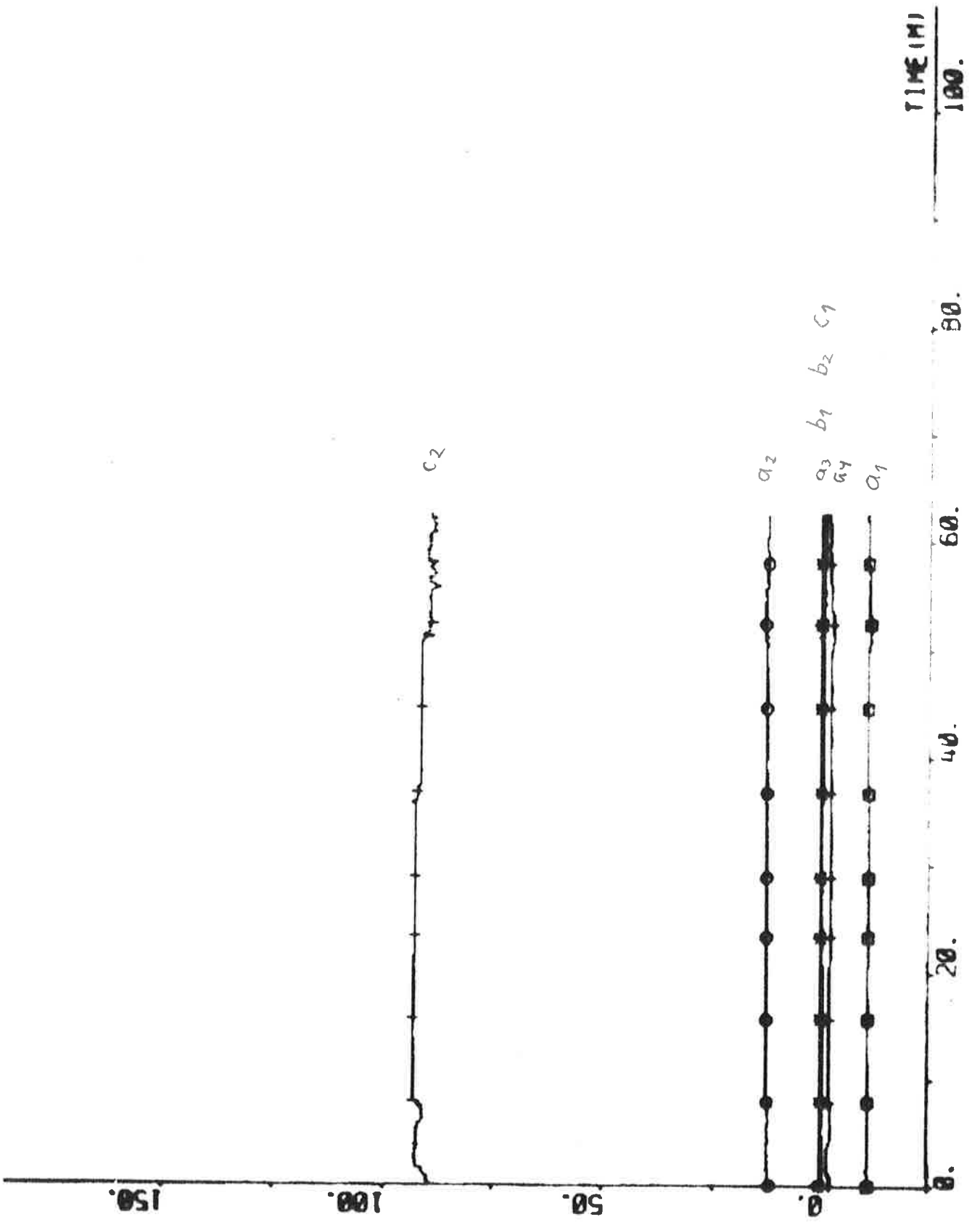
PL0T C3P1(1)-C3P1(2) C3P1(3) ERR1 EPS1 00 020 040 060 -65 15 - DEG



PLOT C3P1(1)-C3P2(3) C3P2(4) C3P2(5) C3P2(6) 00 02 04 06 -6.6 1.6

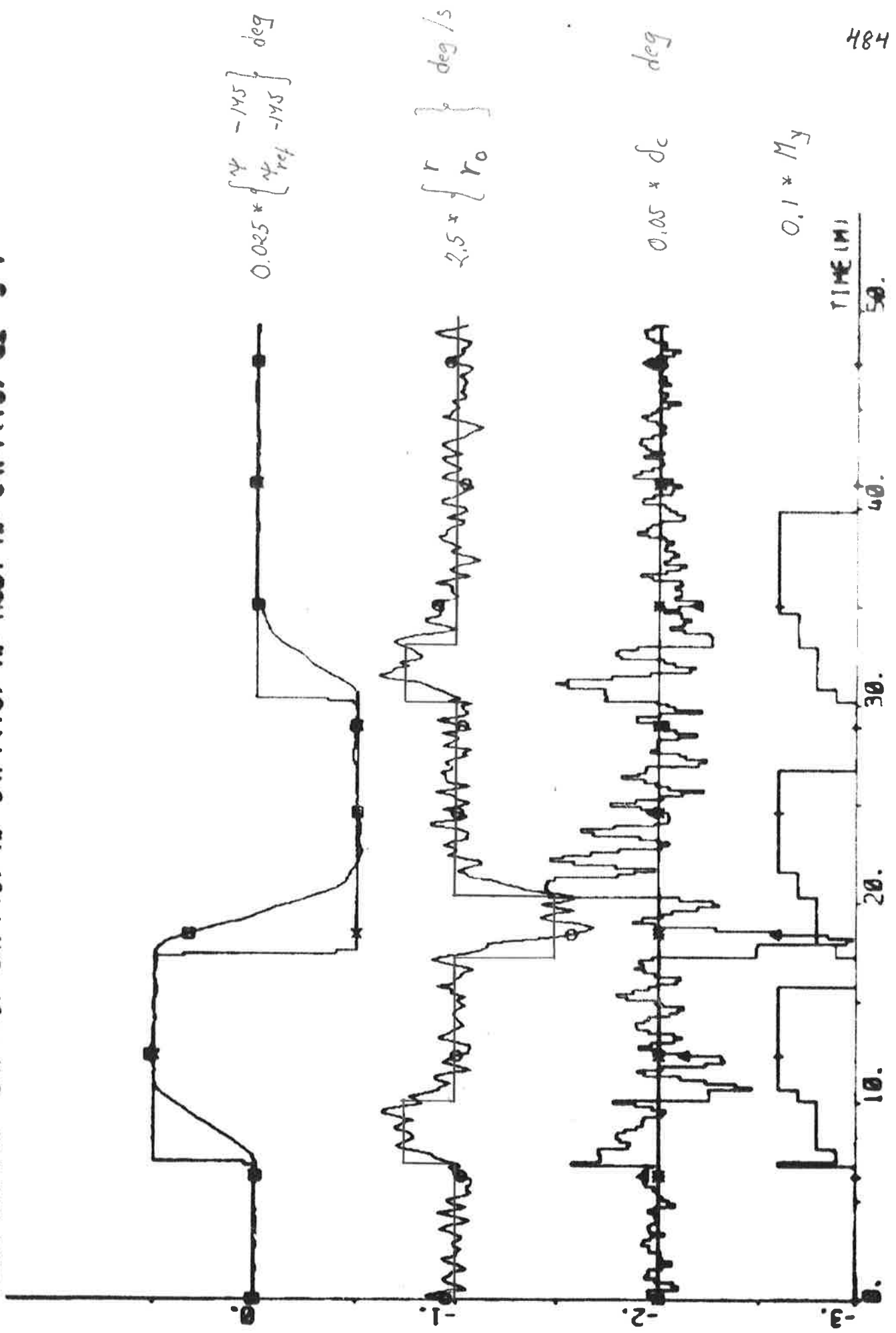


PLOT C3P1(1)-C3P2(7) C3P2(8) C3P2(9) C3P2(10) C3P2(11) C3P2(12) C3P2(13)



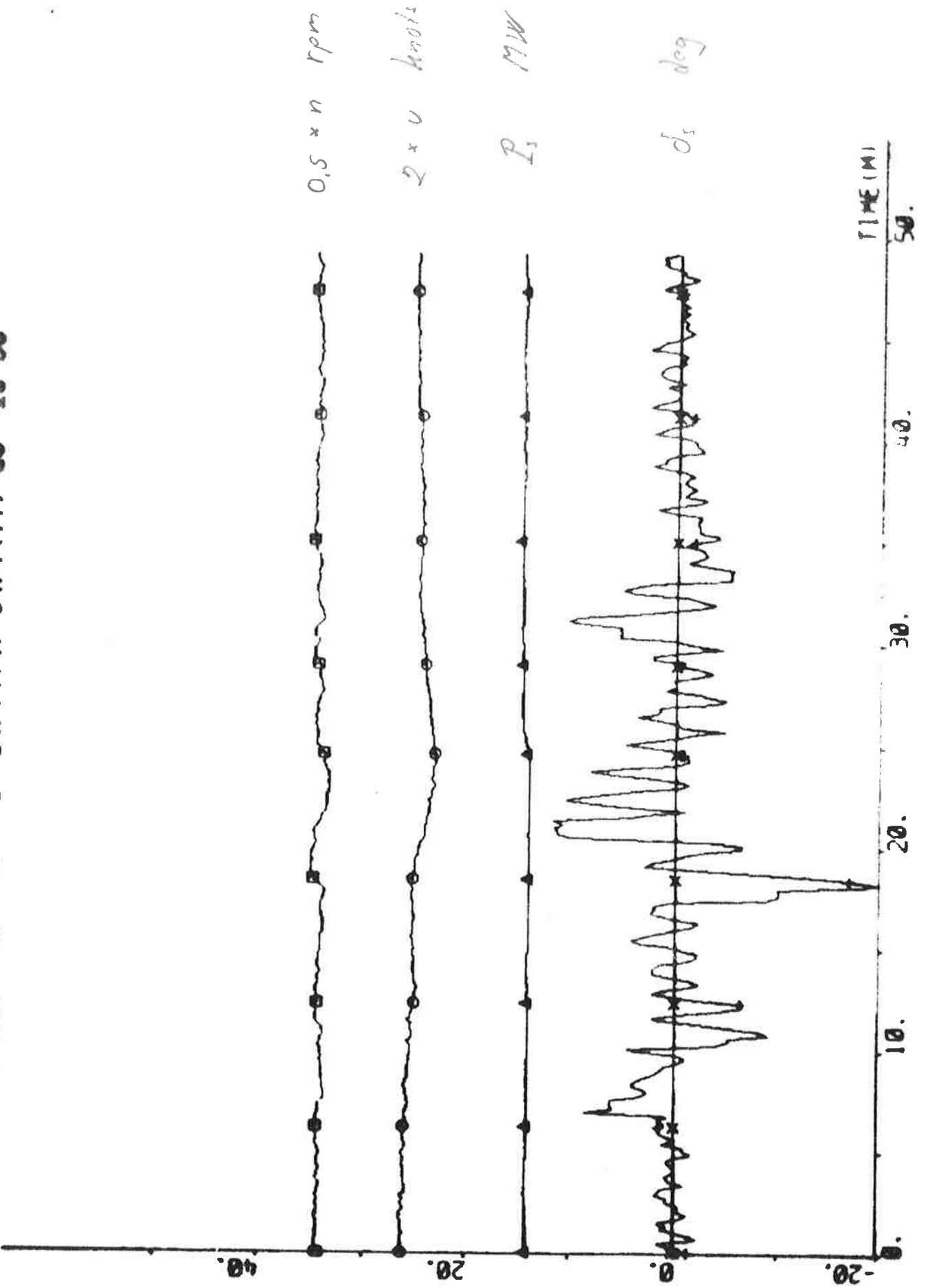


PLOT C4P1(1)-C4P1(8) C4P1(6) HP C4P1(10) HP C4P1(15) HP H00Y HP C4P1(15) 02 -3 1

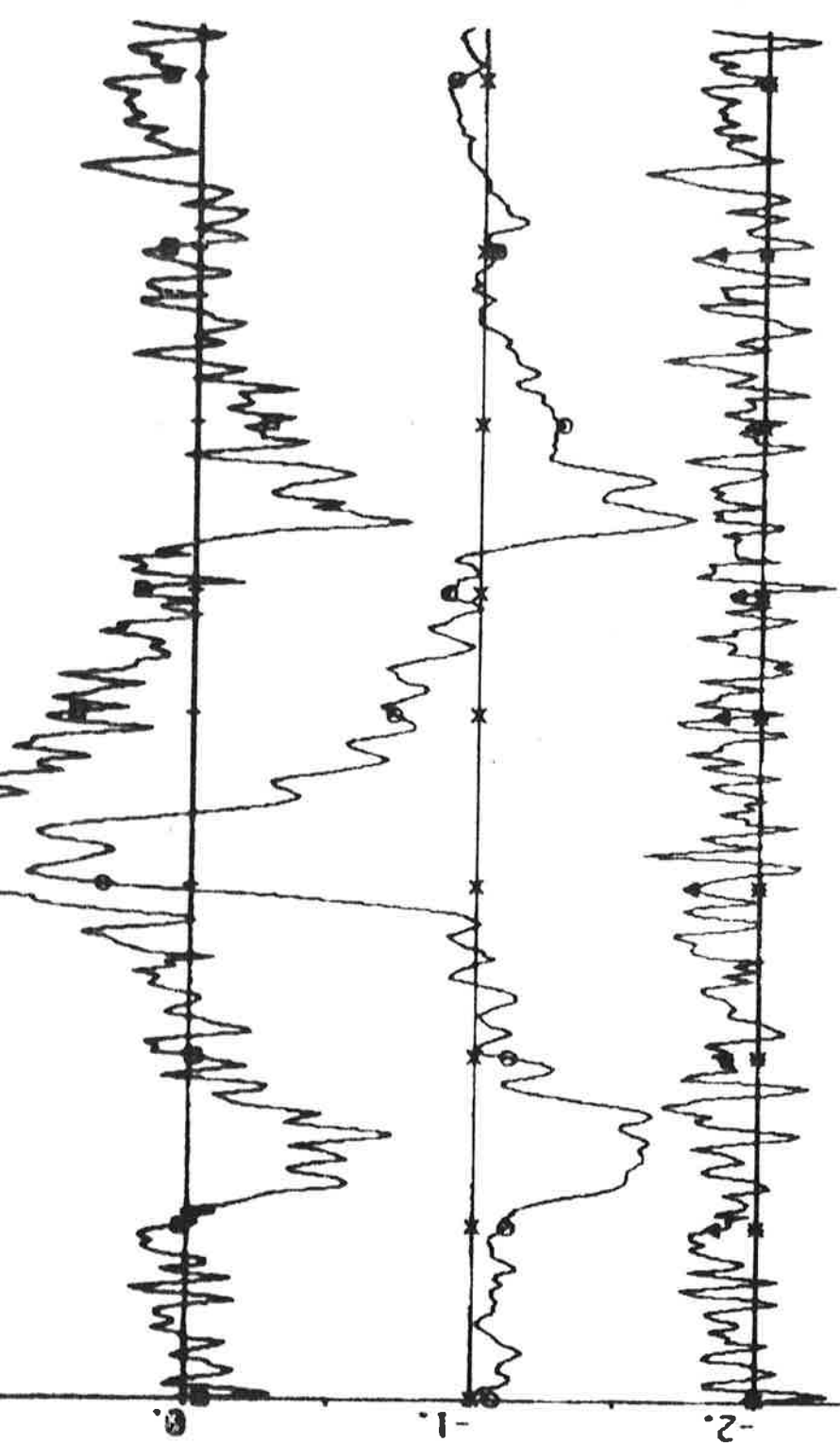




PLOT C4P1(1)-C4P1(13) C4P1(12) C4P1(14) C4P1(11) 00 -20 50

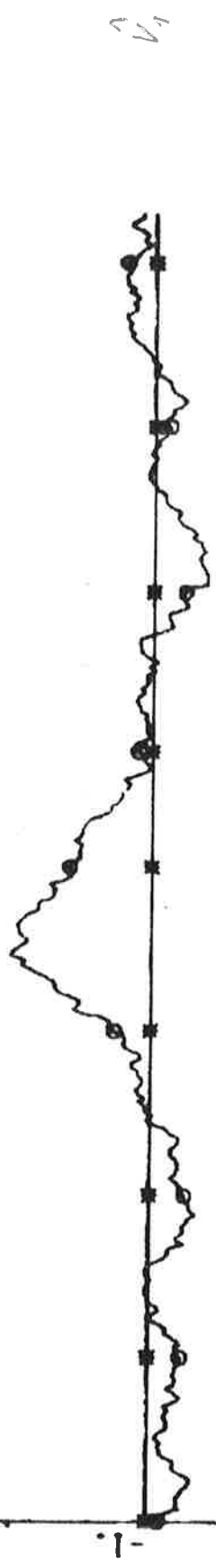
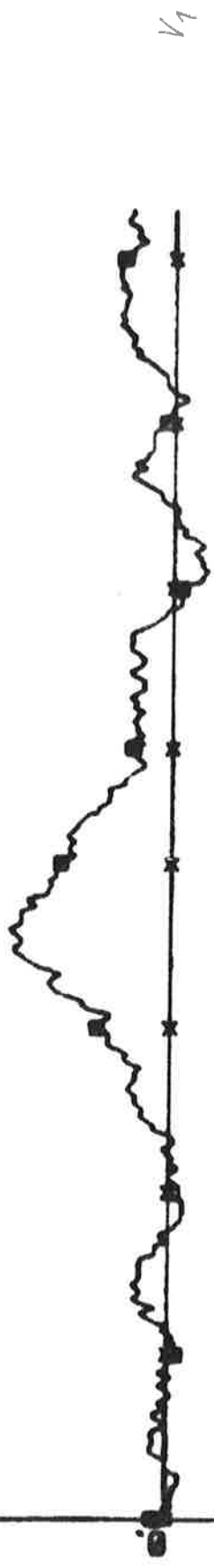


PLOT C4P1(1)-C4P2(1) C4P2(2) ERRS 00 01 02 -3.4 0.0 - KNOTS



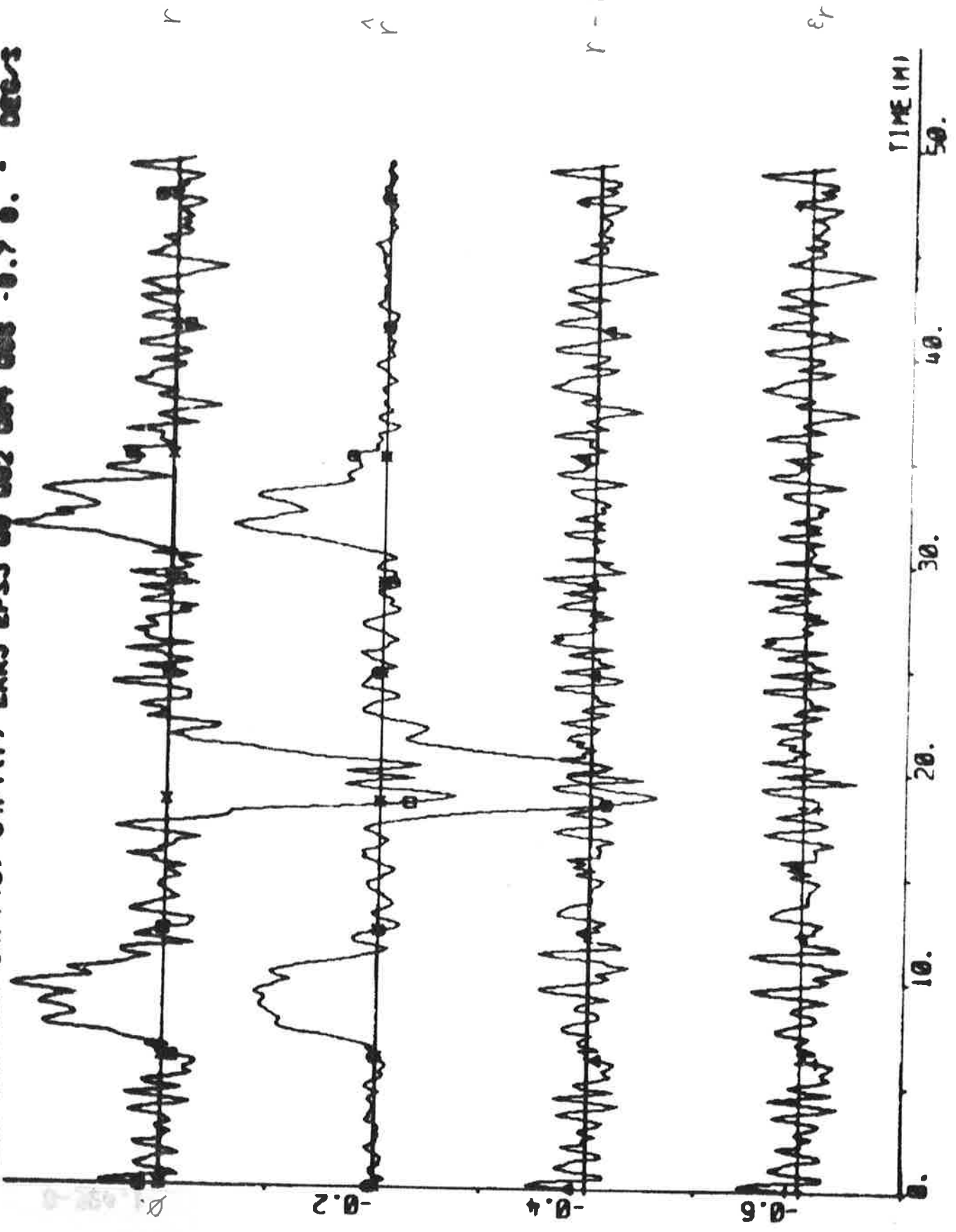
0. 10. 20. 30. 40. 50. TIME (M)

PLOT C4P1(1)-C4P1(4) C4P1(5) ERR2 EPS2 00 01 02 03 -3.4 0.0 - 1000TS

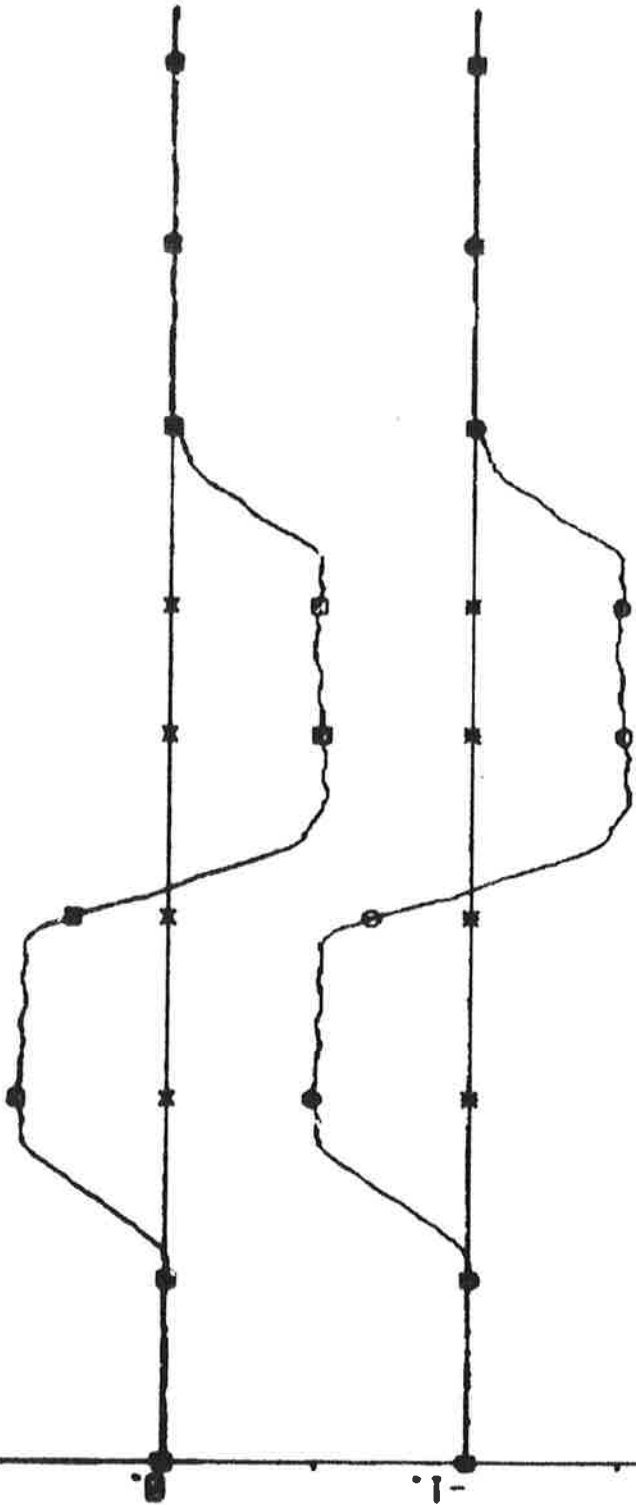


0. 10. 20. 30. 40. 50. TIME (MI)

PLOT CYP1(1)-CYP1(6) CYP1(7) ERR3 EPS3 09 002 004 006 -0.7 0. - DECS



PLOT CYP1(1)-CYP1(8) CYP1(9) ERR4 EPS4 00 01 02 03 -3.4 0.0 - DEC



0,025 \* (7-175)

0,025 \* (7-175)



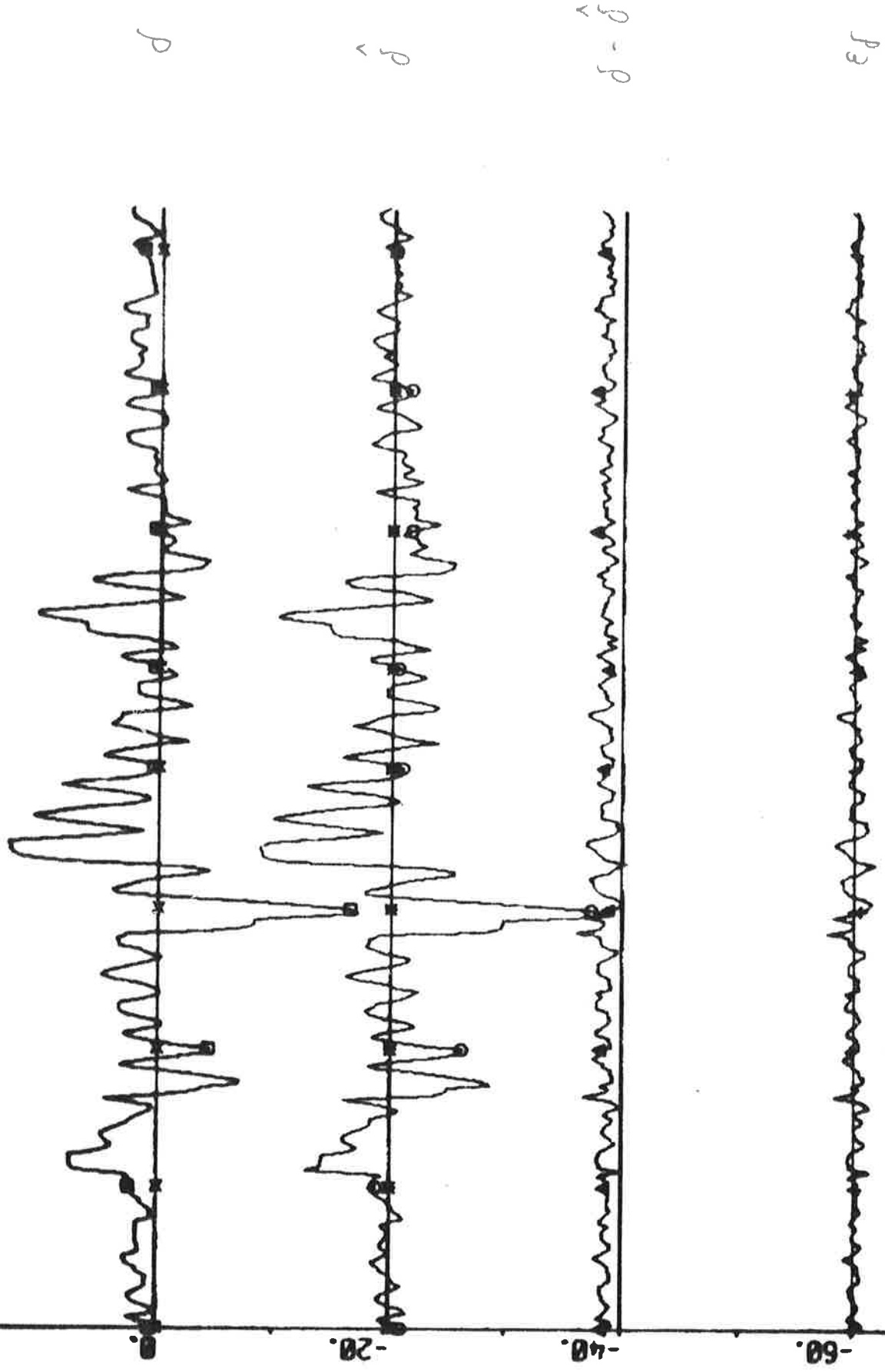
$\eta - \bar{\eta}$



$\epsilon_7$

TIME (M)  
50.

PLOT CWP1(1)-CWP1(2) CWP1(3) ERR1 EPS1 00 020 040 060 -05 15 - DEG



TIME (M)  
50.  
40.  
30.  
20.  
10.  
0.

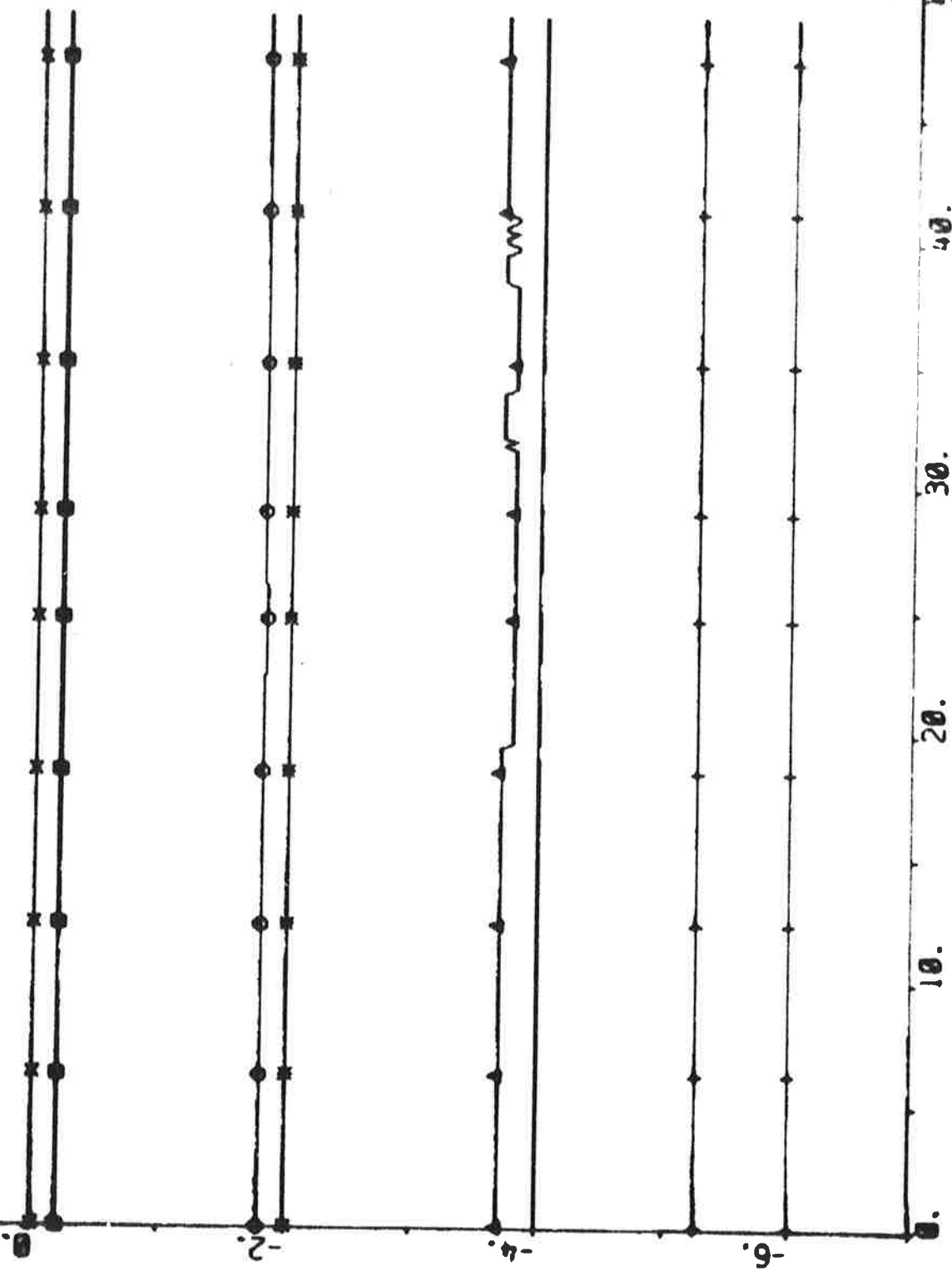
PLOT C4P1(1)-C4P2(3) C4P2(4) C4P2(5) C4P2(6) 00 02 04 06 -0.5 1.5

$0.5 \times \hat{d}_0$  deg

$2 \times \hat{d}_v$  knots

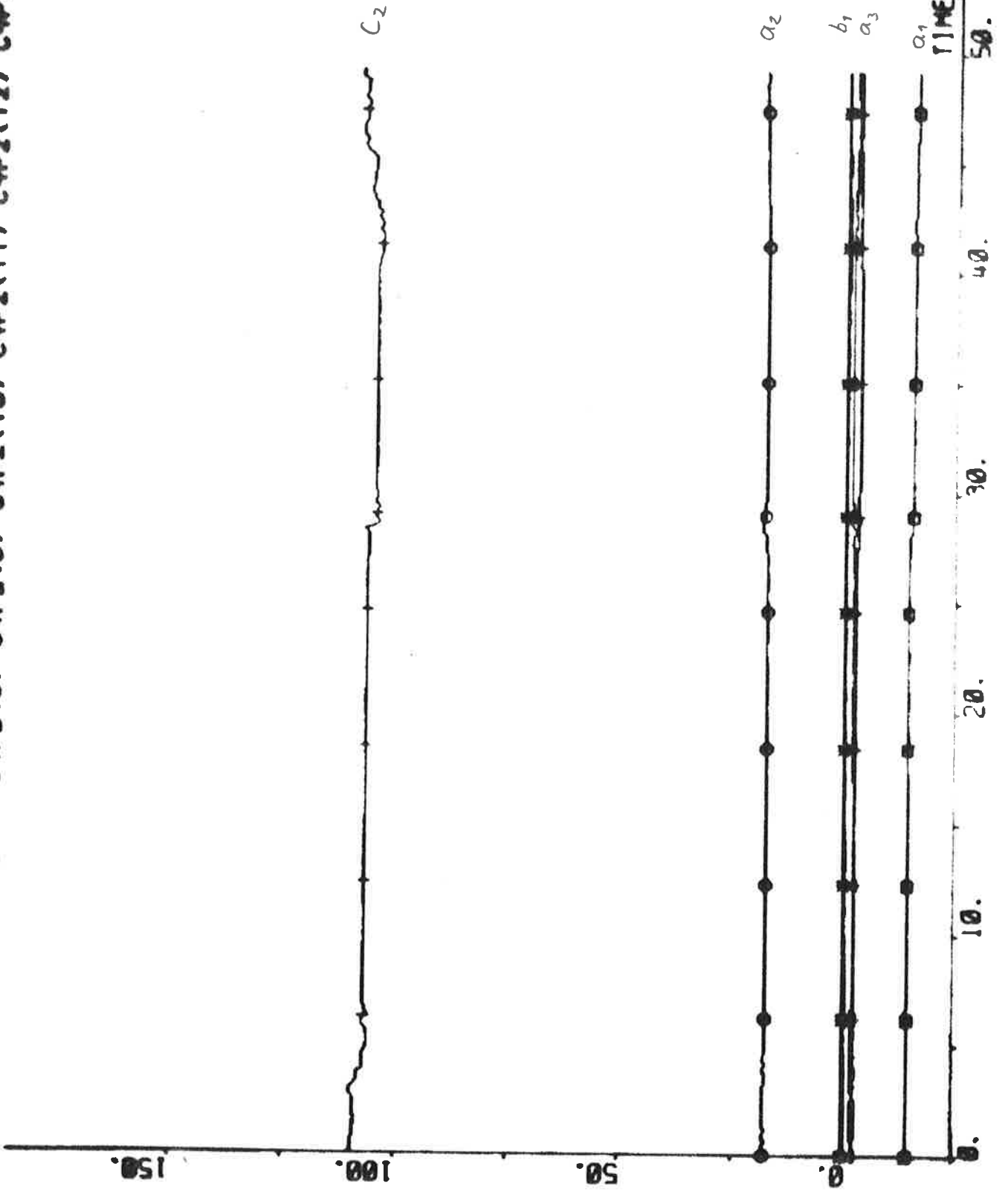
$100 \times \hat{d}_r$  deg/s

$0.5 \times \hat{d}_\delta$  deg



TIME (M)  
50.

PLOT C4P1(1)-C4P2(7) C4P2(8) C4P2(9) C4P2(10) C4P2(11) C4P2(12) C4P2(13)



150.

100.

50.

0.

0.



## EXPERIMENT C5

Date	1976-04-27	Forward draught	10.9 m
Time	14.10	Aft draught	12.9 m
Duration	27 min	Wind direction	SE (1; see App. A)
Position	S 10°27' W 03°14'	Wind velocity	8 m/s (moderate breeze)
$\psi_{\text{ref}}$	144, (150), 149, (147), 139, 144 deg	Wave height	-
$r_{\text{ref}}$	0.1 deg/s		

Self-tuning regulator and yaw regulator using estimates from the Kalman filter.

NC1 = 1	NC2 = 1	k = 7	q = 0
$T_s = 10$ s	$V_0 = 6$ m/s	IVVC = 1	

Final values:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ b_1 \\ b_2 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} -10.07 \\ 8.90 \\ 5.69 \\ -4.90 \\ 0.41 \\ 0.05 \\ -1.54 \\ 48.49 \end{bmatrix} \quad P = \begin{bmatrix} 8.31 & & & & & & & & \\ -13.48 & 35.98 & & & & & & & \\ 3.24 & -29.55 & 47.87 & & & & & & \\ 2.41 & 6.81 & -21.85 & 13.31 & & & & & \\ 0.13 & -0.89 & 1.18 & -0.41 & 0.05 & & & & \\ 0.00 & 0.14 & -0.44 & 0.33 & -0.01 & 0.03 & & & \\ -1.08 & 1.49 & 0.26 & -0.68 & -0.01 & -0.01 & 1.09 & & \\ 23.69 & -42.91 & 0.47 & 18.03 & 1.56 & -0.45 & 17.22 & 1078.29 & \end{bmatrix}$$

$$a_1 + a_2 + a_3 + a_4 = -0.38$$

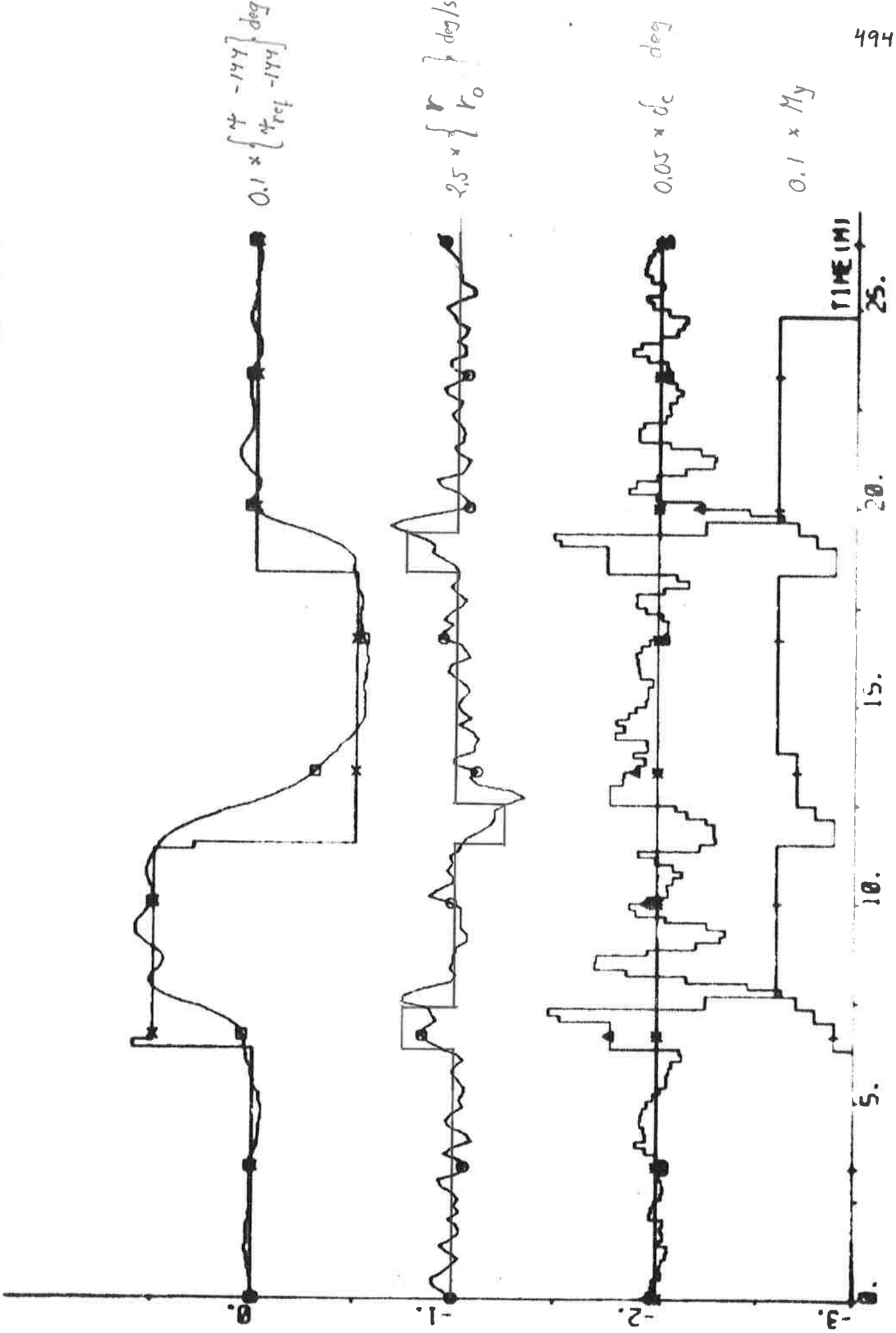
$$\hat{\delta}_0 = 0.1 \text{ deg} \quad \hat{d}_v = 0.13 \text{ knots} \quad \hat{d}_r = 0.002 \text{ deg/s} \quad \hat{d}_\delta = 1.5 \text{ deg}$$

$$\bar{\delta}_c = 0.4 \text{ deg} \quad (\text{Initial value: } 0.3 \text{ deg})$$

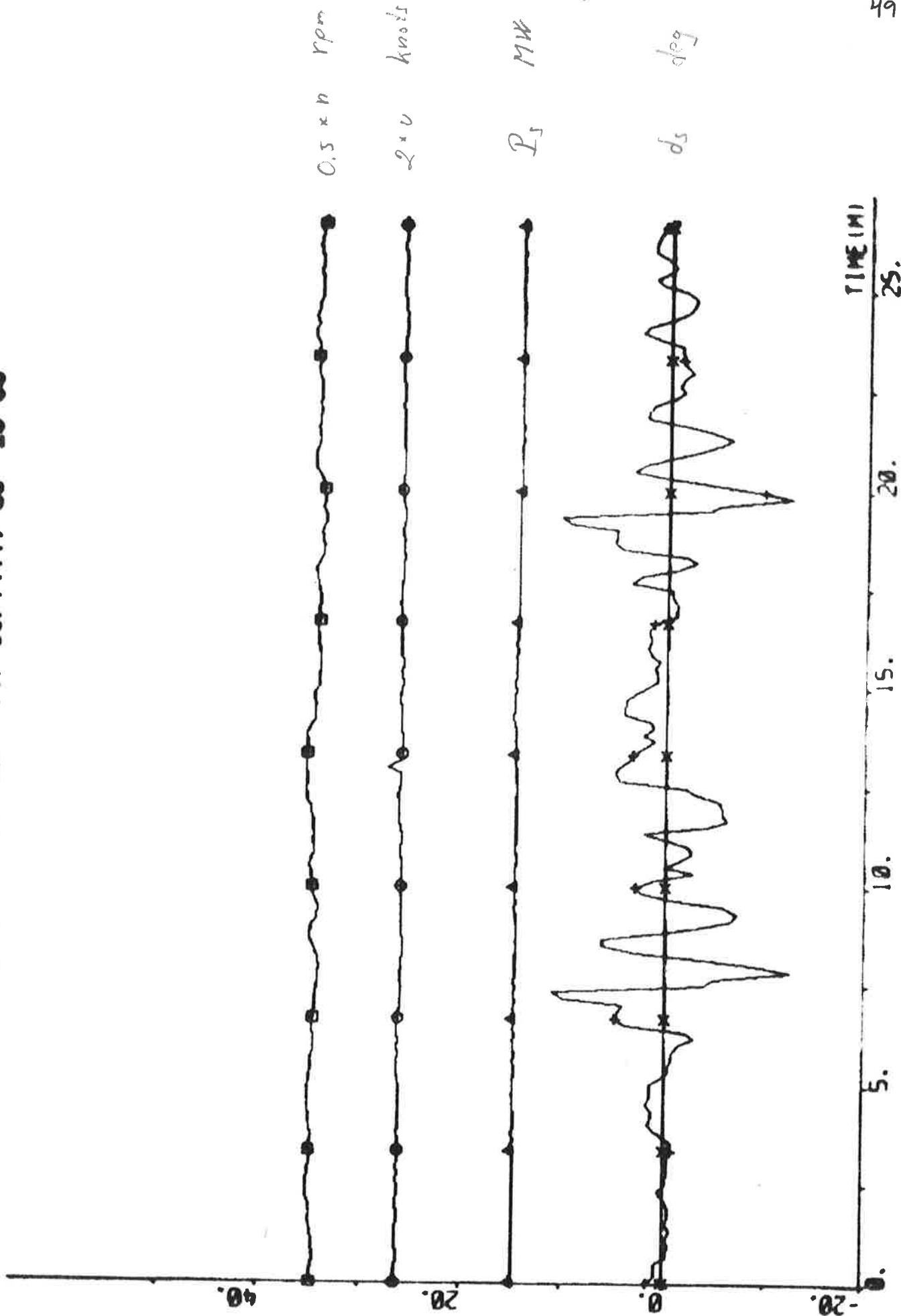
Statistics (mean value and standard deviation)

n	69.7 ± 0.7 rpm	$\varepsilon_v$	0.00 ± 0.03 knots
u	13.2 ± 0.1 knots	$\varepsilon_r$	0.00 ± 0.02 deg/s
$P_s$	14.7 ± 0.2 MW	$\varepsilon_\psi$	0.00 ± 0.03 deg
		$\varepsilon_\delta$	0.0 ± 0.6 deg

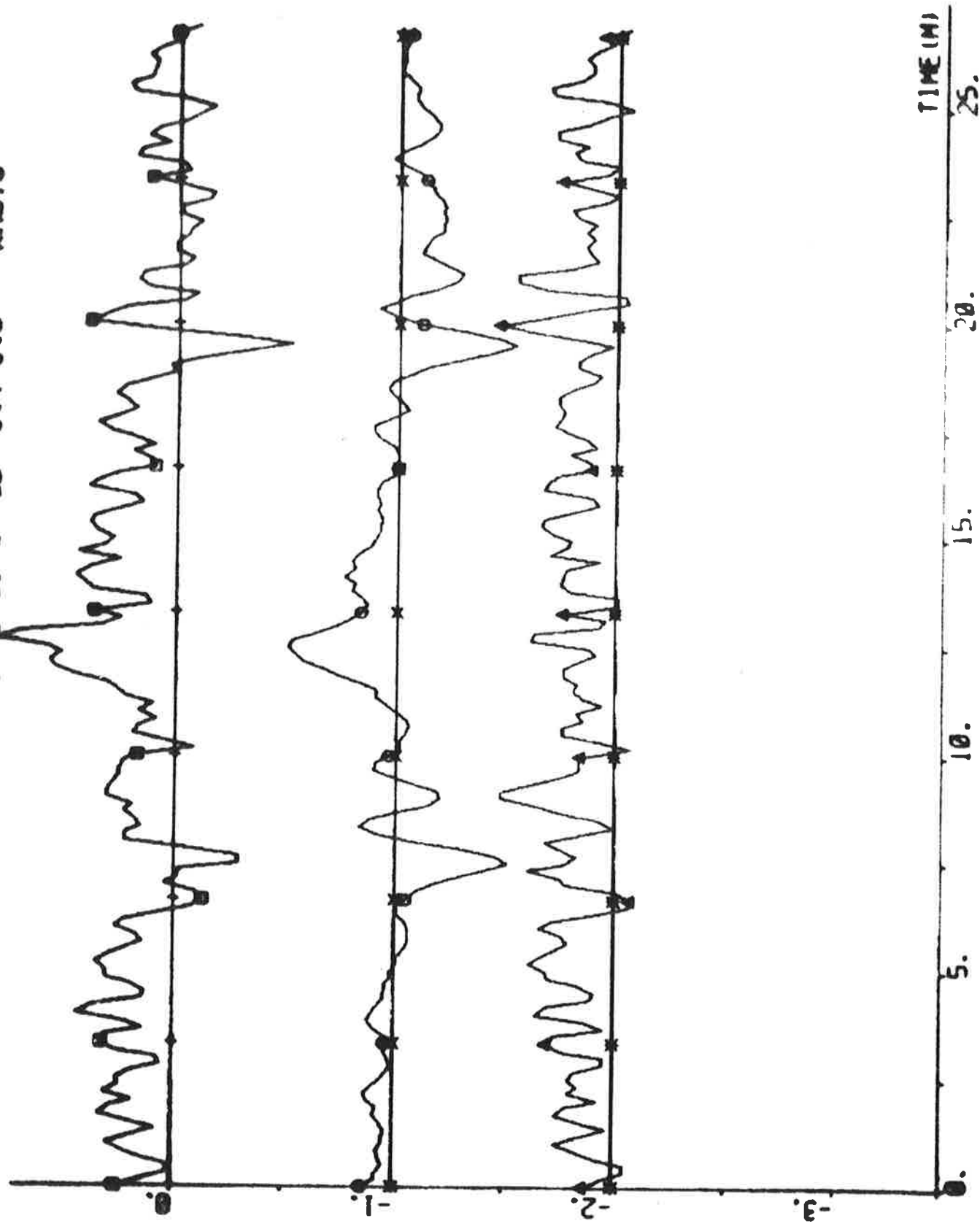
PLOT CSP1(1)-CSP1(8) CSP1(6) HP CSP1(10) HP MOOY HP CSP1(15) 02 -3 1



PLOT CSP1(1)-CSP1(13) CSP1(12) CSP1(14) CSP1(11) 00 -20 50

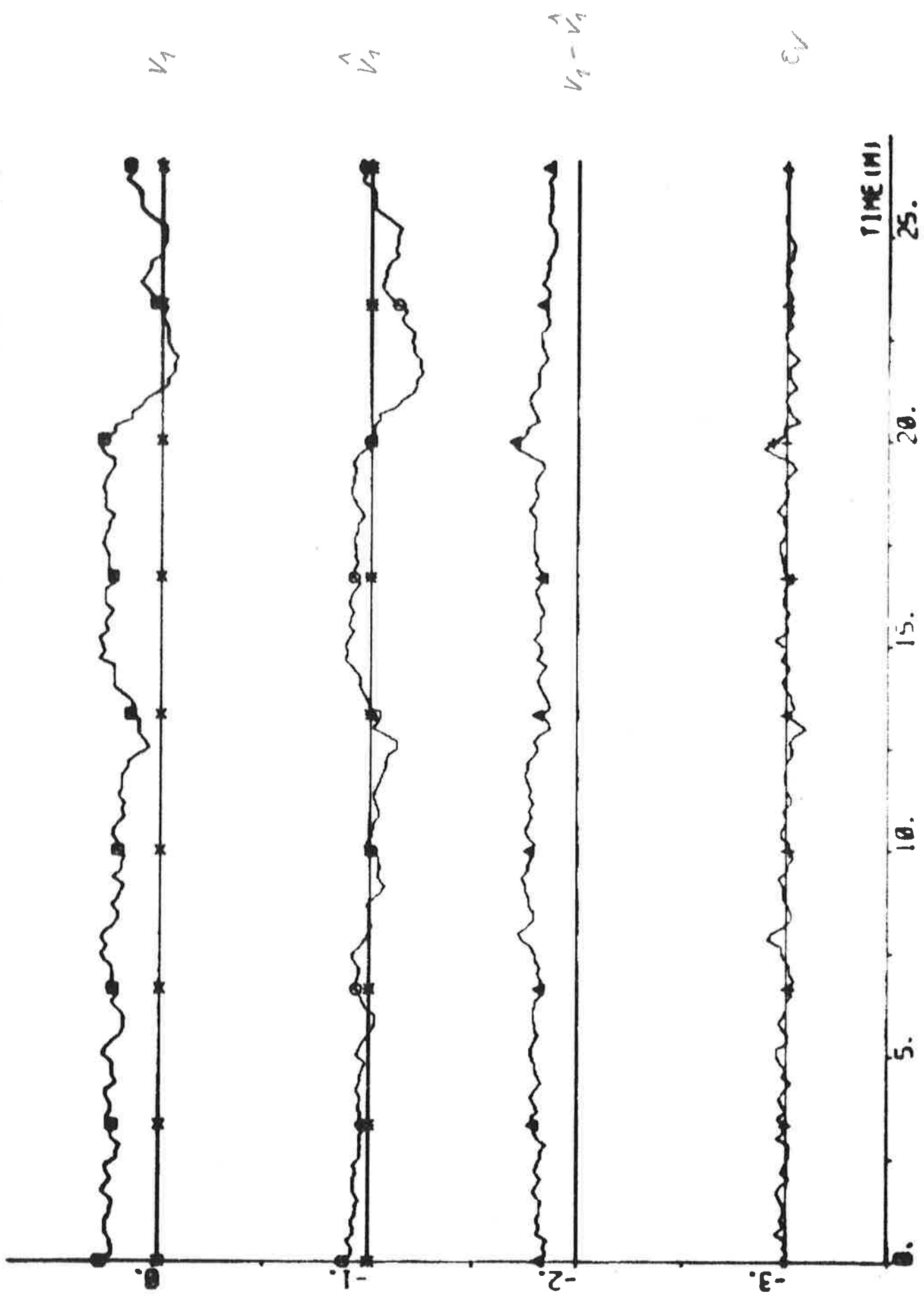


PL0T CSP1(1)-CSP2(1) CSP2(2) ERRS 00 01 02 -3.4 0.0 - KNOTS



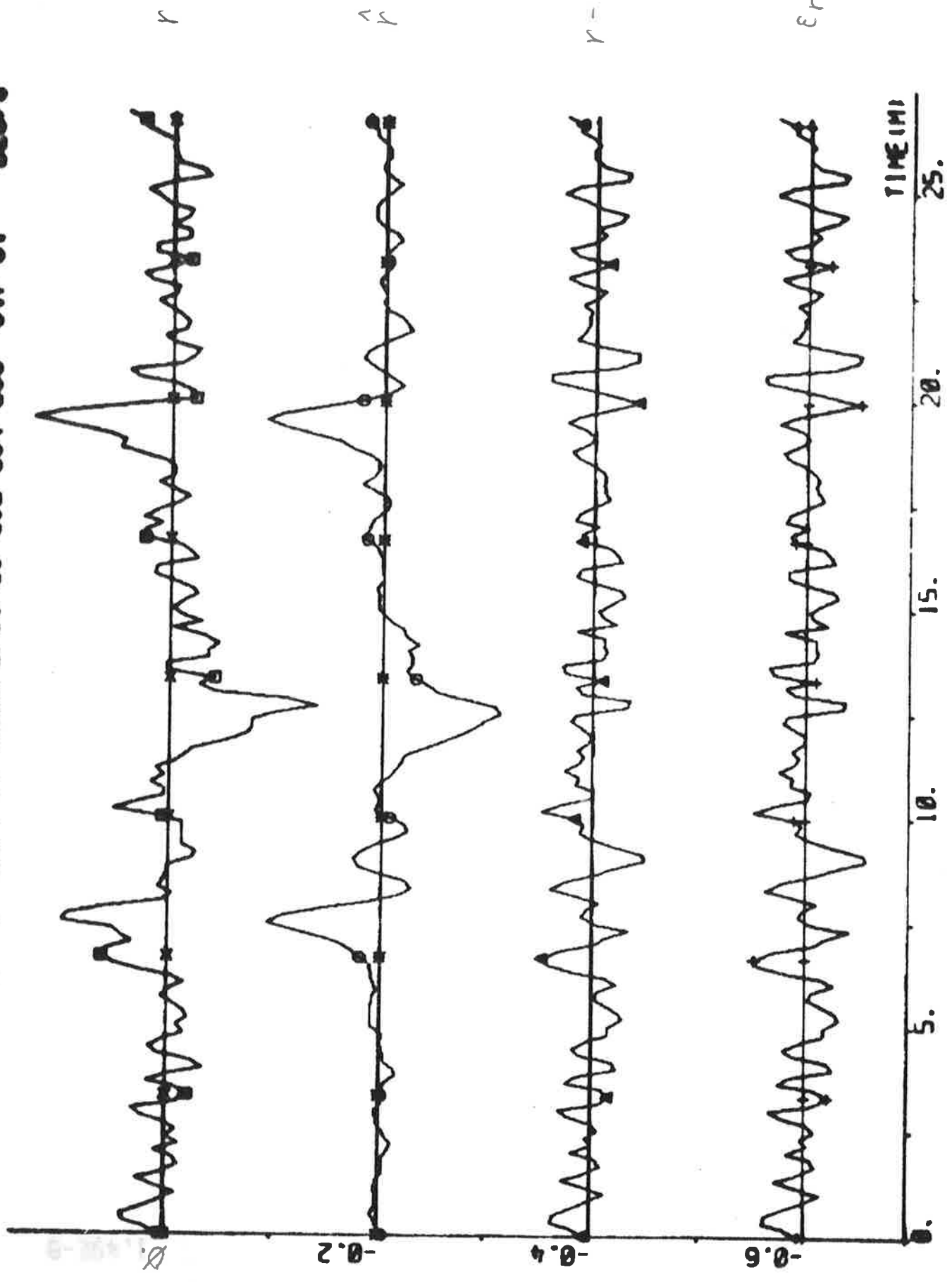
TIME (M)  
25.

PLOT CSP1(1)-CSP1(4) CSP1(5) ERR2 EPS2 00 01 02 03 -3.4 0.0 - 10MOTS



TIME (M)  
25.

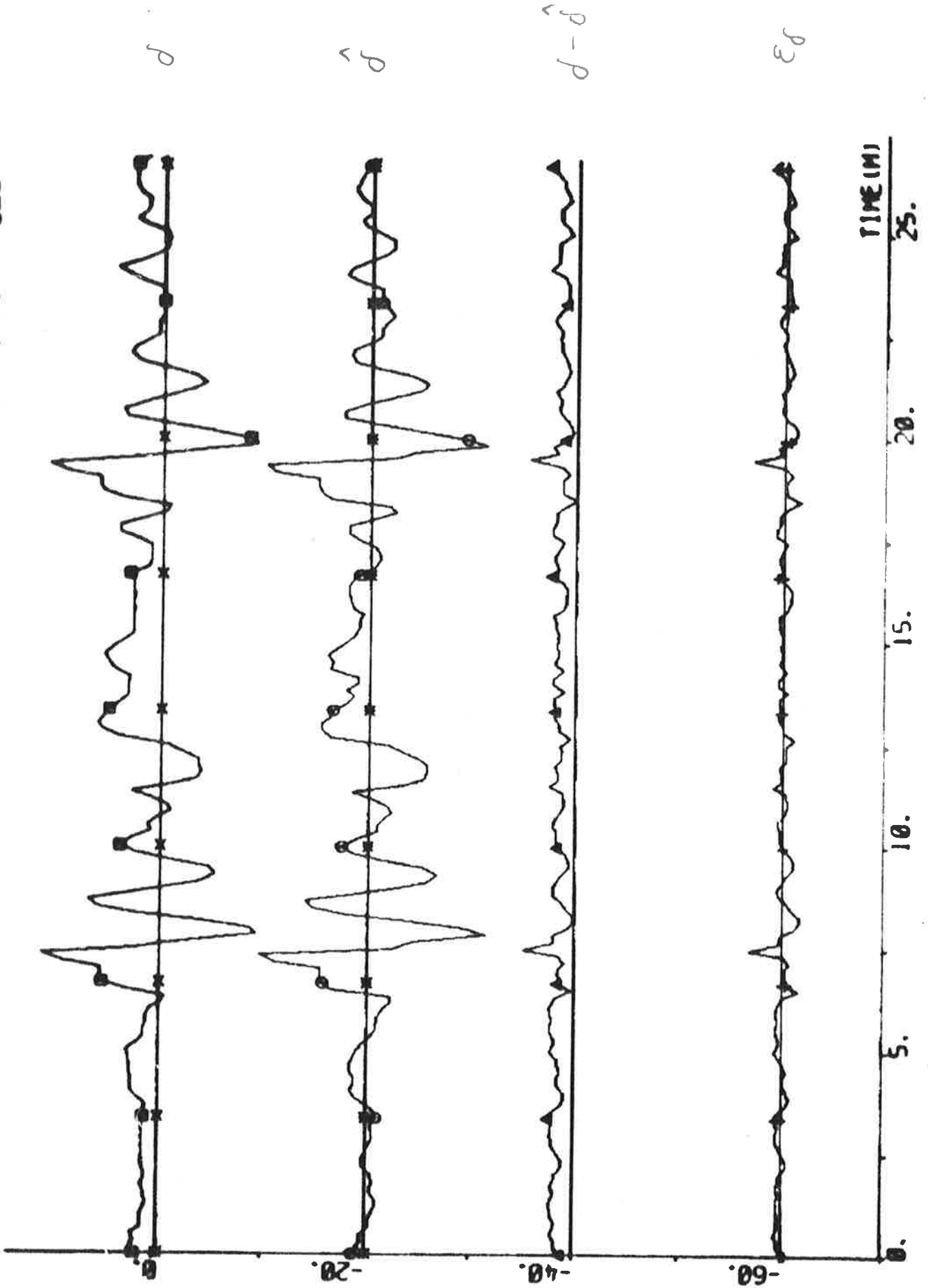
PLOT CSP1(1)-CSP1(6) CSP1(7) ERR3 EPS3 00 002 004 008 -0.7 0. - 000000



PLOT CSP1(1)-CSP1(0) CSP1(9) ERR4 EPS4 00 01 02 03 -3.4 0.0 - DEG

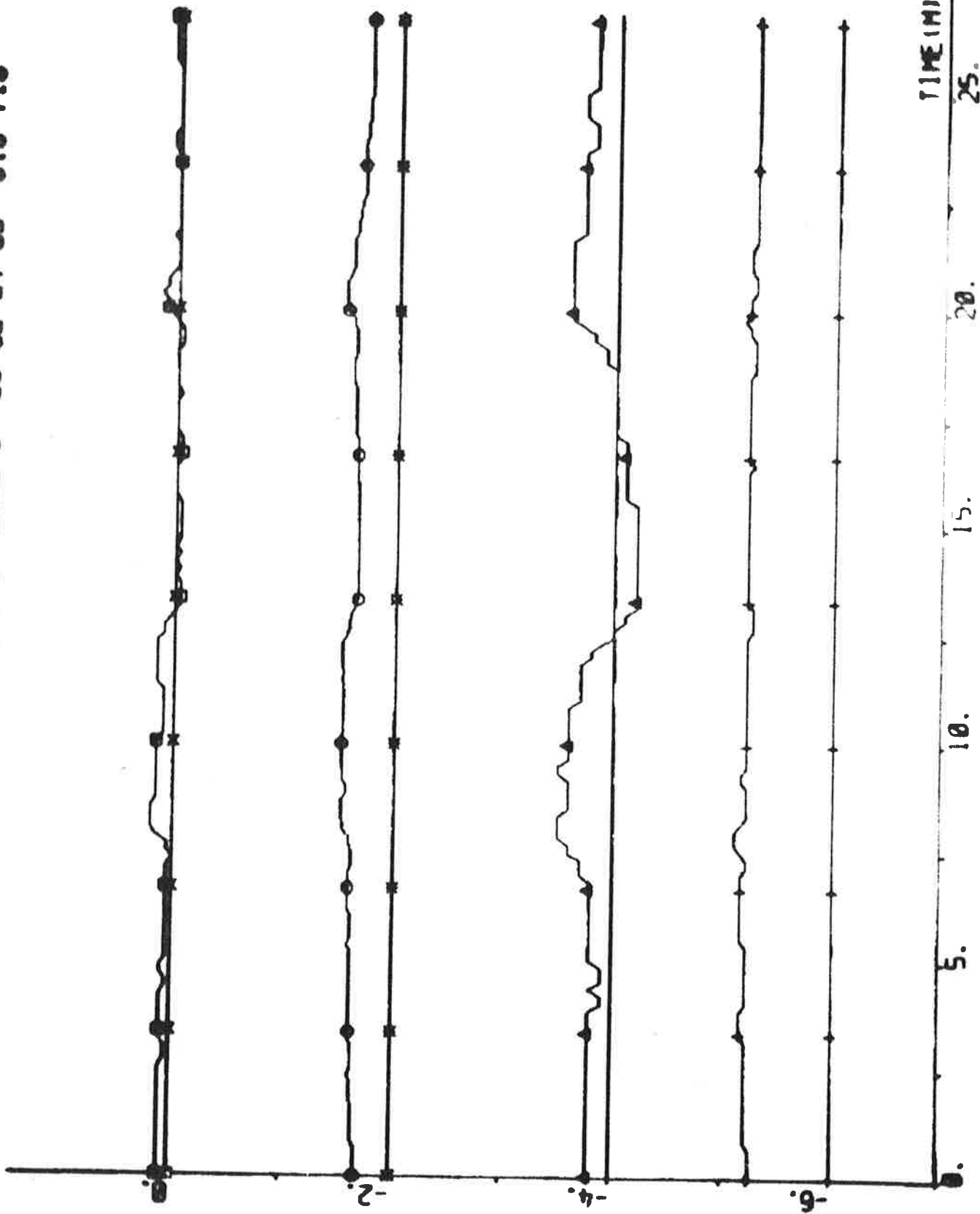


PLOT CSP1(1)-CSP1(2) CSP1(3) ERR1 EPS1 00 020 040 060 .05 15 . 000





PLOT CSP1(1)-CSP2(3) CSP2(4) CSP2(5) CSP2(6) 00 02 04 06 -8.5 1.5



$0.5 \times d_0^1$  deg

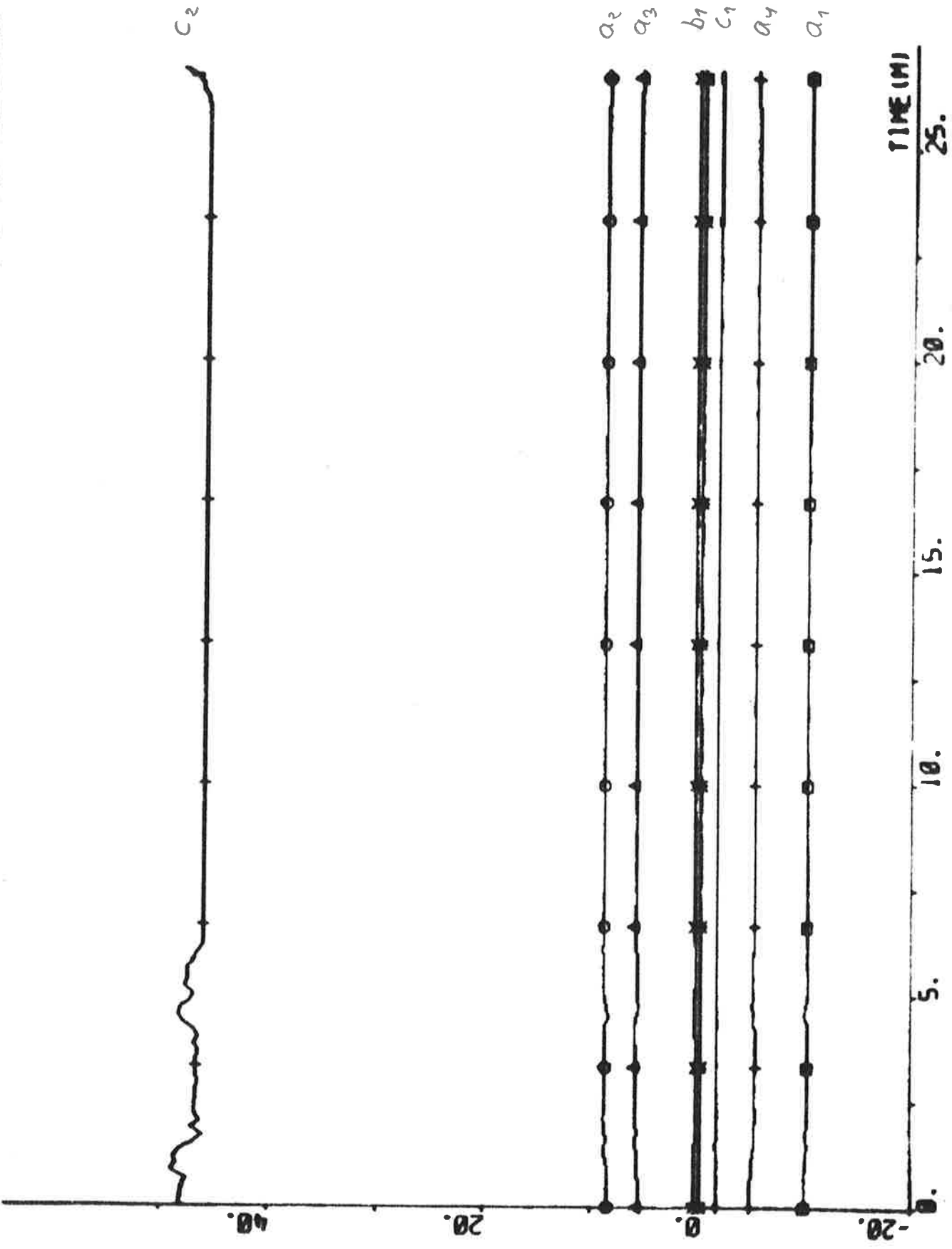
$2 \times d_0^1$  knots

$100 \times d_0^1$  deg/s

$0.5 \times d_0^1$  deg

TIME (H)  
25.

PLOT CSP1(1)-CSP2(7) CSP2(8) CSP2(9) CSP2(10) CSP2(11) CSP2(12) CSP2(13)



## EXPERIMENT C6

Date	1976-04-27	Forward draught	10.9 m
Time	14.51	Aft draught	12.9 m
Duration	72 min	Wind direction	S (1; see App. A)
Position	S 10°34' W 03°09'	Wind velocity	12 m/s (strong breeze)
$\psi_{ref}$	144, (145), 146, 142, 144, 145, 143, 144 deg	Wave height	-
$r_{ref}$	0.1 deg/s		

Self-tuning regulator and yaw regulator using estimates from the Kalman filter

NC1 = 1	NC2 = 1	k = 7	q = 0
$T_s = 10$ s	$V_0 = 6$ m/s	IVVC = 1	

Final values:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ b_1 \\ b_2 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} -11.31 \\ 11.45 \\ 4.34 \\ -4.95 \\ 0.37 \\ 0.08 \\ -0.96 \\ 61.38 \end{bmatrix} \quad P = \begin{bmatrix} 9.13 & & & & & & & & & \\ & -19.56 & 63.44 & & & & & & & \\ & 12.93 & -67.50 & 95.10 & & & & & & \\ & -2.45 & 23.89 & -41.28 & 20.14 & & & & & \\ & 0.33 & -1.71 & 2.28 & -0.91 & 0.07 & & & & \\ & -0.11 & 0.49 & -0.78 & 0.40 & -0.02 & 0.02 & & & \\ & -1.62 & 2.83 & -0.97 & -0.24 & -0.03 & 0.00 & 1.16 & & \\ & 12.86 & -22.71 & -4.52 & 13.10 & 1.31 & -0.45 & 17.41 & 984.28 & \end{bmatrix}$$

$$a_1 + a_2 + a_3 + a_4 = -0.47$$

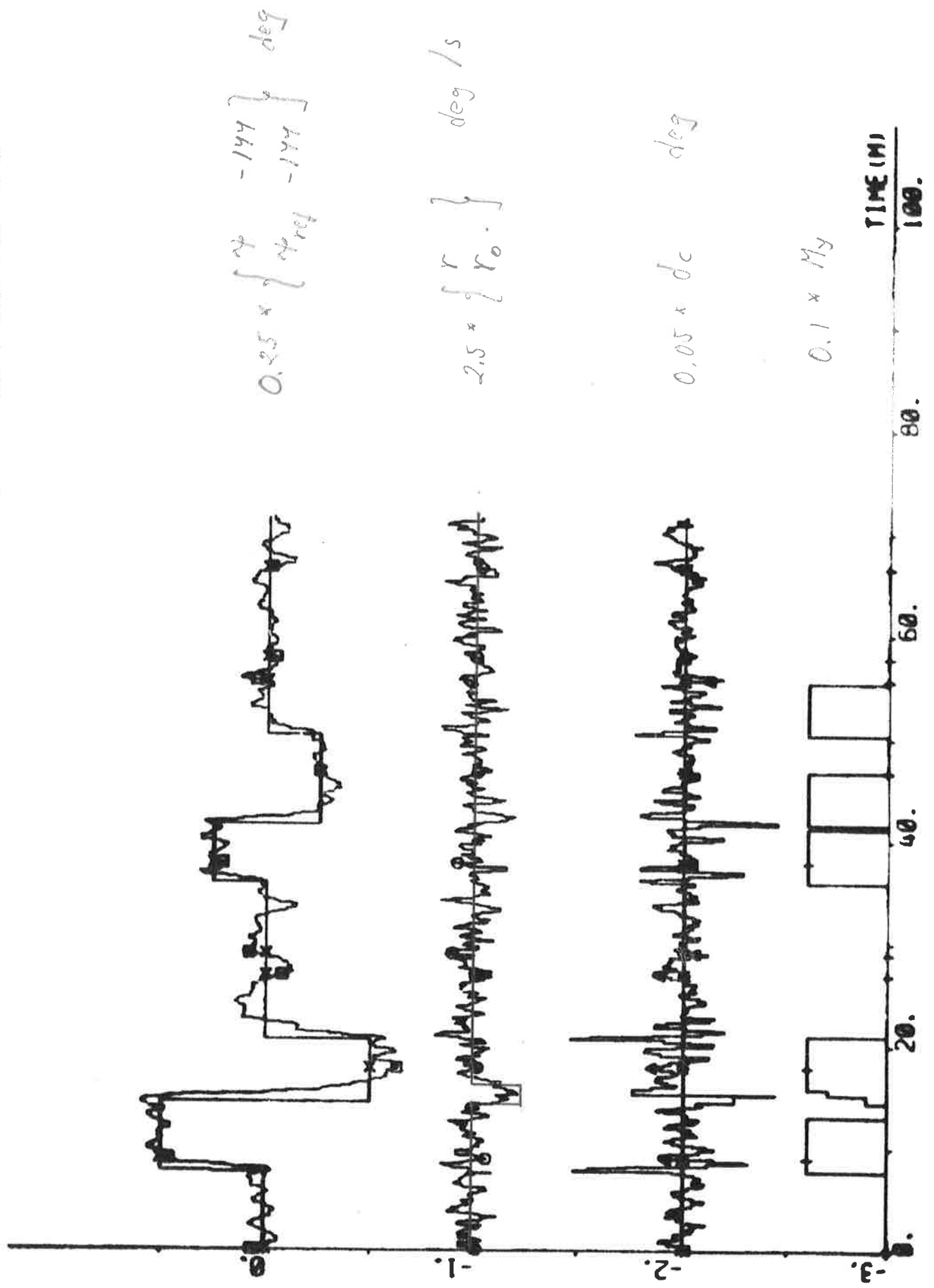
$$\hat{\delta}_0 = -0.1 \text{ deg} \quad \hat{d}_v = 0.19 \text{ knots} \quad \hat{d}_r = 0.001 \text{ deg/s} \quad \hat{d}_\delta = 1.5 \text{ deg}$$

$$\bar{\delta}_c = -0.2 \text{ deg} \quad (\text{Initial value: } 0.2 \text{ deg})$$

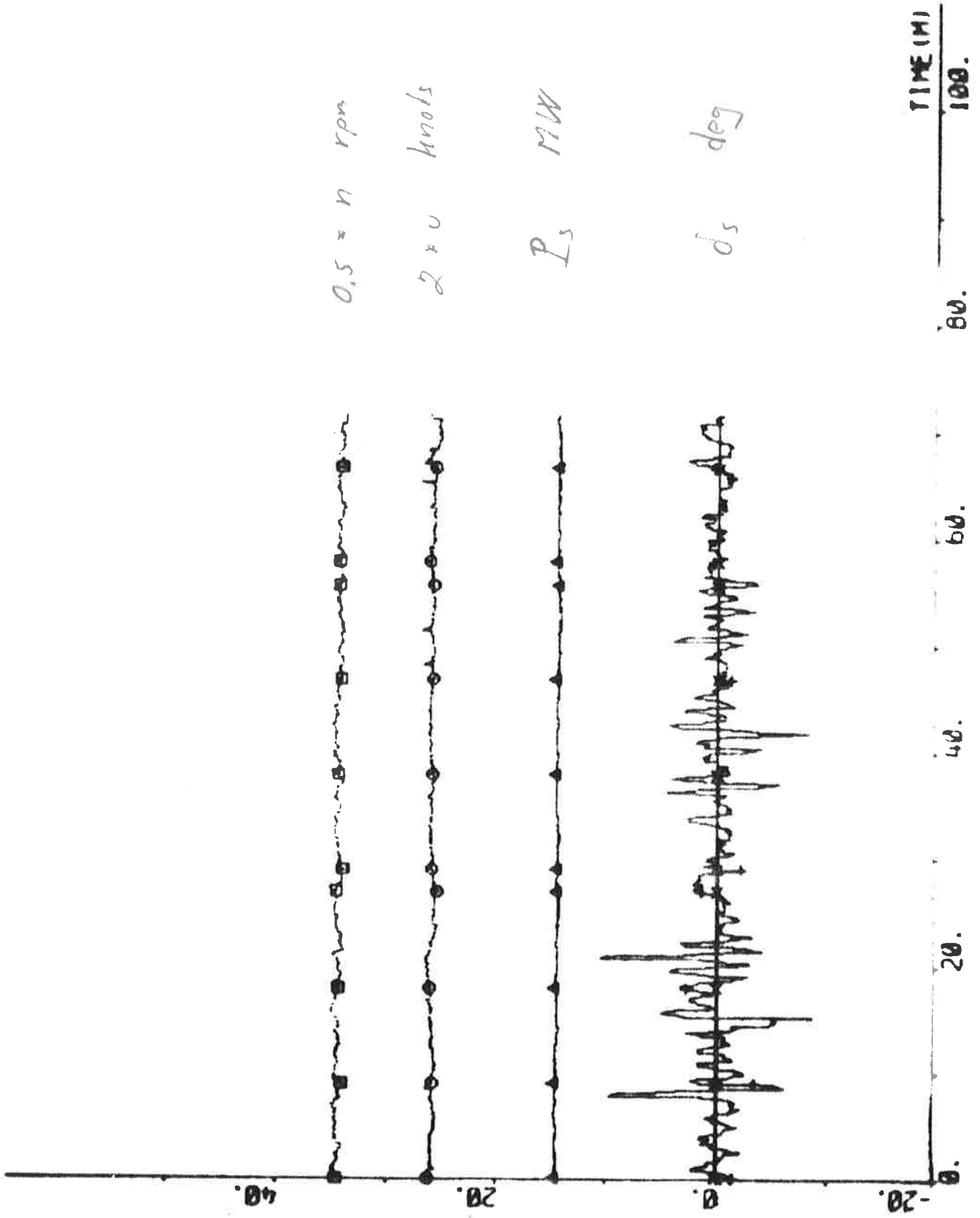
Statistics (mean value and standard deviation)

n	69.0 ± 0.4 rpm	$\epsilon_v$	0.00 ± 0.02 knots
u	13.0 ± 0.1 knots	$\epsilon_r$	0.00 ± 0.02 deg/s
$P_s$	14.4 ± 0.1 MW	$\epsilon_\psi$	0.00 ± 0.03 deg
		$\epsilon_\delta$	0.0 ± 0.6 deg

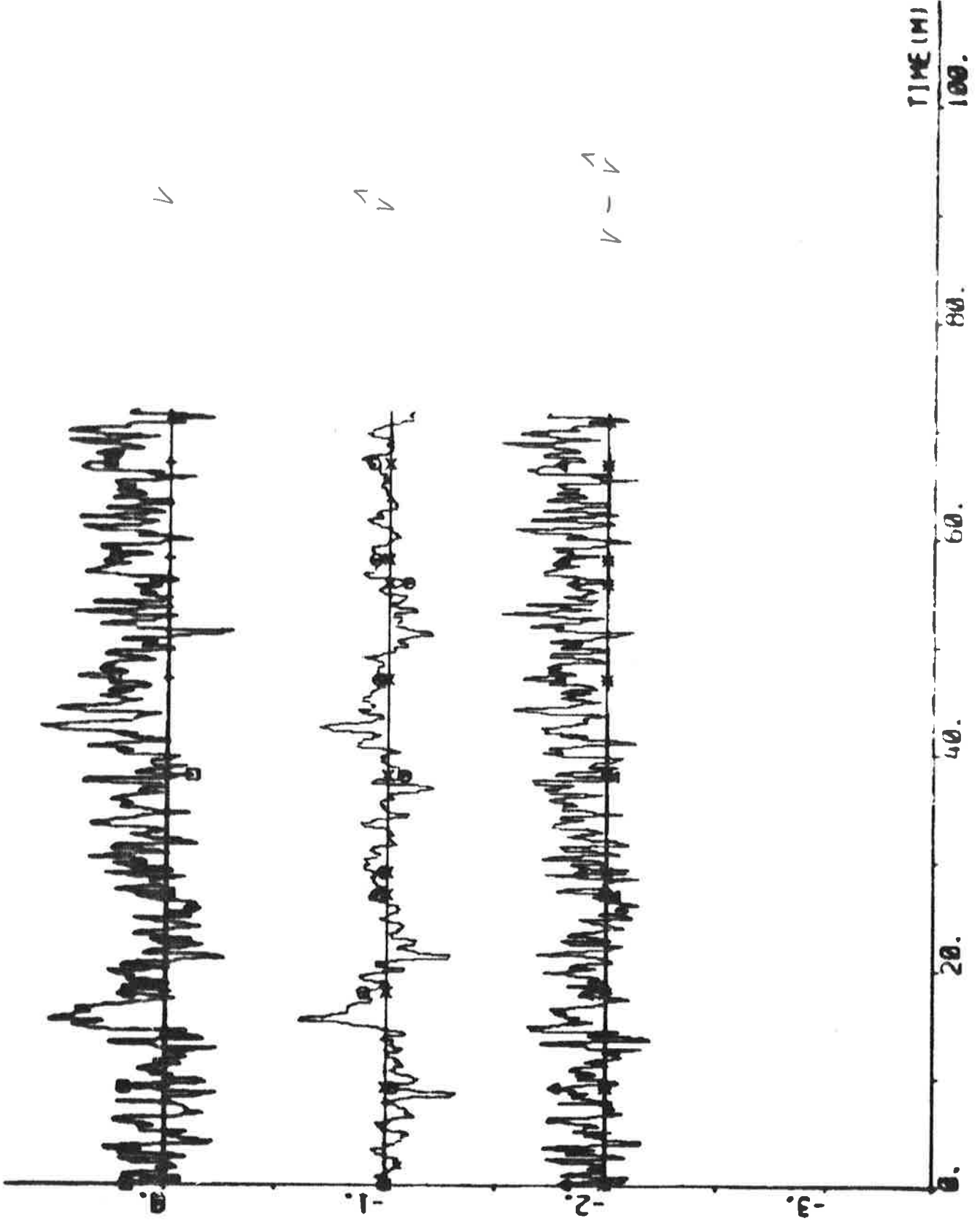
PLOT COP1(1)-COP1(8) COP1(9) COP1(10) MP COP1(15) MP HODY MP COP1(15) 02 -3 1



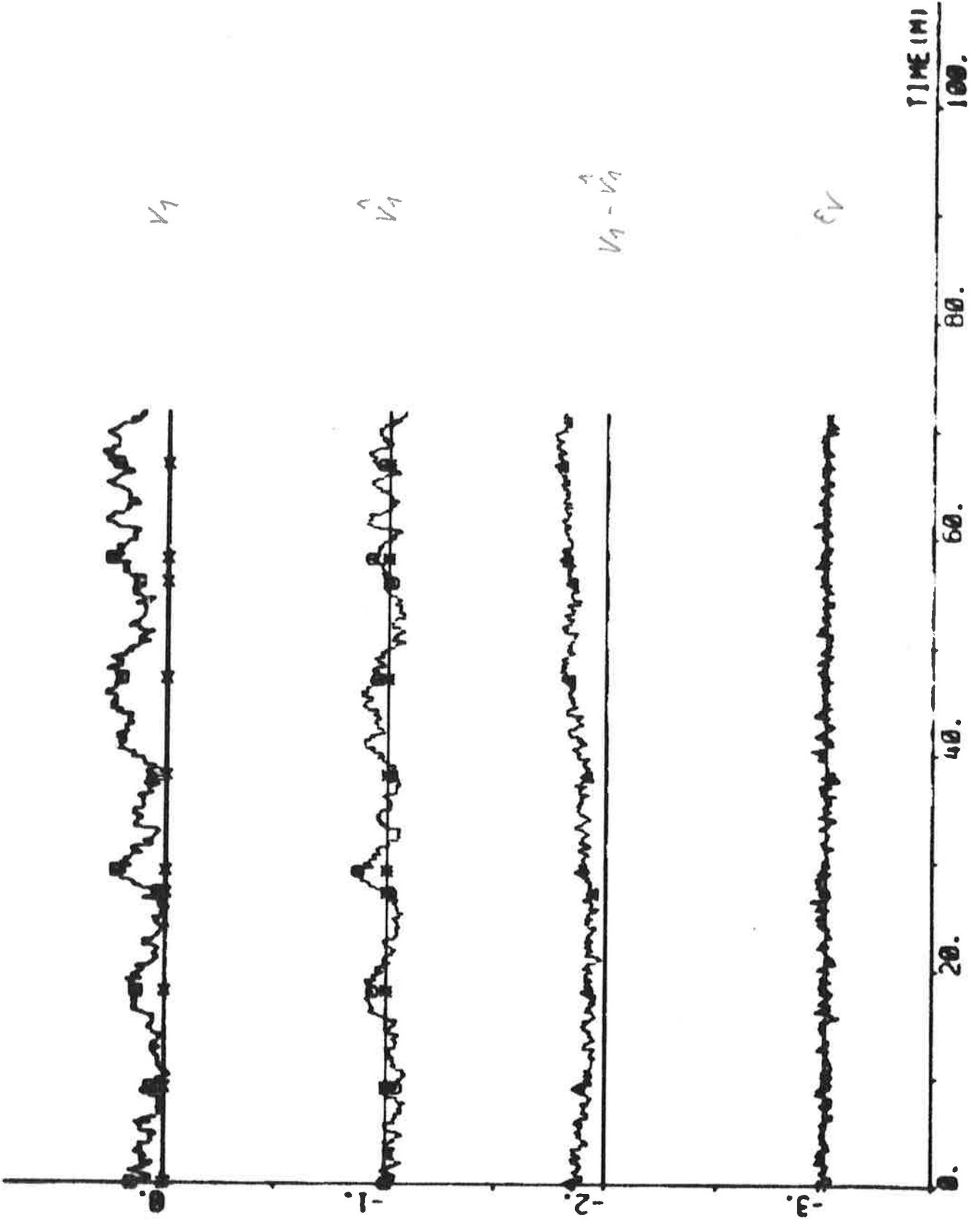
PLOT C6P1(1)-C6P1(13) C6P1(12) C6P1(14) C6P1(11) 00 -20 50



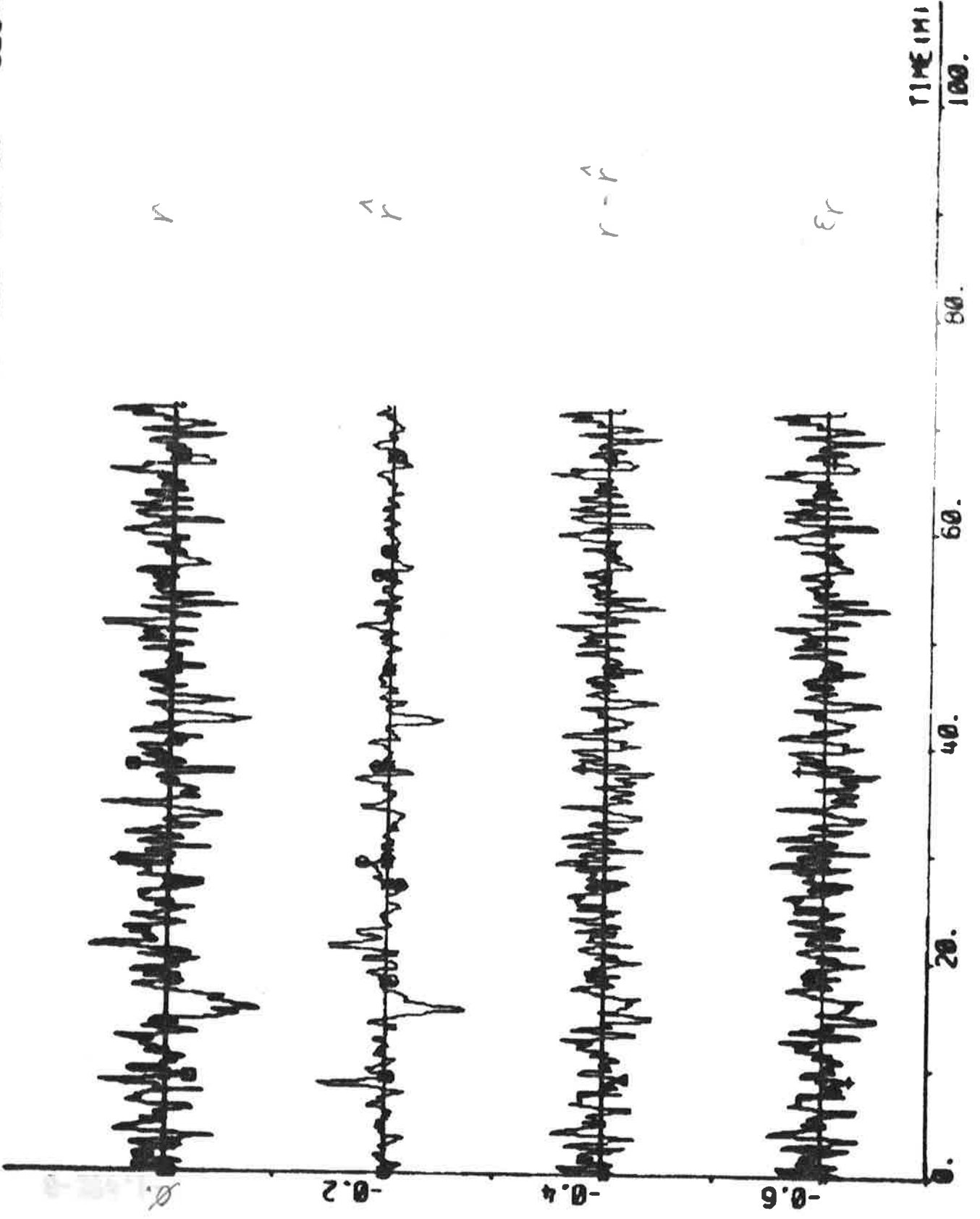
PLOT CSP1(1)-CSP2(1) CSP2(2) ERRS 00 01 02 -3.4 0.0 - KNOTS



PL0T C0P1(1)-C0P1(4) C0P1(5) ER02 EPS2 00 01 02 03 -3.4 0.0 - 1000TS

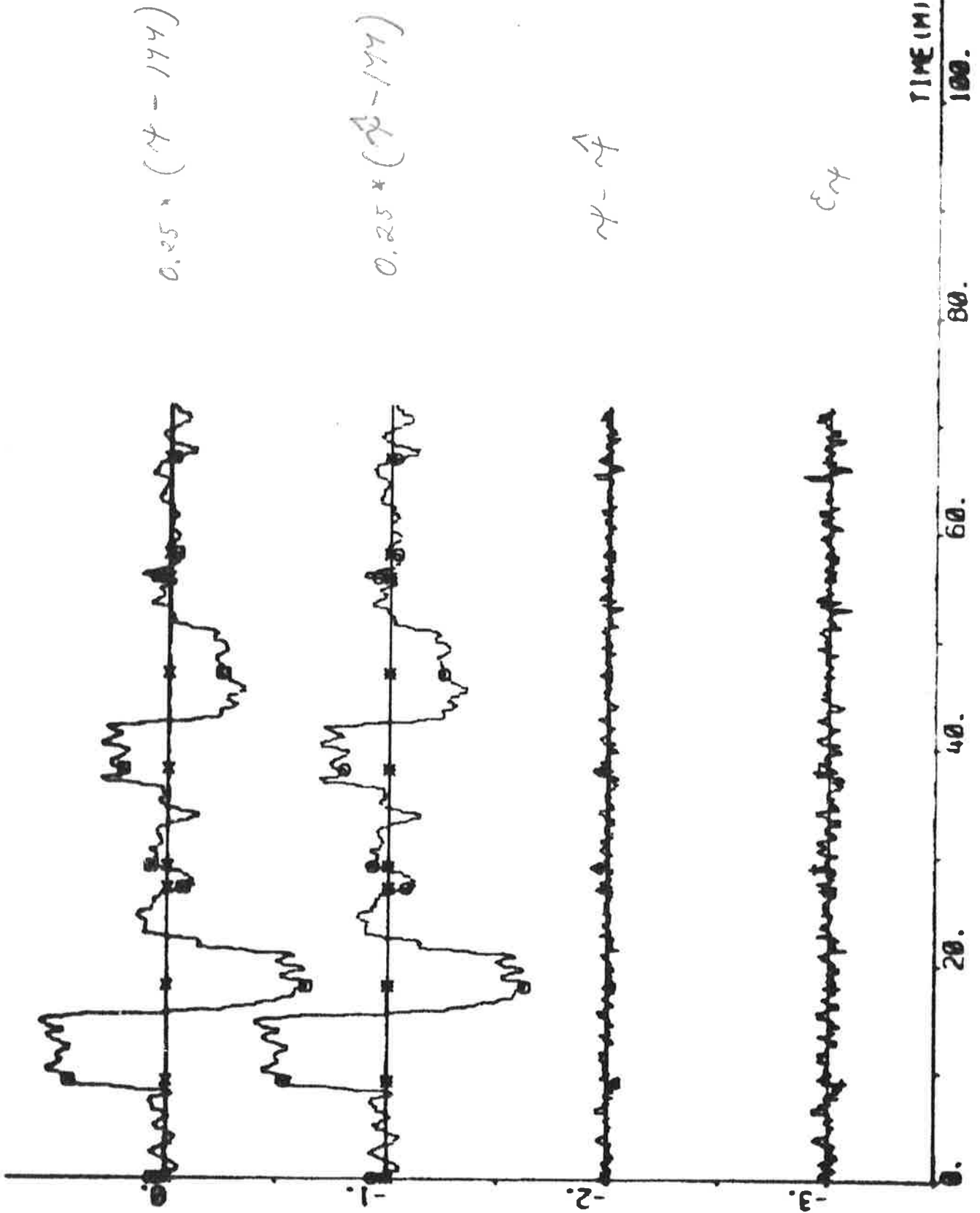


PLOT COP1(1)-COP1(6) COP1(7) ERR3 EPS3 00 002 004 006 -0.7 0. - 000-3

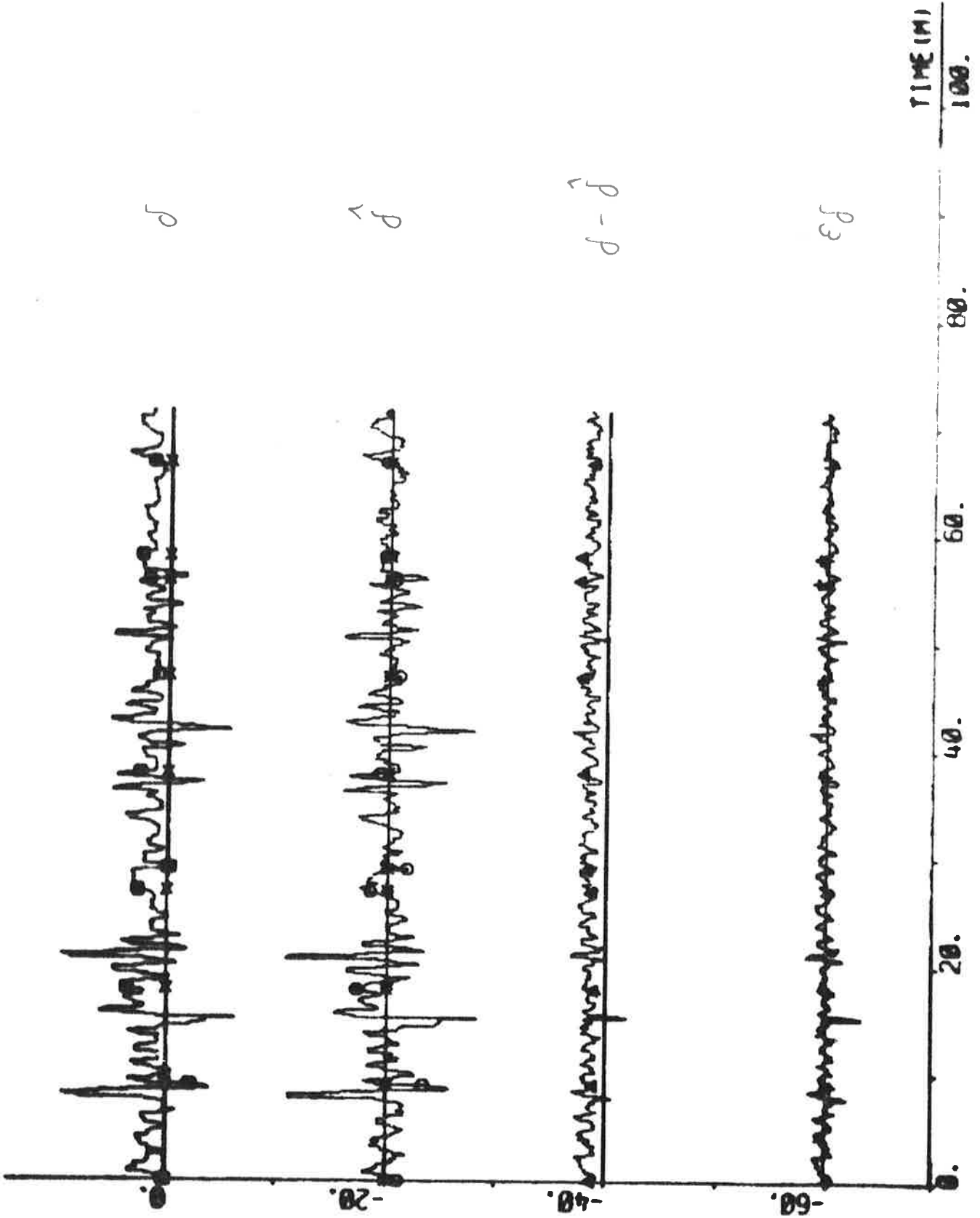




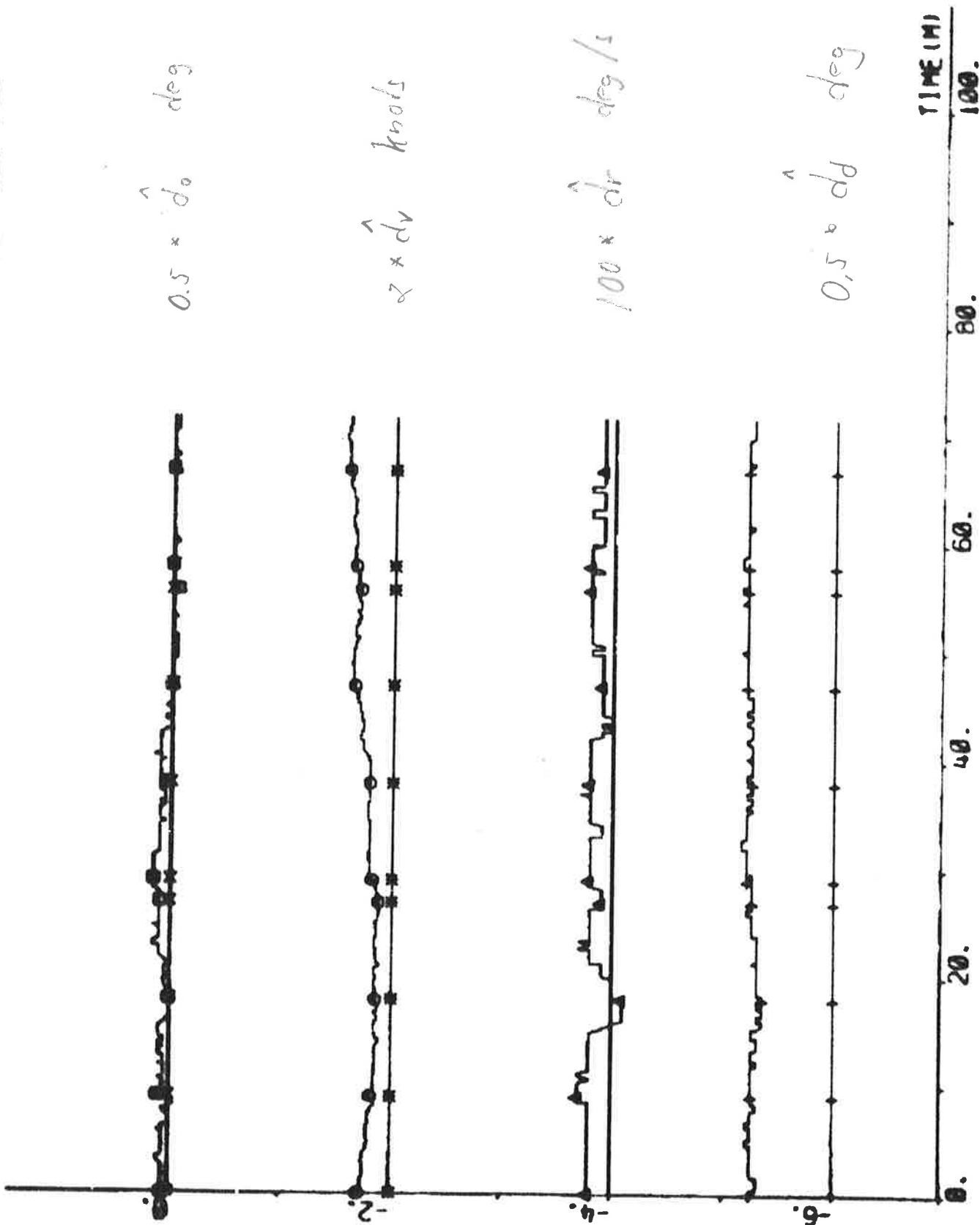
PLOT CDP1(1)-CDP1(8) CDP1(9) ERR4 EPS4 00 01 02 03 -3.4 0.0 - DEC



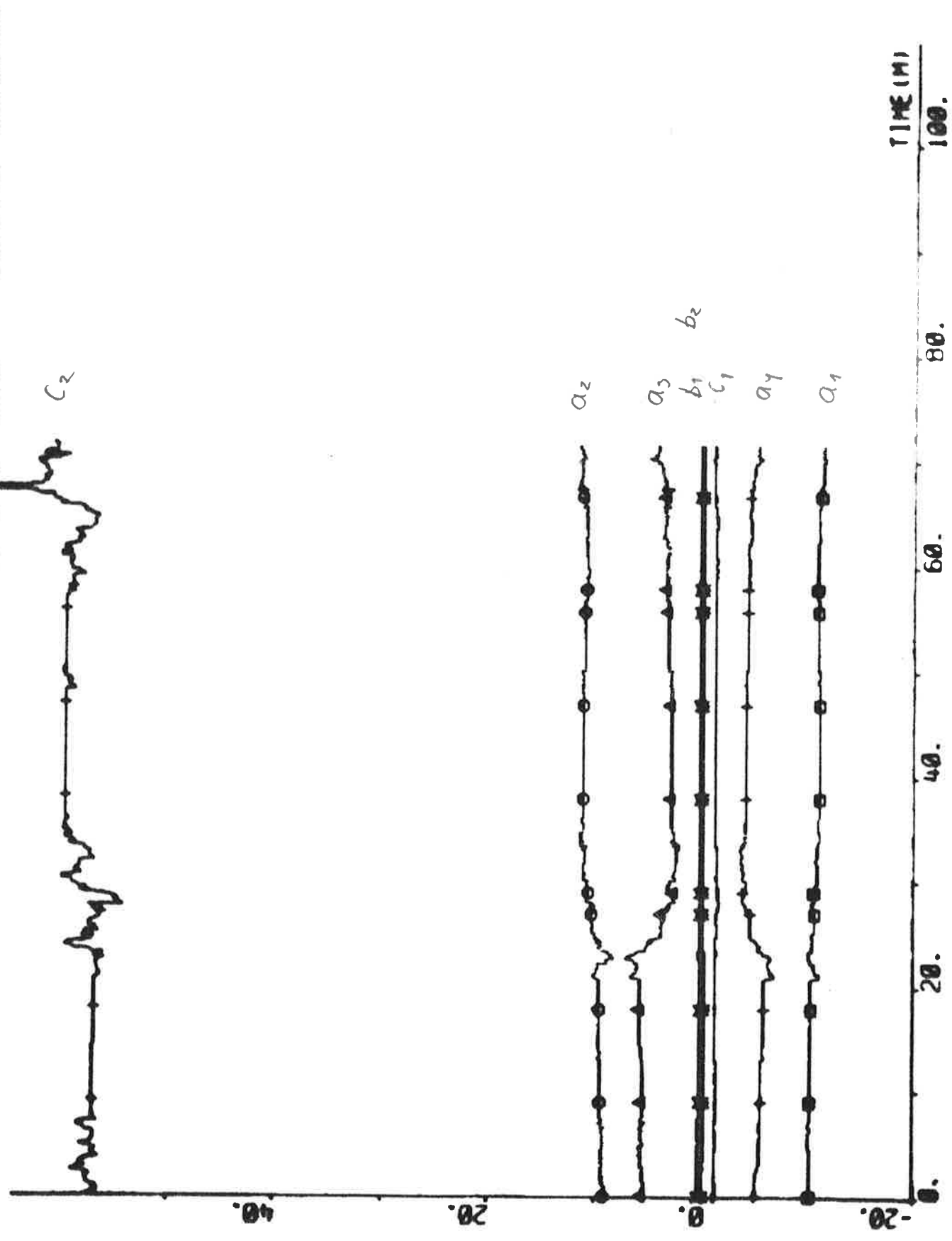
PLOT COP1(1)-COP1(2) COP1(3) ERR1 EPS1 00 020 040 060 -05 15 - DEC



PLOT COP1(1)-COP2(3) COP2(4) COP2(5) COP2(6) 00 02 04 06 -0.6 1.6



PLOT COP1(1)-COP2(7) COP2(8) COP2(9) COP2(10) COP2(11) COP2(12) COP2(13)



TIME (M)  
100.

## EXPERIMENT D1

Date	1976-04-19	Forward draught	8.5 m
Time	13.38	Aft draught	12.5 m
Duration	83 min	Wind direction	E (7; see App. A)
Position	N 28°09' W 15°17'	Wind velocity	5 m/s (gentle breeze)
$\psi_{\text{ref}}$	205, (201), 200, (183), 180, (184), 204, (202), 200 deg	Wave height	Low swell from ESE
$r_{\text{ref}}$	0.1 deg/s		

The forward speed  $u$  was measured by Sperry's log. The sway velocity  $v_1$  was not measured during the experiment. The water depth was about 100 m during the last 15 min of the experiment. The rudder limit was 15 deg during the experiment.

Self-tuning regulator and yaw regulator using estimates from the Kalman filter.

The sway velocity  $v_1$  was not used by the Kalman filter.

Tuning time before the experiment started: > 120 min.

$NC1 = 1$	$NC2 = 1$	$k = 7$	$q = 0$
$T_s = 10 \text{ s}$	$V_0 = 8 \text{ m/s}$	$IVVC = 3$	$V_c = 7 \text{ m/s}$

Final values:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ b_1 \\ b_2 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} -7.32 \\ 6.37 \\ 3.37 \\ -2.70 \\ 0.38 \\ 0.02 \\ 1.41 \\ 112.69 \end{bmatrix} \quad P = \begin{bmatrix} 105.55 & & & & & & & \\ -109.01 & 130.35 & & & & & & \\ -21.87 & -8.40 & 68.77 & & & & & \\ 23.83 & -10.89 & -38.41 & 25.71 & & & & \\ 0.50 & -1.62 & 2.56 & -1.48 & 0.26 & & & \\ 1.22 & -1.26 & -0.17 & 0.16 & 0.12 & 0.15 & & \\ -112.05 & 111.66 & 22.33 & -19.14 & -2.19 & -2.54 & 135.86 & \\ 1859.95 & 1907.60 & 198.28 & -179.47 & -55.06 & -51.19 & 2527.61 & 48427.16 \end{bmatrix}$$

$$\begin{aligned} a_1 + a_2 + a_3 + a_4 &= -0.28 \\ \hat{\delta}_0 &= 1.3 \text{ deg} \quad \hat{d}_v = - \quad \hat{d}_r = -0.002 \text{ deg/s} \quad \hat{d}_\delta = 0.9 \text{ deg} \\ \bar{\delta}_c &= 1.5 \text{ deg} \quad (\text{Initial value: } 0.6 \text{ deg}) \end{aligned}$$

Statistics (mean value and standard deviation)

$n$	$69.6 \pm 0.6 \text{ rpm}$	$\epsilon_v$	-
$u$	$14.2 \pm 0.2 \text{ knots}$	$\epsilon_r$	$0.00 \pm 0.01 \text{ deg/s}$
$P_s$	$14.7 \pm 0.1 \text{ MW}$	$\epsilon_\psi$	$0.00 \pm 0.02 \text{ deg}$
		$\epsilon_\delta$	$0.0 \pm 0.9 \text{ deg}$

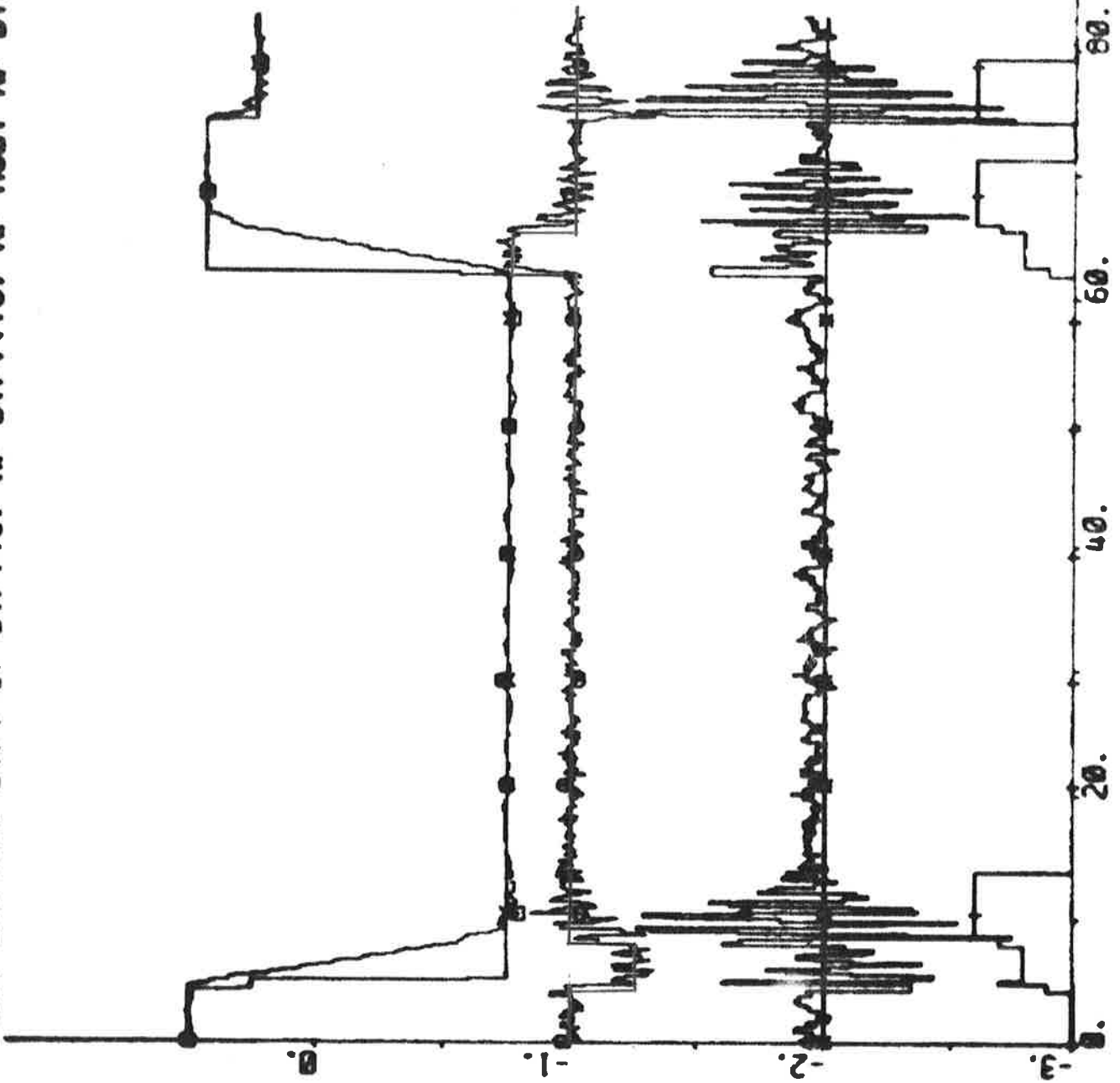
PLOT DIP1(1)-DIP1(8) DIP1(6) MP DIP1(10) MP HOBY MP DIP1(15) 02 -3 1

$$0.05 * \left\{ \begin{array}{l} \gamma - 195 \\ \gamma_{ref} - 195 \end{array} \right\} \text{ deg}$$

$$2.5 * \left\{ \begin{array}{l} r \\ r_0 \end{array} \right\} \text{ deg/s}$$

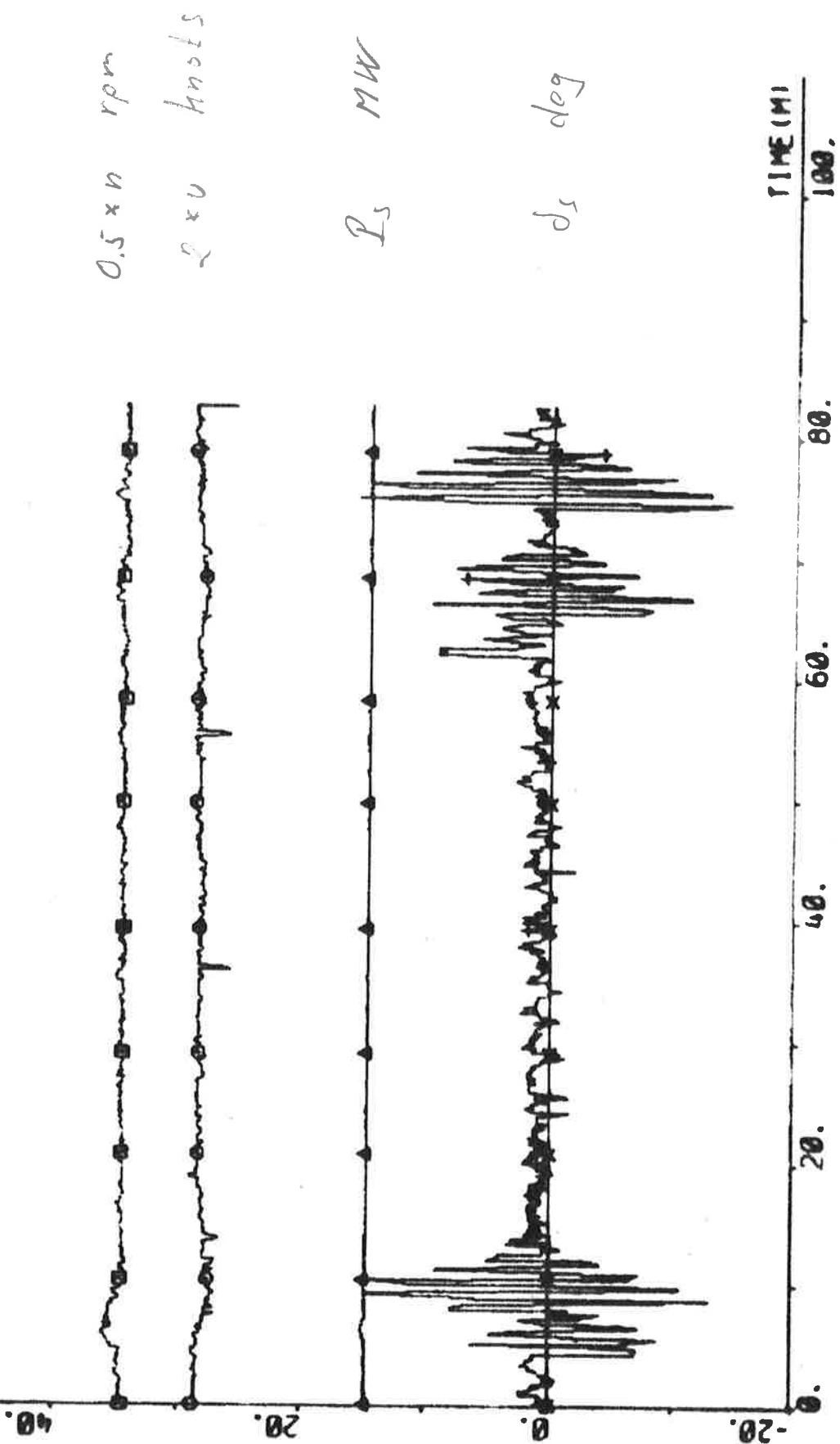
$$0.05 * d_c \text{ deg}$$

$$0.1 * M_y$$

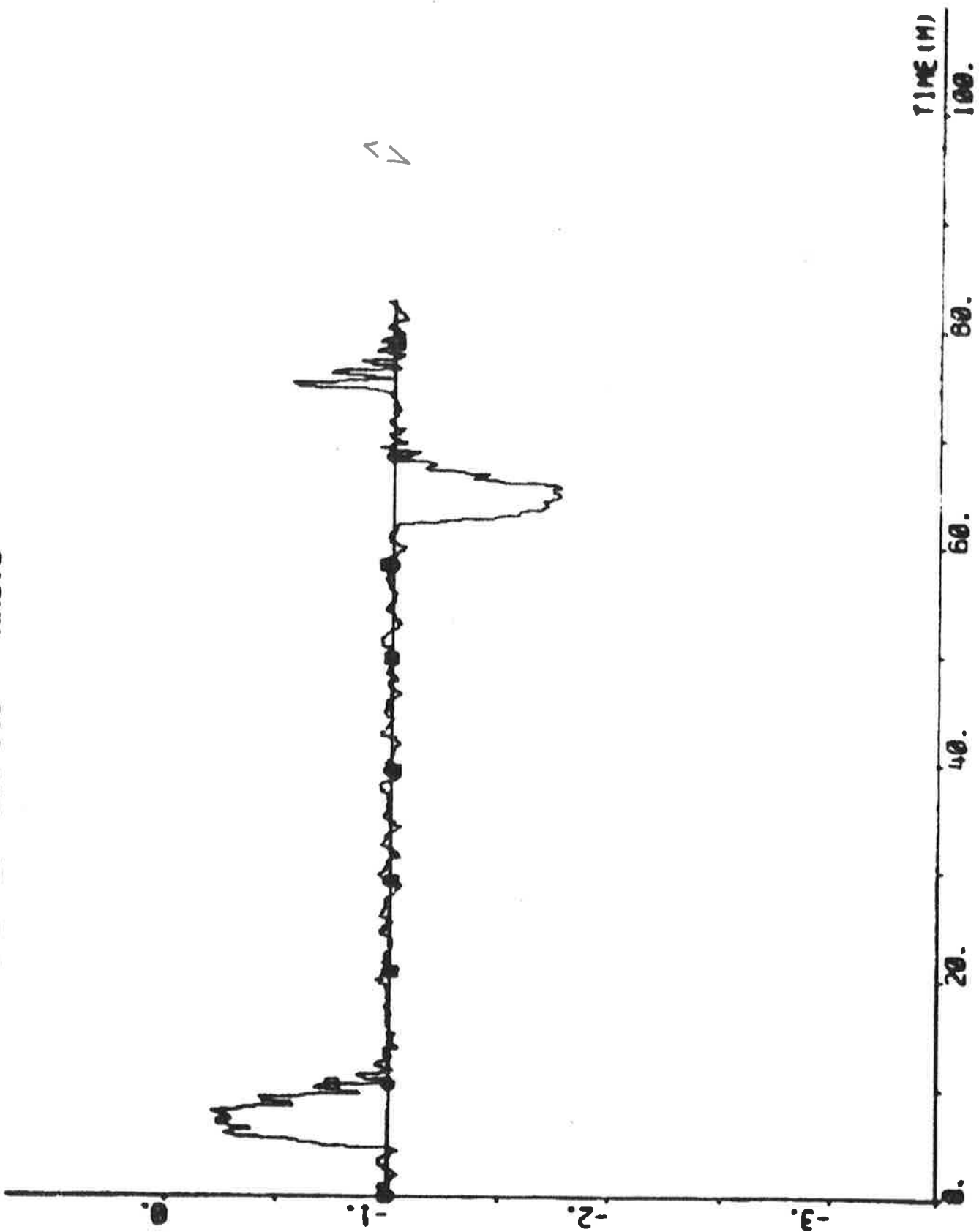


TIME (MI)  
100.

PLOT DIP1(1)-DIP1(13) DIP1(12) DIP1(14) DIP1(11) 00 -20 50

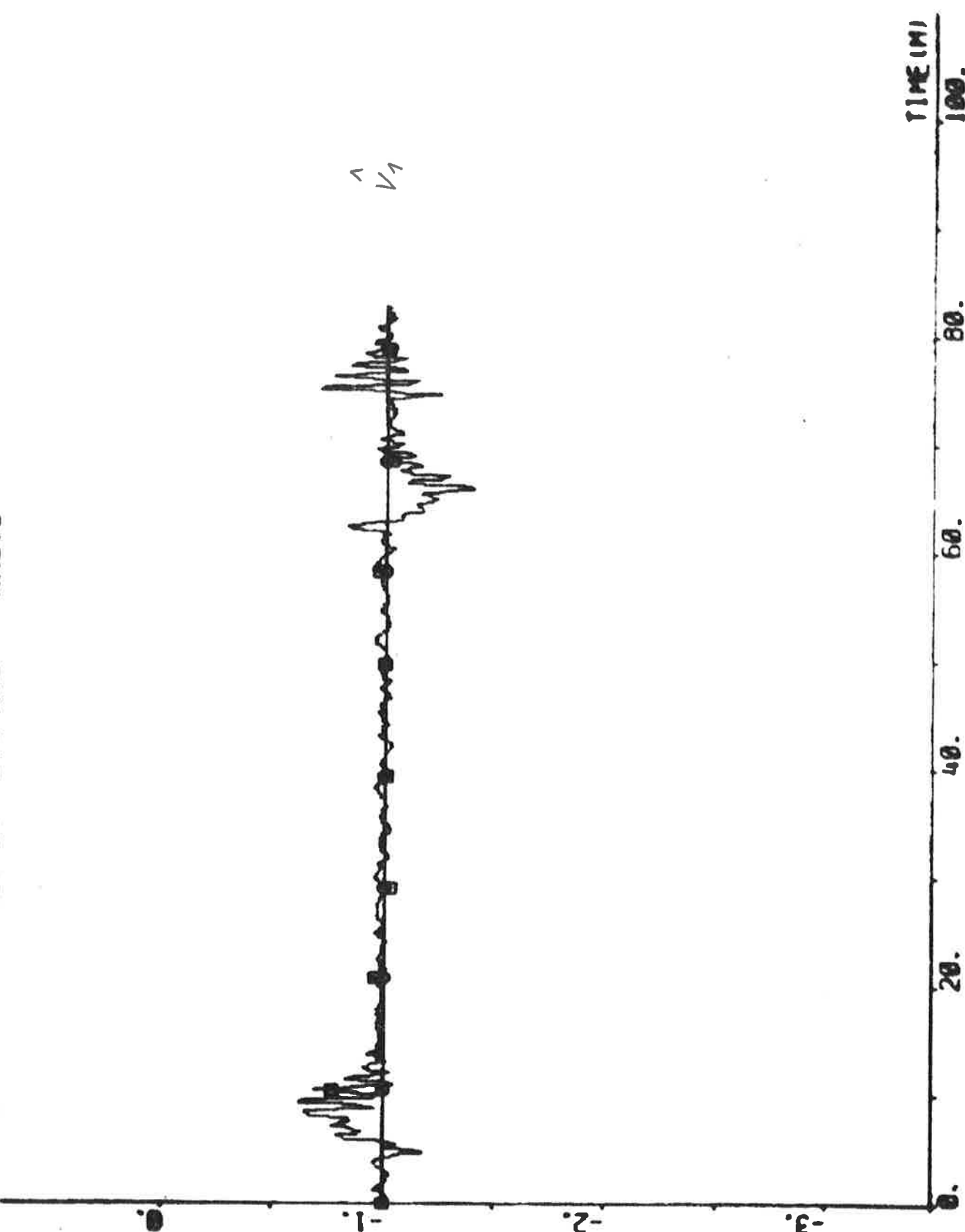


PLOT 01P1-01P2(2) 01 -3.4 0.0 - KNOTS

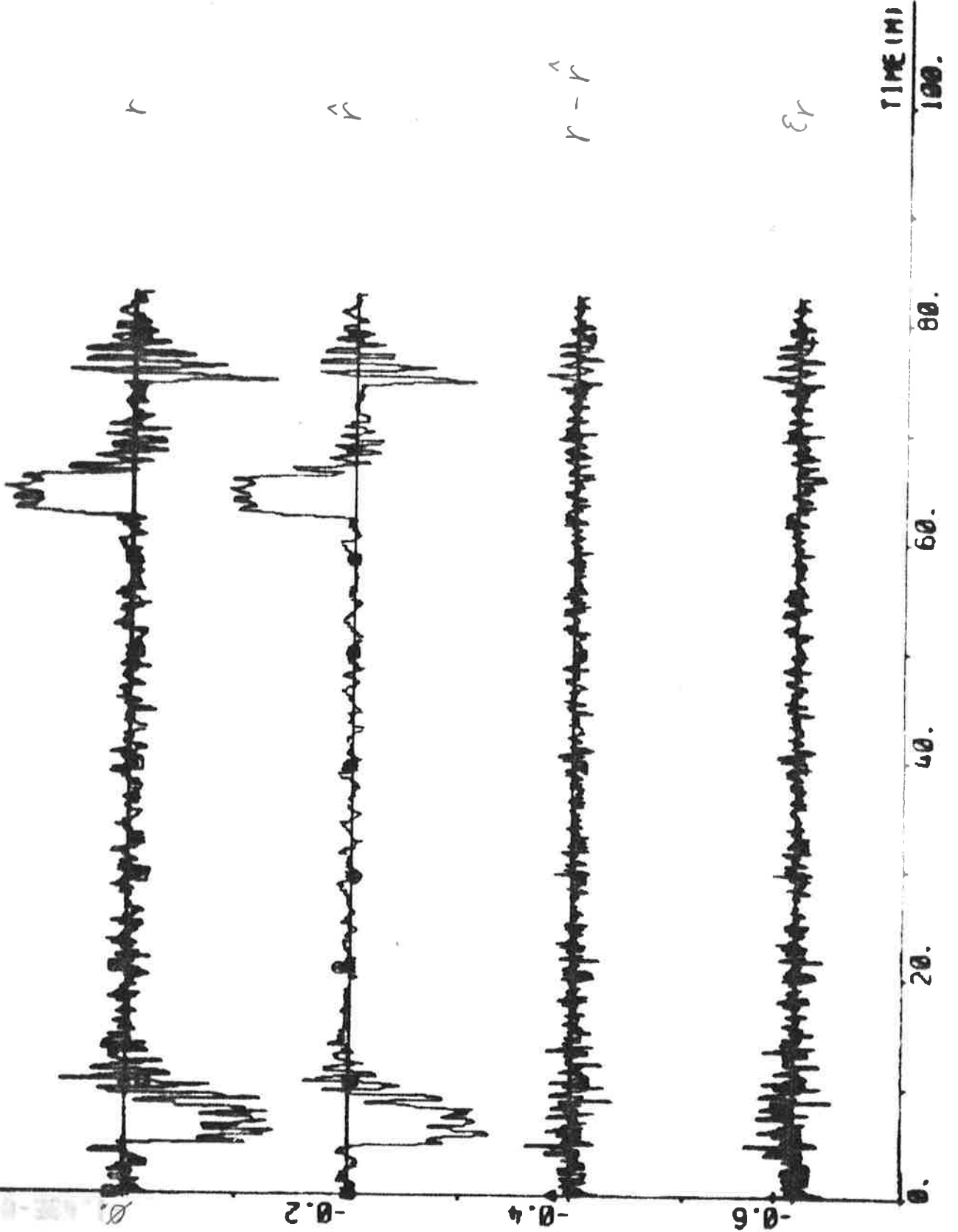




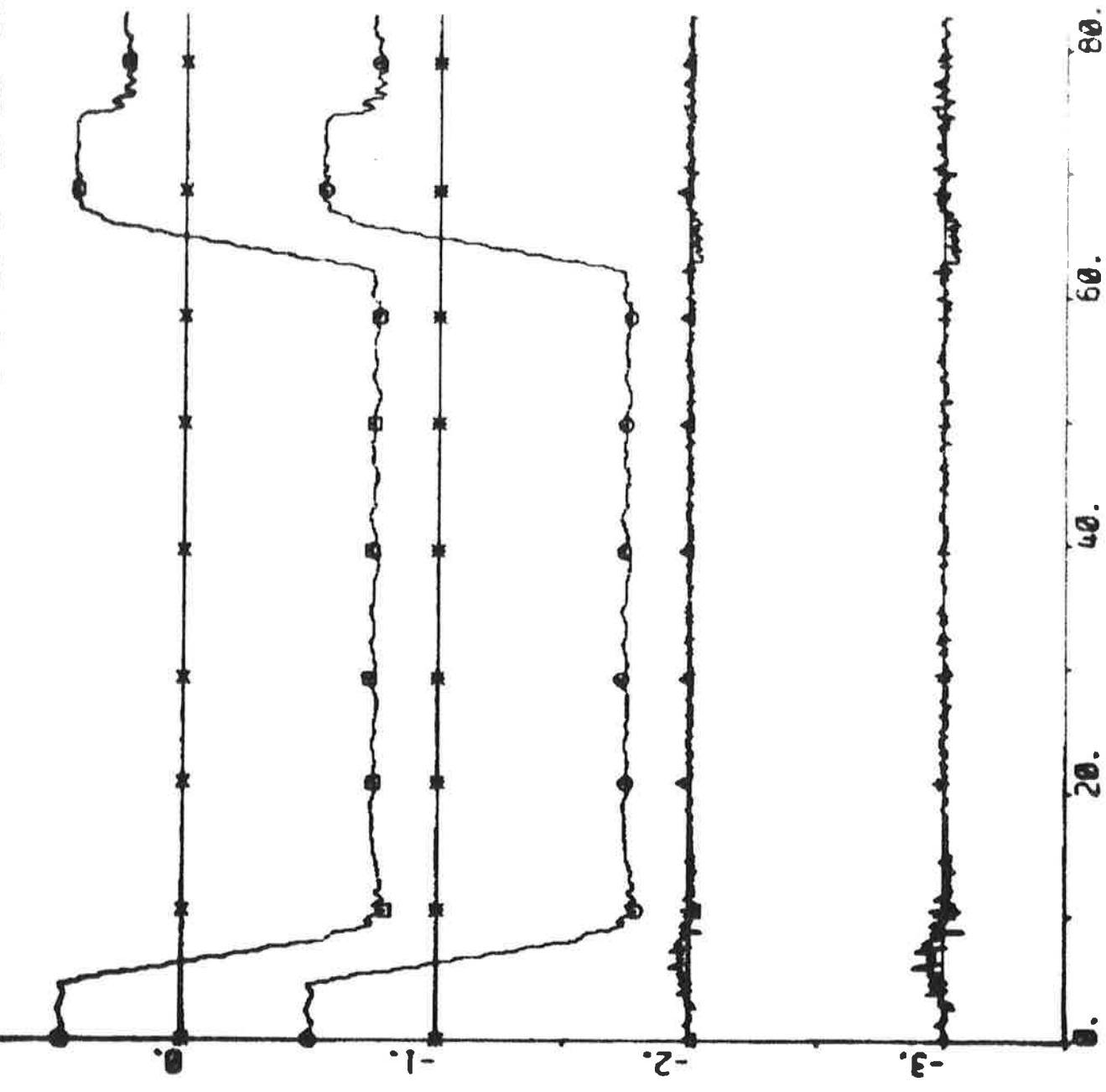
PL0T 01P1(1)-01P1(5) 01 -3.4 0.0 - KNOTS



PLOT DIP1(1)-DIP1(8) DIP1(7) ERR3 EPS3 00 002 004 006 -0.7 0. - DECS



PLOT 01P1(1)-01P1(0) 01P1(9) ERR4 EPS4 00 01 02 03 -3.4 0.0 - DEC



$0.05 \times (7-195)$

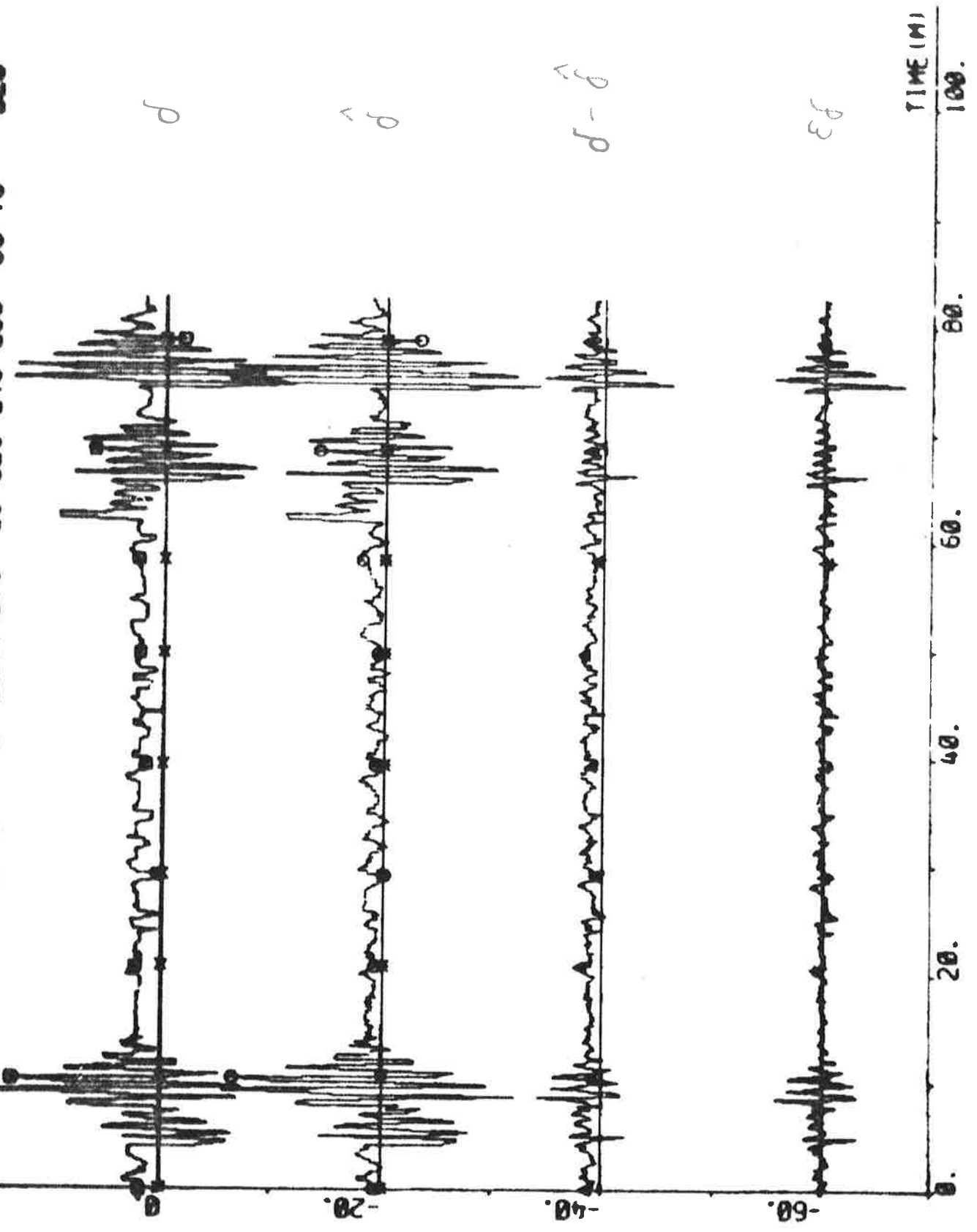
$0.05 \times (7-195)$

7-7

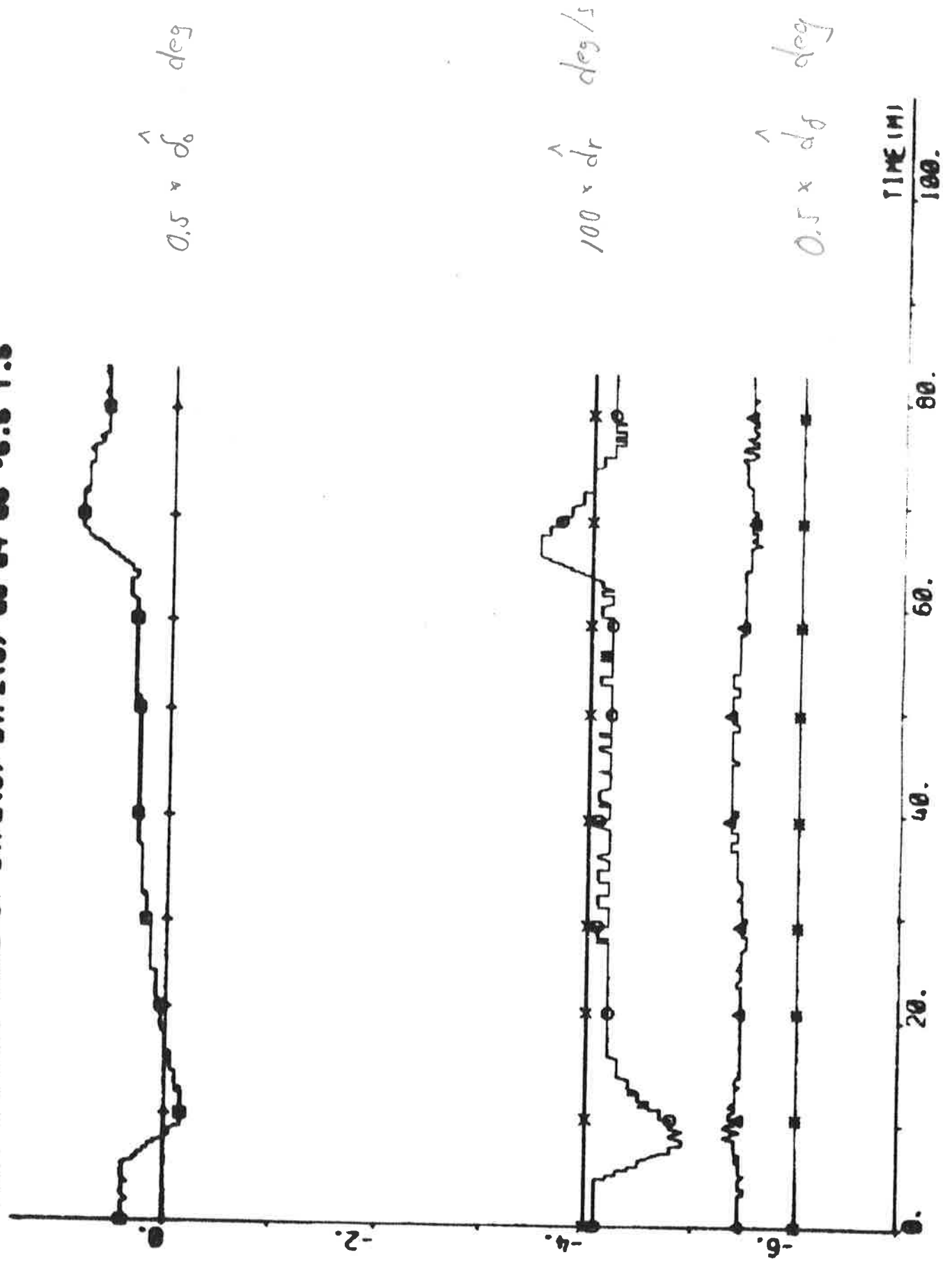
E4

TIME (M)  
100.

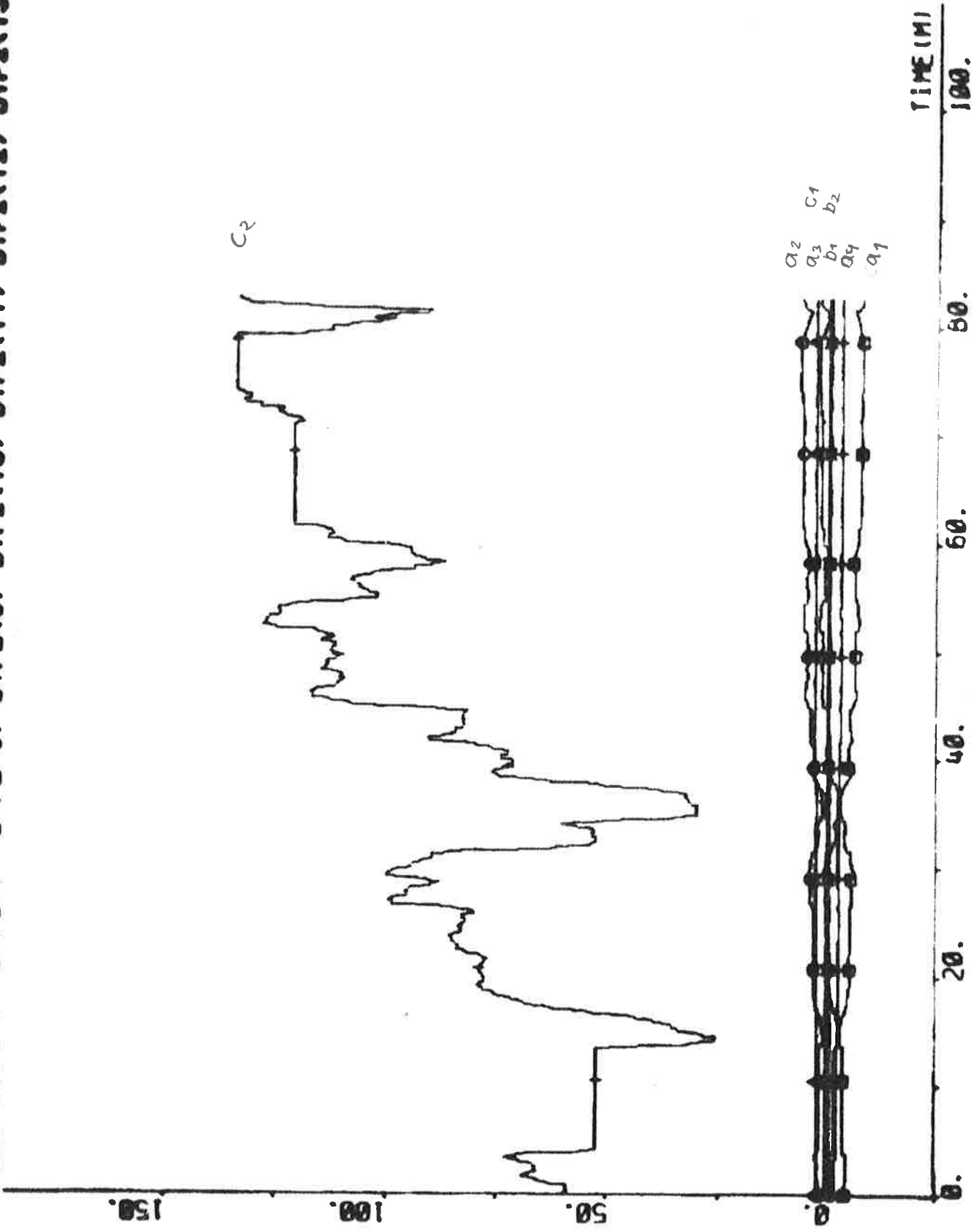
PLOT DIP1(1)-DIP1(2) DIP1(3) ERR1 EPS1 00 020 040 060 -66 15 - DEC



PLOT DIP1(1)-DIP2(3) DIP2(5) DIP2(6) 00 04 08 -6.5 1.5



PLOT DIP1(1)~DIP2(7) DIP2(8) DIP2(9) DIP2(10) DIP2(11) DIP2(12) DIP2(13)





PL0T DZP1(1)-DZP1(8) DZP1(9) DZP1(10) HP DZP1(11) HP DZP1(12) HP DZP1(13) HP DZP1(14) HP DZP1(15) 02 -3 1



$\gamma - \gamma_{ref}$  deg



$2.5 \times \left\{ \frac{r}{r_0} \right\}$  deg/s



$0.05 \times d_c$  deg

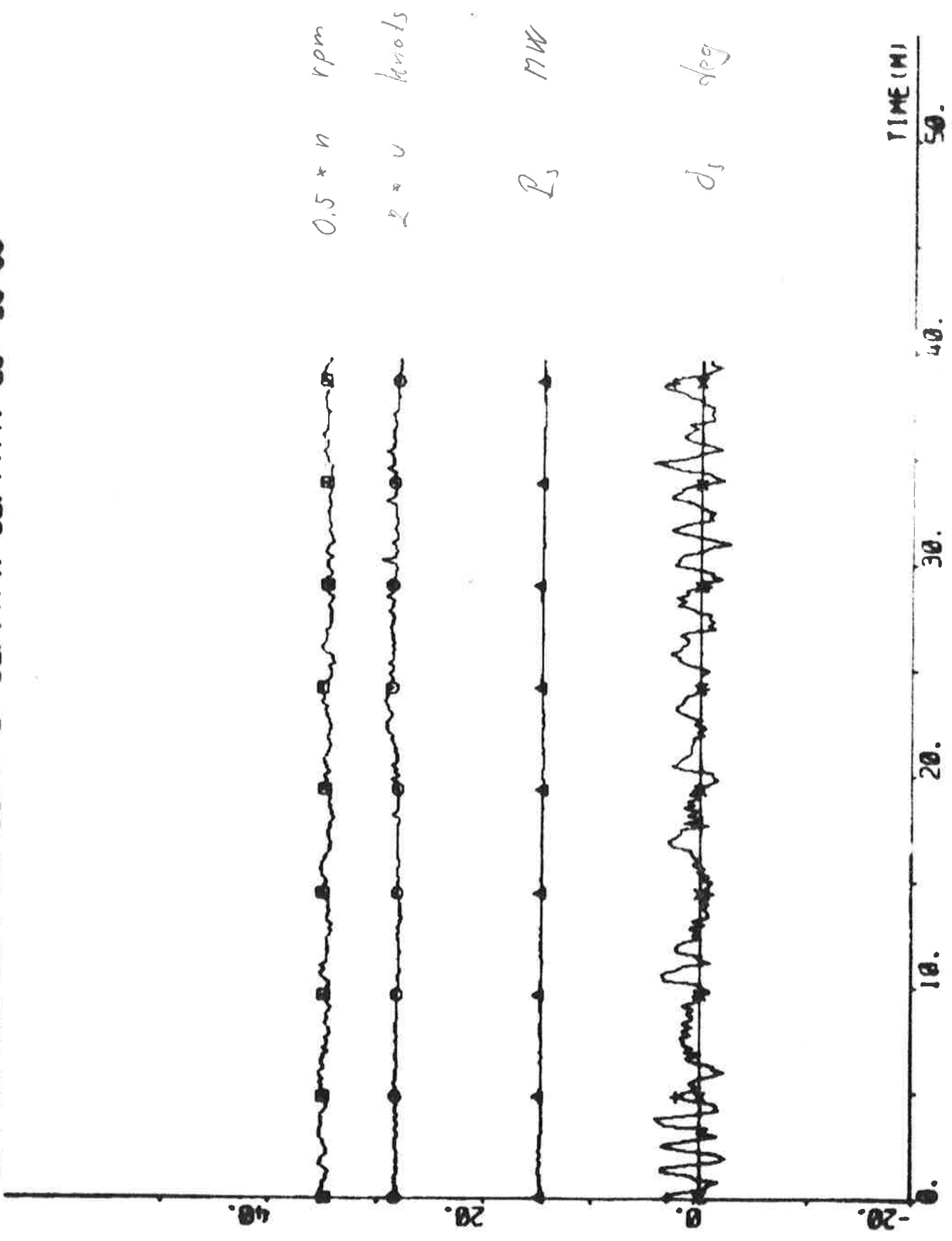
$0.1 * M_y$



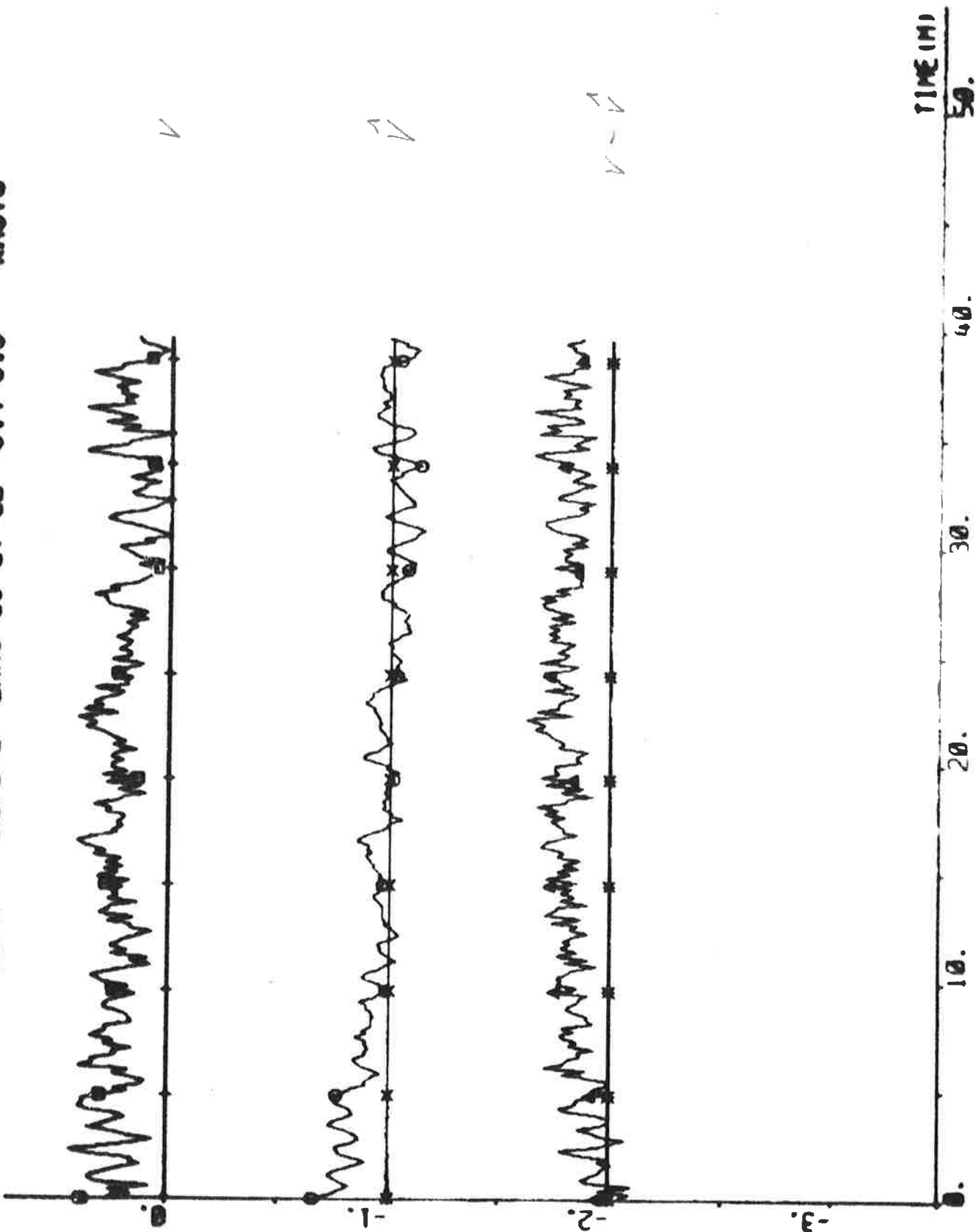
TIME (MI)



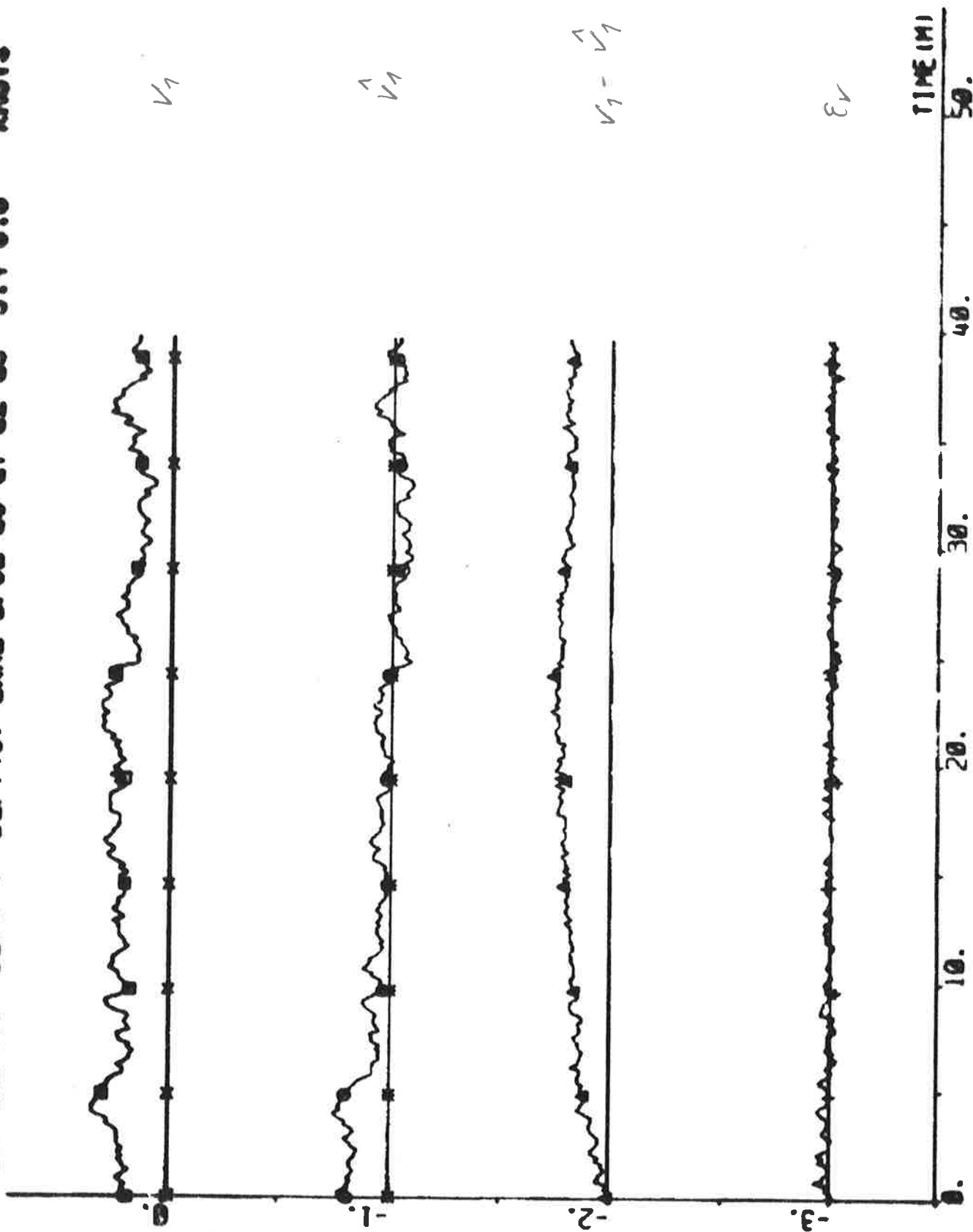
PL0T 02P1(1)-02P1(13) 02P1(12) 02P1(14) 02P1(11) 00 -20 50



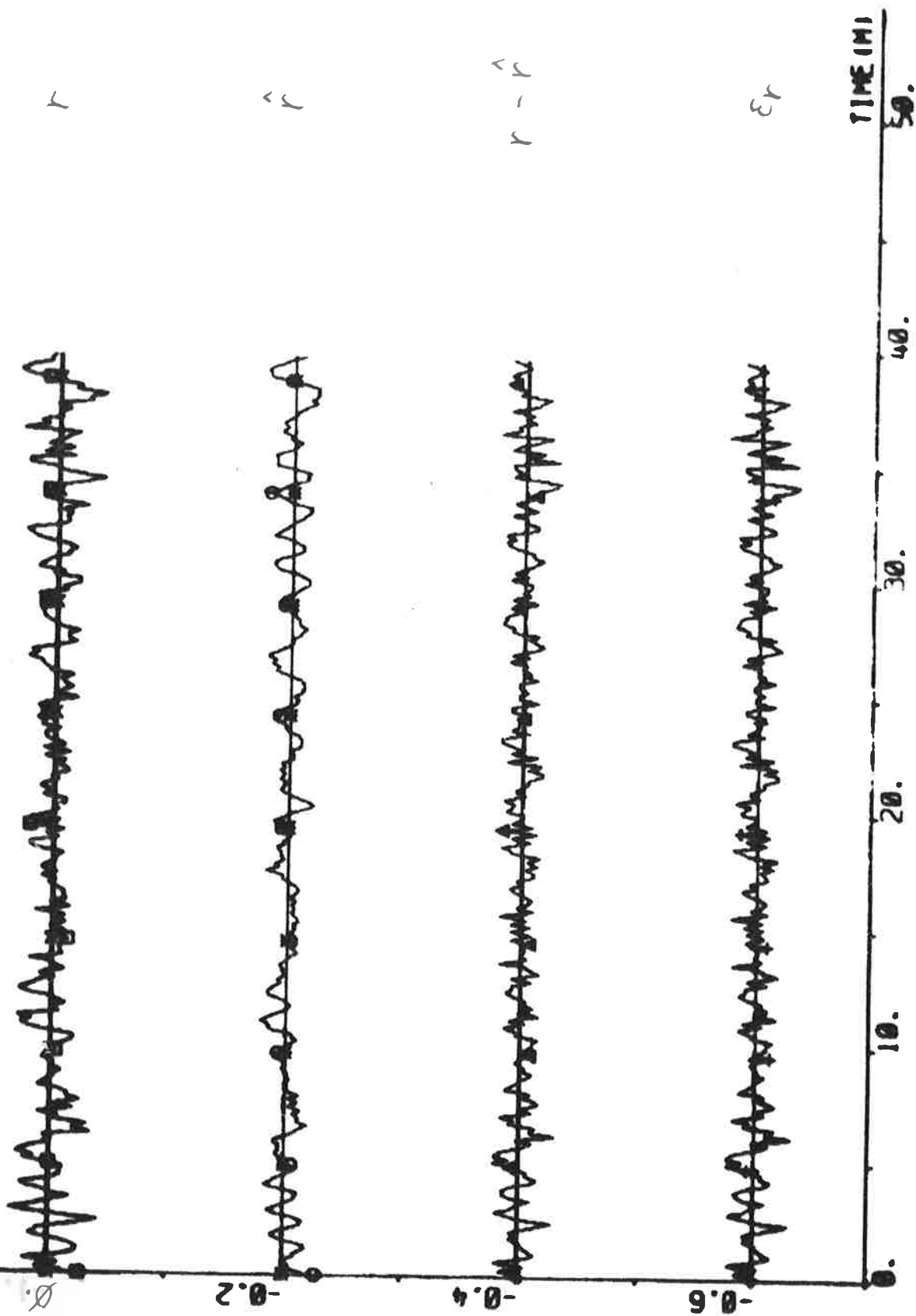
PLOT D2P1(1)-D2P2(1) D2P2(2) ERRS 00 01 02 -3.4 0.0 - KNOTS



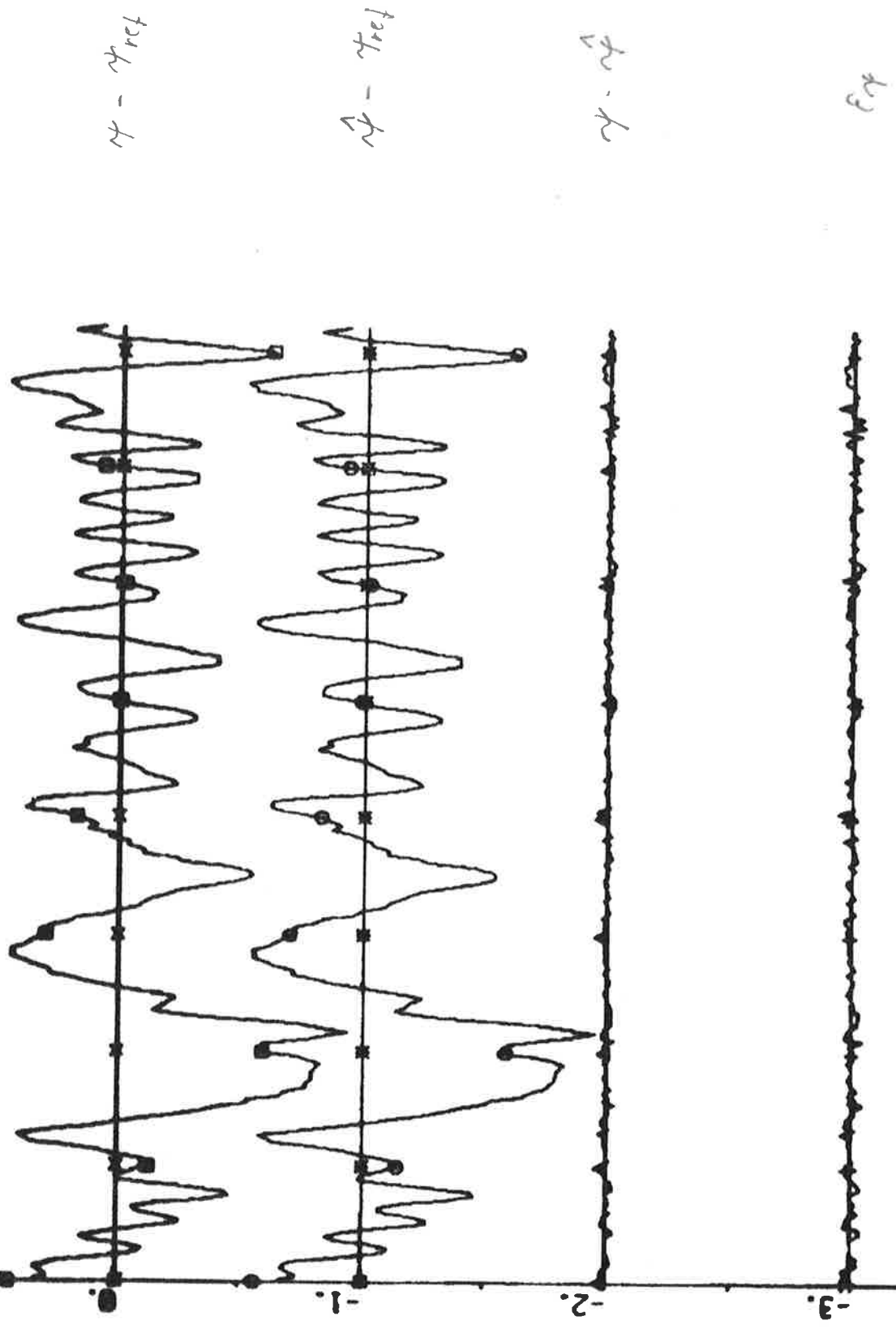
PL0T D2P1(1)-D2P1(4) D2P1(5) ERR2 EPS2 00 01 02 03 -3.4 0.0 - KNOTS



PL07 D2P1(1)-D2P1(6) D2P1(7) ERR3 EP33 00 002 004 006 -0.7 0. • DEC-73



PLOT D2P1(1)-D2P1(0) D2P1(9) ERR4 EPS4 00 01 02 03 -3.4 0.0 - 000



TIME (MI)  
50.

PLOT D2P1(1)-D2P1(2) D2P1(3) ERR1 EPS1 00 020 040 060 .05 16 . DEG



TIME (M)  
50.

PLOT D2P1(1)-D2P2(3) D2P2(4) D2P2(5) D2P2(8) 00 02 04 06 -6.6 1.5

$0.5 \times \hat{d}_0$  deg



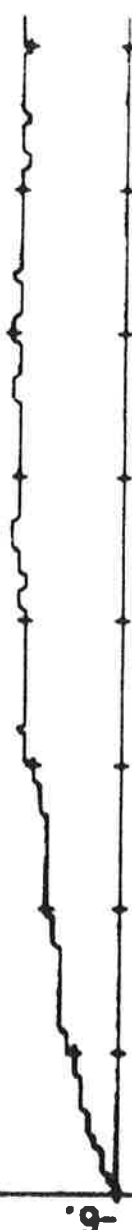
$2 \times \hat{d}_v$  knots



$100 \times \hat{d}_r$  deg/s



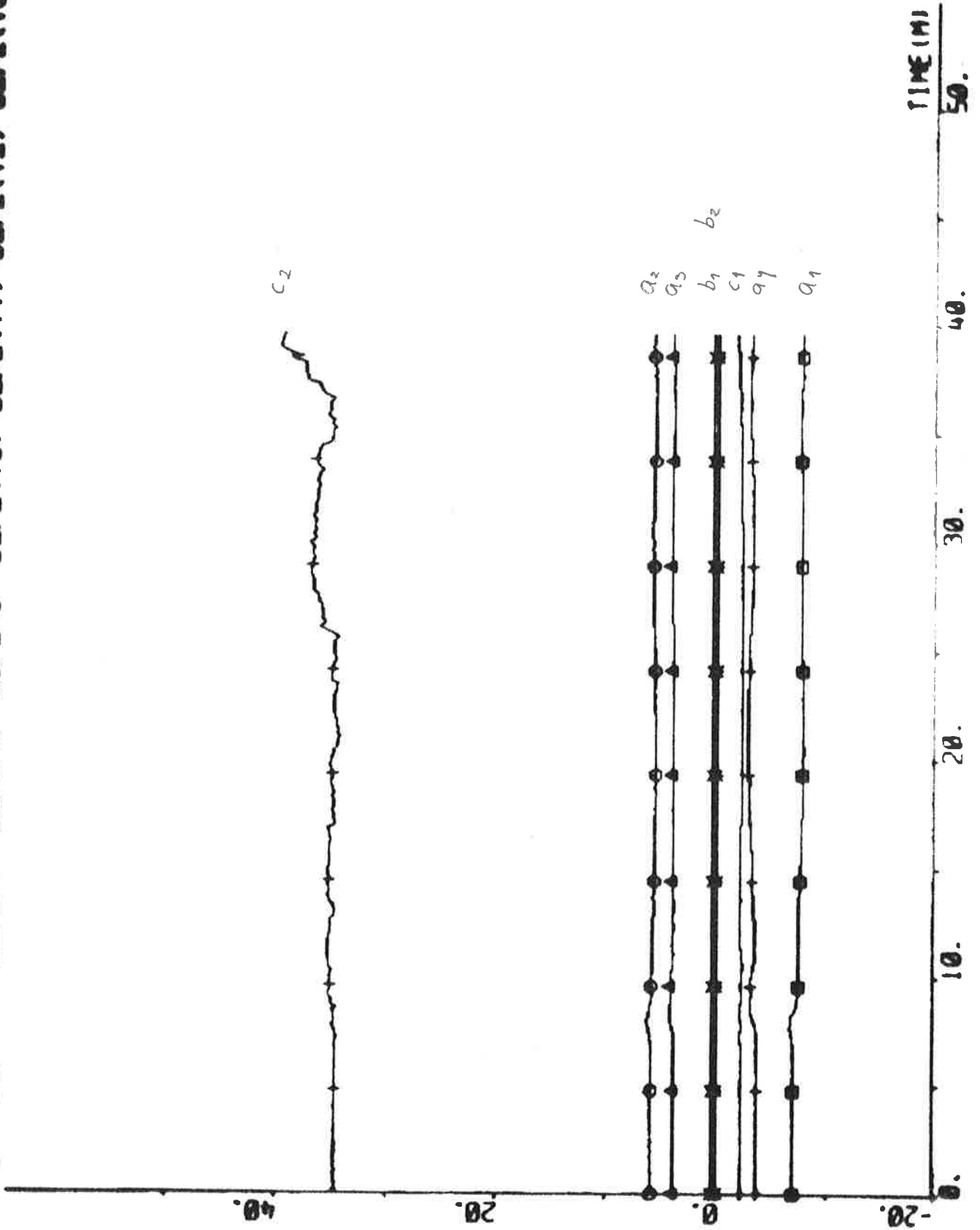
$0.5 \times \hat{d}_\theta$  deg



TIME (MI)

0. 10. 20. 30. 40. 50.

PLOT D2P1(1)-D2P2(7) D2P2(8) D2P2(9) D2P2(10) D2P2(11) D2P2(12) D2P2(13)







PLOT DCP1(1)-DCP1(6) DCP1(6) HP DCP1(10) HP DCP1(15) DCP1(16) 02 -3 1



$$0.25 \times \left\{ \begin{array}{l} \gamma = 175 \\ \gamma_{ref} = 175 \end{array} \right\} \text{ deg}$$



$$2.5 \times \left\{ \begin{array}{l} r \\ r_0 \end{array} \right\} \text{ deg/s}$$

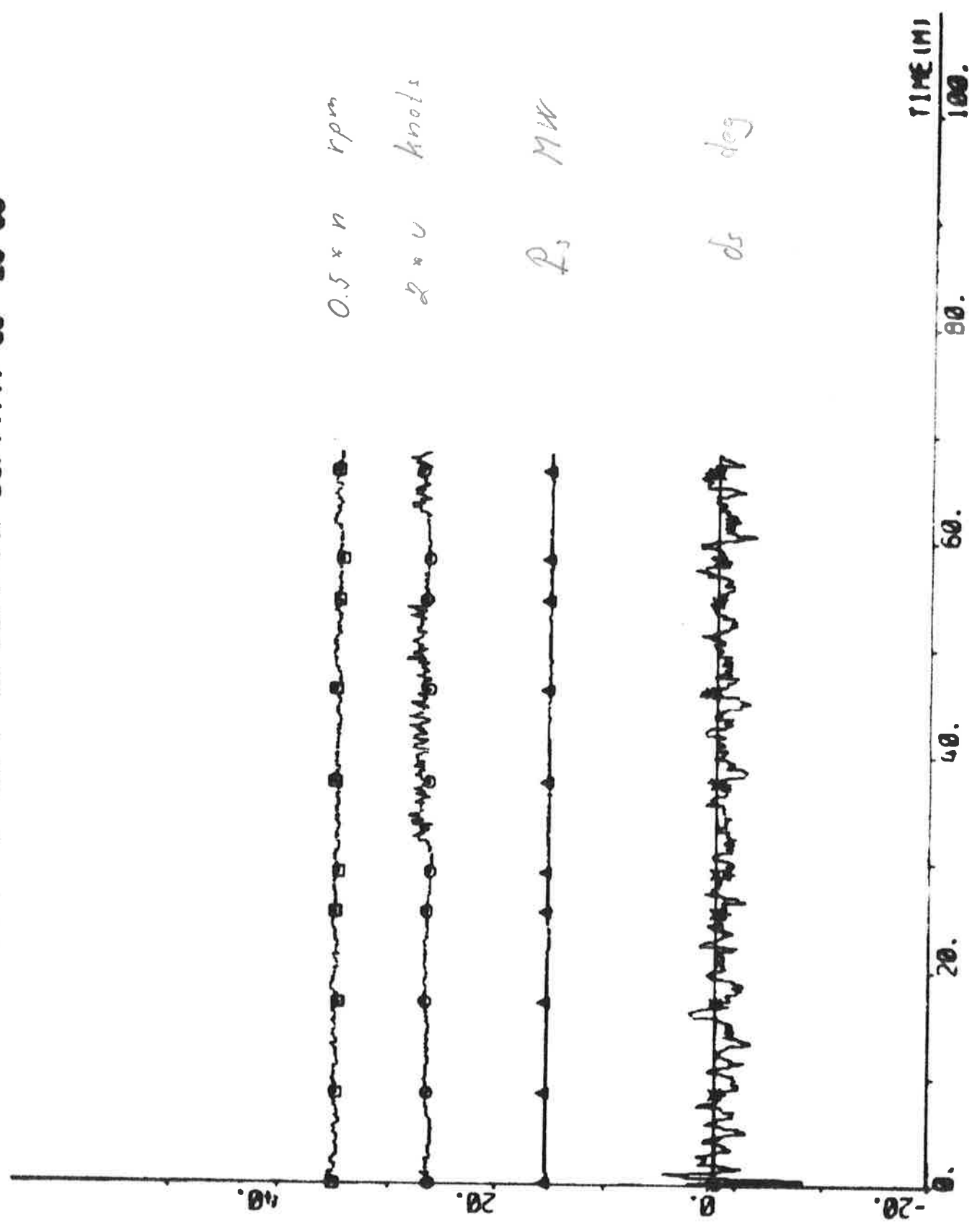


$$0.05 \times d_c \text{ deg}$$

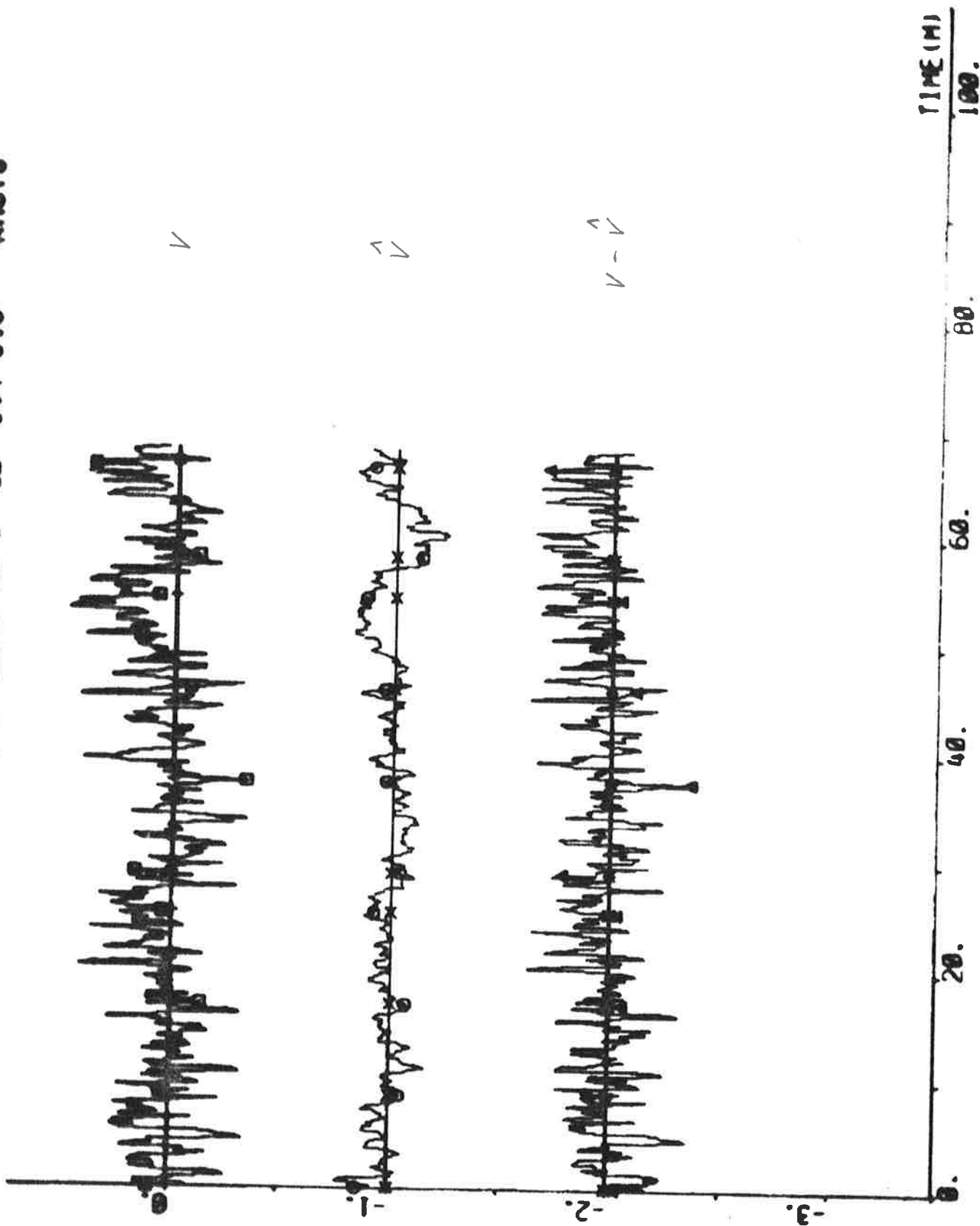
$$0.1 \times M_y$$



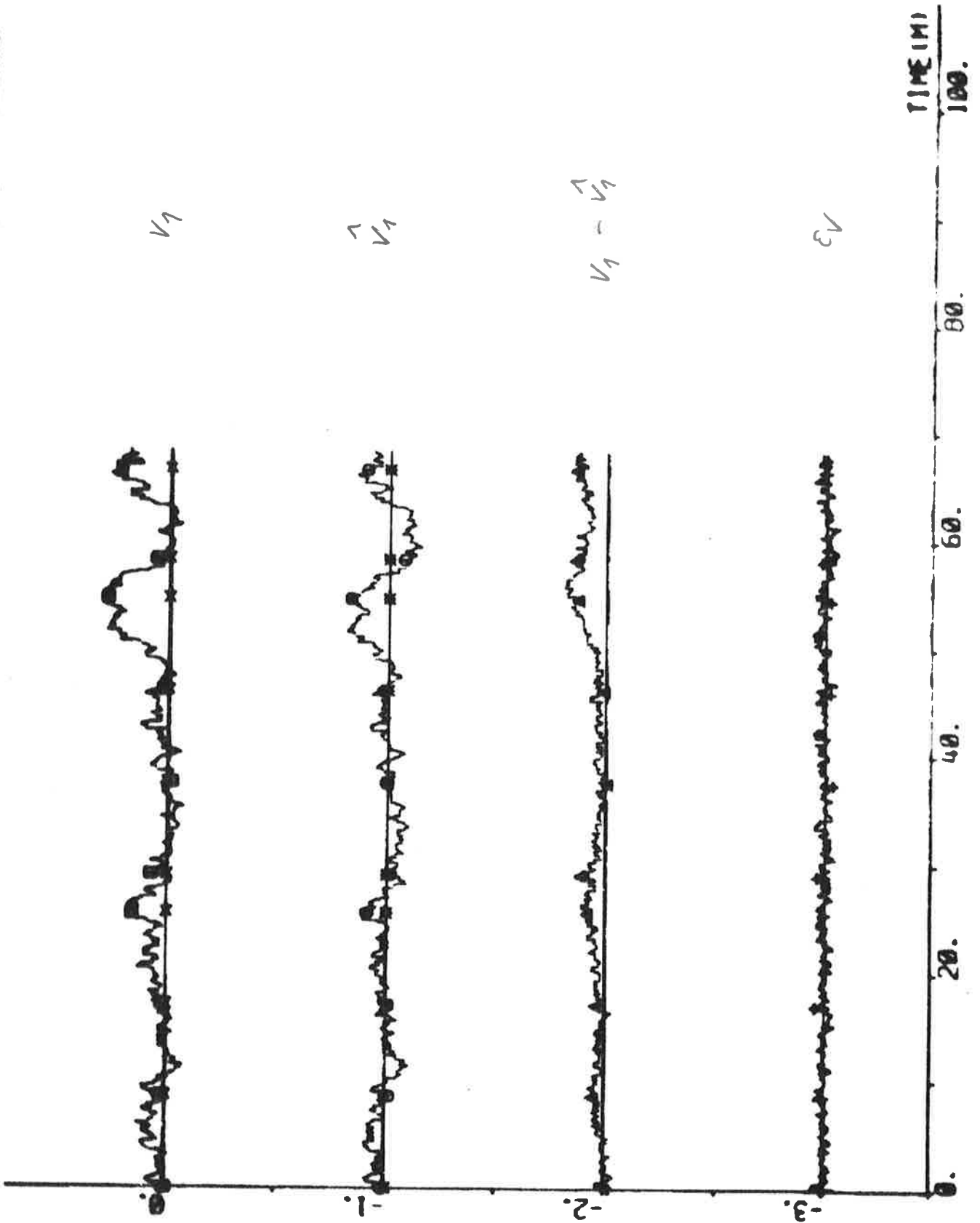
PLOT DDP1(1)-DDP1(13) DDP1(12) DDP1(14) DDP1(11) 00 -20 50



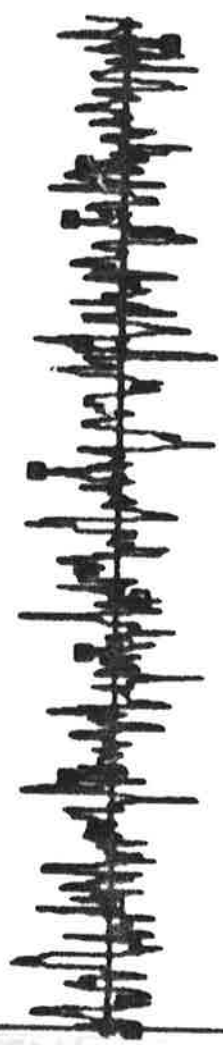
PLOT DCP1(1)-DCP2(1) DCP2(2) ERRS 00 01 02 -3.4 0.0 - KNOTS



PLOT DCP1(1)-DCP1(4) DCP1(5) ERR2 EPS2 00 01 02 03 -3.4 0.0 - KNOTS



PLOT DCP1(1)-DCP1(6) DCP1(7) ERR3 EPS3 60 602 604 606 -0.7 0. - 000/3



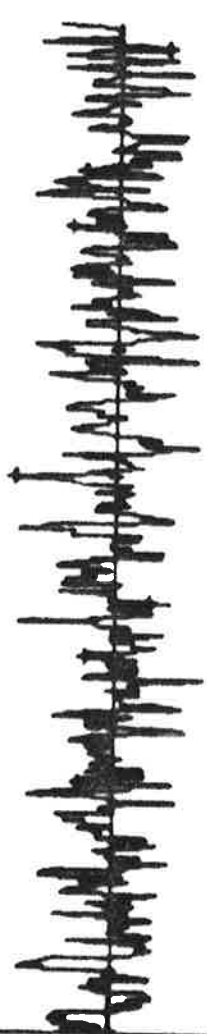
r



r



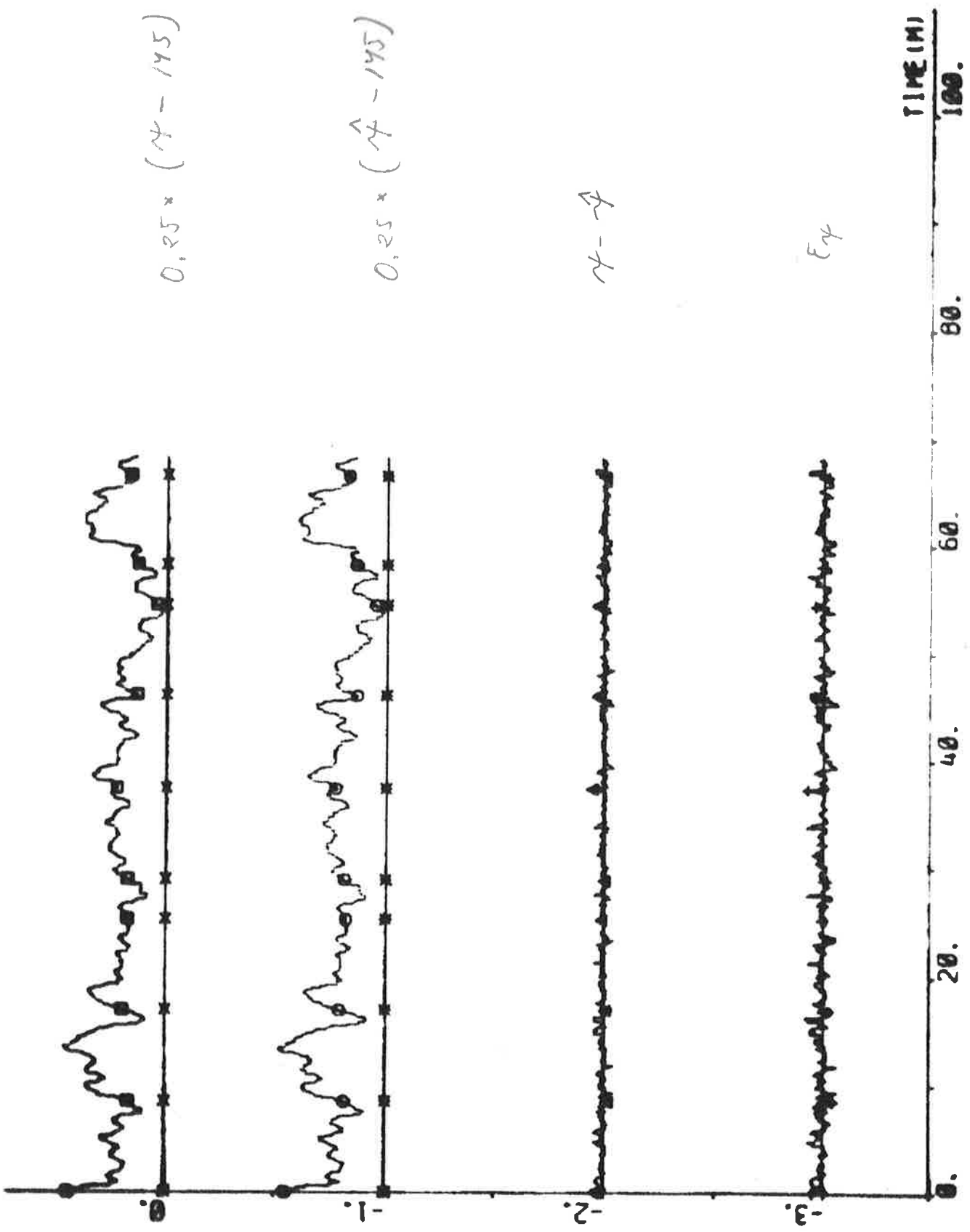
r - r



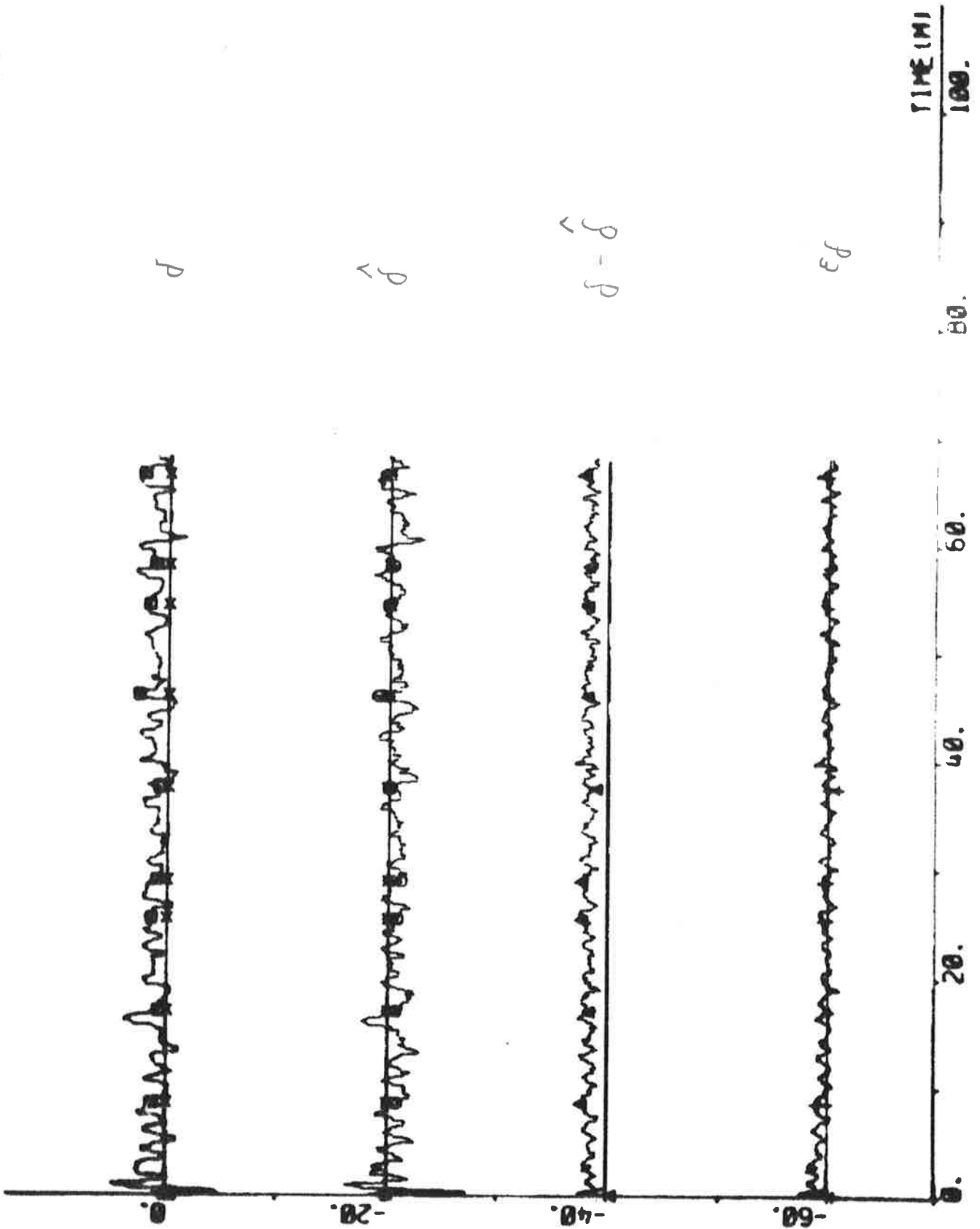
Er



PLOT DSP1(1)-DSP1(8) DSP1(9) ERR4 EPS4 00 01 02 03 -3.4 0.0 - DEG

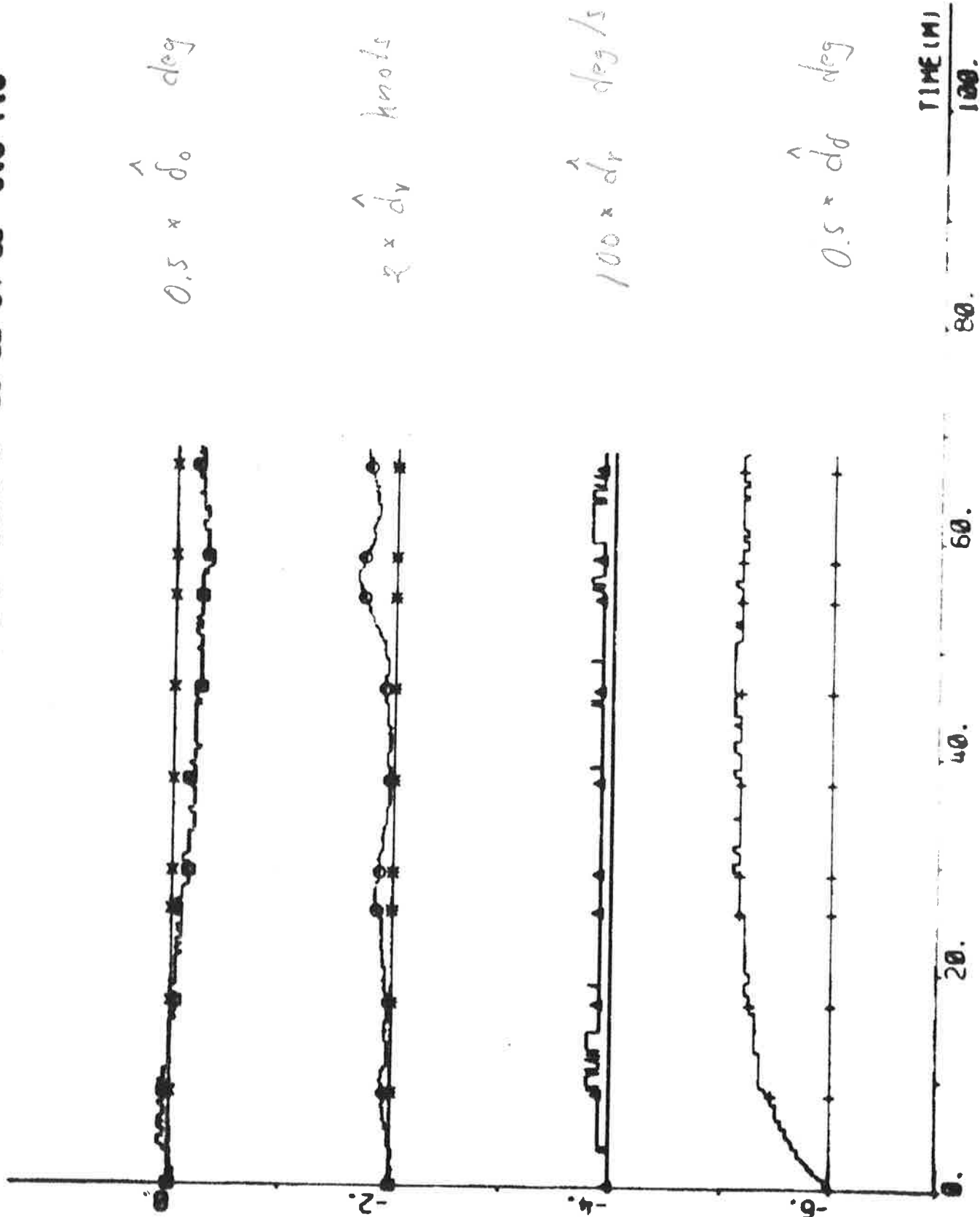


PLOT DSP1(1)-DSP1(2) DSP1(3) ERR1 EPS1 00 020 040 060 -66 16 - DEC

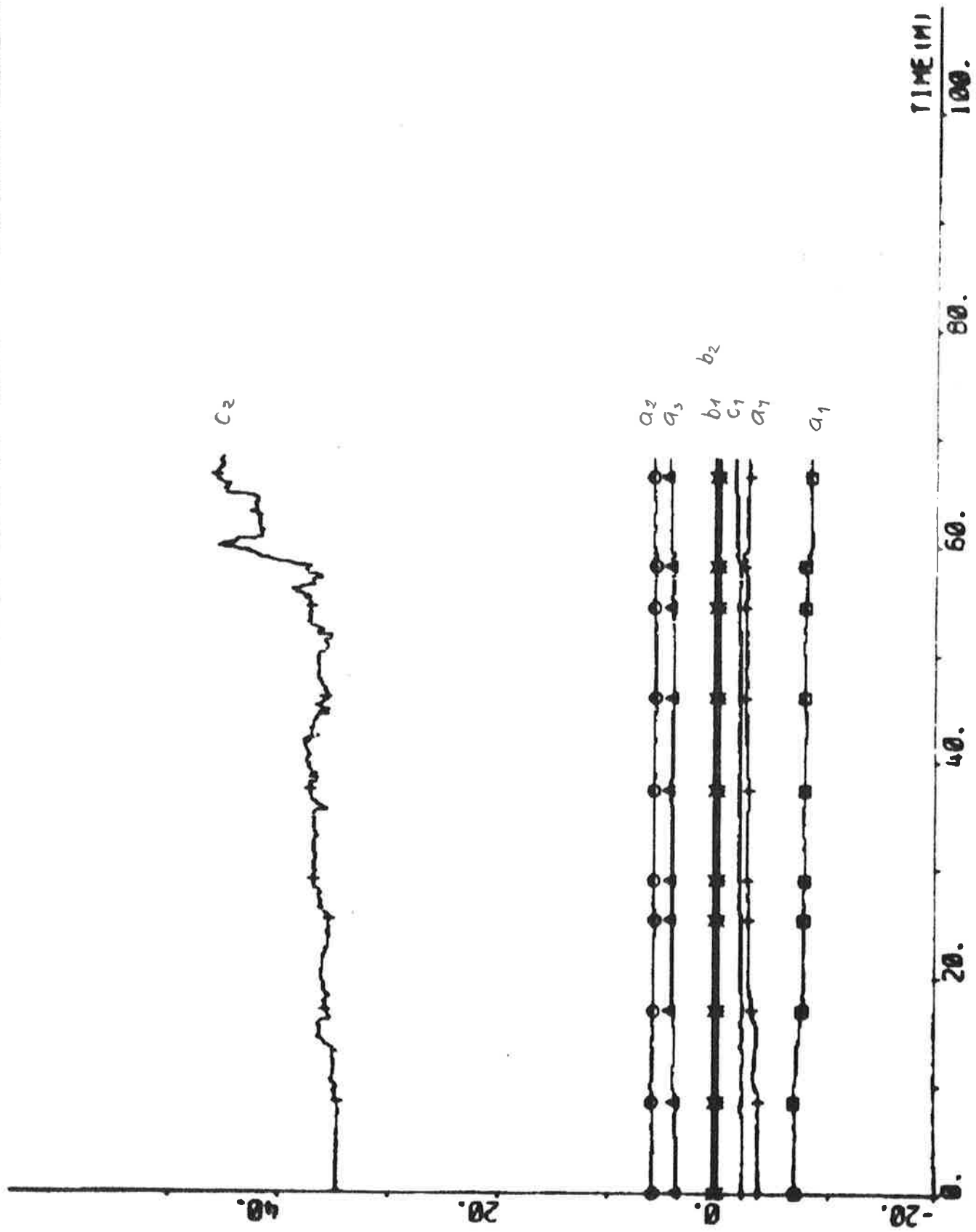




PLOT D3P1(1)-D3P2(3) D3P2(4) D3P2(5) D3P2(6) 00 02 04 06 -6.5 1.5



PLOT DSP1(1)-DSP2(7) DSP2(8) DSP2(9) DSP2(10) DSP2(11) DSP2(12) DSP2(13)



## EXPERIMENT E1

Date	1976-04-30	Forward draught	10.9 m
Time	09.00	Aft draught	12.9 m
Duration	89 min	Wind direction	SE (1; see App. A)
Position	S 21°35' E 07°34'	Wind velocity	11 m/s (strong breeze)
$\psi_{ref}$	146 deg	Wave height	-

Open loop experiment for identification

Tuning time for the Kalman filter before the experiment started: 0 min.

$\delta_{amp} = 3, 5$ deg	$k_{id} = 0$
$T_s = 10$ s	IVVC = 1

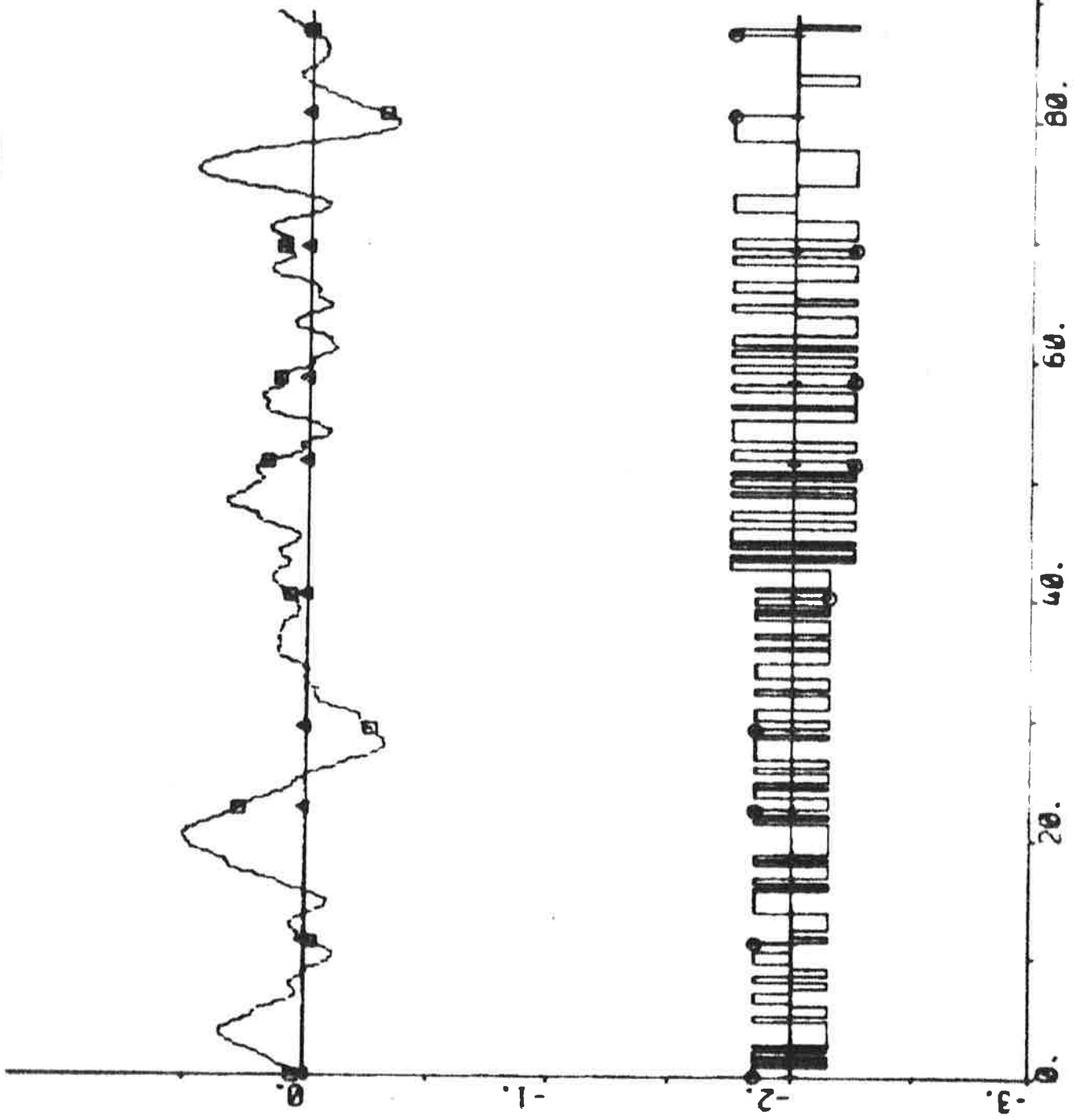
Final values:

$\hat{\delta}_0 = -0.3$  deg     $\hat{d}_v = 0.07$  knots     $\hat{d}_r = 0.005$  deg/s     $\hat{d}_\delta = 1.5$  deg

## Statistics

	Mean value	Stand. dev.		Mean value	Stand. dev.
$\delta_c$ deg	-0.28	3.77	r deg/s	0.003	0.047
$\delta_s$ deg	-0.30	3.80	$\hat{r}$ deg/s	0.000	0.037
n rpm	69.65	0.55	$r-\hat{r}$ deg/s	0.003	0.023
u knots	12.54	0.10	$\epsilon_r$ deg/s	0.001	0.027
$P_s$ MW	15.42	0.08	$\psi-\psi_{ref}$ deg	1.369	3.432
v knots	-0.026	0.253	$\hat{\psi}-\psi_{ref}$ deg	1.370	3.432
$\hat{v}$ knots	0.001	0.211	$\psi-\hat{\psi}$ deg	0.000	0.027
$v-\hat{v}$ knots	-0.026	0.148	$\epsilon_\psi$ deg	0.000	0.043
$v_1$ knots	-0.007	0.096	$\delta$ deg	1.12	3.52
$\hat{v}_1$ knots	0.001	0.088	$\hat{\delta}$ deg	-0.27	3.54
$v_1-\hat{v}_1$ knots	-0.009	0.042	$\delta-\hat{\delta}$ deg	1.38	0.64
$\epsilon_v$ knots	0.001	0.029	$\epsilon_\delta$ deg	0.11	0.68

PLOT EIP1(1)-EIP1(8) HP EIP1(10) 00 02 -3 1 - DEC

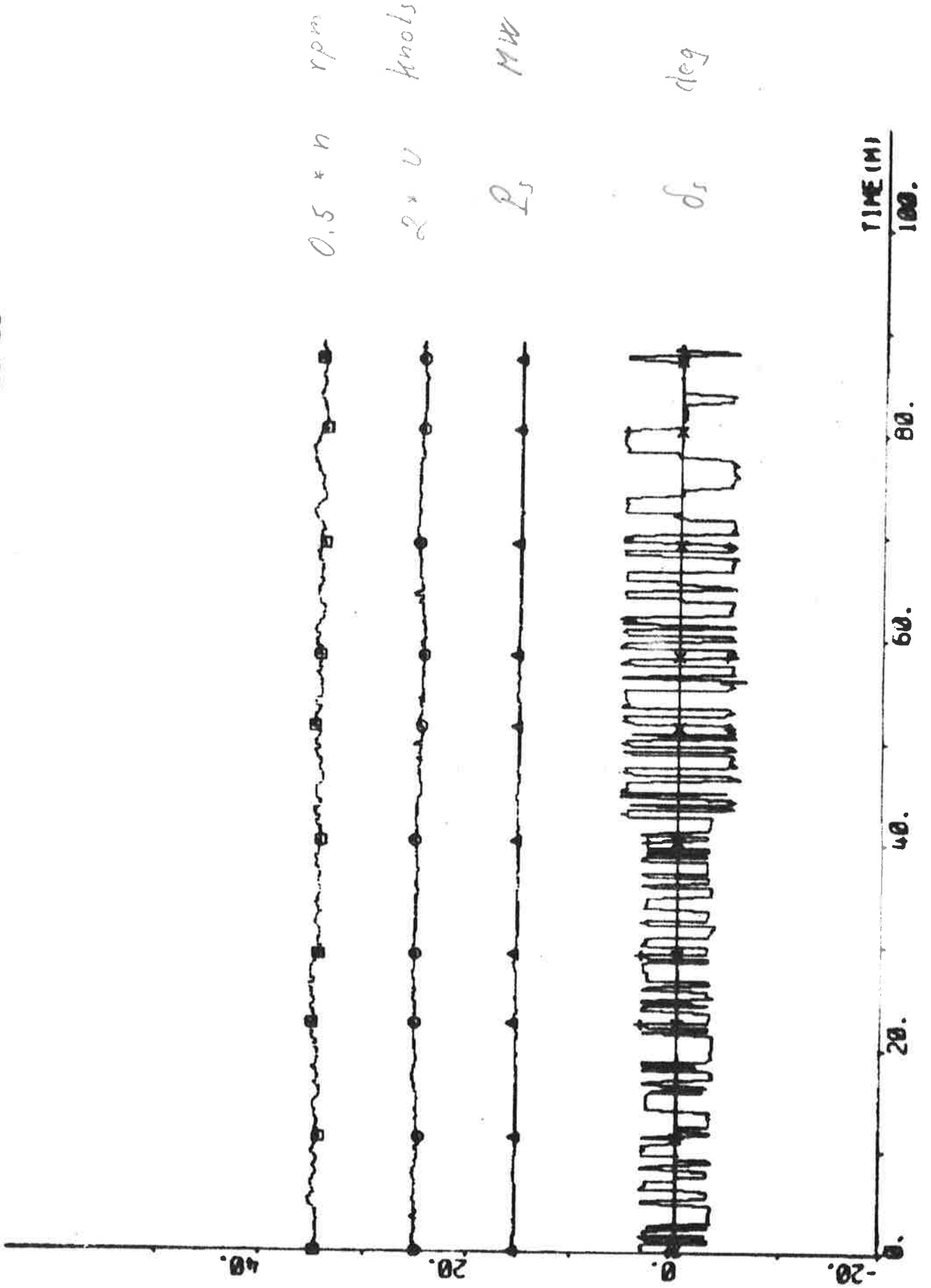


$$0.05 \times (\gamma - \gamma_{ref})$$

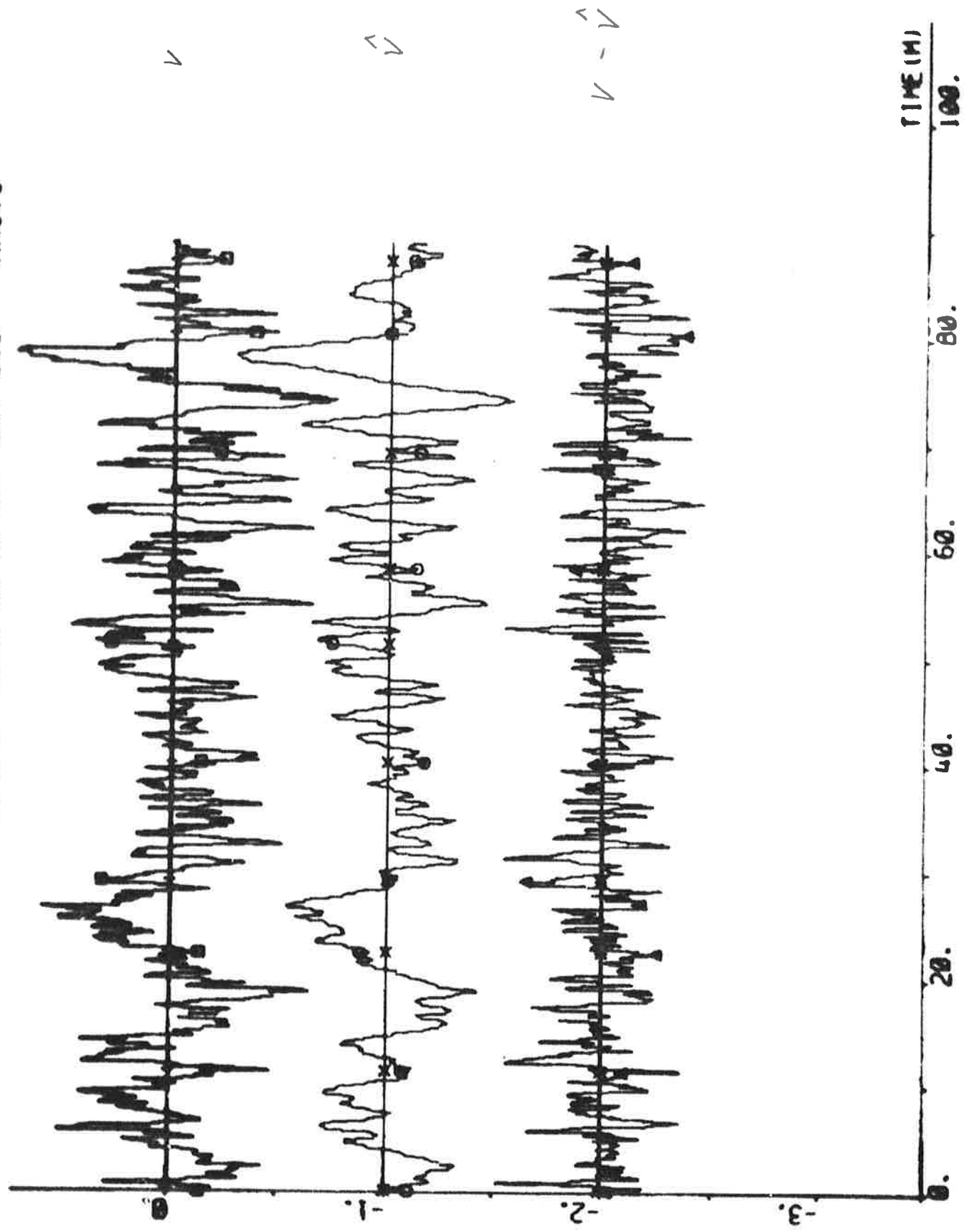
$$0.05 \times \delta_c$$

TIME (M)  
100.

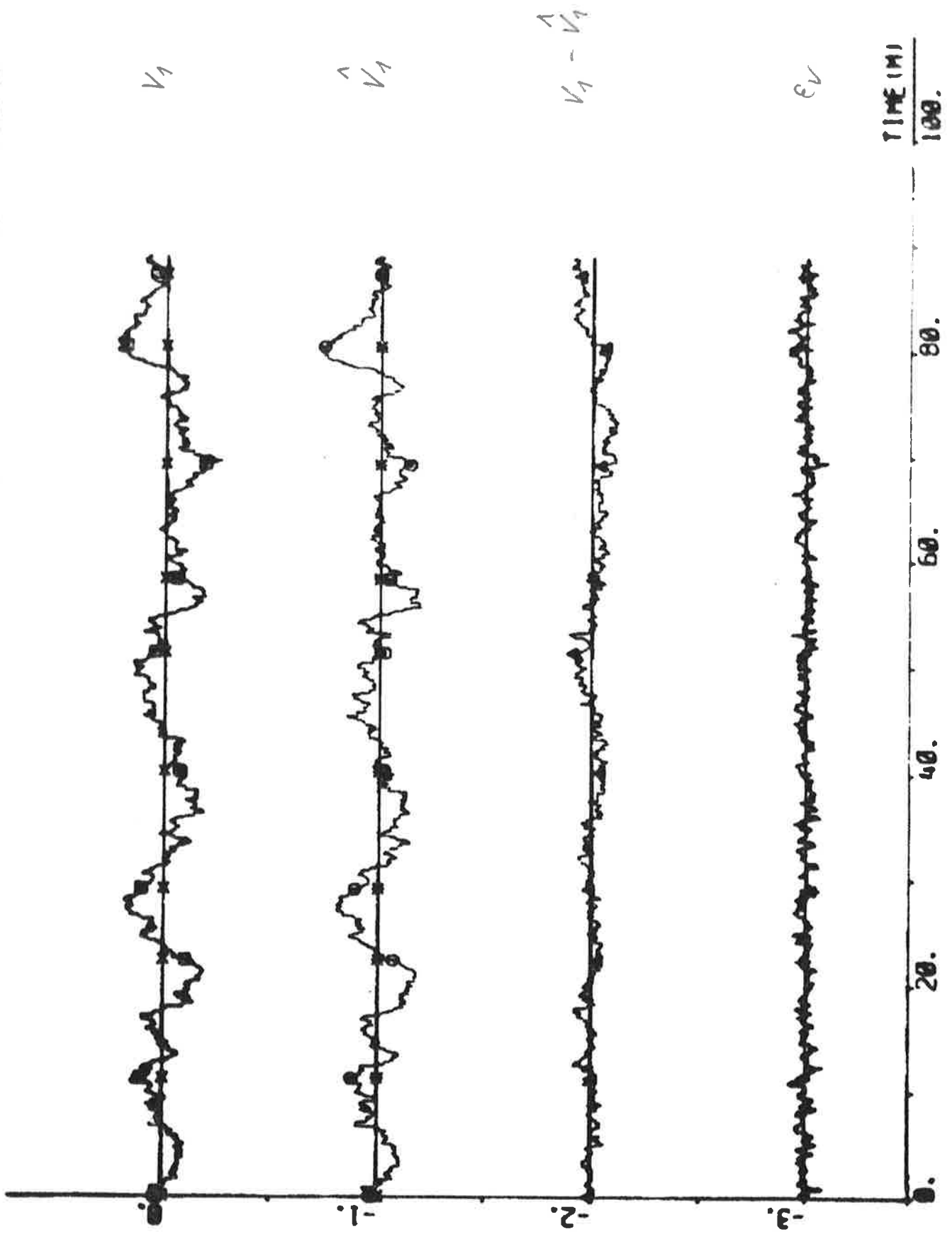
PLOT E1P1(1)-E1P1(13) E1P1(12) E1P1(14) E1P1(11) 00 -20 60



PLOT E1P1(1)-E1P2(1) E1P2(2) ERRS 00 01 02 -3.4 0.0 - KNOTS

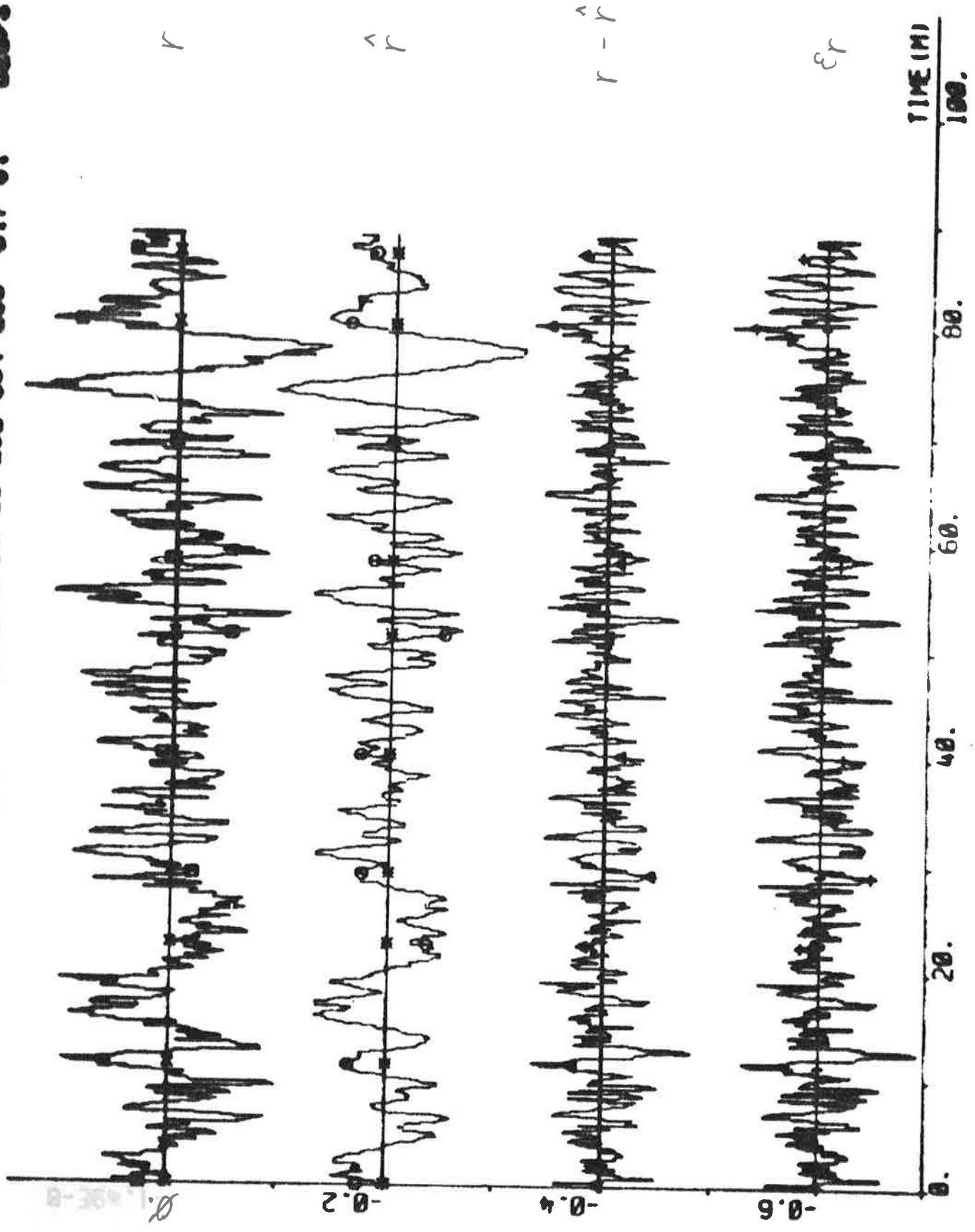


PL0T E1P1(1)-E1P1(4) E1P1(5) ERR2 EPS2 00 01 02 03 -3.4 0.0 - 10NOTS



TIME (M)  
100.

PLOT E1P1(1)-E1P1(6) E1P1(7) ERR3 EPS3 00 002 004 006 -0.7 0. - 000-3





PLOT EIP1(1)-EIP1(0) EIP1(0) ERR4 EPS4 00 01 02 03 -3.4 0.0 - 000



$$0.05 * (\gamma - \gamma_{ref})$$



$$0.05 * (\gamma - \gamma_{ref})$$



$$\gamma - \gamma'$$



Ex

0. TIME (M) 20. 40. 60. 80. 100.

PLOT EIP1(1)-EIP1(2) EIP1(3) ERR1 EPS1 00 020 040 060 -05 15 - 000



$\delta$



$\delta$



$\delta - \delta$



$\epsilon \delta$

0. 20. 40. 60. 80. 100. TIME (M)

PLOT E1P1(1)-E1P2(3) E1P2(4) E1P2(5) E1P2(6) 00 02 04 06 -0.5 1.5

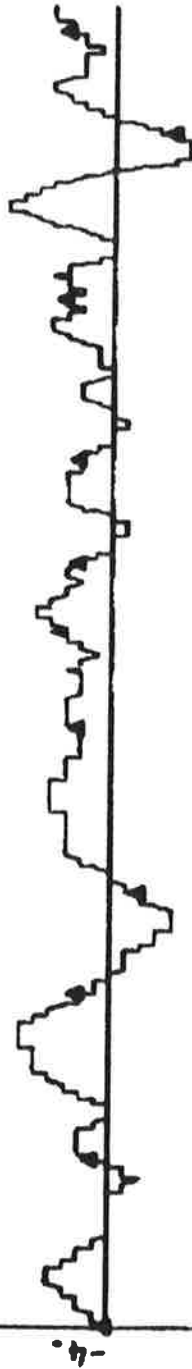
$0.5 \times \dot{\delta}_0$  deg



$2 \times \dot{\delta}_v$  knots



$100 \times \dot{\delta}_r$  deg/s



$0.5 \times \dot{\delta}_d$  deg



0. 20. 40. 60. 80. 100. TIME (M)

## EXPERIMENT E2

Date	1976-04-30	Forward draught	10.9 m
Time	11.03	Aft draught	12.9 m
Duration	19 min	Wind direction	SE (1; see App. A)
Position	S 21°56' E 07°49'	Wind velocity	14 m/s (mod. gale)
$\psi_{ref}$	143 deg	Wave height	-

Open loop experiment for identification (5°/5° zig-zag test)

Tuning time for the Kalman filter before the experiment started: 0 min.

$\delta_{amp} = 5$ deg	$k_{id} = 0$
$T_s = 10$ s	IVVC = 1

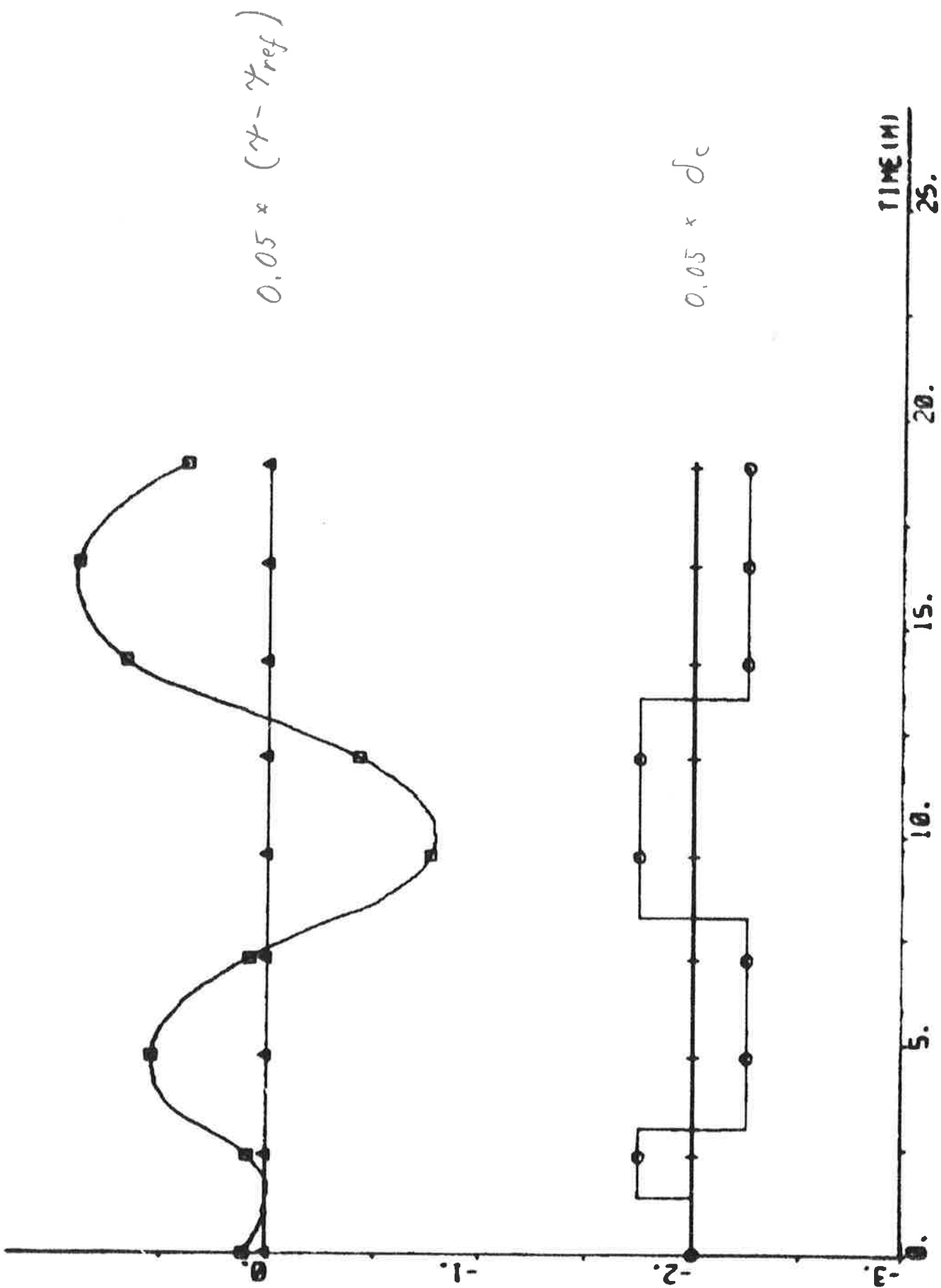
Final values:

$\hat{\delta}_0 = 0.4$  deg     $\hat{d}_v = 0.00$  knots     $\hat{d}_r = -0.001$  deg/s     $\hat{d}_\delta = 1.3$  deg

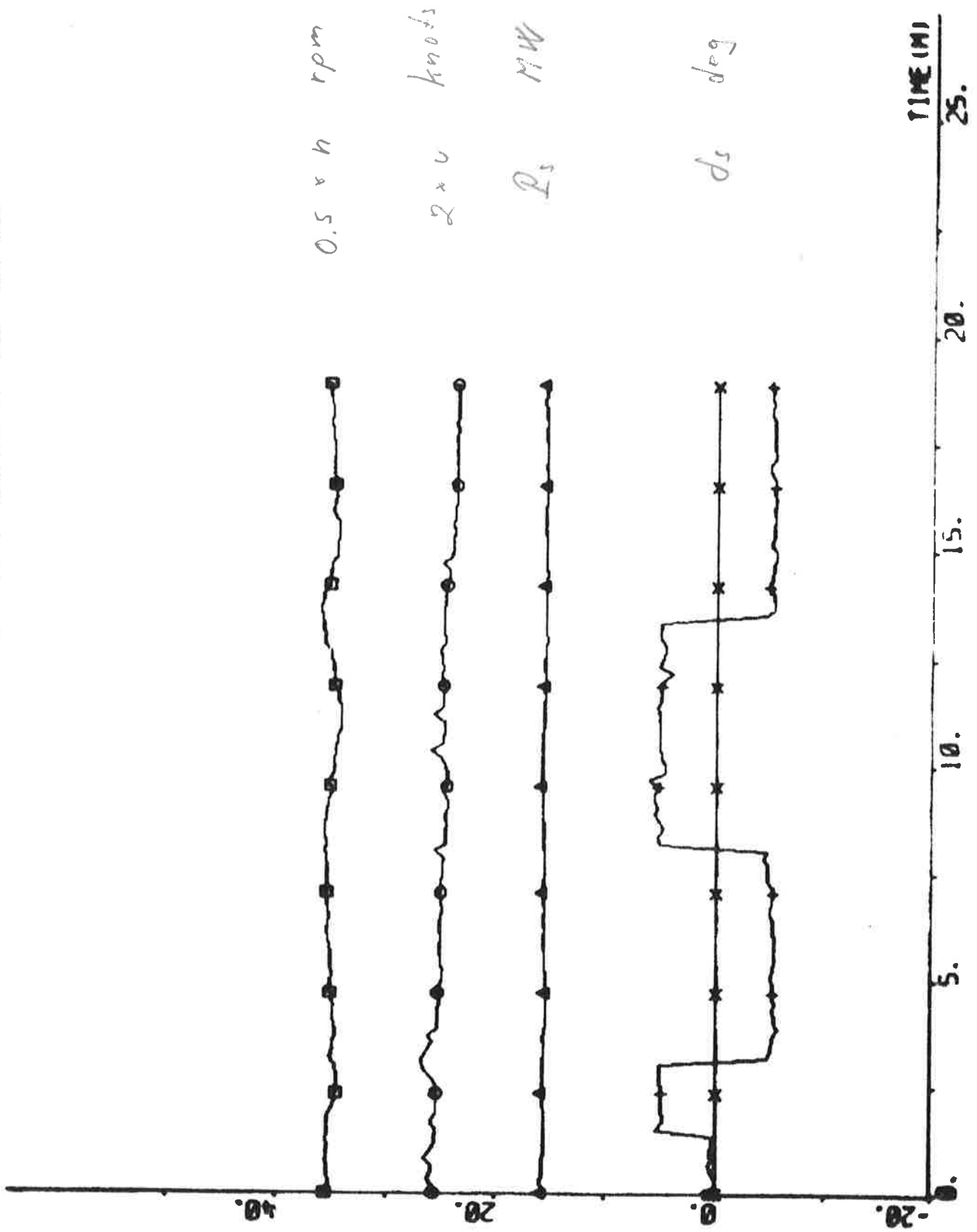
## Statistics

	Mean value	Stand. dev.		Mean value	Stand. dev.
$\delta_c$ deg	-0.93	4.73	$r$ deg/s	0.005	0.093
$\delta_s$ deg	-0.87	4.77	$\hat{r}$ deg/s	0.004	0.088
$n$ rpm	70.53	0.79	$r-\hat{r}$ deg/s	0.002	0.018
$u$ knots	12.48	0.35	$\epsilon_r$ deg/s	0.001	0.023
$P_s$ MW	15.66	0.08	$\psi-\psi_{ref}$ deg	3.545	10.617
$v$ knots	-0.047	0.526	$\hat{\psi}-\psi_{ref}$ deg	3.546	10.611
$\hat{v}$ knots	-0.038	0.509	$\psi-\hat{\psi}$ deg	-0.001	0.051
$v-\hat{v}$ knots	-0.009	0.120	$\epsilon_\psi$ deg	-0.002	0.076
$v_1$ knots	-0.012	0.127	$\delta$ deg	0.60	4.74
$\hat{v}_1$ knots	-0.017	0.155	$\hat{\delta}$ deg	-0.83	4.68
$v_1-\hat{v}_1$ knots	0.005	0.040	$\delta-\hat{\delta}$ deg	1.43	0.49
$\epsilon_v$ knots	0.005	0.028	$\epsilon_\delta$ deg	0.45	0.64

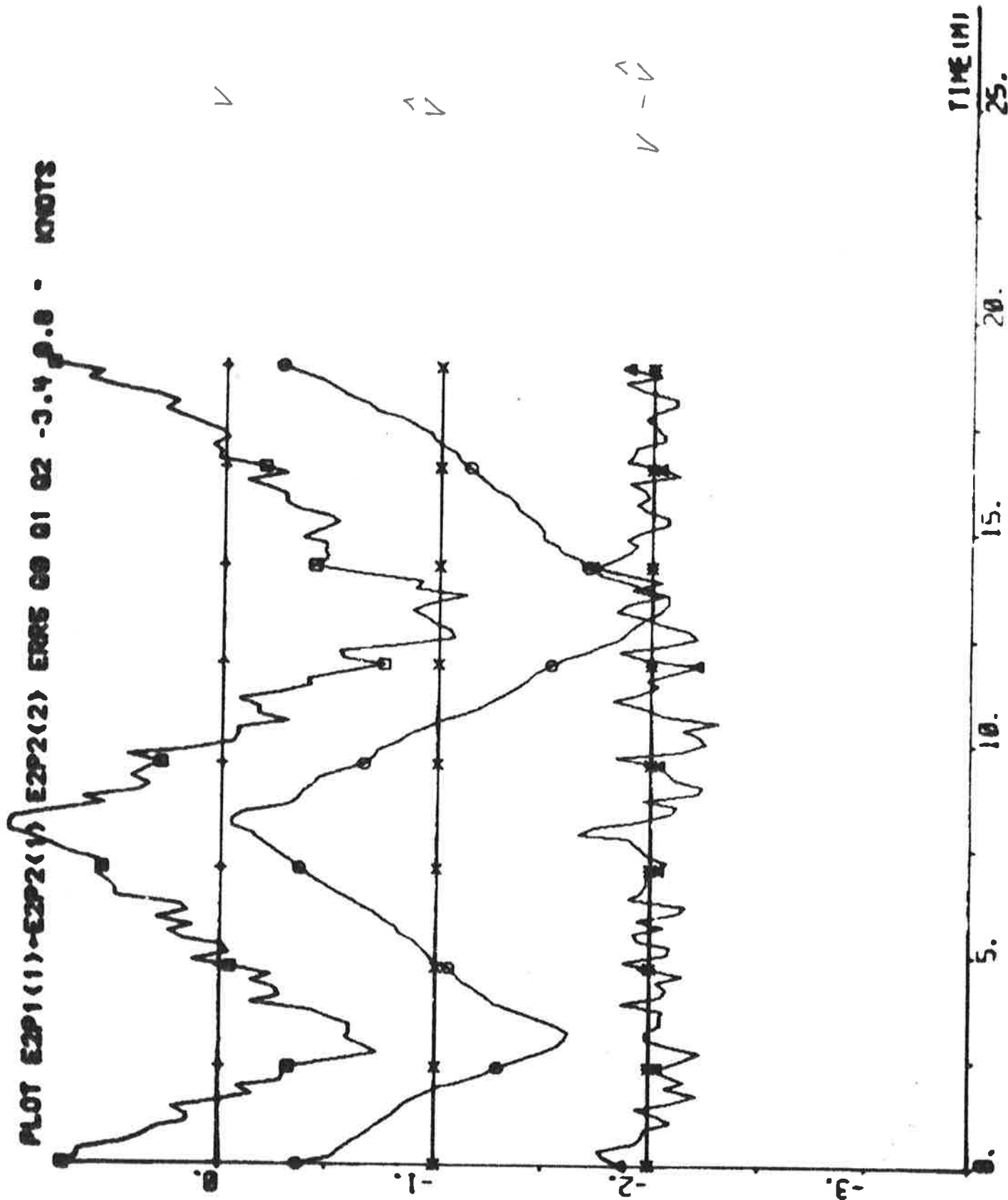
PLOT E2P1(1)-E2P1(0) HP E2P1(10) 00 02 -3 1 - DEG



PLOT E2P1(1)-E2P1(13) E2P1(12) E2P1(14) E2P1(11) 00 -20 50

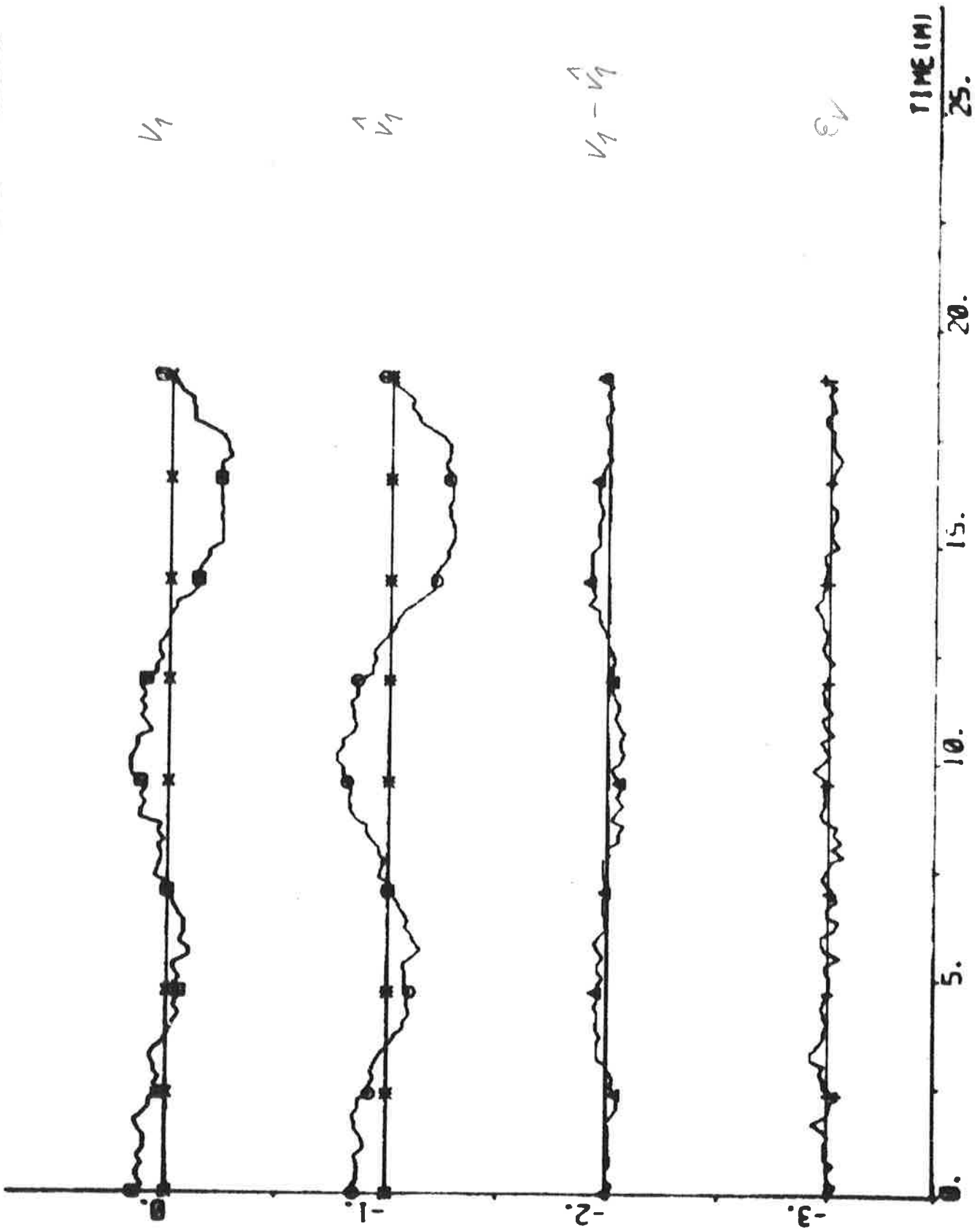


PL0T E2P1(1)-E2P2(1) E2P2(2) ERRS 00 01 02 -3.4 0.0 - KNOTS



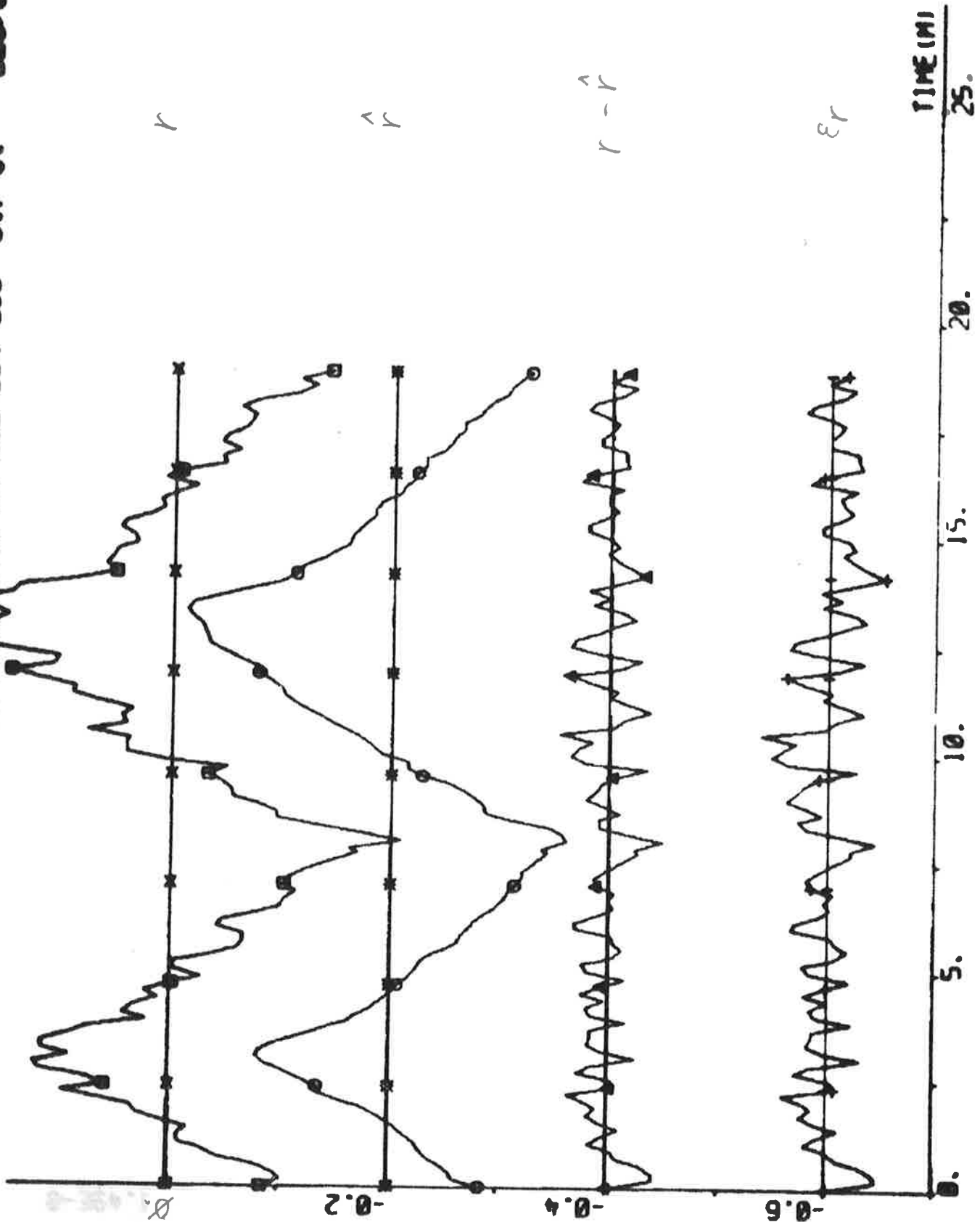
TIME (M)  
25.

PLOT E2P1(1)-E2P1(4) E2P1(5) ERR2 EPS2 00 01 02 03 -3.4 0.0 - KNOTS



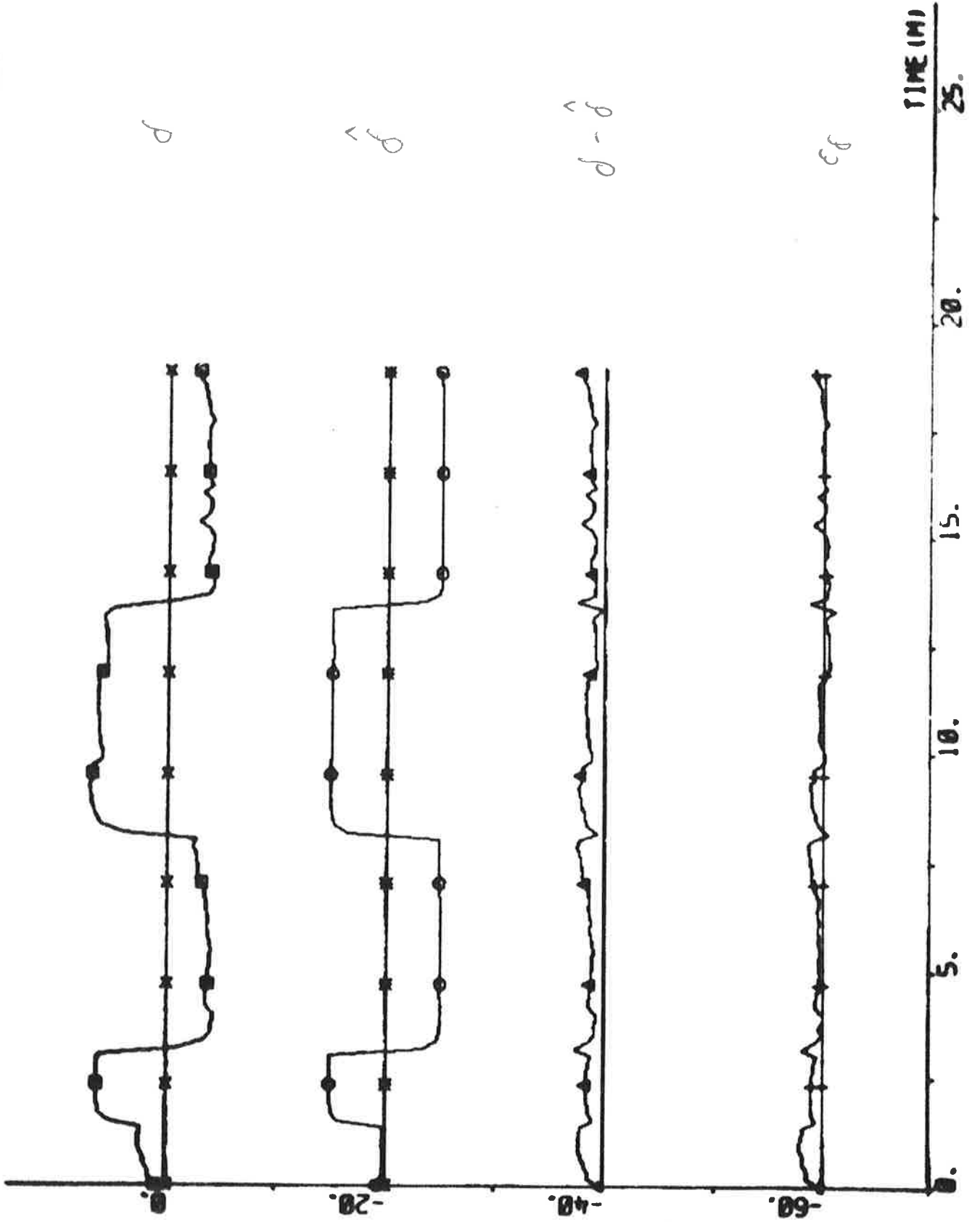


PL0T E2P1(1)-E2P1(6) E2P1(7) E2P1(8) E2P1(9) E2P1(10) E2P1(11) E2P1(12) E2P1(13) E2P1(14) E2P1(15) E2P1(16) E2P1(17) E2P1(18) E2P1(19) E2P1(20) E2P1(21) E2P1(22) E2P1(23) E2P1(24) E2P1(25) E2P1(26) E2P1(27) E2P1(28) E2P1(29) E2P1(30) E2P1(31) E2P1(32) E2P1(33) E2P1(34) E2P1(35) E2P1(36) E2P1(37) E2P1(38) E2P1(39) E2P1(40) E2P1(41) E2P1(42) E2P1(43) E2P1(44) E2P1(45) E2P1(46) E2P1(47) E2P1(48) E2P1(49) E2P1(50) E2P1(51) E2P1(52) E2P1(53) E2P1(54) E2P1(55) E2P1(56) E2P1(57) E2P1(58) E2P1(59) E2P1(60) E2P1(61) E2P1(62) E2P1(63) E2P1(64) E2P1(65) E2P1(66) E2P1(67) E2P1(68) E2P1(69) E2P1(70) E2P1(71) E2P1(72) E2P1(73) E2P1(74) E2P1(75) E2P1(76) E2P1(77) E2P1(78) E2P1(79) E2P1(80) E2P1(81) E2P1(82) E2P1(83) E2P1(84) E2P1(85) E2P1(86) E2P1(87) E2P1(88) E2P1(89) E2P1(90) E2P1(91) E2P1(92) E2P1(93) E2P1(94) E2P1(95) E2P1(96) E2P1(97) E2P1(98) E2P1(99) E2P1(100)

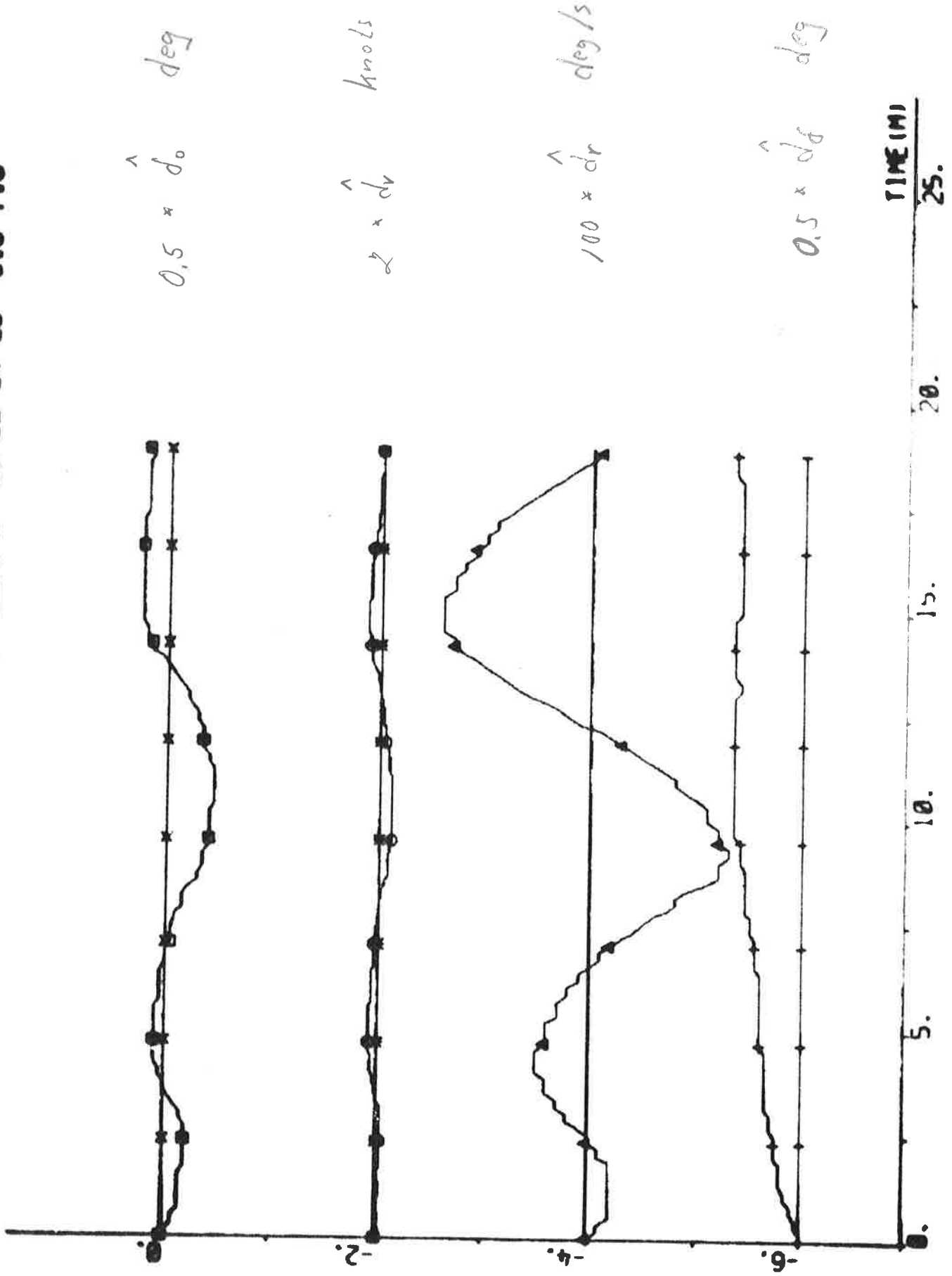




PLOT E2P1(1)-E2P1(2) E2P1(3) ERR1 EPS1 00 020 040 060 -55 15 - 000



PLOT E2P1(1)-E2P2(3) E2P2(4) E2P2(5) E2P2(6) 00 02 04 06 -0.6 1.6



## EXPERIMENT E3

Date	1976-04-30	Forward draught	10.9 m
Time	13.00	Aft draught	12.9 m
Duration	90 min	Wind direction	S (2; see App. A)
Position	S 22°18' E 08°05'	Wind velocity	9 m/s (fresh breeze)
$\psi_{\text{ref}}$	146 deg	Wave height	-

The rudder limit was 15 deg during the experiment.

Closed loop experiment for identification using additive rudder disturbances.

Tuning time for the Kalman filter before the experiment started: 1 min.

$\delta_{\text{amp}} = 5$ deg	$k_{\text{id}} = 2$
$T_s = 10$ s	IVVC = 1

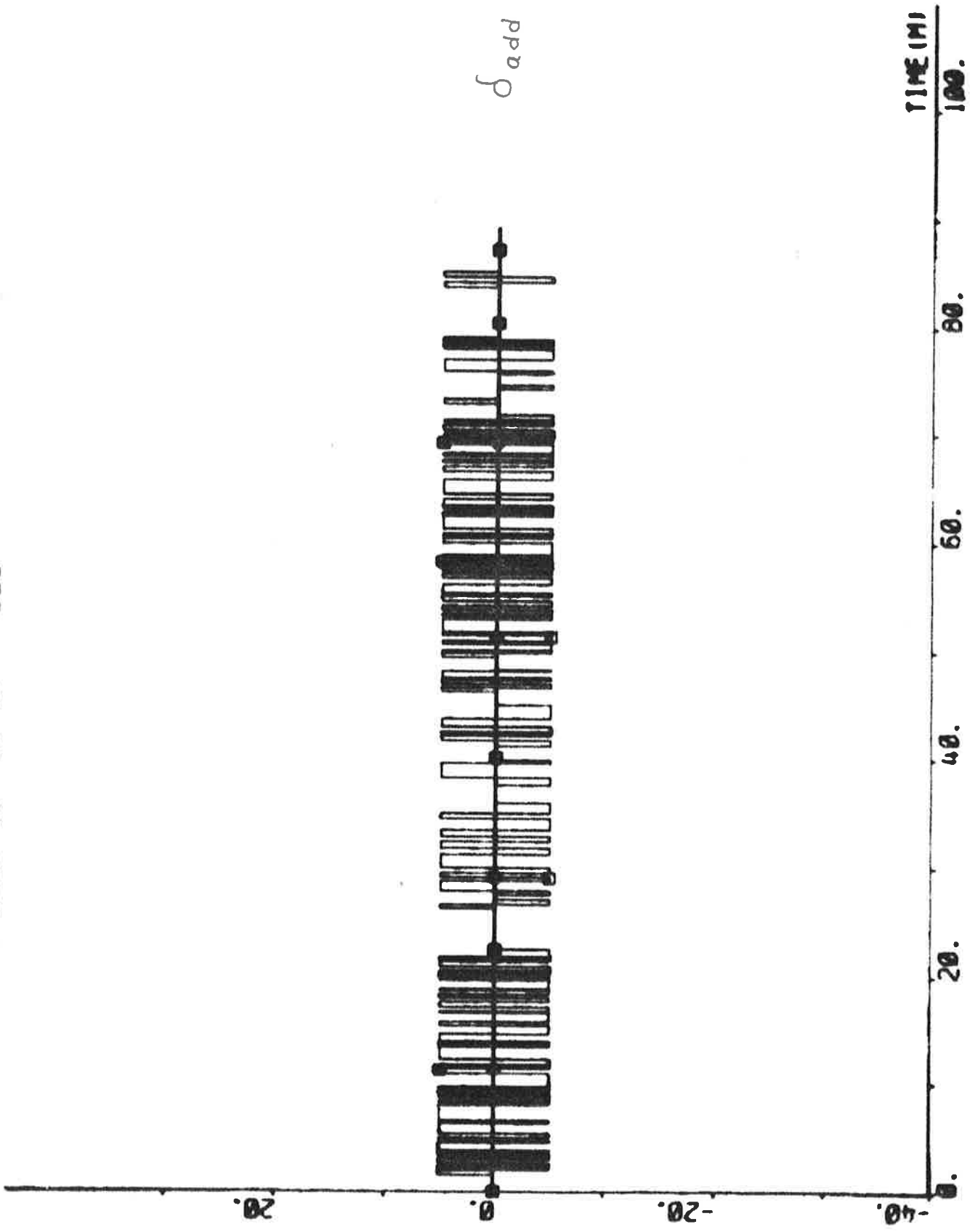
Final values:

$\hat{\delta}_0 = -0.3$  deg     $\hat{d}_v = -0.03$  knots     $\hat{d}_r = 0.002$  deg/s     $\hat{d}_\delta = 1.5$  deg

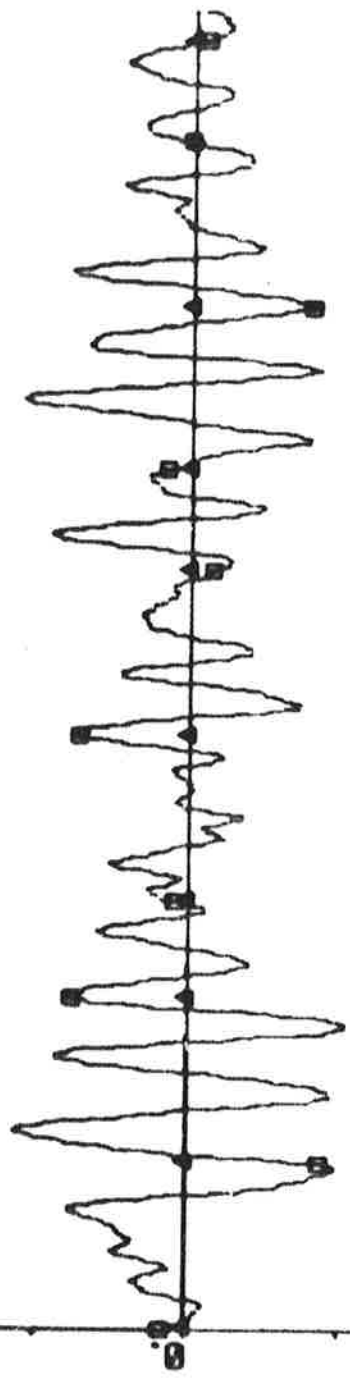
Statistics

	Mean value	Stand. dev.		Mean value	Stand. dev.
$\delta_{\text{add}}$ deg	0.12	4.29	$r$ deg/s	0.001	0.057
$\delta_c$ deg	-0.42	6.21	$\hat{r}$ deg/s	0.000	0.053
$\delta_s$ deg	-0.43	6.17	$r - \hat{r}$ deg/s	0.002	0.021
$n$ rpm	70.03	0.68	$\varepsilon_r$ deg/s	0.000	0.024
$u$ knots	12.89	0.21	$\psi - \psi_{\text{ref}}$ deg	0.273	2.211
$P_s$ MW	15.37	0.08	$\hat{\psi} - \psi_{\text{ref}}$ deg	0.272	2.220
$v$ knots	-0.021	0.290	$\psi - \hat{\psi}$ deg	0.001	0.034
$\hat{v}$ knots	-0.002	0.253	$\varepsilon_\psi$ deg	0.001	0.053
$v - \hat{v}$ knots	-0.019	0.136	$\delta$ deg	1.09	5.59
$v_1$ knots	-0.014	0.076	$\hat{\delta}$ deg	-0.41	5.85
$\hat{v}_1$ knots	-0.004	0.076	$\delta - \hat{\delta}$ deg	1.50	0.95
$v_1 - \hat{v}_1$ knots	-0.010	0.040	$\varepsilon_\delta$ deg	0.10	0.98
$\varepsilon_v$ knots	0.002	0.032			

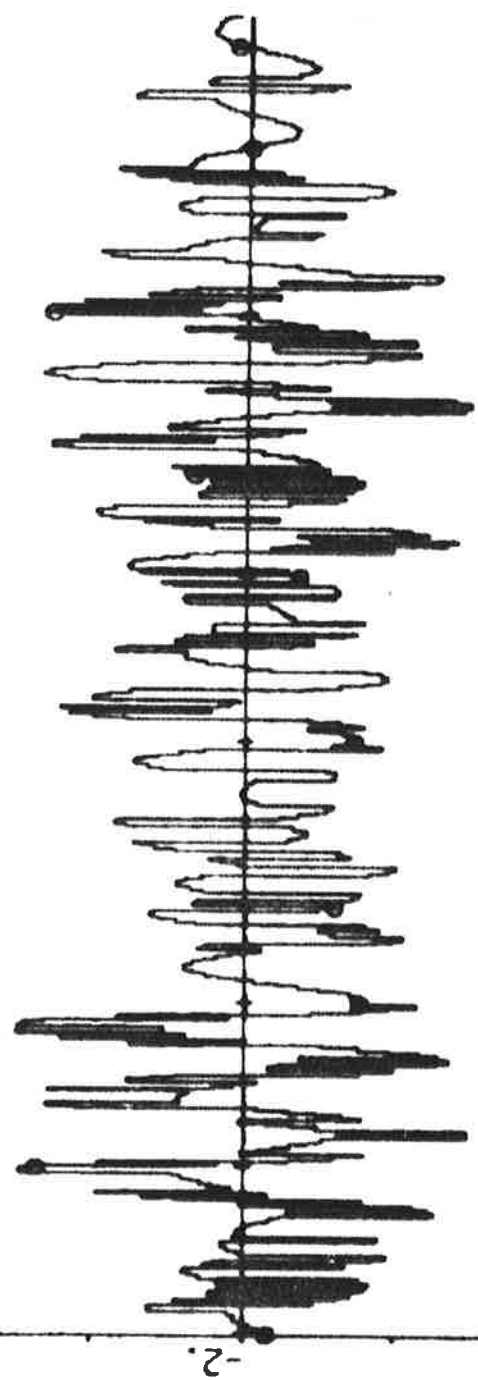
PLOT EXP1(1)-HP DELO 00 -40 40 - DEG



PLOT E3P1(1) - E3P1(8) MP E3P1(10) 00 02 -3 1 - DEC



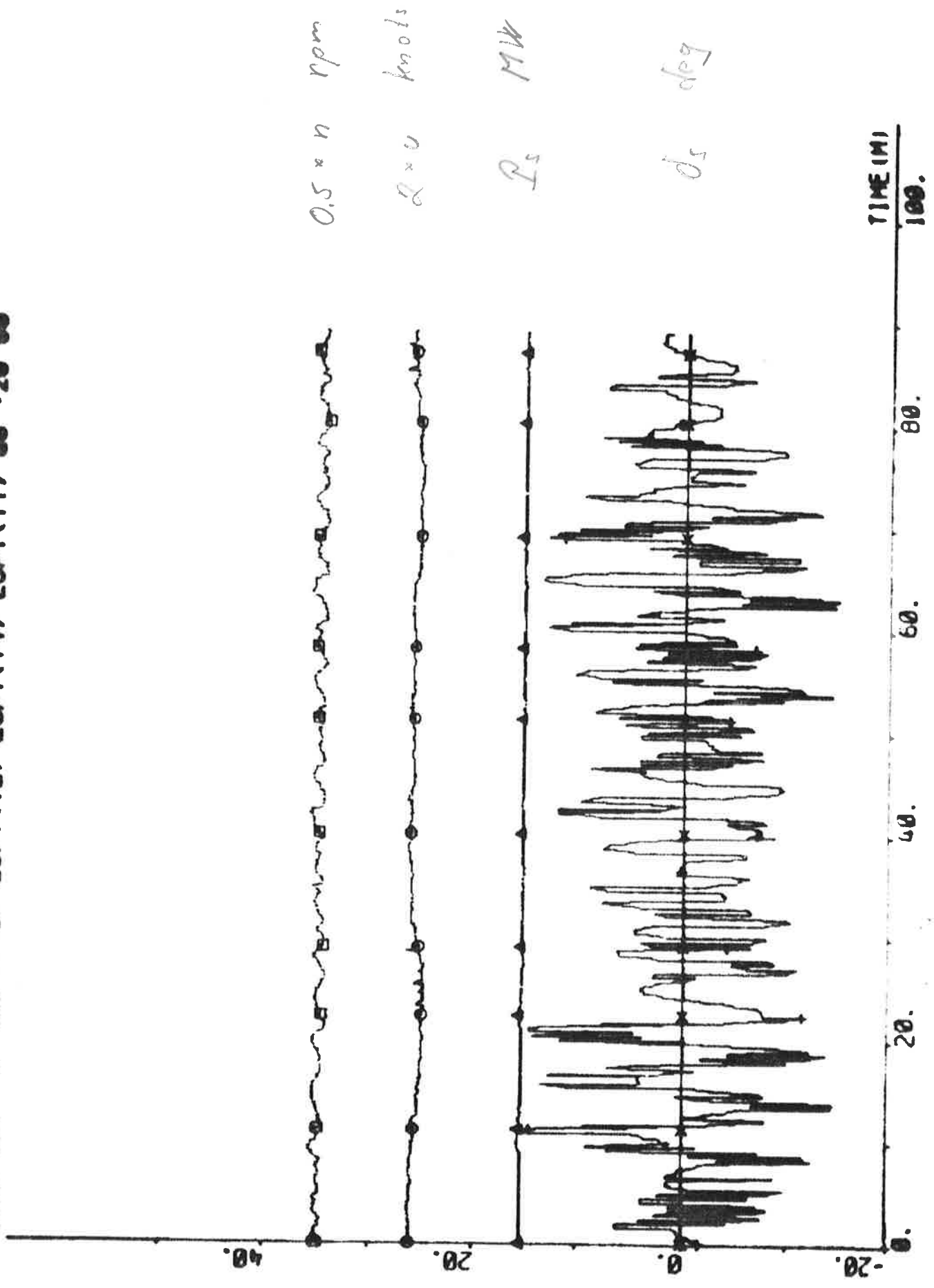
$0.1 * (\gamma - \gamma_{ref})$



$0.05 * d_c$

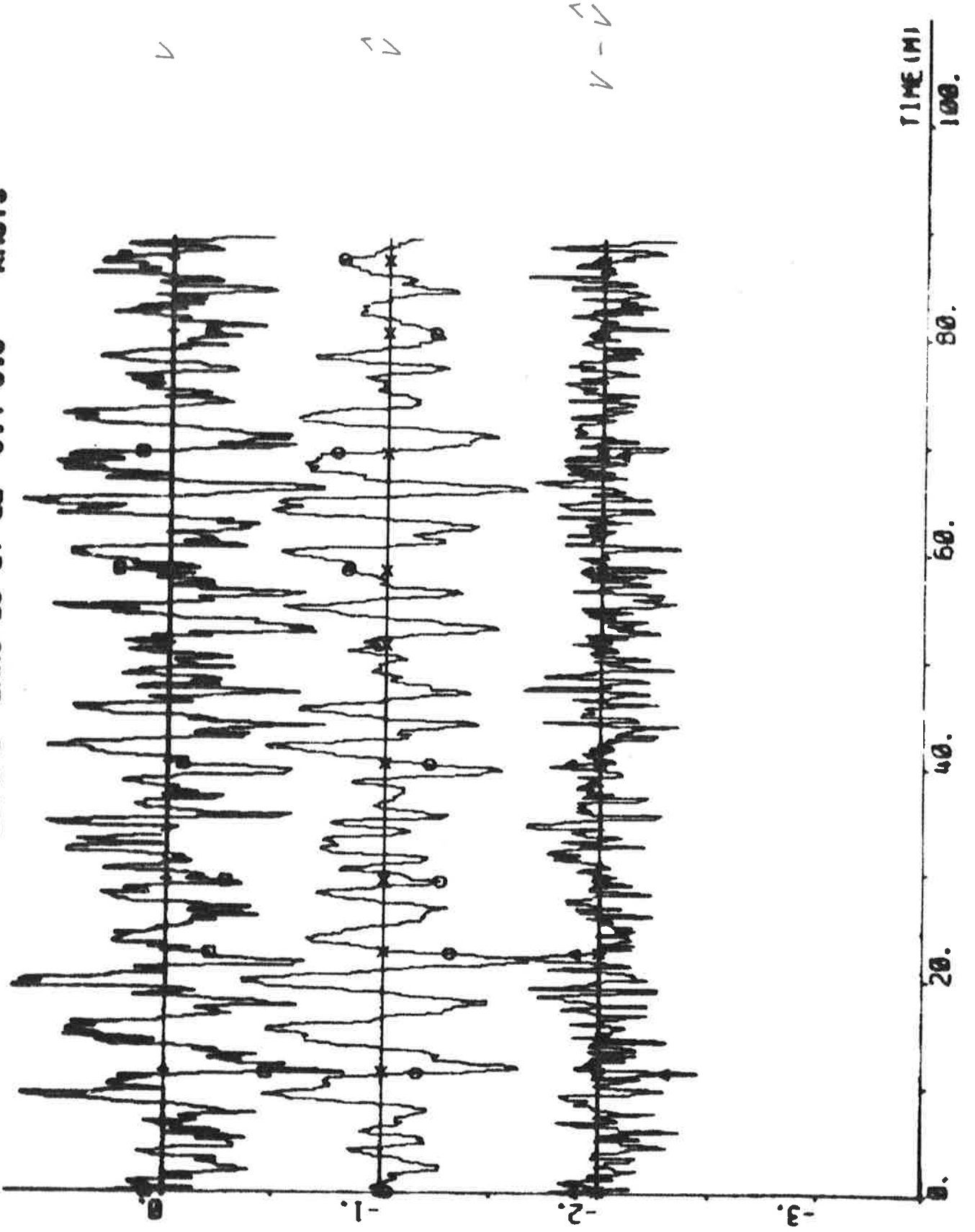


PLOT E3P1(1)-E3P1(13) E3P1(12) E3P1(14) E3P1(11) 00 -20 50

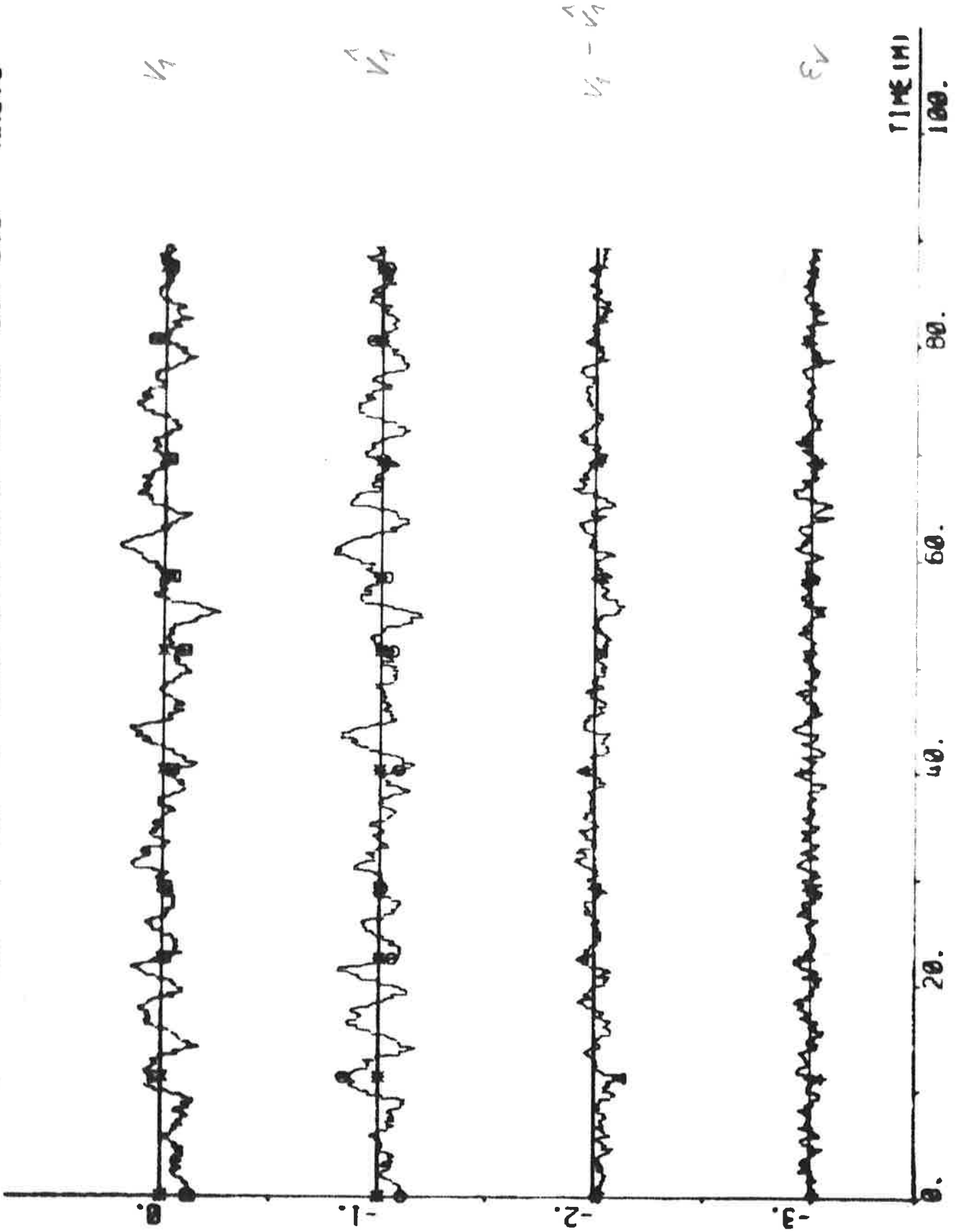




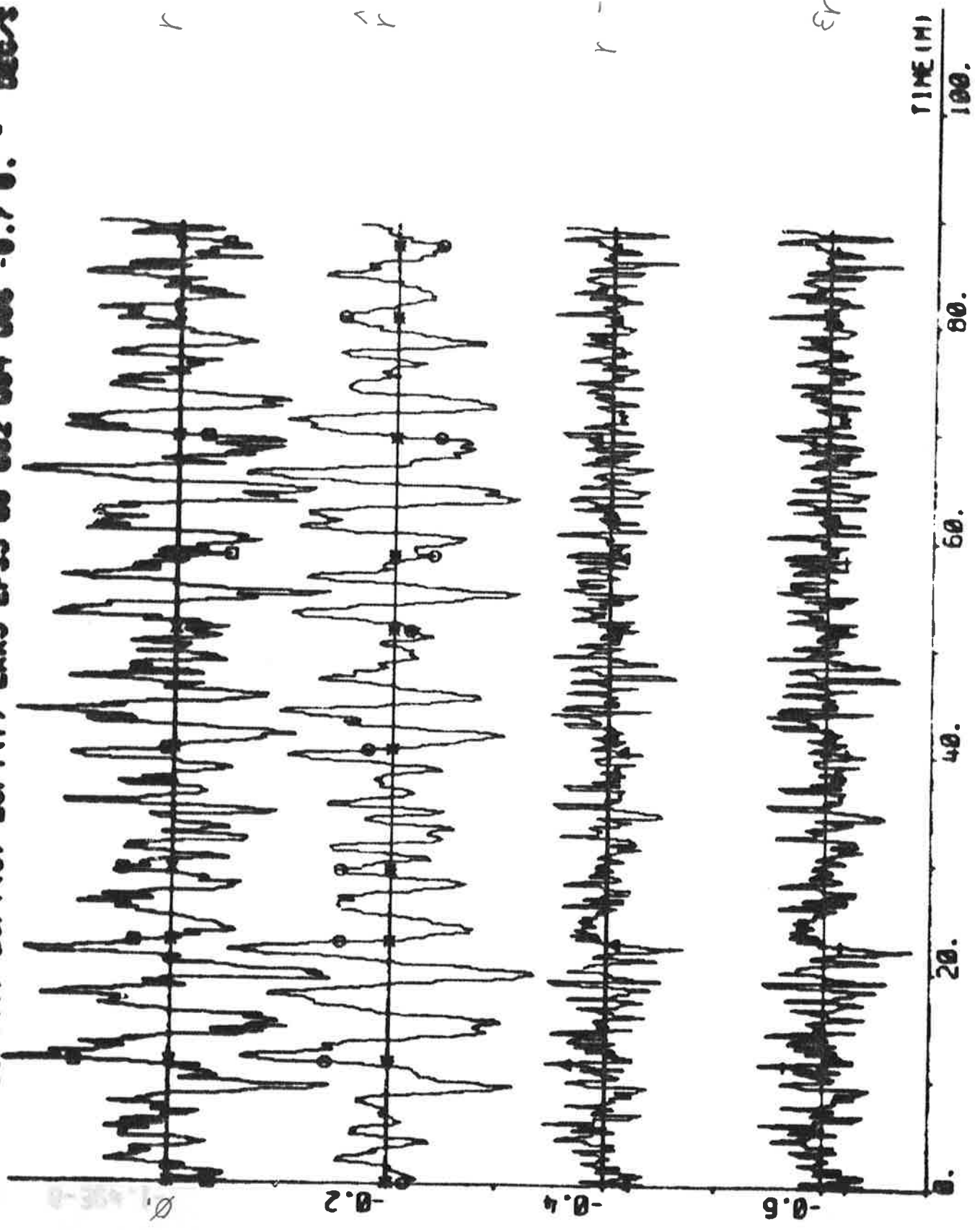
PL0T ESP1(1)-ESP2(1) ESP2(2) ERMS 00 01 02 -3.4 0.0 - KNOTS



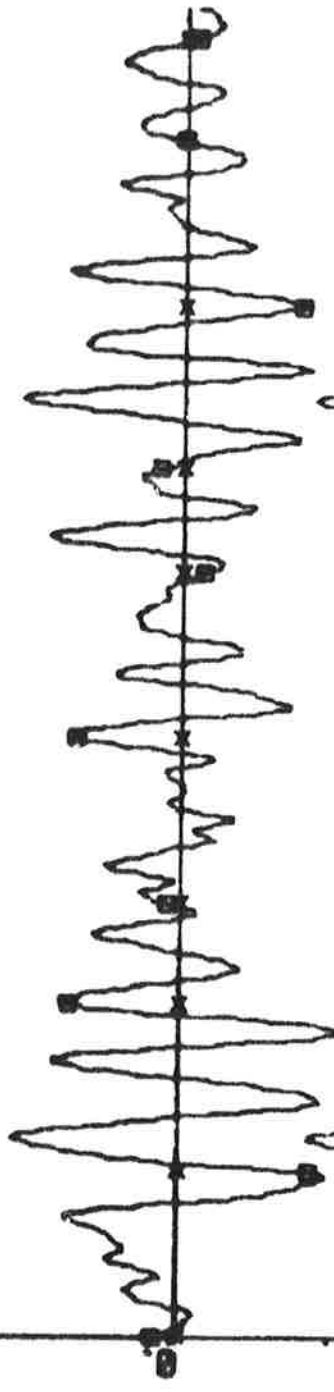
PLOT E3P1(1)-E3P1(4) E3P1(5) ERR2 EPS2 00 01 02 03 -3.4 0.0 - KNOTS



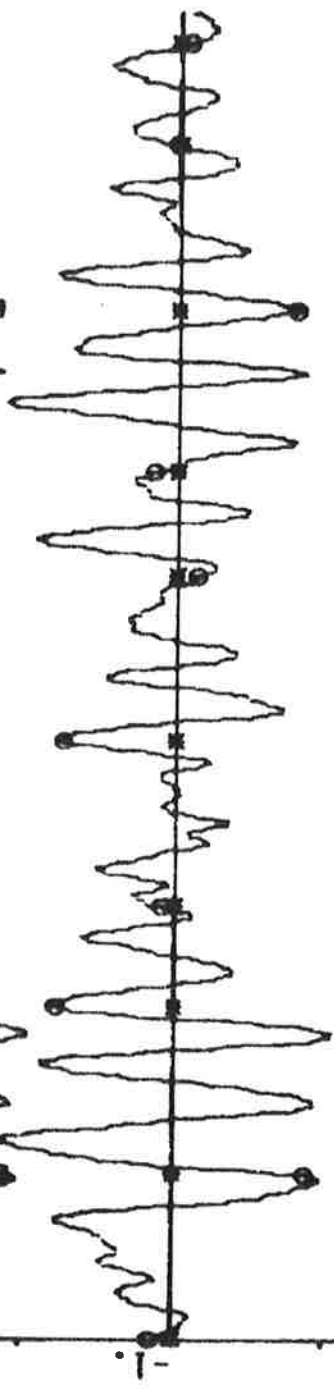
LOT ESP1(1)-ESP1(6) ESP1(7) ERR3 EPS3 00 002 004 006 -0.7 0. - DEG-S



PLOT E3P1(1)-E3P1(8) E3P1(9) ERR4 EPS4 00 01 02 03 -3.4 0.0 - DEG



$$0.1 * (\gamma - \gamma_{ref})$$



$$0.1 * (\gamma - \gamma_{ref})$$



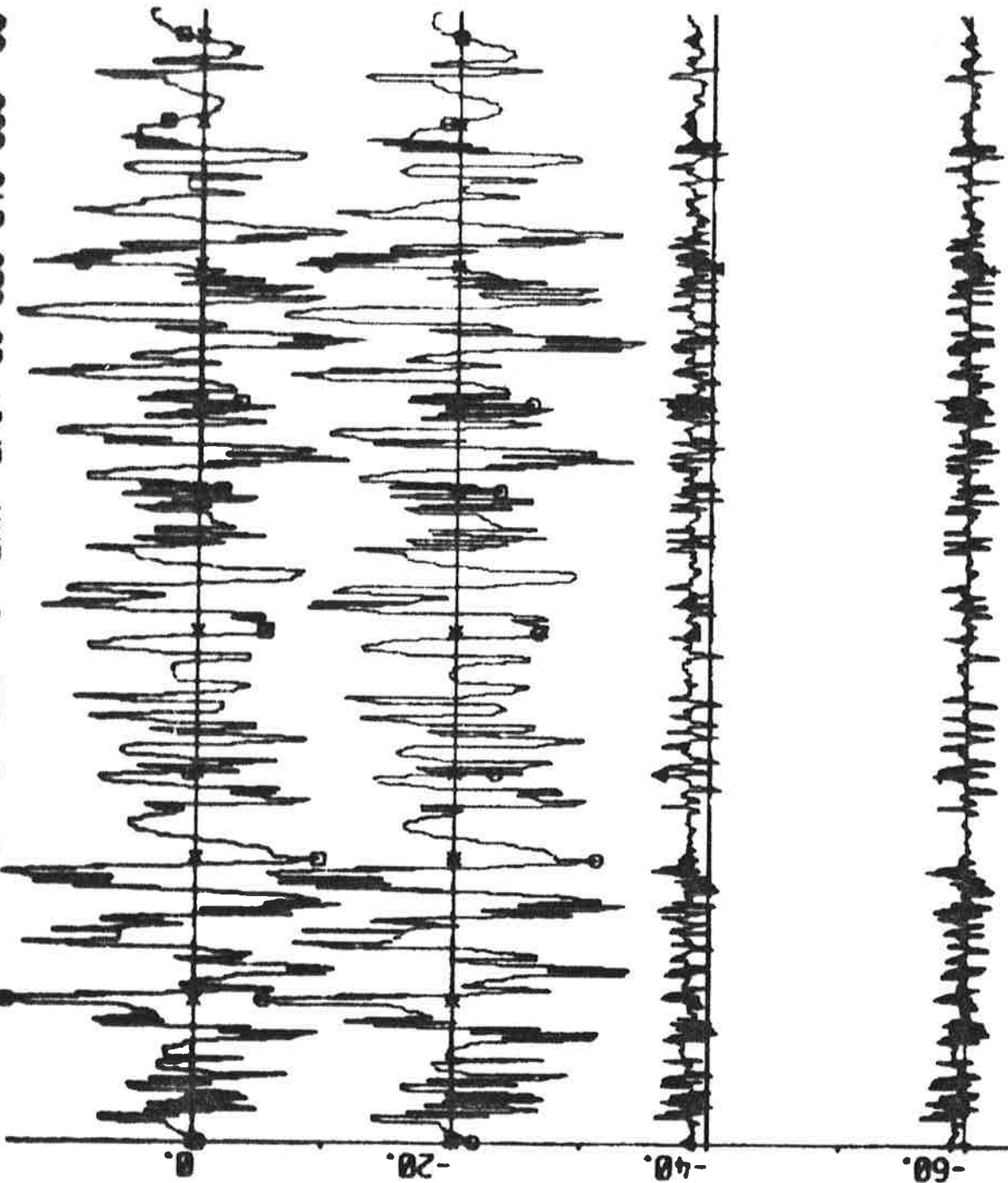
$$\gamma - \gamma^{\wedge}$$



$$E\gamma$$



PLOT EPI(1)-EPI(2) EPI(3) ERR1 EPS1 00 020 040 060 -05 15 - DEG



TIME (M)  
100.

80.

60.

40.

20.

0.

PLOT E3P1(1)-E3P2(3) E3P2(4) E3P2(5) E3P2(6) 00 02 04 06 -6.6 1.6

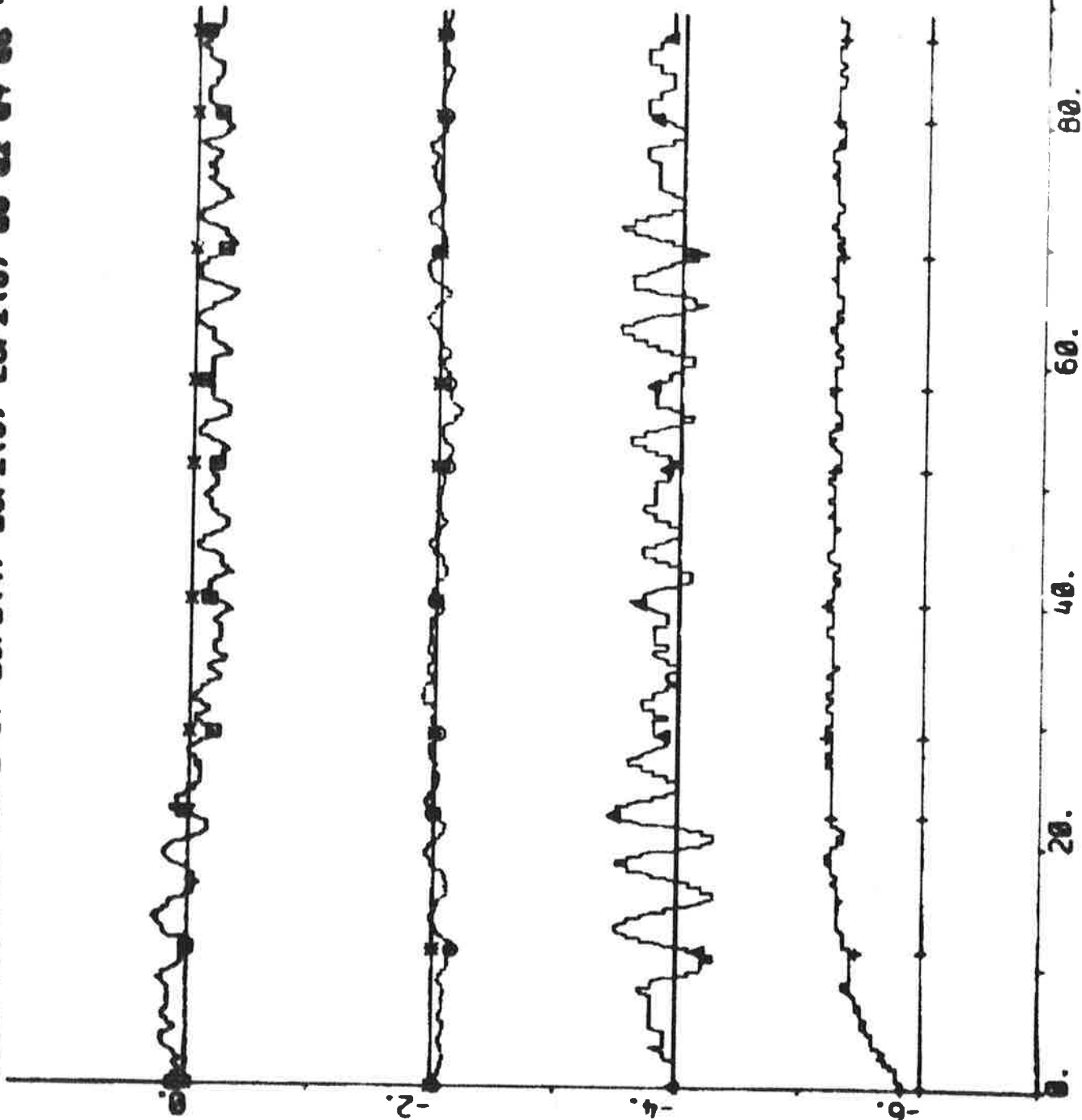
$0.5 \times \hat{d}_0$  deg

$\hat{v} \times dv$  knots

$100 \times \hat{d}_r$  deg/s

$0.5 \times \hat{d}_g$  deg

TIME (M)  
100.



## EXPERIMENT E4

Date	1976-04-30	Forward draught	10.9 m
Time	14.43	Aft draught	12.9 m
Duration	26 min	Wind direction	S (1; see App. A)
Position	S 22°36' E 08°18'	Wind velocity	11 m/s (strong breeze)
$\psi_{ref}$	146 deg	Wave height	-

Open loop experiment for identification

Tuning time for the Kalman filter before the experiment started: 0 min.

$\delta_{amp} = 10$ deg	$k_{id} = 0$
$T_s = 10$ s	IVVC = 1

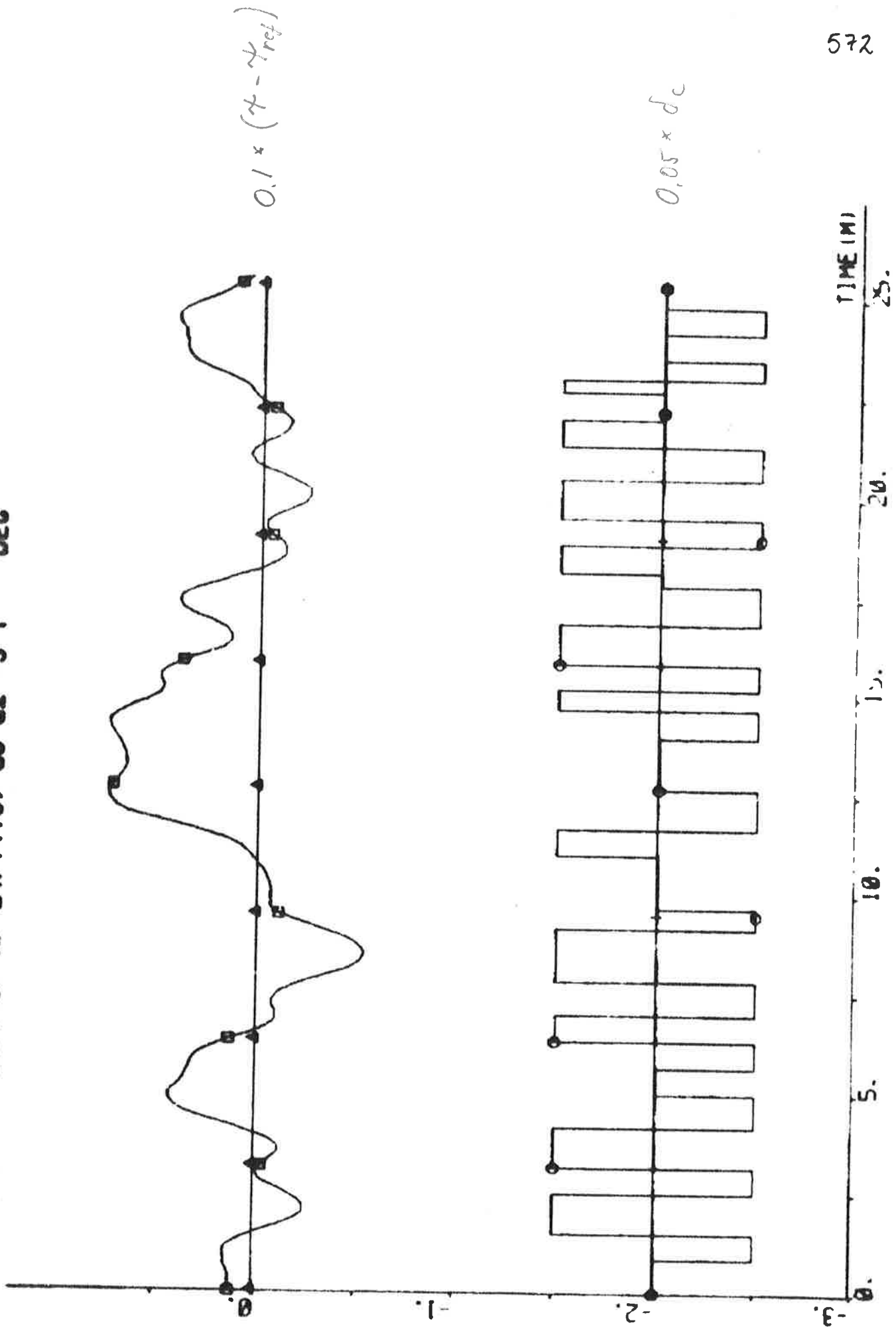
Final values:

$\hat{\delta}_0 = 0.1$  deg     $\hat{d}_v = 0.01$  knots     $\hat{d}_r = 0.001$  deg/s     $\hat{d}_\delta = 1.4$  deg

## Statistics

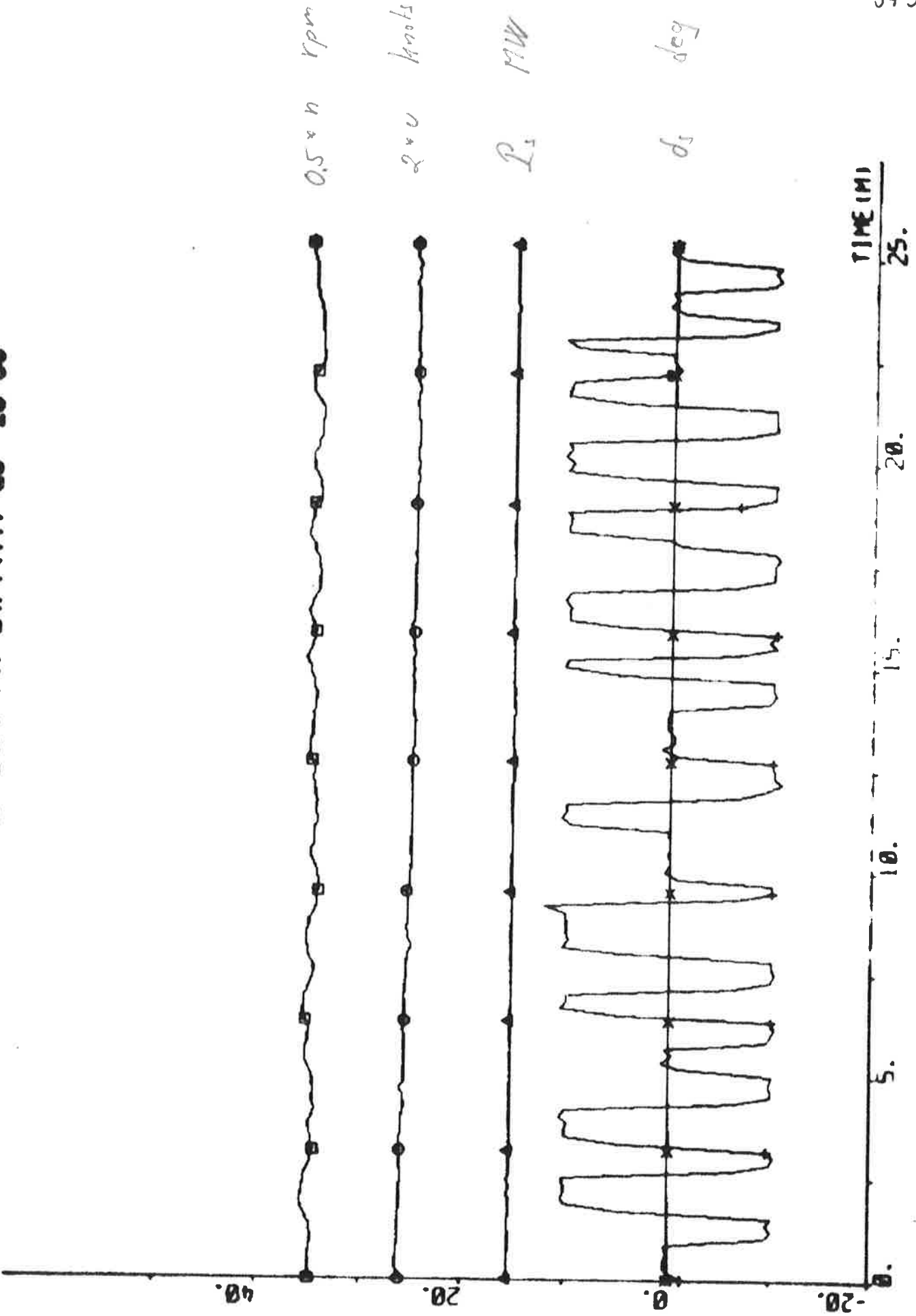
		Mean value	Stand. dev.			Mean value	Stand. dev.
$\delta_c$	deg	-0.52	8.59	r	deg/s	-0.001	0.061
$\delta_s$	deg	-0.57	8.17	$\hat{r}$	deg/s	-0.002	0.054
n	rpm	69.81	0.70	$r - \hat{r}$	deg/s	0.001	0.024
u	knots	12.69	0.20	$\epsilon_r$	deg/s	0.000	0.026
$P_s$	MW	15.34	0.08	$\psi - \psi_{ref}$	deg	1.393	3.058
v	knots	-0.013	0.332	$\hat{\psi} - \psi_{ref}$	deg	1.395	3.061
$\hat{v}$	knots	0.003	0.285	$\psi - \hat{\psi}$	deg	-0.001	0.040
$v - \hat{v}$	knots	-0.016	0.155	$\epsilon_\psi$	deg	-0.002	0.063
$v_1$	knots	-0.017	0.073	$\delta$	deg	0.92	7.35
$\hat{v}_1$	knots	-0.006	0.080	$\hat{\delta}$	deg	-0.48	8.00
$v_1 - \hat{v}_1$	knots	-0.010	0.040	$\delta - \hat{\delta}$	deg	1.40	2.45
$\epsilon_v$	knots	0.002	0.043	$\epsilon_\delta$	deg	0.36	2.49

PLOT E4P1(1)-E4P1(8) HP E4P1(10) 00 02 -3 1 - DEC

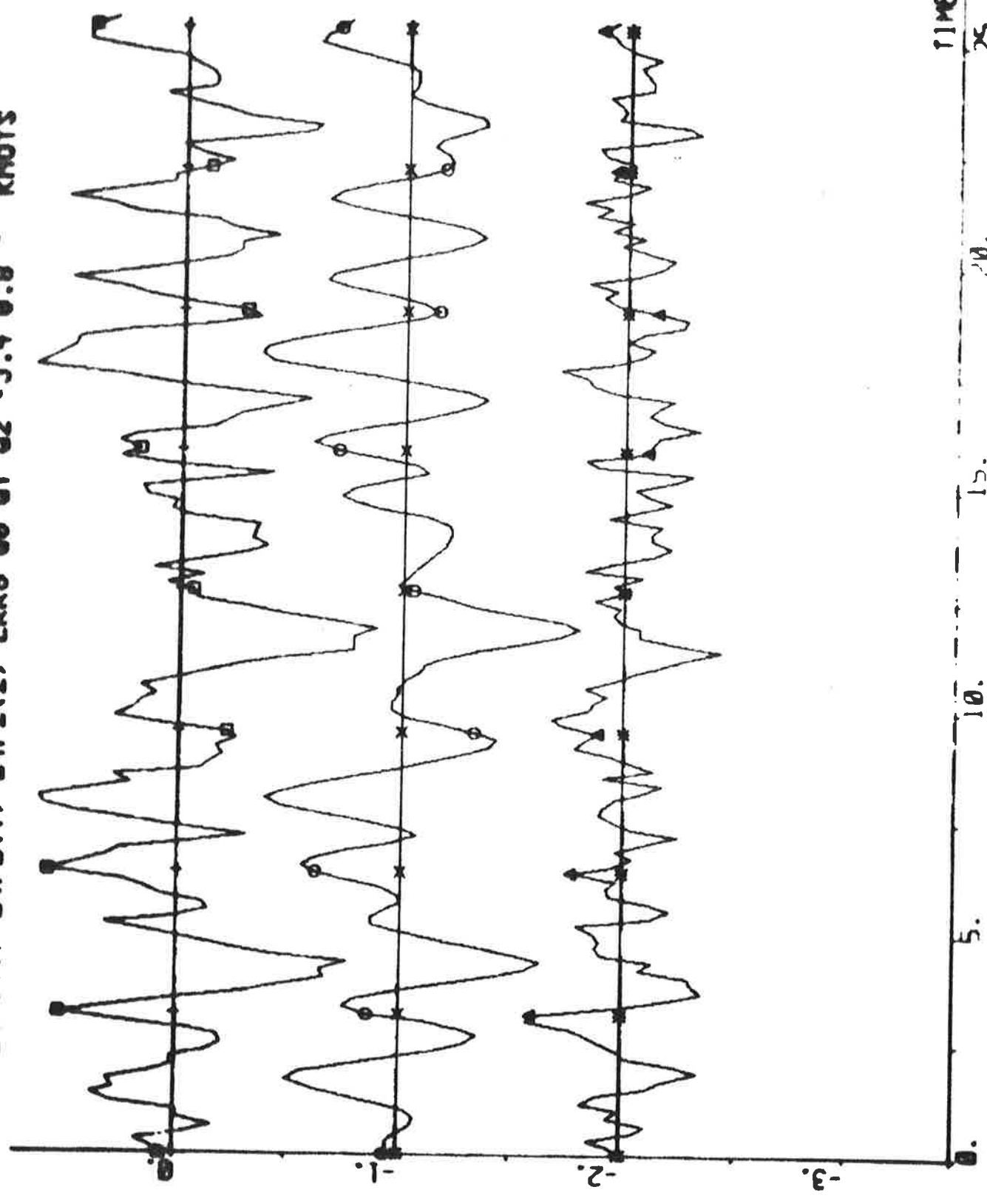




PLOT EXP1(1)-EXP1(13) EXP1(12) EXP1(14) EXP1(11) 00 -20 50



PLOT EYP1(1)-EYP2(1) EYP2(2) ERR6 00 01 02 -3.4 0.8 - KNOTS



TIME (M)  
25.

PLOT EYP1(1)-EYP1(4) EYP1(6) ERR2 EPS2 00 01 02 03 -3.4 0.0 - KNOTS

V1



V1



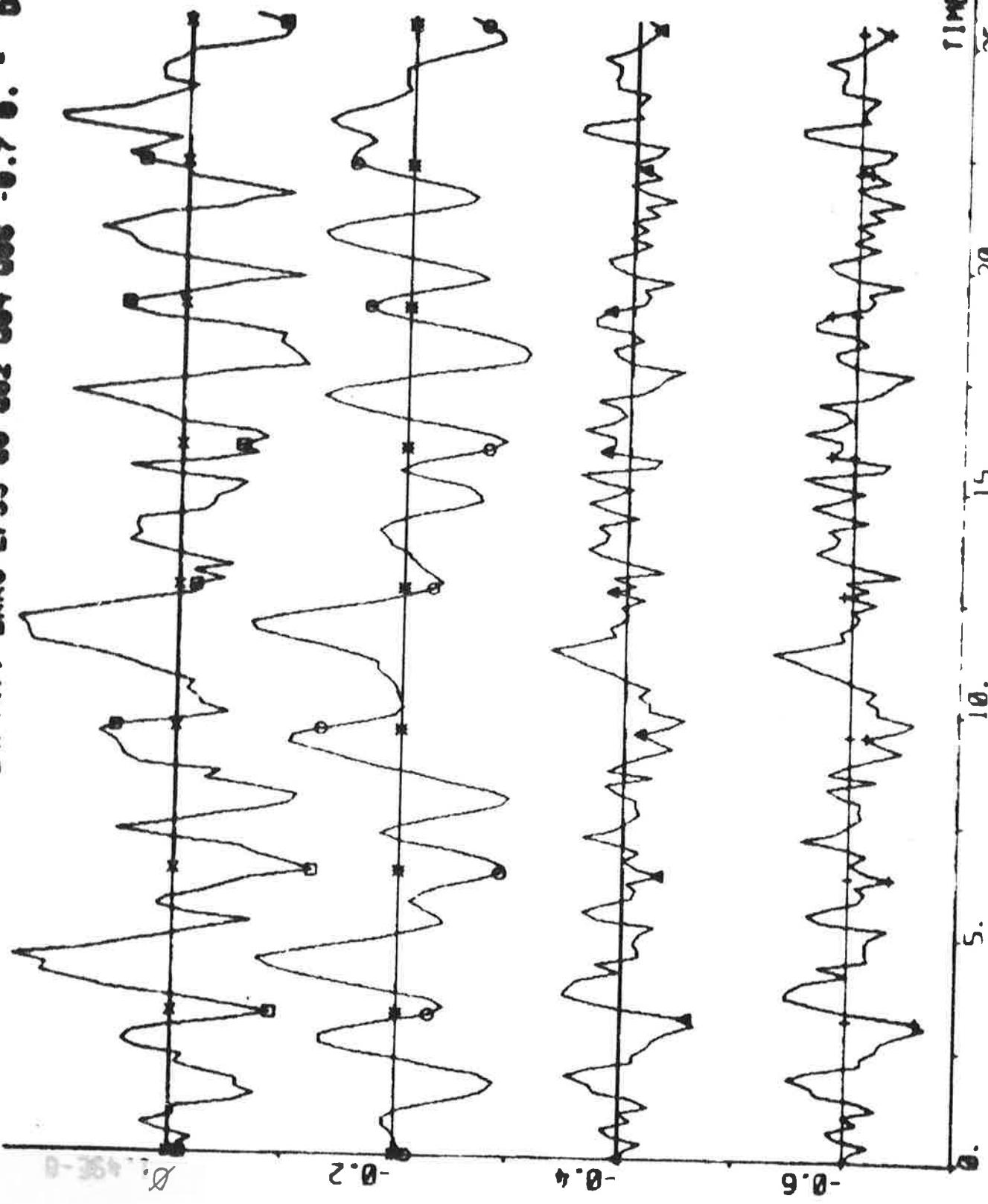
V1 - V1



V1



PLOT E4P1(1)-E4P1(8) E4P1(7) ERR3 EPS3 00 002 004 006 -0.7 0. - DEC 73



TIME (M) 25.

20.

15.

10.

5.

0.

-0.6

-0.4

-0.2

0.

r

r

r - r

Er

0.45E-0

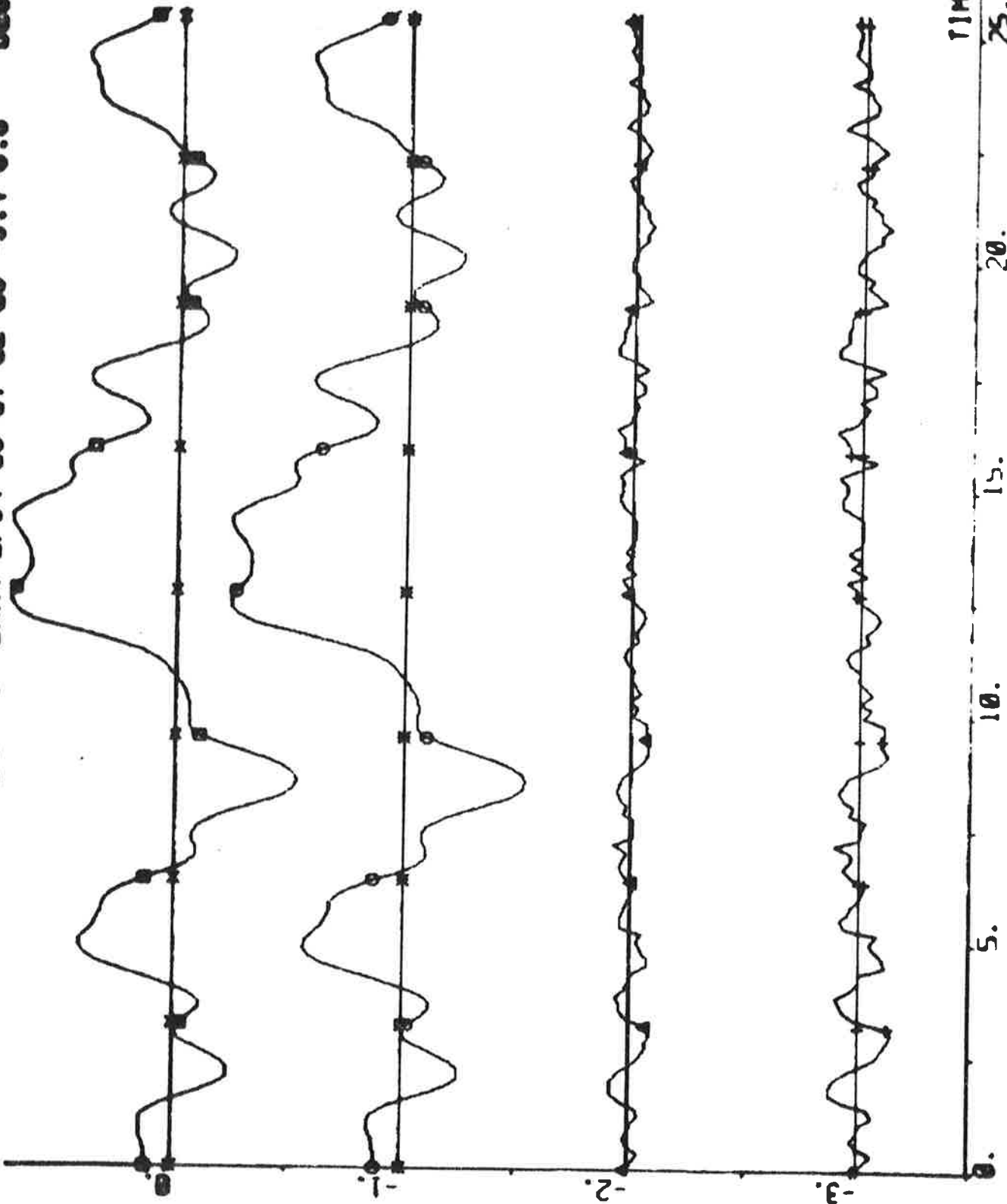
0.7

0.0

DEC 73

976

PLOT EYP1(1)-EYP1(8) EYP1(9) ERR4 EPS4 00 01 02 03 -3.4 0.0 . DEG



$0.1 \times (\gamma - \gamma_{ref})$

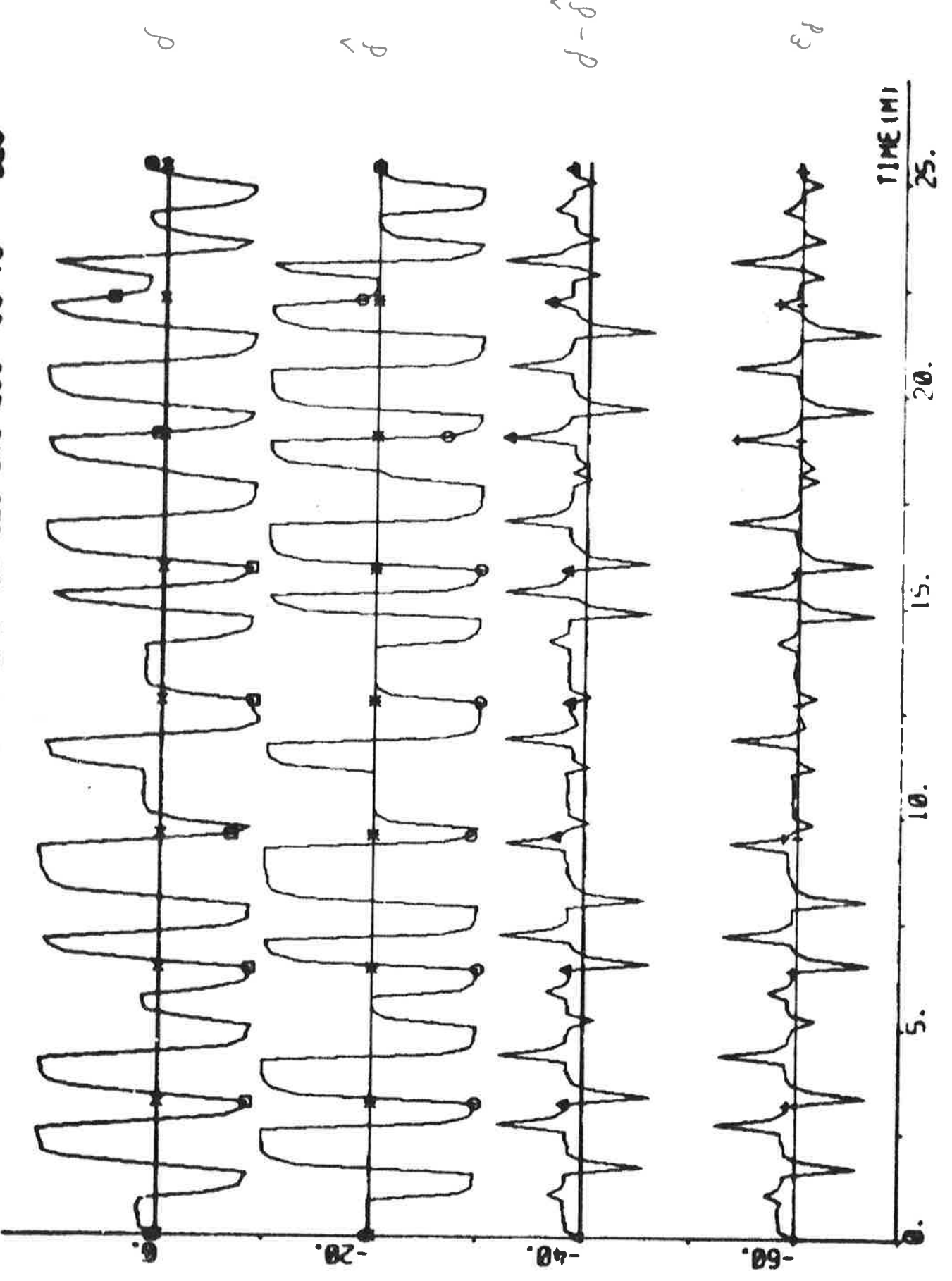
$0.1 \times (\beta - \beta_{ref})$

$\gamma - \gamma_{ref}$

$\epsilon$

TIME (M)  
25.

PLOT EXP1(1)-EXP1(2) EXP1(3) ERR1 EPS1 00 020 040 060 -65 15 - DEG



$\delta$

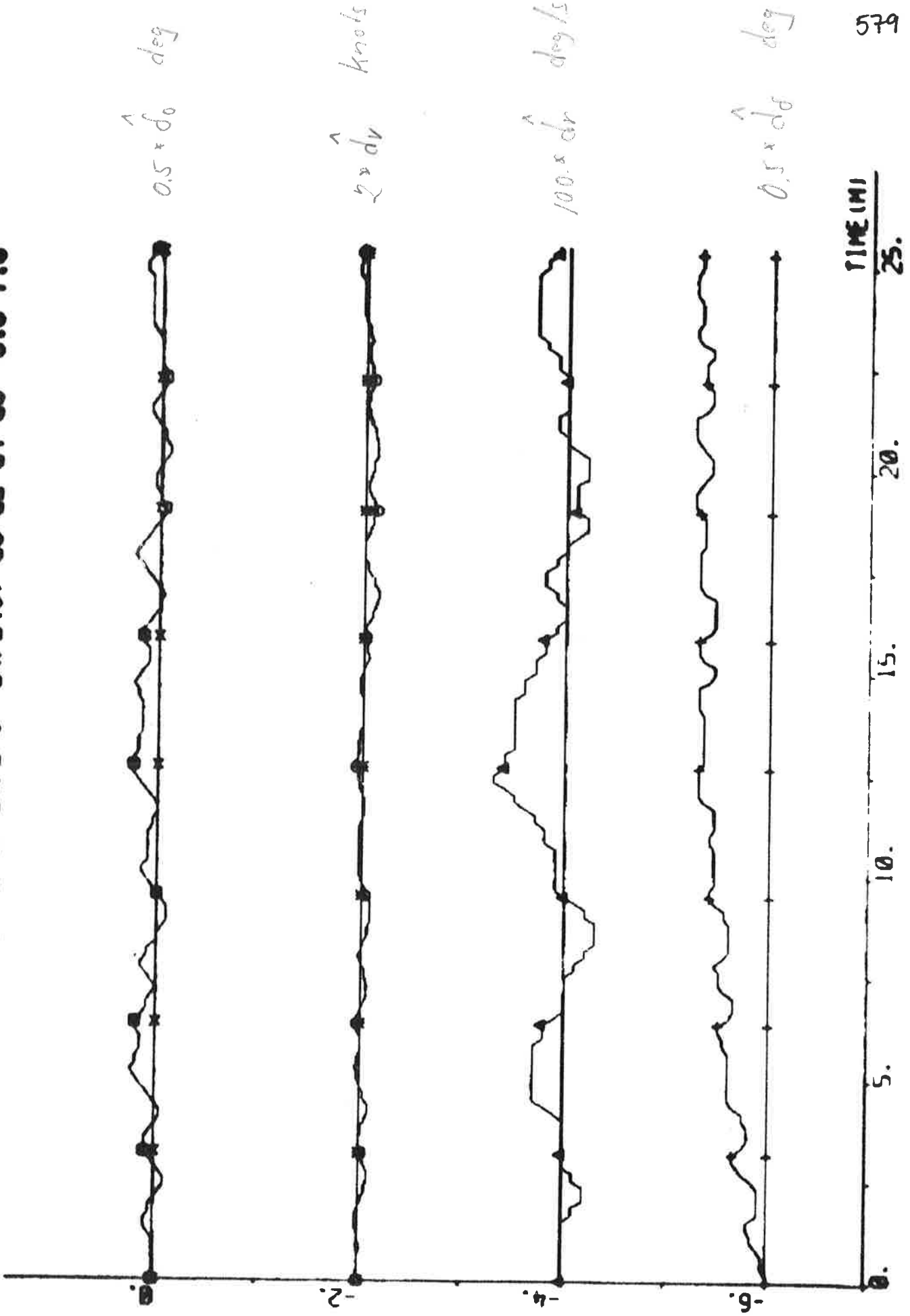
$p$

$\delta - \delta$

$\epsilon$

TIME (MI)  
25.

PLOT E4P1(1)-E4P2(3) E4P2(4) E4P2(5) E4P2(6) 00 02 04 06 -6.6 1.6



## EXPERIMENT F1

Date	1976-04-19	Forward draught	8.5 m
Time	09.03	Aft draught	12.5 m
Duration	60 min	Wind direction	E (6; see App. A)
Position	N 29°11' W 14°46'	Wind velocity	3 m/s (light breeze)
$\psi_{\text{ref}}$	204.4 - 204.5 deg (Sailmaster)	Wave height	-

The rudder limit was 15 deg during the experiment.

Self-tuning regulator using estimates from the Kalman filter

The sway velocity  $v_1$  was not used by the Kalman filter.

NC1 = 1	NC2 = 1	k = 7	q = 0
$T_s = 10$ s	$V_0 = 8$ m/s	IVVC = 3	$V_c = 7$ m/s

Final values:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ b_1 \\ b_2 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} -7.05 \\ 5.88 \\ 3.03 \\ -1.67 \\ 0.46 \\ 0.06 \\ 3.88 \\ 152.75 \end{bmatrix}$$

$$a_1 + a_2 + a_3 + a_4 = 0.19$$

$$\hat{\delta}_0 = 0.4 \text{ deg} \quad \hat{d}_v = - \quad \hat{d}_r = -0.001 \text{ deg/s} \quad \hat{d}_\delta = 1.3 \text{ deg}$$

Statistics

Mean value of $\delta$	= 1.71 deg	$n \approx 69.8$ rpm
Mean value of $\psi - \psi_{\text{ref}}$	= -0.002 deg	$u \approx 14.4$ knots
$V_2$	= 0.038	$P_s \approx 14.7$ MW



## EXPERIMENT F2

Date	1976-04-19	Forward draught	8.5 m
Time	10.20	Aft draught	12.5 m
Duration	49 min	Wind direction	E (7; see App. A)
Position	N 28°54' W 14°55'	Wind velocity	3 m/s (light breeze)
$\psi_{\text{ref}}$	204.1 - 204.3 deg (Sailmaster)	Wave height	-

The rudder limit was 15 deg during the experiment.

PID-regulator using non-filtered measurements

$k_P = 3$	$k_D = 75 \text{ s}$	$k_I = 0.02 \text{ 1/s}$	
$T_S = 10 \text{ s}$	$V_0 = 8 \text{ m/s}$	IVVC = 3	$V_C = 7 \text{ m/s}$

Standard parameter values were used.

Statistics

Mean value of $\delta$	= 1.86 deg	$n \approx 69.6 \text{ rpm}$
Mean value of $\psi - \psi_{\text{ref}}$	= -0.016 deg	$u \approx 14.4 \text{ knots}$
$V_2$	= 0.259	$P_s \approx 14.7 \text{ MW}$

## EXPERIMENT F3

Date	1976-04-29	Forward draught	10.9 m
Time	17.00	Aft draught	12.9 m
Duration	57 min	Wind direction	SE (1; see App. A)
Position	S 18°49' E 05°31'	Wind velocity	11 m/s (strong breeze)
$\psi_{\text{ref}}$	147 deg	Wave height	High sea from SE

The rudder limit was 15 deg during the experiment.

Self-tuning regulator using estimates from the Kalman filter.

NC1 = 1	NC2 = 1	k = 7	q = 0
$T_s = 10$ s	$V_0 = 6$ m/s	IVVC = 1	

Final values:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ b_1 \\ b_2 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} -11.47 \\ 13.11 \\ 0.48 \\ -2.31 \\ 0.58 \\ 0.31 \\ -0.07 \\ 79.27 \end{bmatrix}$$

$$a_1 + a_2 + a_3 + a_4 = -0.19$$

$$\hat{\delta}_0 = 0.2 \text{ deg} \quad \hat{d}_v = 0.13 \text{ knots} \quad \hat{d}_r = 0.001 \text{ deg/s} \quad \hat{d}_\delta = 1.5 \text{ deg}$$

Statistics

Mean value of  $\delta$  = 1.67 deg

$n \approx 69.1$  rpm

Mean value of  $\psi - \psi_{\text{ref}}$  = -0.073 deg

$u \approx 12.1$  knots

$V_2$  = 0.160

$P_s \approx 15.1$  MW

## EXPERIMENT F4

Date	1976-04-29	Forward draught	10.9 m
Time	18.04	Aft draught	12.9 m
Duration	54 min	Wind direction	SE (1; see App. A)
Position	S 18°49' E 05°31'	Wind velocity	11 m/s (strong breeze)
$\psi_{\text{ref}}$	147 deg	Wave height	High sea from SE

The rudder limit was 15 deg during the experiment.

PID-regulator using estimates from the Kalman filter

$k_P = 3$	$k_D = 140 \text{ s}$	$k_I = 0.02 \text{ 1/s}$
$T_S = 10 \text{ s}$	$V_0 = 6 \text{ m/s}$	IVVC = 1

The parameters were manually tuned before the experiment started.

Final values:

$$\hat{\delta}_0 = 0.0 \text{ deg} \quad \hat{d}_v = 0.14 \text{ knots} \quad \hat{d}_r = 0.000 \text{ deg/s} \quad \hat{d}_\delta = 1.5 \text{ deg}$$
Statistics

Mean value of $\delta$	= 1.54 deg	$n \approx 69.7 \text{ rpm}$
Mean value of $\psi - \psi_{\text{ref}}$	= -0.009 deg	$u \approx 12.2 \text{ knots}$
$V_2$	= 0.304	$P_s \approx 15.5 \text{ MW}$