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SIMULATION OF SHIP STEERING

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SIMULATION OF SHIP STEERING

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SIMULATION OF SHIP STEERING

Claes Källström

Abstract

Computer simulations of ship steering are presented. The ship model describes a tanker. An adaptive autopilot, consisting of a Kalman filter, a self-tuning regulator for steady state course keeping, and a turning regulator is tested in different load and speed conditions. Straight course keeping as well as turning is simulated. Comparisons to a conventional autopilot based on a PID-regulator is also performed.

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1. INTRODUCTION

Simulations of straight course keeping and yawing in different load and speed conditions with an autopilot consisting of a stationary Kalman filter, a self-tuning regulator and a yaw regulator are presented in this report. The simulations are performed on the computer UNIVAC 1108 by use of the interactive program SIMNON (see Elmqvist (1975)). The ship model used describes a 350 000 tdw tanker of Kockums' design.

The self-tuning regulator for straight course keeping is based on least squares identification and minimum variance control. A discrete, fixed gain PID-regulator is also used for comparison. The yaw regulator consists of different discrete fixed gain PID-regulators. The reference values used by the yaw regulator are the yaw rate and the heading angle. Either the non-filtered measurements or the Kalman filter estimates are used by the different regulators.

Simulation of ship steering is also discussed in Aspernäs and Foisack (1975), Aspernäs and Källström (1975), Källström (1976a) and (1976b). Full-scale experiments on 255 000 tdw tankers are described in Källström (1974) and (1975).

Listings of the programs used are given in the Appendix.

2. SHIP STEERING DYNAMICS

The following model, which describes a 350 000 tdw tanker of Kockums' design, is used in the simulations (cf. Norrbin (1970)):

$$\dot{\delta} = -\frac{1}{T_r} \delta + \frac{1}{T_r \cdot CRG} \delta_c$$

$$|\dot{\delta}| \leq \frac{1}{CRG} \delta_{lim}$$

$$\begin{aligned} (1 - x_{\dot{u}}'') \dot{u} = & \frac{1}{L} x_{u|u}'' |u| |u| + \frac{1}{L} x_{uu}'' u^2 + (1 + x_{vr}'') vr + \\ & + L(x_{rr}'' + x_G'') r^2 + \frac{1}{gL^2} x_{uvv|v}'' |u| v^2 |v| + \frac{1}{L} x_{u|u|\delta\delta}'' |u| |u| \delta^2 + \\ & + (1-t)(T/m) - F_w \cos\left(\frac{\alpha}{CRG} - \psi\right) \end{aligned}$$

$$\begin{aligned} (1 - y_{\dot{v}}'') \dot{v} = & (y_{ru}'' - 1) ru + \frac{1}{\sqrt{gL}} y_{ru|u}'' |ru| |u| + \frac{1}{L} y_{|u|v}'' |u| |v| + \\ & + \frac{1}{\sqrt{gL^3}} y_{u|u|v}'' |u| |u| |v| + \frac{1}{L} y_{v|v|}'' |v| |v| + y_{r|v|}'' |r| |v| + \\ & + y_{|r|v}'' |r| |v| + \frac{1}{L} y_{uu\delta}'' u^2 \delta + \frac{1}{L} y_{u|u|\delta}'' |u| |u| \delta + \\ & + y_{T\delta}'' (T/m) \delta + k_{TY} (T/m) - F_w \sin\left(\frac{\alpha}{CRG} - \psi\right) + w_1 \end{aligned}$$

$$\begin{aligned} (k_{zz}''^2 - N_{\dot{r}}'') \dot{r} = & \frac{1}{L} (N_{r|u}'' - x_G'') r |u| + \frac{1}{\sqrt{gL^3}} N_{ru|u}'' |ru| |u| + \\ & + \frac{1}{L^2} N_{uv}'' uv + \frac{1}{\sqrt{gL^5}} N_{u|u|v}'' |u| |u| |v| + \frac{1}{L^2} N_{v|v|}'' |v| |v| + \\ & + N_{r|r|}'' |r| |r| + \frac{1}{L} N_{r|v|}'' |r| |v| + \frac{1}{L} N_{|r|v}'' |r| |v| + \\ & + \frac{1}{L^2} N_{uu\delta}'' u^2 \delta + \frac{1}{L^2} N_{u|u|\delta}'' |u| |u| \delta + \frac{1}{L} N_{T\delta}'' (T/m) \delta + \\ & + \frac{1}{L} k_{TN} (T/m) + \frac{1}{L^2} F_w \ell_w \sin\left(\frac{\alpha}{CRG} - \psi\right) + w_2 \end{aligned}$$

$$\dot{\psi} = r \quad (2.1)$$

It is assumed that the number of propeller revolutions n is kept constant to a specified value n_0 by a regulator during all the simulations. The propeller thrust per mass unit (T/m) is computed as:

$$J = \frac{u (1-w) \cdot 60}{nD}$$

$$J' = \frac{J}{\sqrt{1 + J^2}}$$

$$K_T' = -0.33 \cdot J'^2 - 0.38 \cdot J' + 0.35 \quad (2.2)$$

$$T = K_T' \left(\frac{J}{J'} \right)^2 \rho_s n^2 D^4 / 3600$$

$$(T/m) = \frac{T}{\rho_s \nabla}$$

Notice that the terms $(T/m)\delta$ in (2.1) always are limited by the value $(T/m)_0\delta$, where $(T/m)_0$ is computed from (2.2) with the stationary forward speed corresponding to $n = n_0$ rpm.

Input signals:

rudder command (or rudder servo position)	δ_c	[deg]
number of propeller revolutions	n	[rpm]

States:

rudder angle	δ	[rad]
forward velocity	u	[m/s]
sway velocity	v	[m/s]
yaw rate	r	[rad/s]
heading angle	ψ	[rad]

Disturbances:

sway acceleration disturbance	w_1	$[m/s^2]$
disturbance of yaw angle acceleration	w_2	$[rad/s^2]$

Other notations:

time constant of rudder servo	T_r	$[s]$
limit of rudder turning rate	δ_{lim}	$[deg/s]$
length of ship	L	$[m]$
acceleration of gravity	g	$[m/s^2]$
propeller thrust per mass unit	T/m	$[m/s^2]$
wind force per mass unit	F_w	$[m/s^2]$
lever arm of wind force	ℓ_w	$[m]$
angle of wind direction	α	$[deg]$
conversion factor rad $\rightarrow$ deg	CRG	$[deg]$

The following parameter values are used:

$$\begin{aligned}
 T_r &= 5 \text{ s} \\
 \delta_{lim} &= 2.32 \text{ deg/s} \\
 L &= 350 \text{ m} \\
 g &= 9.80665 \text{ m/s}^2 \\
 F_w &= 0.002 \text{ m/s}^2 \\
 \ell_w &= 25 \text{ m} \\
 \alpha &= 90 \text{ deg} \\
 CRG &= 57.2958 \text{ deg}
 \end{aligned}$$

The values of the other parameters are given in Dyne and Trägårdh (1975). Two different load conditions are considered corresponding to the mean draught $T = 22.3 \text{ m}$ (full load, forward and aft draught equal to 22.3 m) and $T = 10.5 \text{ m}$ (ballast, forward and aft draught equal to

9.0 m and 12.0 m, resp.). The forward speed u which corresponds to $n = 87.6$ rpm is equal to 15.8 knots when $T = 22.3$ m and equal to 17.25 knots when $T = 10.5$ m. If the model (2.1) and (2.2) is linearized, the following transfer function relating the yaw rate r to the rudder angle δ is obtained:

$$G(s) = \frac{K(1 + sT_3)}{(1 + sT_1)(1 + sT_2)} \quad (2.3)$$

If the forward speed u is assumed to be constant and equal to 15.8 knots, then the following parameter values of (2.3) are obtained when $T = 22.3$ m:

$$\begin{aligned} K &= -0.0161 \text{ 1/s} \\ T_1 &= -110.1 \text{ s} \\ T_2 &= 18.3 \text{ s} \\ T_3 &= 54.3 \text{ s} \end{aligned} \quad (2.4)$$

The corresponding values when $u = 17.25$ knots and $T = 10.5$ m are:

$$\begin{aligned} K &= -0.0707 \text{ 1/s} \\ T_1 &= -337.1 \text{ s} \\ T_2 &= 19.9 \text{ s} \\ T_3 &= 69.5 \text{ s} \end{aligned} \quad (2.5)$$

Notice that the sign of the rudder angle in the model is chosen in such a way that a positive rudder angle (starboard rudder) gives a positive yaw rate (starboard yaw). From (2.4) and (2.5) it can be concluded that the tanker is unstable in full load condition as well as in ballast condition.

The disturbance signals w_1 and w_2 are obtained as white, gaussian noise. The covariance matrix of the white noise vector, which generates w_1 and w_2 , is

$$R_w = \begin{pmatrix} 0.8 \cdot 10^{-4} & 0 \\ 0 & 0.53 \cdot 10^{-10} \end{pmatrix} \quad (2.6)$$

The measured outputs from the model (2.1) and (2.2) are

$$\begin{aligned} \delta_m &= \text{CRG} \cdot \delta + d_\delta + e_1 \\ v_1 &= \bar{v}_1 + d_v + e_2, & \bar{v}_1 &= \text{CMK} \cdot (v + \ell_1 r) \\ r_m &= \text{CRG} \cdot r + d_r + e_3 \\ \psi_m &= \text{CRG} \cdot \psi + e_4 \\ u_m &= \text{CMK} \cdot u \\ n_m &= n \end{aligned}$$

where e_1, e_2, e_3 and e_4 are white, gaussian measurement noise with covariance matrix

$$R_e = \begin{pmatrix} 0.04 & 0 & 0 & 0 \\ 0 & 0.0025 & 0 & 0 \\ 0 & 0 & 0.0004 & 0 \\ 0 & 0 & 0 & 0.0025 \end{pmatrix} \quad (2.7)$$

The measured rudder angle δ_m [deg], sway velocity of bow v_1 [knots], yaw rate r_m [deg/s], heading angle ψ_m [deg], forward speed u_m [knots] and number of propeller revolutions n_m [rpm] are used by the autopilot. The conversion factor from m/s to knots CMK is equal to 1.943844 and the lever arm ℓ_1 is equal to 164.35 m. The measurement biases are assigned the following values:

$$\begin{aligned} d_\delta &= 2 \text{ deg} \\ d_v &= 0.5 \text{ knots} \\ d_r &= 0.05 \text{ deg/s} \end{aligned} \quad (2.8)$$

It should be pointed out that the model of the disturbances is extremely simplified. A more realistic approach is given in Berlekom, Trägårdh, and Dellhag (1975). Notice, however,

that the disturbances applied in the simulations describe a rather rough weather condition.

The program of the ship model, TANK3, is given in the Appendix. Almost the same model was used in the simulations of Källström (1976b).

3. AUTOPILOT

The structure of the autopilot is shown in Fig. 3.1. To obtain a good performance in all speeds, the autopilot performs a speed scaling using the speed V_s [m/s]. V_s is computed every second according to

$$V_s = 0.18 \cdot n_m / \text{CMK} \quad (3.1)$$

The signals δ_m , v_1 , r_m , ψ_m , u_m and n_m are measured every second. The Kalman filtering as well as the computation of v_m [knots] according to

$$v_m = v_1 - \text{CMK} \cdot \ell_1 \cdot r_m / \text{CRG} \quad (3.2)$$

are performed every second. Either the estimates from the Kalman filter $\hat{v}$, $\hat{r}$ and $\hat{\psi}$ or the measurements v_m , r_m and ψ_m are used by the self-tuning regulator, the PID-regulator, and the yaw regulator. The rudder command δ_c is computed with sampling interval 10 s or 15 s. The PID-regulator for straight course keeping is only used for comparison. The reference course ψ_{ref} and the reference yaw rate r_{ref} for yawing as well as the rudder limit δ_{ℓ} are also used by the autopilot. The complete autopilot is implemented by the Fortran subroutines AUTP3 and STUR given in the Appendix.

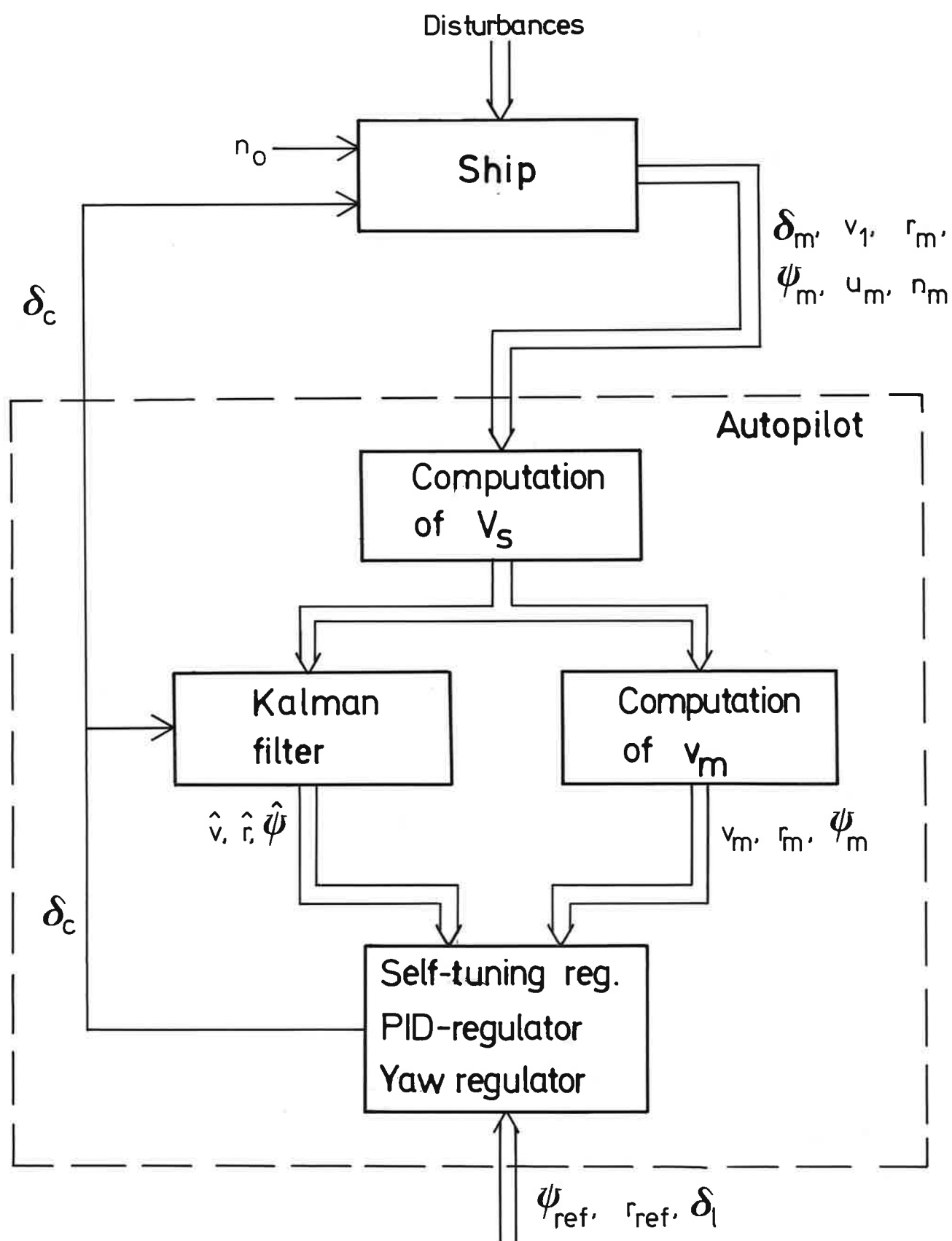


Fig. 3.1 - Structure of the autopilot.

3.1 Kalman Filter

The following linear model is used when designing the Kalman filter:

$$\begin{cases} dx = Ax dt + Bu dt + dw \\ y(t_k) = \theta x(t_k) + \tilde{e}(t_k), \quad k = 0, 1, 2, \dots \end{cases} \quad (3.3)$$

where $\{ w(t), t_0 \leq t \leq \infty \}$ is assumed to be a wiener process with incremental covariance $R_1 dt$ and the measurement errors $\{ \tilde{e}(t_k) \}$ are assumed to be independent and gaussian with zero mean and covariance $\tilde{R}_2$. It is furthermore assumed that the measurement errors are independent of $\{ w(t), t_0 \leq t \leq \infty \}$. The vectors and matrices of (3.3) are explained by:

$$x^T = \begin{bmatrix} v & r & \psi & \delta_d & \delta_0 & d_v / \text{CMK} & d_r / \text{CRG} & d_\delta / \text{CRG} \end{bmatrix}$$

$$u = \delta_c / \text{CRG}$$

$$y^T = \begin{bmatrix} \delta_m / \text{CRG} & v_1 / \text{CMK} & r_m / \text{CRG} & \psi_m / \text{CRG} \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} \frac{V}{L} & a_{12} V & 0 & b_{11} \frac{V^2}{L} & 0 & 0 & 0 & 0 \\ a_{21} \frac{V}{L^2} & a_{22} \frac{V}{L} & 0 & b_{21} \frac{V^2}{L^2} & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{T_r} & -\frac{1}{T_r} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{T_r} & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\theta = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & \ell_1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{diag}(R_1) = \begin{bmatrix} r_1 & r_2 & r_3 & r_4 & r_5 & r_6 & r_7 & r_8 \end{bmatrix}$$

$$\text{diag}(\tilde{R}_2) = \begin{bmatrix} (\sigma_\delta / \text{CRG})^2 & (\sigma_v / \text{CMK})^2 & (\sigma_r / \text{CRG})^2 & (\sigma_\psi / \text{CRG})^2 \end{bmatrix}$$

where δ_0 [rad] is the rudder bias due to disturbances, δ_d [rad] is equal to $\delta - \delta_0$ and d_v [knots], d_r [deg/s] and d_δ [deg] are measurement biases (cf (2.8)). The total speed is denoted V [m/s]. The parameters are assigned the following values (cf Chapter 2):

$$\text{CRG} = 57.2958 \text{ deg}$$

$$\text{CMK} = 1.943844 \text{ knots} \cdot \text{s/m}$$

$$L = 350 \text{ m}$$

$$\ell_1 = 164.35 \text{ m}$$

$$T_r = 5 \text{ s}$$

$$a_{11} = Y''_{|u|v} / (1 - Y''_v) = -0.385$$

$$a_{12} = (Y''_{ru} - 1) / (1 - Y''_v) = -0.451$$

$$a_{21} = N''_{uv} / (k''_{zz} - N''_r) = -3.398$$

$$a_{22} = (N''_{r|u|} - x''_G) / (k''_{zz} - N''_r) = -1.583$$

$$b_{11} = (Y''_{uu\delta} + Y''_{u|u|\delta}) / (1 - Y''_v) = -0.0967$$

$$b_{21} = (N''_{uu\delta} + N''_{u|u|\delta}) / (k''_{zz} - N''_r) = 0.806$$

$$r_1 = 2.5 \cdot 10^{-5} \text{ m}^2/\text{s}^3$$

$$r_2 = 5 \cdot 10^{-9} \text{ 1/s}^3$$

$$r_3 = 8 \cdot 10^{-8} \text{ 1/s}$$

$$\begin{aligned}
r_4 &= 8 \cdot 10^{-8} \text{ 1/s} \\
r_5 &= 3 \cdot 10^{-9} \text{ 1/s} \\
r_6 &= 9 \cdot 10^{-8} \text{ m}^2/\text{s}^3 \\
r_7 &= 3 \cdot 10^{-12} \text{ 1/s}^3 \\
r_8 &= 8 \cdot 10^{-11} \text{ 1/s} \\
\left. \begin{aligned}
\sigma_\delta &= 0.2 \text{ deg} \\
\sigma_v &= 0.05 \text{ knots} \\
\sigma_r &= 0.02 \text{ deg/s} \\
\sigma_\psi &= 0.05 \text{ deg}
\end{aligned} \right\} \text{ cf. (2.7)}
\end{aligned}$$

The dynamics of (3.3) is equivalent to the linearization of the model (2.1) when parameter values for full load condition are used. By performing some calculations it can be concluded that the model (3.3) is observable, if the heading measurement ψ_m is one of the outputs. This means that one or more of the measurement signals δ_m , v_1 and r_m can be rejected and still it is possible to obtain good state estimates, at least if the dynamics and the disturbances are modelled reasonably well.

The model (3.3) is now normalized using the length of the ship L as unit of length and the time to cover the length L , i.e. L/V , as unit of time:

$$\begin{cases} dx' = A'x'dt' + B'u'dt' + dw' \\ y'(t'_k) = \theta'x'(t'_k) + \tilde{e}'(t'_k), \quad k = 0, 1, 2, \dots \end{cases} \quad (3.4)$$

where

$$(x')^T = \begin{bmatrix} v' & r' & \psi' & \delta'_d & \delta'_0 & d'_v & d'_r & d'_\delta \end{bmatrix}$$

$$u' = \delta'_c$$

$$(Y')^T = \begin{bmatrix} \delta'_m & v'_1 & r'_m & \psi'_m \end{bmatrix}$$

$$A' = \begin{bmatrix} a_{11} & a_{12} & 0 & b_{11} & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & b_{21} & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{L}{V} \cdot \frac{1}{T_r} & -\frac{L}{V} \cdot \frac{1}{T_r} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$(B')^T = \begin{bmatrix} 0 & 0 & 0 & \frac{L}{V} \cdot \frac{1}{T_r} & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\theta' = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & \ell_1/L & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{diag } (R'_1) = \begin{bmatrix} \frac{L}{V^3} r_1 & \frac{L^3}{V^3} r_2 & \frac{L}{V} r_3 & \frac{L}{V} r_4 & \frac{L}{V} r_5 & \frac{L}{V^3} r_6 & \frac{L^3}{V^3} r_7 & \frac{L}{V} r_8 \end{bmatrix}$$

$$\text{diag } (\tilde{R}'_2) = \begin{bmatrix} (\sigma_\delta/\text{CRG})^2 & (\sigma_v/(\text{CMK} \cdot V))^2 & (\sigma_r \cdot L/(\text{CRG} \cdot V))^2 & (\sigma_\psi/\text{CRG})^2 \end{bmatrix}$$

The normalized variables are obtained as

$$\begin{aligned}
t' &= \frac{V}{L} \cdot t \\
t'_k &= \frac{V}{L} \cdot t_k \\
v' &= \frac{1}{V} v \\
r' &= \frac{L}{V} r \\
\psi' &= \psi \\
\delta'_d &= \delta_d \\
\delta'_0 &= \delta_0 \\
d'_v &= \frac{1}{CMK \cdot V} d_v \\
d'_r &= \frac{L}{CRG \cdot V} d_r \\
d'_\delta &= d_\delta / CRG \\
\delta'_c &= \delta_c / CRG \\
\delta'_m &= \delta_m / CRG \\
v'_1 &= \frac{1}{CMK \cdot V} v_1 \\
r'_m &= \frac{L}{CRG \cdot V} r_m \\
\psi'_m &= \psi_m / CRG
\end{aligned} \tag{3.5}$$

The normalized model (3.4) is now transformed to a discrete model with sampling interval $h' = \frac{V}{L} h$, where $h = 1$ s:

$$\begin{cases} x'(t'+h') = \Phi' x'(t') + \Gamma' u'(t') + \tilde{w}'(t') \\ y'(t') = \Theta' x'(t') + \tilde{e}'(t') \end{cases} \tag{3.6}$$

Since the value of h' is rather small (e.g. $h' = 0.023$ if $V = 8$ m/s), the following approximations are not too bad:

$$\begin{aligned}
\Phi' &\approx I + A'h' \\
\Gamma' &\approx B'h' \\
\tilde{R}_1 &\approx R_1'h'
\end{aligned} \tag{3.7}$$

It can be concluded using (3.7) that the speed dependence of Φ' , Γ' and Θ' in (3.6) is rather insignificant. Some

elements of the covariance matrices $\tilde{R}_1'$ and $\tilde{R}_2'$, however, are dependent on the speed V .

If it is assumed that $V = 8$ m/s, the following matrices are obtained (no approximations):

$$\Phi' = \begin{bmatrix} 0.99163 & -1.00810 \cdot 10^{-2} & 0 & -2.08207 \cdot 10^{-3} & 2.12358 \cdot 10^{-4} & 0 & 0 & 0 \\ -7.59537 \cdot 10^{-2} & 0.96485 & 0 & 1.64706 \cdot 10^{-2} & -1.70991 \cdot 10^{-3} & 0 & 0 & 0 \\ -8.74514 \cdot 10^{-4} & 2.24515 \cdot 10^{-2} & 1 & 1.95418 \cdot 10^{-4} & -1.32723 \cdot 10^{-5} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.81873 & -0.18127 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$(\Gamma')^T = \begin{bmatrix} -2.12358 \cdot 10^{-4} & 1.70991 \cdot 10^{-3} & 1.32723 \cdot 10^{-5} & 0.18127 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Theta' = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0.46957 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\tilde{R}_1' = \text{(see page 16)}$$

$$\text{diag} (\tilde{R}_2') = \begin{bmatrix} 1.21847 \cdot 10^{-5} & 1.03380 \cdot 10^{-5} & 2.33223 \cdot 10^{-4} & 7.61544 \cdot 10^{-7} \end{bmatrix}$$

$$\tilde{\mathbf{R}}_1' = \begin{bmatrix} 3.87637 \cdot 10^{-7} & -6.23157 \cdot 10^{-8} & -8.42885 \cdot 10^{-10} & -7.45031 \cdot 10^{-11} & 2.14470 \cdot 10^{-13} & 0 & 0 & 0 \\ -6.23157 \cdot 10^{-8} & 9.23544 \cdot 10^{-6} & 1.05534 \cdot 10^{-7} & 6.00259 \cdot 10^{-10} & -1.74199 \cdot 10^{-12} & 0 & 0 & 0 \\ -8.42885 \cdot 10^{-10} & 1.05534 \cdot 10^{-7} & 8.16225 \cdot 10^{-8} & 4.57698 \cdot 10^{-12} & -1.00657 \cdot 10^{-14} & 0 & 0 & 0 \\ -7.45031 \cdot 10^{-11} & 6.00259 \cdot 10^{-10} & 4.57698 \cdot 10^{-12} & 6.59705 \cdot 10^{-8} & -2.80961 \cdot 10^{-10} & 0 & 0 & 0 \\ 2.14470 \cdot 10^{-13} & -1.74199 \cdot 10^{-12} & -1.00657 \cdot 10^{-14} & -2.80961 \cdot 10^{-10} & 3.00000 \cdot 10^{-9} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.40625 \cdot 10^{-9} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 5.74218 \cdot 10^{-9} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8.00000 \cdot 10^{-11} \end{bmatrix}$$

Using (3.6) the Kalman filter equations are obtained as (cf. Åström (1970)):

$$\begin{aligned}\hat{\mathbf{x}}'(t'|t'-h') &= \Phi' \hat{\mathbf{x}}'(t'-h'|t'-h') + \Gamma' u'(t'-h') \\ \hat{\mathbf{x}}'(t'|t') &= \hat{\mathbf{x}}'(t'|t'-h') + K \varepsilon'(t') \\ y'(t') &= \theta' \hat{\mathbf{x}}'(t'|t'-h') + \varepsilon'(t')\end{aligned}\quad (3.8)$$

where

$$(\varepsilon')^T = \begin{bmatrix} \varepsilon'_\delta & \varepsilon'_v & \varepsilon'_r & \varepsilon'_\psi \end{bmatrix} \quad (3.9)$$

and

$$K = \begin{bmatrix} -1.29 \cdot 10^{-4} & 9.73 \cdot 10^{-2} & -2.40 \cdot 10^{-2} & -0.369 \\ 5.31 \cdot 10^{-4} & 0.481 & 9.61 \cdot 10^{-2} & 1.01 \\ 1.22 \cdot 10^{-5} & 7.90 \cdot 10^{-3} & 3.12 \cdot 10^{-3} & 0.326 \\ 1.60 \cdot 10^{-2} & -2.13 \cdot 10^{-4} & 1.65 \cdot 10^{-3} & 1.89 \cdot 10^{-2} \\ -5.82 \cdot 10^{-12} & 3.08 \cdot 10^{-4} & -1.63 \cdot 10^{-3} & -1.88 \cdot 10^{-2} \\ -3.69 \cdot 10^{-5} & 9.39 \cdot 10^{-3} & 1.61 \cdot 10^{-4} & 8.17 \cdot 10^{-5} \\ -1.40 \cdot 10^{-6} & -1.56 \cdot 10^{-3} & 3.01 \cdot 10^{-3} & -5.78 \cdot 10^{-2} \\ 2.54 \cdot 10^{-3} & 2.57 \cdot 10^{-6} & 3.31 \cdot 10^{-6} & 3.66 \cdot 10^{-5} \end{bmatrix} \quad (3.10)$$

The speed V_s (cf. (3.1)) is used to normalize the measurement vector, i.e.

$$(y')^T = \begin{bmatrix} \delta_m / \text{CRG} & \frac{1}{\text{CMK} \cdot V_s} v_l & \frac{L}{\text{CRG} \cdot V_s} r_m & \psi_m / \text{CRG} \end{bmatrix} \quad (3.11)$$

The state estimate vector

$$(\hat{\mathbf{x}}')^T = \begin{bmatrix} \hat{v}' & \hat{r}' & \hat{\psi}' & \hat{\delta}'_d & \hat{\delta}'_0 & \hat{d}'_v & \hat{d}'_r & \hat{d}'_\delta \end{bmatrix} \quad (3.12)$$

is updated every second. The initial state estimate vector is equal to

$$(\hat{x}'(t_0))^T = \begin{bmatrix} \frac{1}{CMK \cdot V_S} v_m & \frac{L}{CRG \cdot V_S} r_m & \psi_m/CRG & \delta_m/CRG & 0 & 0 & 0 & 0 \end{bmatrix}$$

where v_m is obtained from (3.2) and all the measurements are from the time t_0 .

If

$$|\varepsilon'_\delta| > t'_\delta,$$

$$|\varepsilon'_V| > t'_V,$$

$$|\varepsilon'_r| > t'_r,$$

or

$$|\varepsilon'_\psi| > t'_\psi,$$

where

$$t'_\delta = 0.75$$

$$t'_V = 0.06$$

$$t'_r = 0.25$$

$$t'_\psi = 0.015,$$

(3.13)

then the corresponding measurement or measurements are rejected, when the state estimate vector is updated. A measurement signal is definitively rejected when 10 consecutive measurements are rejected. However, during the first 900 s after the Kalman filter is initialized, no measurements are rejected because the bias states of the Kalman filter must be fairly estimated to avoid incorrect rejectings. The rejecting of a measurement is performed by putting the corresponding column of K (see (3.10)) equal to zero. Notice that the values of (3.13) are chosen very large to avoid rejectings when the Kalman filter is tested.

The non-normalized state estimate vector is obtained as (cf. (3.12)):

$$(\hat{\mathbf{x}})^T = \begin{bmatrix} \hat{v} & \hat{r} & \hat{\psi} & \hat{\delta}_d & \hat{\delta}_0 & \hat{d}_v & \hat{d}_r & \hat{d}_\delta \end{bmatrix} \quad (3.14)$$

where

$$\hat{v} = \text{CMK} \cdot V_s \cdot \hat{v}', \quad [\text{knots}]$$

$$\hat{r} = \frac{\text{CRG} \cdot V_s}{L} \hat{r}', \quad [\text{deg/s}]$$

$$\hat{\psi} = \text{CRG} \cdot \hat{\psi}', \quad [\text{deg}]$$

$$\hat{\delta}_d = \text{CRG} \cdot \hat{\delta}_d', \quad [\text{deg}]$$

$$\hat{\delta}_0 = \text{CRG} \cdot \hat{\delta}_0', \quad [\text{deg}]$$

$$\hat{d}_v = \text{CMK} \cdot V_s \cdot \hat{d}_v', \quad [\text{knots}]$$

$$\hat{d}_r = \frac{\text{CRG} \cdot V_s}{L} \hat{d}_r', \quad [\text{deg/s}]$$

$$\hat{d}_\delta = \text{CRG} \cdot \hat{d}_\delta', \quad [\text{deg}]$$

Notice the following expressions, too:

$$\hat{v}_1 = \hat{v} + \text{CMK} \cdot \ell_1 \cdot \hat{r} / \text{CRG} \quad [\text{knots}]$$

$$\hat{\delta} = \hat{\delta}_d + \hat{\delta}_0 \quad [\text{deg}]$$

3.2 Self-tuning Regulator

A simple self-tuning regulator based on least squares identification and minimum variance control is used for straight course keeping. The basic self-tuning regulator is described in Wittenmark (1973).

The following model of the ship is used by the self-tuning regulator:

$$\begin{aligned}
 & \left(\hat{\psi}(t) - \psi_{\text{ref}} \right) + a_1 \left(\hat{\psi}(t-k-1) - \psi_{\text{ref}} \right) + \dots + \\
 & + a_{\text{NA}} \left(\hat{\psi}(t-k-\text{NA}) - \psi_{\text{ref}} \right) = \\
 & = \left(V_s(t-k-1)/V_0 \right)^2 b_0 \nabla \delta_c(t-k-1) + \\
 & + \left(V_s(t-k-2)/V_0 \right)^2 b_0 b_1 \nabla \delta_c(t-k-2) + \dots + \\
 & + \left(V_s(t-k-\text{NB}-1)/V_0 \right)^2 b_0 b_{\text{NB}} \nabla \delta_c(t-k-\text{NB}-1) + \\
 & + V_s(t-k-1) c_1 \nabla \hat{v}(t-k-1) + c_2 \nabla \hat{r}(t-k-1) + \varepsilon(t)
 \end{aligned} \tag{3.15}$$

where the design speed is denoted V_0 [m/s] and the speed V_s [m/s] is defined by (3.1). The minimum variance control is given by

$$\begin{aligned}
 \nabla \delta_c(t) = & \left(\frac{V_0}{V_s(t)} \right)^2 \frac{1}{b_0} \left[a_1 \left(\hat{\psi}(t) - \psi_{\text{ref}} \right) + \dots + \right. \\
 & + a_{\text{NA}} \left(\hat{\psi}(t - \text{NA} + 1) - \psi_{\text{ref}} \right) - \\
 & - \left(V_s(t-1)/V_0 \right)^2 b_0 b_1 \nabla \delta_c(t-1) - \dots - \\
 & - \left(V_s(t-\text{NB})/V_0 \right)^2 b_0 b_{\text{NB}} \nabla \delta_c(t-\text{NB}) - \\
 & \left. - V_s(t) c_1 \nabla \hat{v}(t) - c_2 \nabla \hat{r}(t) \right]
 \end{aligned} \tag{3.16}$$

where

$$\nabla \delta_c(t) = \delta_c(t) - \delta_c(t-1)$$

$$\nabla \hat{v}(t) = \hat{v}(t) - \hat{v}(t-1)$$

$$\nabla \hat{r}(t) = \hat{r}(t) - \hat{r}(t-1)$$

Notice that the speed scaling of (3.15) and (3.16) is introduced in such a way that the parameters $a_1, \dots, a_{NA}$, $b_1, \dots, b_{NB}$, c_1 and c_2 are approximately independent of the forward speed. This fact will, of course, simplify the mission of the self-tuning regulator.

The following standard values are used:

$$NA = 4$$

$$NB = 2$$

$$k = 7$$

$$T_s = 10 \text{ s}$$

$$\lambda_f = 0.99$$

$$b_0 = 1$$

$$V_0 = 8 \text{ m/s}$$

where T_s is the sampling interval and λ_f the exponential forgetting factor. Other values of k and T_s are sometimes used. The following initial values of the parameters and of the covariance matrix P are used when the Kalman filter estimates are fed into the self-tuning regulator:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ b_1 \\ b_2 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} -6.91 \\ 5.95 \\ 3.88 \\ -3.57 \\ 0.48 \\ 0.11 \\ -2.10 \\ 34.73 \end{bmatrix} \quad P = \begin{bmatrix} 1 & & & & & & & \\ & 1 & & & & & & \\ & & 1 & & & & & \\ & & & 1 & & & & \\ & & & & 1 & & & \\ & & & & & 0.01 & & \\ & & & & & & 0.01 & \\ & & & & & & & 1 \\ & & & & & & & & 100 \end{bmatrix} \quad (3.17)$$

When non-filtered measurements are used instead of Kalman filter estimates, then $c_1 = c_2 = 0$, i.e. no feedforward signals. The following initial values are then used:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} -7.64 \\ 8.44 \\ 1.74 \\ -3.15 \\ 0.00834 \\ 0.195 \end{bmatrix} \quad P = \begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & 1 & \\ & & & & & 0.01 \\ & & & & & & 0.01 \end{bmatrix} \quad (3.18)$$

By use of the minimum variance control (3.16) the following criterion is minimized:

$$J_1 = \sum_{n=k+1}^N \left(\hat{\psi}(nT_s) - \psi_{ref} \right)^2 \quad (3.19)$$

If the criterion

$$J_2 = \sum_{n=k+1}^N \left[\left(\hat{\psi}(nT_s) - \psi_{ref} \right)^2 + q_2 \left(\nabla \delta_c((n-k-1)T_s) \right)^2 \right] \quad (3.20)$$

is minimized instead, a penalty on the rudder motions is introduced by the parameter q_2 . However, a proper solution

of this problem requires the solving of a Riccati equation. A self-tuning regulator, which performs this, is used in Källström (1976a).

If the criterion (3.20) is modified to read

$$J_3(n) = \left(\hat{\psi}((n+k+1)T_s) - \psi_{ref} \right)^2 + q_2 \left(\nabla \delta_c(nT_s) \right)^2 \quad (3.21)$$

$$n = 0, 1, \dots, N-k-1$$

and if (3.21) is minimized at every sample event, then a simpler regulator is obtained. By inserting (3.15) into (3.21) and then performing the minimization, the following control is obtained:

$$\overline{\nabla \delta_c(t)} = \frac{(v_s(t)/v_0)^4 b_0^2}{(v_s(t)/v_0)^4 b_0^2 + q_2} \nabla \delta_c(t) \quad (3.22)$$

where $\nabla \delta_c(t)$ is the minimum variance control given by (3.16). If $q_2 = 0$, then minimization of (3.21) gives the same result as minimization of (3.19) and consequently the controls (3.22) and (3.16) are equivalent. Notice that (3.22) only is a very small modification of (3.16) and that the identification part of the self-tuning regulator is unchanged. However, the control (3.22) has the serious disadvantage that no guarantee of closed loop stability is obtained in the general case.

The minimum variance control (3.16) is approximately scaled by $(v_0/v_s(t))^2$ when the speed changes, i.e. (3.22) may be re-written

$$\overline{\nabla \delta_c(t)} = \frac{b_0^2}{b_0^2 + \left(\frac{v_0}{v_s(t)} \right)^4 q_2} \left(\frac{v_0}{v_s(t)} \right)^2 \left[\nabla \delta_c(t) \right]_{v_0} \quad (3.23)$$

where $[\nabla \delta_c(t)]_{v_0}$ denotes the minimum variance control when $v_s = v_0$. By introducing $q_2^* = (v_0/v_s(t))^4 q_2$ we obtain

$$\begin{aligned}
 \overline{\nabla \delta_c(t)} &= \frac{b_0^2}{b_0^2 + q_2^*} \left(\frac{v_0}{v_s(t)} \right)^2 [\nabla \delta_c(t)]_{v_0} = \\
 &= \frac{b_0^2}{b_0^2 + q_2^*} \nabla \delta_c(t)
 \end{aligned} \tag{3.24}$$

which is the actual control used in the autopilot. The standard value of q_2^* is equal to zero.

The estimates from the Kalman filter are used in all formulas of this section. Notice, however, that it is possible to use the non-filtered measurements instead.

3.3 PID-regulator

The following discrete PID-regulator for straight course keeping is also implemented for comparison:

$$\delta_c(nT_s) = - \left(\frac{V_0}{V_s(nT_s)} \right)^2 \left[k_P (\hat{\psi}(nT_s) - \psi_{ref}) + k_D \hat{r}(nT_s) + k_I T_s \sum_{i=0}^{n-1} (\hat{\psi}(iT_s) - \psi_{ref}) \right] \quad (3.25)$$

$$n = 0, 1, 2, \dots$$

The following standard values are used:

$$\begin{aligned} k_P &= 3 \\ k_D &= 75 \text{ s} \\ k_I &= 0.02 \text{ 1/s} \\ T_s &= 10 \text{ s} \\ V_0 &= 8 \text{ m/s} \end{aligned} \quad (3.26)$$

The scaling speed V_s is obtained from (3.1). Notice that it is possible to use the non-filtered measurements instead of the Kalman filter estimates in (3.25). The special speed scaling used in (3.25) will approximately give the same course keeping performance independent of the forward speed. The rudder deviations, however, are increased proportional to $(V_0/V_s)^2$ when the speed is decreased.

3.4 Yaw Regulator

A yaw performed by the yaw regulator consists of four different phases, viz. the initial phase (phase 1), the phase of constant yaw rate (phase 2), the checking rudder phase (phase 3) and the terminating phase (phase 4). However, if the requested heading change $\Delta\psi_{\text{ref}}$ is small, one or more of the phases may be skipped. The Kalman filter estimates used by the yaw regulator are the yaw rate $\hat{r}$ and the heading $\hat{\psi}$, and the reference values used are the requested yaw rate r_{ref} and the new requested heading ψ_{ref} . The phase of straight course keeping is denoted phase 0.

Modified discrete, fixed gain PID-regulators are used in the different phases (note that $n = 0, 1, 2, \dots$):

Phase 1:

$$\delta_c(nT_s) = - \left(\frac{v_0}{v_s(nT_s)} \right)^2 k_4 \left[\hat{r}(nT_s) - r_0 \right] + \bar{\delta}_c$$

$$\left| k_4 \left[\hat{r}(nT_s) - r_0 \right] \right| \leq \left| \bar{c}_1 r_0 \right|$$

Phase 2:

$$\delta_c(nT_s) = - \left(\frac{v_0}{v_s(nT_s)} \right)^2 k_5 \left[\hat{r}(nT_s) - r_0 \right] -$$

$$- \left(\frac{v_0}{v_s(nT_s)} \right)^2 k_6 T_s \sum_{i=0}^{n-1} \left[\hat{r}(iT_s) - r_0 \right] + \bar{\delta}_c$$

Phase 3:

$$\begin{aligned}\delta_c(nT_s) &= - \left(\frac{V_0}{V_s(nT_s)} \right)^2 k_7 \left[\hat{\psi}(nT_s) - \psi_{\text{ref}} \right] - \\ &\quad - \left(\frac{V_0}{V_s(nT_s)} \right)^2 k_8 \hat{r}(nT_s) \\ \left| k_7 \left[\hat{\psi}(nT_s) - \psi_{\text{ref}} \right] + k_8 \hat{r}(nT_s) \right| &\leq \left| \bar{c}_3 r_0 \right|\end{aligned}$$

Phase 4:

$$\begin{aligned}\delta_c(nT_s) &= - \left(\frac{V_0}{V_s(nT_s)} \right)^2 k_1 \left[\hat{\psi}(nT_s) - \psi_{\text{ref}} \right] - \\ &\quad - \left(\frac{V_0}{V_s(nT_s)} \right)^2 k_2 \hat{r}(nT_s) - \\ &\quad - \left(\frac{V_0}{V_s(nT_s)} \right)^2 k_3 T_s \sum_{i=0}^{n-1} \left[\hat{\psi}(iT_s) - \psi_{\text{ref}} \right]\end{aligned}$$

The scaling speed V_s is obtained from (3.1). The moving average $\bar{\delta}_c$ of the rudder commands δ_c is only updated during phase 0 and phase 4:

$$\begin{aligned}\bar{\delta}_c((k+1)T_s) &= \bar{\delta}_c(kT_s) + \left(\frac{1-\gamma}{k+1} + \gamma \right) \left(\delta_c(kT_s) - \bar{\delta}_c(kT_s) \right), \\ k &= 0, 1, 2, \dots\end{aligned}$$

$$\bar{\delta}_c(0) = 0$$

The reference yaw rate r_0 including sign is computed once, when the yaw is initiated, as

$$r_0 = r_{\text{ref}} \quad \text{if} \quad \hat{\psi} - \psi_{\text{ref}} \leq 0$$

or as

$$r_0 = -r_{\text{ref}} \quad \text{if} \quad \hat{\psi} - \psi_{\text{ref}} > 0$$

Notice that the value of r_{ref} always is positive.

The conditions to jump from one phase to another read:

Phase 0 → phase 4:

$$\psi_1 < \Delta\psi_{\text{ref}} \leq \psi_2$$

Phase 0 → phase 1:

$$\Delta\psi_{\text{ref}} > \psi_2$$

Phase 1 → phase 2:

$$r_0 \geq 0 \quad \text{and} \quad \hat{r} - r_0 > -\varepsilon_1$$

or

$$r_0 < 0 \quad \text{and} \quad \hat{r} - r_0 < \varepsilon_1$$

or

$$(\text{time in phase 1}) > T_1$$

Phase 1 or 2 → phase 3:

$$r_0 \geq 0 \quad \text{and} \quad -\bar{c}_2 \hat{r} < \hat{\psi} - \psi_{\text{ref}}$$

or

$$r_0 < 0 \quad \text{and} \quad -\bar{c}_2 \hat{r} > \hat{\psi} - \psi_{\text{ref}}$$

Phase 3 → phase 4:

$$|\hat{r}| < \varepsilon_2$$

or

$$r_0 \geq 0 \quad \text{and} \quad \hat{\psi} - \psi_{\text{ref}} > -\varepsilon_3$$

or

$$r_0 < 0 \quad \text{and} \quad \hat{\psi} - \psi_{\text{ref}} < \varepsilon_3$$

or

$$(\text{time in phase 3}) > T_3$$

Phase 4 → phase 0:

$$(\text{time in phase 4}) > T_4$$

The condition to remain in phase 0 is:

$$\Delta\psi_{\text{ref}} \leq \psi_1$$

If the reference yaw rate r_{ref} is changed during a yaw, the new value is immediately used, but no other changes. It is also possible to change the reference course ψ_{ref} during a yaw, and then a new yaw is initiated by entering phase 1, if

$$|\Delta\psi_{\text{ref}}| > \psi_3$$

and

(3.27)

the actual phase is 3 or 4

or if

$$|\Delta\psi_{\text{ref}}| > \psi_3,$$

the actual phase is 1 or 2,

and one of the two conditions

(3.28)

$$r_0 > 0, \quad \hat{\psi} - \psi_{\text{ref}} > 0$$

and

$$r_0 < 0, \quad \hat{\psi} - \psi_{\text{ref}} < 0$$

is satisfied.

If neither condition (3.27) nor condition (3.28) is fulfilled, the new value of ψ_{ref} is used, but no other changes.

The following parameter values of the yaw regulator are used:

$k_1 = 5$	$\bar{c}_1 = 60 \text{ s}$
$k_2 = 200 \text{ s}$	$\bar{c}_2 = 50 \text{ s}$
$k_3 = 0.005 \text{ 1/s}$	$\bar{c}_3 = 60 \text{ s}$
$k_4 = 200 \text{ s}$	$T_1 = 30 \text{ s}$
$k_5 = 200 \text{ s}$	$T_3 = 100 \text{ s}$
$k_6 = 8$	$T_4 = 300 \text{ s}$
$k_7 = 2$	$\psi_1 = 0.35 \text{ deg}$
$k_8 = 200 \text{ s}$	$\psi_2 = 2.5 \text{ deg}$
$\varepsilon_1 = 0 \text{ deg/s}$	$\psi_3 = 2.5 \text{ deg}$
$\varepsilon_2 = 0.02 \text{ deg/s}$	$V_0 = 8 \text{ m/s}$
$\varepsilon_3 = 1 \text{ deg}$	$T_s = 10 \text{ s}$
	$\gamma = 0.05$

A special indicator M_y is used to describe the actual yaw phase, i.e. $M_y = 0, 1, 2, 3, 4$ corresponds to phase 0, 1, 2, 3, 4 respectively. Notice that it is possible to use the non-filtered measurements instead of the Kalman filter estimates in the yaw regulator. The special speed scaling used in the yaw regulator will approximately give the same performance of the yaw rate and the heading independent of the forward speed. The rudder deviations, however, are increased proportional to $(V_0/V_s)^2$ when the speed is decreased.

4. SIMULATIONS

To make it possible to compare the steering quality of different autopilot structures, a loss function is now introduced:

$$V_{\ell} = \frac{1}{\tau} \int_0^{\tau} \left[(\psi(t) - \psi_{\text{ref}})^2 + \lambda (\delta(t) - m_{\delta})^2 \right] dt \quad (4.1)$$

where m_{δ} is the mean value of the rudder angle δ and the weighting factor λ is equal to 1/12 or 0. The duration of a simulation is denoted τ . The loss function is approximated by:

$$V_{\ell} = \frac{1}{N} \sum_{n=0}^{N-1} \left[(\psi(nh) - \psi_{\text{ref}})^2 + \lambda (\delta(nh) - m_{\delta})^2 \right] \quad (4.2)$$

where $Nh = \tau$ and the sampling interval h always is equal to 1 s.

In the sequel the mean values m_{δ} and m_{ψ} , and the standard deviations σ_{δ} and σ_{ψ} , of the rudder angle δ and the course error $\psi - \psi_{\text{ref}}$, resp, will be presented as well as the loss function V_{ℓ} . Notice that the rudder angle δ and the heading angle ψ without measurement noise and without measurement bias are used. If nothing else is remarked, the standard values given in Chapters 2 and 3 are used in the simulations. Several plots are often shown in the same figure, and the plots are then slided in relation to each other. The corresponding straight line is the level zero. Notice that the initial forward speed is denoted u_0 .

4.1 Kalman Filter Testing

Simulations of straight course keeping ($\psi_{\text{ref}} = 0$ deg) with the self-tuning regulator using estimates from the Kalman filter are presented in this section. The performance of the Kalman filter and the self-tuning regulator for different load conditions and different speeds are shown in Figs 4.1 - 4.6.

In Fig. 4.7 the initial covariance matrix of the self-tuning regulator is given as (cf.(3.17)):

$$\text{diag} (P) = [10 \quad 10 \quad 10 \quad 10 \quad 0.1 \quad 0.1 \quad 10 \quad 500] \quad (4.3)$$

The only measurement signal used by the Kalman filter in Figs. 4.8 and 4.9 is the heading angle. The first three columns of the filter gain K (cf.(3.10)) are cancelled in Fig. 4.8, while the following filter gain, designed correctly for the case of heading measurements only, is used in Fig. 4.9:

$$K^T = [-0.481 \quad 2.4 \quad 0.39 \quad 0.072 \quad -0.0716 \quad 0 \quad 0 \quad 0] \quad (4.4)$$

The process noise is increased in Fig. 4.10 by use of the following covariance matrix (cf.(2.6)):

$$R_w = \begin{bmatrix} 3.2 \cdot 10^{-4} & 0 \\ 0 & 2.12 \cdot 10^{-10} \end{bmatrix} \quad (4.5)$$

A modified filter gain is used in Fig. 4.11. The tuning rate of the state estimates $\hat{\delta}_0$ and $\hat{d}_v$ is increased by changing two elements of R_1 :

$$r_5 = 1.2 \cdot 10^{-8} \quad 1/s$$

$$r_6 = 4 \cdot 10^{-7} \quad m^2/s^3$$

The corresponding filter gain is (cf.(3.10)):

$$K = \begin{bmatrix} -9.27 \cdot 10^{-5} & 8.82 \cdot 10^{-2} & -2.40 \cdot 10^{-2} & -0.367 \\ 6.30 \cdot 10^{-4} & 0.481 & 9.65 \cdot 10^{-2} & 1.02 \\ 1.69 \cdot 10^{-5} & 7.86 \cdot 10^{-3} & 3.14 \cdot 10^{-3} & 0.326 \\ 1.66 \cdot 10^{-2} & -2.90 \cdot 10^{-3} & 5.65 \cdot 10^{-3} & 6.51 \cdot 10^{-2} \\ 1.03 \cdot 10^{-3} & 2.98 \cdot 10^{-3} & -5.63 \cdot 10^{-3} & -6.49 \cdot 10^{-2} \\ -1.26 \cdot 10^{-4} & 2.00 \cdot 10^{-2} & -6.69 \cdot 10^{-5} & -5.06 \cdot 10^{-3} \\ 1.63 \cdot 10^{-6} & -1.68 \cdot 10^{-3} & 3.02 \cdot 10^{-3} & -5.77 \cdot 10^{-2} \\ 2.54 \cdot 10^{-3} & 7.61 \cdot 10^{-6} & 1.24 \cdot 10^{-5} & 1.41 \cdot 10^{-4} \end{bmatrix} \quad (4.6)$$

All the simulations are summarized in Table 4.1. The reason why $\lambda = 0$ (cf.(4.1)) when the initial speed u_0 is equal to 4 knots is that the course keeping is much more important than the rudder deviations at such a low speed.

From the simulations it can be concluded that the performance of the Kalman filter is very good for the initial speeds 15.8 knots and 10 knots in full load condition as well as in ballast condition (cf. Figs 4.1 - 4.4). Notice, however, that the tuning rate of the state estimates $\hat{\delta}_0$ and $\hat{d}_v$ is rather small. The performance of the Kalman filter for the initial speed $u_0 = 4$ knots (Figs 4.5 - 4.7) is not as good as for larger speeds, but quite acceptable. The small tuning rate of $\hat{d}_v$, however, is emphasized. Another difficulty is that the limited rudder turning rate has not been considered in the Kalman filter. This means large values of the residuals ε'_δ (see Figs 4.6j and 4.7j) when large rudder changes are requested, which is the case quite often when the speed is low. To avoid incorrect rejections of the rudder angle measurements, the test value t'_δ (cf.(3.13)) must be chosen that large, that the checking is quite meaningless.

T [m]	n_0 [rpm]	u_0 [knots]	δ_ℓ [deg]	m_δ [deg]	σ_δ [deg]	m_ψ [deg]	σ_ψ [deg]	V_ℓ [deg ²]	λ	Fig.	Remarks
22.3	87.6	15.8	10	- 1.28	2.04	-0.09	0.47	0.55	1/12	4.1	
10.5	87.6	15.8	10	- 0.15	1.55	-0.01	0.29	0.29	1/12	4.2	
22.3	55.443	10	35	- 3.71	4.61	-0.16	0.51	2.05	1/12	4.3	
10.5	55.443	10	35	- 1.18	3.36	-0.01	0.35	1.06	1/12	4.4	
22.3	22.1772	4	45	-21.62	19.96	0.66	1.01	1.46	0	4.5	
10.5	22.1772	4	45	- 8.93	19.24	0.30	0.61	0.46	0	4.6	
22.3	22.1772	4	45	-22.29	22.53	0.90	1.14	2.11	0	4.7	Initial P given by (4.3).
22.3	87.6	15.8	10	- 1.30	2.58	0.03	0.48	0.79	1/12	4.8	Only ψ_m used by Kalman filter.
22.3	87.6	15.8	10	- 1.30	2.52	0.03	0.46	0.74	1/12	4.9	Only ψ_m used by Kalman filter. Correct K (4.4).
22.3	87.6	15.8	10	- 1.38	3.83	-0.14	0.82	1.91	1/12	4.10	R_w given by (4.5).
22.3	87.6	15.8	10	- 1.28	2.04	-0.06	0.44	0.54	1/12	4.11	K given by (4.6).

Table 4.1 – Summary of simulations for Kalman filter testing. The duration of each simulation is 2400 s, but the values of m_δ , σ_δ , m_ψ , σ_ψ , and V_ℓ are computed for the part 1200 – 2400 s. The straight course keeping ($\psi_{ref} = 0$ deg) is performed by the self-tuning regulator using estimates from the Kalman filter.

If the only measurement signal used by the Kalman filter is the heading angle, the state estimates $\hat{v}$, $\hat{r}$, $\hat{\psi}$, $\hat{\delta}$ and $\hat{\delta}_0$ are nevertheless good, which can be concluded from Figs 4.8 and 4.9. The difference between cancelling the corresponding columns of K (Fig. 4.8) and designing a correct K (Fig. 4.9) is very small.

The quality of the Kalman filter estimates is only decreased very little, when the standard deviations of the process disturbances are doubled (see Fig. 4.10).

Finally a filter gain designed for increased tuning rate of $\hat{\delta}_0$ and $\hat{d}_v$ is tested in the simulation of Fig. 4.11. It can be concluded that the tuning rate of $\hat{d}_v$ still is small, although an improvement is obtained compared to Fig. 4.1.

The final parameter values of the self-tuning regulator are summarized in Table 4.7 of the next section. Some examples of the performance of the Kalman filter during yaws are shown in Section 4.3.

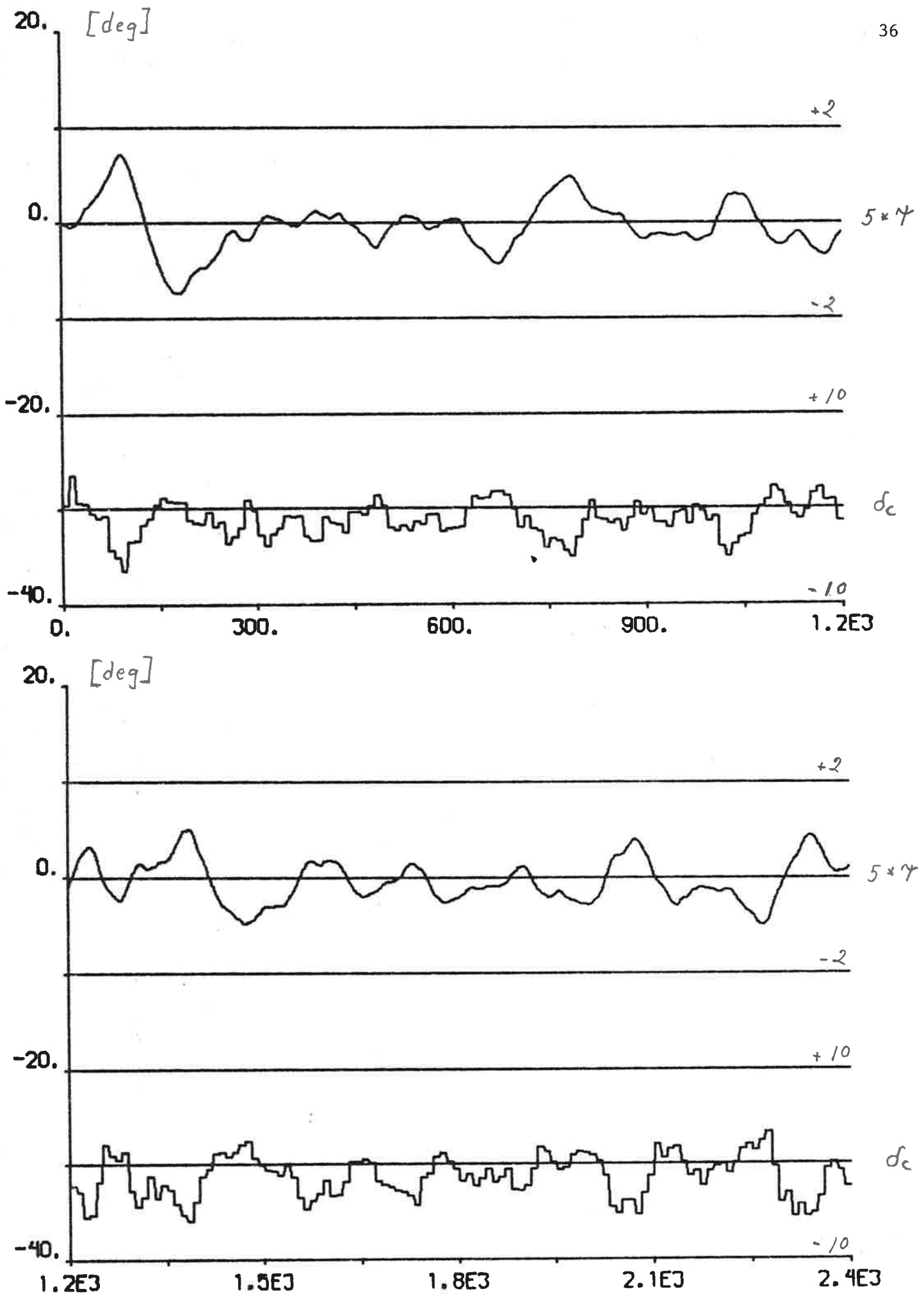


Fig. 4.1a - $T = 22.3$ m, $n_0 = 87.6$ rpm, $u_0 = 15.8$ knots, $\delta_\ell = 10$ deg, self-tuning regulator using estimates from the Kalman filter.

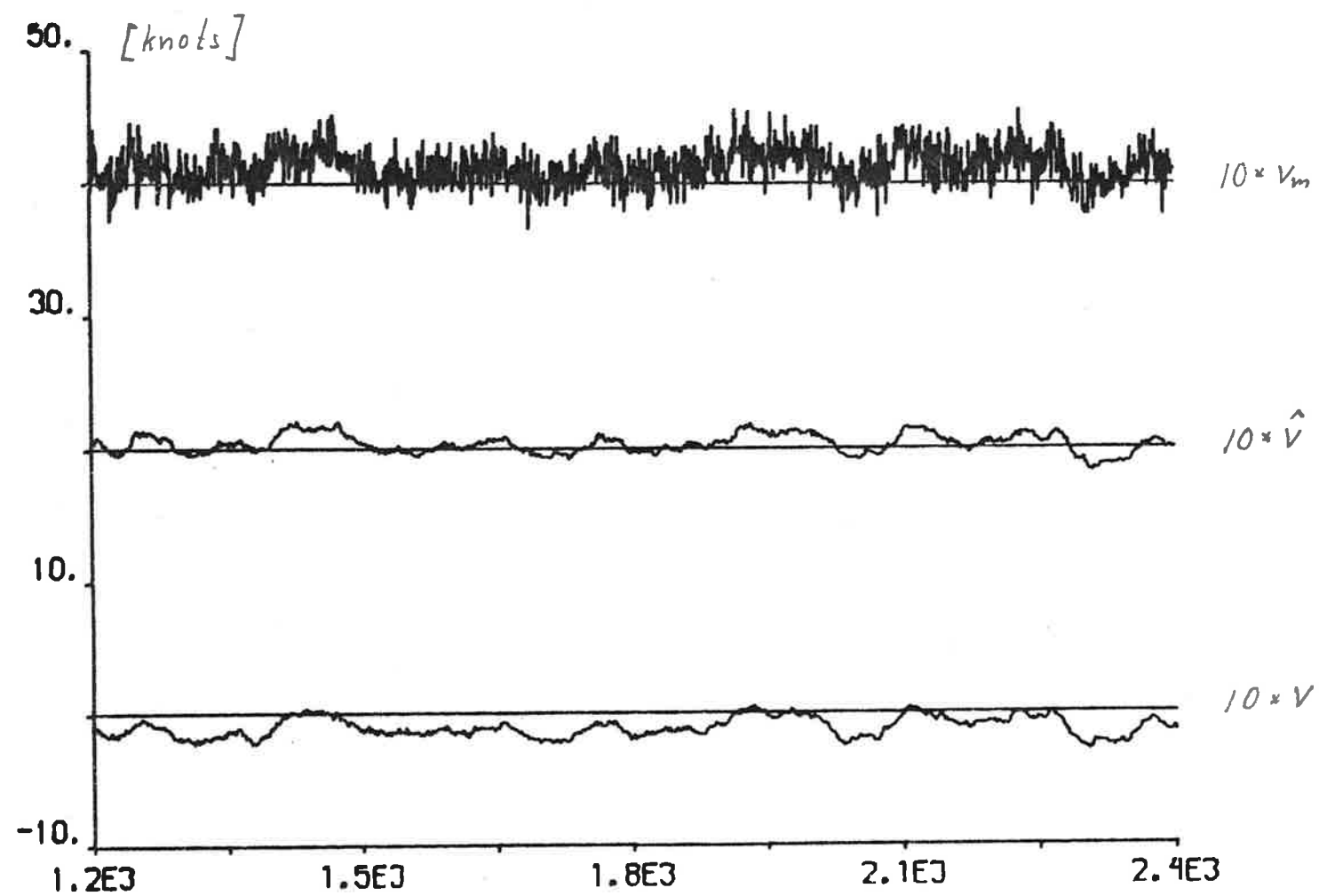
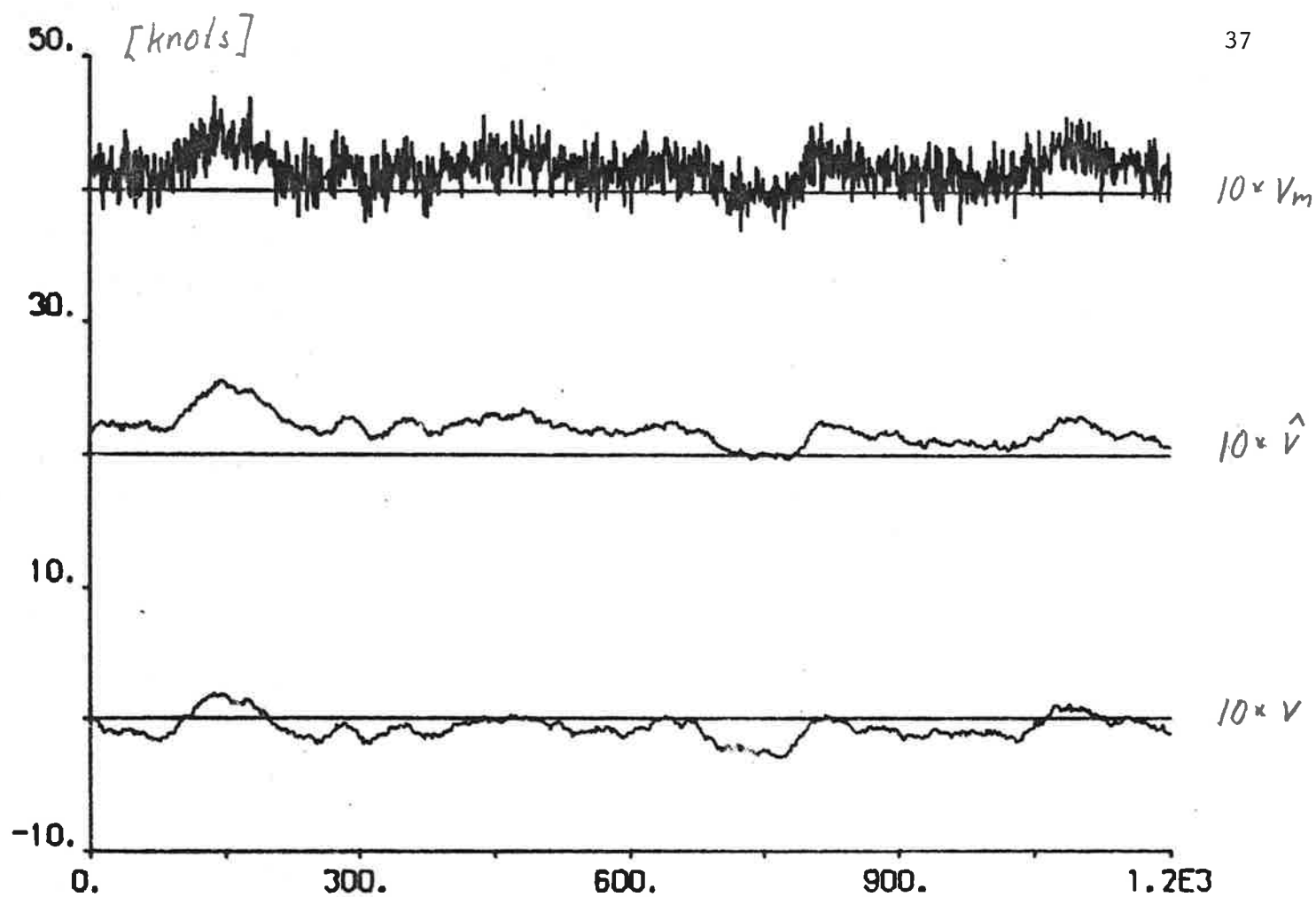


Fig. 4.1b

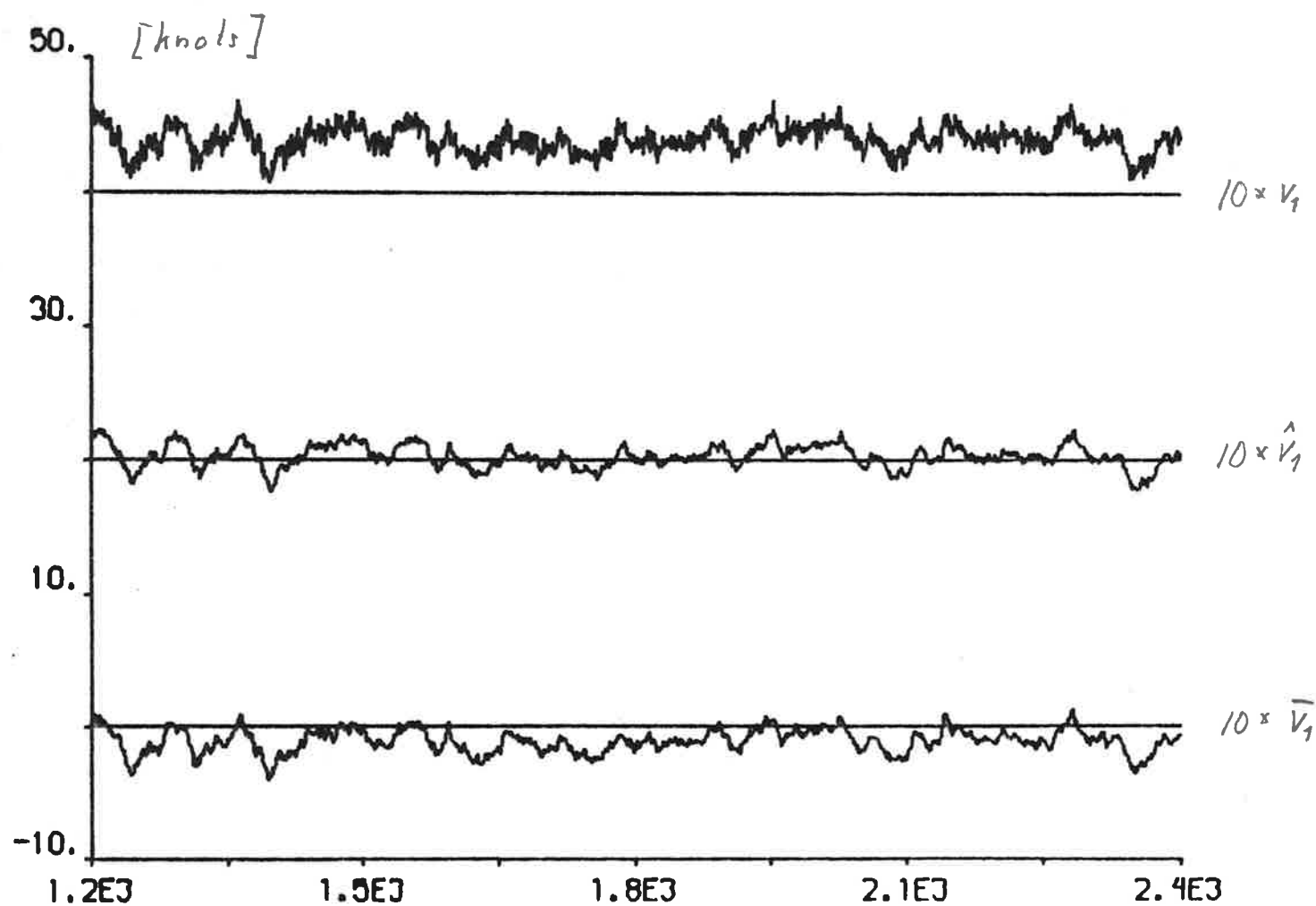
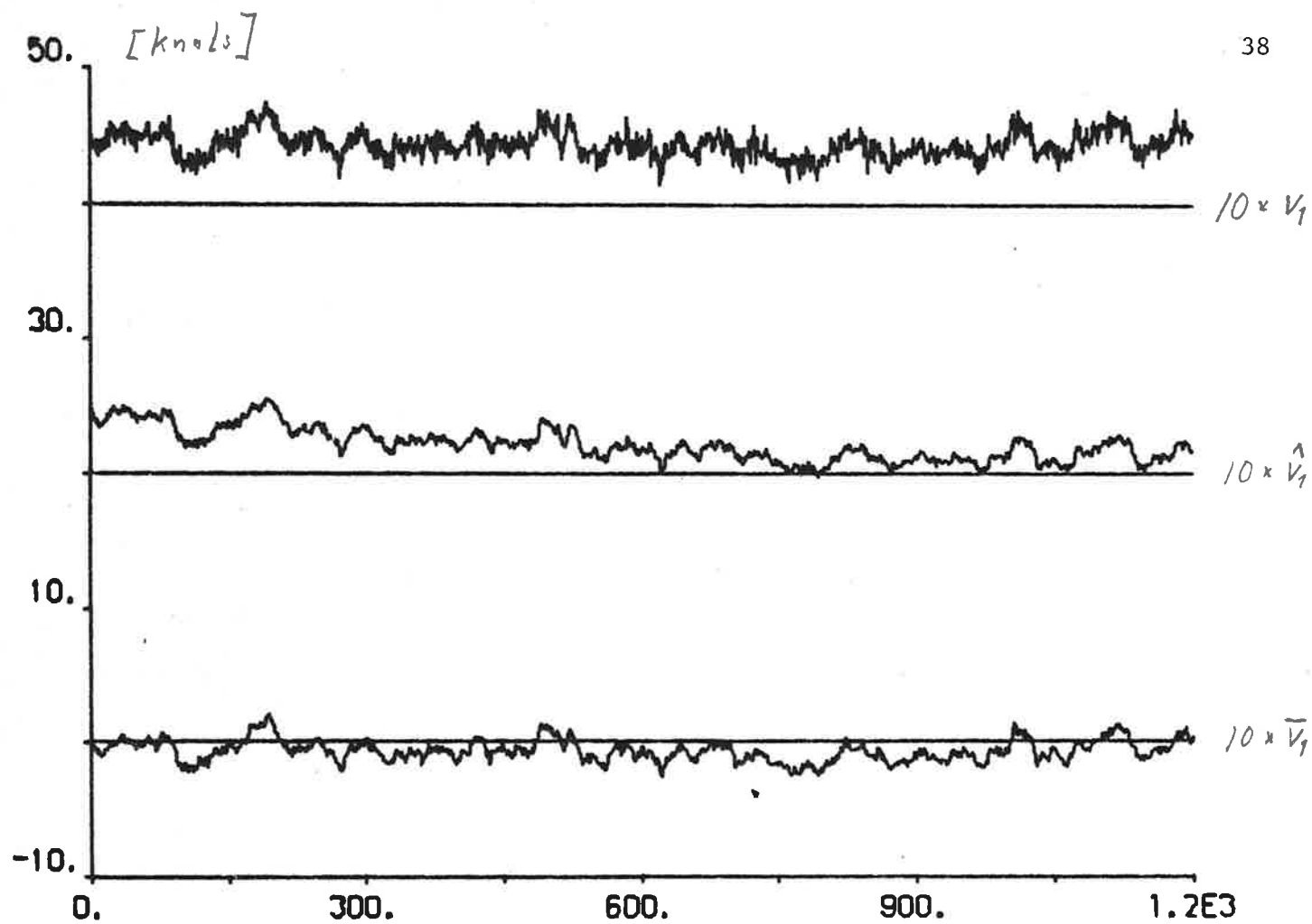


Fig. 4.1c

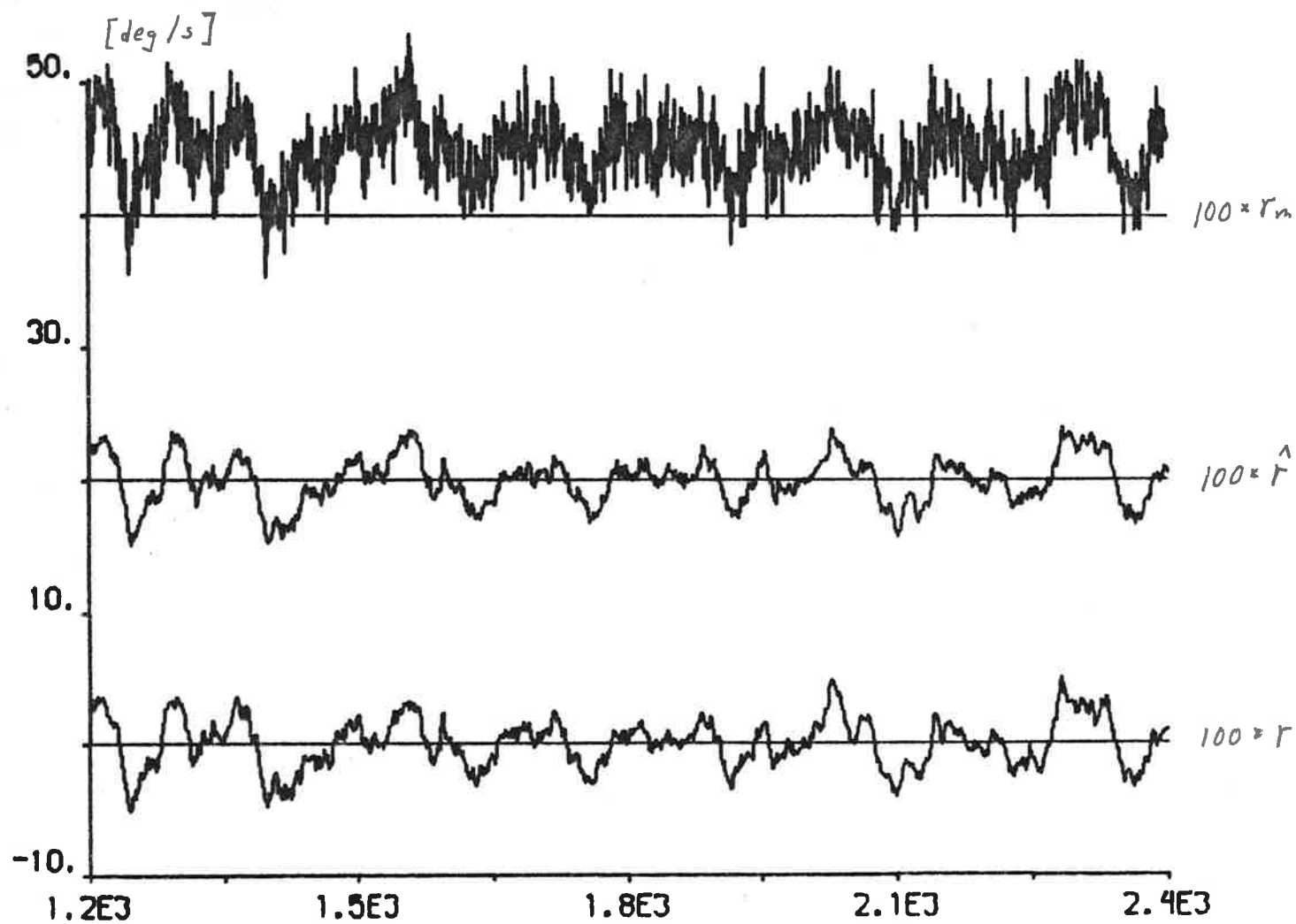
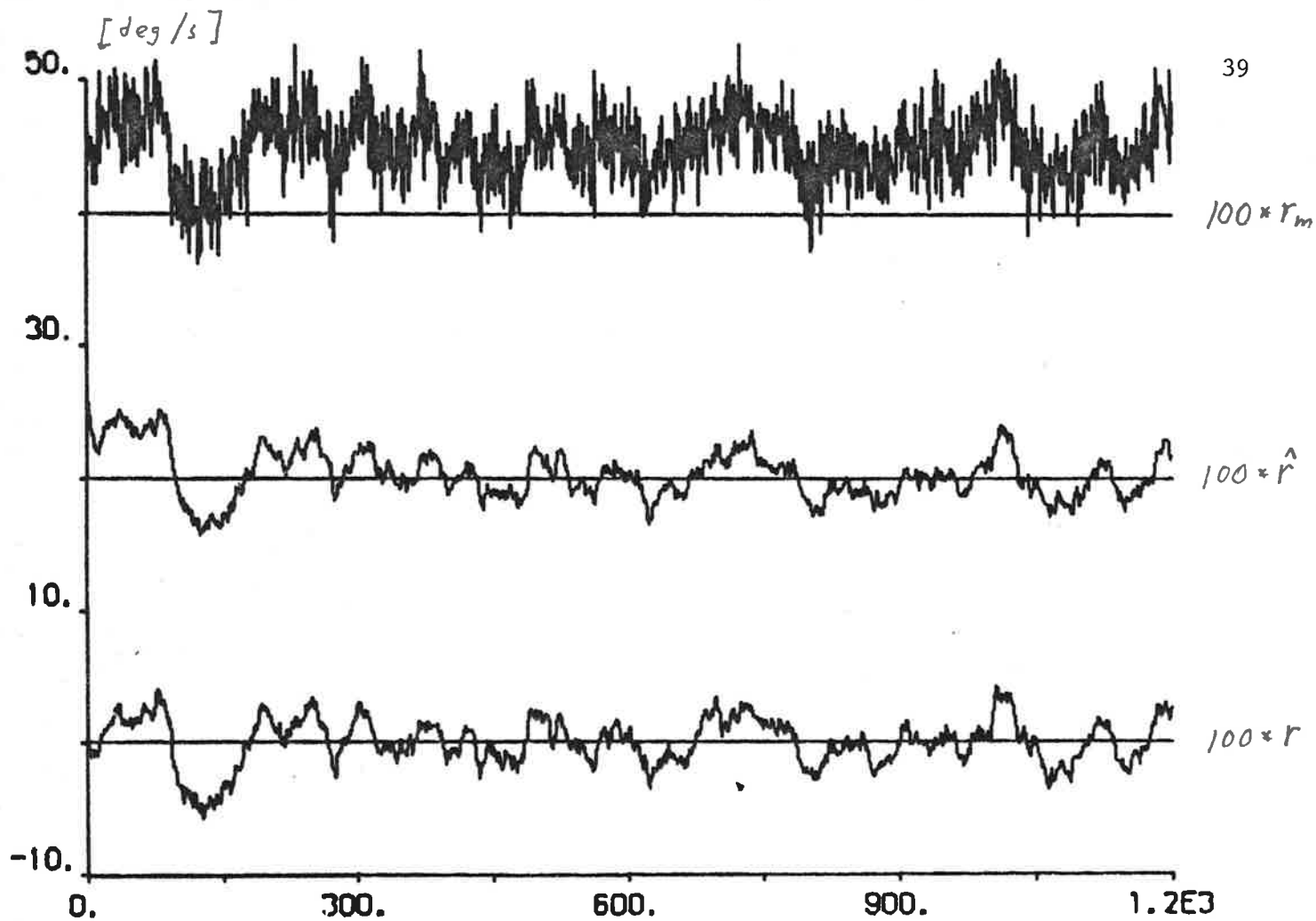


Fig. 4.1d

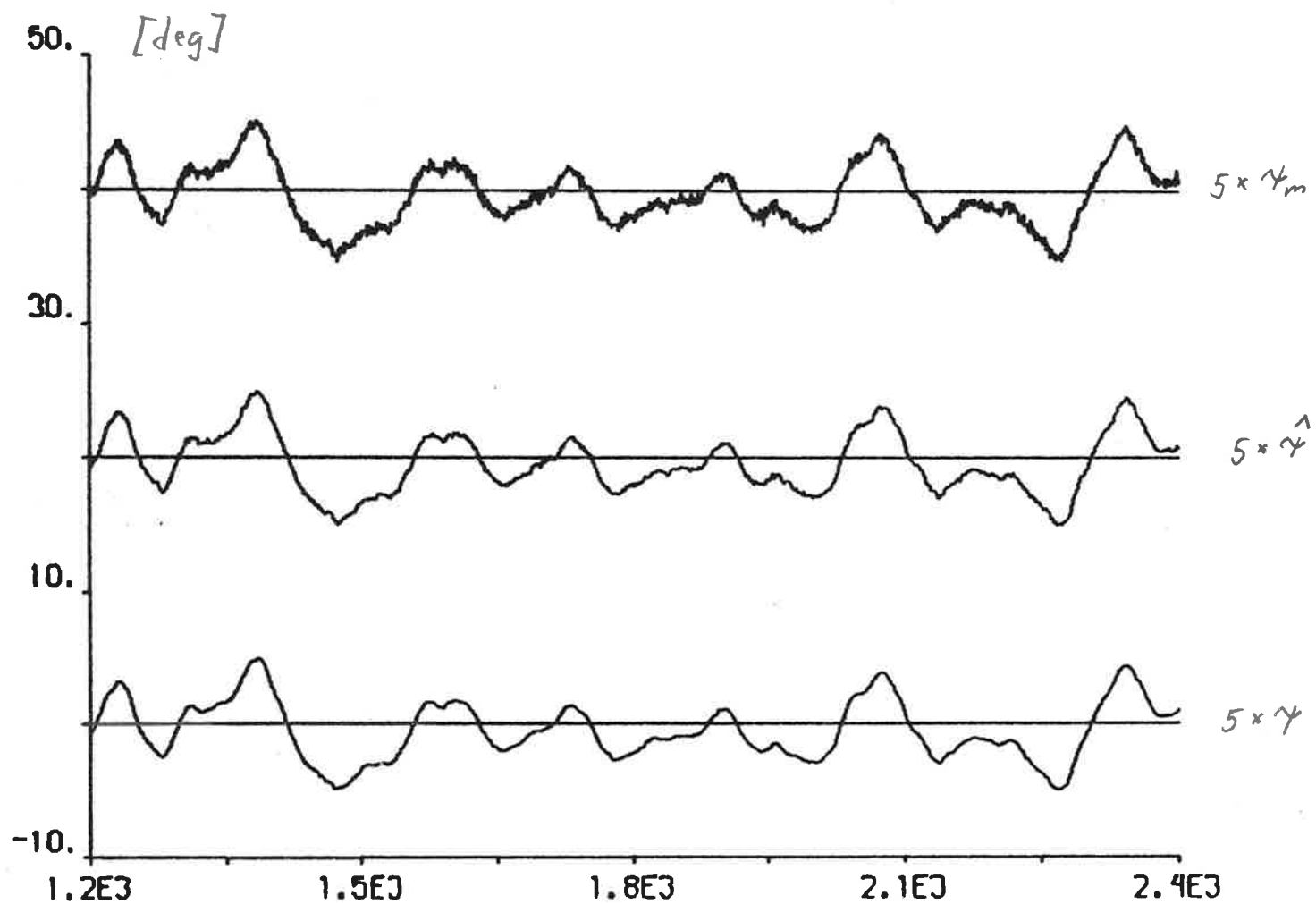
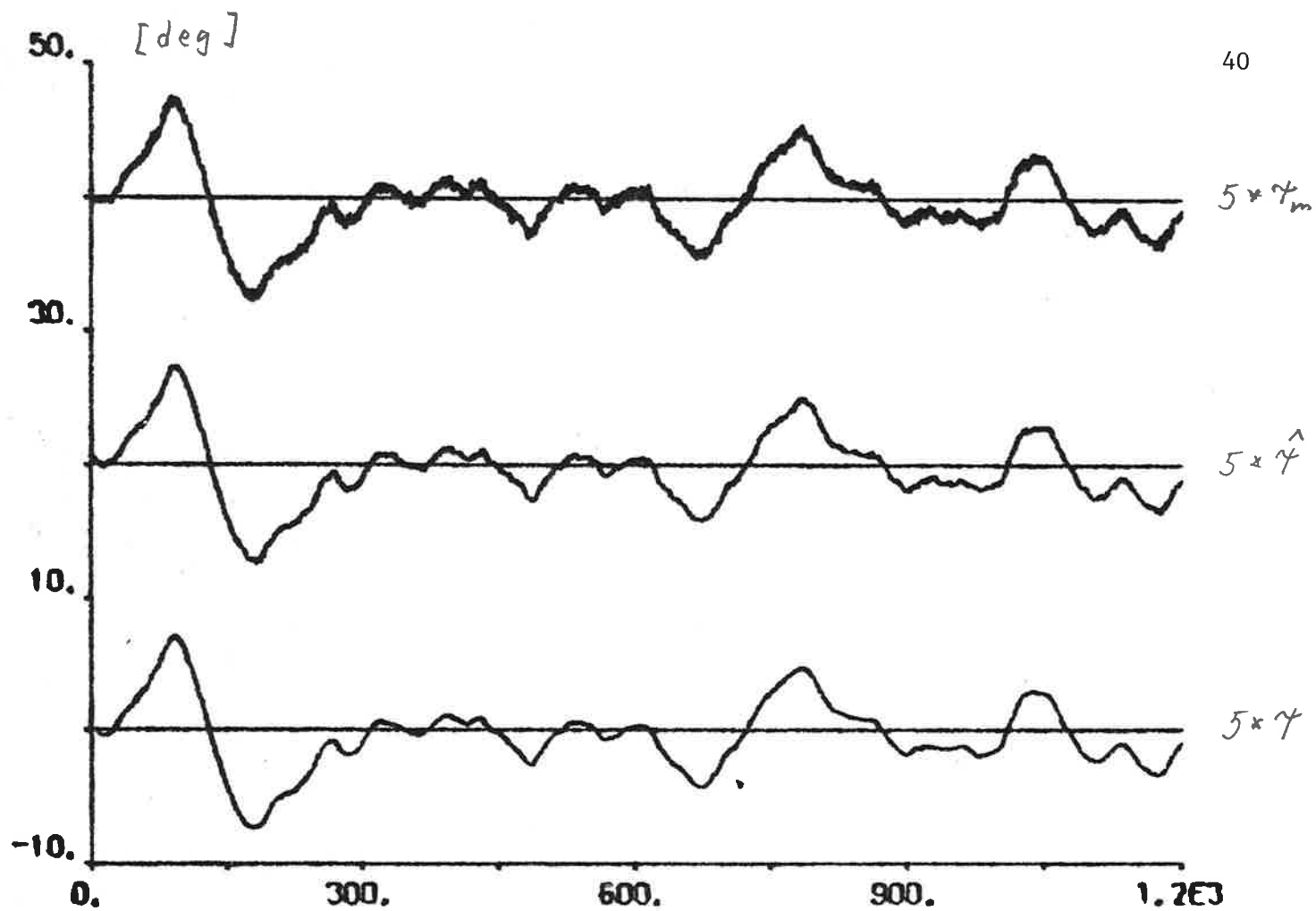


Fig. 4.1e

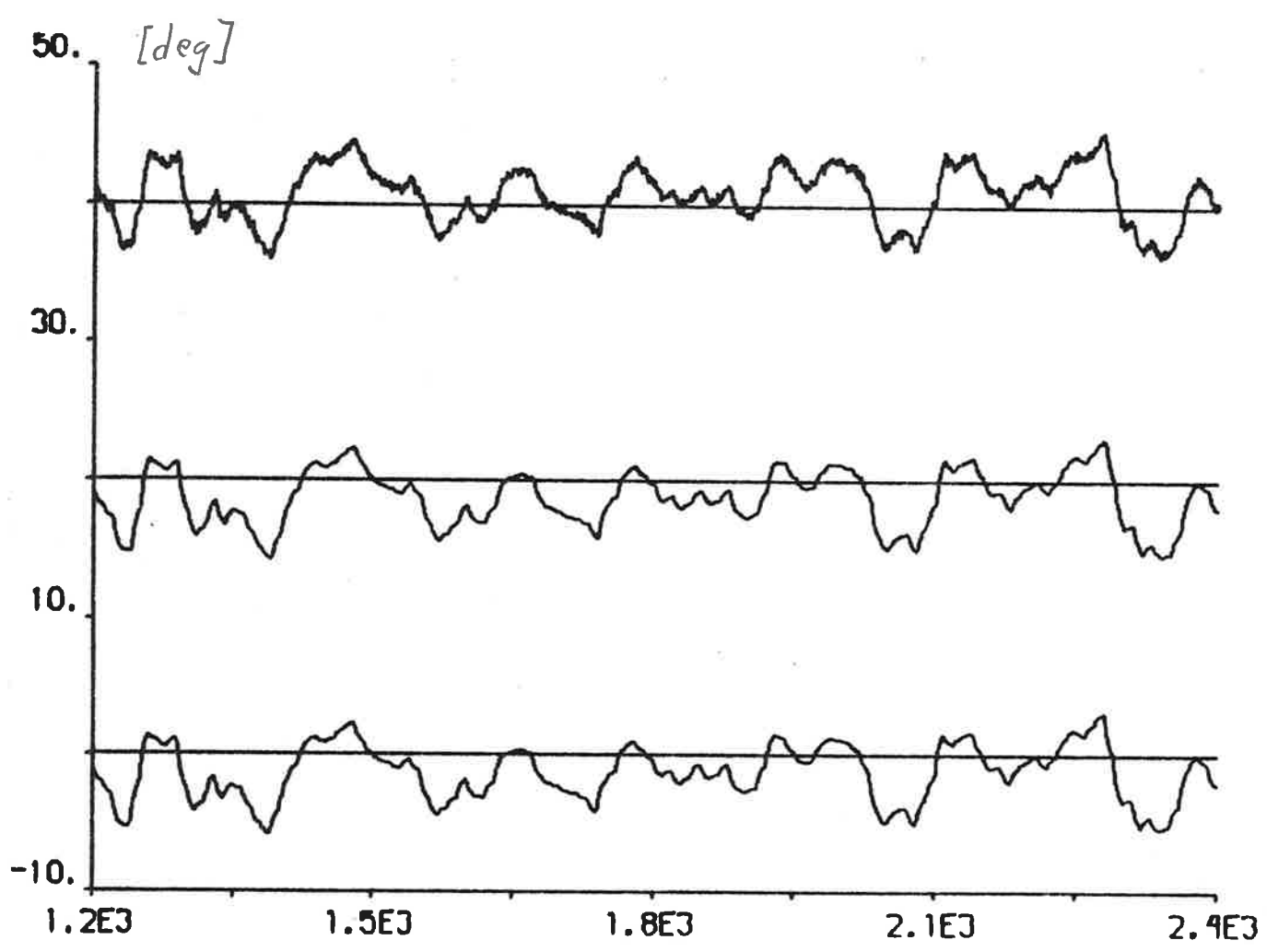
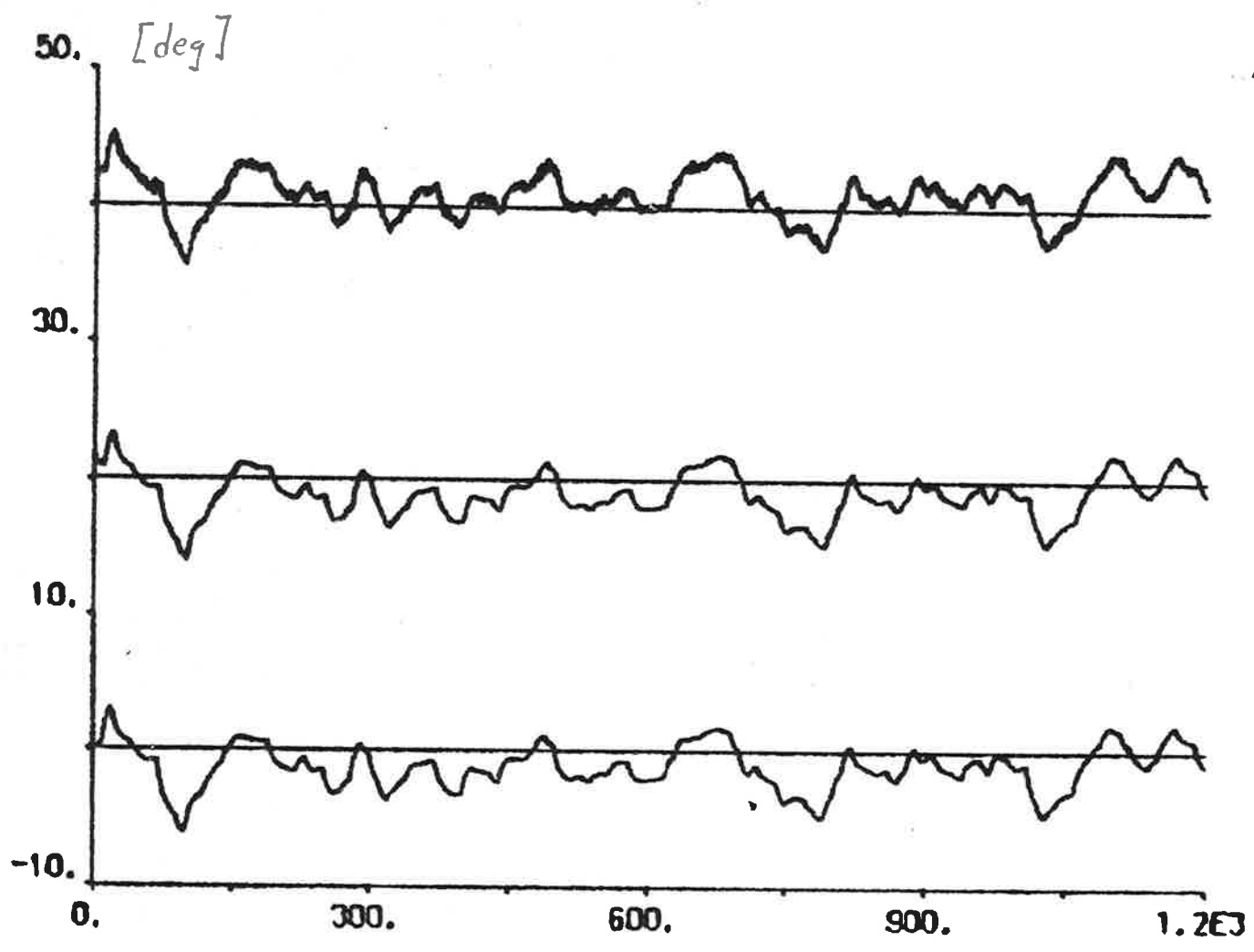


Fig. 4.1f

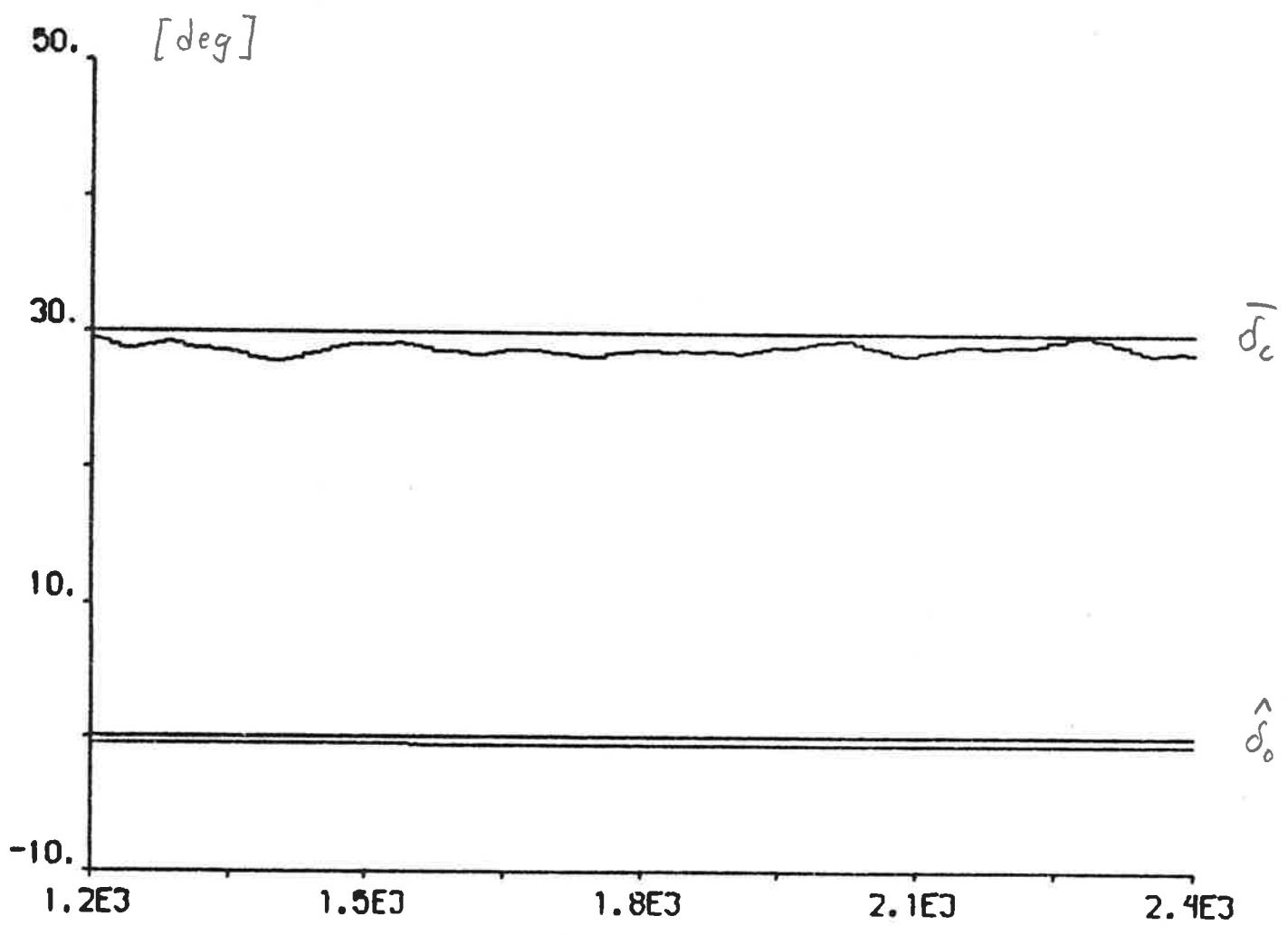
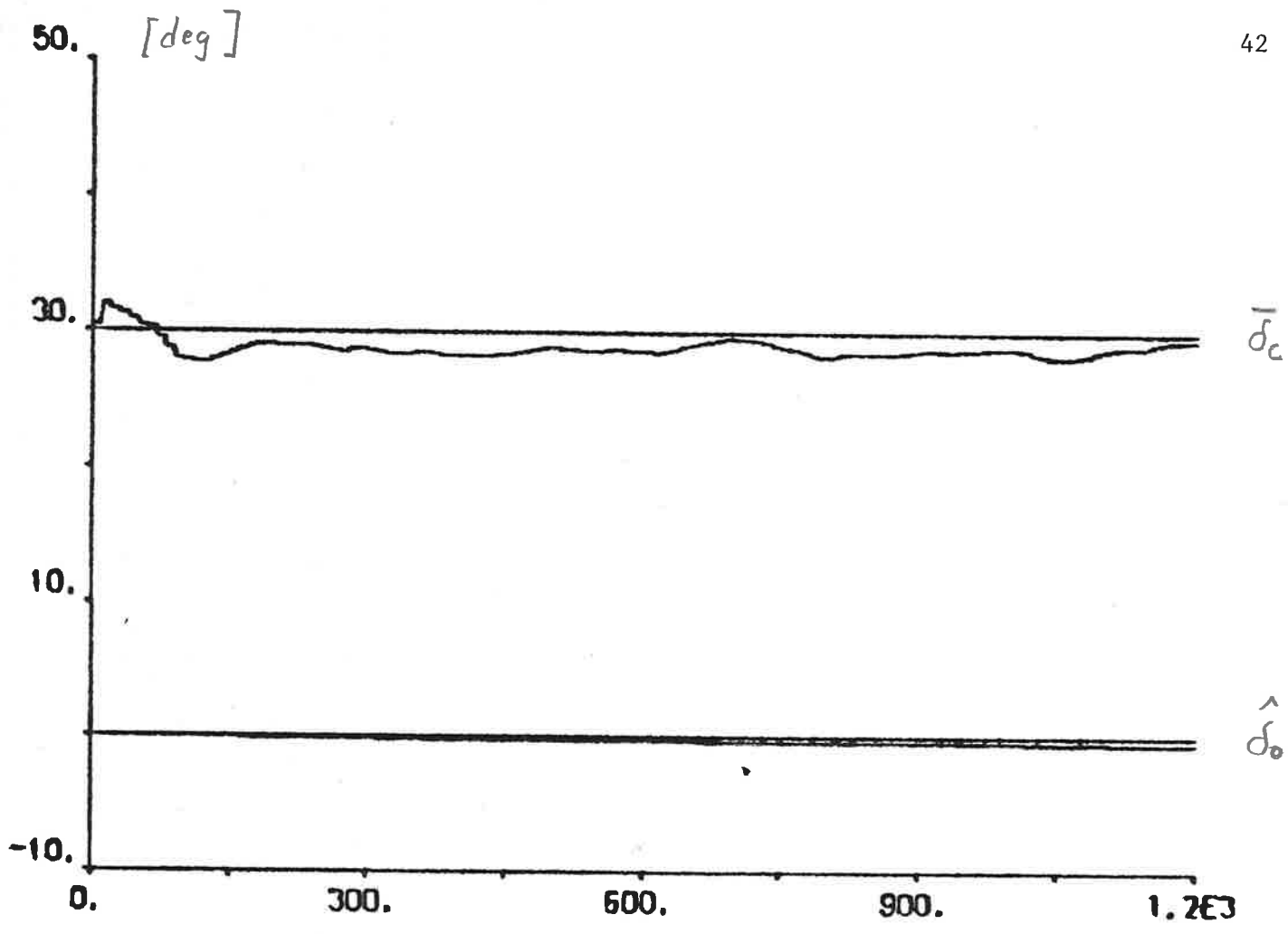


Fig. 4.1g

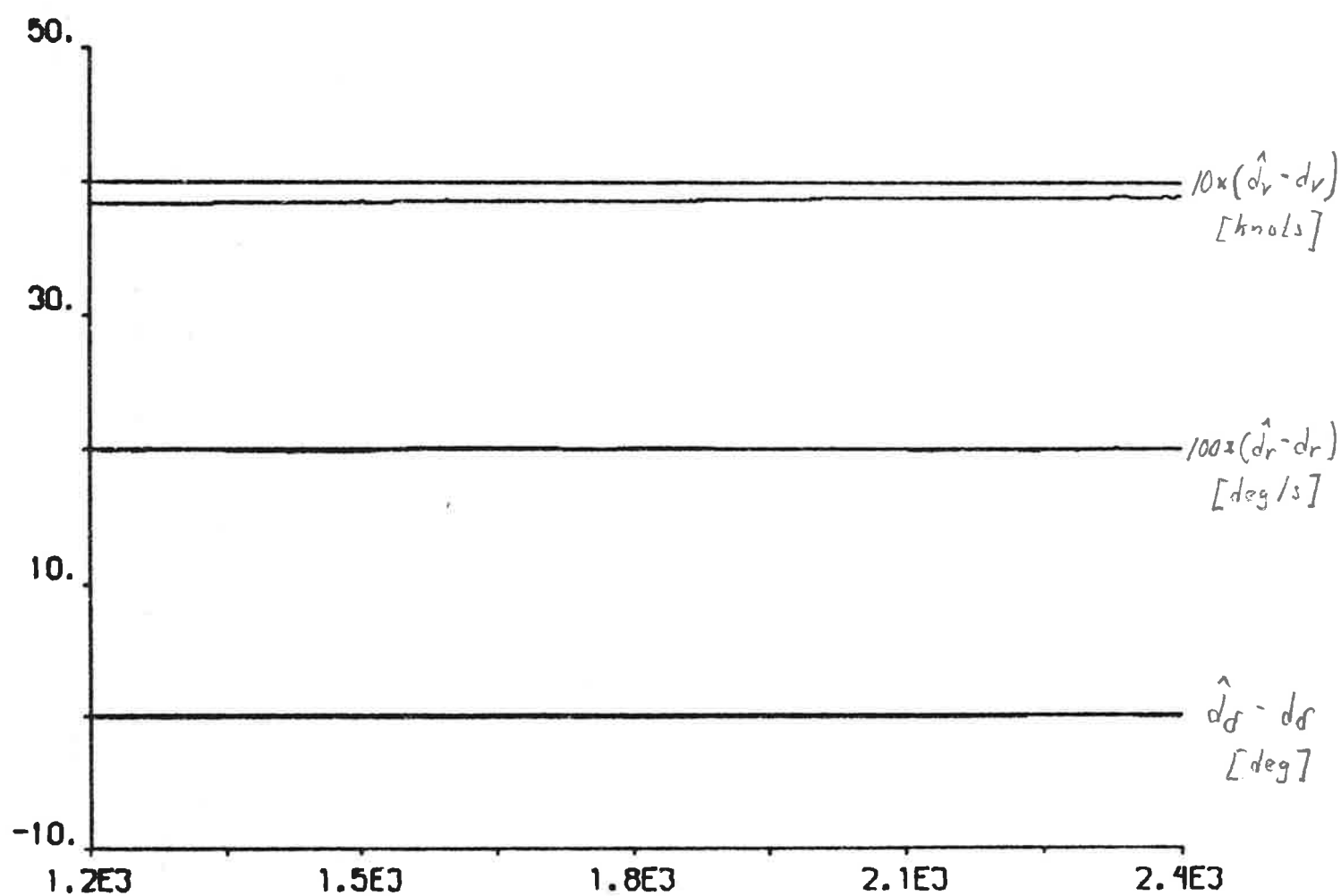
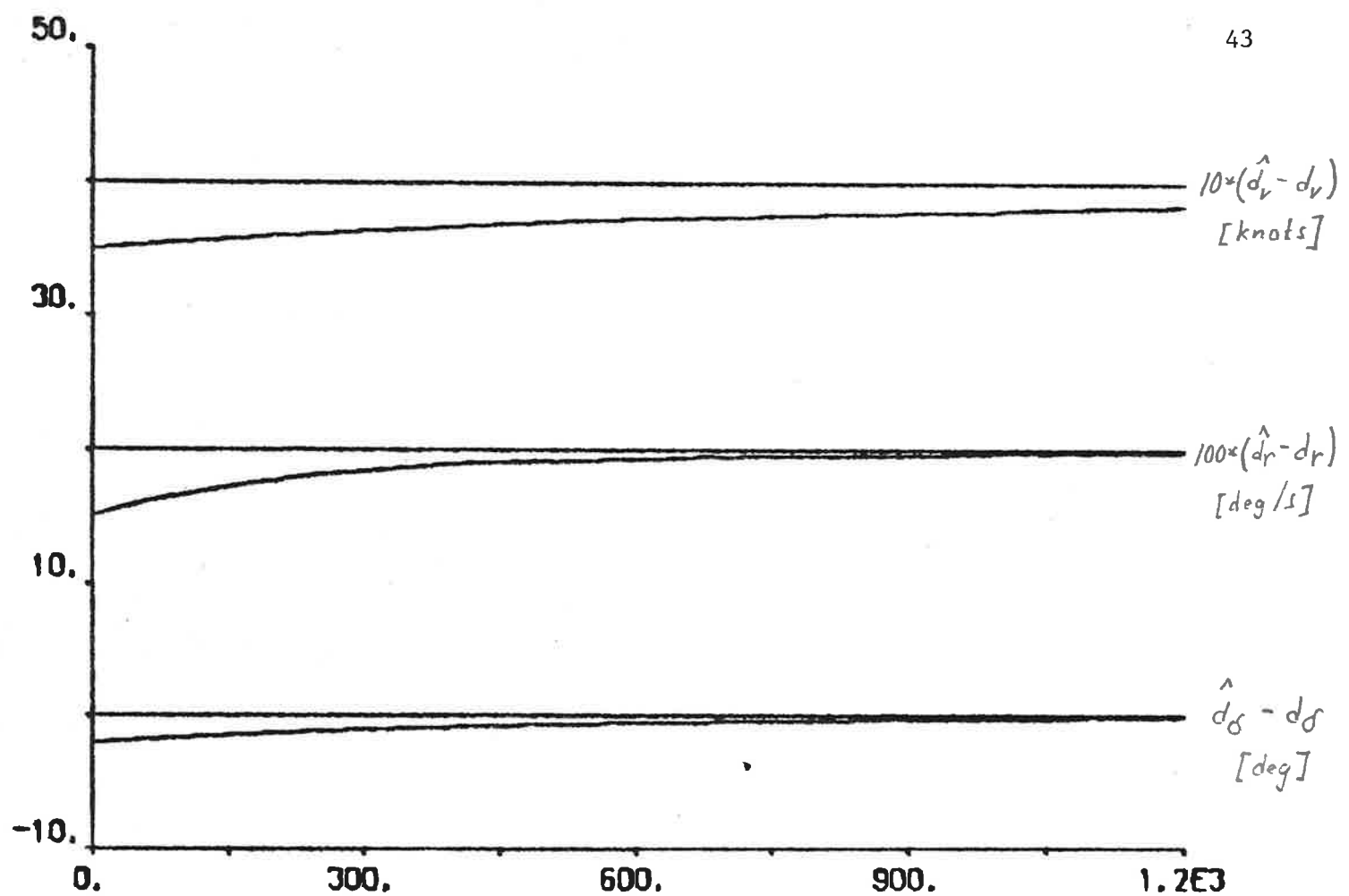
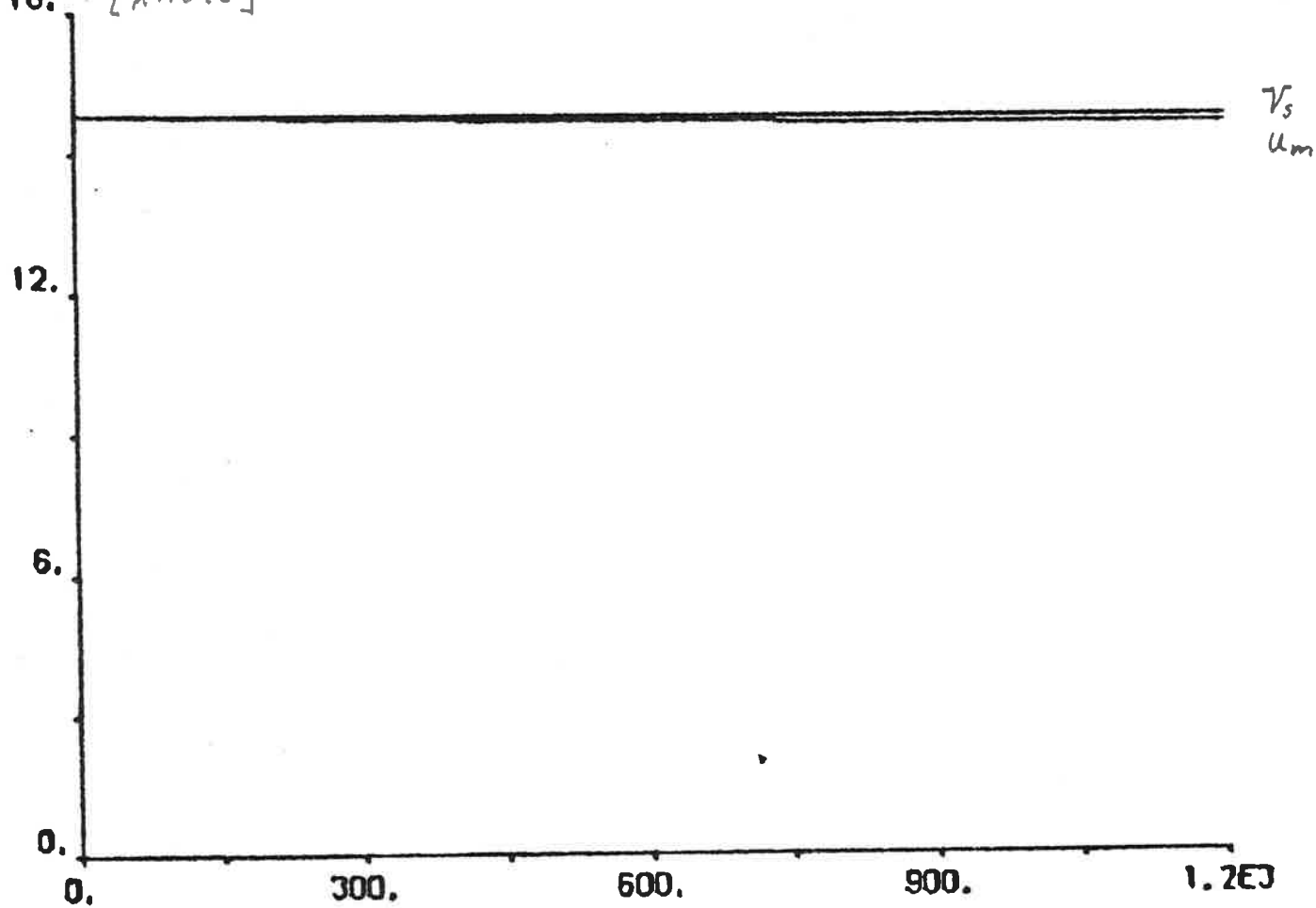


Fig. 4.1h

18. [knots]

44



18. [knots]

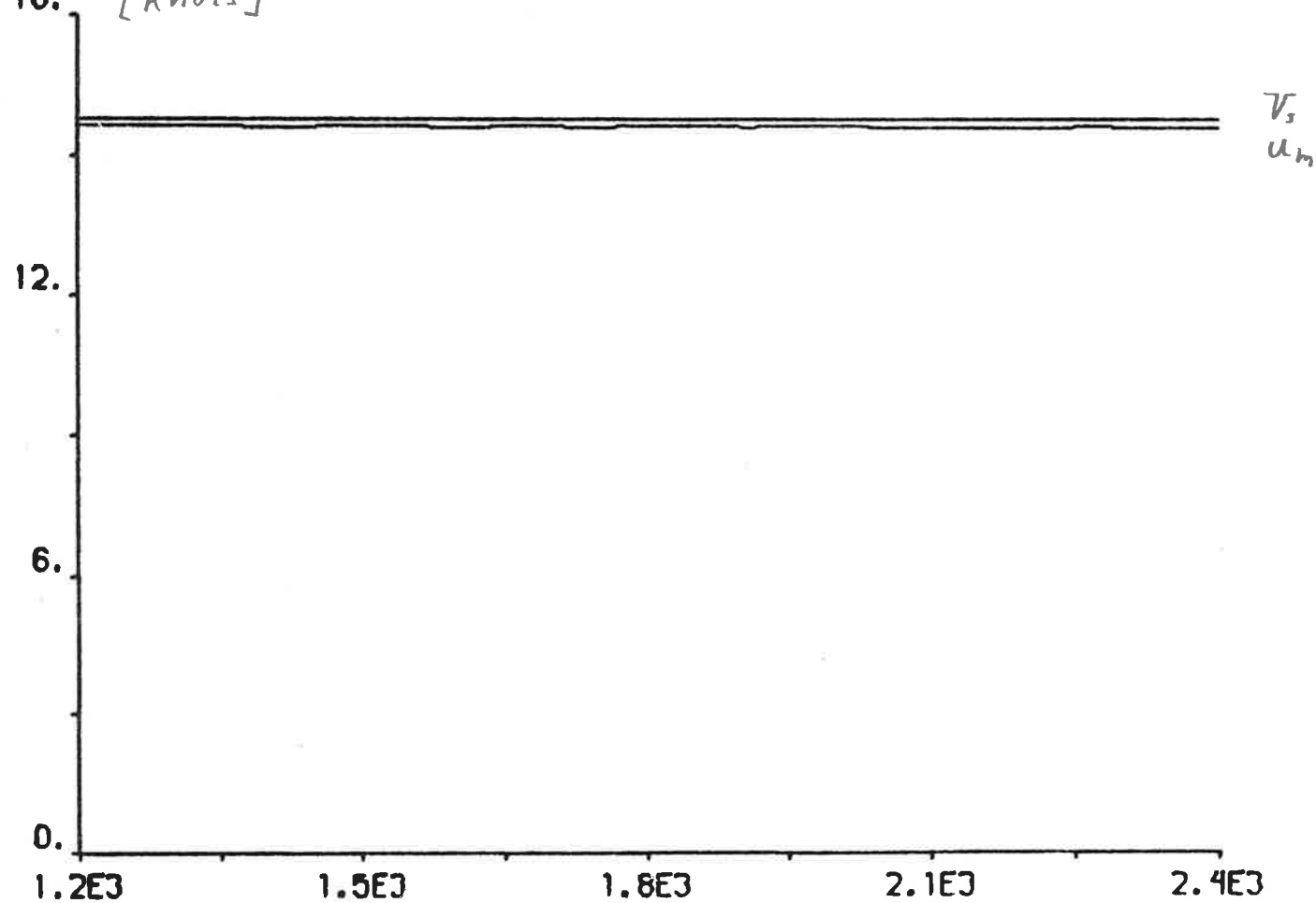


Fig. 4.1i

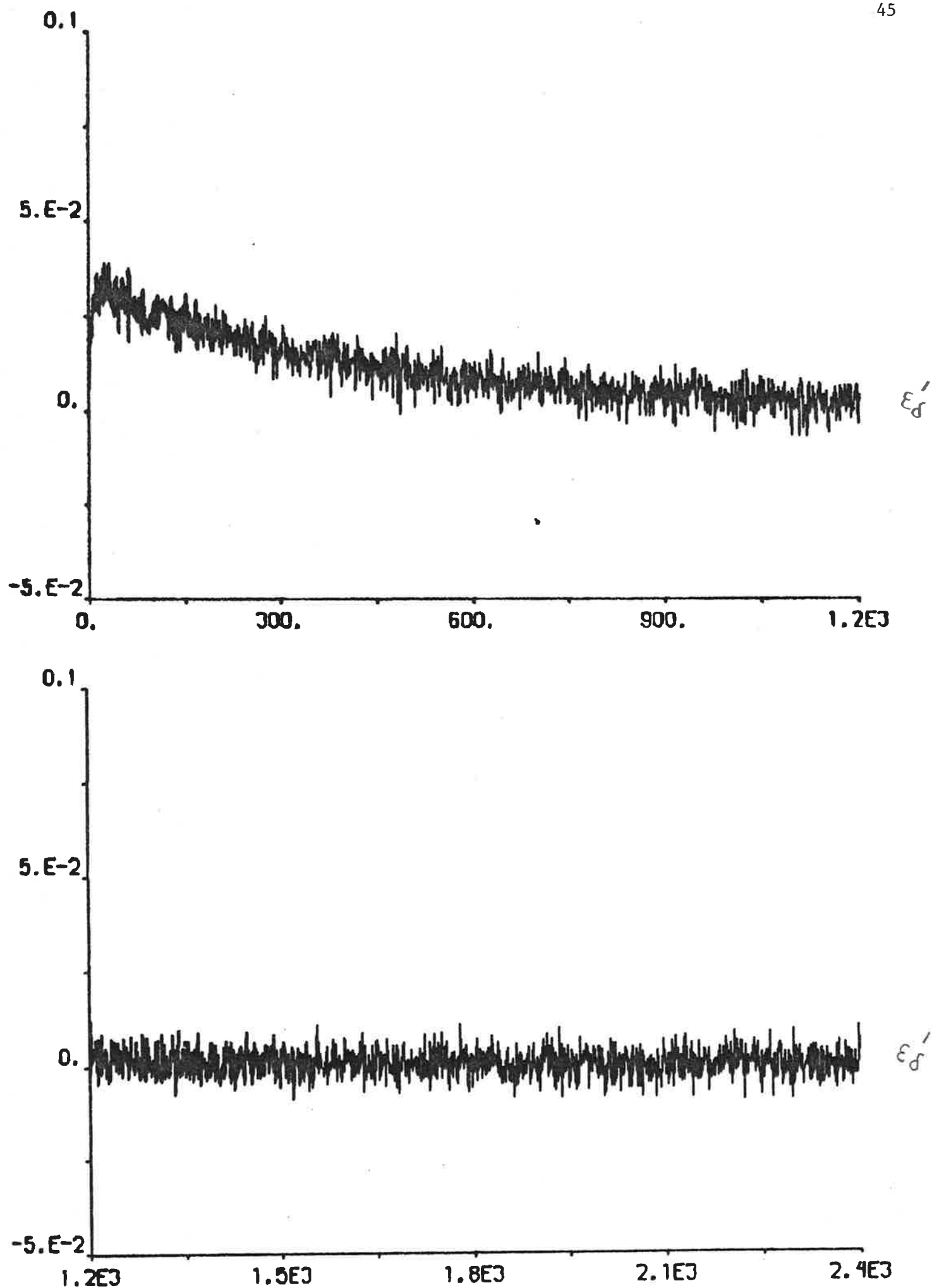


Fig. 4.1j

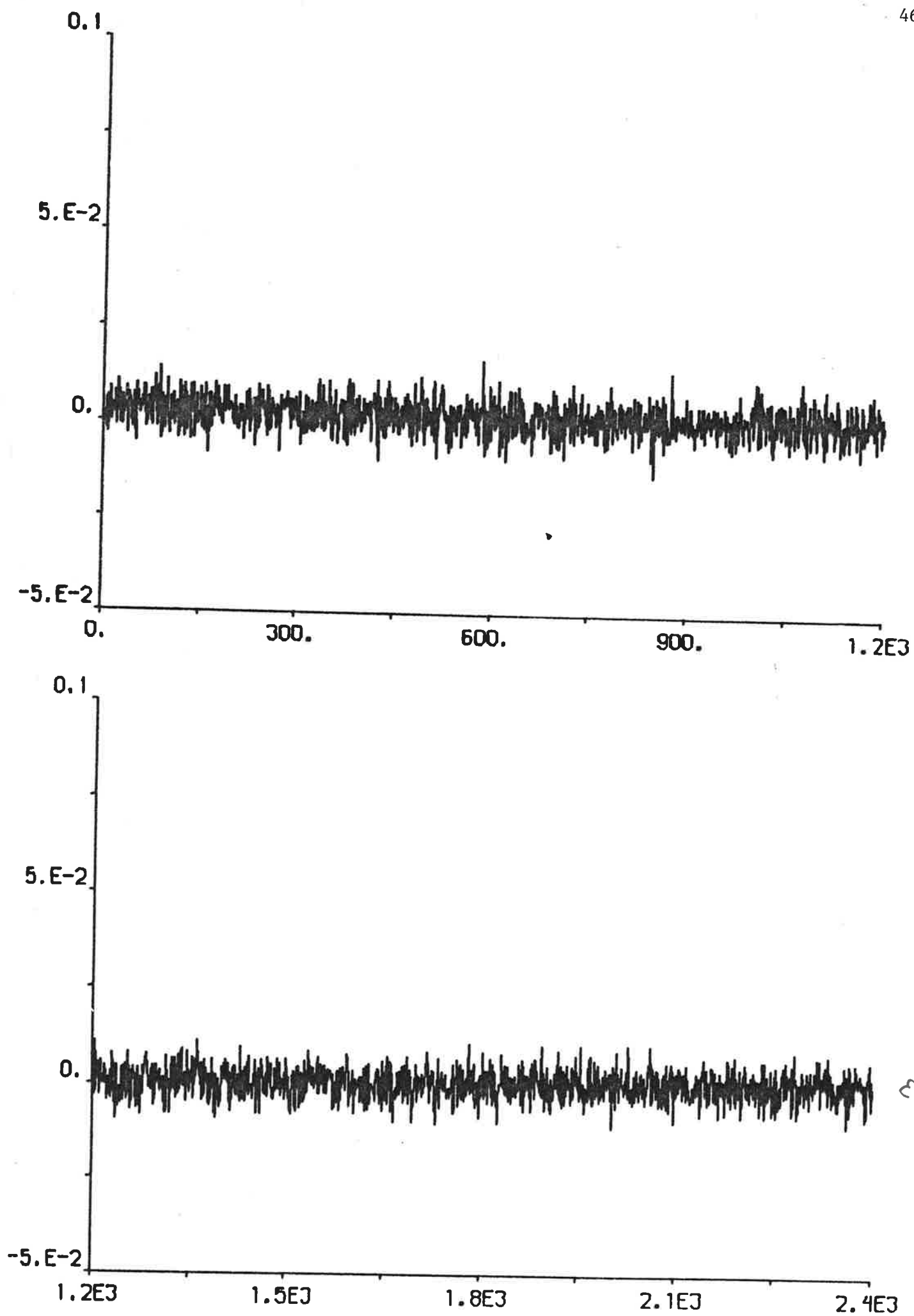


Fig. 4.1k

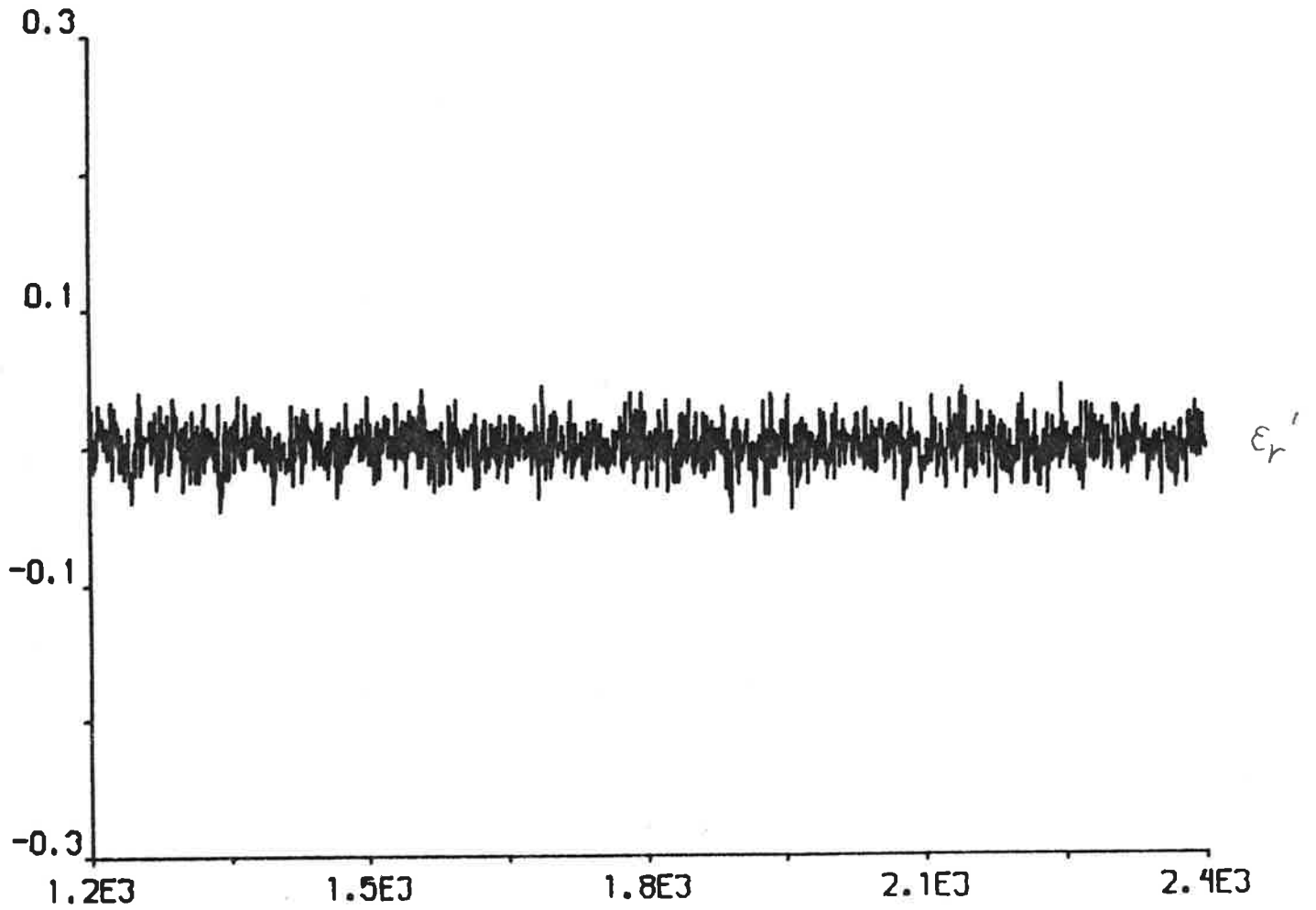
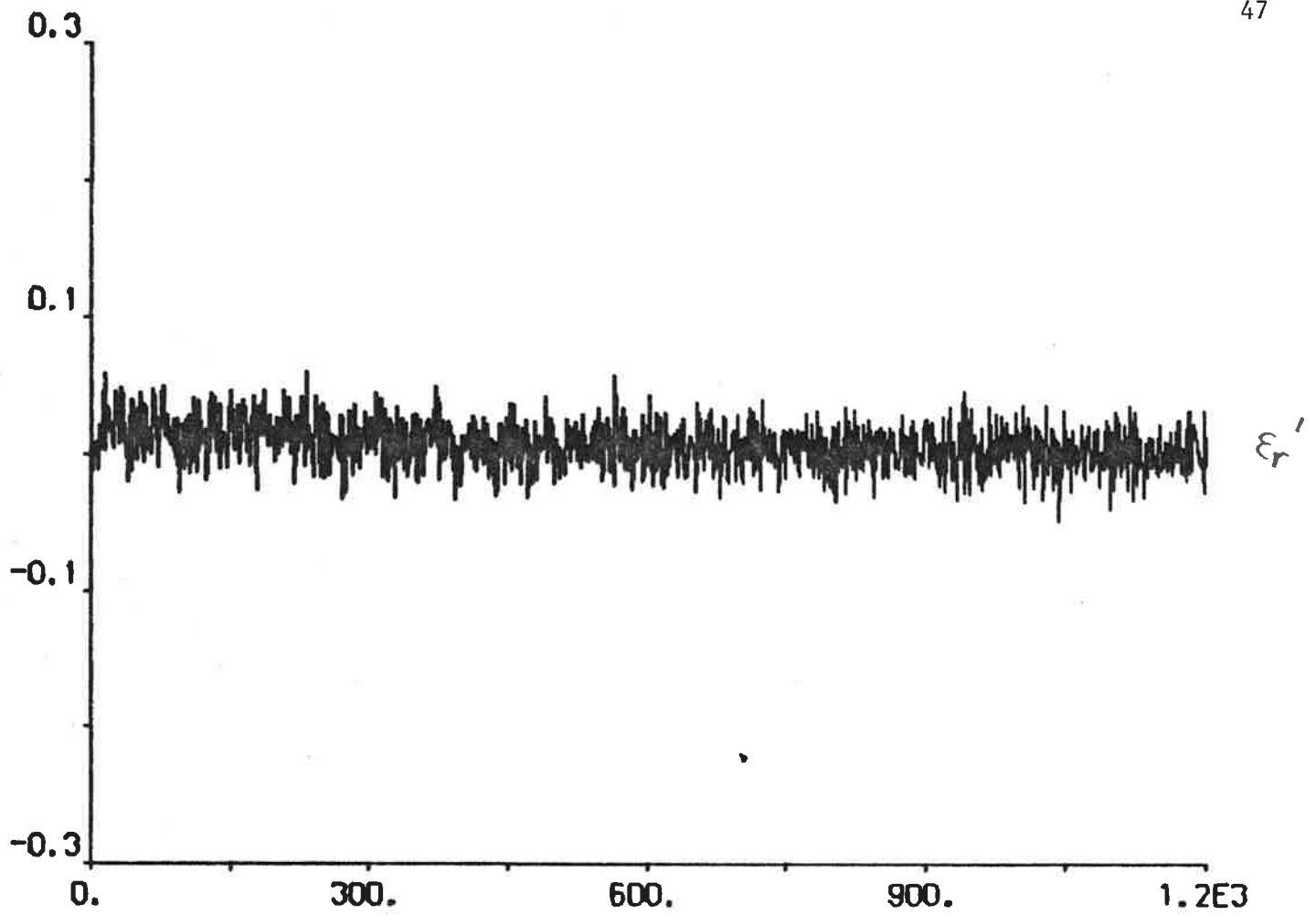


Fig. 4.1ℓ

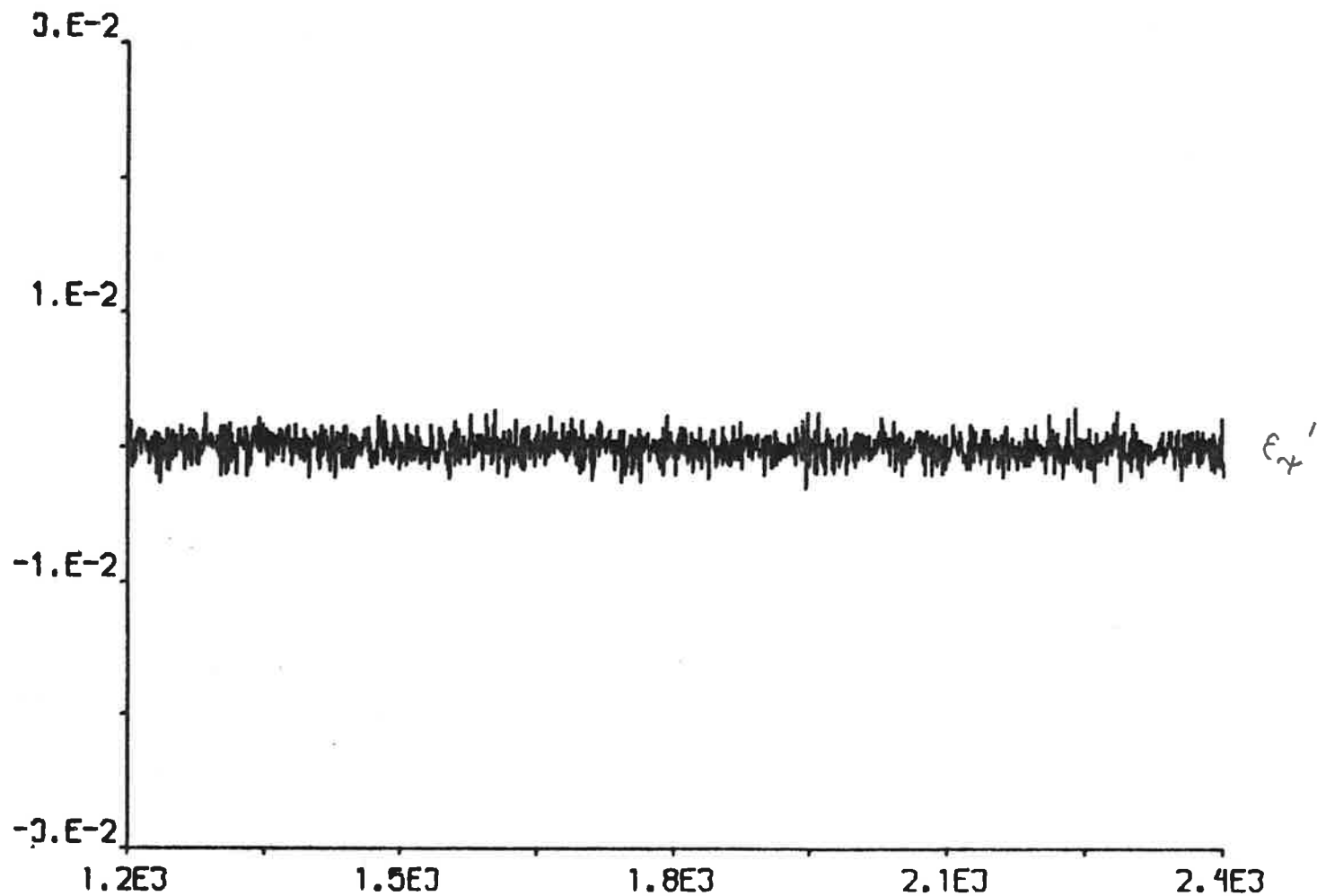
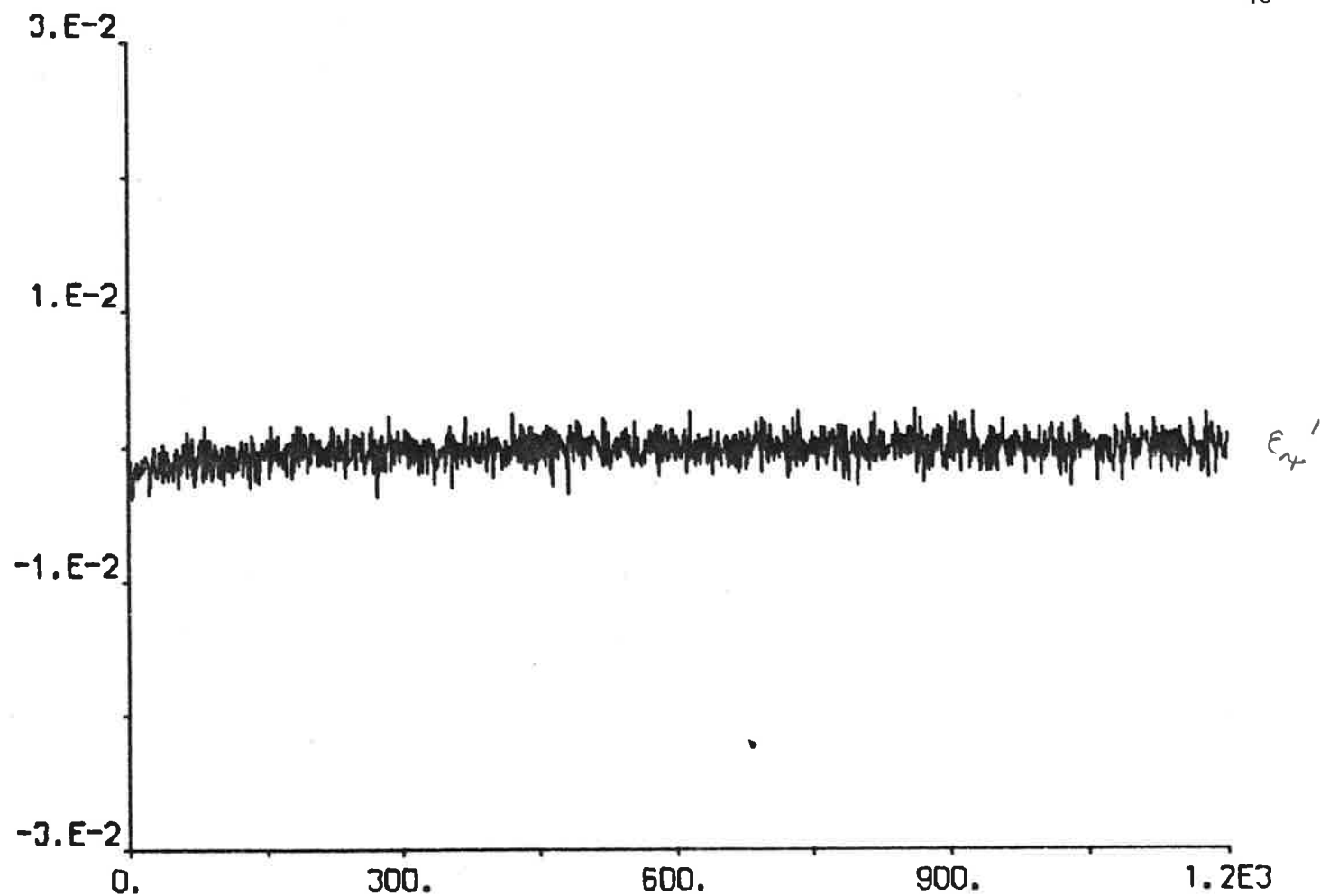


Fig. 4.1m

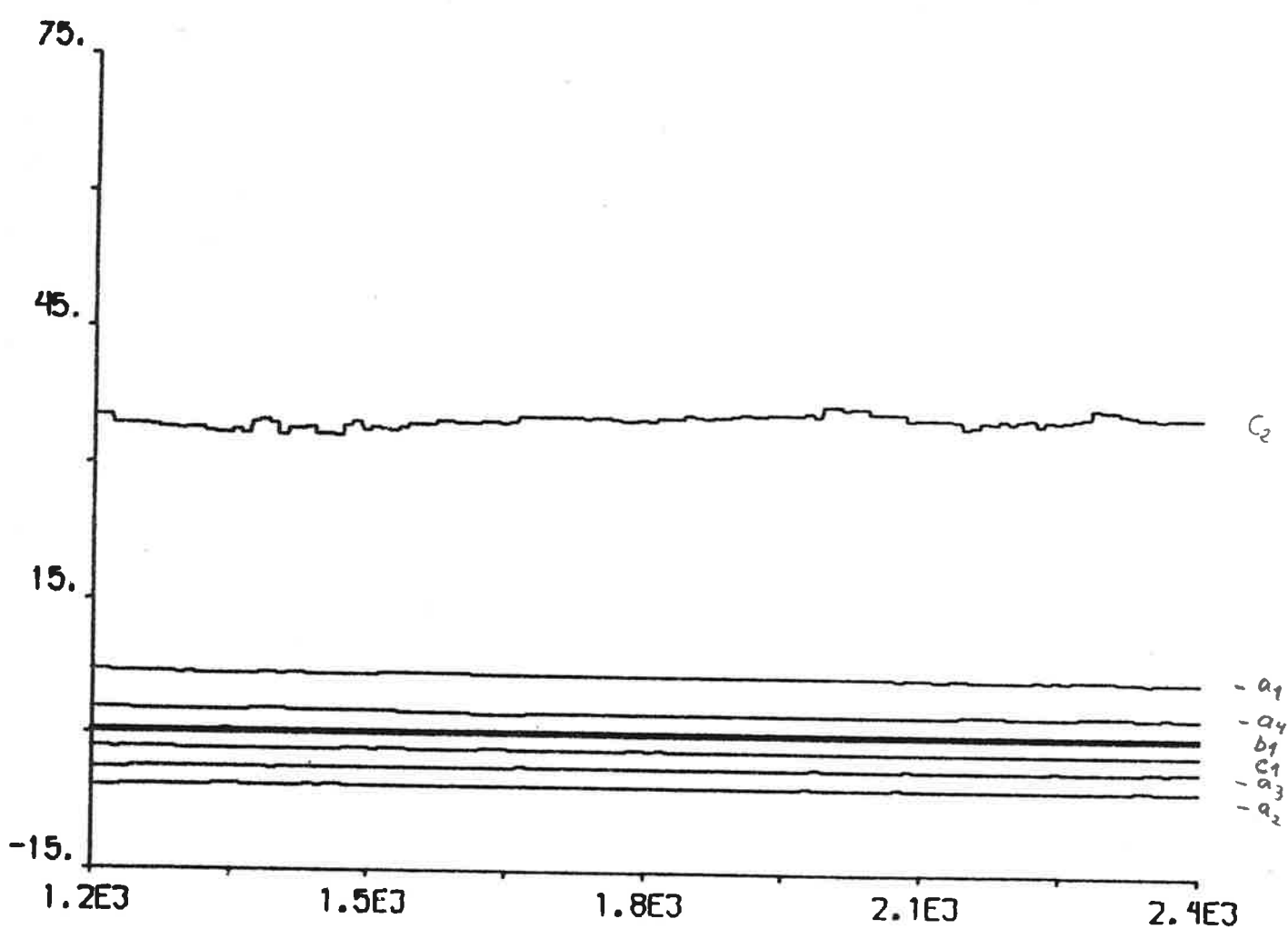
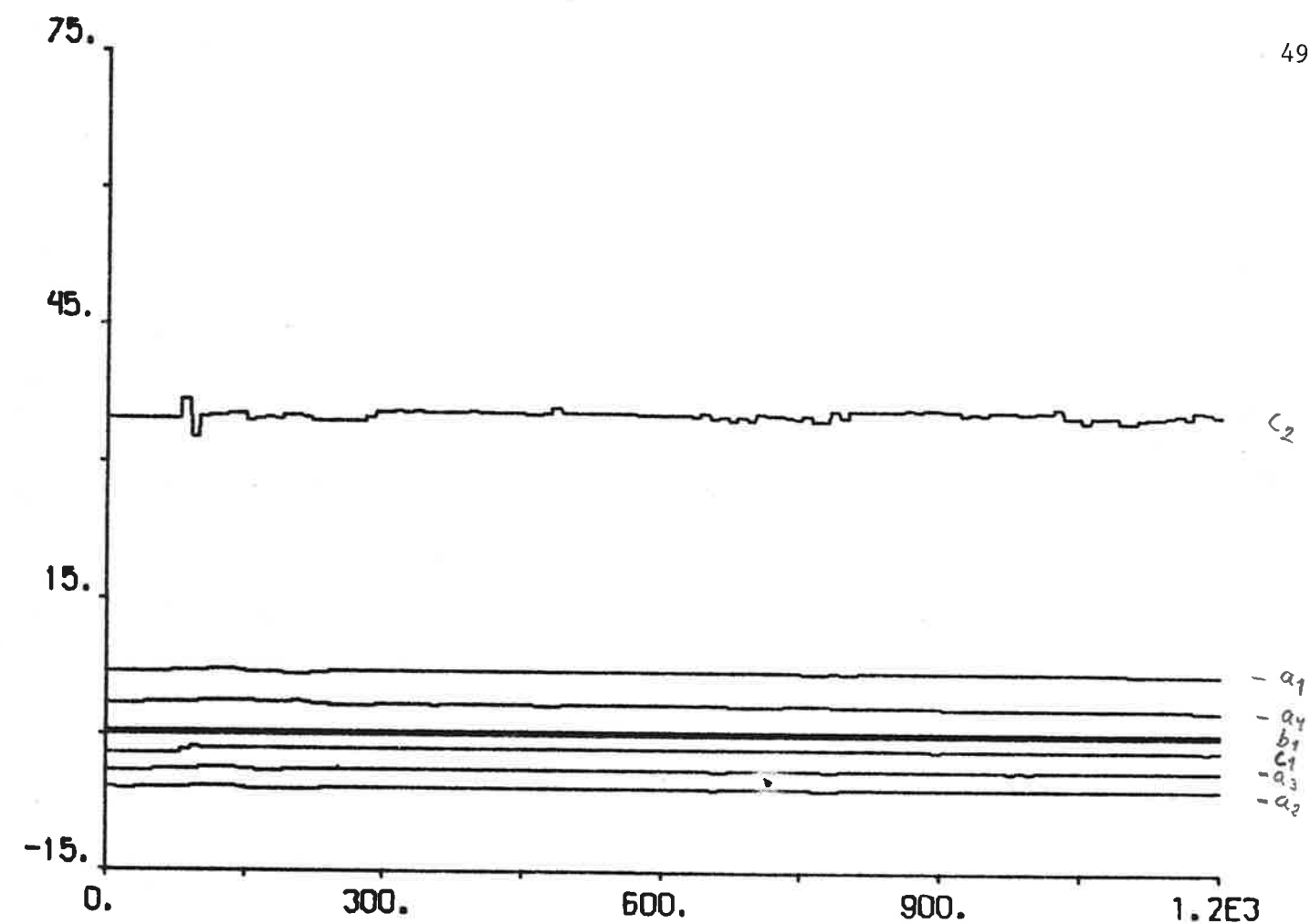


Fig. 4.1n

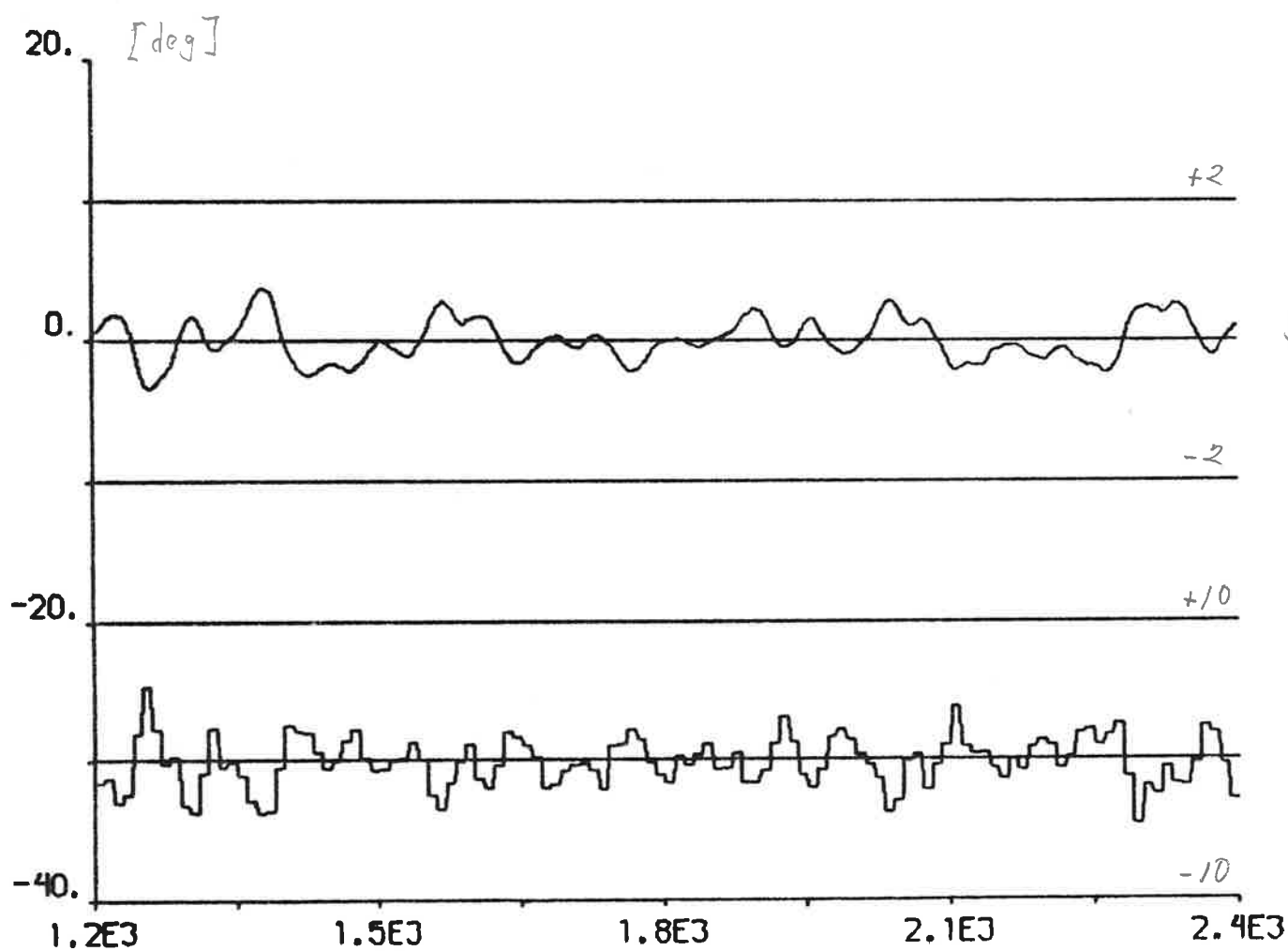
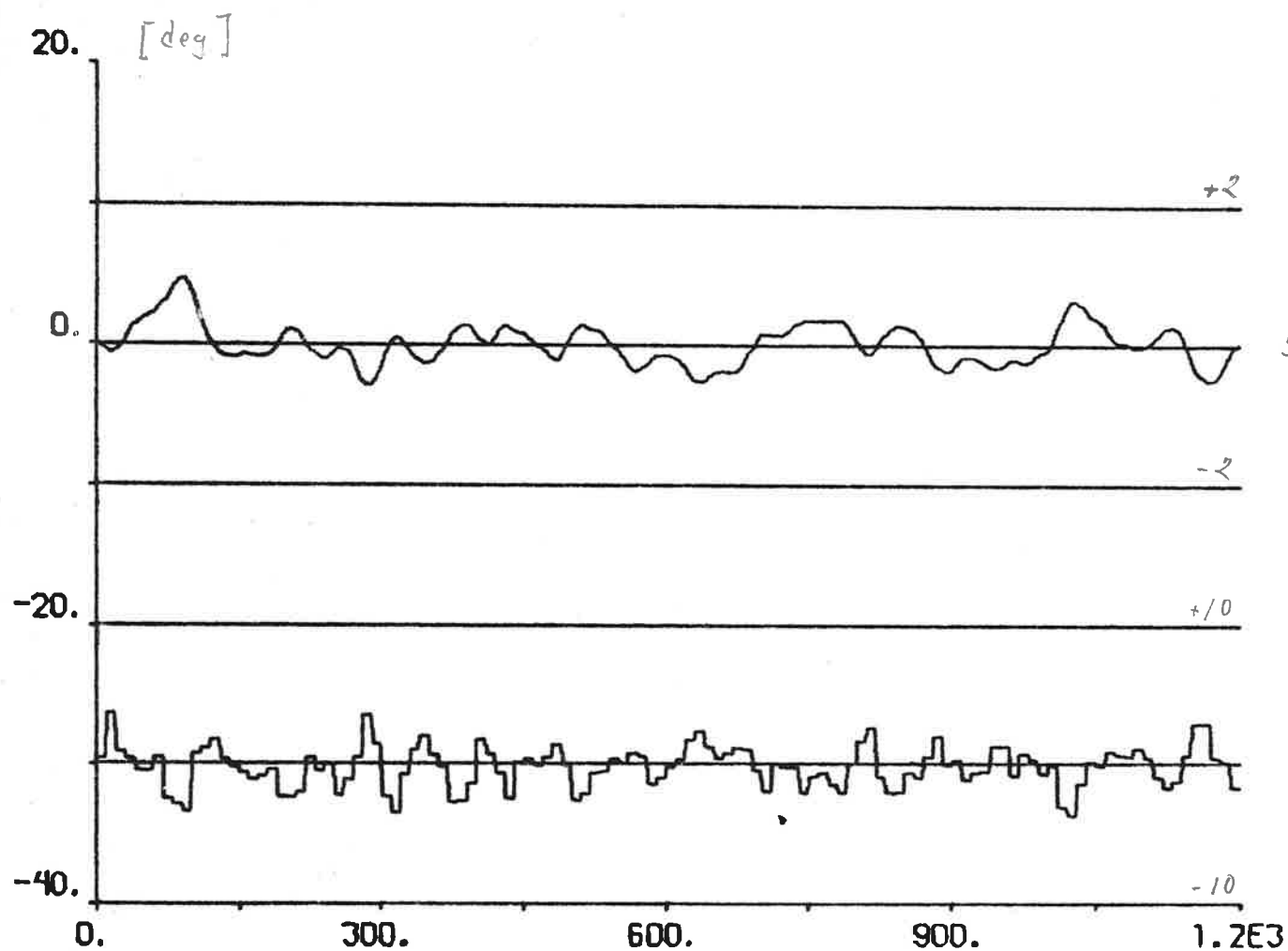


Fig. 4.2a - $T = 10.5$ m, $n_0 = 87.6$ rpm, $u_0 = 15.8$ knots, $\delta_\ell = 10$ deg, self-tuning regulator using estimates from the Kalman filter.

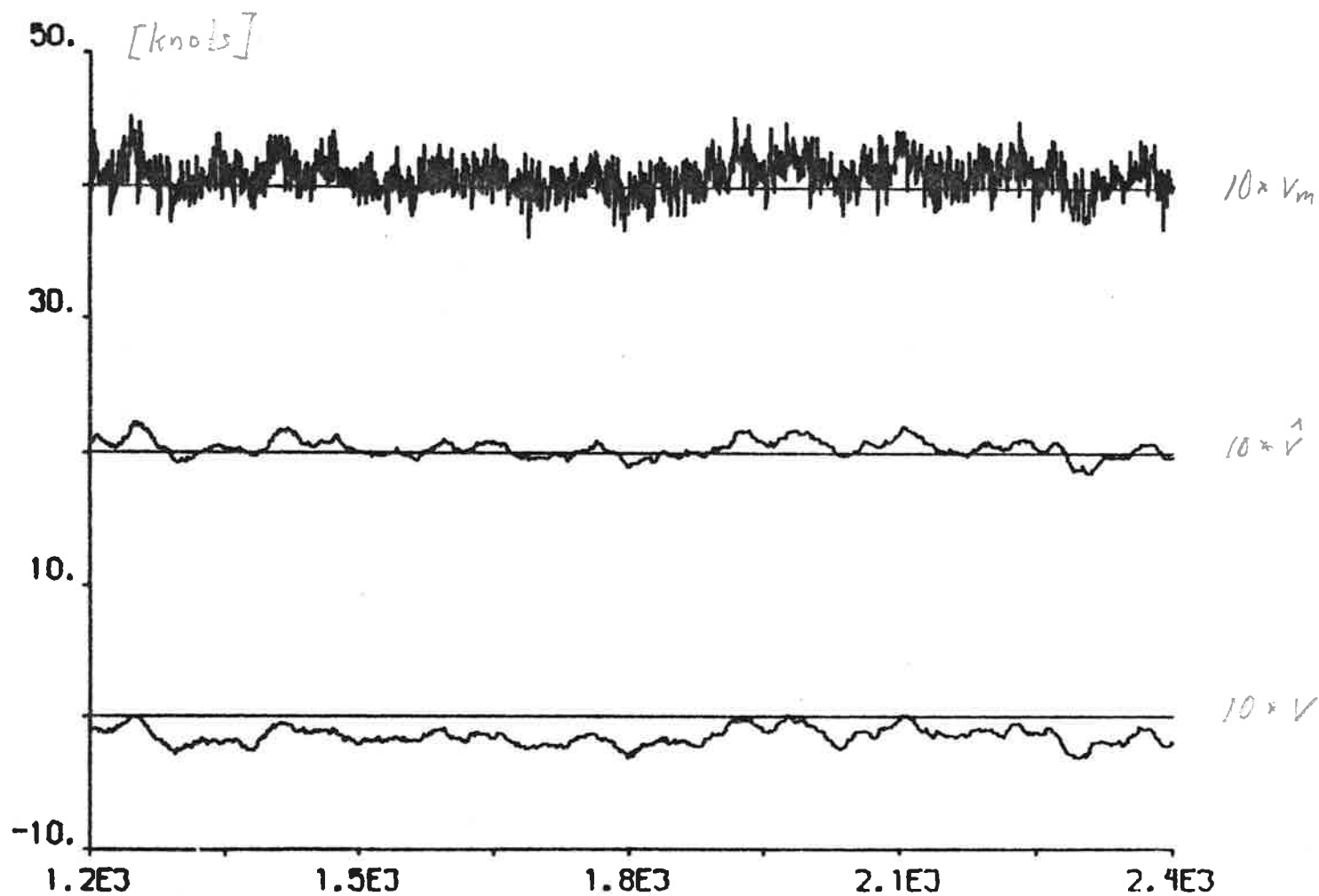
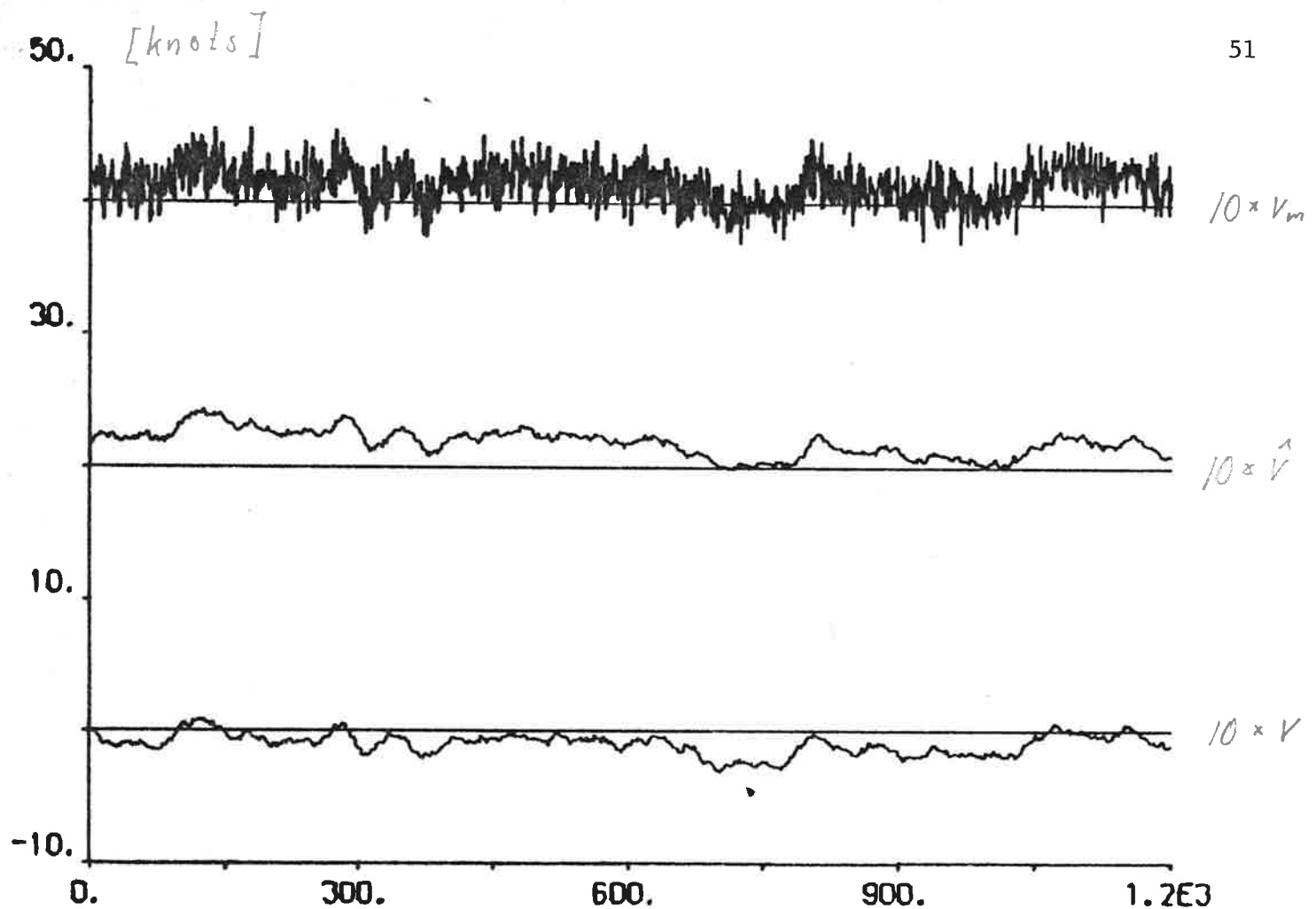


Fig. 4.2b

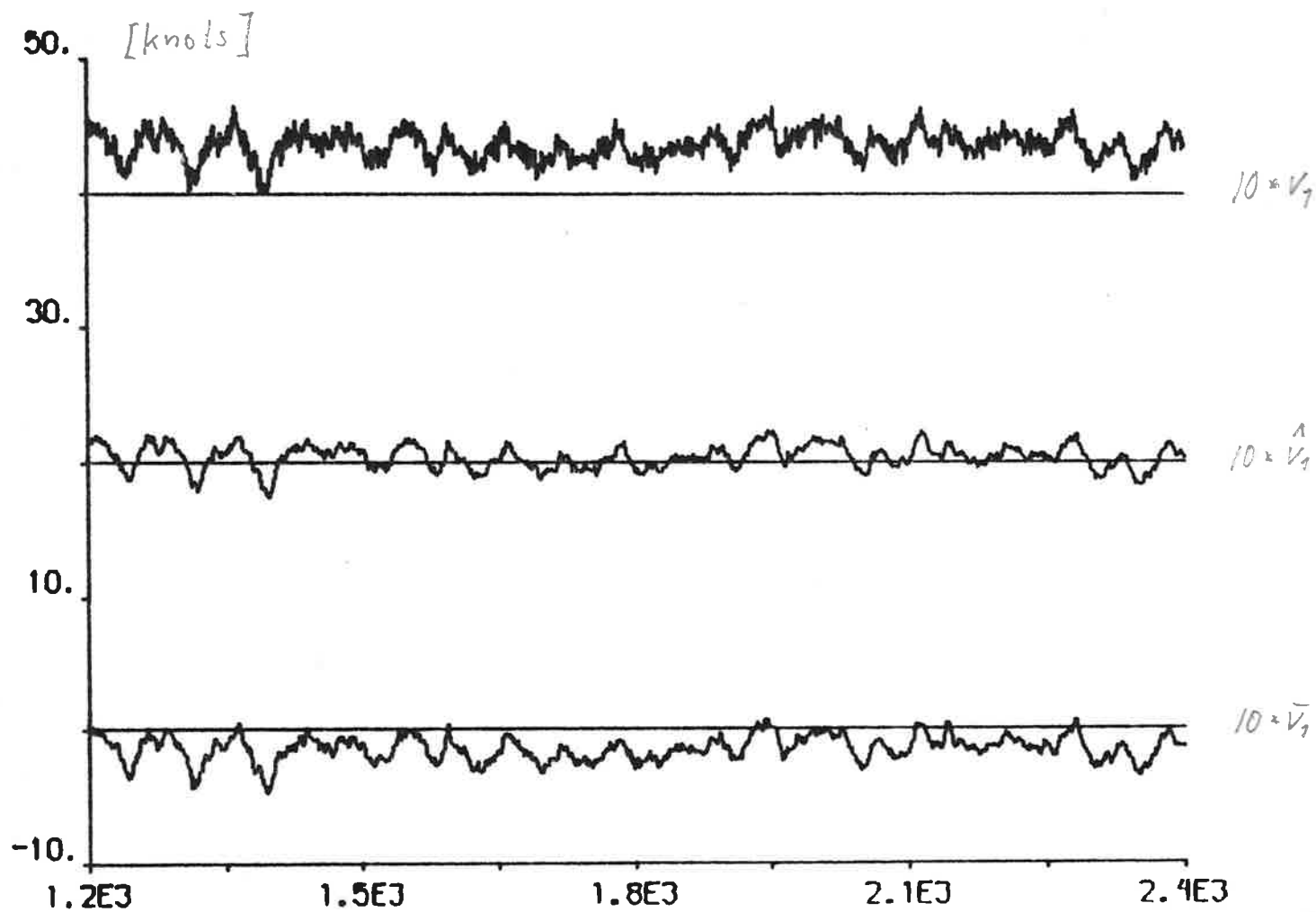
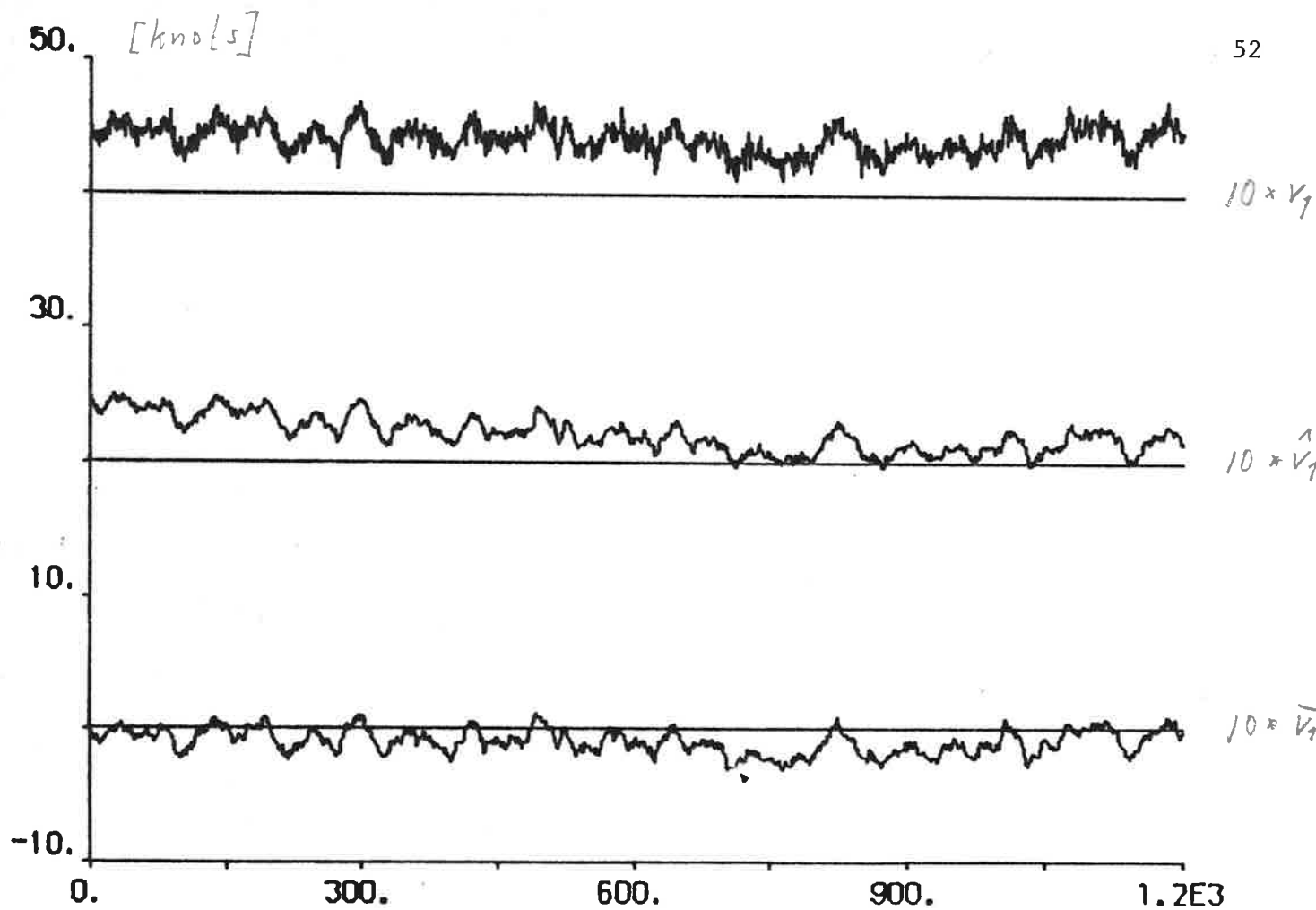


Fig. 4.2c

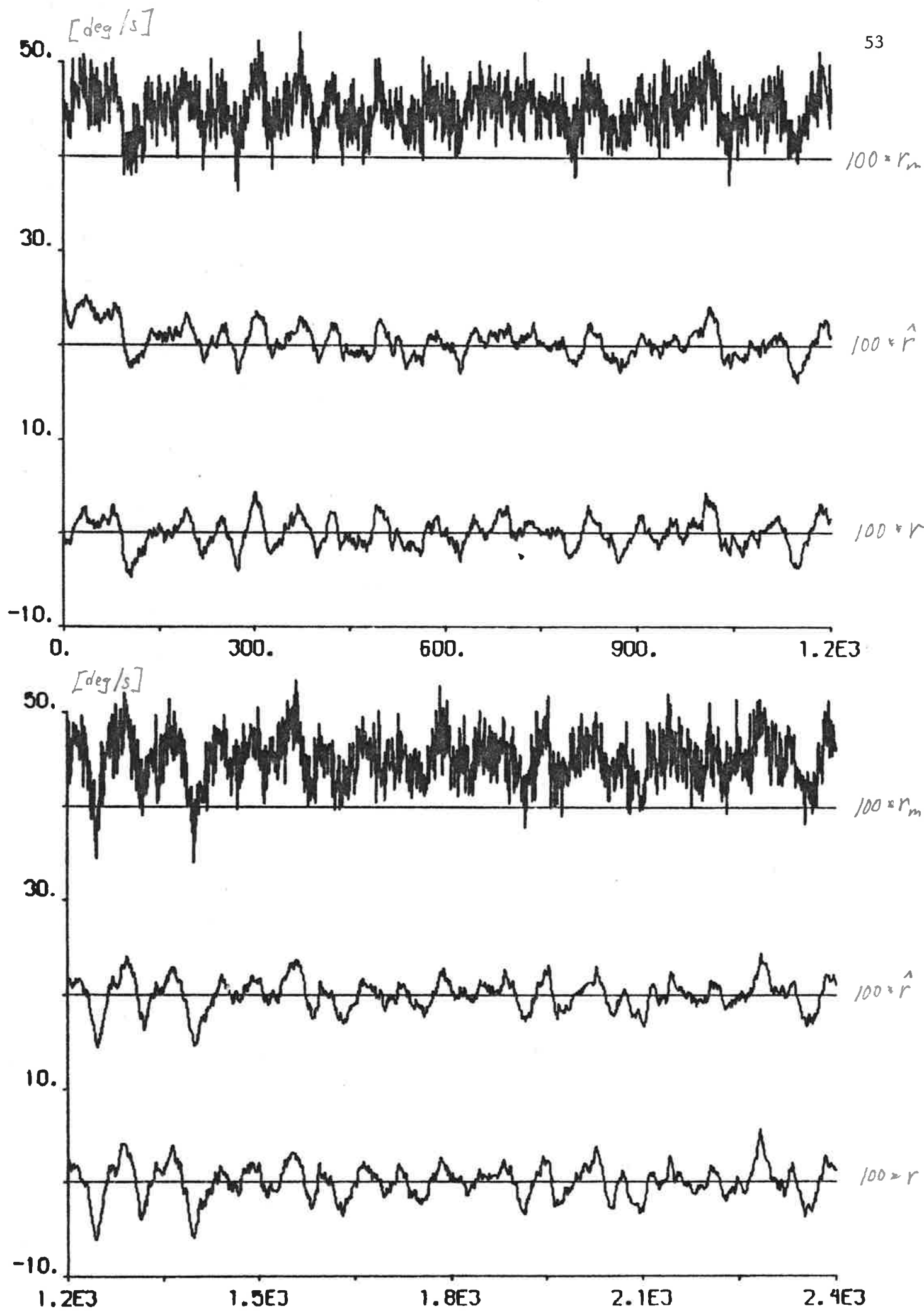


Fig. 4.2d

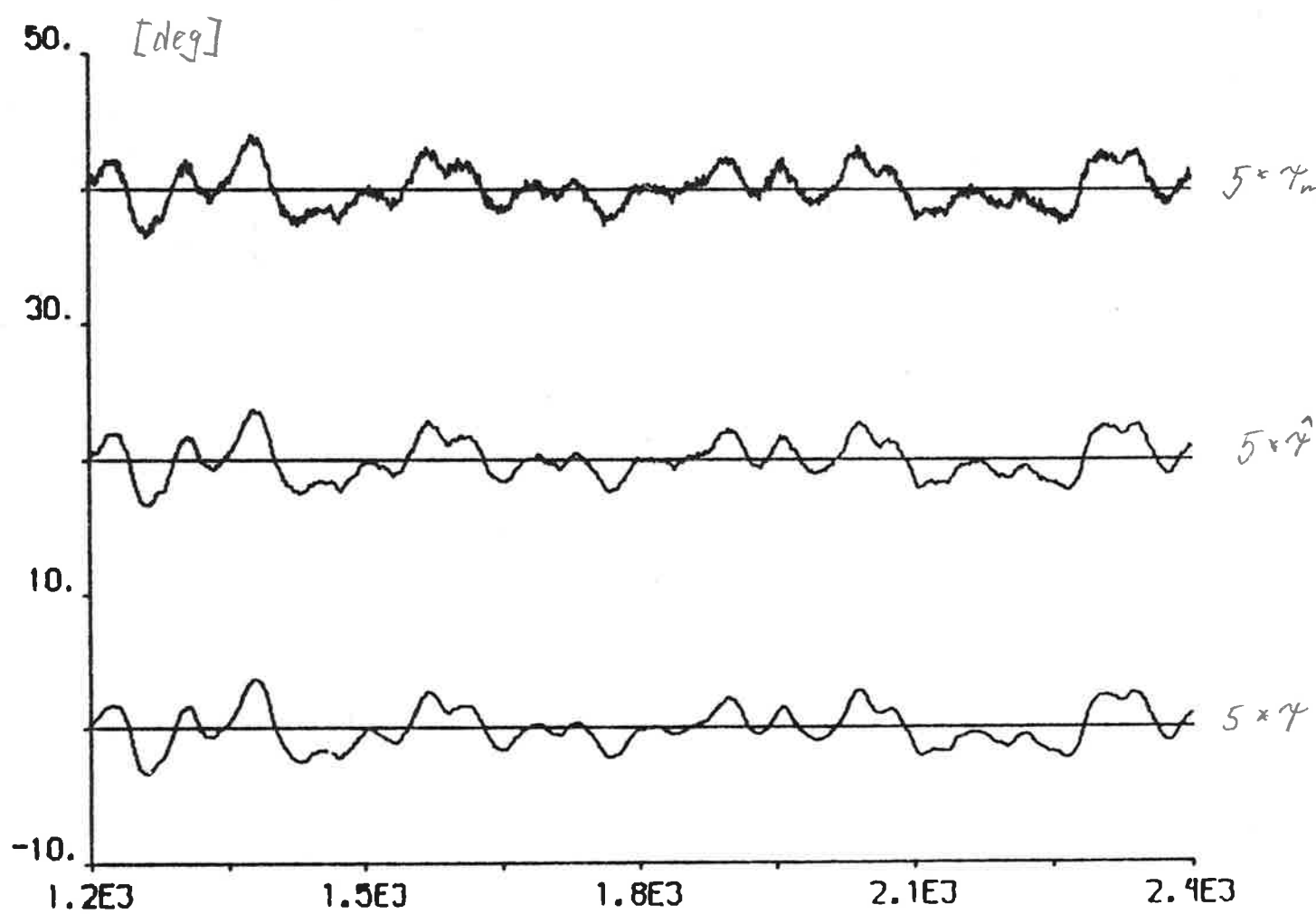
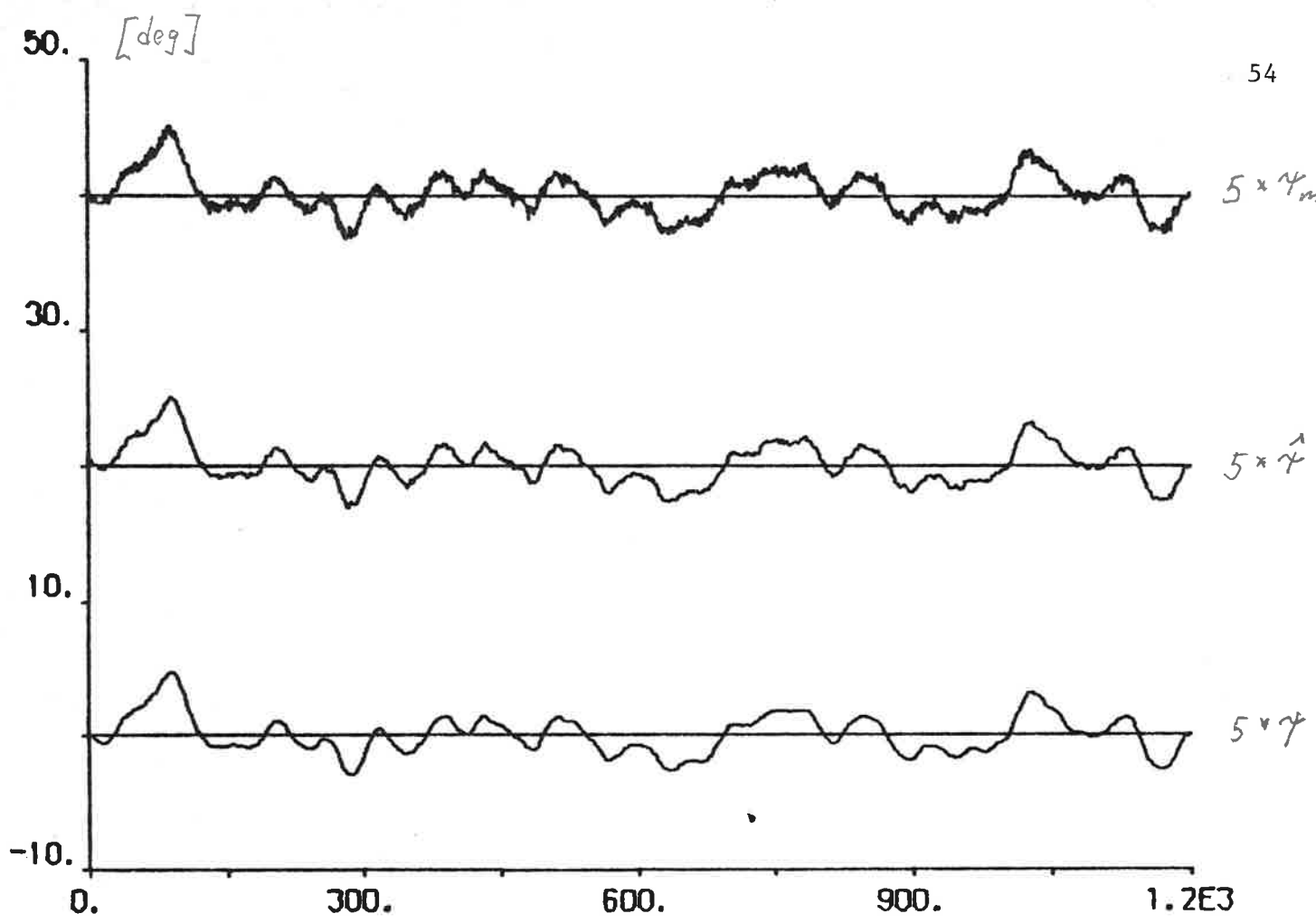


Fig. 4.2e

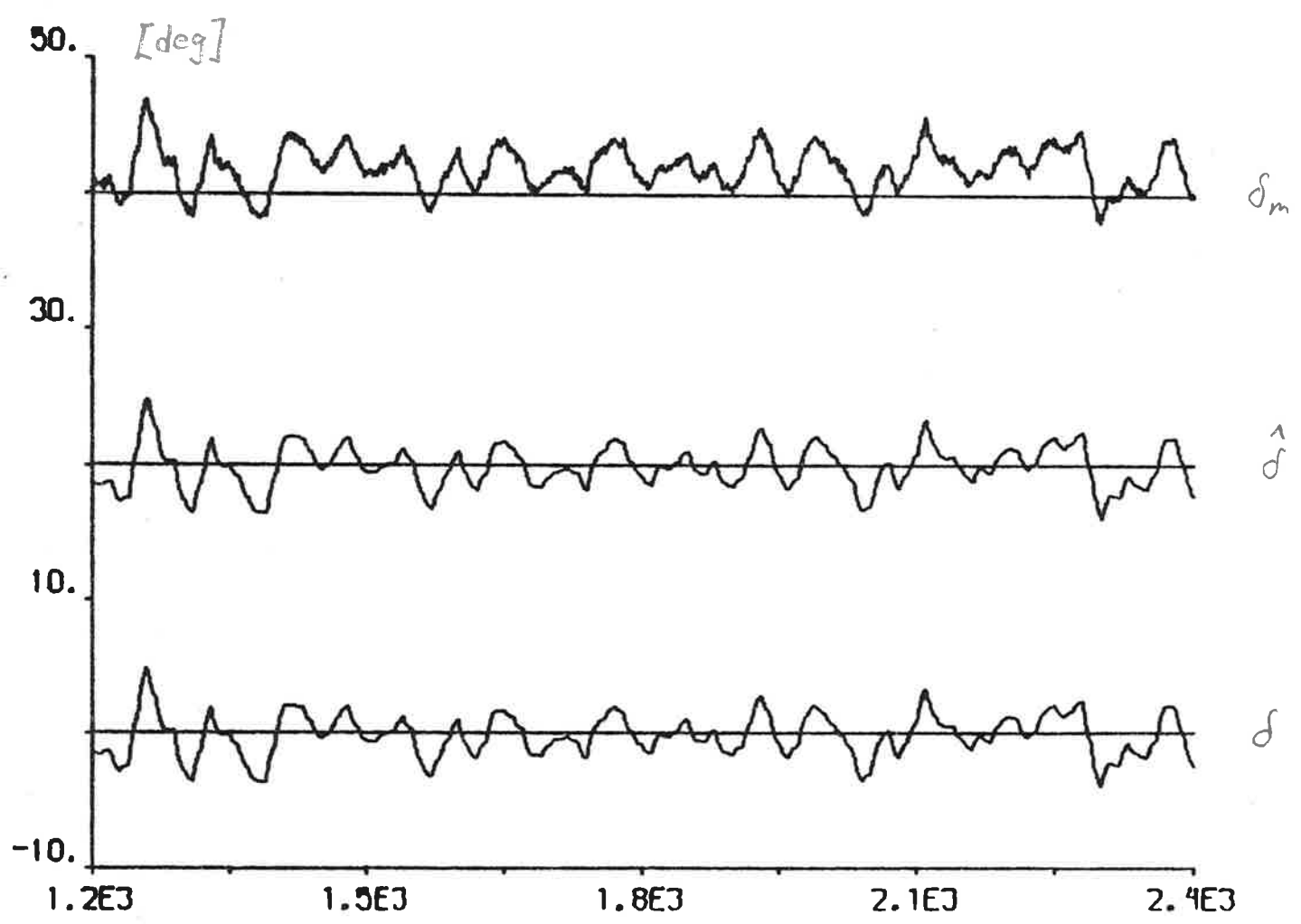
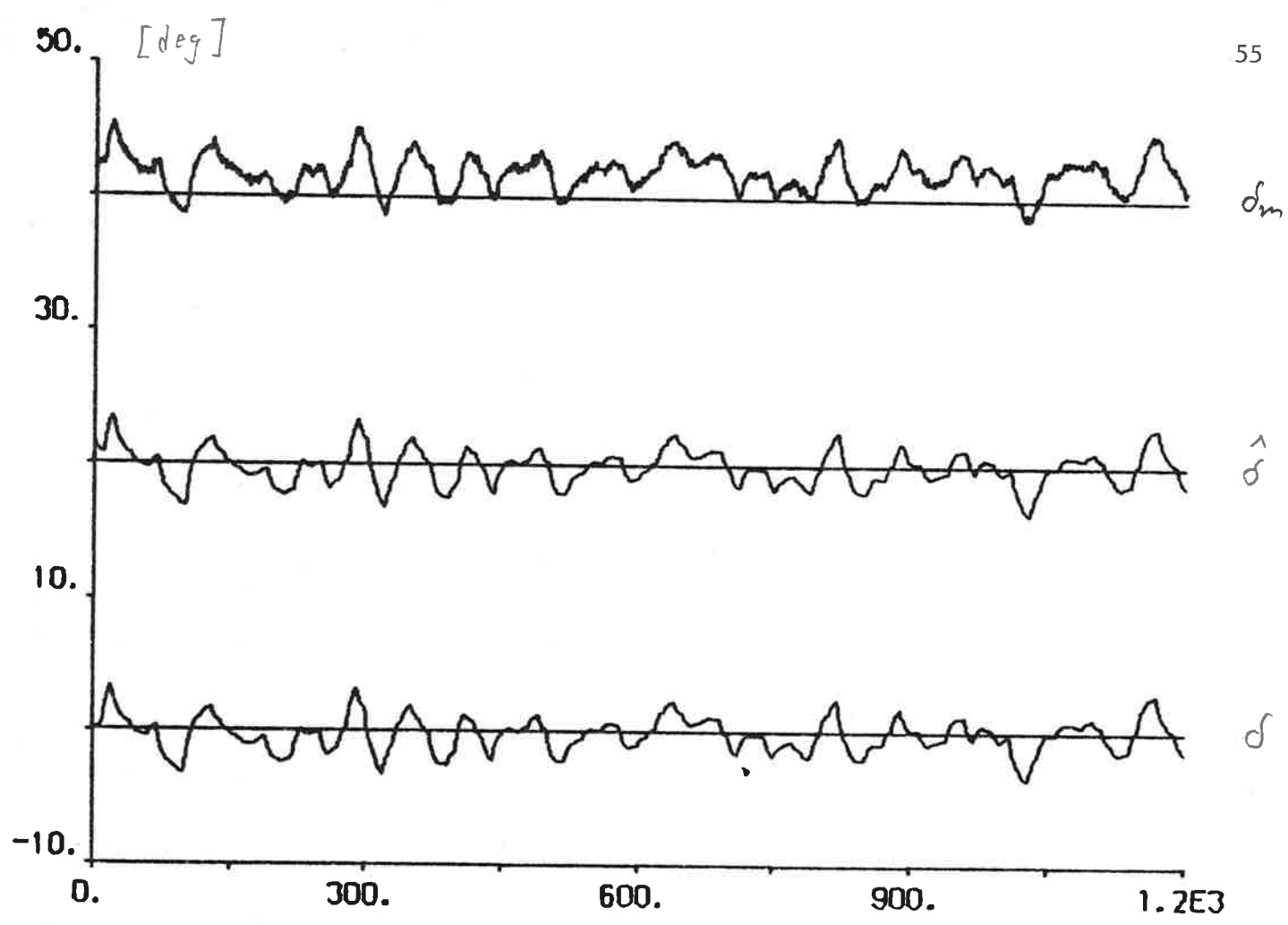


Fig. 4.2f

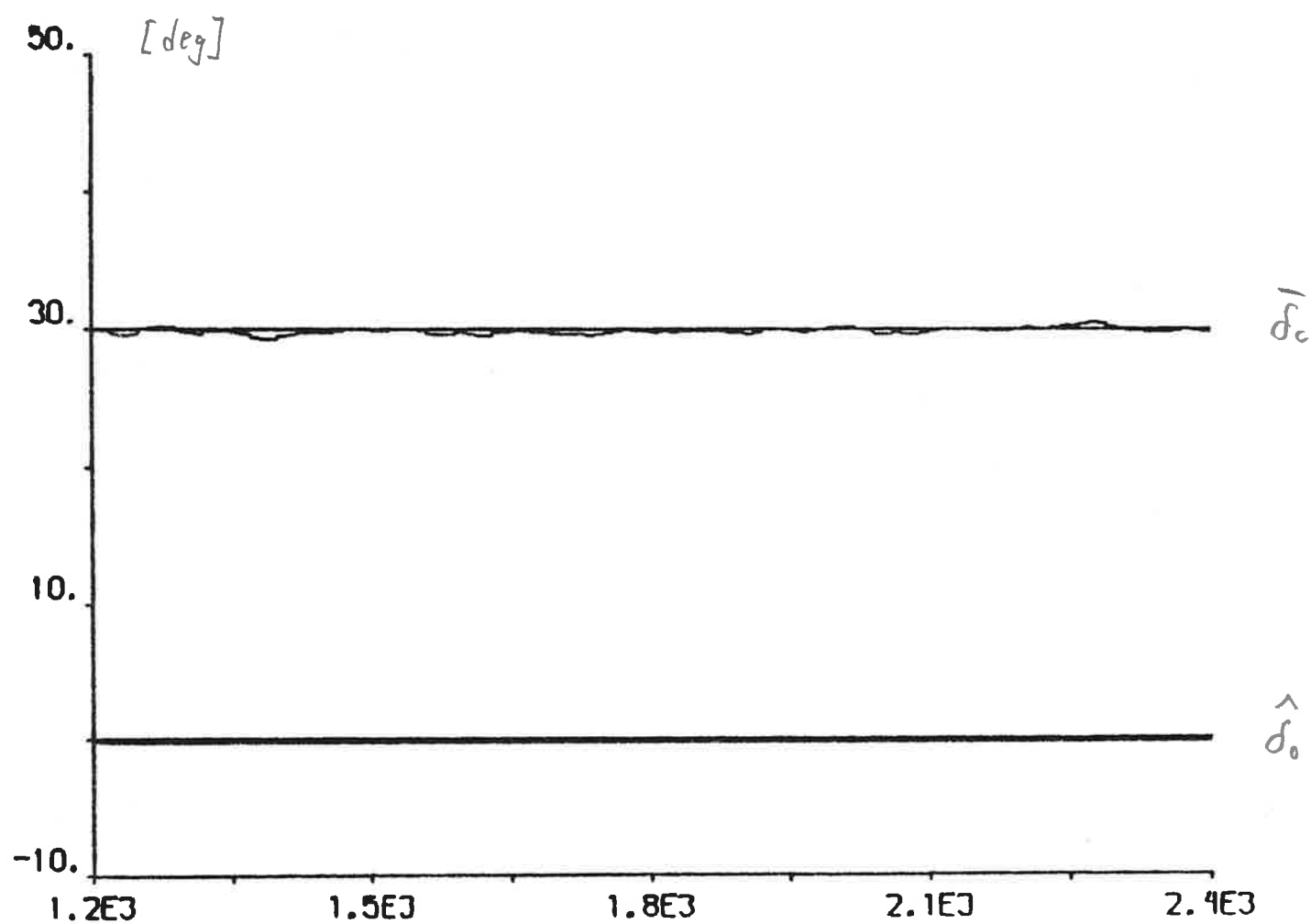
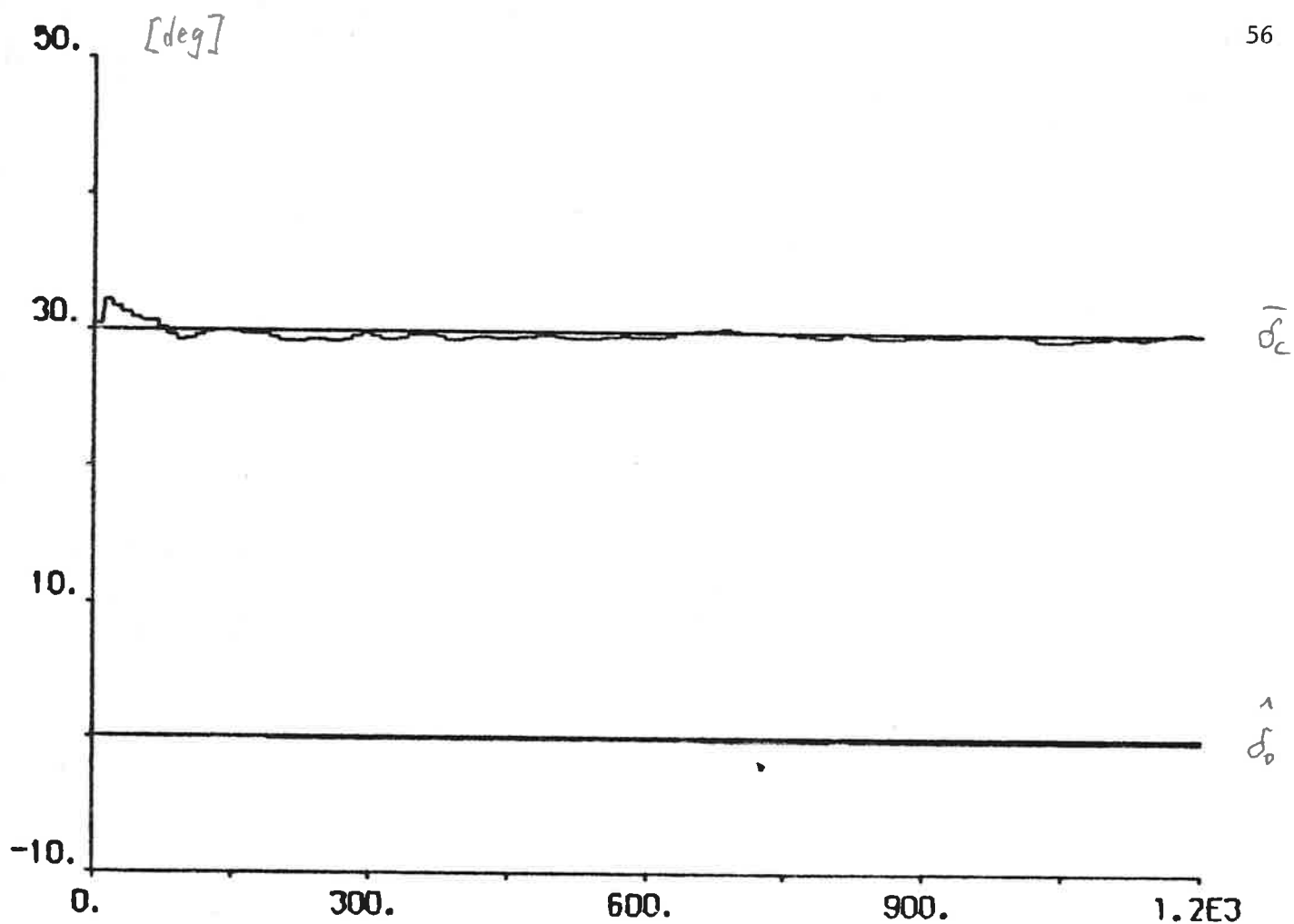


Fig. 4.2g

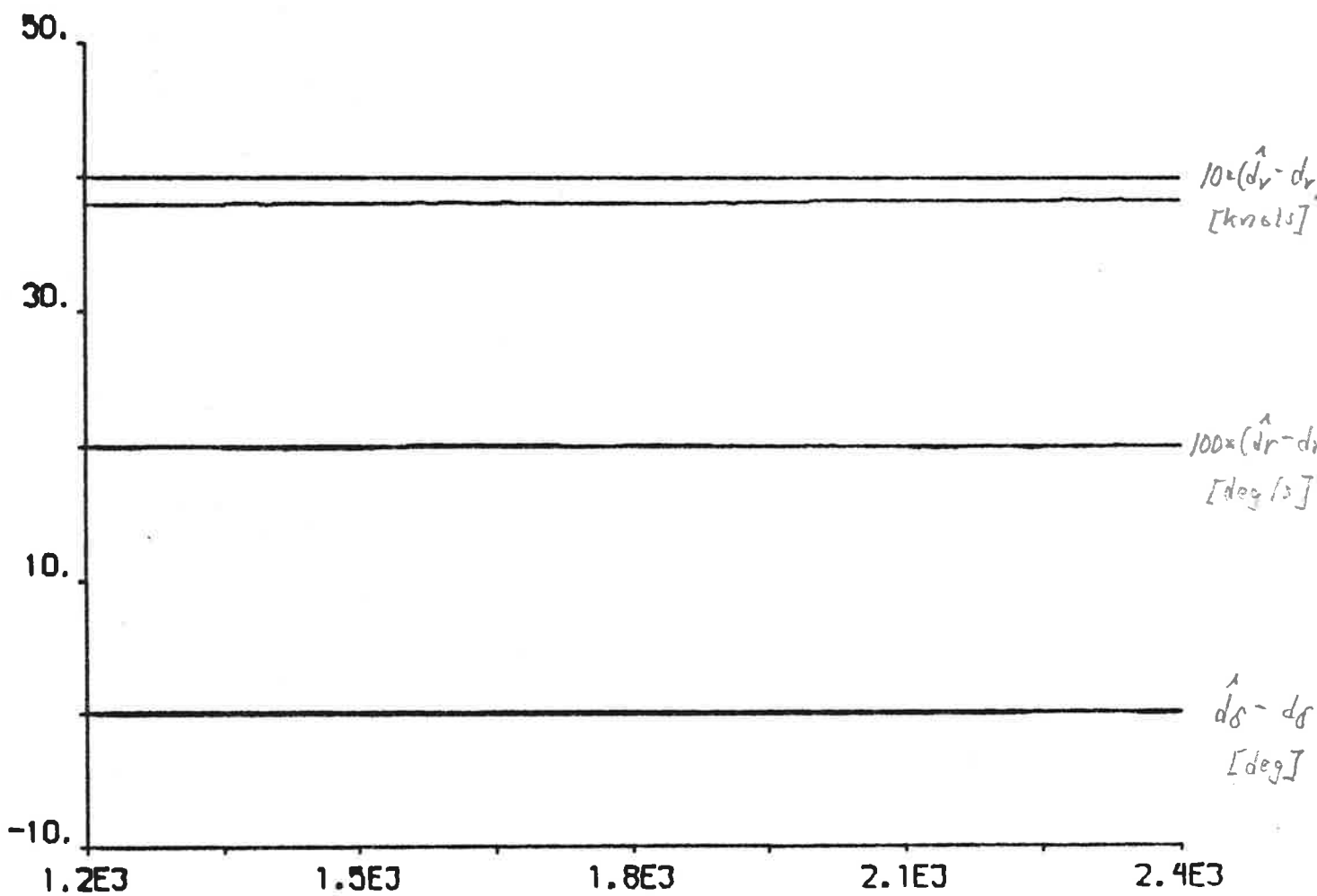
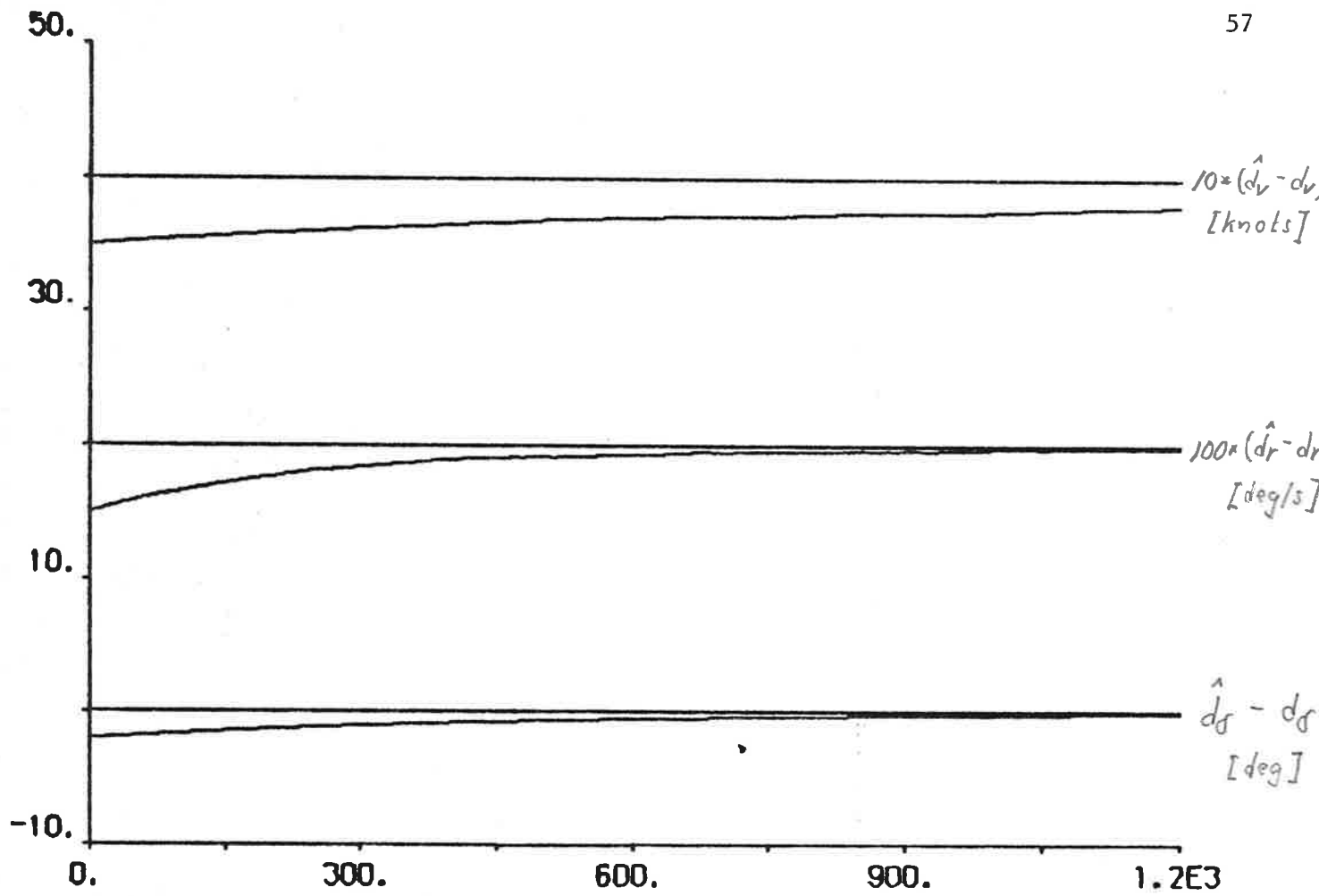


Fig. 4.2h

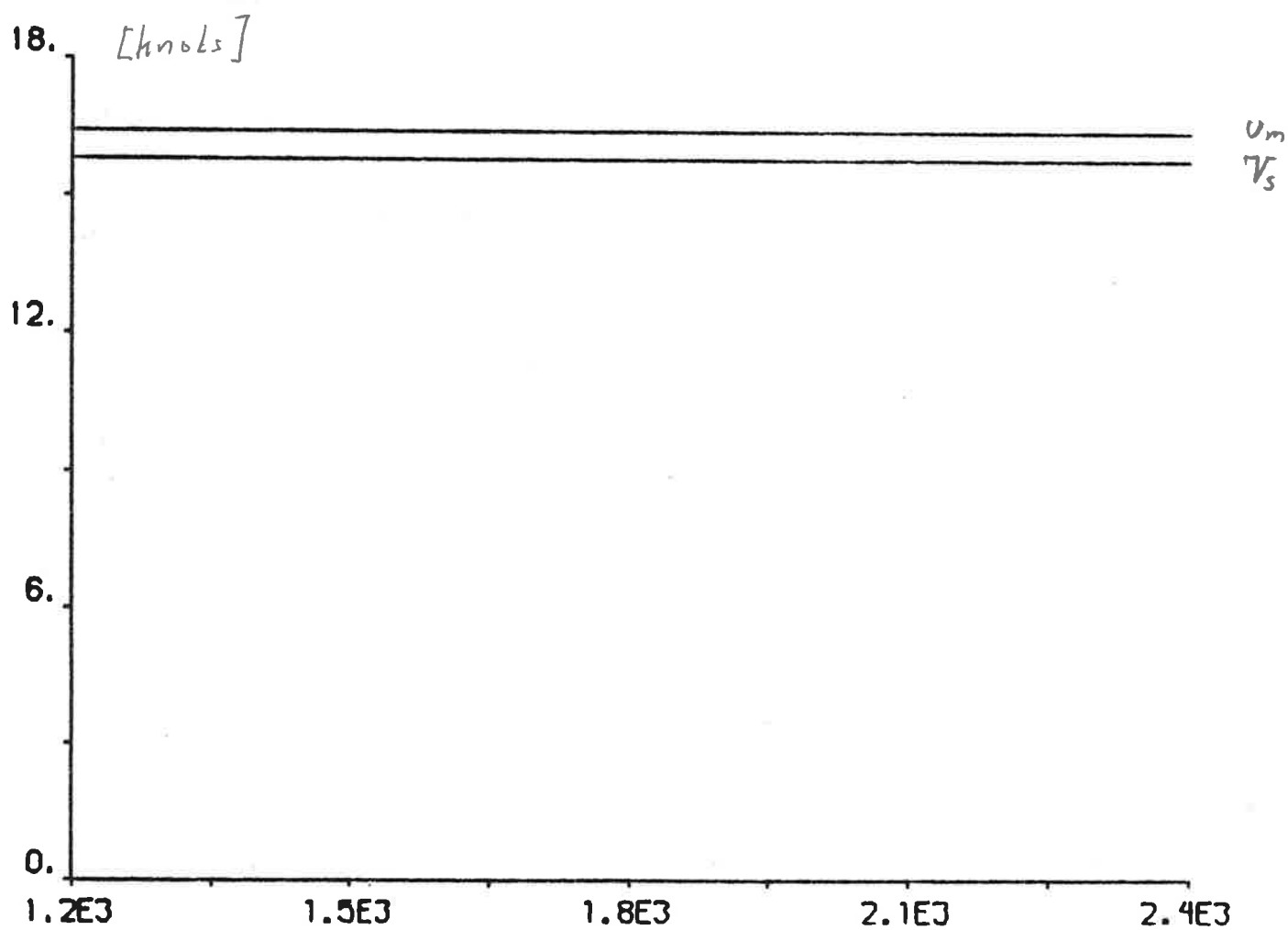
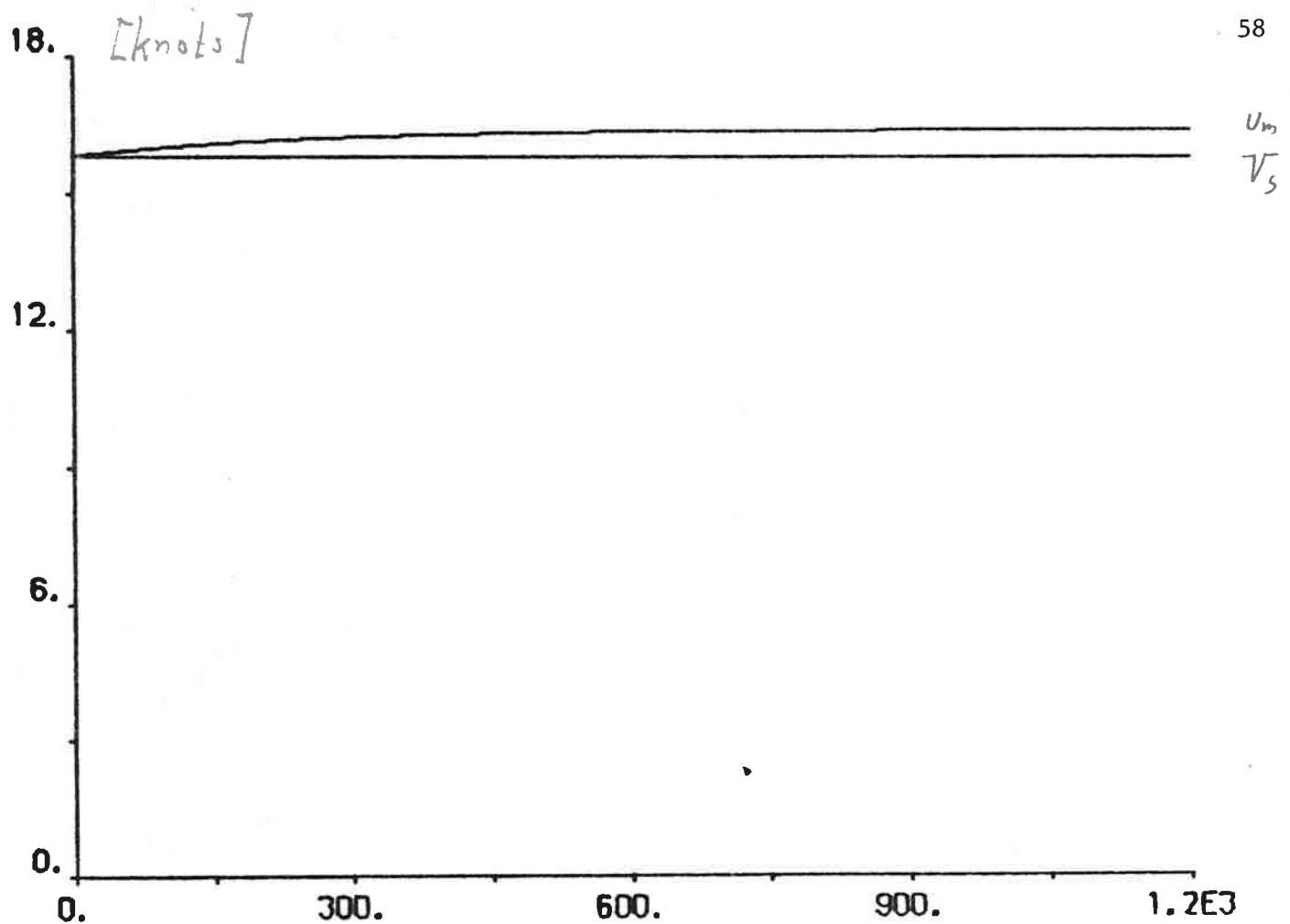


Fig. 4.2i

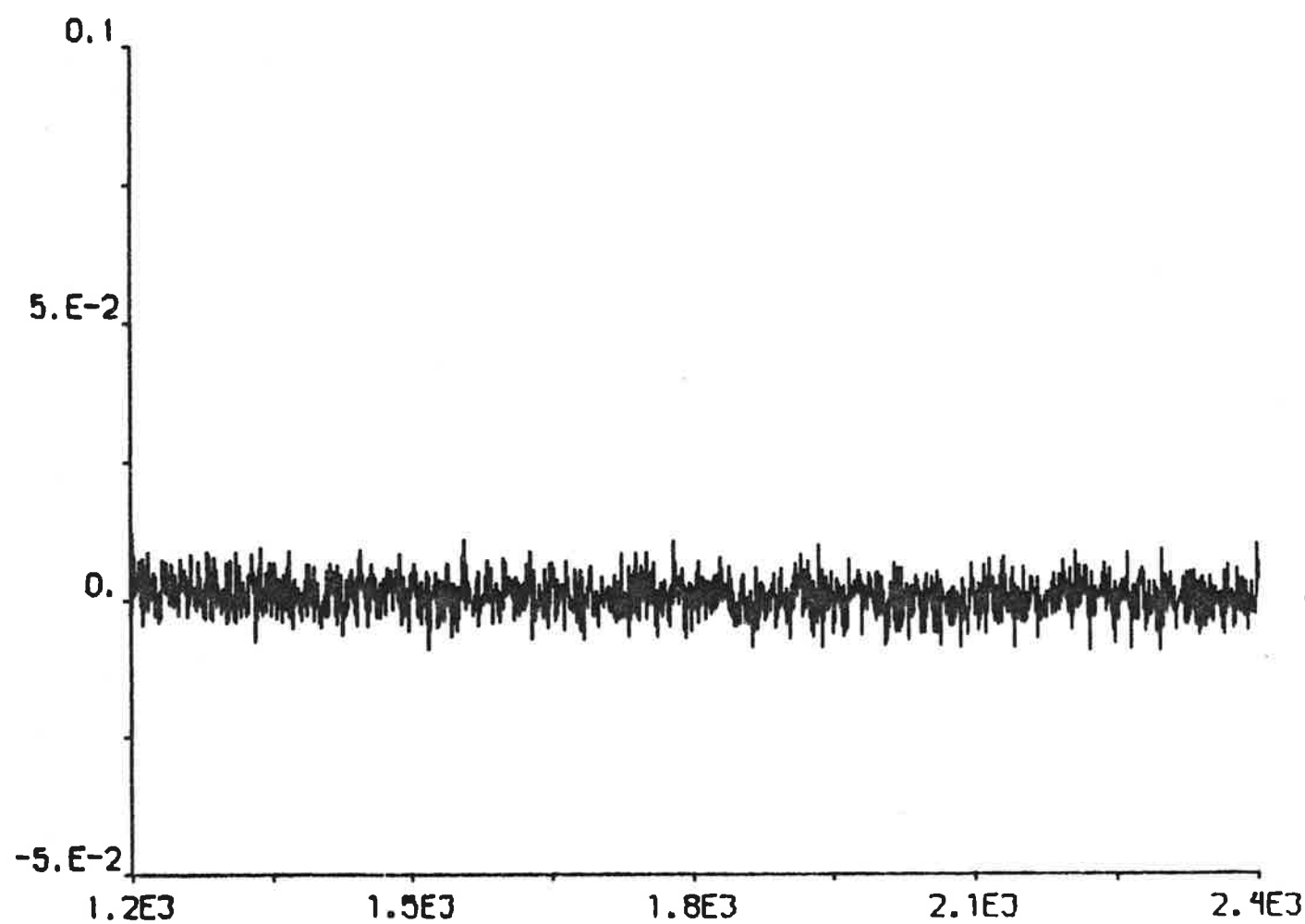
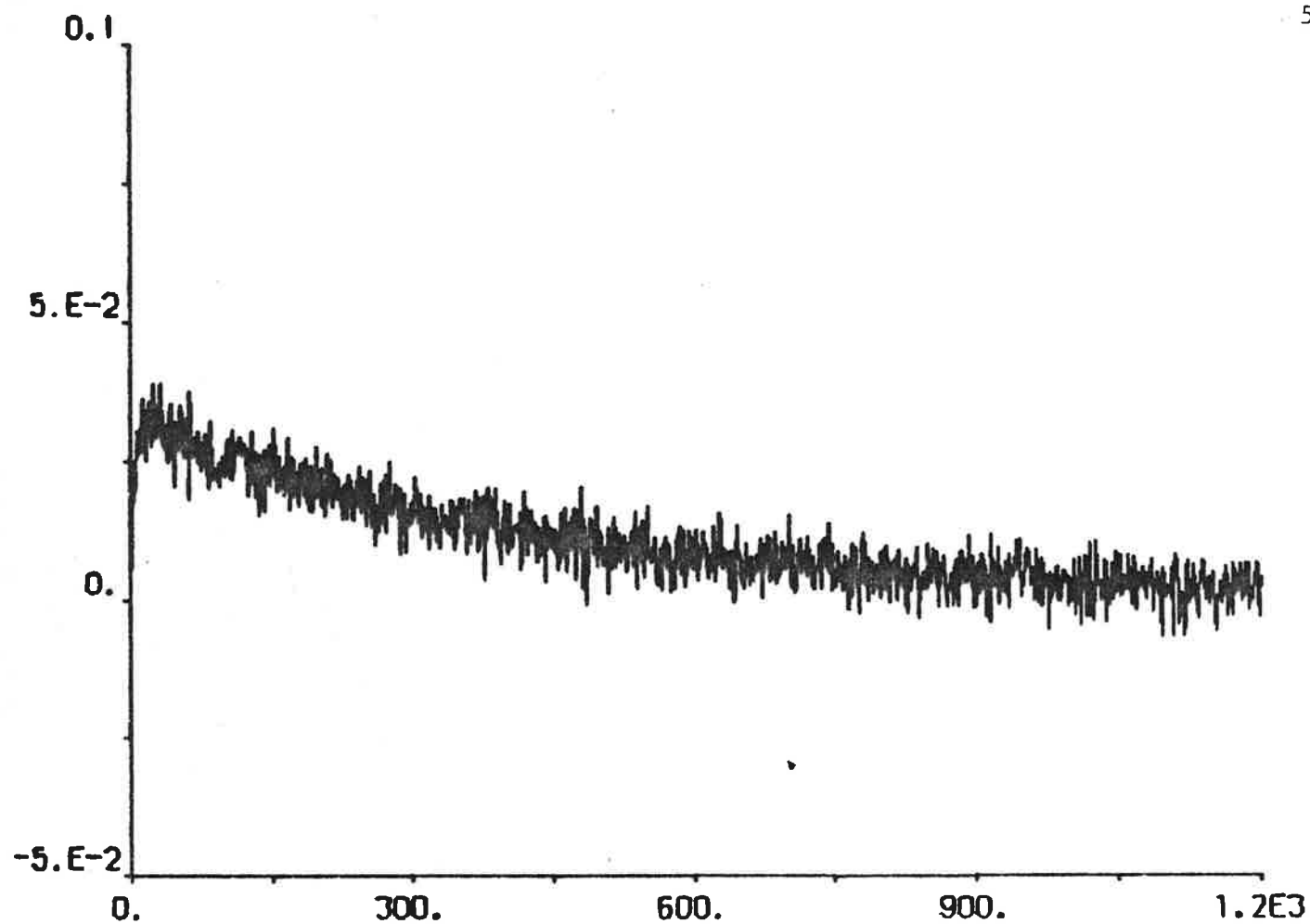
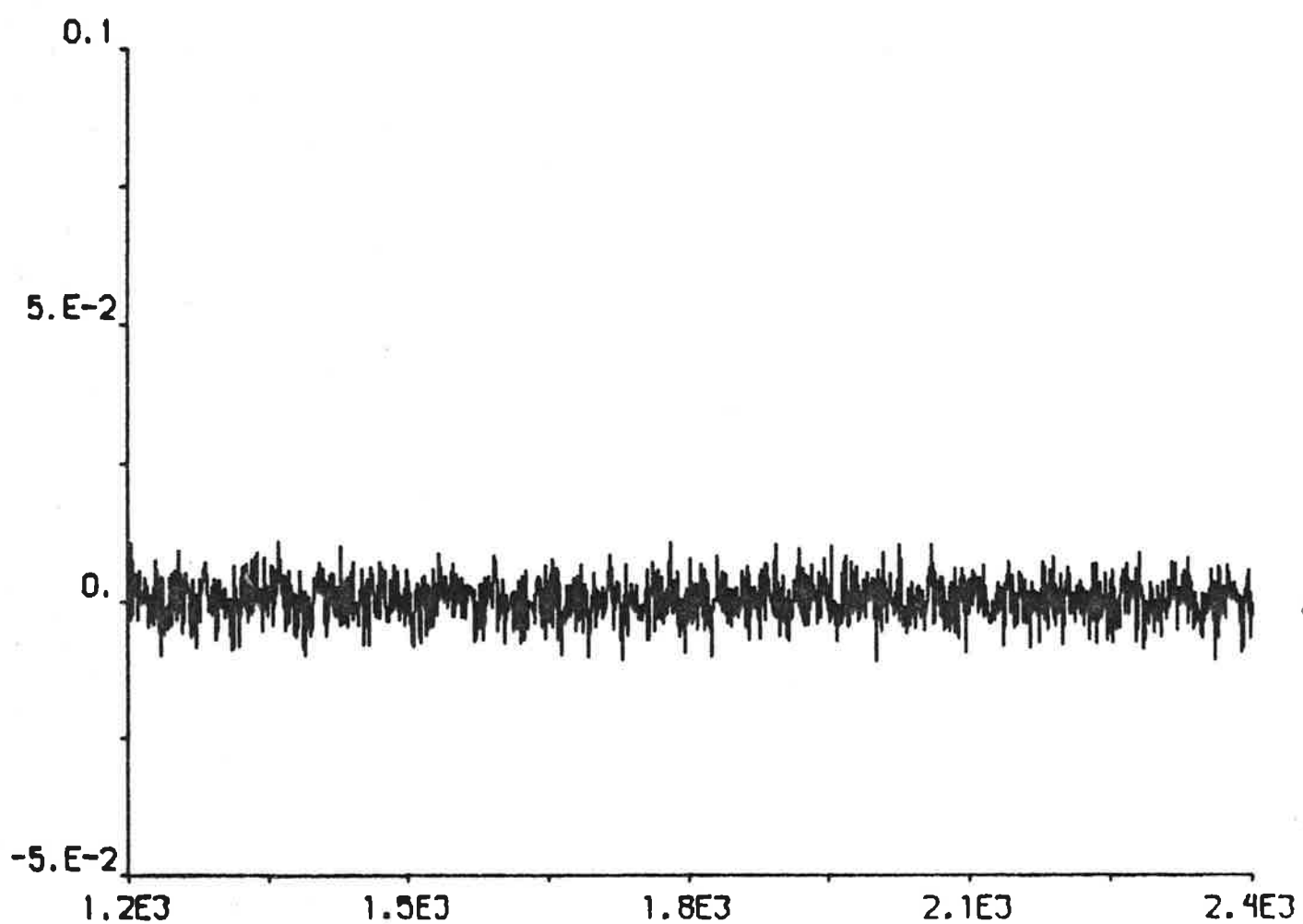
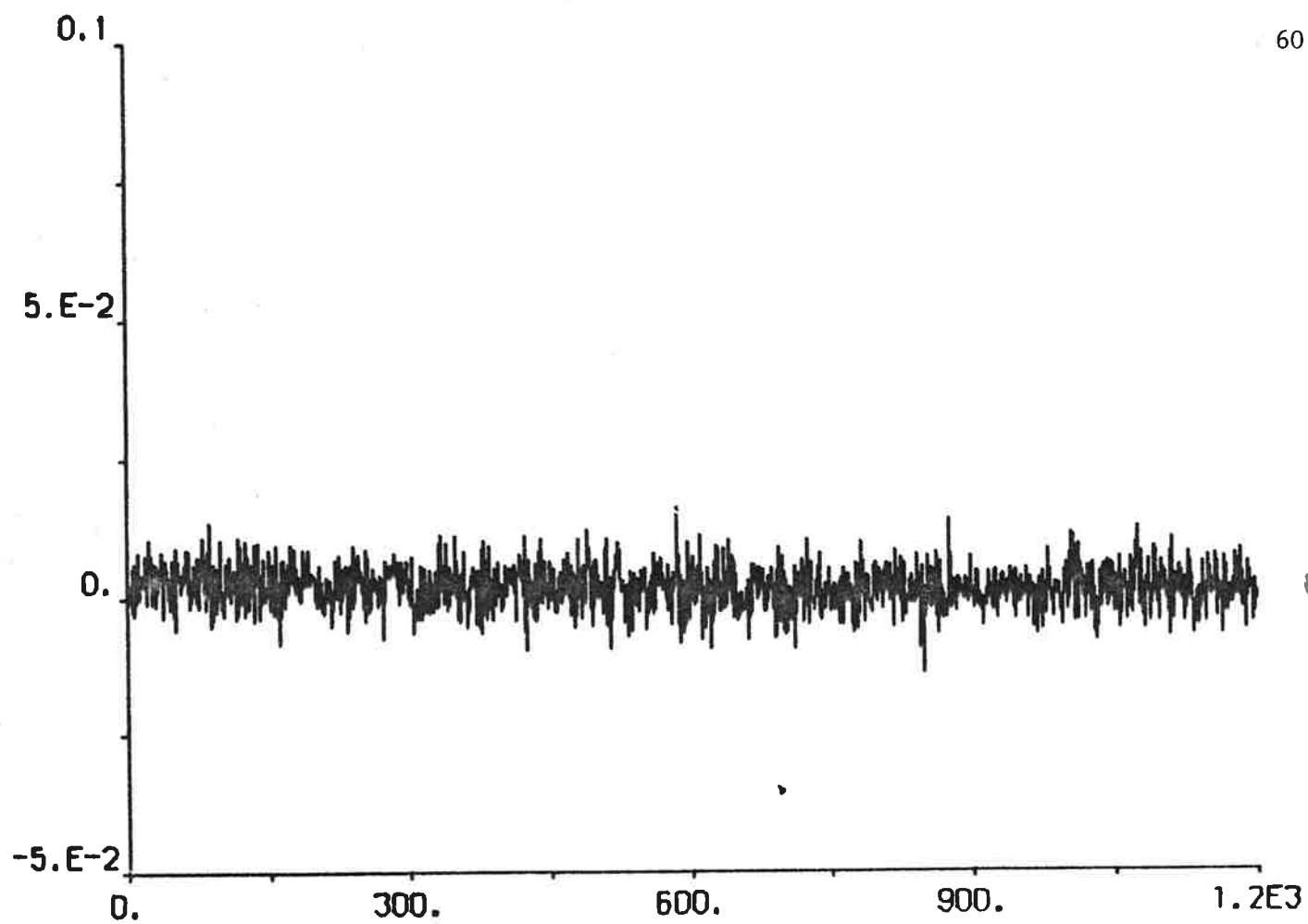


Fig. 4.2j

Fig. 4.2k

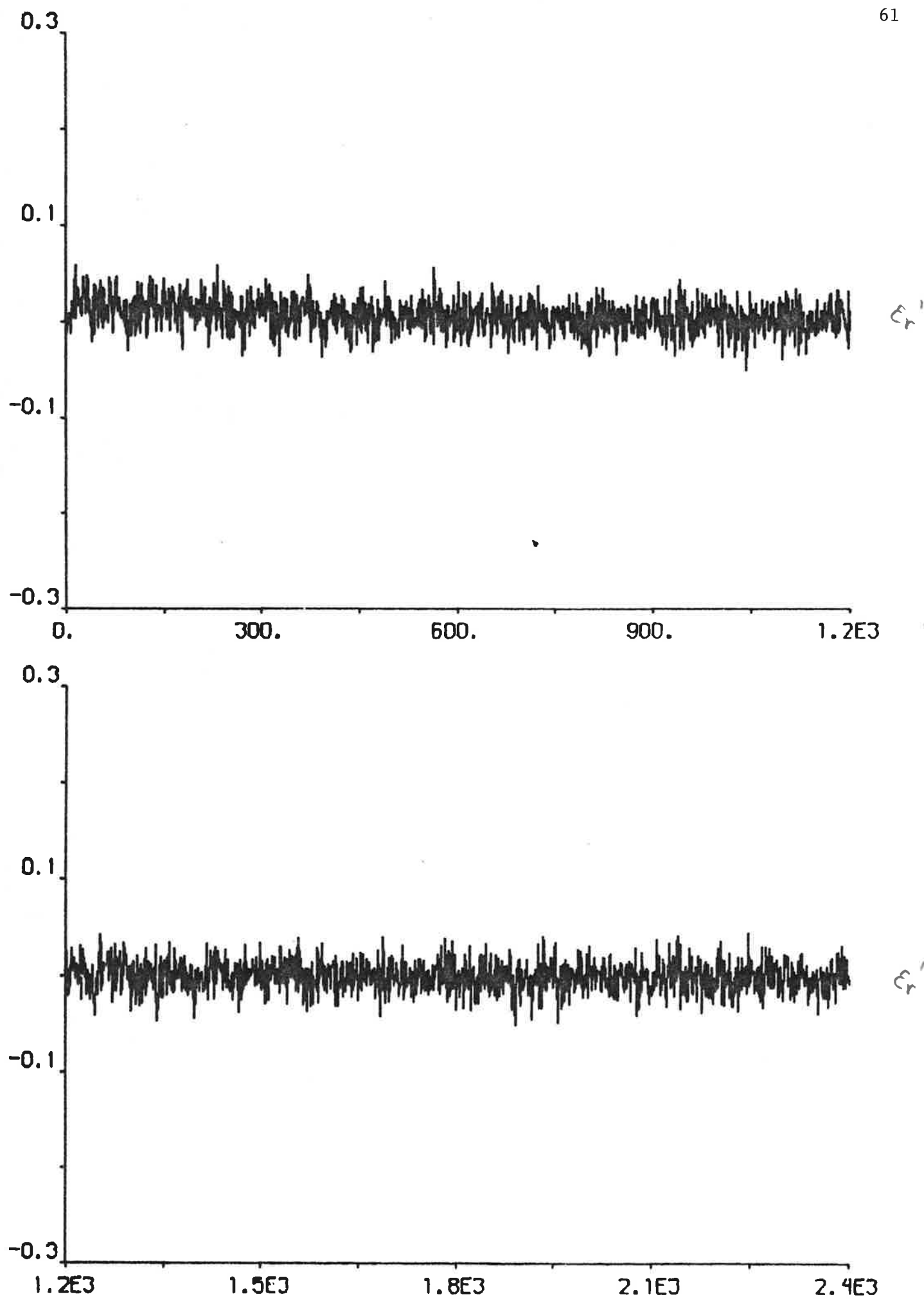


Fig. 4.2ℓ

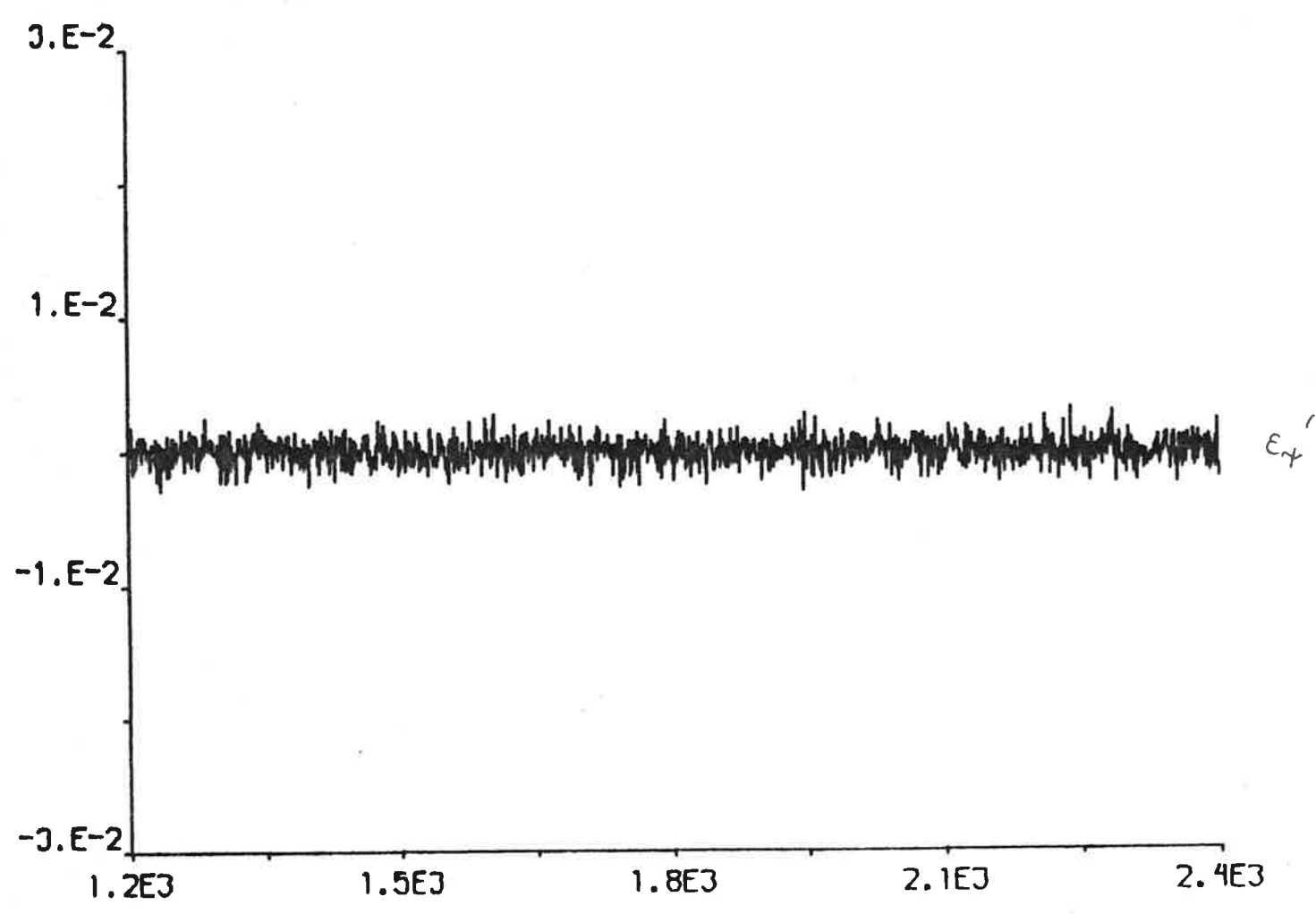
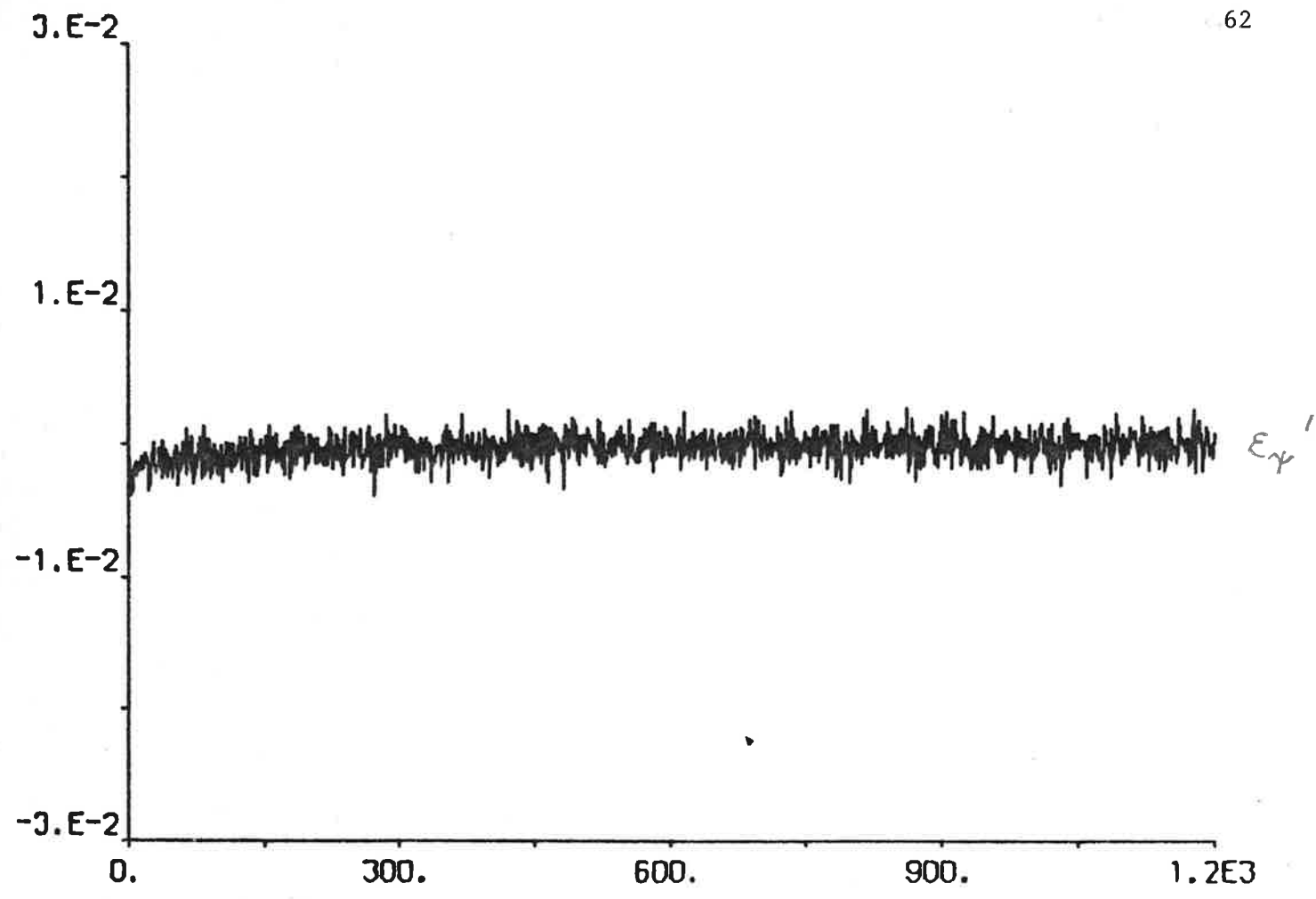


Fig. 4.2m

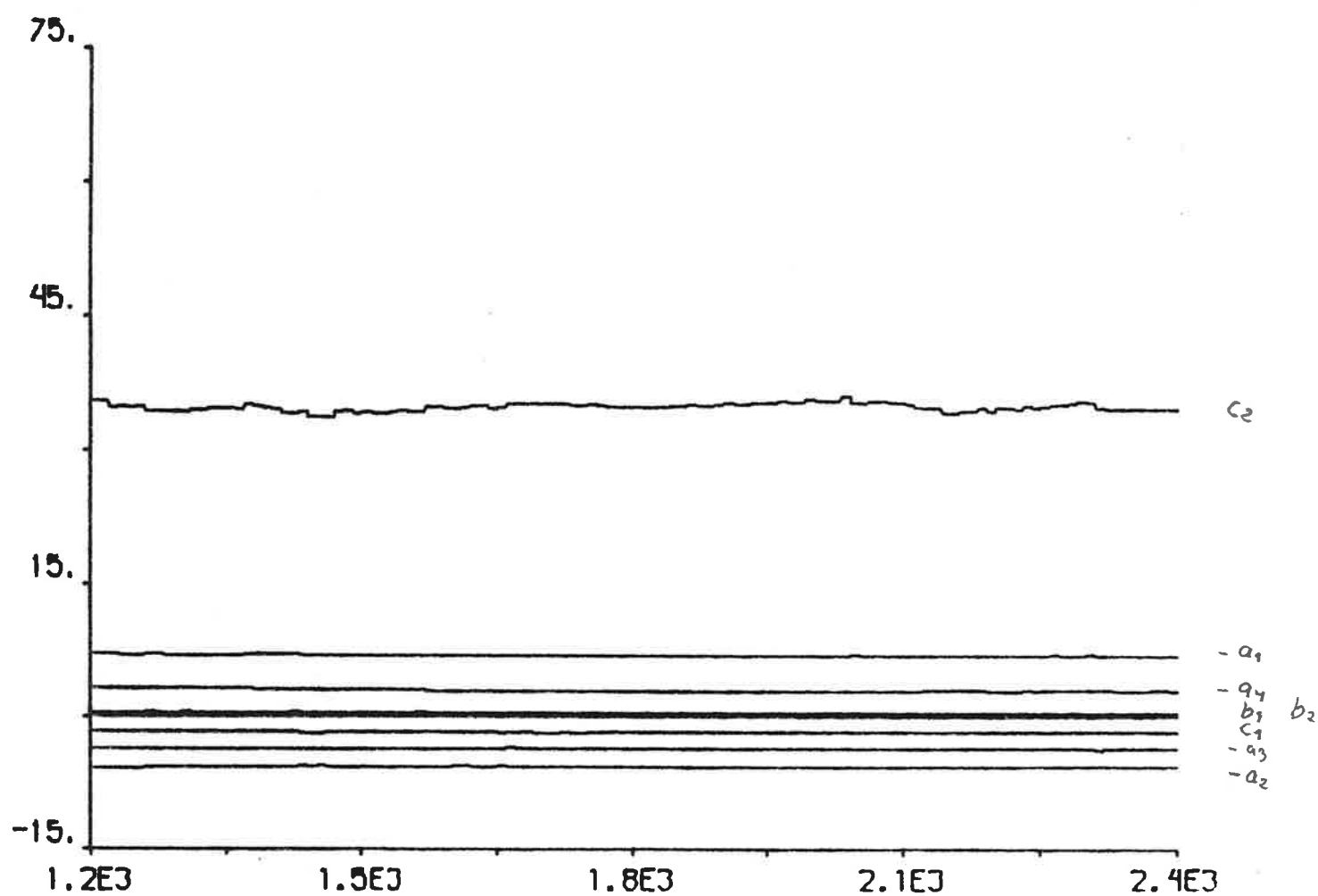
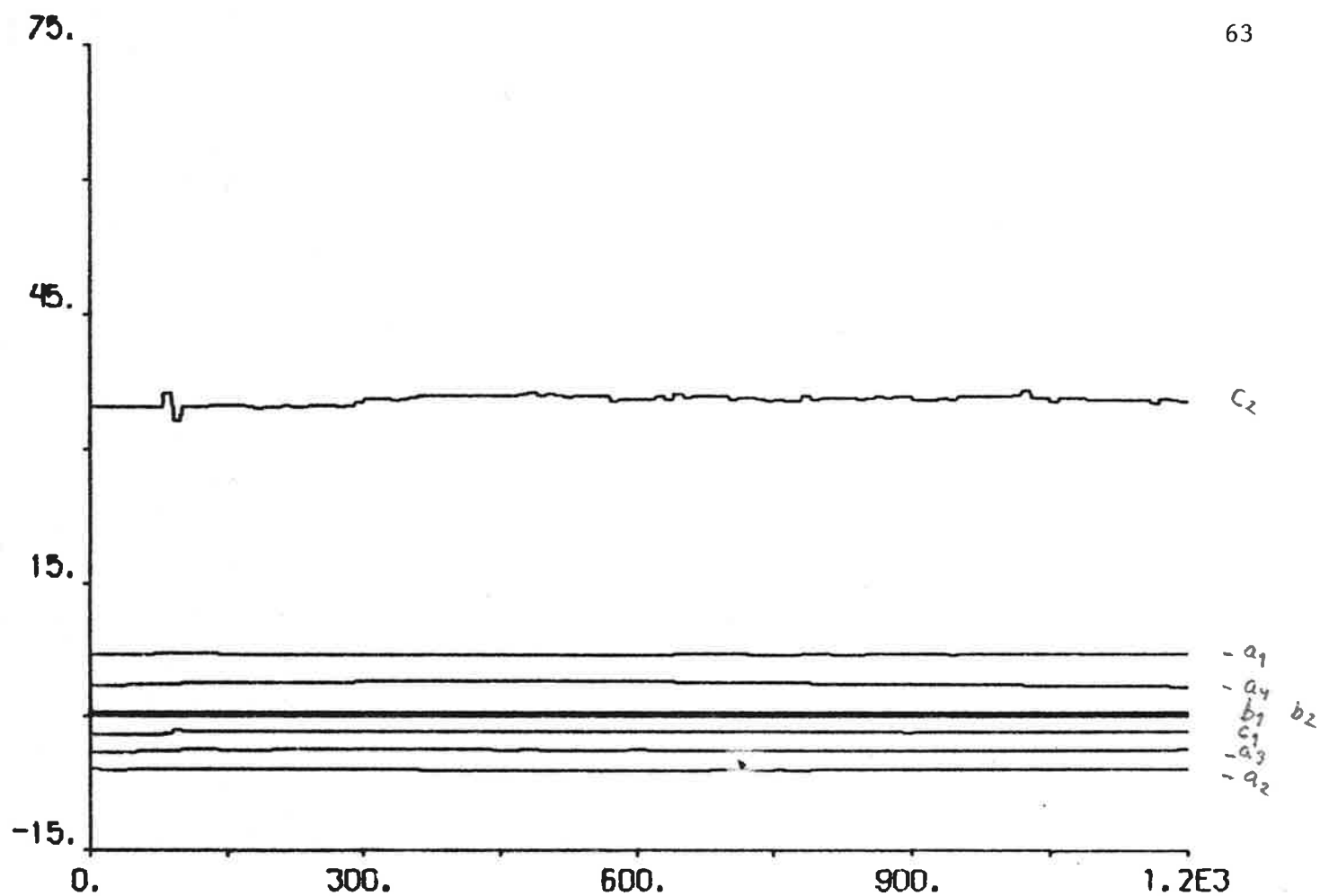


Fig. 4.2n

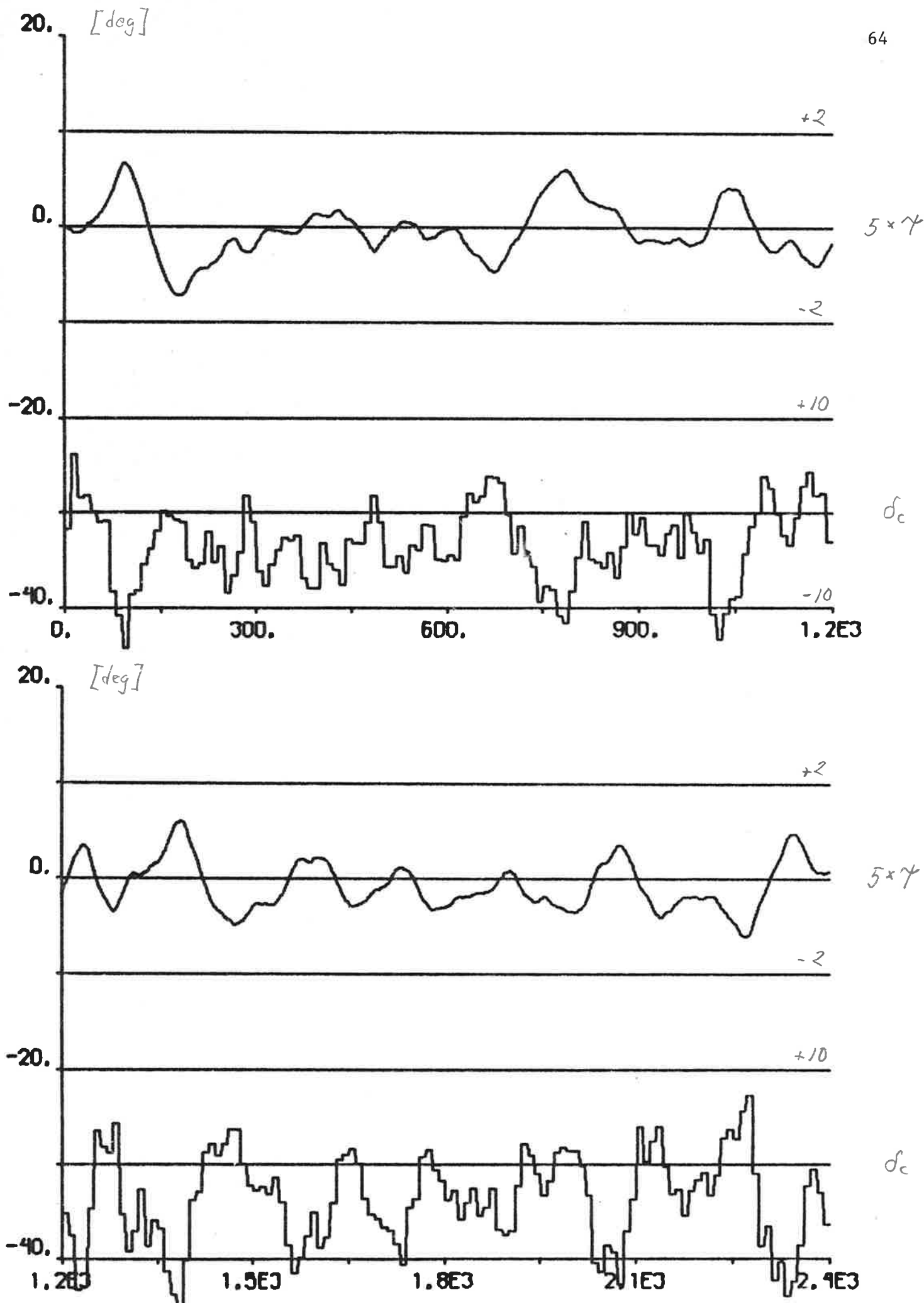


Fig. 4.3a - $T = 22.3$ m, $n_0 = 55.443$ rpm, $u_0 = 10$ knots, $\delta_\ell = 35$ deg, self-tuning regulator using estimates from the Kalman filter.

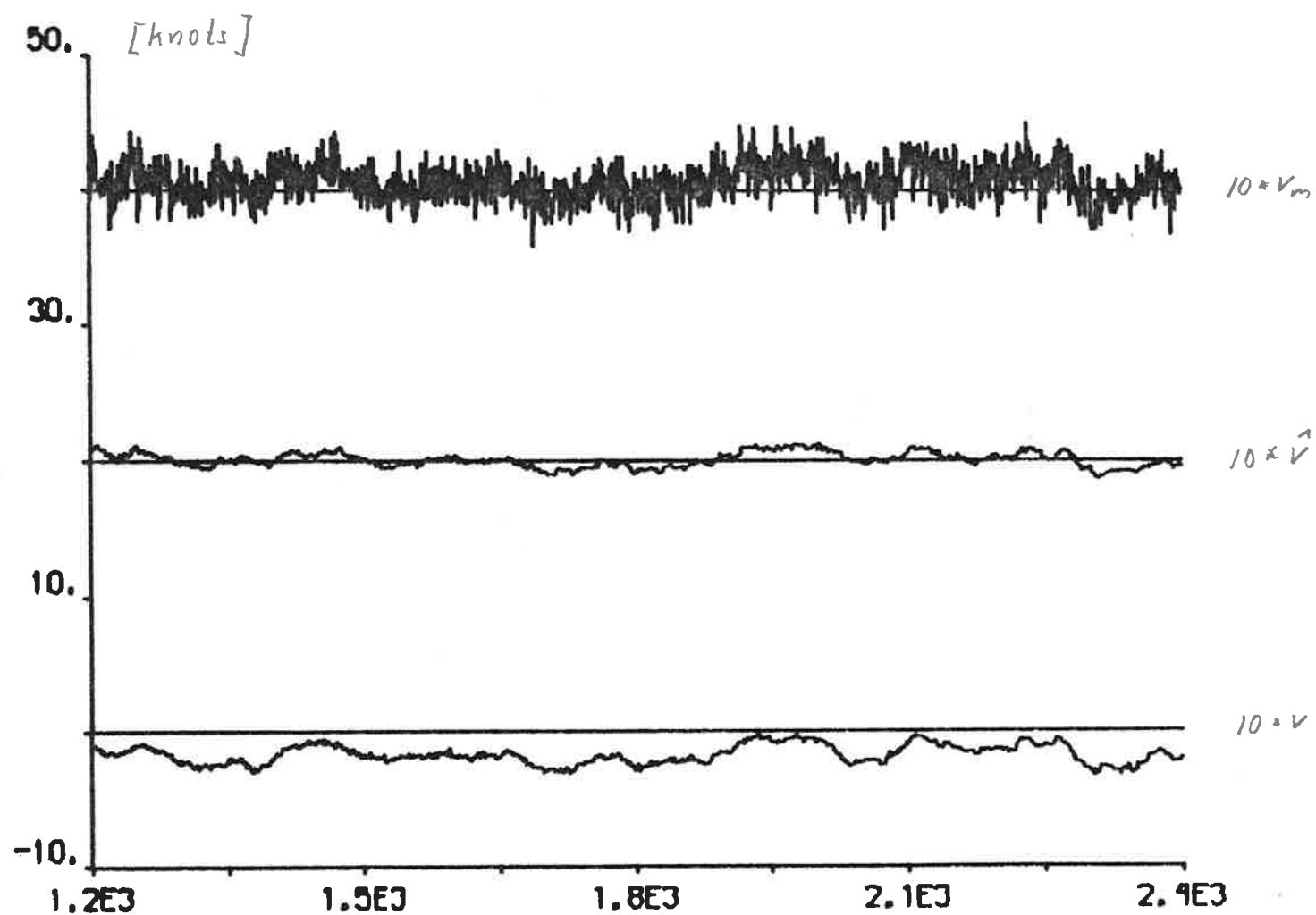
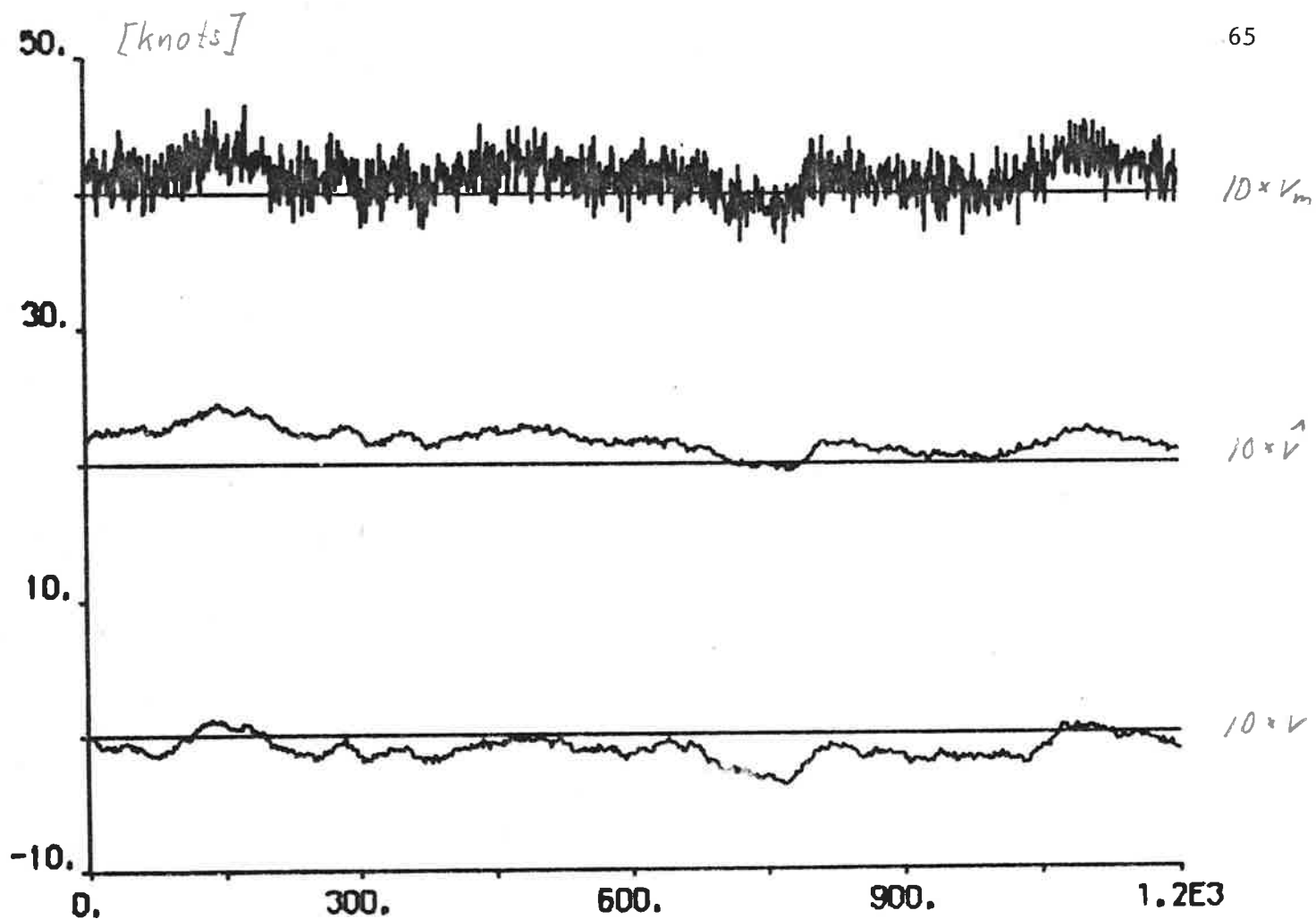


Fig. 4.3b

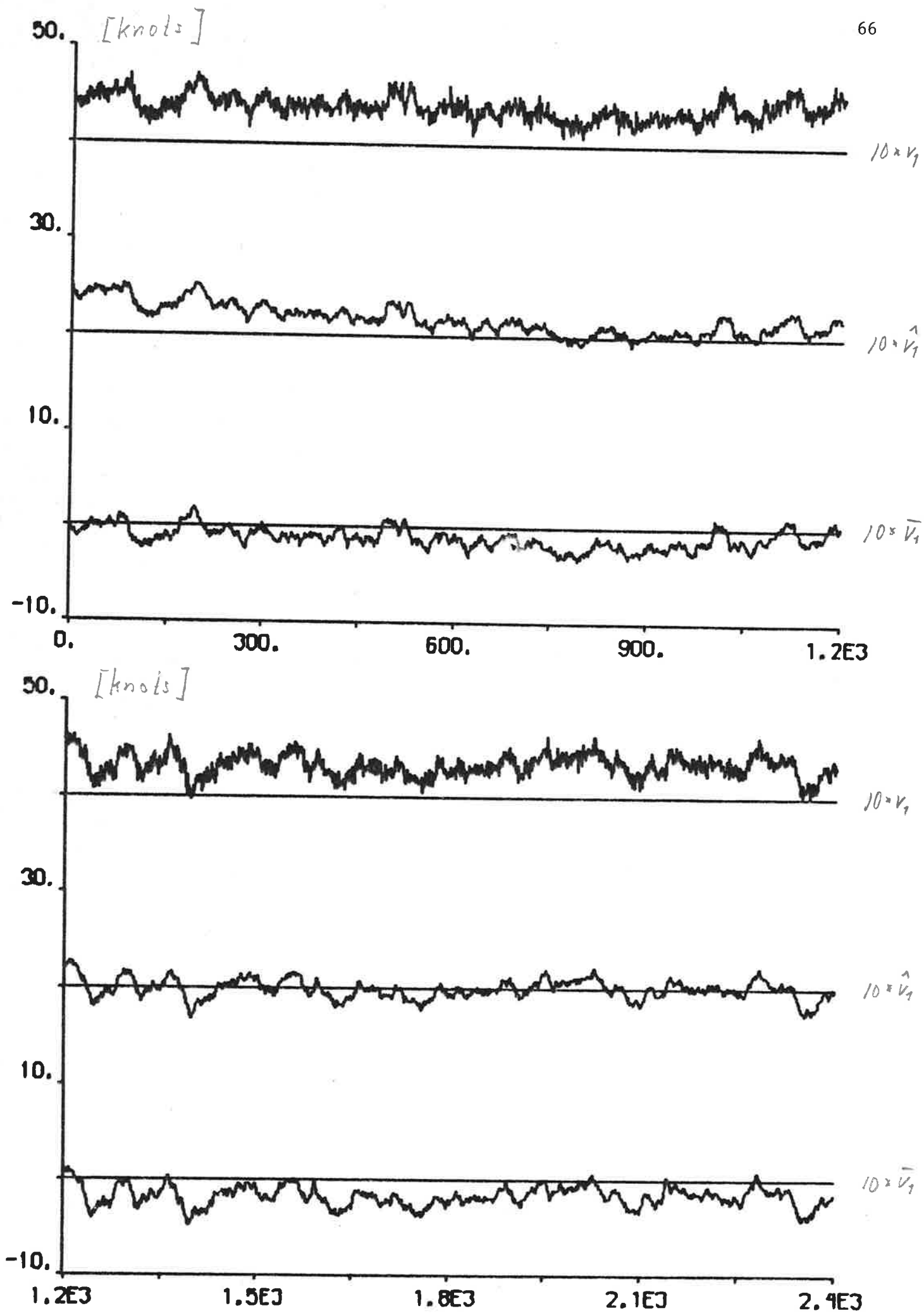


Fig. 4.3c

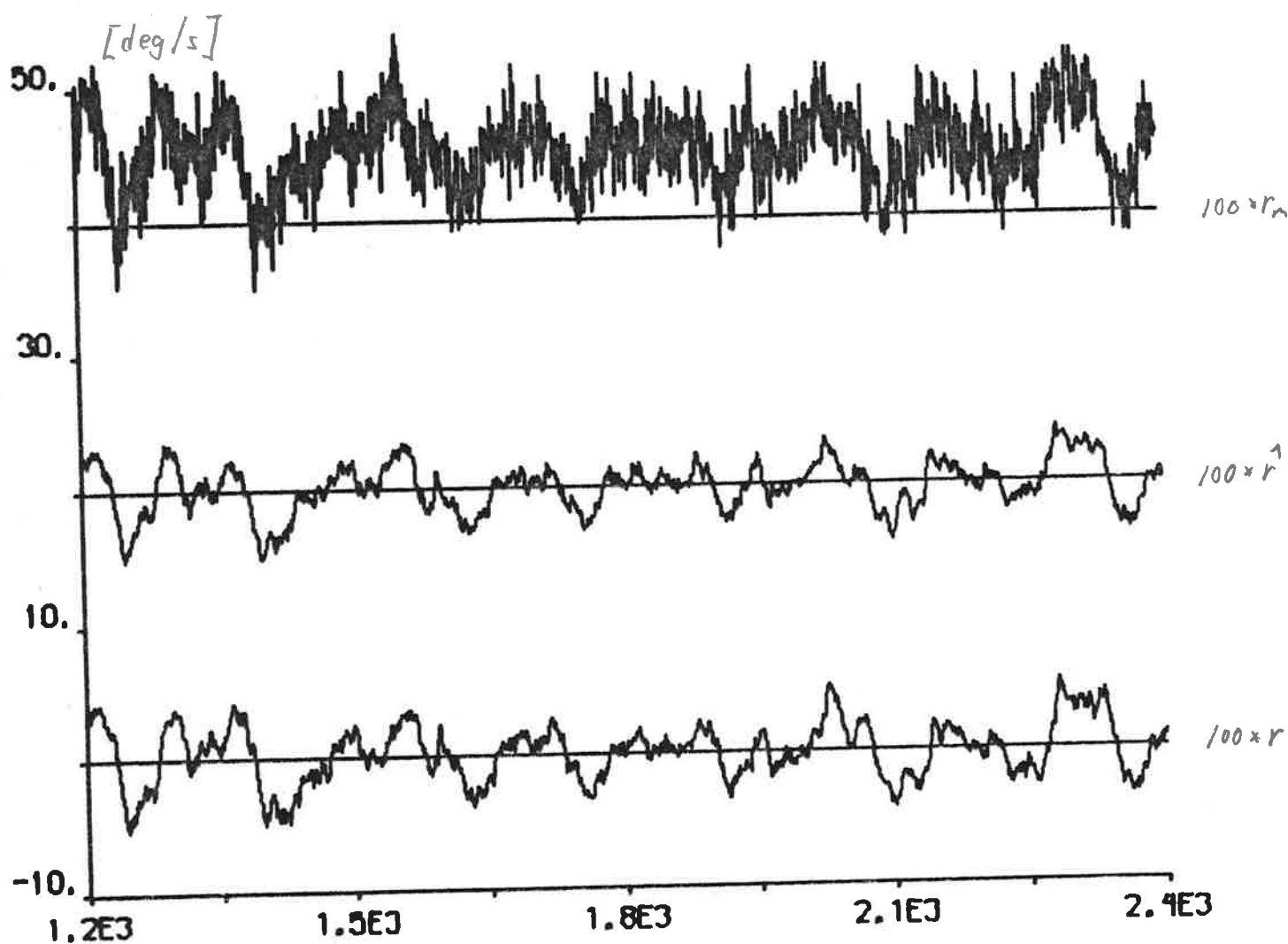
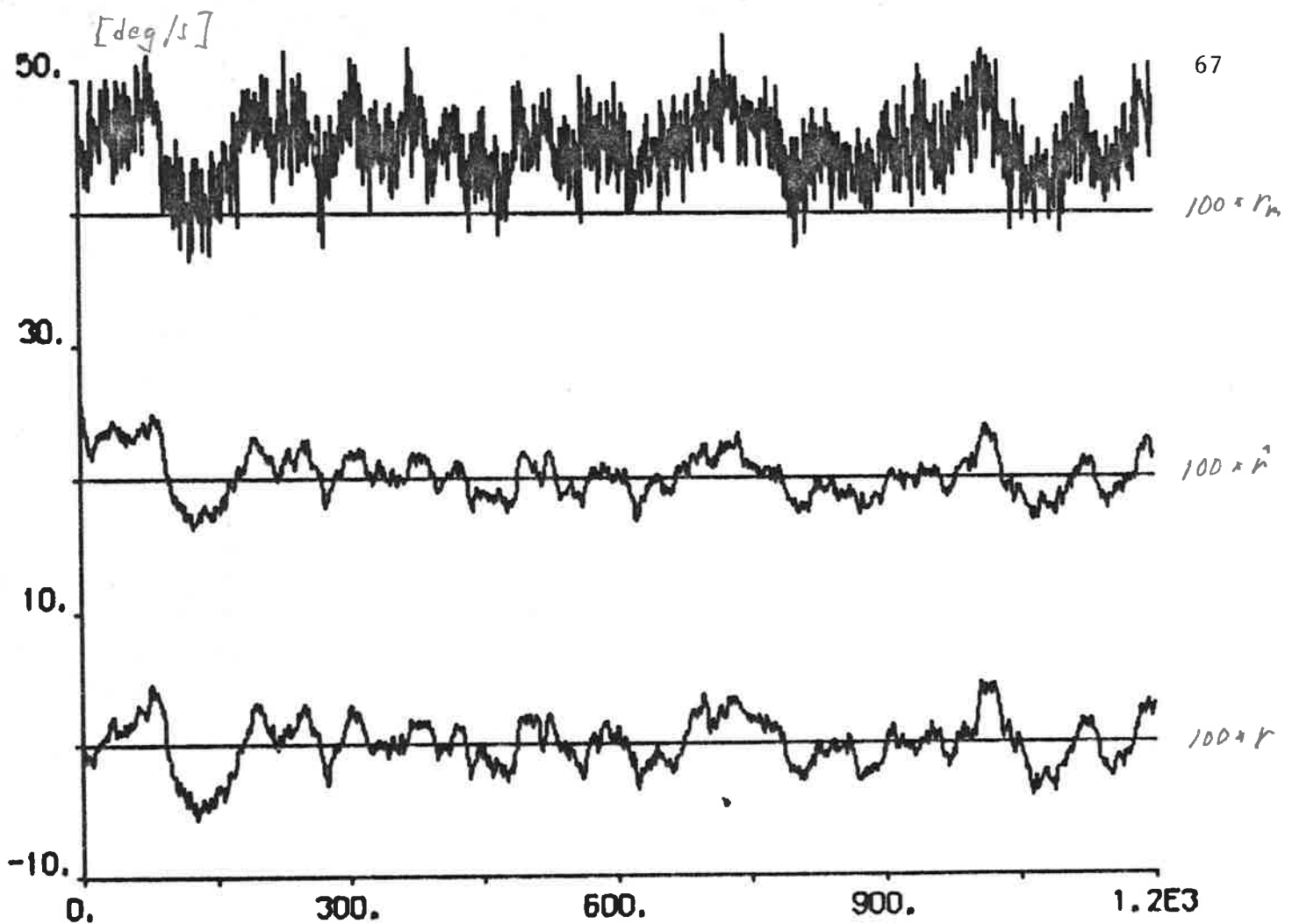


Fig. 4.3d

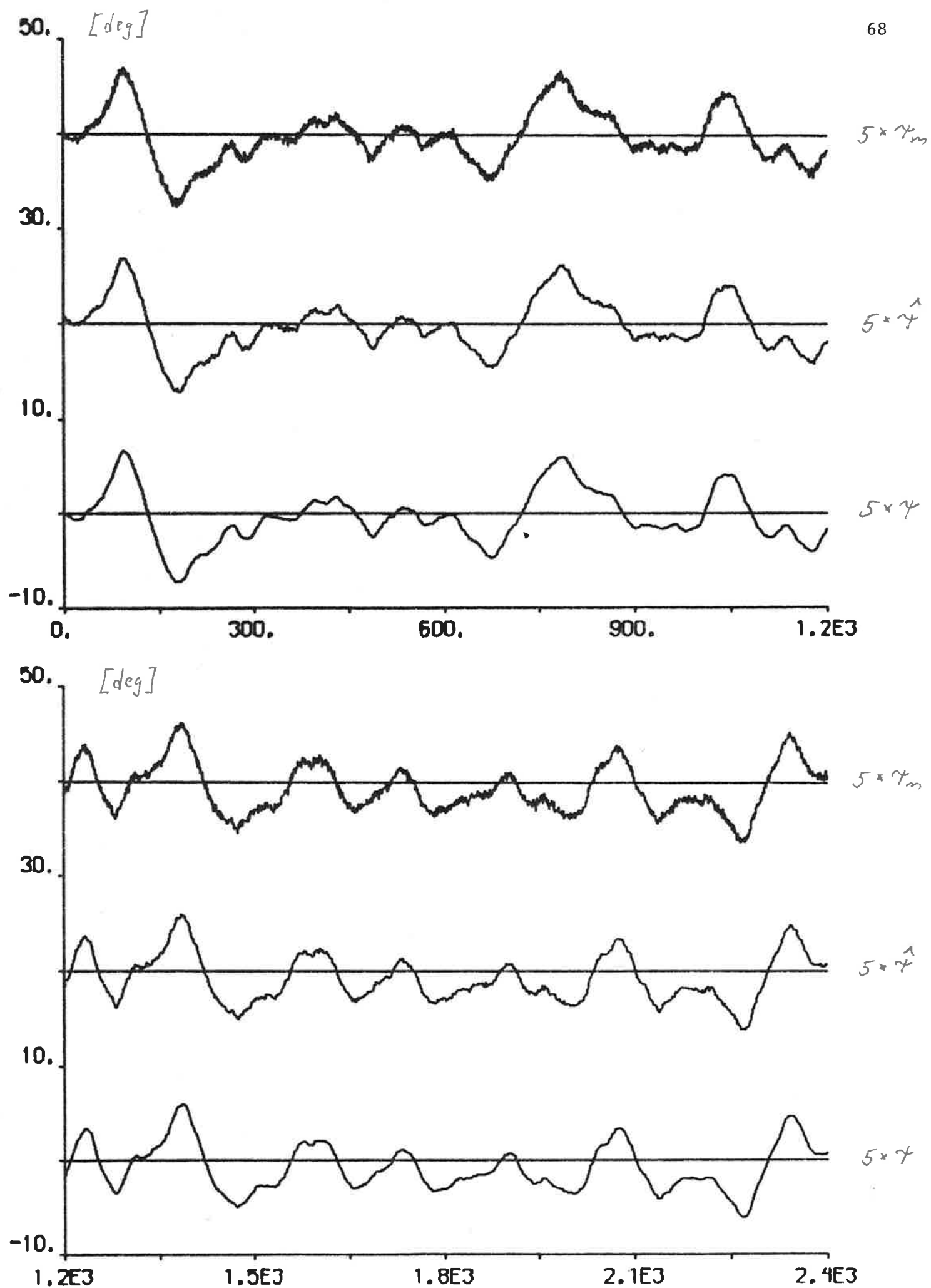


Fig. 4.3e

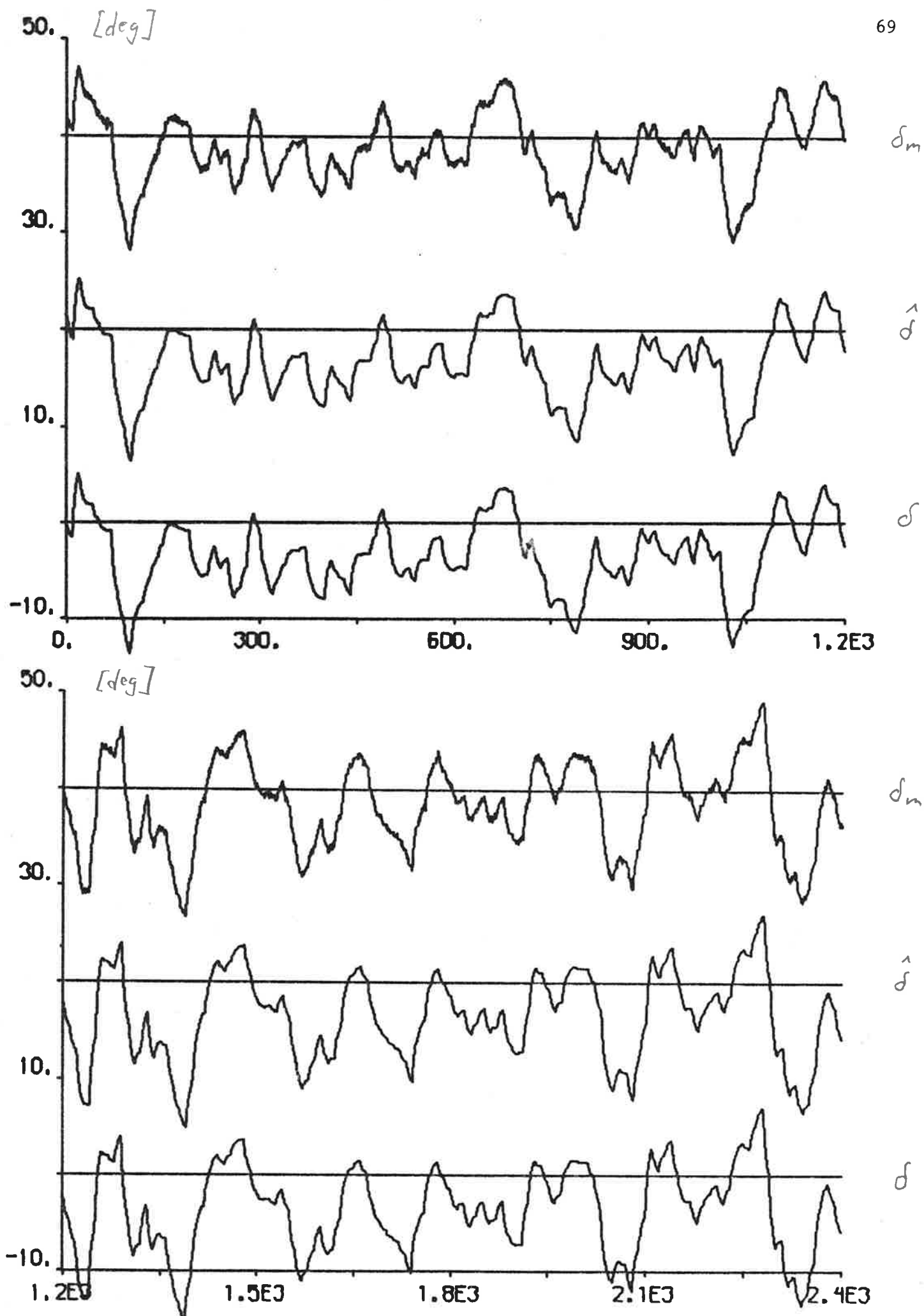


Fig. 4.3f

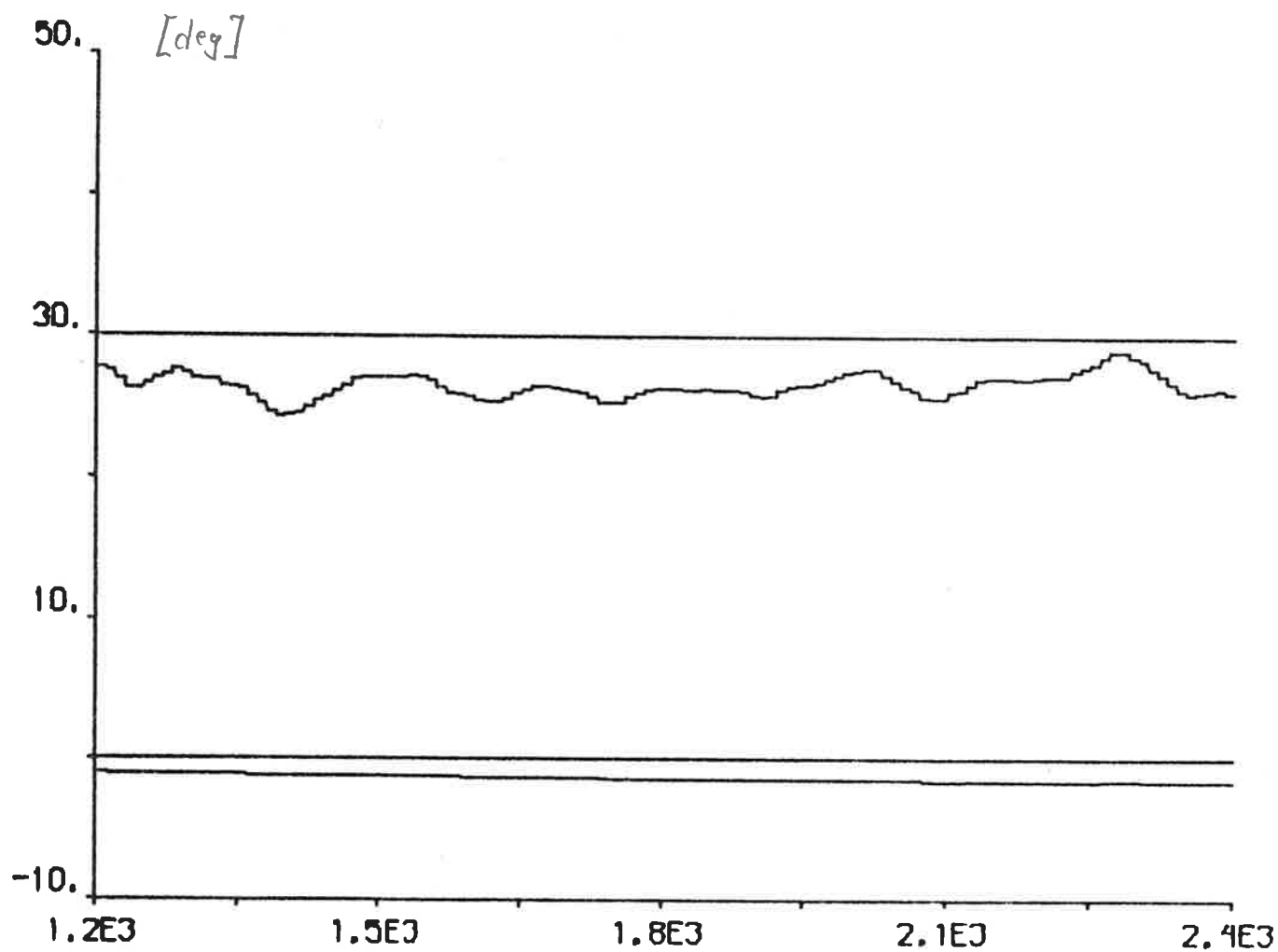
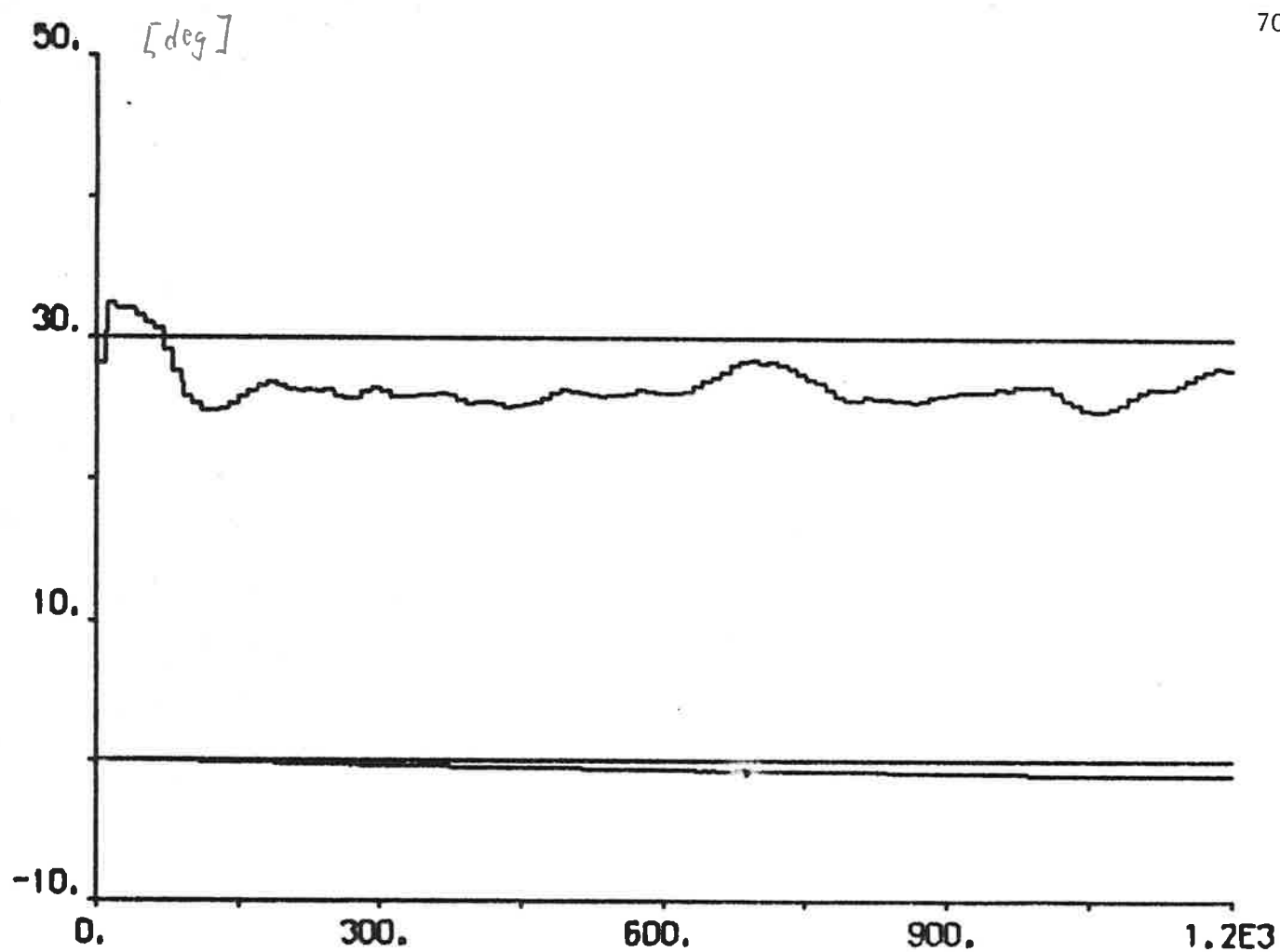


Fig. 4.3g

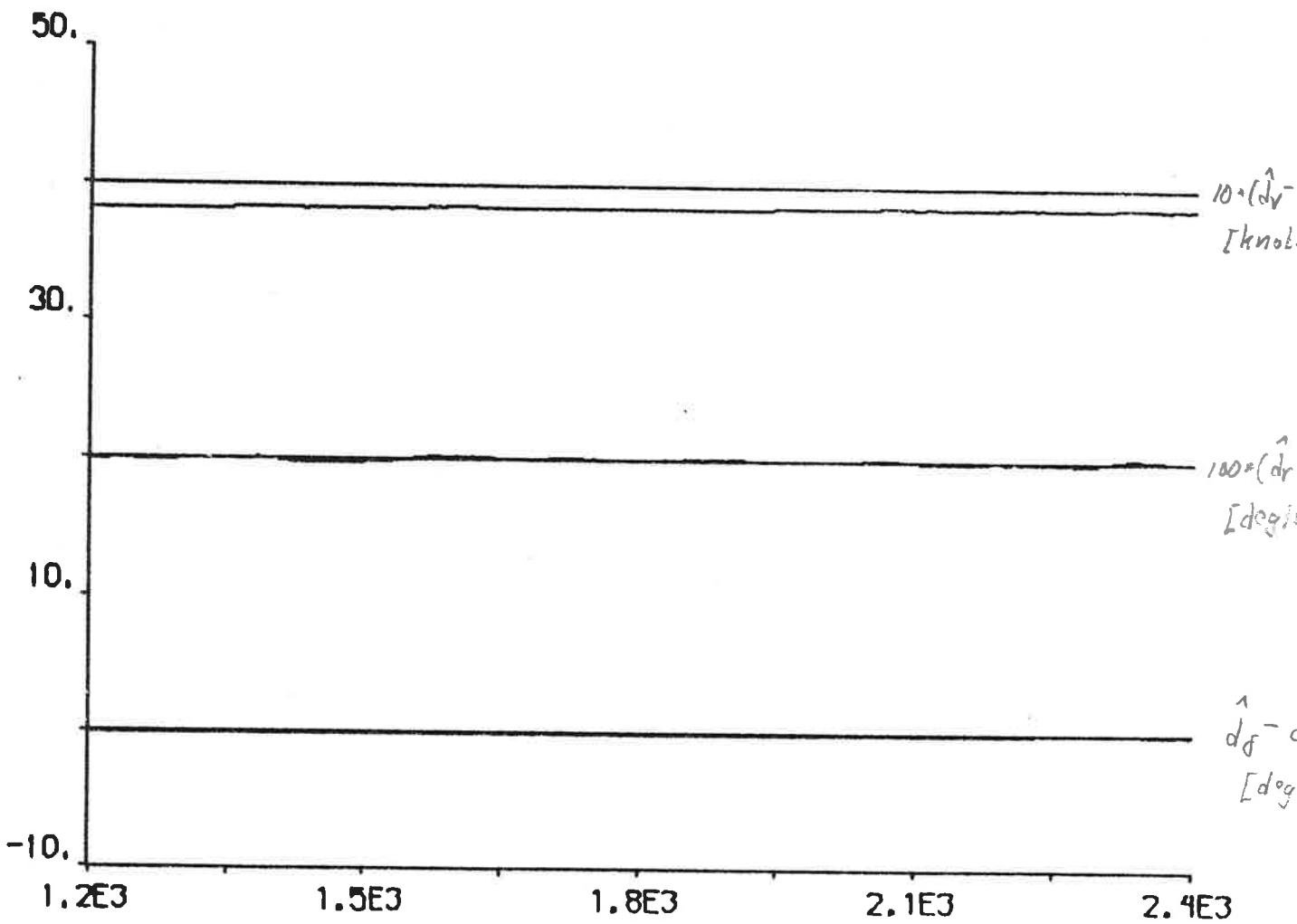
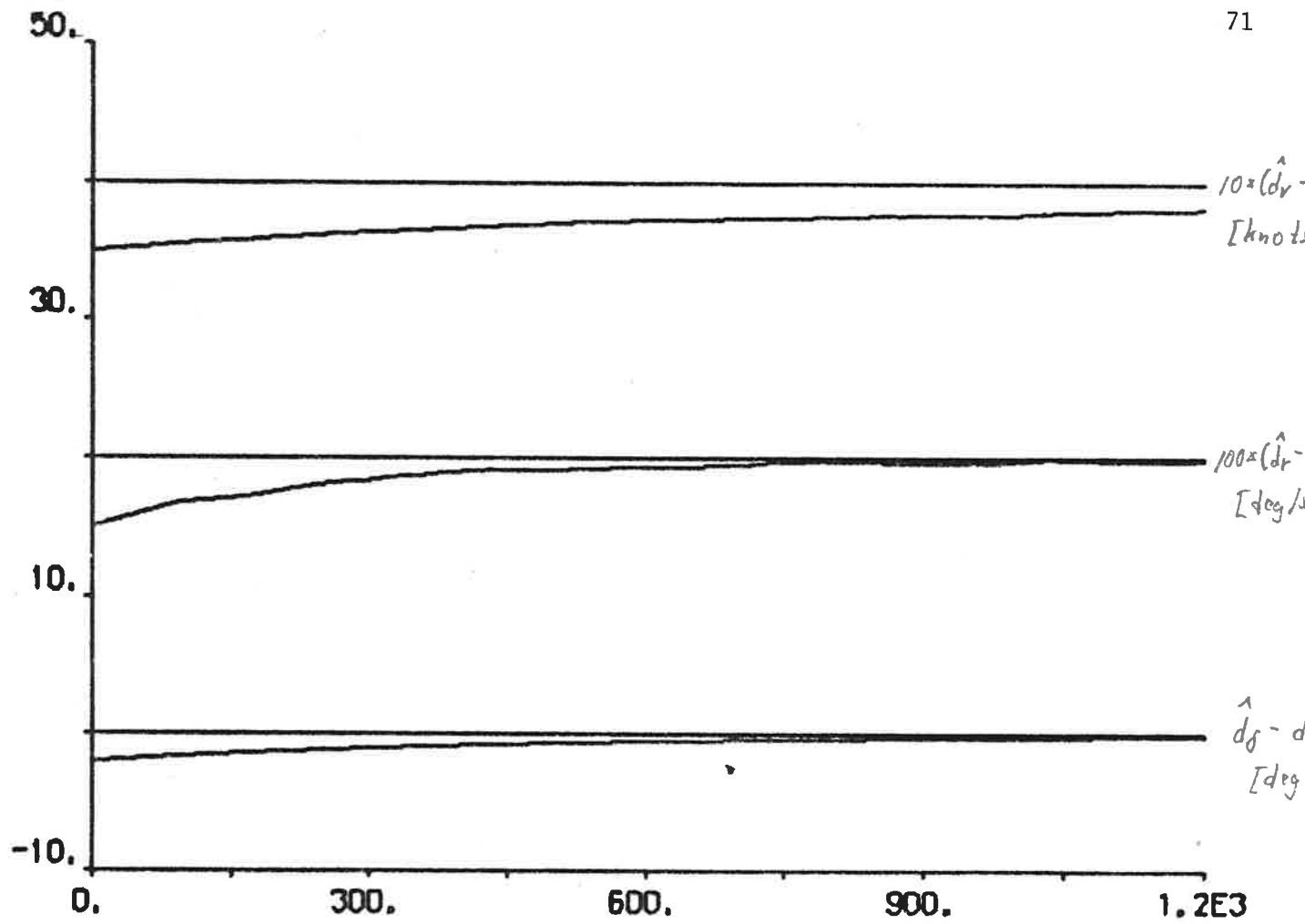


Fig. 4.3h

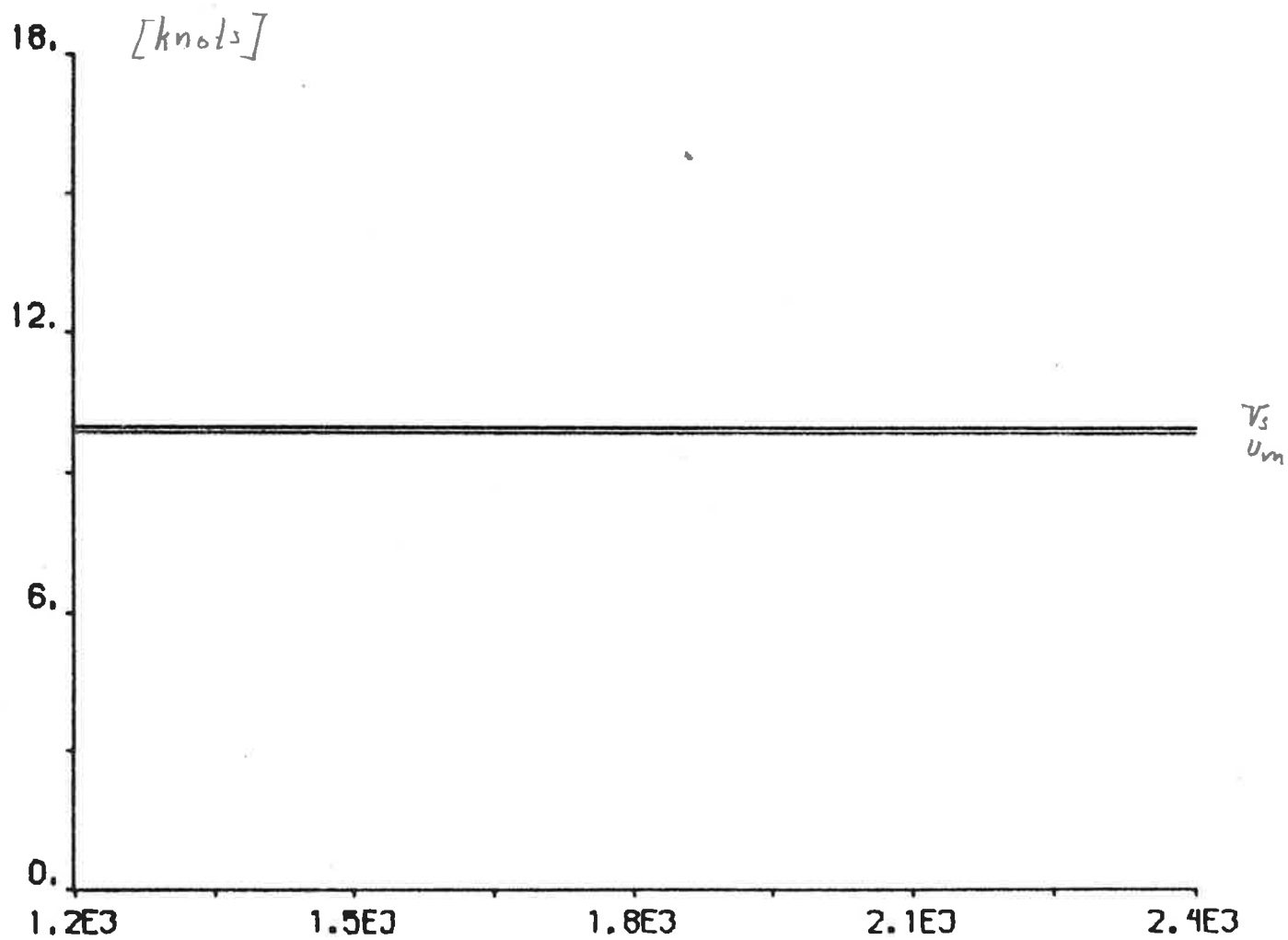
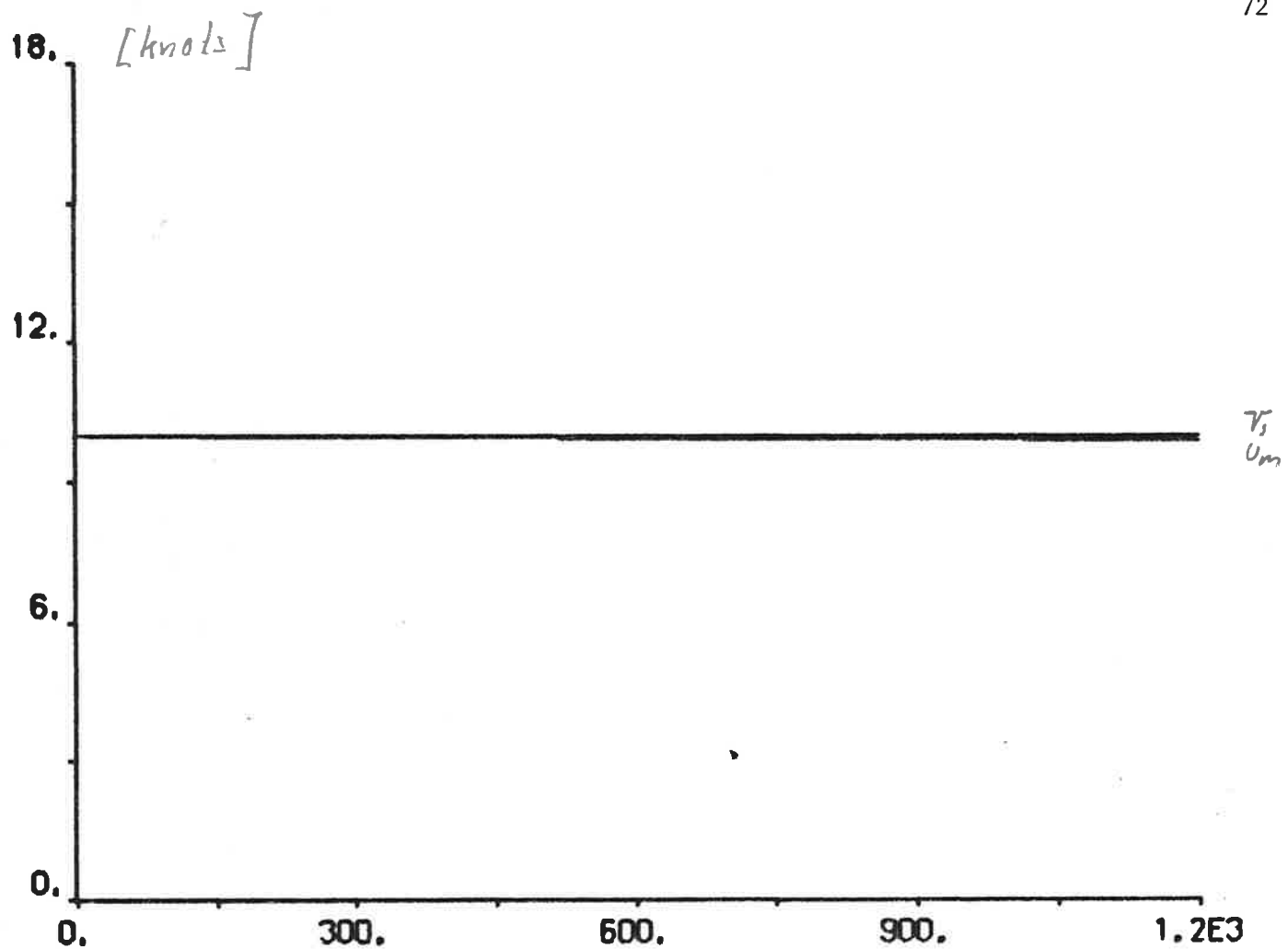


Fig. 4.3i

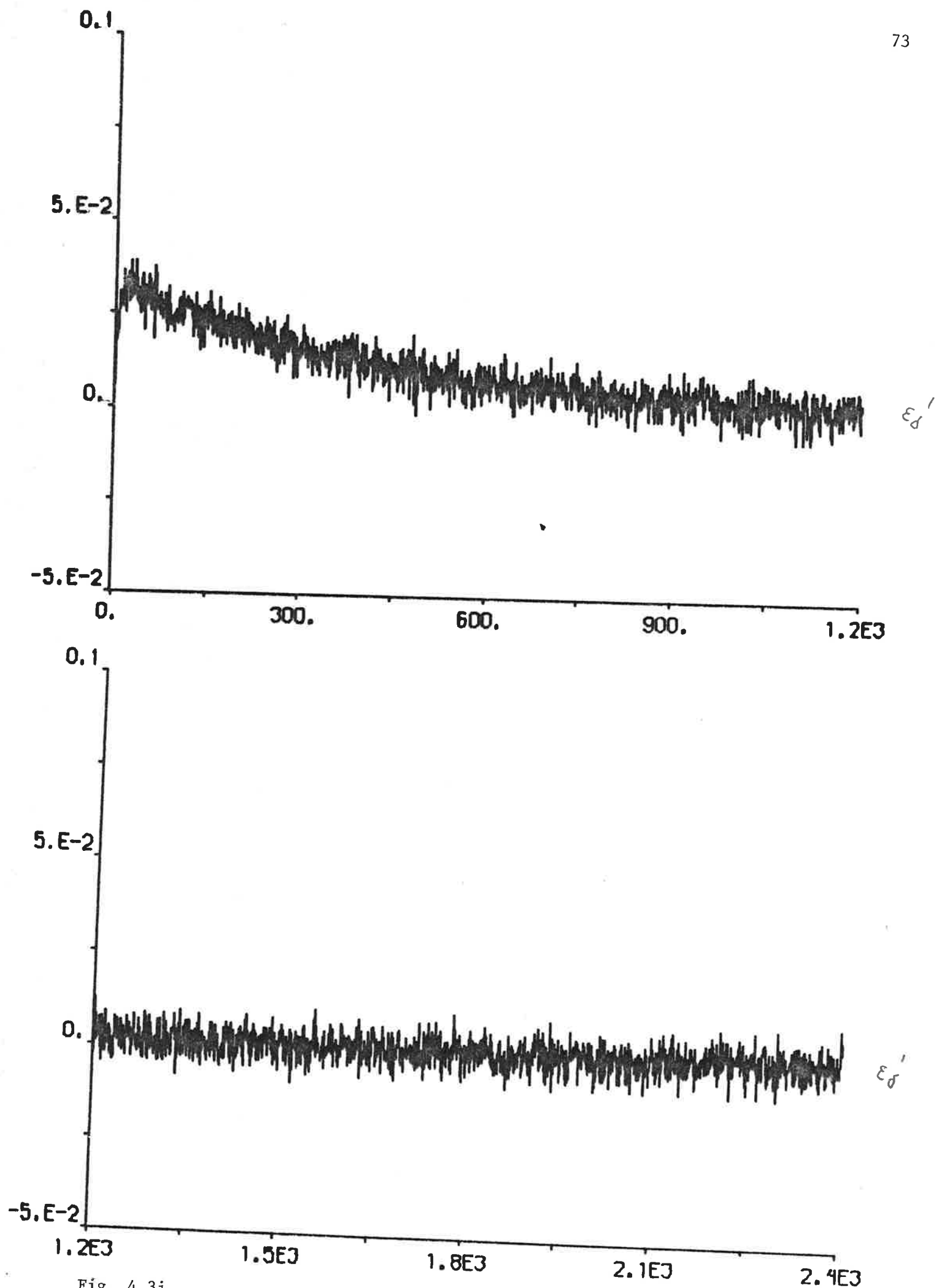


Fig. 4.3j

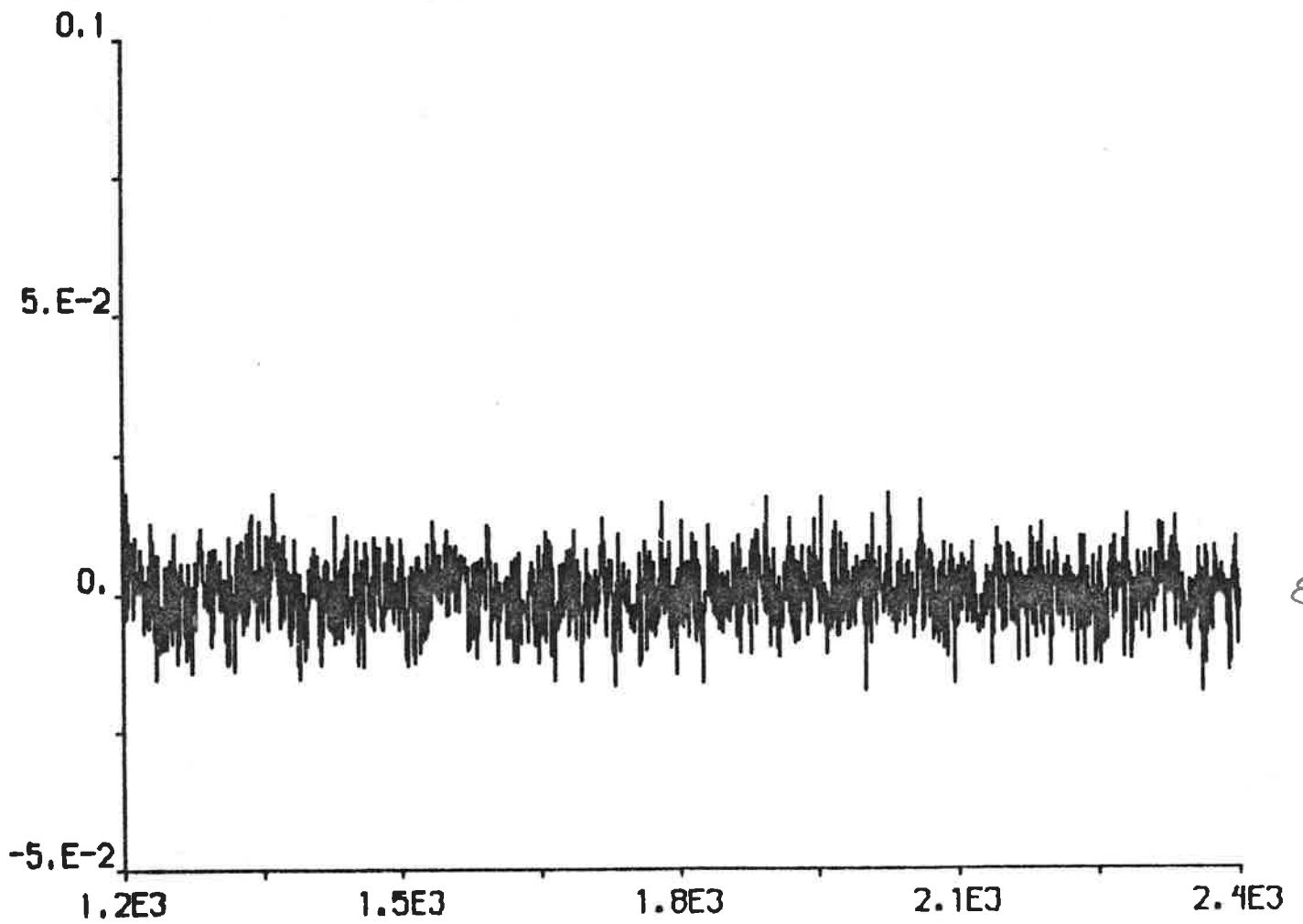
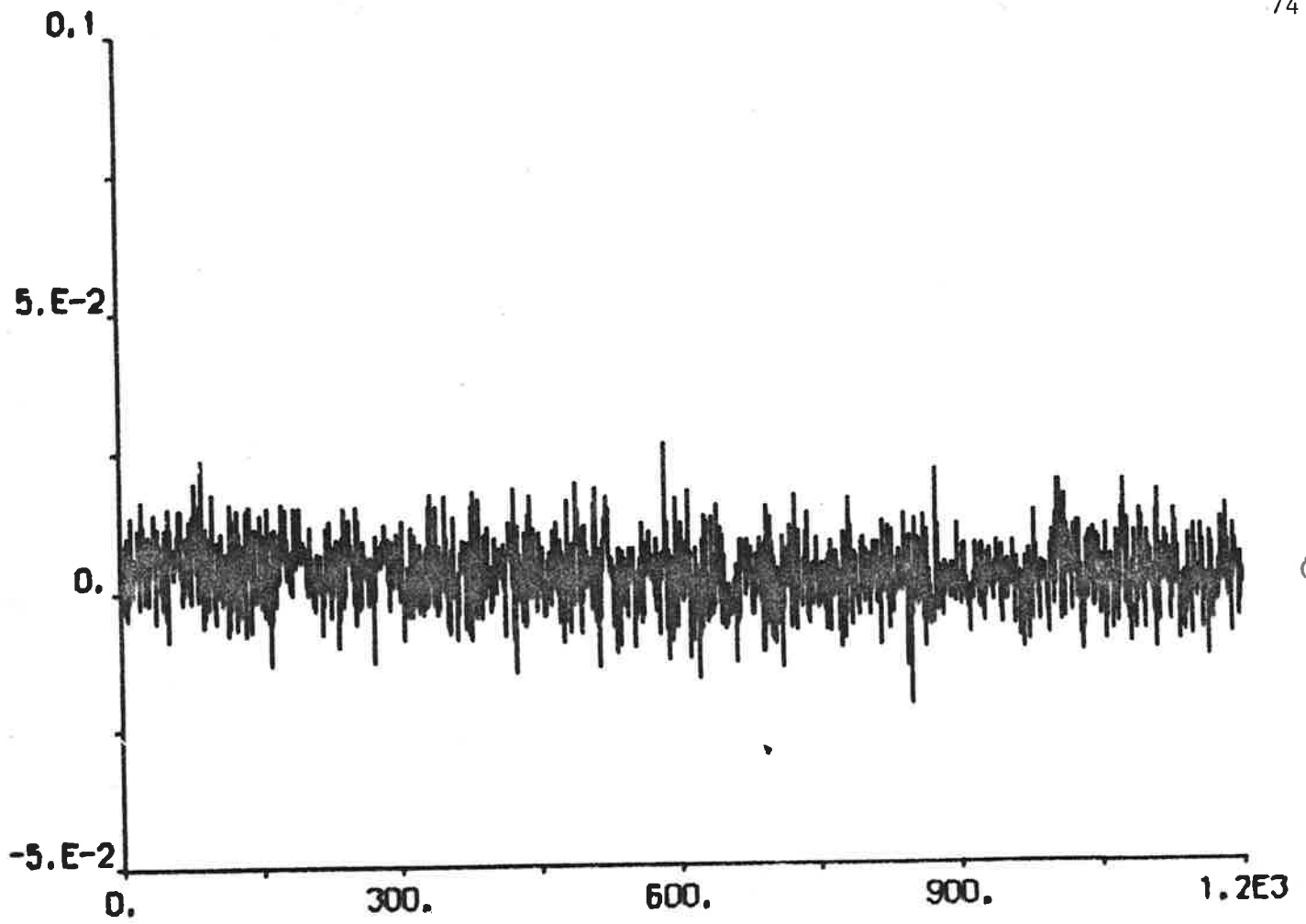


Fig. 4.3k

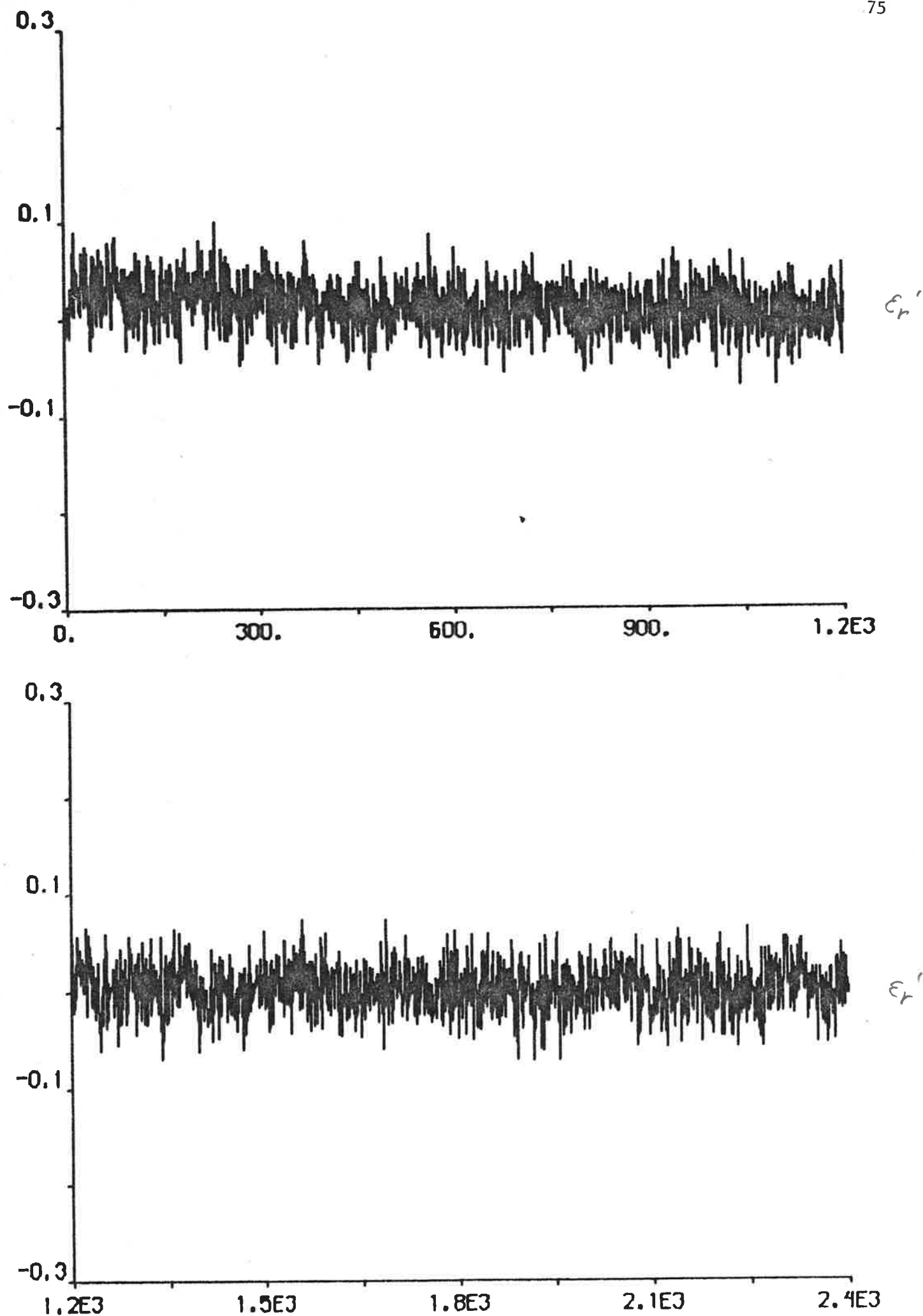


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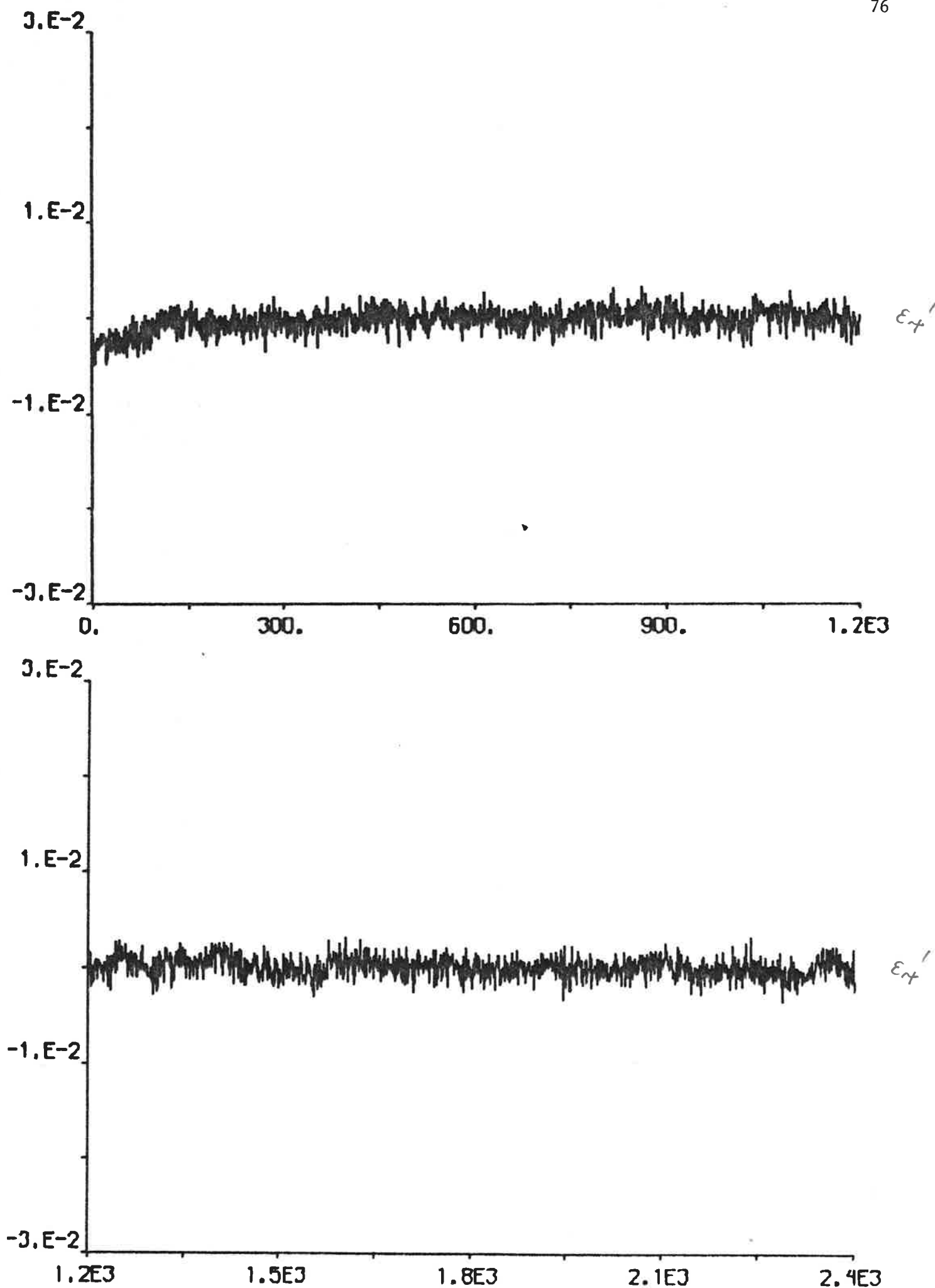


Fig. 4.3m

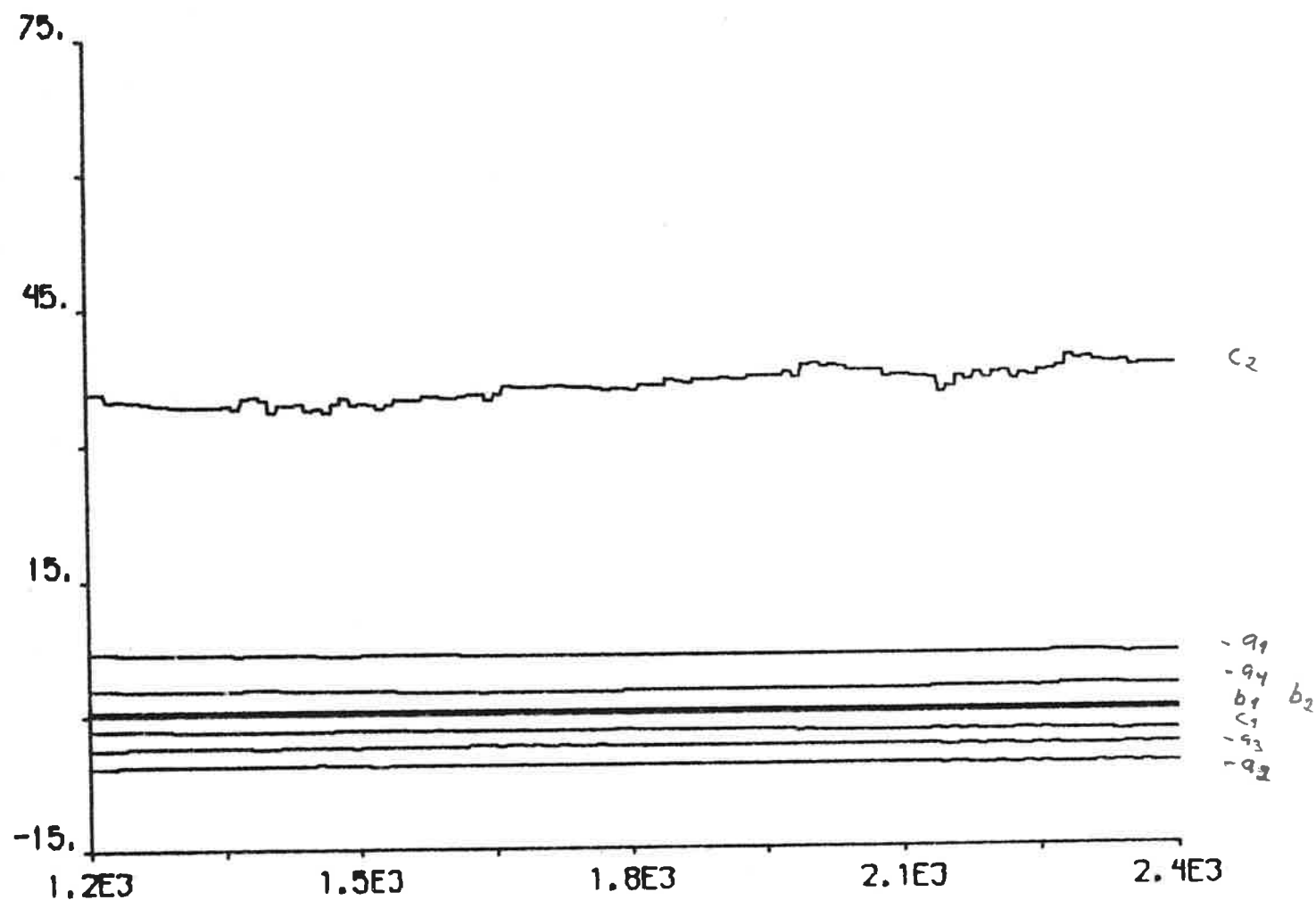
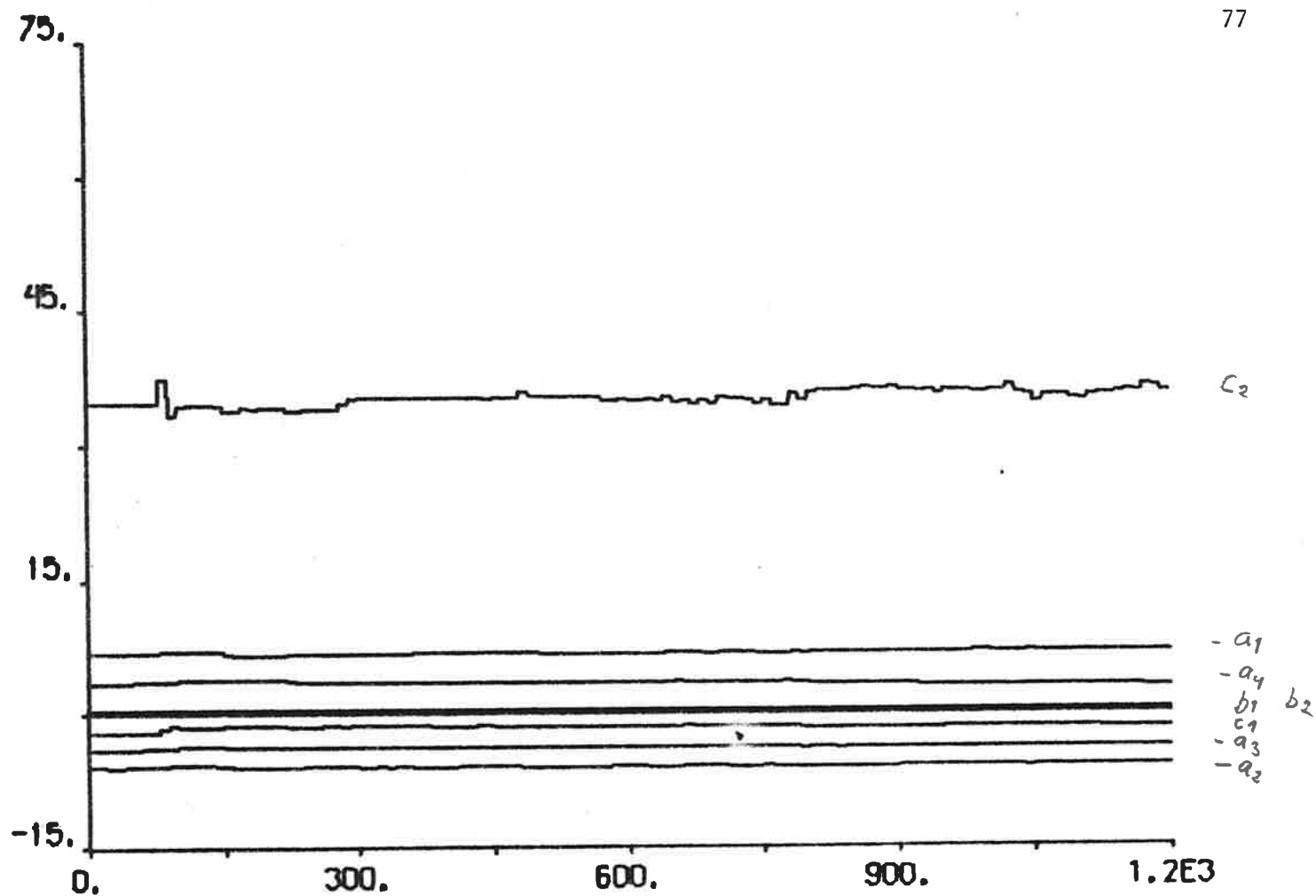


Fig. 4.3n

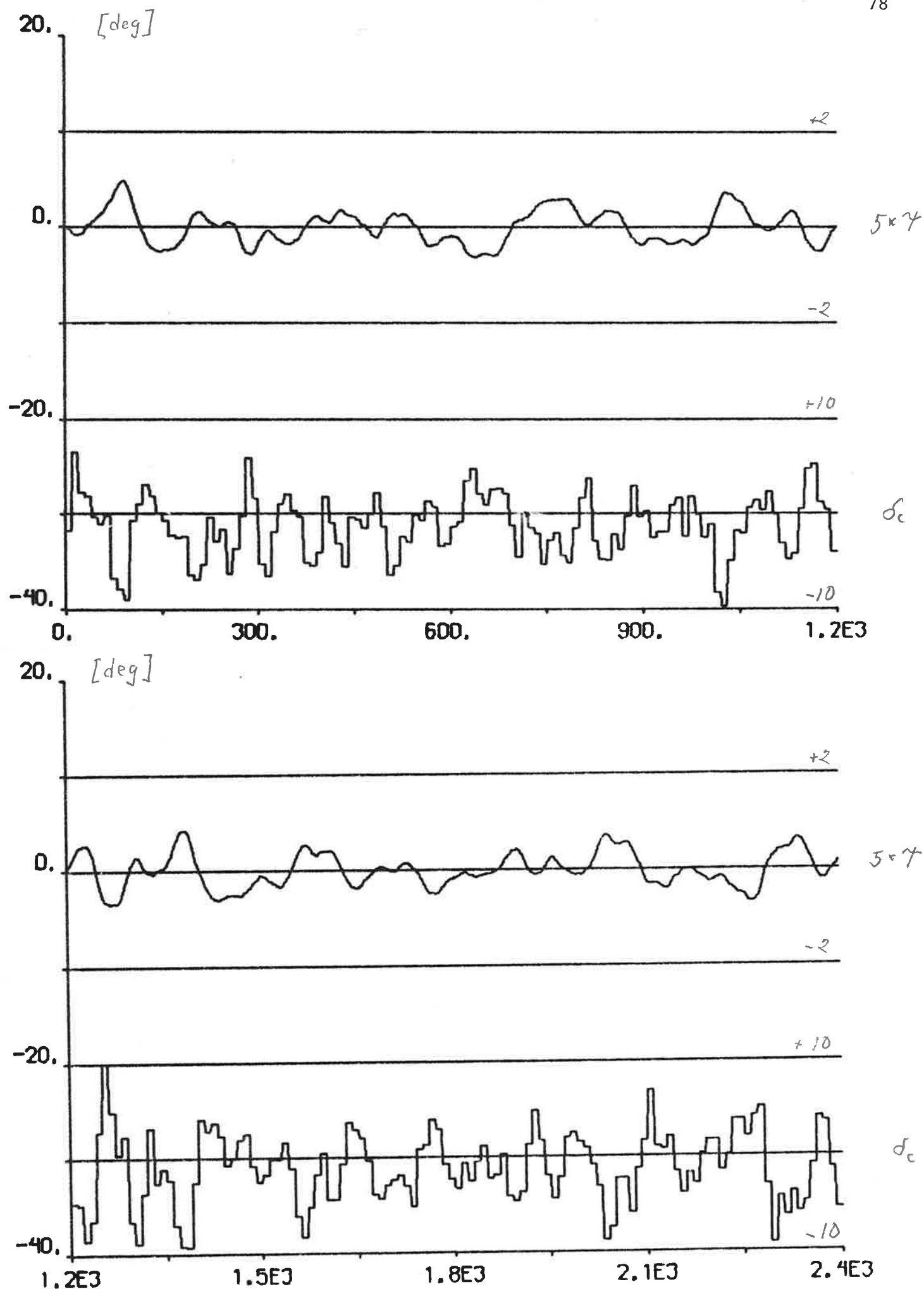


Fig. 4.4a - $T = 10.5$ m, $n_0 = 55.443$ rpm, $u_0 = 10$ knots, $\delta_\ell = 35$ deg, self-tuning regulator using estimates from the Kalman filter.

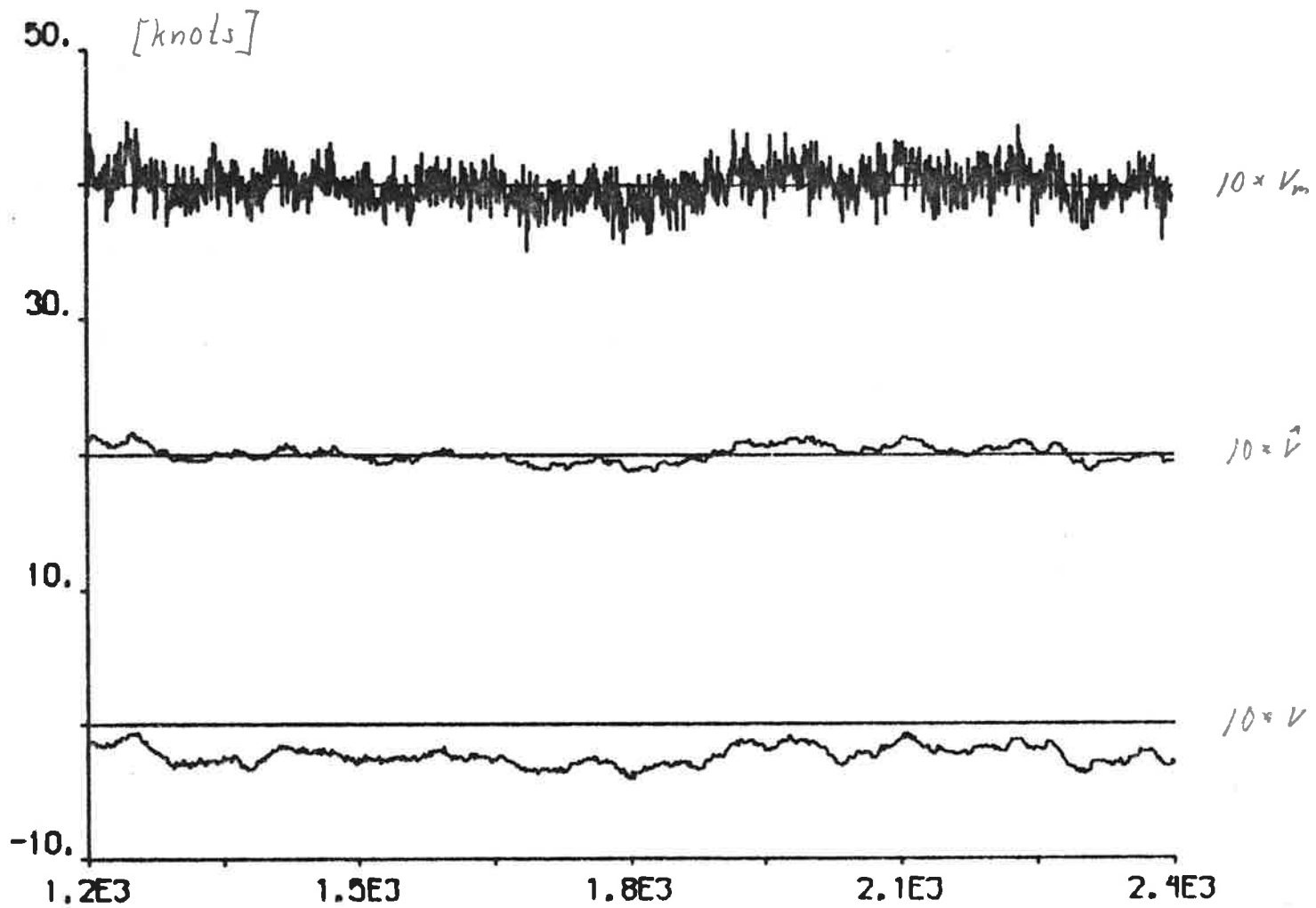
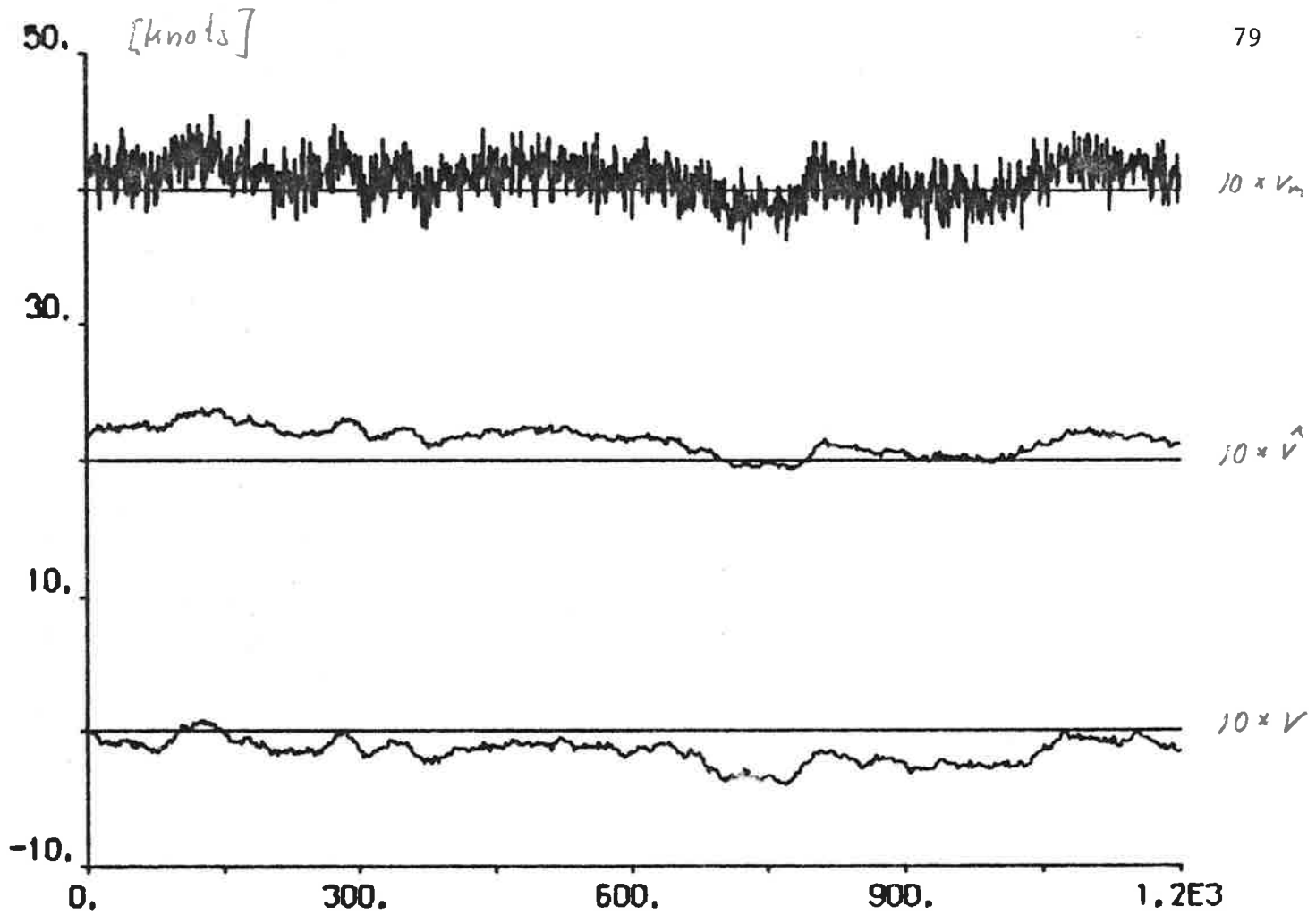


Fig. 4.4b

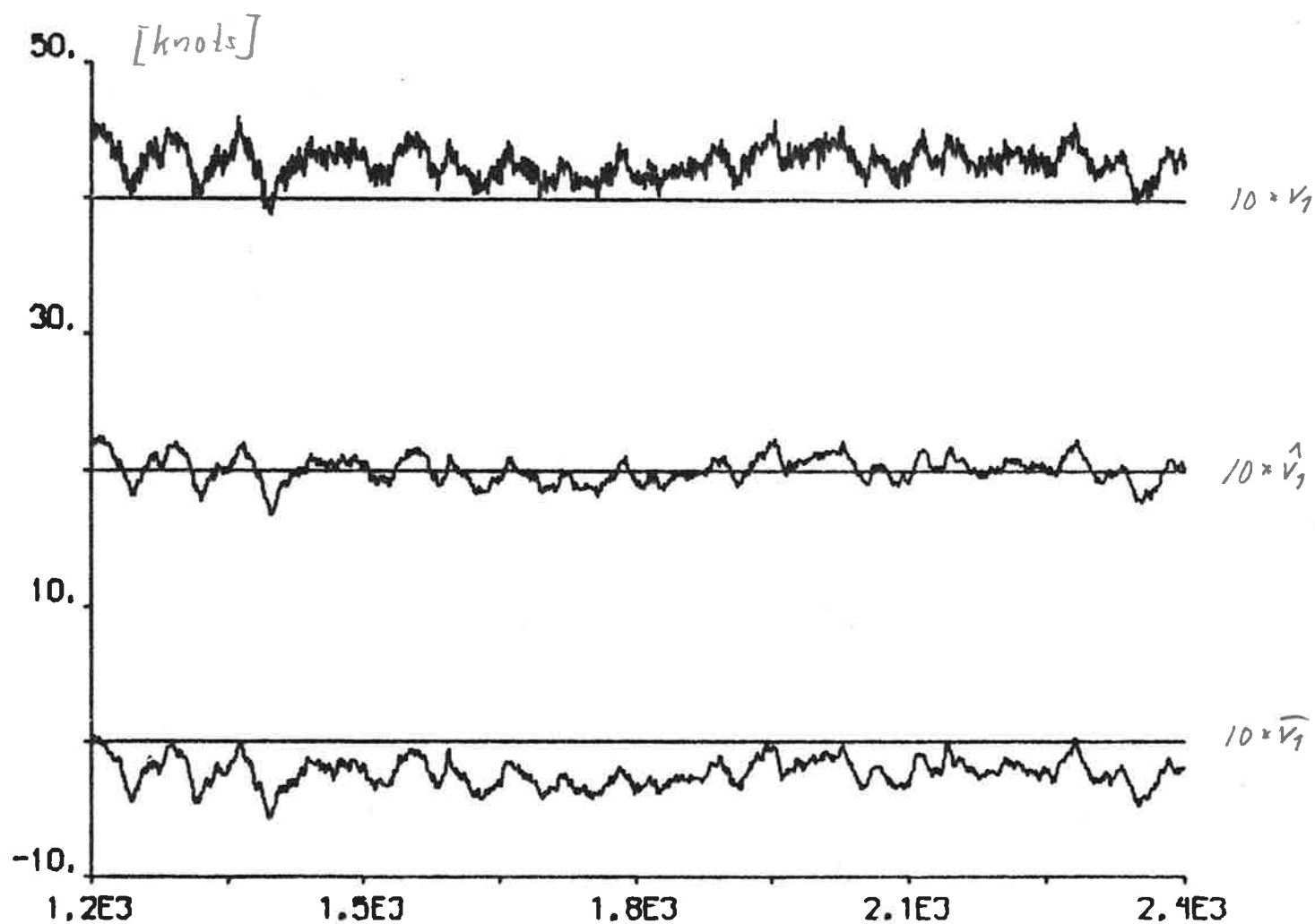
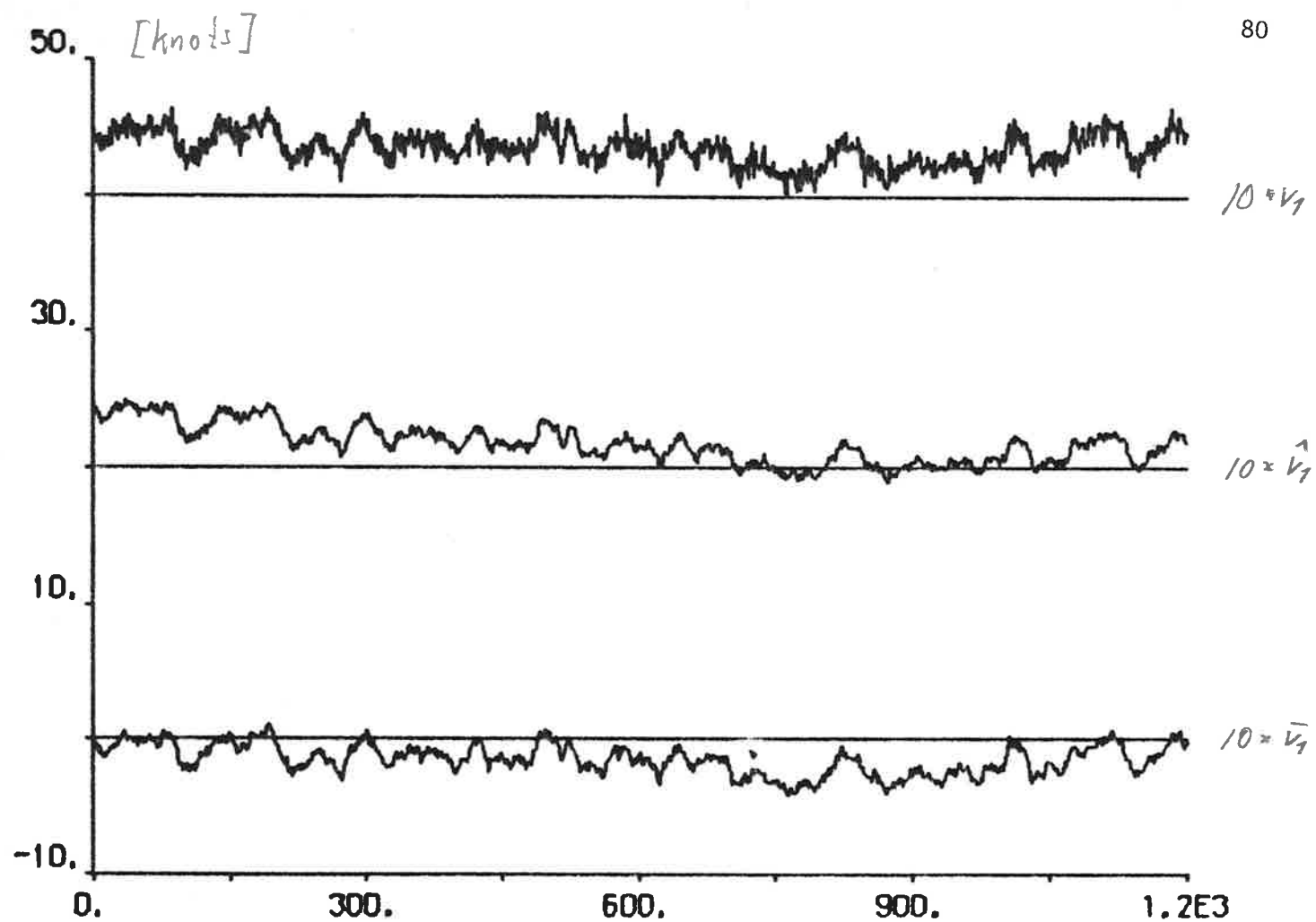


Fig. 4.4c

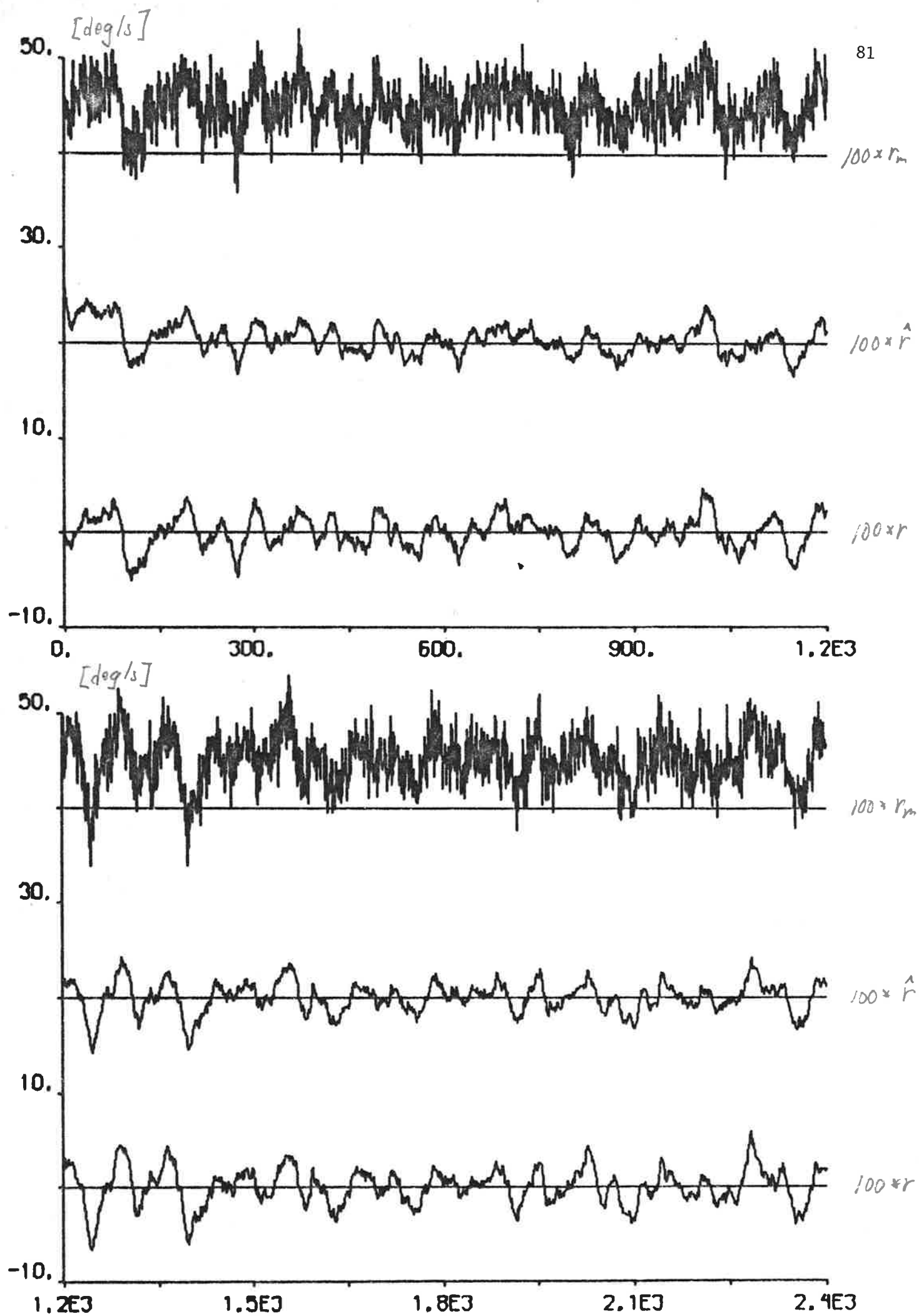


Fig. 4.4d

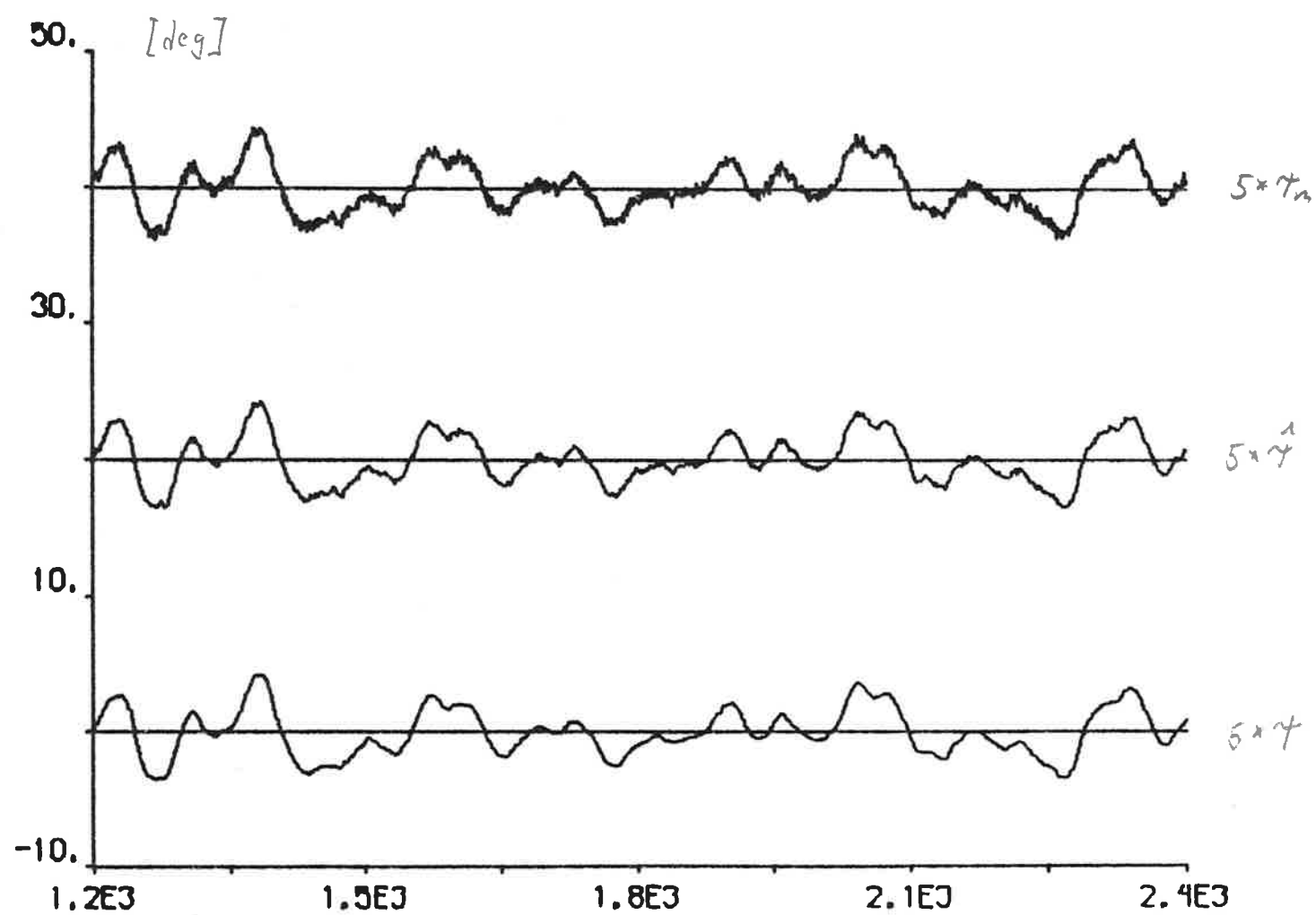
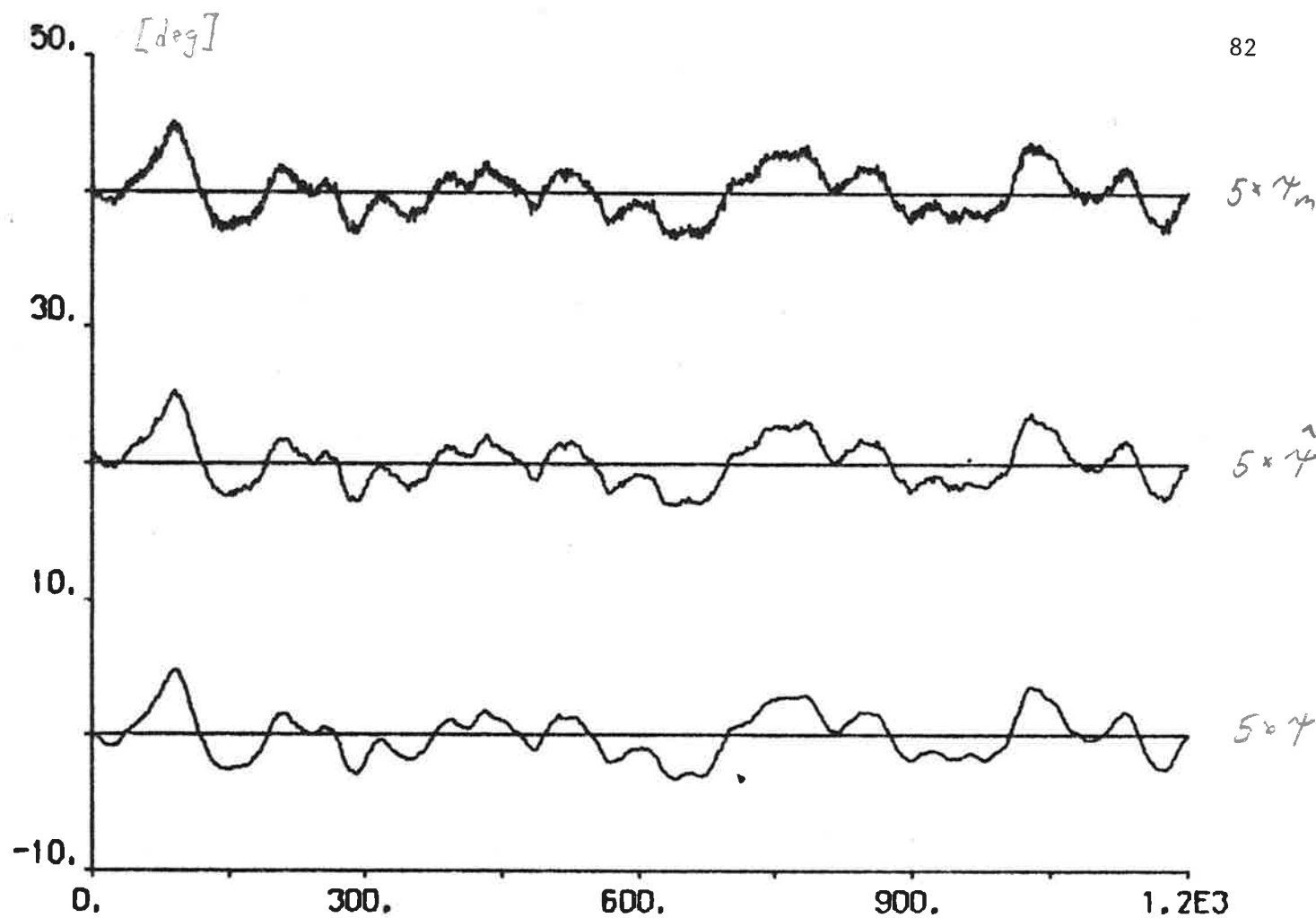


Fig. 4.4e

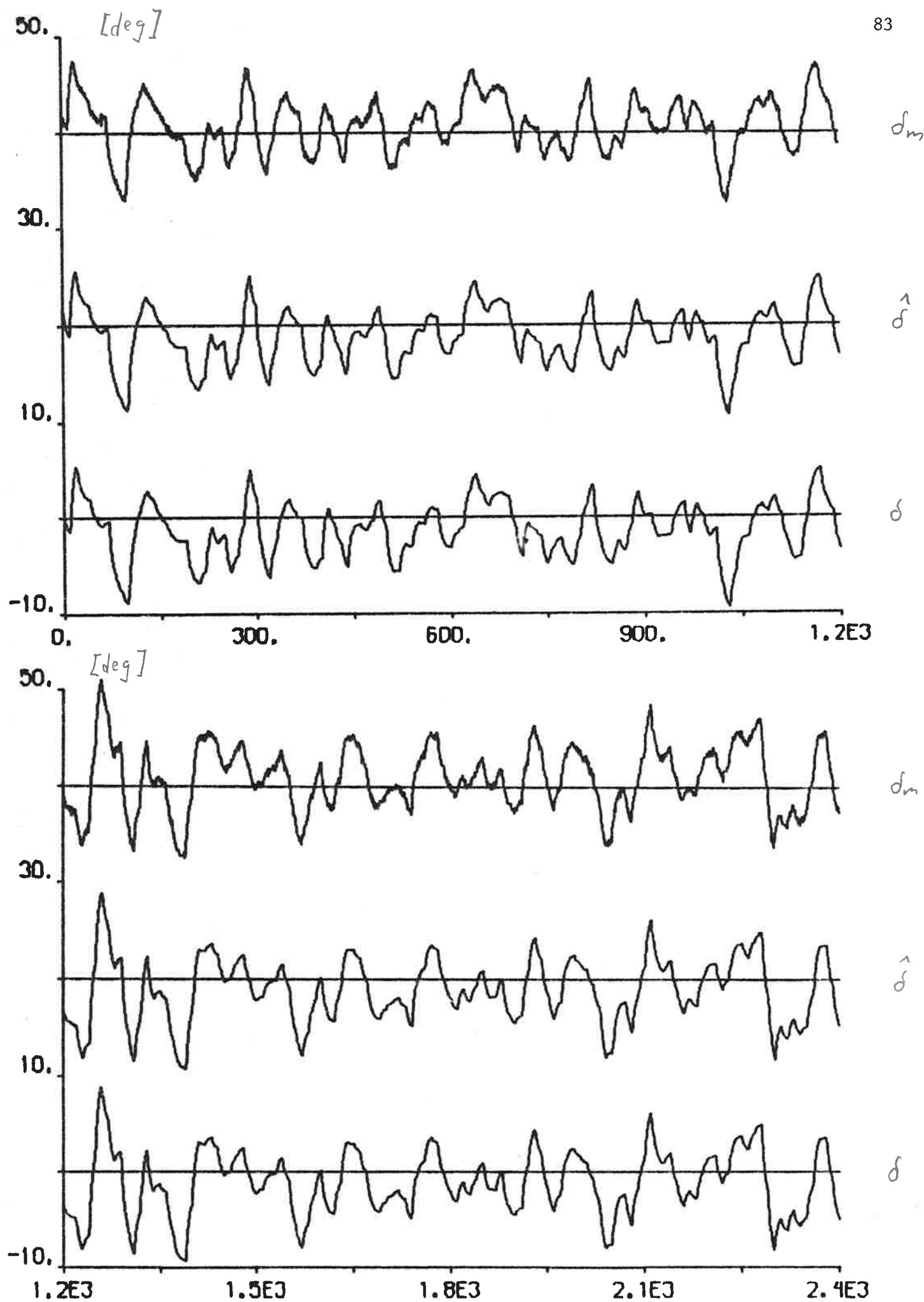


Fig. 4.4f

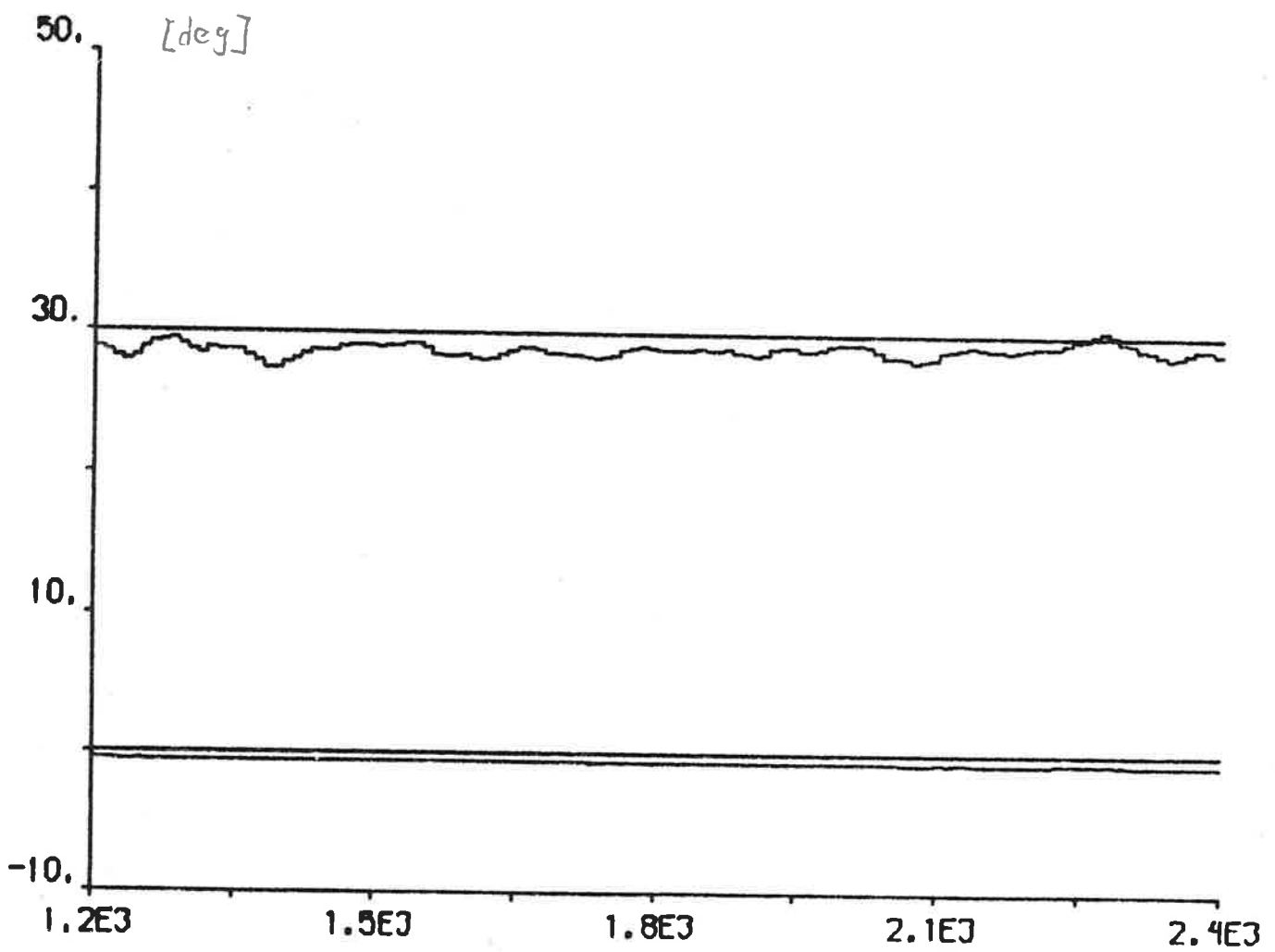
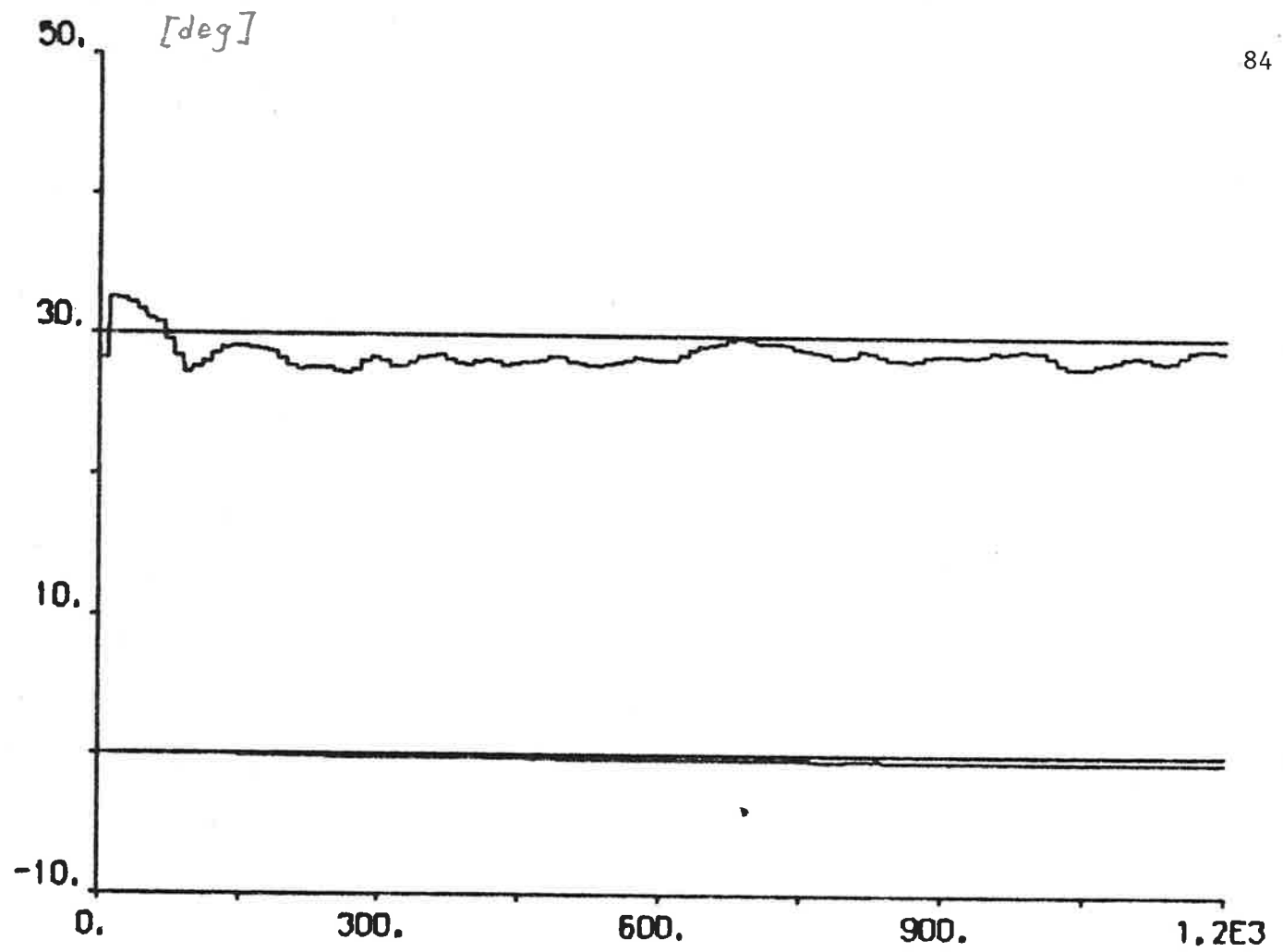


Fig. 4.4g

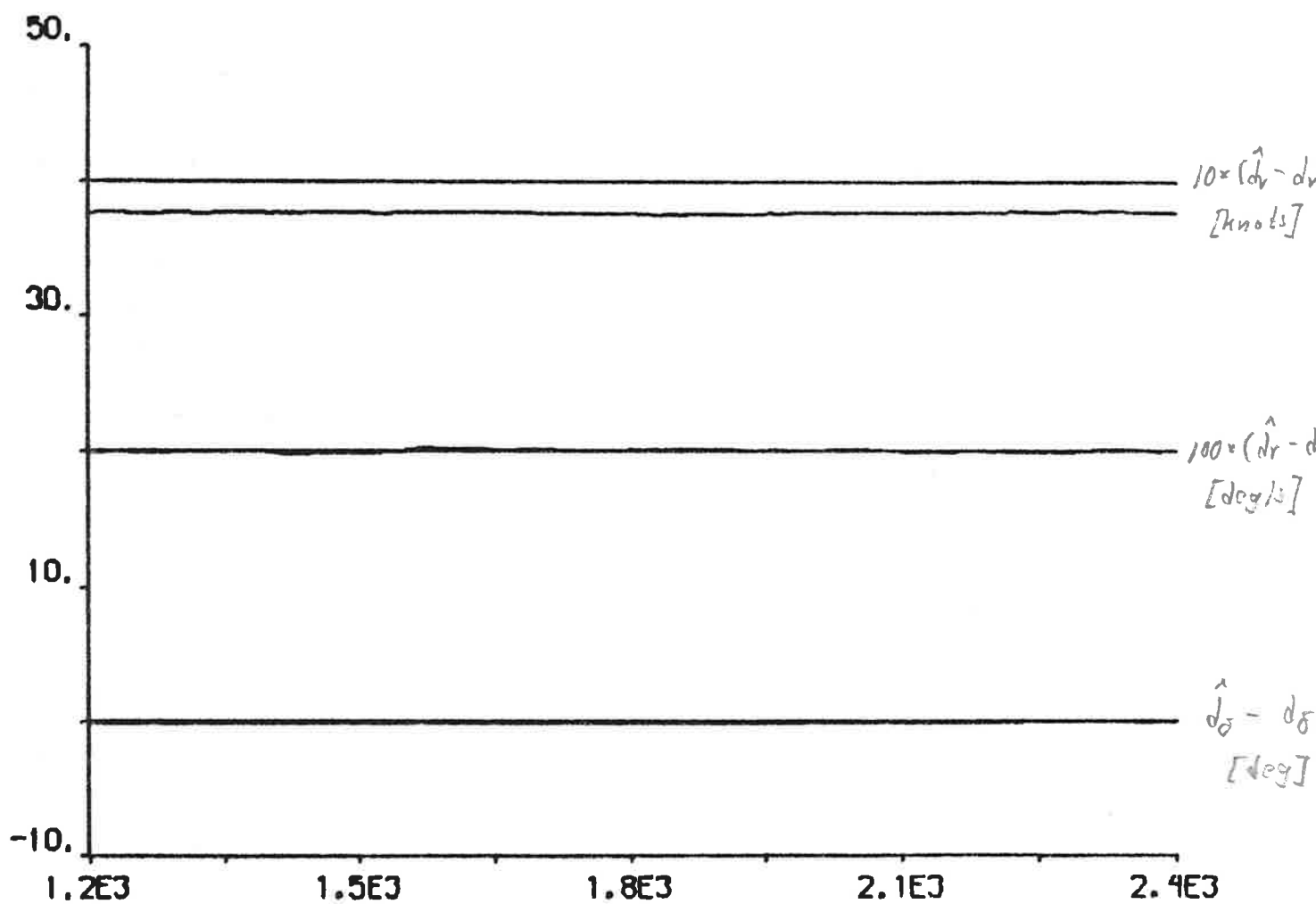
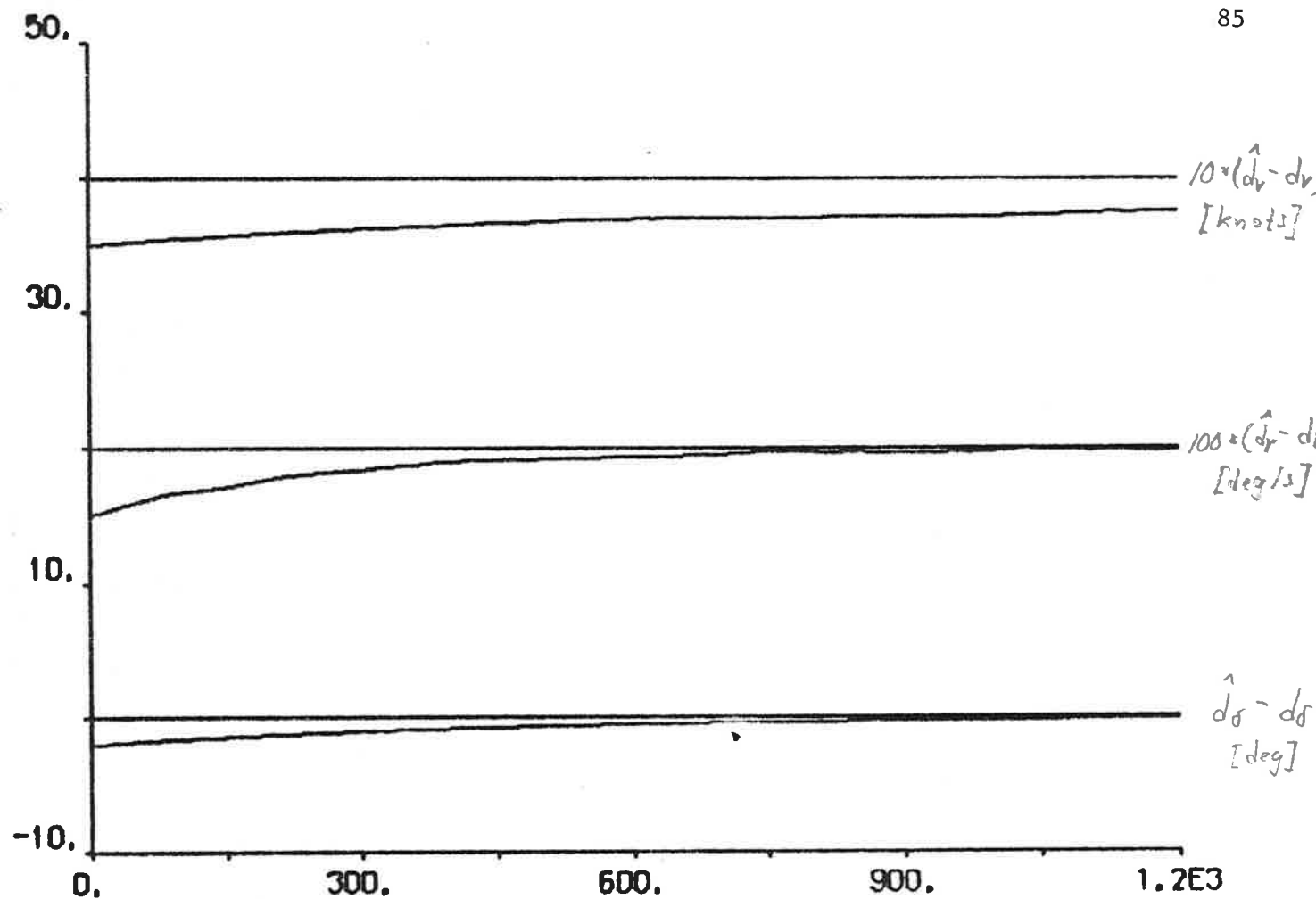


Fig. 4.4h

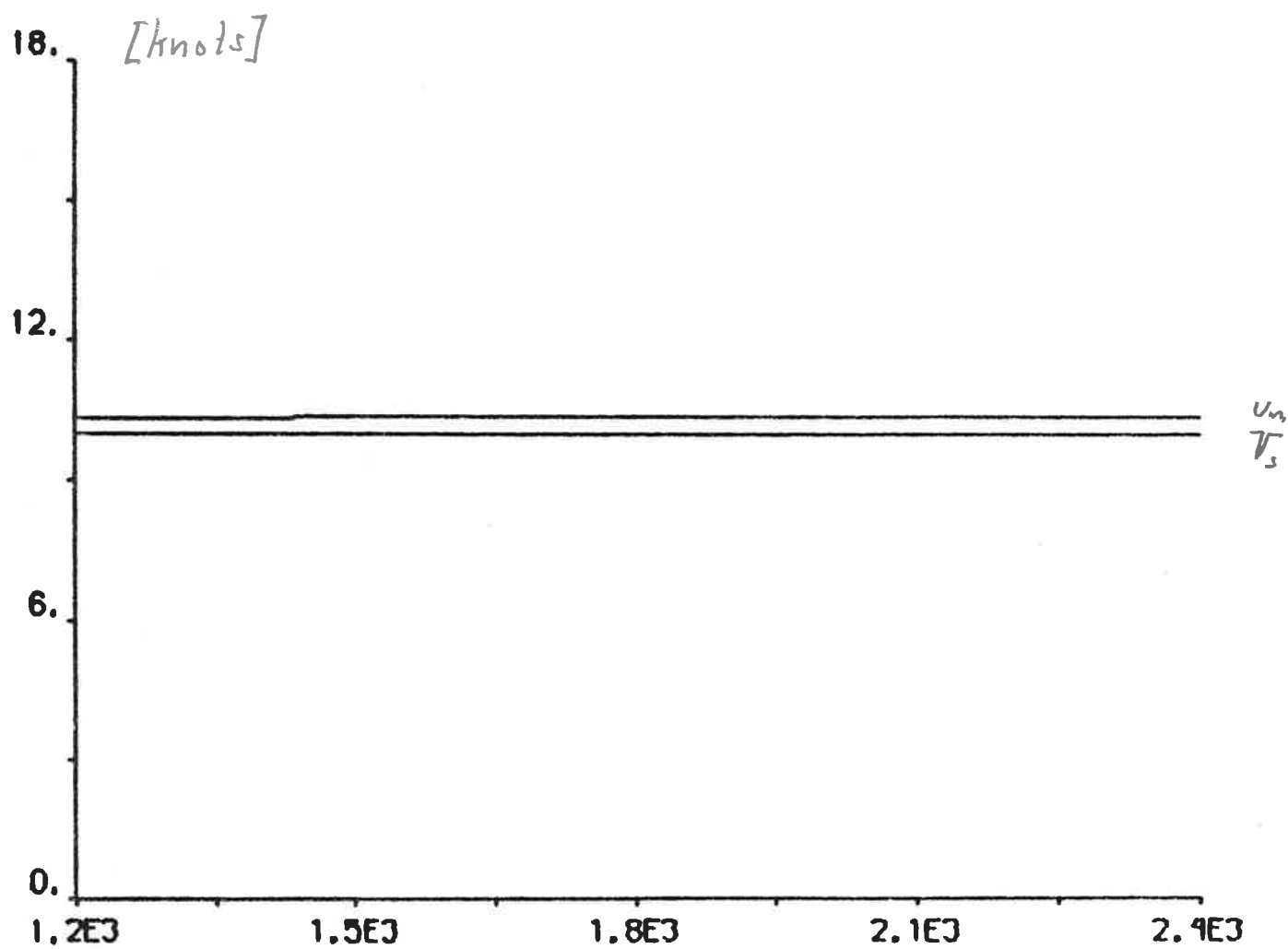
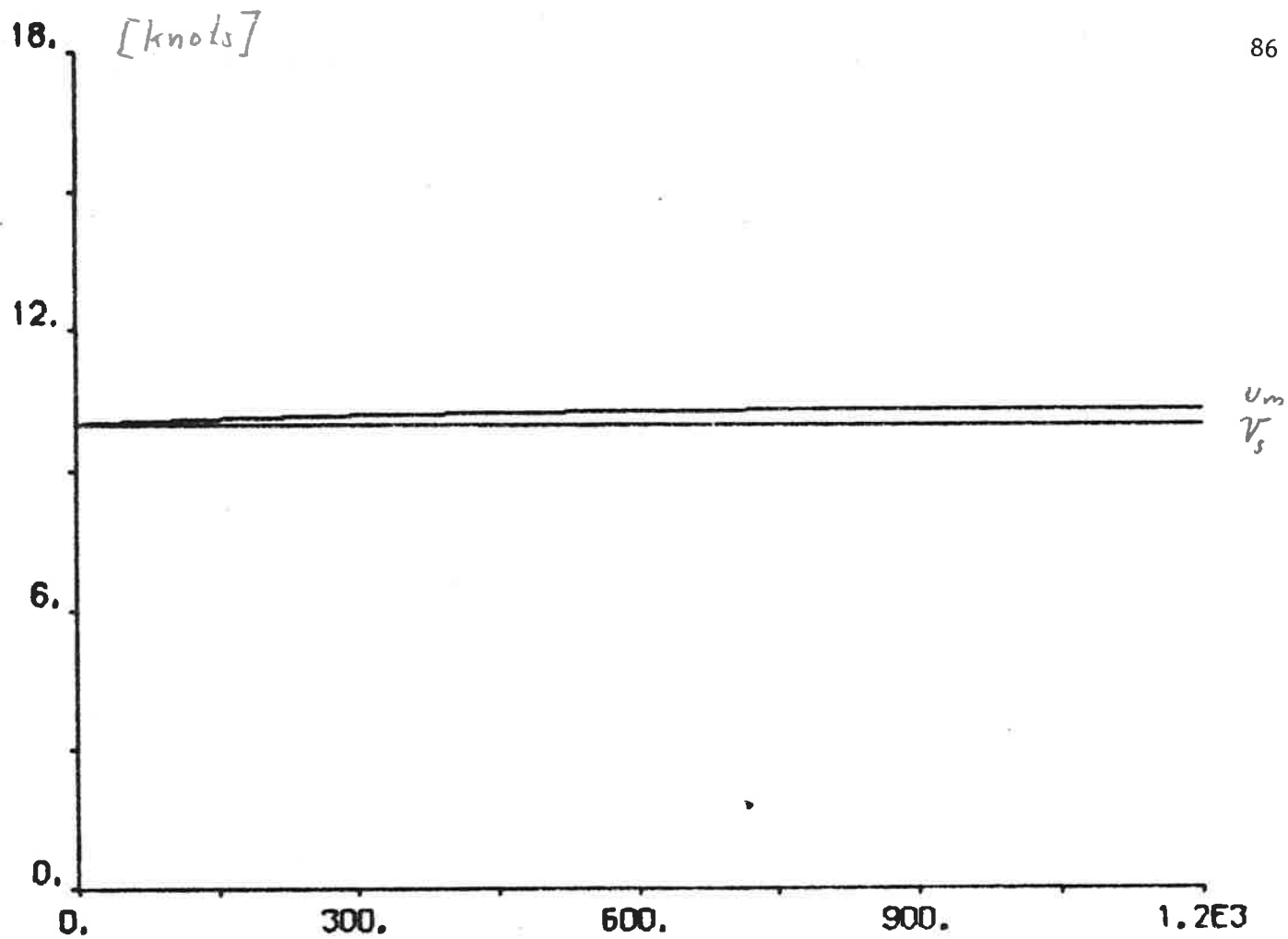


Fig. 4.4i

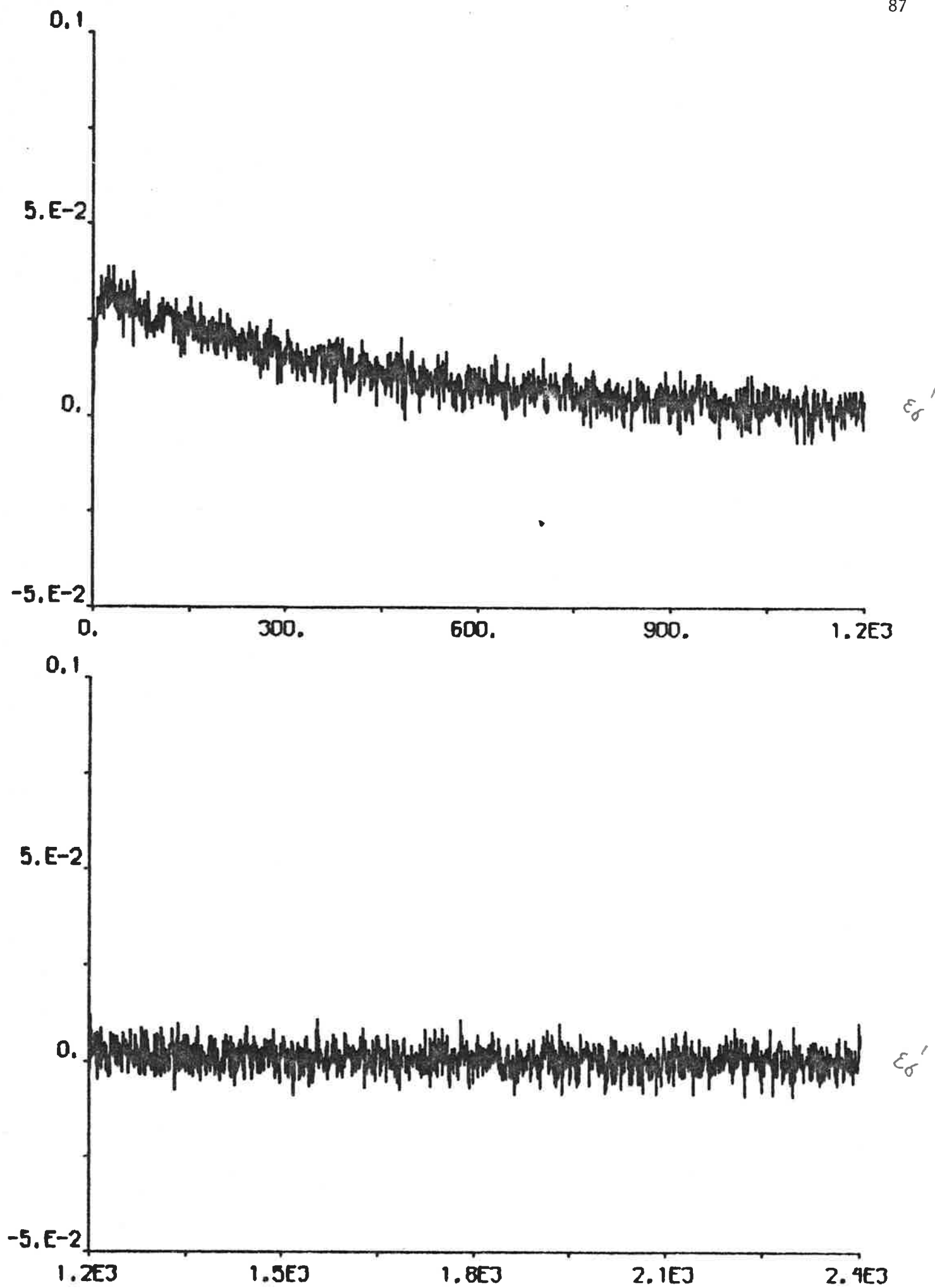


Fig. 4.4j

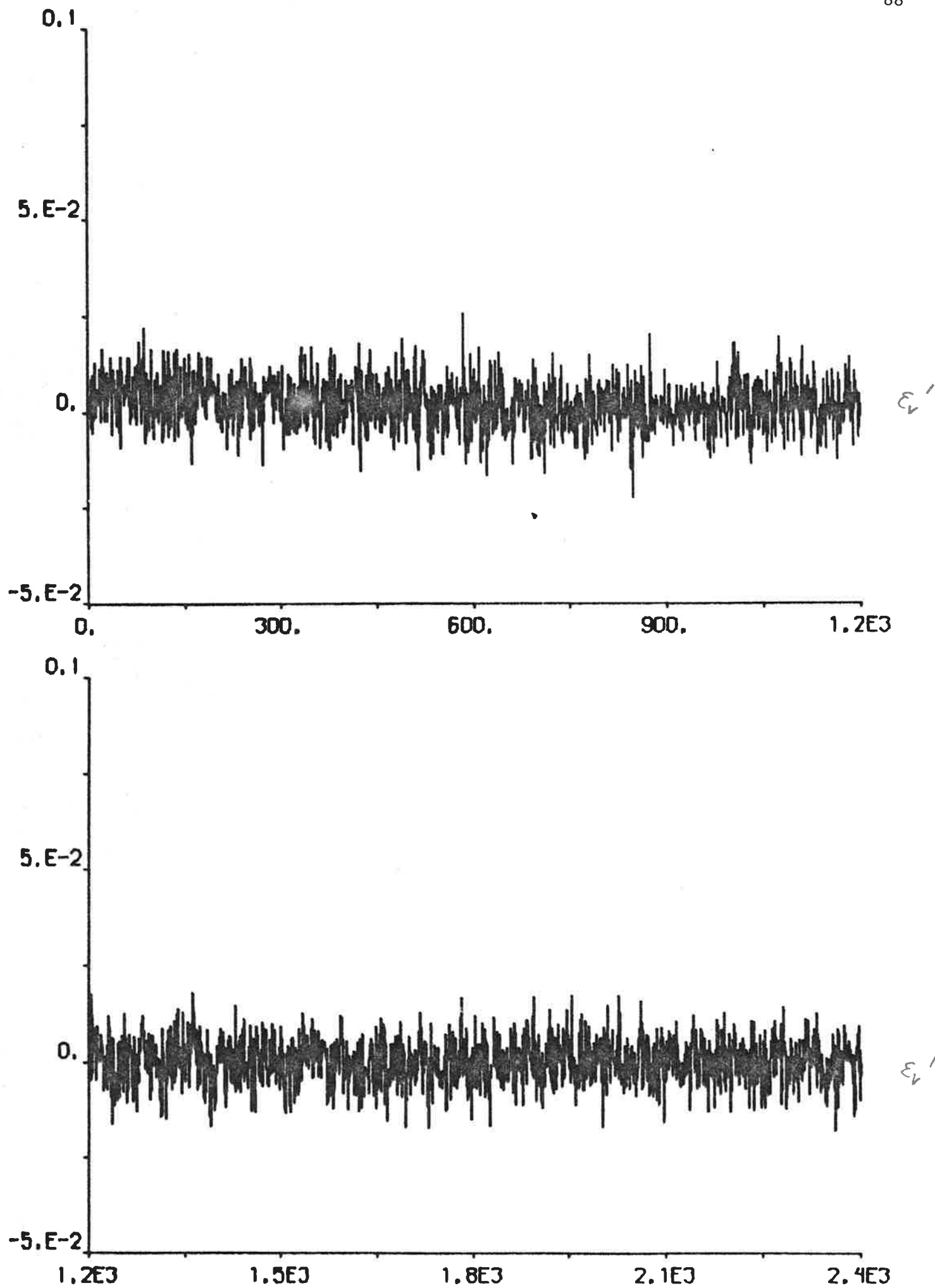
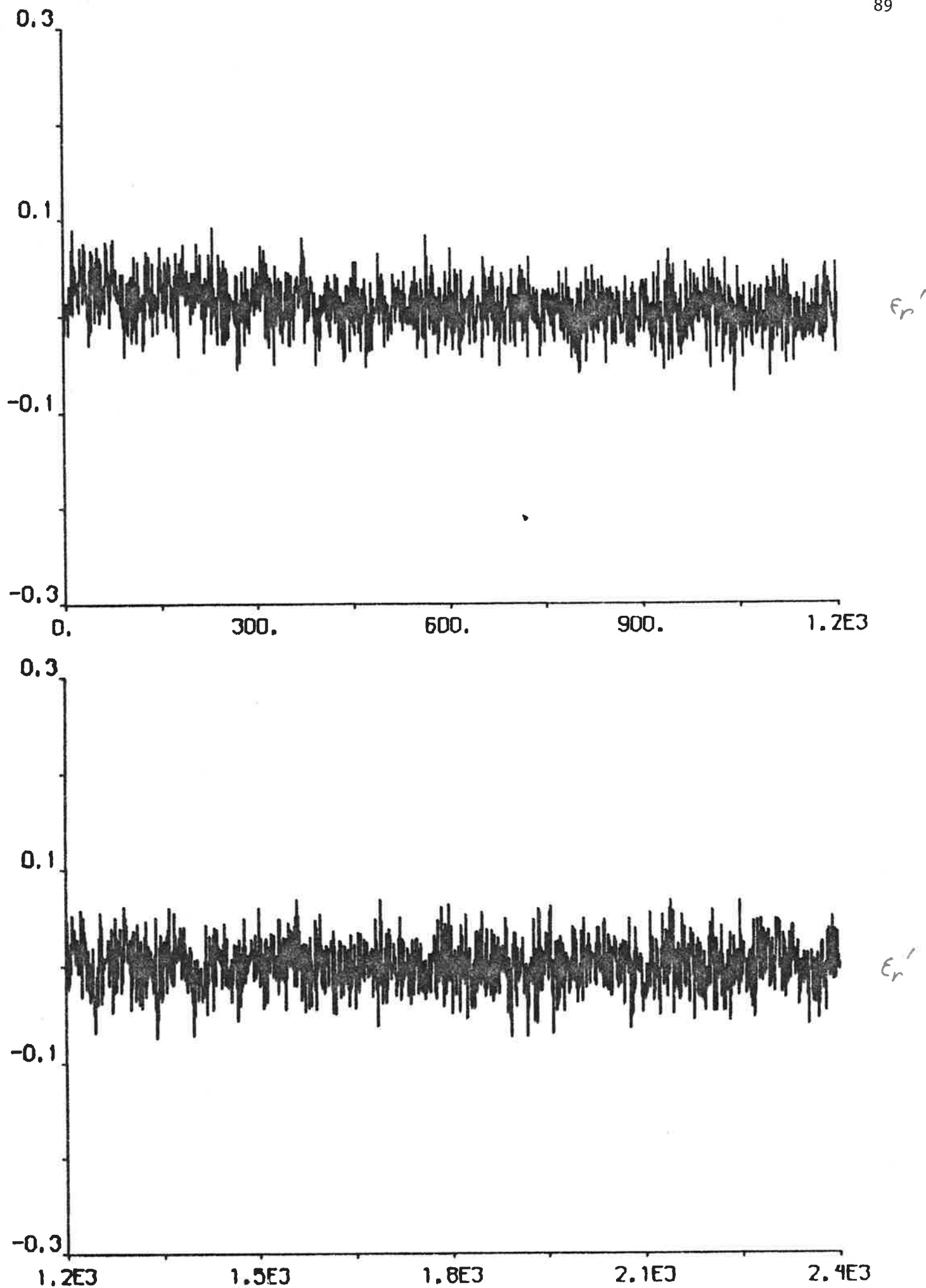


Fig. 4.4k

Fig. 4.4*l*

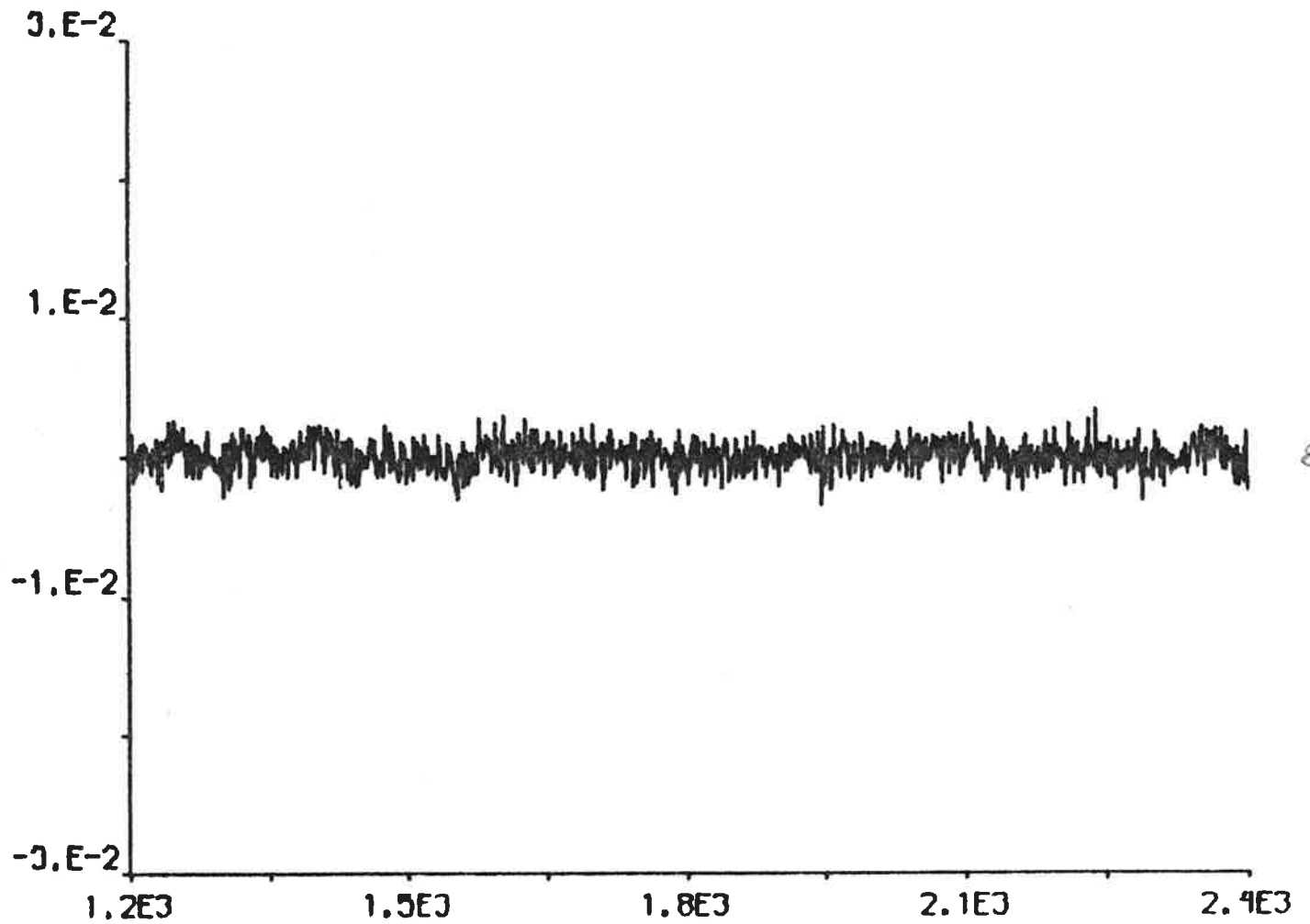
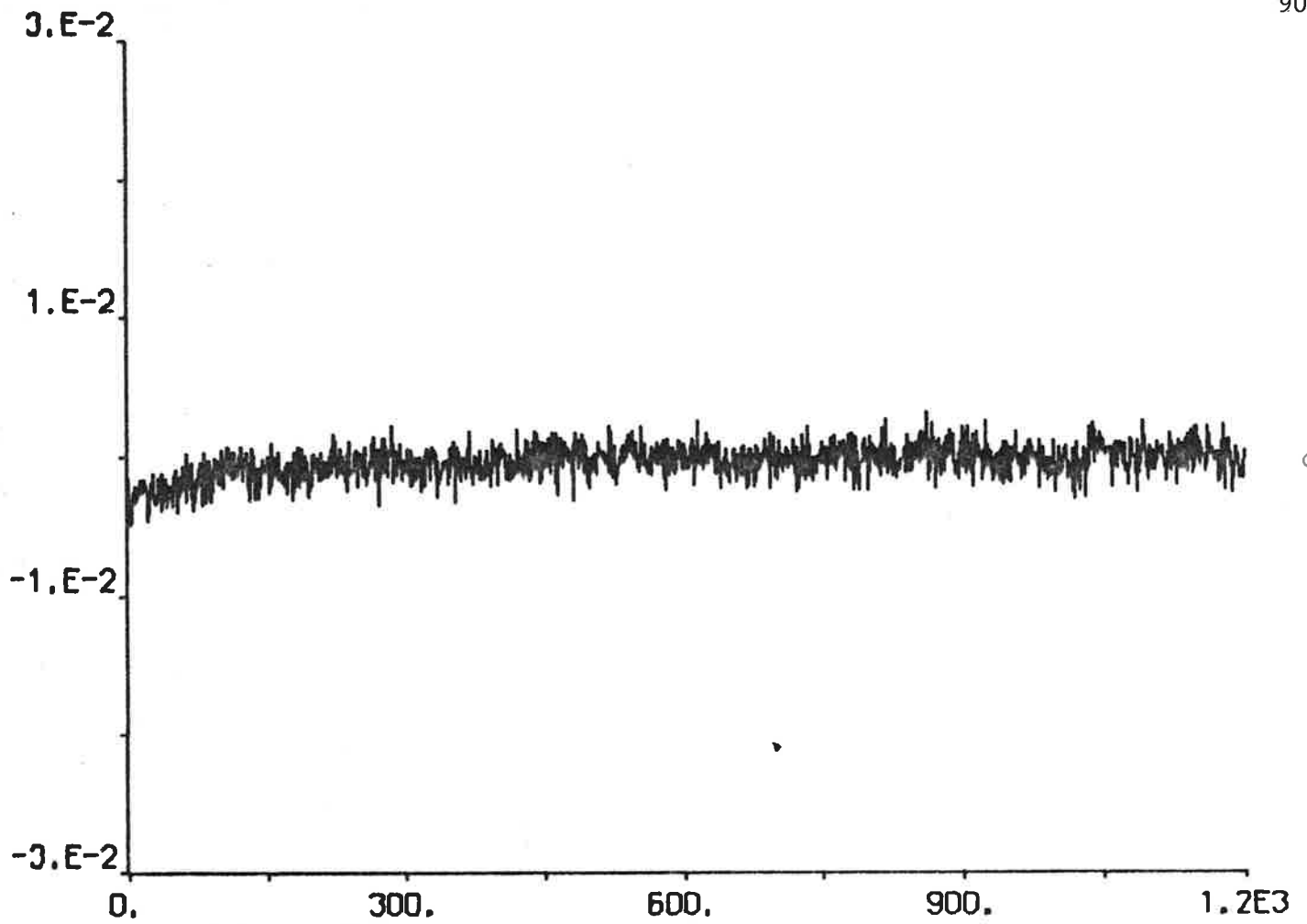


Fig. 4.4m

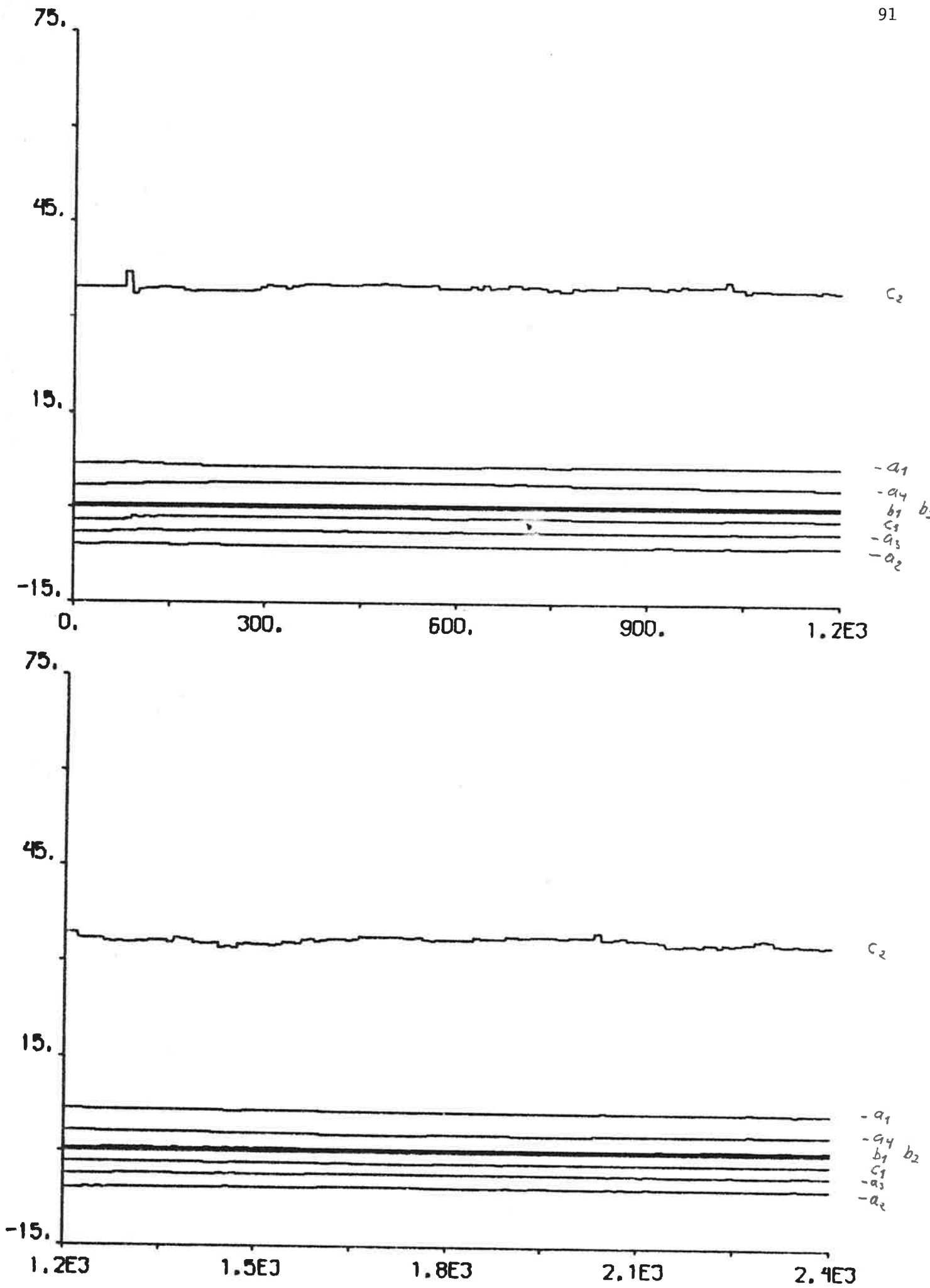


Fig. 4.4n

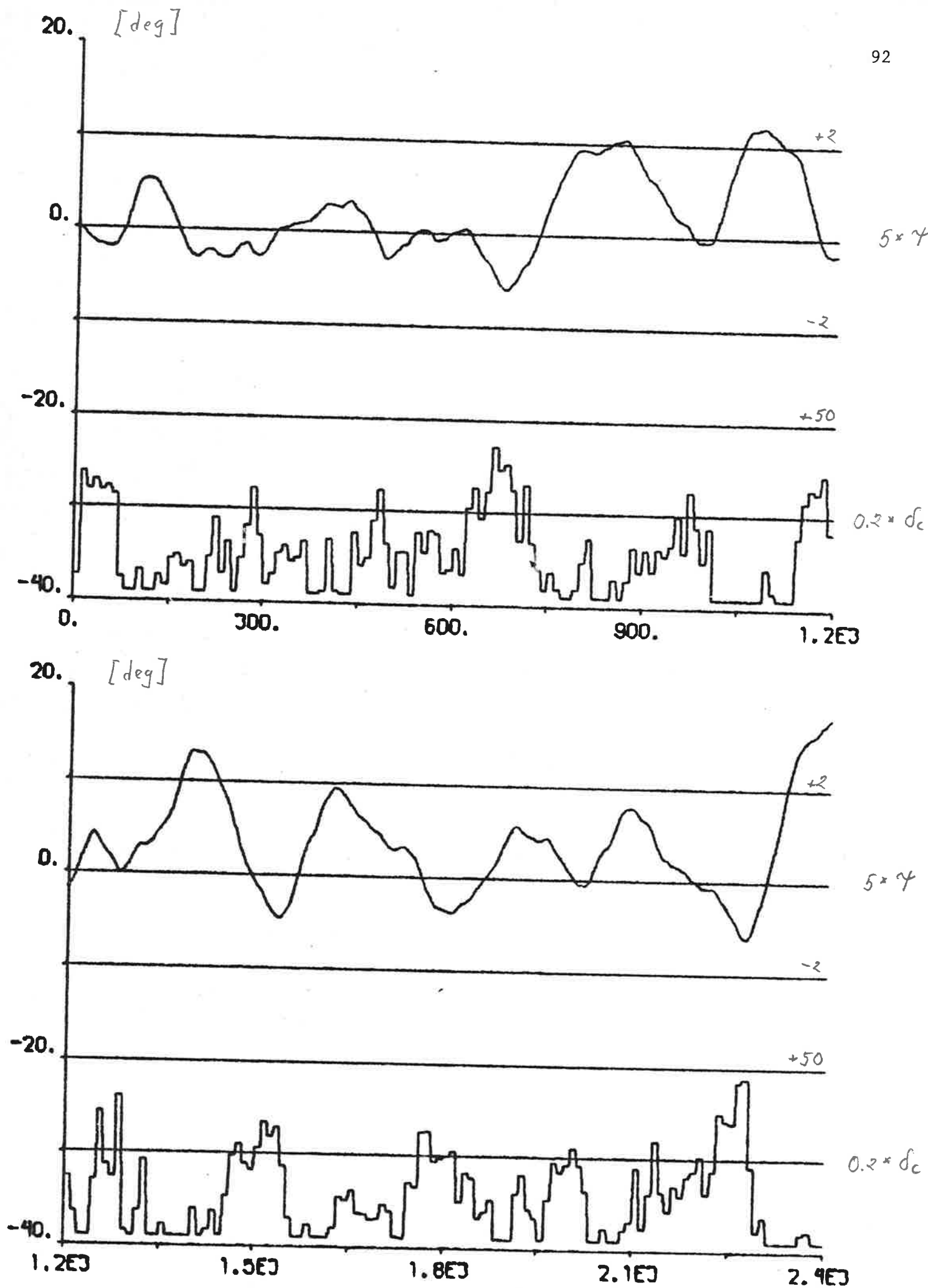


Fig. 4.5a - $T = 22.3$ m, $n_0 = 22.1772$ rpm, $u_0 = 4$ knots, $\delta_\ell = 45$ deg, self-tuning regulator using estimates from the Kalman filter.

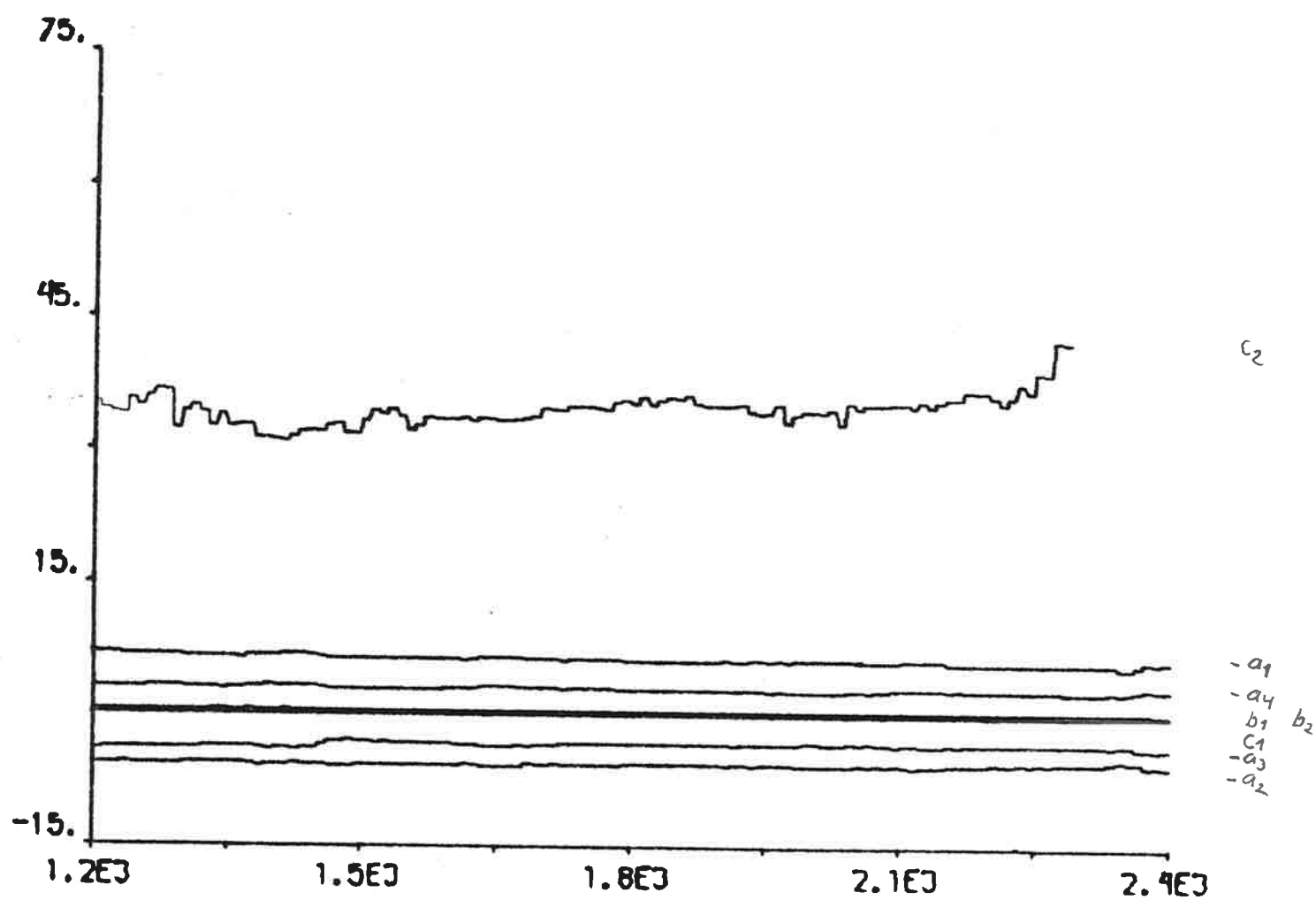
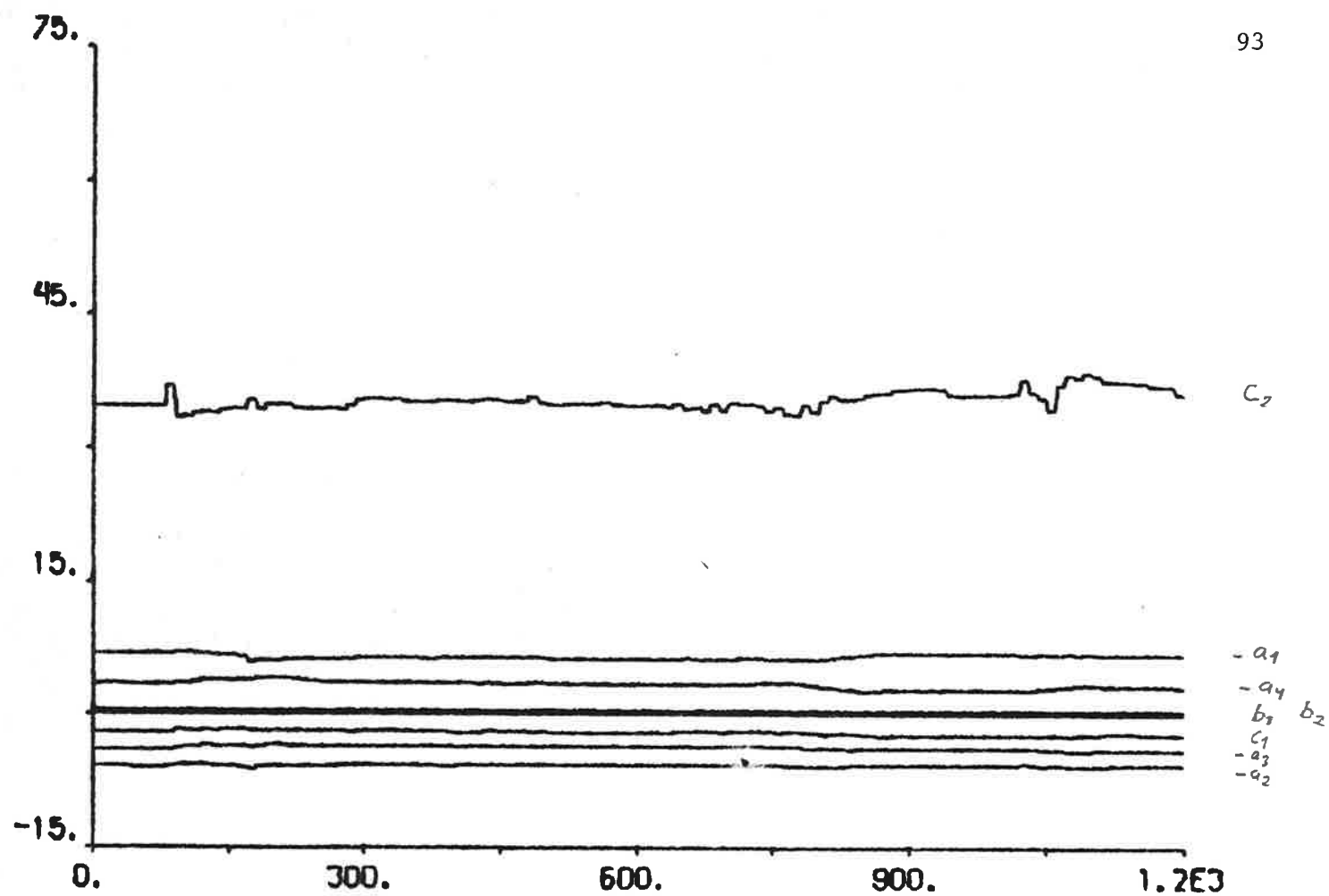


Fig. 4.5b

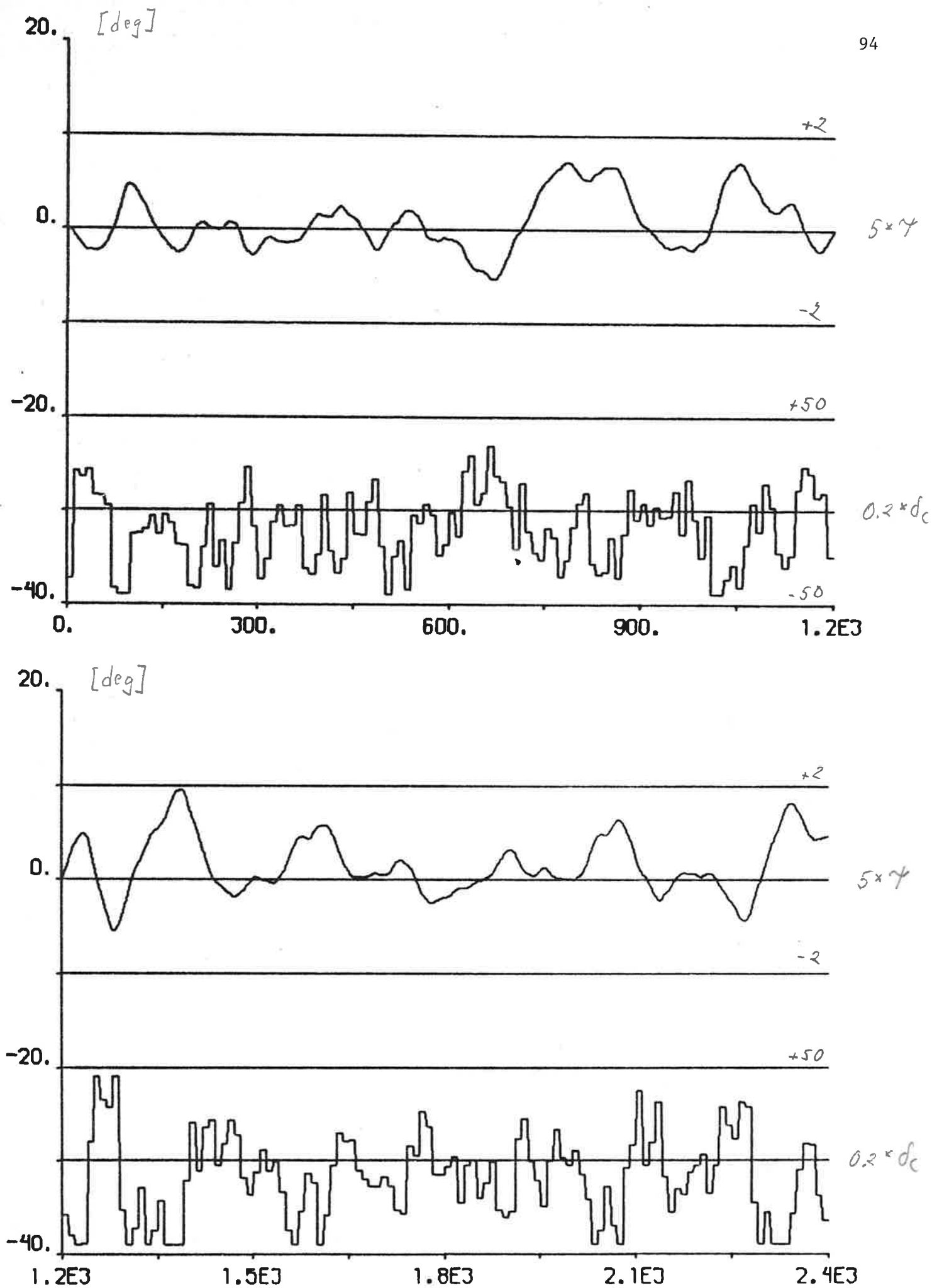


Fig. 4.6a - $T = 10.5$ m, $n_0 = 22.1772$ rpm, $u_0 = 4$ knots, $\delta_\ell = 45$ deg, self-tuning regulator using estimates from the Kalman filter.

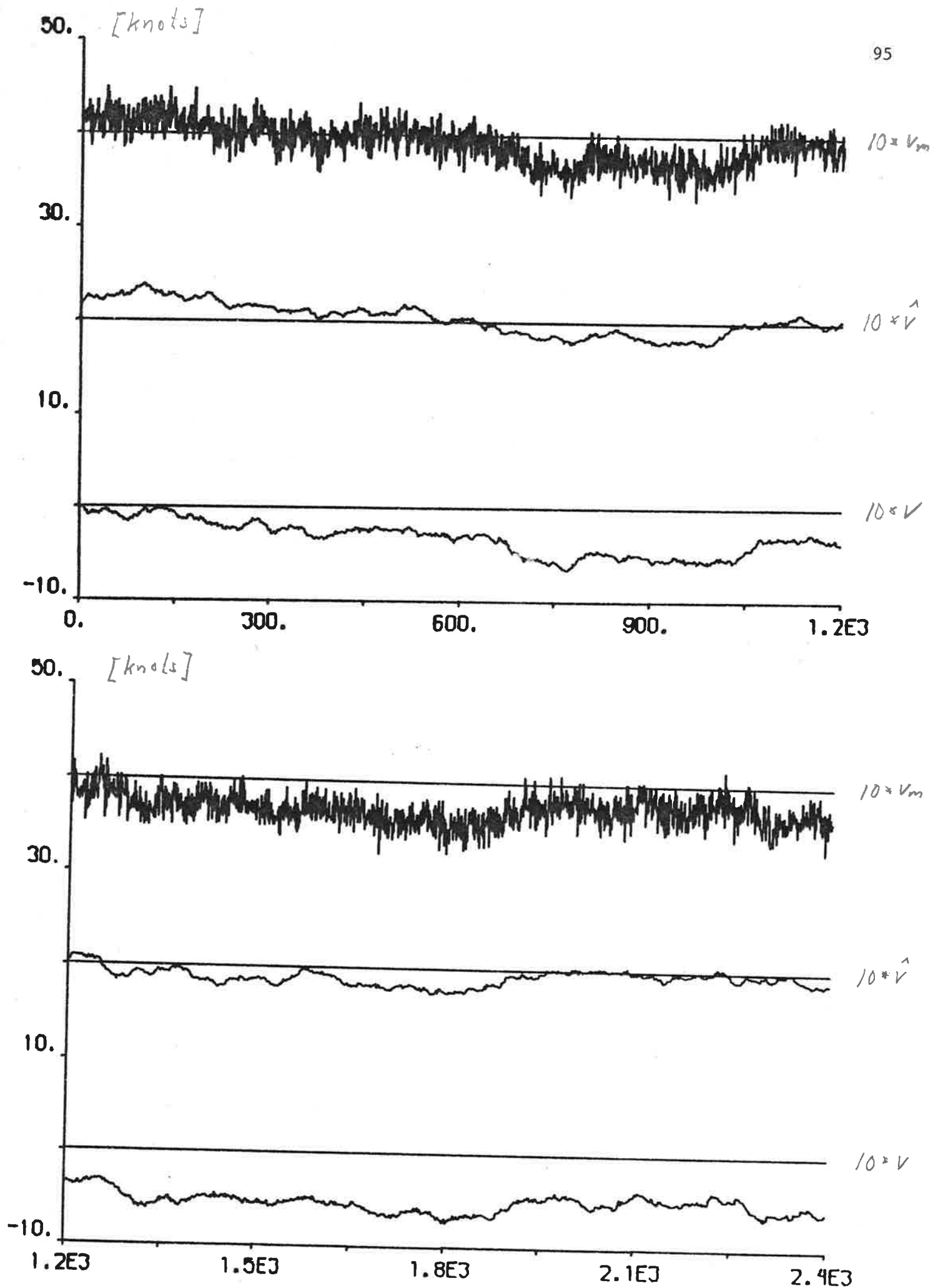


Fig. 4.6b

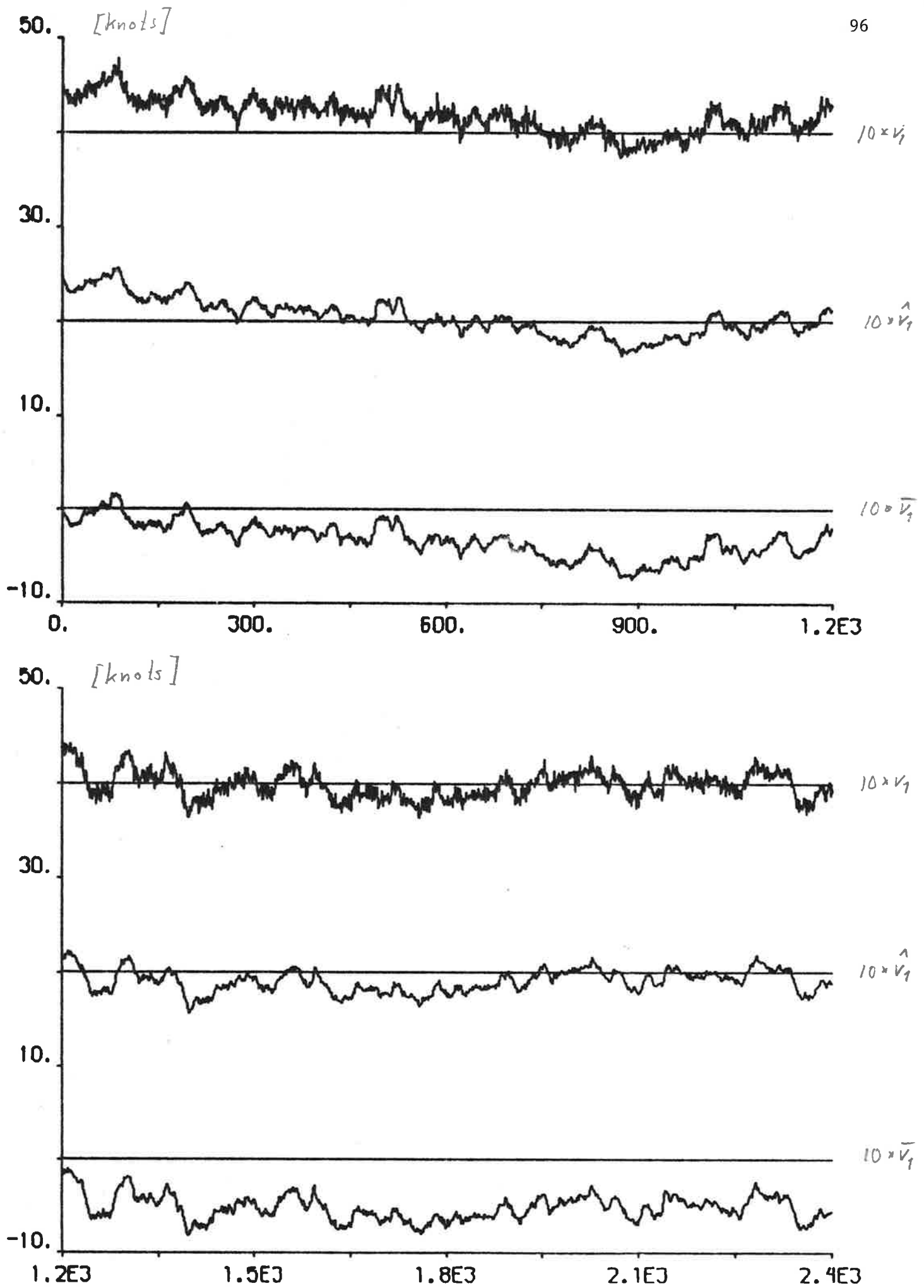


Fig. 4.6c

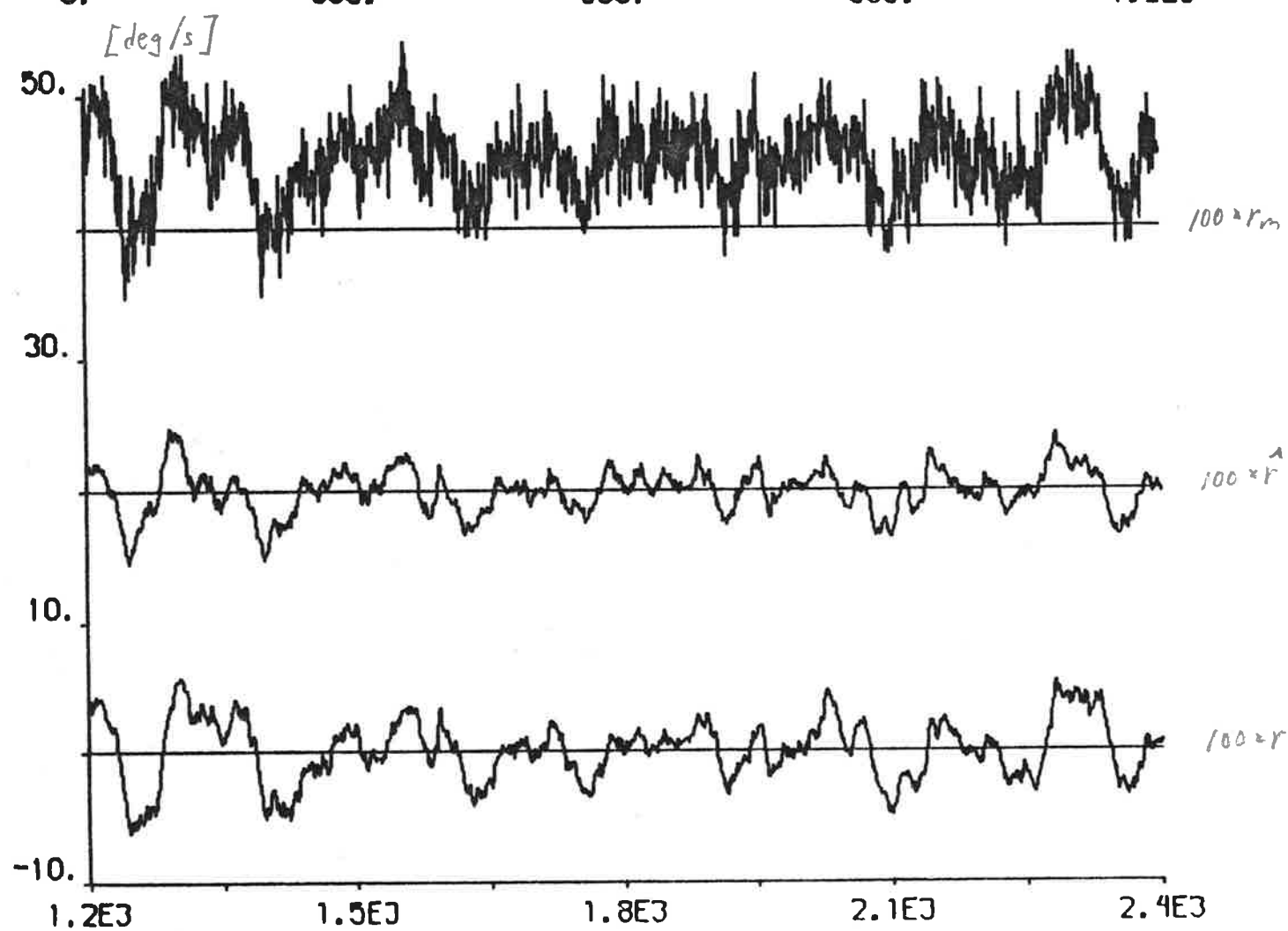
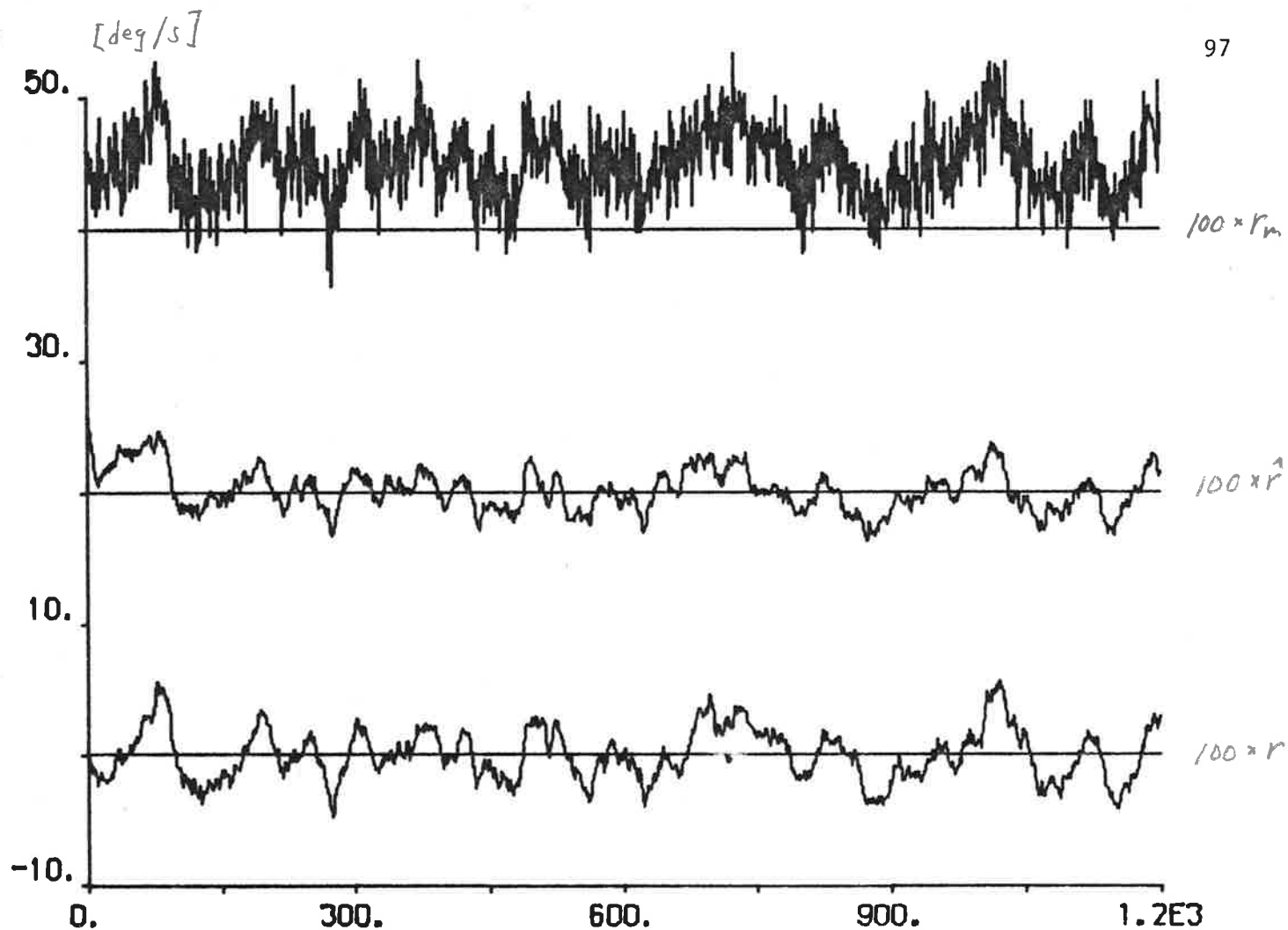


Fig. 4.6d

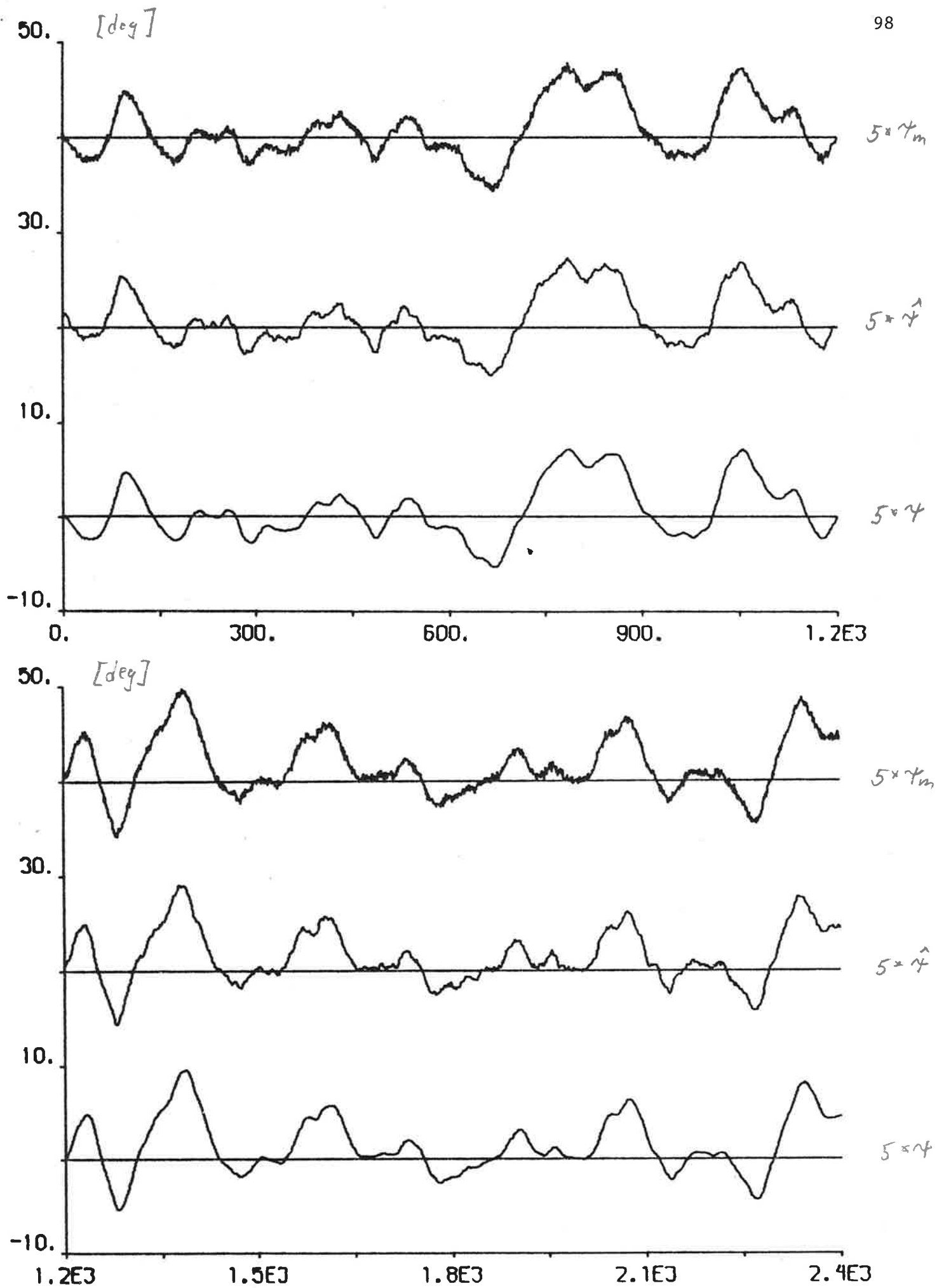


Fig. 4.6e

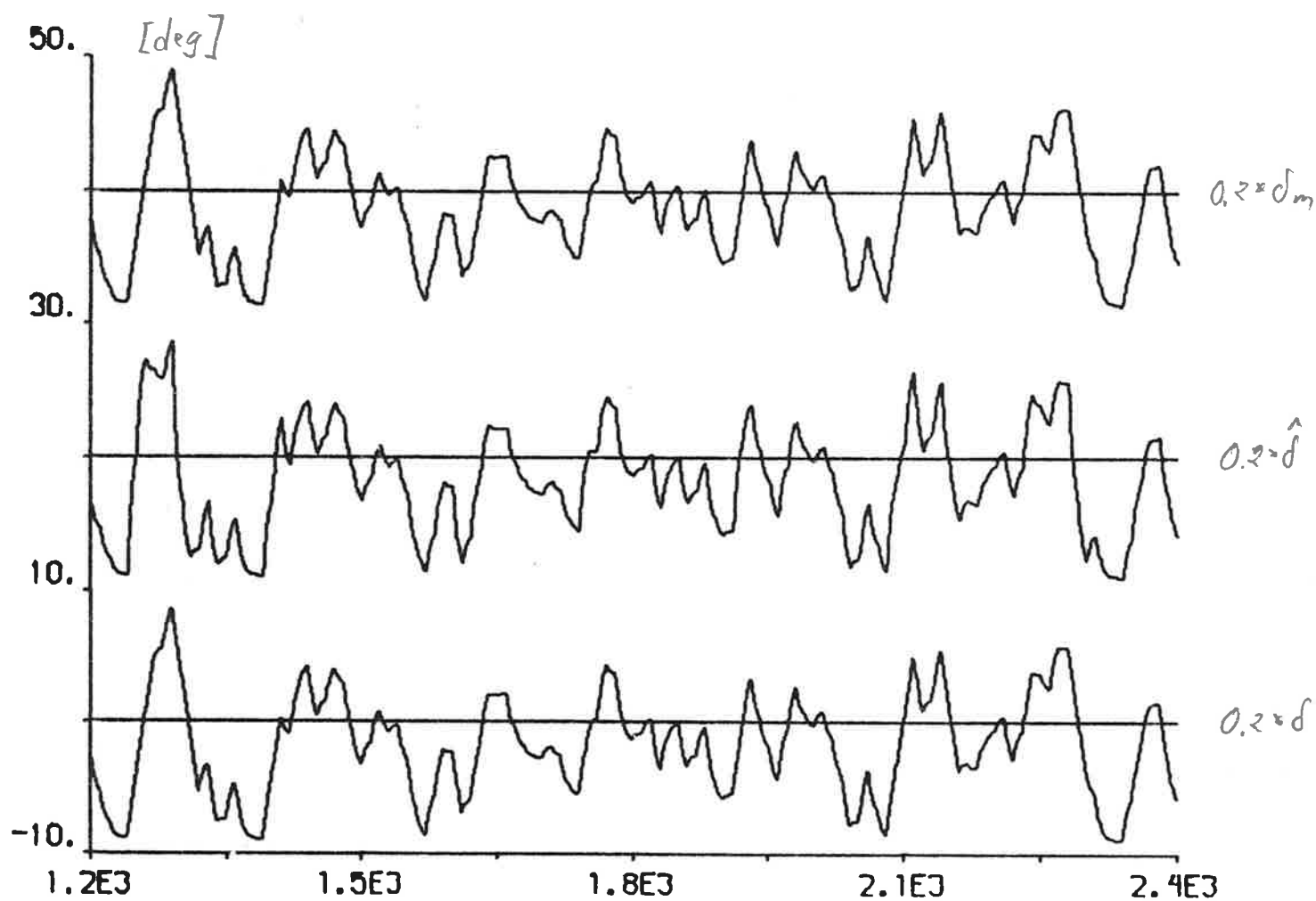
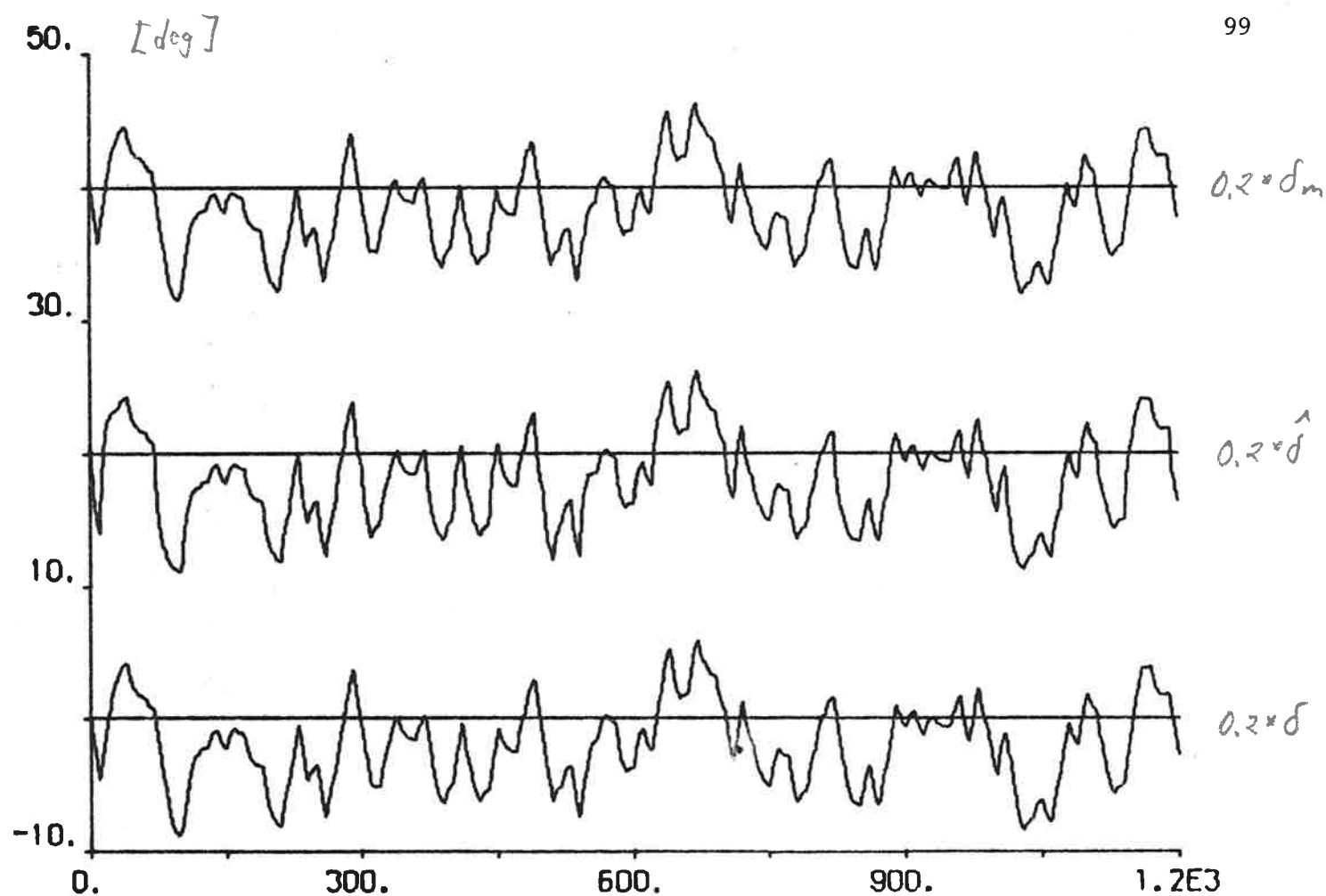


Fig. 4.6f

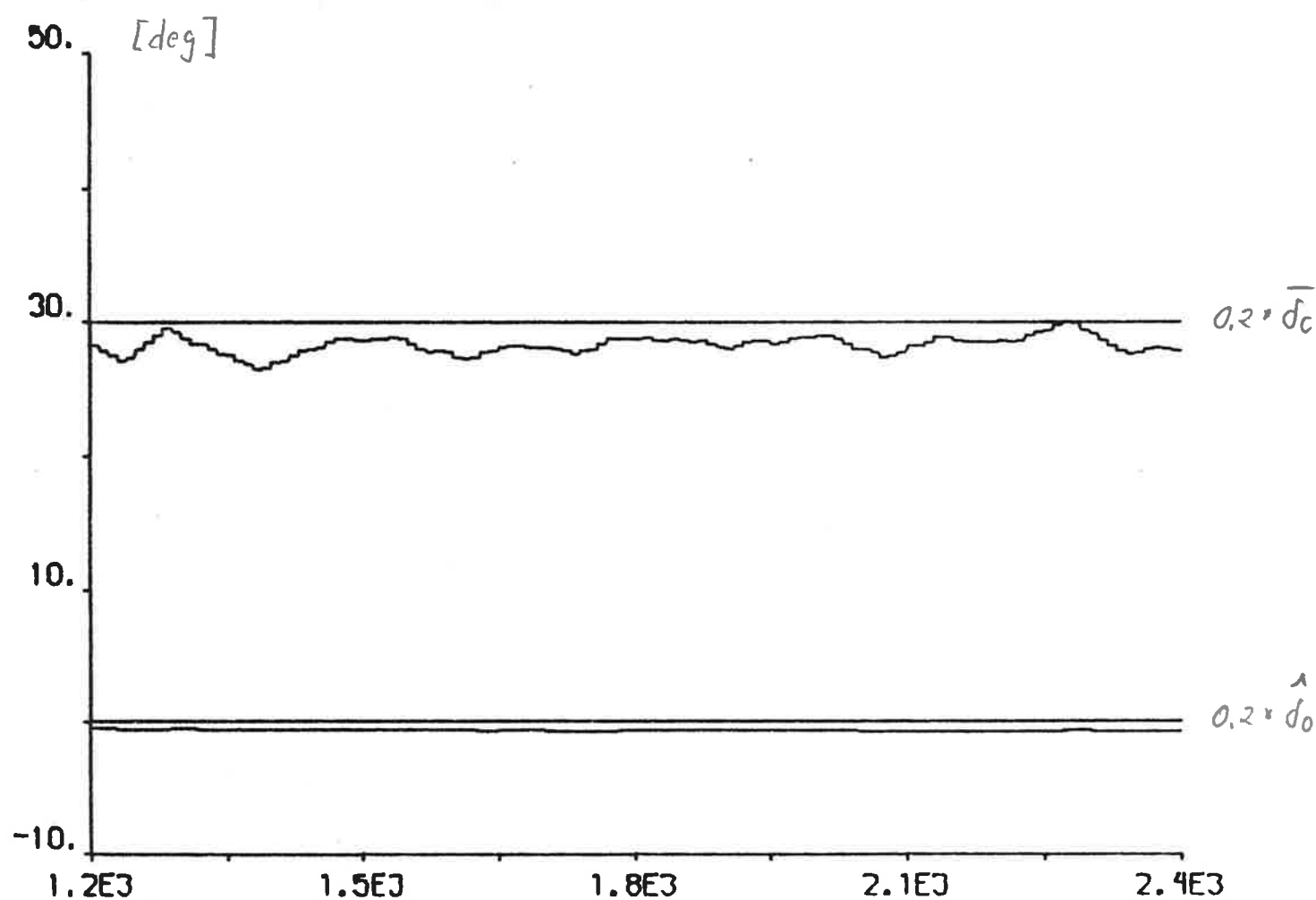
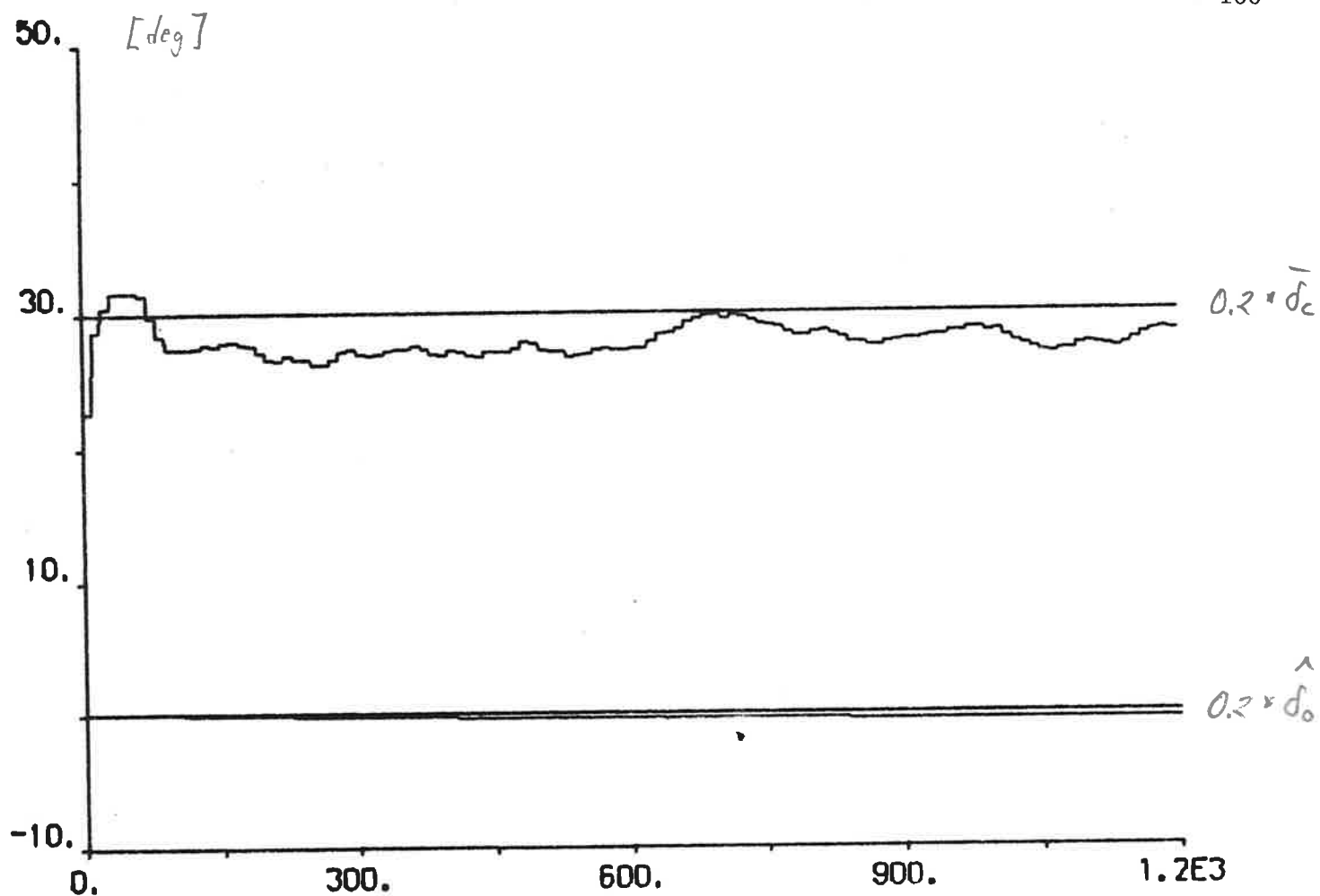


Fig. 4.6g

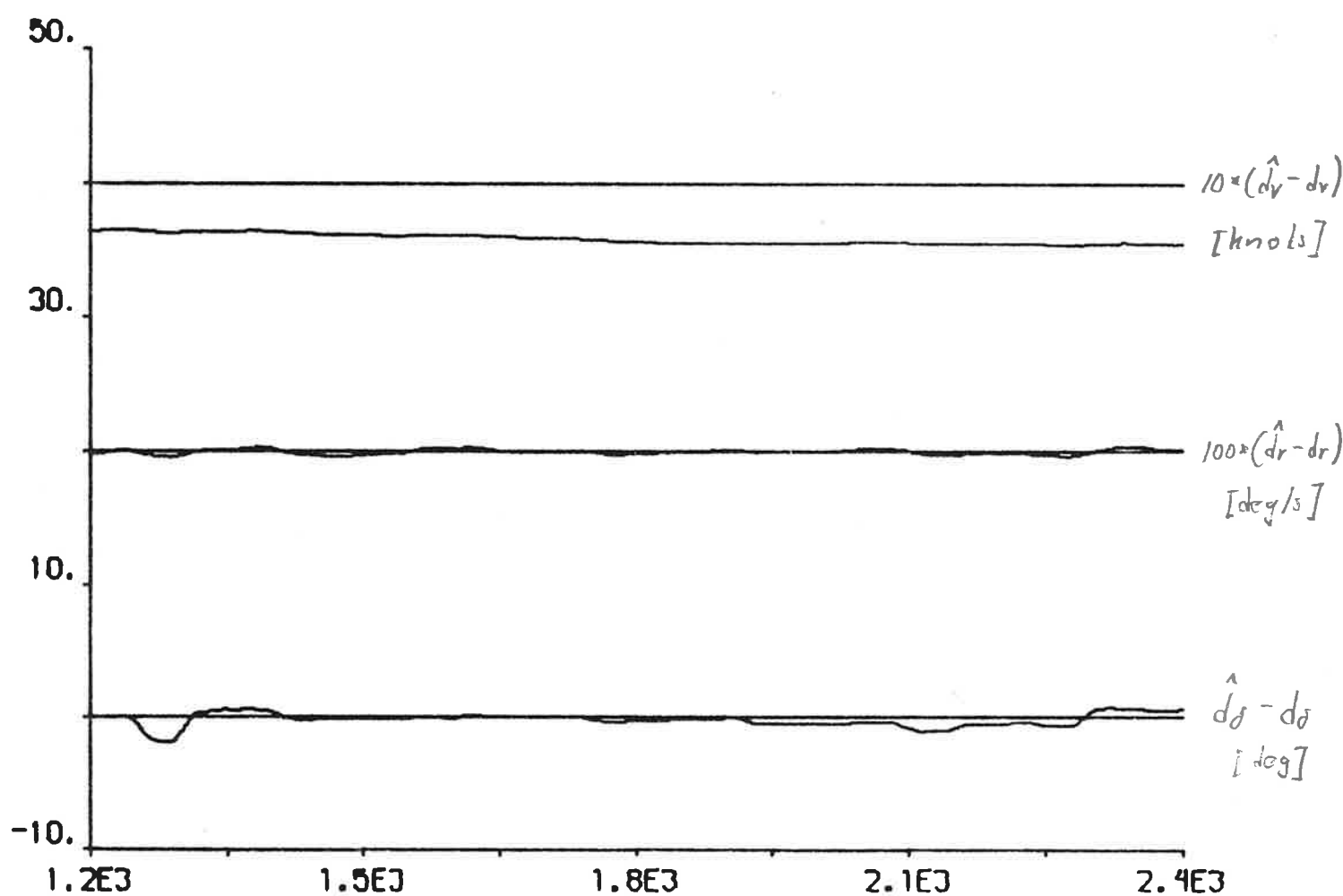
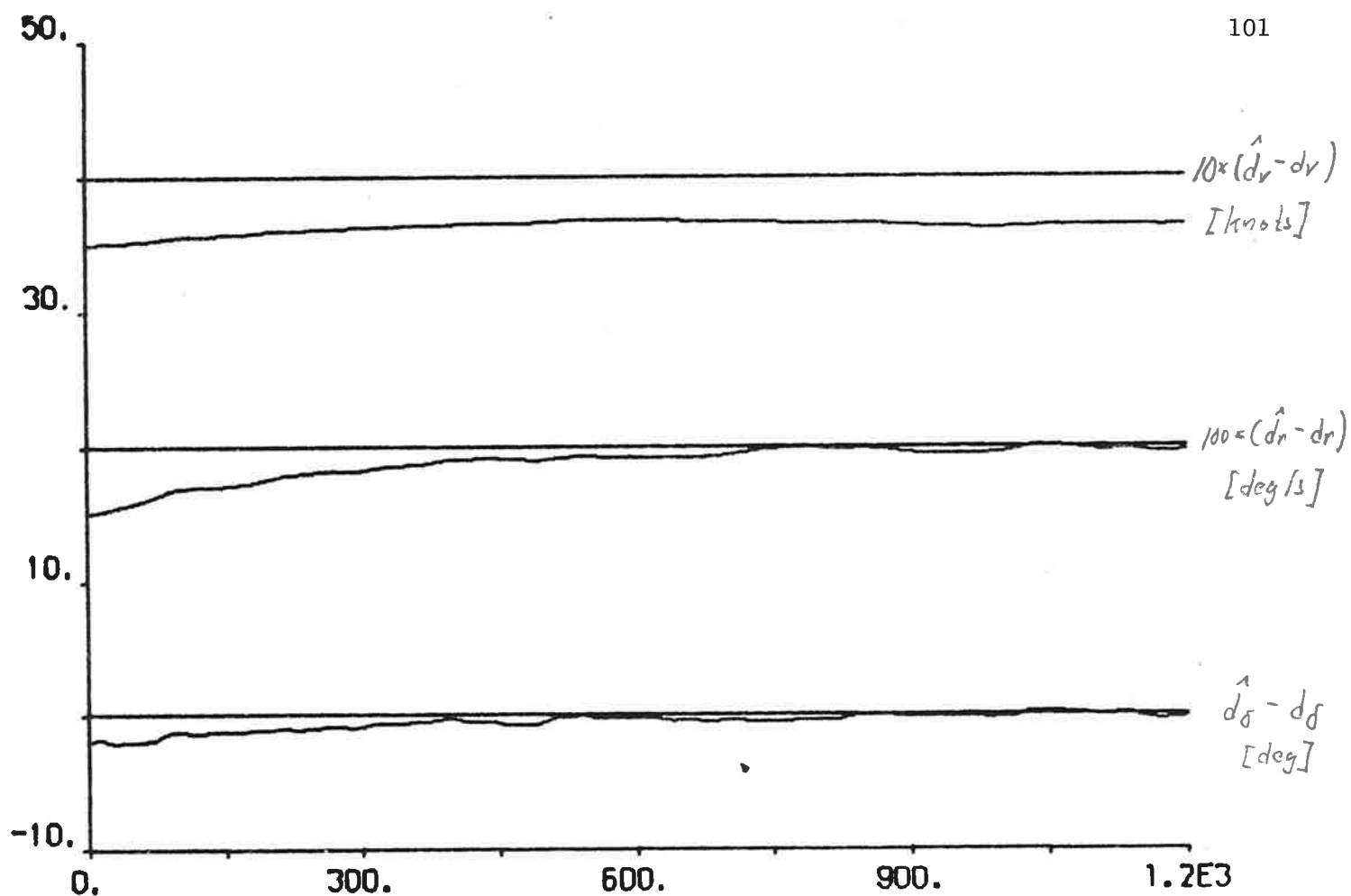


Fig. 4.6h

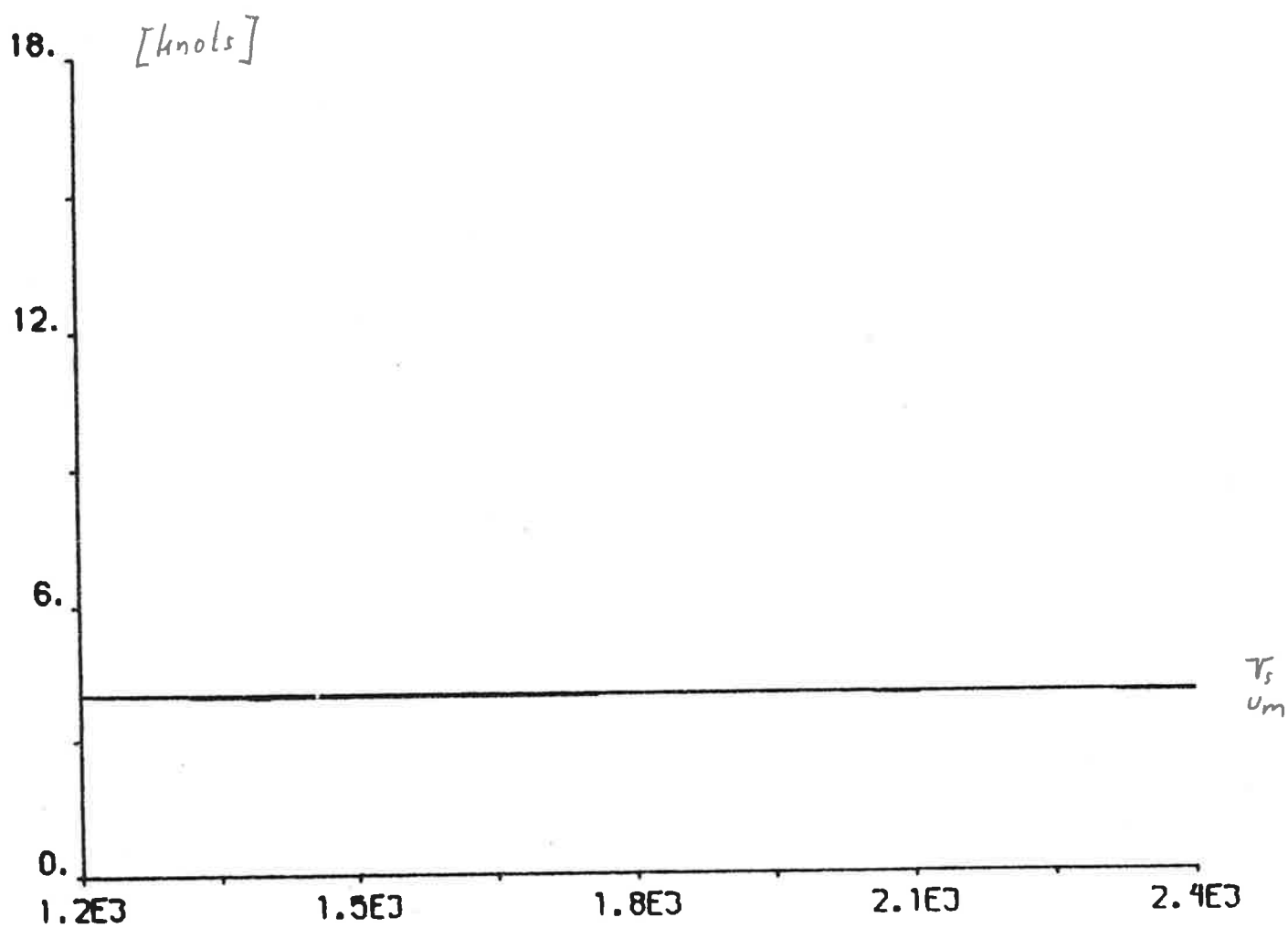
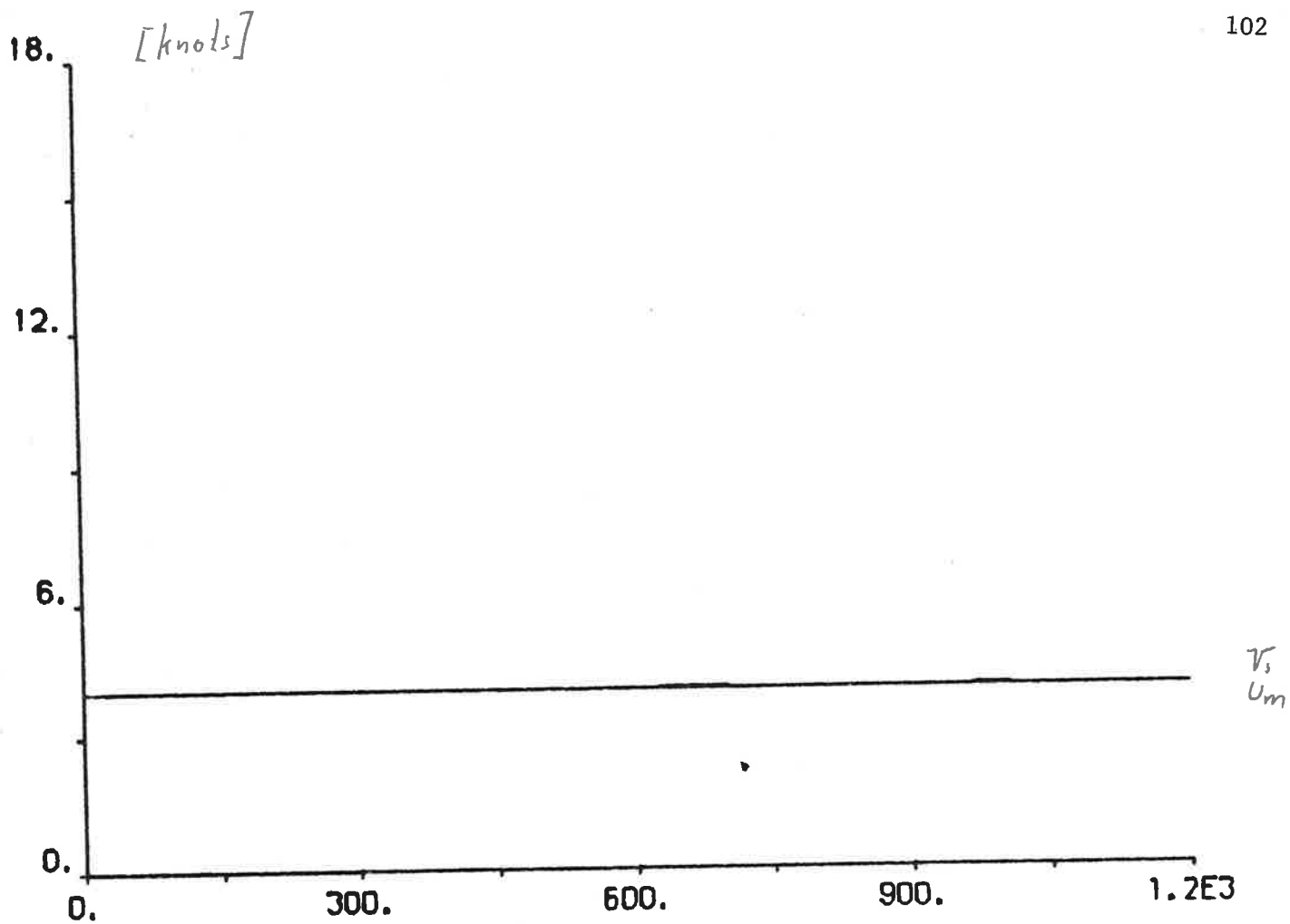


Fig. 4.6i

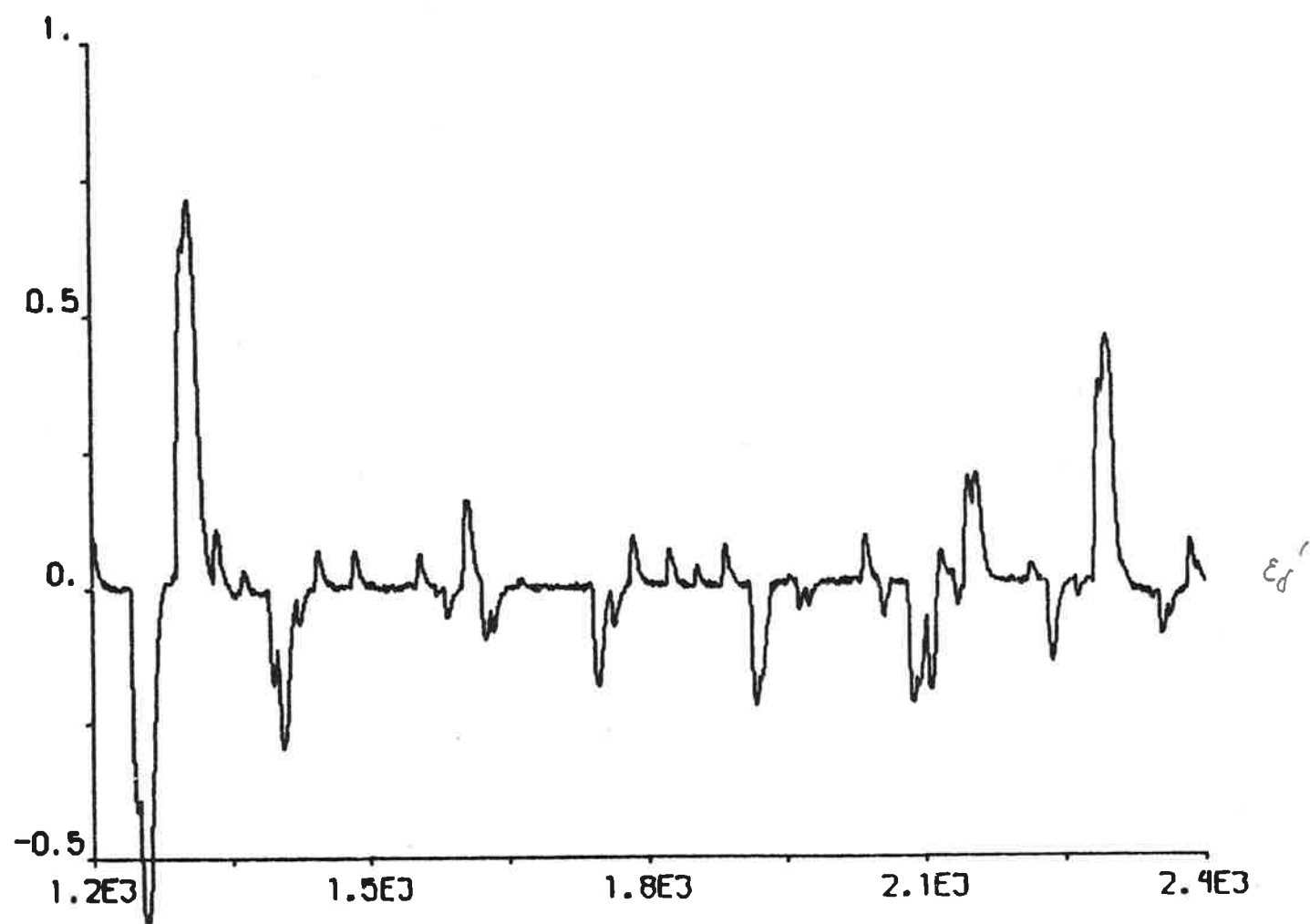
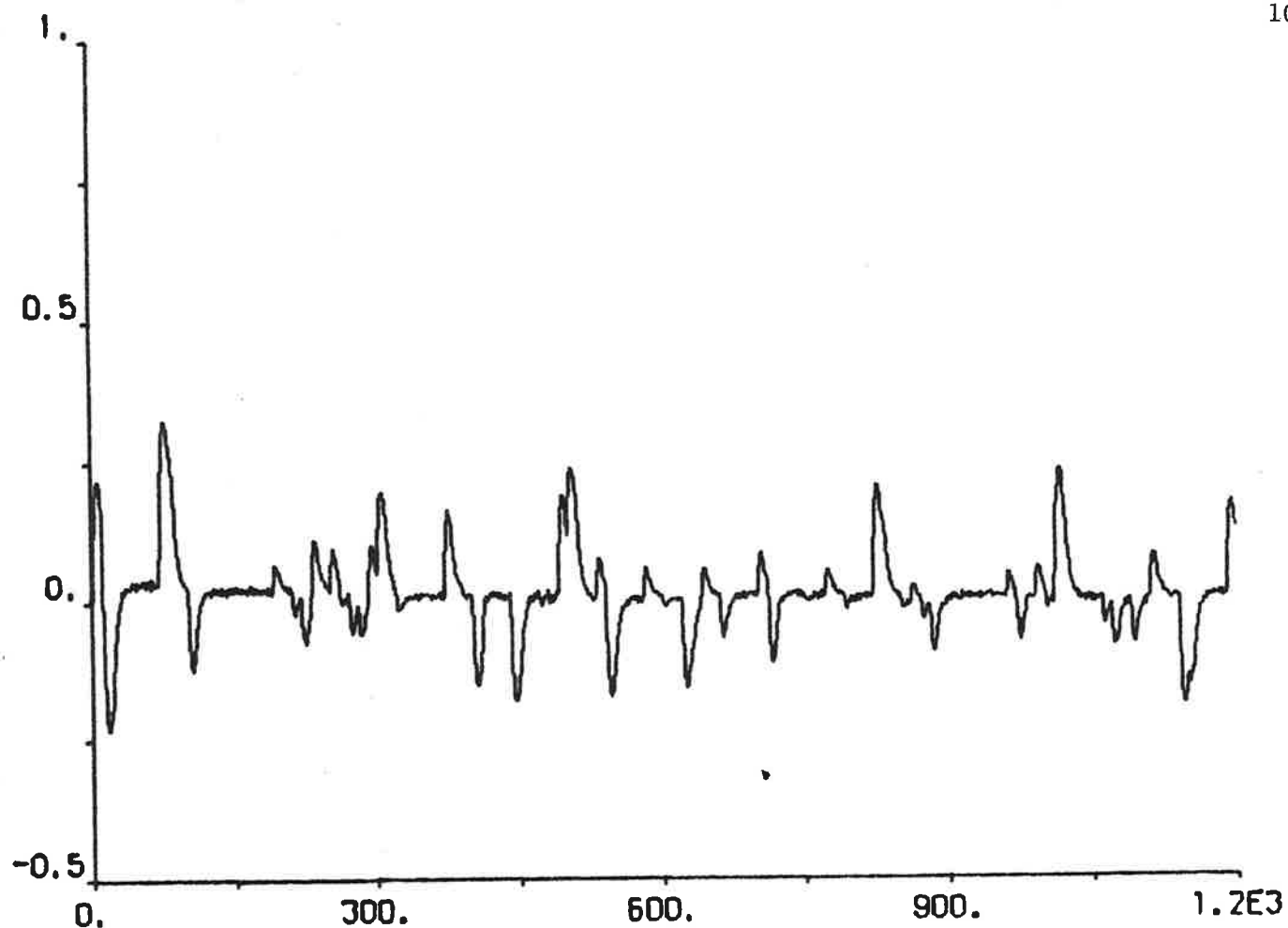


Fig. 4.6j

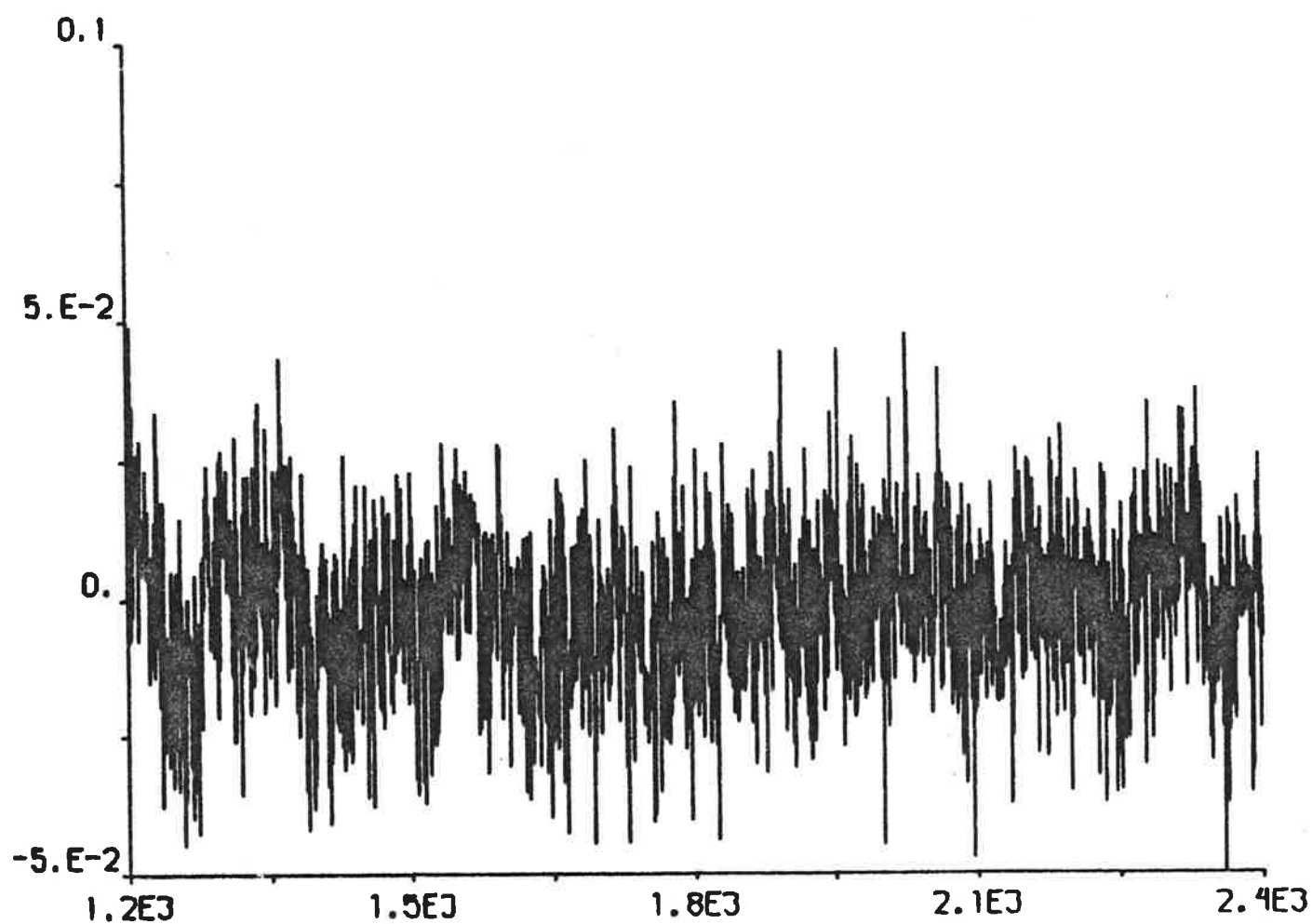
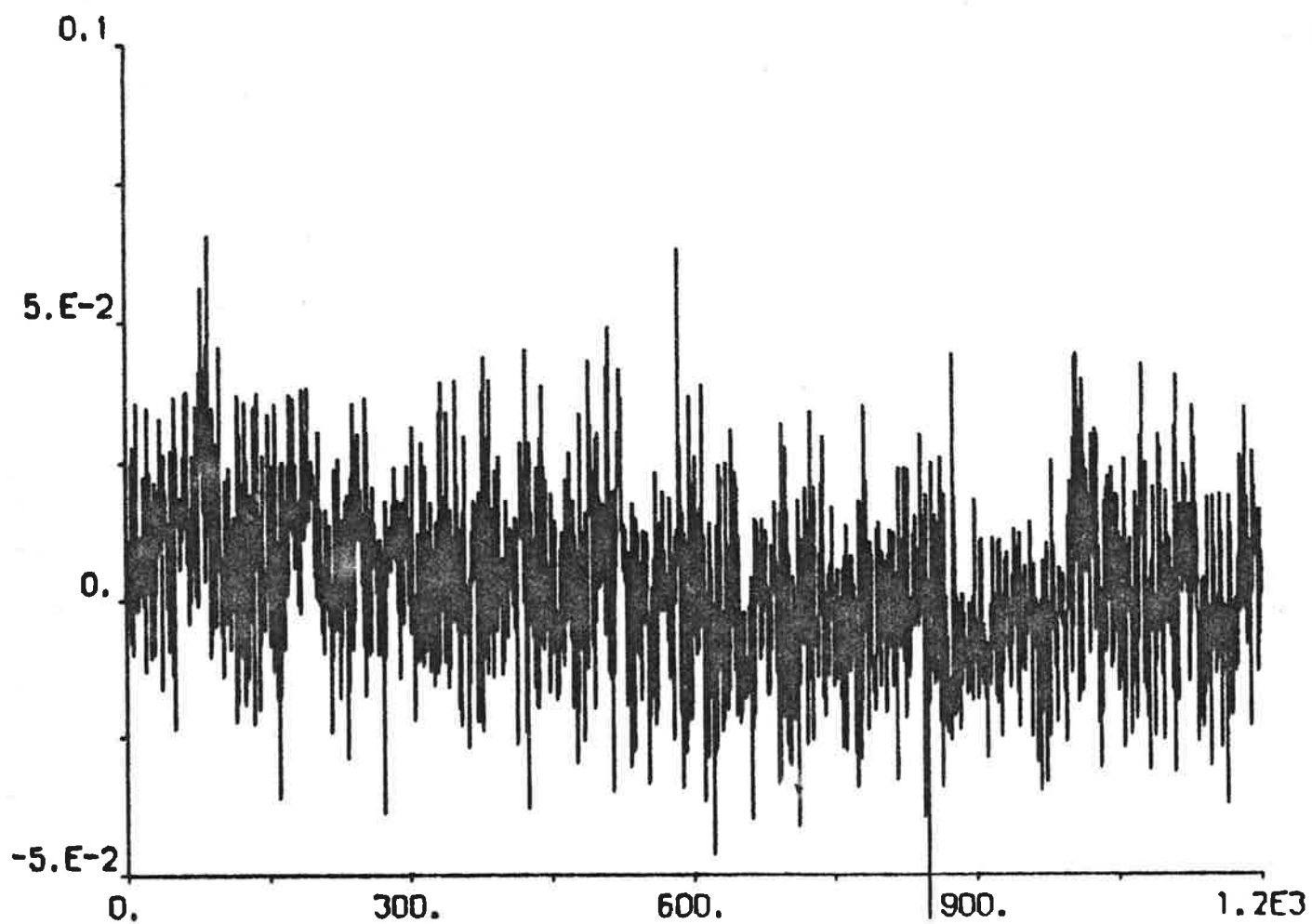


Fig. 4.6k

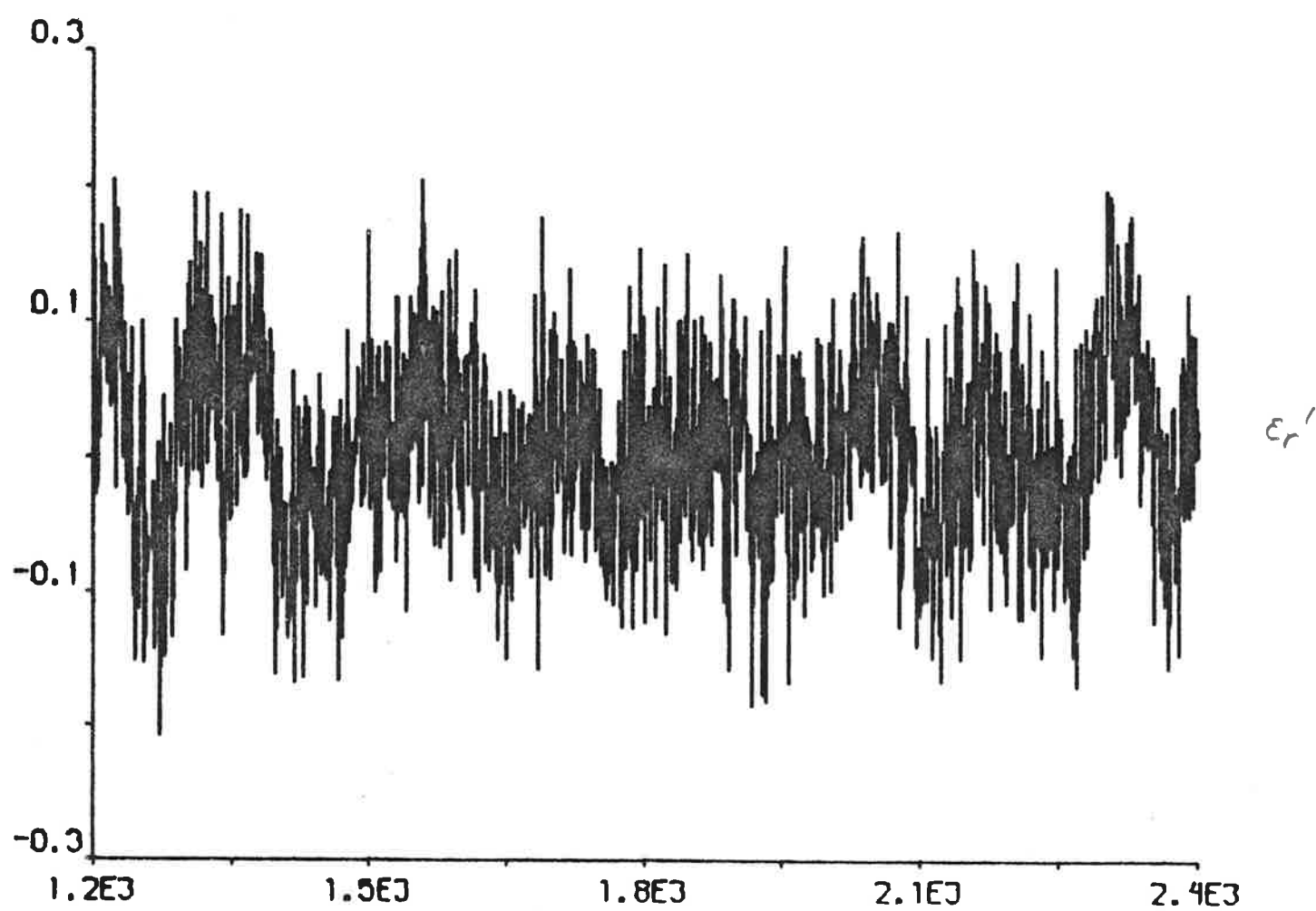
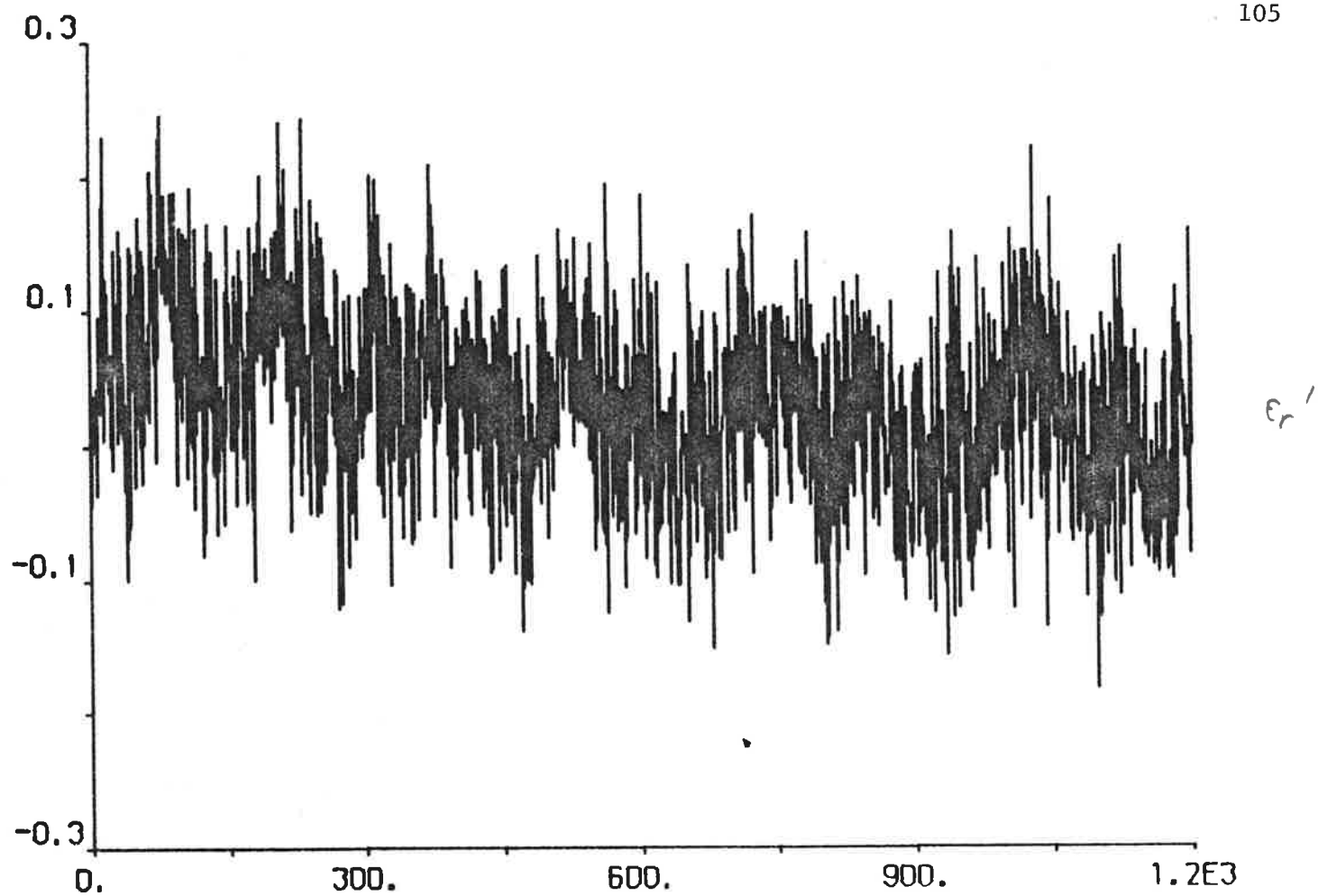


Fig. 4.6*l*

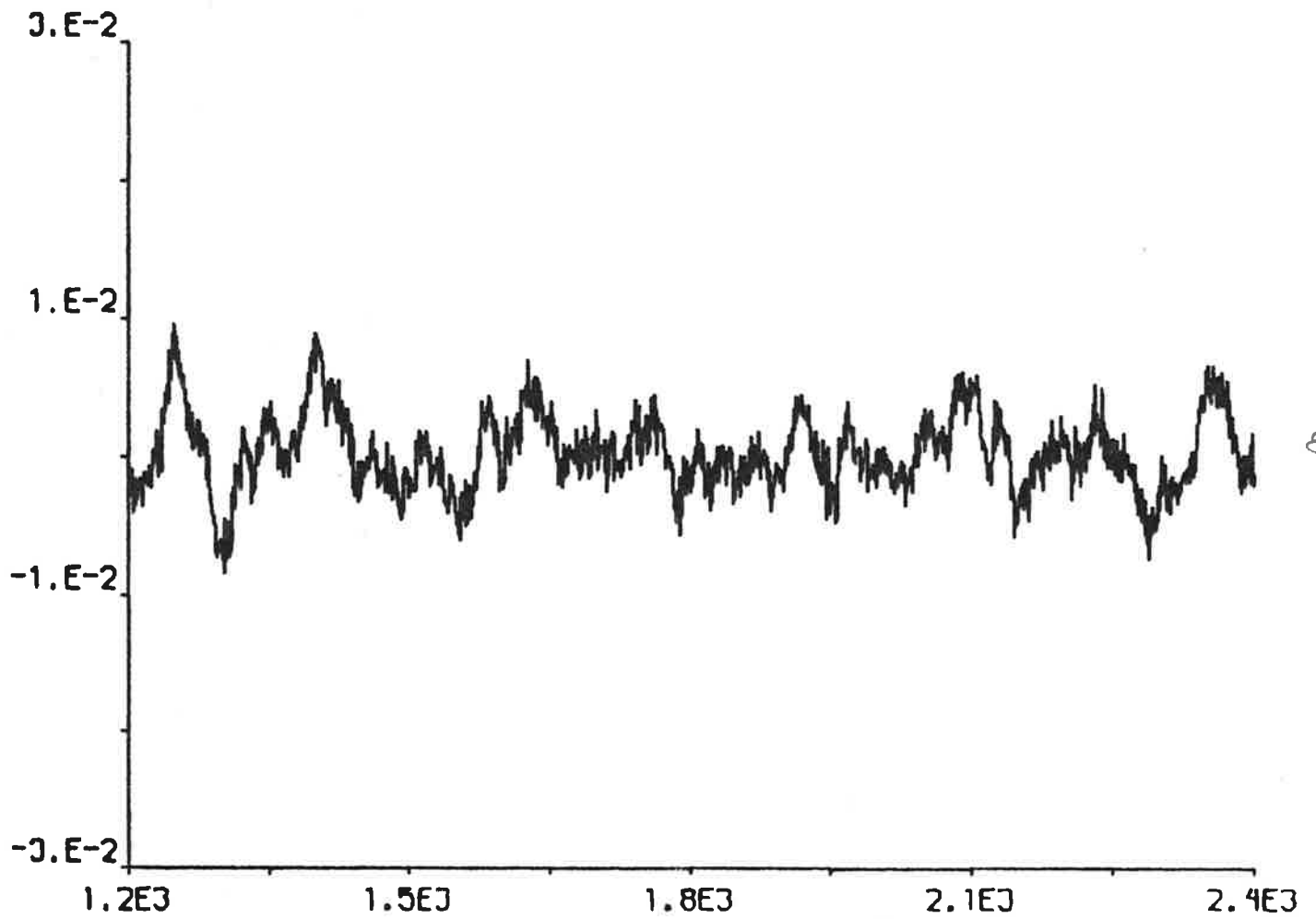
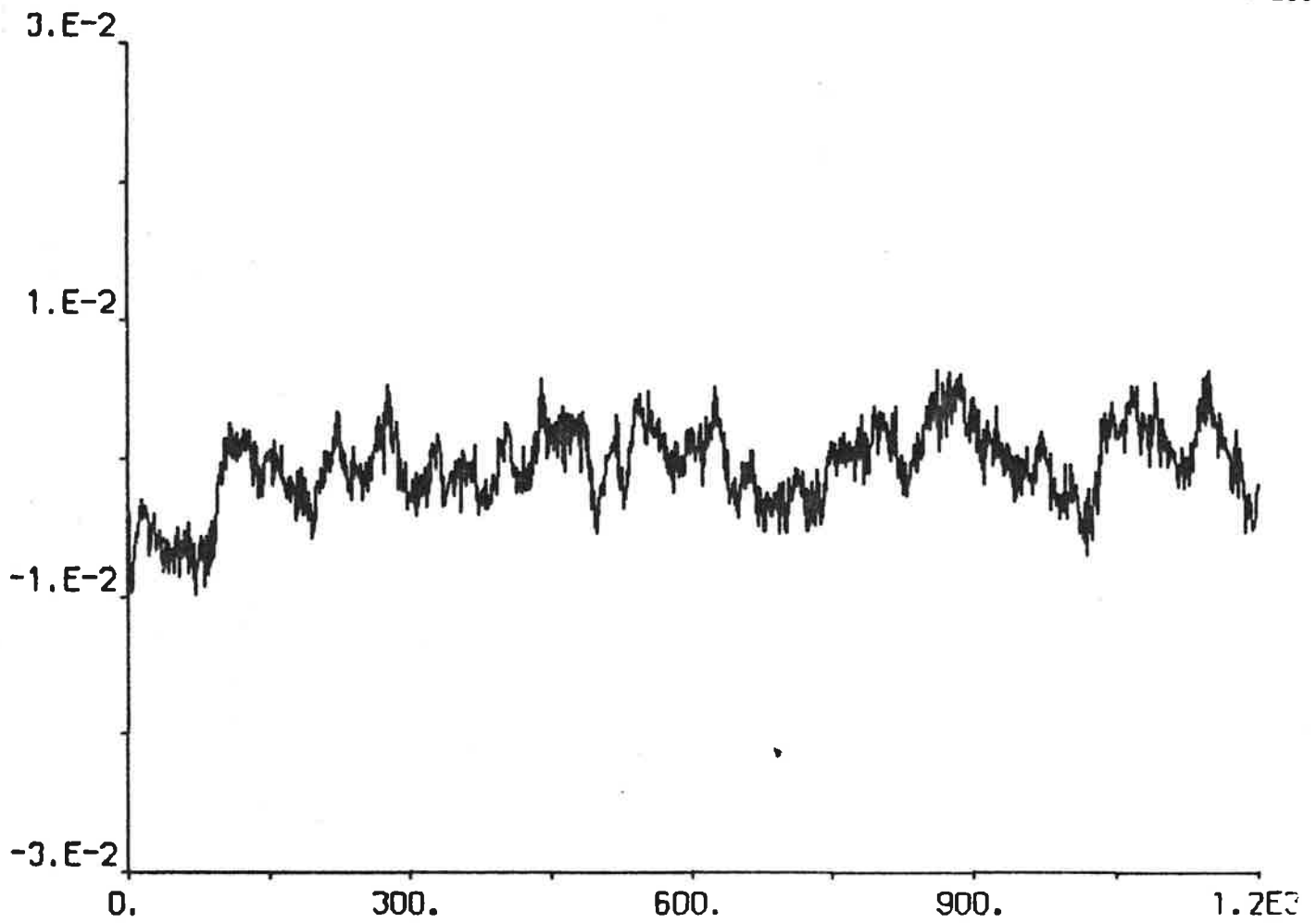


Fig. 4.6m

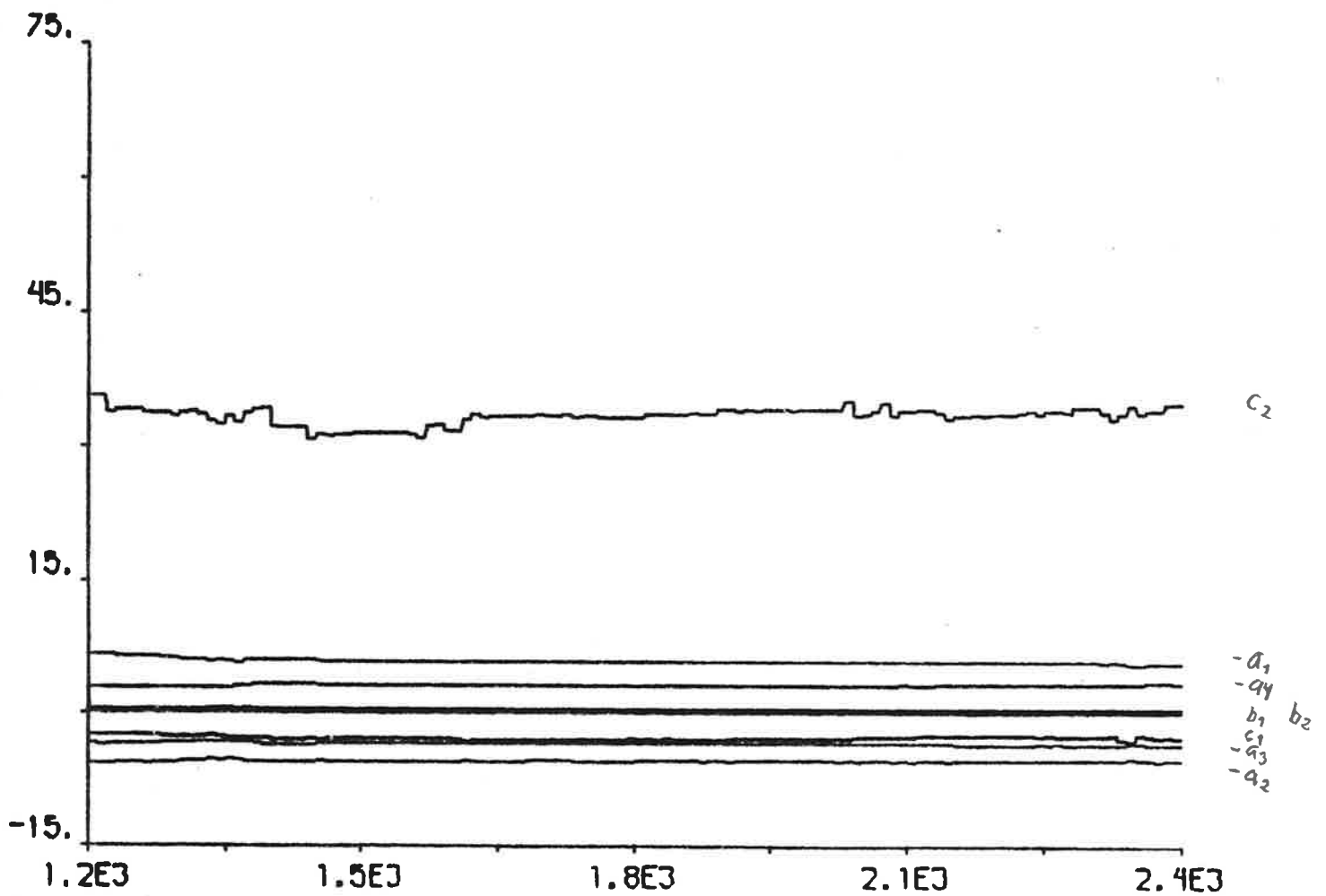
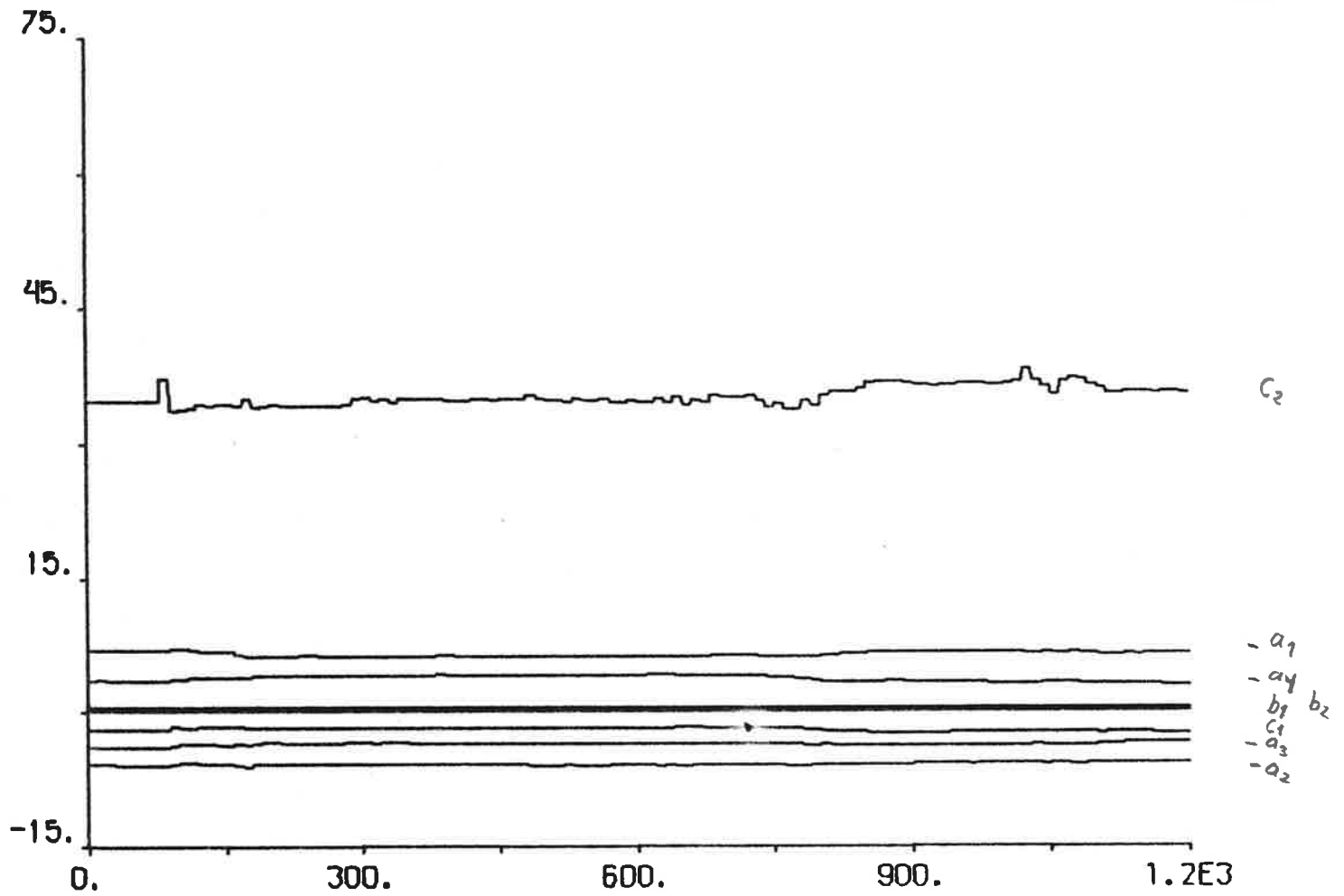


Fig. 4.6n

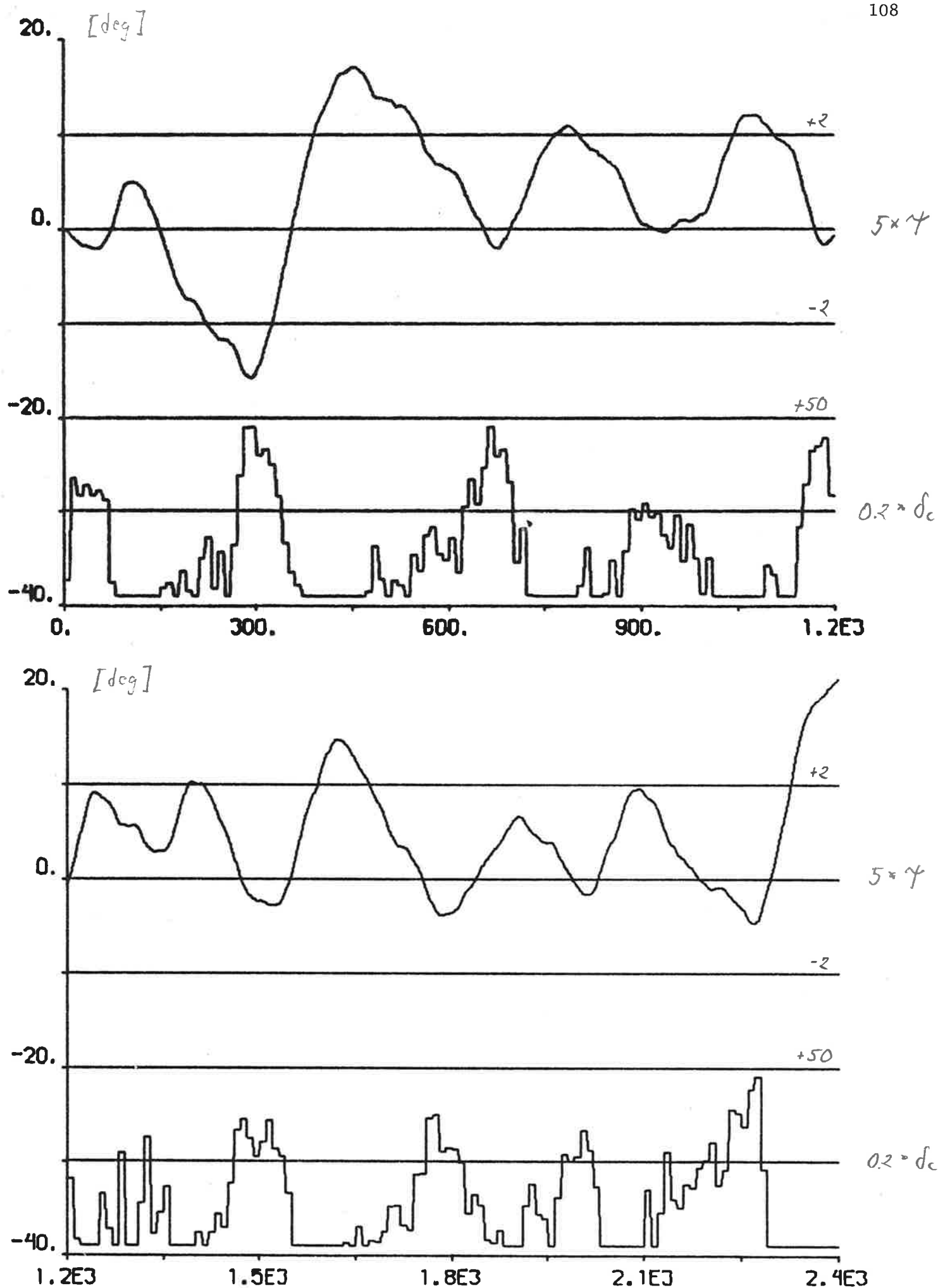


Fig. 4.7a - $T = 22.3$ m, $n_0 = 22.1772$ rpm, $u_0 = 4$ knots, $\delta_0 = 45$ deg, self-tuning regulator using estimates from the Kalman filter. The initial covariance matrix P of the self-tuning regulator is given by (4.3).

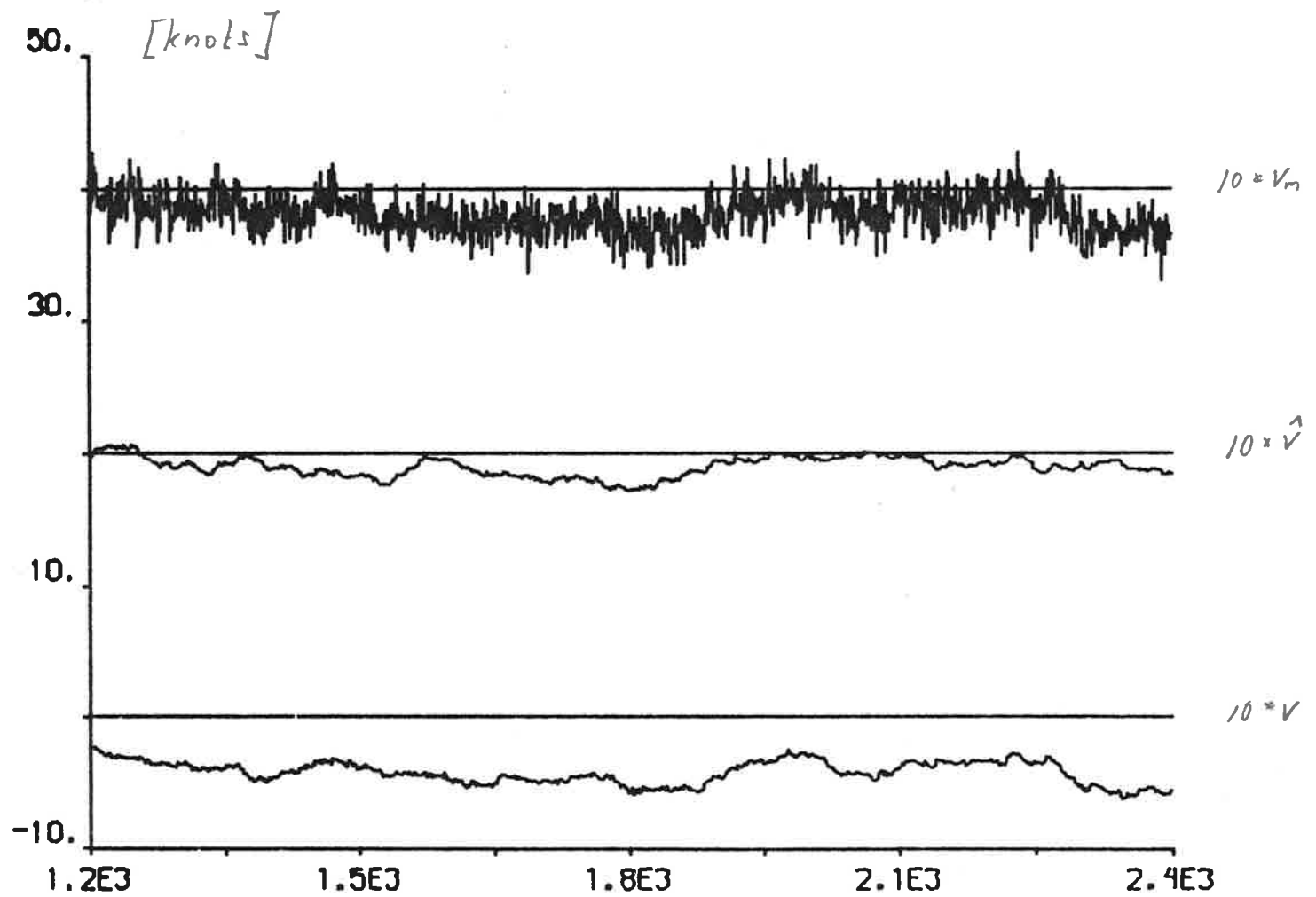
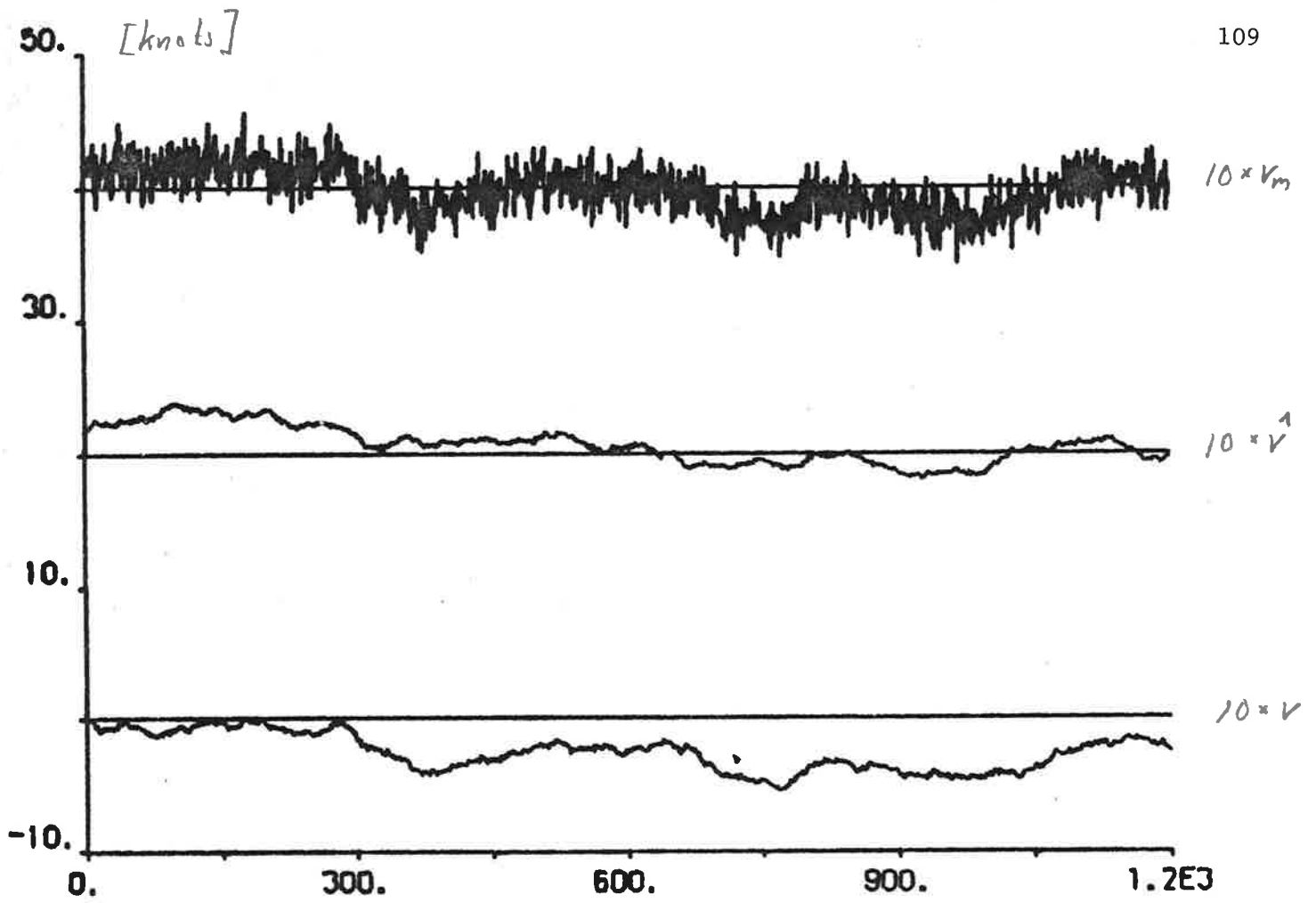


Fig. 4.7b

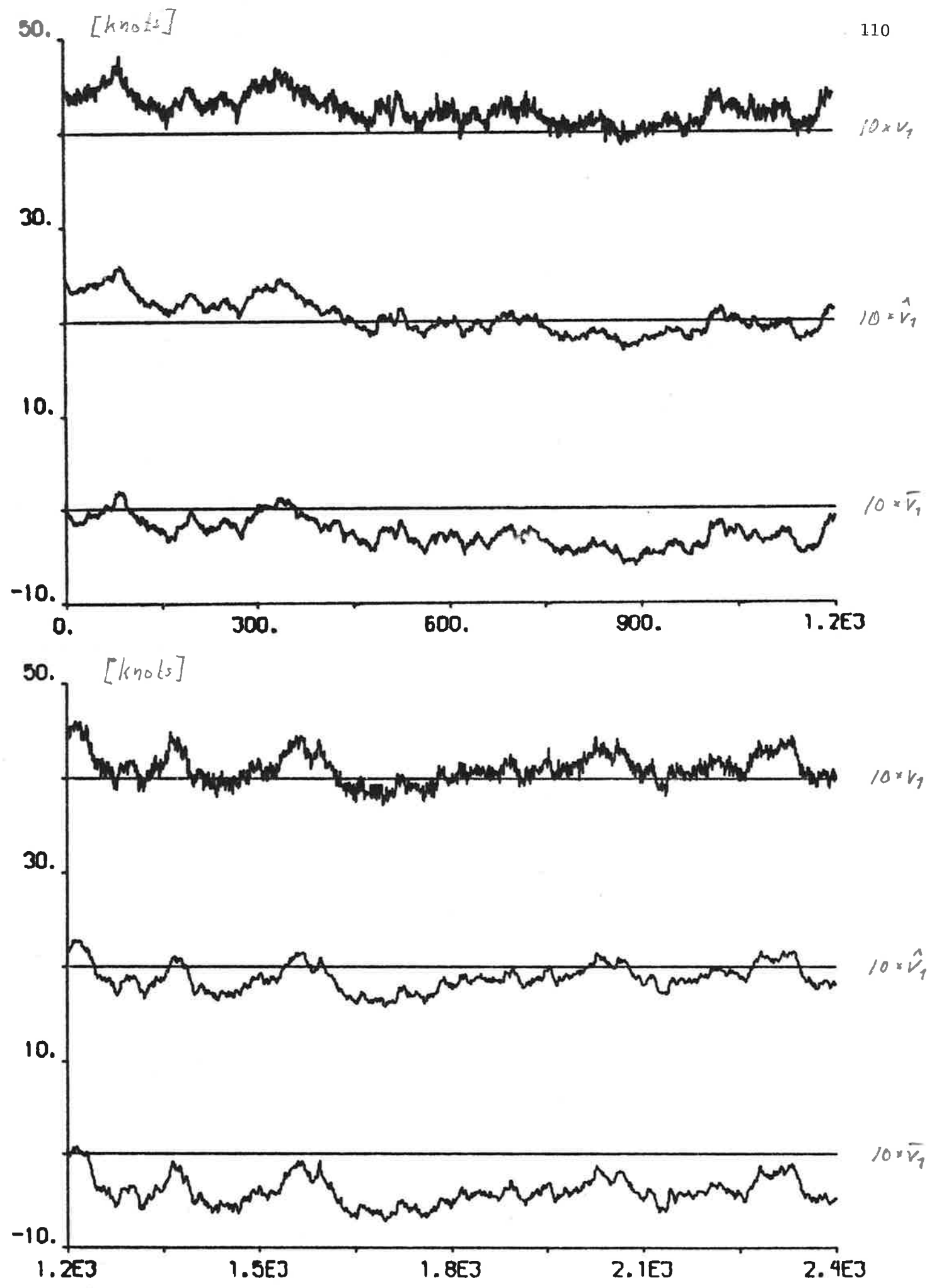


Fig. 4.7c

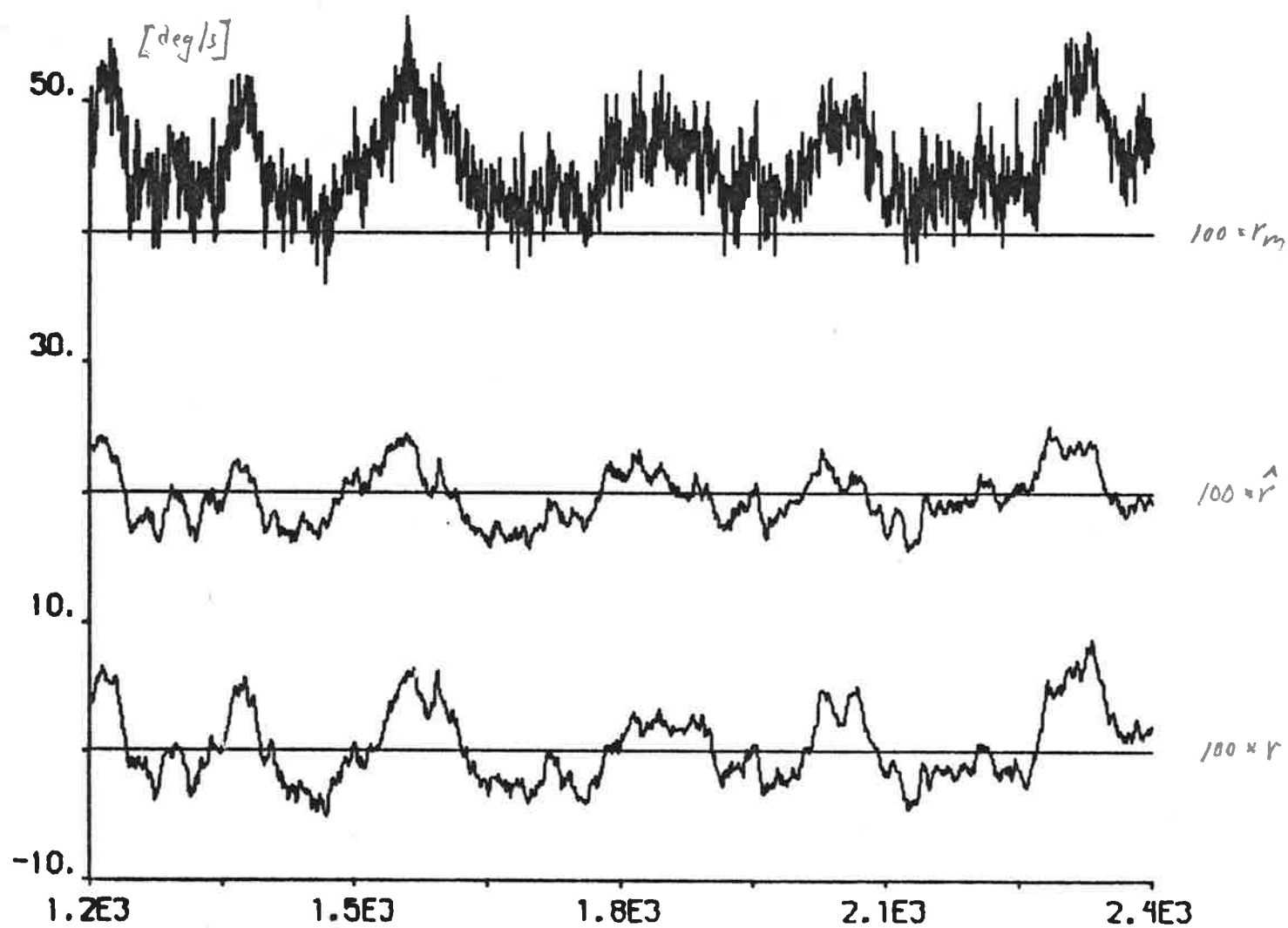
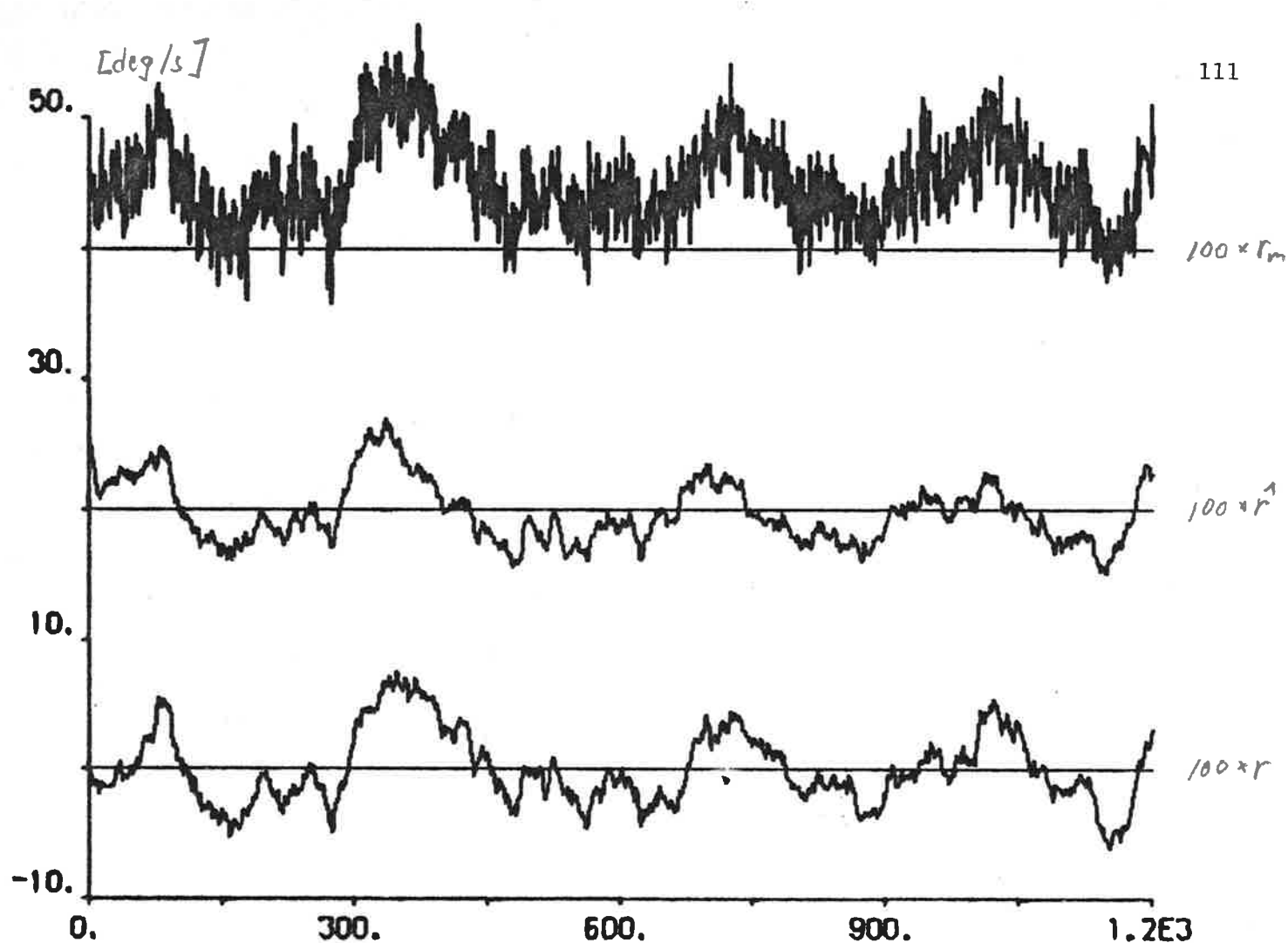


Fig. 4.7d

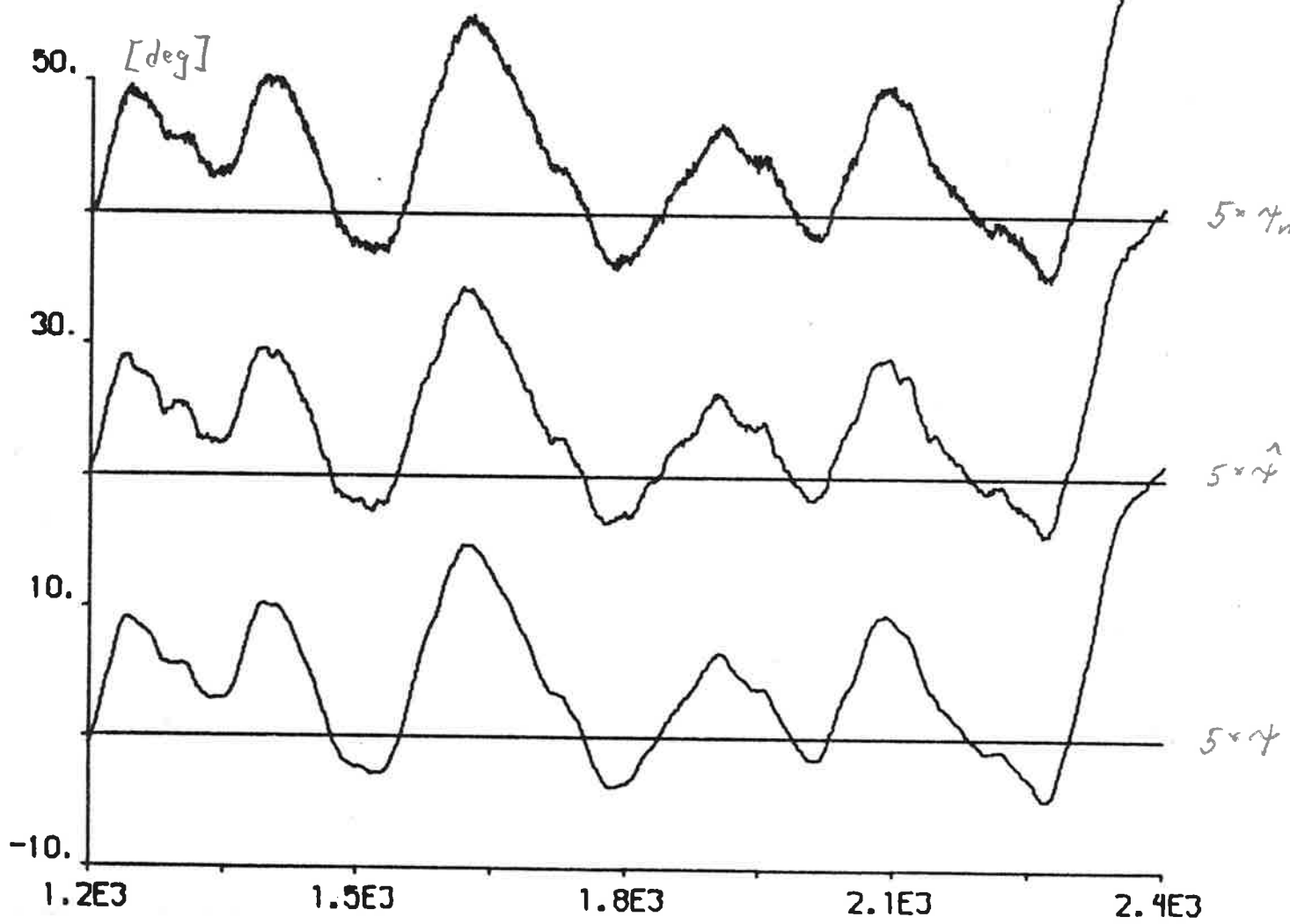
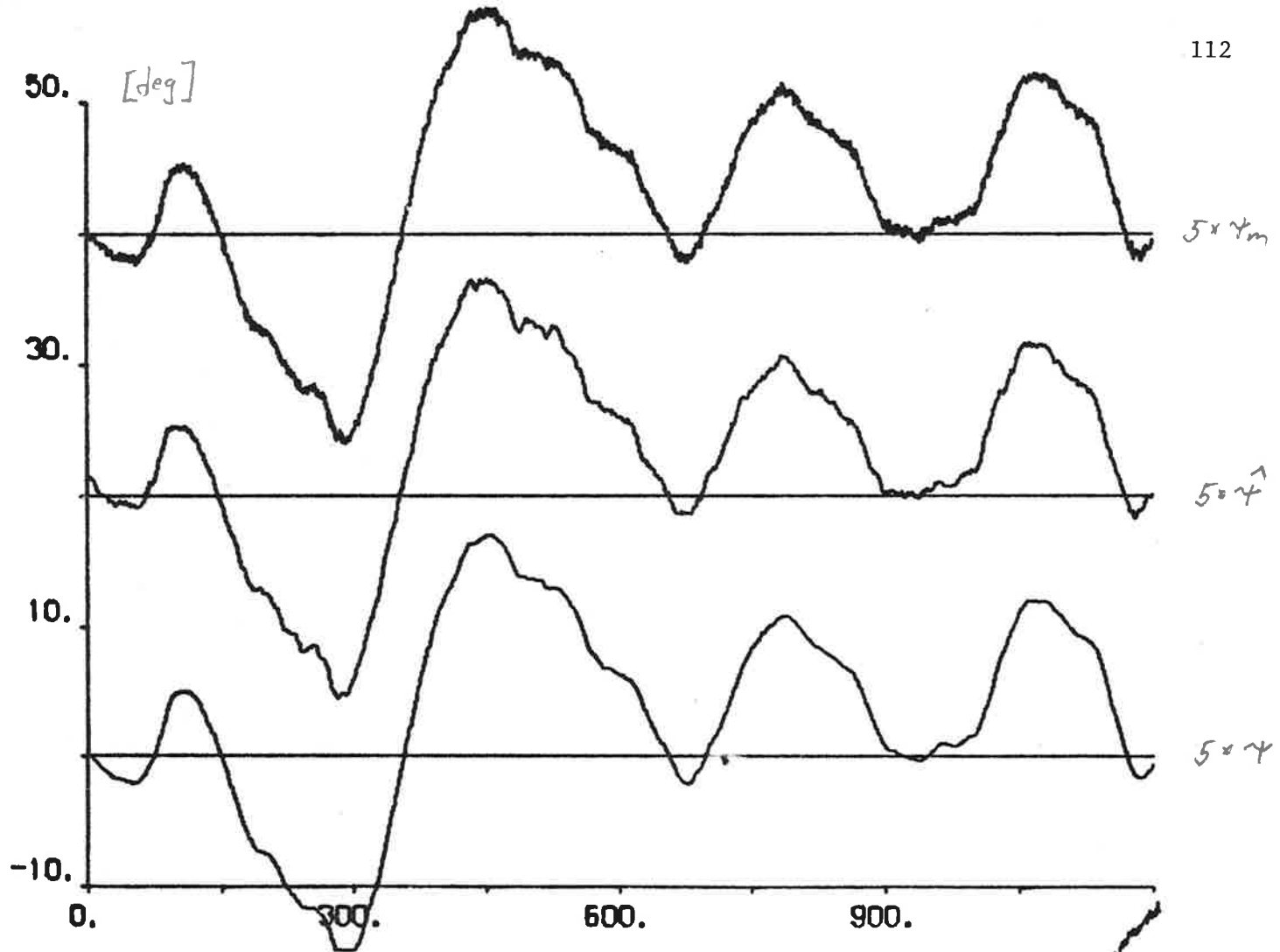


Fig. 4.7e

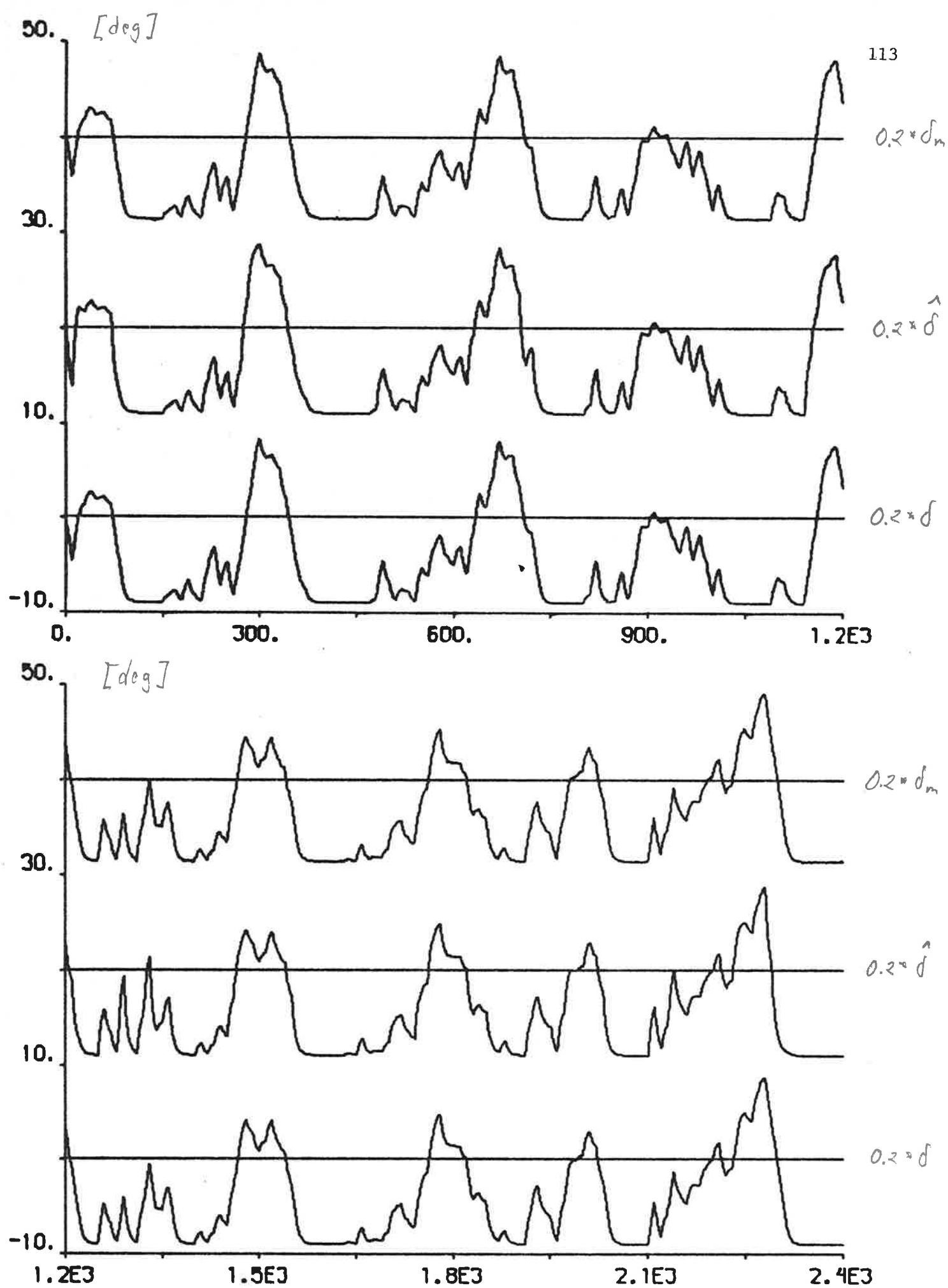


Fig. 4.7f

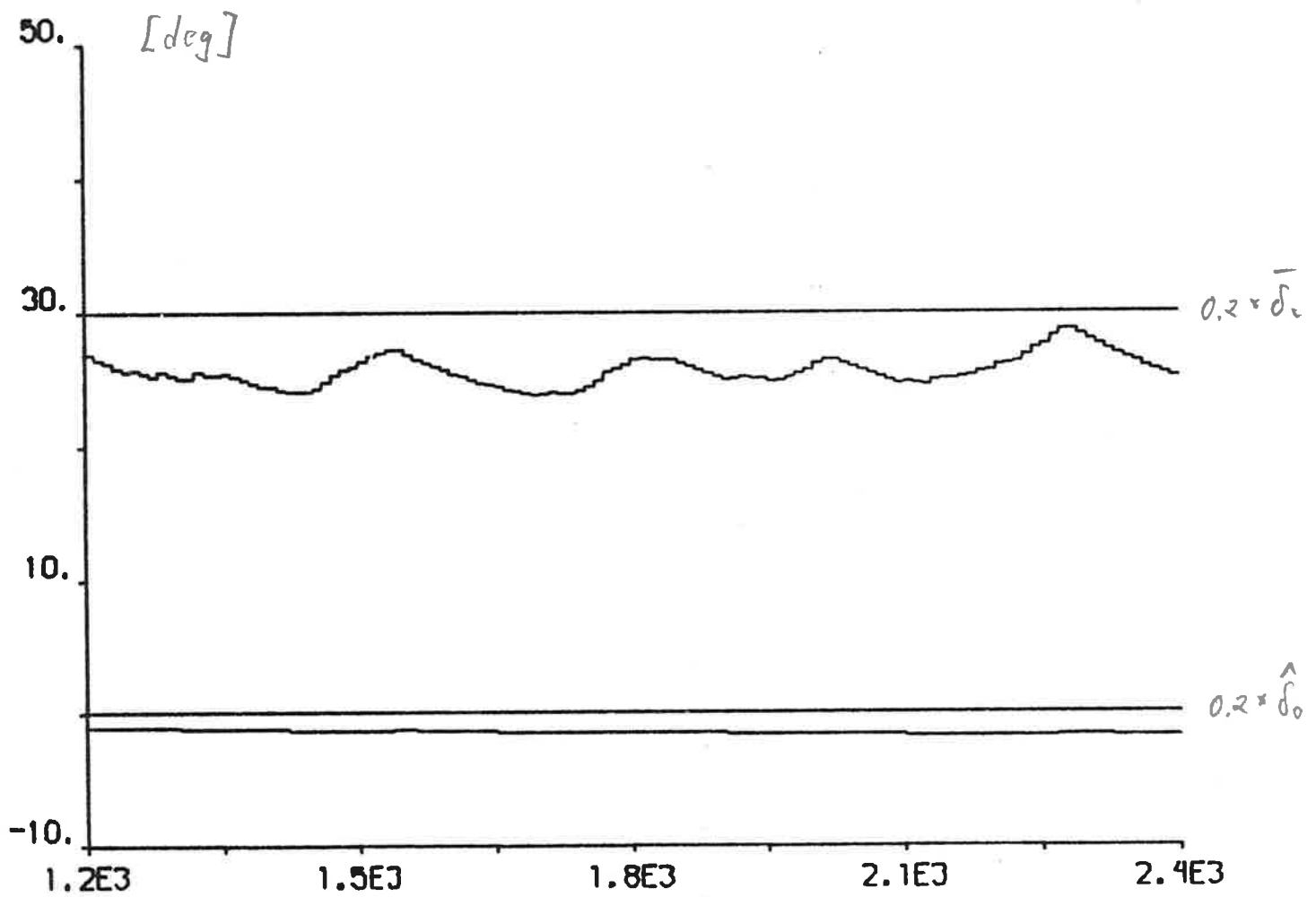
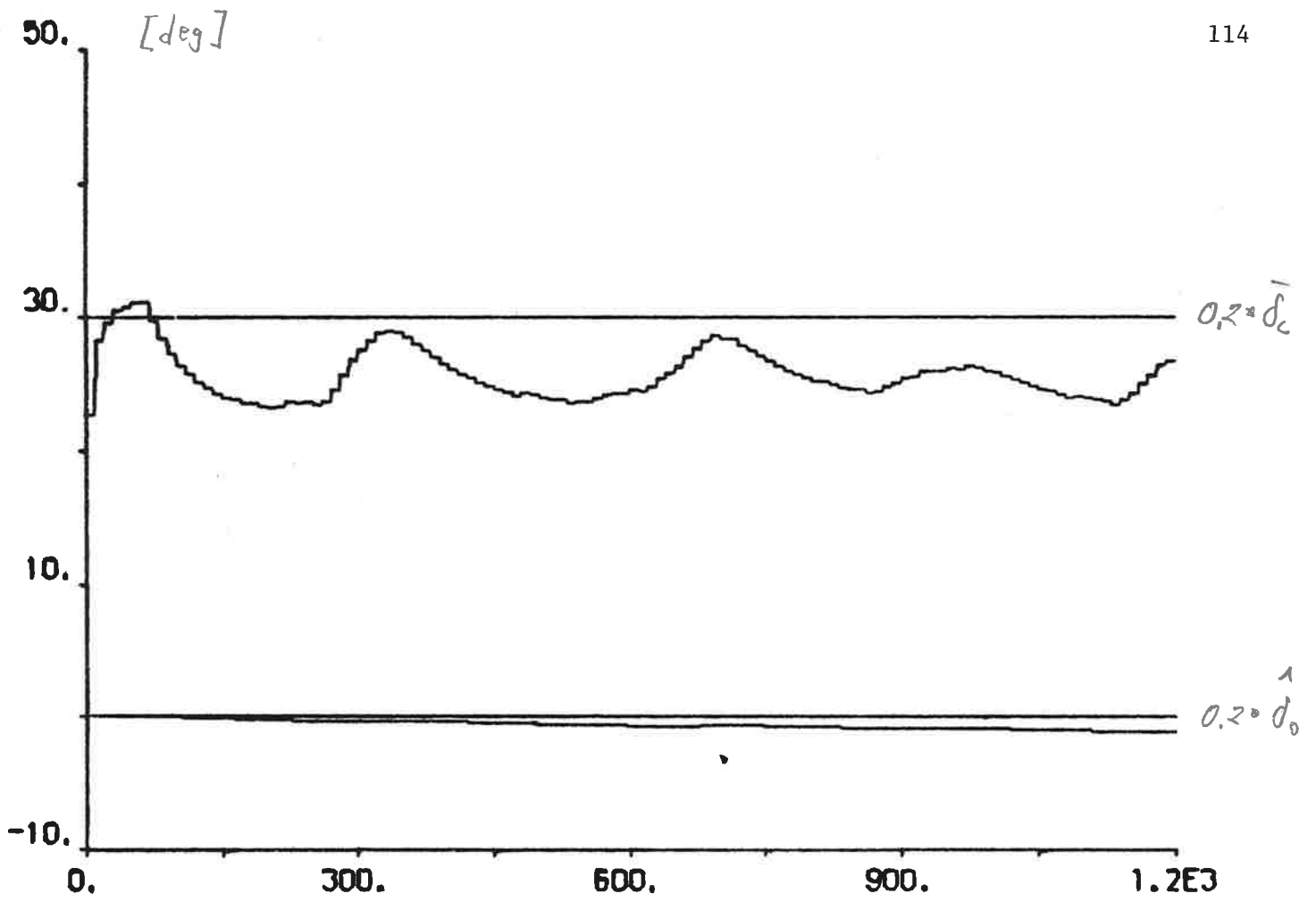


Fig. 4.7g

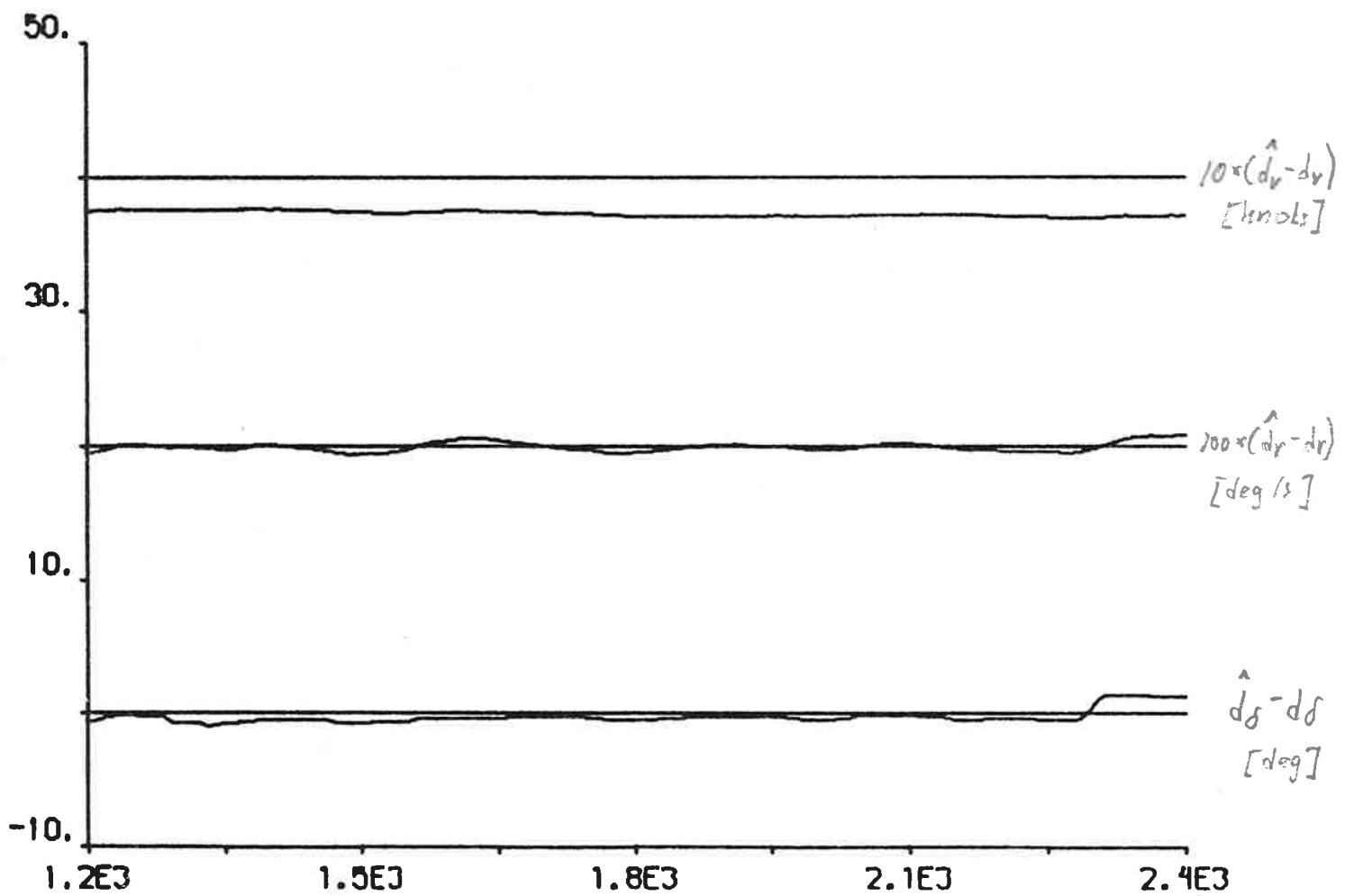
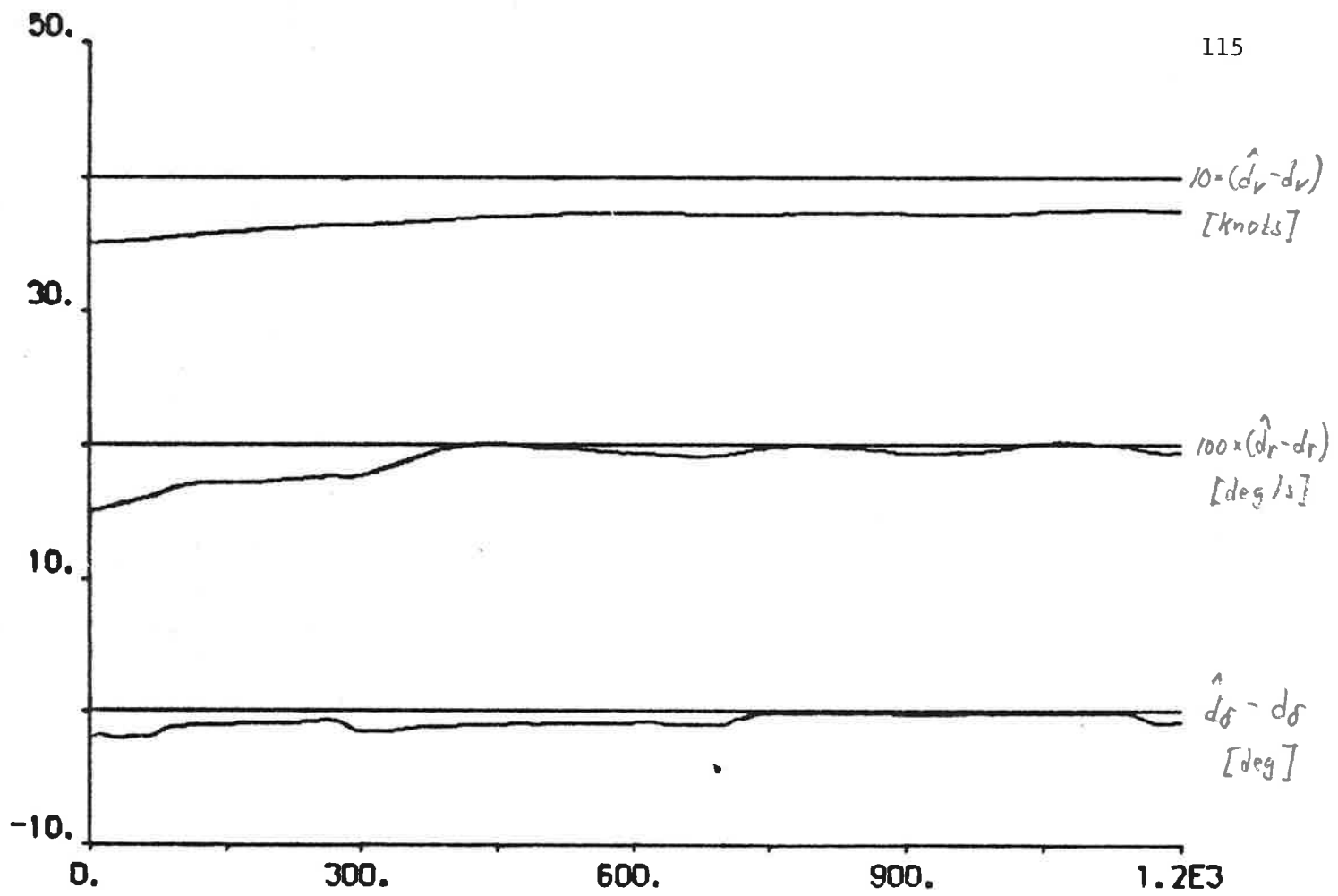


Fig. 4.7h

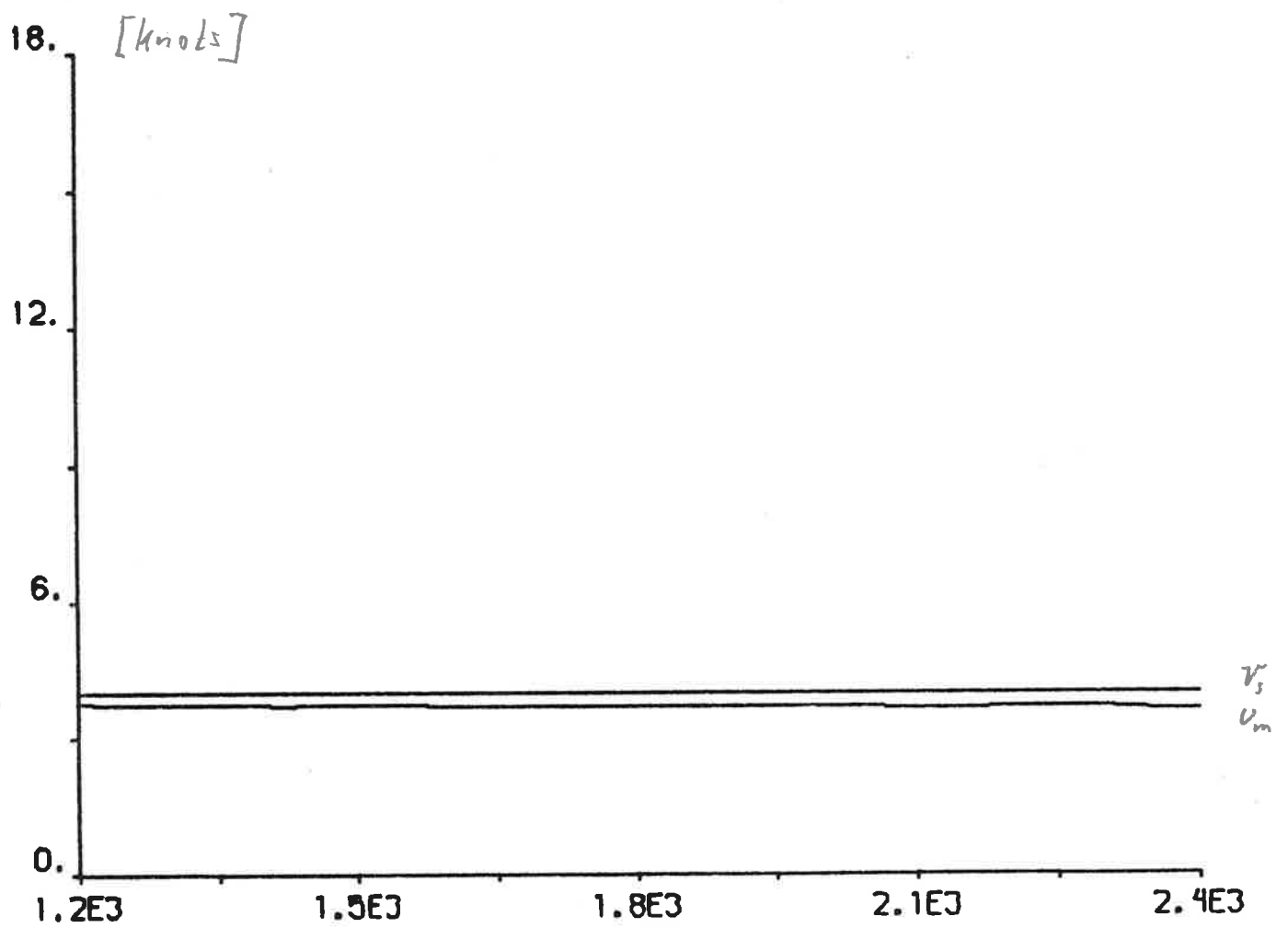
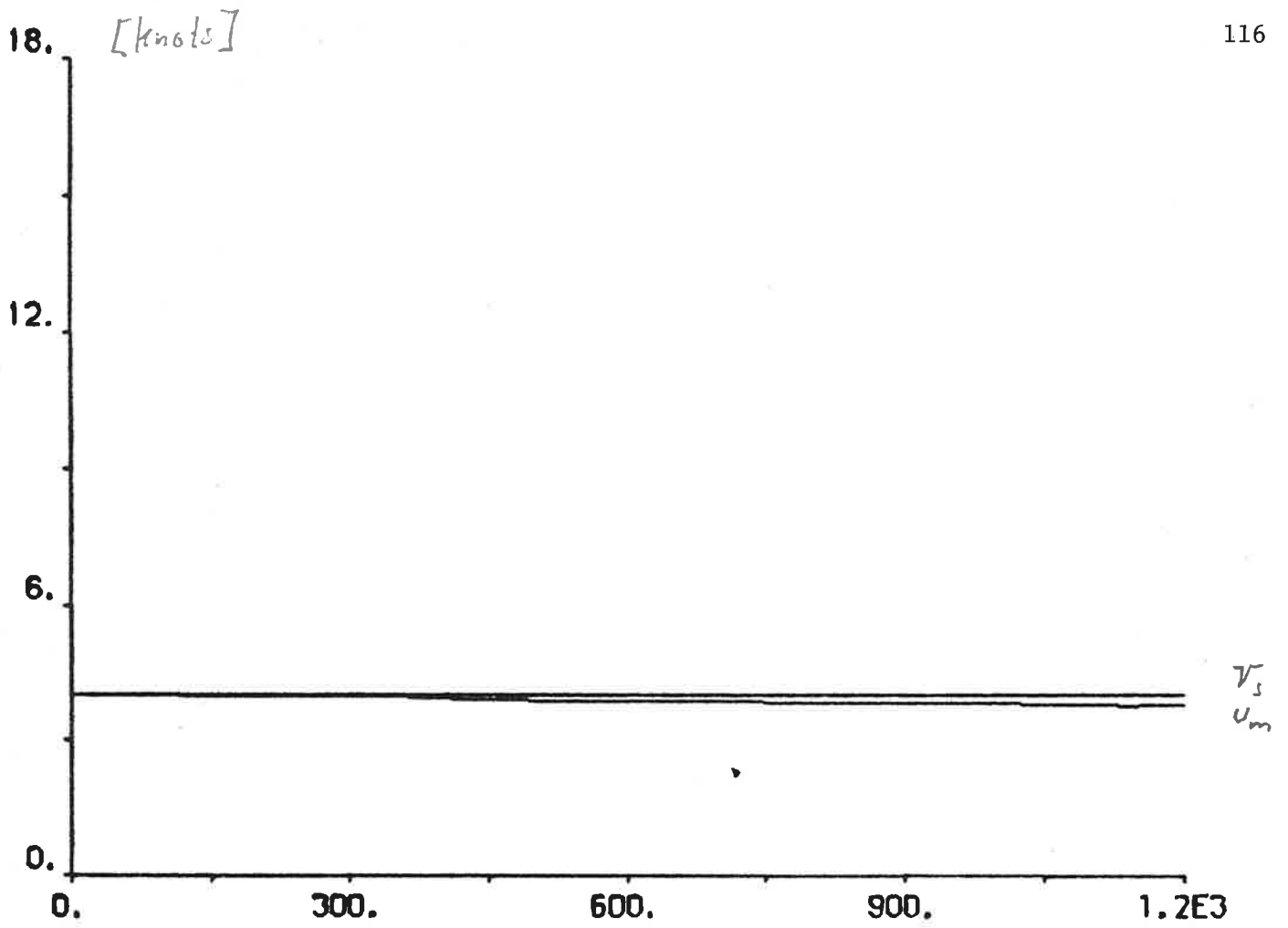


Fig. 4.7i

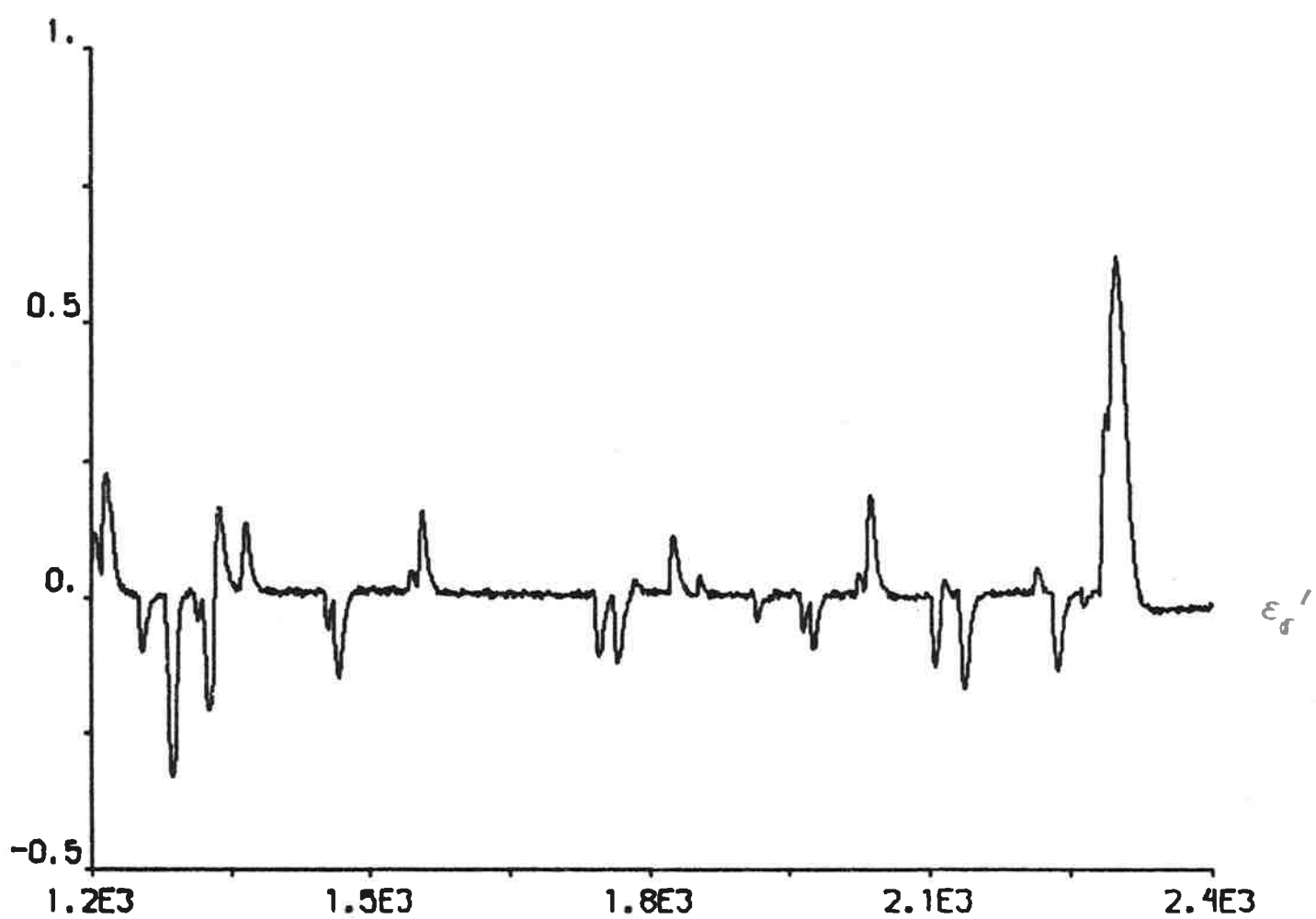
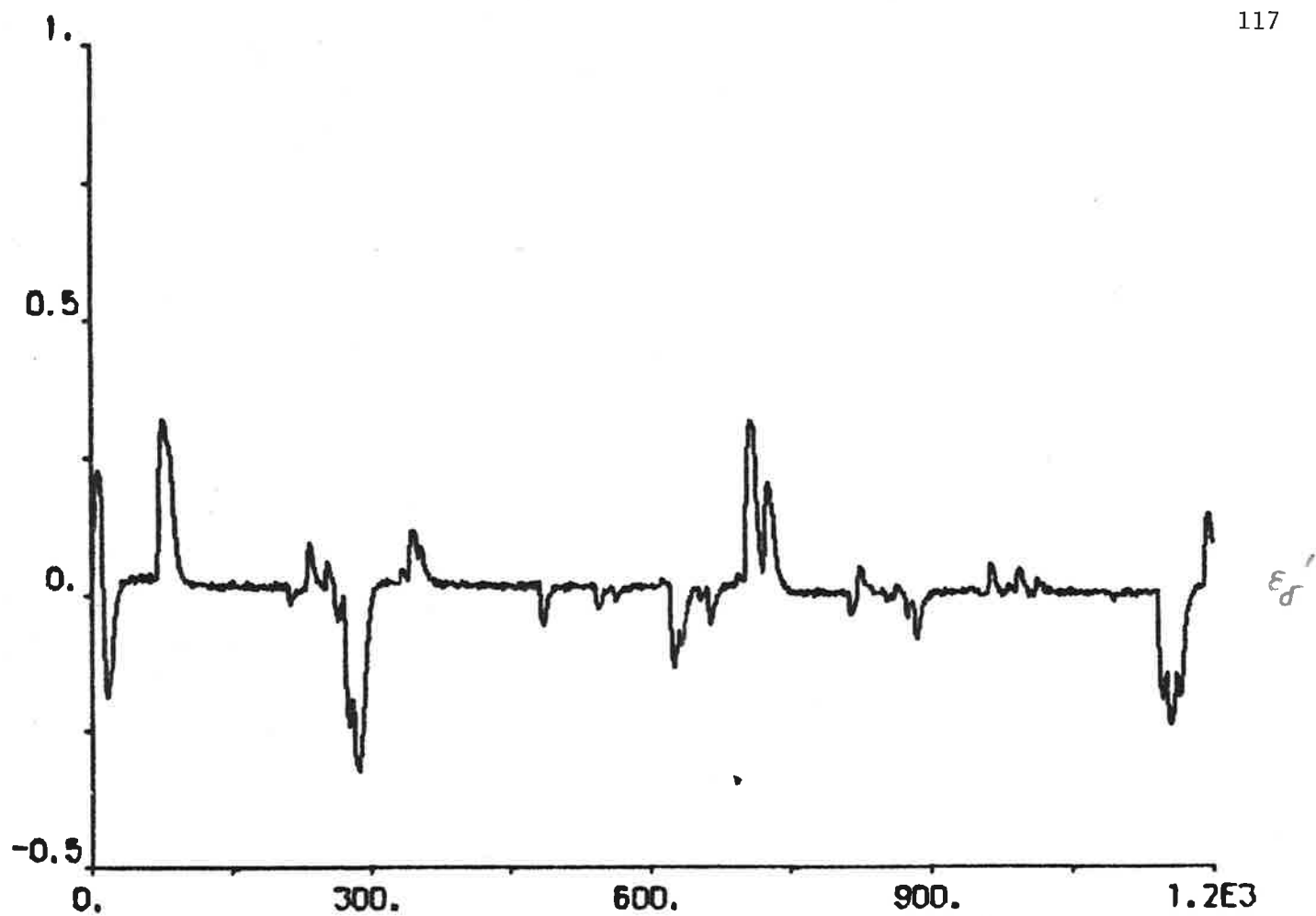


Fig. 4.7j

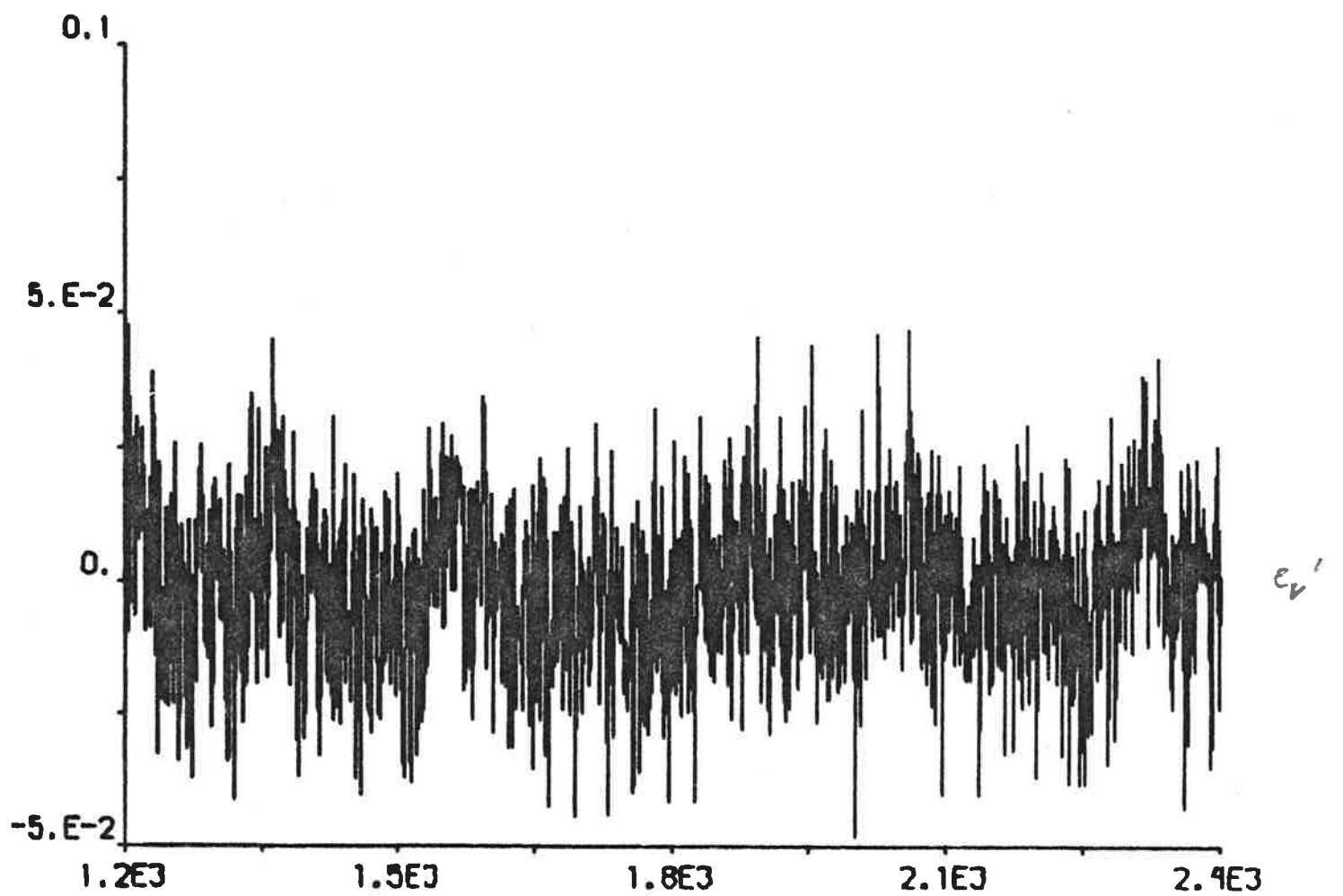
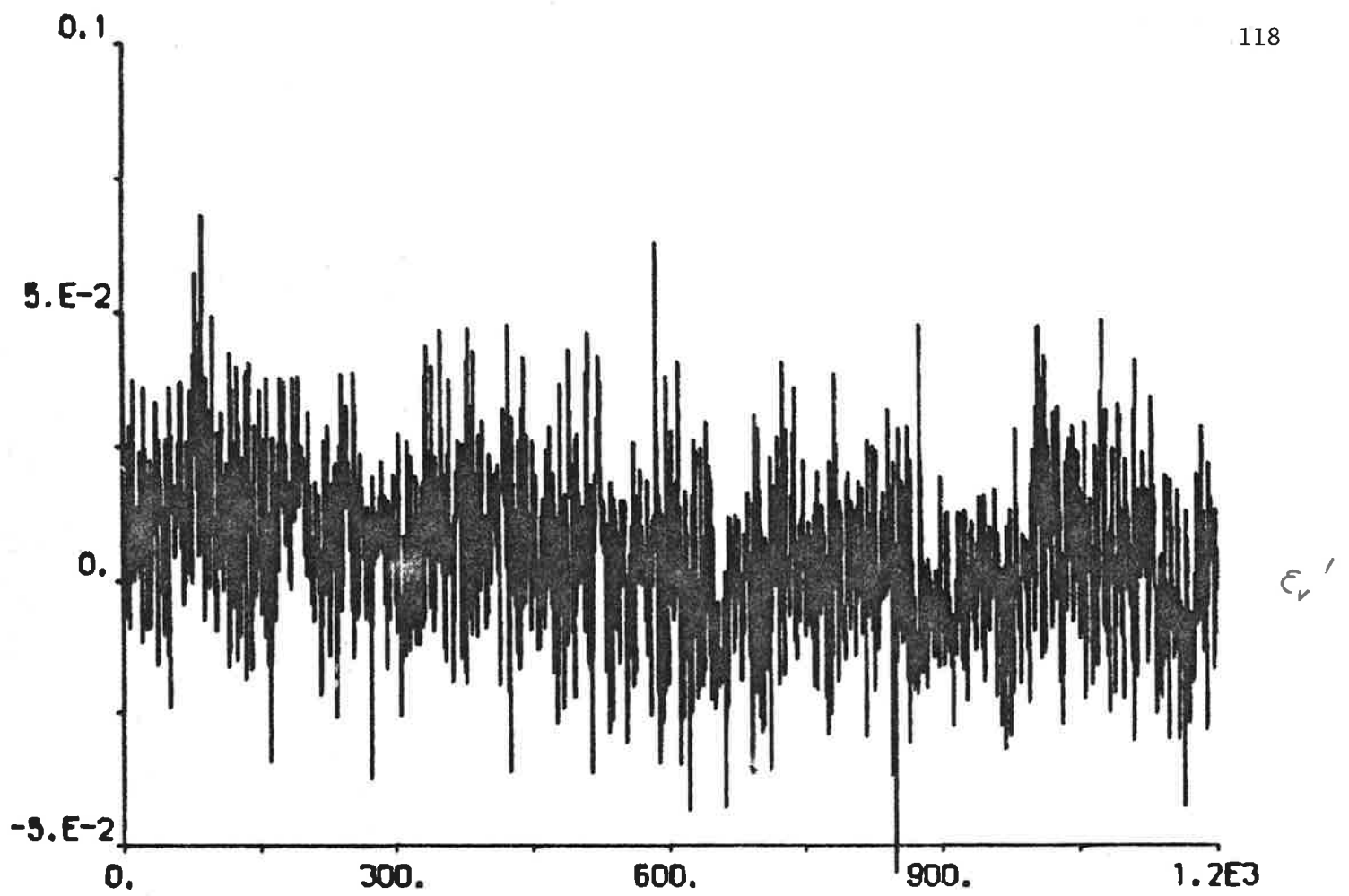


Fig. 4.7k

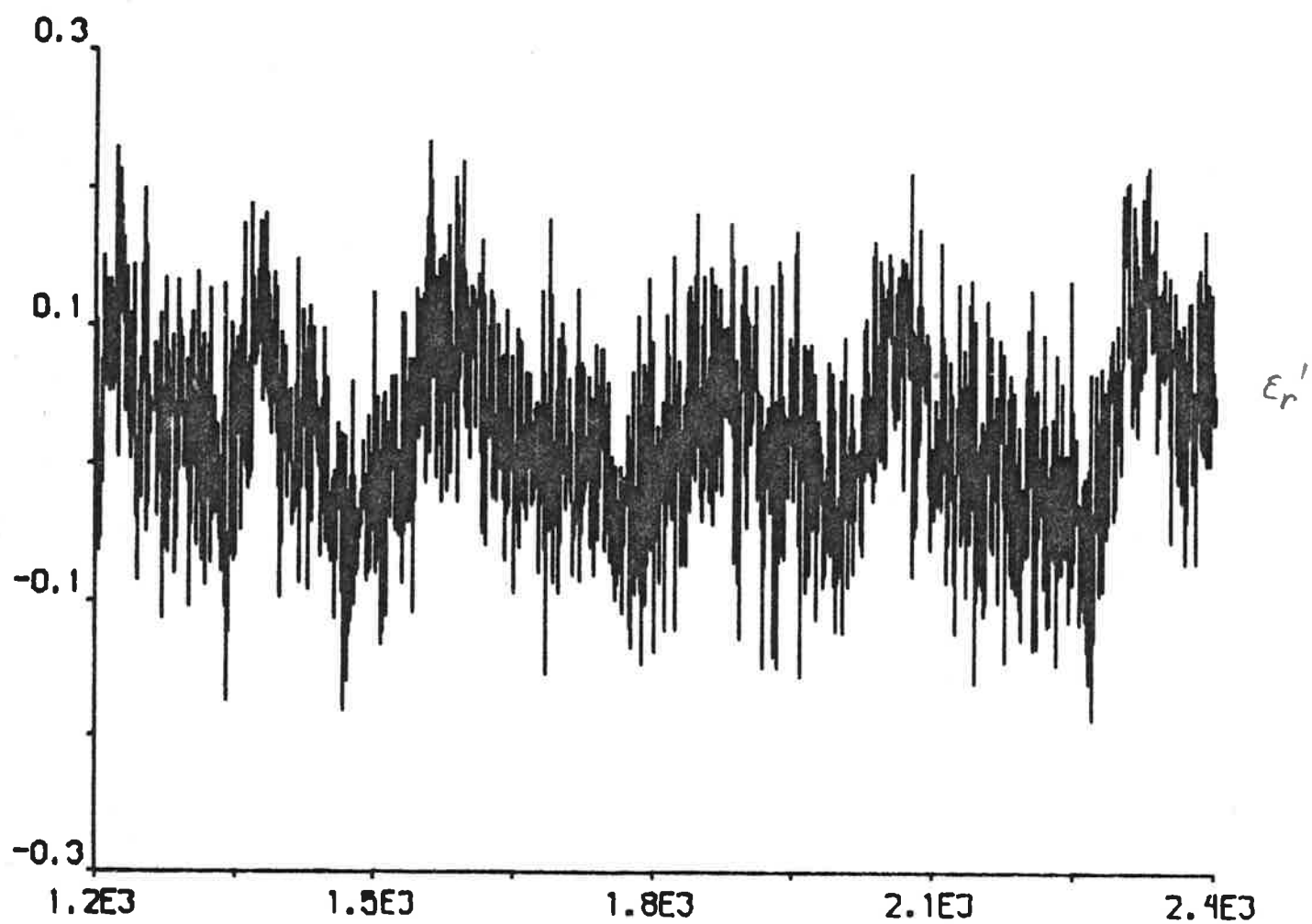
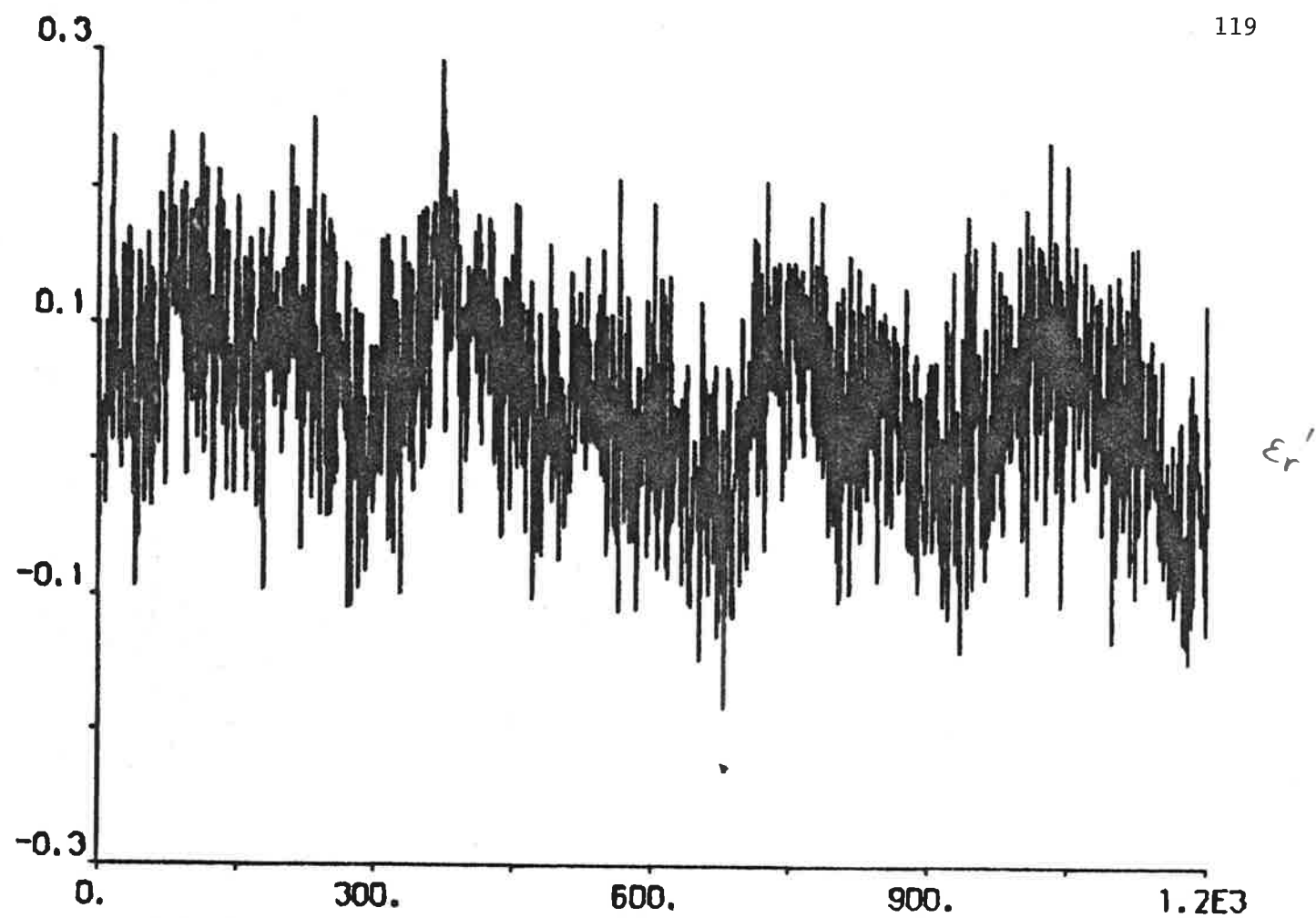


Fig. 4.7 ℓ

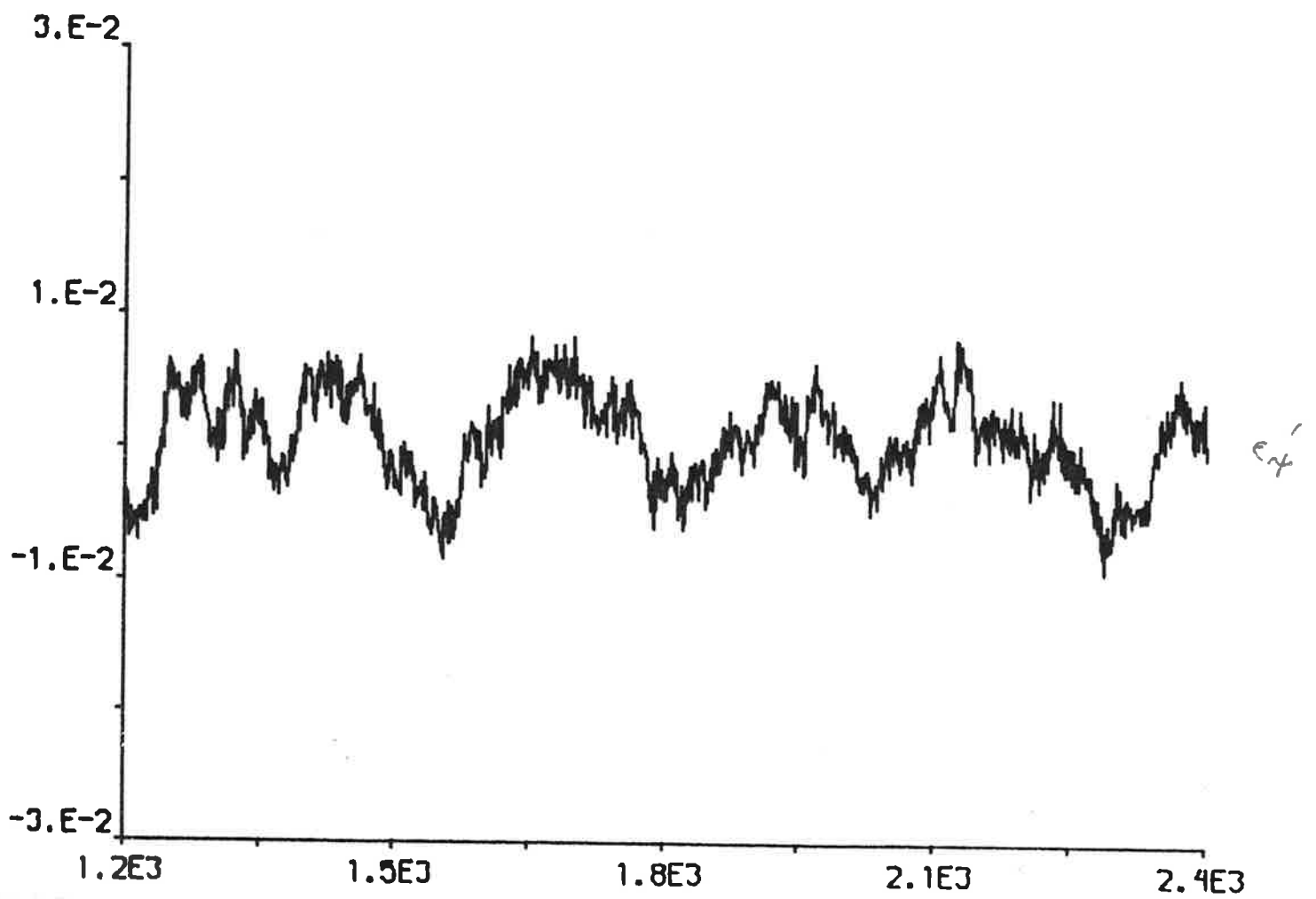
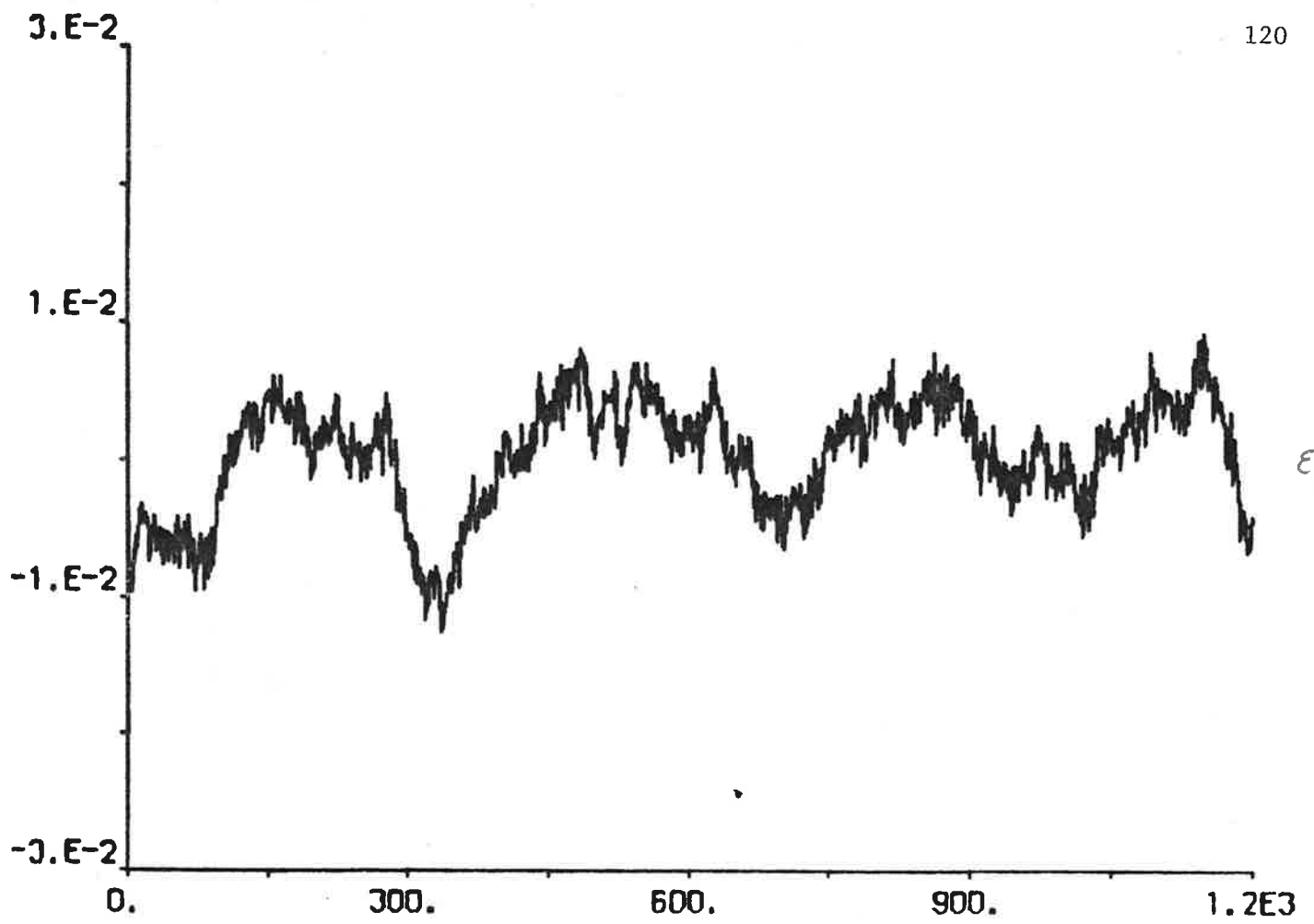


Fig. 4.7m

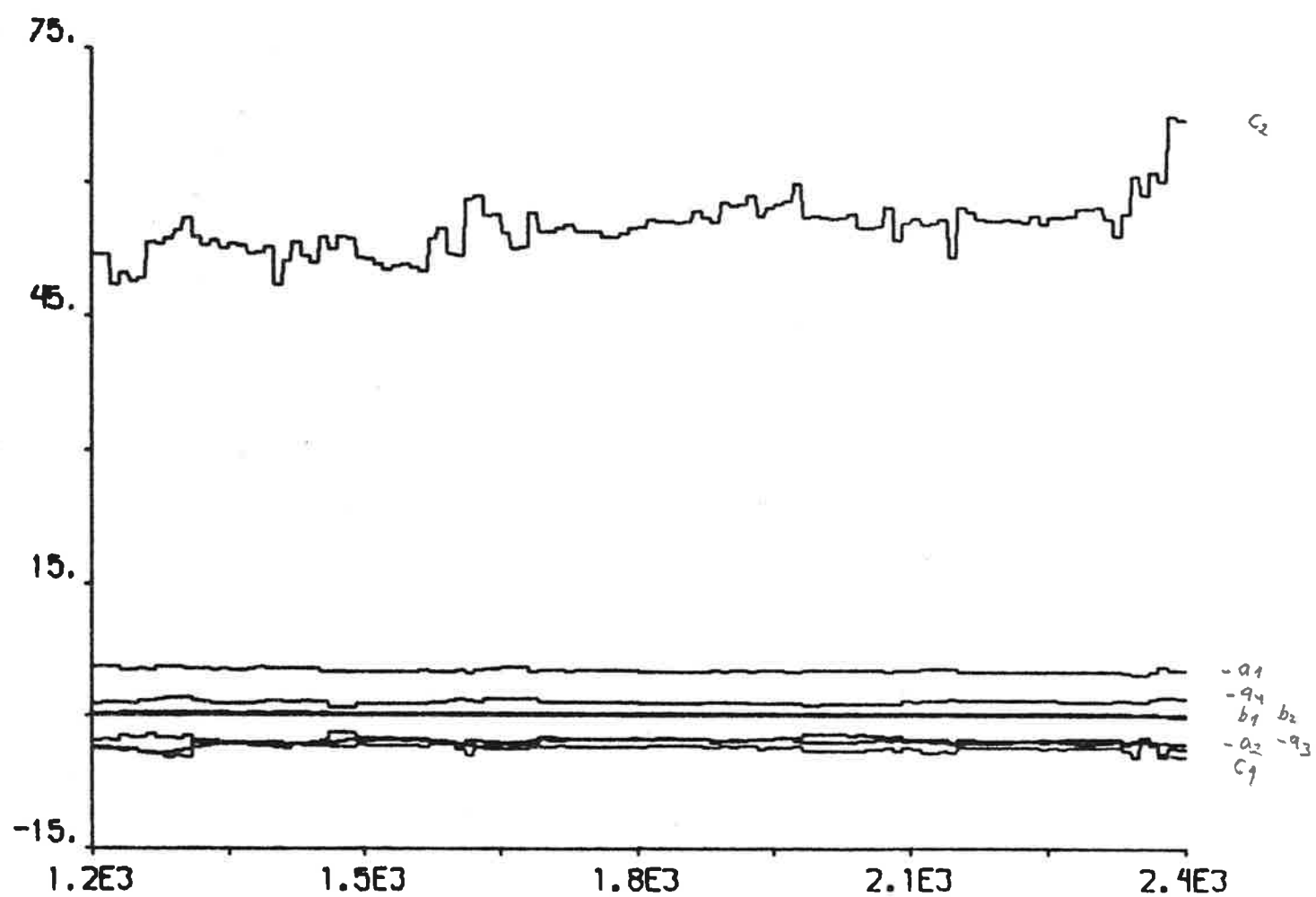
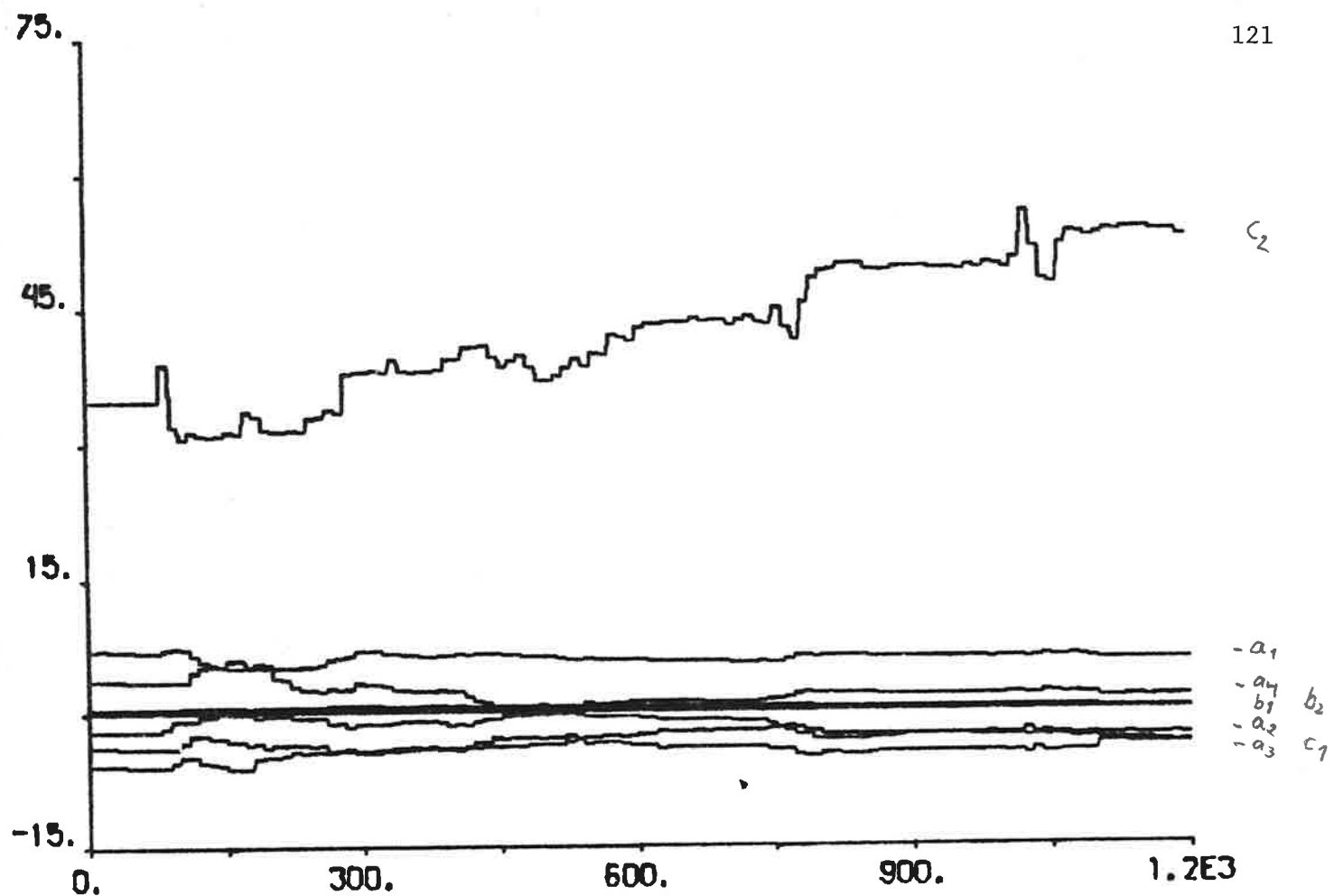


Fig. 4.7n

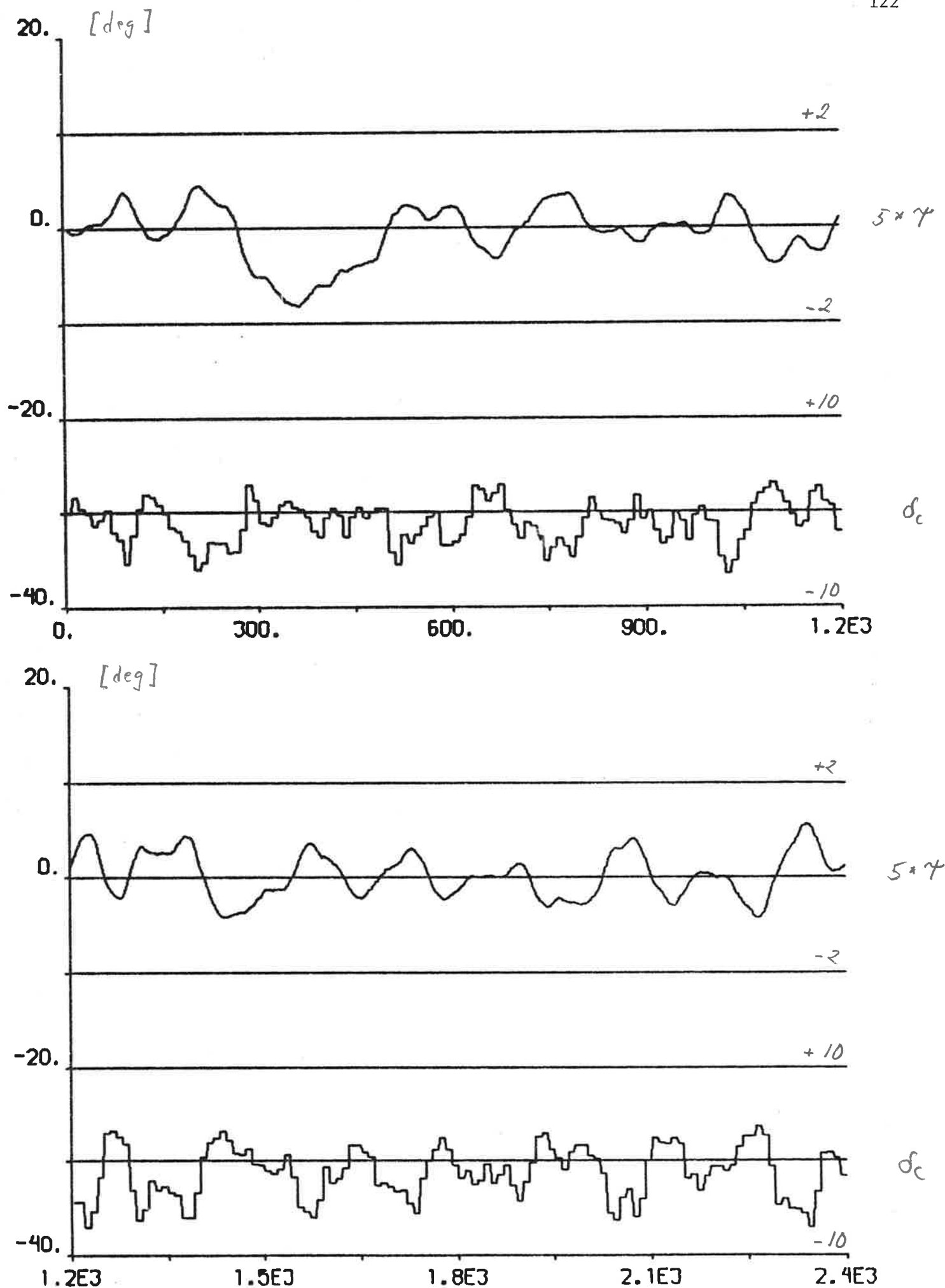


Fig. 4.8a - $T = 22.3$ m, $n_0 = 87.6$ rpm, $u_0 = 15.8$ knots, $\delta_l = 10$ deg, self-tuning regulator using estimates from the Kalman filter. The only measurement signal used by the filter is the heading angle.

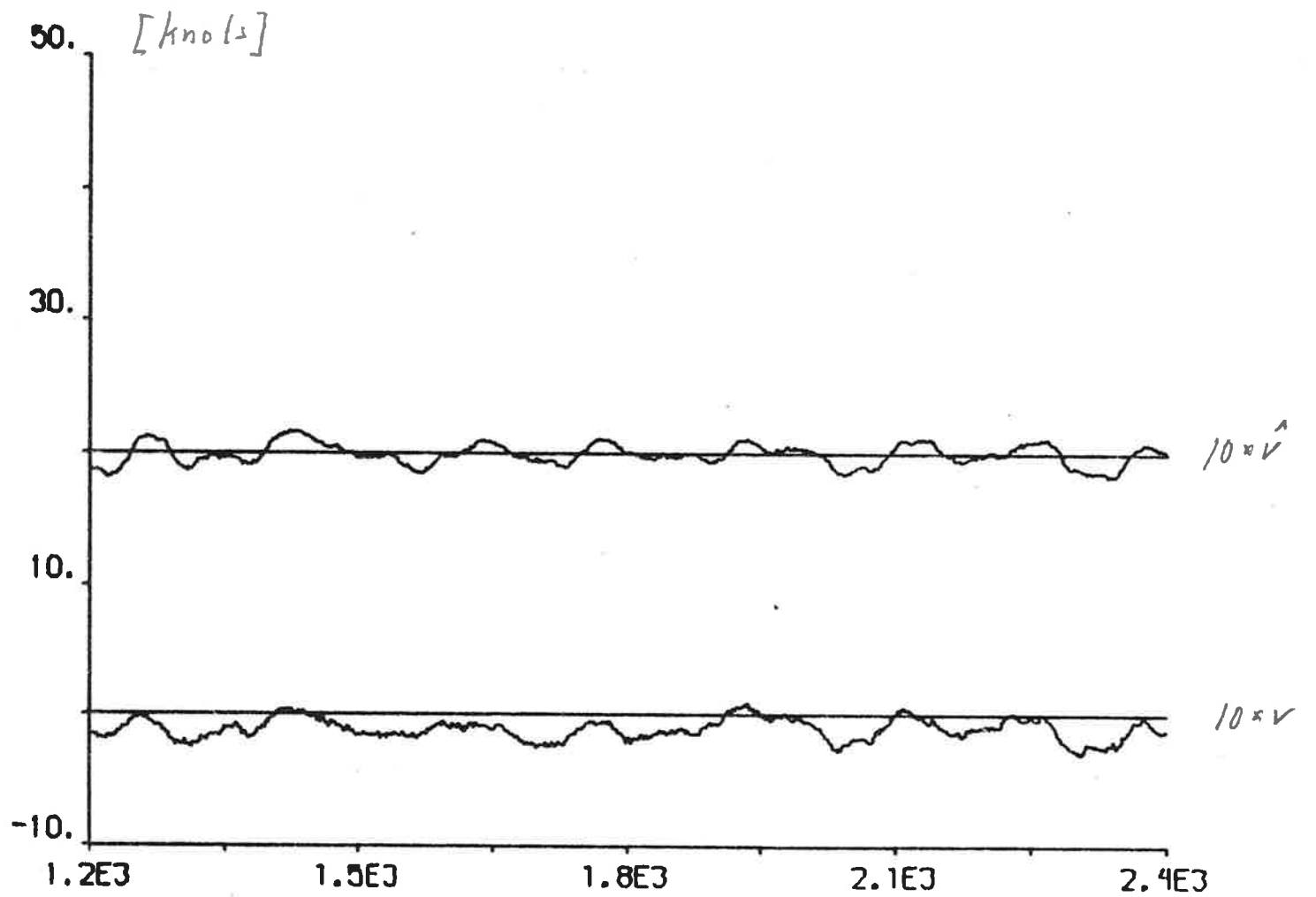
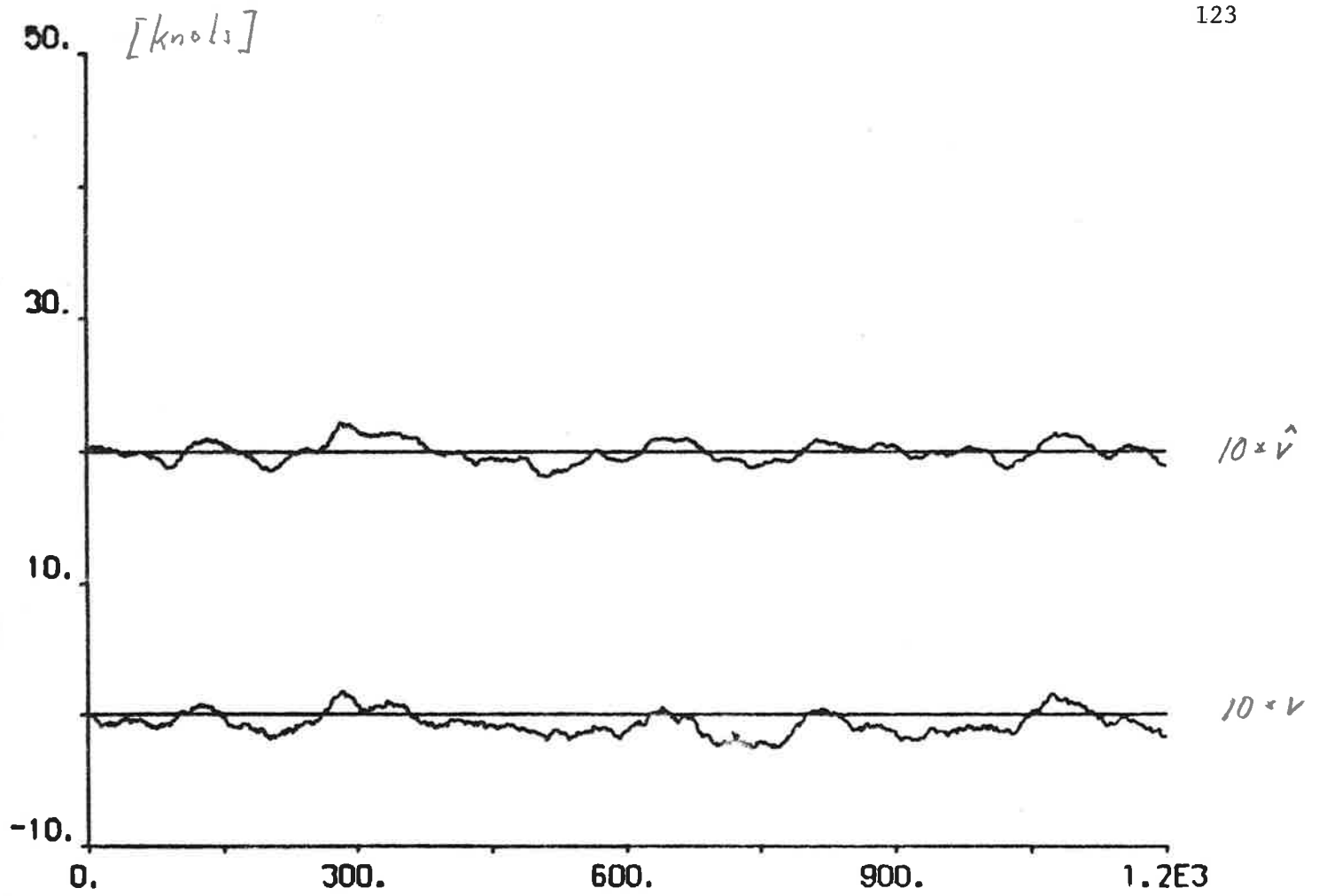


Fig. 4.8b

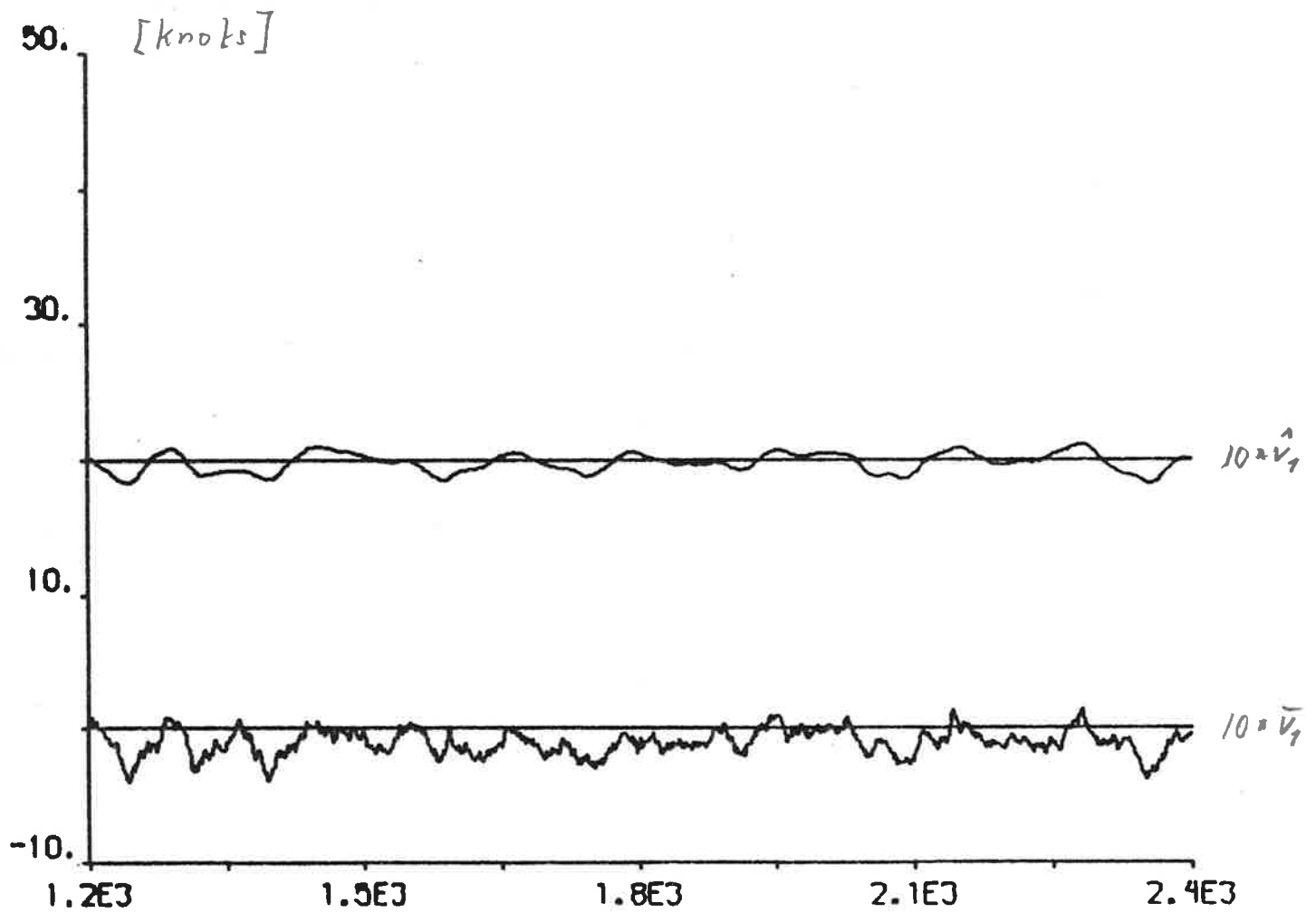
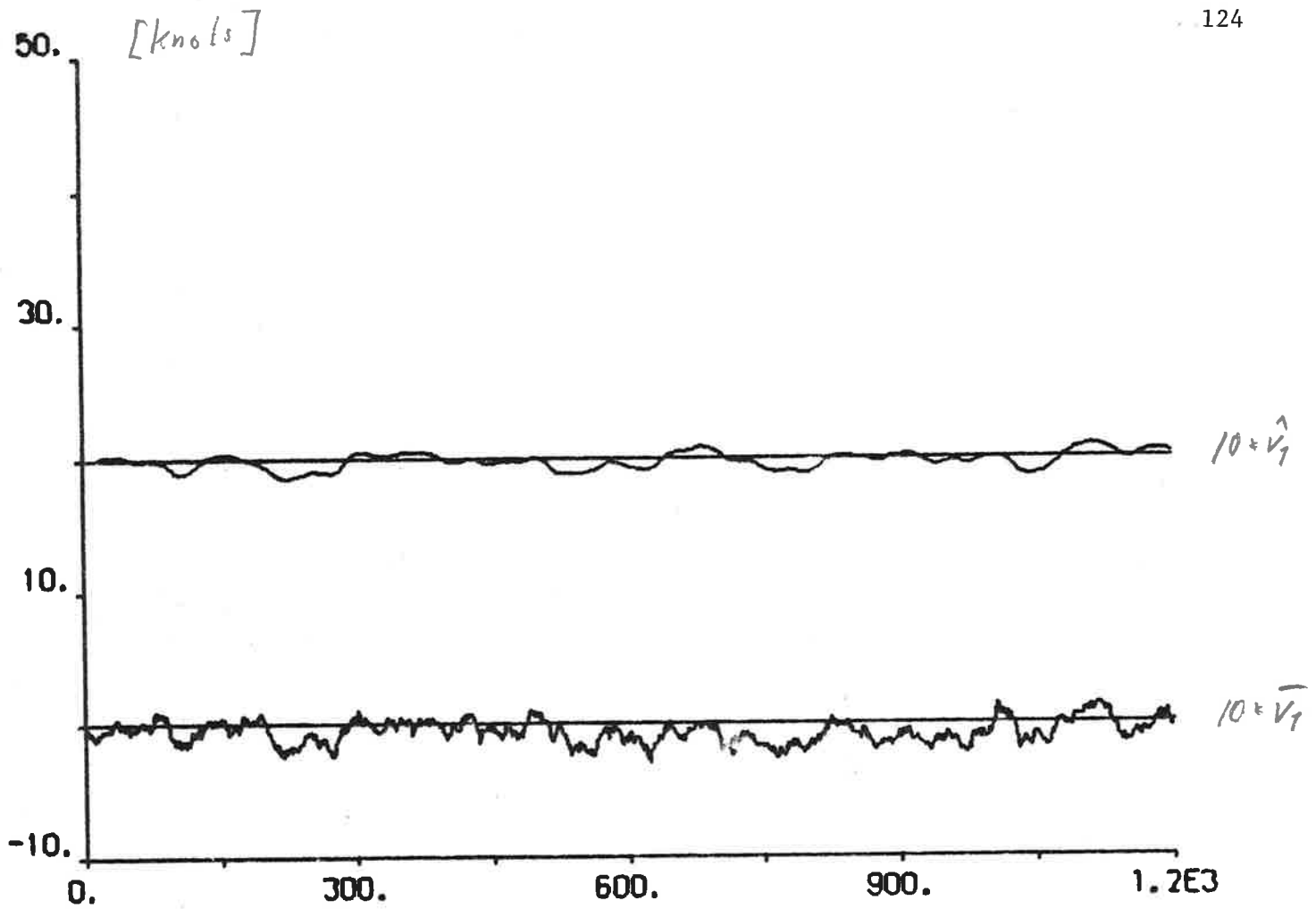


Fig. 4.8c

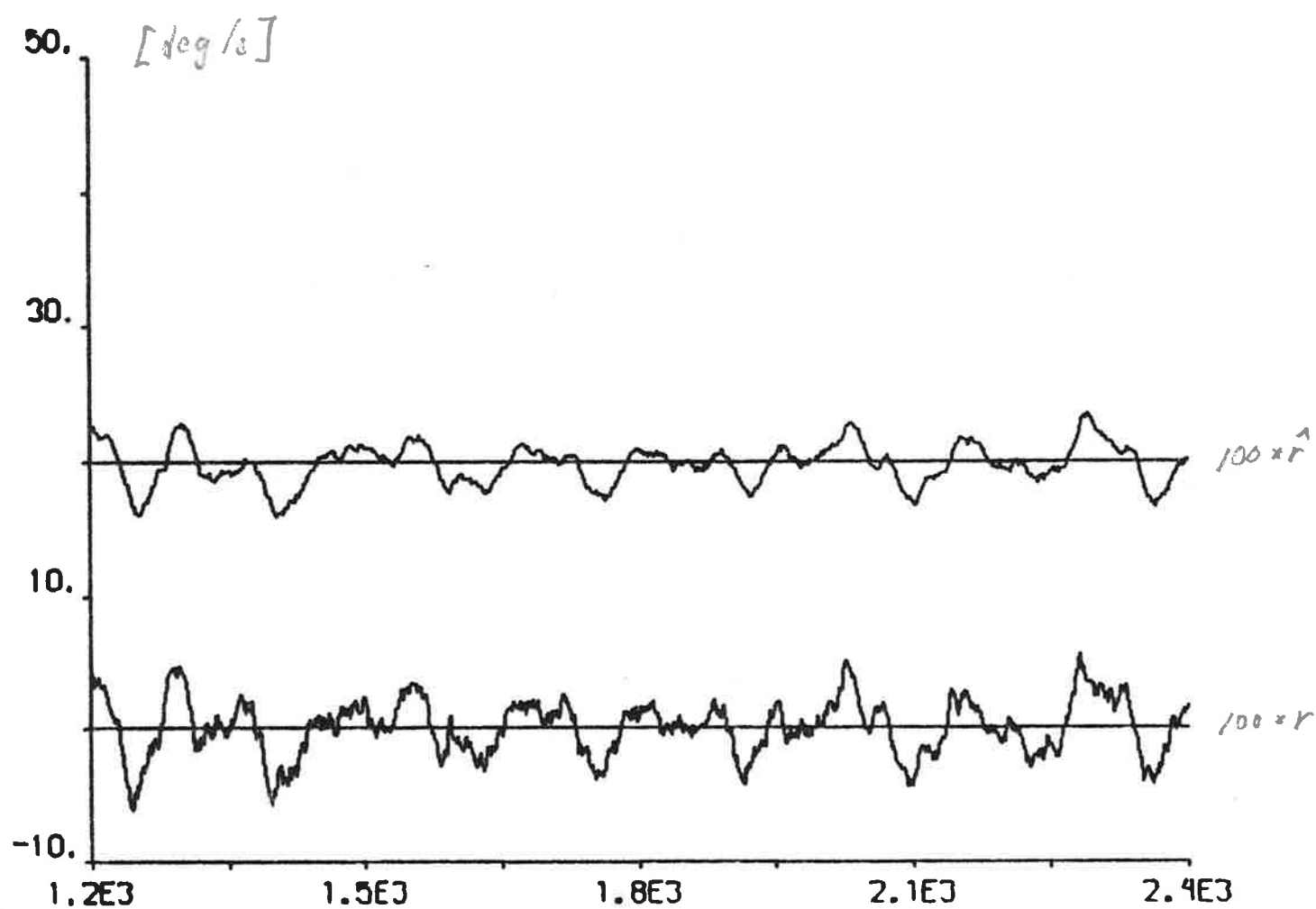
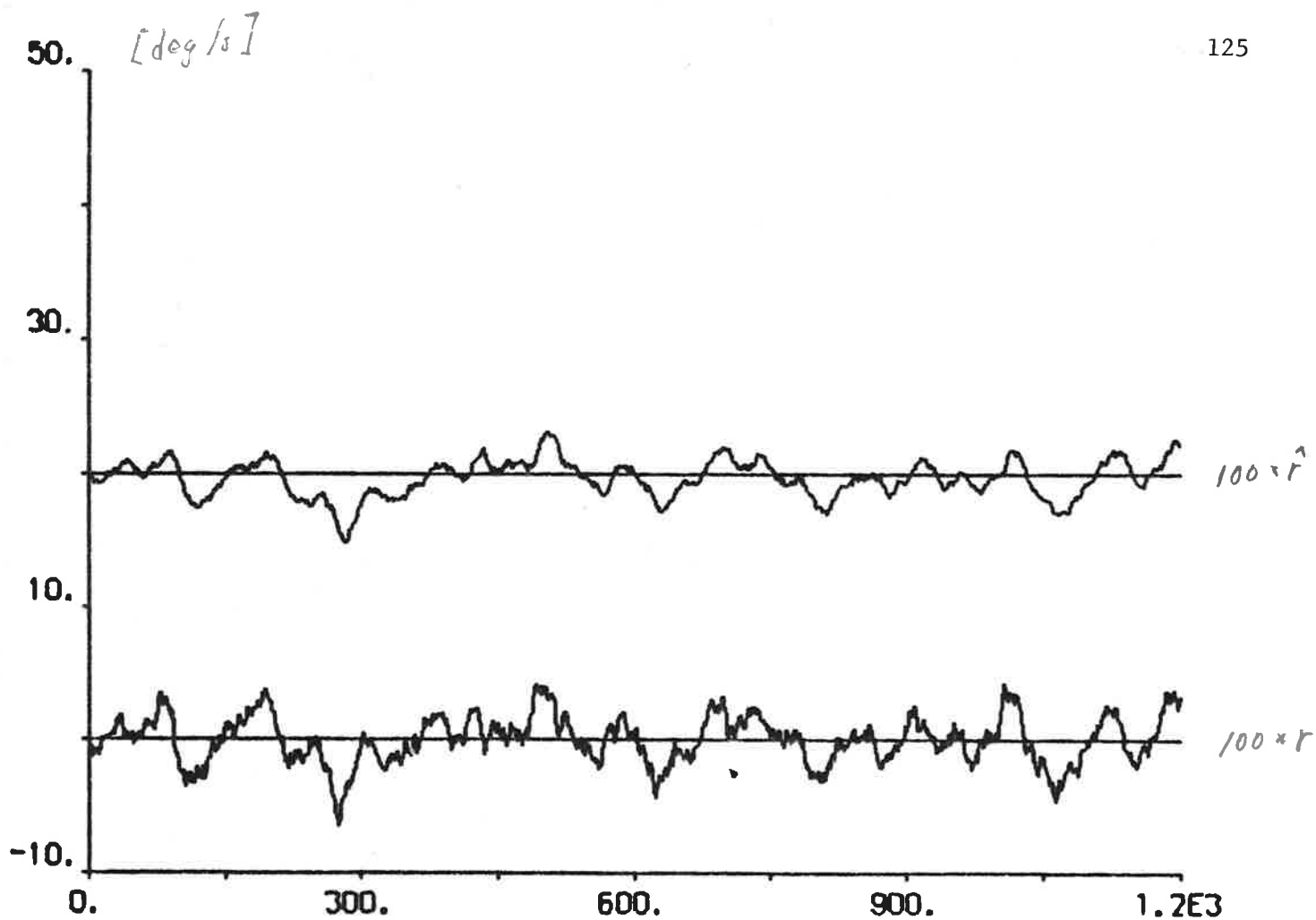


Fig. 4.8d

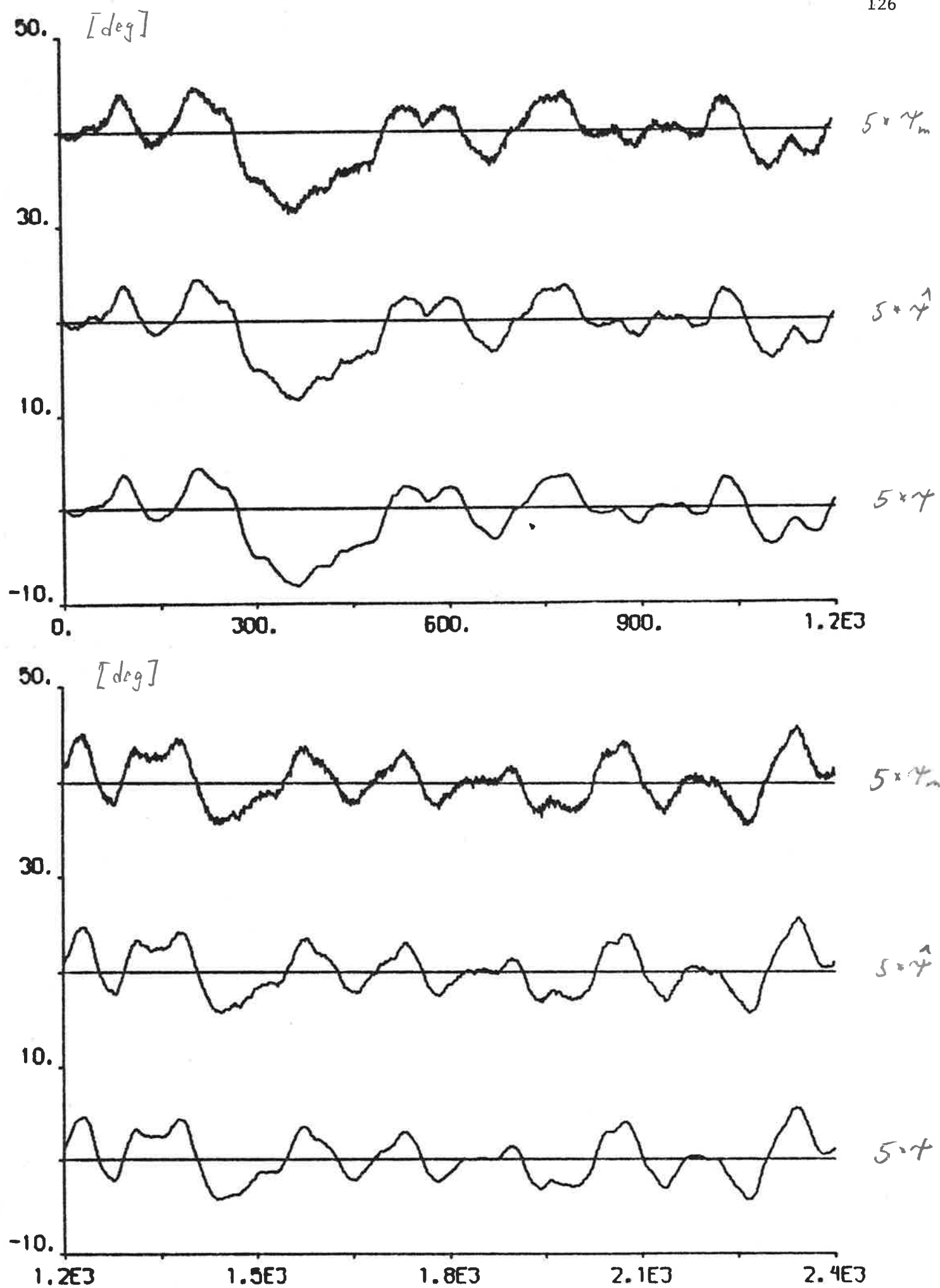


Fig. 4.8e

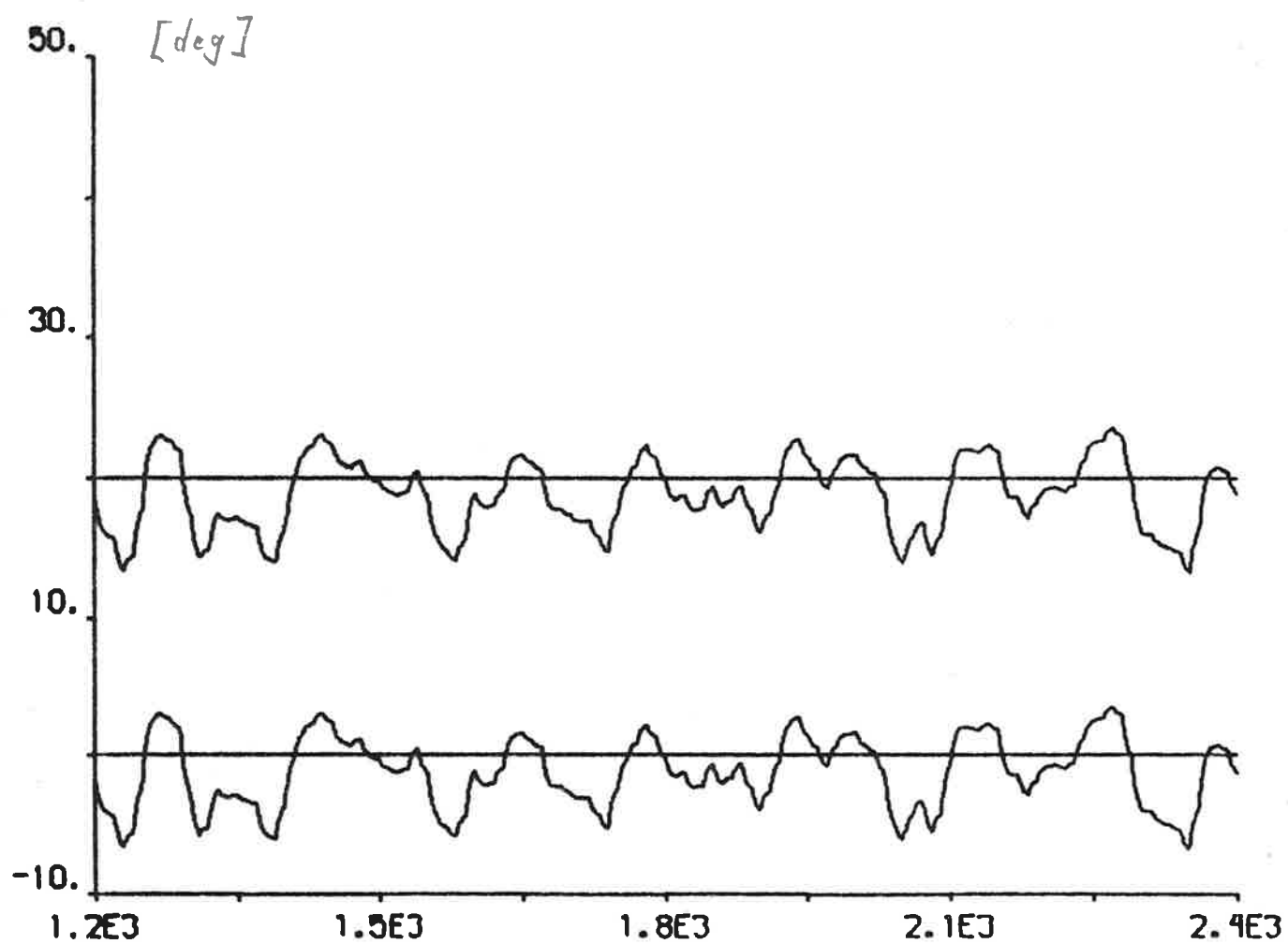
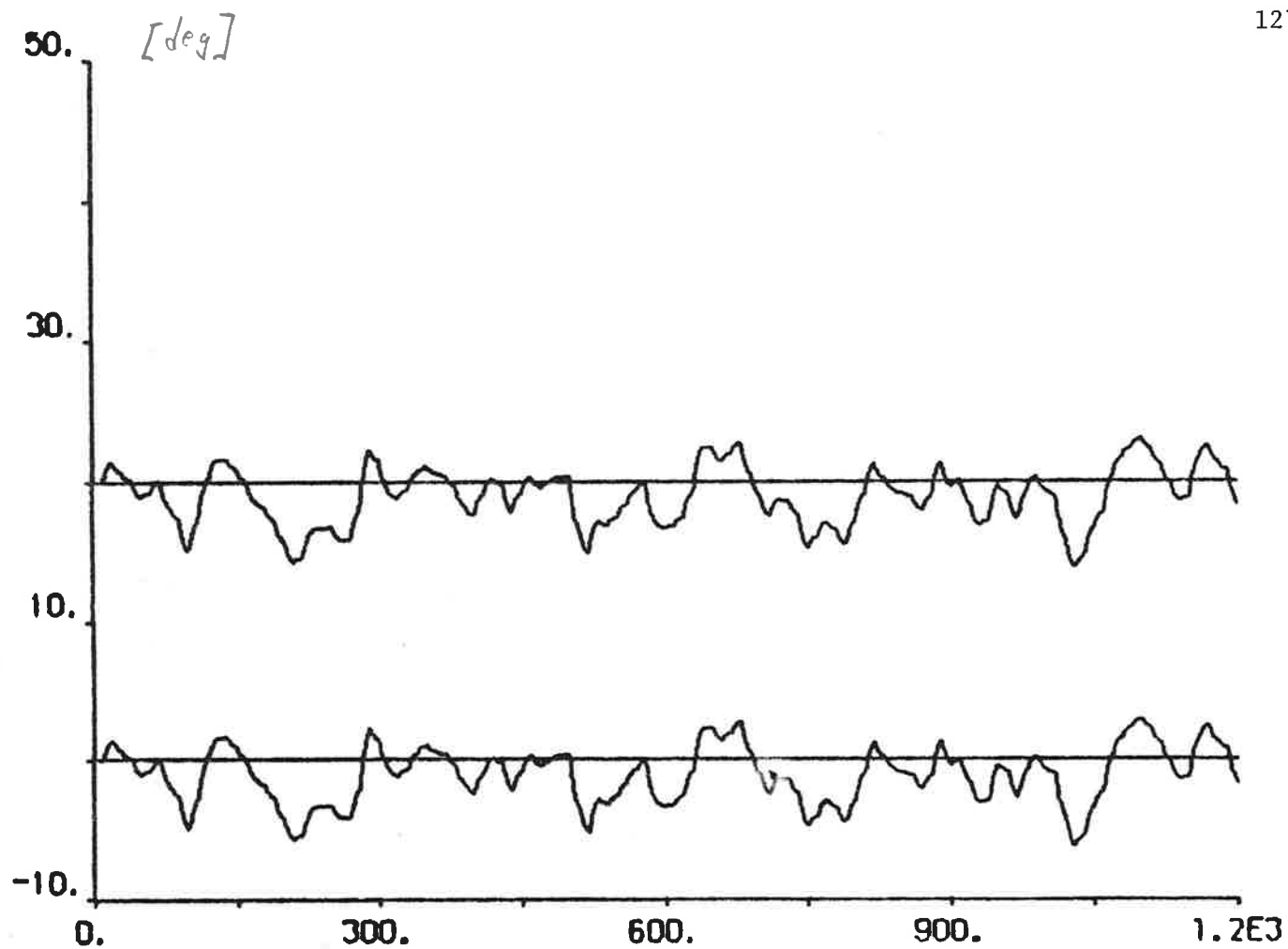


Fig. 4.8f

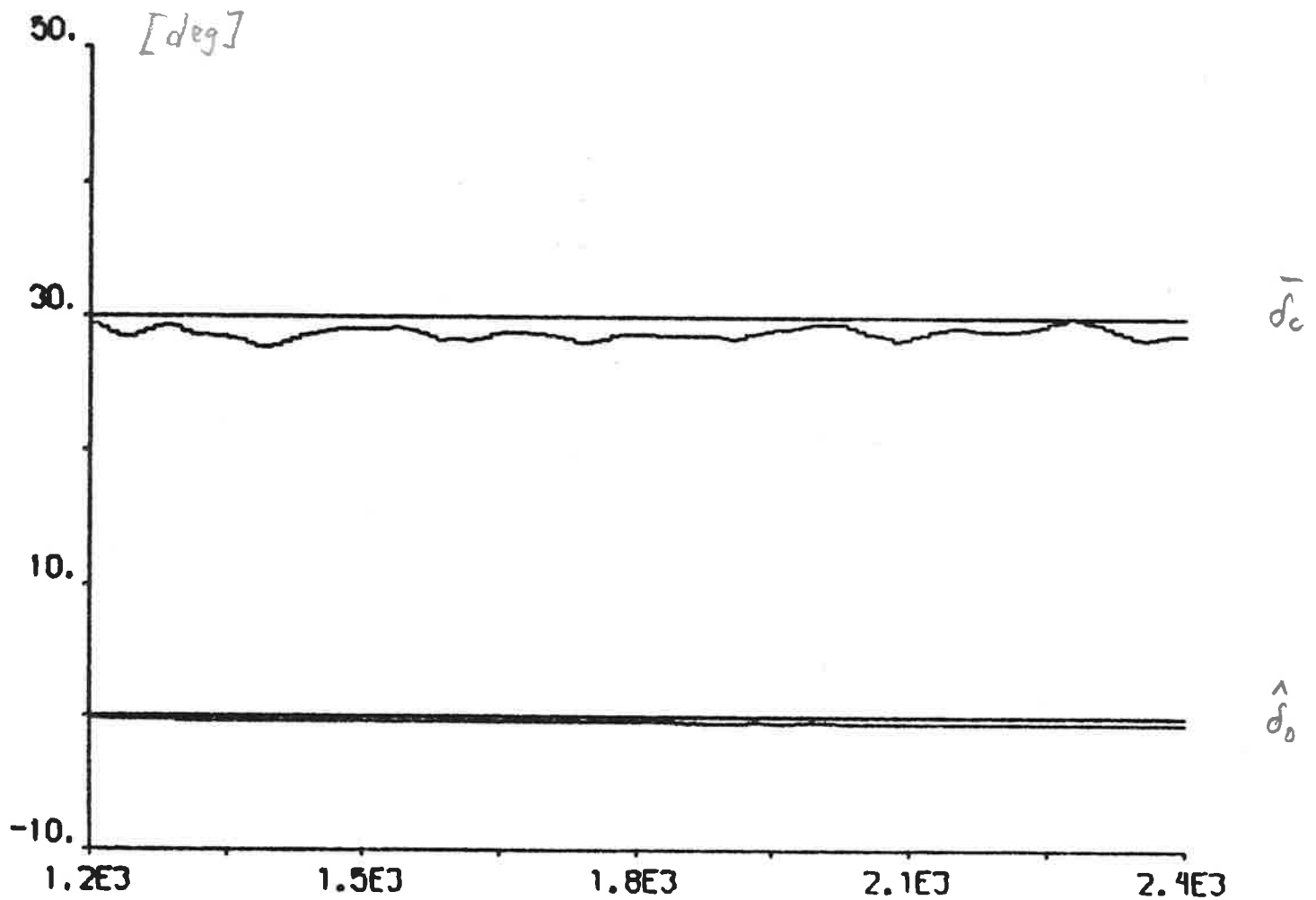
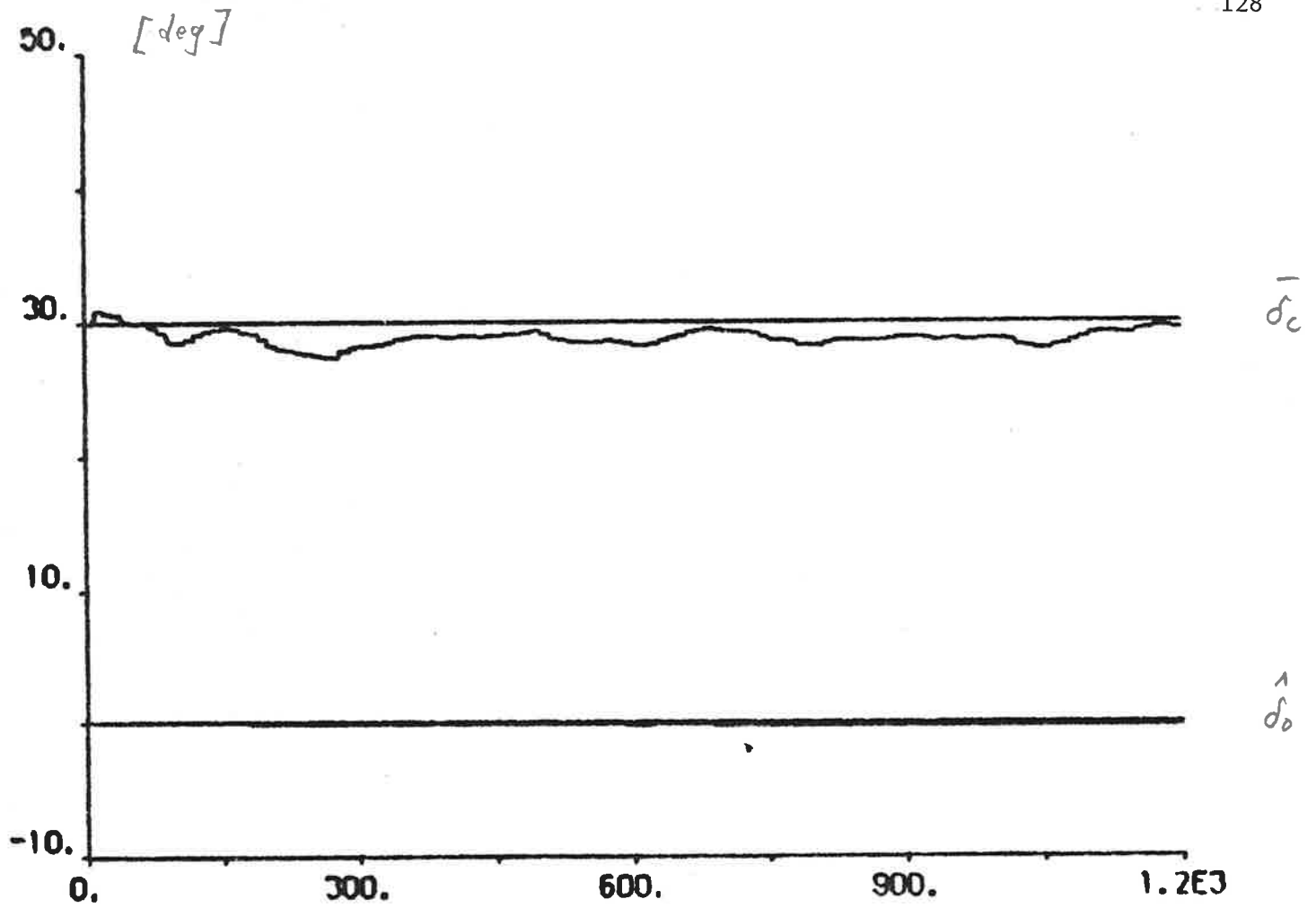


Fig. 4.8g

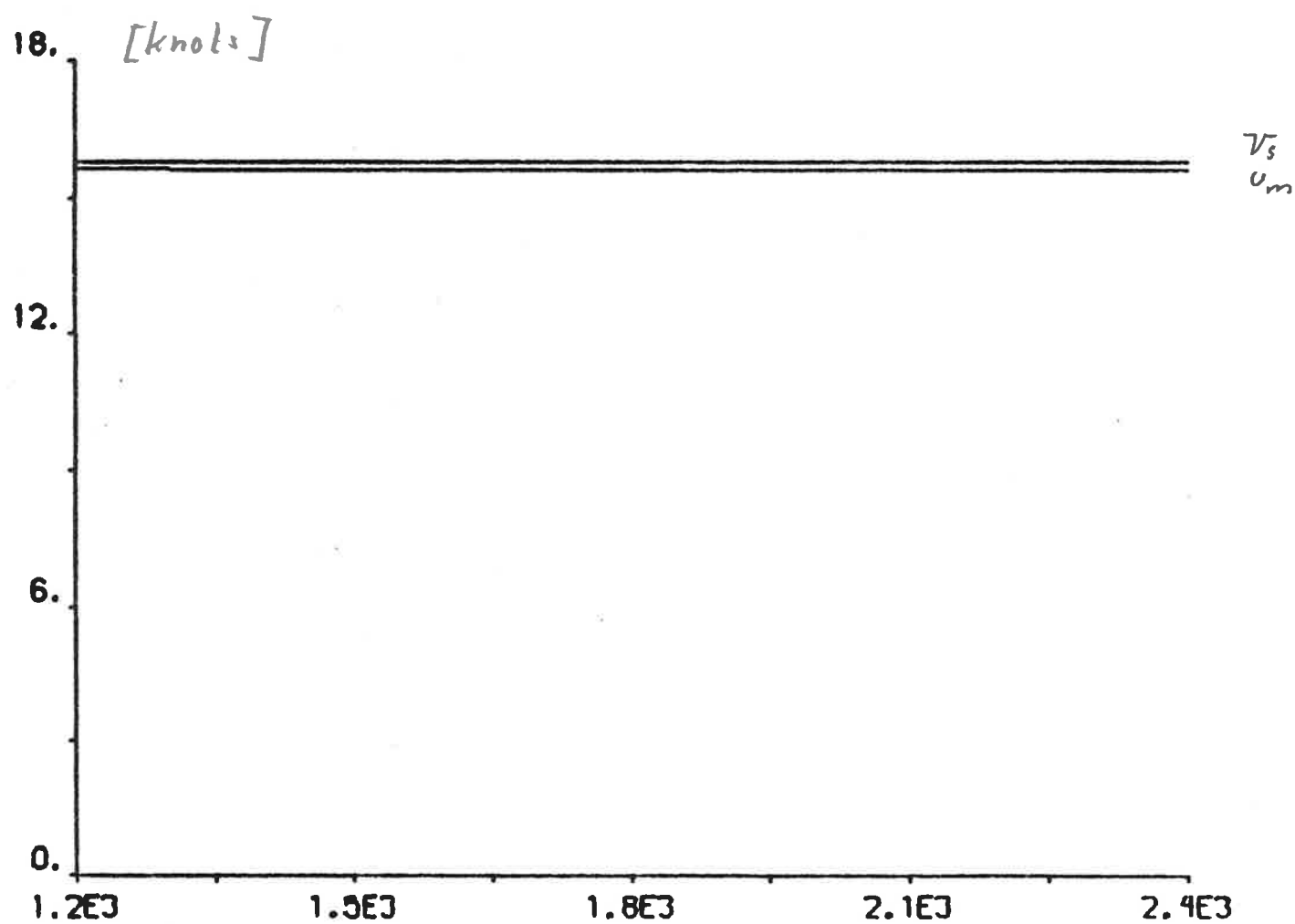
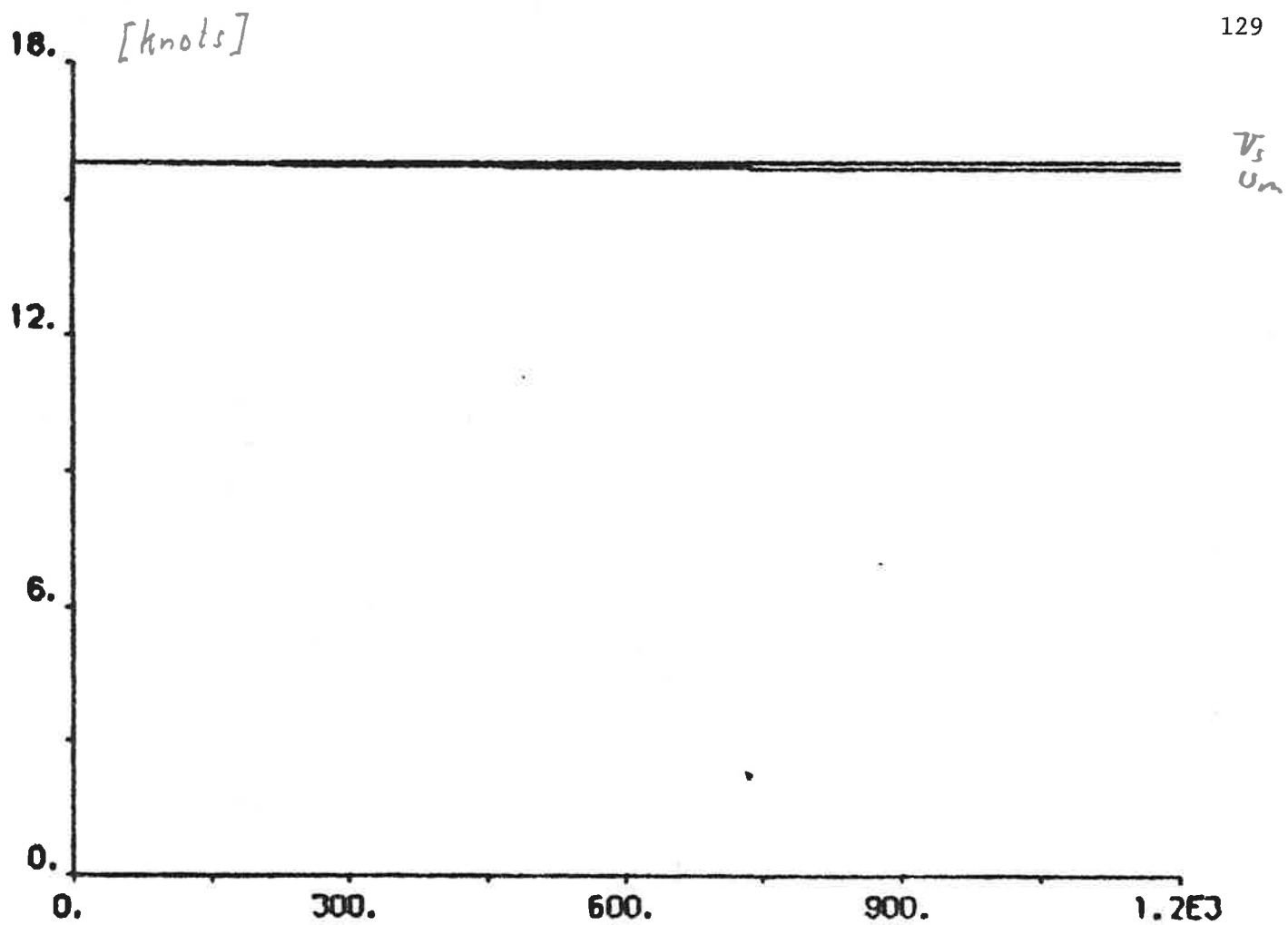


Fig. 4.8h

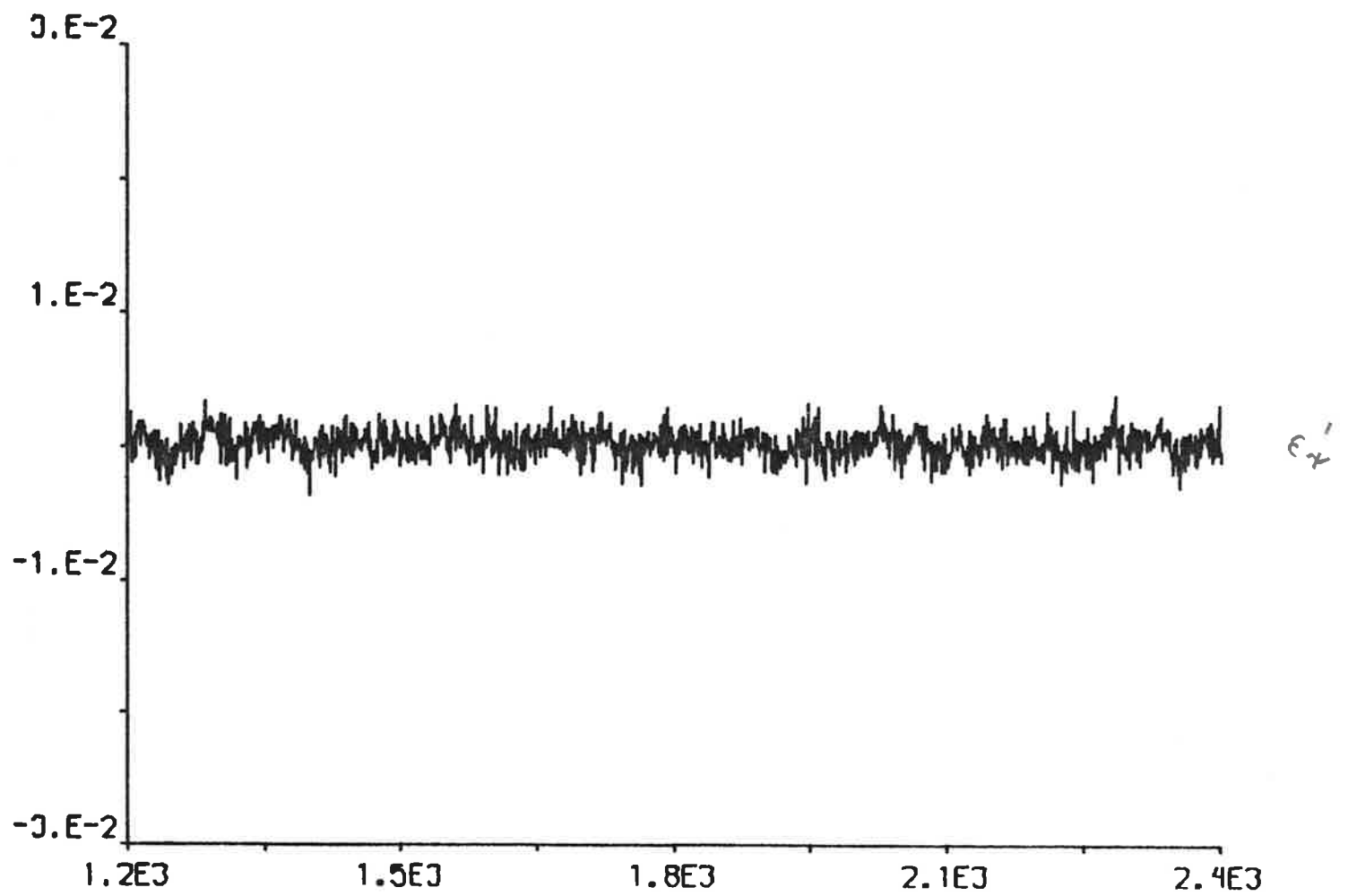
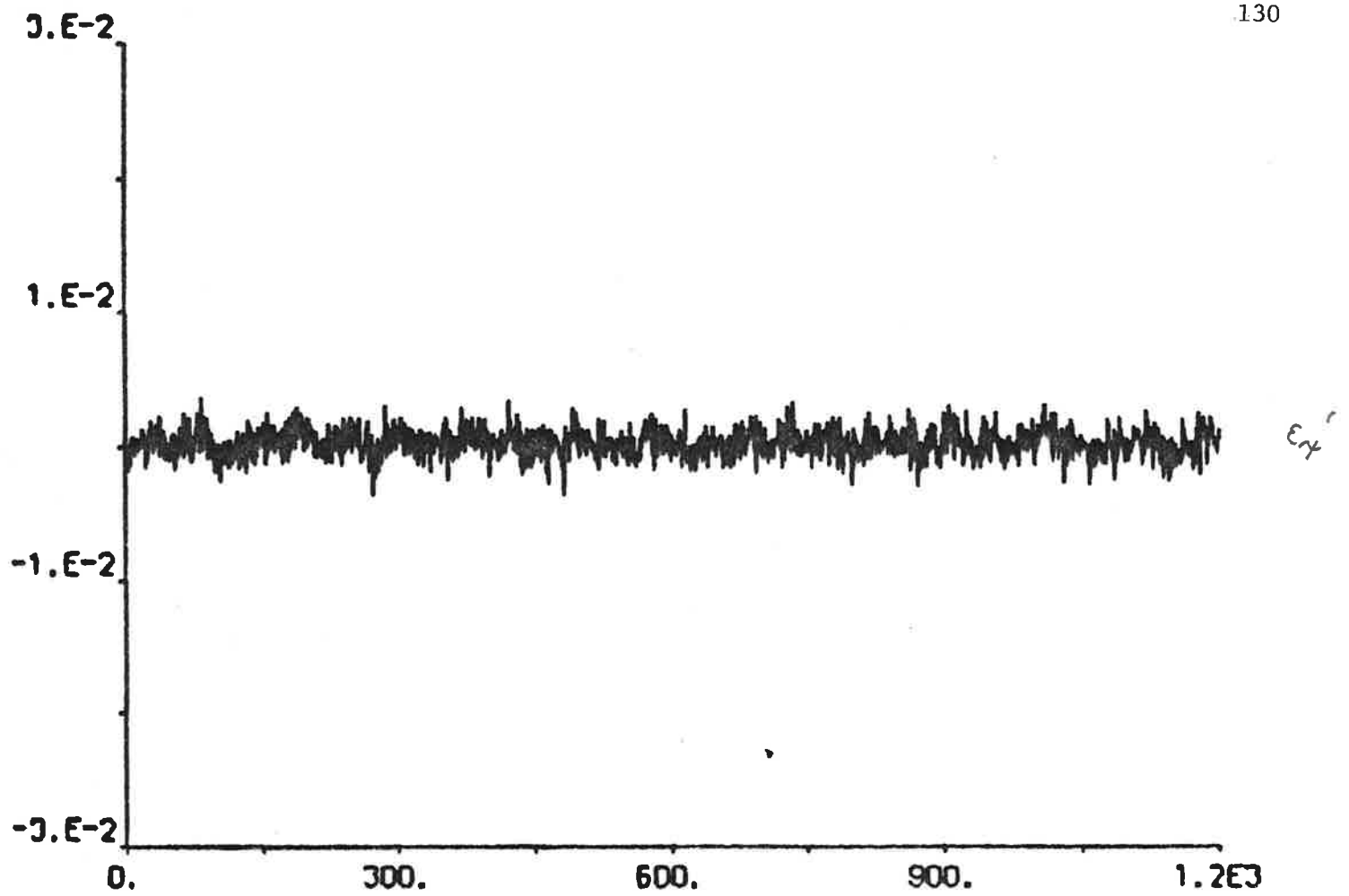


Fig. 4.8i

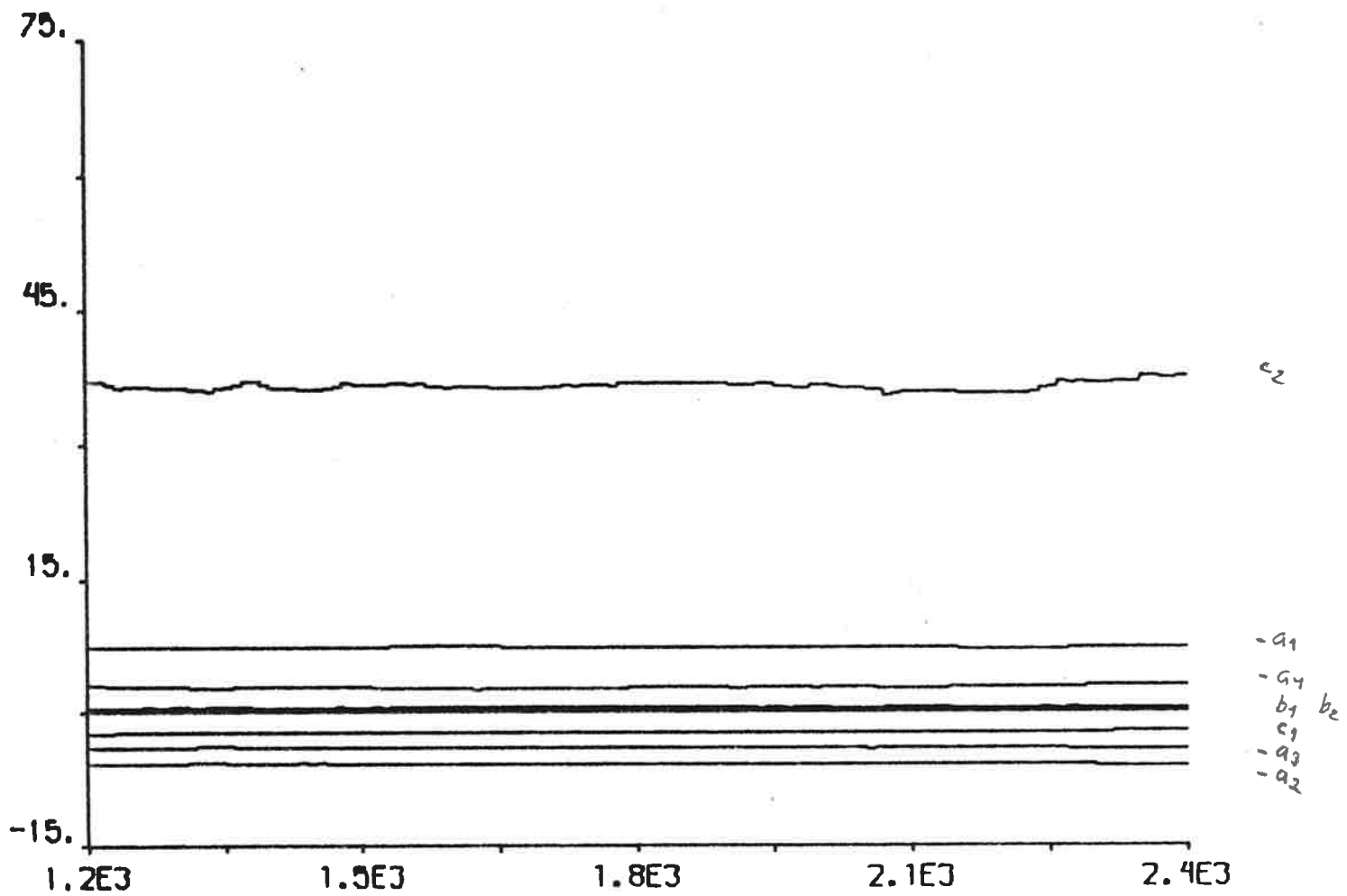
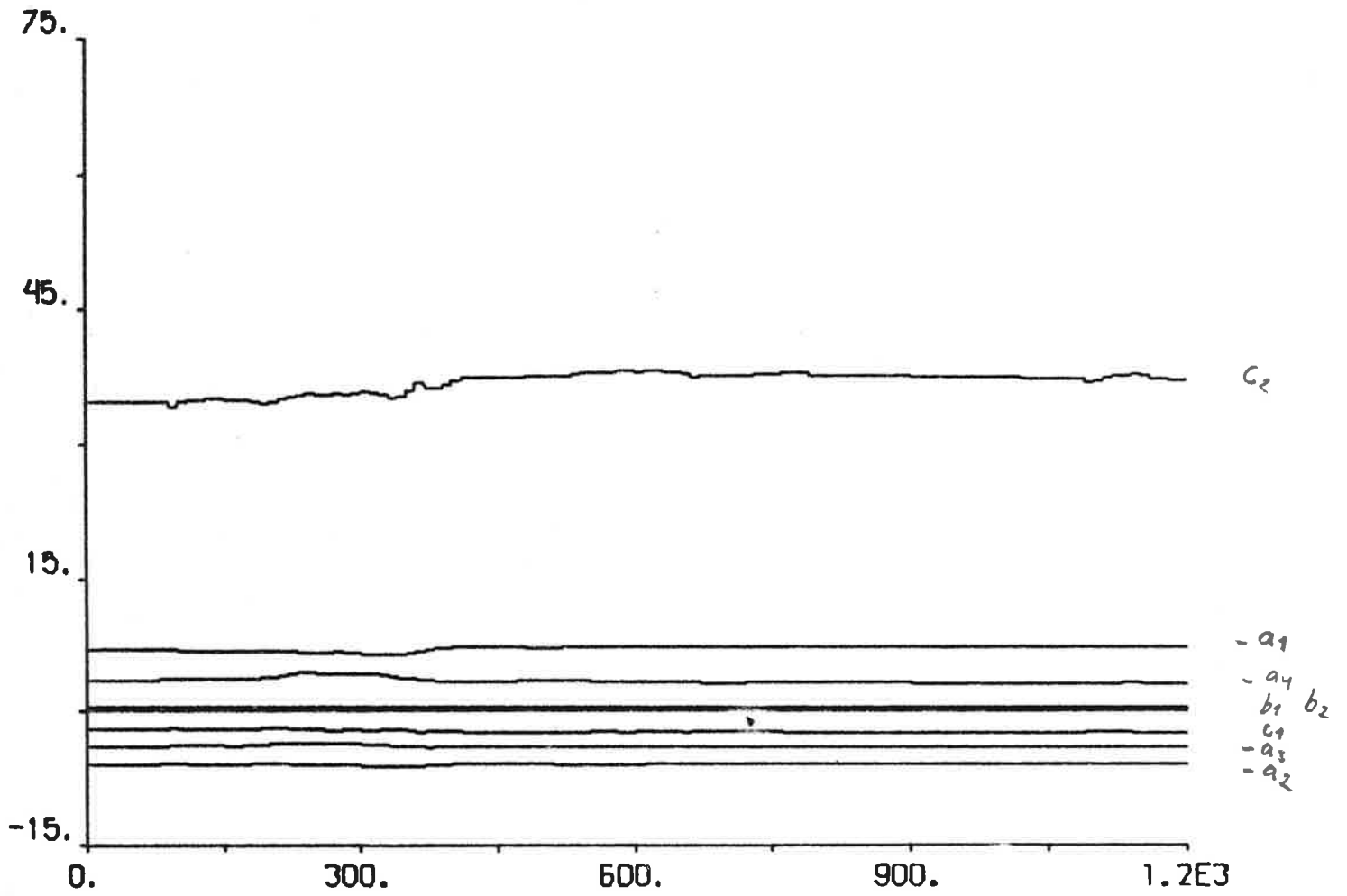


Fig. 4.8j

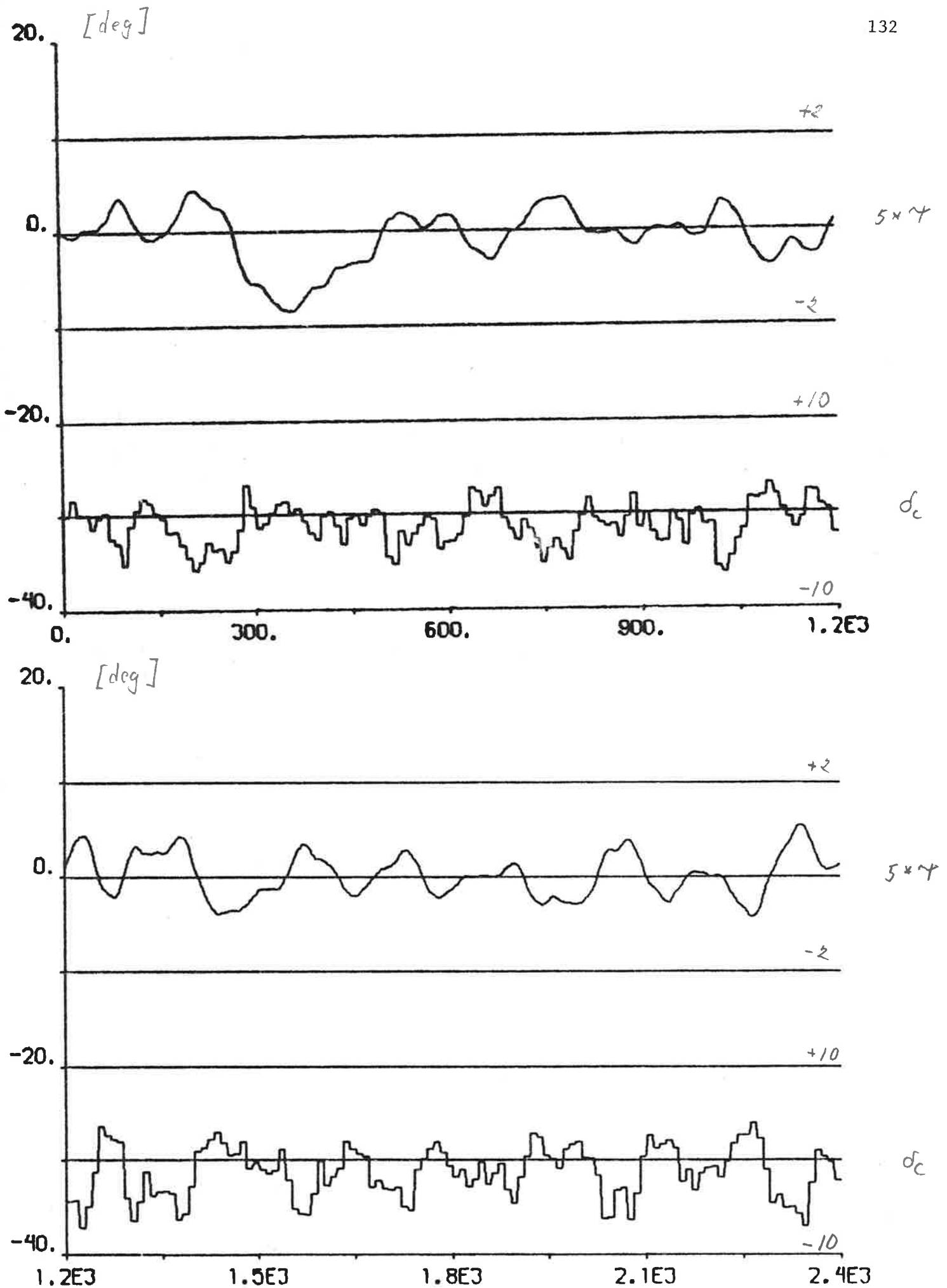


Fig. 4.9a - $T = 22.3$ m, $n_0 = 87.6$ rpm, $u_0 = 15.8$ knots, $\delta_0 = 10$ deg, self-tuning regulator using estimates from the Kalman filter. The only measurement signal used by the filter is the heading angle. The correct filter gain K given by (4.4) is used.

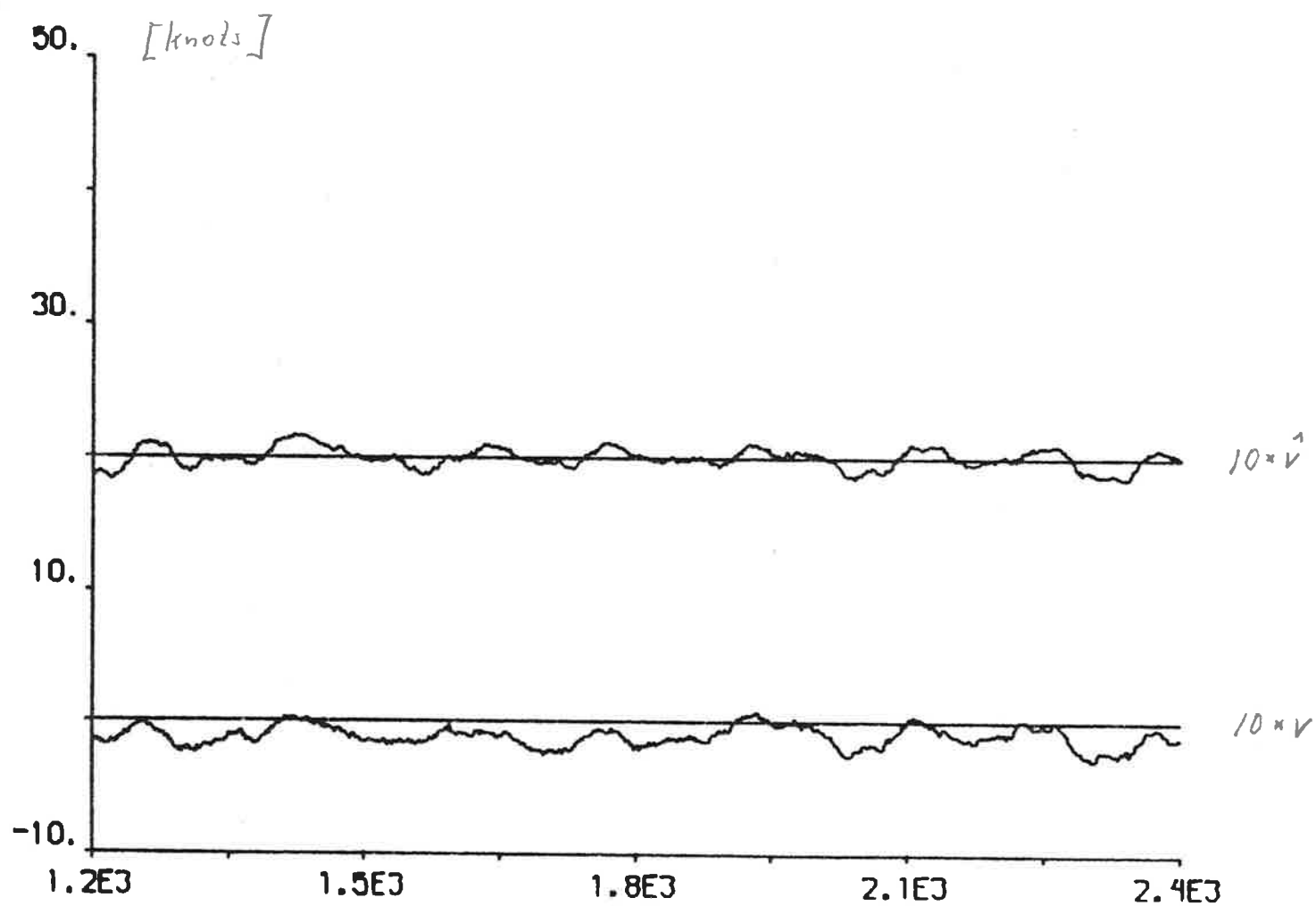
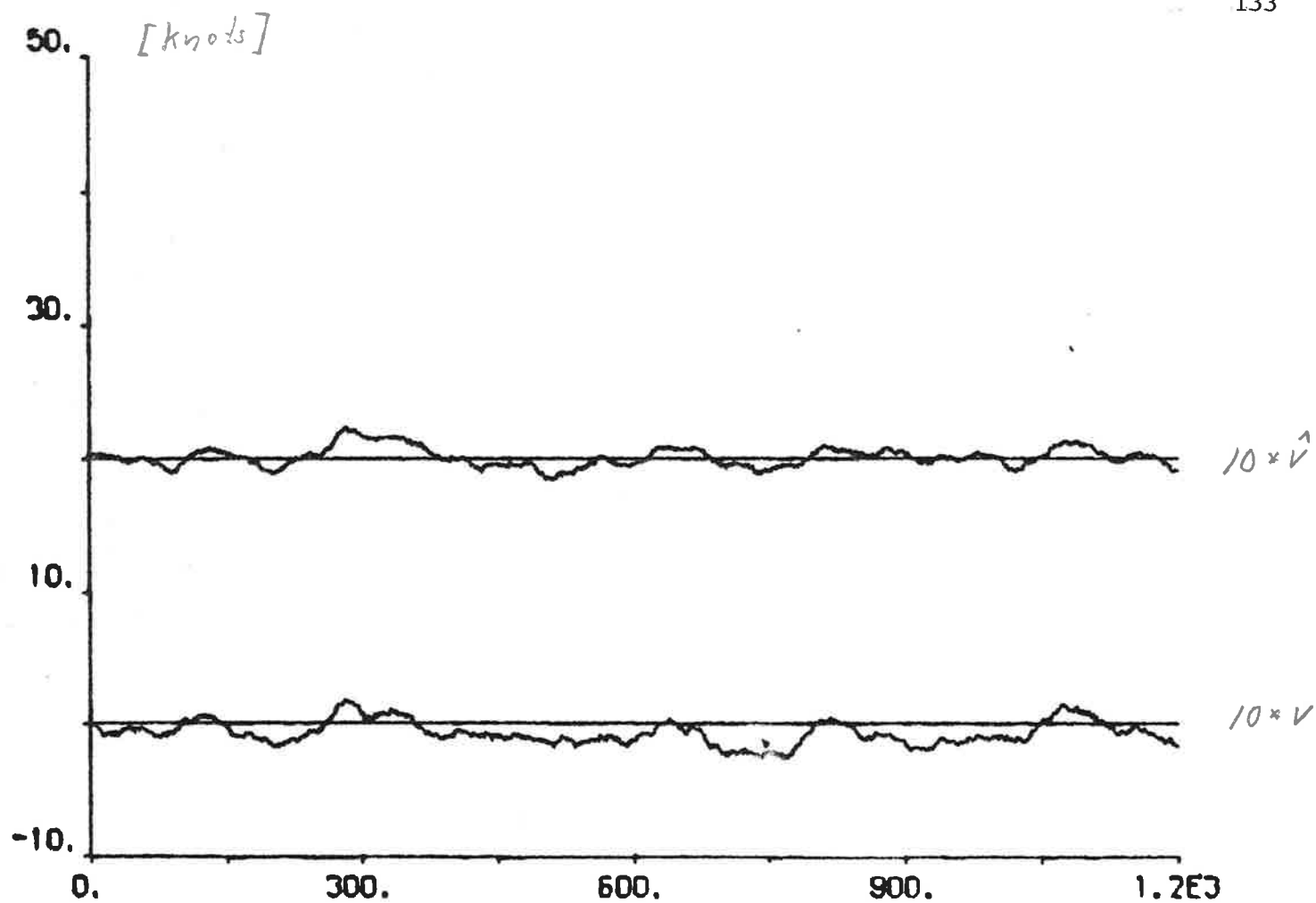


Fig. 4.9b

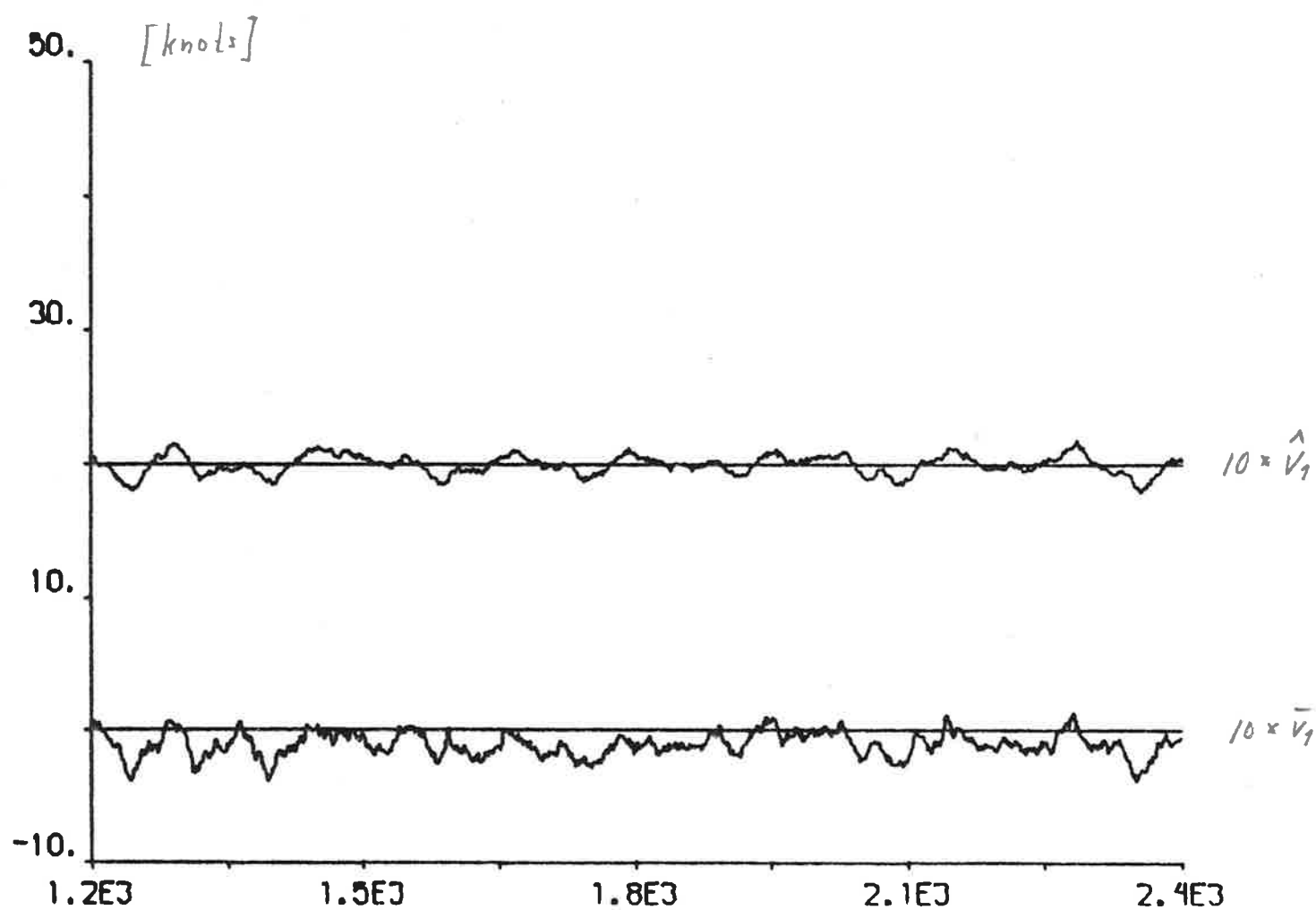
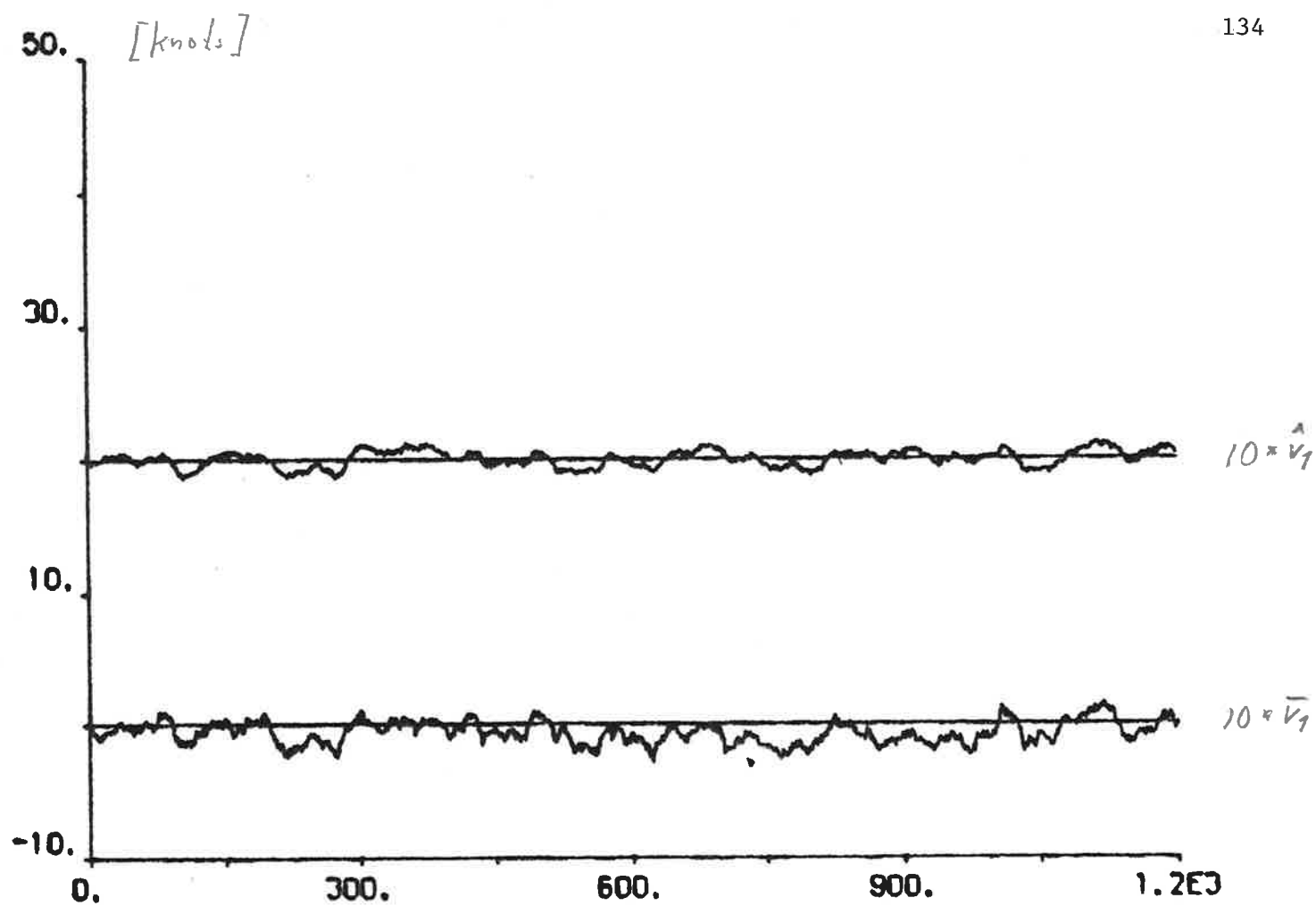


Fig. 4.9c

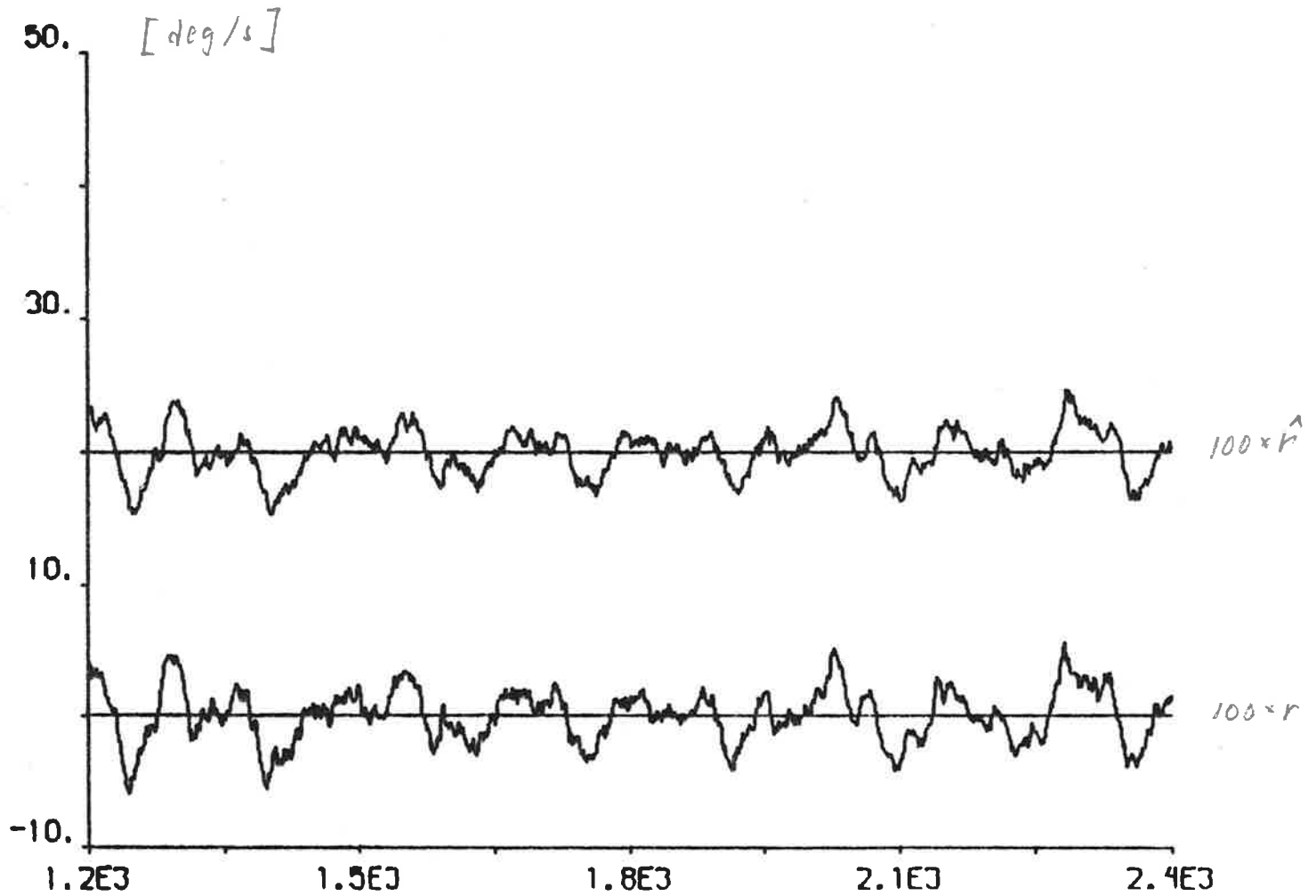
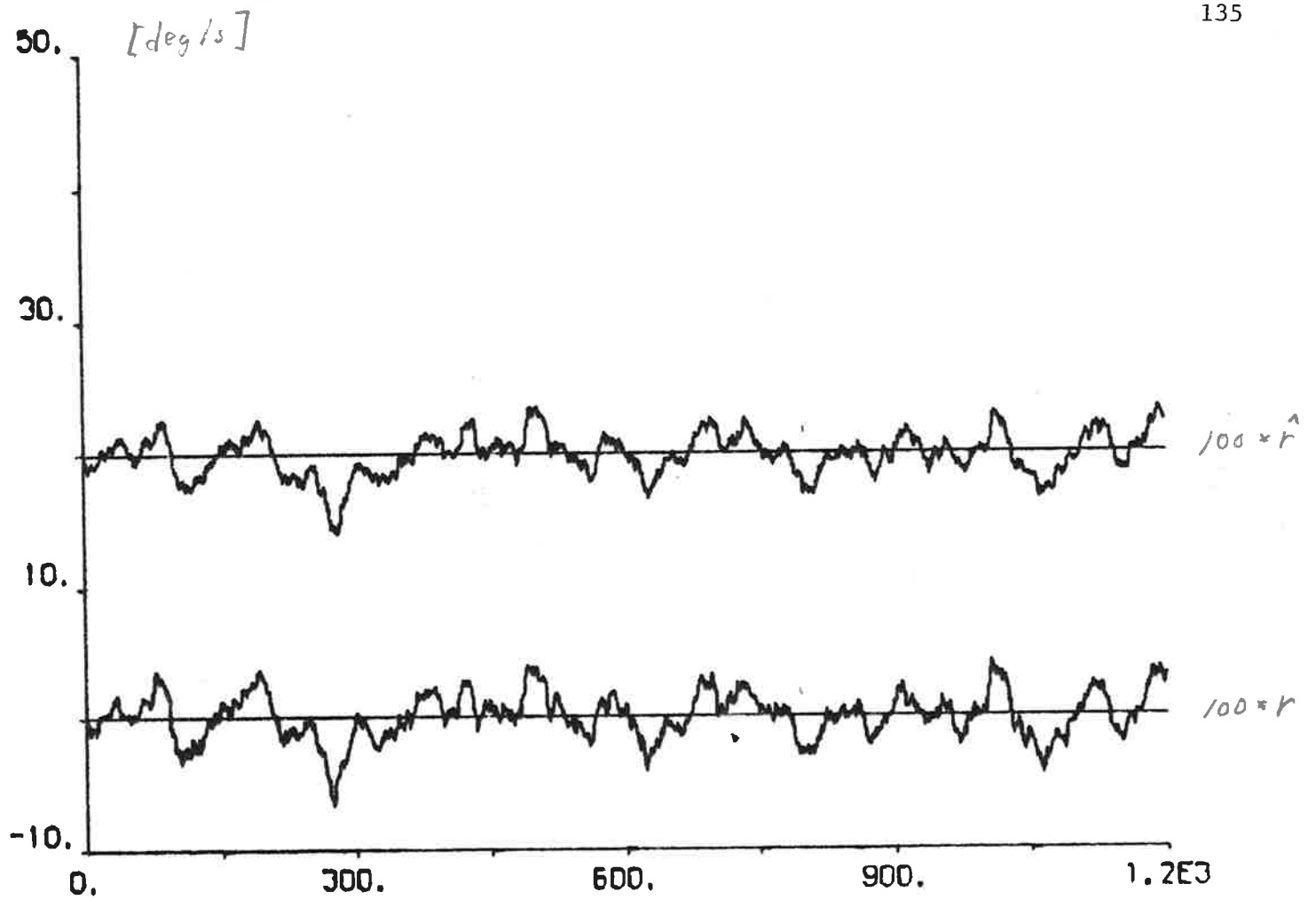


Fig. 4.9d

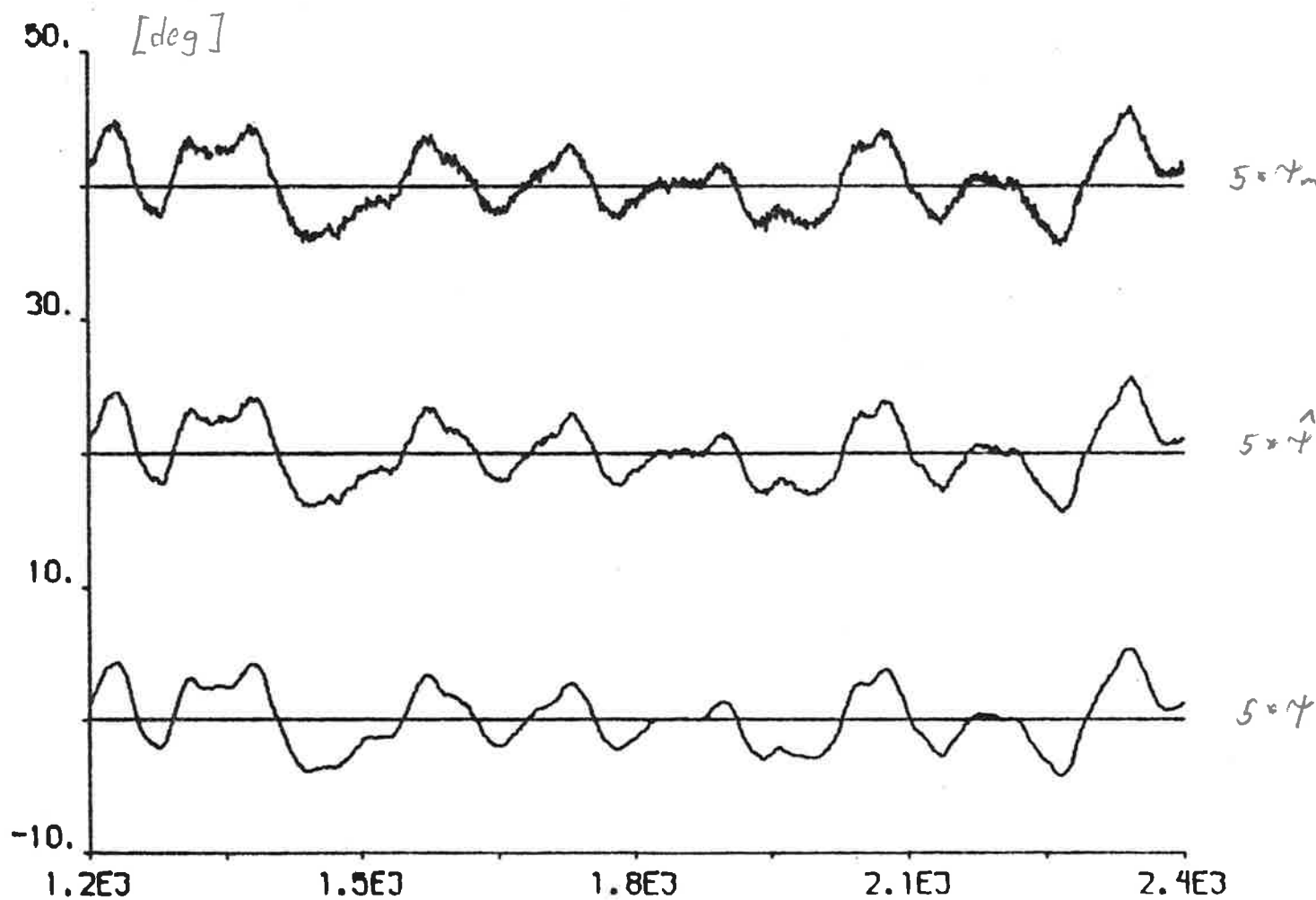
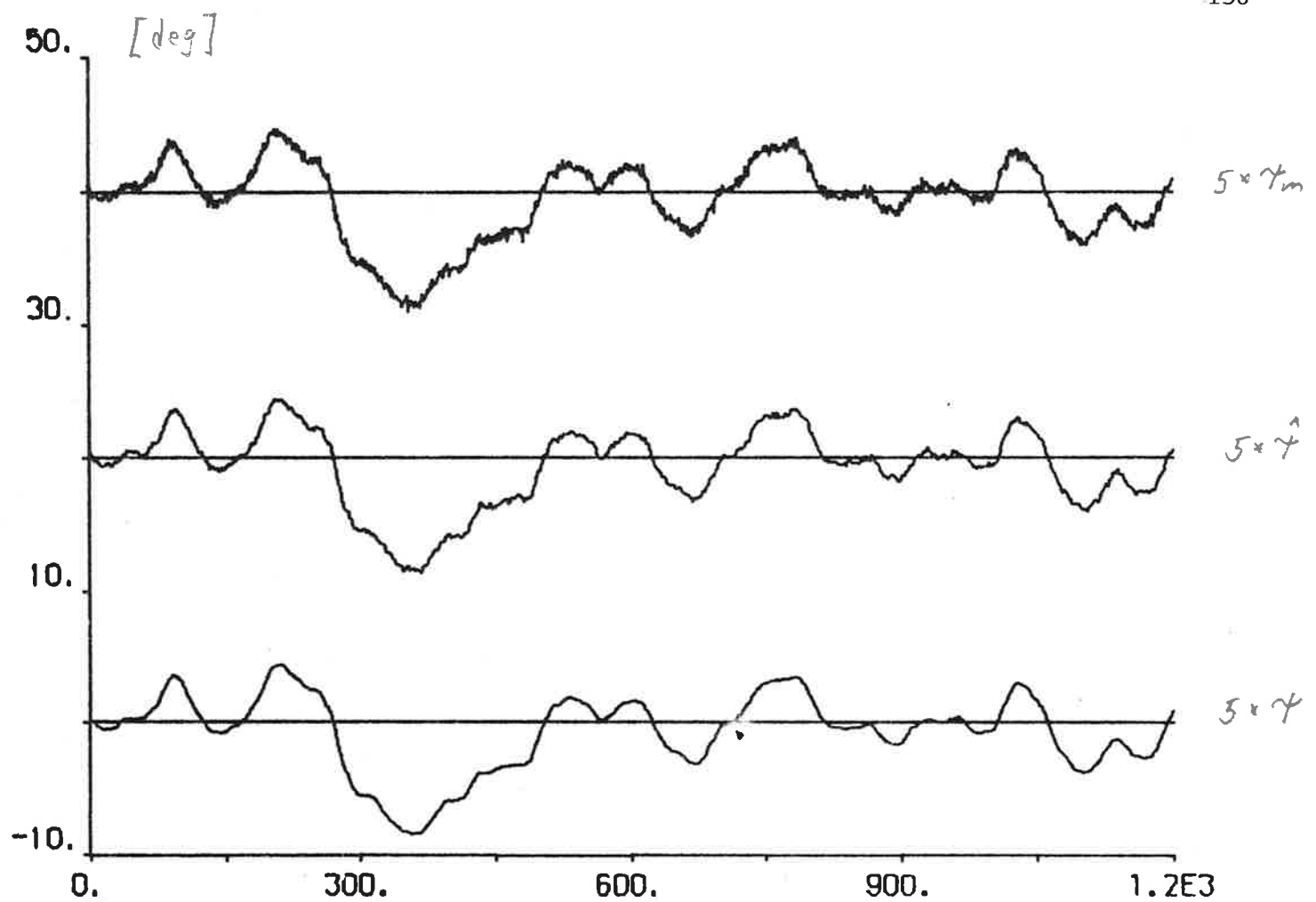


Fig. 4.9e

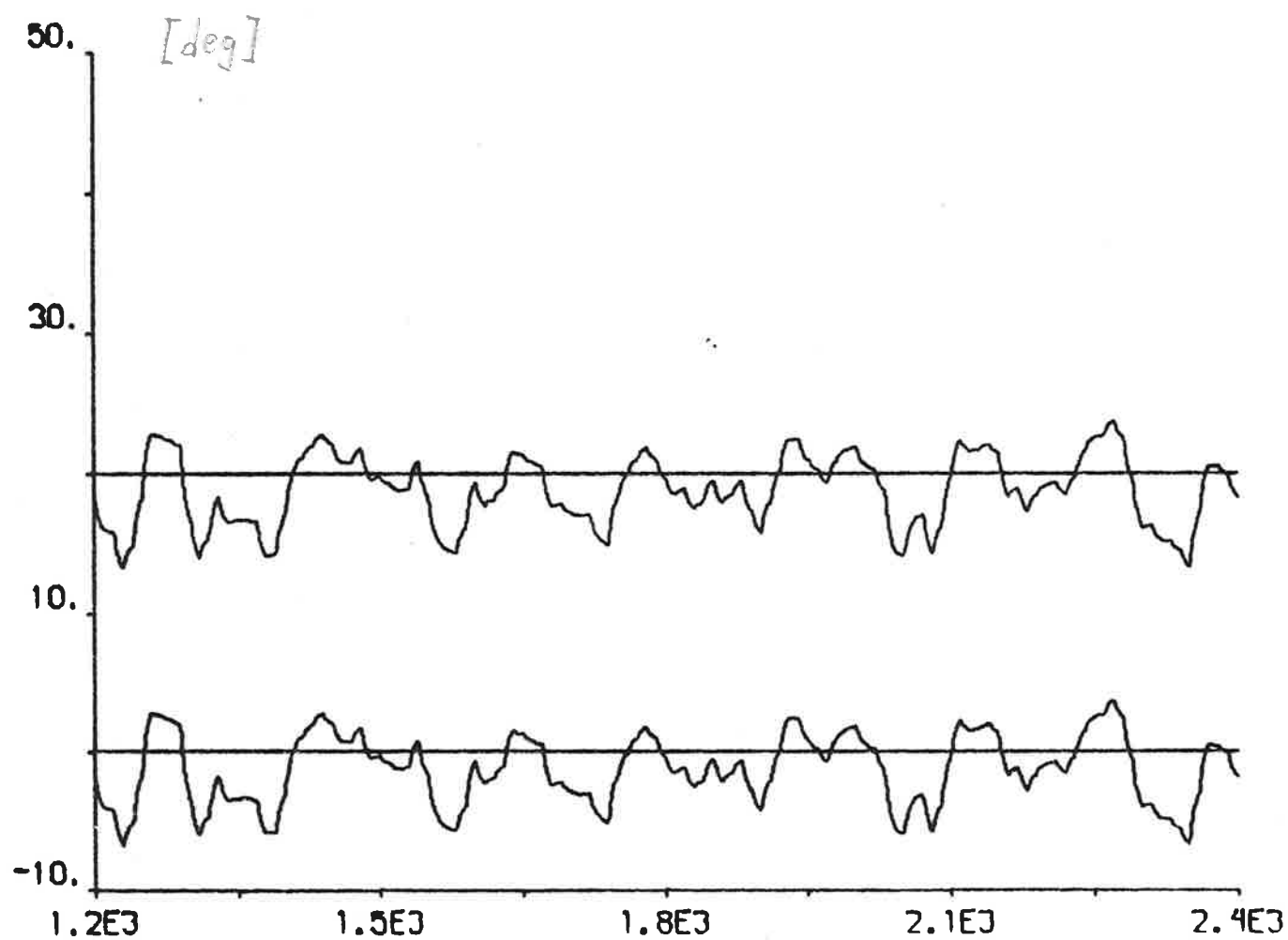
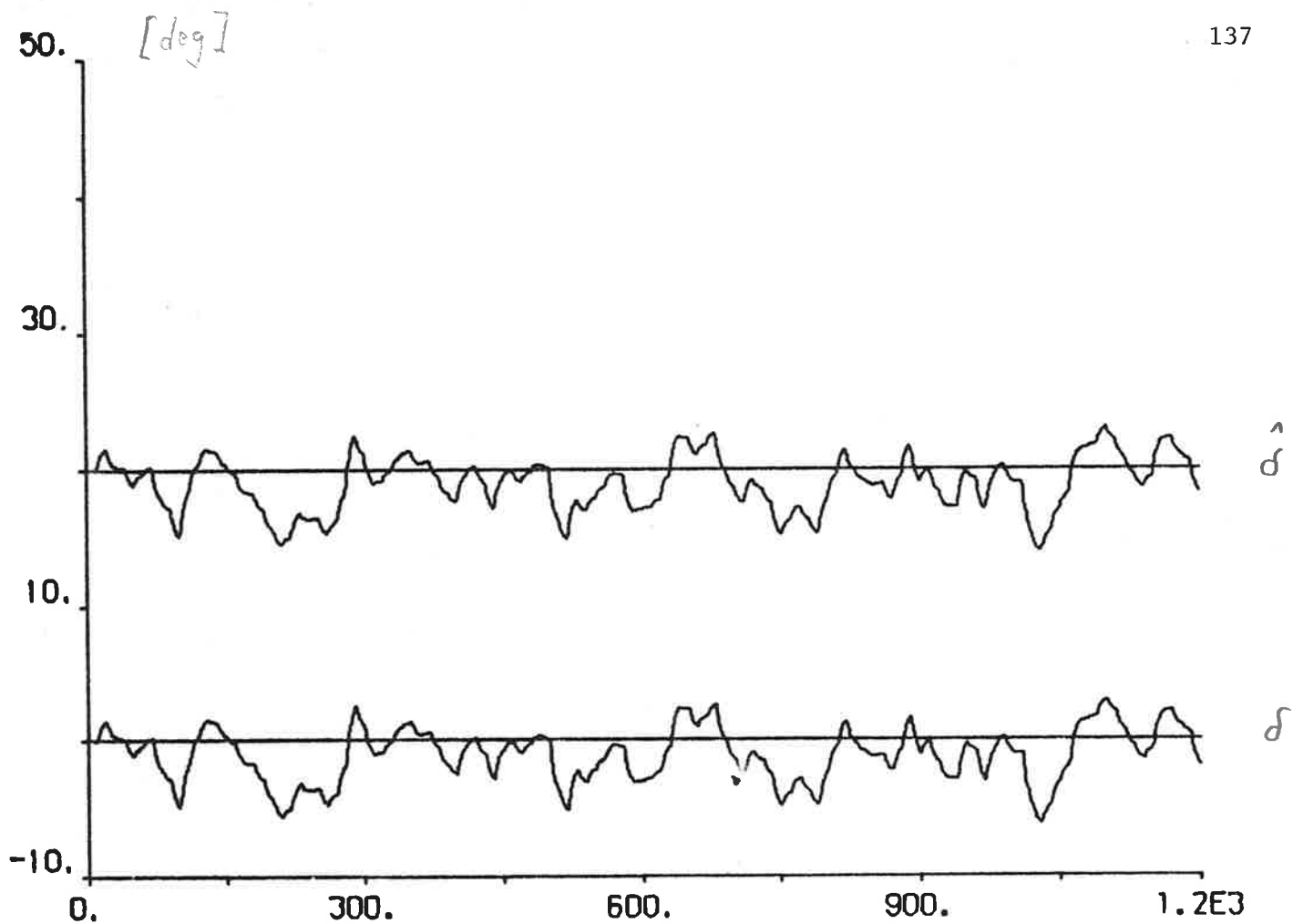


Fig. 4.9f

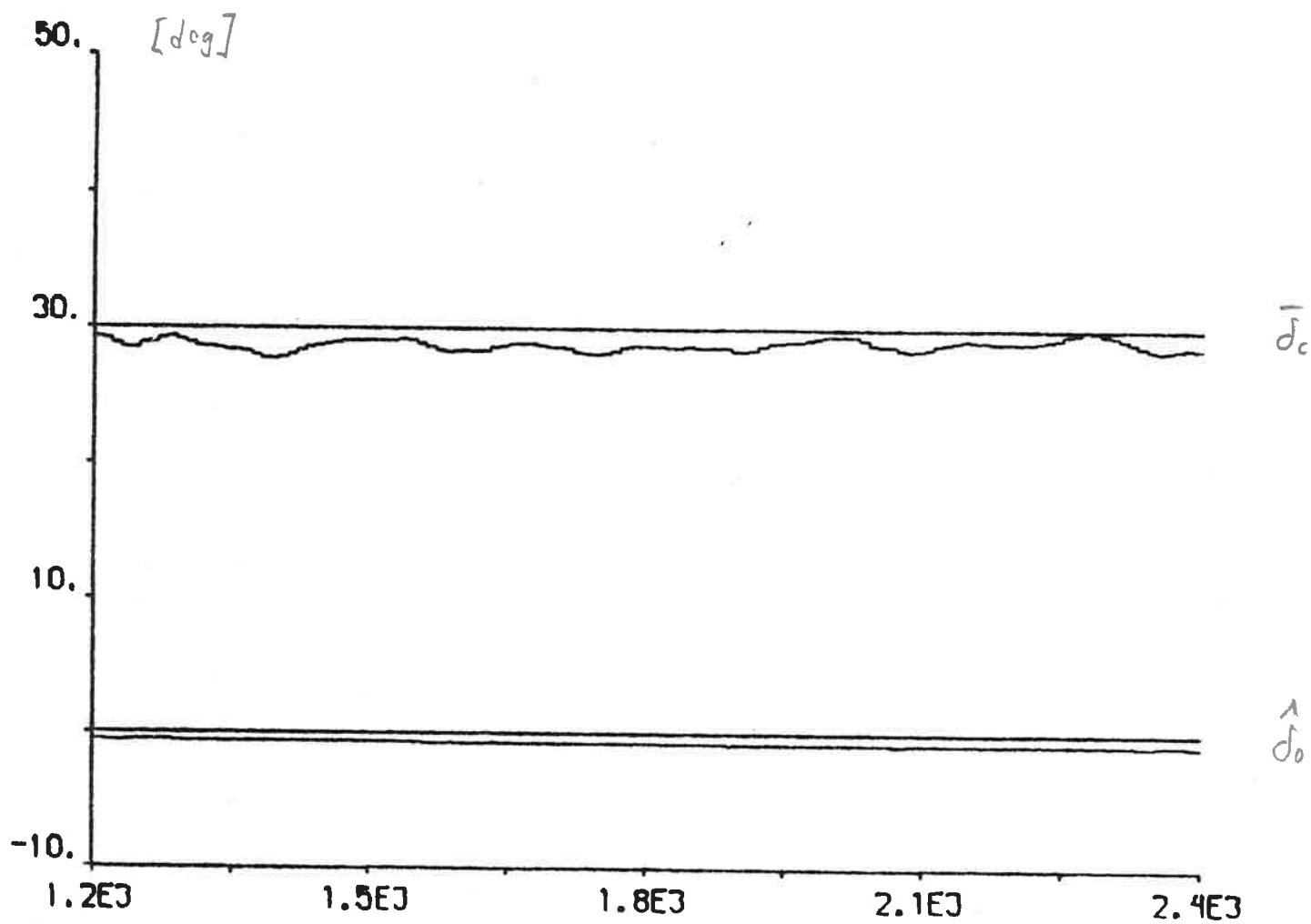
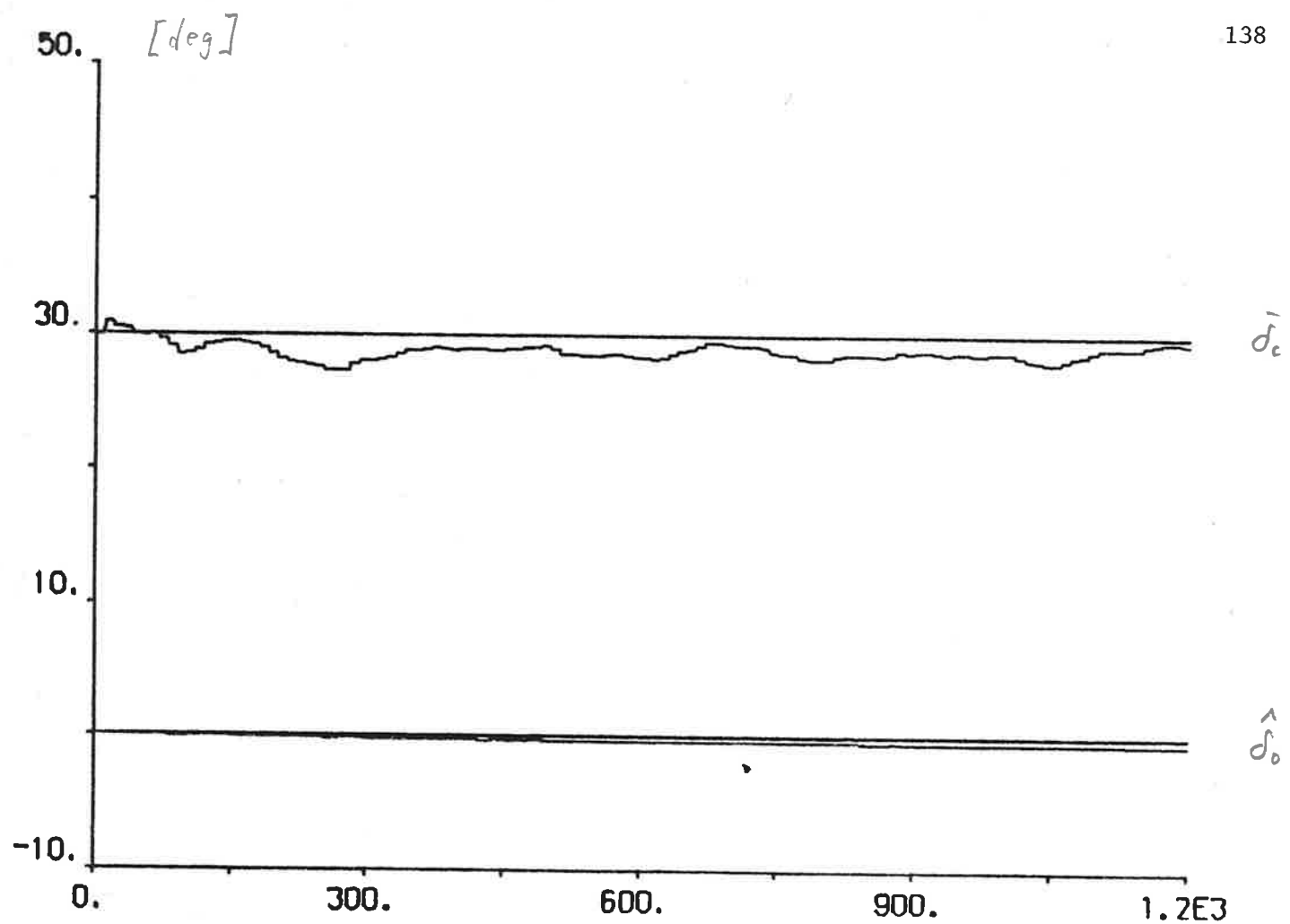


Fig. 4.9g

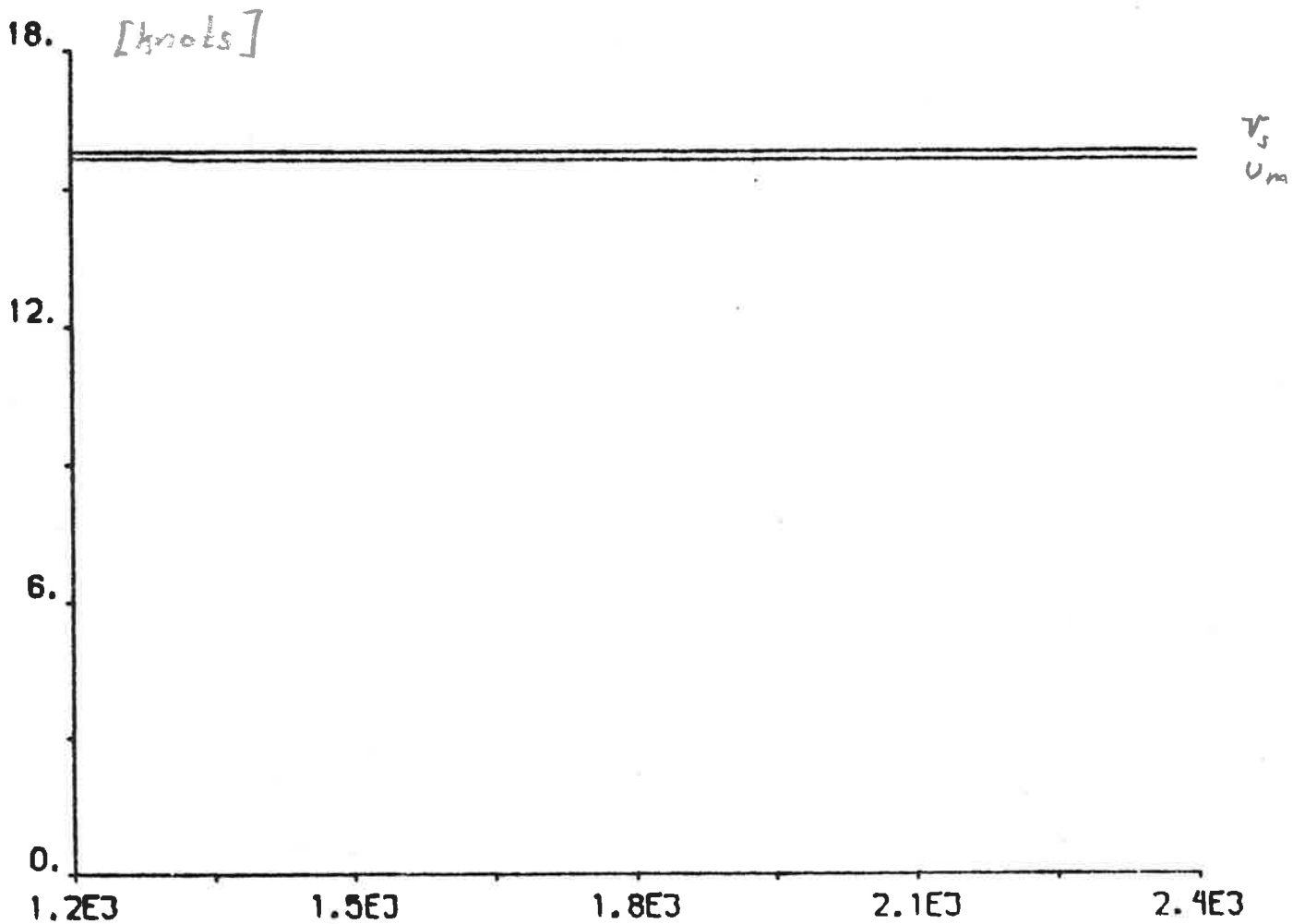
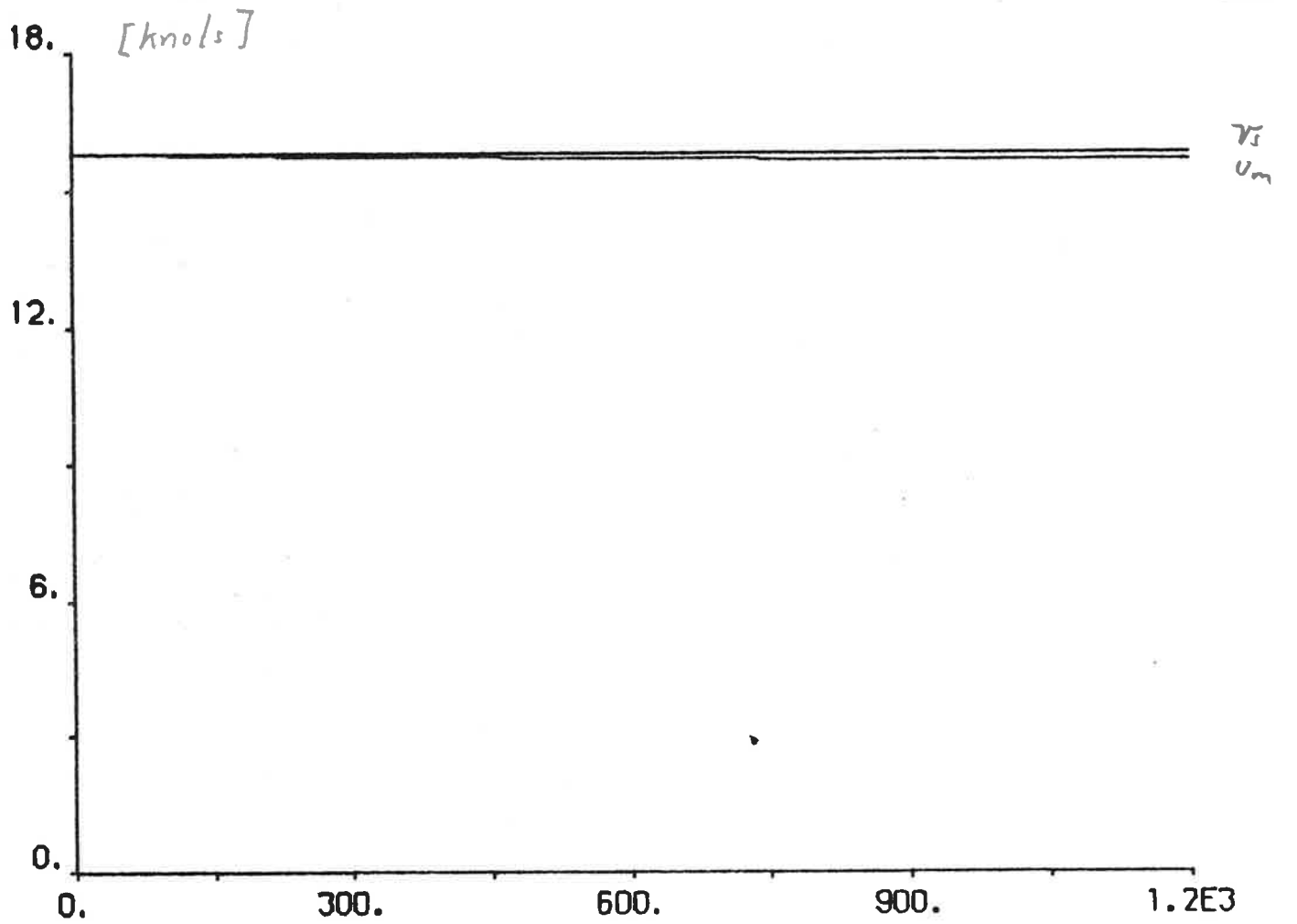


Fig. 4.9h

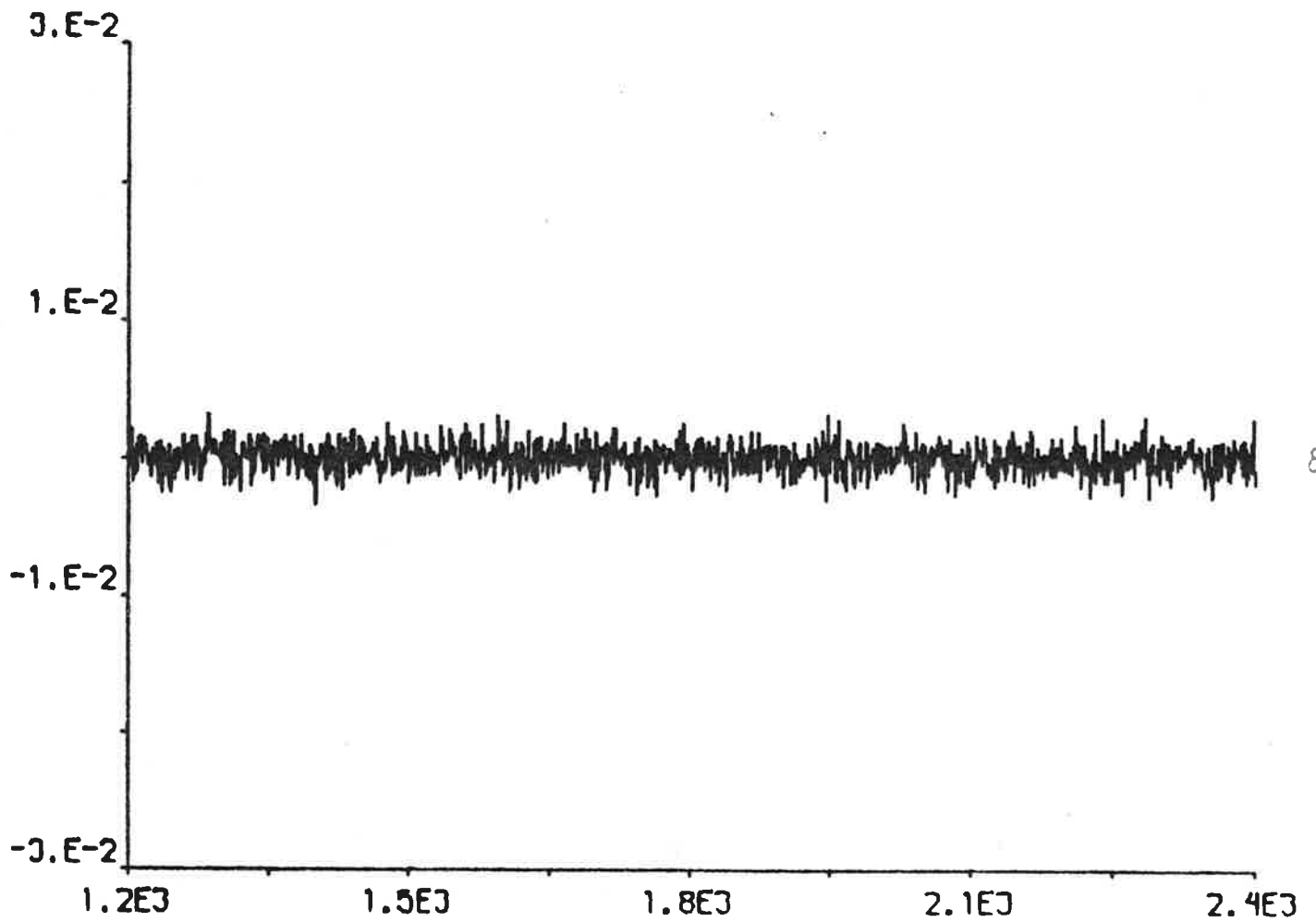
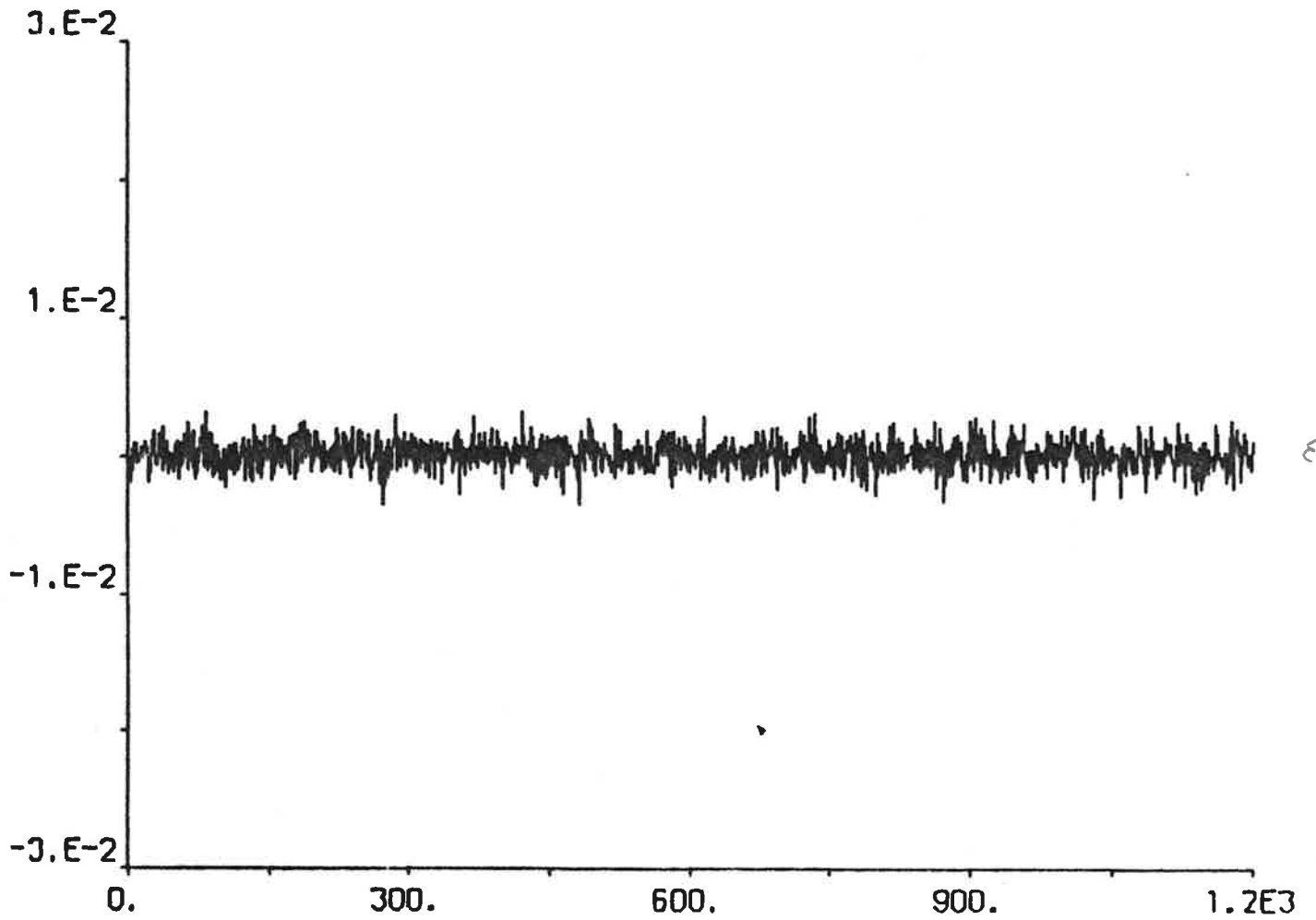


Fig. 4.9i

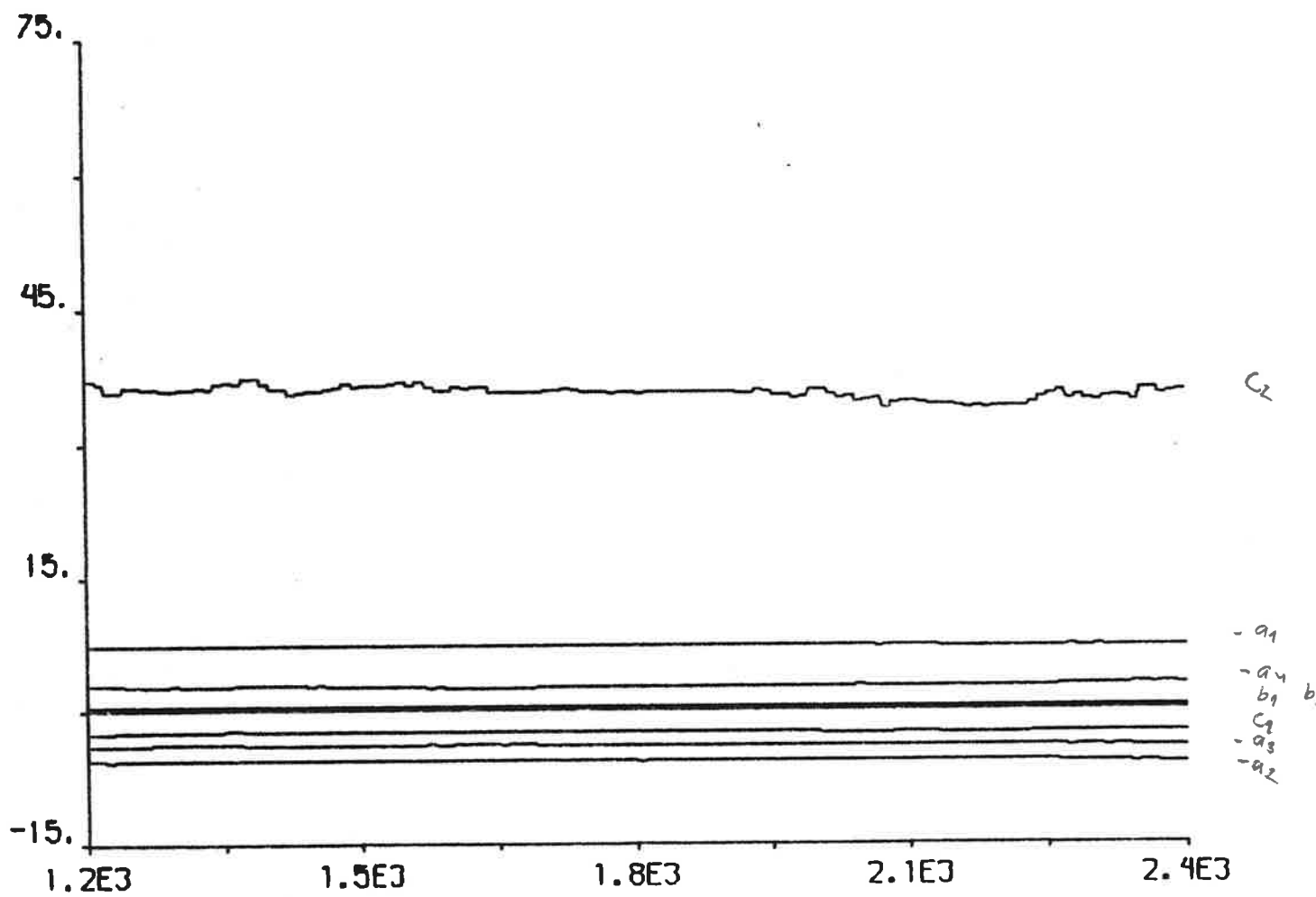
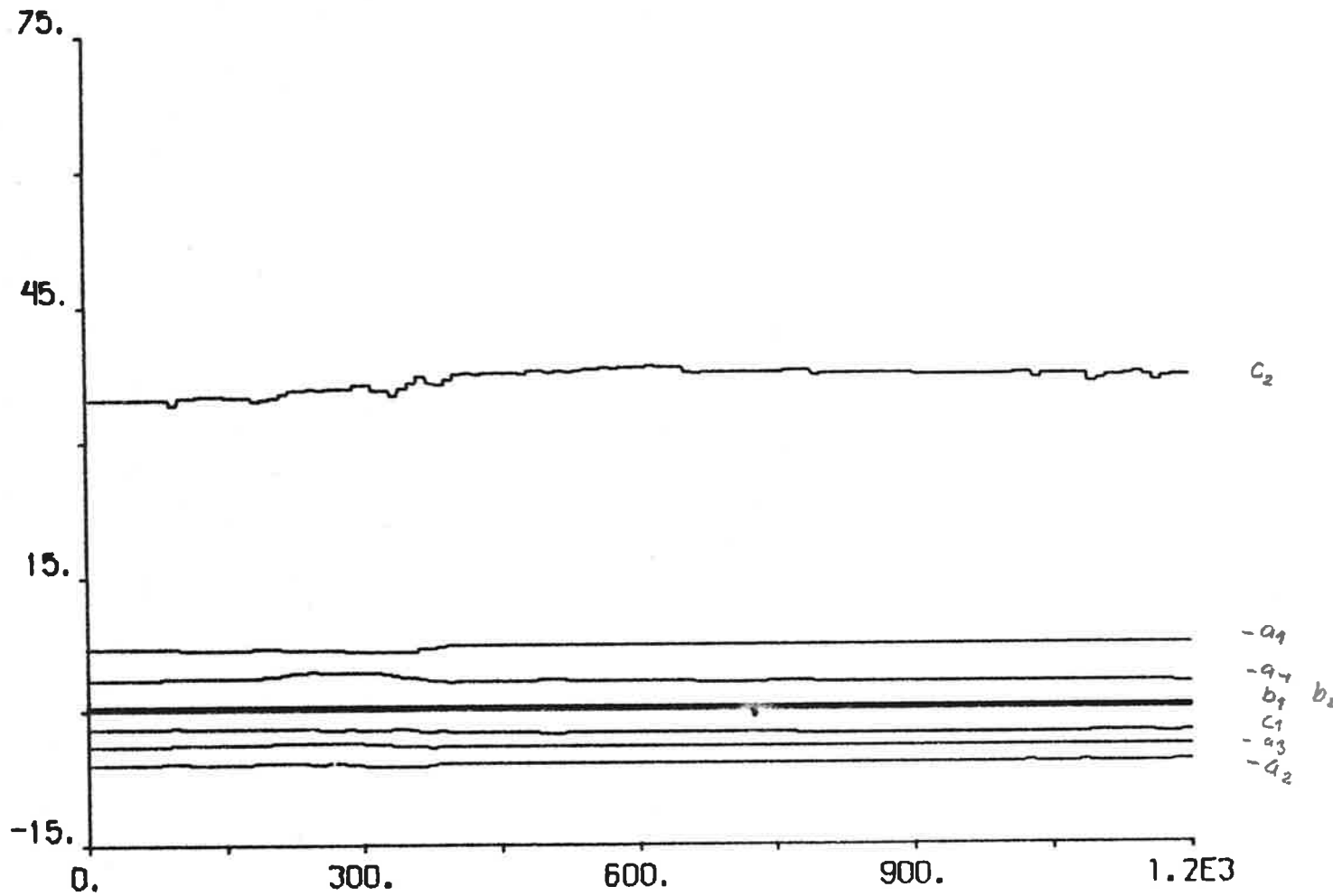


Fig. 4.9j

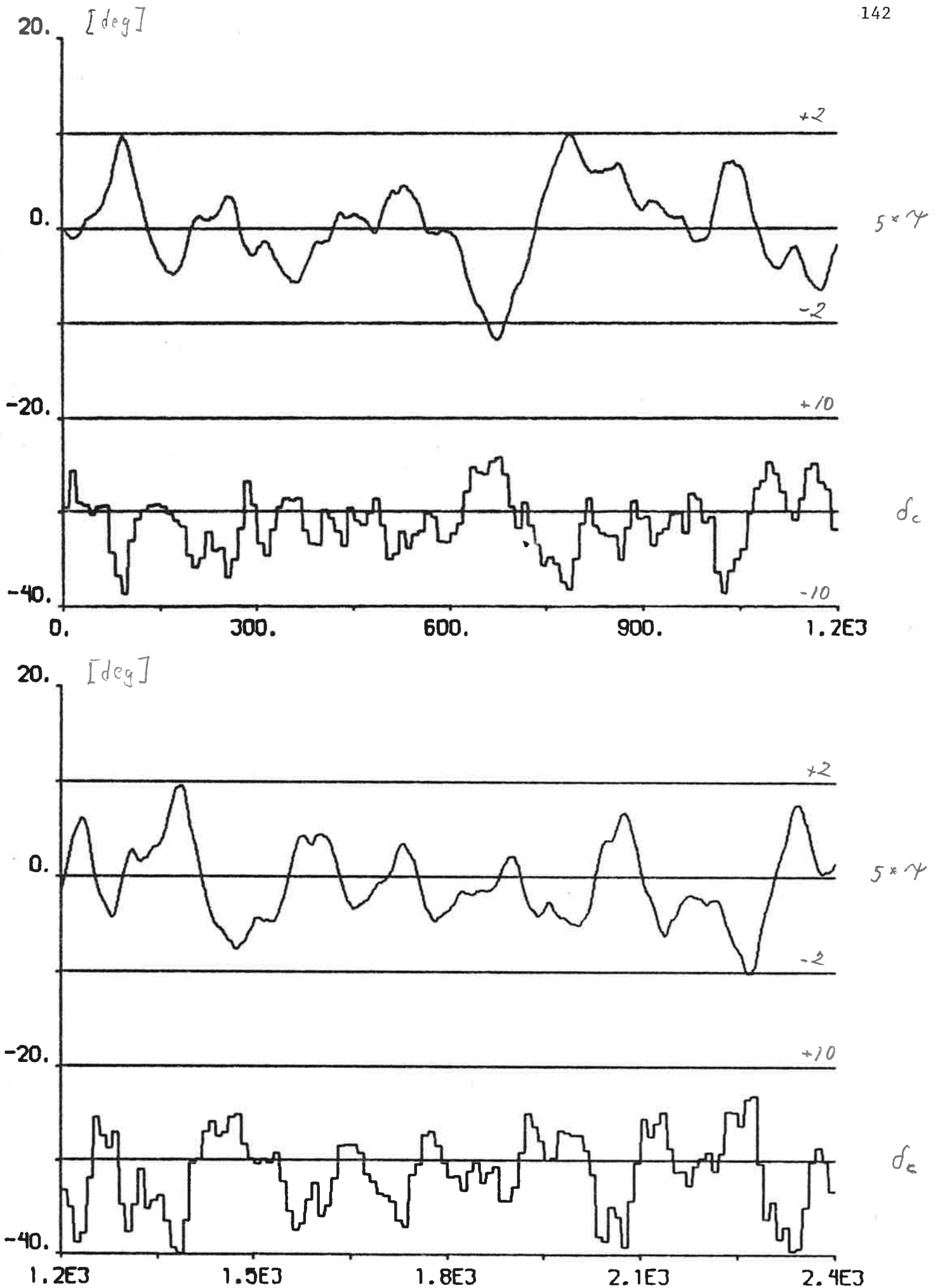


Fig. 4.10a - $T = 22.3$ m, $n_0 = 87.6$ rpm, $u_0 = 15.8$ knots, $\delta_\ell = 10$ deg, self-tuning regulator using estimates from the Kalman filter. The covariance matrix R_w is given by (4.5).

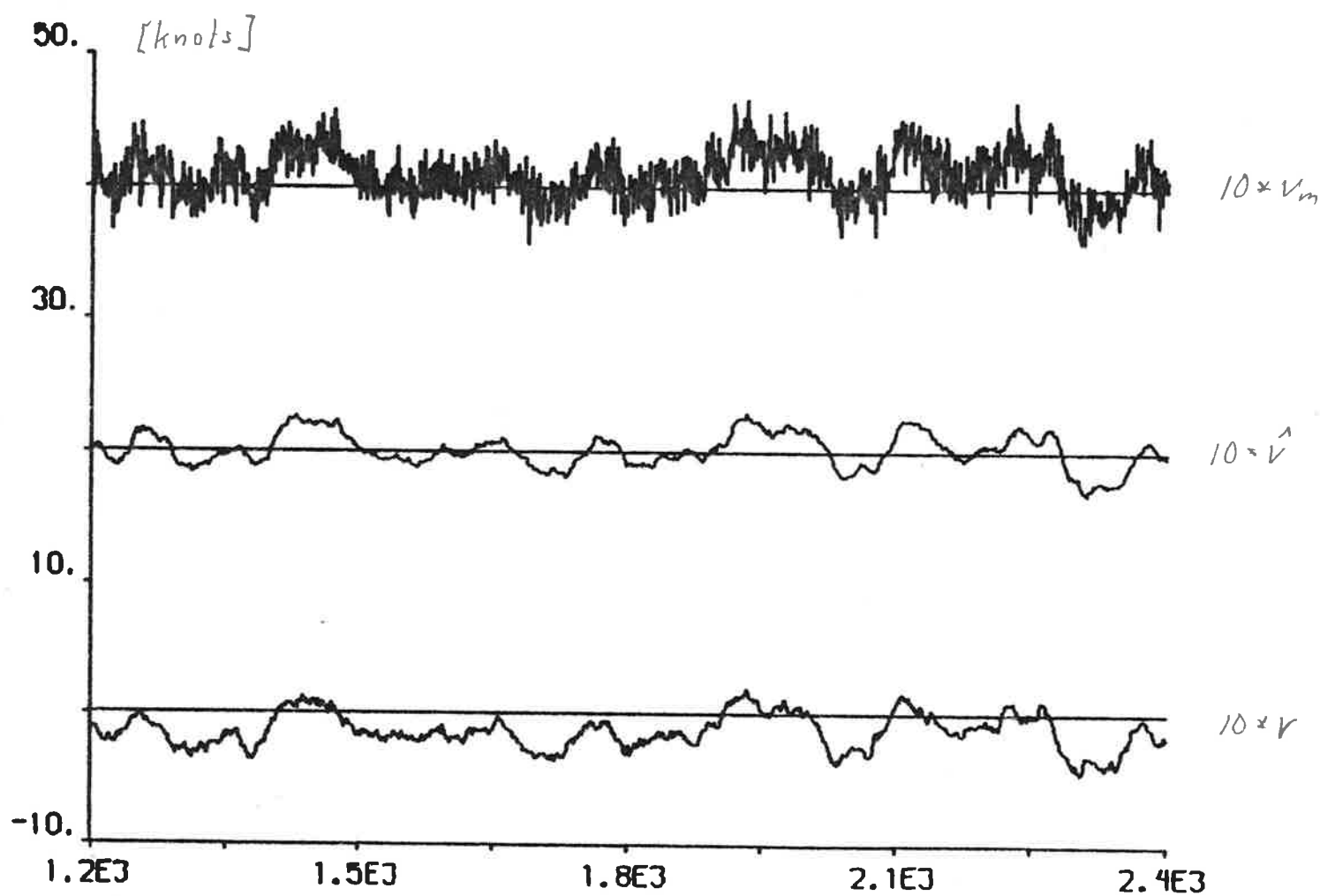
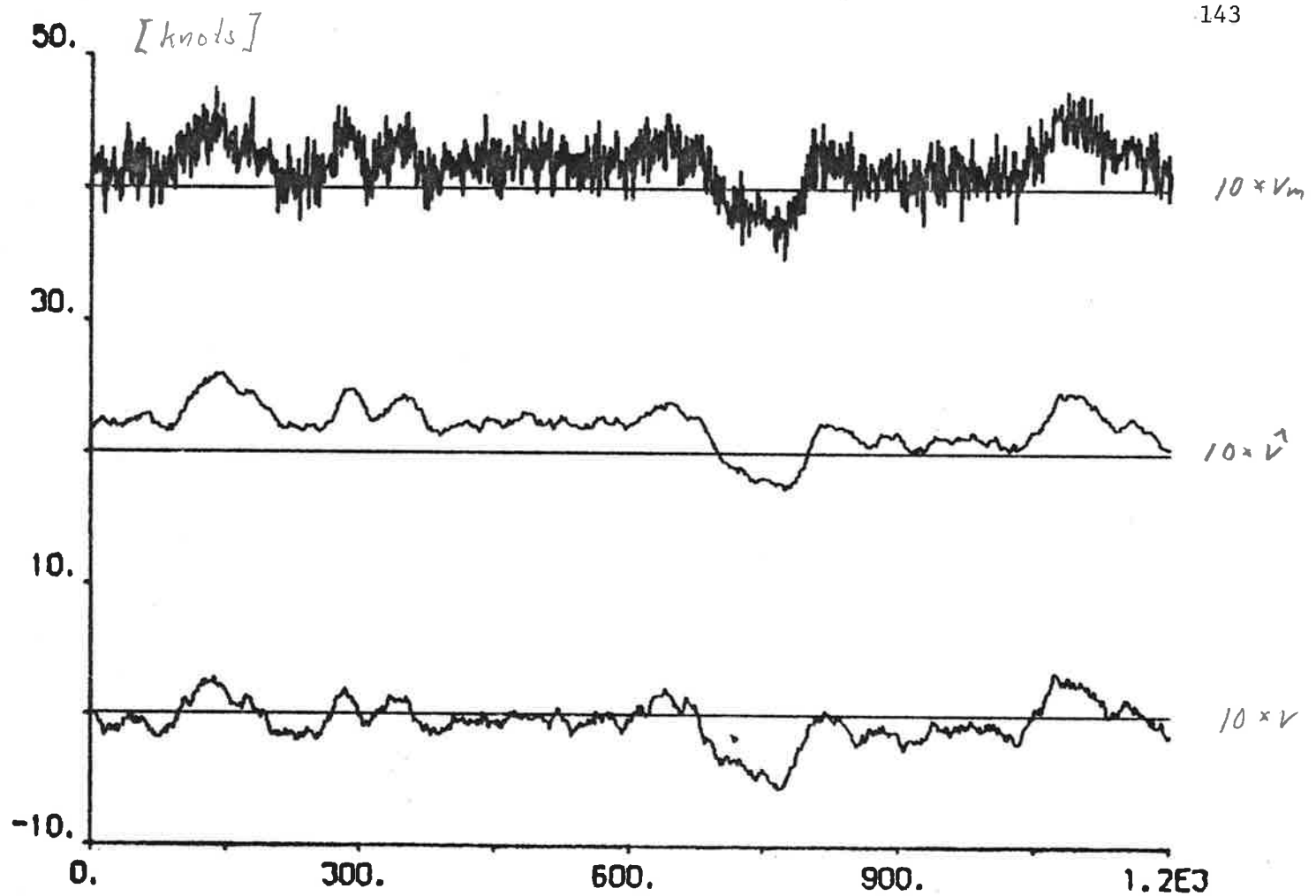


Fig. 4.10 b

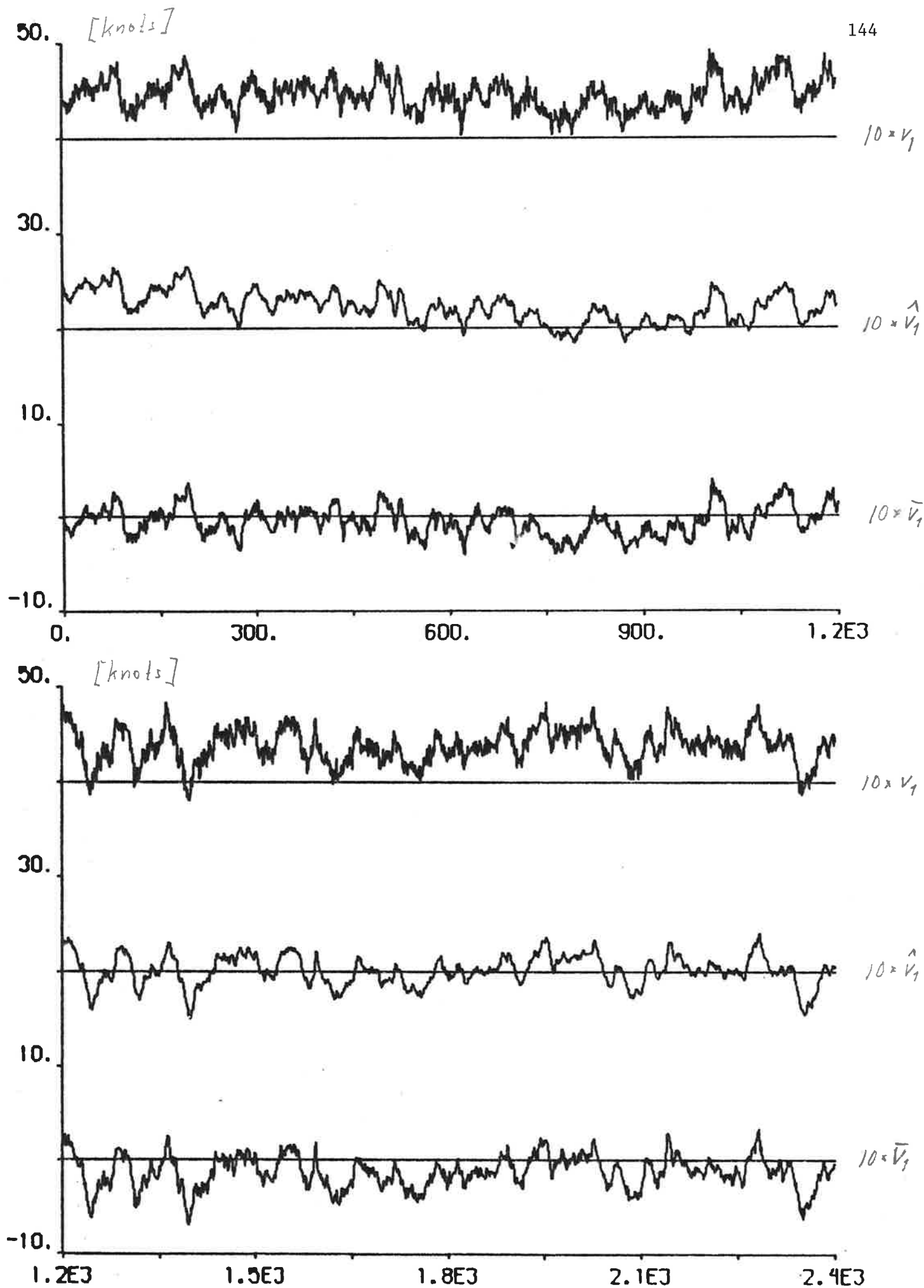


Fig. 4.10 c

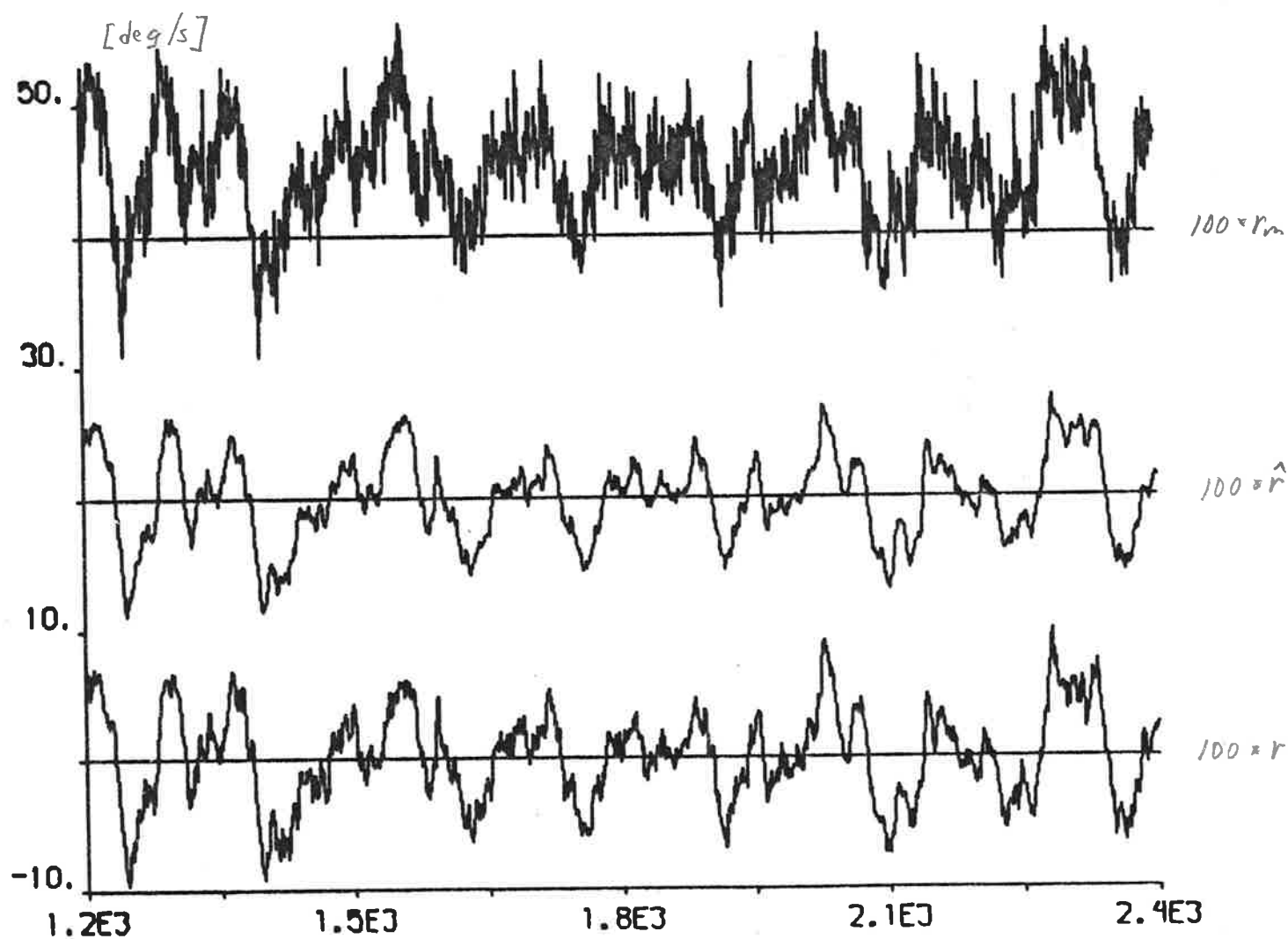
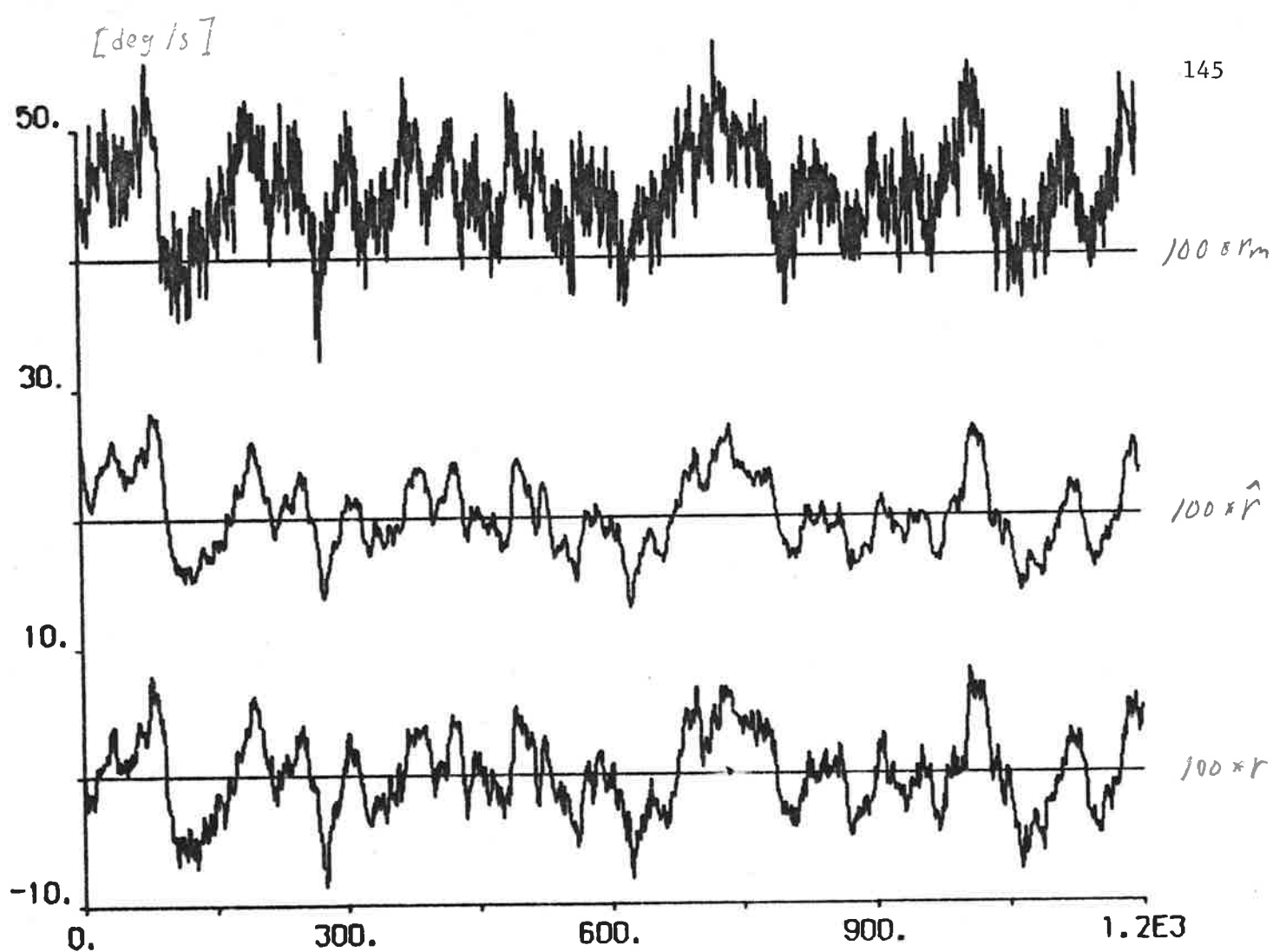


Fig. 4.10 d

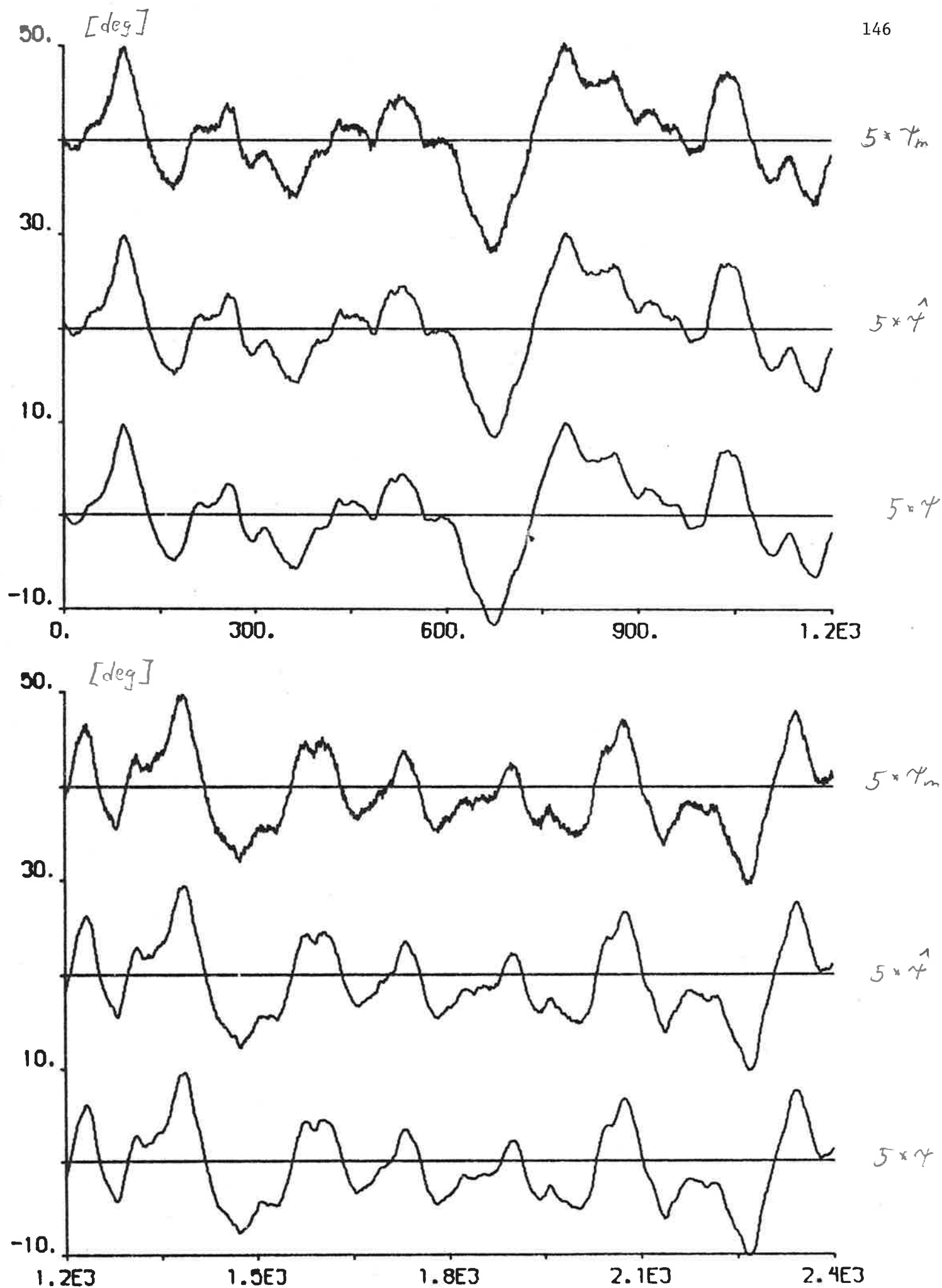


Fig. 4.10 e

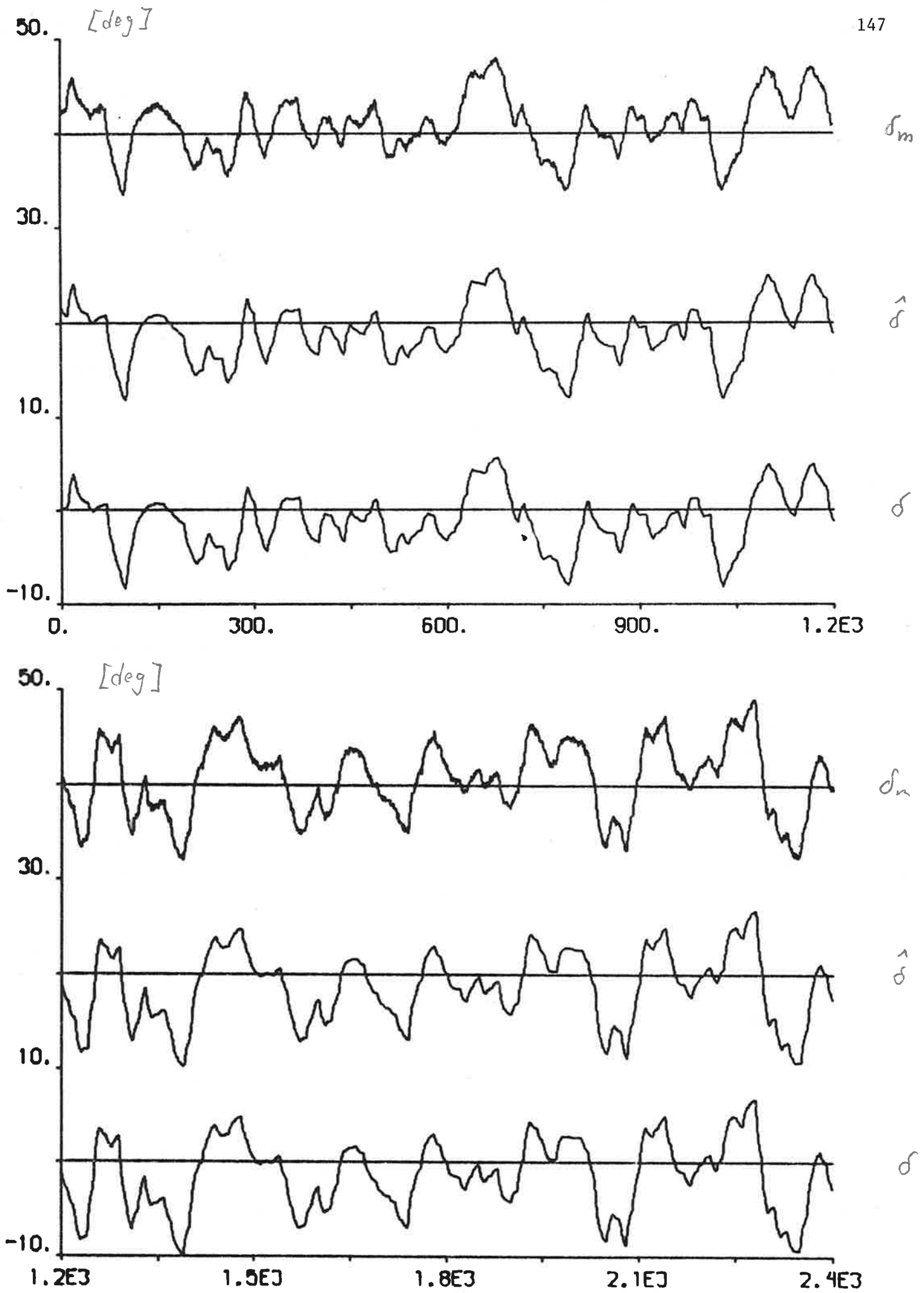


Fig. 4.10 f

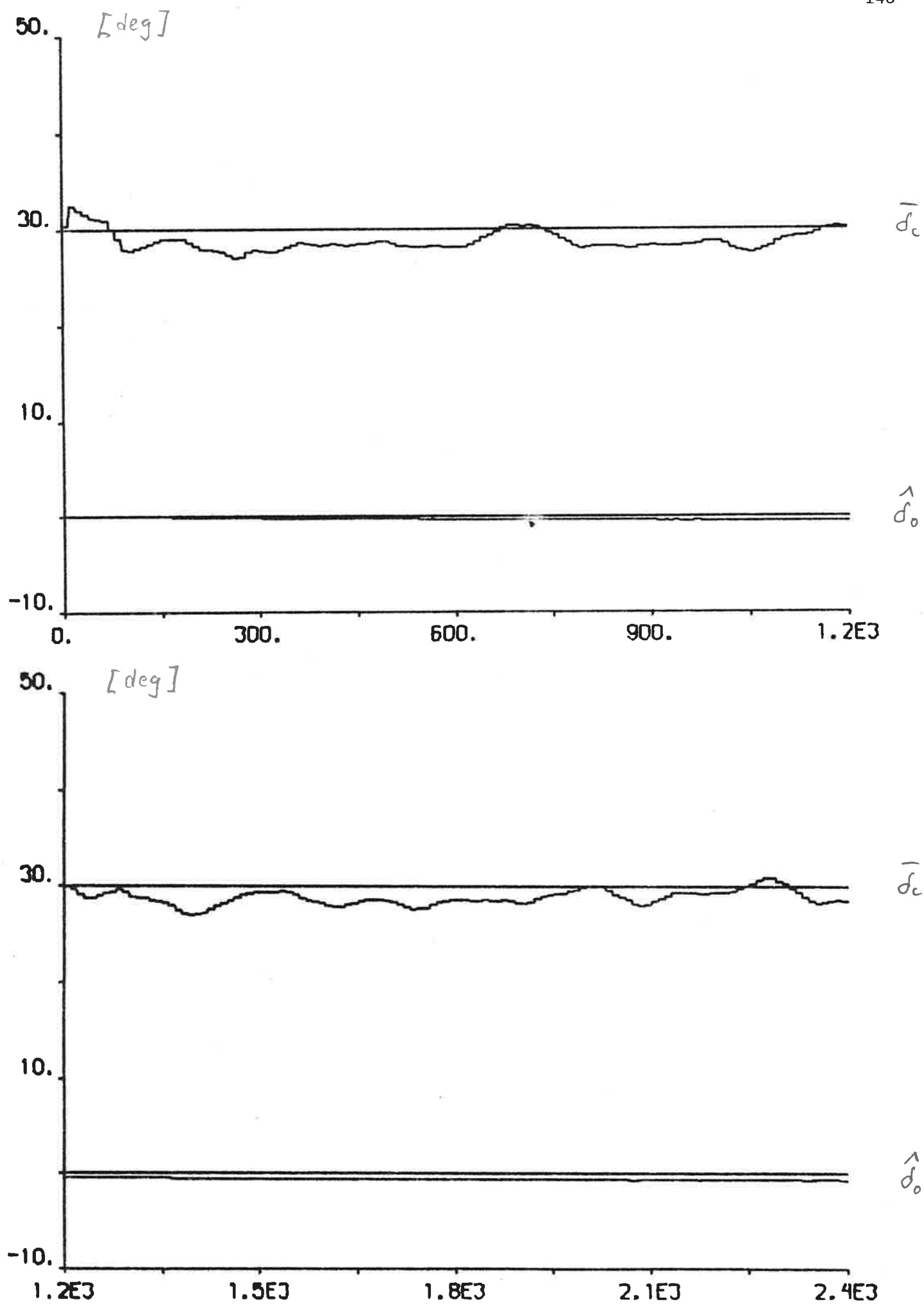


Fig. 4.10 g

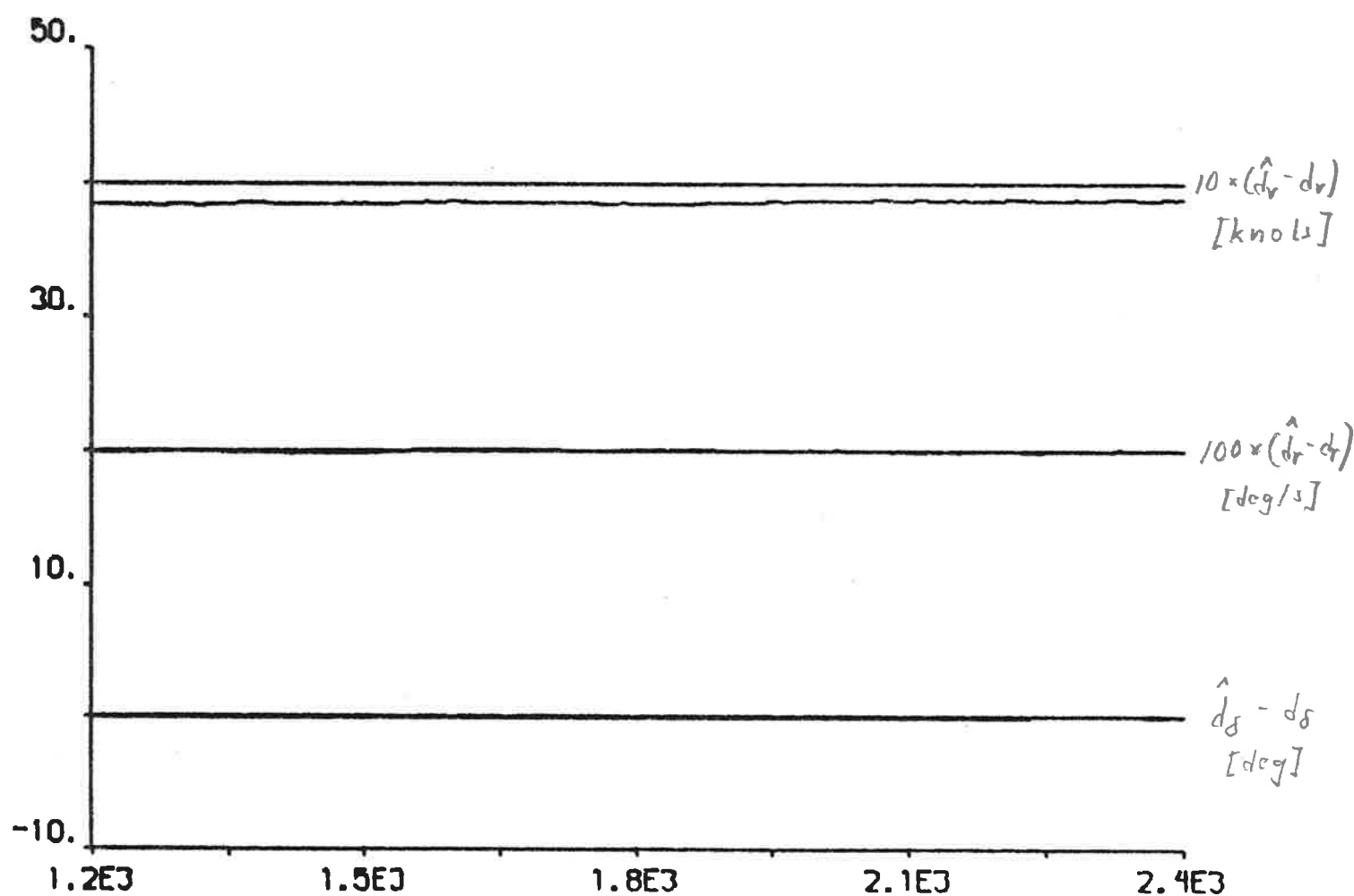
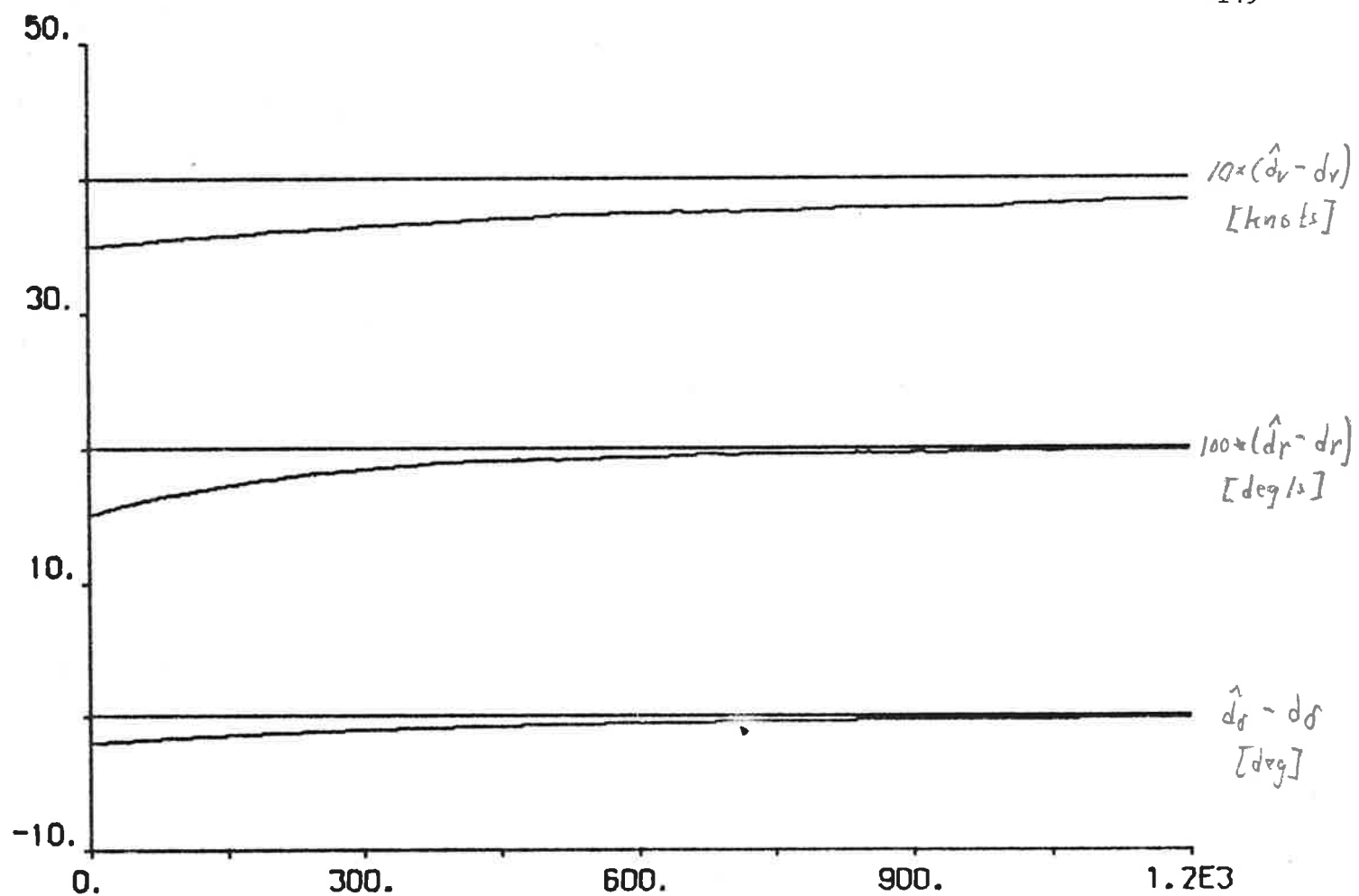


Fig. 4.10 h

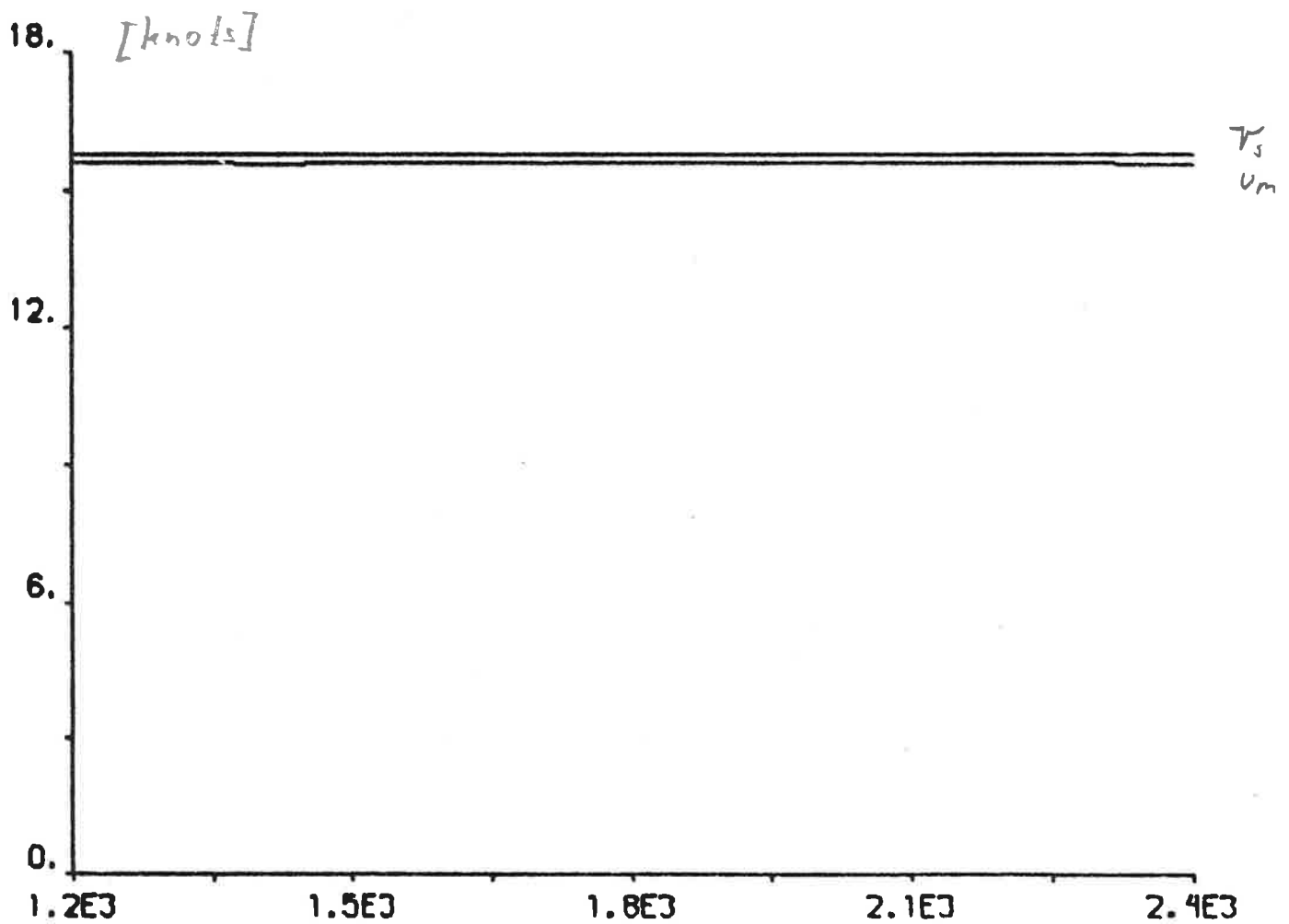
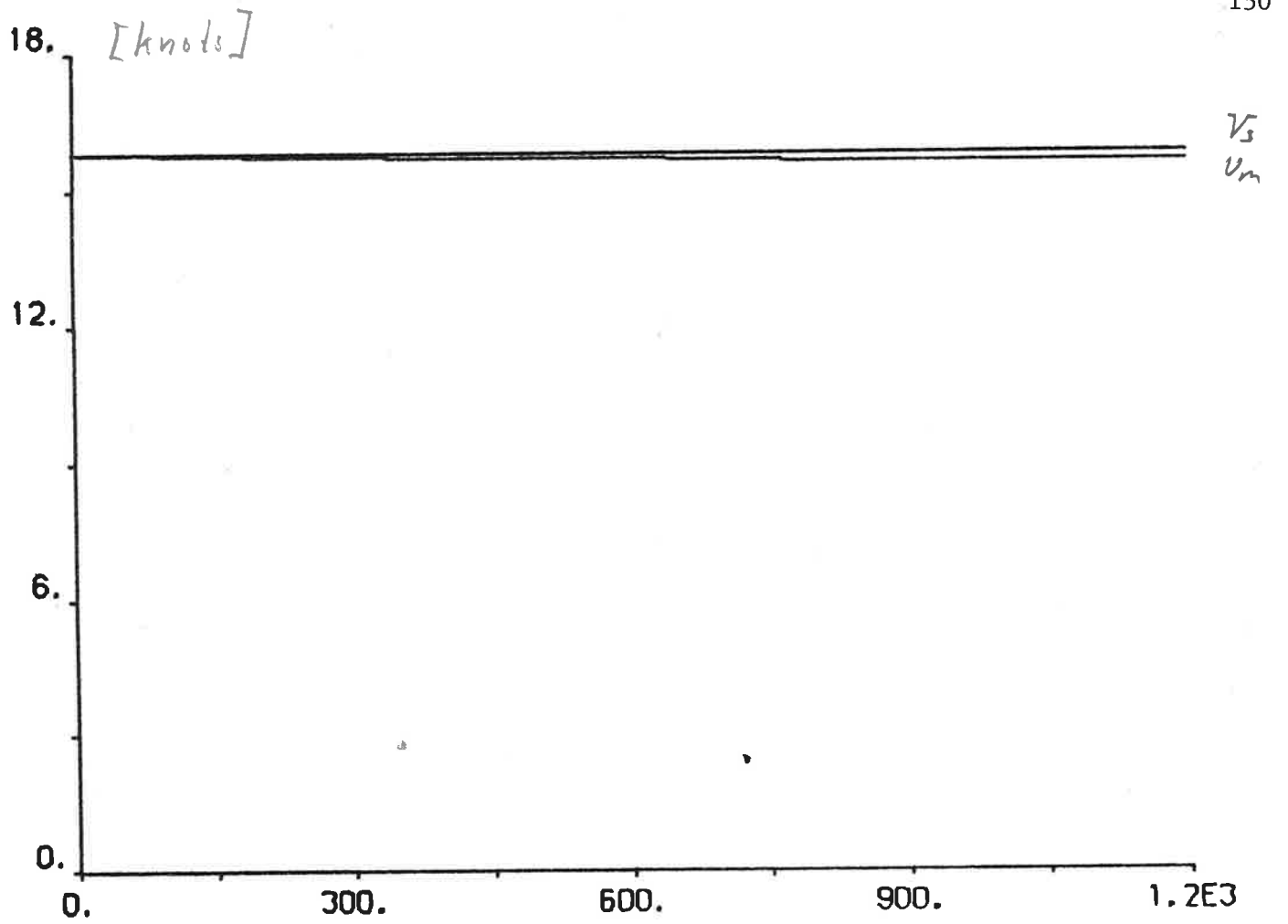


Fig. 4.10 i

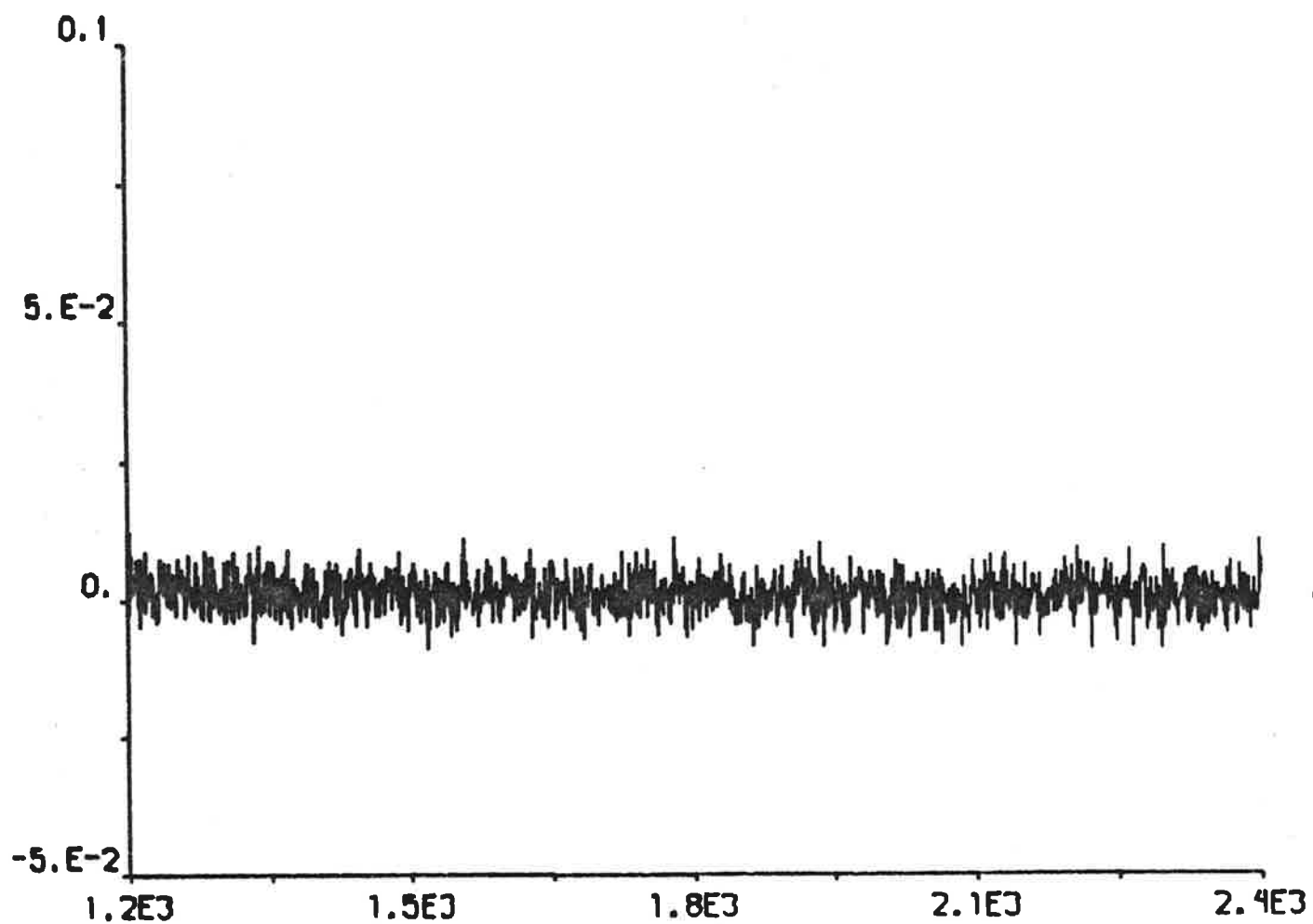
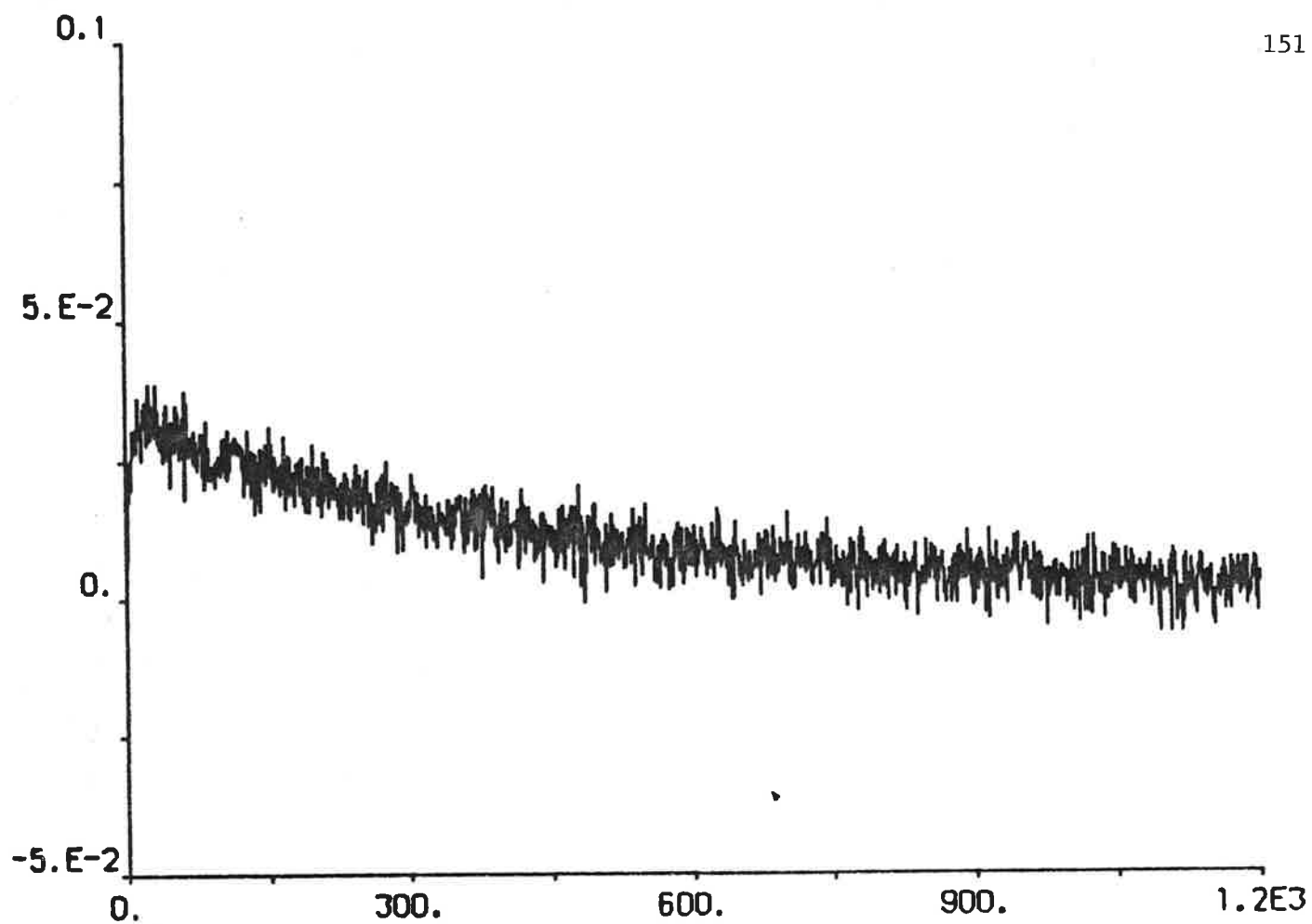


Fig. 4.10 j

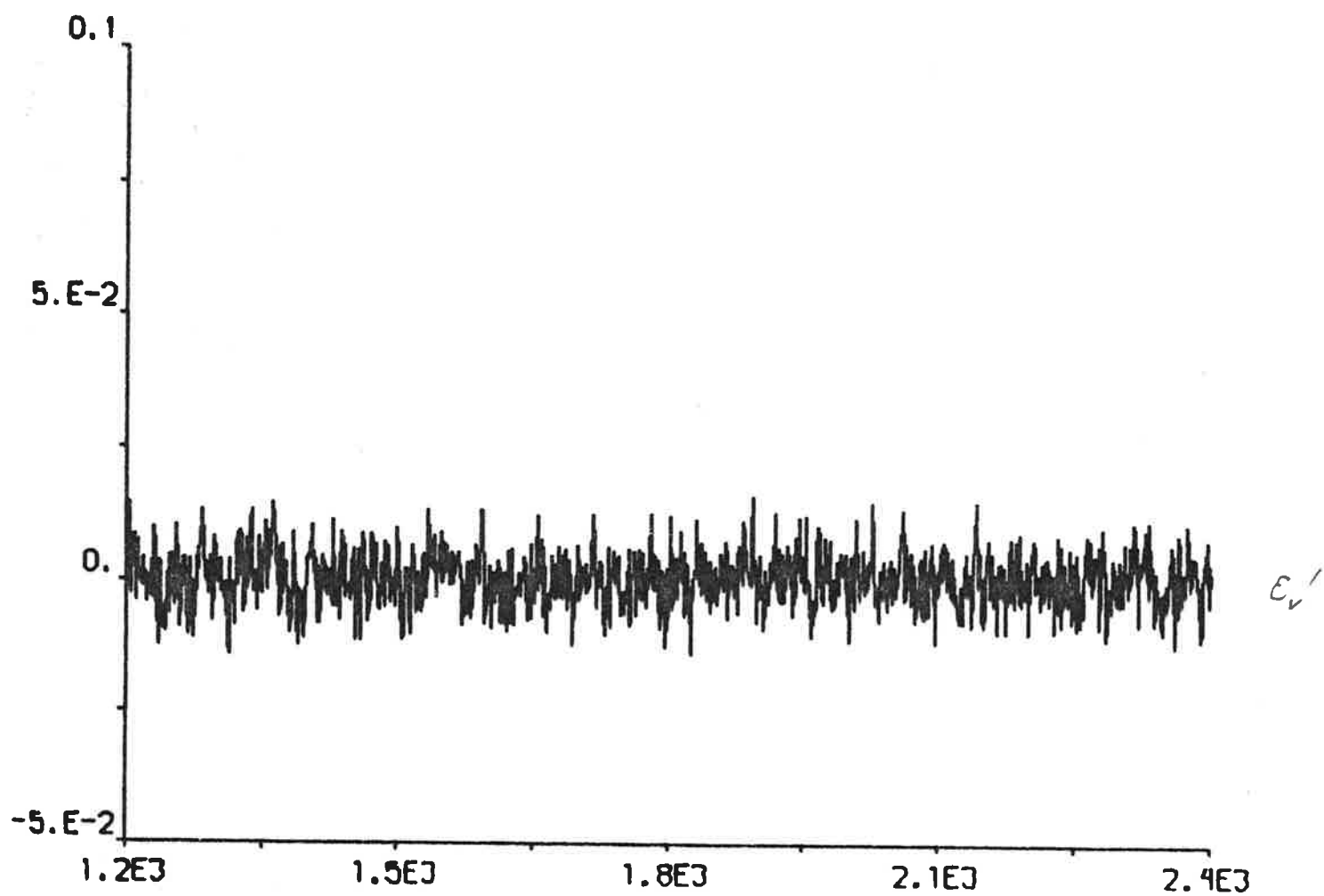
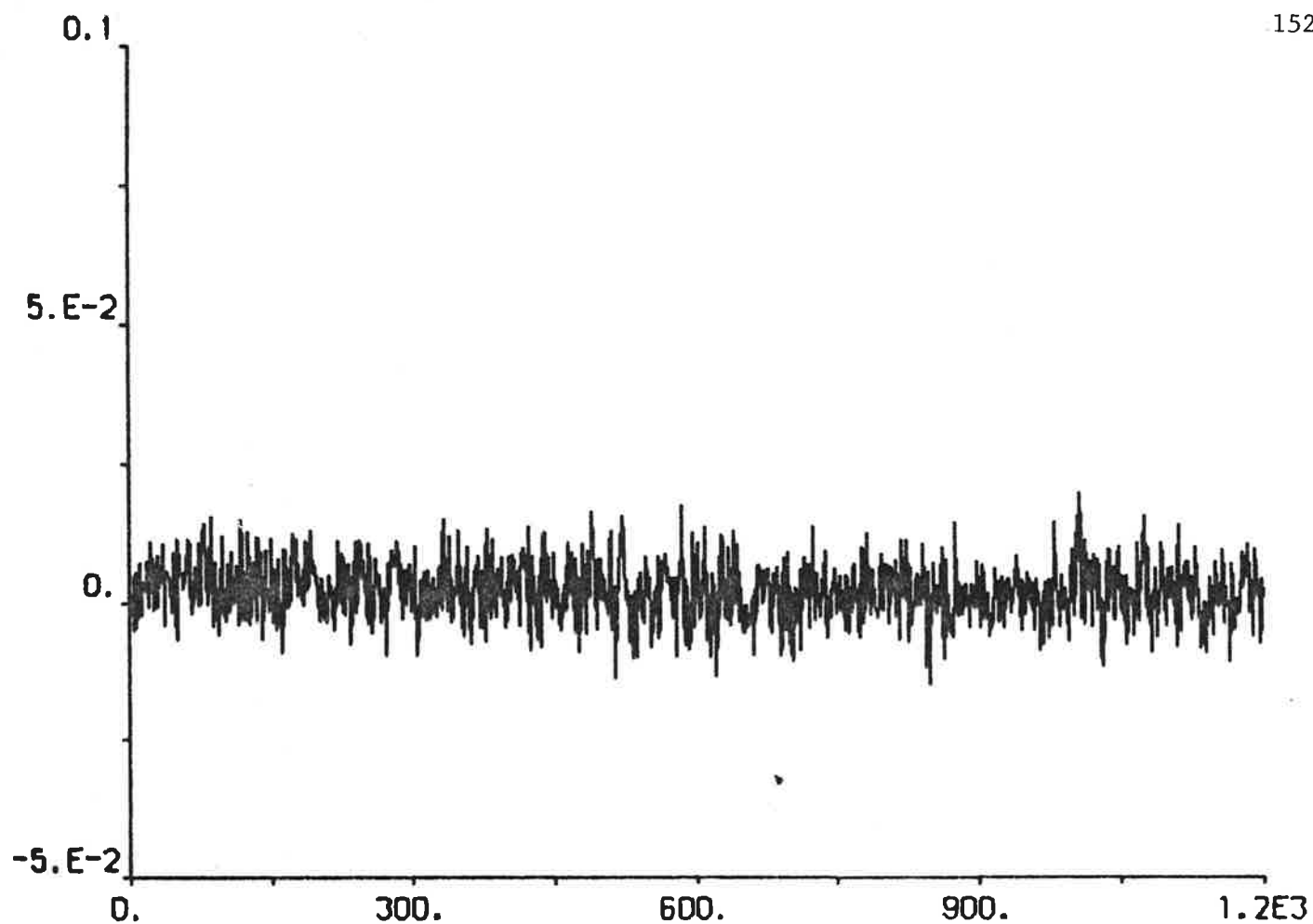


Fig. 4.10 k

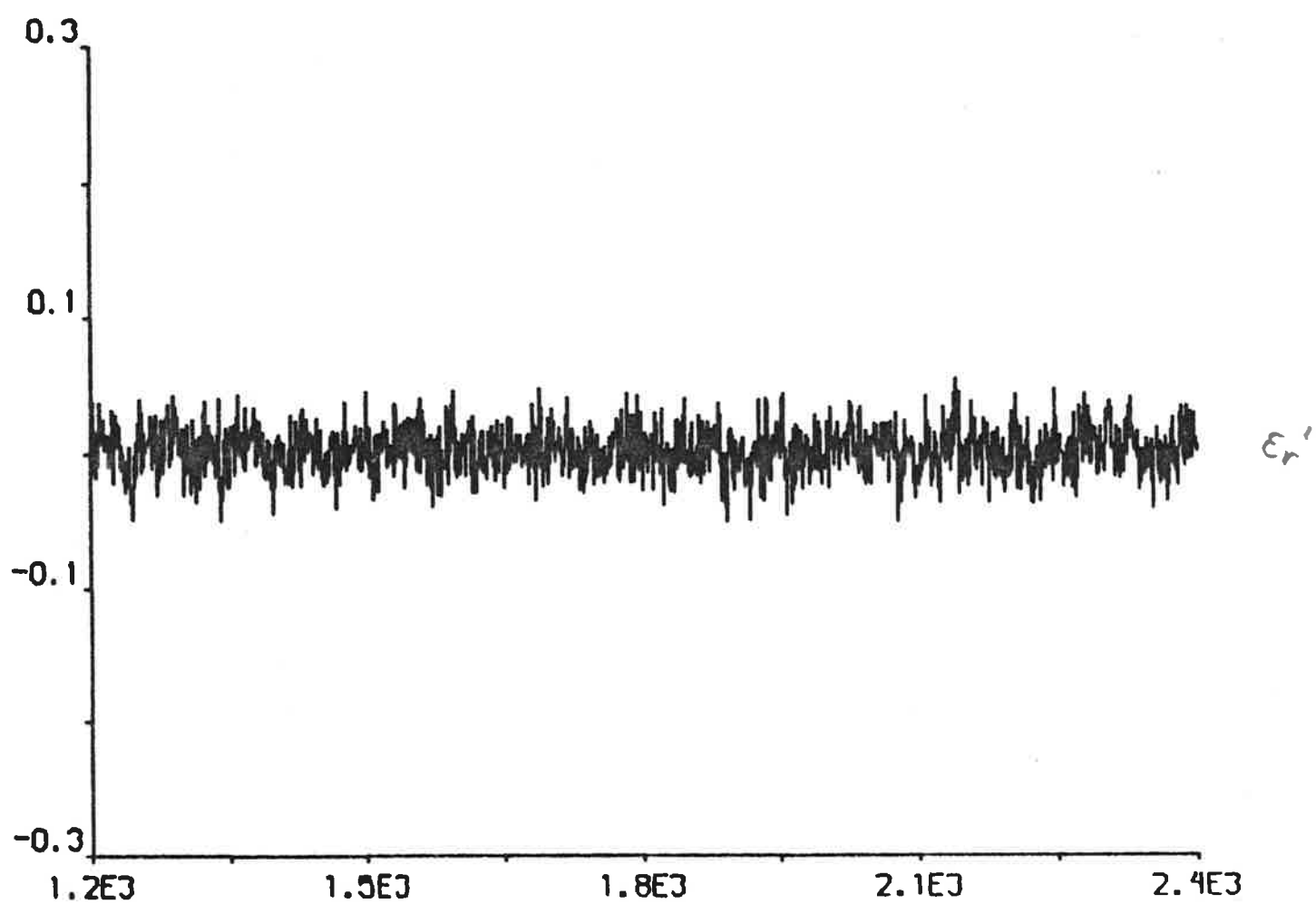
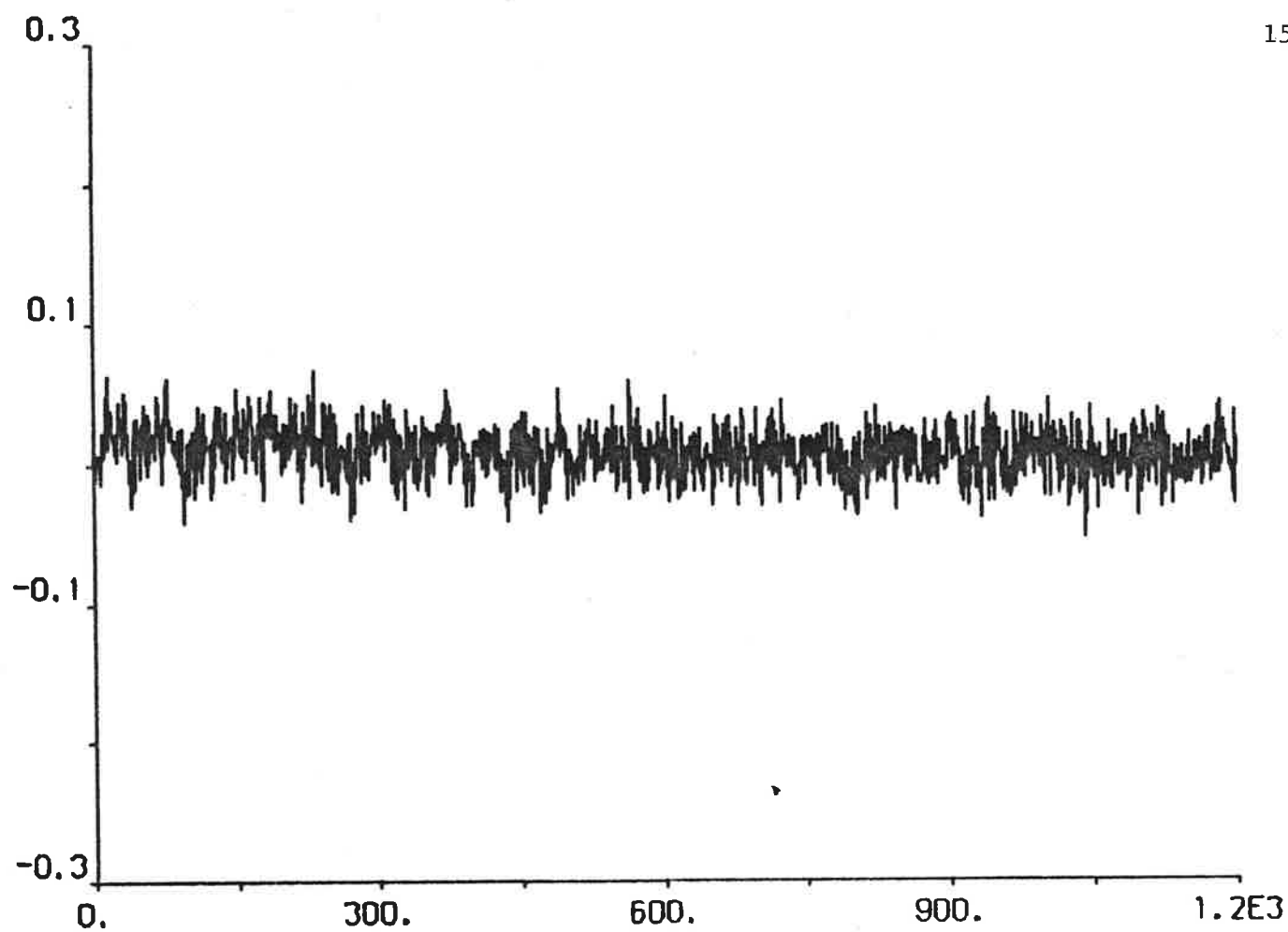


Fig. 4.10 ℓ

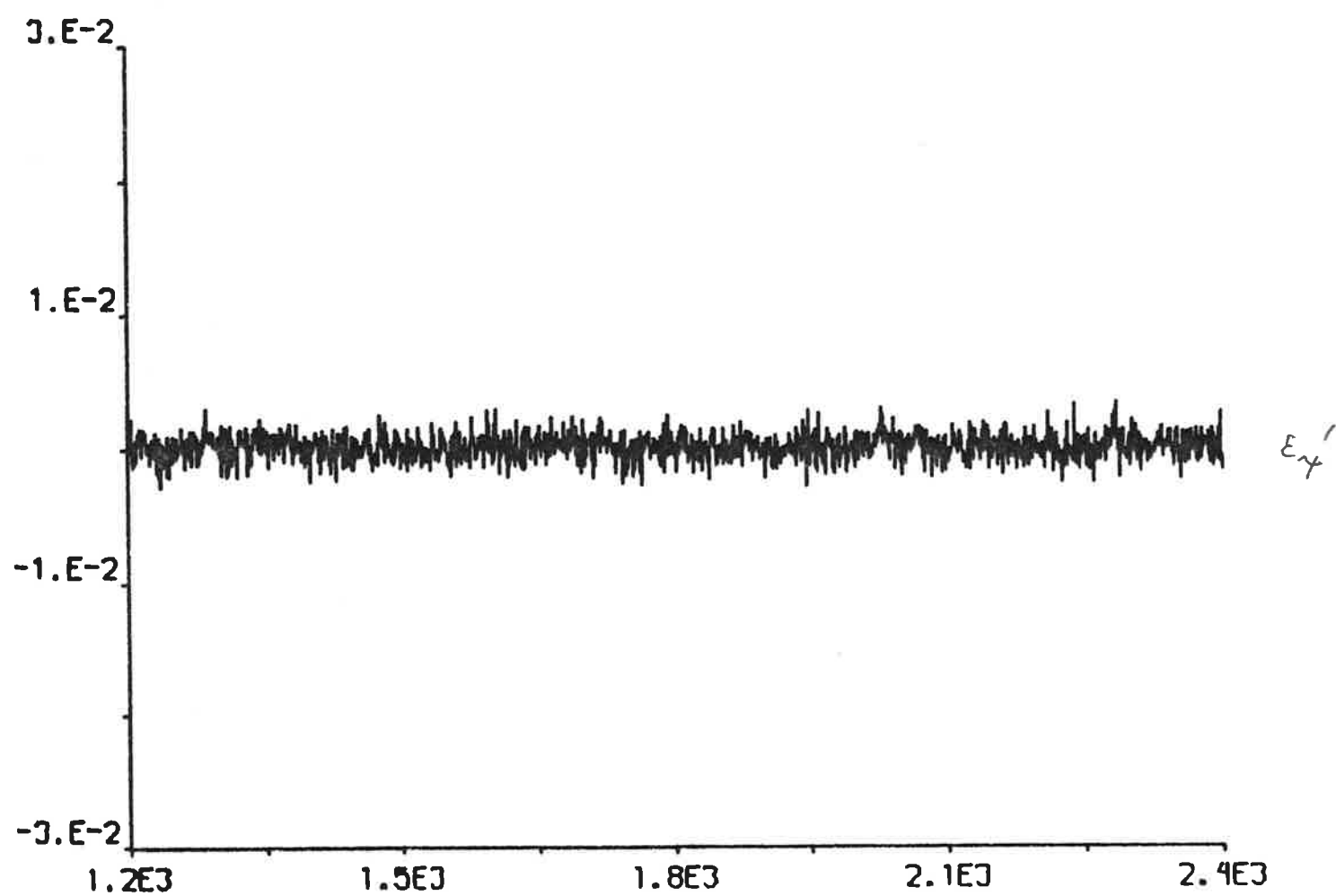
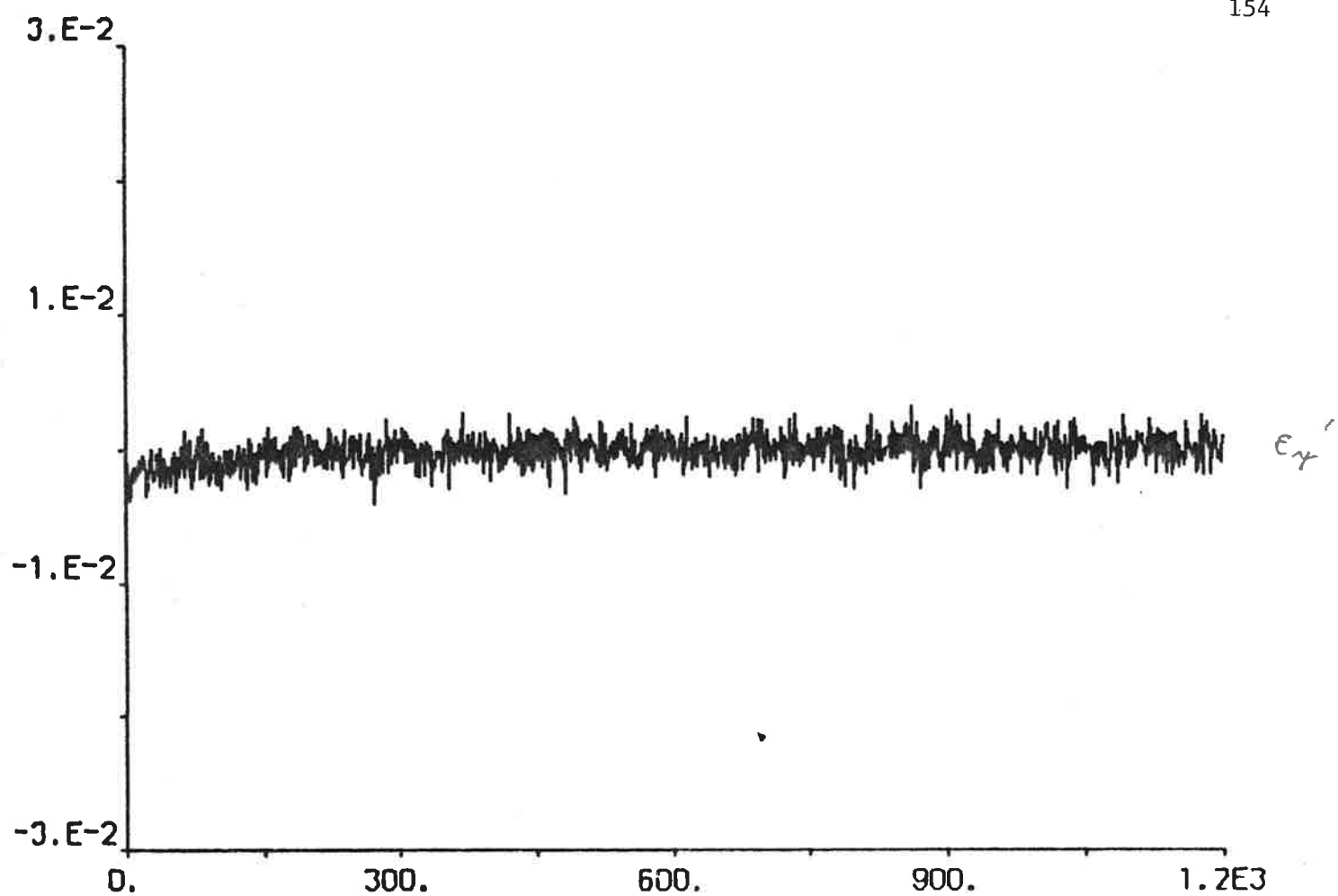


Fig. 4.10 m

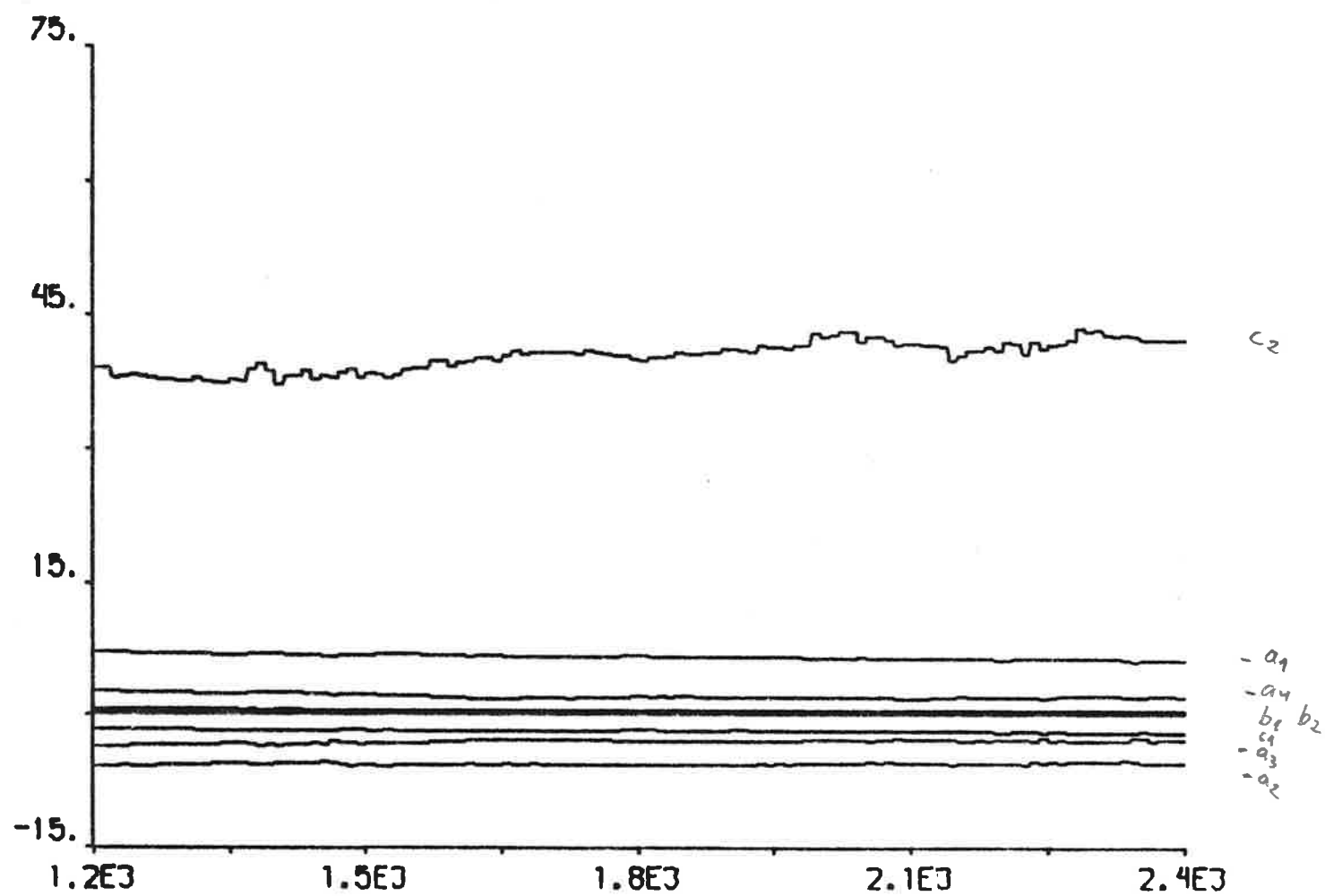
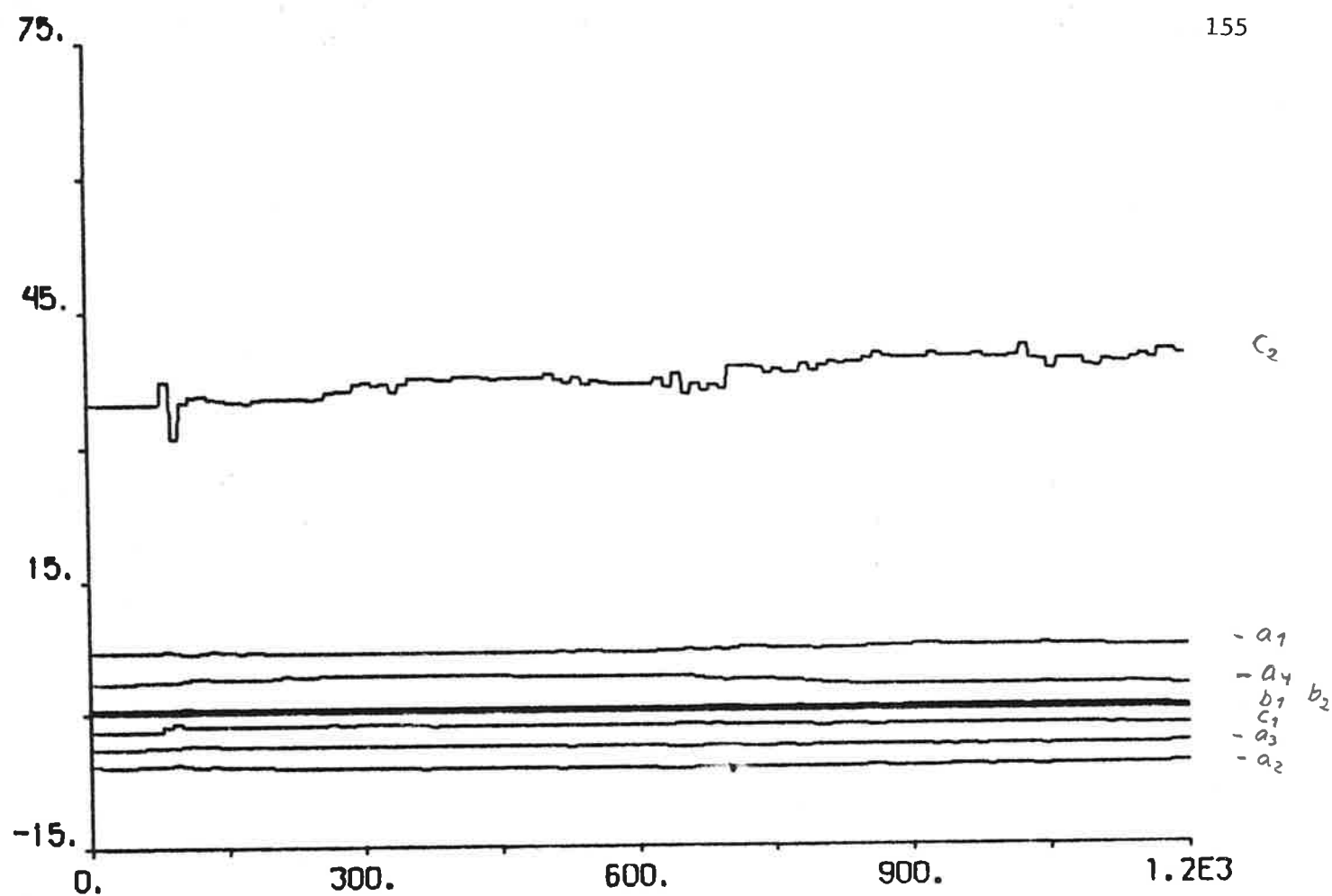


Fig. 4.10 n

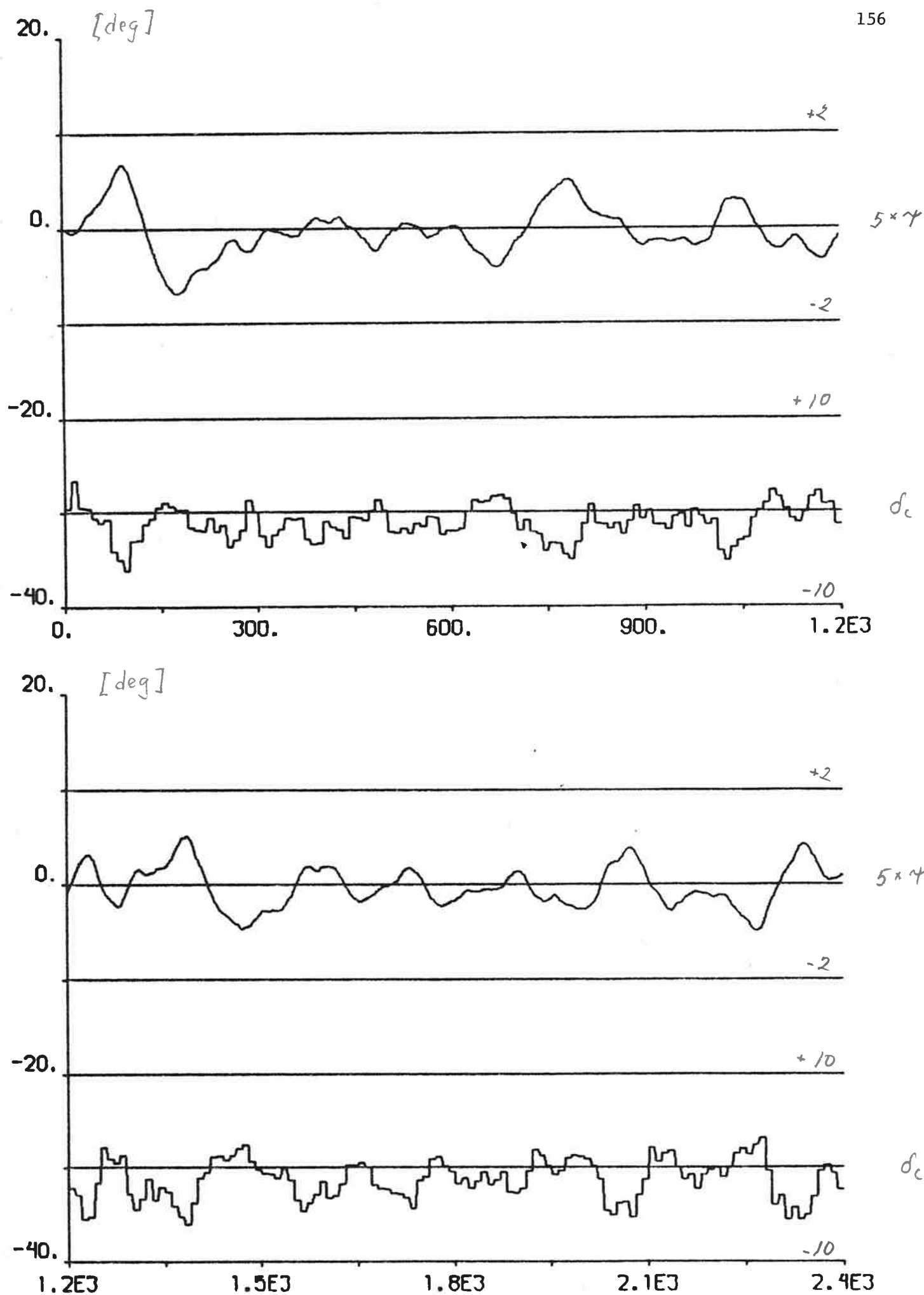


Fig. 4.11 a - $T = 22.3$ m, $n_0 = 87.6$ rpm, $u_0 = 15.8$ knots, $\delta_\ell = 10$ deg, self-tuning regulator using estimates from the Kalman filter. The filter gain K is given by (4.6).

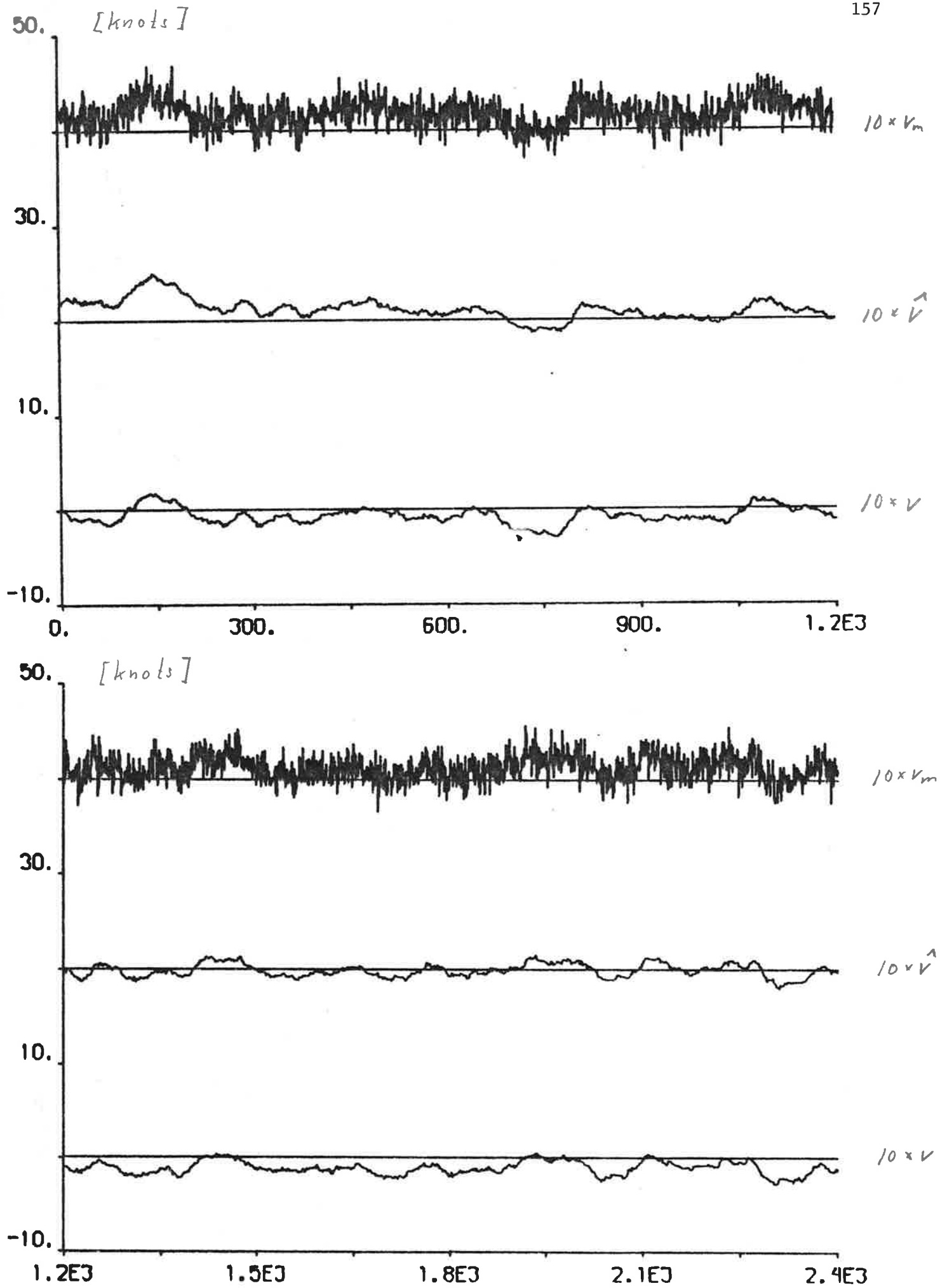


Fig. 4.11 b

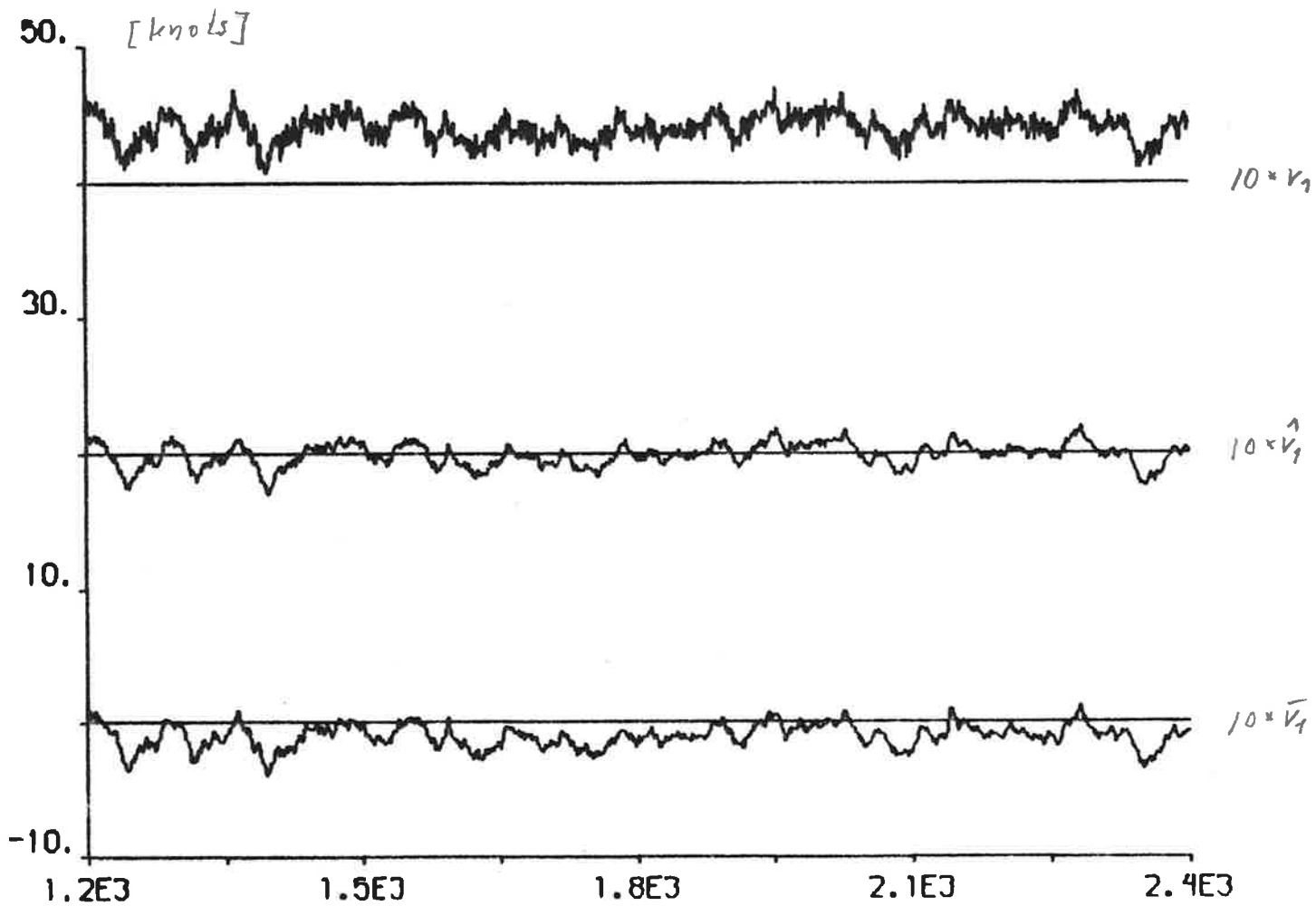
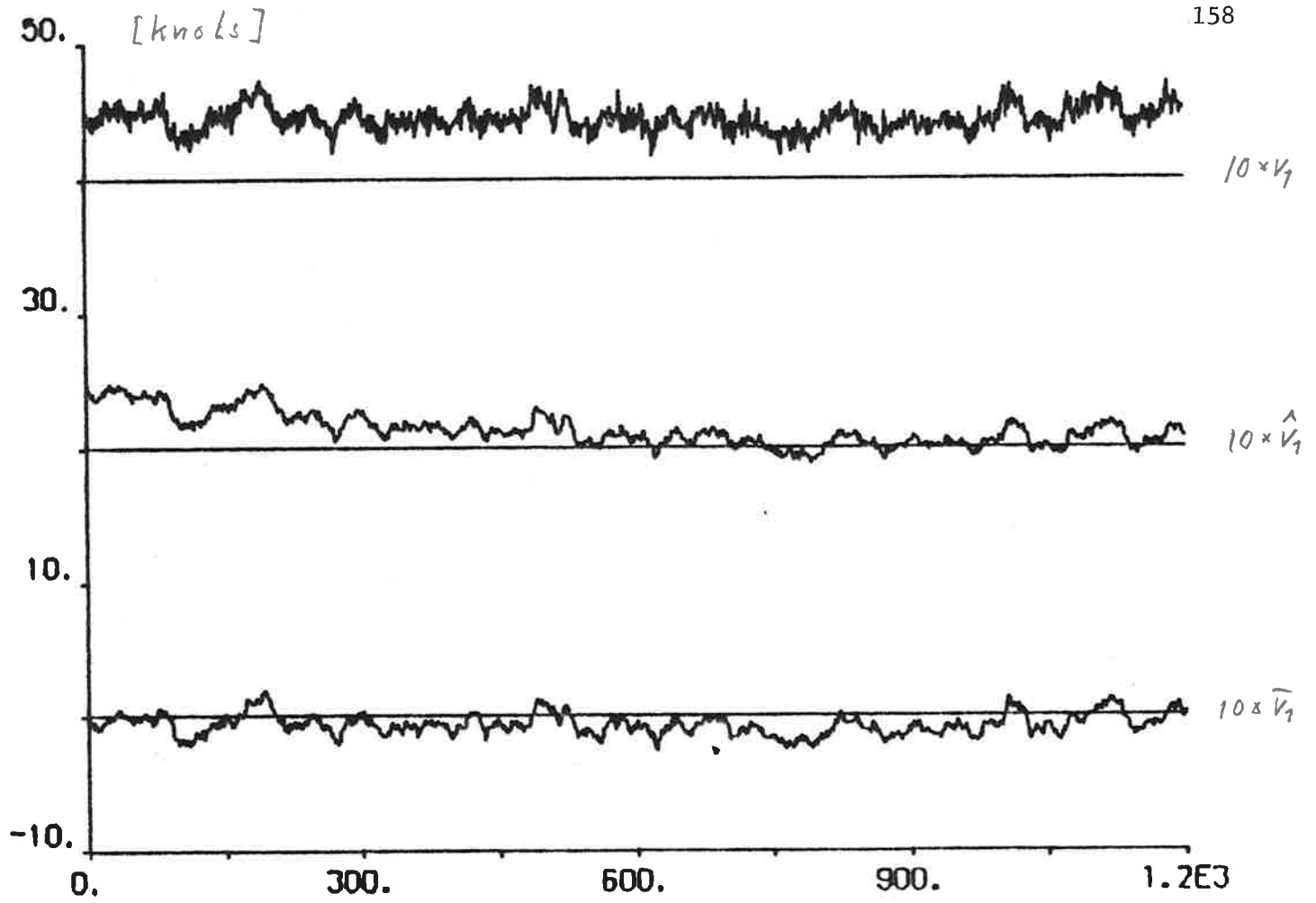


Fig. 4.11 c

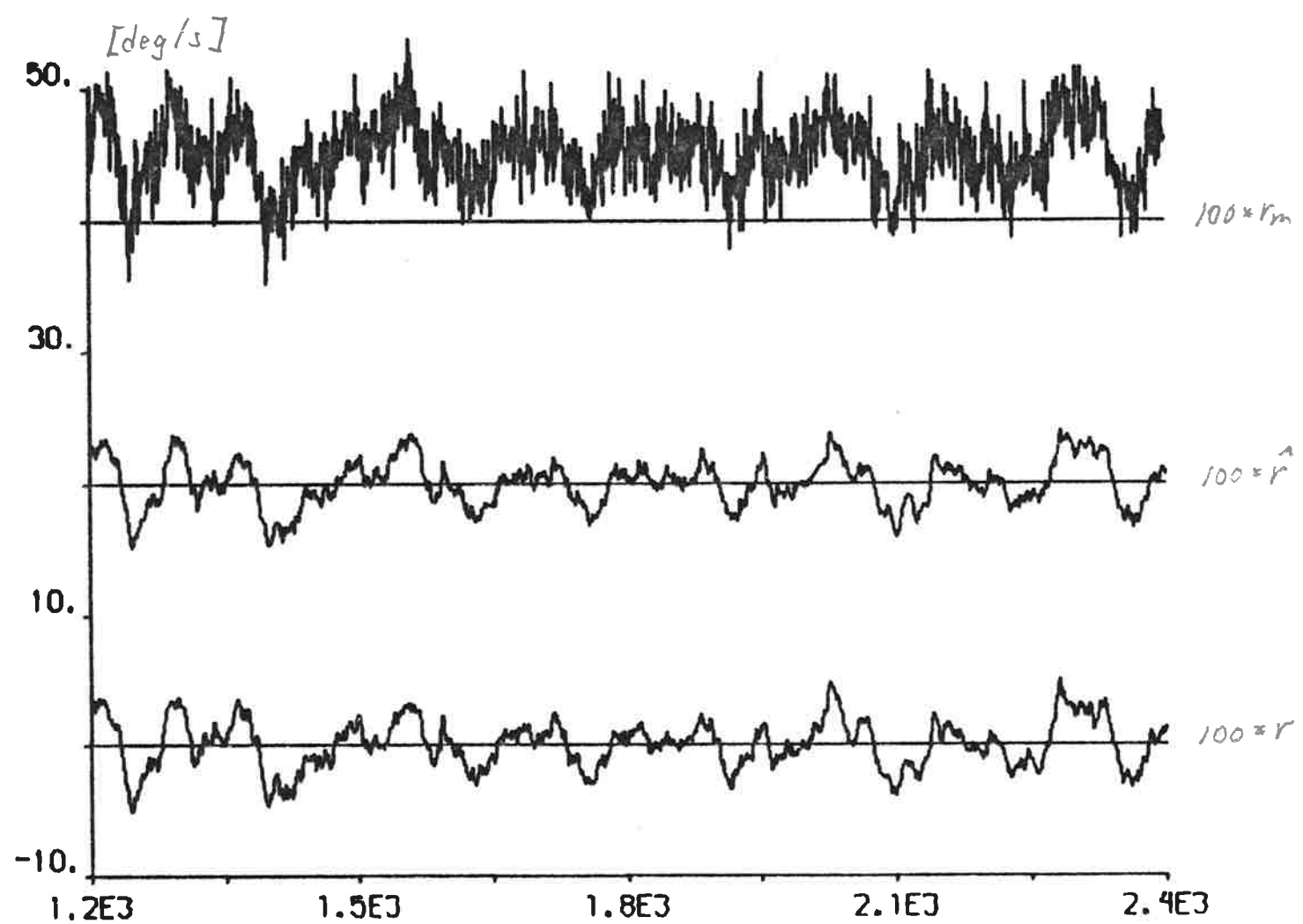
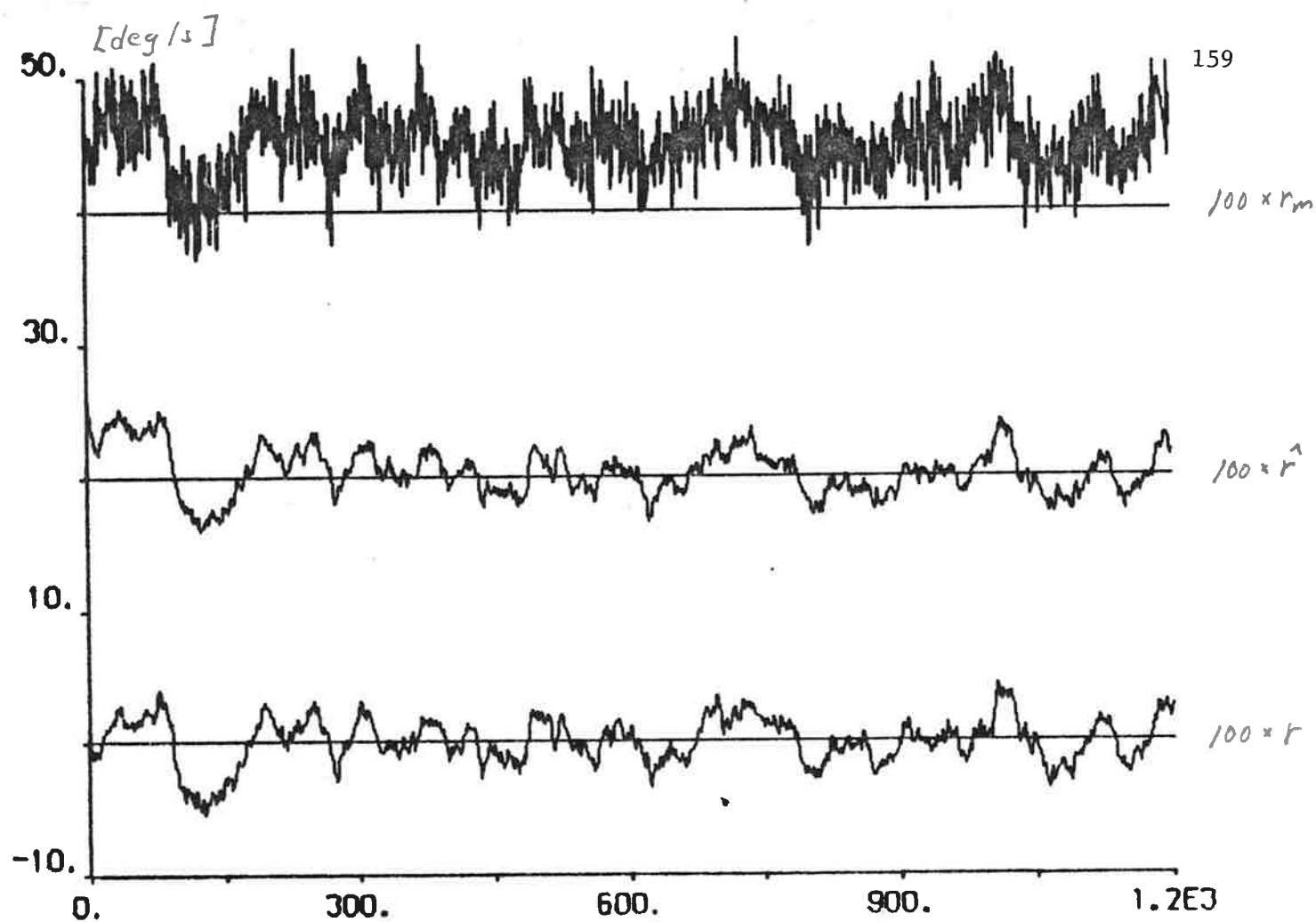


Fig. 4.11 d

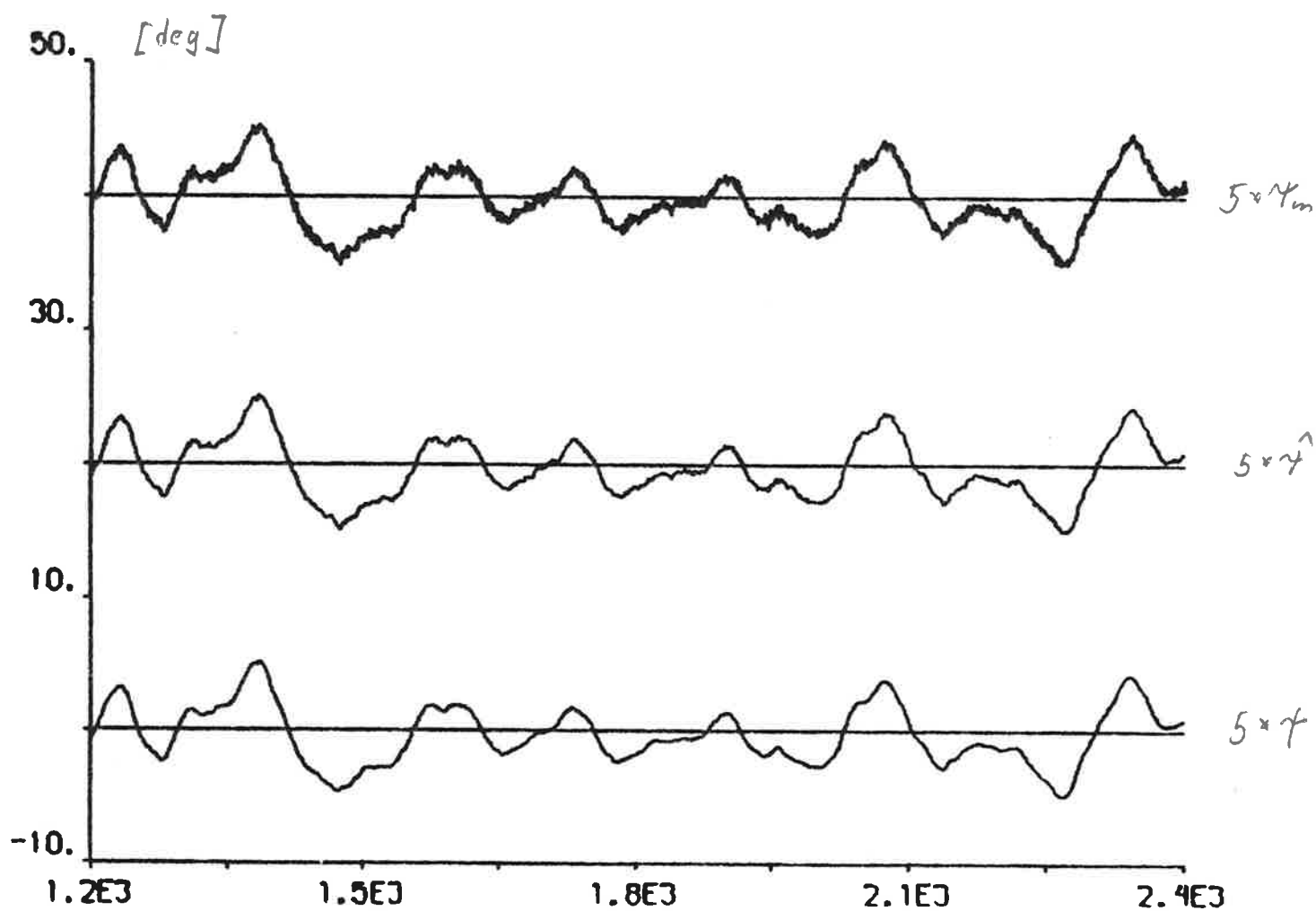
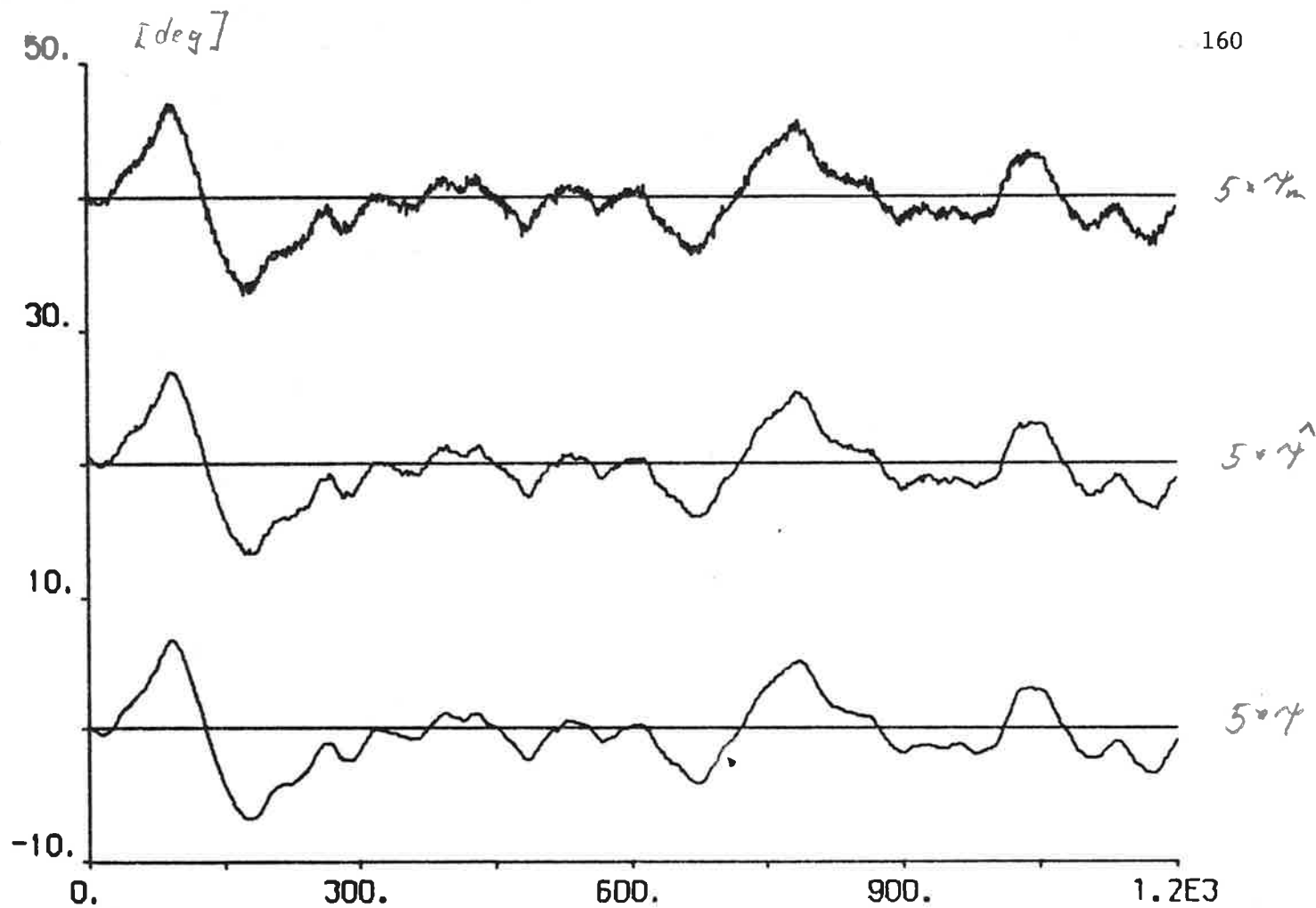


Fig. 4.11 e

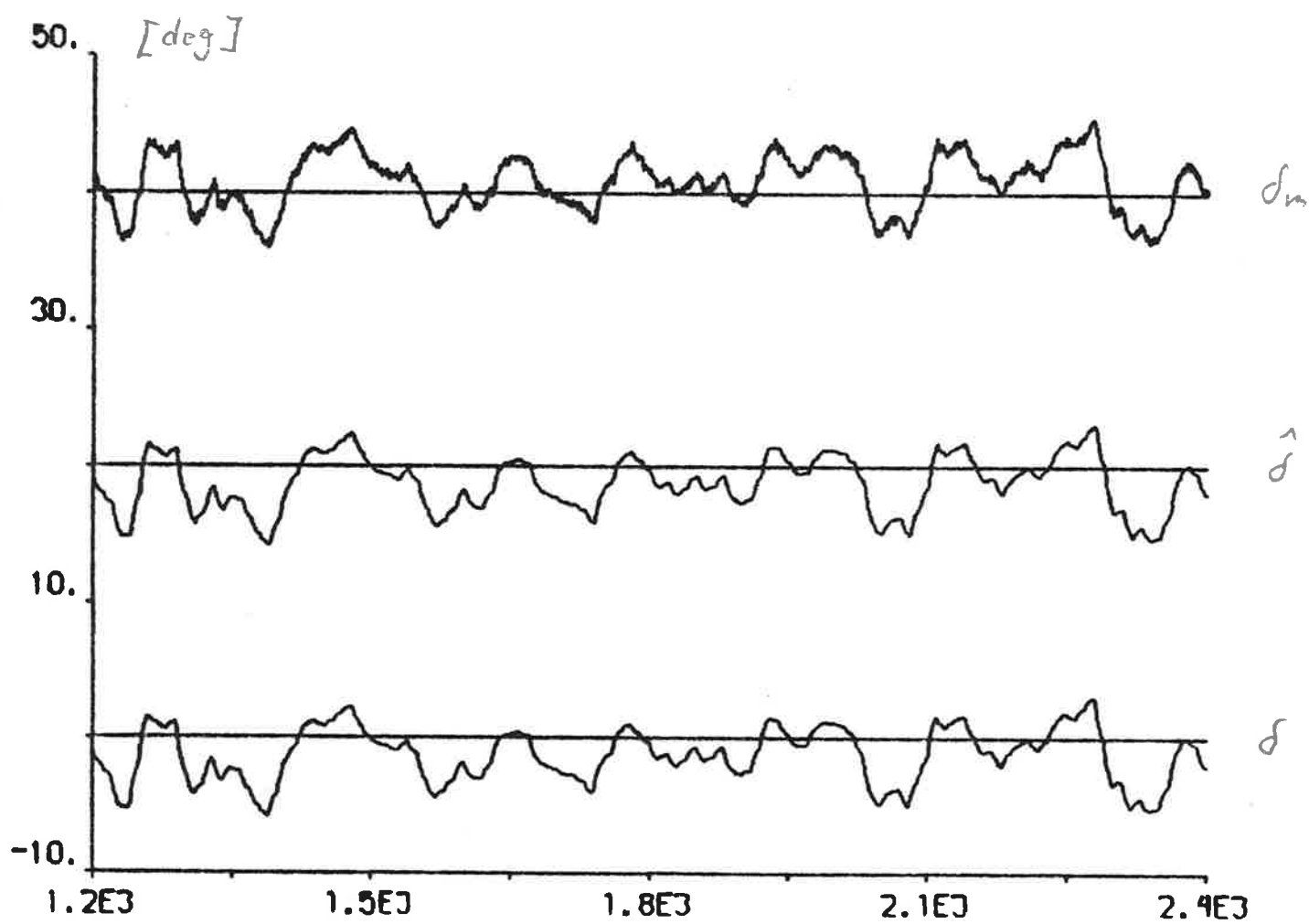
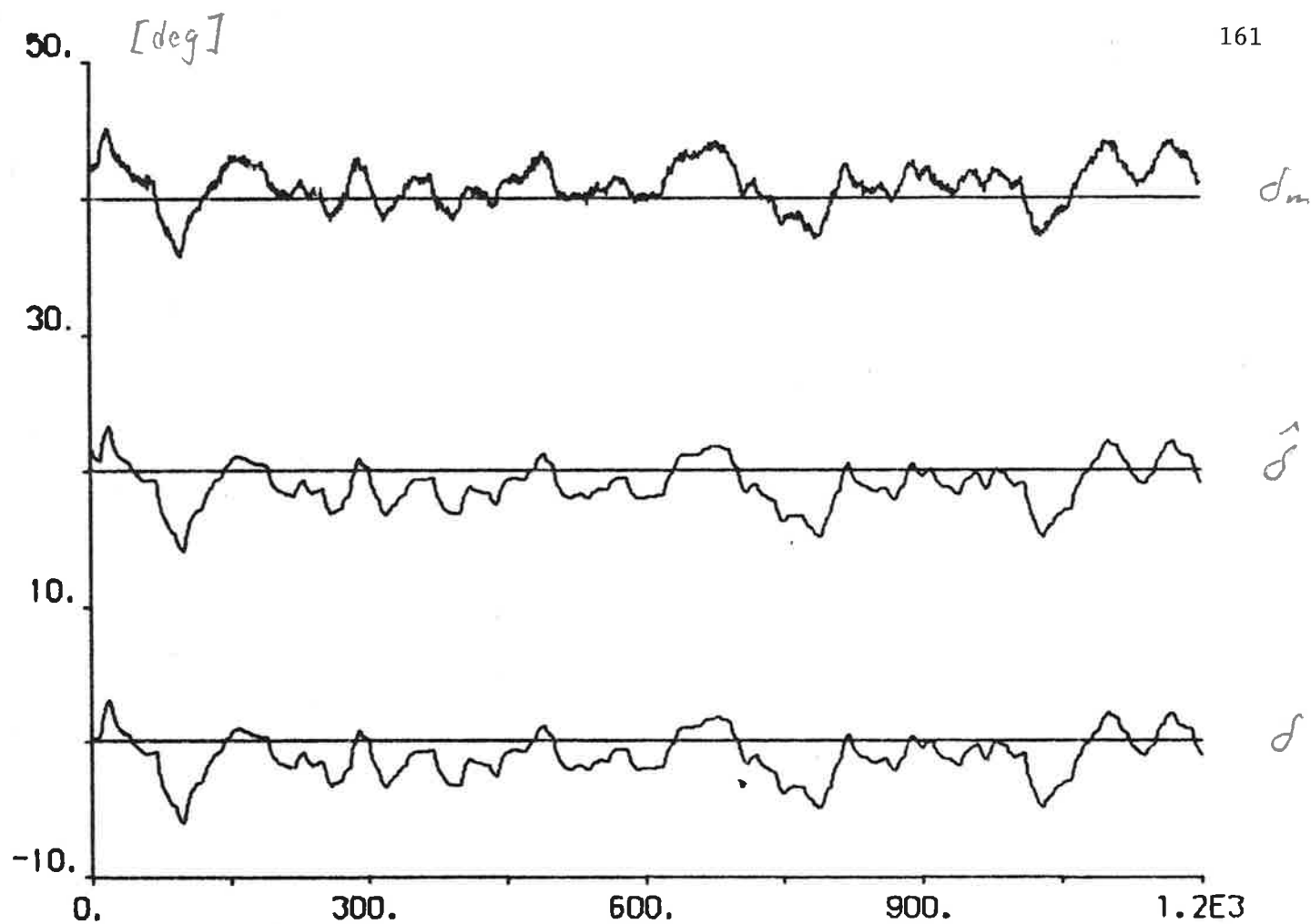


Fig. 4.11 f

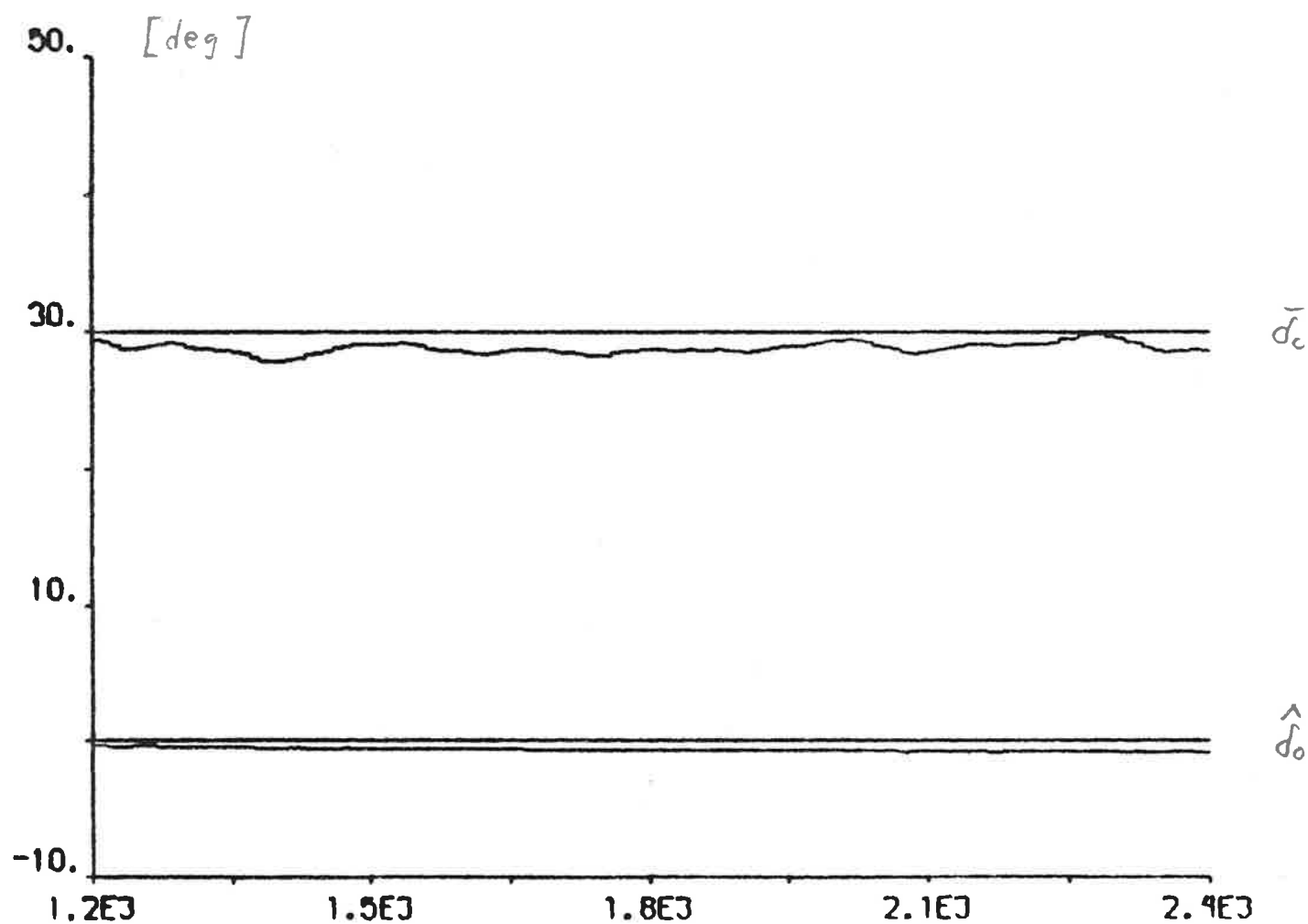
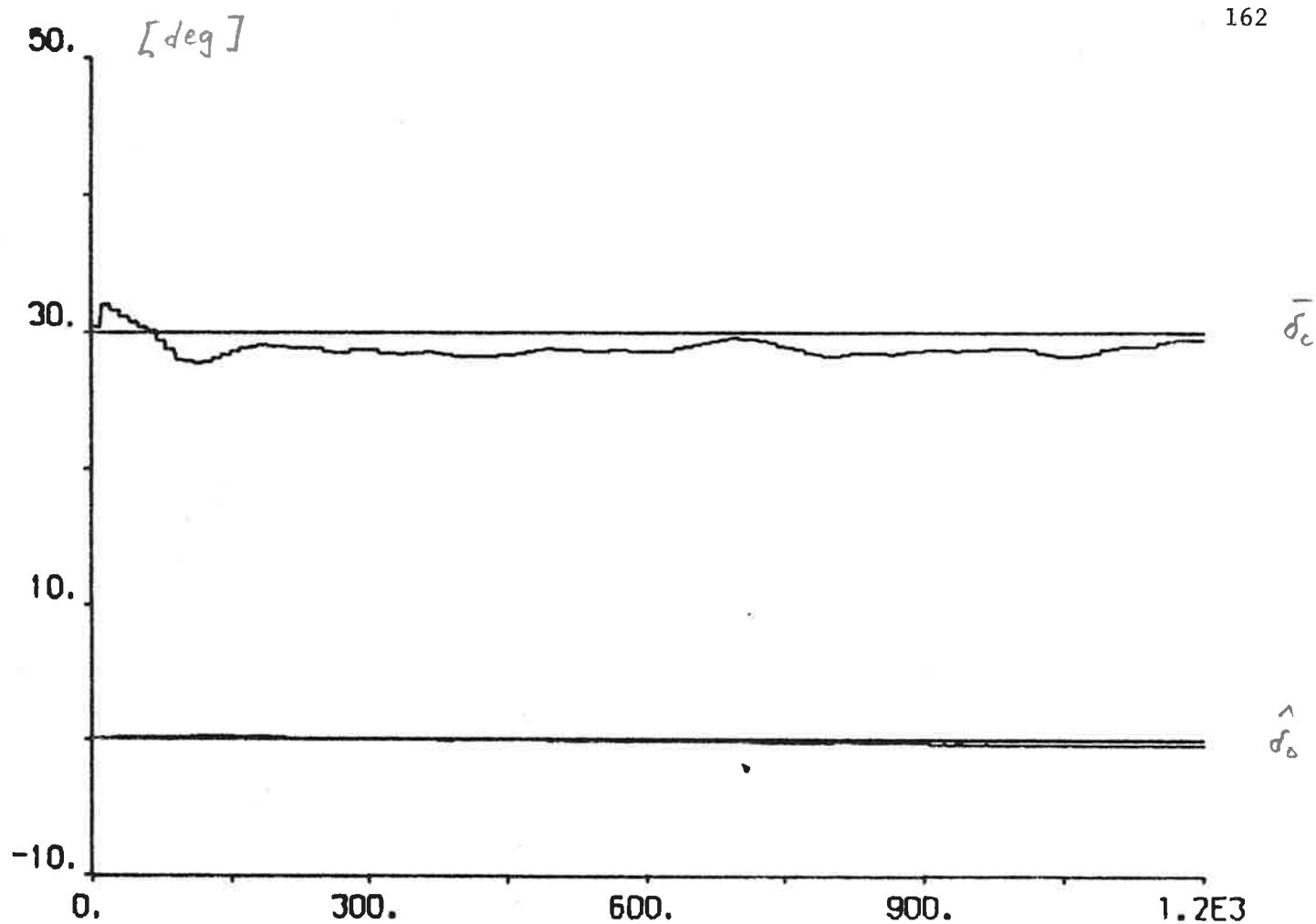


Fig. 4.11 g

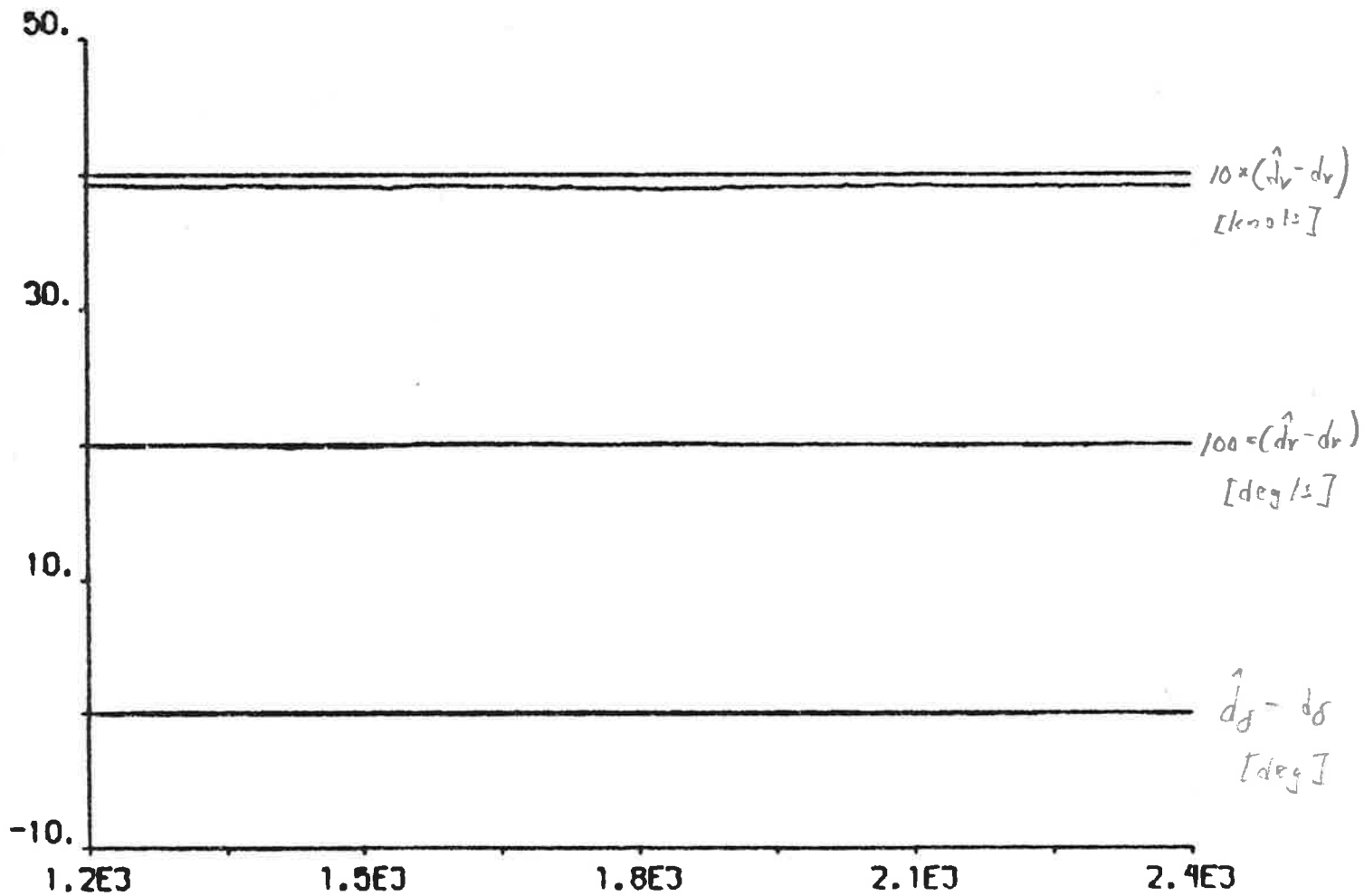
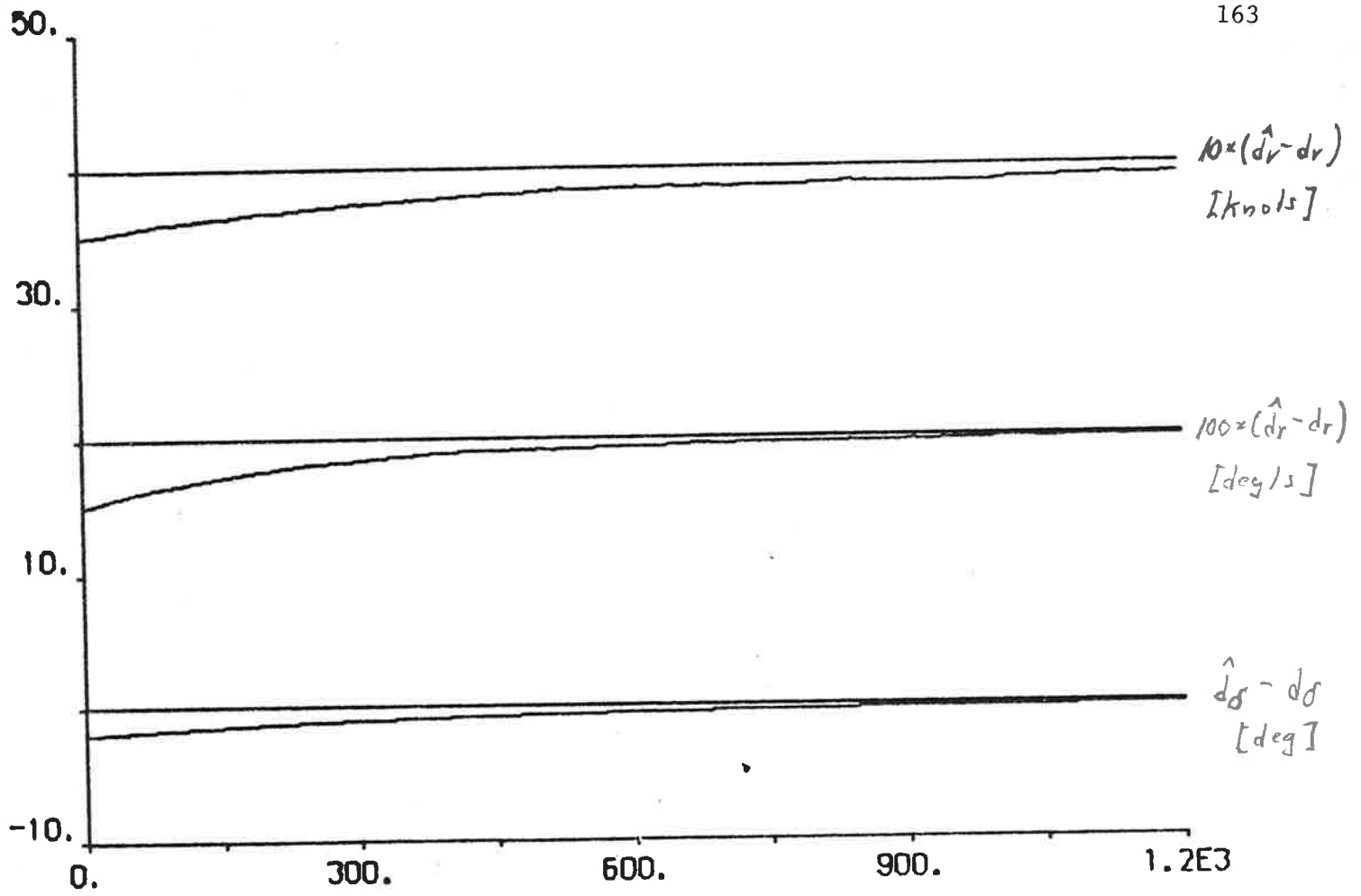


Fig. 4.11 h

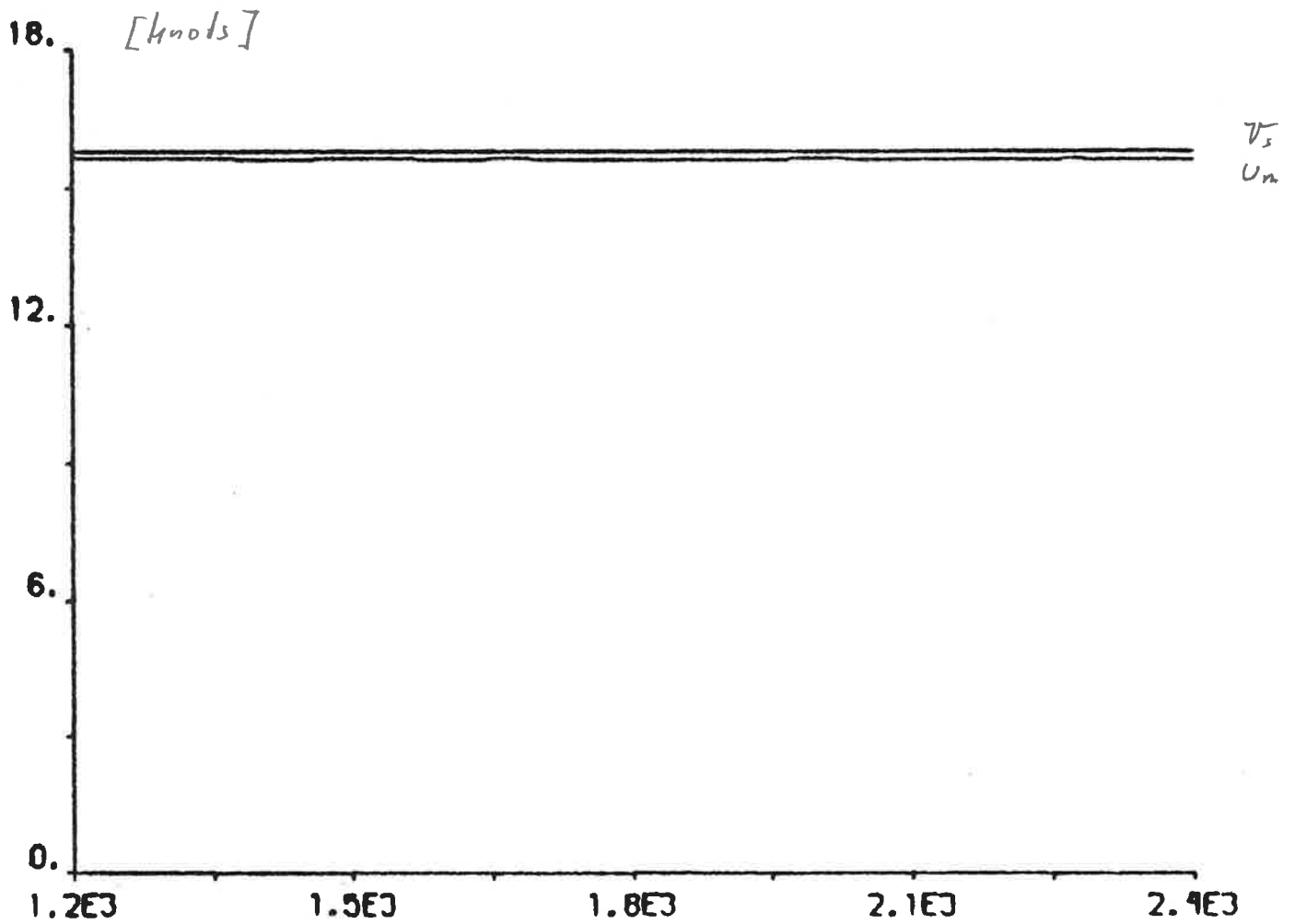
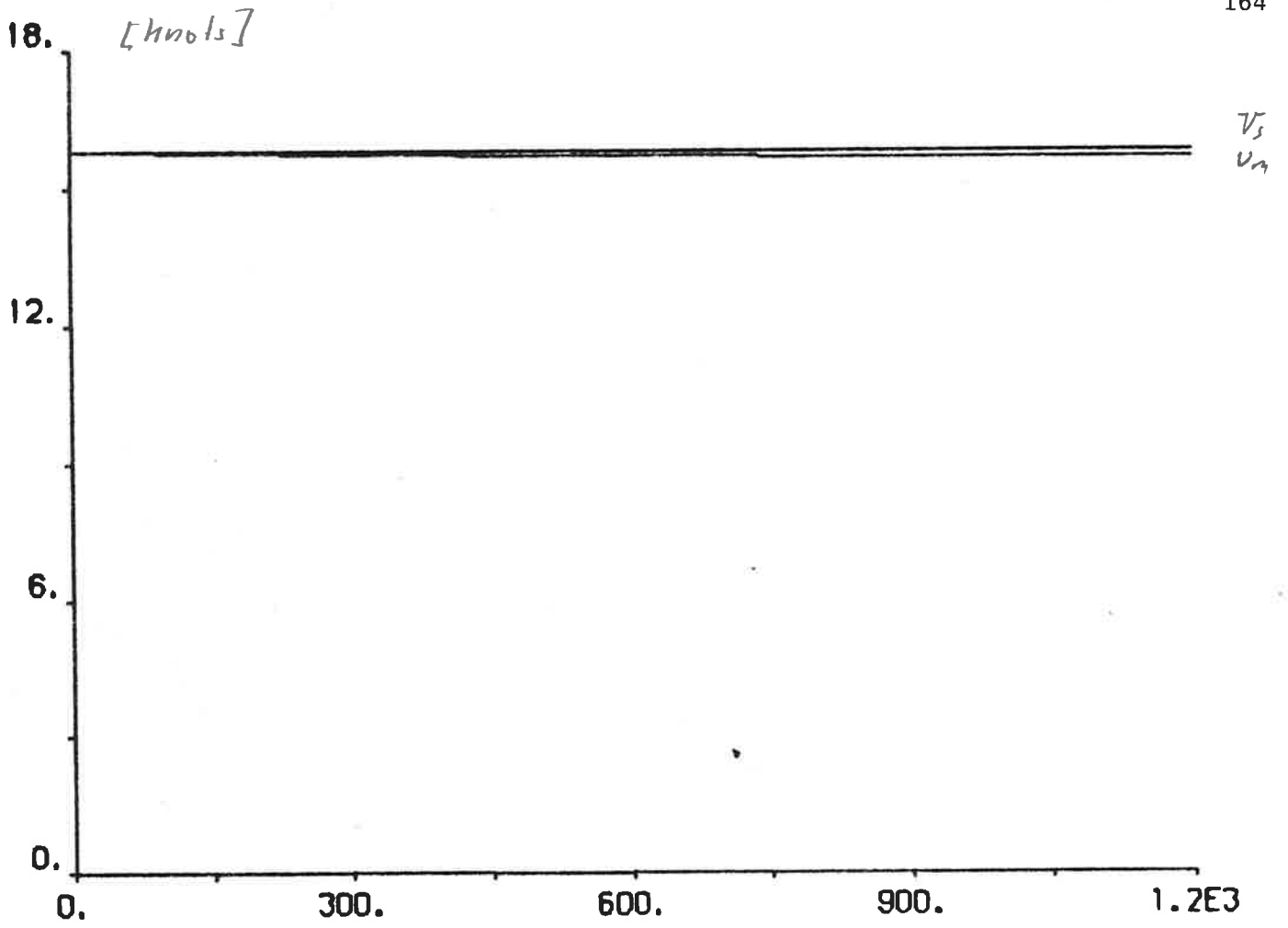


Fig. 4.11 i

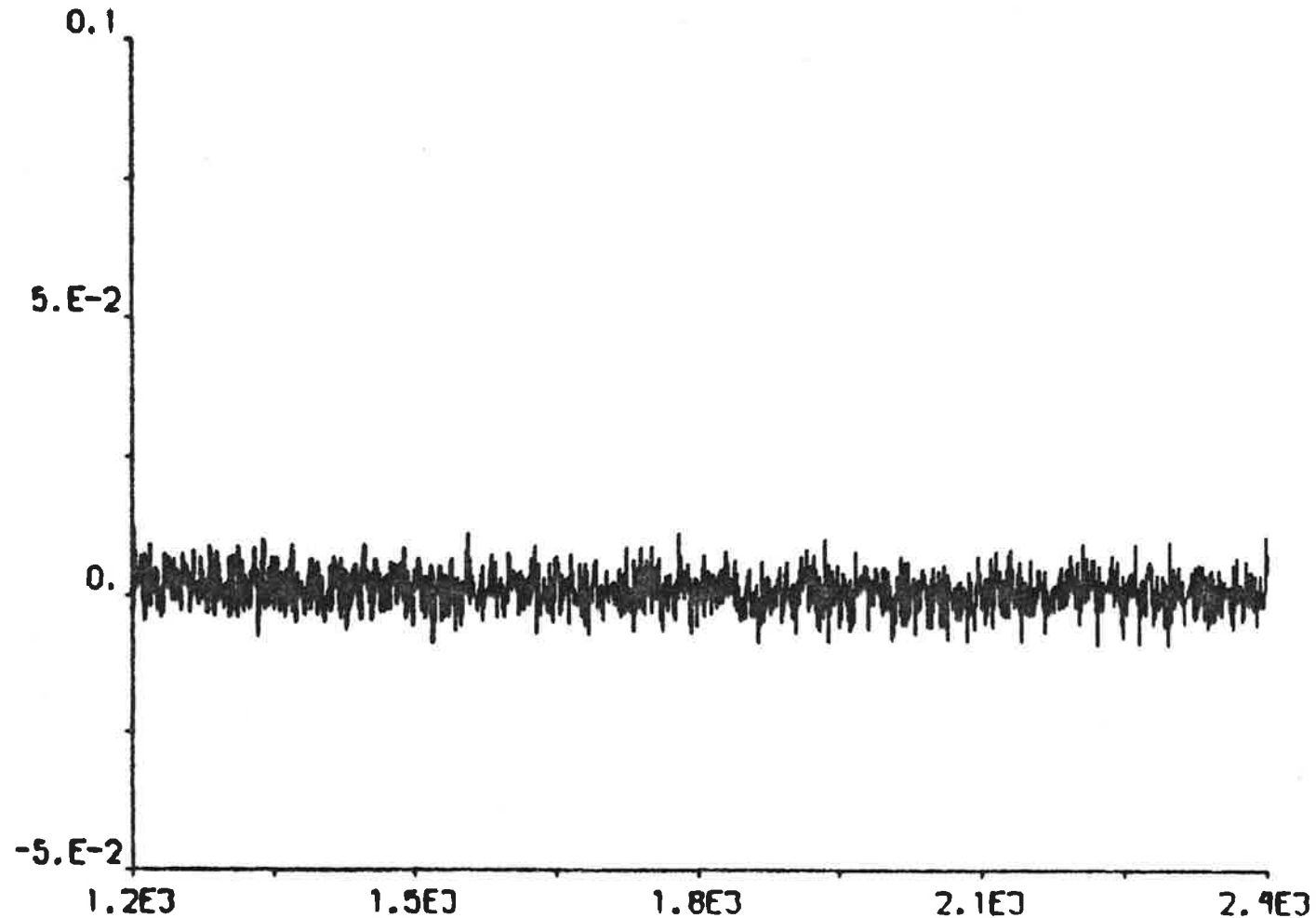
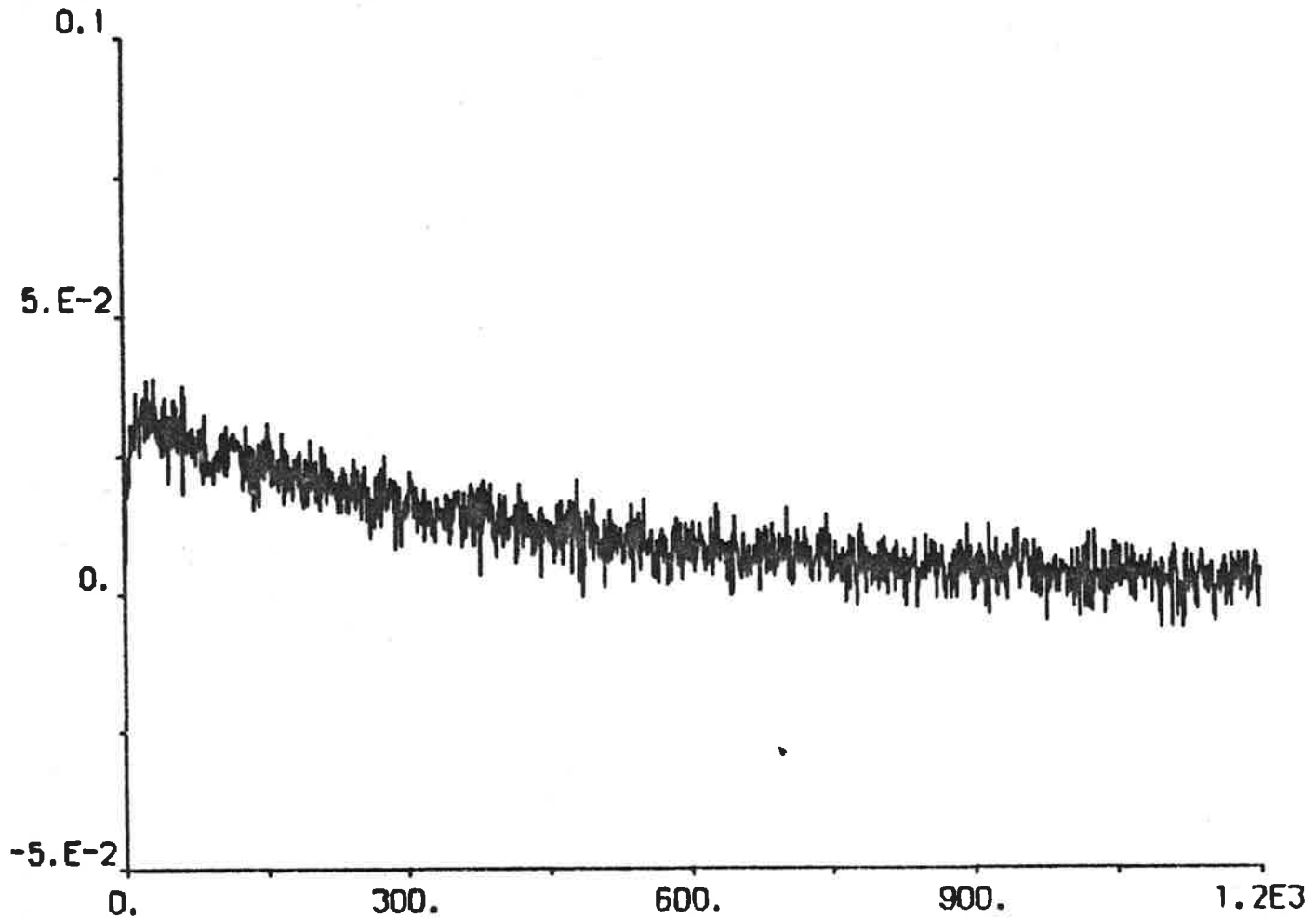


Fig. 4.11 j

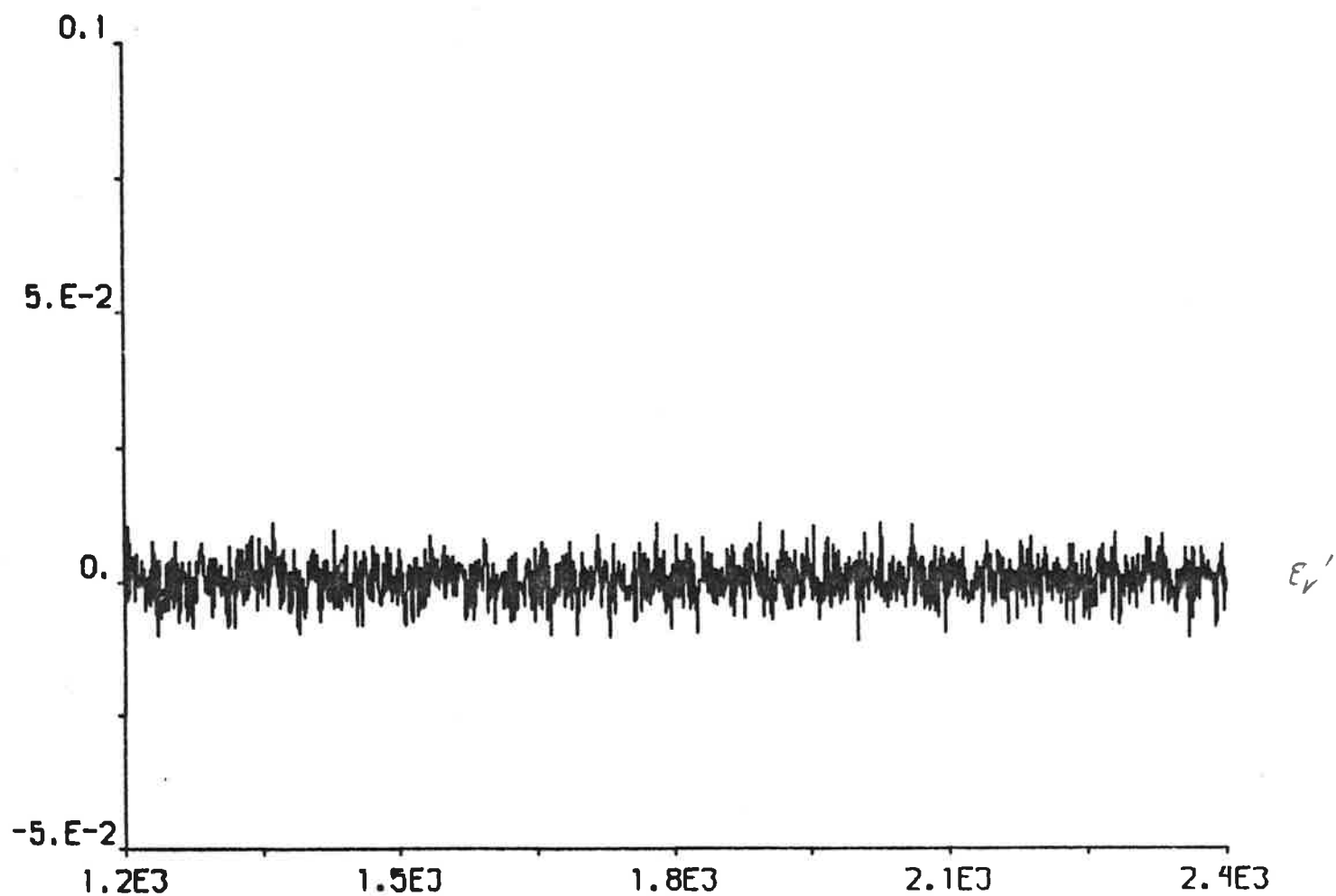
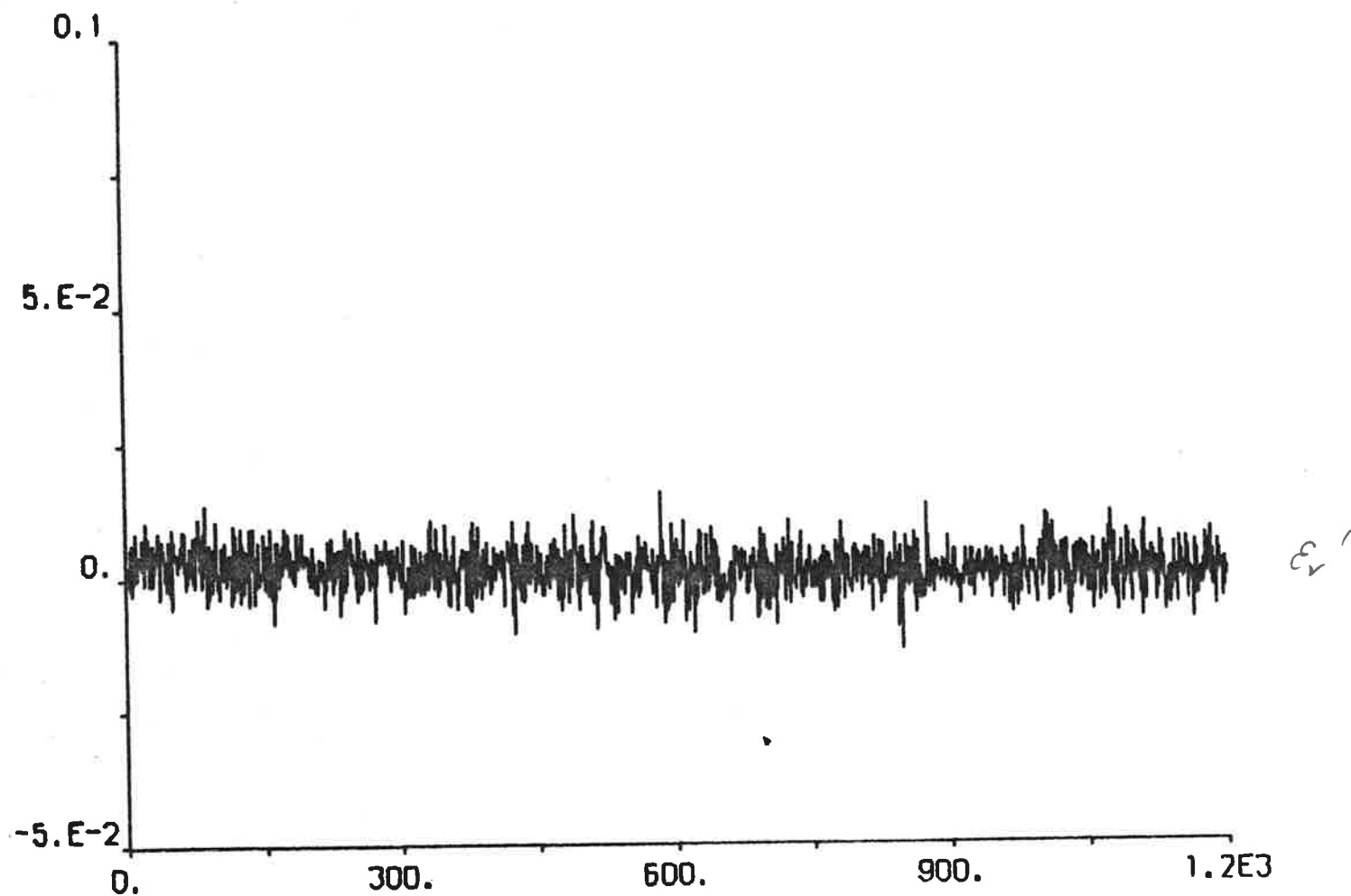


Fig. 4.11 k

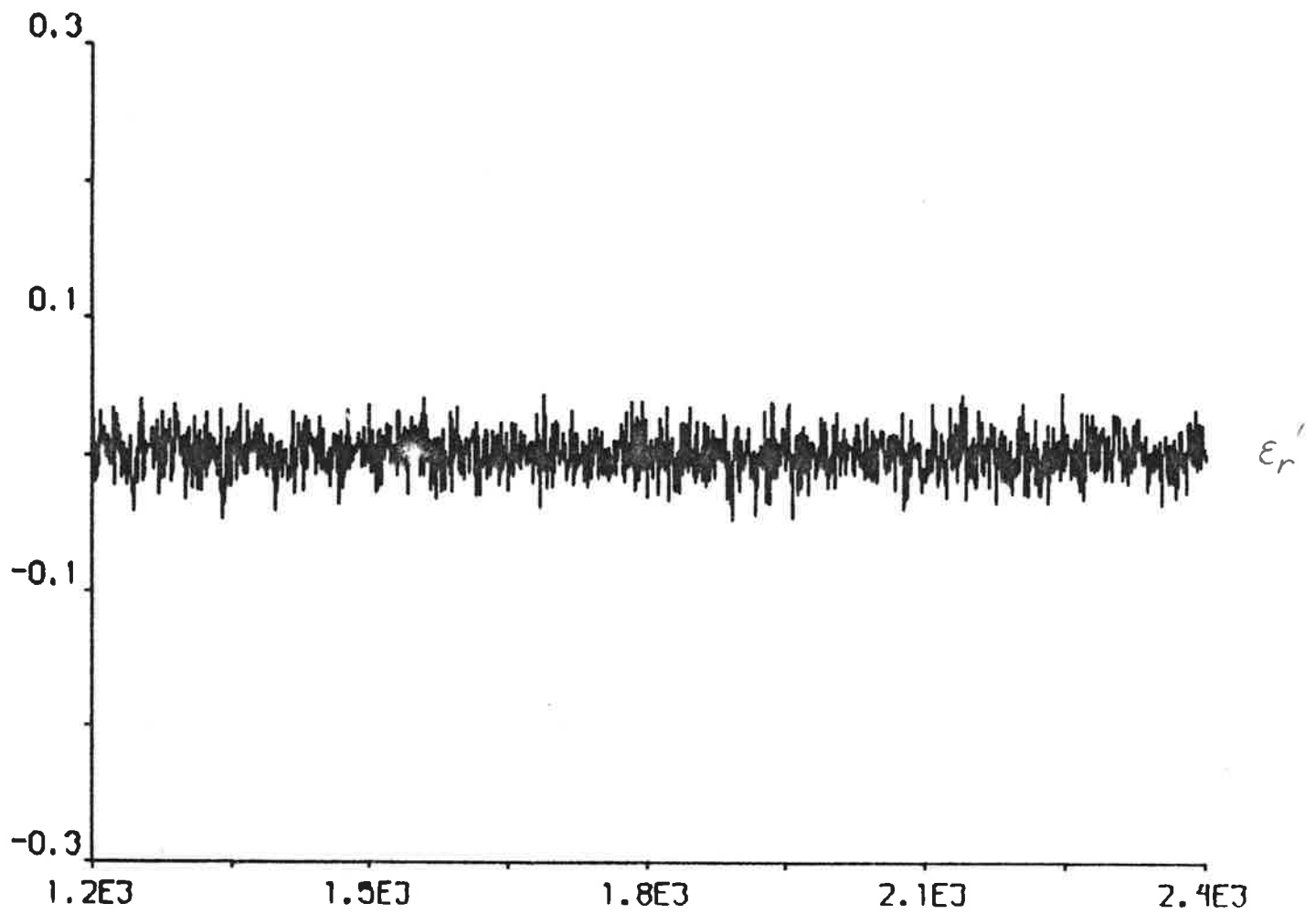
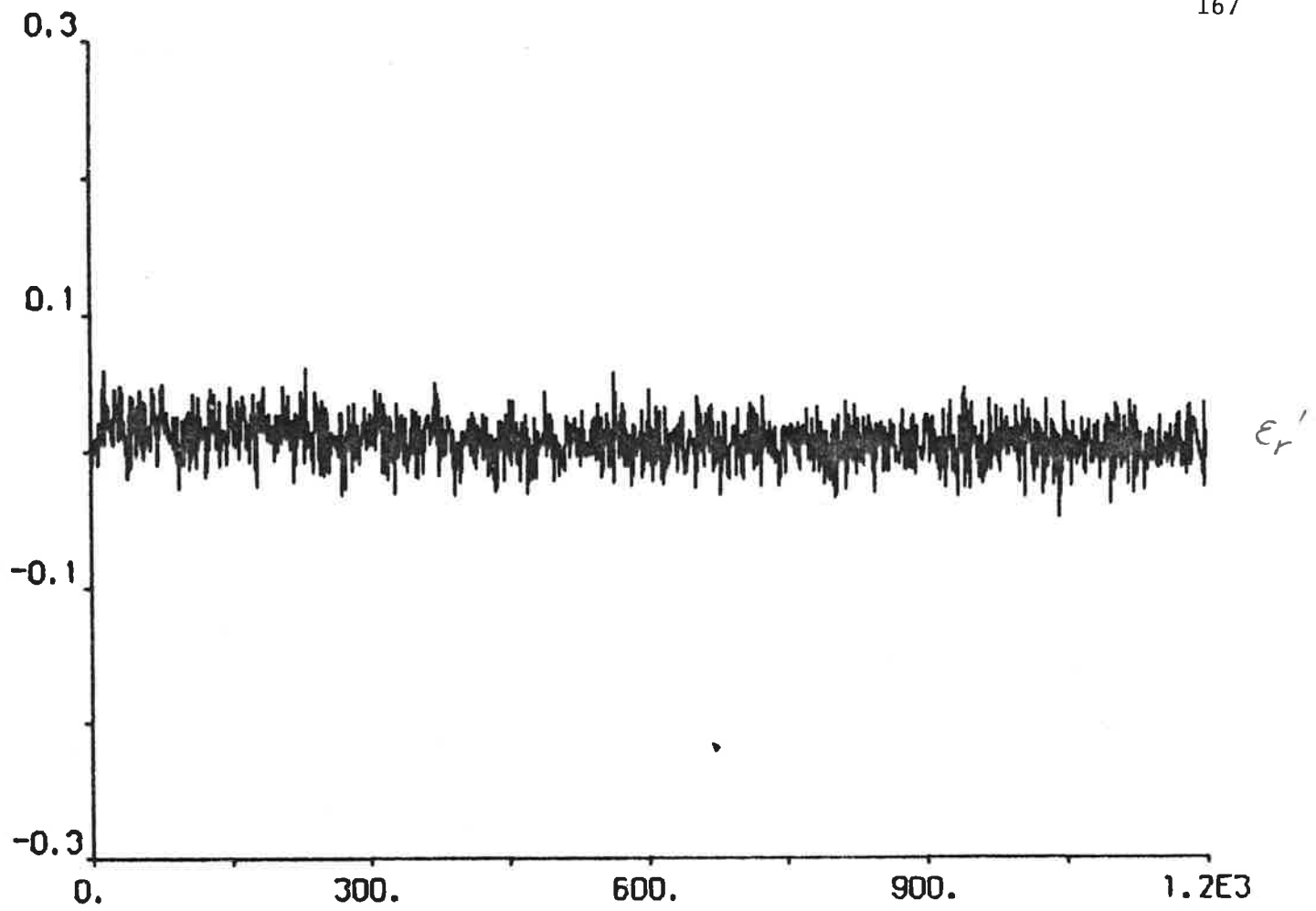


Fig. 4.11 ℓ

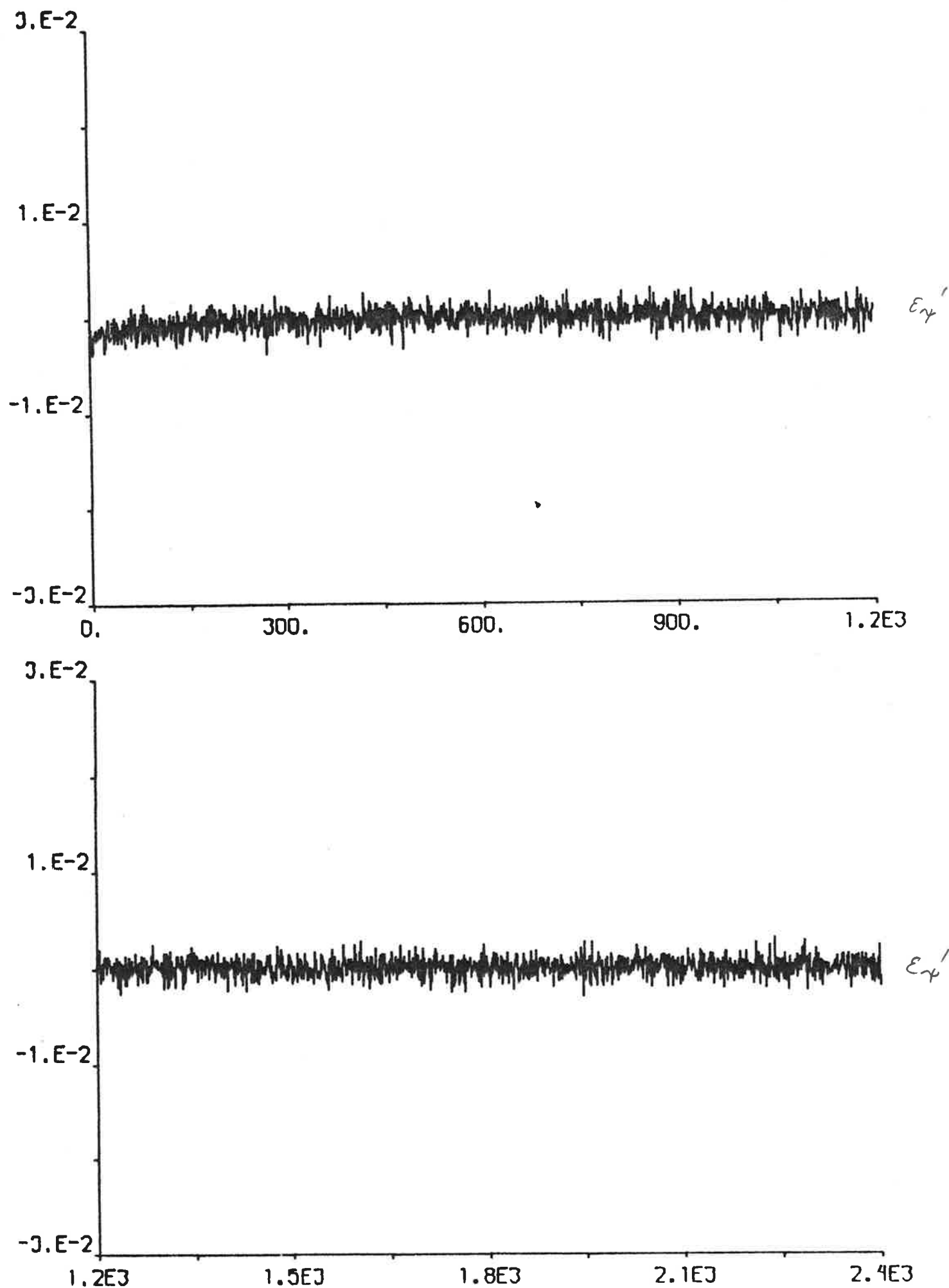


Fig. 4.11 m

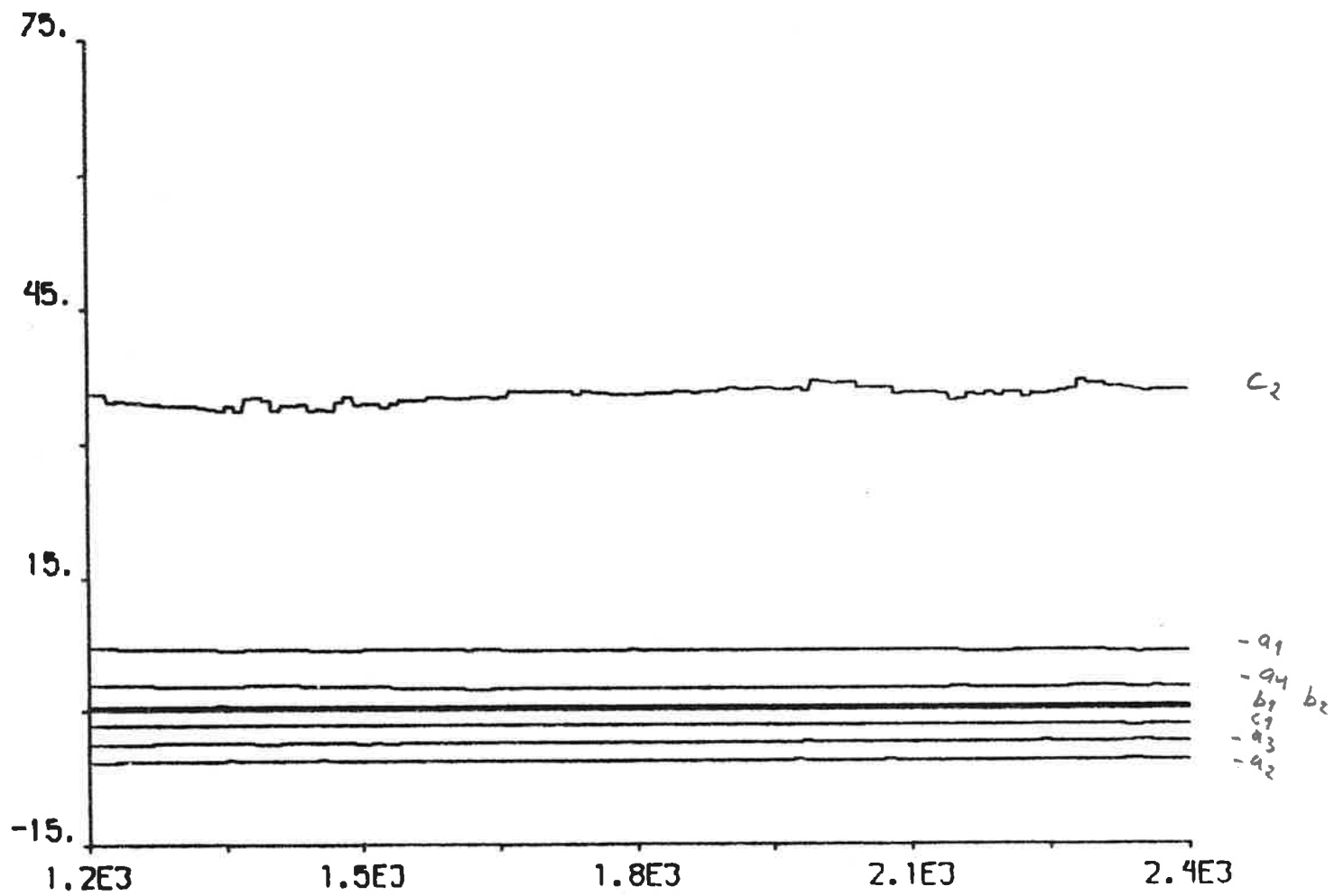
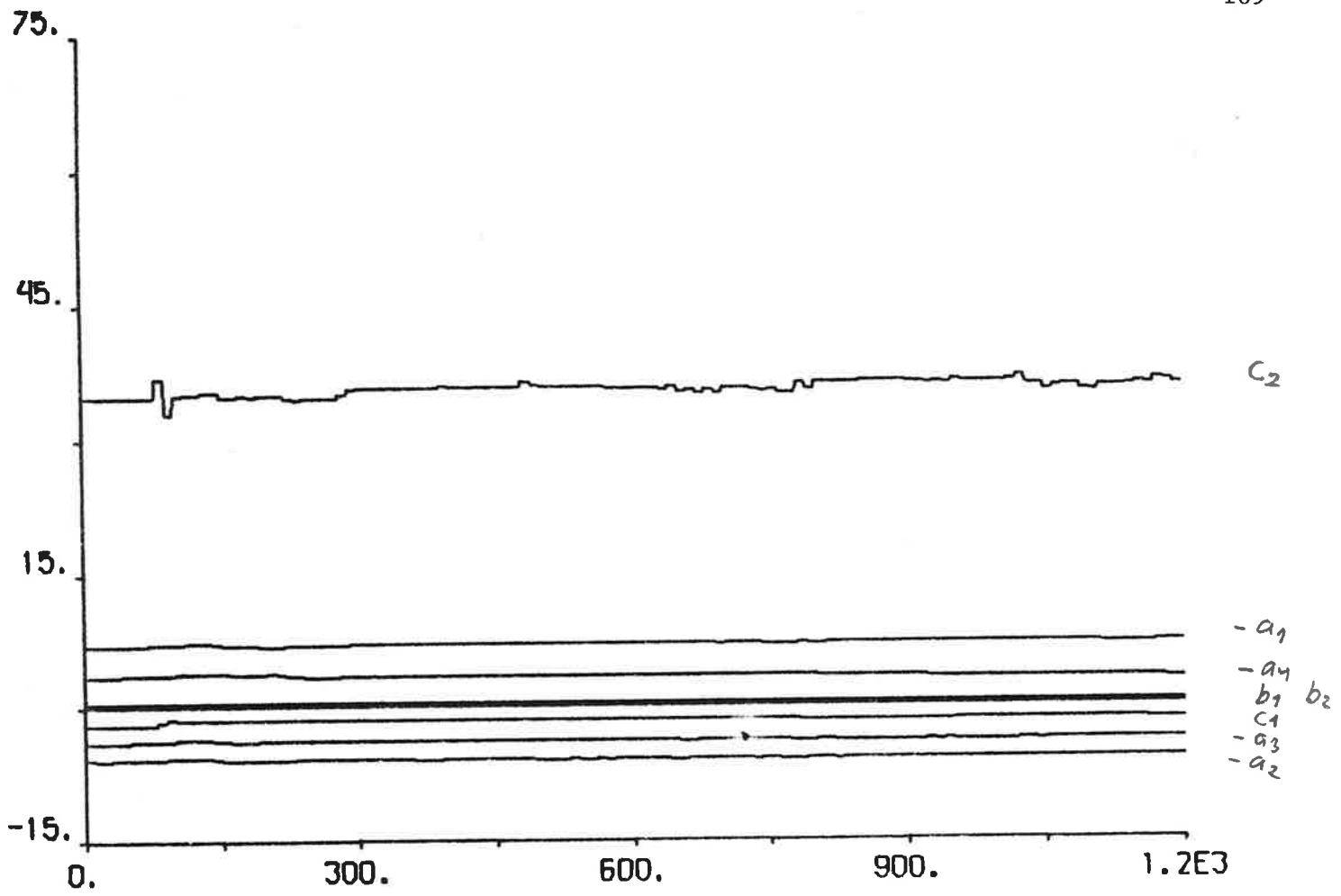


Fig. 4.11 n

4.2 Straight Course Keeping

Simulations of straight course keeping with the self-tuning regulator using non-filtered measurements are shown in Figs 4.12 - 4.25, where the mean draught T is equal to 22.3 m and k , T_s and q_2^* are varied. Two different initial speeds, $u_0 = 15.8$ knots and $u_0 = 4$ knots, are used. The initial parameter values and the initial covariance matrix are

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} -10.92 \\ 10.38 \\ 4.717 \\ -4.871 \\ 0.8884 \\ 0.2482 \end{bmatrix} \quad P = \begin{bmatrix} 100 & & & & & \\ & 100 & & & & \\ & & 100 & & & \\ & & & 100 & & \\ & & & & 1 & \\ & & & & & 1 \end{bmatrix} \quad (4.7)$$

or

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} -13.88 \\ 12.85 \\ 7.866 \\ -7.302 \\ 0.6101 \\ -0.01241 \end{bmatrix} \quad P = \begin{bmatrix} 100 & & & & & \\ & 100 & & & & \\ & & 100 & & & \\ & & & 100 & & \\ & & & & 1 & \\ & & & & & 1 \end{bmatrix} \quad (4.8)$$

or

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} -8.105 \\ 8.189 \\ 2.845 \\ -3.605 \\ 0.4698 \\ 0.2522 \end{bmatrix} \quad P = \begin{bmatrix} 10 & & & & & \\ & 10 & & & & \\ & & 10 & & & \\ & & & 10 & & \\ & & & & 0.1 & \\ & & & & & 0.1 \end{bmatrix} \quad (4.9)$$

or

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} -9.47 \\ 9.67 \\ 3.55 \\ -4.42 \\ 0.150 \\ 0.266 \end{bmatrix} \quad P = \begin{bmatrix} 1 & & & & & \\ & 10 & & & & \\ & & 10 & & & \\ & & & 1 & & \\ & & & & 0.1 & \\ & & & & & 0.1 \end{bmatrix} \quad (4.10)$$

Notice that $c_1 = c_2 = 0$. The speed V_s (cf (3.1)) is computed according to

$$V_s = V_c \quad (4.11)$$

The simulations are summarized in Table 4.2. When $u_0 = 15.8$ knots, it can be concluded that the lowest value of V_ℓ (0.75) is obtained for $k = 7$, $T_s = 10$ s and $q_2^* = 0$. These values are also chosen as standard values. Notice, however, that $k = 6$, $T_s = 10$ s, $q_2^* = 0$ ($V_\ell = 0.76$) and $k = 5$, $T_s = 15$ s, $q_2^* = 0$ ($V_\ell = 0.79$) are good alternatives. It can also be concluded that the performance of the self-tuning regulator is not improved when $q_2^* \neq 0$. When $u_0 = 4$ knots, the best choice is $k = 8$, $T_s = 10$ s, $q_2^* = 0$. The only difference compared to the standard values is that $k = 8$ instead of $k = 7$.

The performance of the self-tuning regulator using estimates from the Kalman filter, when the speed is decreased and increased, is shown in Figs 4.26 and 4.27, resp. A summary is given in Table 4.3. The standard parameter values are used, but the speed V_s (cf (3.1)) is computed according to

$$V_s = u_m / CMK \quad (4.12)$$

Notice that the parameters of the self-tuning regulator are changed very little and that the performance of the regulator is very good in both cases. This may be an indication that the speed scaling introduced in the self-tuning regulator is acceptable.

n_0 [rpm]	u_0 [knots]	δ_ℓ [deg]	k	T_s [s]	q_2^*	V_c [m/s]	m_δ [deg]	σ_δ [deg]	m_ψ [deg]	σ_ψ [deg]	V_ℓ [deg ²]	λ	Fig.	Remarks
87.6	15.8	10	4	15	0	8	- 1.31	2.71	0.05	0.51	0.87	1/12	4.12	Initial values (4.7)
87.6	15.8	10	5	15	0	8	- 1.32	2.37	0.05	0.57	0.79	1/12	4.13	" -
87.6	15.8	10	6	15	0	8	- 1.27	3.73	0.06	0.64	1.57	1/12	4.14	" -
87.6	15.8	10	5	10	0	8	- 1.28	3.12	0.01	0.44	1.01	1/12	4.15	Initial values (4.8)
87.6	15.8	10	6	10	0	8	- 1.29	2.58	-0.05	0.45	0.76	1/12	4.16	" -
87.6	15.8	10	7	10	0	8	- 1.30	2.58	0.00	0.45	0.75	1/12	4.17	" -
87.6	15.8	10	8	10	0	8	- 1.29	3.62	0.02	0.64	1.50	1/12	4.18	" -
87.6	15.8	10	5	10	0.05	8	- 1.29	3.04	0.01	0.52	1.04	1/12	4.19	" -
87.6	15.8	10	6	10	0.05	8	- 1.30	2.43	-0.10	0.53	0.78	1/12	4.20	" -
87.6	15.8	10	6	10	0.1	8	- 1.31	2.33	-0.16	1.61	0.84	1/12	4.21	" -
22.1772	4	45	6	10	0	2	-21.34	28.57	1.21	1.48	3.65	0	4.22	Initial values (4.9)
22.1772	4	45	7	10	0	2	-22.04	26.28	1.22	1.49	3.71	0	4.23	" -
22.1772	4	45	8	10	0	2	-22.56	25.86	1.18	1.30	3.08	0	4.24	" -
22.1772	4	45	6	10	0.05	2	-22.28	29.46	1.24	1.39	3.47	0	4.25	Initial values (4.10)

Table 4.2 - Summary of straight course keeping simulations ($\psi_{ref} = 0$ deg) with the self-tuning regulator using non-filtered measurements, when k , T_s , and q_2^* are varied. The mean draught T is equal to 22.3 m. The speed V_s is computed according to (4.11). The duration of each simulation is 2400 s, but the values of m_δ , σ_δ , m_ψ , σ_ψ , and V_ℓ are computed for the part 1200 - 2400 s.

T [m]	n_0 [rpm]	u_0 [knots]	δ_ℓ [deg]	m_δ [deg]	σ_δ [deg]	m_ψ [deg]	σ_ψ [deg]	V_ℓ [deg ²]	λ	Fig.
22.3	22.1772	15.8	45	-4.47	5.60	-0.11	0.85	3.34	1/12	4.26
22.3	87.6	4	45	-1.85	9.36	-0.07	0.71	7.81	1/12	4.27

Table 4.3 - Summary of straight course keeping simulations ($\psi_{\text{ref}} = 0$ deg) with the self-tuning regulator using estimates from the Kalman filter, when the speed is changed significantly. The speed V_s is computed according to (4.12).

Simulations of straight course keeping with the self-tuning regulator and the PID-regulator, when the Kalman filter estimates as well as the non-filtered measurements are used, are shown in Figs 4.28 - 4.53. The initial speed u_0 is equal to 15.8, 10 or 4 knots and the mean draught T is equal to 22.3 or 10.5 m. A summary is given in Tables 4.4, 4.5 and 4.6, where also simulations from Section 4.1 are included.

The speed scaling used in the PID-regulator is $(V_0/V_s)^2$ (cf (3.25)). By putting $V_0 = 6.3$ m/s when $V_s = 5$ m/s, and $V_0 = 4$ m/s when $V_s = 2$ m/s, the speed scaling V_0/V_s may be simulated (cf Figs 4.38, 4.43, 4.48 and 4.53). The speed scaling used in the self-tuning regulator is also $(V_0/V_s)^2$ (cf (3.24)), since $q_2^* = 0$. It is, however, possible to obtain the speed scaling V_0/V_s instead, if $q_2^* = 0.6$ when $V_s = 5$ m/s and if $q_2^* = 3$ when $V_s = 2$ m/s (cf Figs 4.37, 4.42, 4.47, and 4.52).

T [m]	Kalman filter estimates used		Non-filtered measurements used		m_δ [deg]	σ_δ [deg]	m_ψ [deg]	σ_ψ [deg]	V_ℓ [deg ²]	λ	Fig.
	Self-tuning regulator	PID- regulator	Self-tuning regulator	PID- regulator							
22.3	X				-1.28	2.04	-0.09	0.47	0.55	1/12	4.1
22.3		X			-1.36	2.08	0.07	0.66	0.80	1/12	4.28
22.3			X		-1.32	2.34	-0.22	0.64	0.91	1/12	4.29
22.3				X	-1.28	2.40	0.03	0.80	1.12	1/12	4.30
10.5	X				-0.15	1.55	-0.01	0.29	0.29	1/12	4.2
10.5		X			-0.16	1.29	0.01	0.32	0.24	1/12	4.31
10.5			X		-0.17	1.94	0.01	0.43	0.50	1/12	4.32
10.5				X	-0.13	1.63	-0.01	0.42	0.39	1/12	4.33

Table 4.4 - Summary of straight course keeping simulations ($\psi_{\text{ref}} = 0$ deg), when $n_0 = 87.6$ rpm, $u_0 = 15.8$ knots, and $\delta_\ell = 10$ deg. The duration of each simulation is 2400 s, but the values of m_δ , σ_δ , m_ψ , σ_ψ , and V_ℓ are computed for the part 1200-2400 s.

T [m]	Kalman filter estimates used		Non-filtered measurements used		m_δ [deg]	σ_δ [deg]	m_ψ [deg]	σ_ψ [deg]	V_ℓ [deg ²]	λ	Fig.	Remarks
	Self-tuning regulator	PID- regulator	Self-tuning regulator	PID- regulator								
22.3	X				-3.71	4.61	-0.16	0.51	2.05	1/12	4.3	$q_2^* = 0.6$ $V_0 = 6.3 \text{ m/s}$
22.3		X			-3.82	4.62	0.07	0.65	2.21	1/12	4.34	
22.3			X		-3.76	5.64	-0.19	0.66	3.12	1/12	4.35	
22.3				X	-3.65	5.33	0.01	0.65	2.79	1/12	4.36	
22.3	X				-3.77	4.43	-0.26	0.83	2.38	1/12	4.37	
22.3		X			-3.64	4.46	0.06	1.08	2.83	1/12	4.38	
10.5	X				-1.18	3.36	-0.01	0.35	1.06	1/12	4.4	$q_2^* = 0.6$ $V_0 = 6.3 \text{ m/s}$
10.5		X			-1.19	3.32	0.01	0.37	1.06	1/12	4.39	
10.5			X		-1.19	5.54	0.00	0.53	2.83	1/12	4.40	
10.5				X	-1.12	4.47	-0.01	0.42	1.84	1/12	4.41	
10.5	X				-1.19	2.31	-0.18	0.70	0.97	1/12	4.42	
10.5		X			-1.22	2.56	0.03	0.52	0.81	1/12	4.43	

Table 4.5 - Summary of straight course keeping simulations ($\psi_{\text{ref}} = 0 \text{ deg}$), when $n_0 = 55.443 \text{ rpm}$, $u_0 = 10 \text{ knots}$, and $\delta_\ell = 35 \text{ deg}$. The duration of each simulation is 2400 s, but the values of m_δ , σ_δ , m_ψ , σ_ψ , and V_ℓ are computed for the part 1200 - 2400 s.

T [m]	Kalman filter estimates used		Non-filtered measurements used		m_δ [deg]	σ_δ [deg]	m_ψ [deg]	σ_ψ [deg]	V_ℓ [deg ²]	λ	Fig.	Remarks
	Self-tuning regulator	PID- regulator	Self-tuning regulator	PID- regulator								
22.3	X				-21.62	19.96	0.66	1.01	1.46	0	4.5	
22.3		X			-20.44	22.44	-0.03	1.54	2.37	0	4.44	
22.3			X		-21.89	27.55	1.25	1.48	3.75	0	4.45	
22.3				X	-18.00	31.25	-0.17	2.05	4.23	0	4.46	
22.3	X				-20.02	24.21	0.83	1.87	4.19	0	4.47	$q_2^* = 3$
22.3		X			-19.55	13.95	-0.19	1.31	1.75	0	4.48	$V_0 = 4 \text{ m/s}$
10.5	X				-8.93	19.24	0.30	0.61	0.46	0	4.6	
10.5		X			-9.23	22.26	0.05	0.68	0.46	0	4.49	
10.5			X		-9.08	23.44	0.38	0.82	0.82	0	4.50	
10.5				X	-9.31	31.12	0.03	1.21	1.47	0	4.51	
10.5	X				-8.47	13.73	-0.48	1.41	2.22	0	4.52	$q_2^* = 3$
10.5		X			-7.83	12.05	-0.04	1.24	1.54	0	4.53	$V_0 = 4 \text{ m/s}$

Table 4.6 - Summary of straight course keeping simulations ($\psi_{\text{ref}} = 0 \text{ deg}$), when $n_0 = 22.1772 \text{ rpm}$, $u_0 = 4 \text{ knots}$, and $\delta_\ell = 45 \text{ deg}$. The duration of each simulation is 2400 s, but the values of m_δ , σ_δ , m_ψ , σ_ψ , and V_ℓ are computed for the part 1200 - 2400 s.

The simulations show that the performance of both the self-tuning regulator and the PID-regulator is improved when Kalman filter estimates are used instead of non-filtered measurements. This is true for each of the three different initial speeds. When Kalman filter estimates are used, the performance of the self-tuning regulator is significantly better than the performance of the PID-regulator in full load condition ($T = 22.3$ m). However, when $T = 10.5$ m, the difference between the two regulators is hardly noticable. When non-filtered measurements are used, a comparison between the self-tuning regulator and the PID-regulator is rather confusing; sometimes the self-tuning regulator is to prefer in front of the PID-regulator and sometimes vice versa. However, the performance of the self-tuning regulator seems to be somewhat better compared to the PID-regulator when $T = 22.3$ m. It can also be concluded that the self-tuning regulator using Kalman filter estimates always is significantly better than the PID-regulator using non-filtered measurements, and that the PID-regulator using Kalman filter estimates always is significantly better than the self-tuning regulator using non-filtered measurements.

In general, the rudder deviations are decreased when the speed scaling V_0/V_S is used instead of $(V_0/V_S)^2$. When $u_0 = 10$ knots, the speed scaling V_0/V_S is to prefer if $T = 10.5$ m, but the speed scaling $(V_0/V_S)^2$ is to prefer if $T = 22.3$ m. When $u_0 = 4$ knots, the speed scaling $(V_0/V_S)^2$, with one exception, is preferable.

The final parameter values of the self-tuning regulator for all the simulations presented in Sections 4.1 and 4.2 are shown in Table 4.7. It can be concluded that the values don't change very much when the speed and the draught are changed.

Fig.	a_1	a_2	a_3	a_4	b_1	b_2	c_1	c_2	Σa_i
4.1	-6.53	5.67	3.43	-2.56	0.50	0.15	-1.65	36.28	0.01
4.2	-6.93	5.70	3.58	-2.97	0.53	0.20	-1.64	34.93	-0.62
4.3	-6.49	5.76	3.61	-2.94	0.47	0.12	-2.08	38.72	-0.06
4.4	-6.28	5.61	3.45	-2.90	0.53	0.20	-1.60	33.07	-0.12
4.5	-6.34	5.31	4.01	-3.24	0.36	-0.01	-3.42	43.48	-0.26
4.6	-5.60	5.11	3.79	-3.37	0.51	0.11	-2.56	35.57	-0.07
4.7	-5.32	3.33	3.83	-2.29	0.08	-0.04	-4.32	67.13	-0.45
4.8	-7.26	6.10	4.11	-3.01	0.53	0.18	-2.07	37.39	-0.06
4.9	-7.20	5.95	4.06	-2.96	0.52	0.18	-2.36	35.55	-0.15
4.10	-6.20	5.46	2.80	-2.02	0.48	0.23	-1.97	42.28	0.04
4.11	-6.53	5.69	3.46	-2.55	0.50	0.16	-1.69	35.92	0.07
4.12	-12.15	13.10	2.62	-4.14	0.83	0.22	-	-	-0.57
4.13	-10.68	11.86	2.18	-3.63	0.86	0.28	-	-	-0.27
4.14	-11.18	12.08	1.01	-2.77	0.62	-0.02	-	-	-0.86
4.15	-14.45	15.38	4.81	-5.94	0.47	0.10	-	-	-0.20
4.16	-13.80	15.07	5.55	-6.78	0.54	0.06	-	-	0.04
4.17	-12.66	12.82	5.84	-5.88	0.61	0.15	-	-	0.12
4.18	-12.76	17.64	-3.86	-1.75	0.09	0.23	-	-	-0.73
4.19	-12.31	11.34	6.77	-5.95	0.52	0.09	-	-	-0.15
4.20	-11.43	11.22	6.52	-6.29	0.55	0.05	-	-	0.02
4.21	-9.91	9.11	6.43	-5.67	0.53	0.03	-	-	-0.04
4.22	-9.47	9.67	3.55	-4.42	0.15	0.27	-	-	-0.67
4.23	-9.24	10.28	2.30	-3.86	0.26	0.20	-	-	-0.52
4.24	-9.62	10.96	2.09	-3.97	0.38	0.19	-	-	-0.54
4.25	-8.08	8.55	2.24	-3.24	-0.14	0.33	-	-	-0.53
4.26	-7.57	5.76	3.81	-2.39	0.47	0.15	-1.32	36.42	-0.39
4.27	-6.19	5.42	3.26	-2.81	0.52	0.19	-2.09	35.16	-0.32
4.29	-8.21	9.19	2.44	-3.46	0.04	0.17	-	-	-0.04
4.32	-6.75	7.14	1.31	-1.84	0.09	0.27	-	-	-0.14
4.35	-7.66	8.77	2.25	-3.46	0.07	0.18	-	-	-0.10
4.37	-4.63	3.26	3.31	-2.00	0.45	0.07	-1.25	22.26	-0.06
4.40	-6.16	6.76	1.02	-1.71	0.08	0.29	-	-	-0.09
4.42	-3.63	2.94	2.47	-1.84	0.52	0.09	-0.64	12.48	-0.06
4.45	-8.35	9.41	1.83	-3.31	0.04	0.19	-	-	-0.42
4.47	-4.61	1.90	3.01	-0.66	0.53	0.20	-2.54	28.74	-0.36
4.50	-6.38	8.24	0.59	-2.56	0.22	0.31	-	-	-0.11
4.52	-3.46	1.72	2.61	-1.01	0.64	0.25	-2.87	21.31	-0.14

Table 4.7 - Final parameter values of the self-tuning regulator.

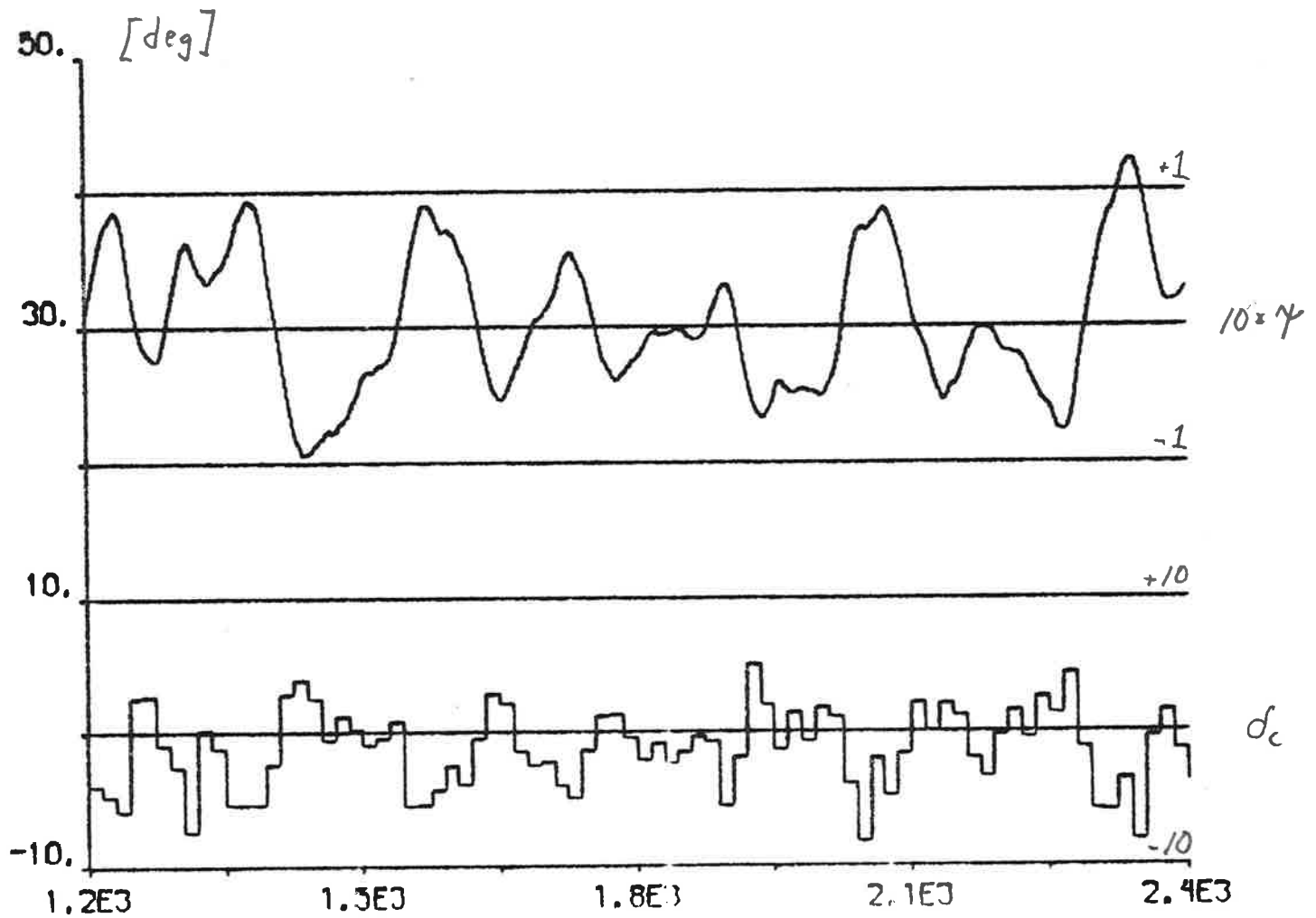


Fig. 4.12 - $T = 22.3$ m, $n_0 = 87.6$ rpm, $u_0 = 15.8$ knots,
 $\delta_\ell = 10$ deg, self-tuning regulator using non-filtered
 measurements ($k = 4$, $T_s = 15$ s, $q_2^* = 0$, $V_c = 8$ m/s).

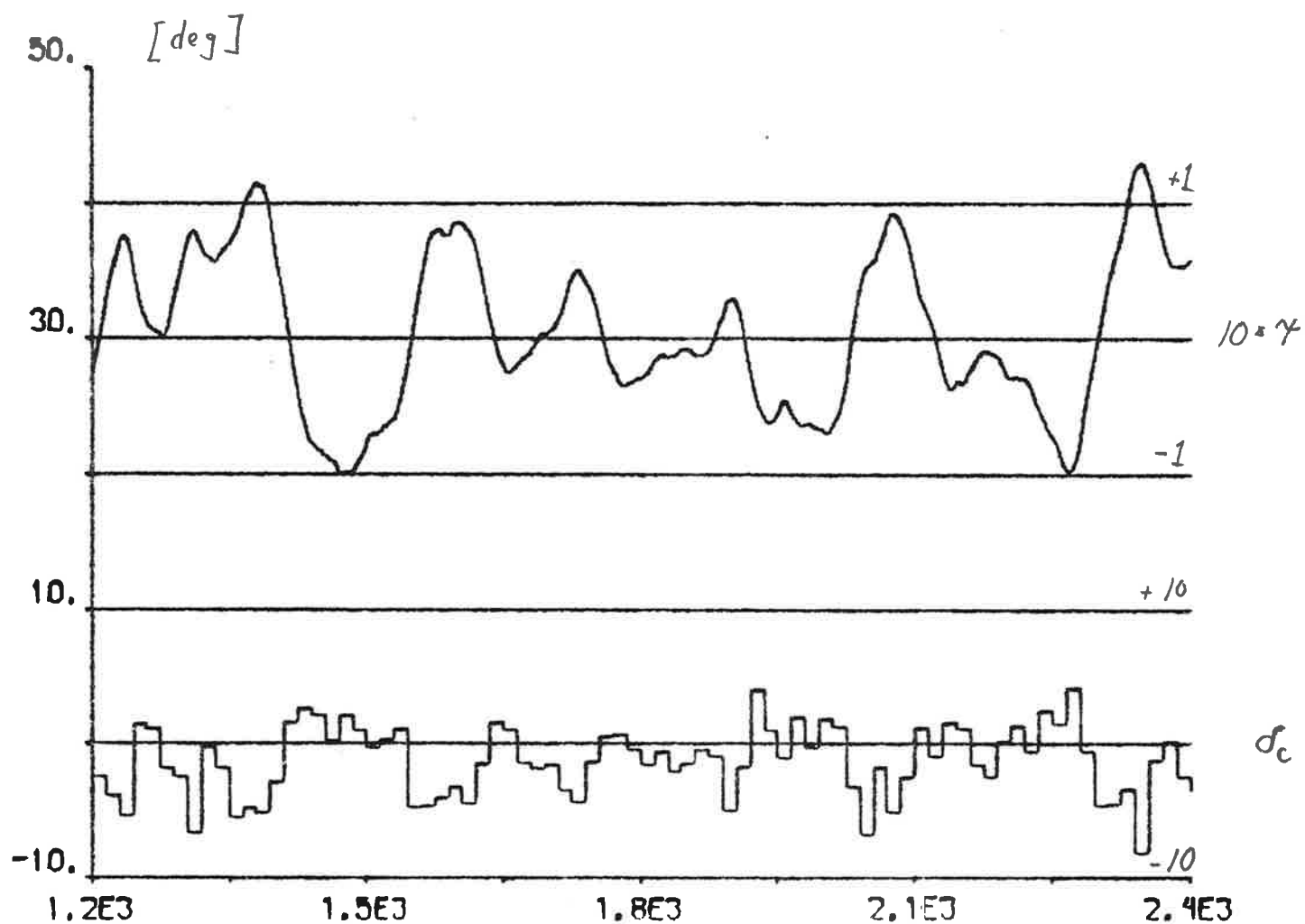


Fig. 4.13 - $T = 22.3$ m, $n_0 = 87.6$ rpm, $u_0 = 15.8$ knots,
 $\delta_\ell = 10$ deg, self-tuning regulator using
 non-filtered measurements ($k = 5$, $T_s = 15$ s,
 $q_2^* = 0$, $V_c = 8$ m/s).

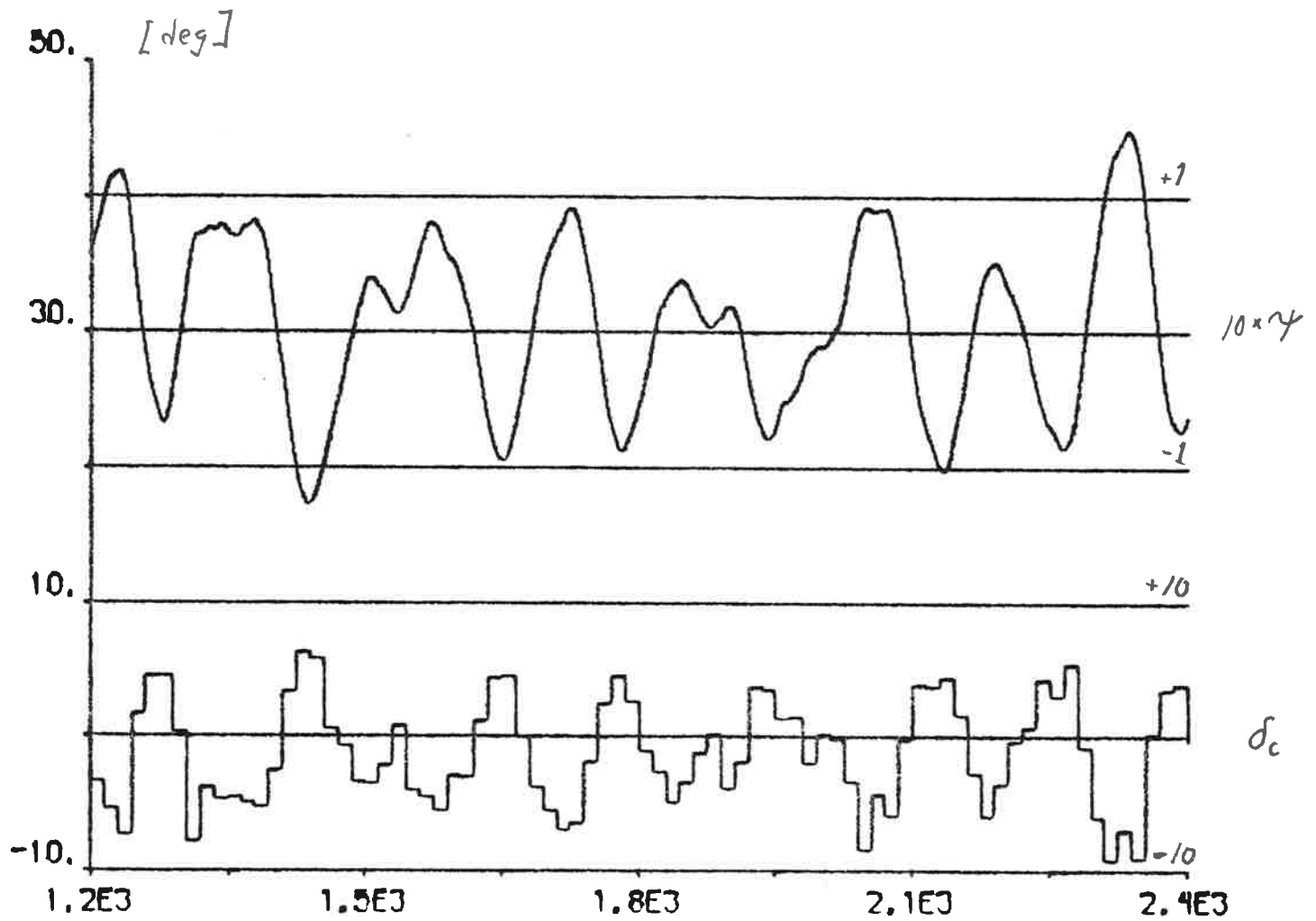


Fig. 4.14 - $T = 22.3$ m, $n_0 = 87.6$ rpm, $u_0 = 15.8$ knots,
 $\delta_\ell = 10$ deg, self-tuning regulator using
 non-filtered measurements ($k = 6$, $T_s = 15$ s,
 $q_2^* = 0$, $V_c = 8$ m/s).

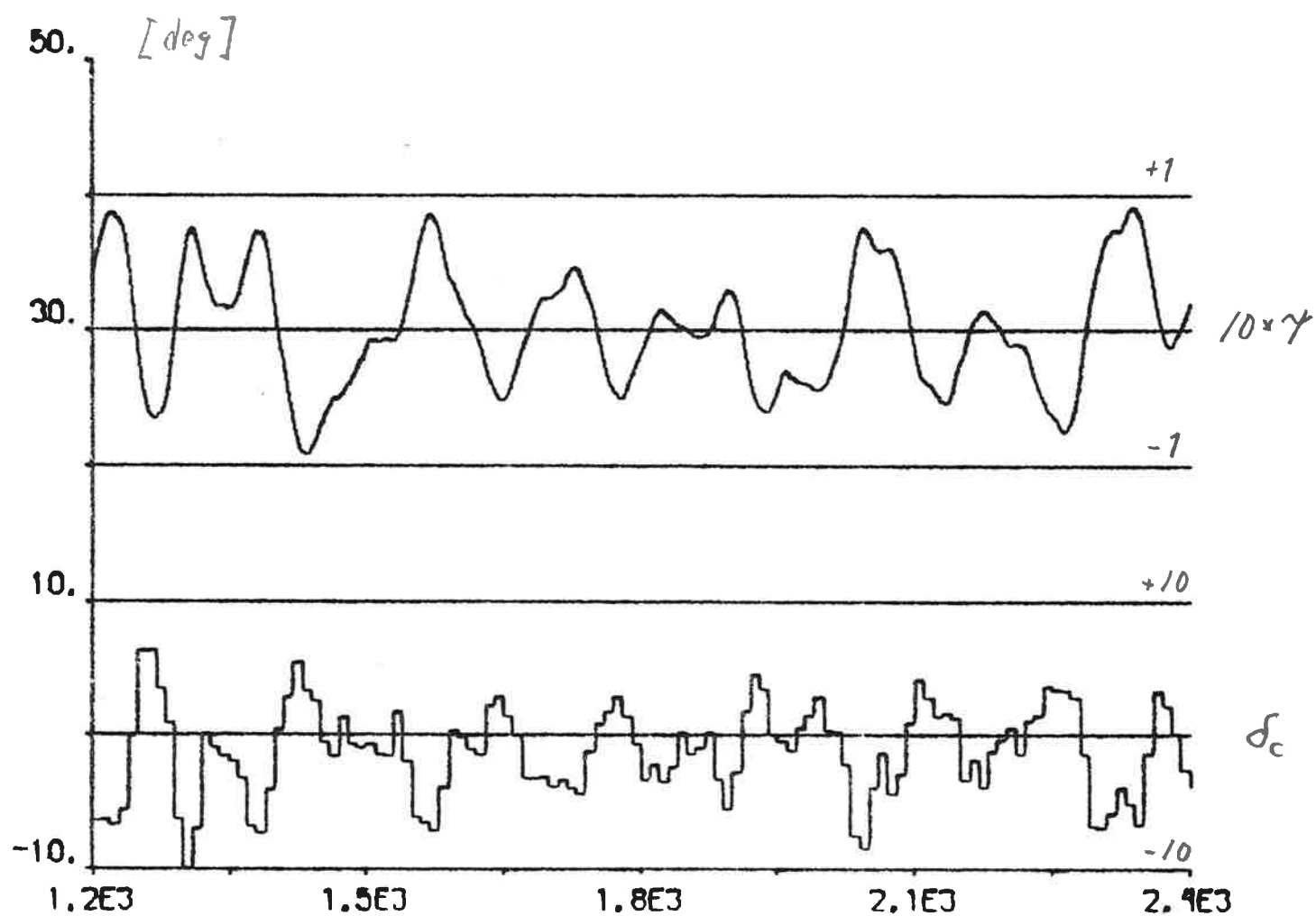


Fig. 4.15 - $T = 22.3$ m, $n_0 = 87.6$ rpm, $u_0 = 15.8$ knots,
 $\delta_\ell = 10$ deg, self-tuning regulator using
 non-filtered measurements ($k = 5$, $T_s = 10$ s,
 $q_2^* = 0$, $V_c = 8$ m/s).

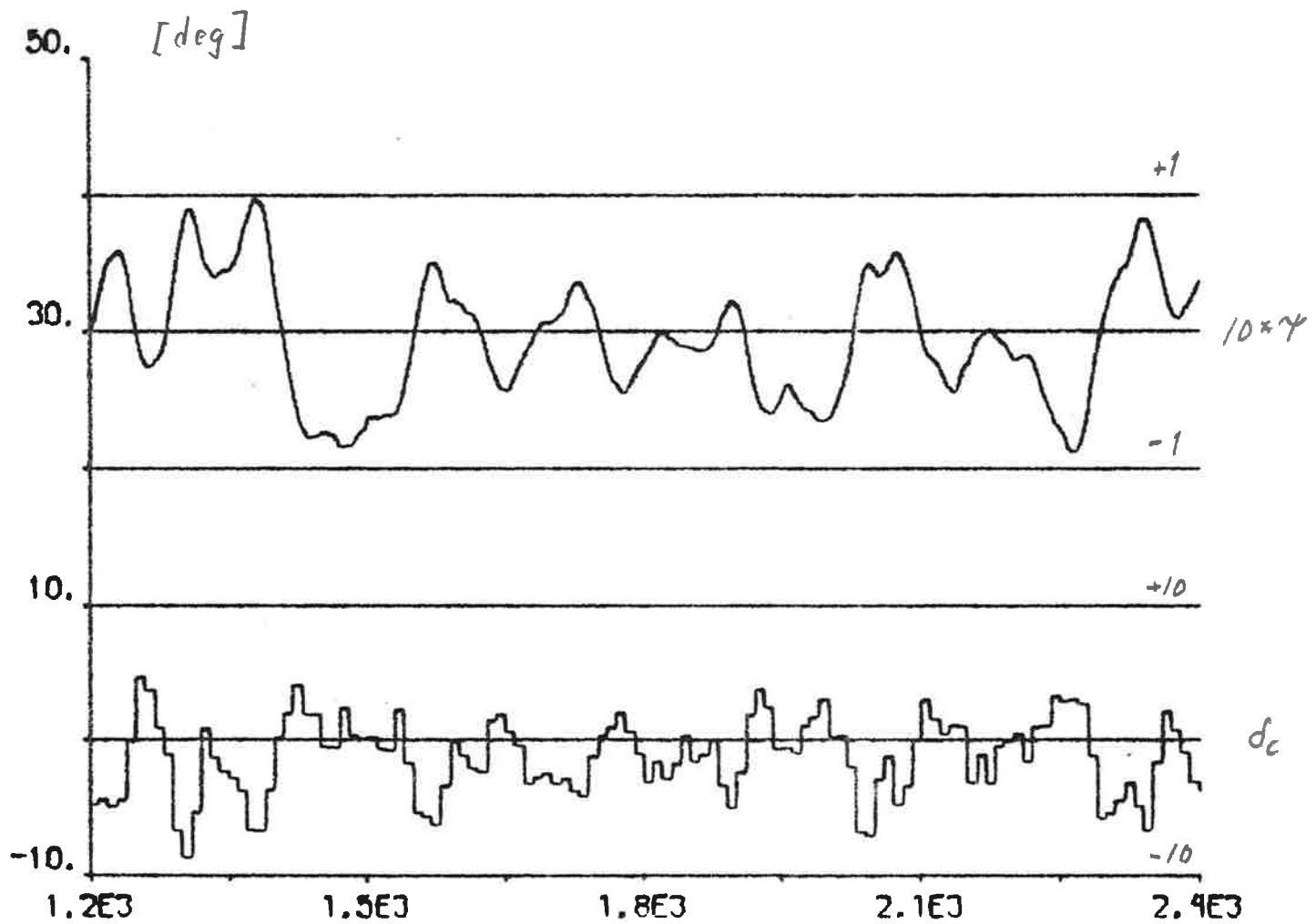


Fig. 4.16 - $T = 22.3$ m, $n_0 = 87.6$ rpm, $u_0 = 15.8$ knots,
 $\delta_\ell = 10$ deg, self-tuning regulator using
 non-filtered measurements ($k = 6$, $T_s = 10$ s,
 $q_2^* = 0$, $V_c = 8$ m/s).

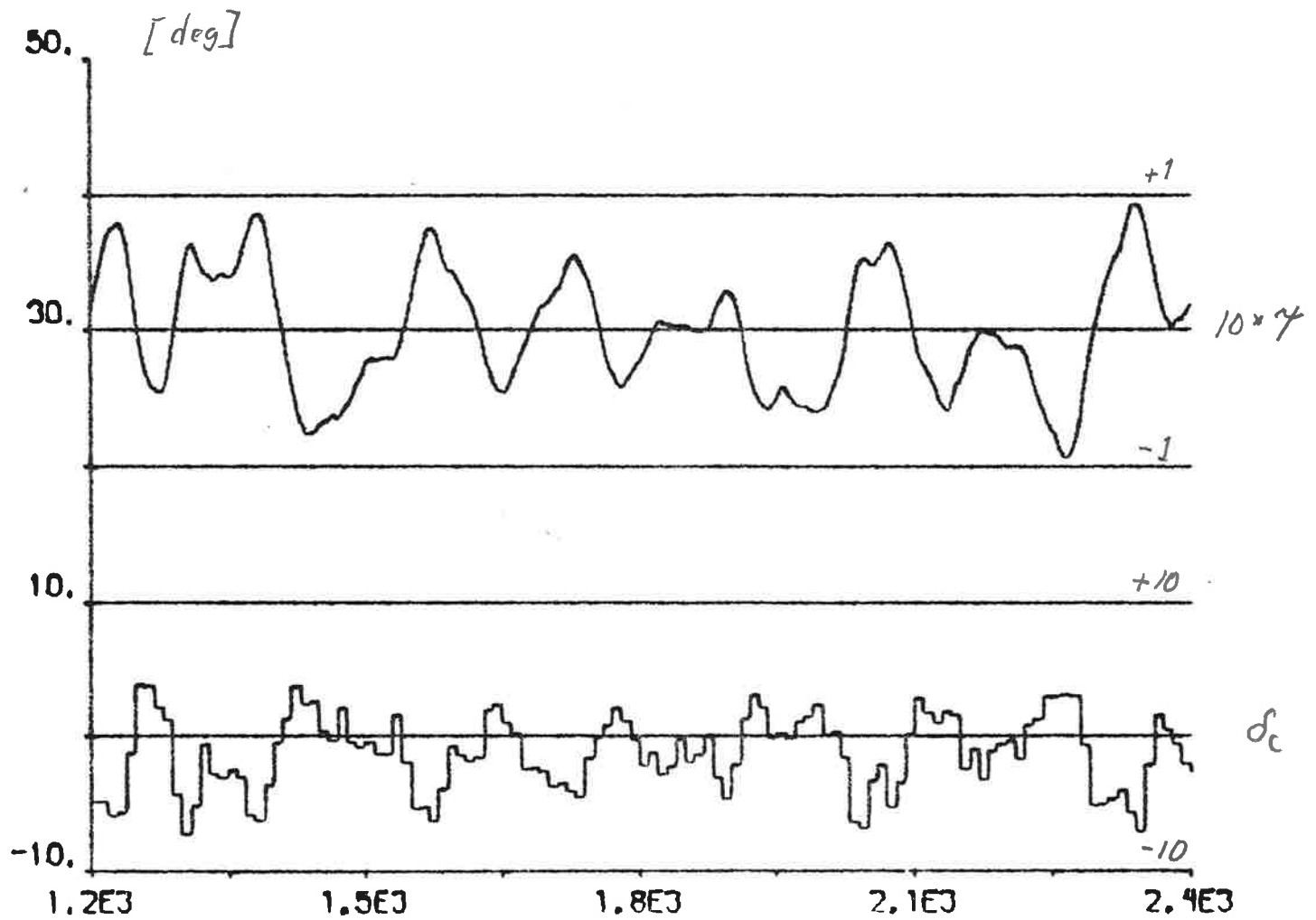


Fig. 4.17 - $T = 22.3$ m, $n_0 = 87.6$ rpm, $u_0 = 15.8$ knots,
 $\delta_\ell = 10$ deg, self-tuning regulator using
 non-filtered measurements ($k = 7$, $T_s = 10$ s,
 $q_2^* = 0$, $V_c = 8$ m/s).

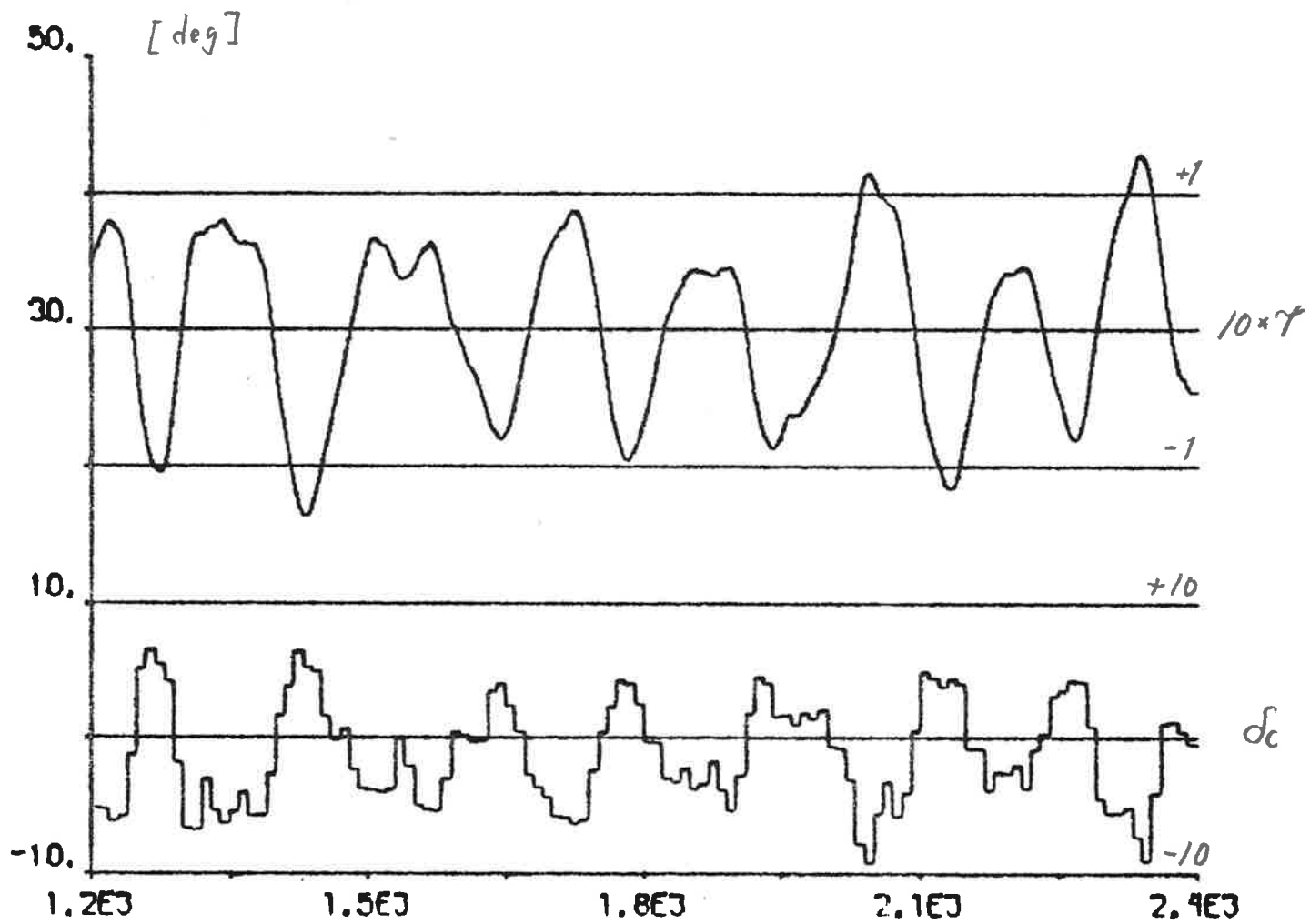


Fig. 4.18 - $T = 22.3$ m, $n_0 = 87.6$ rpm, $u_0 = 15.8$ knots,
 $\delta_\ell = 10$ deg, self-tuning regulator using
 non-filtered measurements ($k = 8$, $T_s = 10$ s,
 $q_2^* = 0$, $V_c = 8$ m/s).

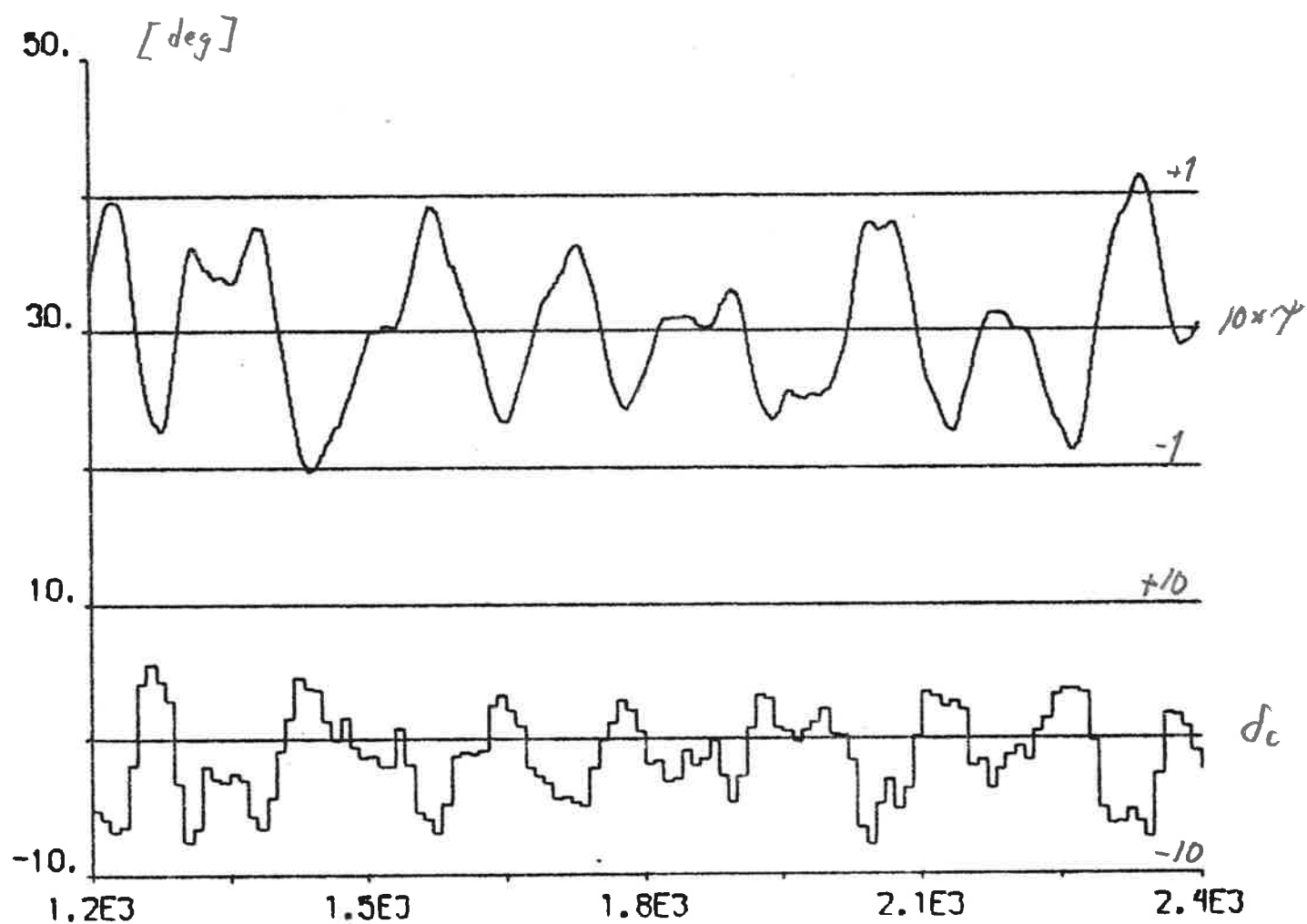


Fig. 4.19 - $T = 22.3$ m, $n_0 = 87.6$ rpm, $u_0 = 15.8$ knots,
 $\delta_\ell = 10$ deg, self-tuning regulator using
 non-filtered measurements ($k = 5$, $T_s = 10$ s,
 $q_2^* = 0.05$, $V_c = 8$ m/s).

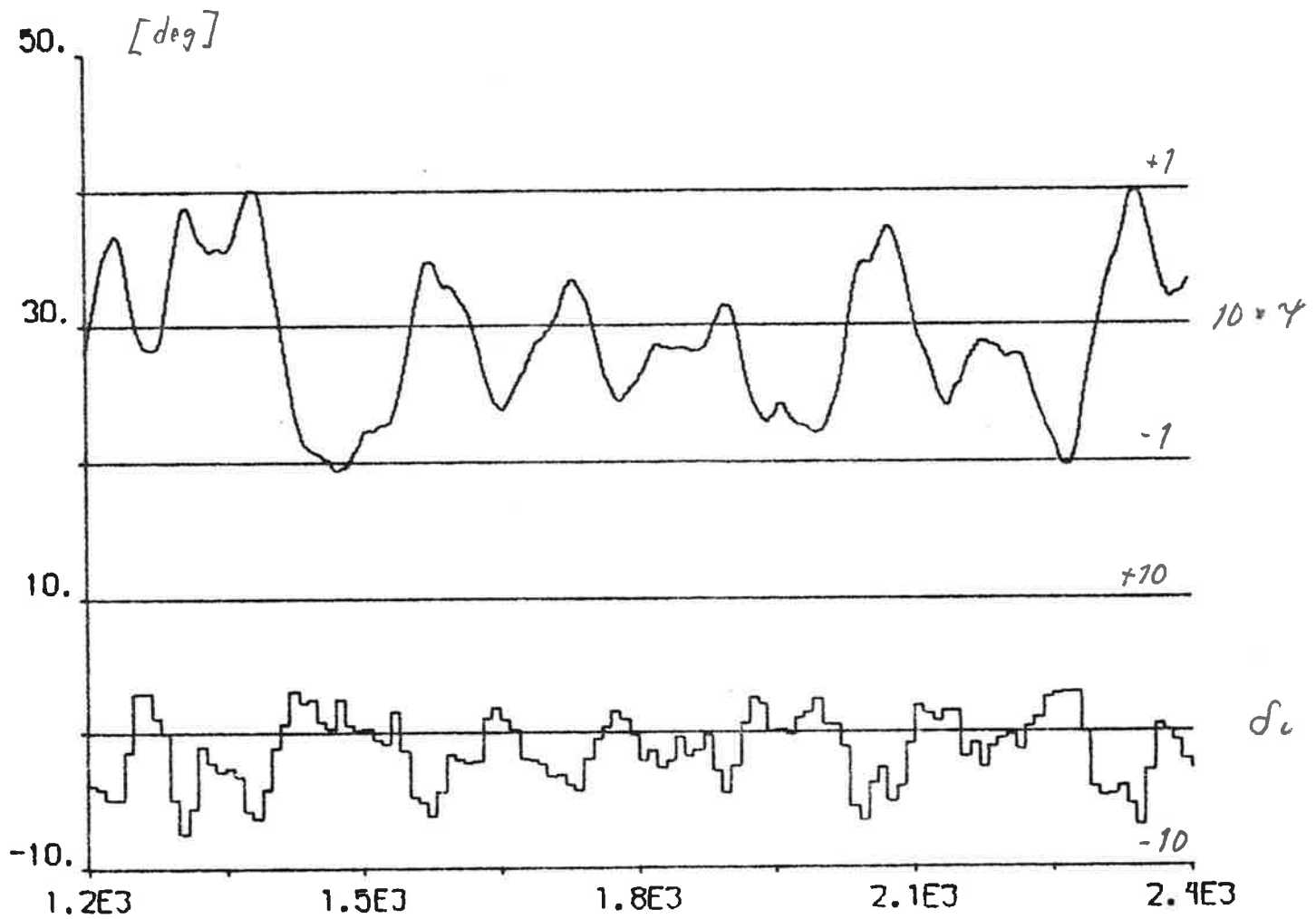


Fig. 4.20 - $T = 22.3$ m, $n_0 = 87.6$ rpm, $u_0 = 15.8$ knots,
 $\delta_\ell = 10$ deg, self-tuning regulator using
 non-filtered measurements ($k = 6$, $T_s = 10$ s,
 $q_2^* = 0.05$, $V_C = 8$ m/s).

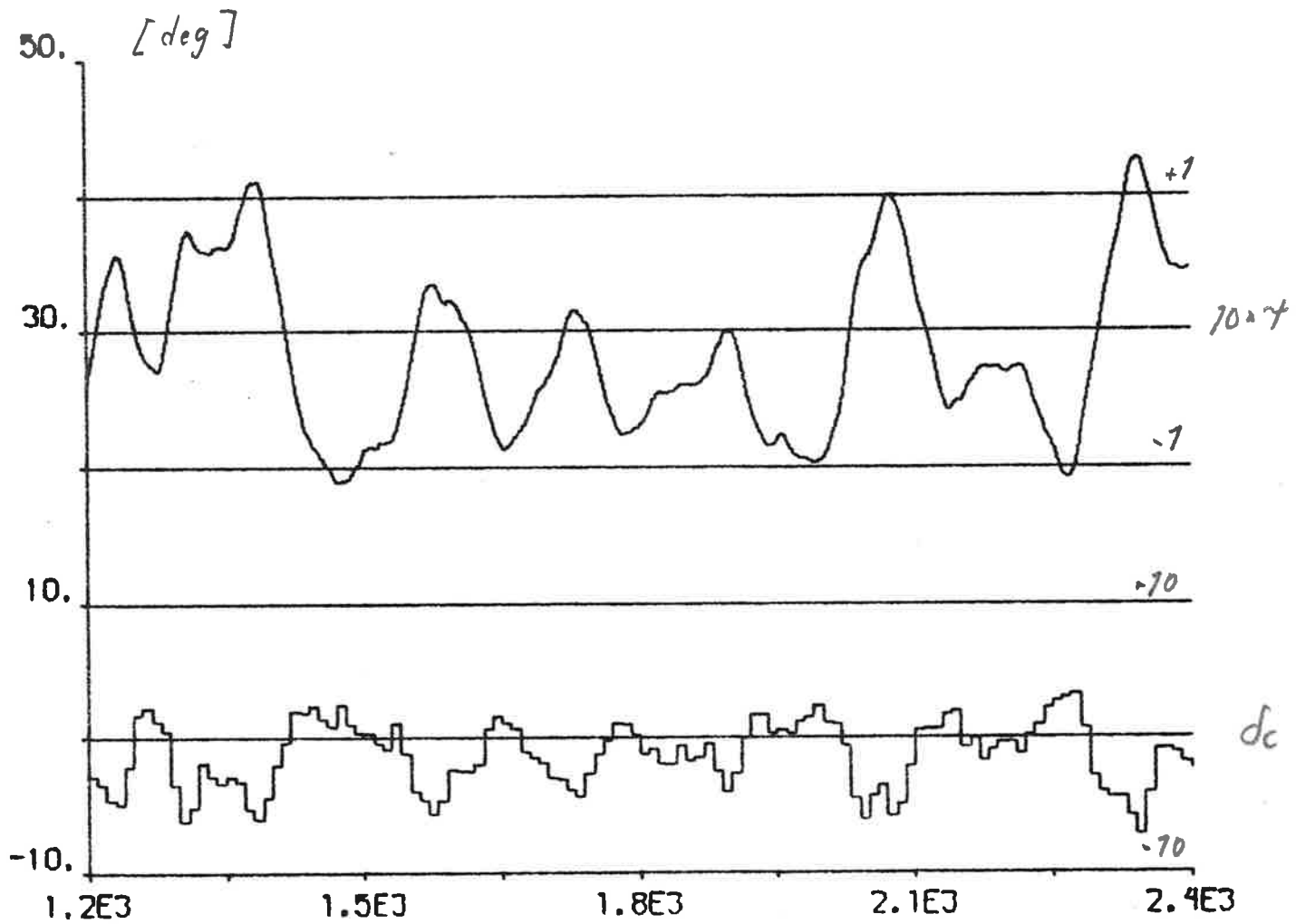


Fig. 4.21 - $T = 22.3$ m, $n_0 = 87.6$ rpm, $u_0 = 15.8$ knots,
 $\delta_\ell = 10$ deg, self-tuning regulator using
 non-filtered measurements ($k = 6$, $T_s = 10$ s,
 $q_2^* = 0.1$, $V_c = 8$ m/s).

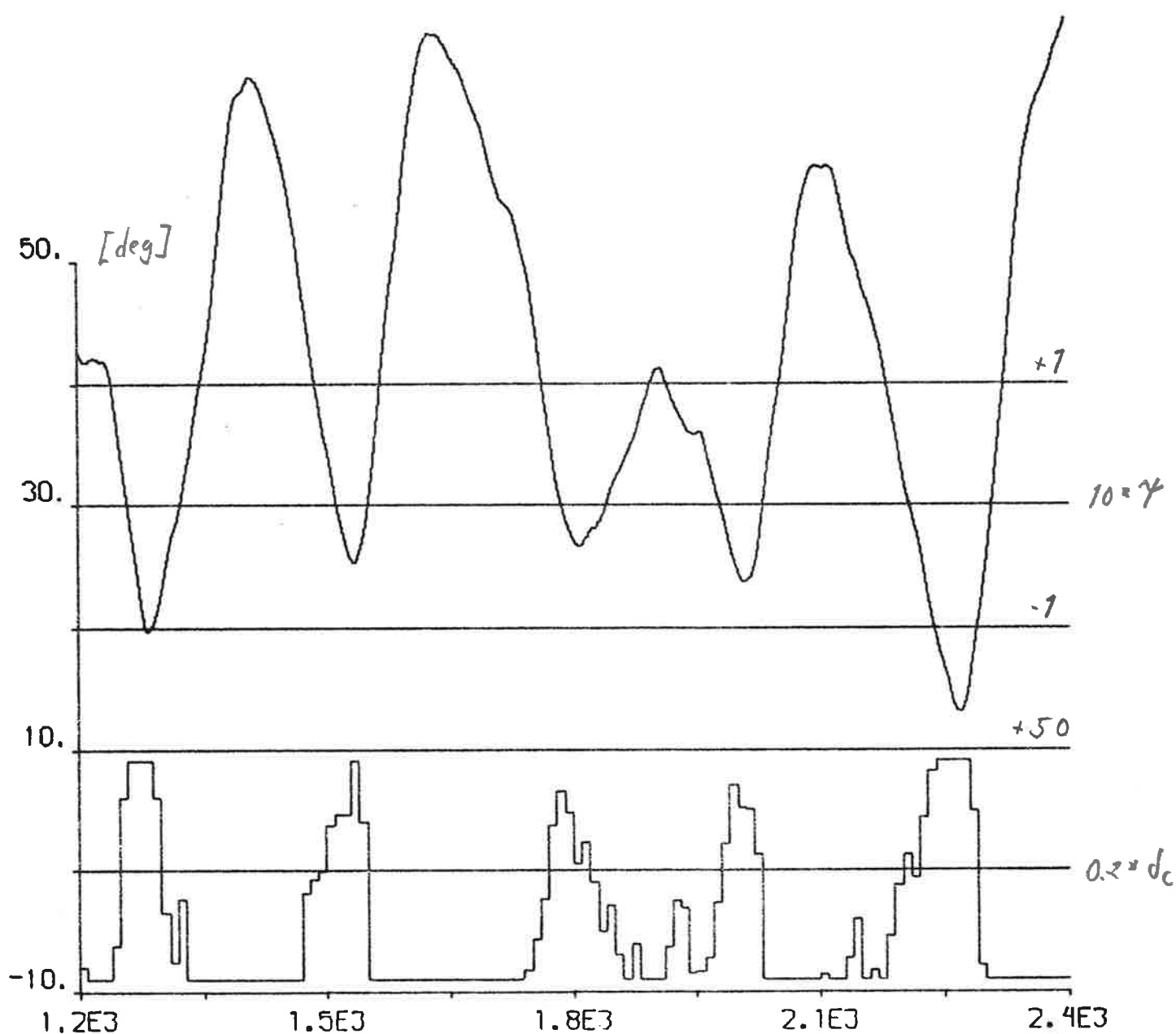


Fig. 4.22 - $T = 22.3$ m, $n_0 = 22.1772$ rpm, $u_0 = 4$ knots, $\delta_\ell = 45$ deg, self-tuning regulator using non-filtered measurements ($k = 6$, $T_s = 10$ s, $q_2^* = 0$, $V_c = 2$ m/s).

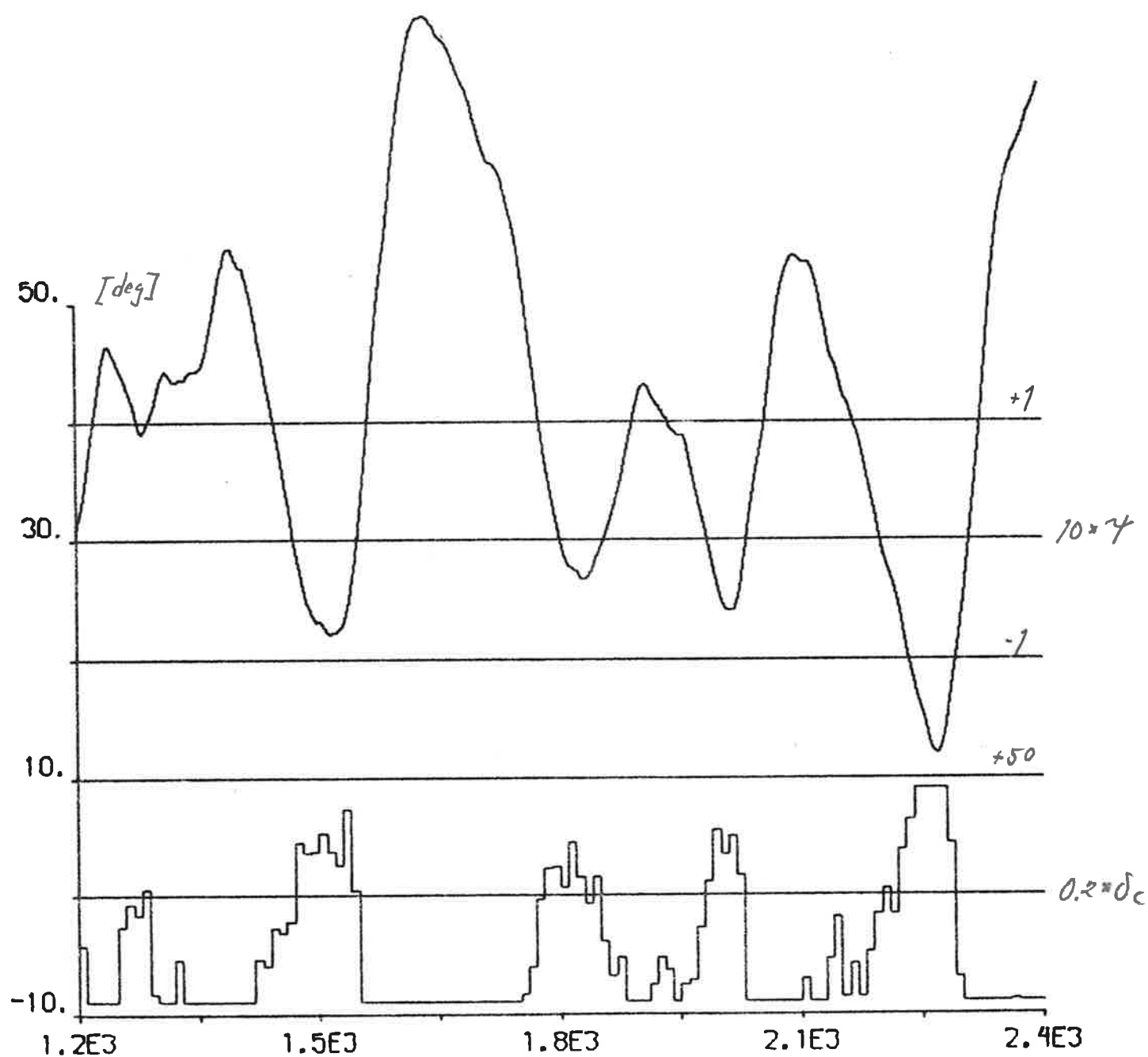


Fig. 4.23 - $T = 22.3$ m, $n_0 = 22.1772$ rpm, $u_0 = 4$ knots, $\delta_\ell = 45$ deg, self-tuning regulator using non-filtered measurements ($k = 7$, $T_s = 10$ s, $q_2^* = 0$, $v_c = 2$ m/s).

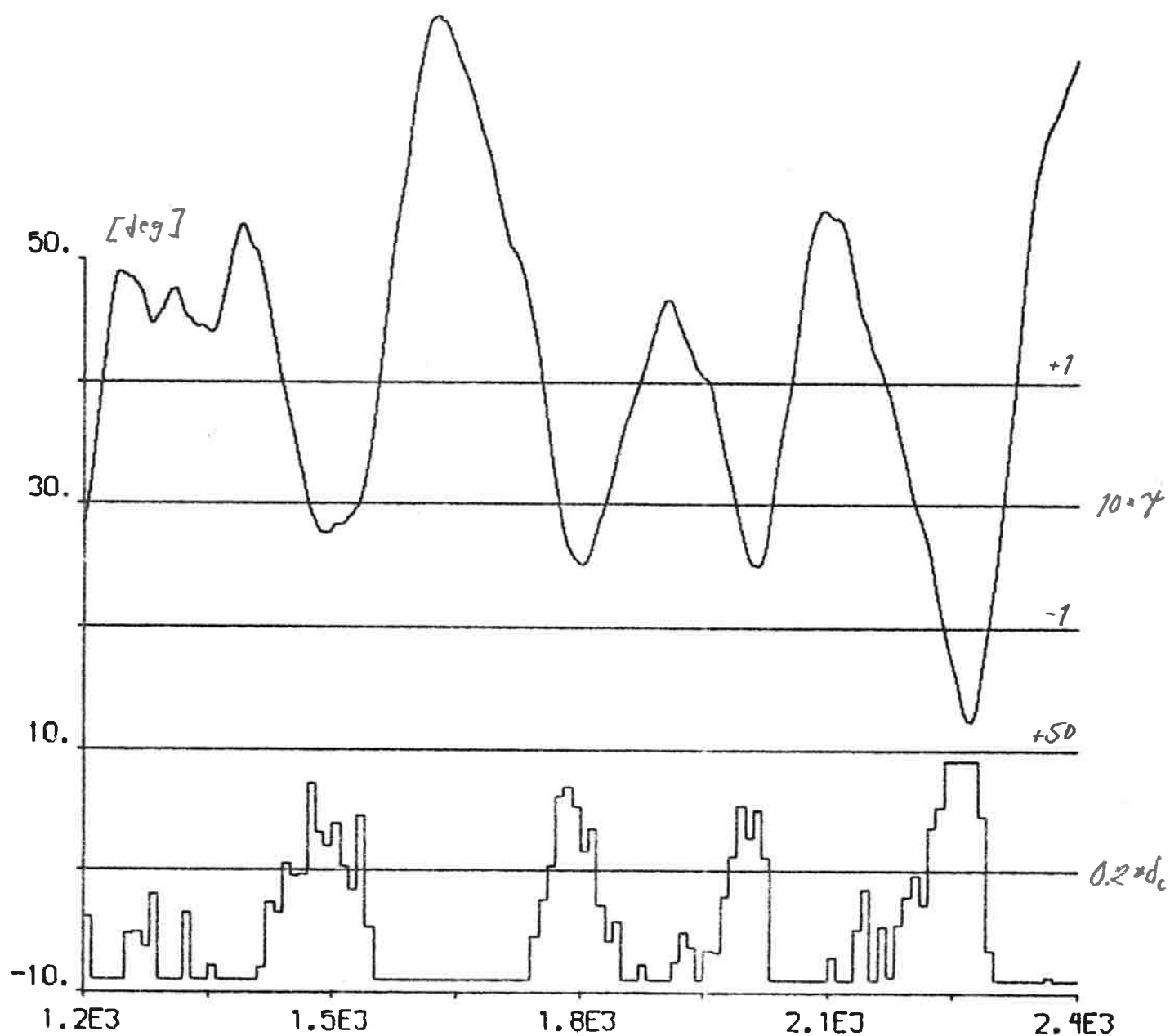


Fig. 4.24 - $T = 22.3$ m, $n_0 = 22.1772$ rpm, $u_0 = 4$ knots, $\delta_\ell = 45$ deg, self-tuning regulator using non-filtered measurements ($k = 8$, $T_s = 10$ s, $q_2^* = 0$, $V_C = 2$ m/s).

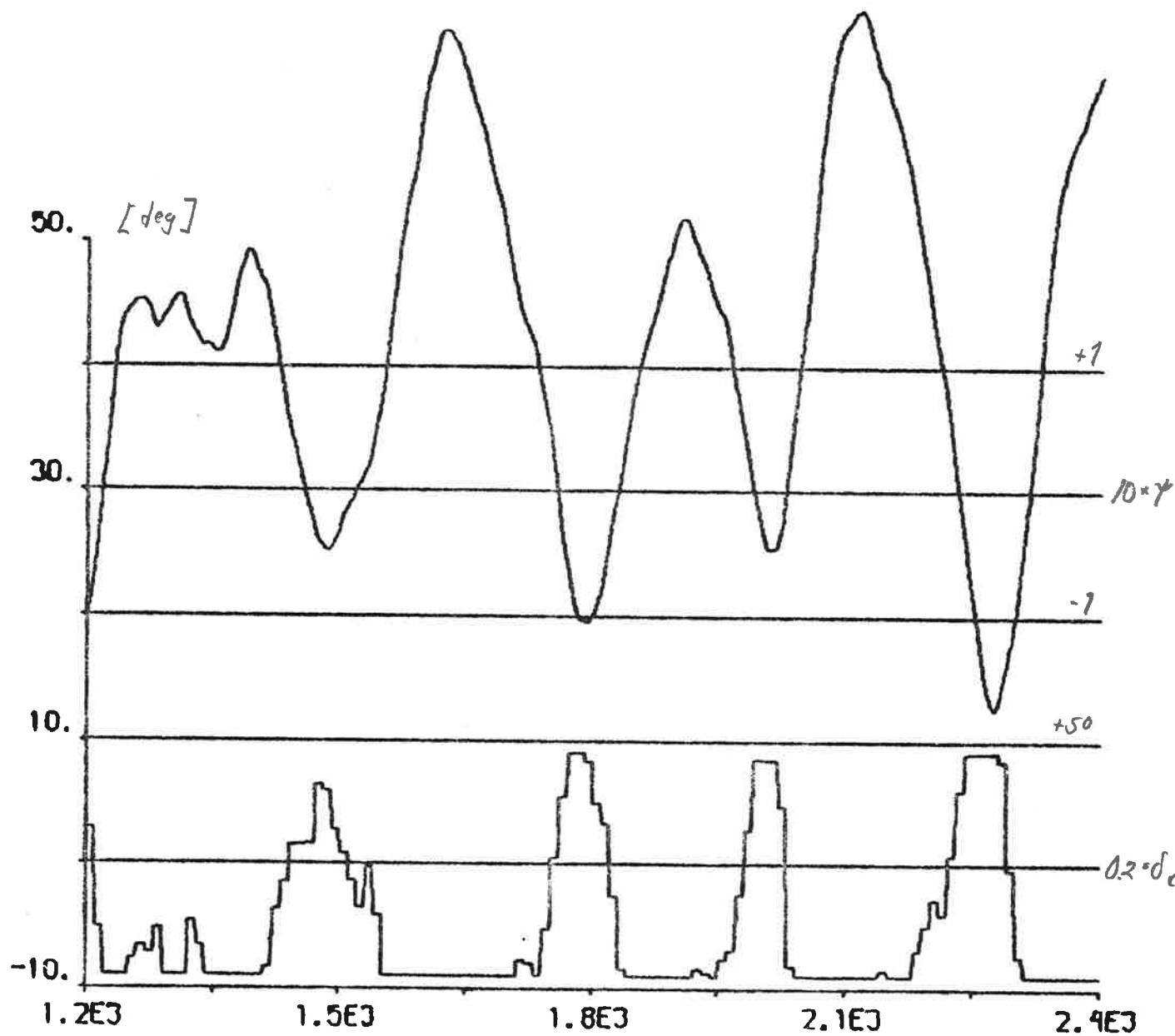


Fig. 4.25 - $T = 22.3$ m, $n_0 = 22.1772$ rpm, $u_0 = 4$ knots, $\delta_\ell = 45$ deg, self-tuning regulator using non-filtered measurements ($k = 6$, $T_s = 10$ s, $q_2^* = 0.05$, $V_c = 2$ m/s).

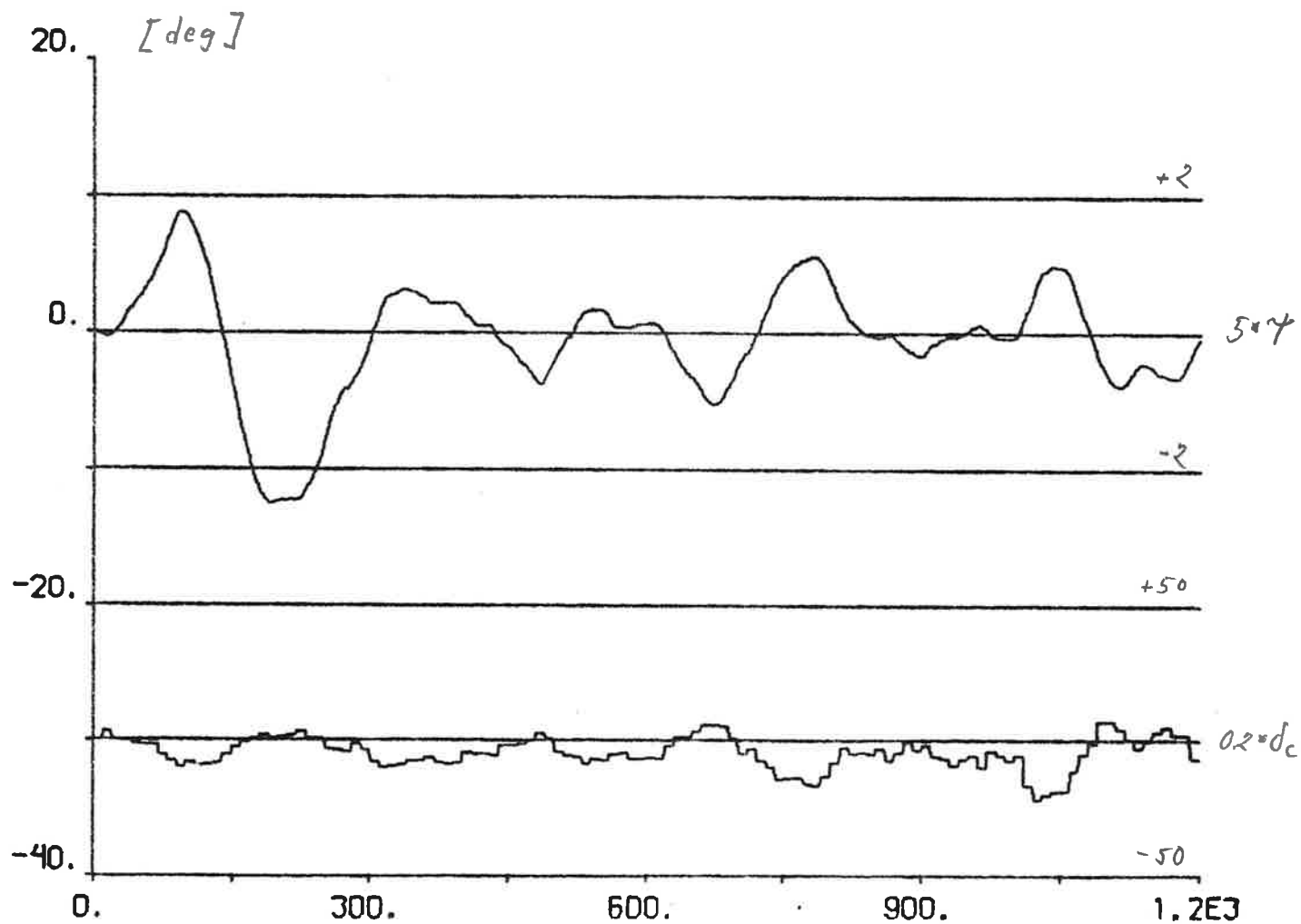


Fig. 4.26 a - $T = 22.3$ m, $n_0 = 22.1772$ rpm, $u_0 = 15.8$ knots,
 $\delta_\ell = 45$ deg, self-tuning regulator using
 estimates from the Kalman filter.

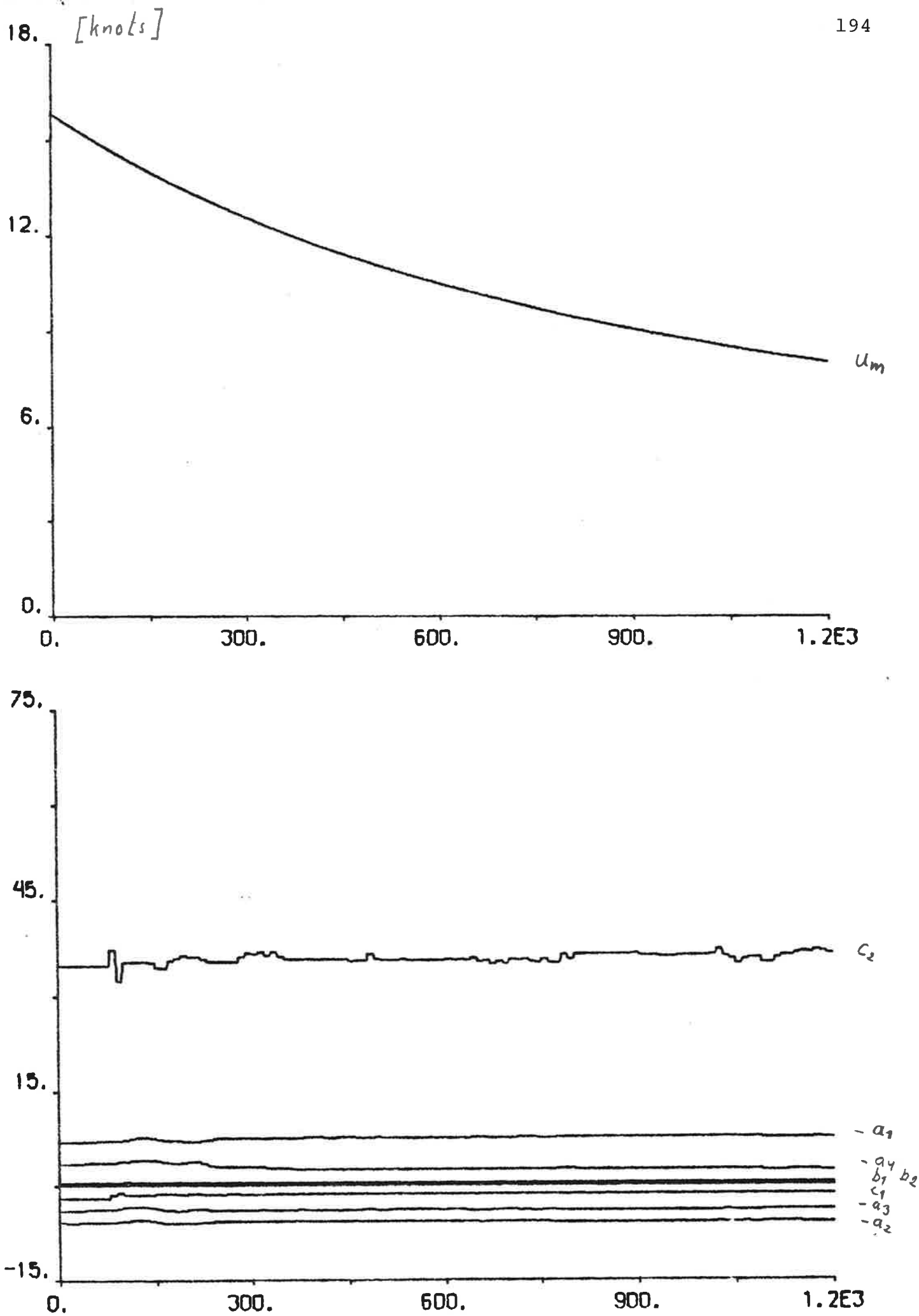


Fig. 4.26 b

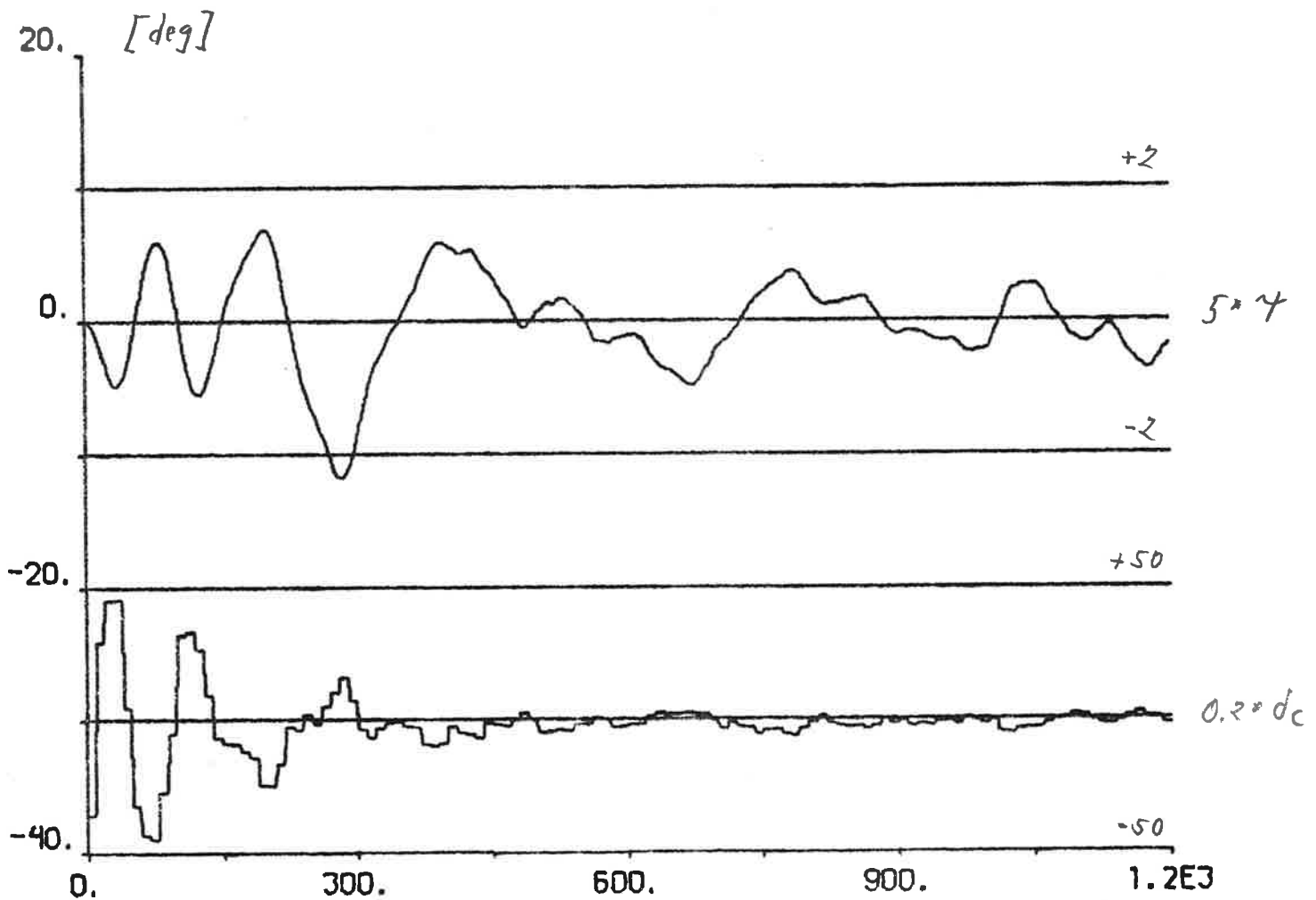


Fig. 4.27 a - $T = 22.3$ m, $n_0 = 87.6$ rpm, $u_0 = 4$ knots,
 $\delta_\ell = 45$ deg, self-tuning regulator using
 estimates from the Kalman filter.

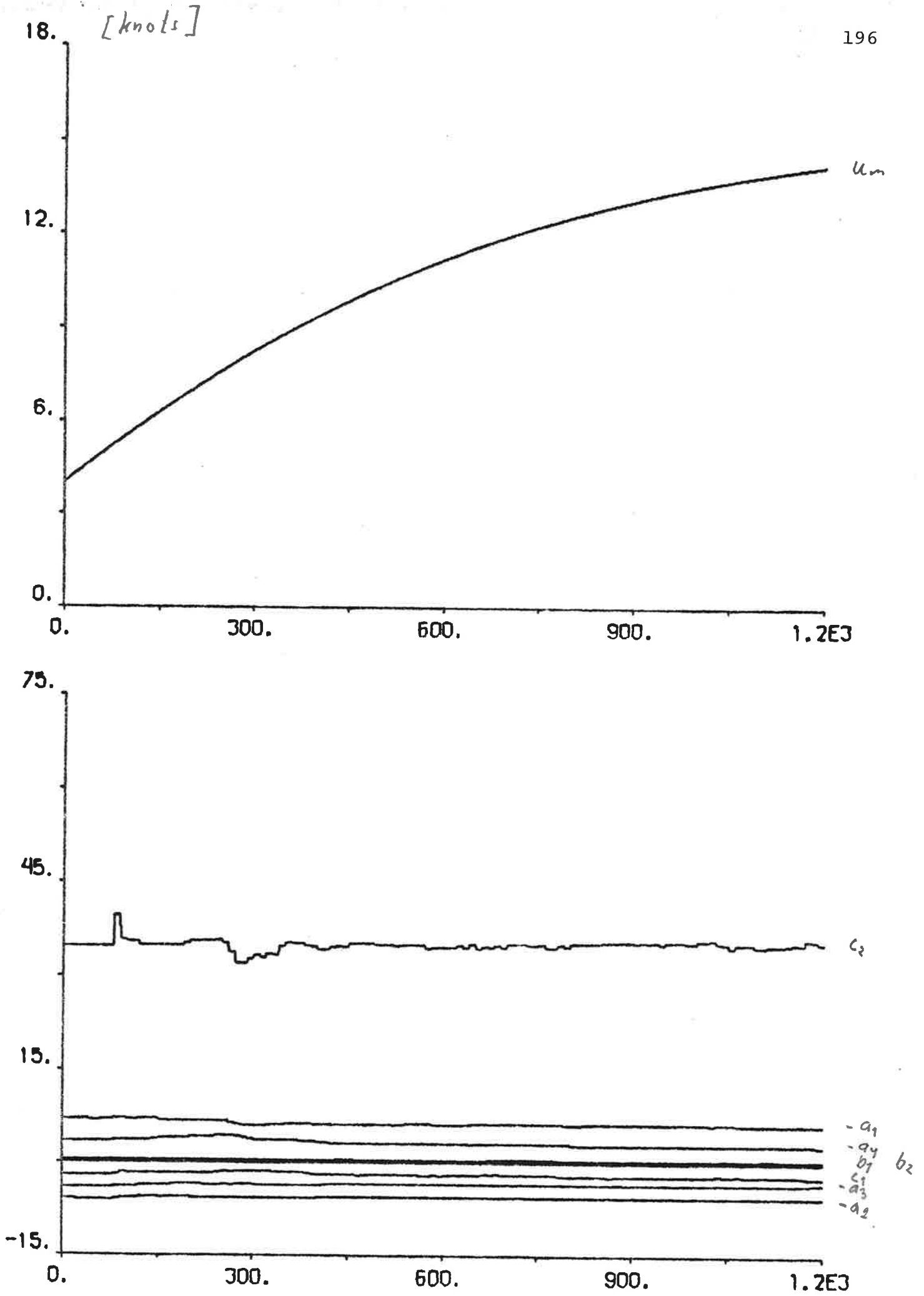


Fig. 4.27 b

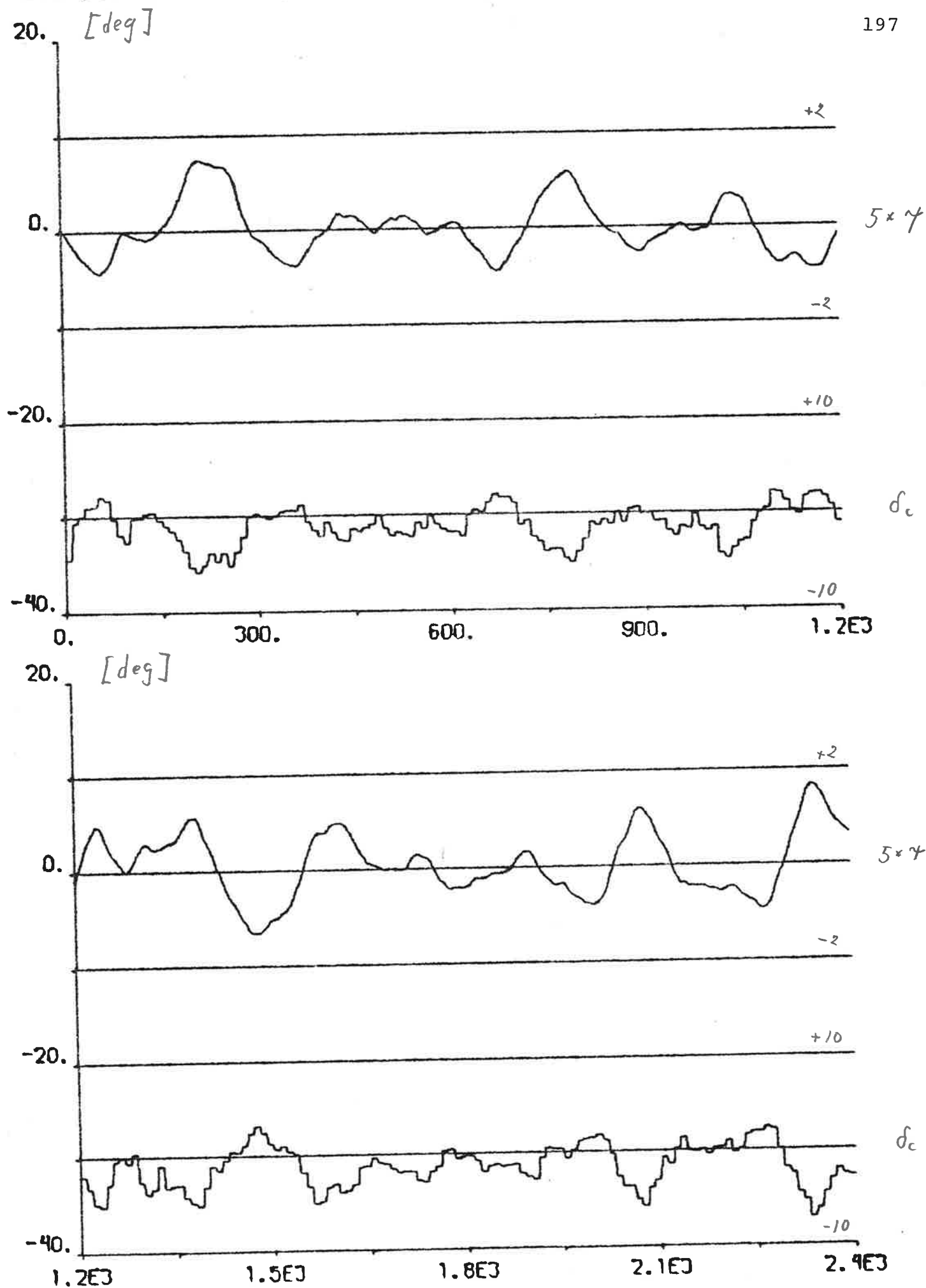


Fig. 4.28 - $T = 22.3$ m, $n_0 = 87.6$ rpm, $u_0 = 15.8$ knots, $\delta_\ell = 10$ deg, PID-regulator using estimates from the Kalman filter.

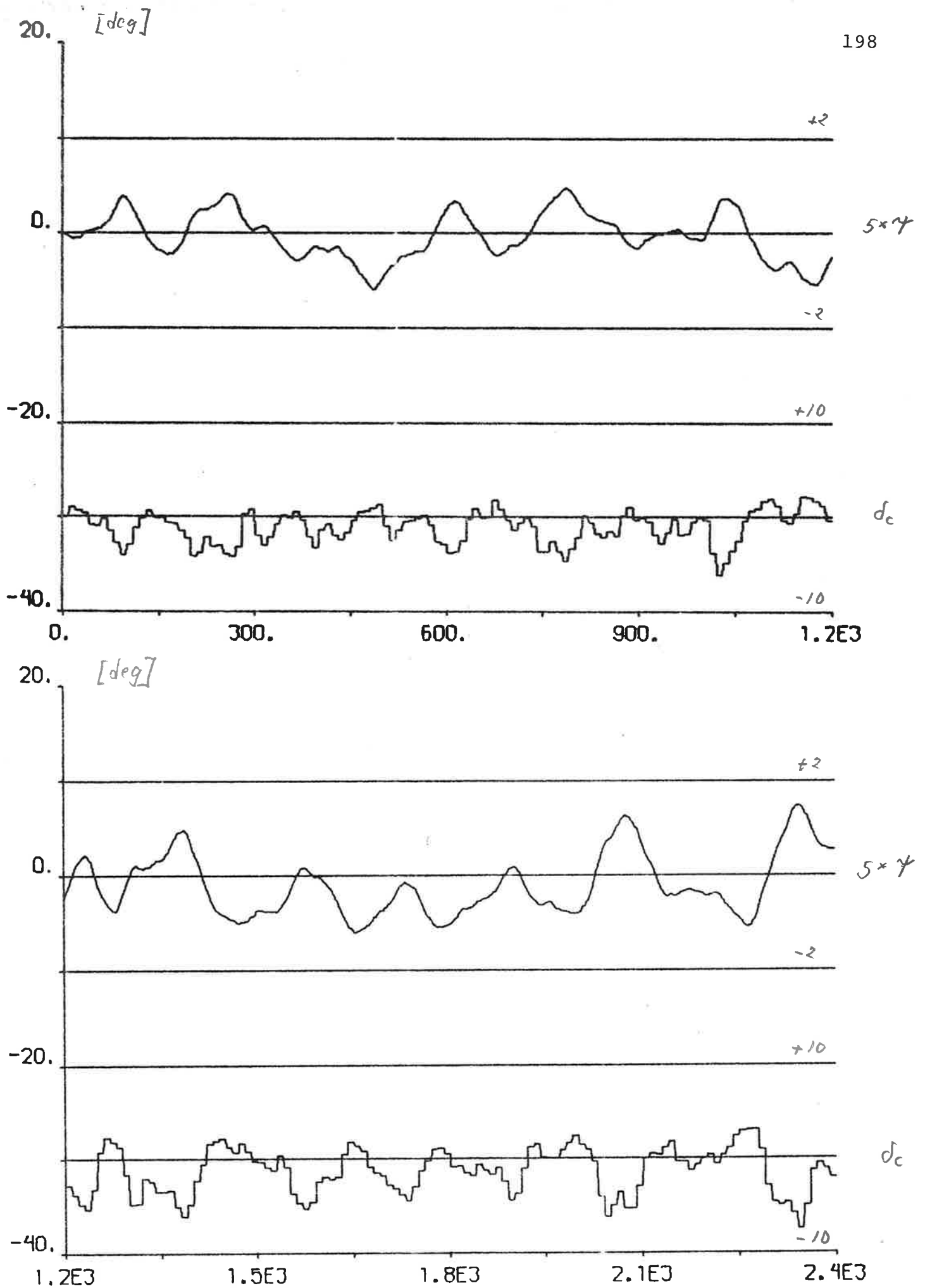


Fig. 4.29 a - $T = 22.3$ m, $n_0 = 87.6$ rpm, $u_0 = 15.8$ knots, $\delta_\ell = 10$ deg, self-tuning regulator using non-filtered measurements.

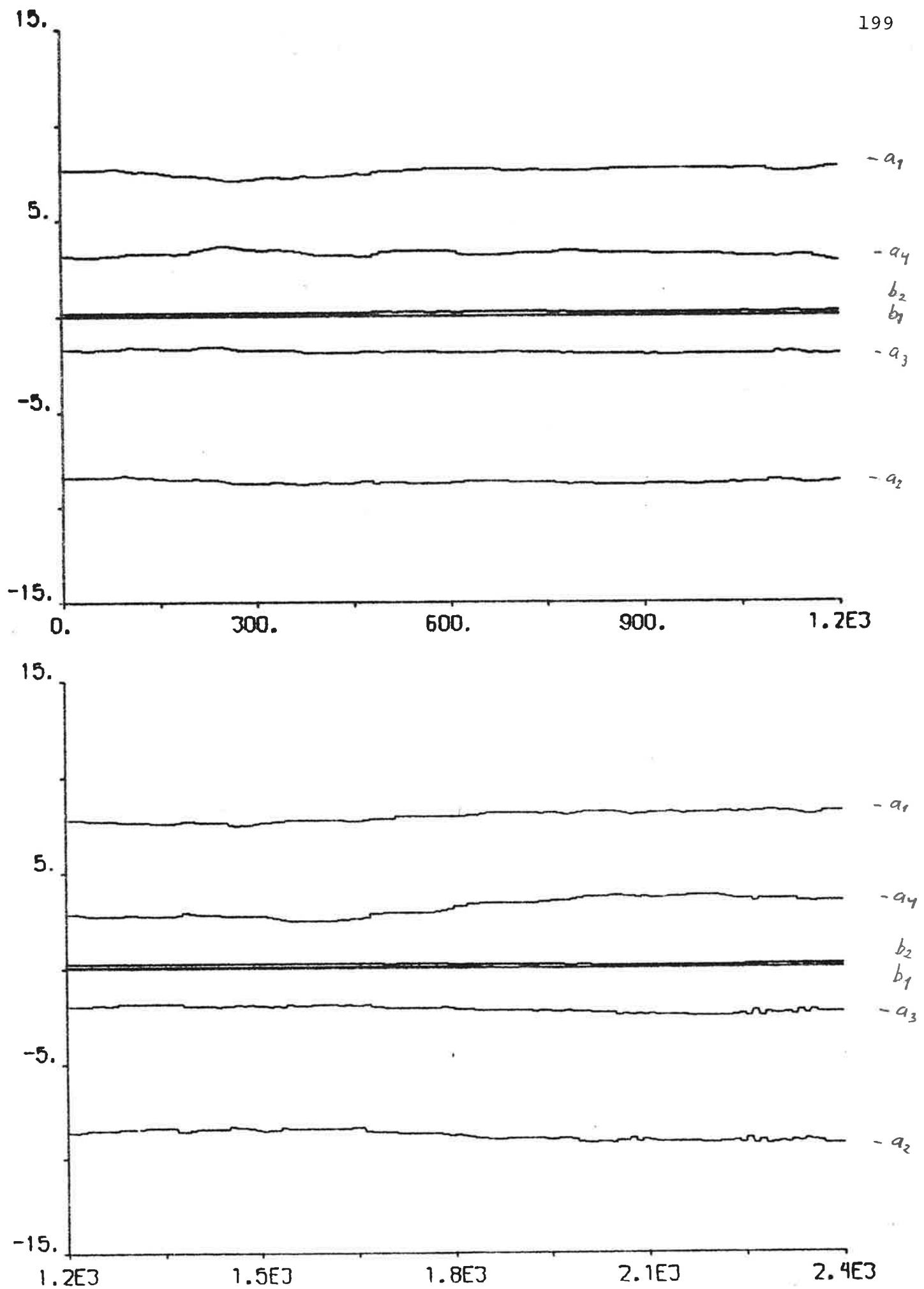


Fig. 4.29 b

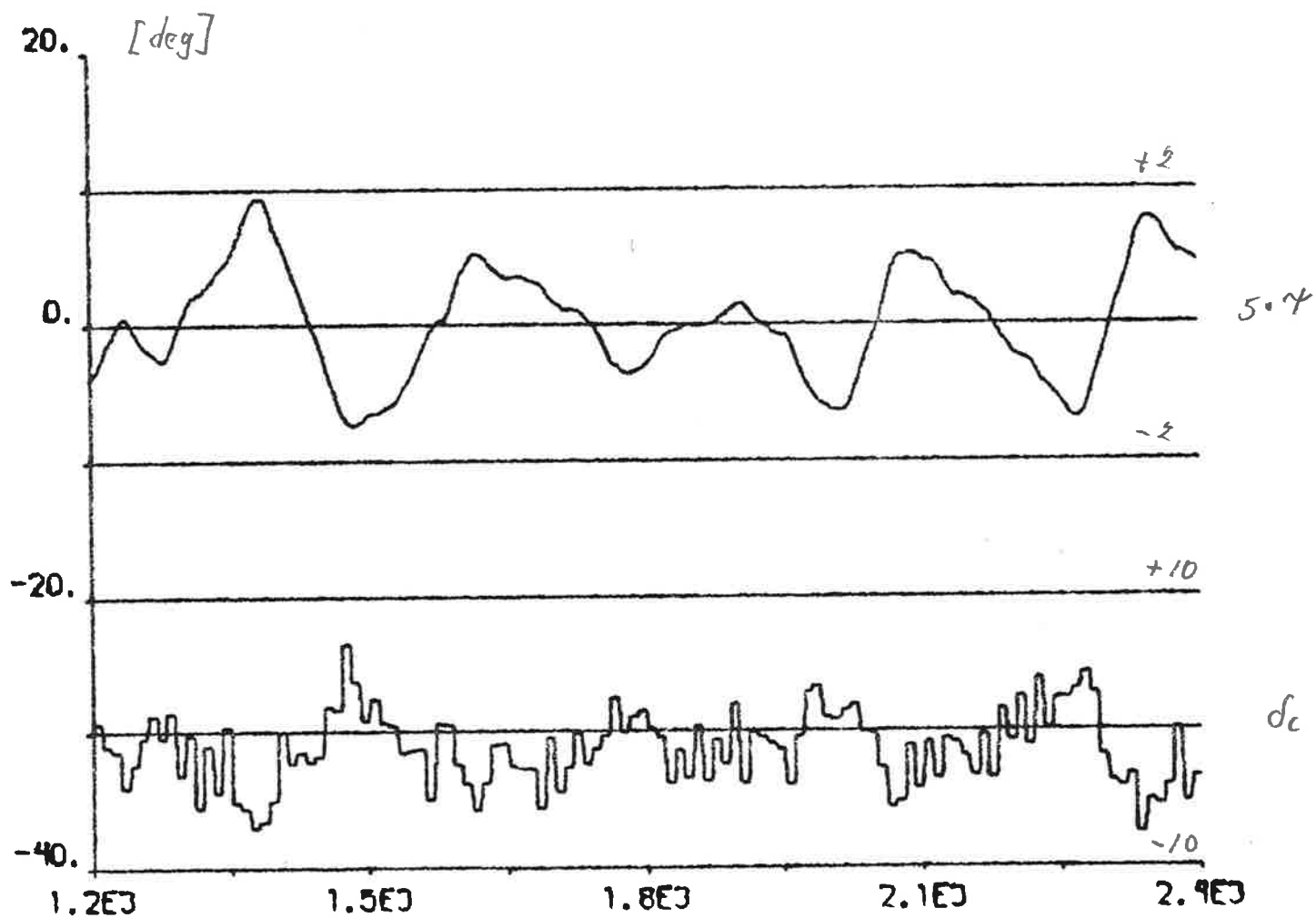
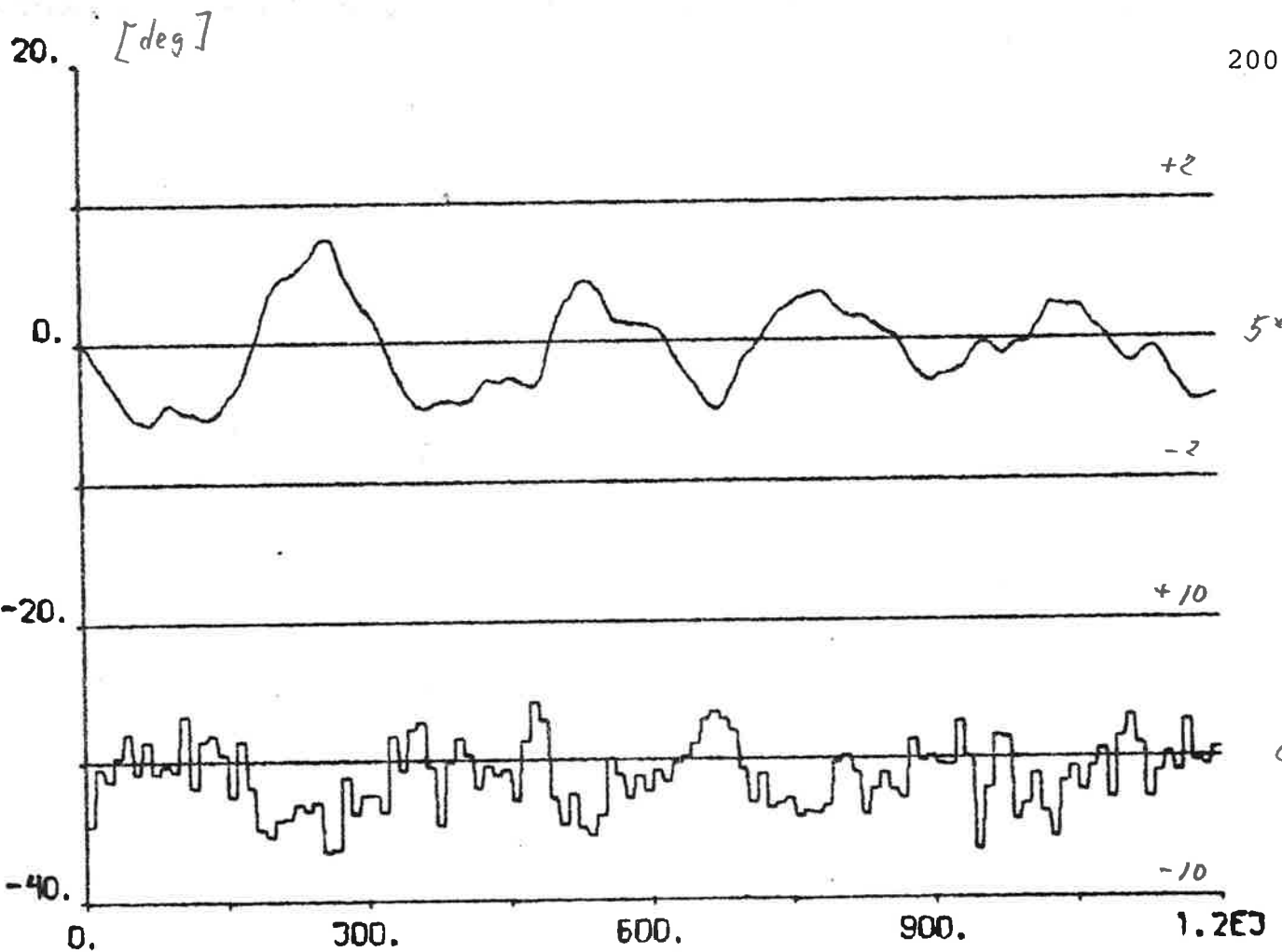


Fig. 4.30 - $T = 22.3$ m, $n_0 = 87.6$ rpm, $u_0 = 15.8$ knots, $\delta_\ell = 10$ deg, PID-regulator using non-filtered measurements.

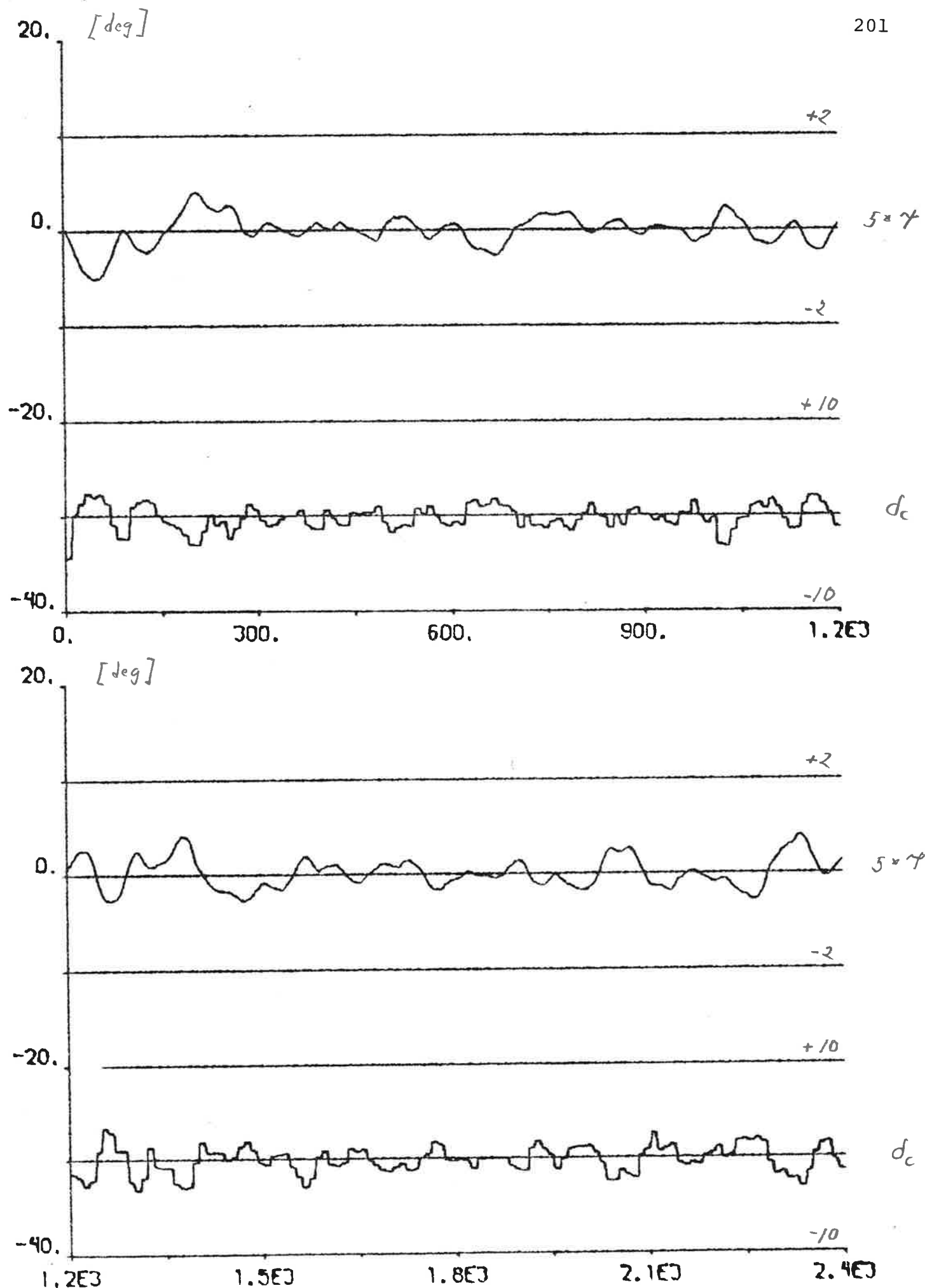


Fig. 4.31 - $T = 10.5$ m, $n_0 = 87.6$ rpm, $u_0 = 15.8$ knots, $\delta_\ell = 10$ deg, PID-regulator using estimates from the Kalman filter.

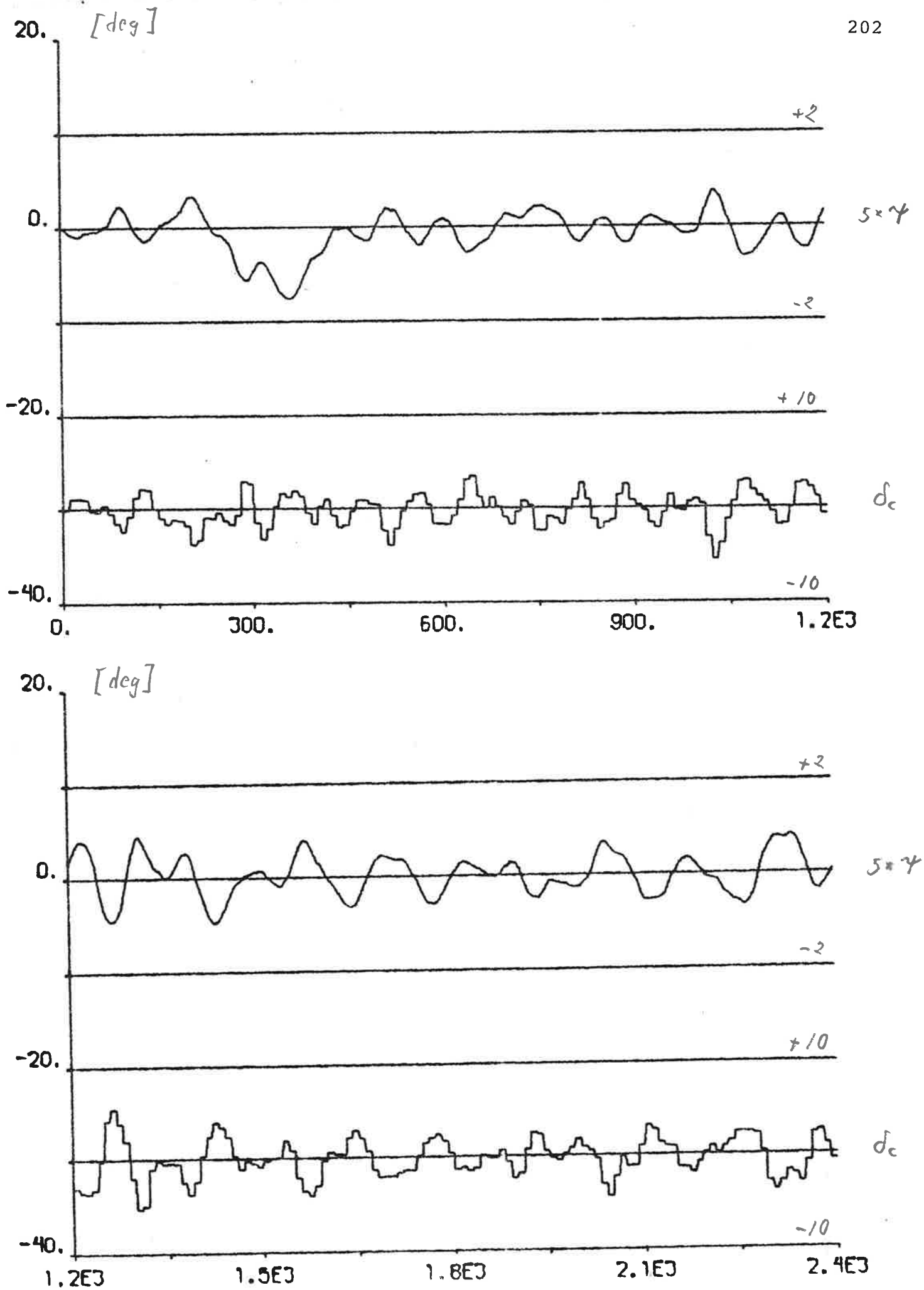


Fig. 4.32 a - $T = 10.5$ m, $n_0 = 87.6$ rpm, $u_0 = 15.8$ knots, $\delta_\ell = 10$ deg, self-tuning regulator using non-filtered measurements.

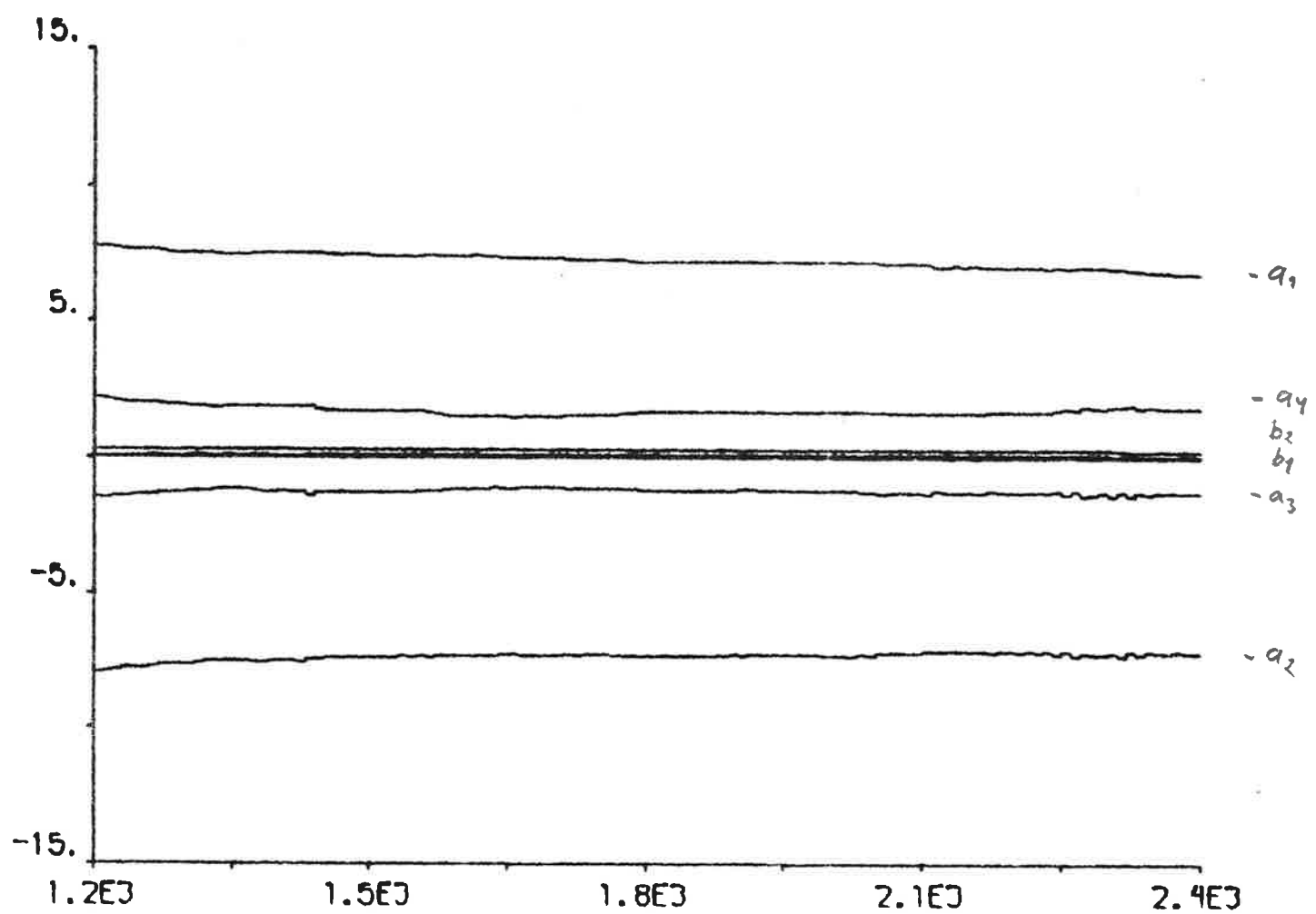
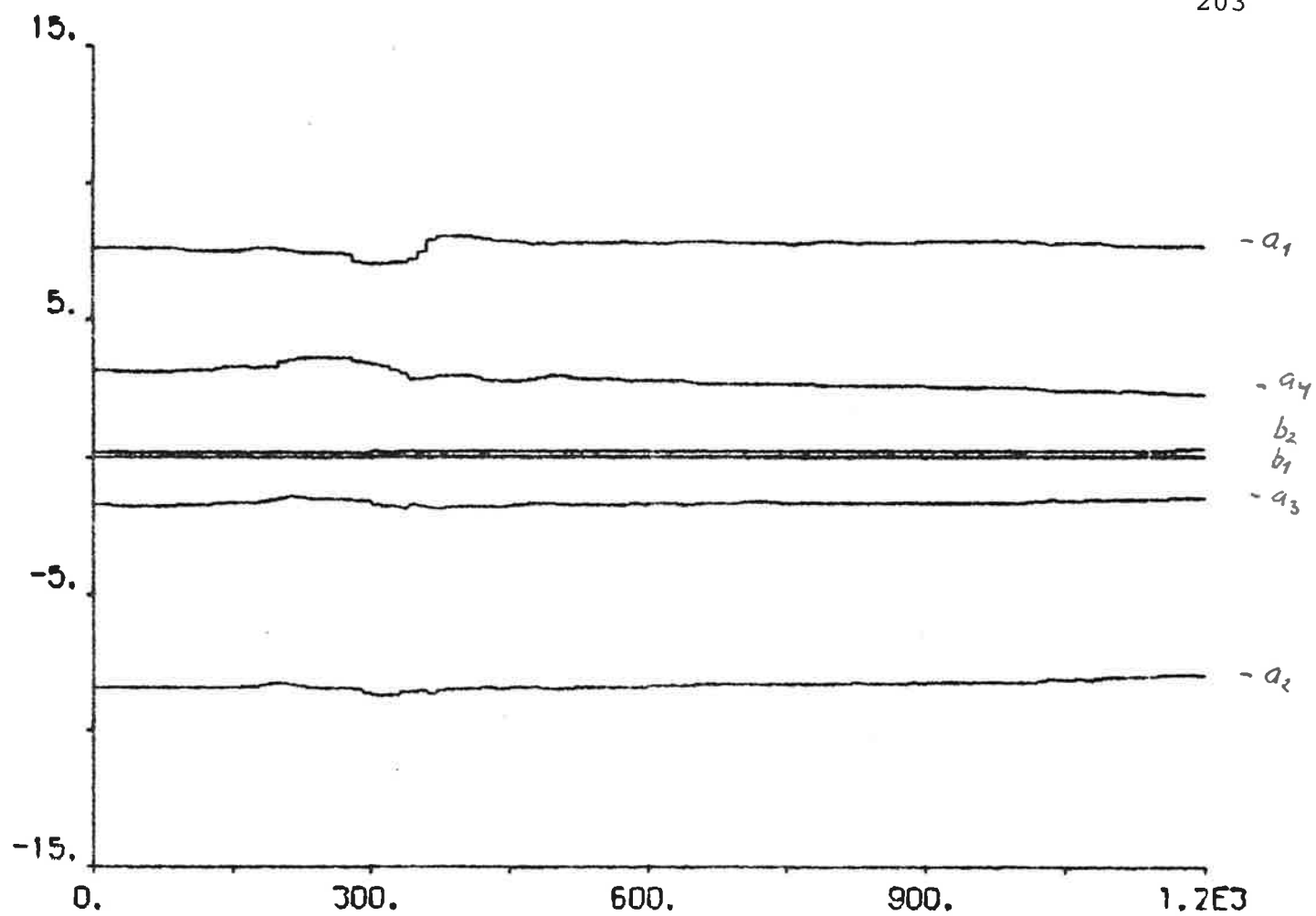


Fig. 4.32 b

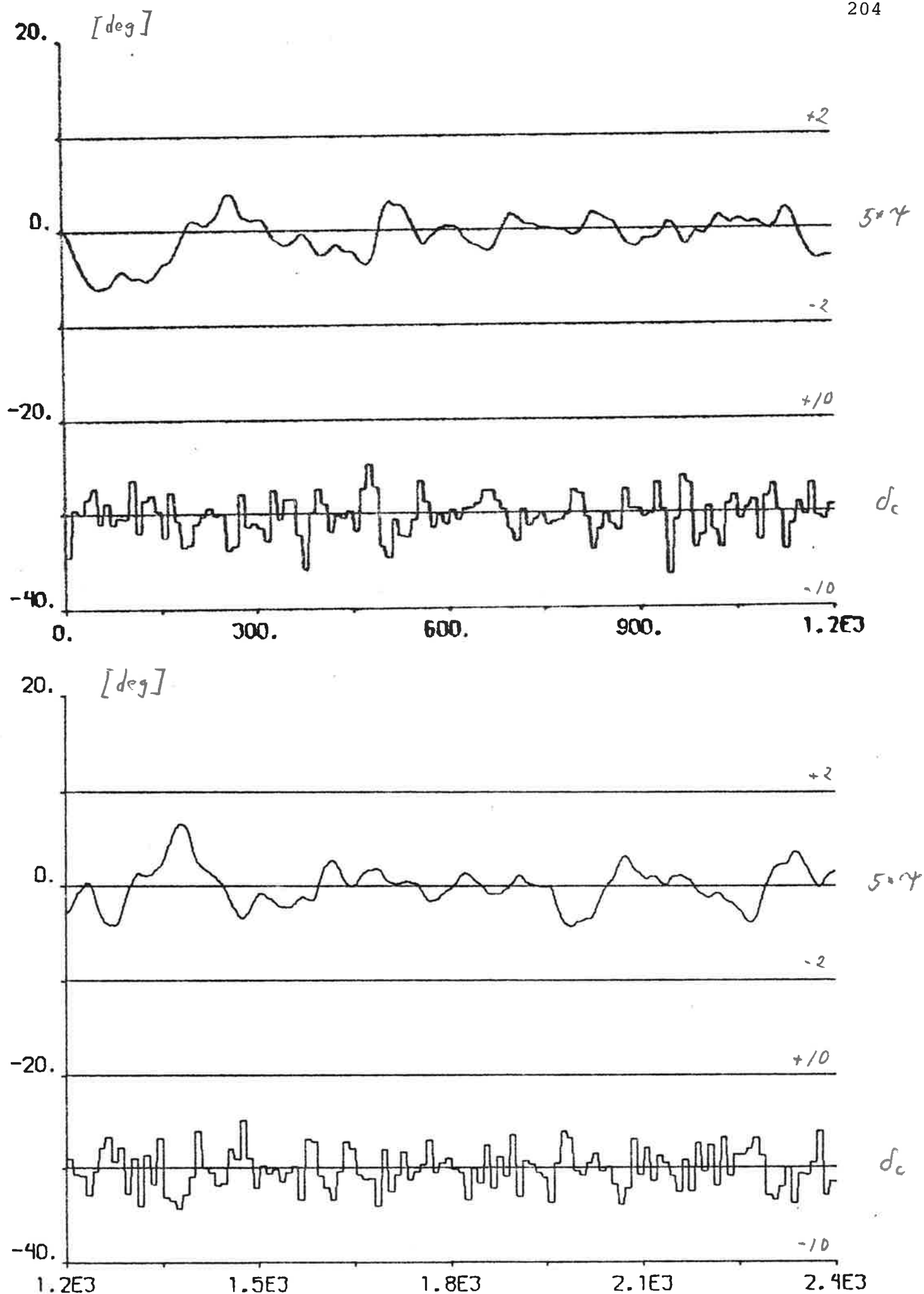


Fig. 4.33 - $T = 10.5$ m, $n_0 = 87.6$ rpm, $u_0 = 15.8$ knots, $\delta_\ell = 10$ deg, PID-regulator using non-filtered measurements.

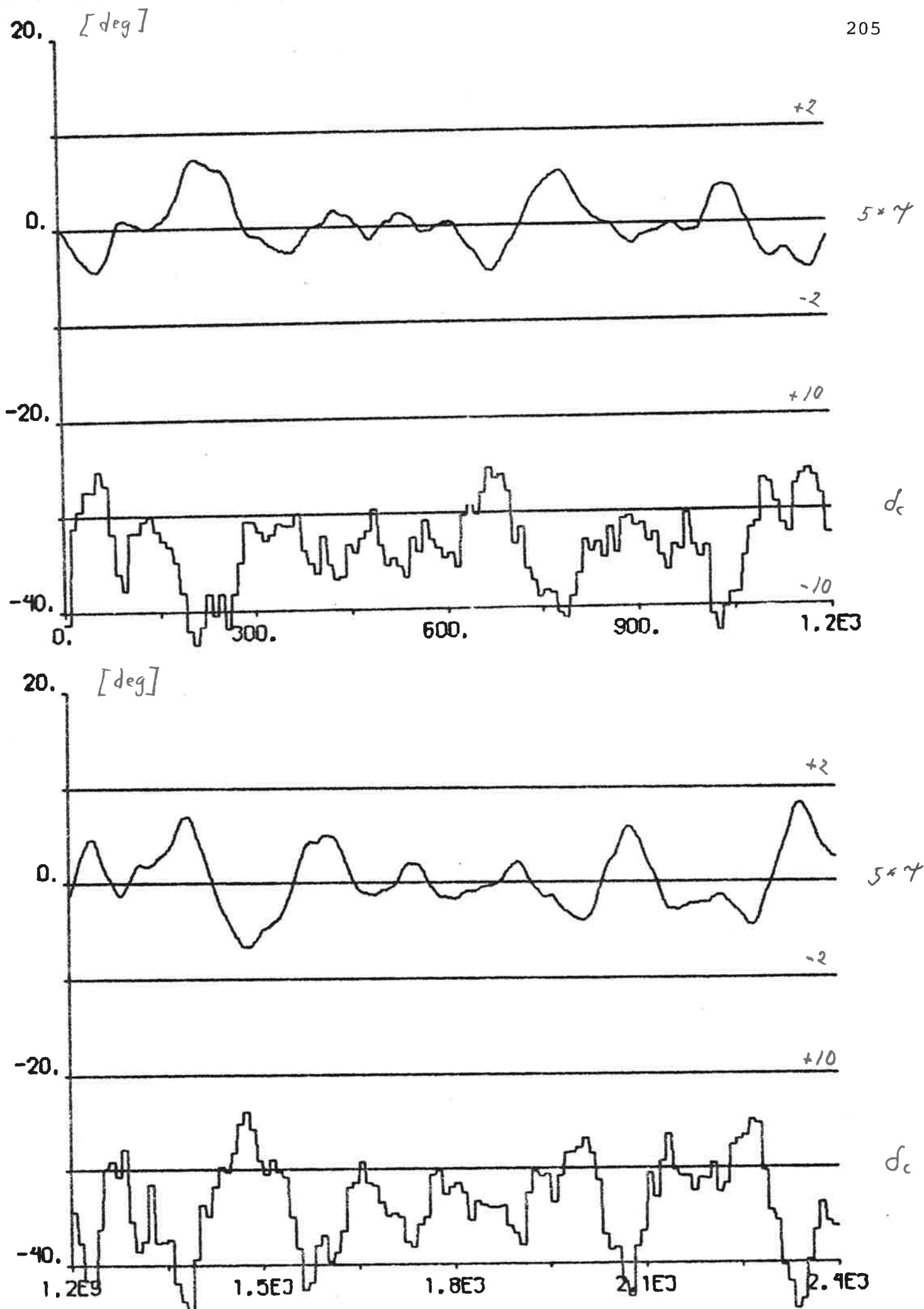


Fig. 4.34 - $T = 22.3$ m, $n_0 = 55.443$ rpm, $u_0 = 10$ knots, $\delta_{\ell} = 35$ deg, PID-regulator using estimates from the Kalman filter.

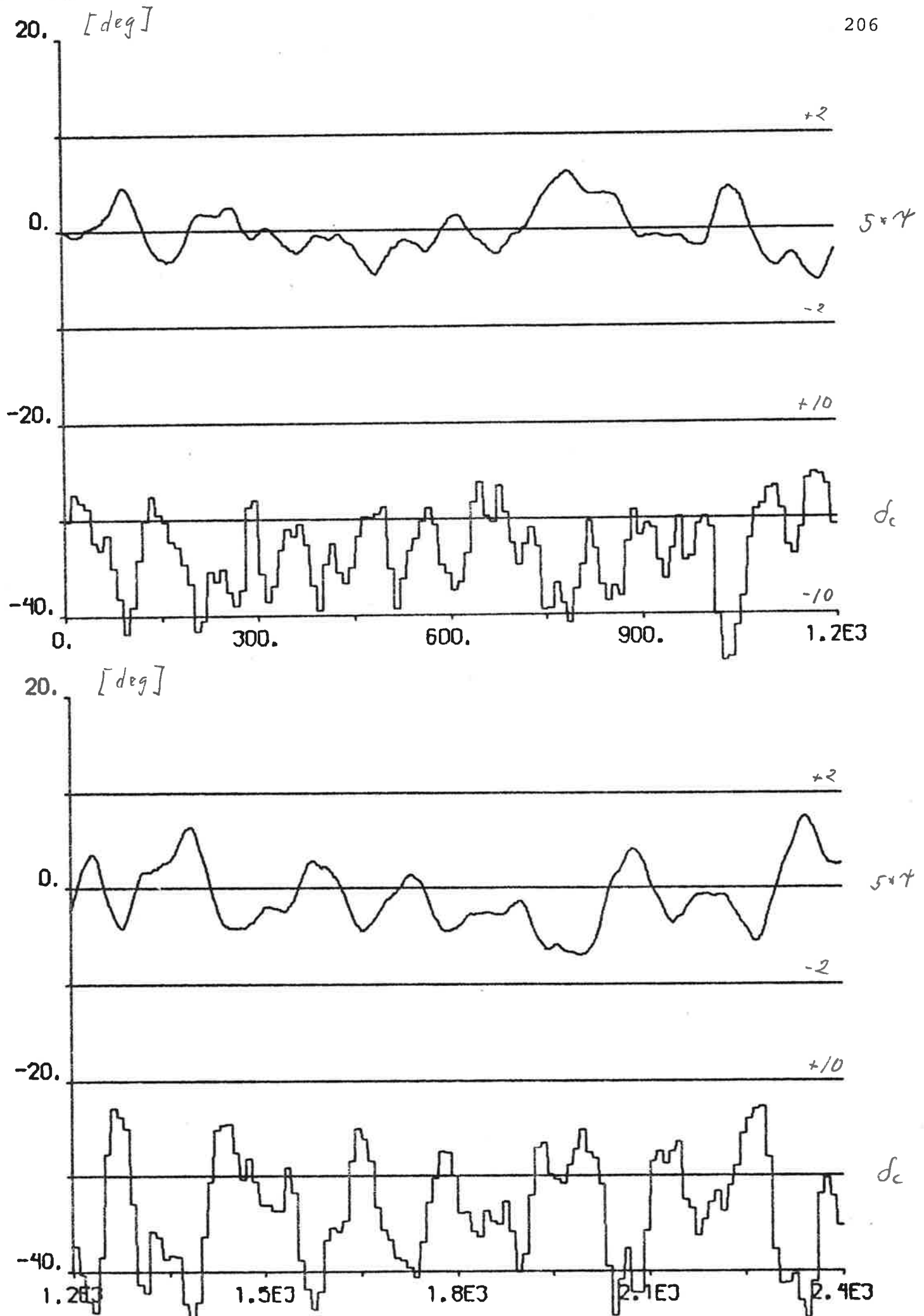


Fig. 4.35 a - $T = 22.3$ m, $n_0 = 55.443$ rpm, $u_0 = 10$ knots, $\delta_\ell = 35$ deg, self-tuning regulator using non-filtered measurements.

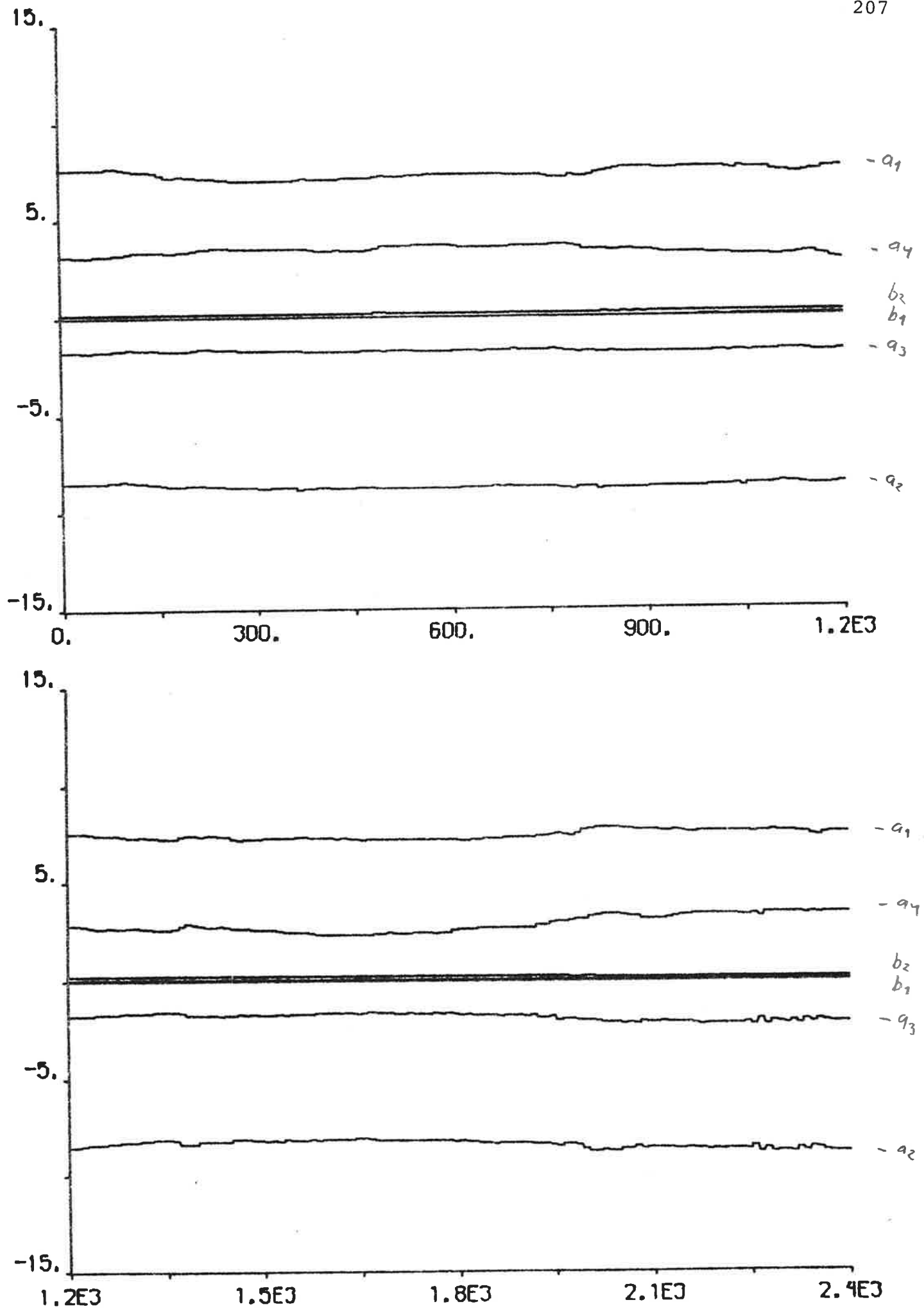


Fig. 4.35 b

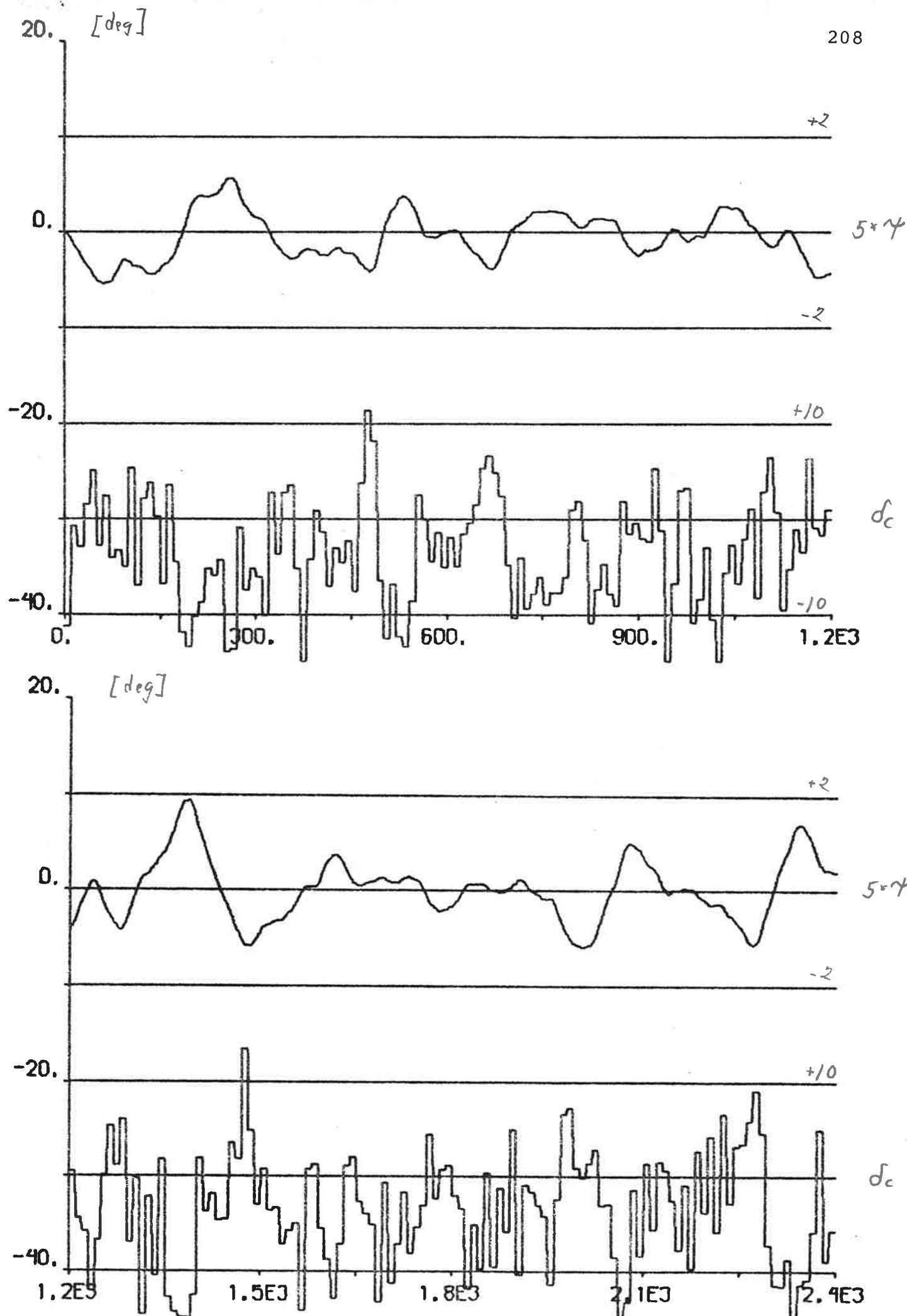


Fig. 4.36 - $T = 22.3$ m, $n_0 = 55.443$ rpm, $u_0 = 10$ knots, $\delta_\ell = 35$ deg, PID-regulator using non-filtered measurements.

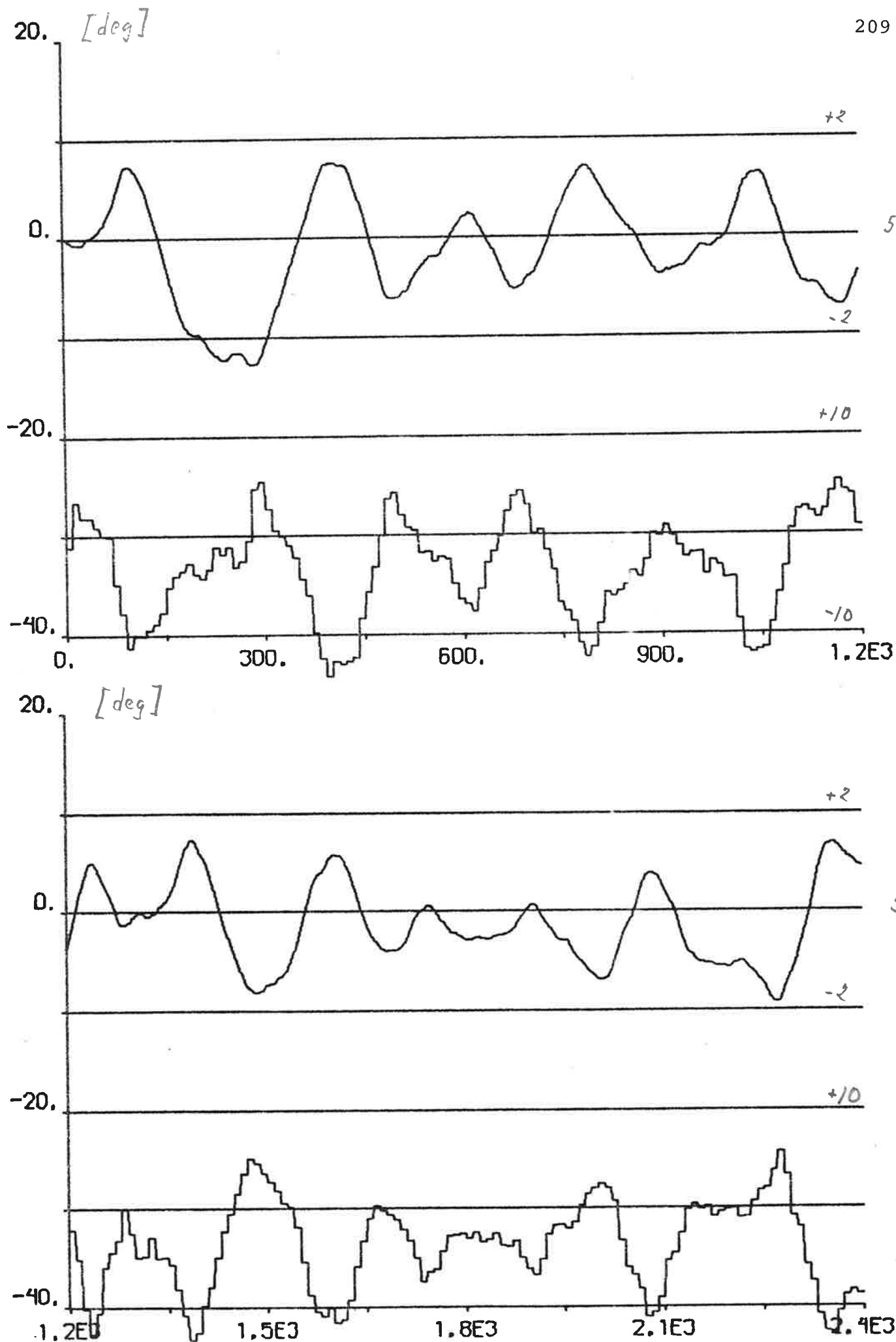


Fig. 4.37 a - $T = 22.3$ m, $n_0 = 55.443$ rpm, $u_0 = 10$ knots, $\delta_\ell = 35$ deg, self-tuning regulator using estimates from the Kalman filter ($q_2^* = 0.6$).

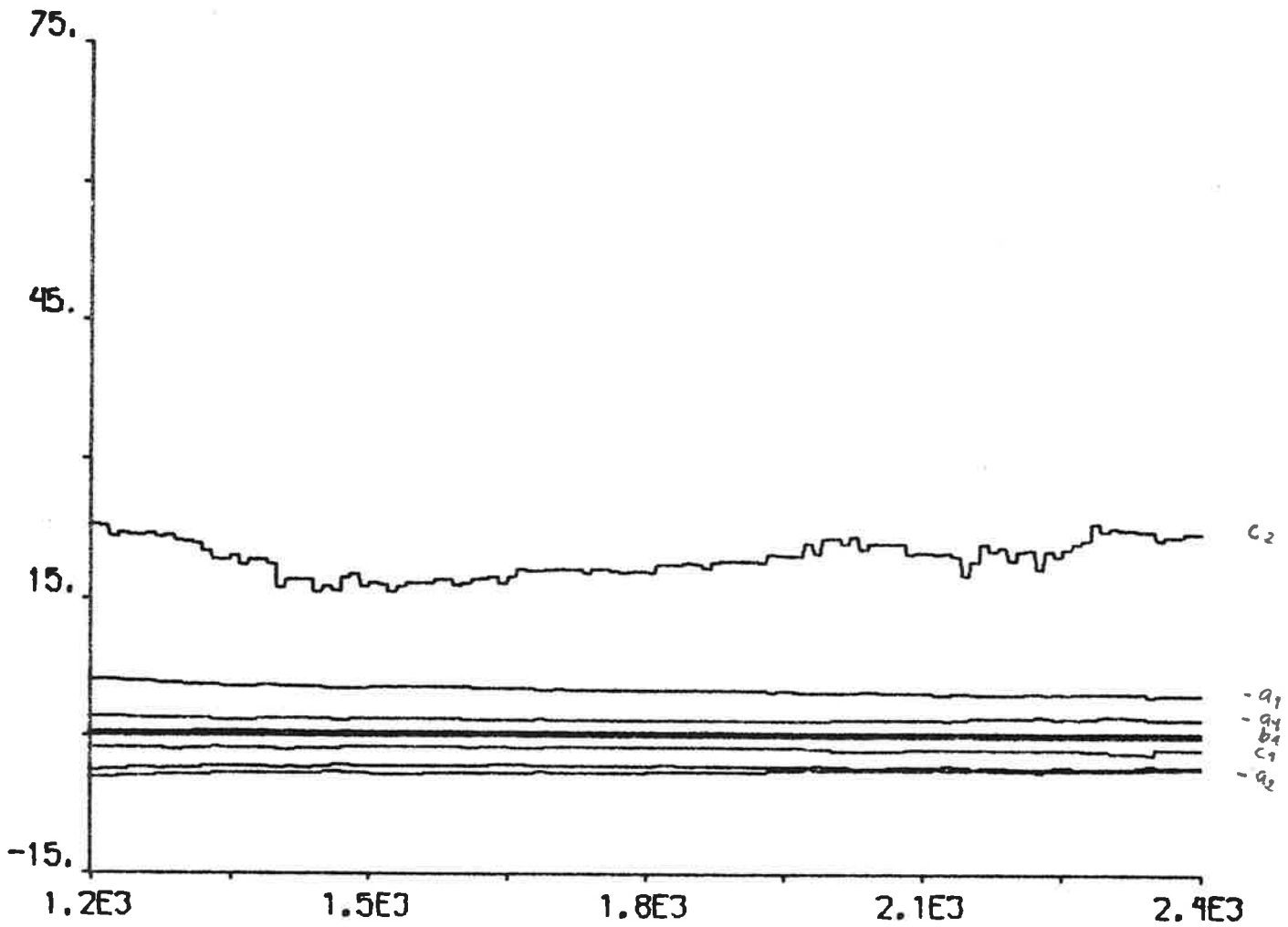
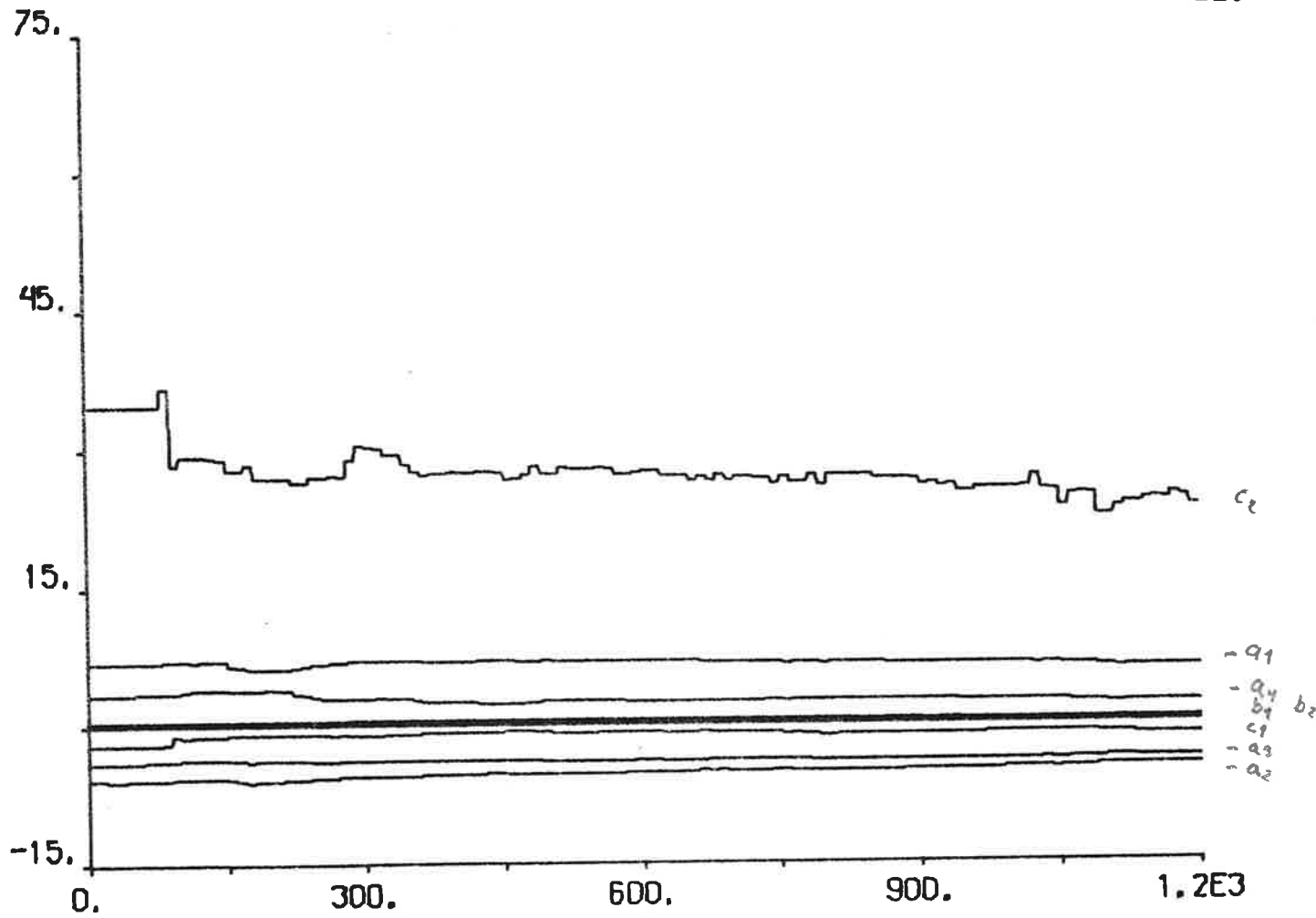


Fig. 4.37 b

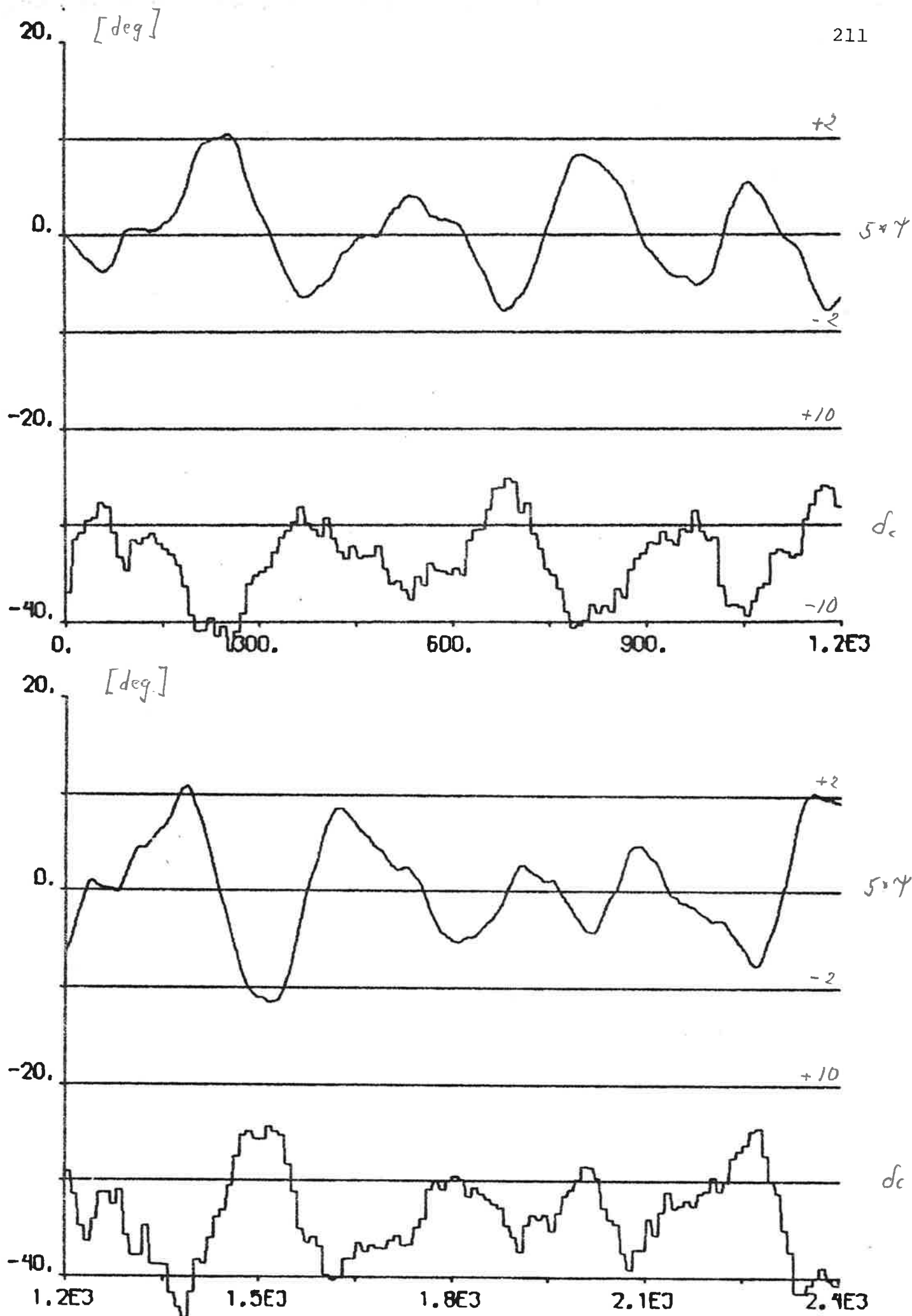


Fig. 4.38 - $T = 22.3$ m, $n_0 = 55.443$ rpm, $u_0 = 10$ knots, $\delta_l = 35$ deg, PID-regulator using estimates from the Kalman filter ($V_0 = 6.3$ m/s).

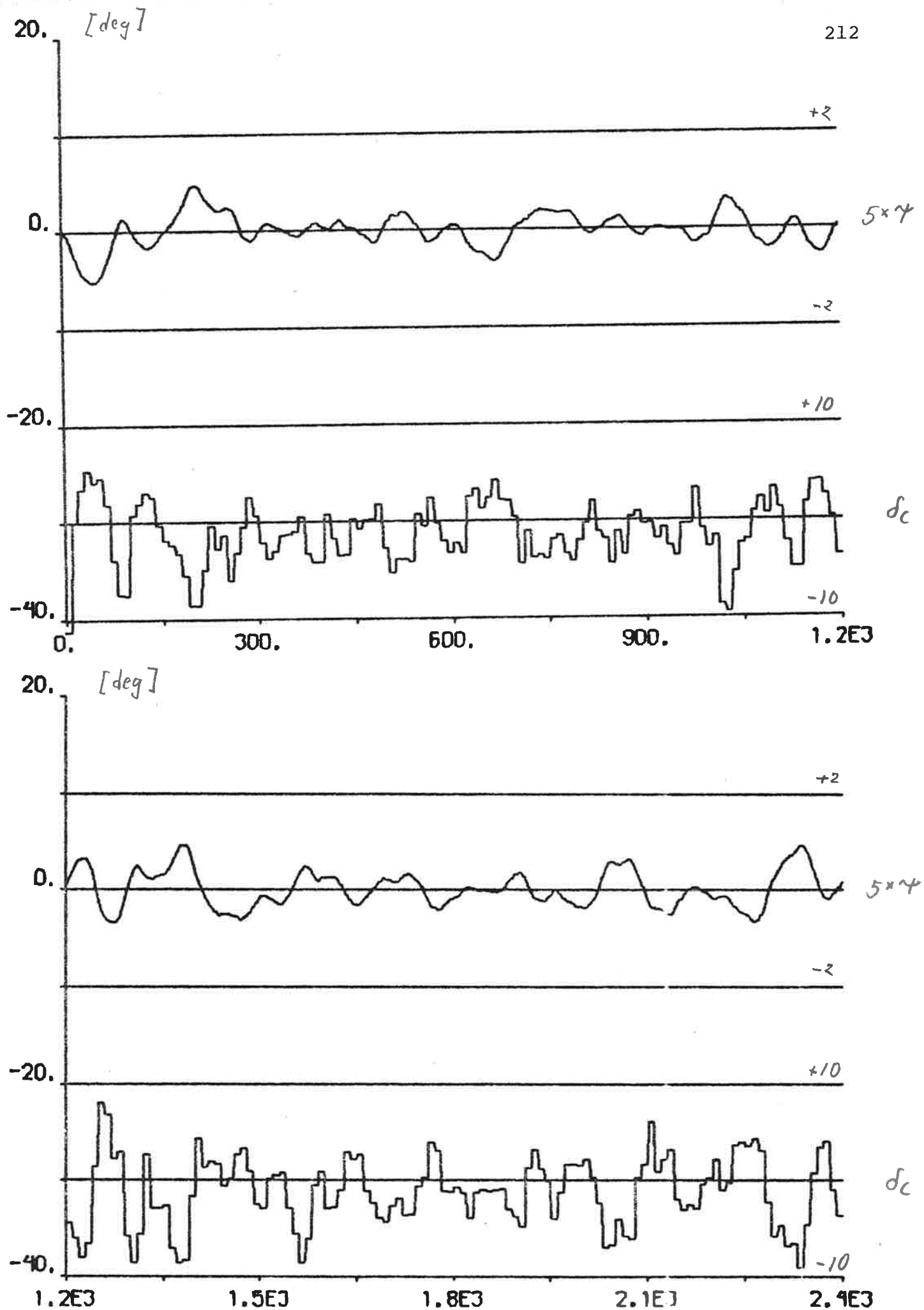


Fig. 4.39 - $T = 10.5$ m, $n_0 = 55.443$ rpm, $u_0 = 10$ knots, $\delta_\ell = 35$ deg, PID-regulator using estimates from the Kalman filter.

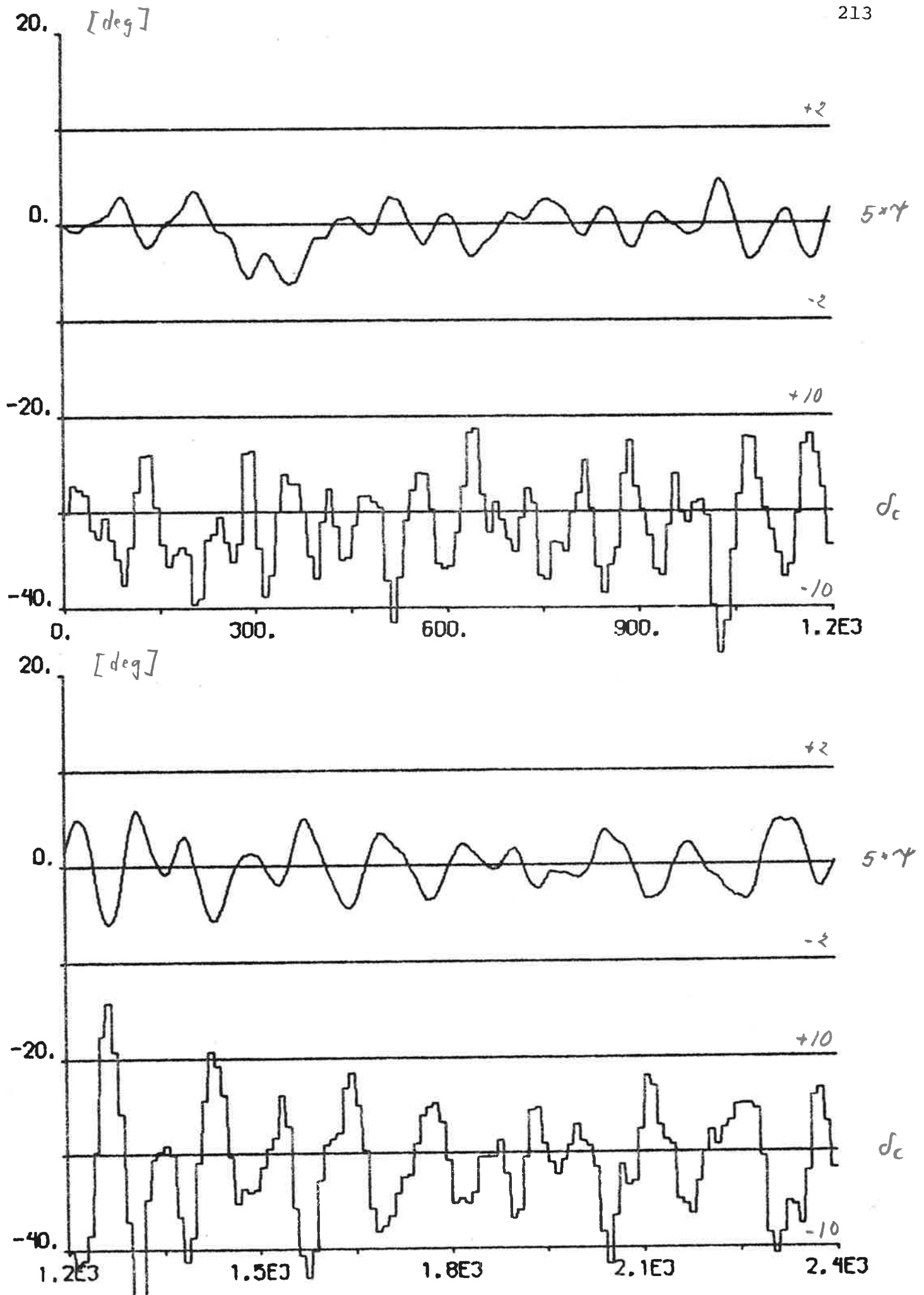


Fig. 4.40 a - $T = 10.5$ m, $n_0 = 55.443$ rpm, $u_0 = 10$ knots, $\delta_\ell = 35$ deg, self-tuning regulator using non-filtered measurements.

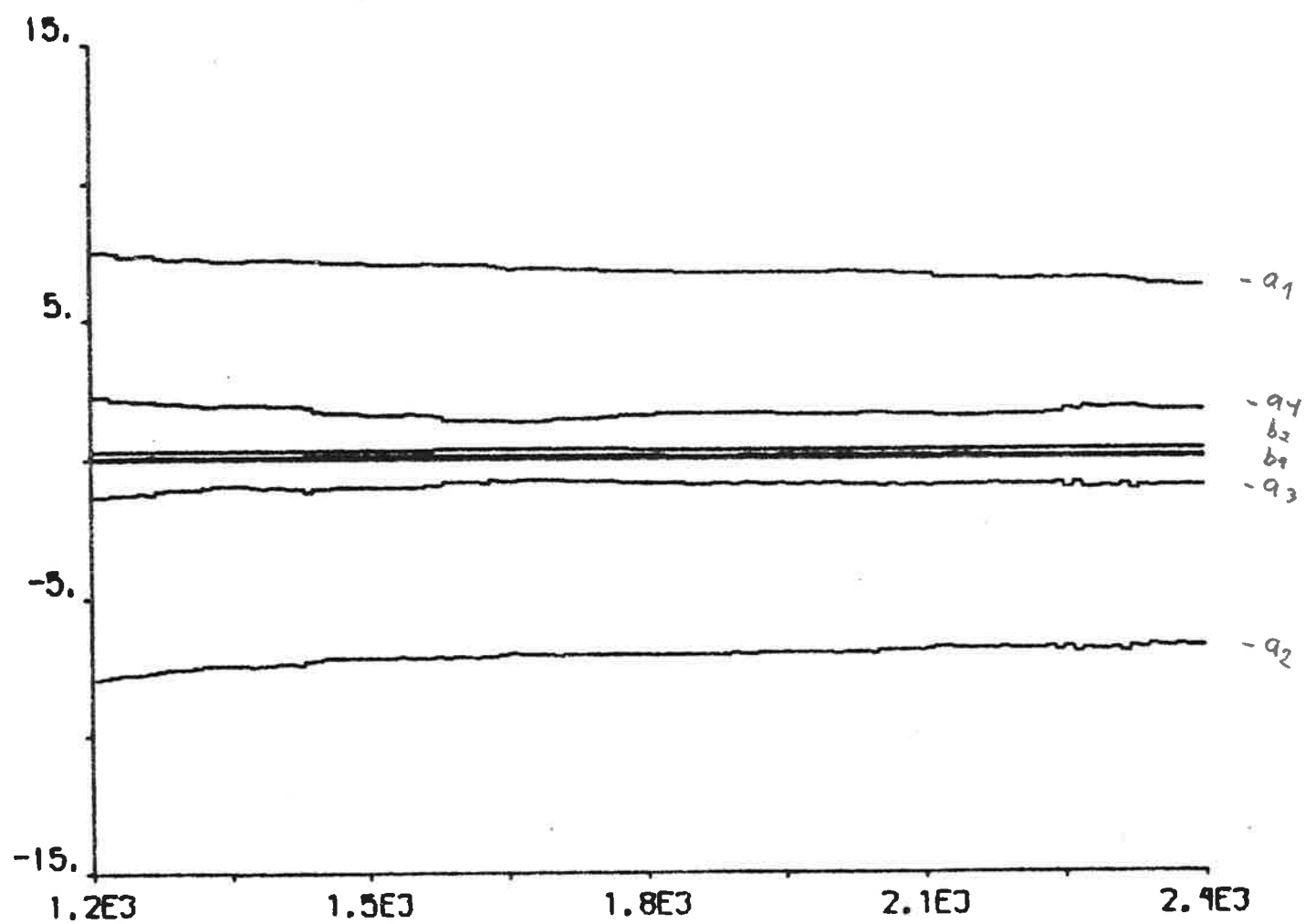
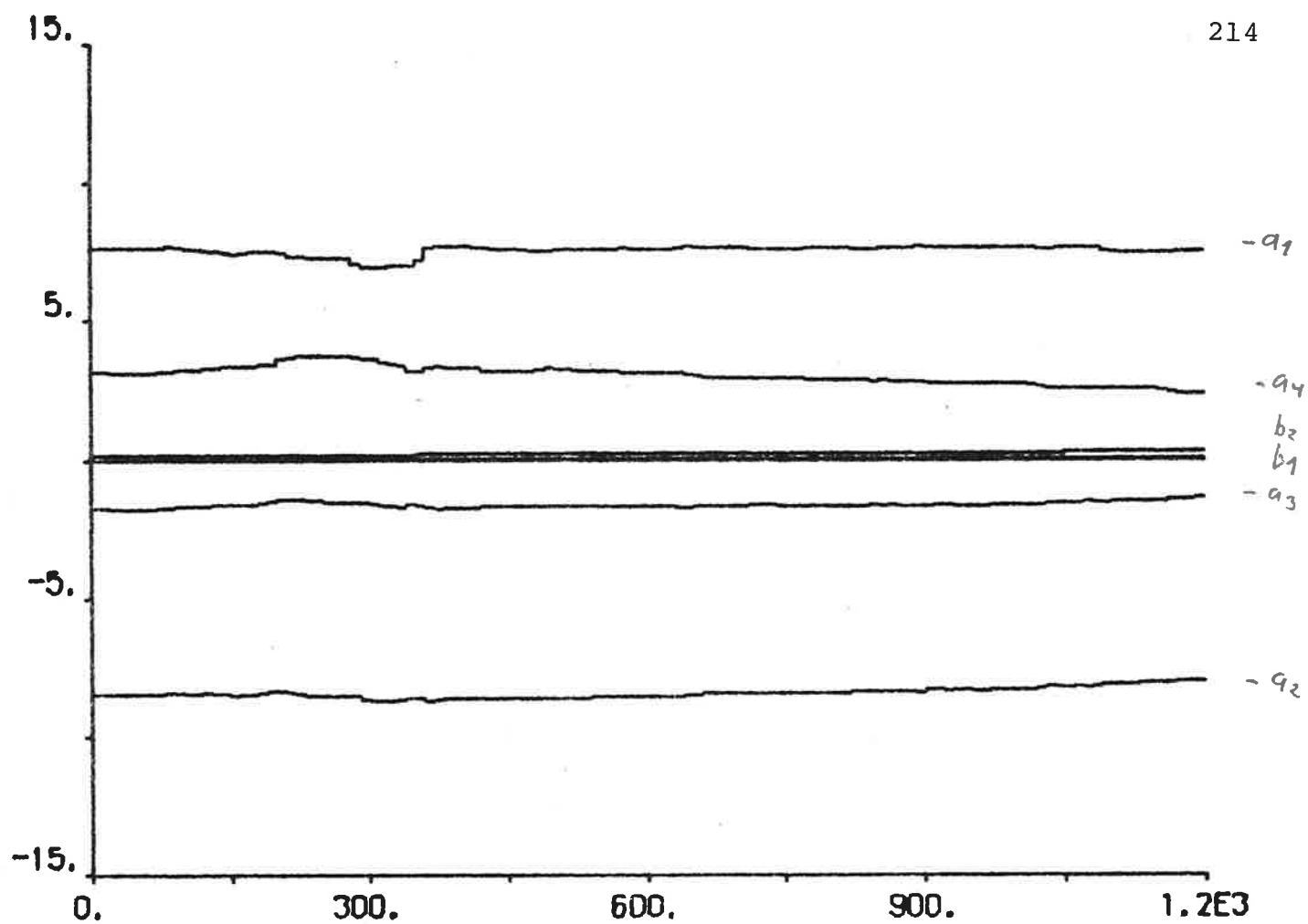


Fig. 4.40 b

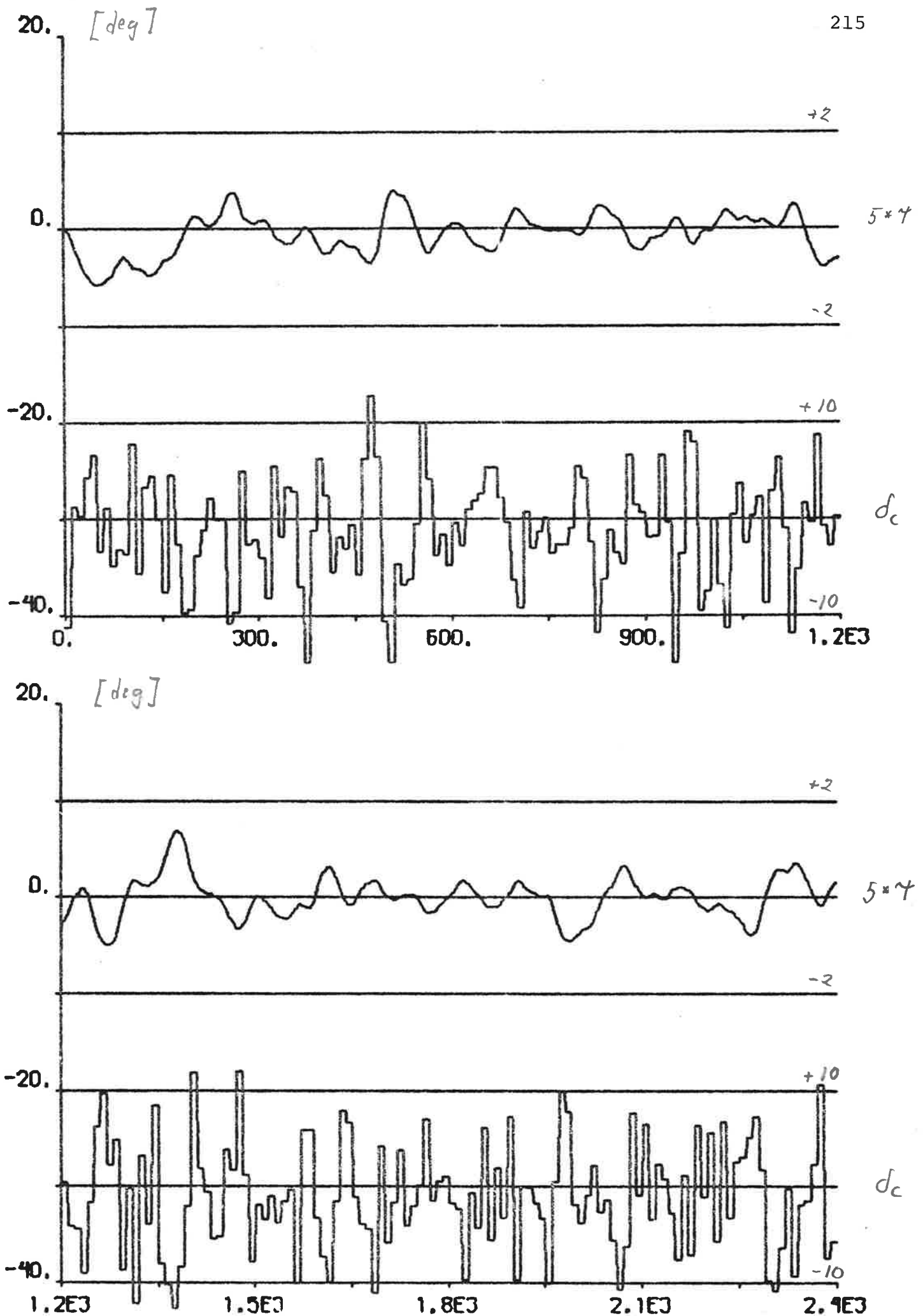


Fig. 4.41 - $T = 10.5$ m, $n_0 = 55.443$ rpm, $u_0 = 10$ knots, $\delta_\ell = 35$ deg, PID-regulator using non-filtered measurements.

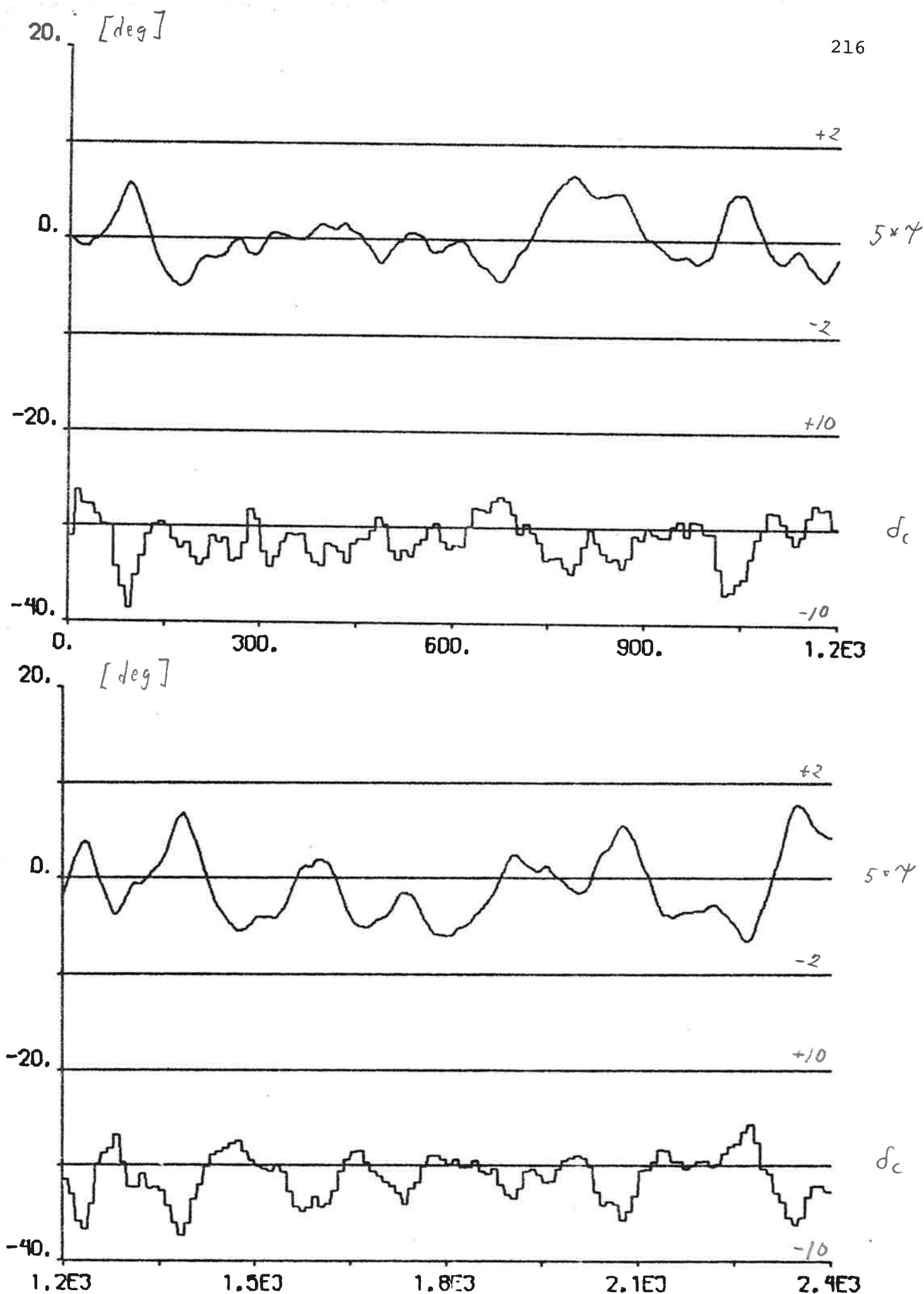


Fig. 4.42 a - $T = 10.5$ m, $n_0 = 55.443$ rpm, $u_0 = 10$ knots, $\delta_{\ell} = 35$ deg, self-tuning regulator using estimates from the Kalman filter ($q_2^* = 0.6$).

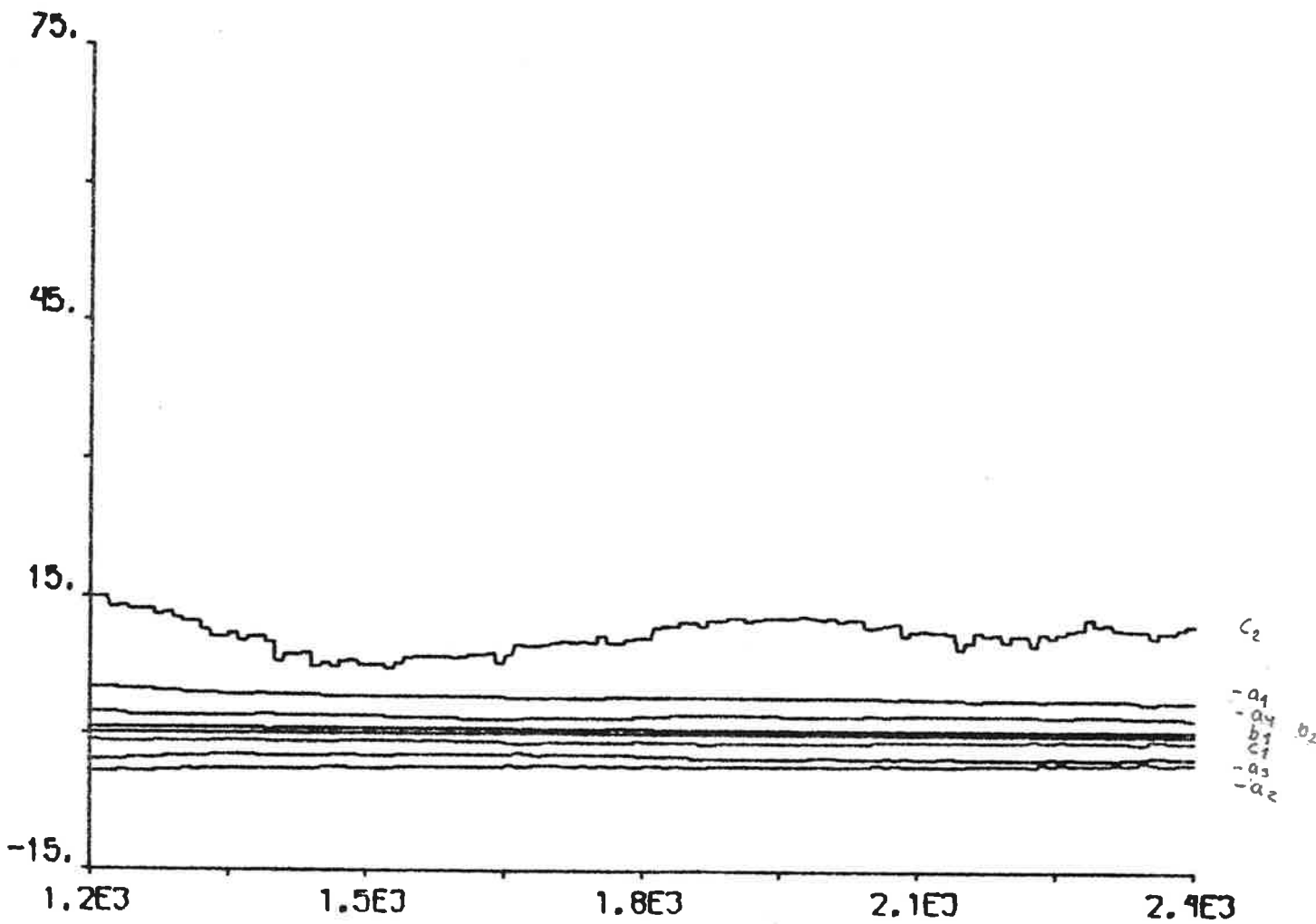
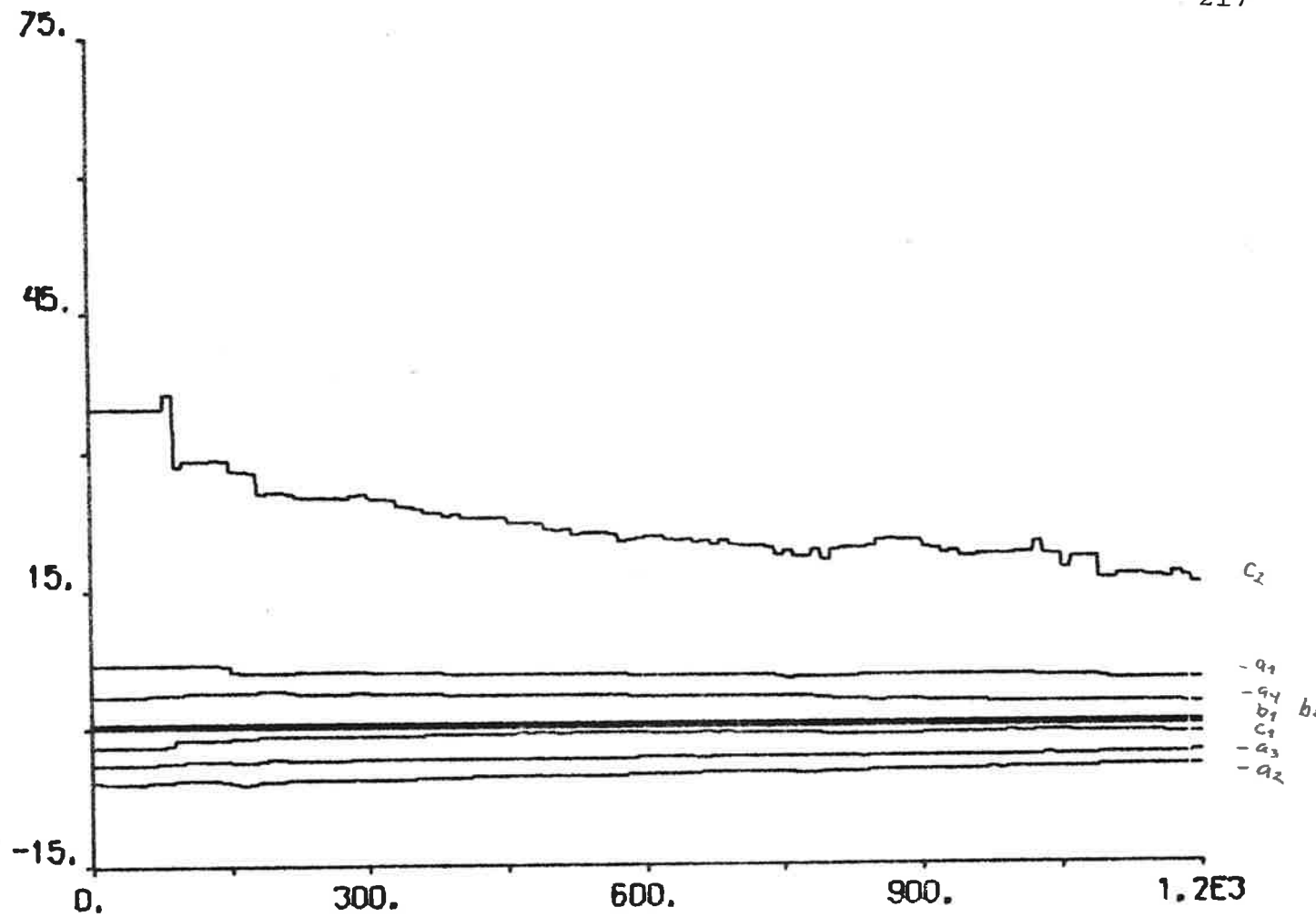


Fig. 4.42 b

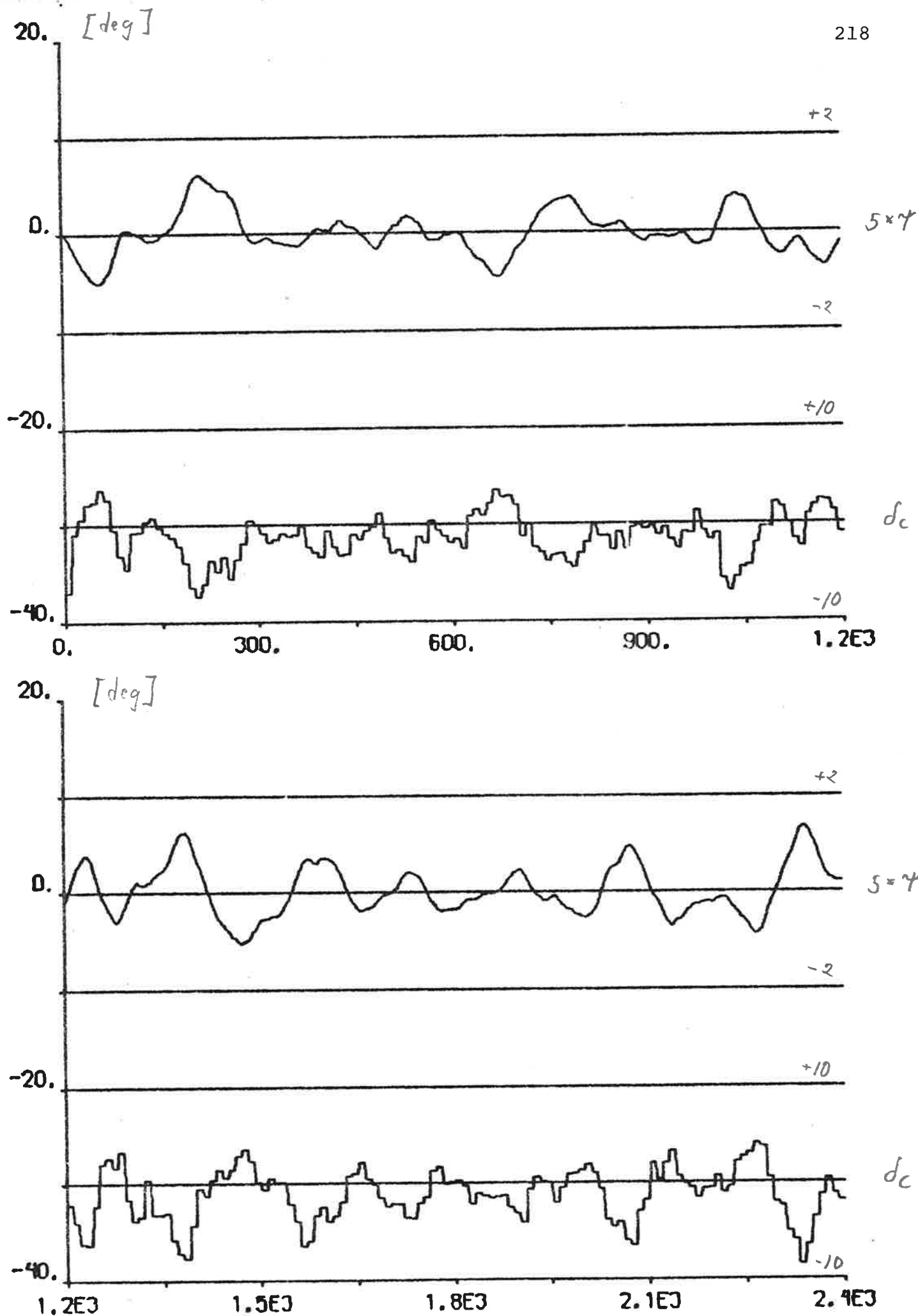


Fig. 4.43 - $T = 10.5$ m, $n_0 = 55.443$ rpm, $u_0 = 10$ knots, $\delta_l = 35$ deg, PID-regulator using estimates from the Kalman filter ($V_0 = 6.3$ m/s).

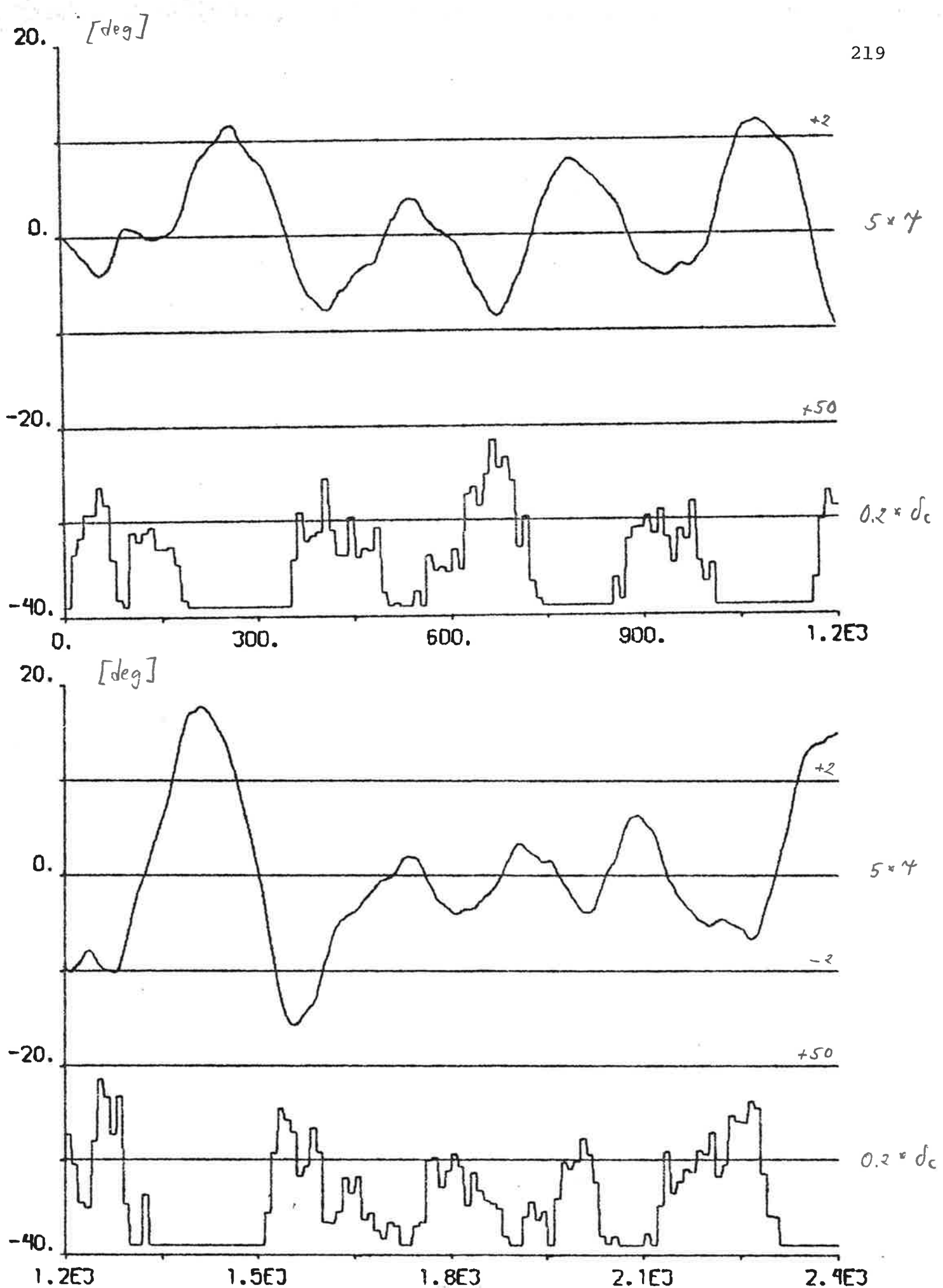


Fig. 4.44 - $T = 22.3$ m, $n_0 = 22.1772$ rpm, $u_0 = 4$ knots, $\delta_\ell = 45$ deg, PID-regulator using estimates from the Kalman filter.

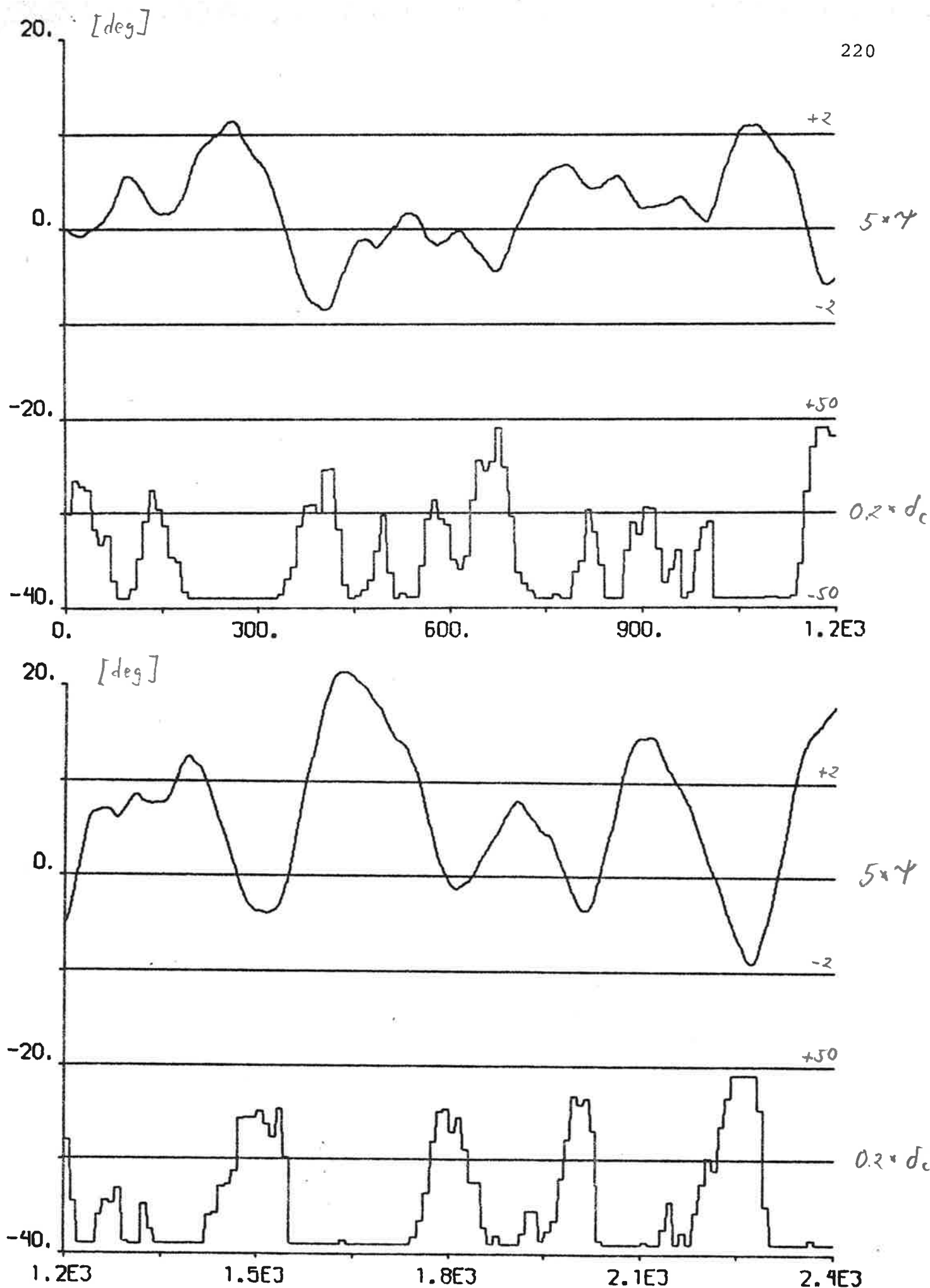


Fig. 4.45 a - $T = 22.3$ m, $n_0 = 22.1772$ rpm, $u_0 = 4$ knots, $\delta_\ell = 45$ deg, self-tuning regulator using non-filtered measurements.

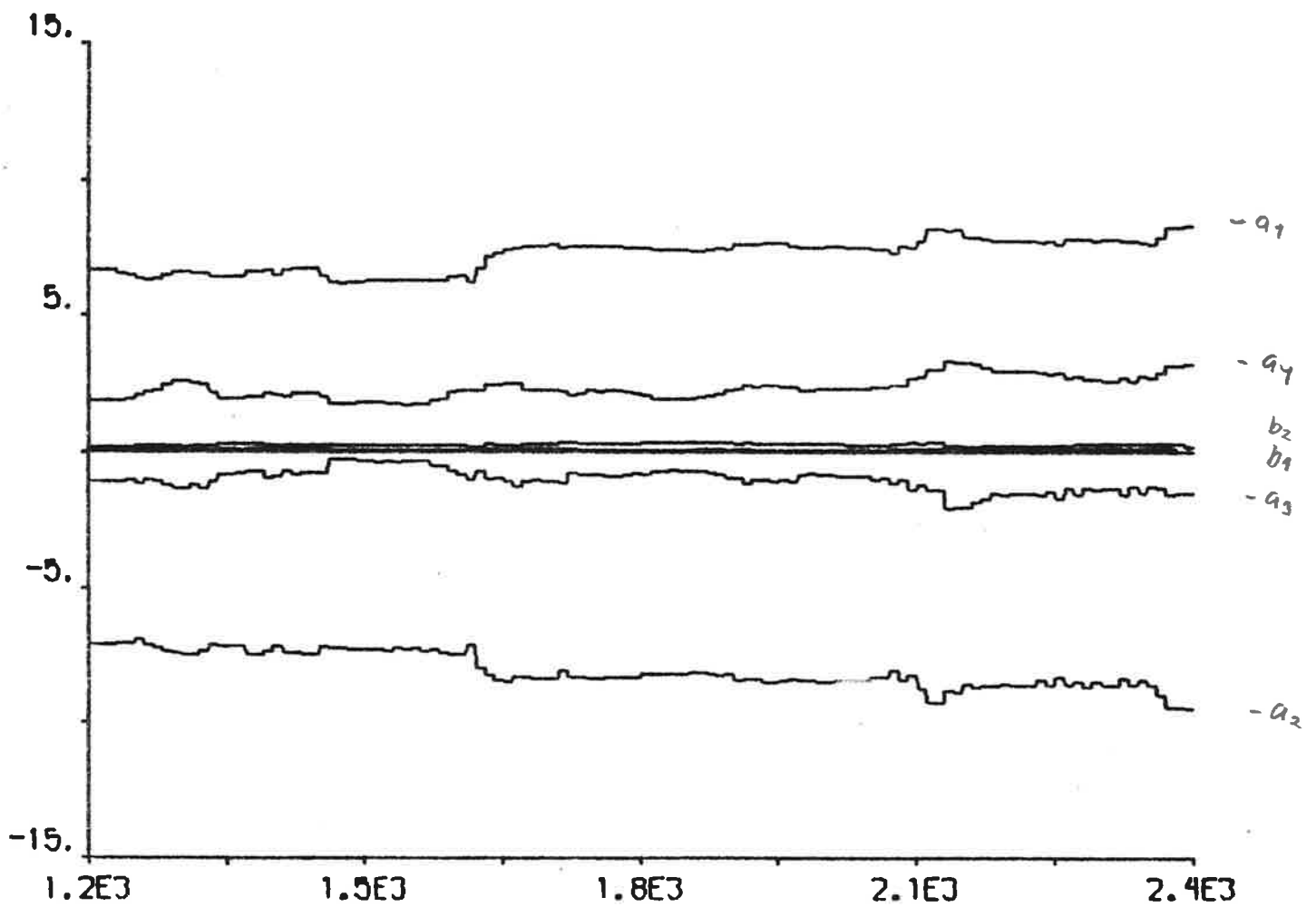
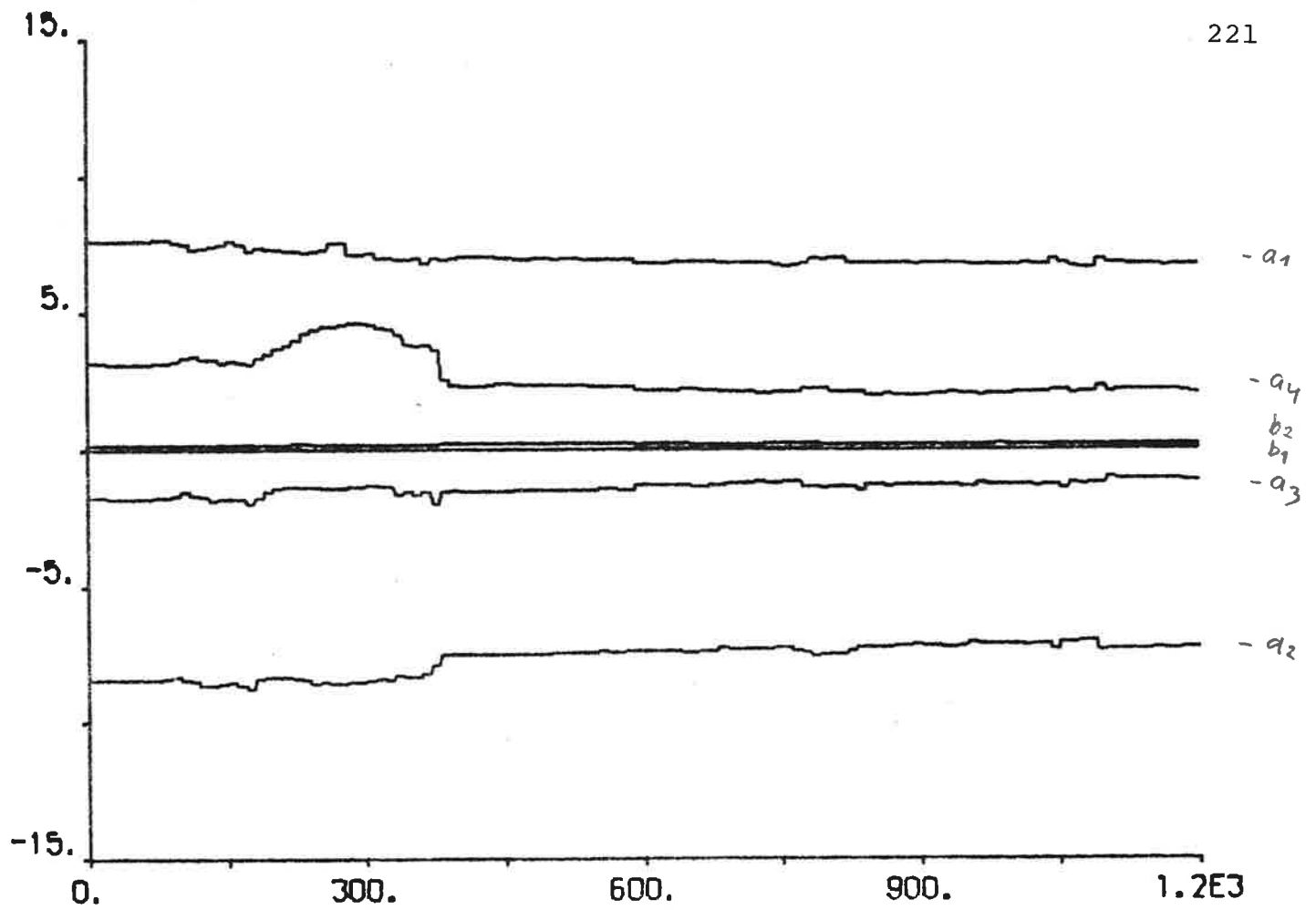


Fig. 4.45 b

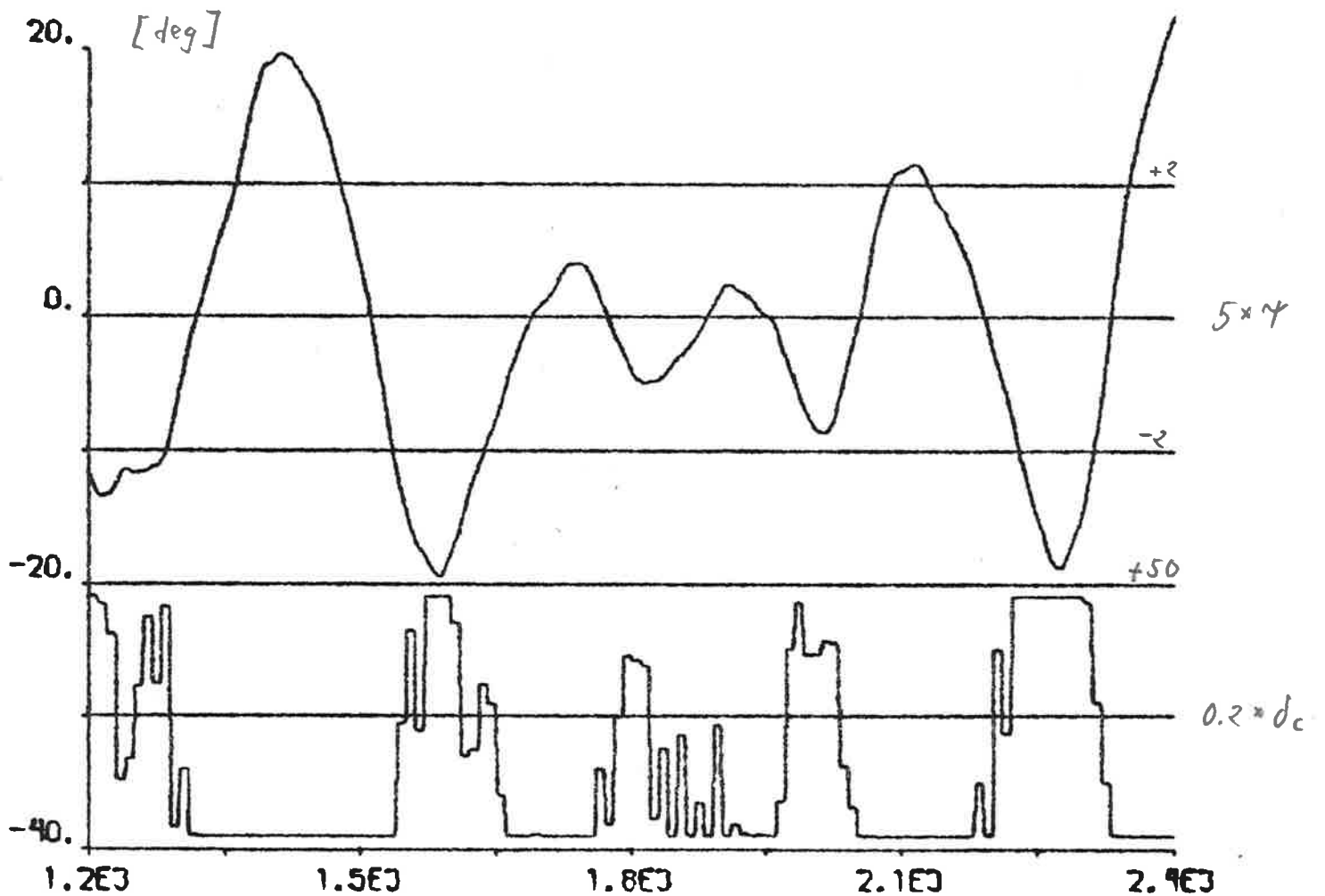
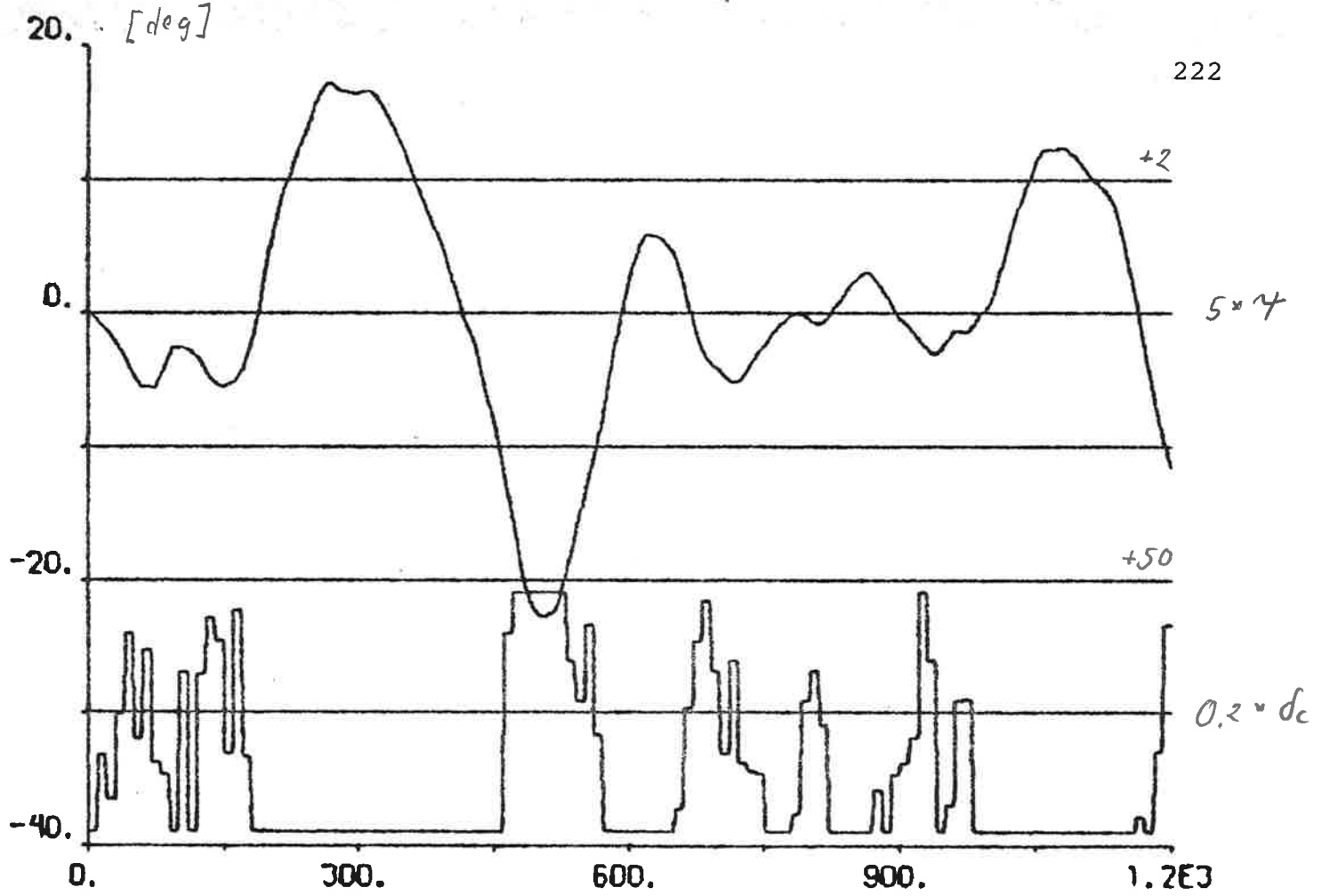


Fig. 4.46 - $T = 22.3$ m, $n_0 = 22.1772$ rpm, $u_0 = 4$ knots, $\delta_l = 45$ deg, PID-regulator using non-filtered measurements.

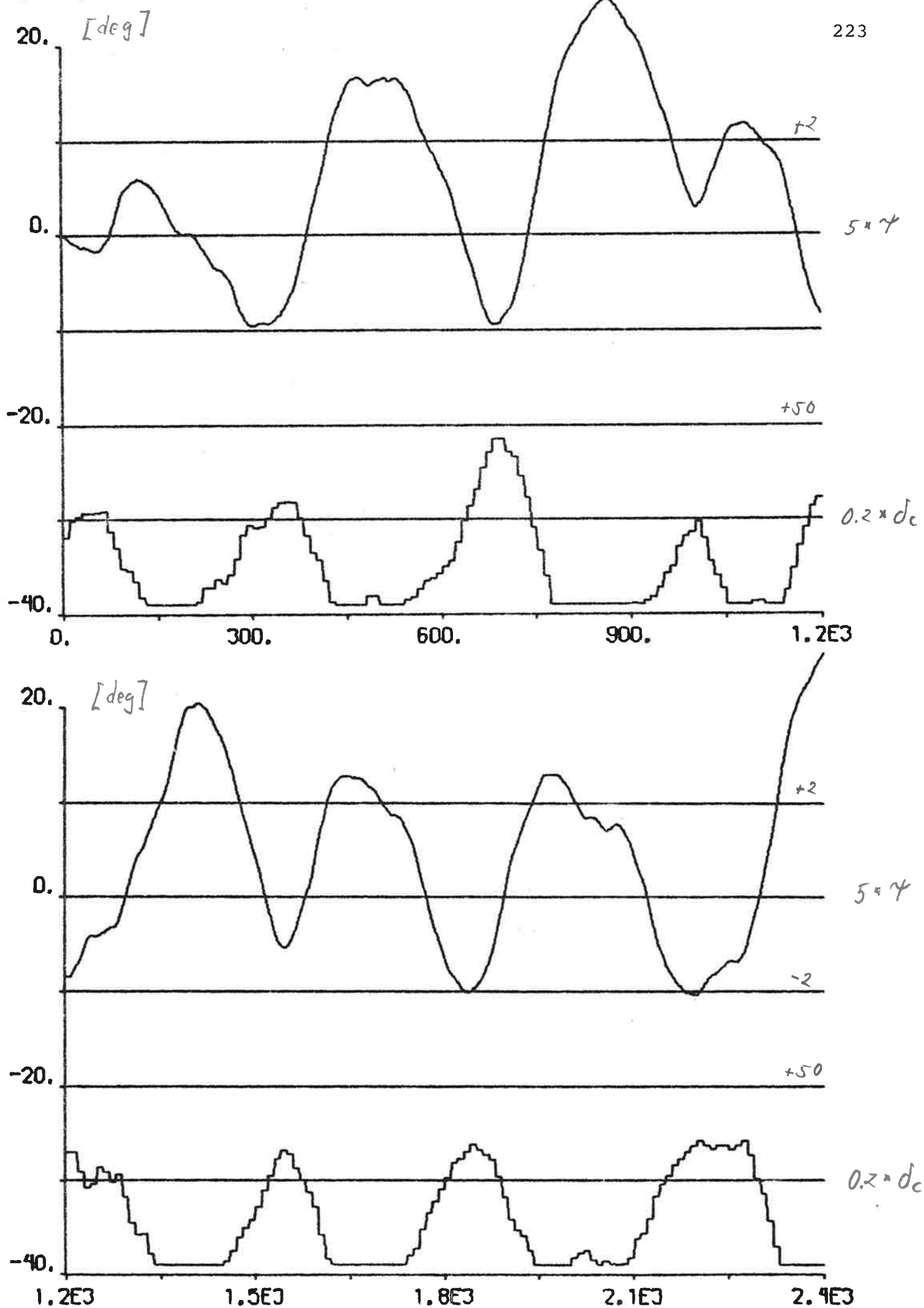


Fig. 4.47 a - $T = 22.3$ m, $n_0 = 22.1772$ rpm, $u_0 = 4$ knots, $\delta_\ell = 45$ deg, self-tuning regulator using estimates from the Kalman filter ($q_2^* = 3$).

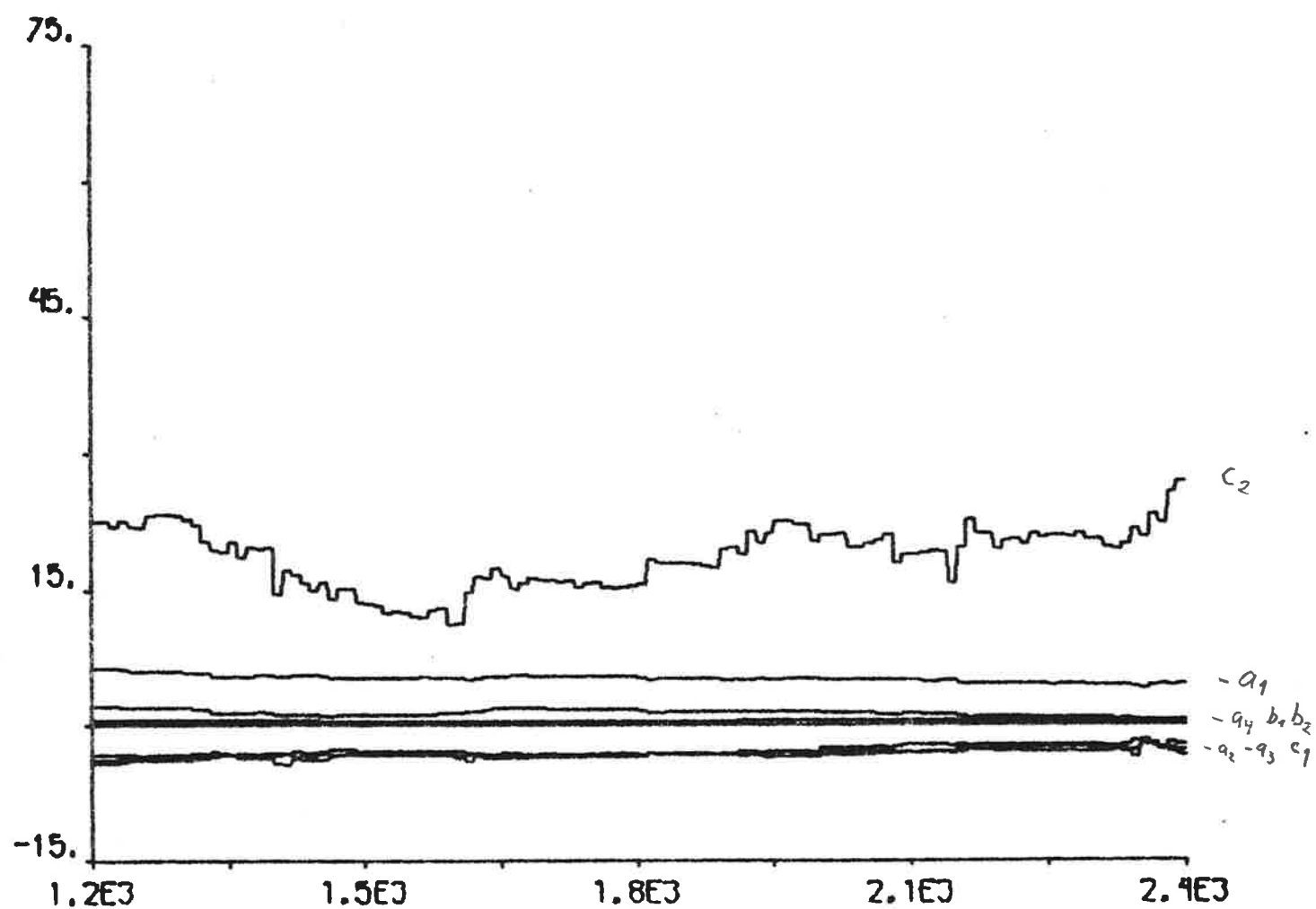
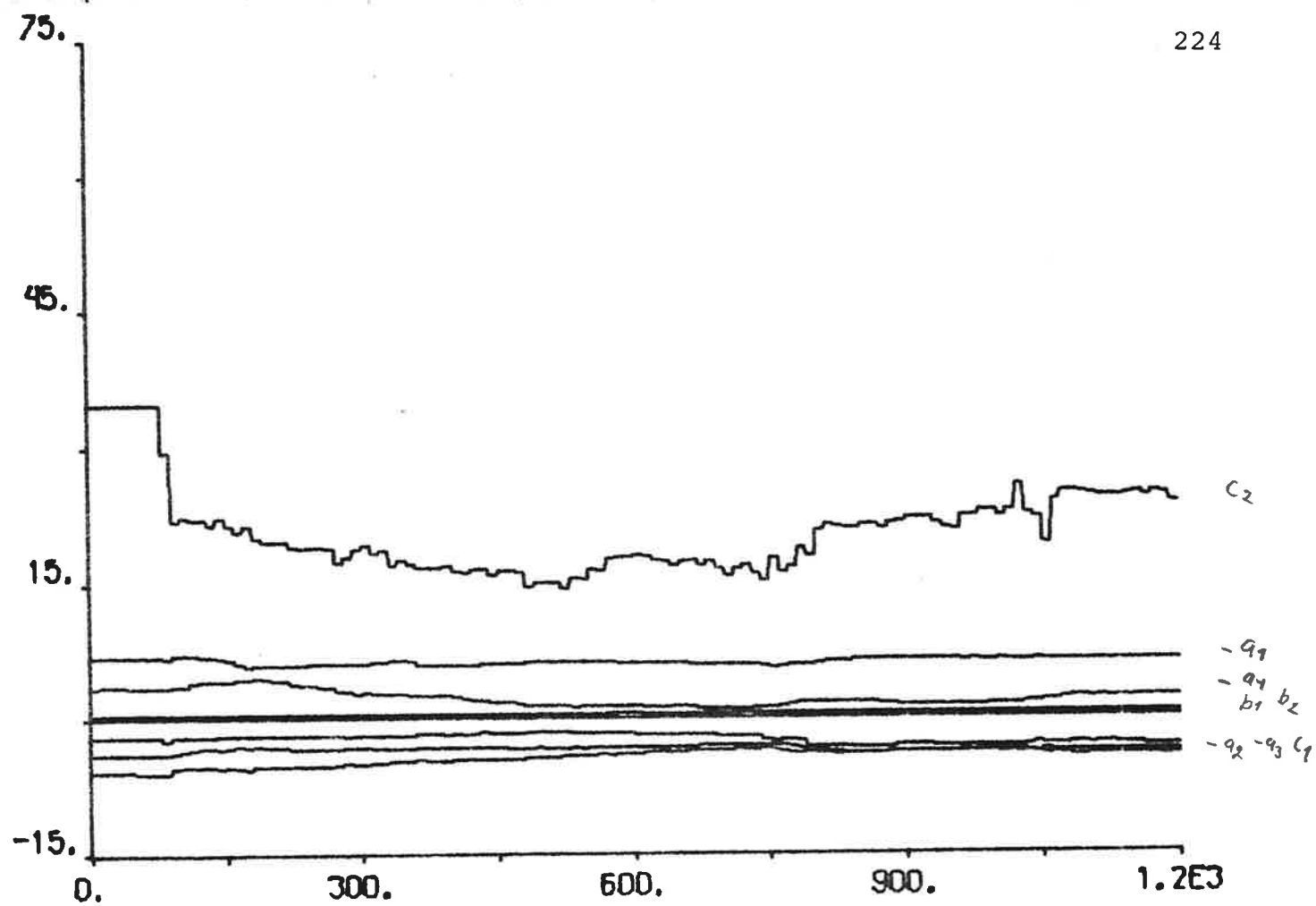


Fig. 4.47 b

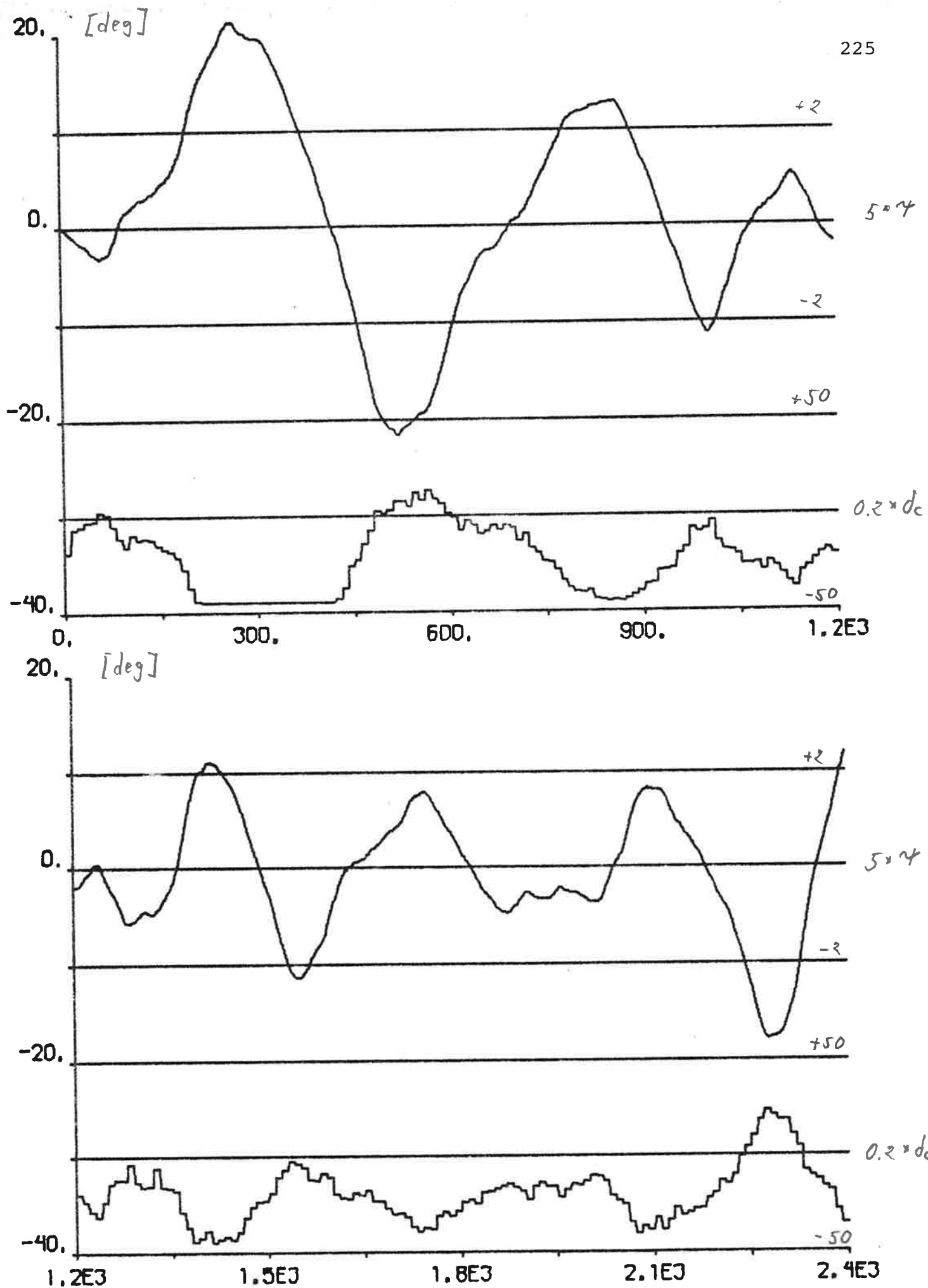


Fig. 4.48 - $T = 22.3$ m, $n_0 = 22.1772$ rpm, $u_0 = 4$ knots, $\delta_\ell = 45$ deg, PID-regulator using estimates from the Kalman filter ($V_0 = 4$ m/s).

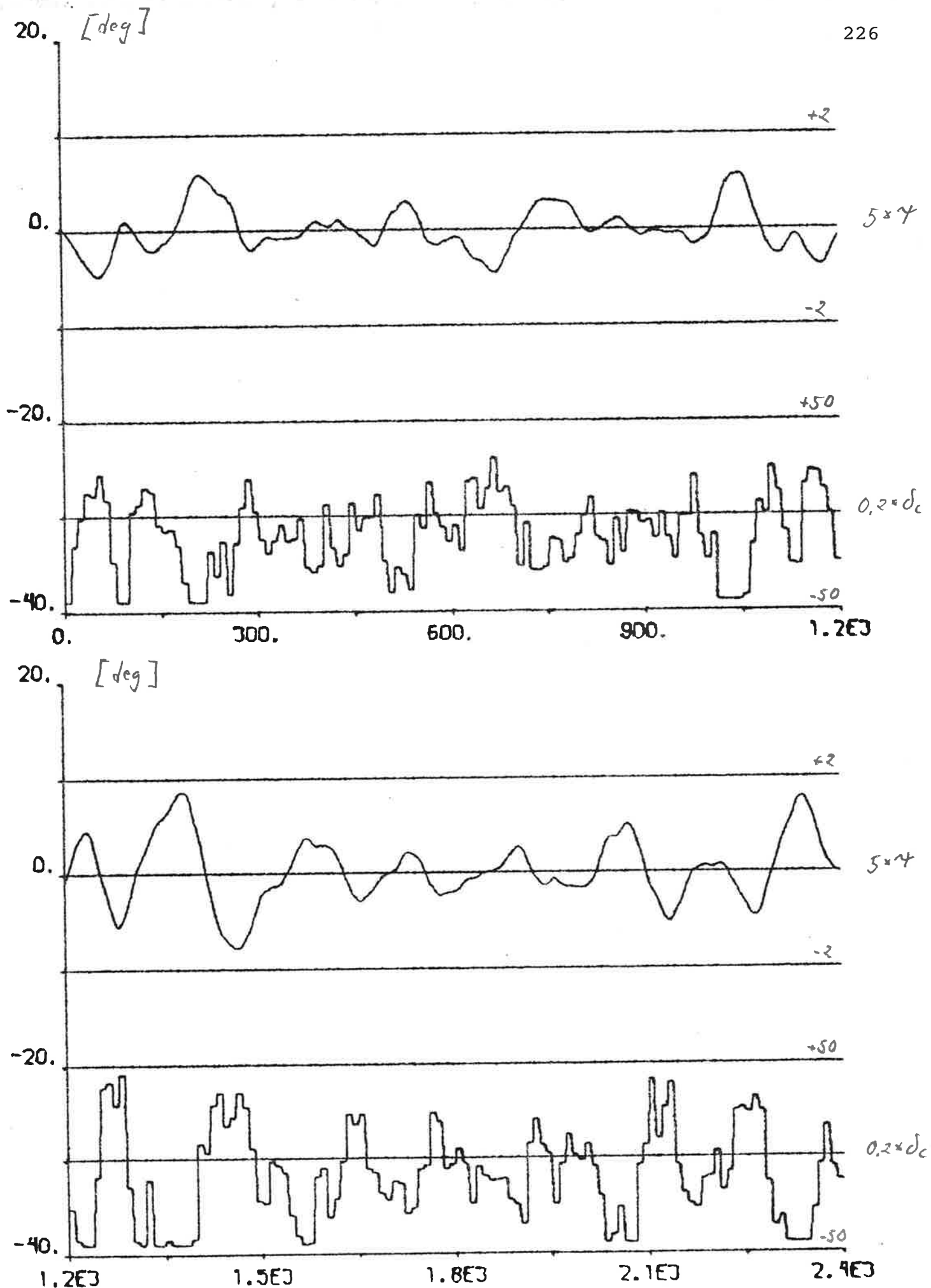


Fig. 4.49 - $T = 10.5$ m, $n_0 = 22.1772$ rpm, $u_0 = 4$ knots, $\delta_\ell = 45$ deg, PID-regulator using estimates from the Kalman filter.

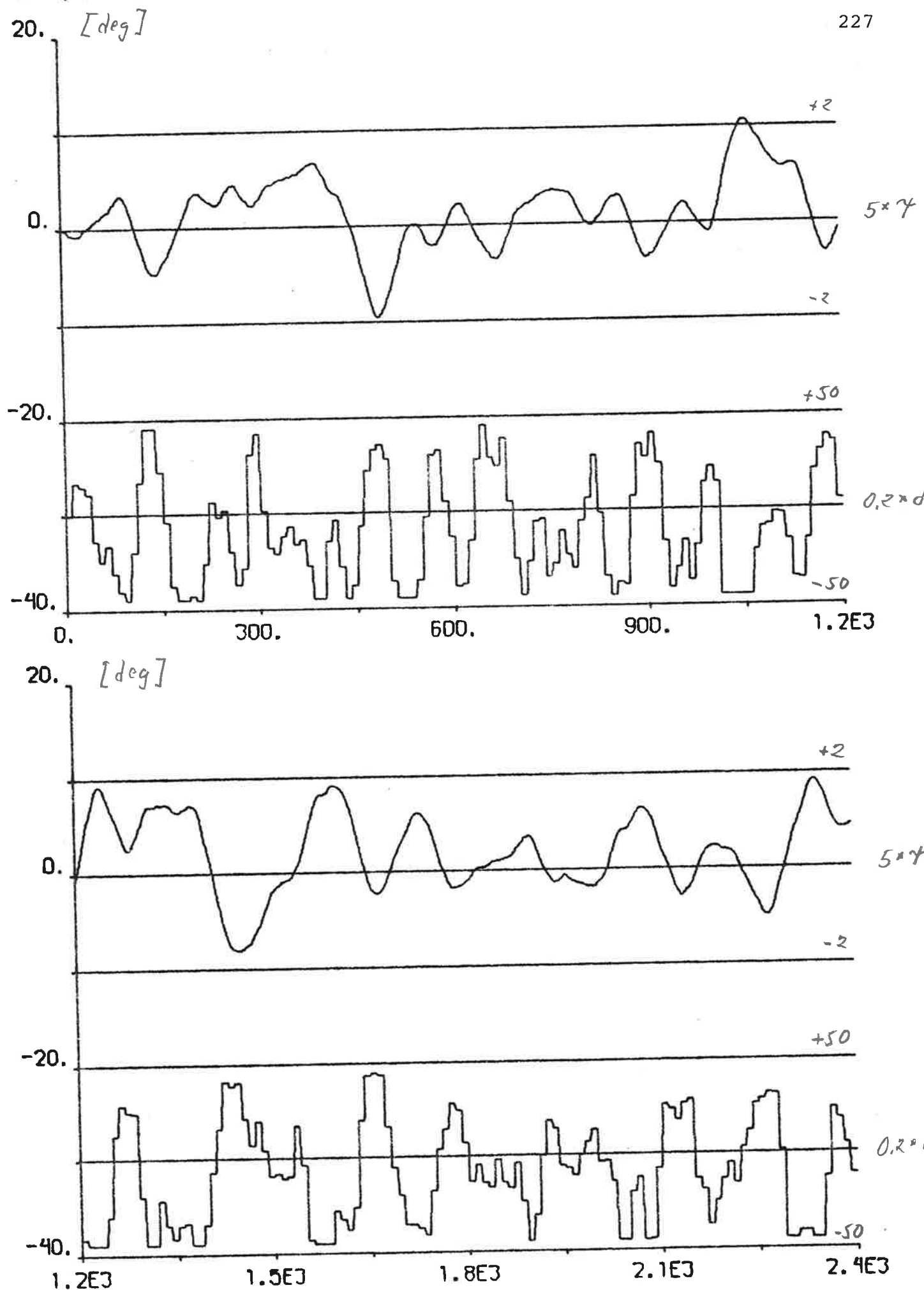


Fig. 4.50 a - $T = 10.5$ m, $n_0 = 22.1772$ rpm, $u_0 = 4$ knots, $\delta_\ell = 45$ deg
self-tuning regulator using non-filtered measurements.

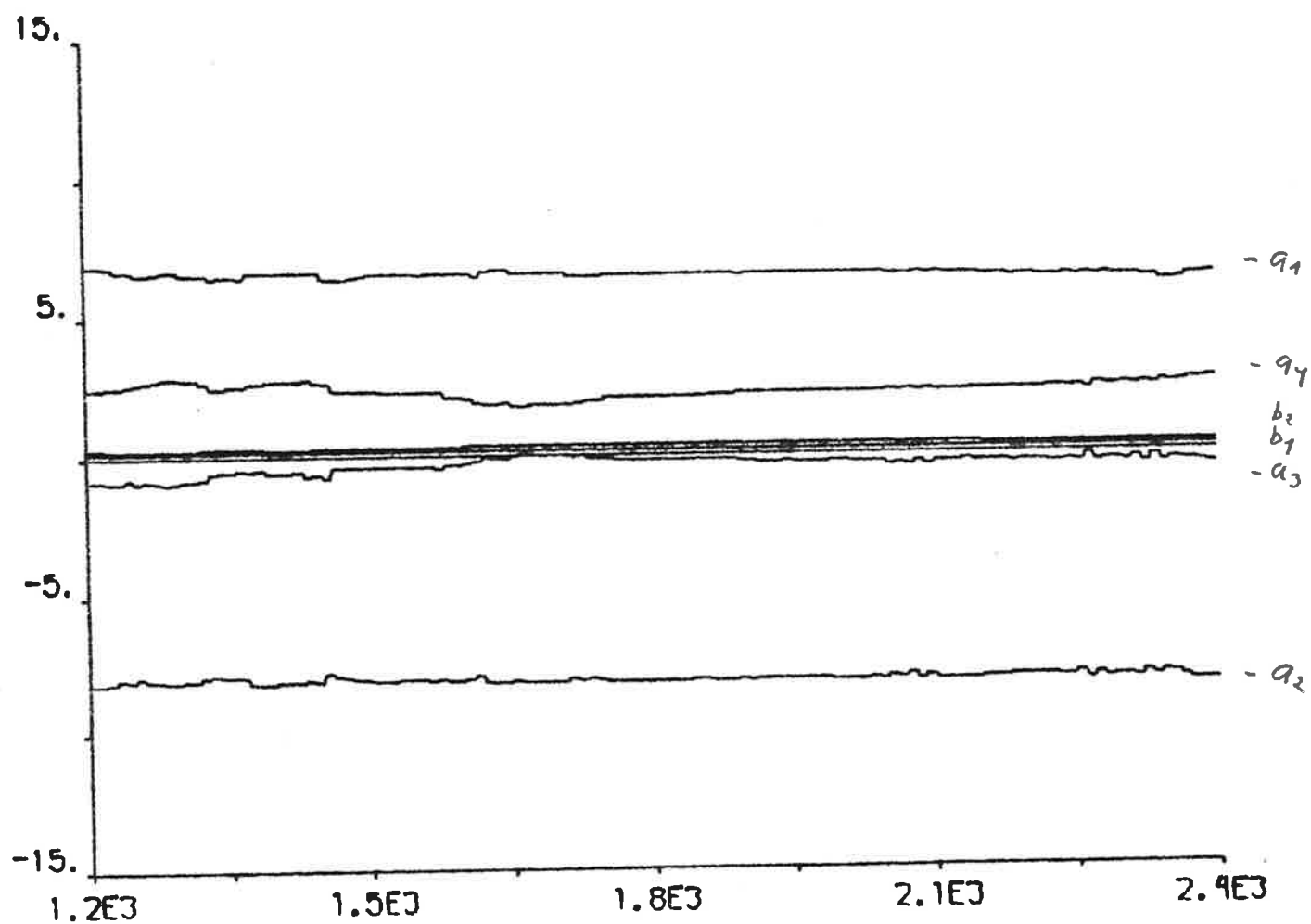
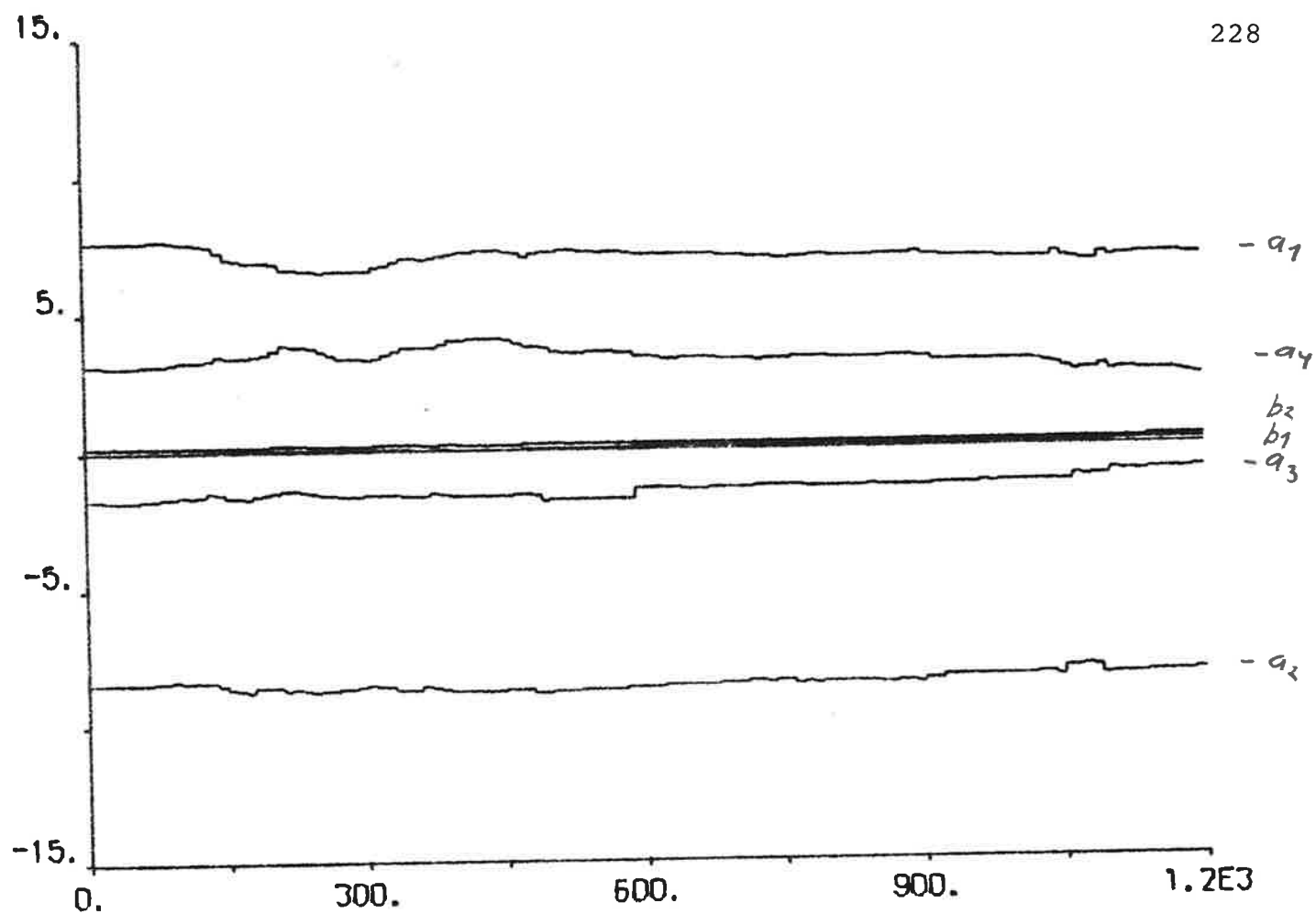


Fig. 4.50 b

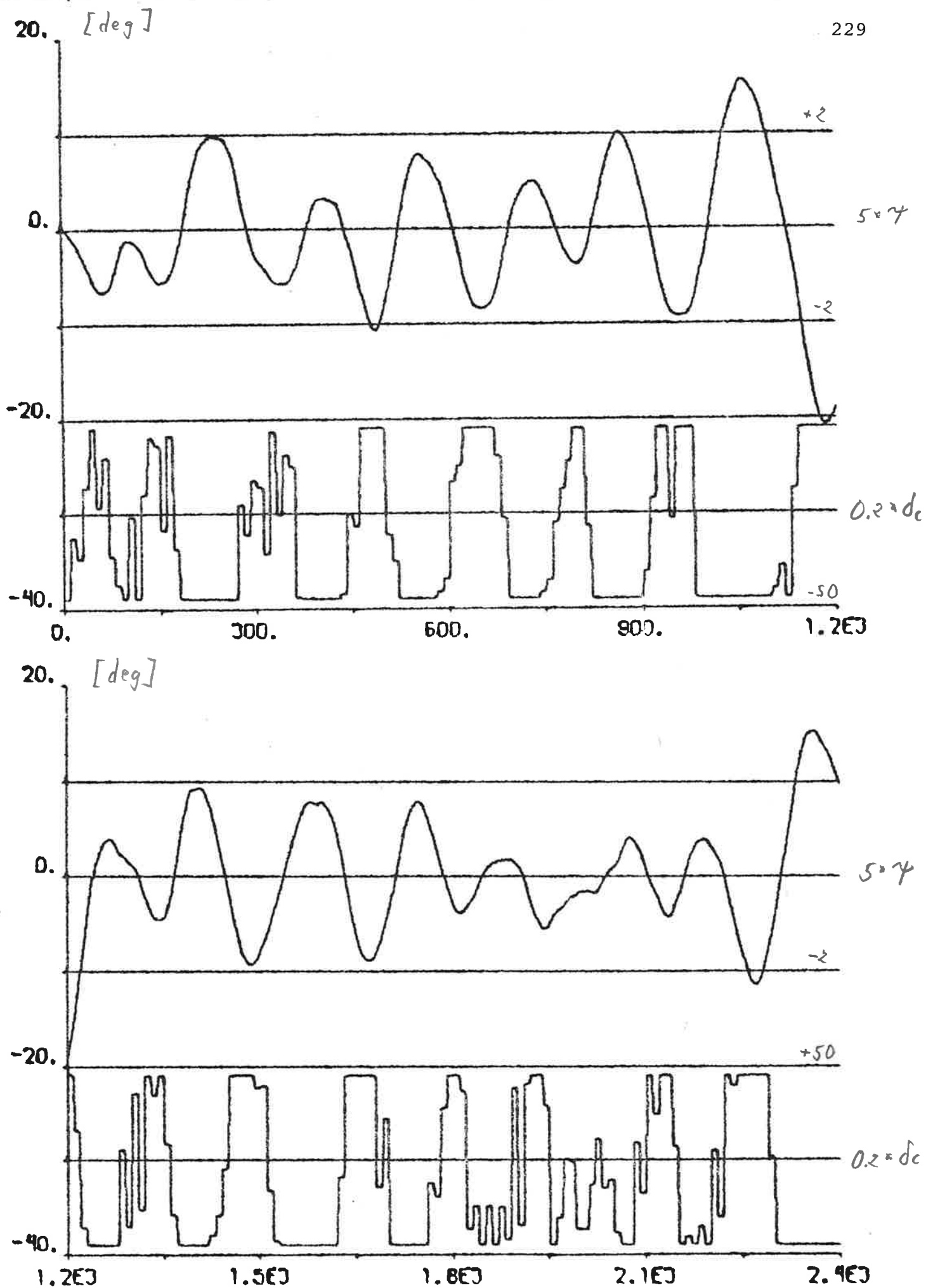


Fig. 4.51 - $T = 10.5 \text{ m}$, $n_0 = 22.1772 \text{ rpm}$, $u_0 = 4 \text{ knots}$, $\delta_l = 45 \text{ deg}$, PID-regulator using non-filtered measurements.

20. [deg]

230

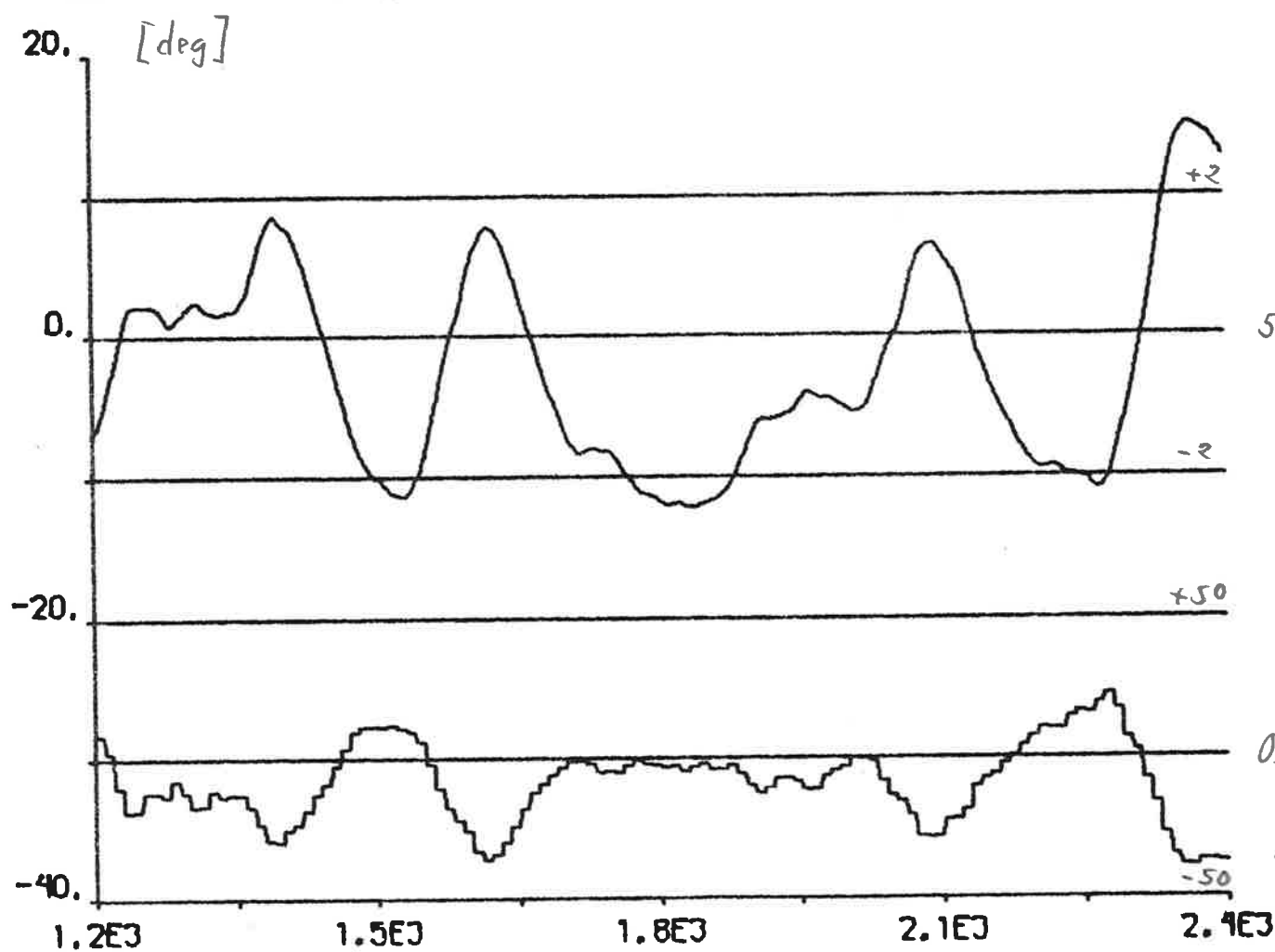
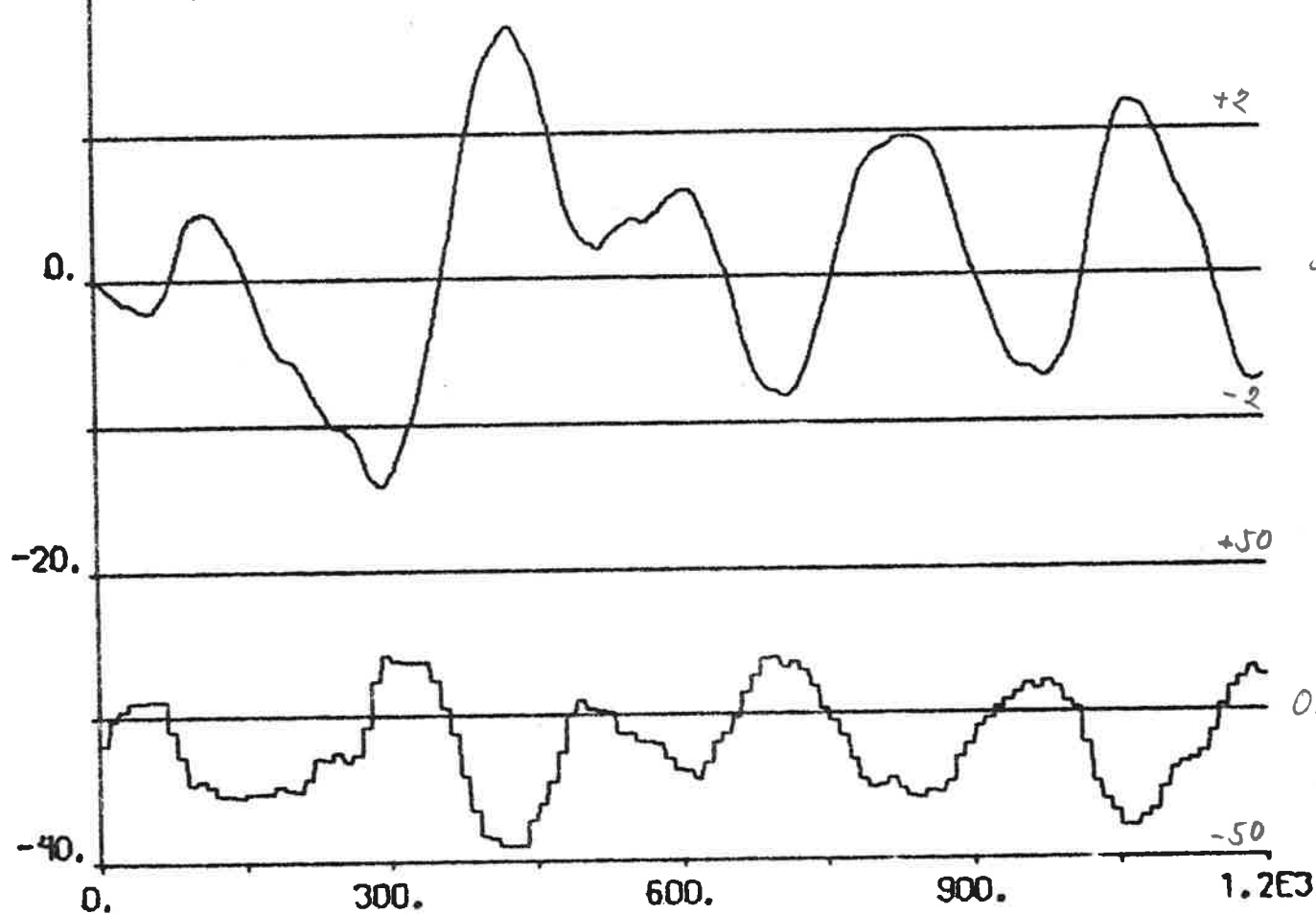


Fig. 4.52 a - $T = 10.5$ m, $n_0 = 22.1772$ rpm, $u_0 = 4$ knots, $\delta_\ell = 45$ deg, self-tuning regulator using estimates from the Kalman filter ($q_2^* = 3$).

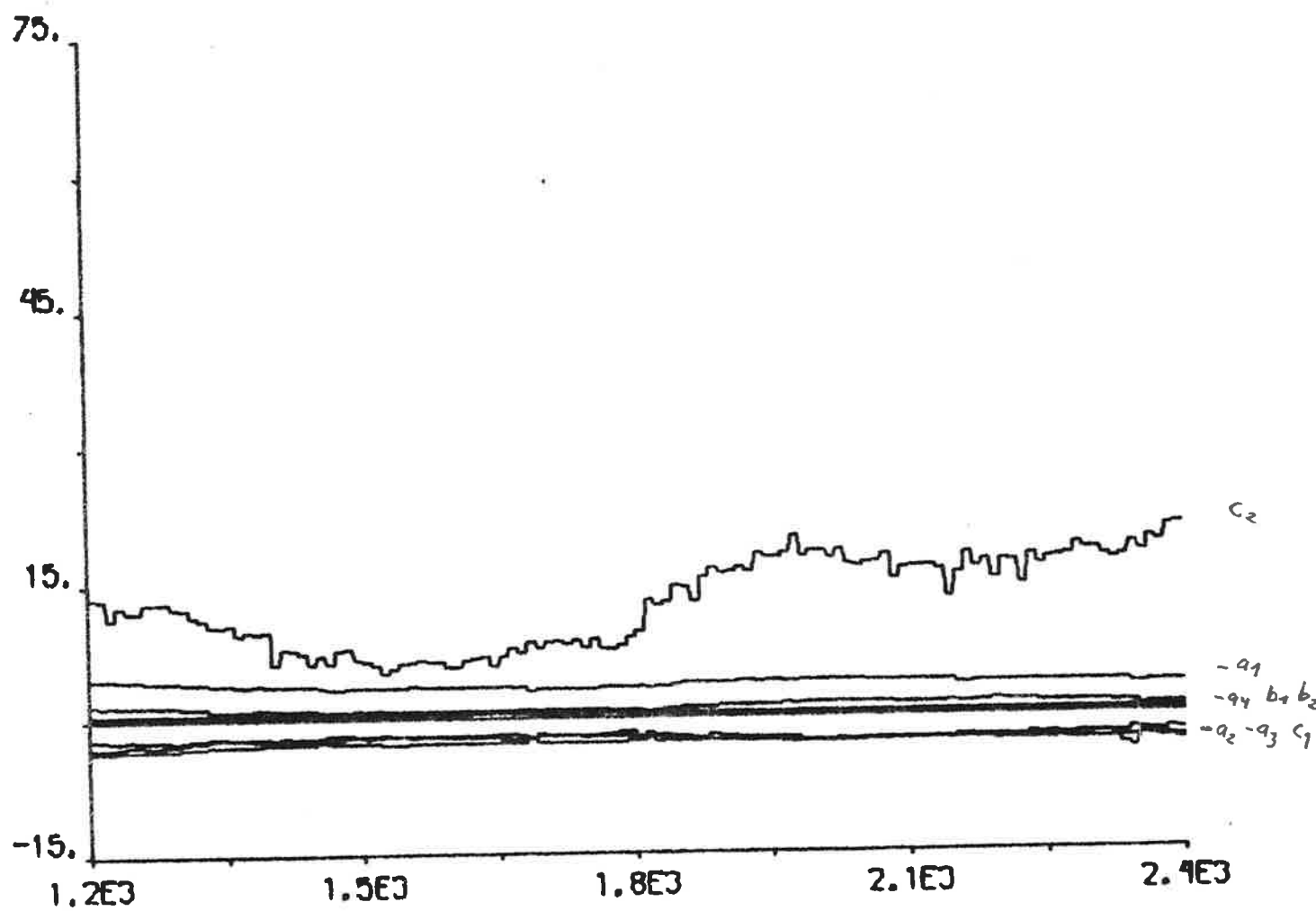
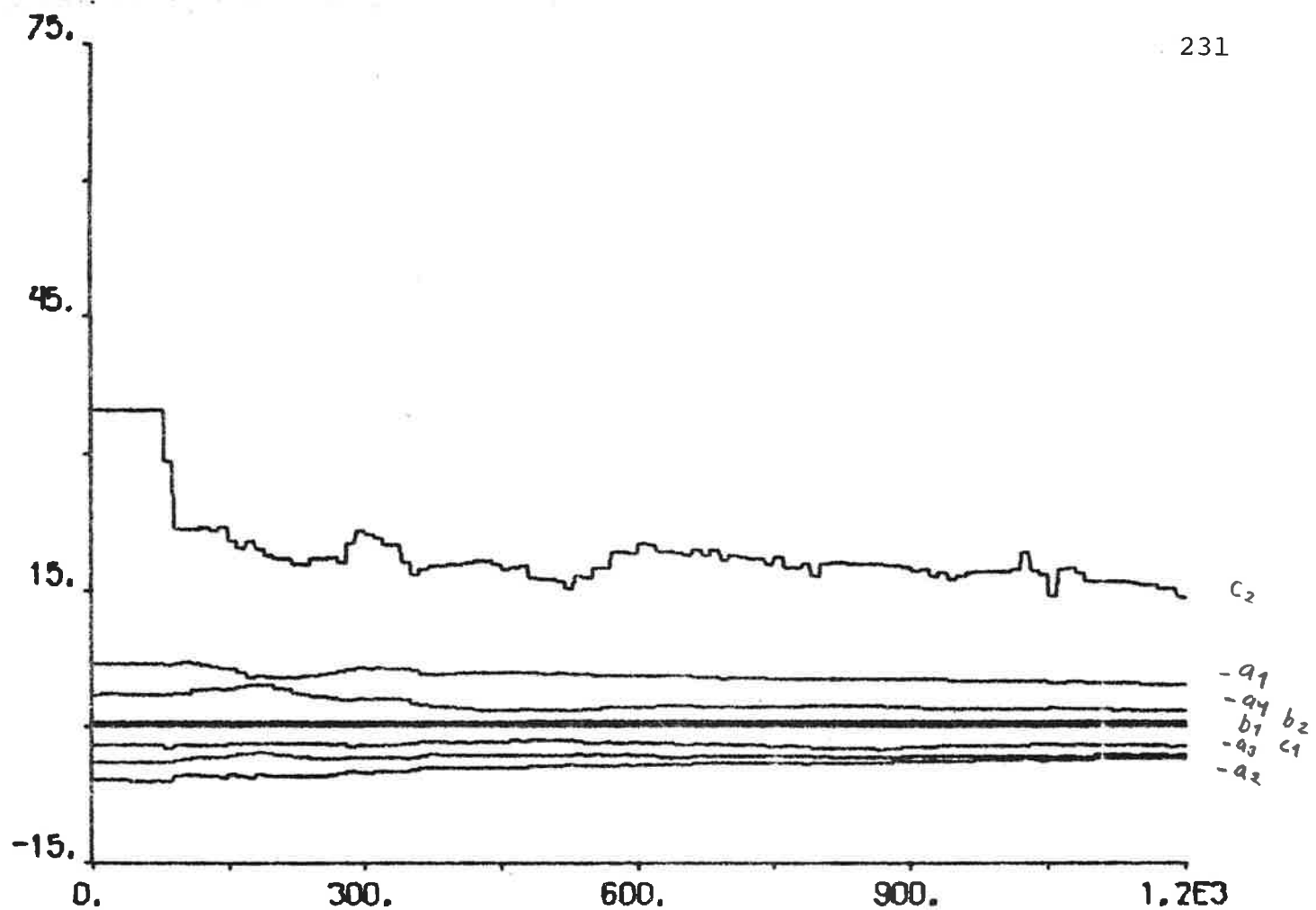


Fig. 4.52 b

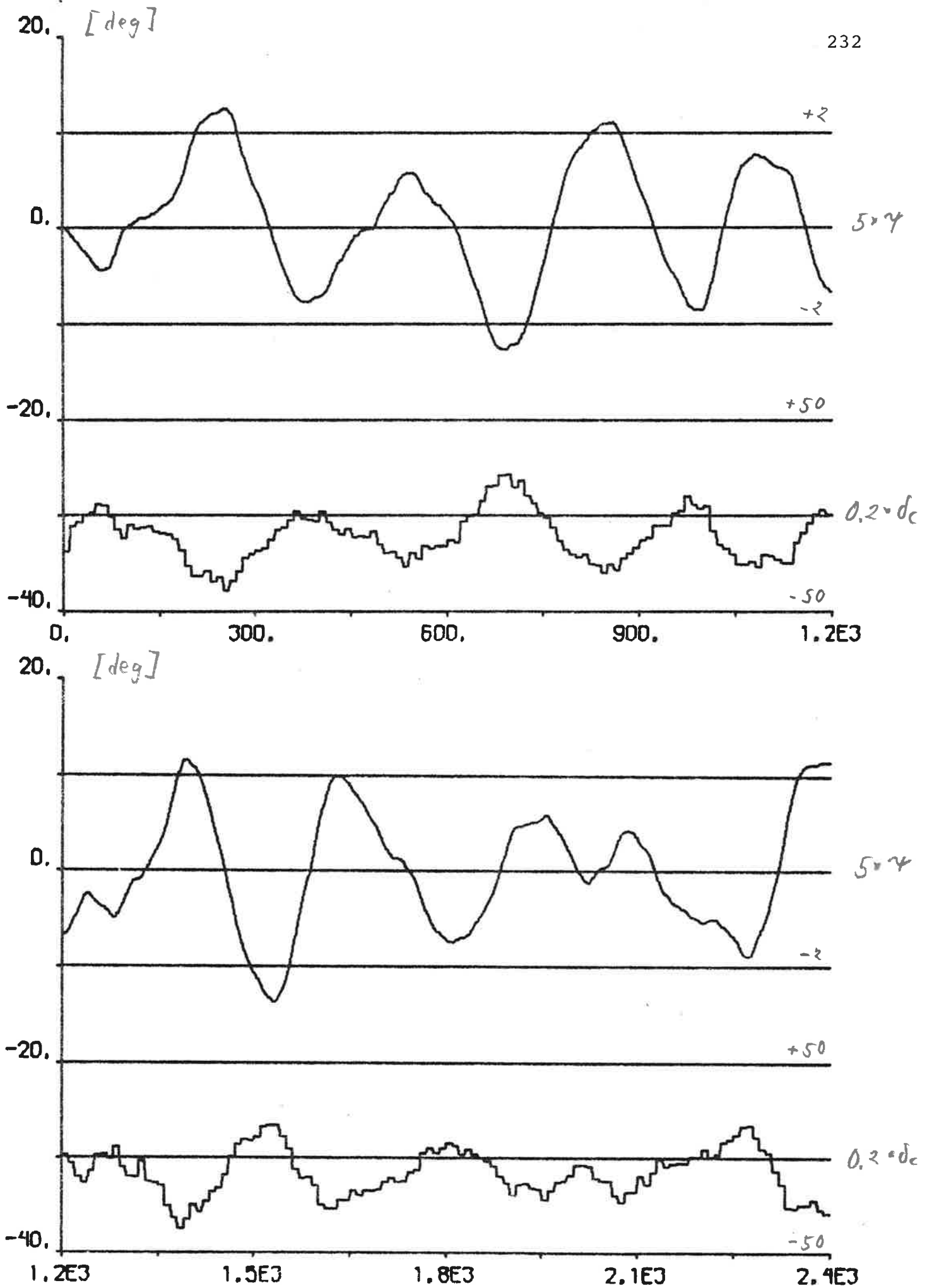


Fig. 4.53 - $T = 10.5$ m, $n_0 = 22.1772$ rpm, $u_0 = 4$ knots, $\delta_\ell = 45$ deg, PID-regulator using estimates from the Kalman filter ($V_0 = 4$ m/s).

4.3 Yawing

Yawing simulations with the yaw regulator, using either the Kalman filter estimates or the non-filtered measurements, are shown in Figs 4.54 - 4.95. The initial speed u_0 is equal to 15.8, 10 or 4 knots and the mean draught T is equal to 22.3 or 10.5 m. Course changes $\Delta\psi_{\text{ref}}$ of 2, 4, 45 and 120 deg are requested and the reference yaw rate r_{ref} is equal to 0, 0.1 or 0.3 deg/s. The rudder limit δ_{ℓ} is always equal to 45 deg. Examples of Kalman filter estimates during yaws are shown in Figs 4.57, 4.58, 4.65, 4.66, 4.88 and 4.93. A summary of the simulations is given in Table 4.8.

It can be concluded that the performance of the Kalman filter, when the draught and the initial speed are varied, is approximately of the same quality during yawing as during straight course keeping (cf Section 4.1). Notice, however, that the estimates $\hat{\delta}_0$, $\hat{d}_v$, $\hat{d}_r$ and $\hat{d}_\delta$ are disturbed rather much during yaws. This implies that the correct yaw rate is not always kept when the yaw regulator uses the Kalman filter estimates. Of course, an incorrect yaw rate is always kept when non-filtered measurements are used, since the yaw rate measurement is biased in the simulations. A straightforward way to improve the Kalman filter is to skip the updating of $\hat{\delta}_0$, $\hat{d}_v$, $\hat{d}_r$ and $\hat{d}_\delta$ during yaws.

The only parameter of the yaw regulator, that is changed when the speed is changed, is $\bar{c}_2$. The following relation is found empirically:

$$\bar{c}_2 = \sqrt{\frac{V_0}{V_s}} c_2^*$$

where $V_0 = 8$ m/s and $c_2^* = 50$ s. This relation is used in the simulations of this section.

T [m]	n_0 [rpm]	u_0 [knots]	Self-tuning reg. and yaw reg. using Kalman filter estimates	Self-tuning reg. and yaw reg. using non-filtered measurements	$\bar{c}_2$ [s]	$\Delta\psi_{\text{ref}}$ [deg]	r_{ref} [deg/s]	Fig.	Remarks
22.3	87.6	15.8	x		50	2	0	4.54	x) x)
22.3	87.6	15.8	x		50	4	0.1	4.55	
22.3	87.6	15.8	x		50	4	0.3	4.56	
22.3	87.6	15.8	x		50	45	0.1	4.57	
22.3	87.6	15.8	x		50	120	0.3	4.58	
22.3	87.6	15.8		x	50	4	0.1	4.59	
22.3	87.6	15.8		x	50	45	0.1	4.60	
22.3	87.6	15.8		x	50	120	0.3	4.61	
10.5	87.6	15.8	x		50	2	0	4.62	x) x)
10.5	87.6	15.8	x		50	4	0.1	4.63	
10.5	87.6	15.8	x		50	4	0.3	4.64	
10.5	87.6	15.8	x		50	45	0.1	4.65	
10.5	87.6	15.8	x		50	120	0.3	4.66	
10.5	87.6	15.8		x	50	4	0.1	4.67	
10.5	87.6	15.8		x	50	45	0.1	4.68	
10.5	87.6	15.8		x	50	120	0.3	4.69	
22.3	55.443	10	x		63.25	2	0	4.70	
22.3	55.443	10	x		63.25	4	0.1	4.71	
22.3	55.443	10	x		63.25	4	0.3	4.72	
22.3	55.443	10	x		63.25	45	0.1	4.73	
22.3	55.443	10	x		63.25	120	0.3	4.74	
22.3	55.443	10		x	63.25	4	0.1	4.75	
22.3	55.443	10		x	63.25	45	0.1	4.76	
22.3	55.443	10		x	63.25	120	0.3	4.77	
10.5	55.443	10	x		63.25	2	0	4.78	
10.5	55.443	10	x		63.25	4	0.1	4.79	
10.5	55.443	10	x		63.25	4	0.3	4.80	
10.5	55.443	10	x		63.25	45	0.1	4.81	
10.5	55.443	10	x		63.25	120	0.3	4.82	
10.5	55.443	10		x	63.25	4	0.1	4.83	
10.5	55.443	10		x	63.25	45	0.1	4.84	
10.5	55.443	10		x	63.25	120	0.3	4.85	
22.3	22.1772	4	x		100	2	0	4.86	x)
22.3	22.1772	4	x		100	4	0.1	4.87	
22.3	22.1772	4	x		100	45	0.1	4.88	
22.3	22.1772	4		x	100	4	0.1	4.89	
22.3	22.1772	4		x	100	45	0.1	4.90	
10.5	22.1772	4	x		100	2	0	4.91	x)
10.5	22.1772	4	x		100	4	0.1	4.92	
10.5	22.1772	4	x		100	45	0.1	4.93	
10.5	22.1772	4		x	100	4	0.1	4.94	
10.5	22.1772	4		x	100	45	0.1	4.95	

x) The Kalman filter estimates are shown.

Table 4.8 - Summary of yawing simulations. The initial reference course ψ_{ref} is equal to 0 deg and the course change $\Delta\psi_{\text{ref}}$ is requested after 100 s. The rudder limit δ_ℓ is equal to 45 deg.

It can be concluded from the simulations that the performance of the yaw regulator, when Kalman filter estimates are used, is very good for different load conditions and speeds, with one exception: the performance when $T = 22.3$ m and $u_0 = 4$ knots is rather bad (cf Figs 4.86 - 4.88). Maybe it is possible to increase the yawing quality in this case by introducing more speed dependent parameters of the yaw regulator. Notice, however, that the forward speed u_m is decreased to the approximate value 1 knot, when $T = 22.3$ m, $u_0 = 4$ knots, $\Delta\psi_{\text{ref}} = 45$ deg and $r_{\text{ref}} = 0.1$ deg/s (cf Fig. 4.88 e). It is, of course, rather difficult to obtain a good performance in such an extremely small speed. When the initial speed u_0 is equal to 10 knots, rather large rudder deviations are required compared to the case when $u_0 = 15.8$ knots. This is necessary, however, to obtain a good performance.

The performance of the yaw regulator, when non-filtered measurements are used, is not at all as good as the case when Kalman filter estimates are used. The rudder deviations are in general very large due to the noisy yaw rate measurements. It is probably possible to improve the performance by decreasing the values of the gain factors of the yaw regulator. See Källström (1976b). It can also be concluded that it is highly desired to filter the yaw rate in some way, if the measurements are very noisy. If the resolution of the heading measurements is good, it is also possible to perform a difference approximation to obtain a yaw rate estimate.

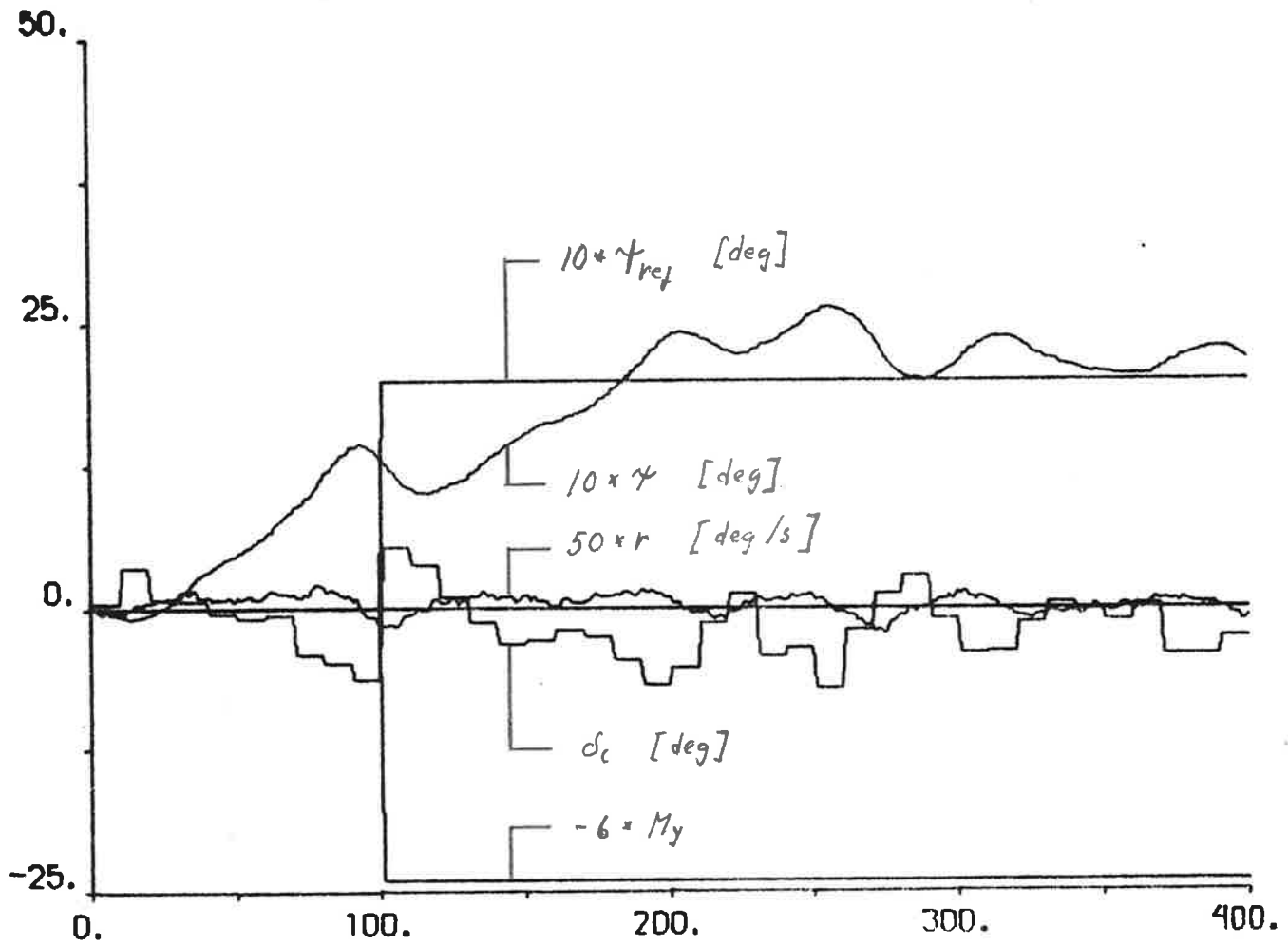


Fig. 4.54 - $T = 22.3$ m, $n_0 = 87.6$ rpm, $u_0 = 15.8$ knots,
 $\Delta\psi_{ref} = 2$ deg, $r_{ref} = 0$ deg/s, self-tuning regulator
 and yaw regulator using estimates from the Kalman
 filter ($\bar{c}_2 = 50$ s).

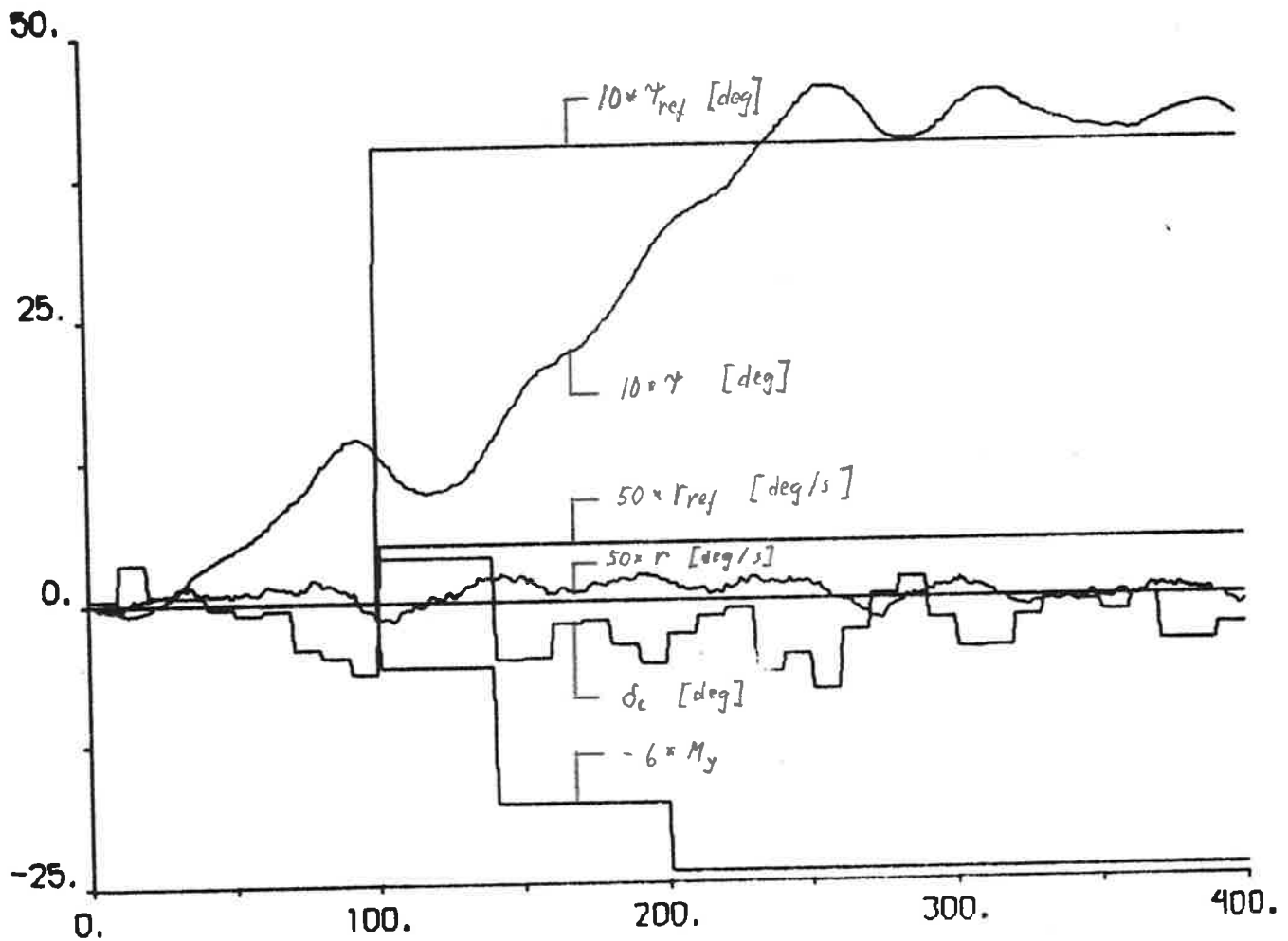


Fig. 4.55 - $T = 22.3$ m, $n_0 = 87.6$ rpm, $u_0 = 15.8$ knots, $\Delta\psi_{ref} = 4$ deg, $r_{ref} = 0.1$ deg/s, self-tuning regulator and yaw regulator using estimates from the Kalman filter ($\bar{c}_2 = 50$ s).

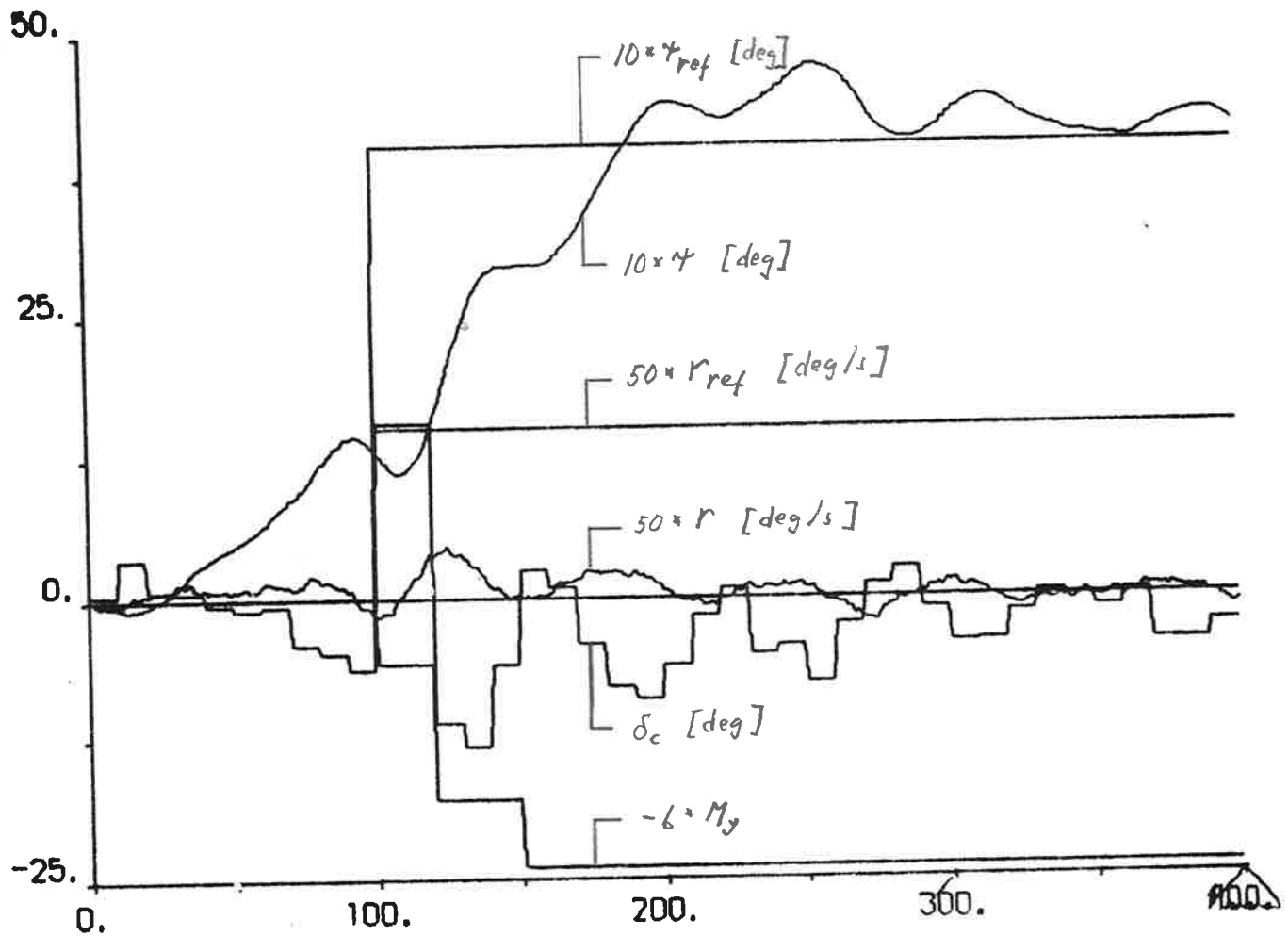


Fig. 4.56 - $T = 22.3$ m, $n_0 = 87.6$ rpm, $u_0 = 15.8$ knots,
 $\Delta\psi_{ref} = 4$ deg, $r_{ref} = 0.3$ deg/s, self-tuning
 regulator and yaw regulator using estimates from
 the Kalman filter ($\bar{c}_2 = 50$ s).

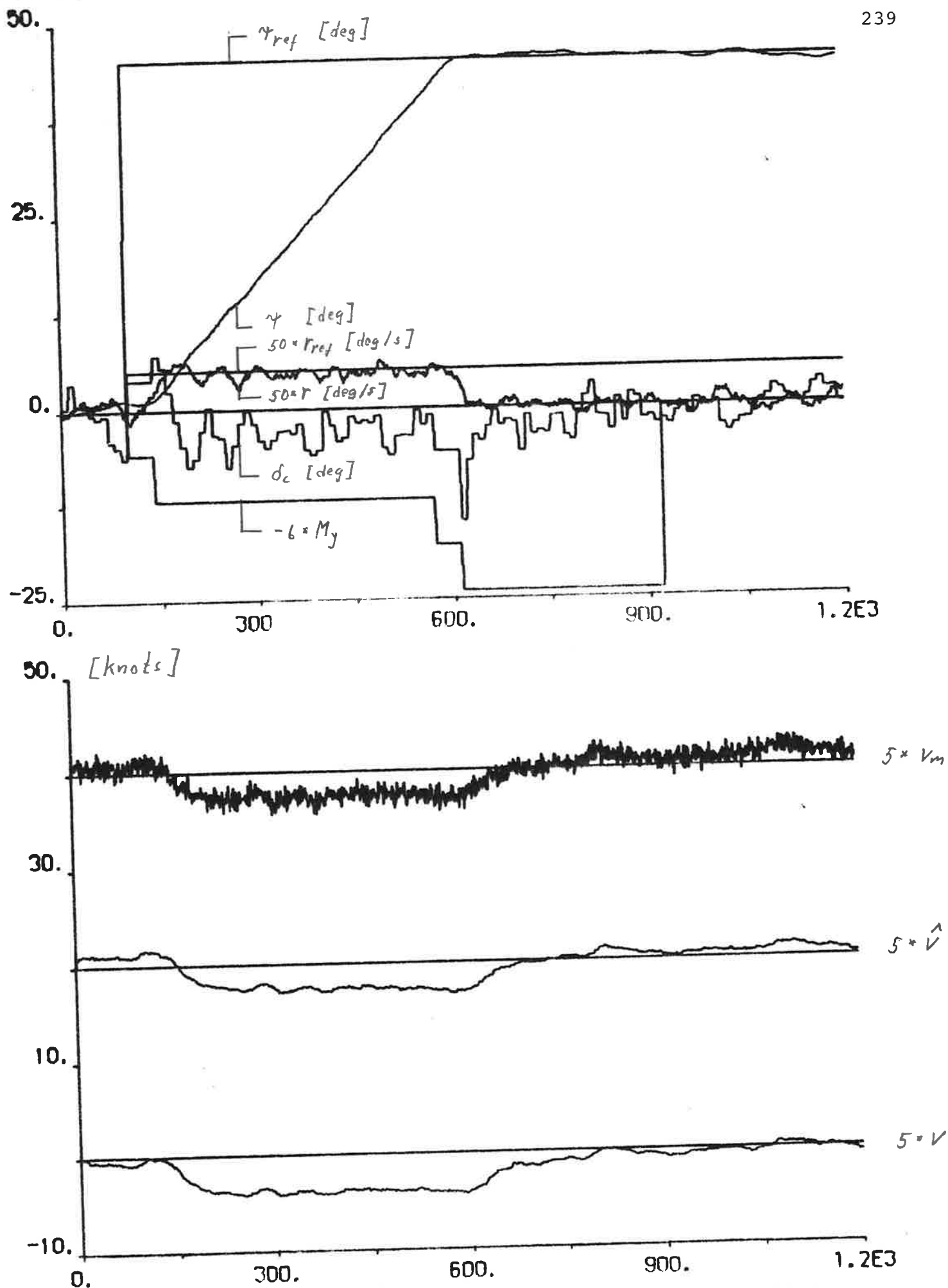


Fig. 4.57 a - $T = 22.3$ m, $n_0 = 87.6$ rpm, $u_0 = 15.8$ knots, $\Delta\psi_{ref} = 45$ deg, $\dot{\gamma}_{ref} = 0.1$ deg/s, self-tuning regulator and yaw regulator using estimates from the Kalman filter ($\bar{c}_2 = 50$ s).

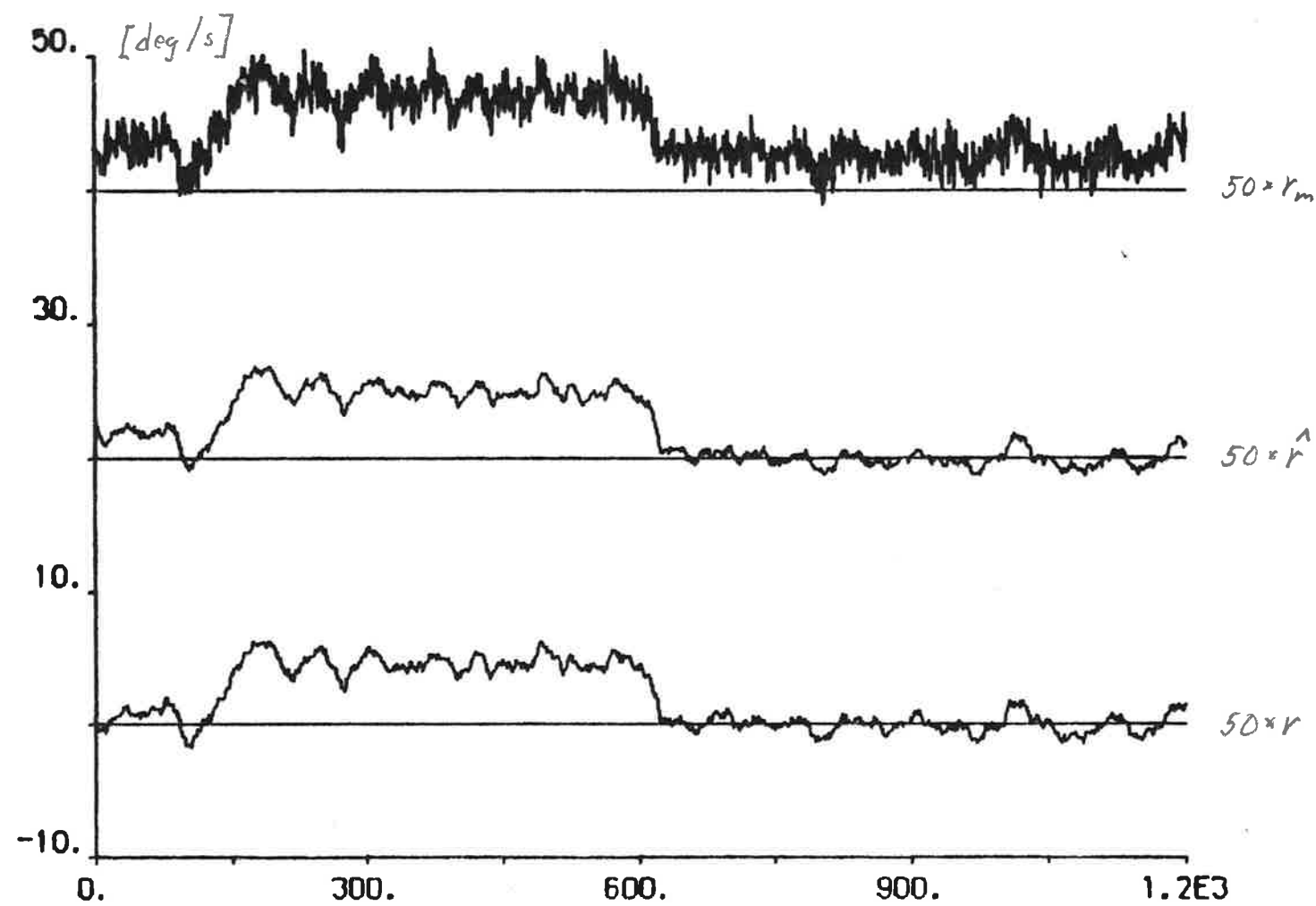
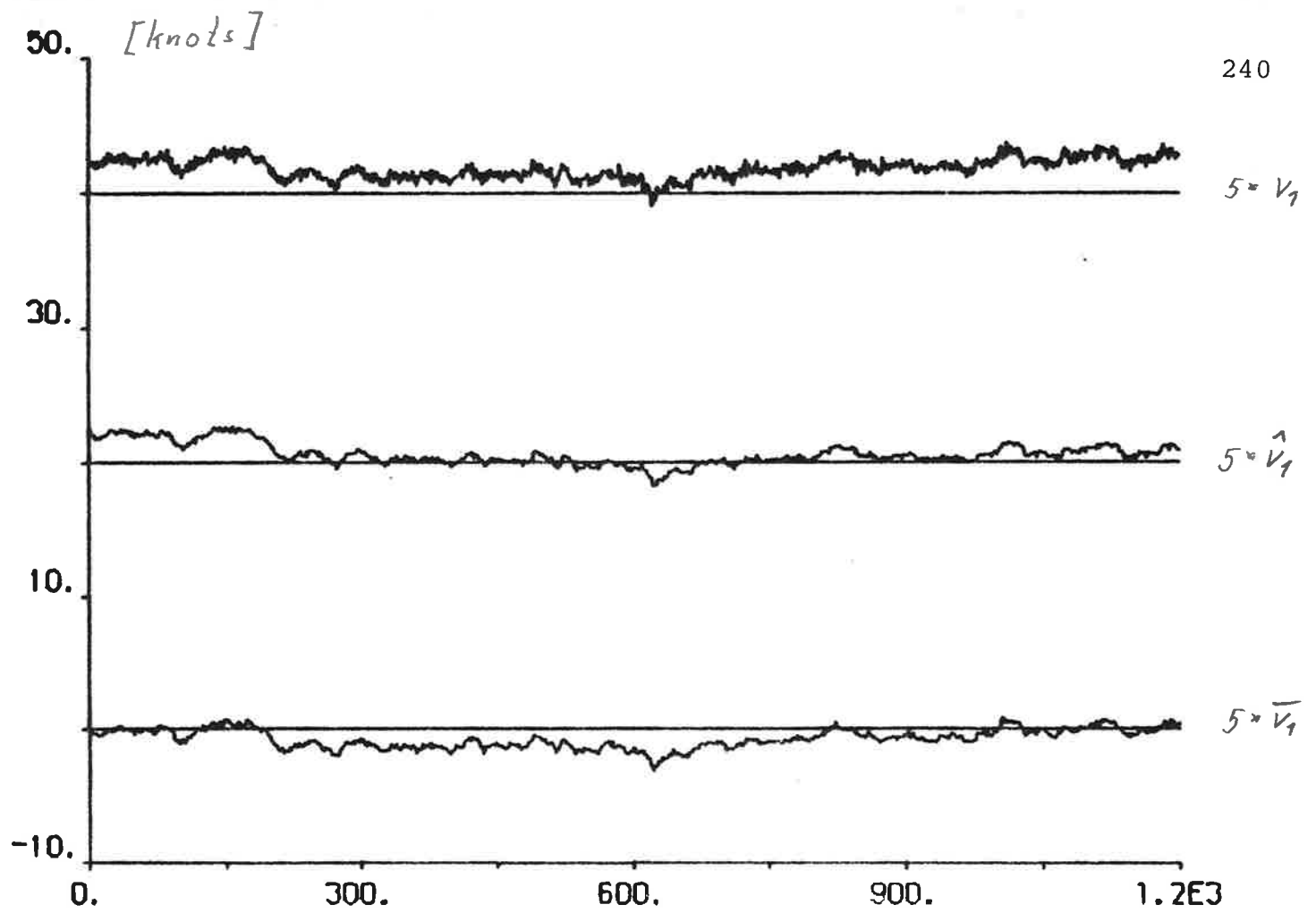


Fig. 4.57 b

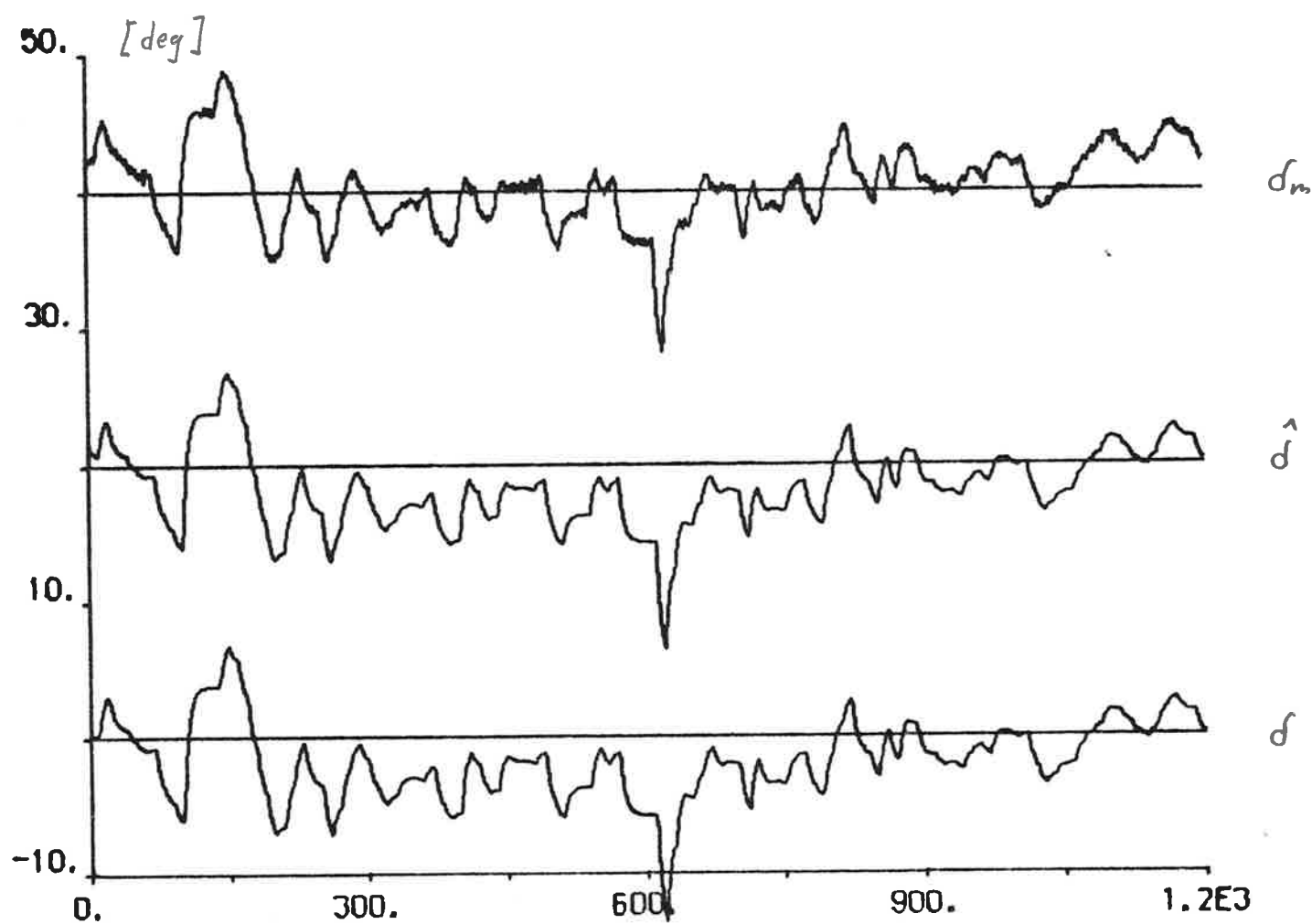
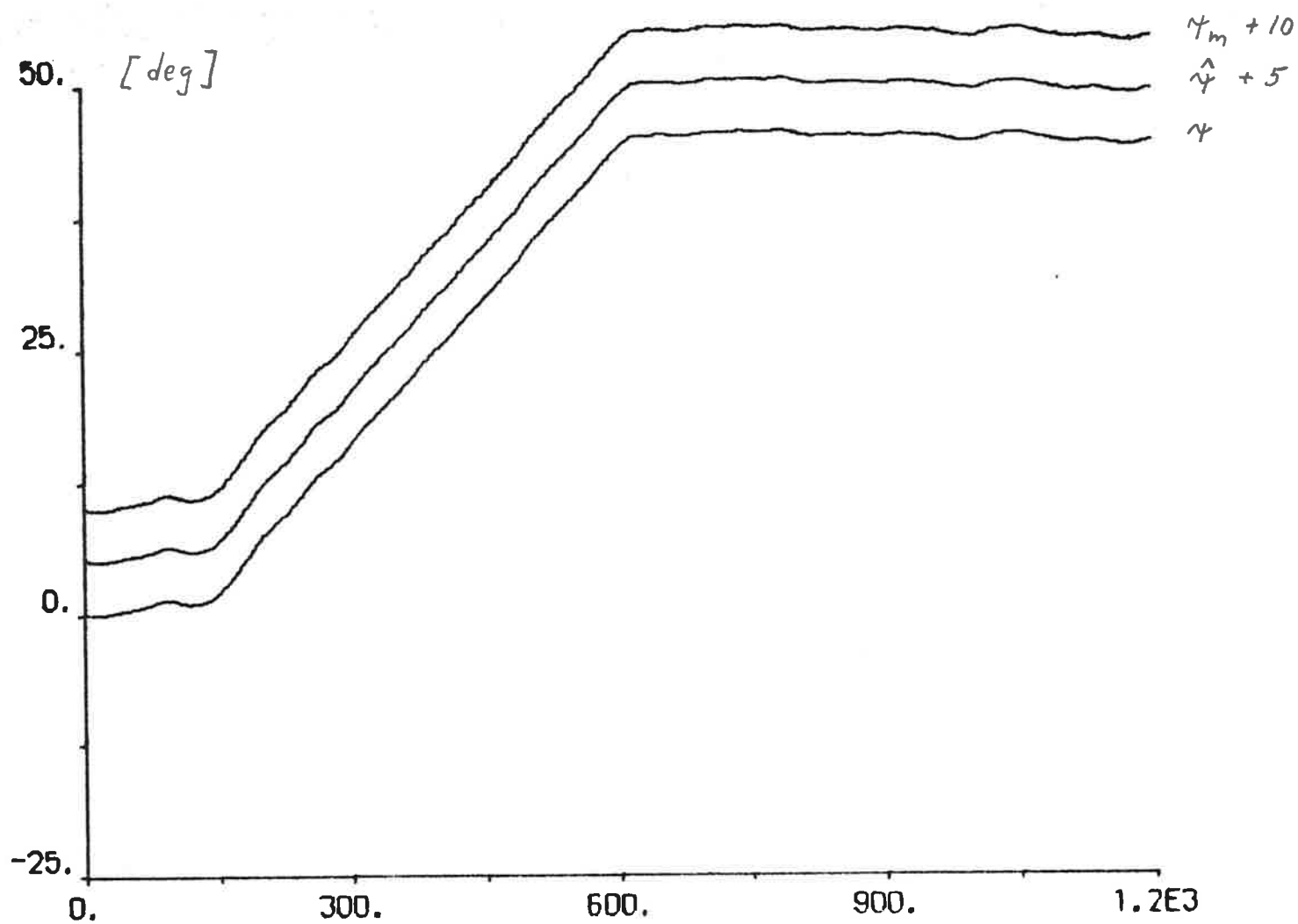


Fig. 4.57 c

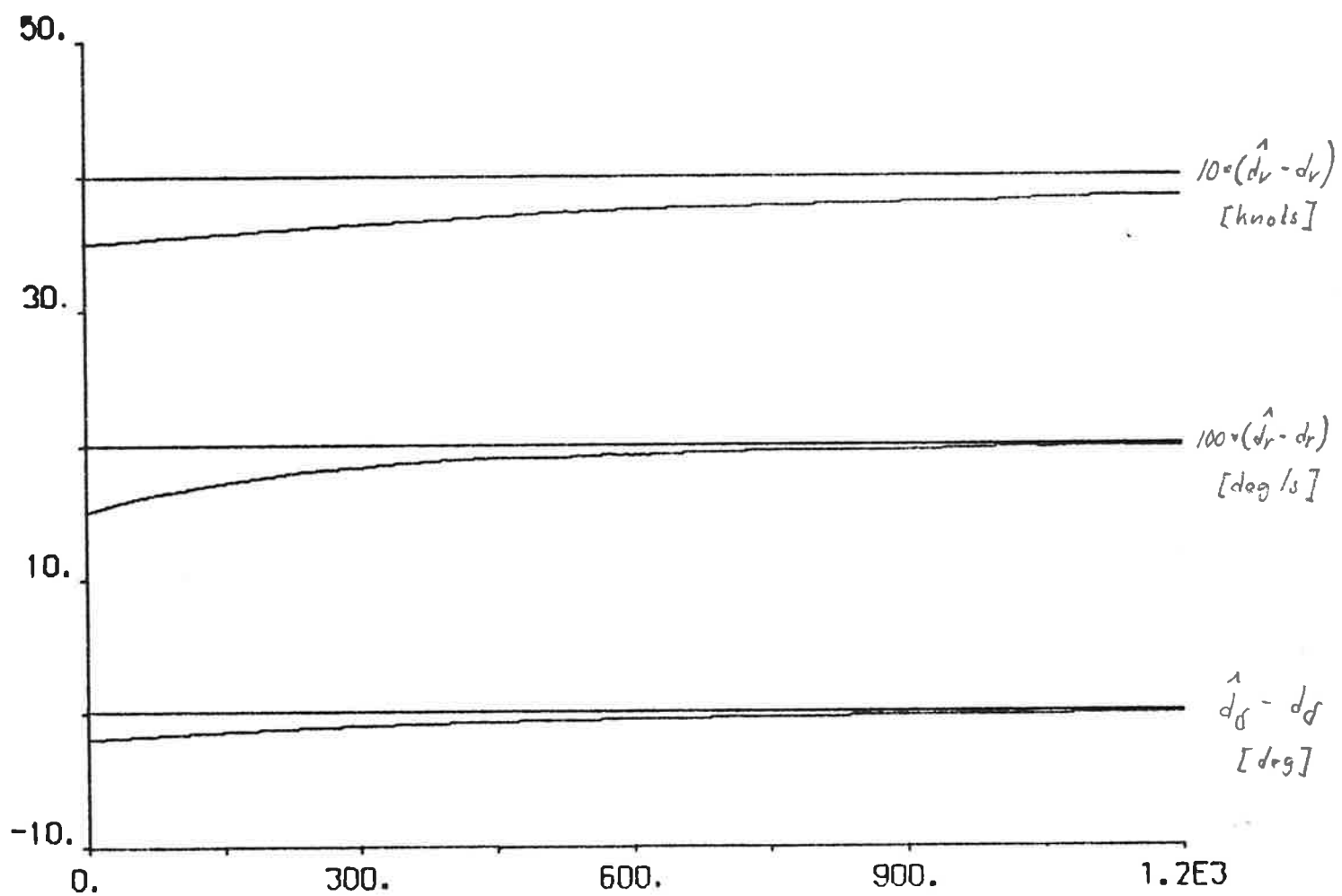
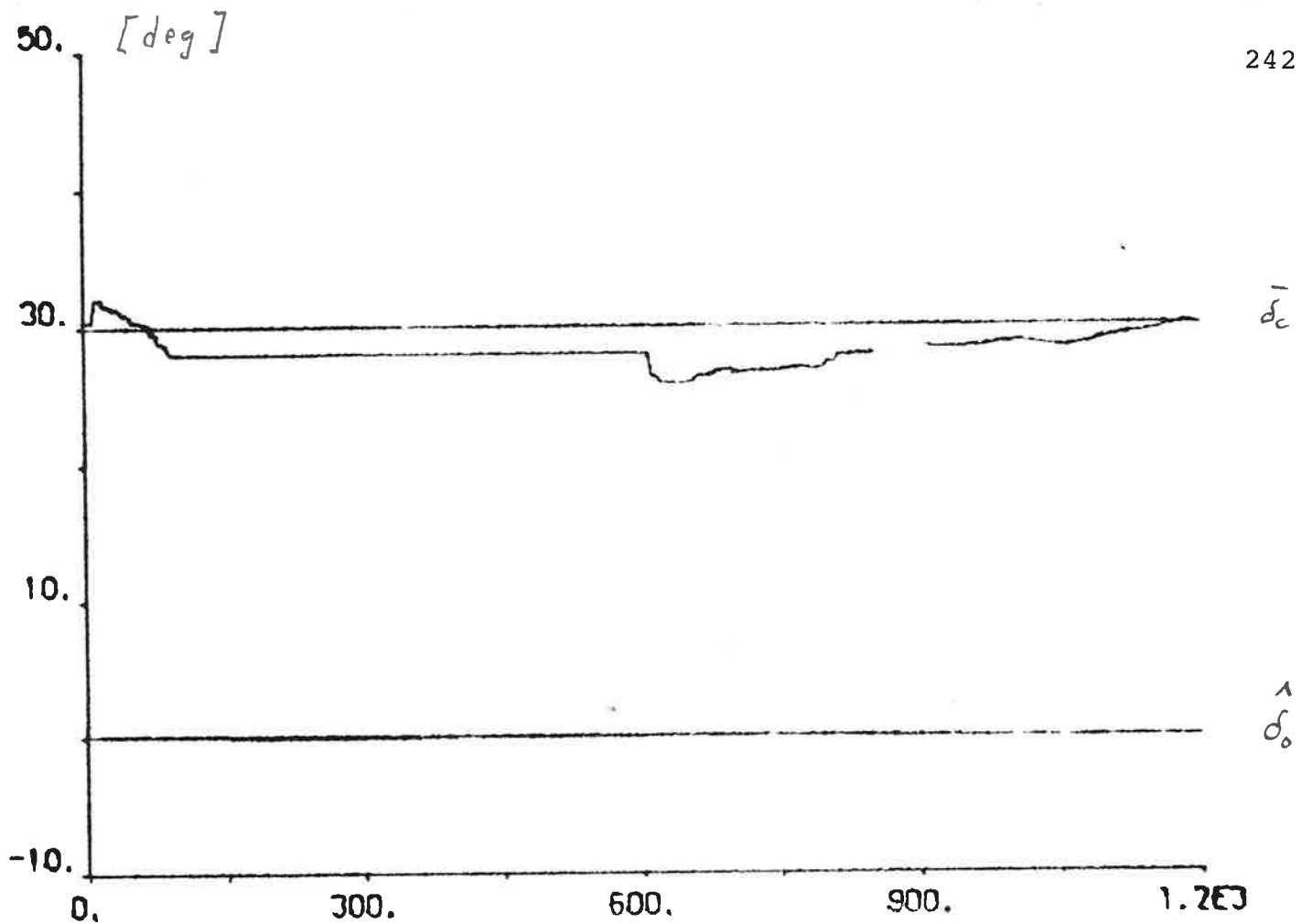


Fig. 4.57 d

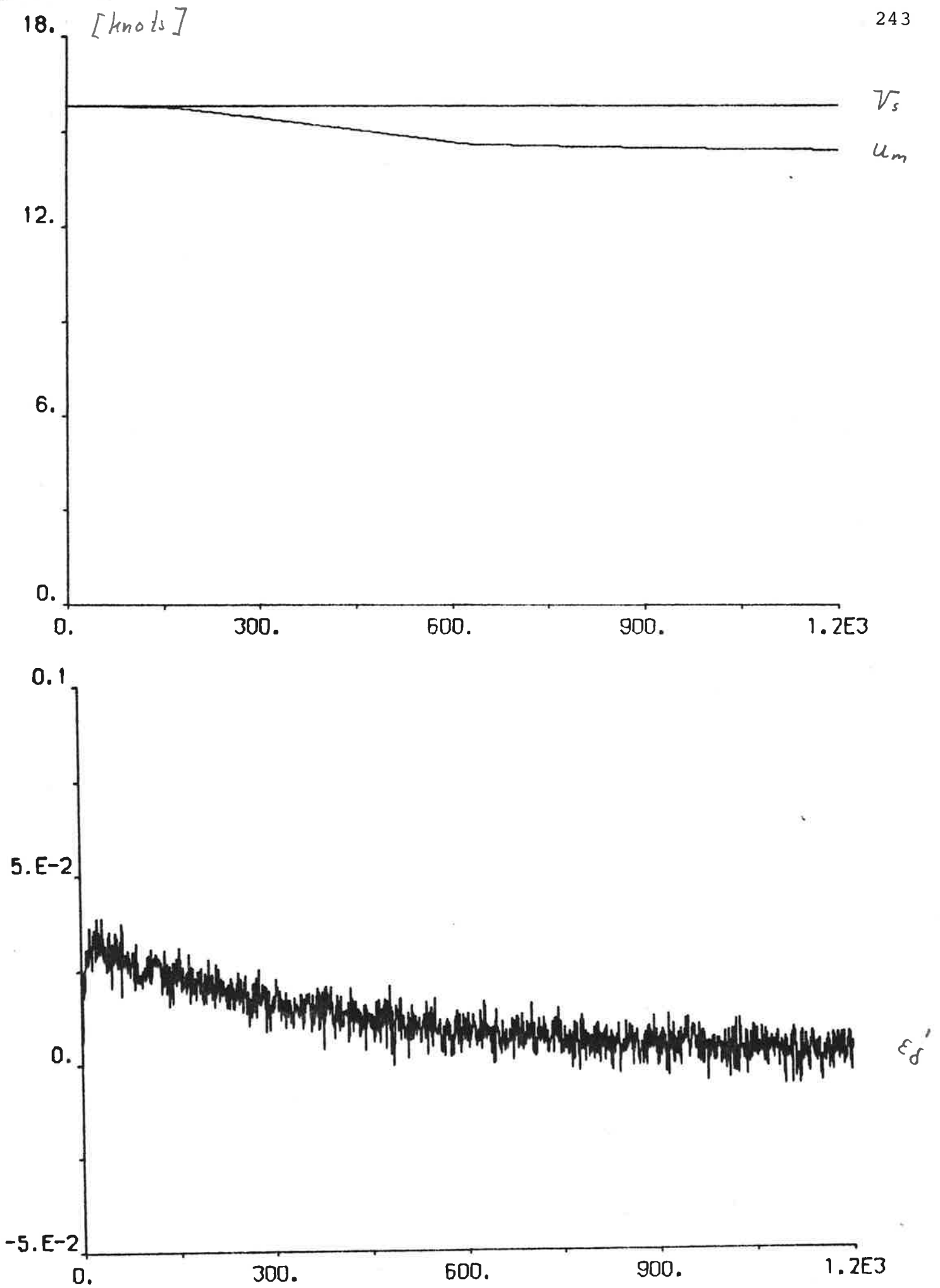


Fig. 4.57 e

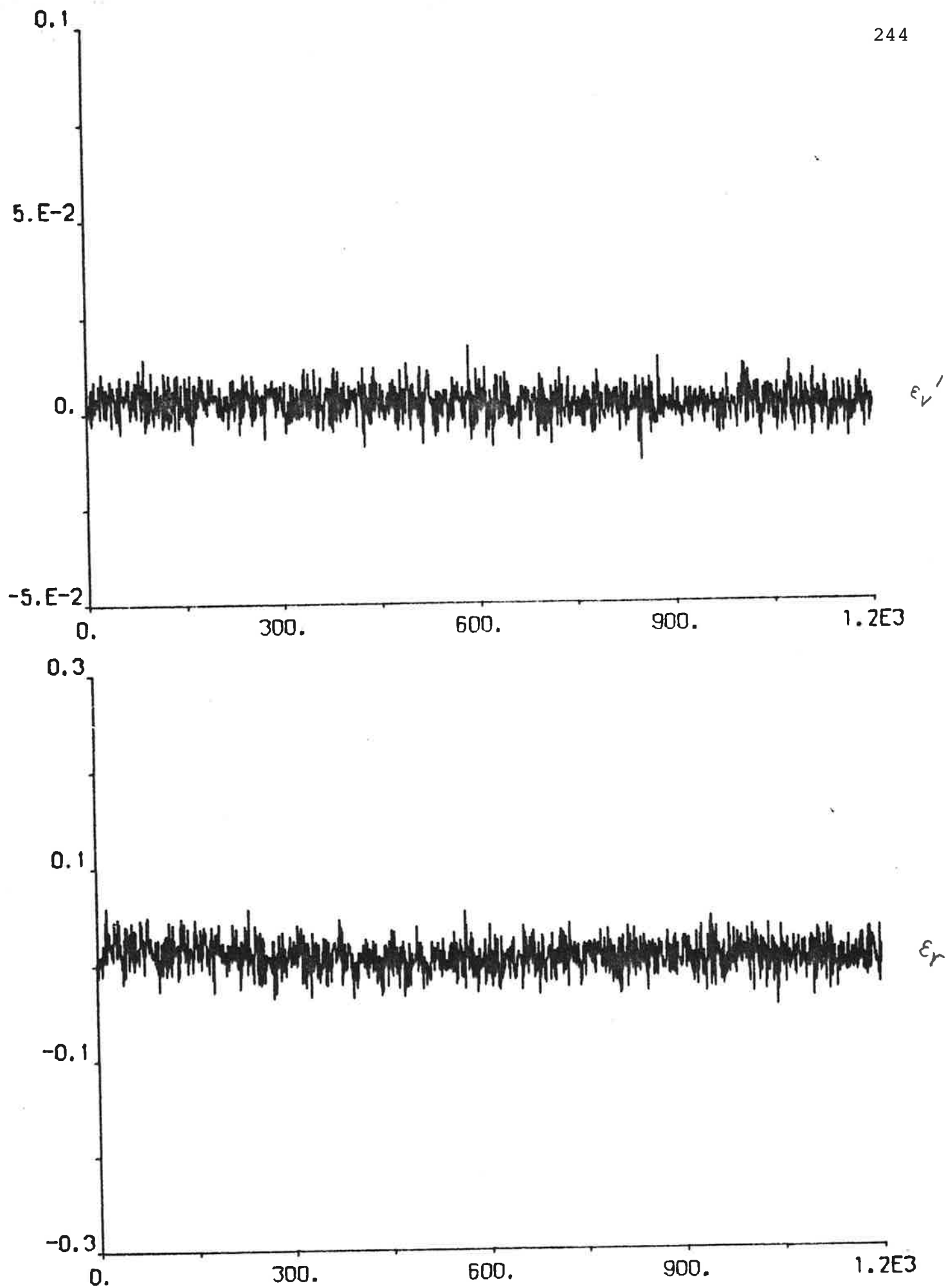


Fig. 4.57 f

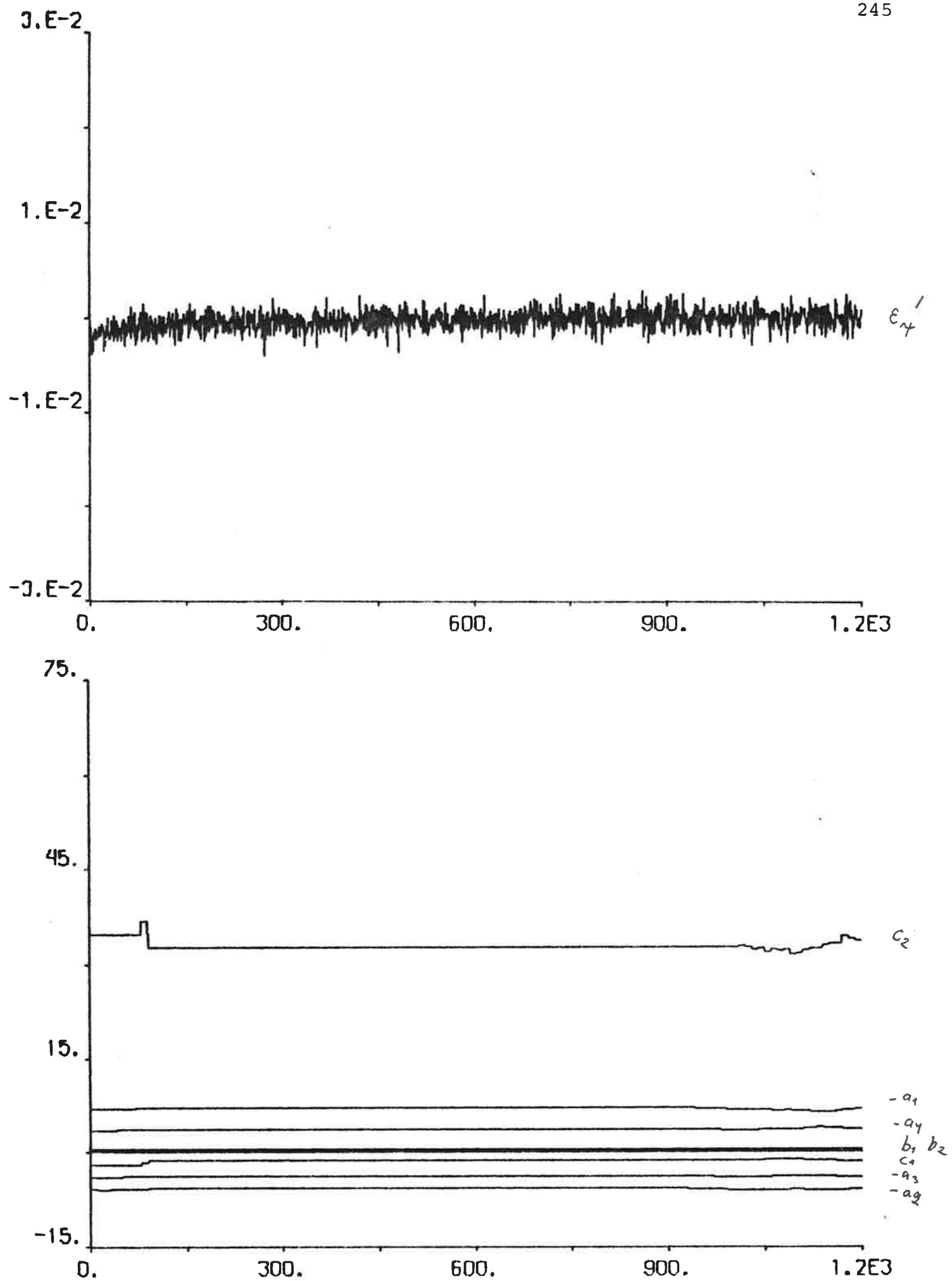


Fig. 4.57 g

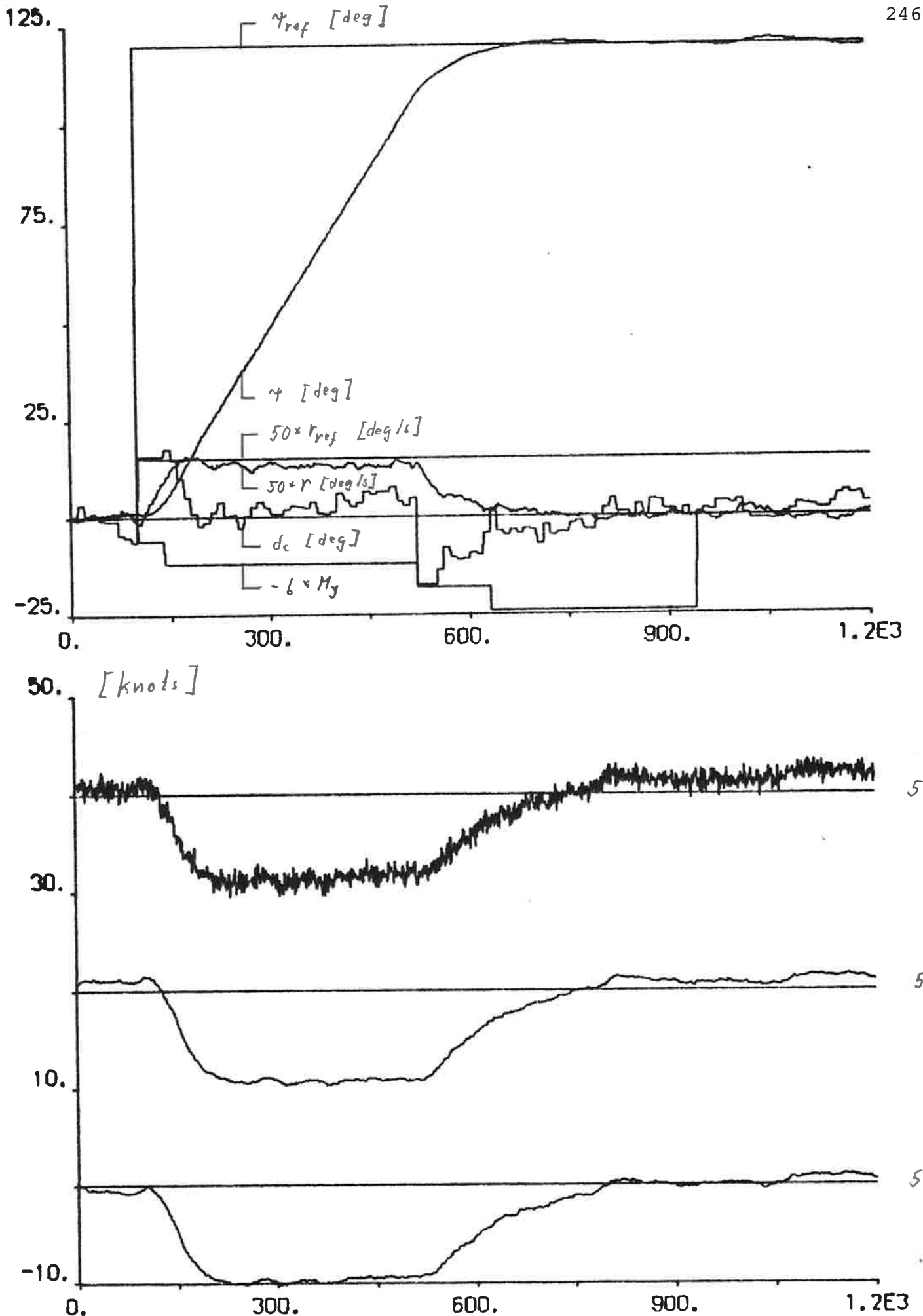


Fig. 4.58 a - $T = 22.3$ m, $n_0 = 87.6$ rpm, $u_0 = 15.8$ knots, $\Delta\psi_{ref} = 120$ deg, $\dot{\gamma}_{ref} = 0.3$ deg/s, self-tuning regulator and yaw regulator using estimates from the Kalman filter ($\bar{c}_2 = 50$ s).

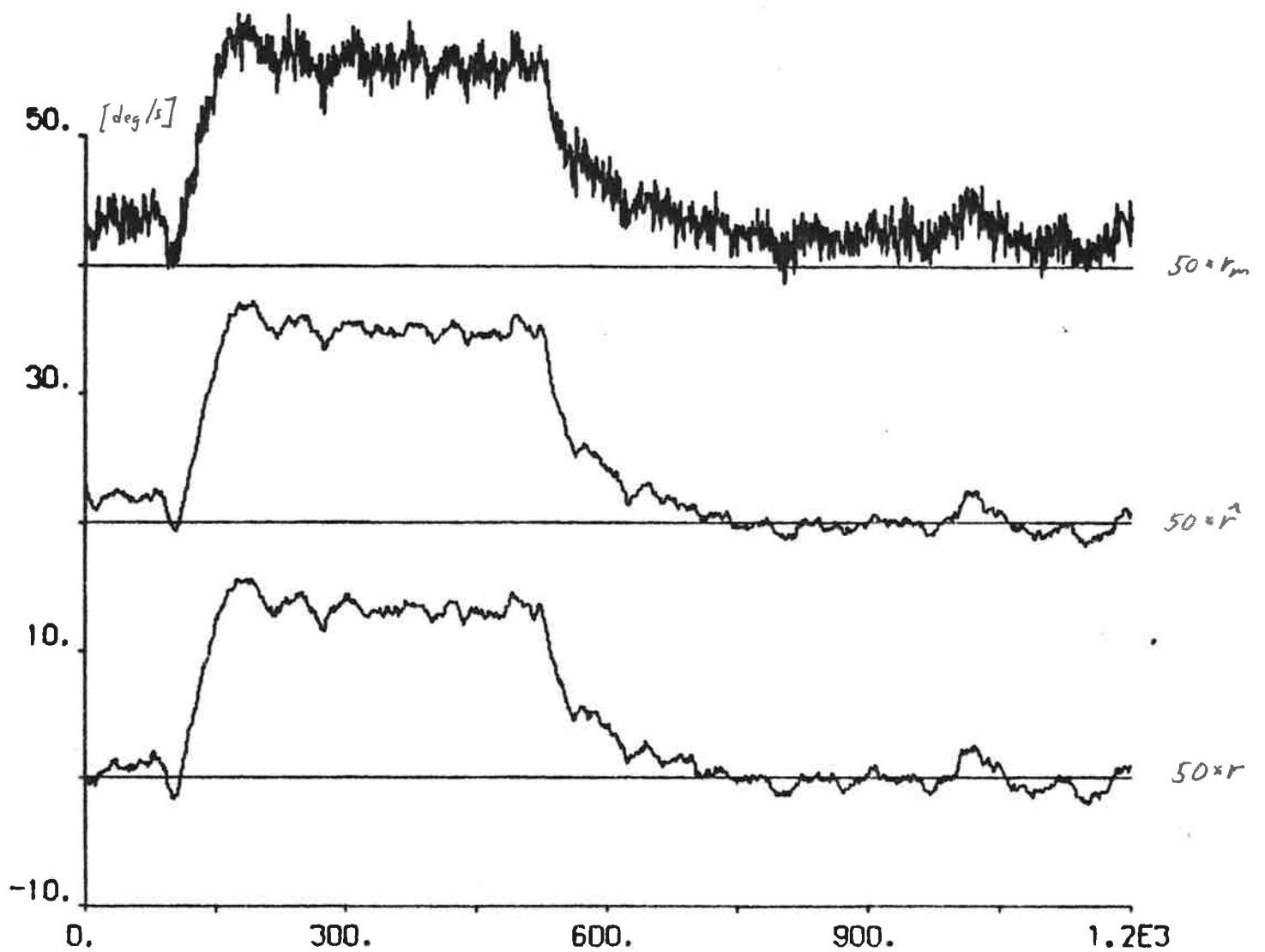
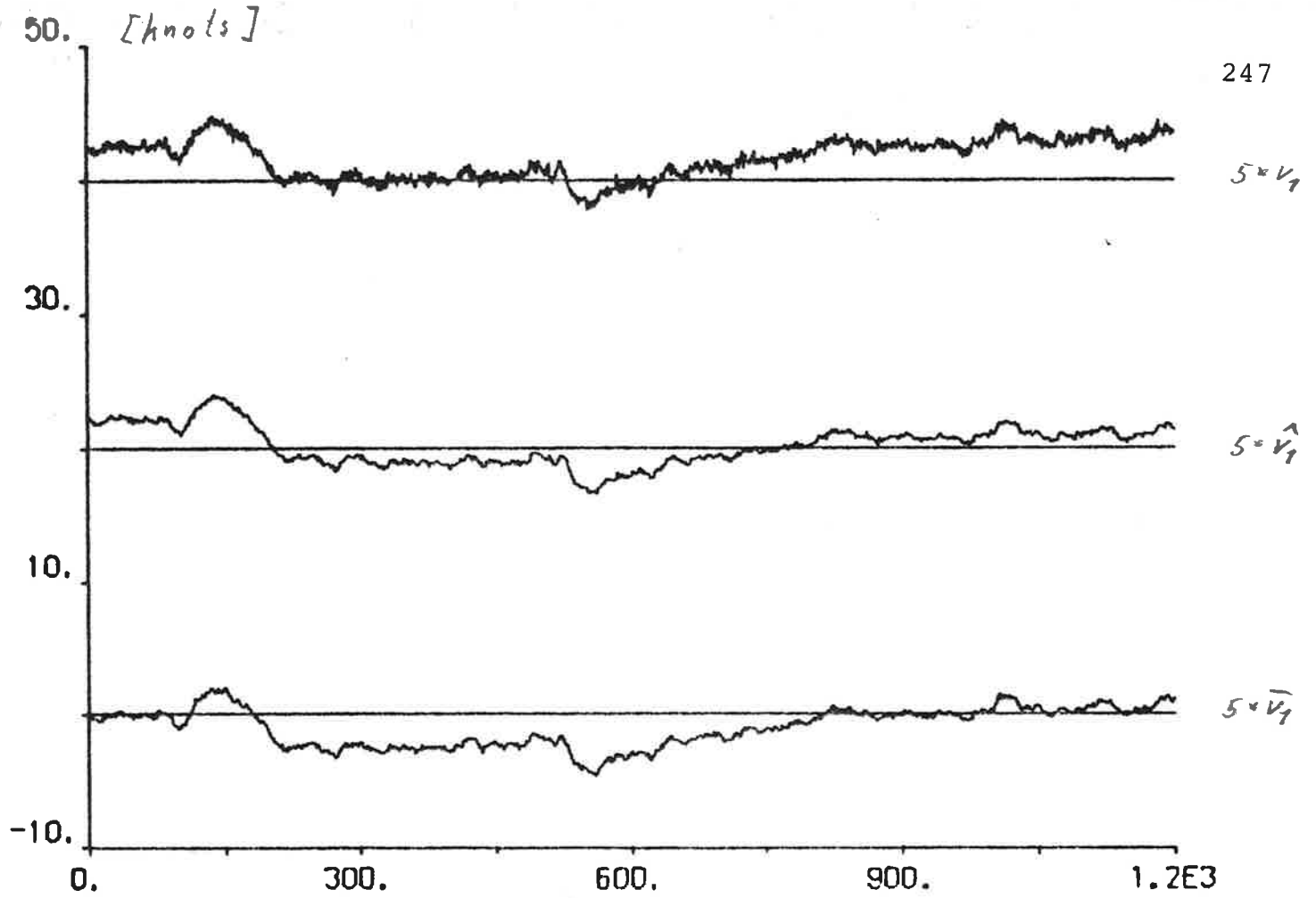


Fig. 4.58 b

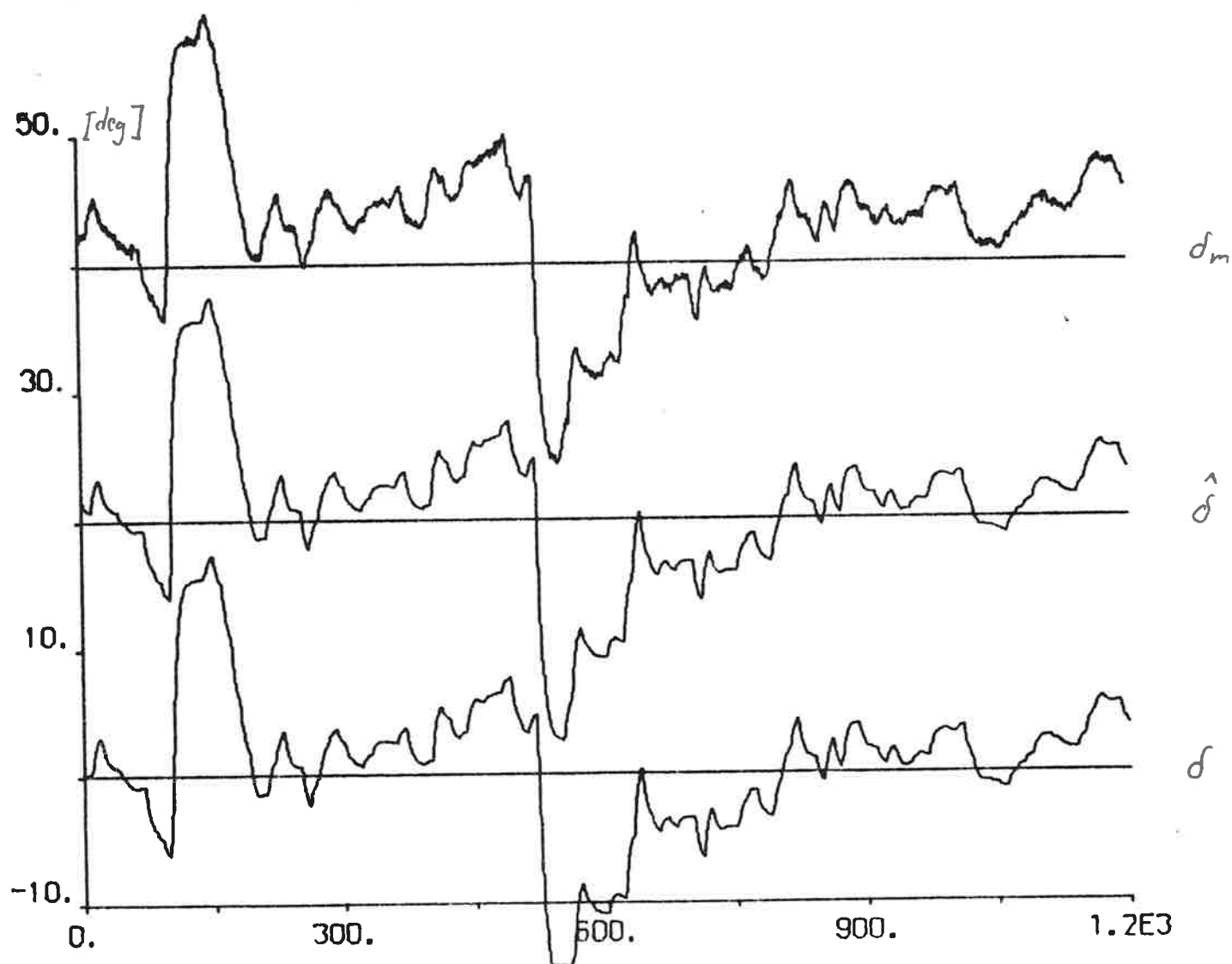
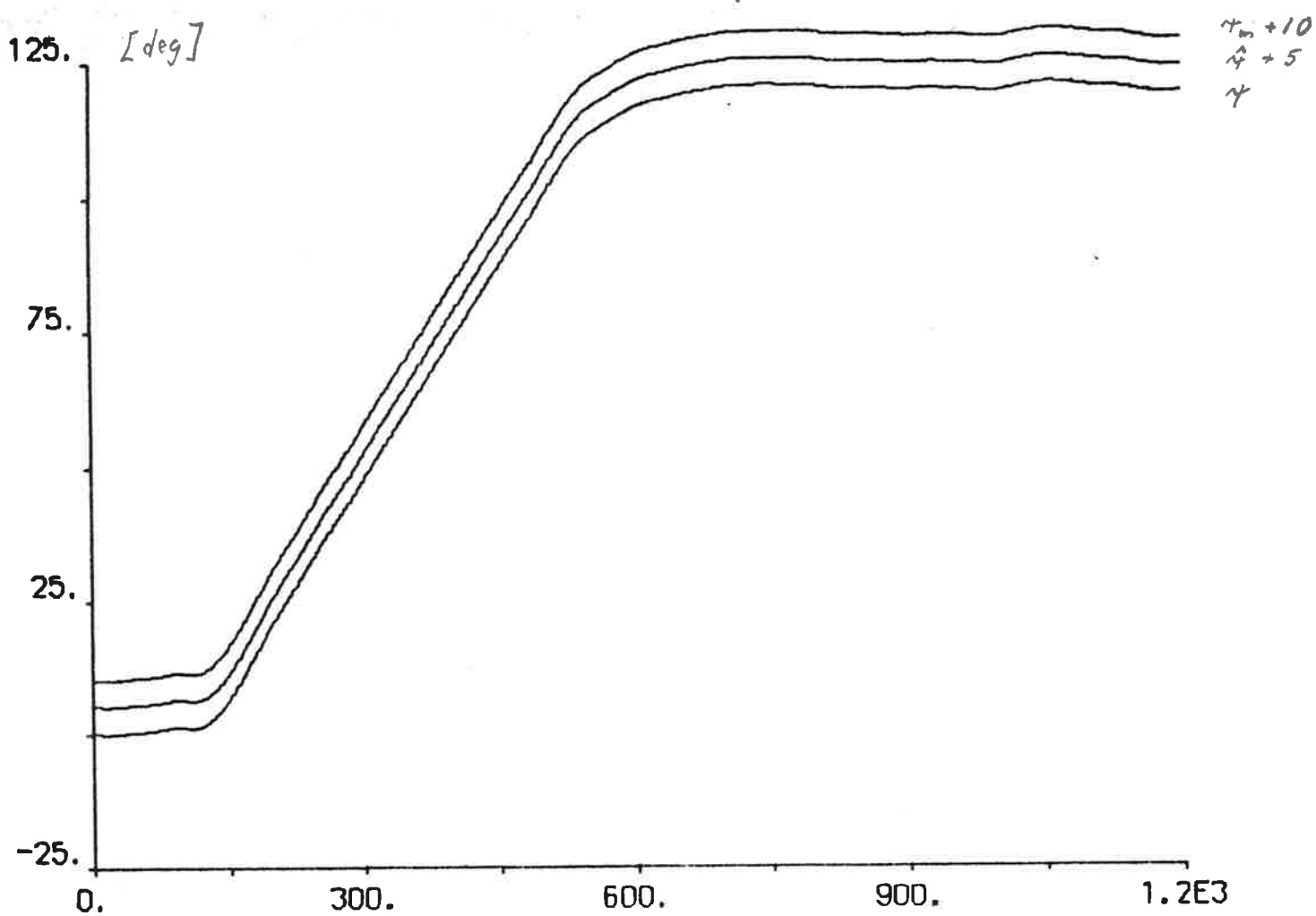


Fig. 4.58 c

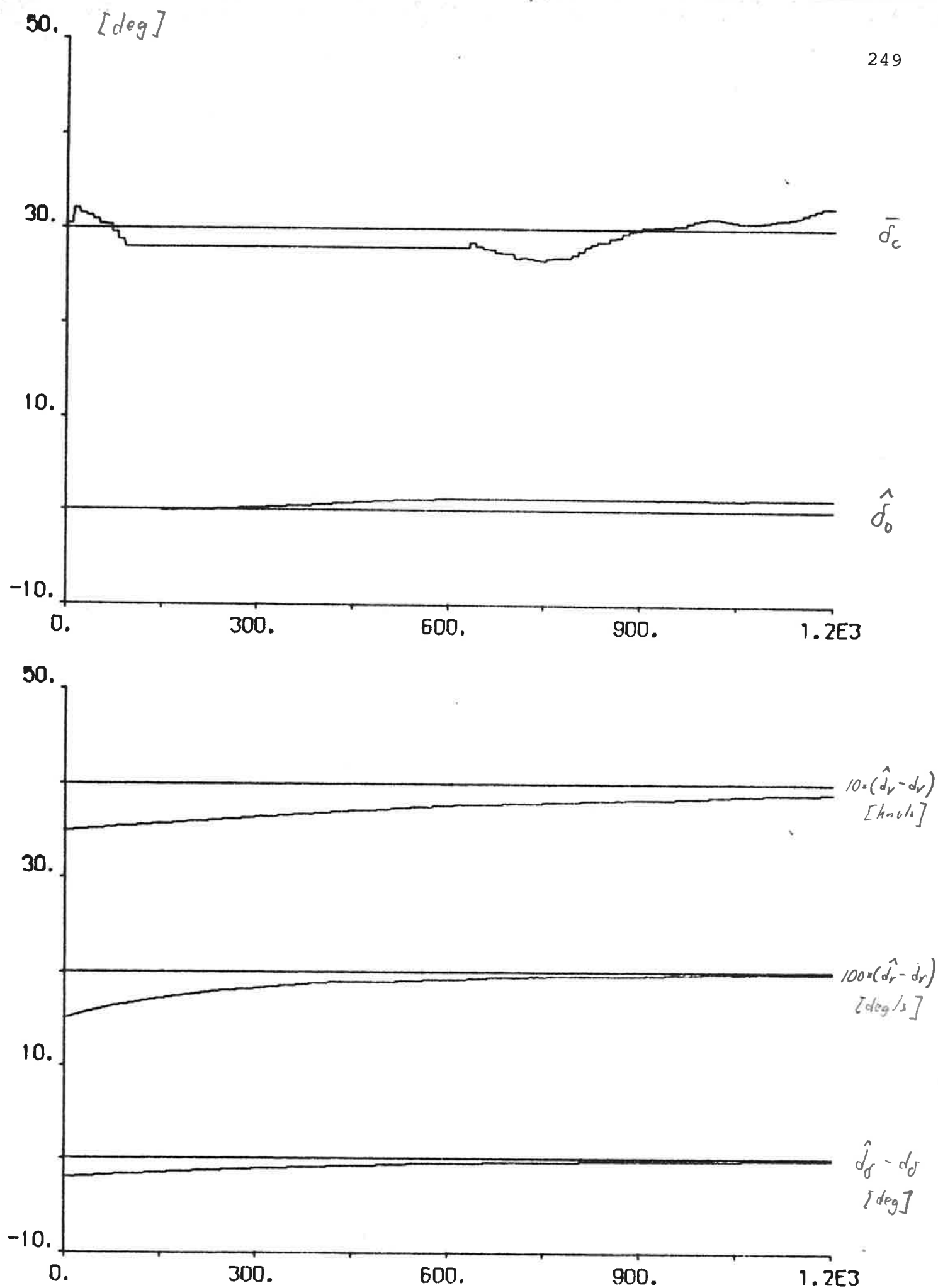


Fig. 4.58 d

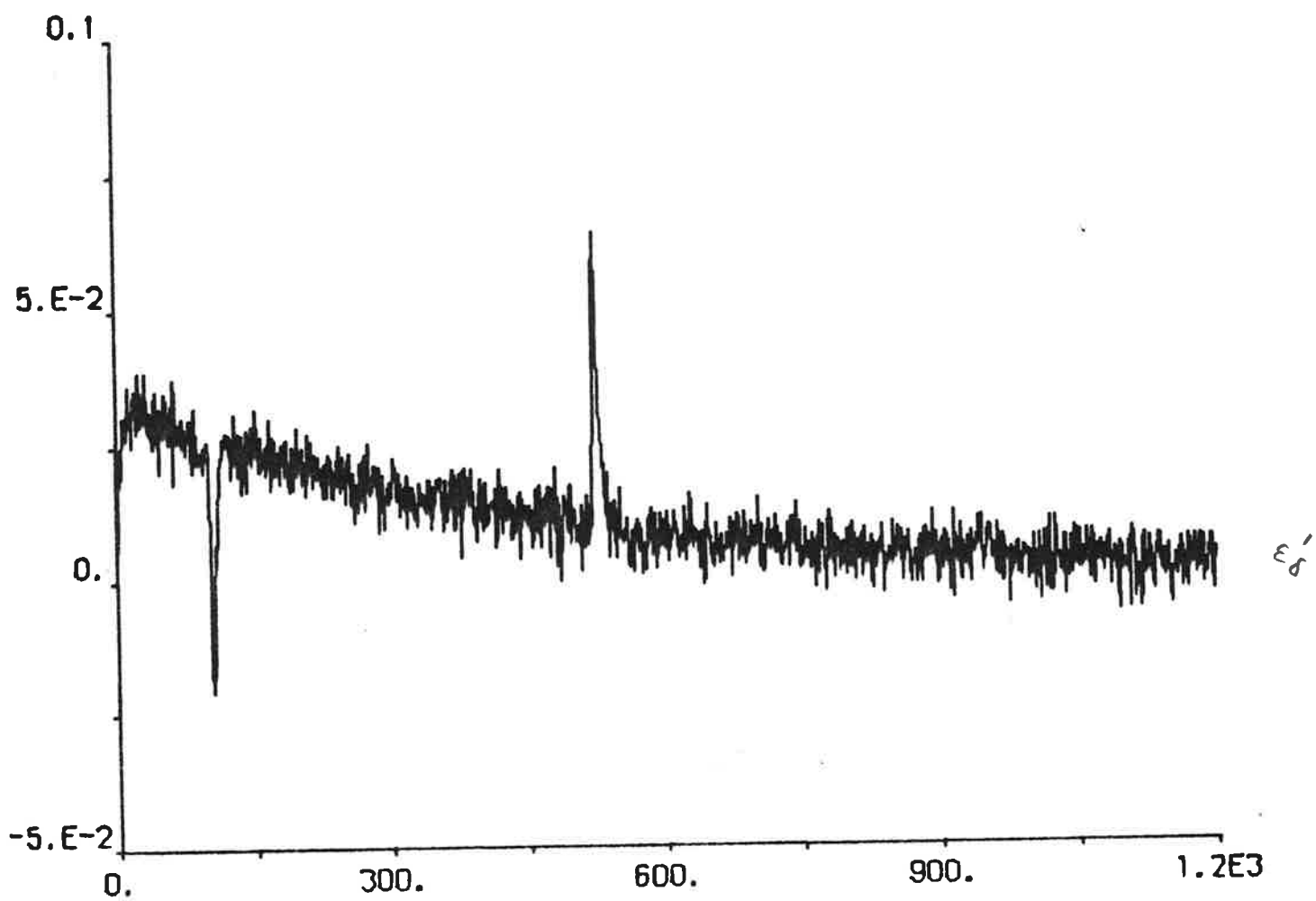
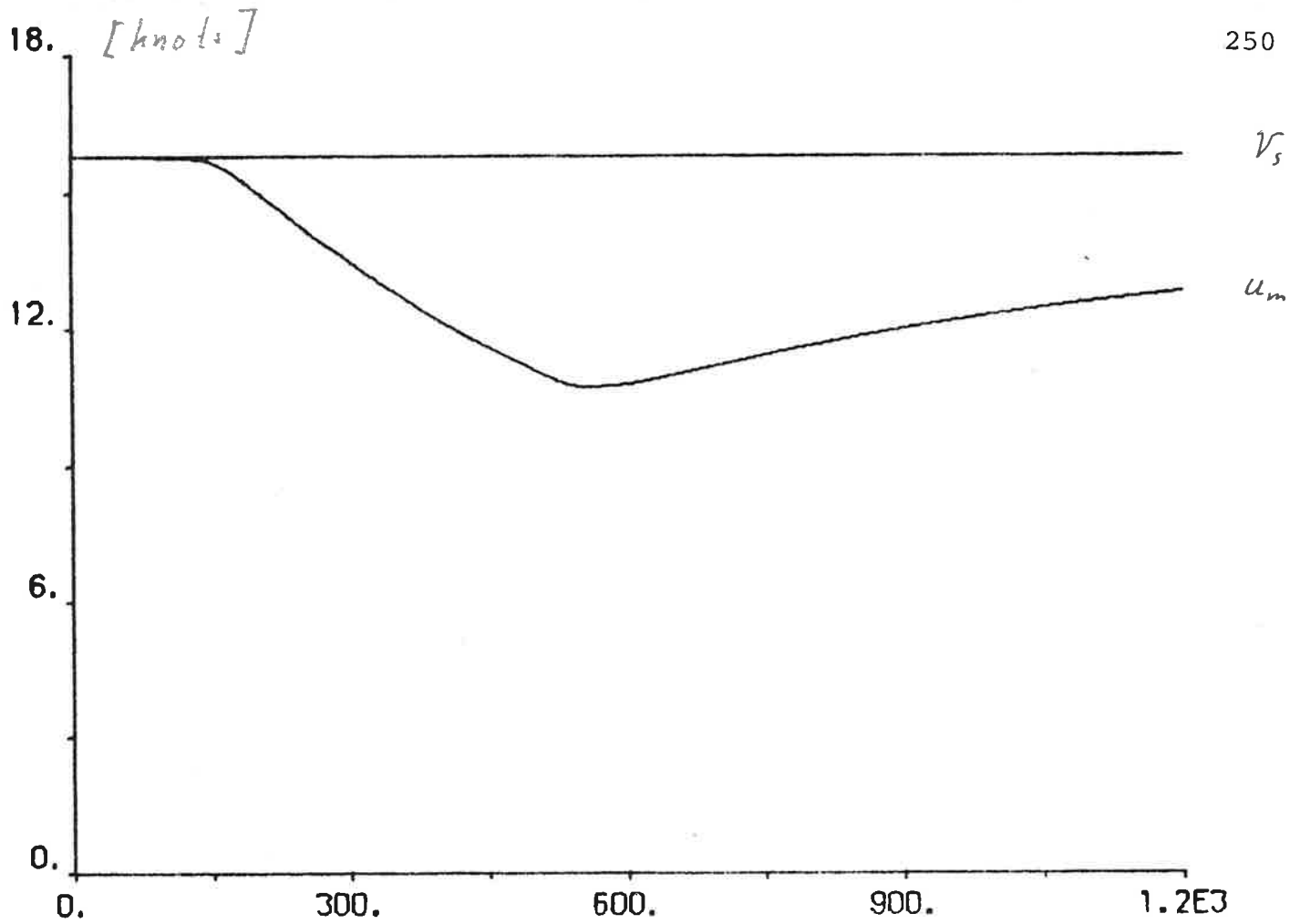


Fig. 4.58 e

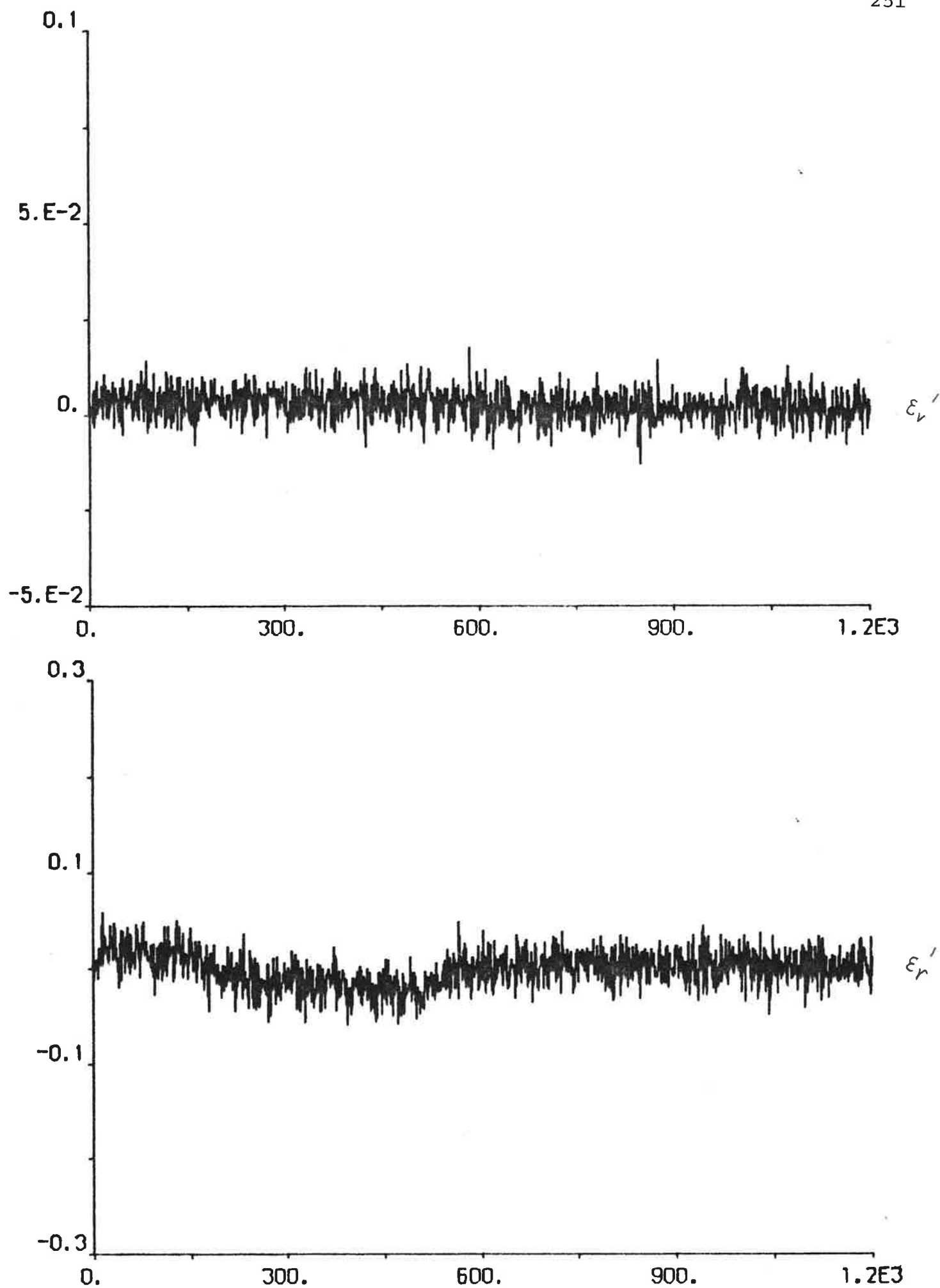


Fig. 4.58 f

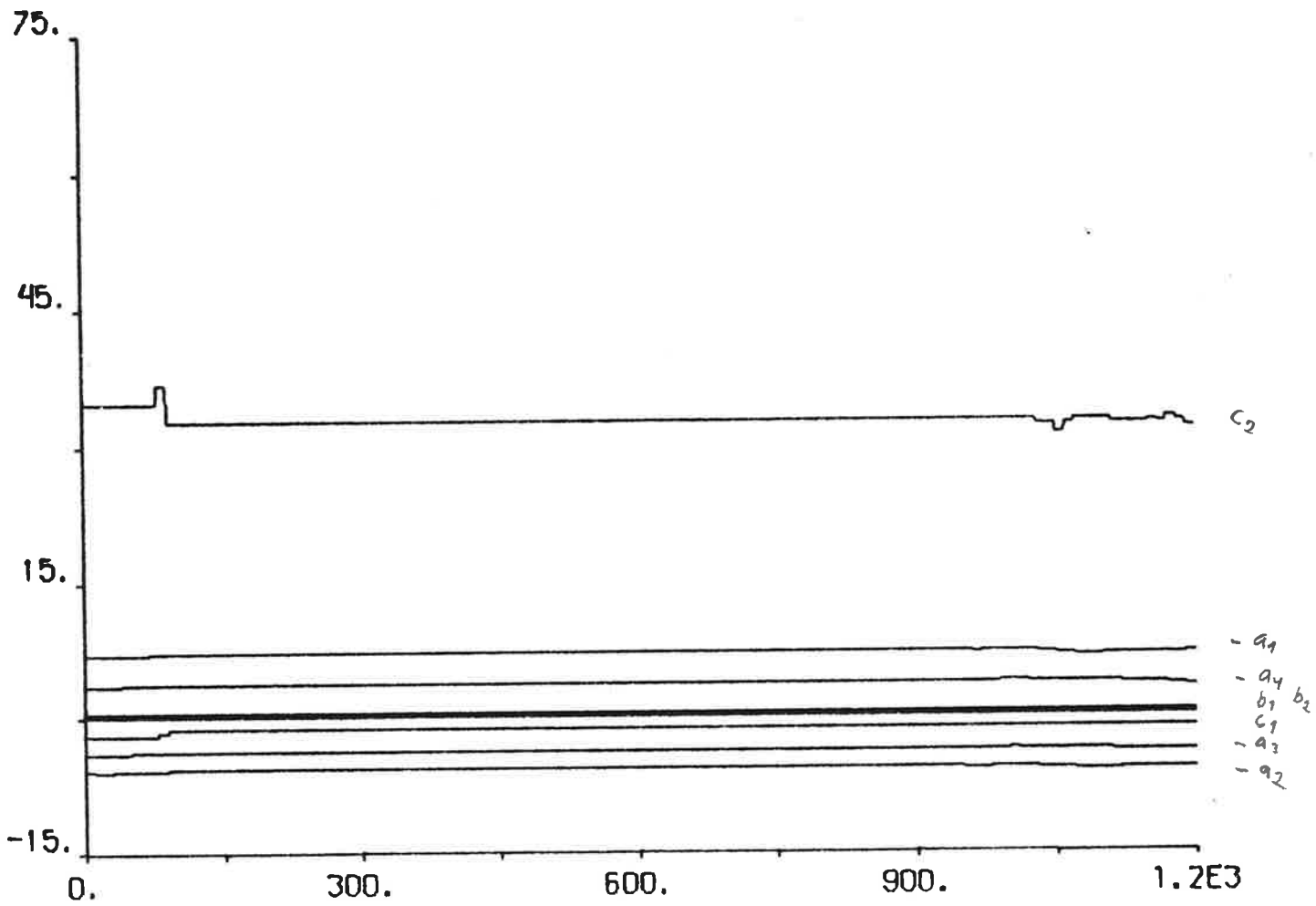
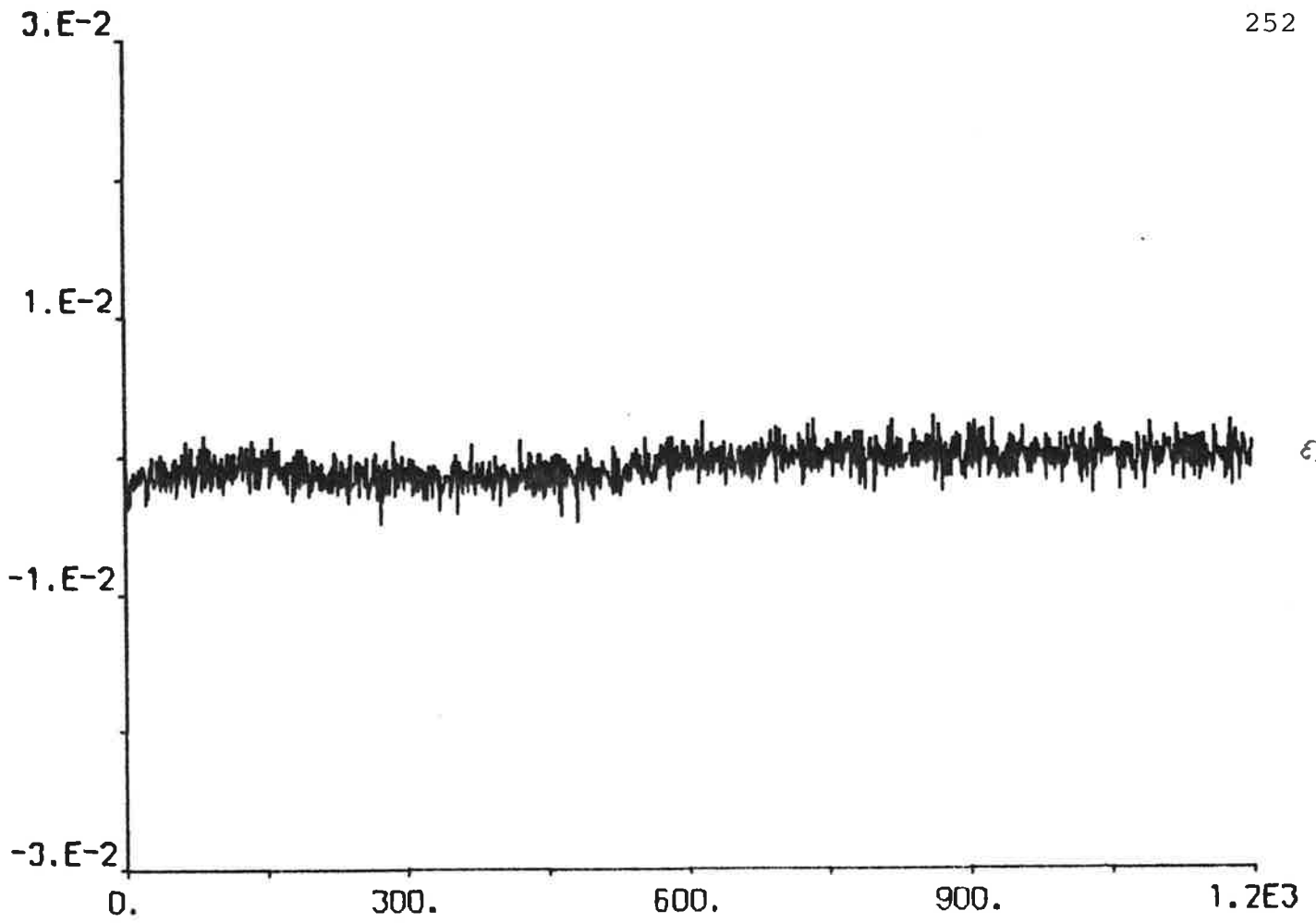


Fig. 4.58 g

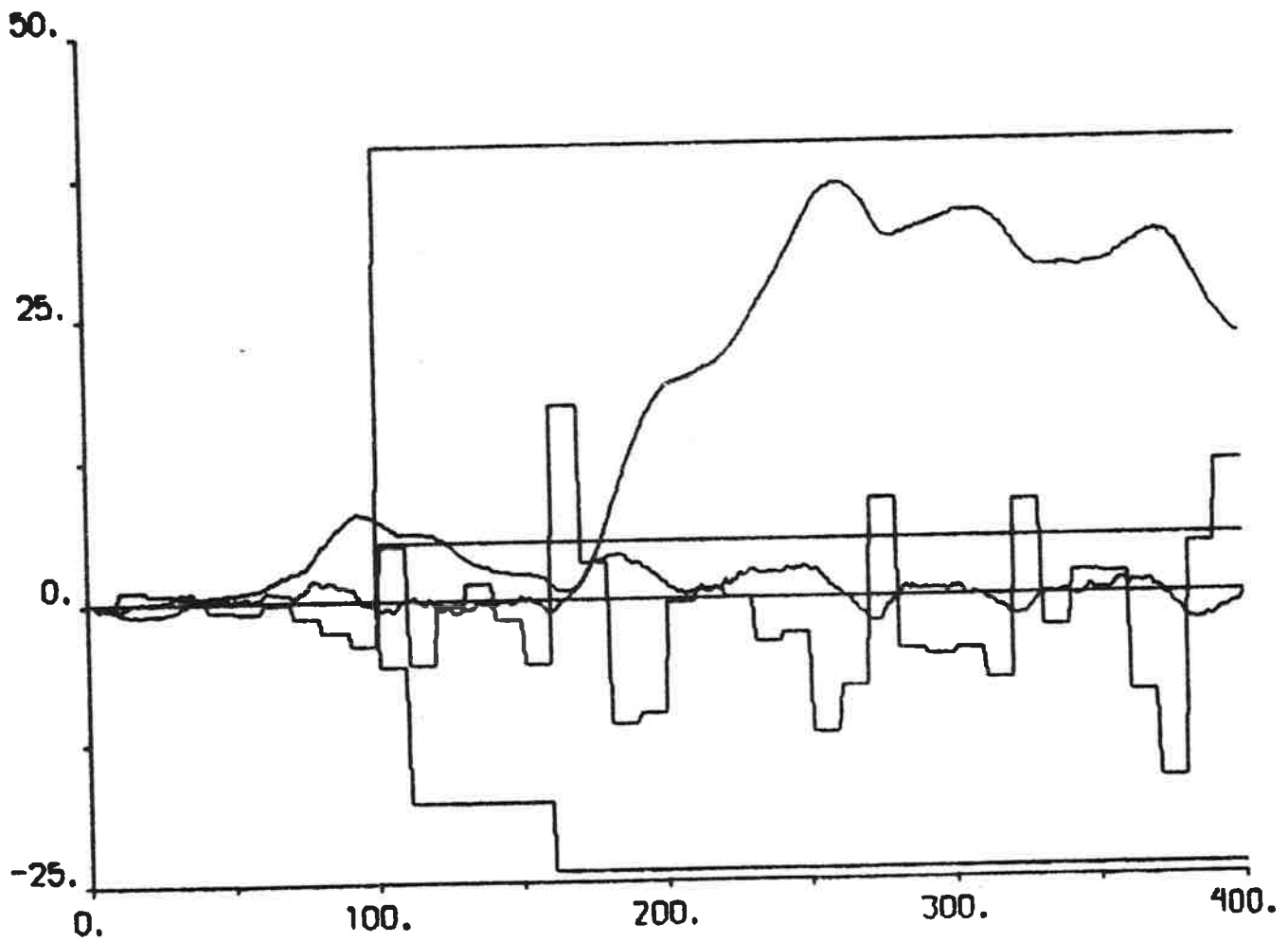


Fig. 4.59 - $T = 22.3$ m, $n_0 = 87.6$ rpm, $u_0 = 15.8$ knots,
 $\Delta\psi_{\text{ref}} = 4$ deg, $r_{\text{ref}} = 0.1$ deg/s, self-tuning
 regulator and yaw regulator using non-filtered
 measurements ($\bar{c}_2 = 50$ s).

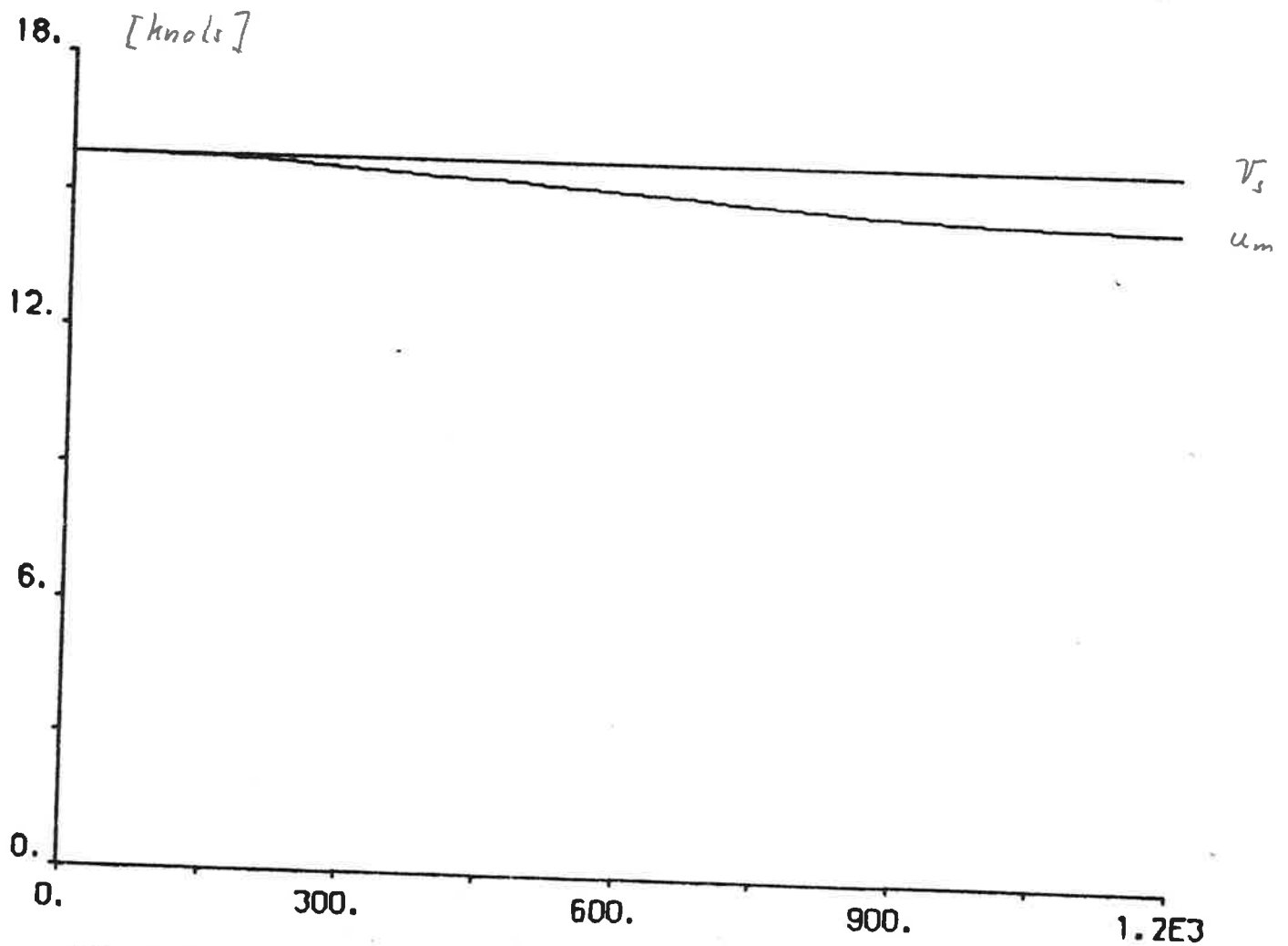
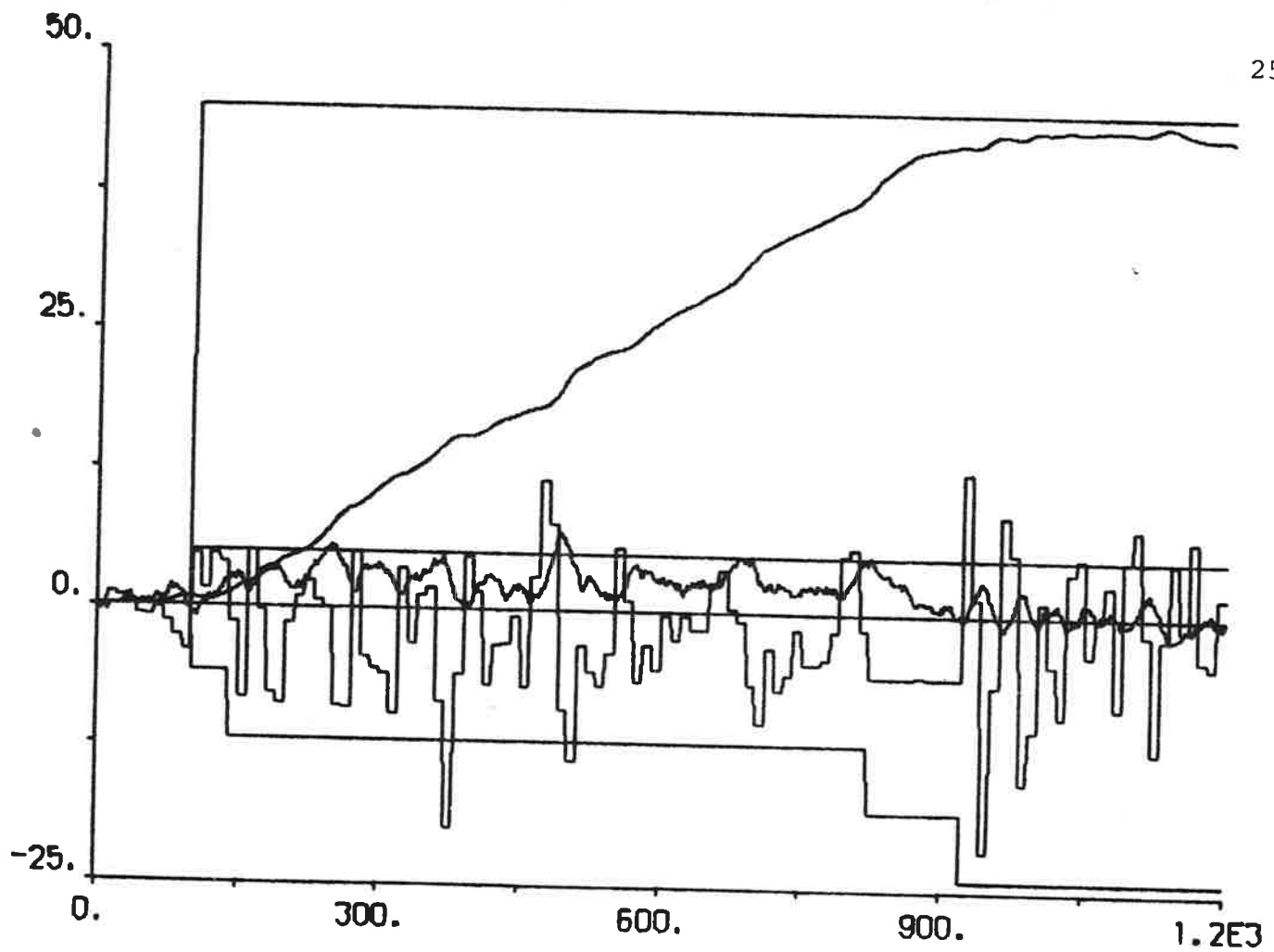


Fig. 4.60 - $T = 22.3$ m, $n_0 = 87.6$ rpm, $u_0 = 15.8$ knots, $\Delta\psi_{\text{ref}} = 45$ deg, $r_{\text{ref}} = 0.1$ deg/s, self-tuning regulator and yaw regulator using non-filtered measurements ($\tau_2 = 50$ s).

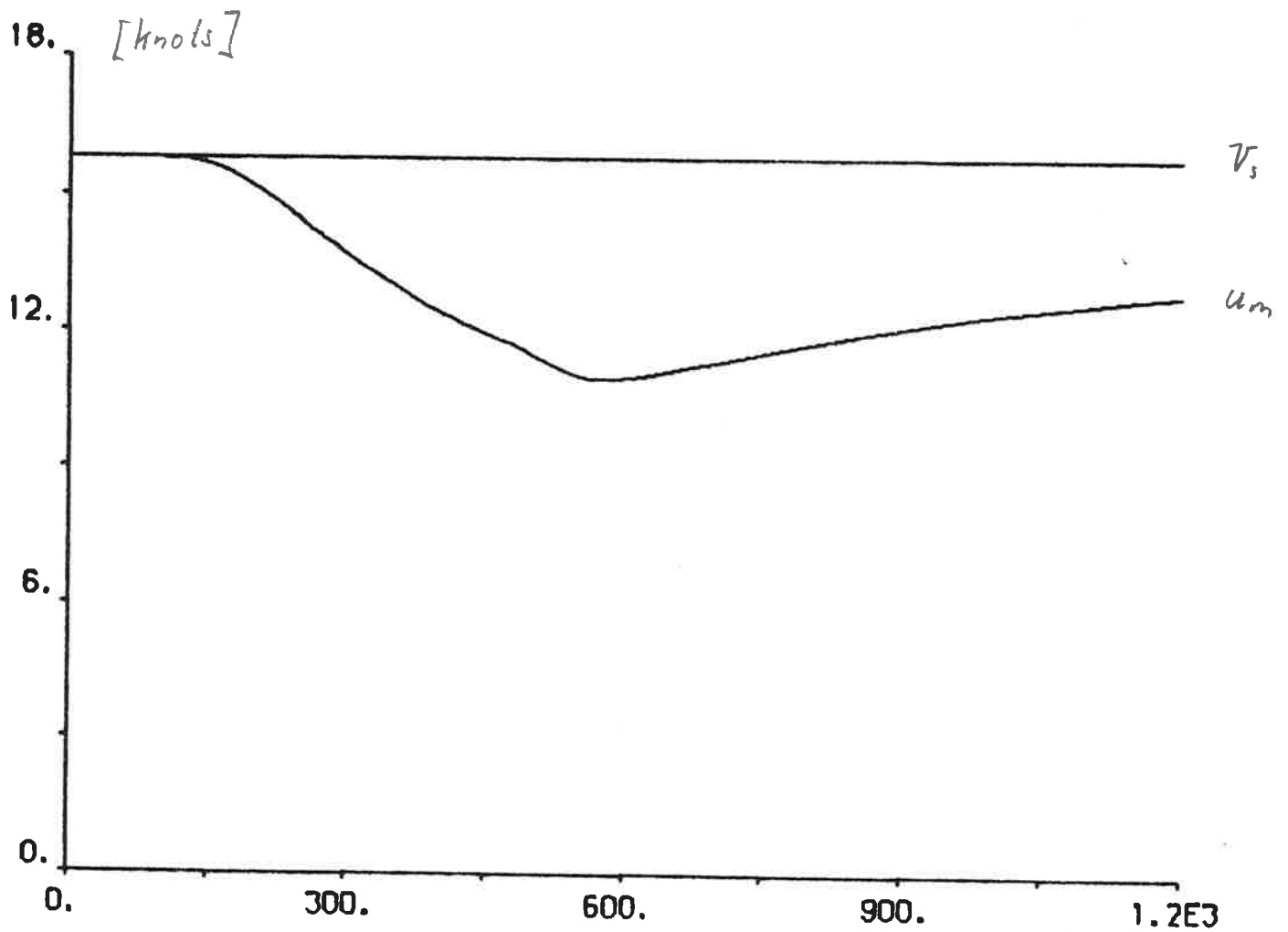
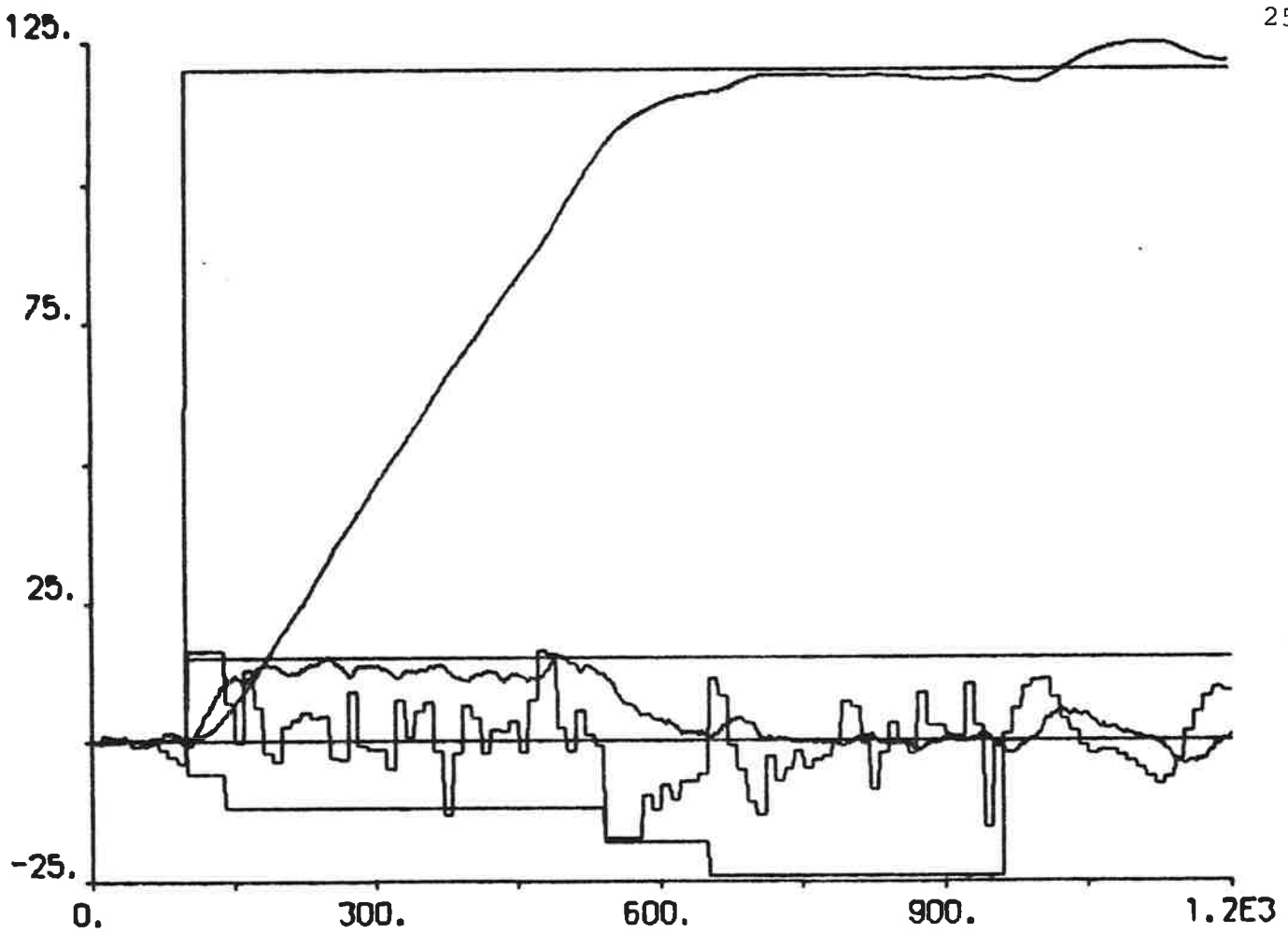


Fig. 4.61 - $T = 22.3$ m, $n_0 = 87.6$ rpm, $u_0 = 15.8$ knots, $\Delta\psi_{\text{ref}} = 120$ deg, $r_{\text{ref}} = 0.3$ deg/s, self-tuning regulator and yaw regulator using non-filtered measurements ($\bar{\tau}_2 = 50$ s).

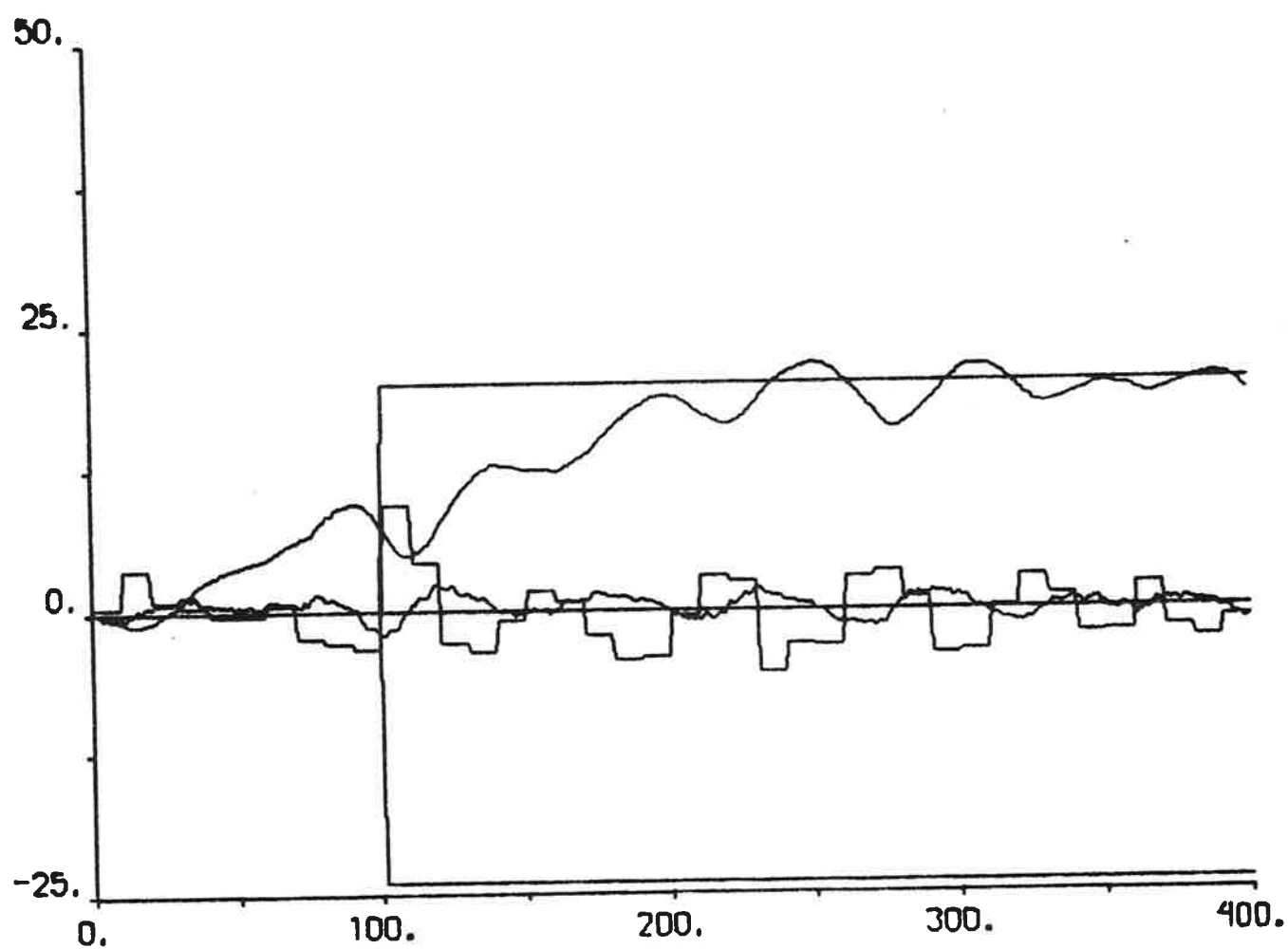


Fig. 4.62 - $T = 10.5$ m, $n_0 = 87.6$ rpm, $u_0 = 15.8$ knots, $\Delta\psi_{\text{ref}} = 2$ deg, $r_{\text{ref}} = 0$ deg/s, self-tuning regulator and yaw regulator using estimates from the Kalman filter ($\bar{c}_2 = 50$ s).

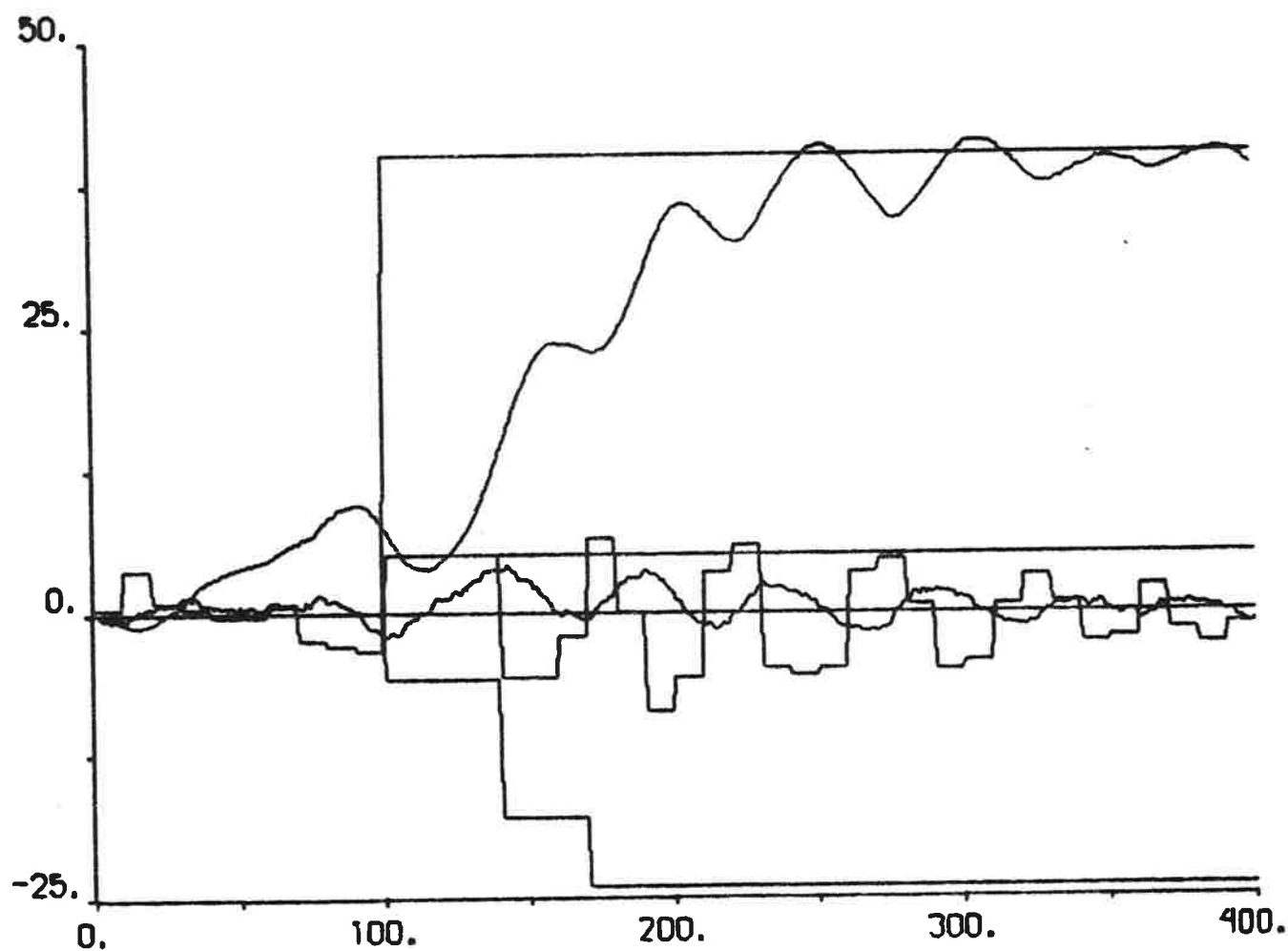


Fig. 4.63 - $T = 10.5$ m, $n_0 = 87.6$ rpm, $u_0 = 15.8$ knots,
 $\Delta\psi_{\text{ref}} = 4$ deg, $r_{\text{ref}} = 0.1$ deg/s, self-tuning
 regulator and yaw regulator using estimates from
 the Kalman filter ($\bar{c}_2 = 50$ s).

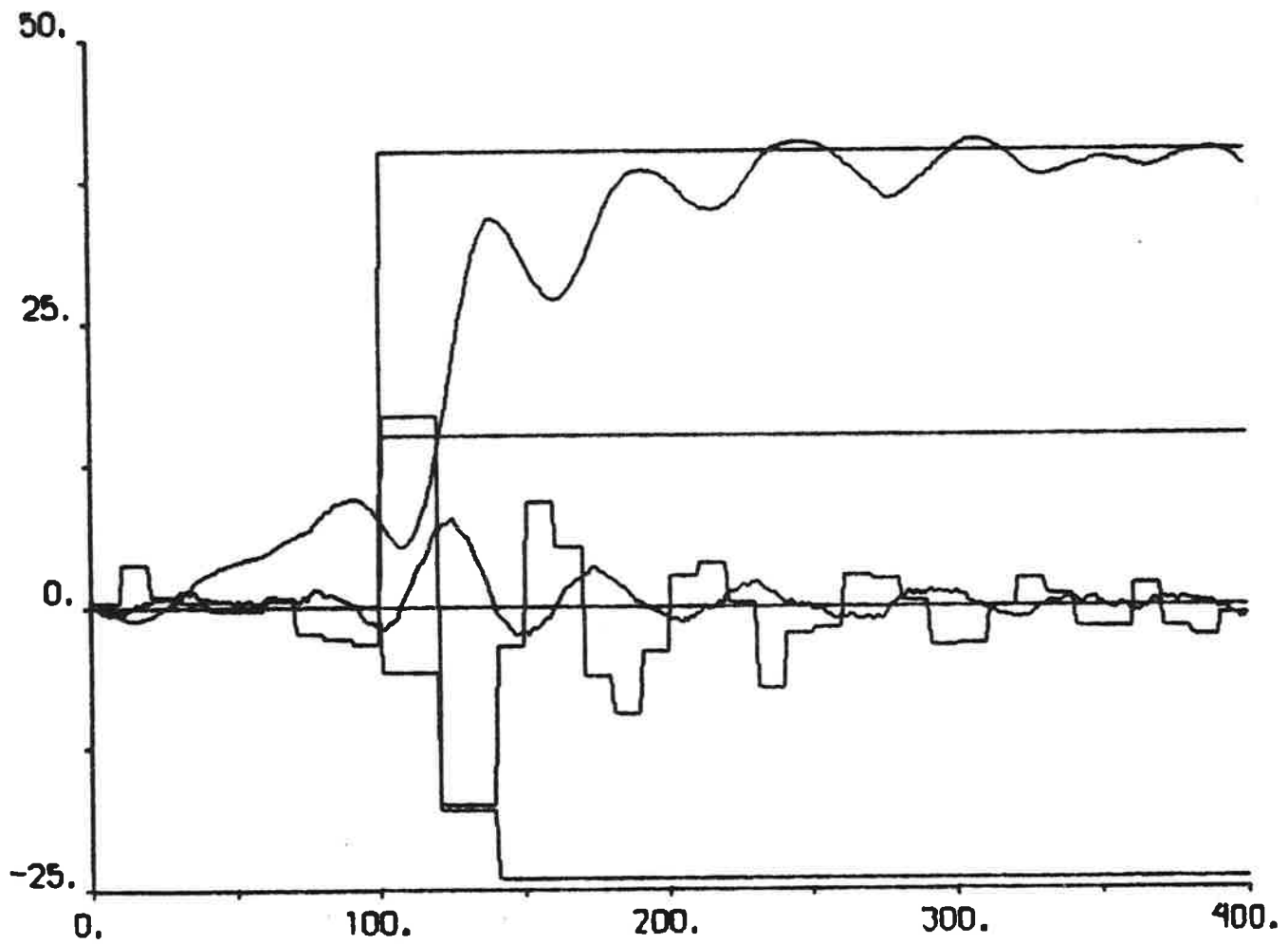


Fig. 4.64 - $T = 10.5$ m, $n_0 = 87.6$ rpm, $u_0 = 15.8$ knots,
 $\Delta\psi_{\text{ref}} = 4$ deg, $r_{\text{ref}} = 0.3$ deg/s, self-tuning
 regulator and yaw regulator using estimates from
 the Kalman filter ($\bar{c}_2 = 50$ s).

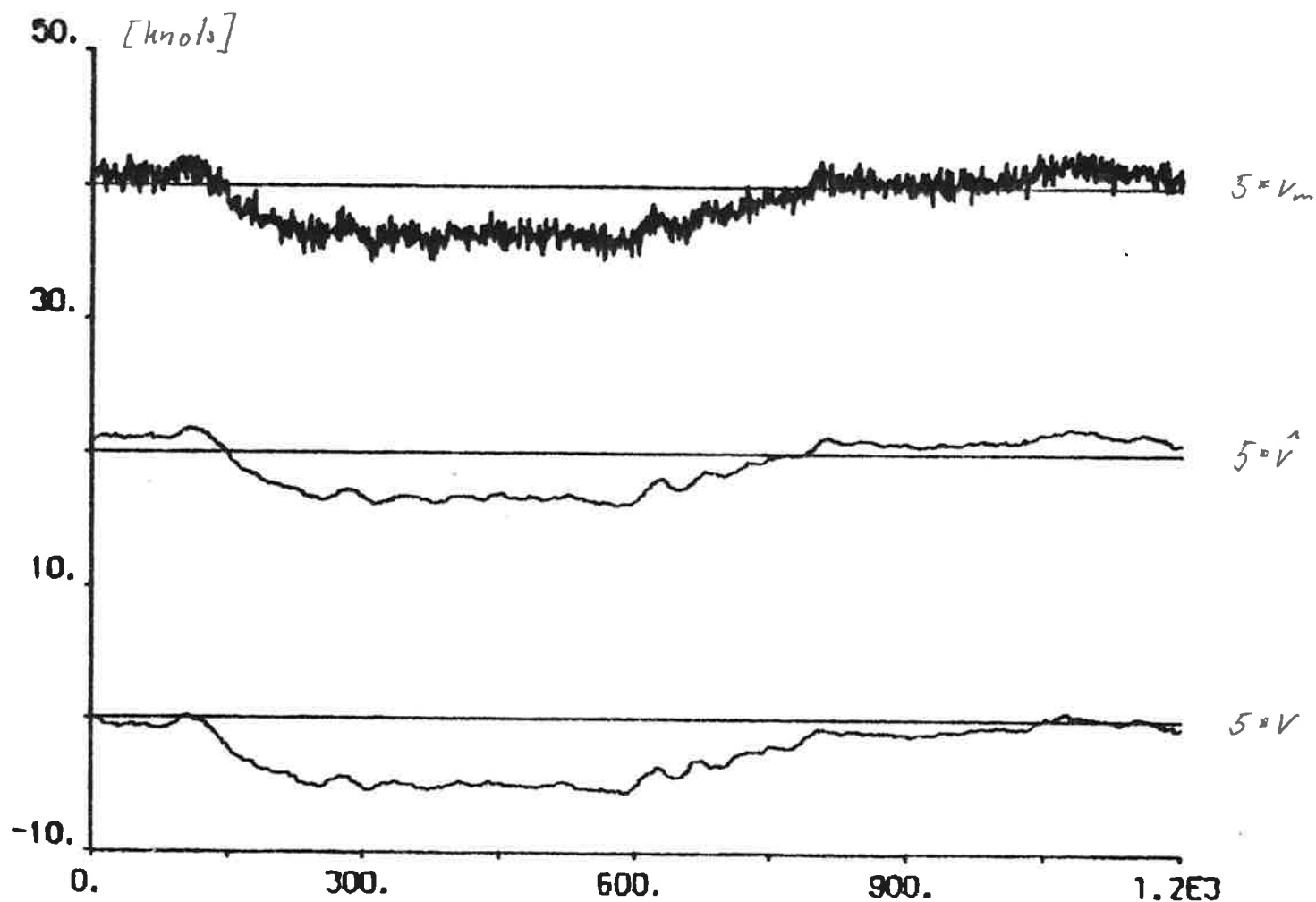
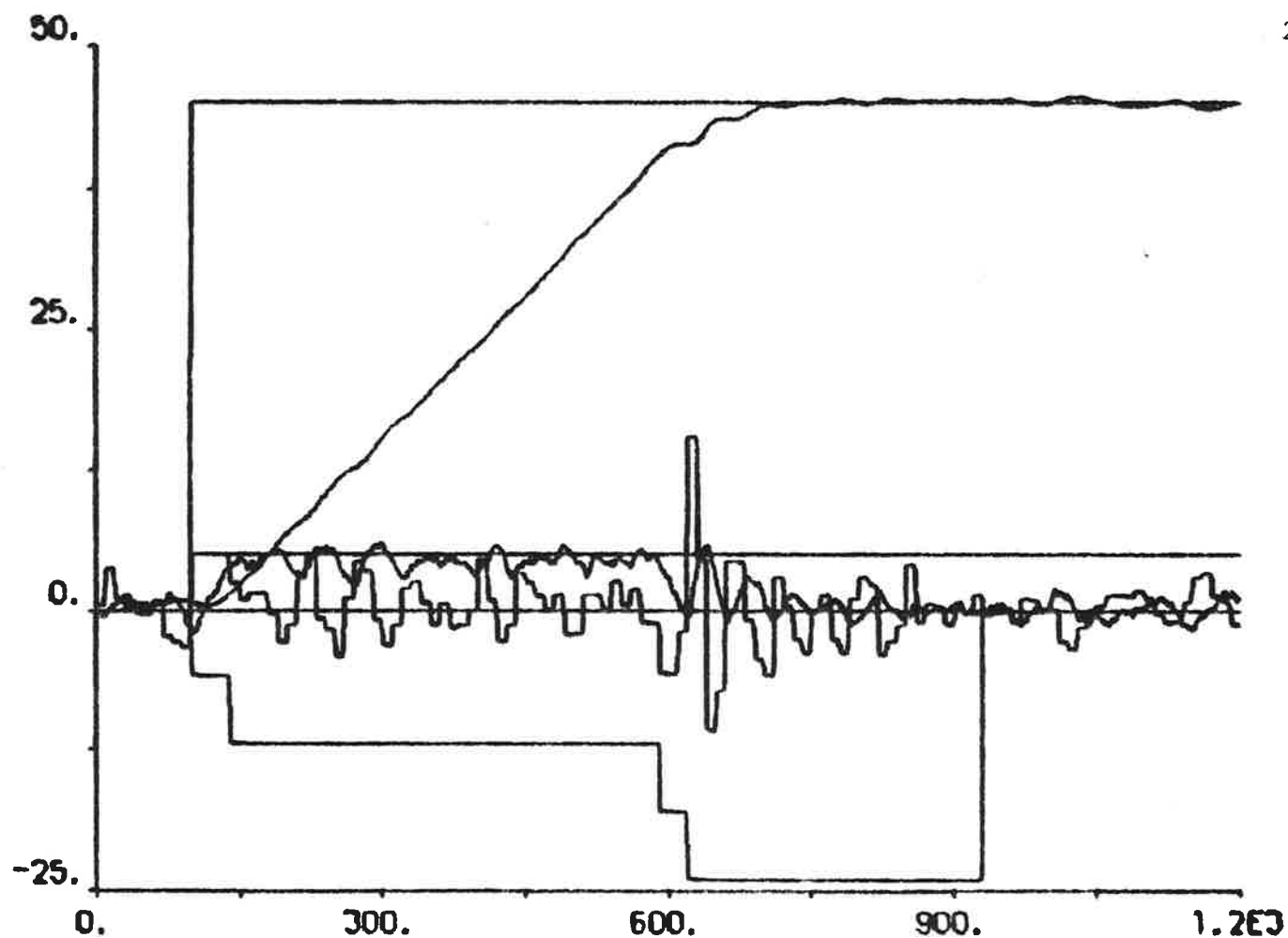


Fig. 4.65 a - $T = 10.5$ m, $n_0 = 87.6$ rpm, $u_0 = 15.8$ knots, $\Delta\psi_{\text{ref}} = 45$ deg, $r_{\text{ref}} = 0.1$ deg/s, self-tuning regulator and yaw regulator using estimates from the Kalman filter ($\bar{c}_2 = 50$ s).

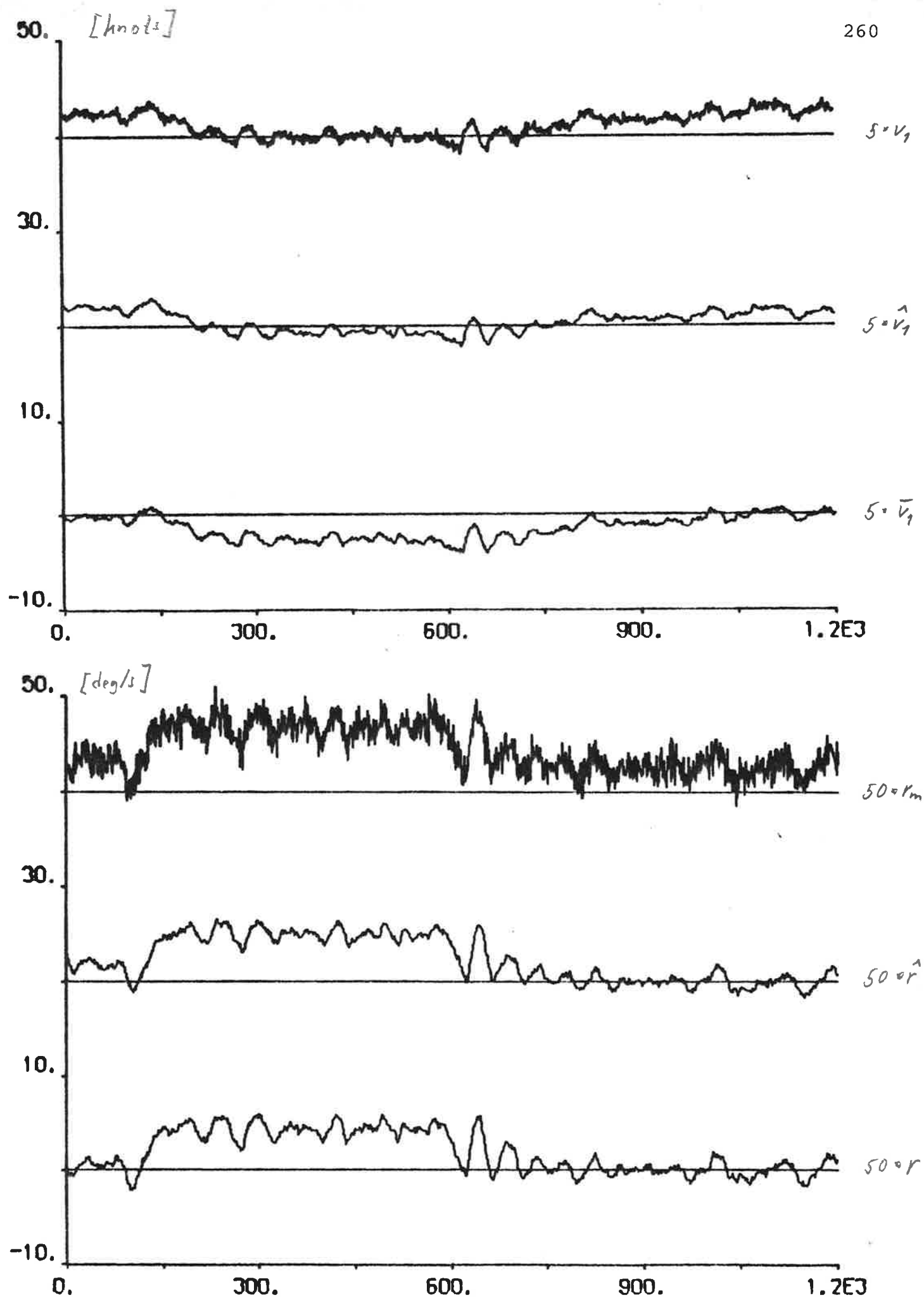


Fig. 4.65 b

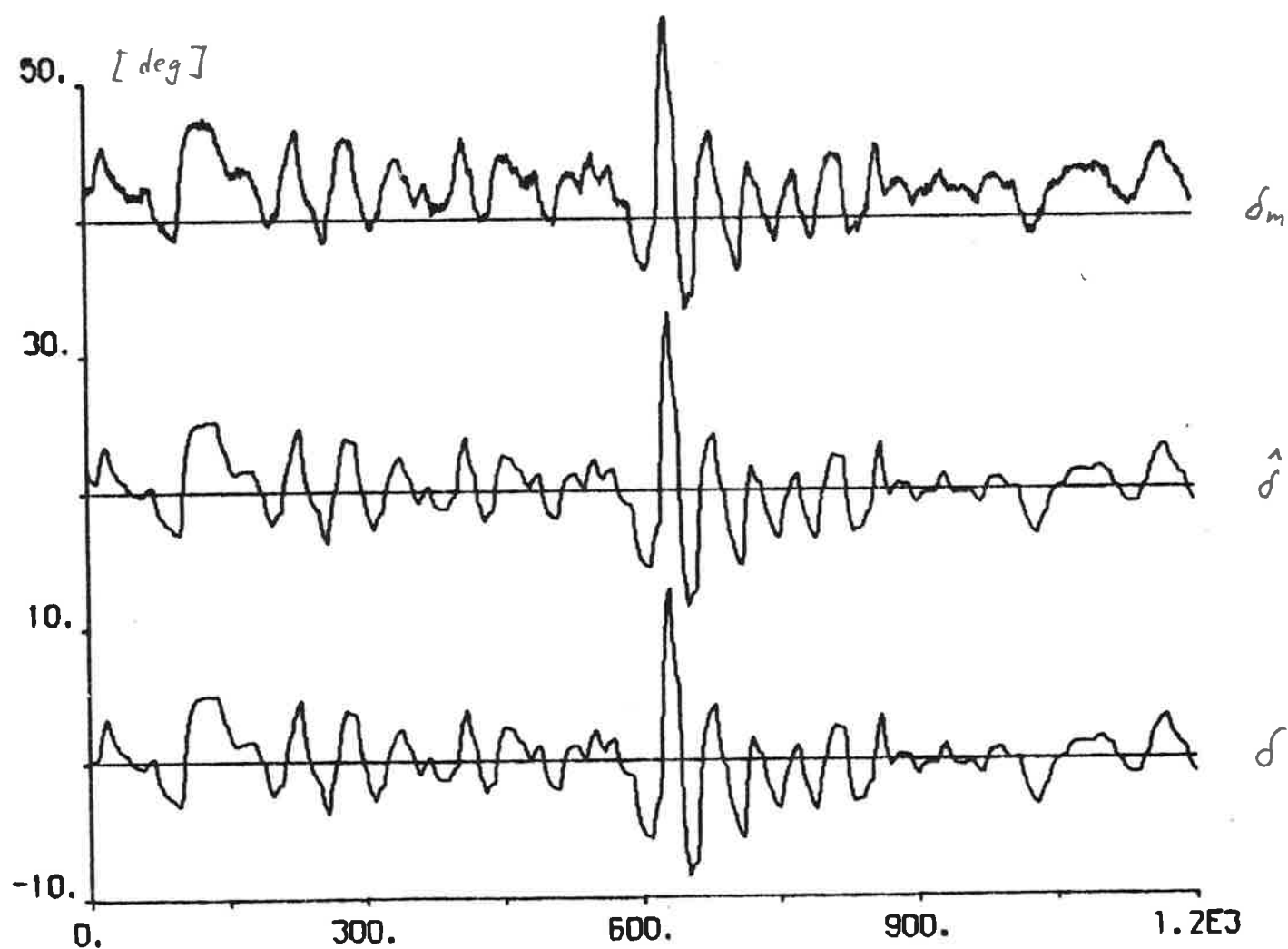
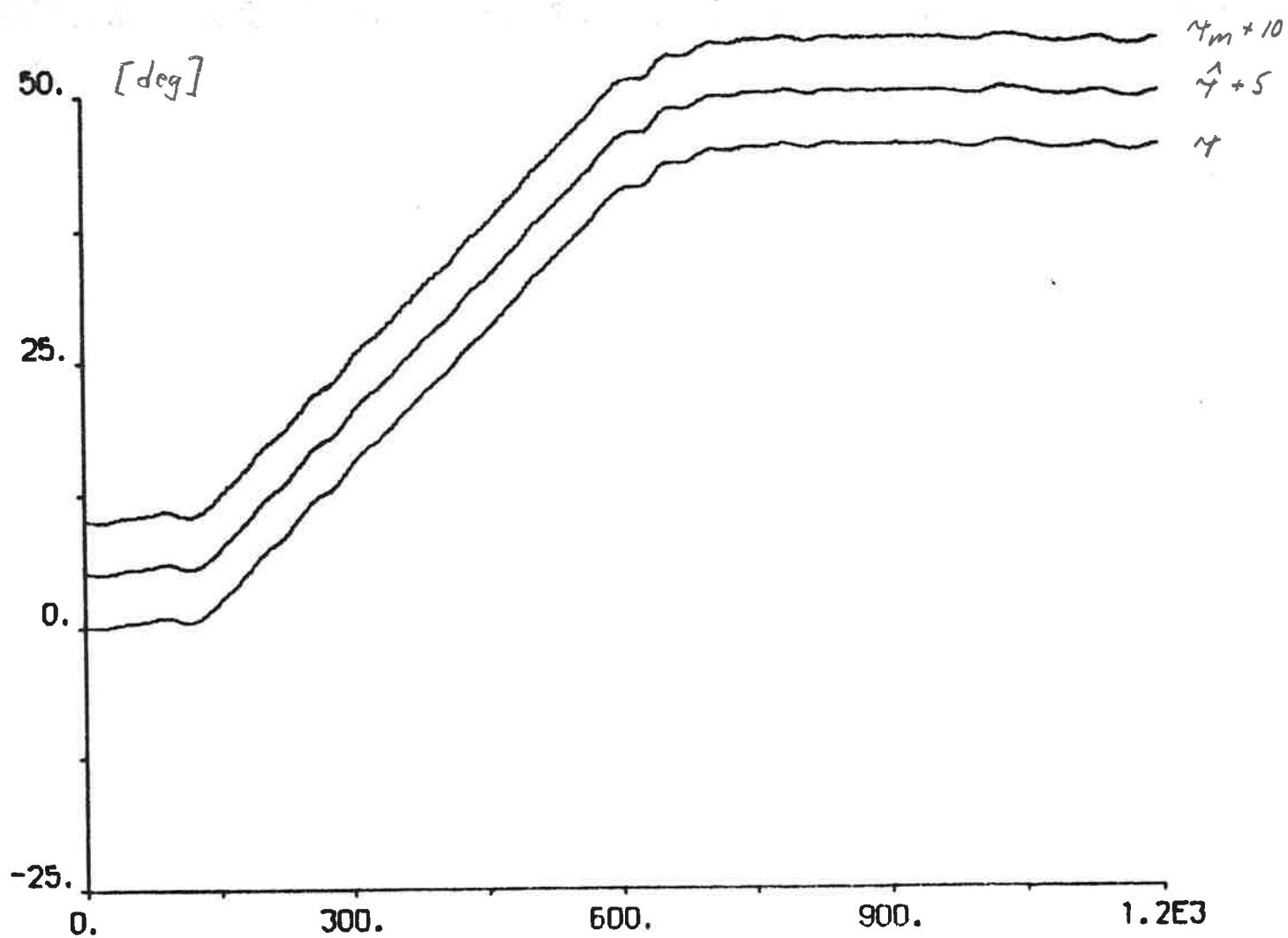


Fig. 4.65 c

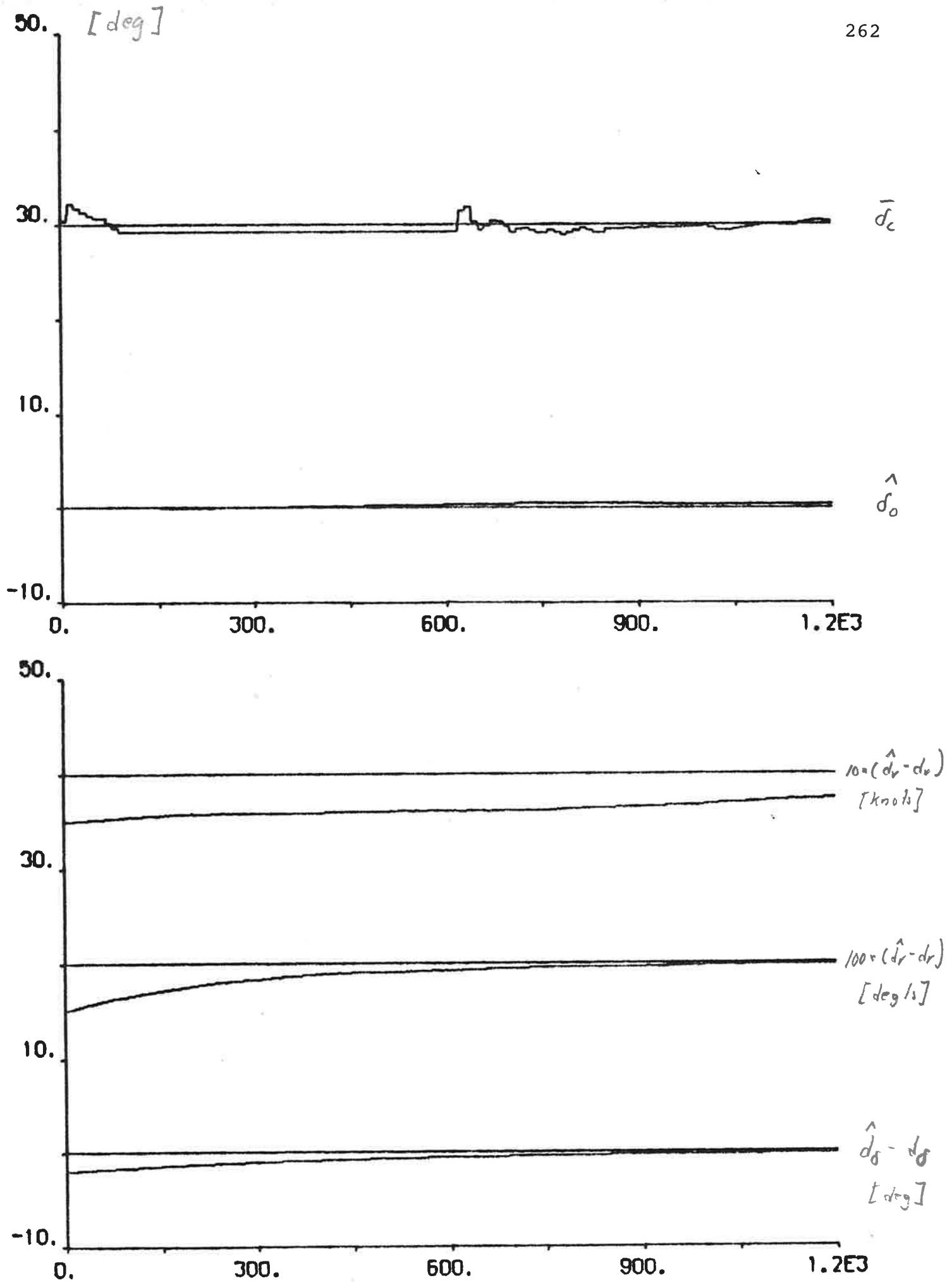


Fig. 4.65 d

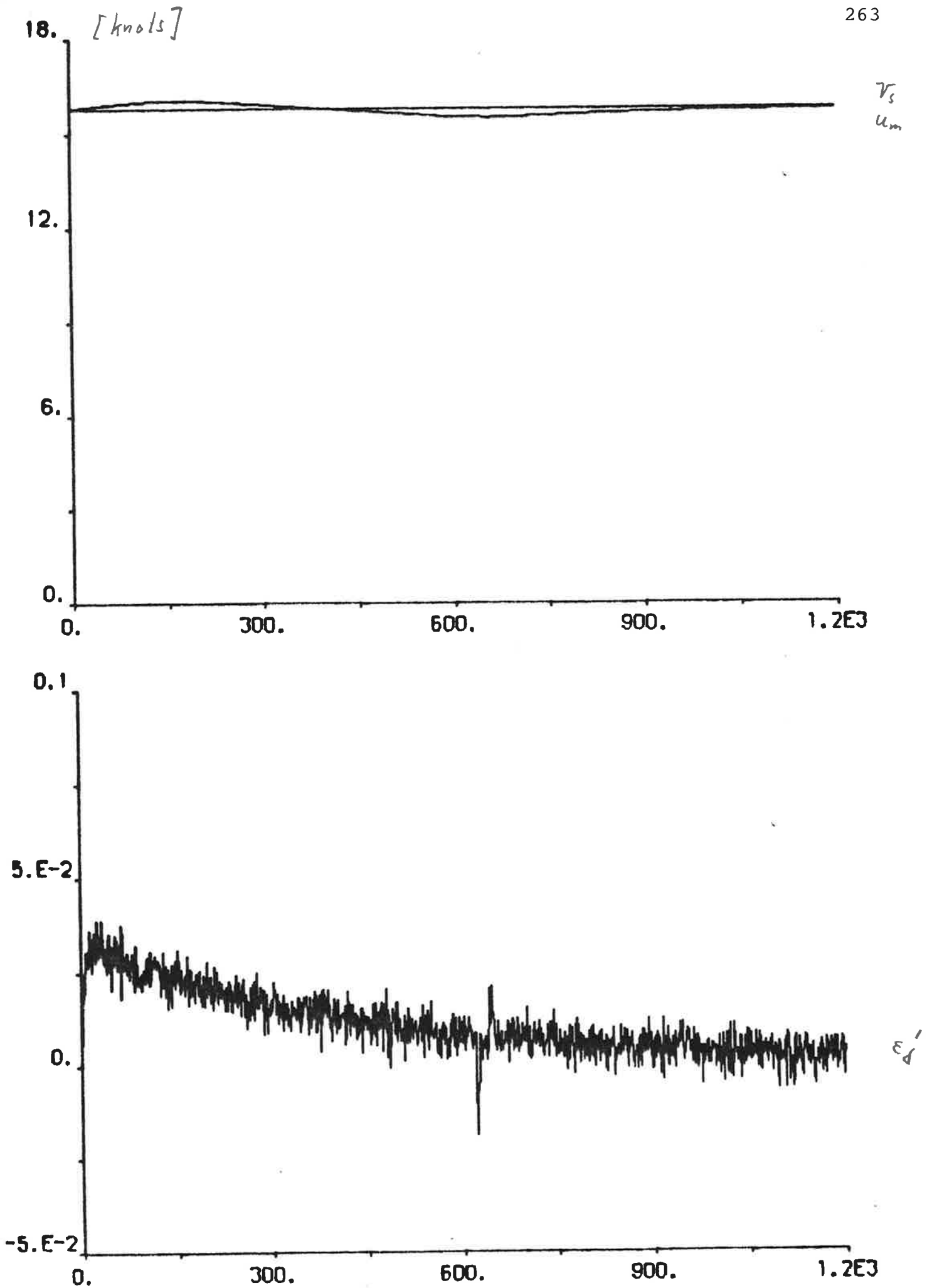


Fig. 4.65 e

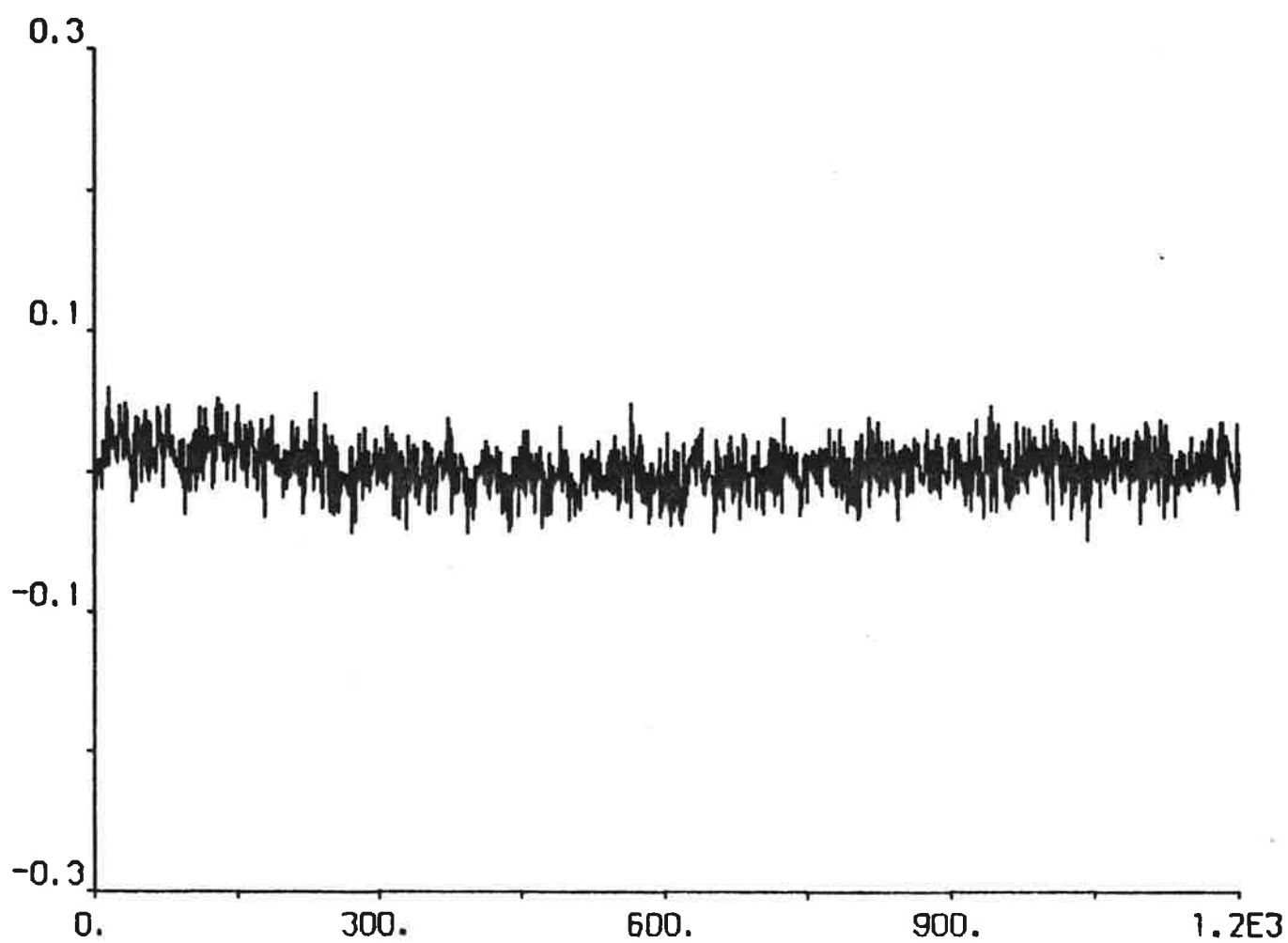
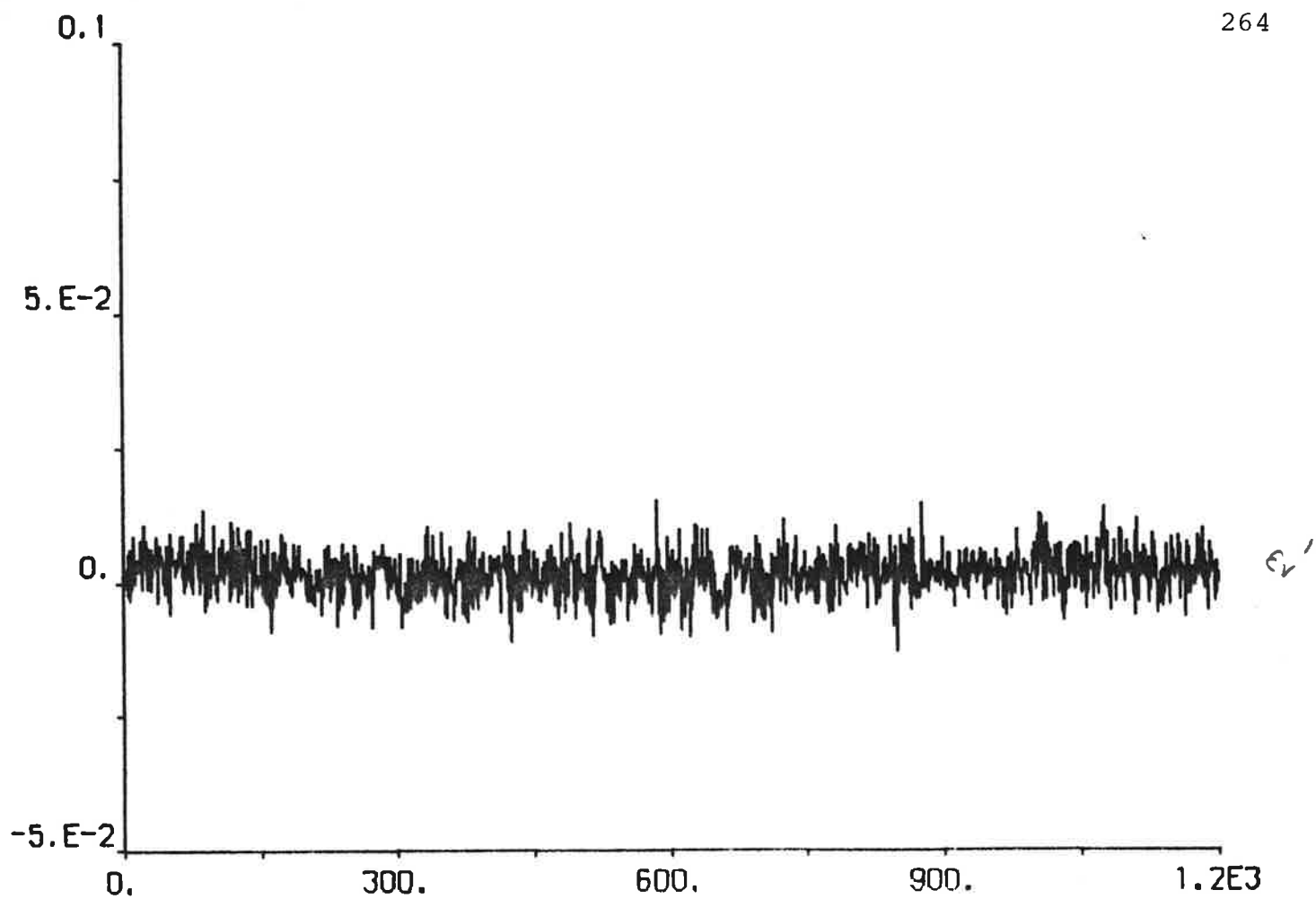


Fig. 4.65 f

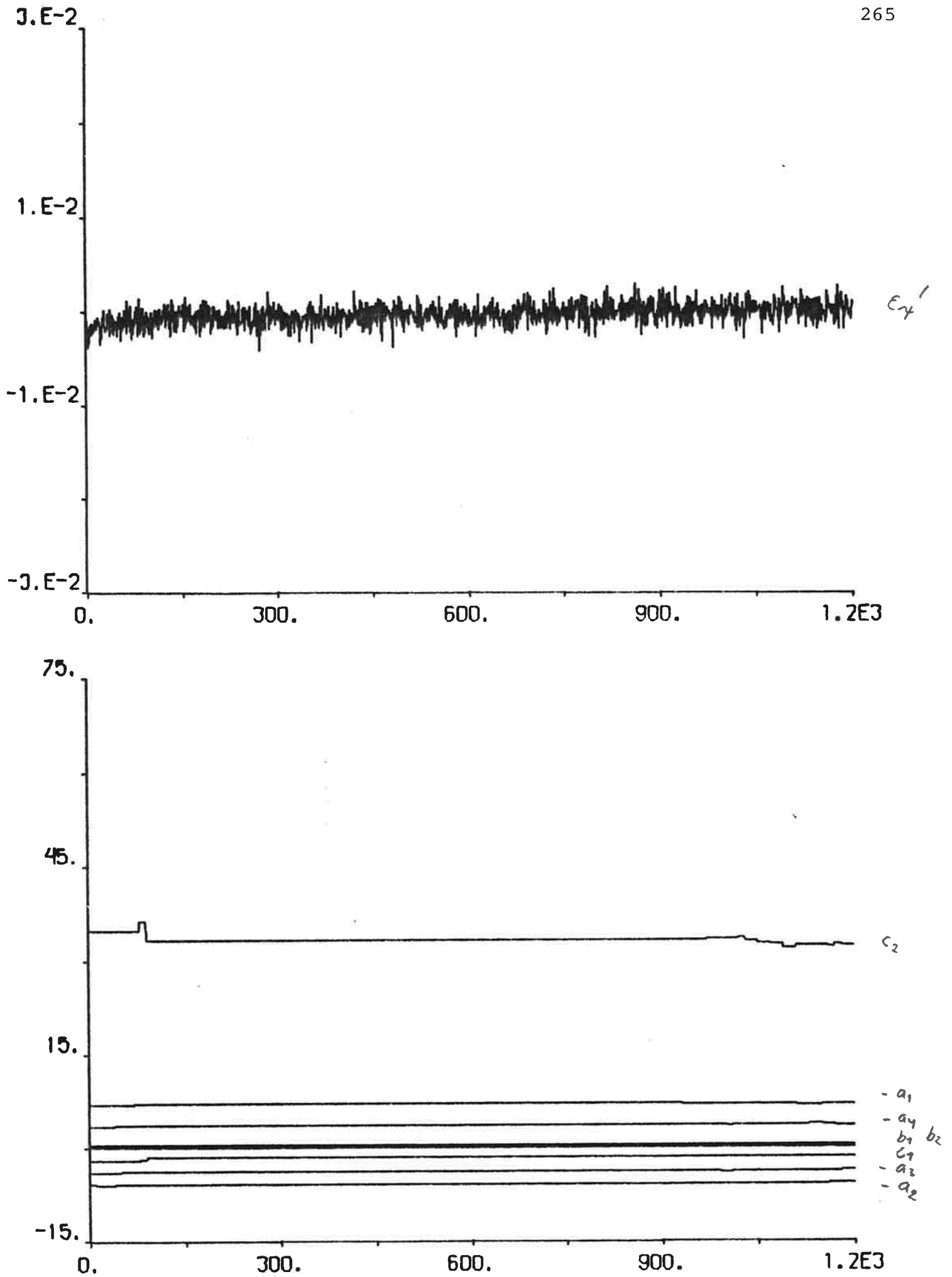


Fig. 4.65 g

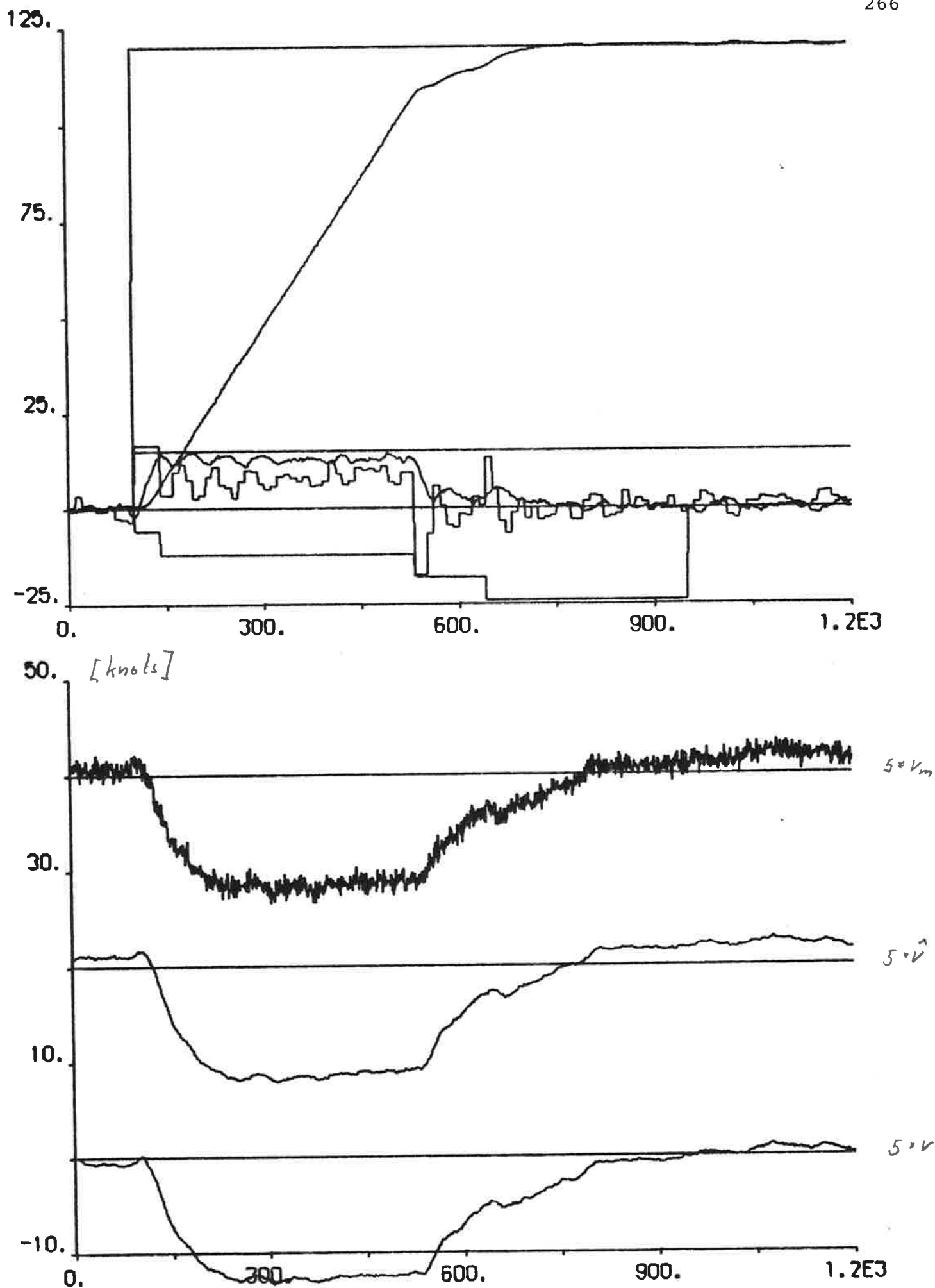


Fig. 4.66 a - $T = 10.5$ m, $n_0 = 87.6$ rpm, $u_0 = 15.8$ knots, $\Delta\psi_{\text{ref}} = 120$ deg, $r_{\text{ref}} = 0.3$ deg/s, self-tuning regulator and yaw regulator using estimates from the Kalman filter ($\bar{\tau}_2 = 50$ s).

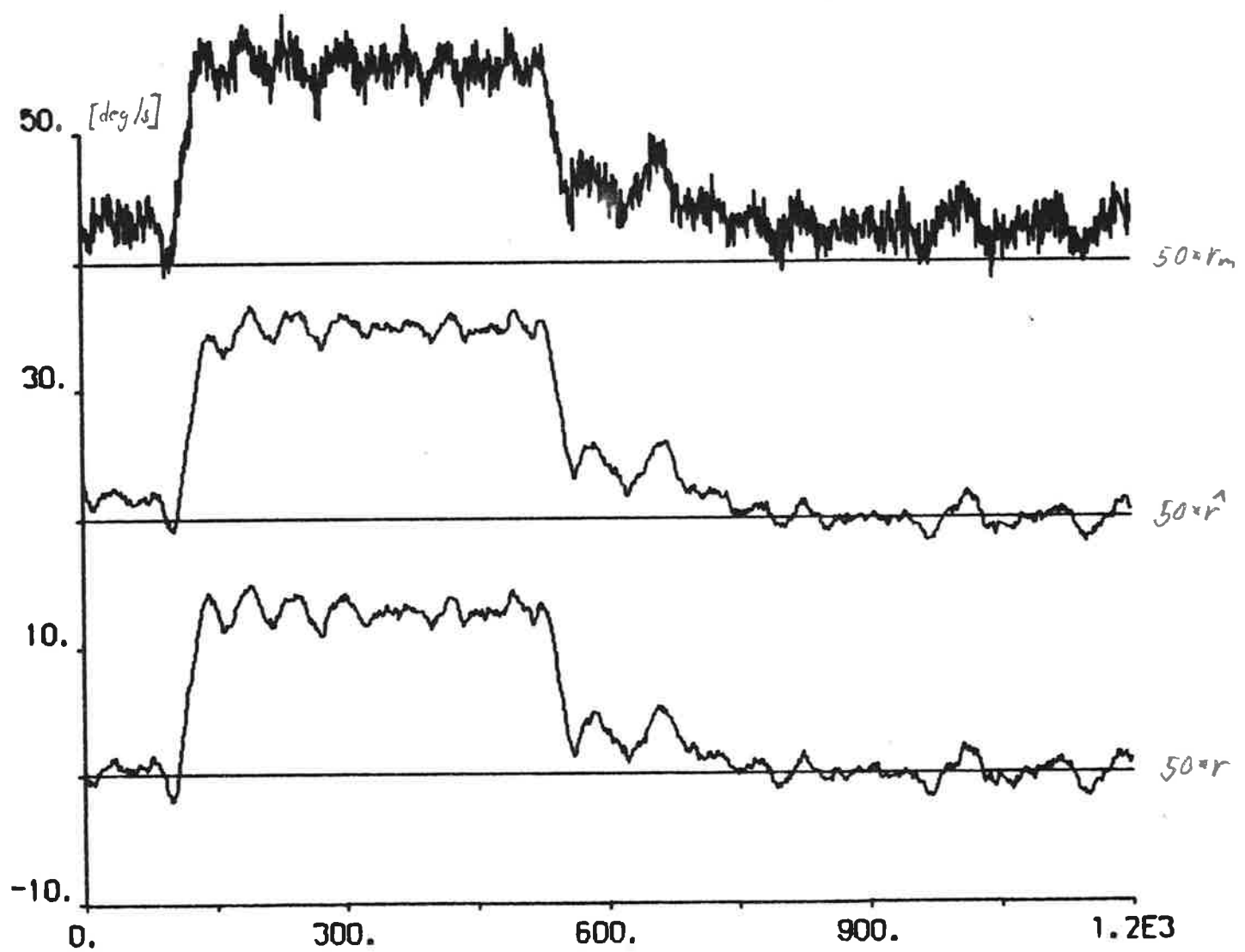
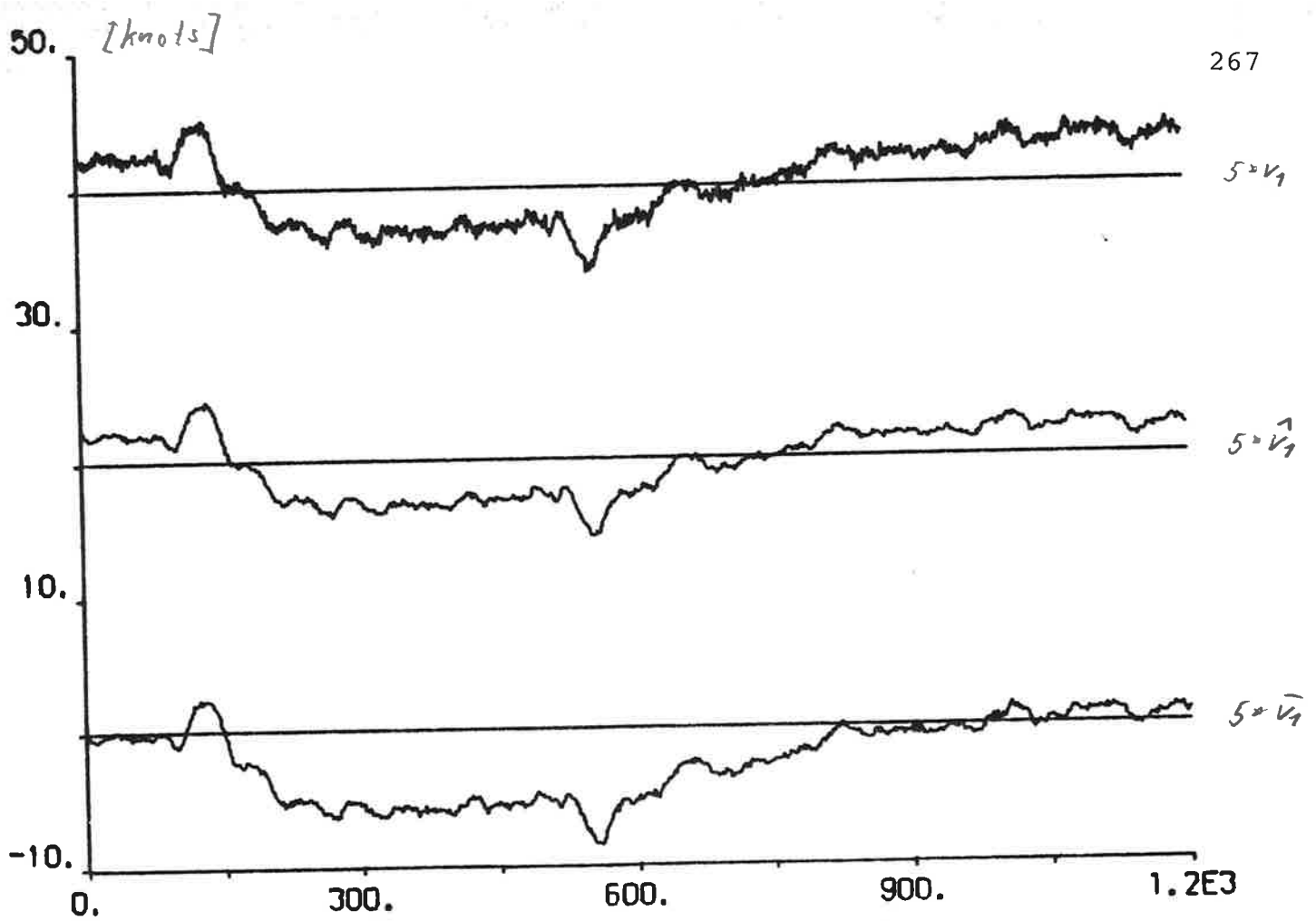


Fig. 4.66 b

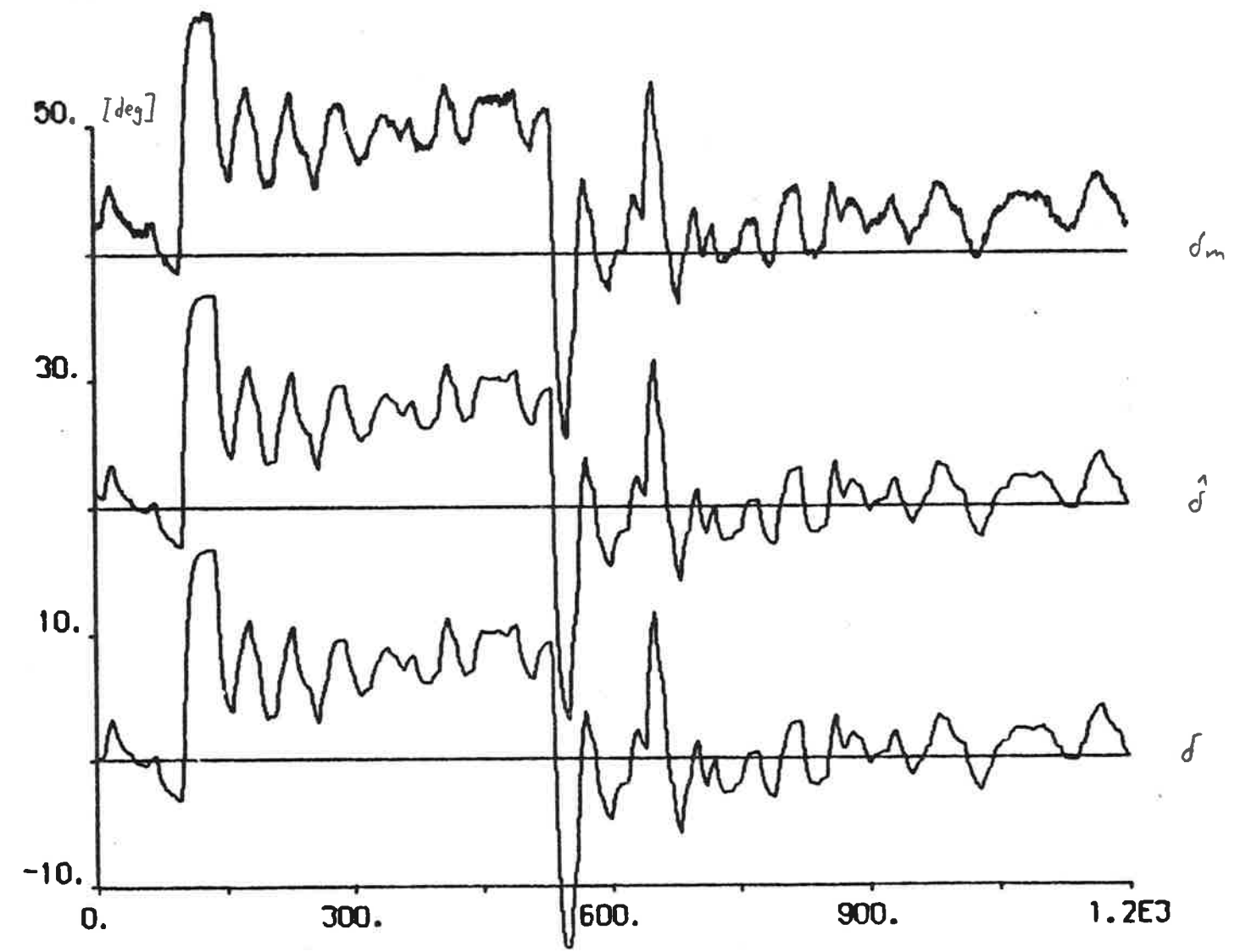
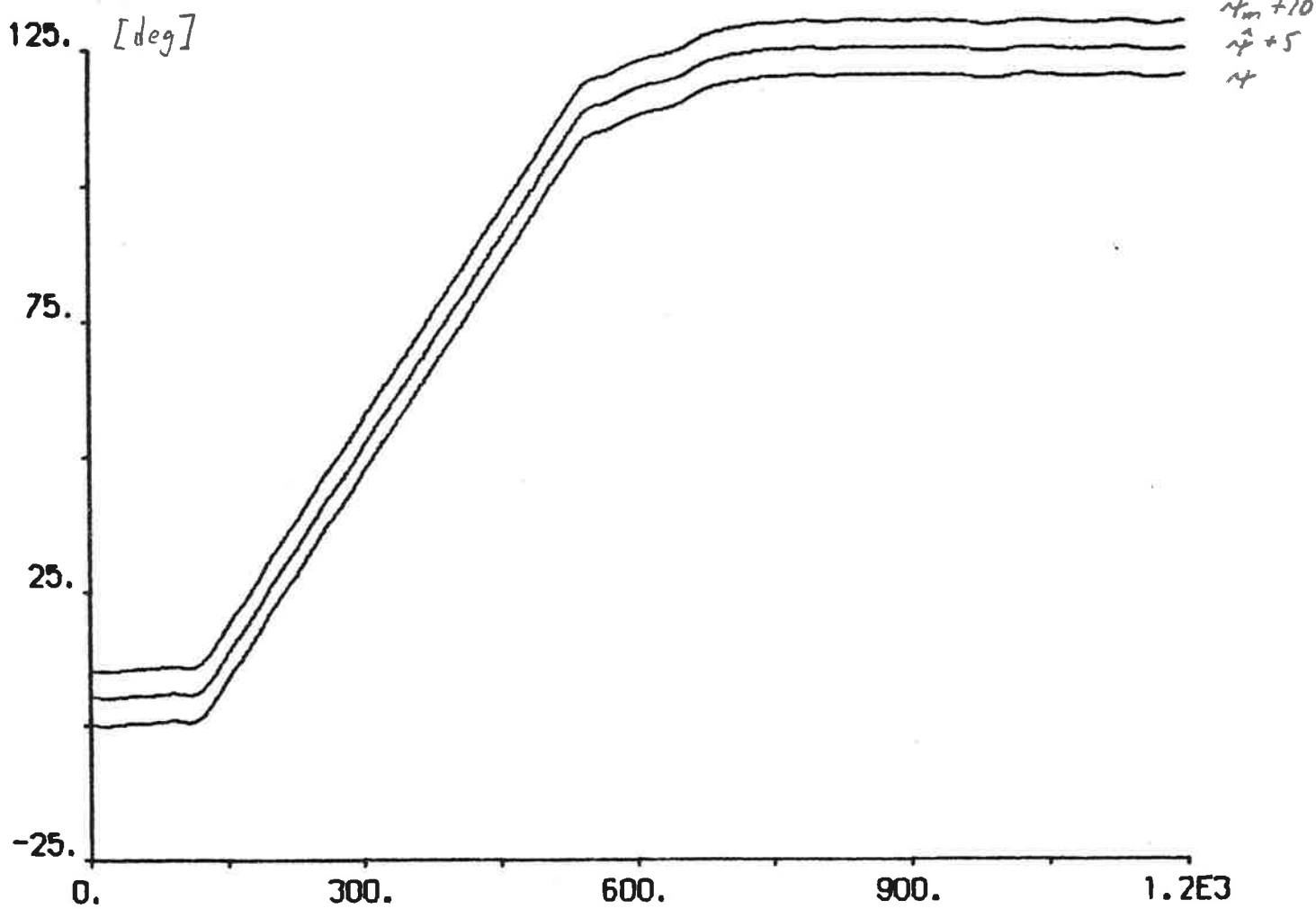


Fig. 4.66 c

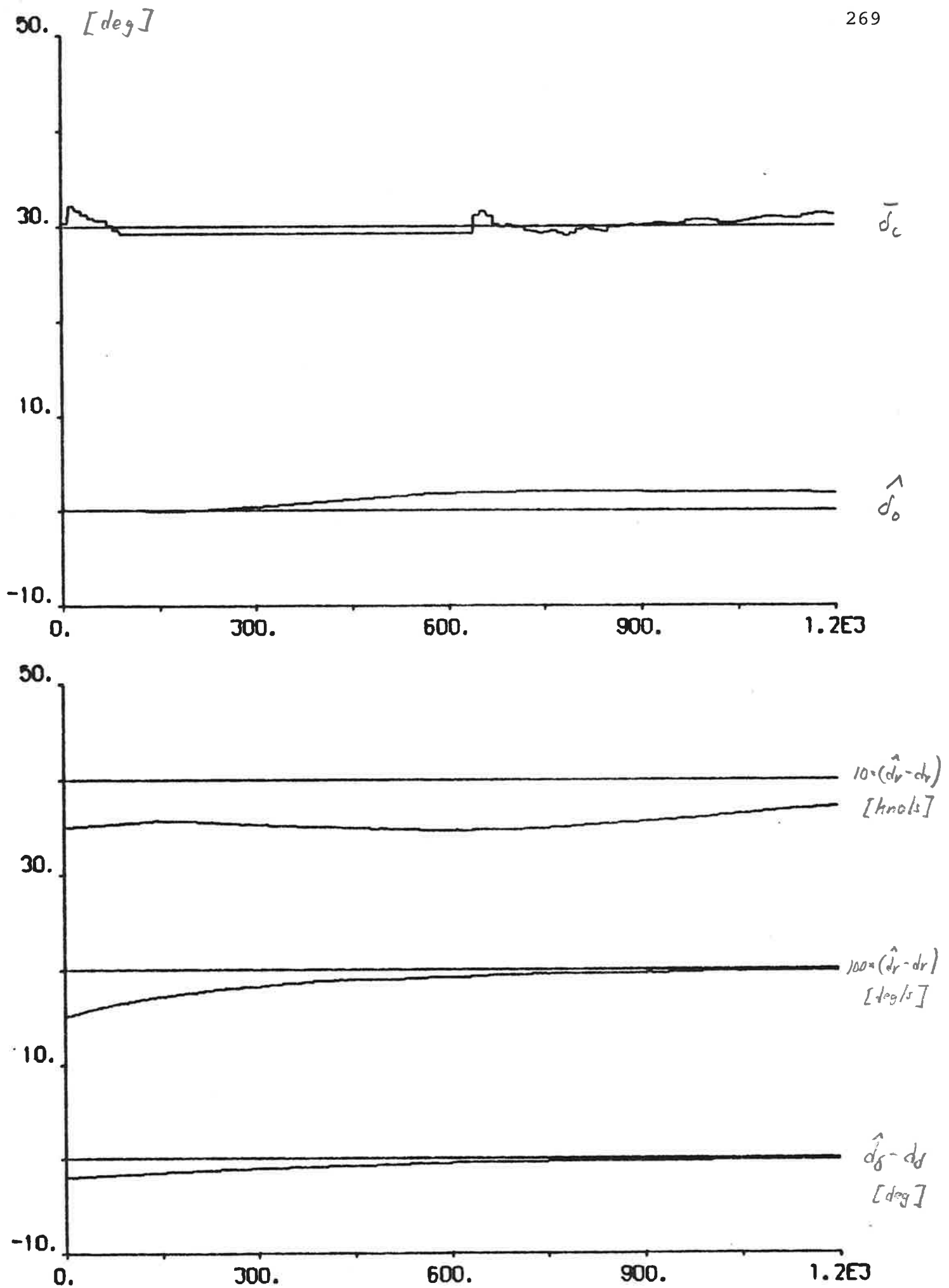


Fig. 4.66 d

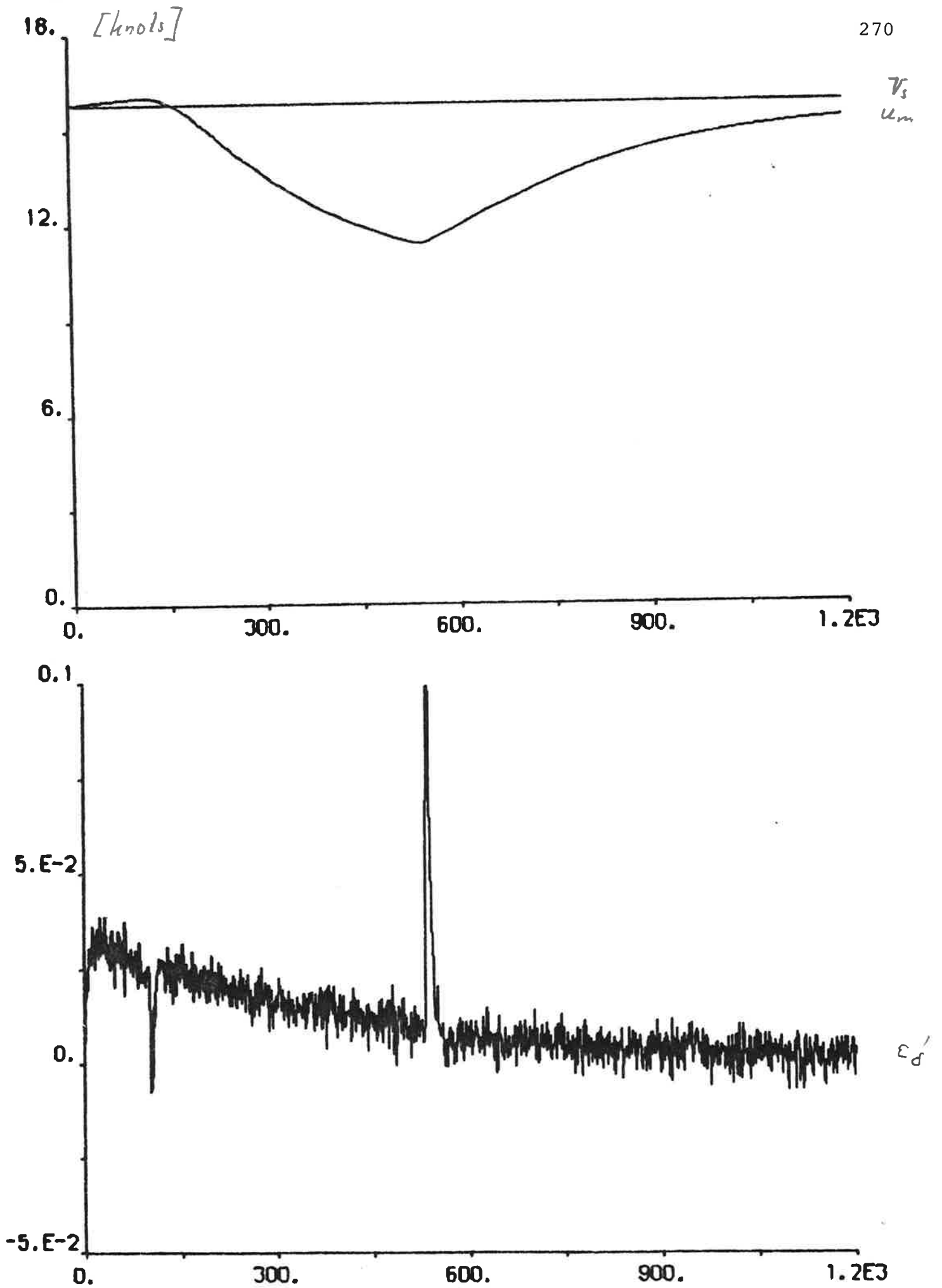


Fig. 4.66 e

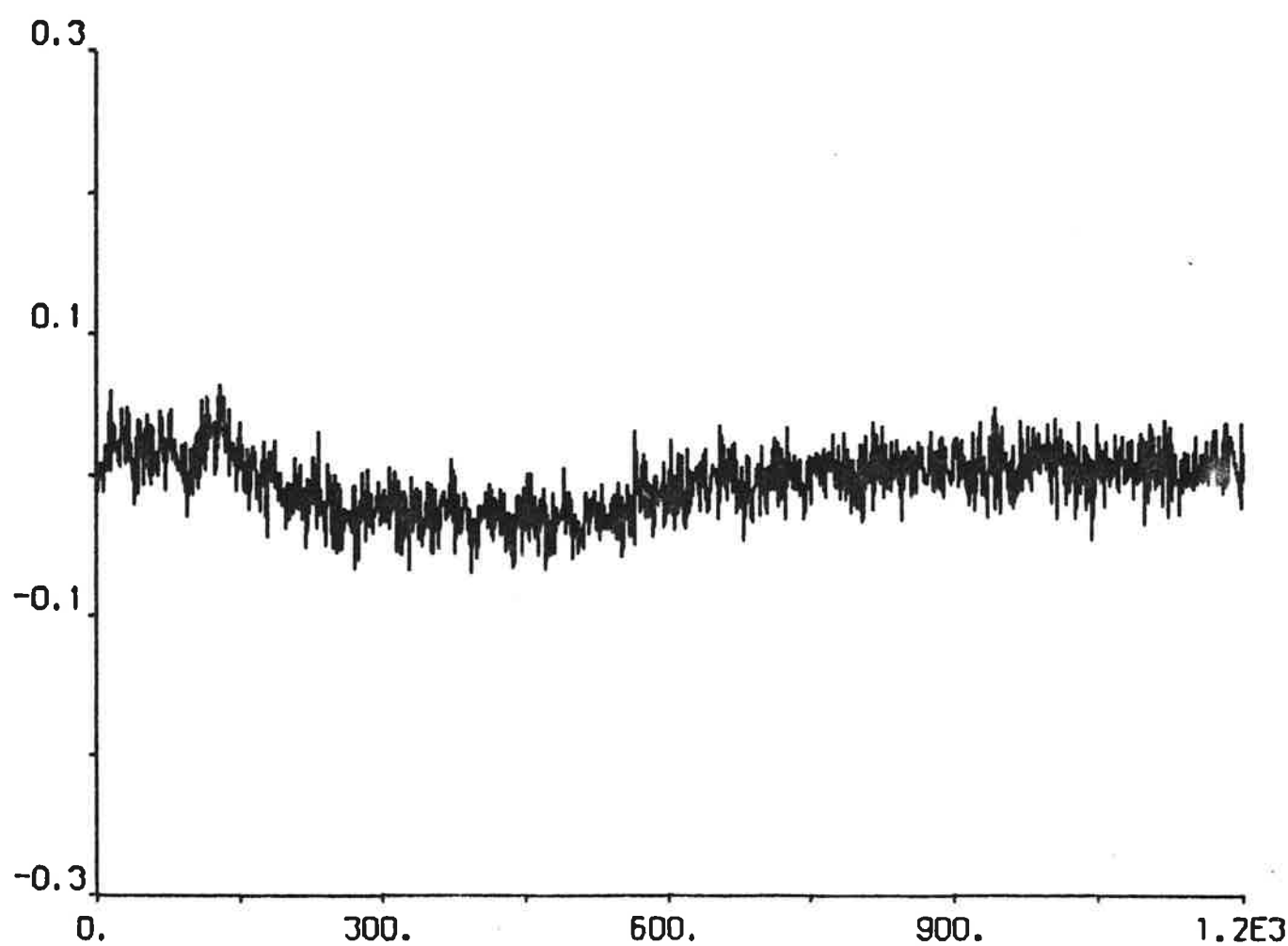
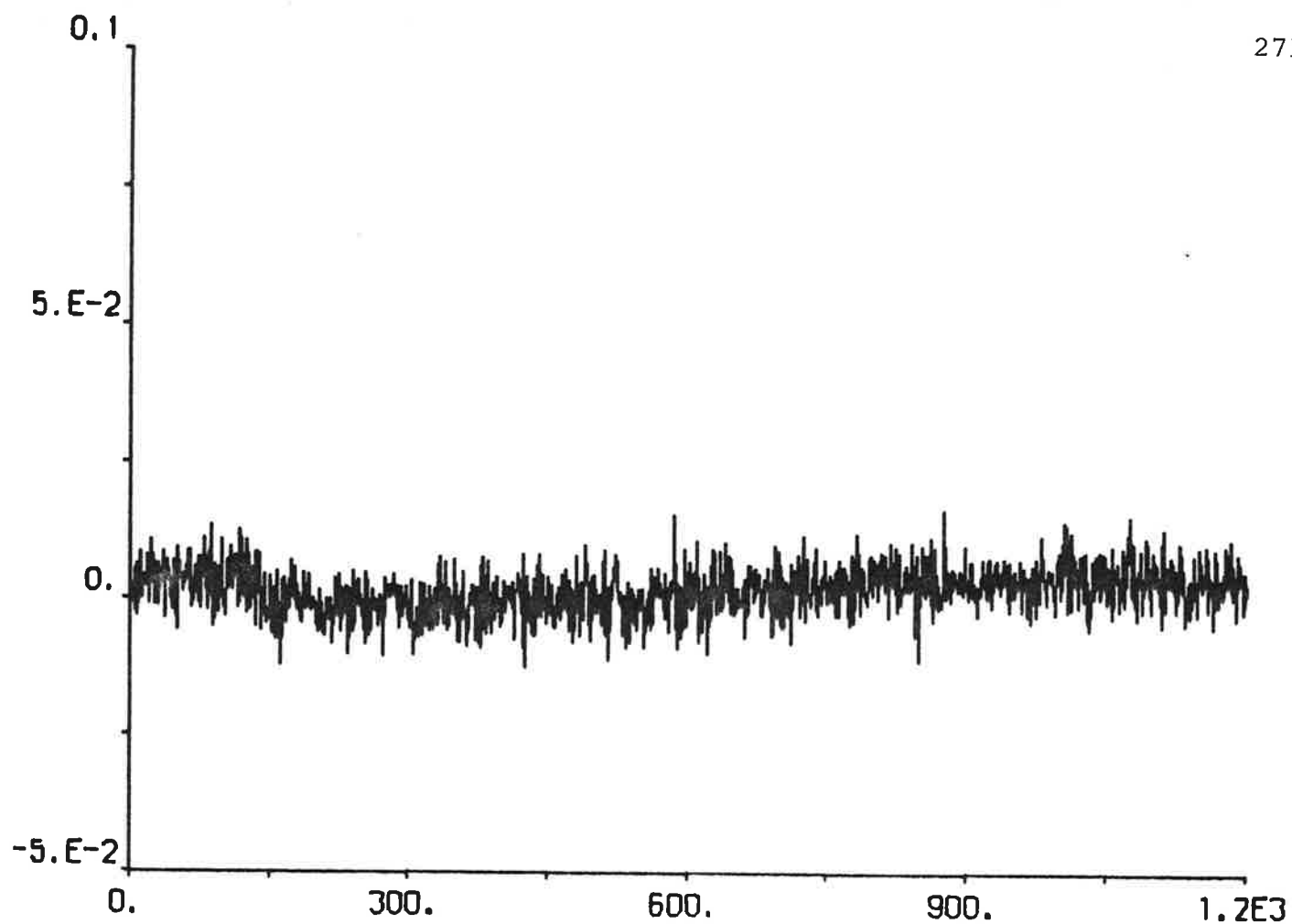


Fig. 4.66 f

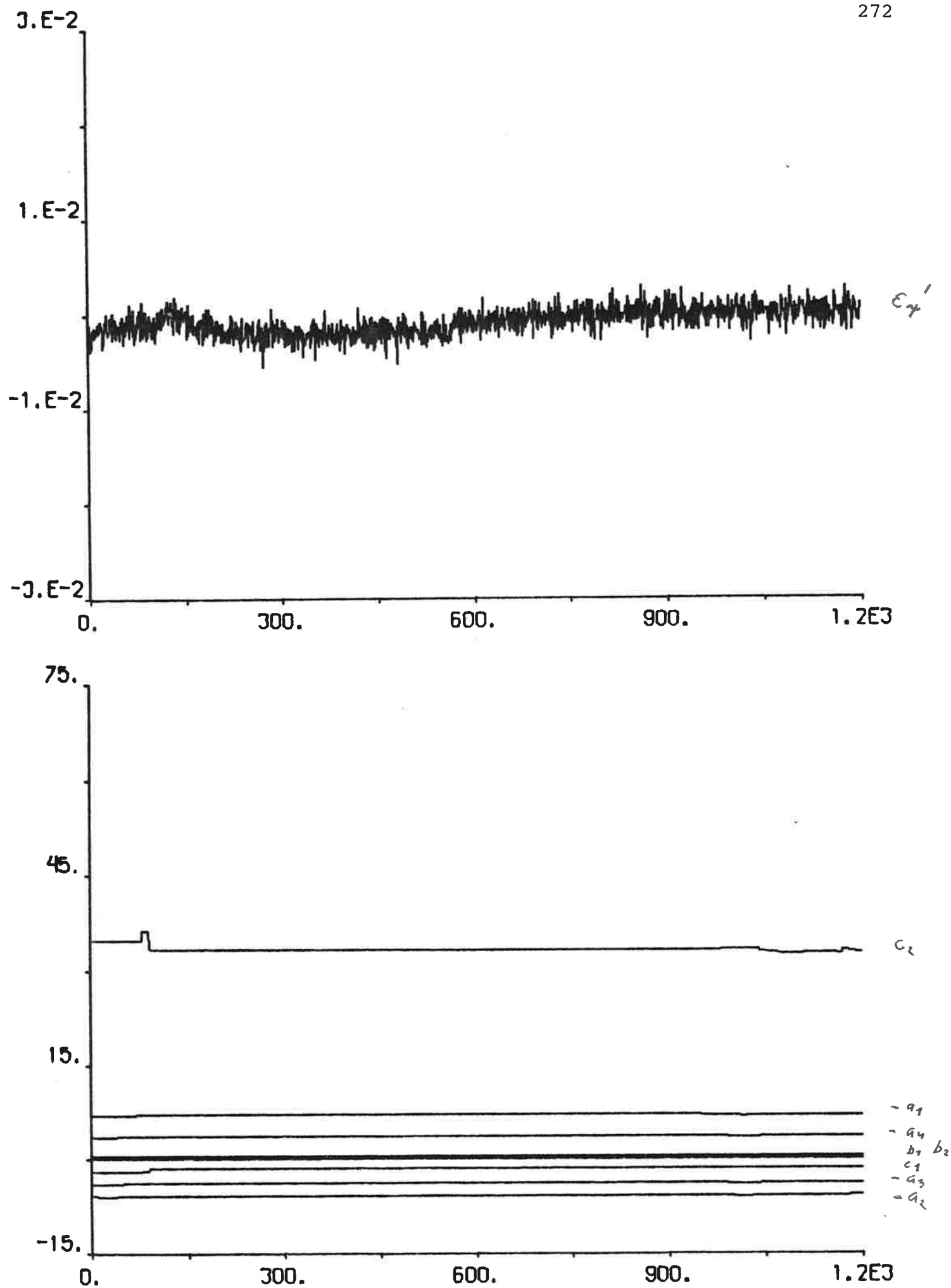


Fig. 4.66 g

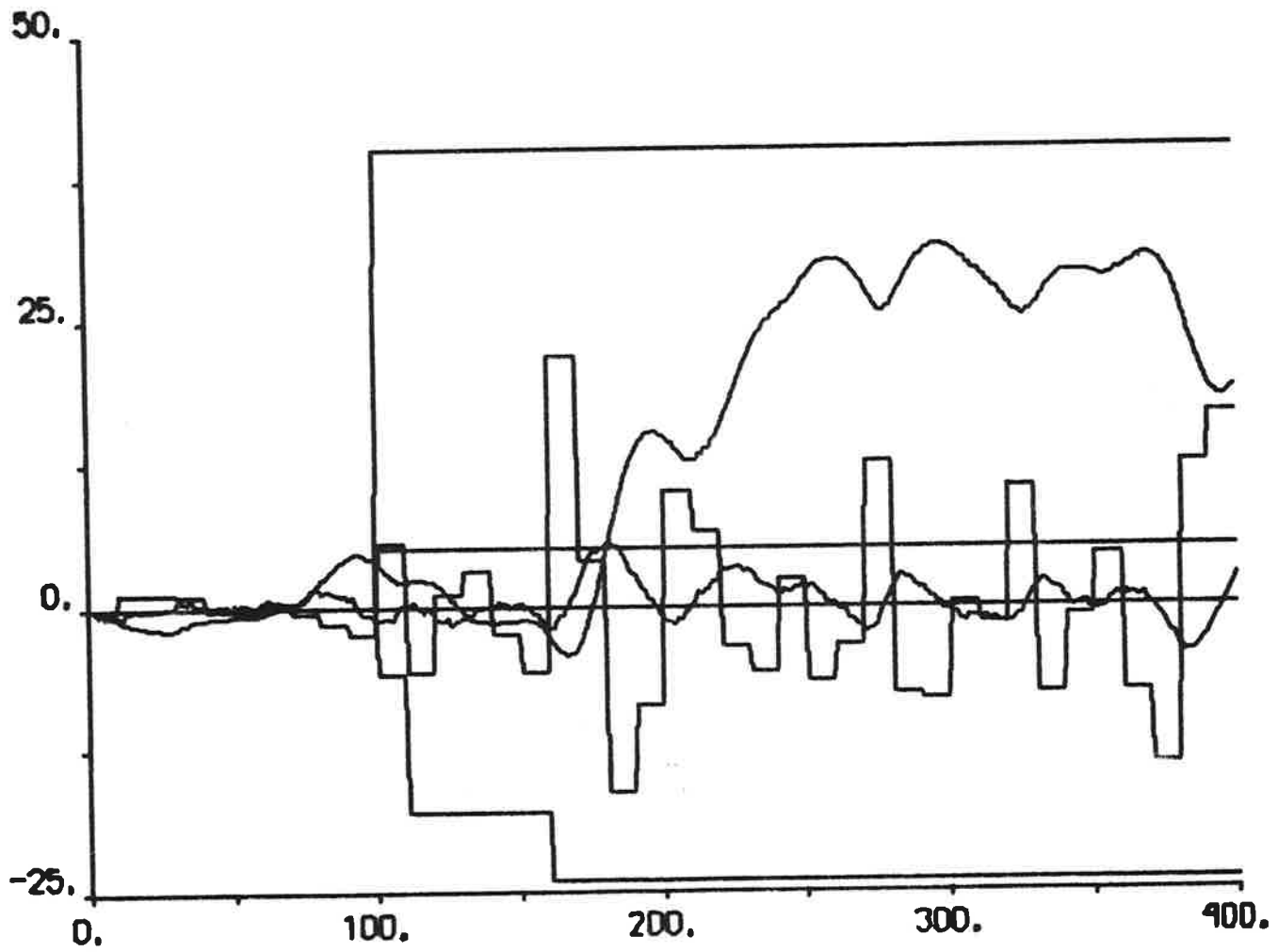


Fig. 4.67 - $T = 10.5$ m, $n_0 = 87.6$ rpm, $u_0 = 15.8$ knots,
 $\Delta\psi_{\text{ref}} = 4$ deg, $r_{\text{ref}} = 0.1$ deg/s, self-tuning
 regulator and yaw regulator using non-filtered
 measurements ($\bar{c}_2 = 50$ s).

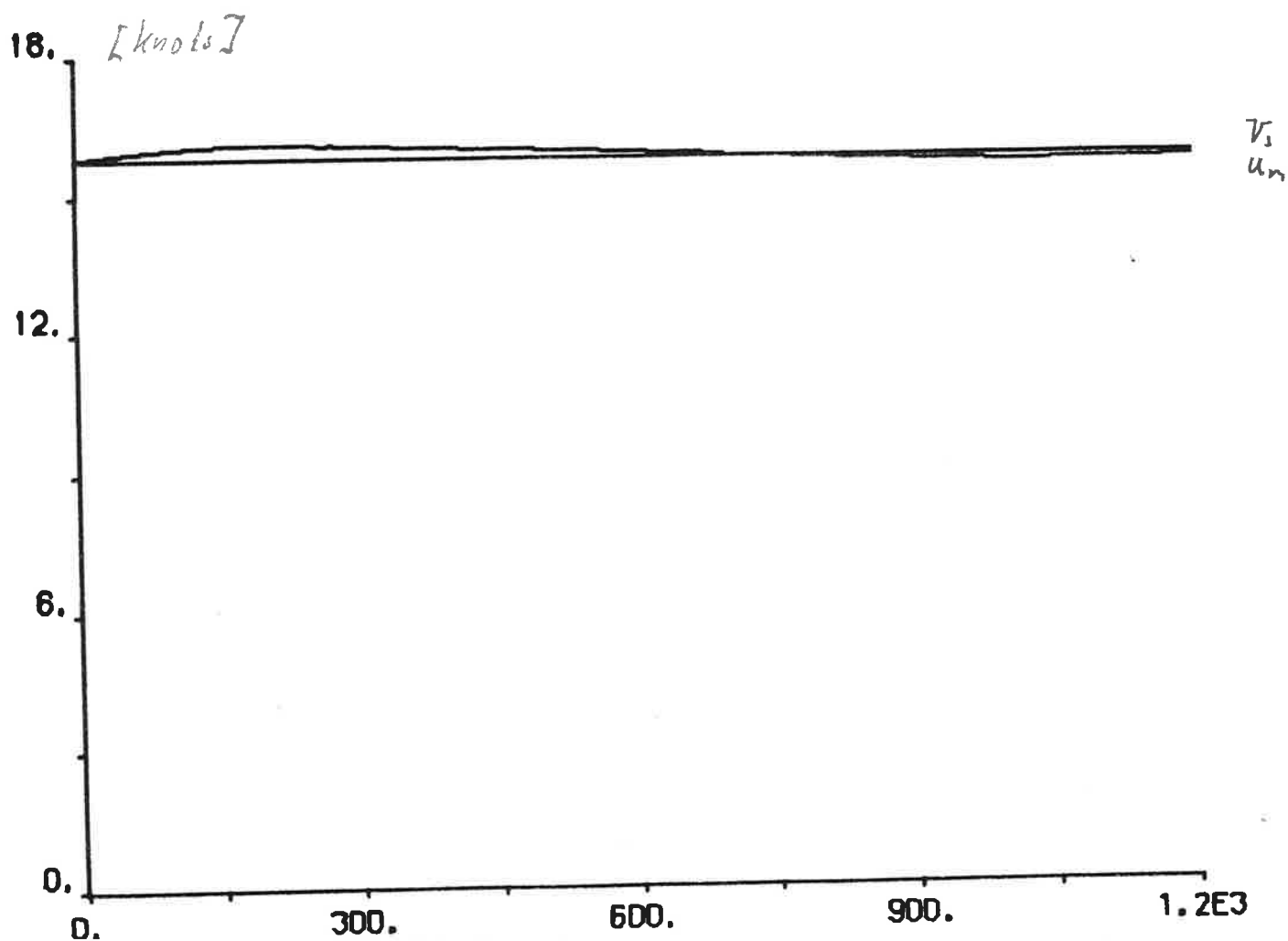
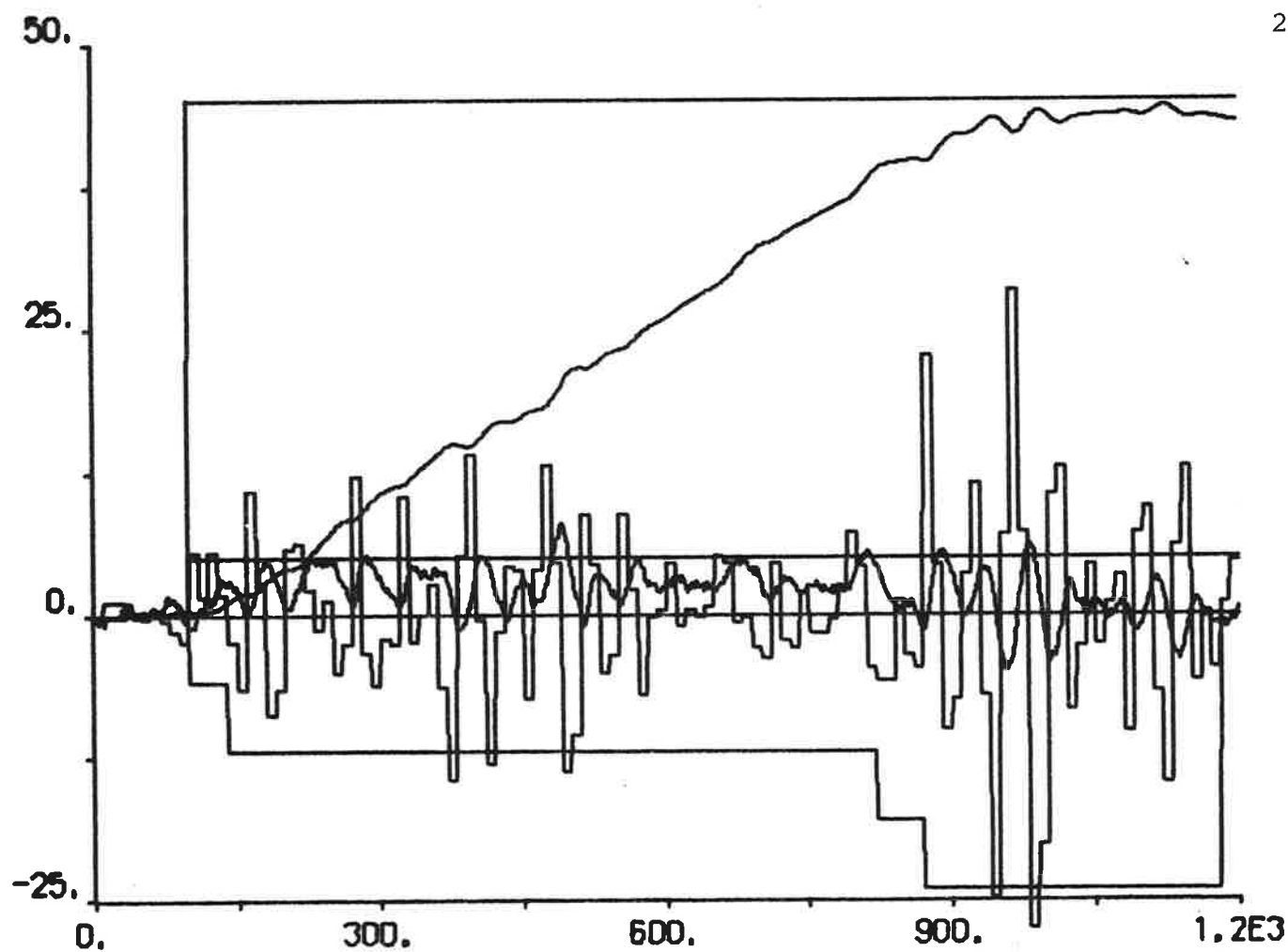


Fig. 4.68 - $T = 10.5$ m, $n_0 = 87.6$ rpm, $u_0 = 15.8$ knots, $\Delta\psi_{\text{ref}} = 45$ deg, $r_{\text{ref}} = 0.1$ deg/s, self-tuning regulator and yaw regulator using non-filtered measurements ($\bar{c}_2 = 50$ s).

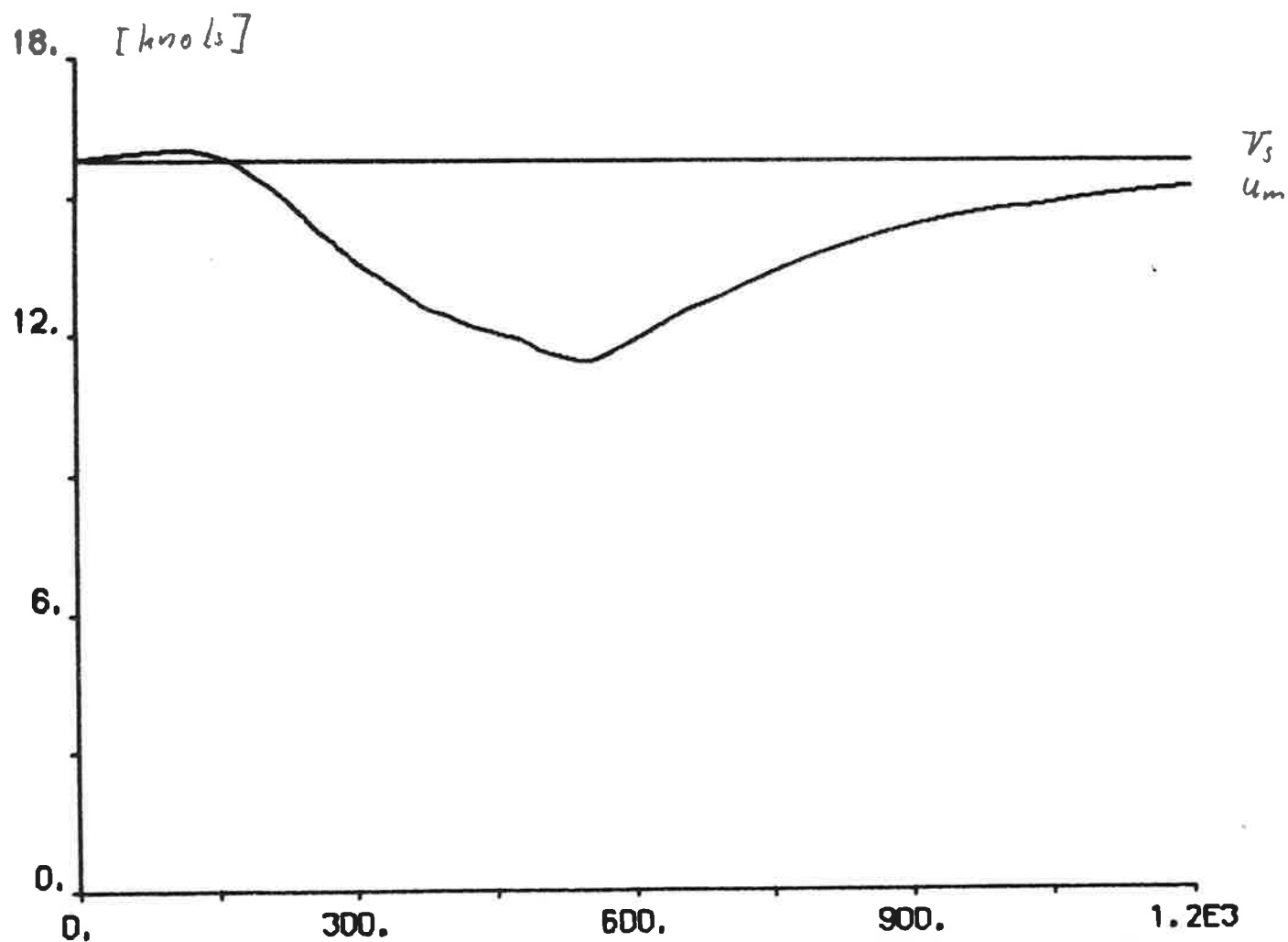
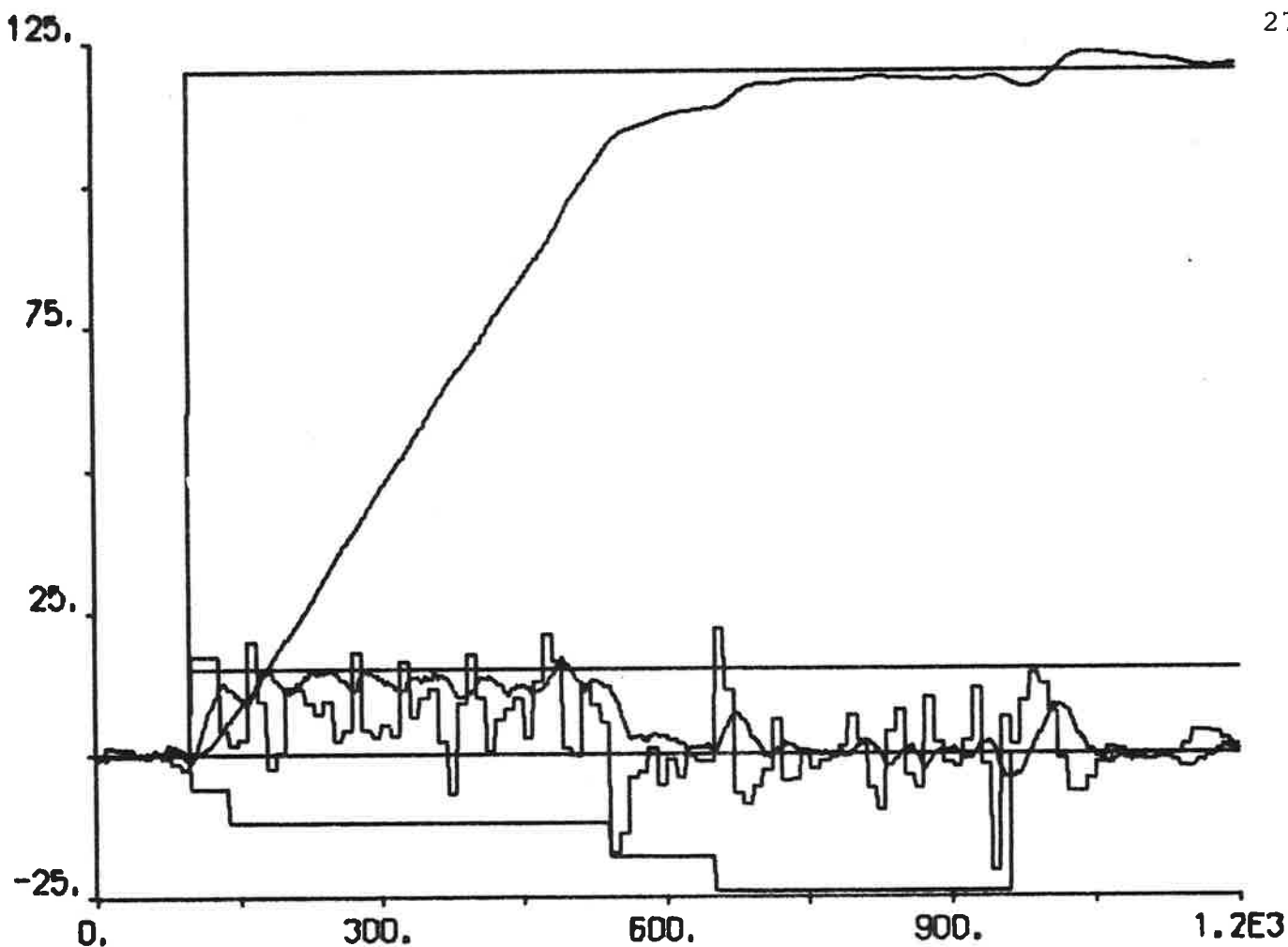


Fig. 4.69 - $T = 10.5$ m, $n_0 = 87.6$ rpm, $u_0 = 15.8$ knots, $\Delta\psi_{\text{ref}} = 120$ deg, $r_{\text{ref}} = 0.3$ deg/s, self-tuning regulator and yaw regulator using non-filtered measurements ($\bar{c}_2 = 50$ s).

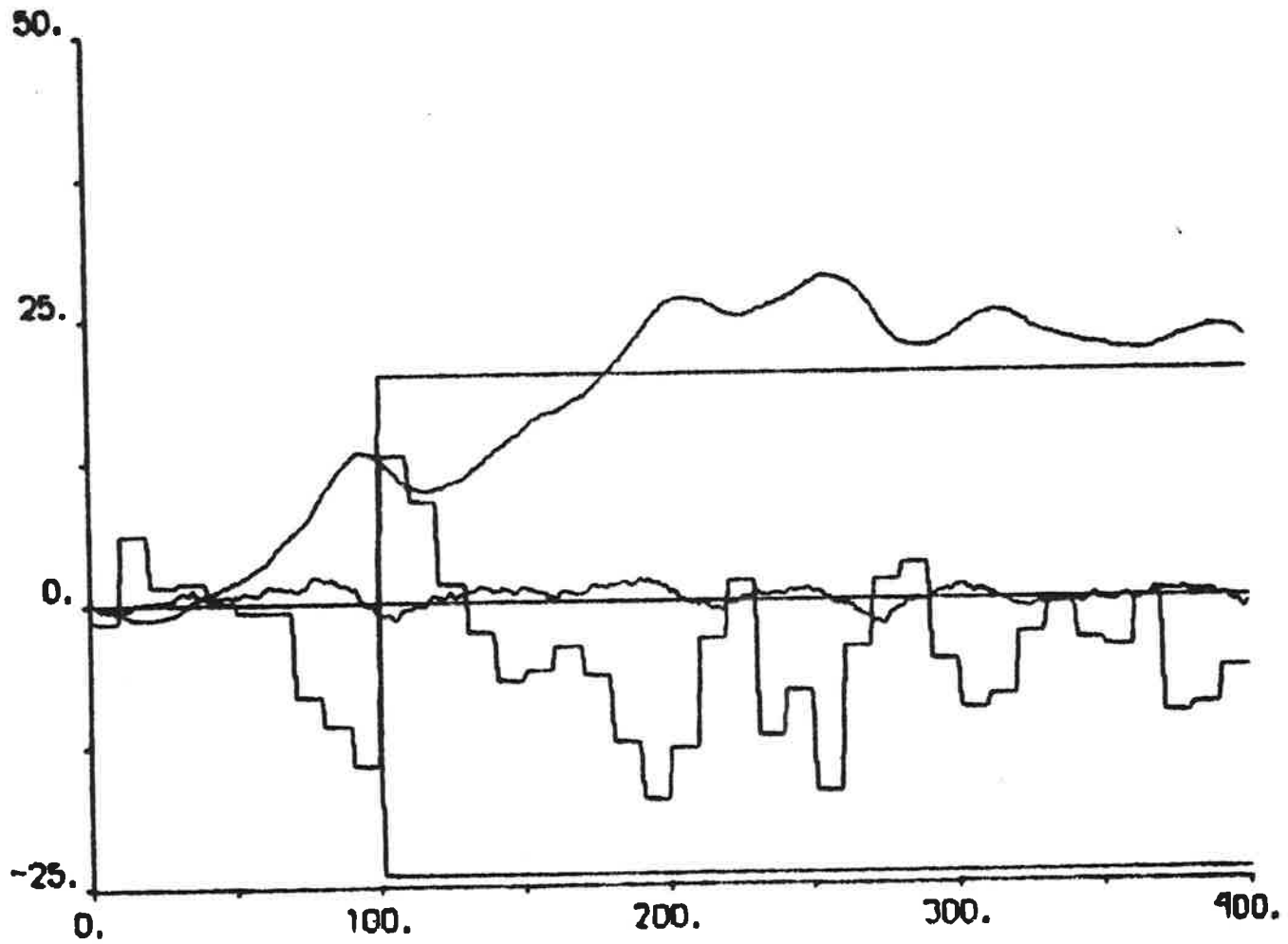


Fig. 4.70 - $T = 22.3$ m, $n_0 = 55.443$ rpm, $u_0 = 10$ knots,
 $\Delta\psi_{\text{ref}} = 2$ deg, $r_{\text{ref}} = 0$ deg/s, self-tuning
 regulator and yaw regulator using estimates
 from the Kalman filter ($\bar{c}_2 = 63.25$ s).

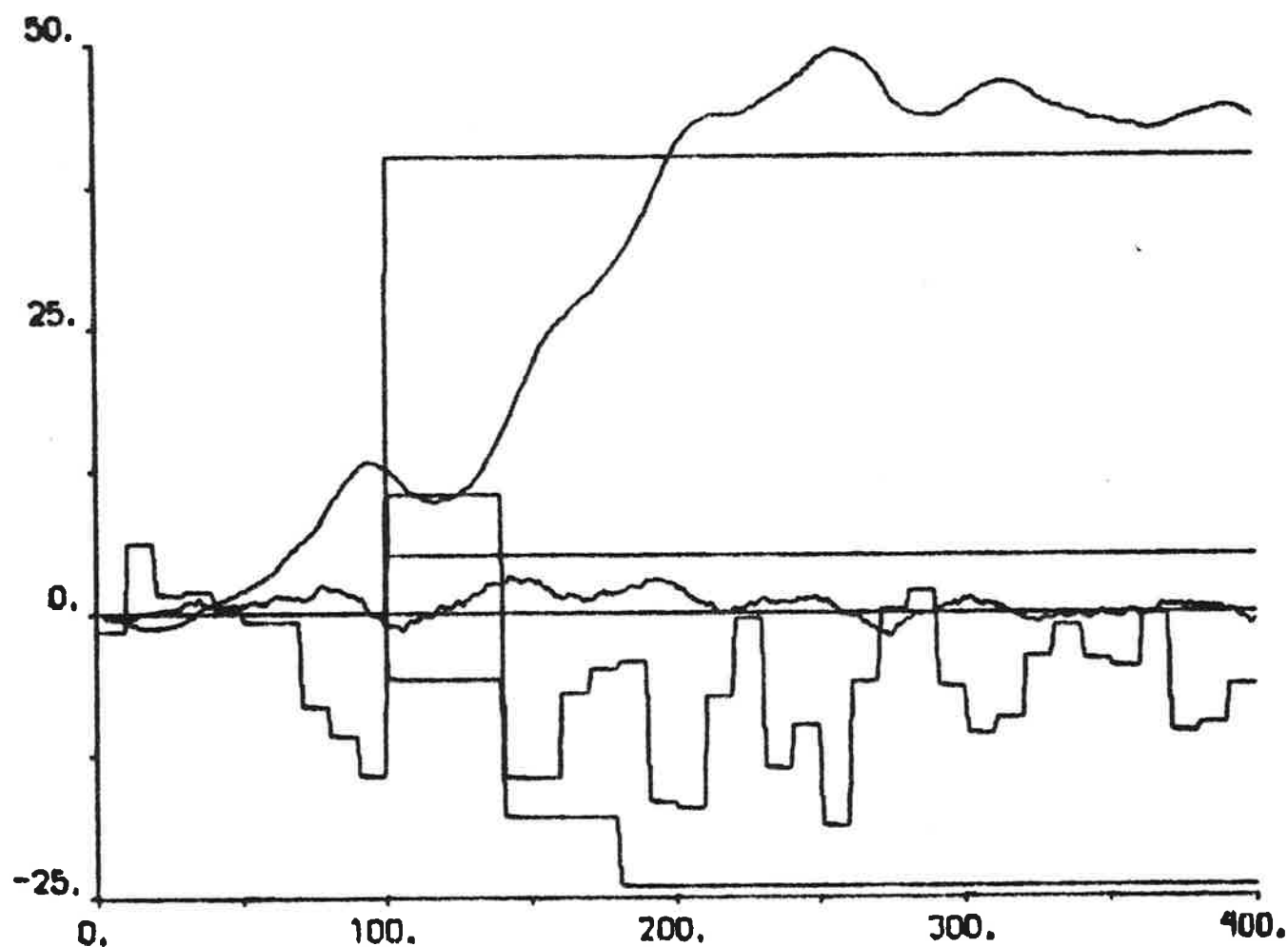


Fig. 4.71 - $T = 22.3$ m, $n_0 = 55.443$ rpm, $u_0 = 10$ knots,
 $\Delta\psi_{\text{ref}} = 4$ deg, $r_{\text{ref}} = 0.1$ deg/s, self-tuning
 regulator and yaw regulator using estimates
 from the Kalman filter ($\bar{c}_2 = 63.25$ s).

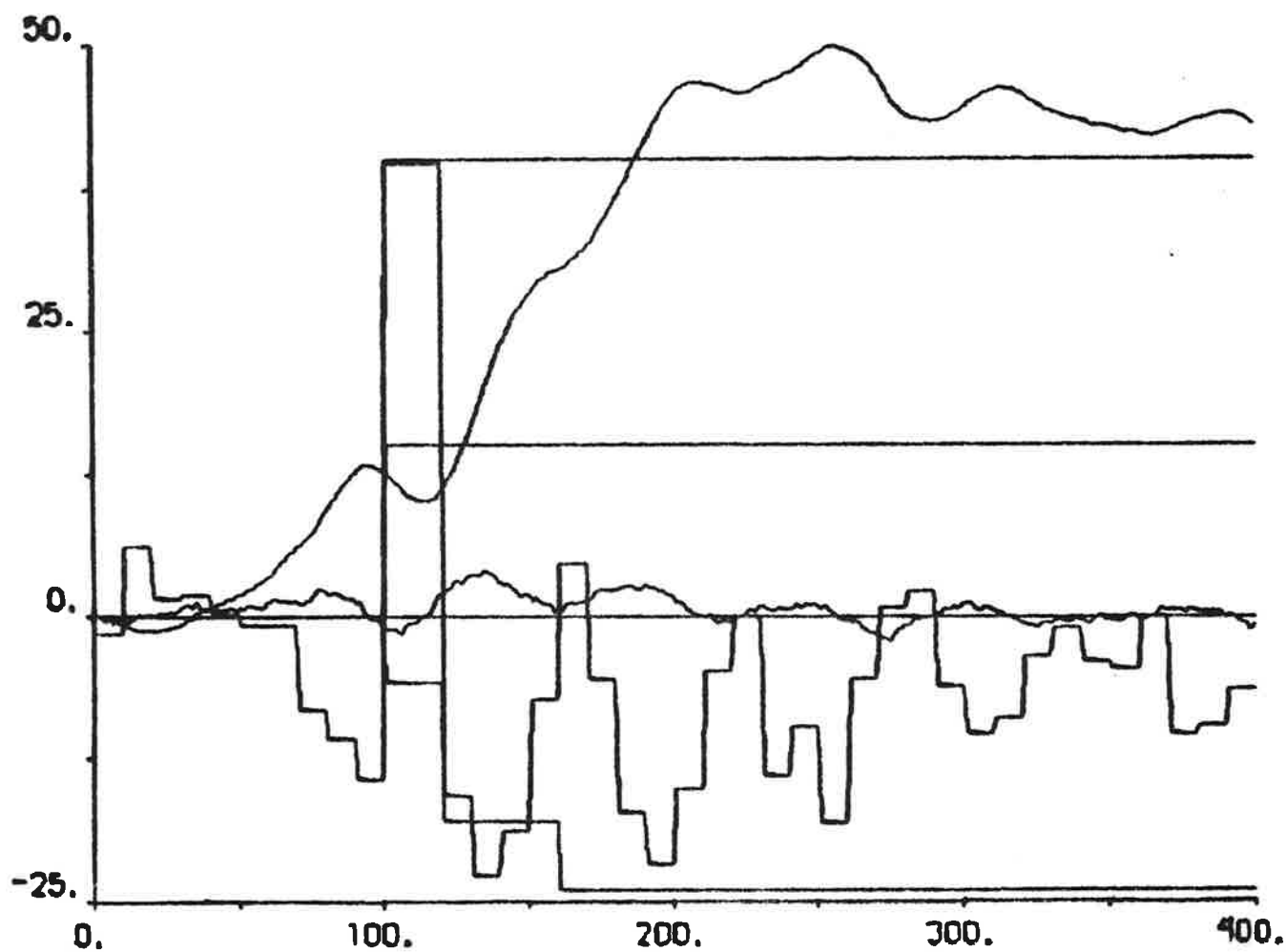


Fig. 4.72 - $T = 22.3$ m, $n_0 = 55.443$ rpm, $u_0 = 10$ knots,
 $\Delta\psi_{\text{ref}} = 4$ deg, $r_{\text{ref}} = 0.3$ deg/s, self-tuning
 regulator and yaw regulator using estimates
 from the Kalman filter ($\bar{c}_2 = 63.25$ s).

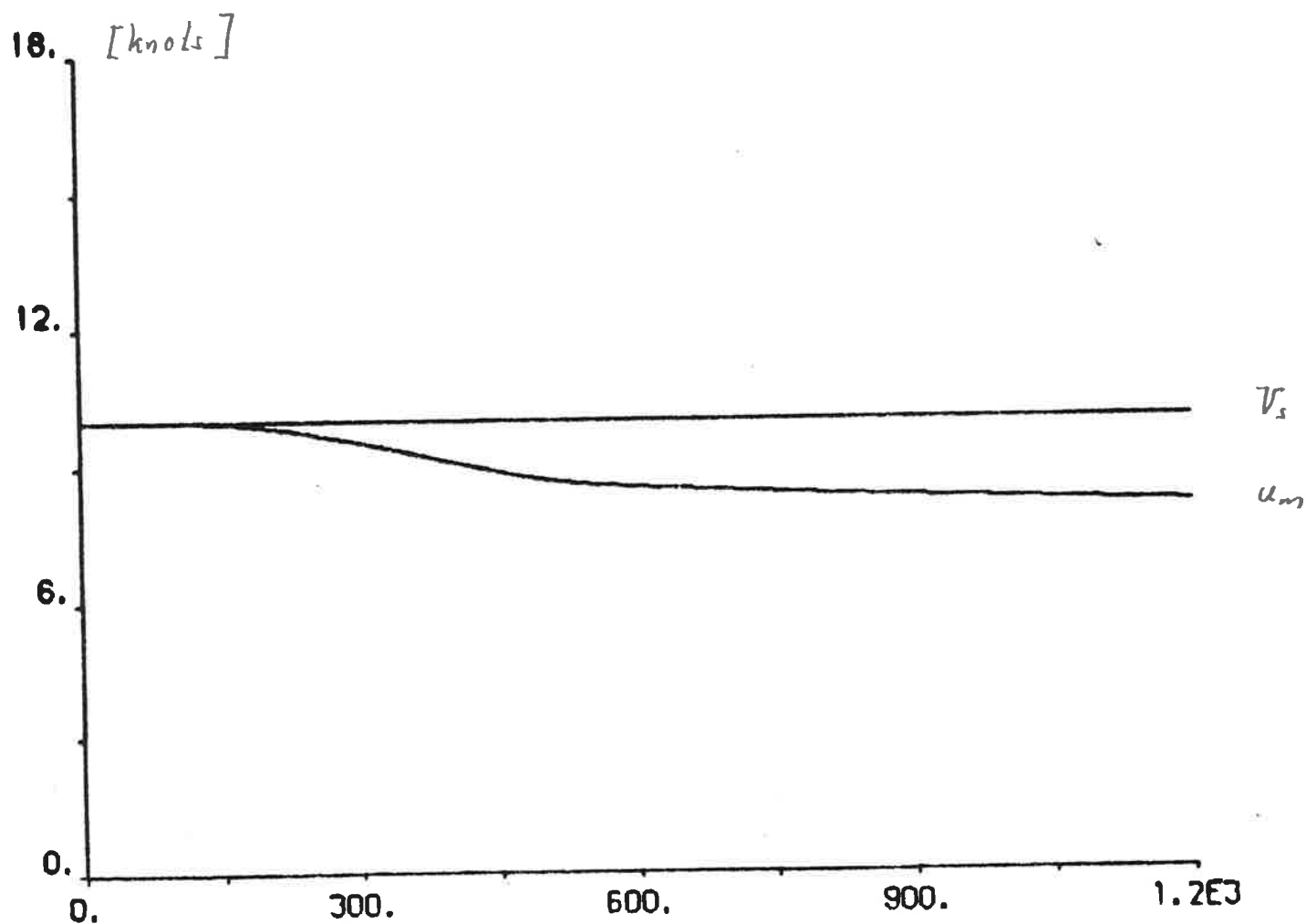
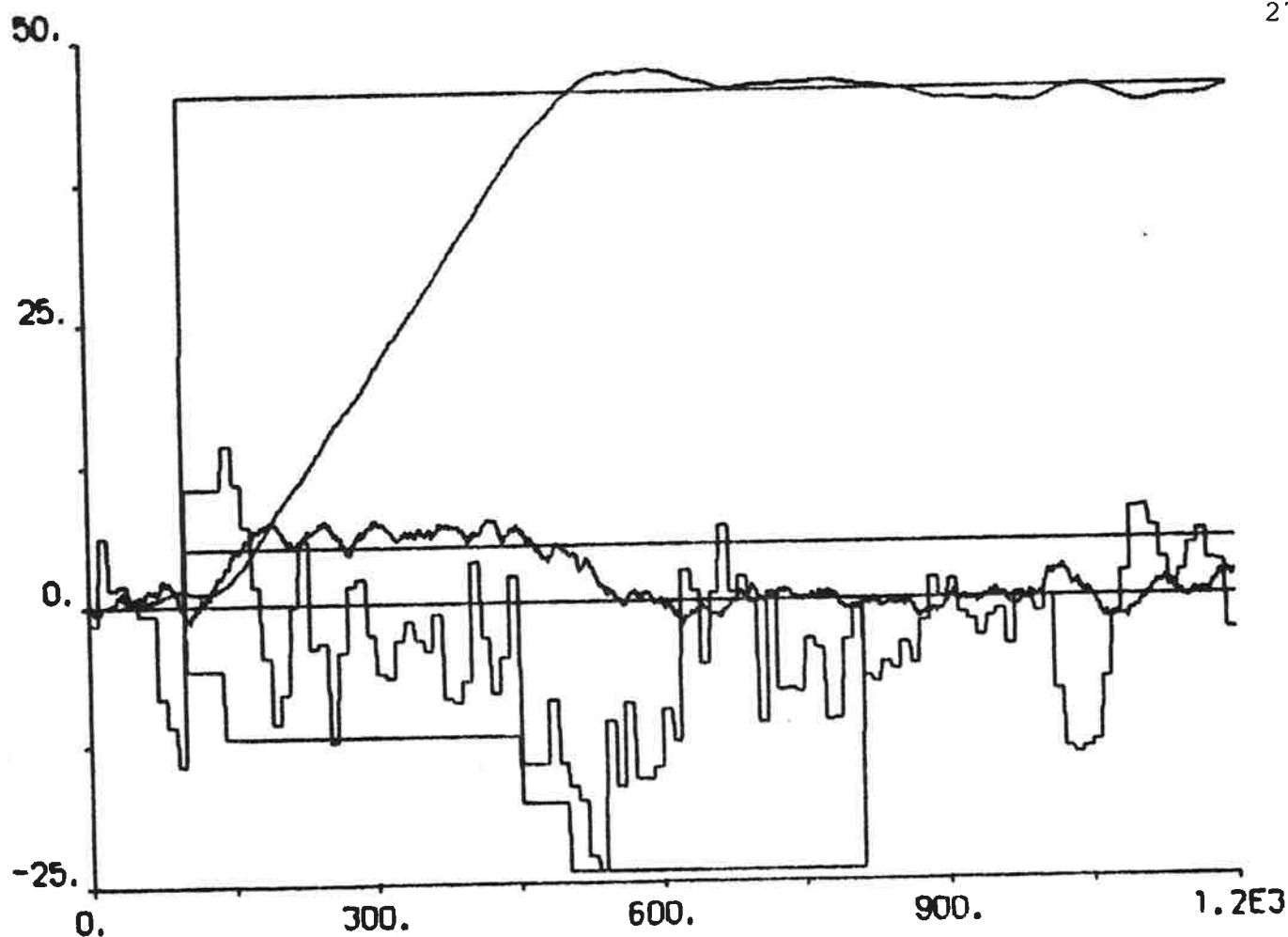


Fig. 4.73 - $T = 22.3$ m, $n_0 = 55.443$ rpm, $u_0 = 10$ knots, $\Delta\psi_{\text{ref}} = 45$ deg, $r_{\text{ref}} = 0.1$ deg/s, self-tuning regulator and yaw regulator using estimates from the Kalman filter ($\bar{c}_2 = 63.25$ s).

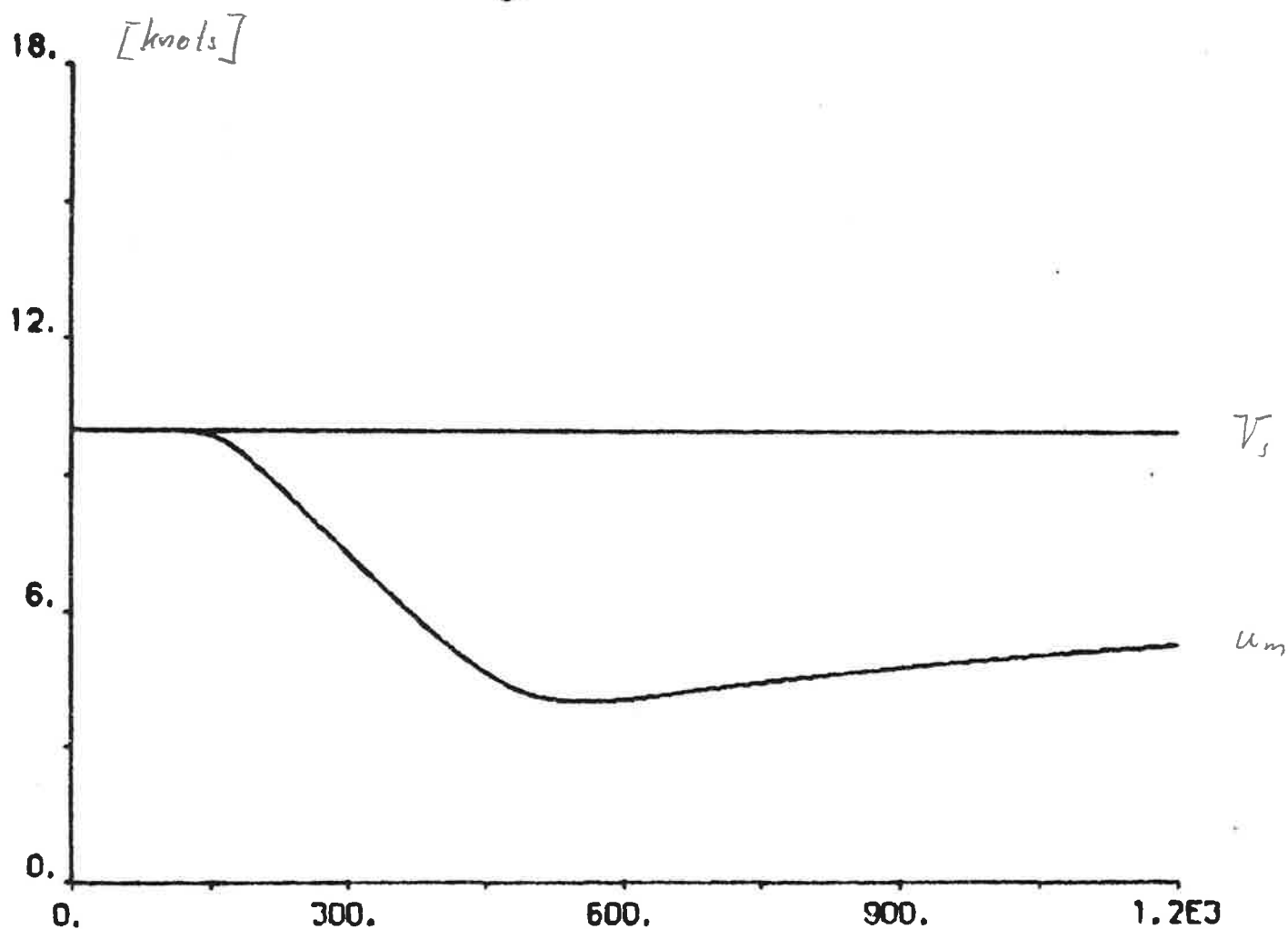
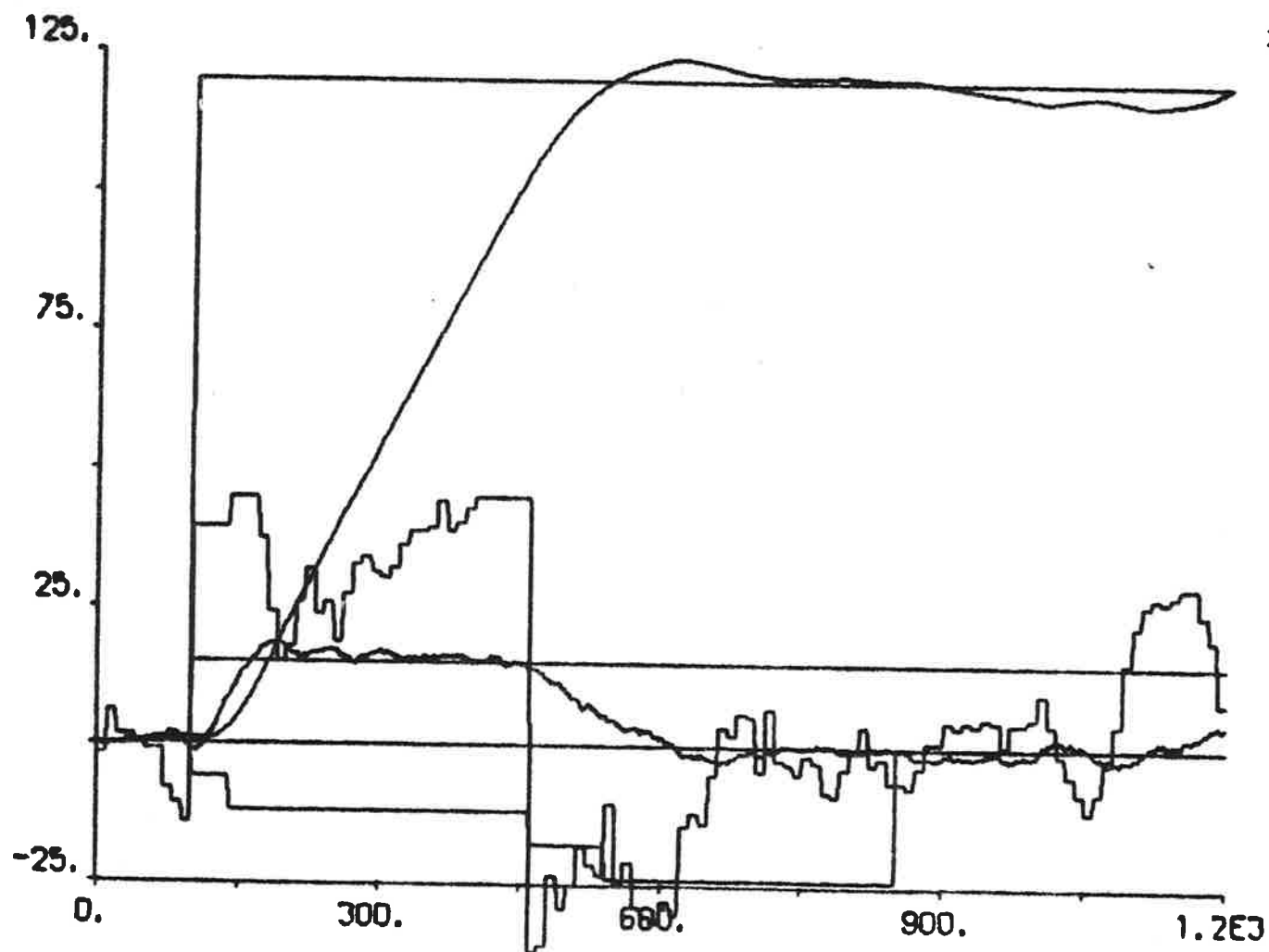


Fig. 4.74 - $T = 22.3$ m, $n_0 = 55.443$ rpm, $u_0 = 10$ knots, $\Delta\psi_{\text{ref}} = 120$ deg, $r_{\text{ref}} = 0.3$ deg/s, self-tuning regulator and yaw regulator using estimates from the Kalman filter ($\bar{\tau}_2 = 63.25$ s).

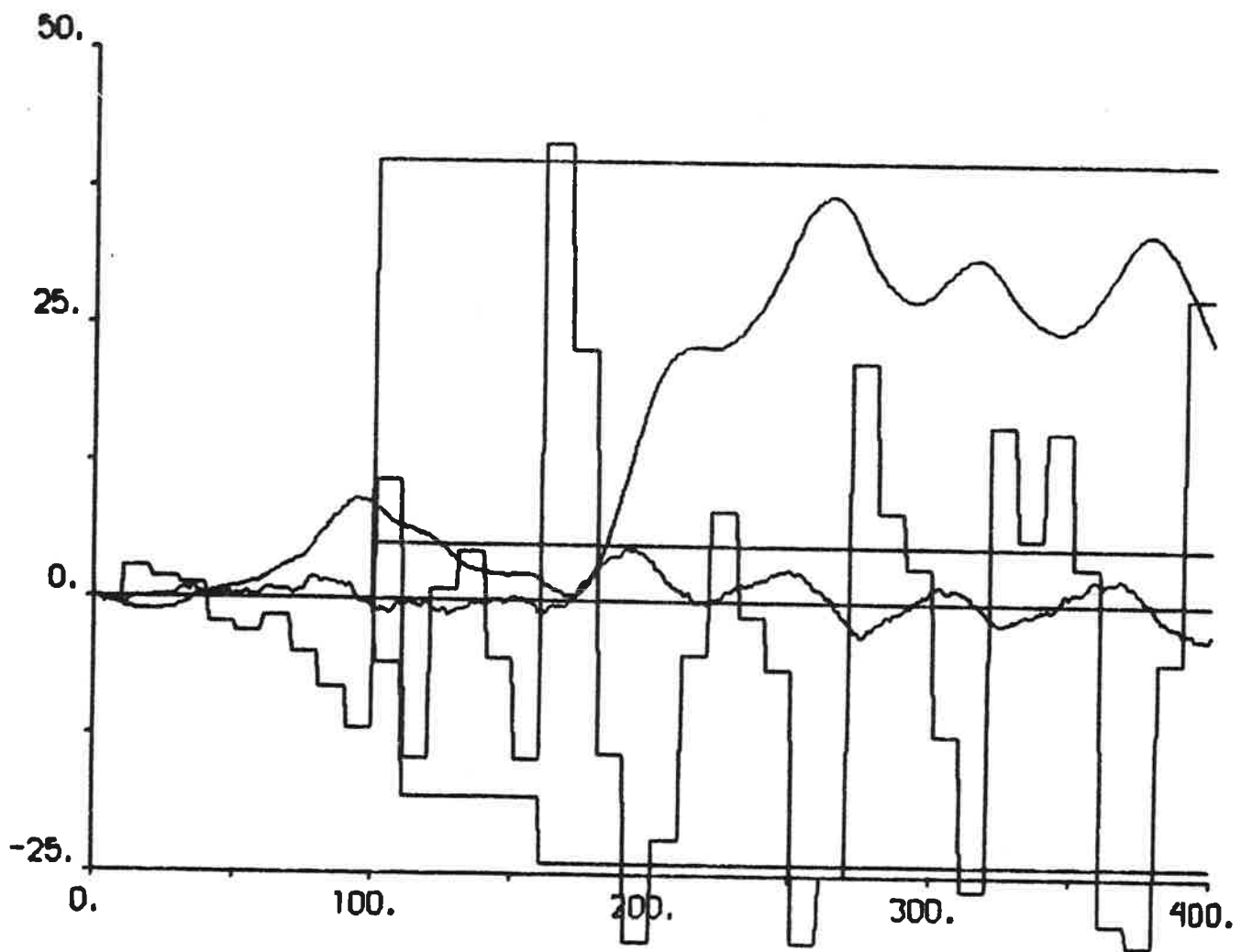


Fig. 4.75 - $T = 22.3$ m, $n_0 = 55.443$ rpm, $u_0 = 10$ knots,
 $\Delta\psi_{\text{ref}} = 4$ deg, $r_{\text{ref}} = 0.1$ deg/s, self-tuning
 regulator and yaw regulator using non-filtered
 measurements ($\bar{c}_2 = 63.25$ s).

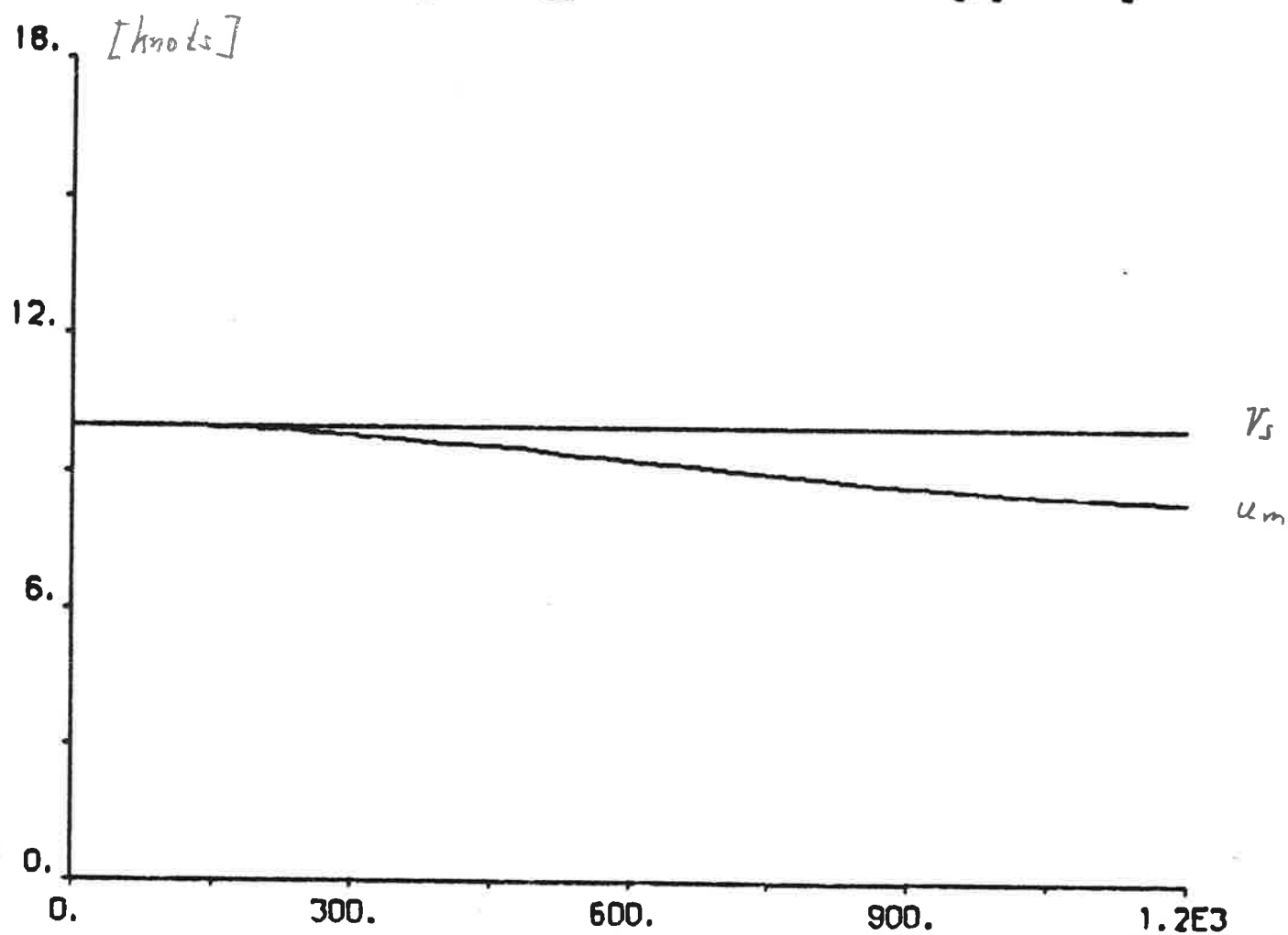
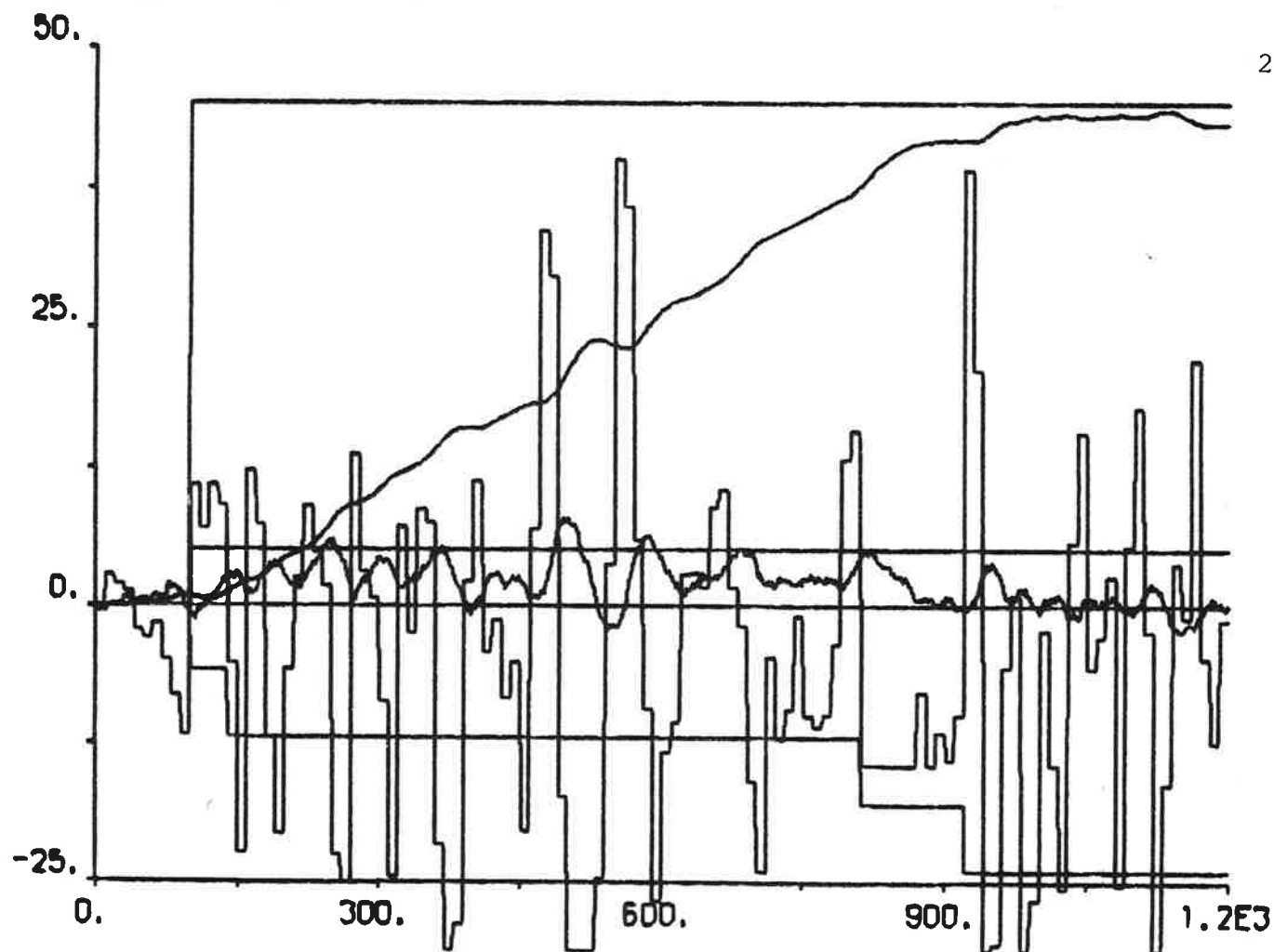
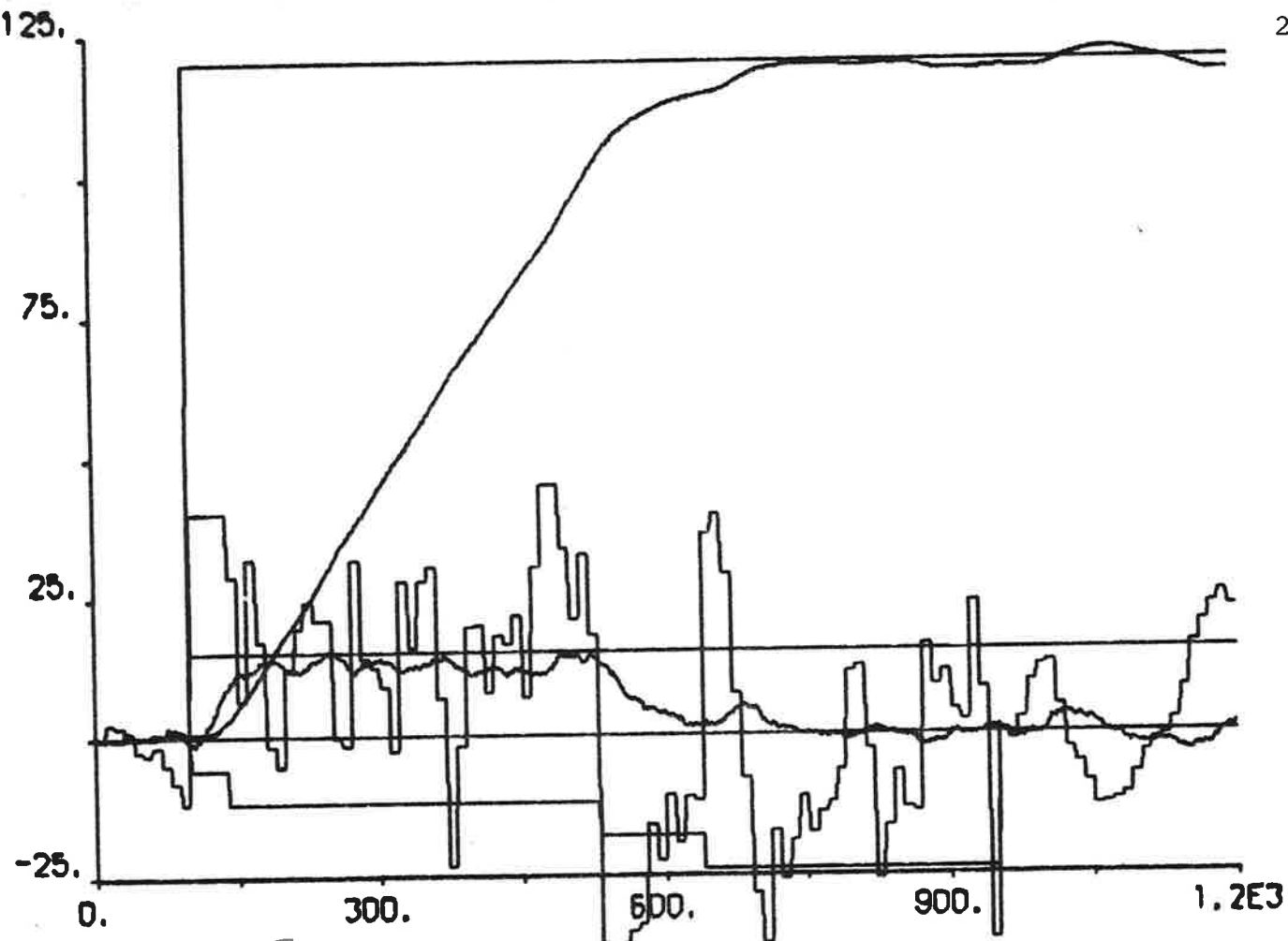


Fig. 4.76 - $T = 22.3$ m, $n_0 = 55.443$ rpm, $u_0 = 10$ knots, $\Delta\psi_{\text{ref}} = 45$ deg, $r_{\text{ref}} = 0.1$ deg/s, self-tuning regulator and yaw regulator using non-filtered measurements ($\bar{c}_2 = 63.25$ s).



[knots]

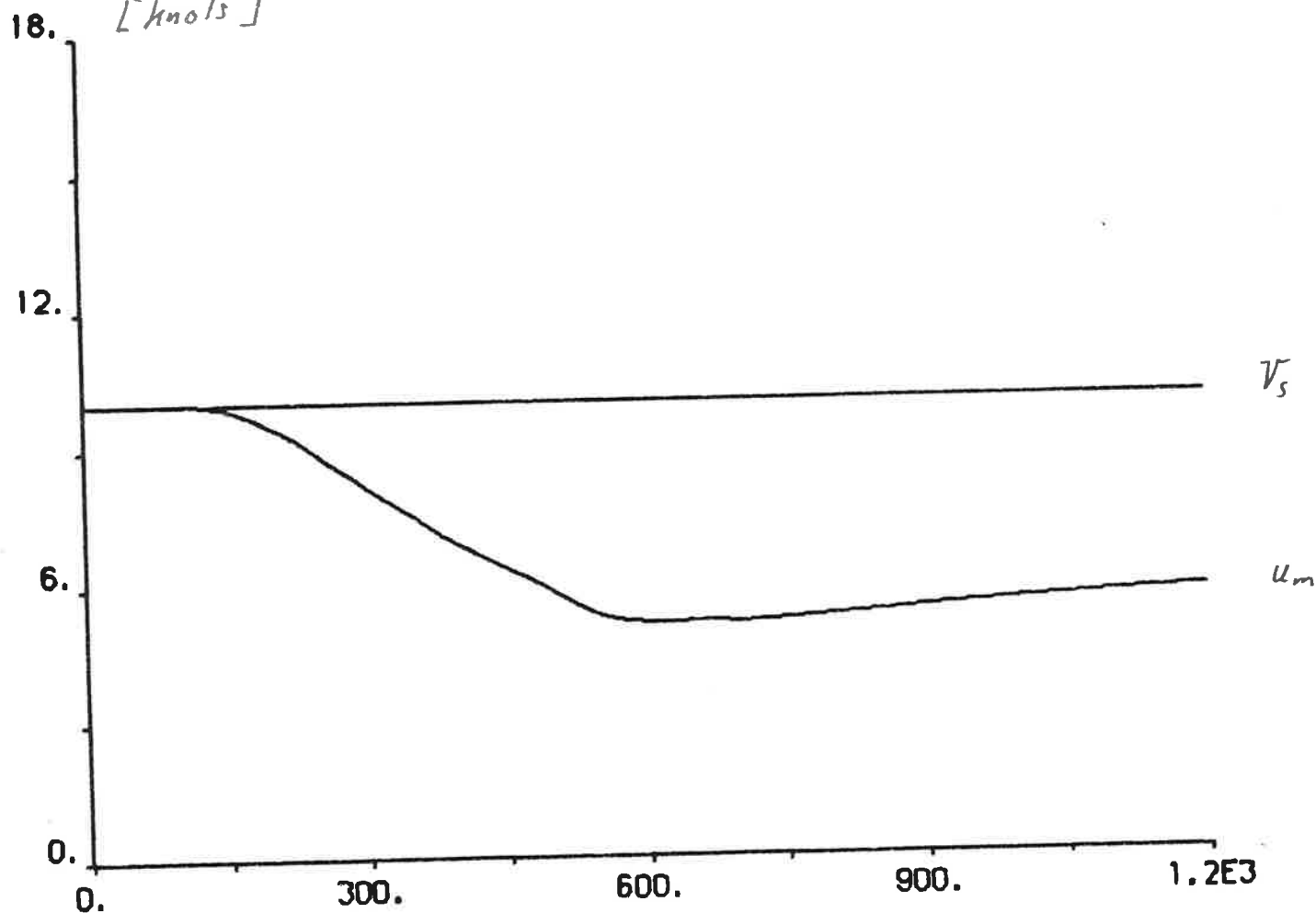


Fig. 4.77 - $T = 22.3$ m, $n_0 = 55.443$ rpm, $u_0 = 10$ knots, $\Delta\psi_{\text{ref}} = 120$ deg, $r_{\text{ref}} = 0.3$ deg/s, self-tuning regulator and yaw regulator using non-filtered measurements ($\bar{c}_2 = 63.25$ s).

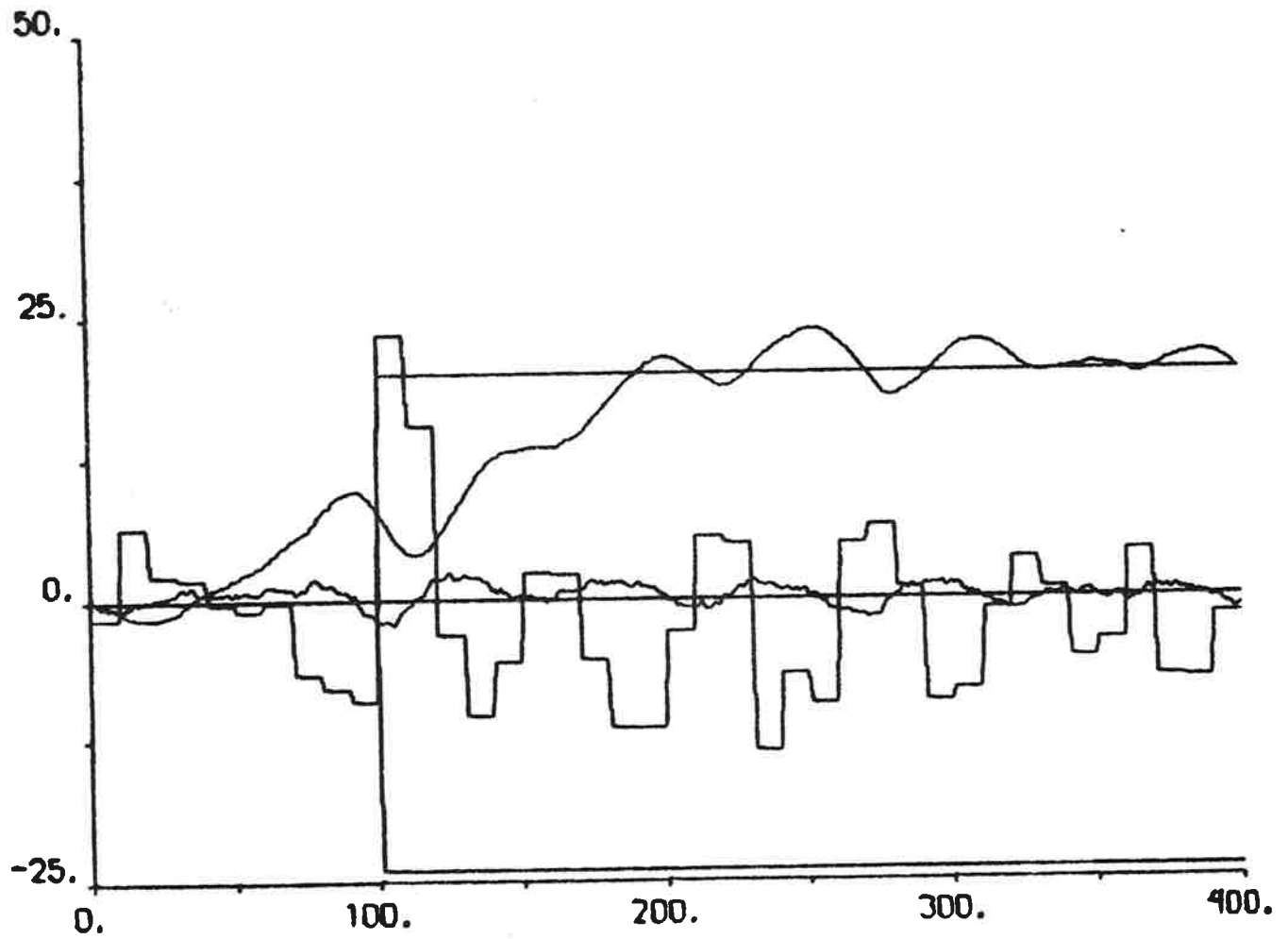


Fig. 4.78 - $T = 10.5$ m, $n_0 = 55.443$ rpm, $u_0 = 10$ knots, $\Delta\psi_{\text{ref}} = 2$ deg, $r_{\text{ref}} = 0$ deg/s, self-tuning regulator and yaw regulator using estimates from the Kalman filter ($\bar{c}_2 = 63.25$ s).

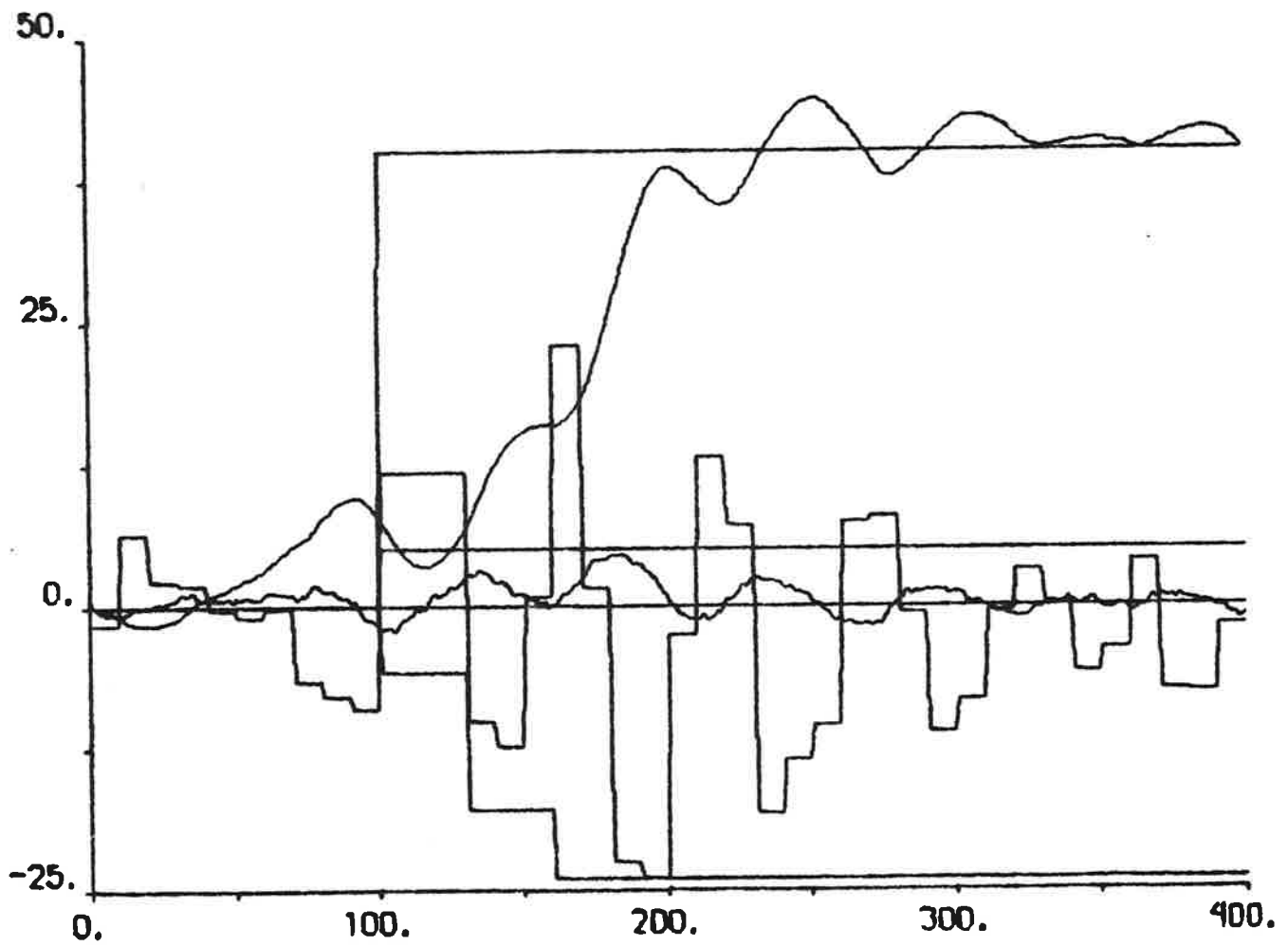


Fig. 4.79 - $T = 10.5$ m, $n_0 = 55.443$ rpm, $u_0 = 10$ knots,
 $\Delta\psi_{\text{ref}} = 4$ deg, $r_{\text{ref}} = 0.1$ deg/s, self-tuning
 regulator and yaw regulator using estimates from
 the Kalman filter ($\bar{c}_2 = 63.25$ s).

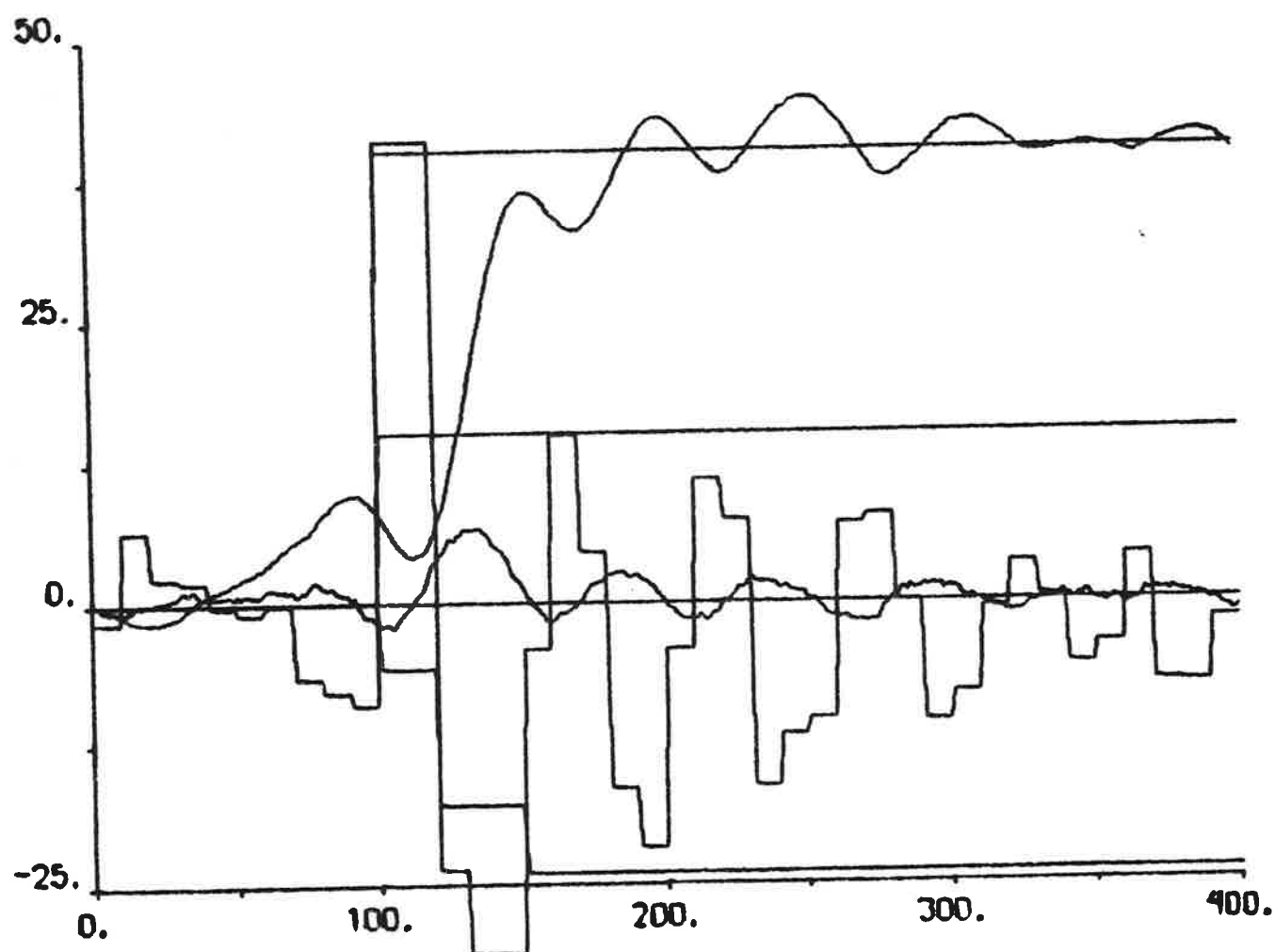


Fig. 4.80 - $T = 10.5$ m, $n_0 = 55.443$ rpm, $u_0 = 10$ knots,
 $\Delta\psi_{\text{ref}} = 4$ deg, $r_{\text{ref}} = 0.3$ deg/s, self-tuning
 regulator and yaw regulator using estimates
 from the Kalman filter ($\tau_2 = 63.25$ s).

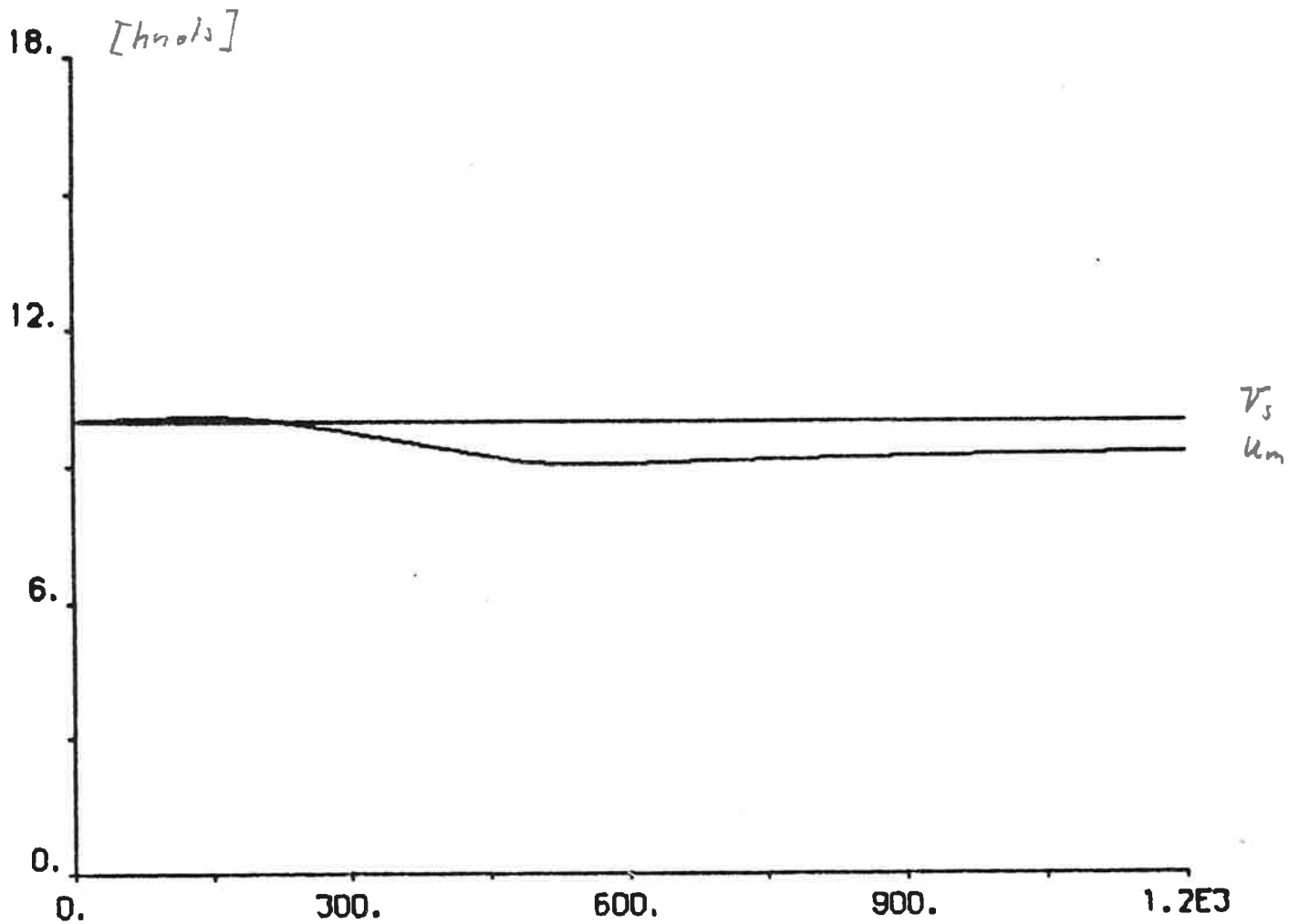
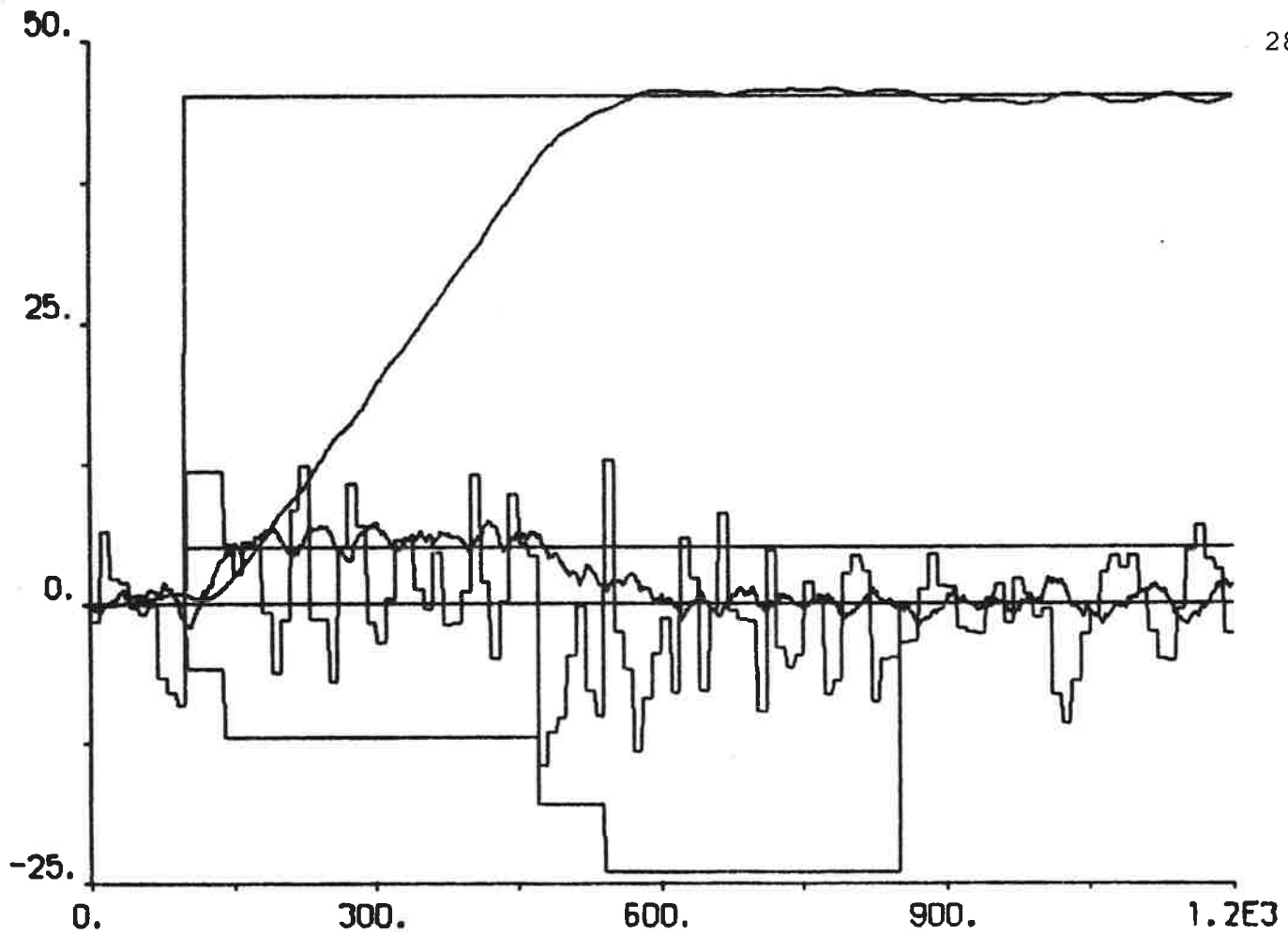


Fig. 4.81 - $T = 10.5$ m, $n_0 = 55.443$ rpm, $u_0 = 10$ knots, $\Delta\psi_{\text{ref}} = 45$ deg, $r_{\text{ref}} = 0.1$ deg/s, self-tuning regulator and yaw regulator using estimates from the Kalman filter ($\bar{c}_2 = 63.25$ s).

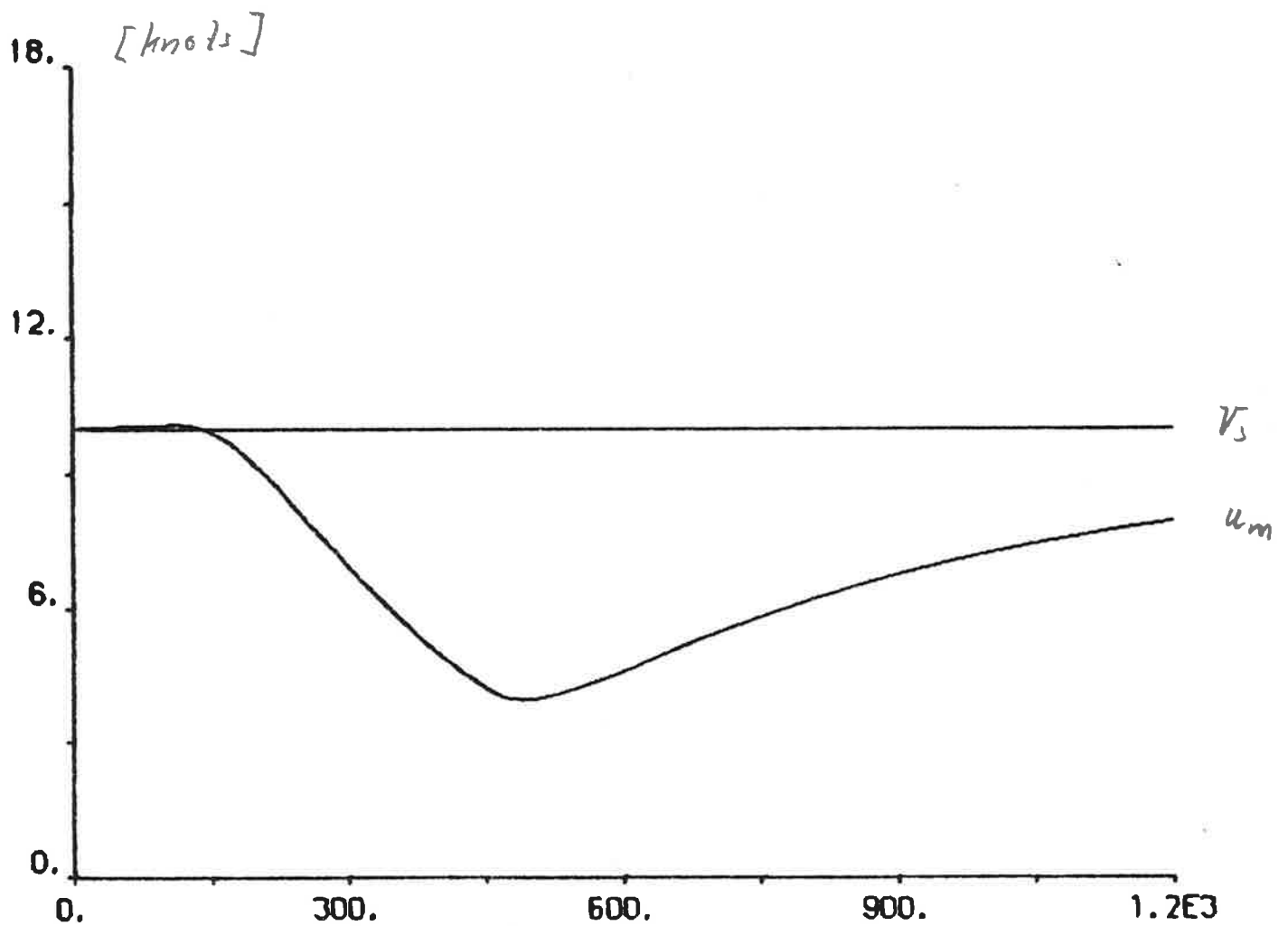
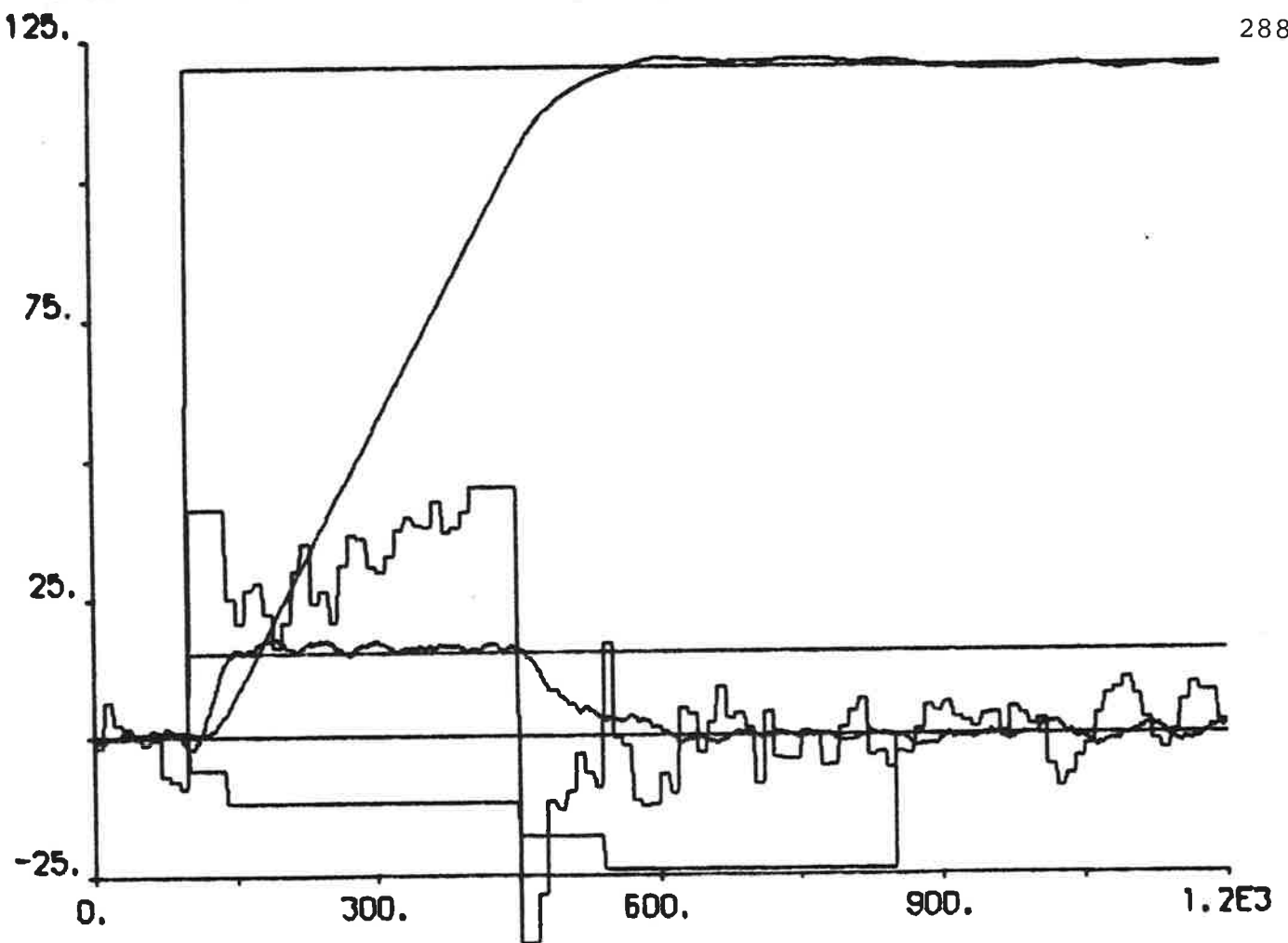


Fig. 4.82 - $T = 10.5$ m, $n_0 = 55.443$ rpm, $u_0 = 10$ knots, $\Delta\psi_{\text{ref}} = 120$ deg, $r_{\text{ref}} = 0.3$ deg/s, self-tuning regulator and yaw regulator using estimates from the Kalman filter ($\bar{c}_2 = 63.25$ s).

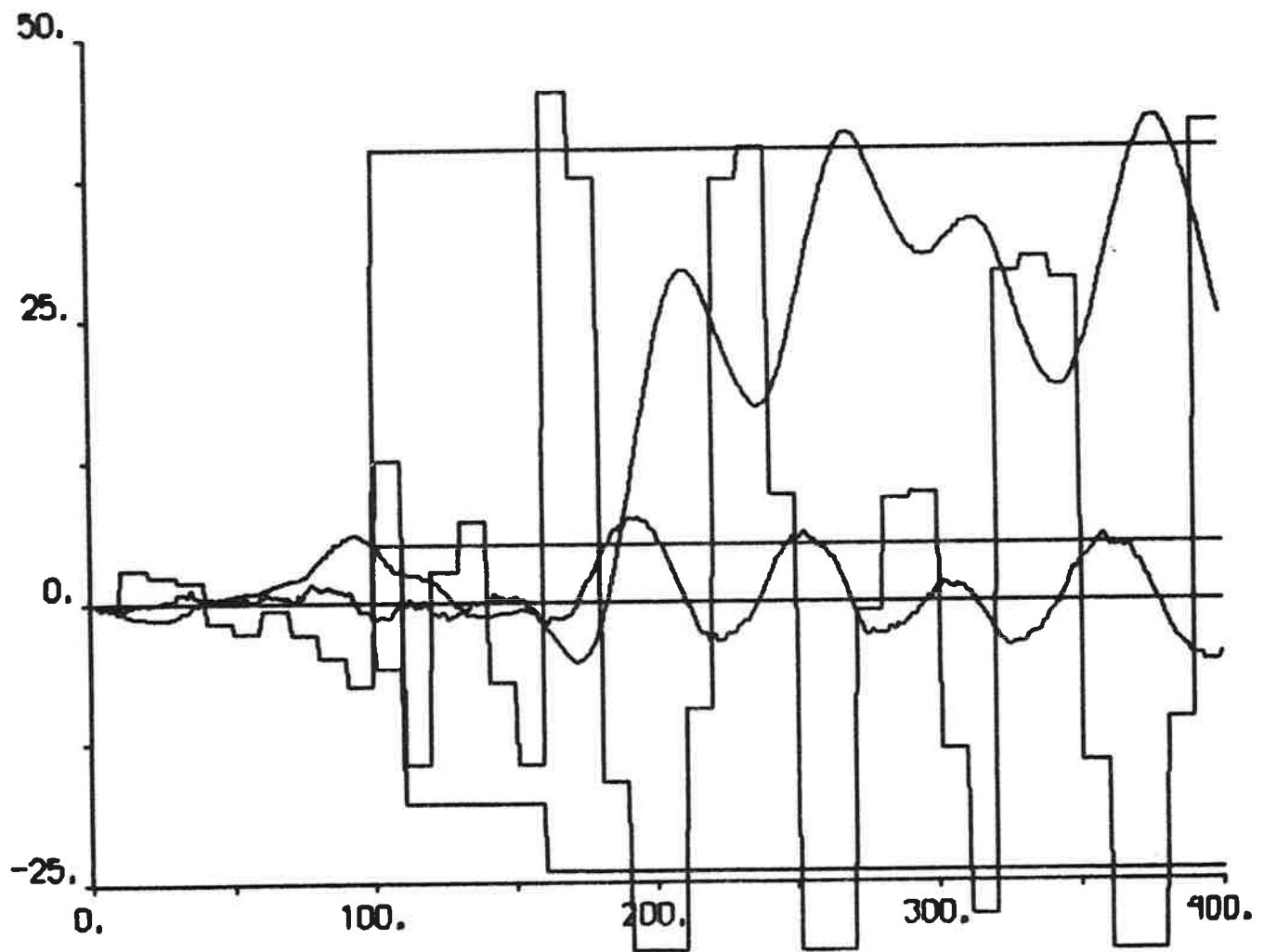


Fig. 4.83 - $T = 10.5$ m, $n_0 = 55.443$ rpm, $u_0 = 10$ knots,
 $\Delta\psi_{\text{ref}} = 4$ deg, $r_{\text{ref}} = 0.1$ deg/s, self-tuning
 regulator and yaw regulator using non-filtered
 measurements ($\bar{c}_2 = 63.25$ s).

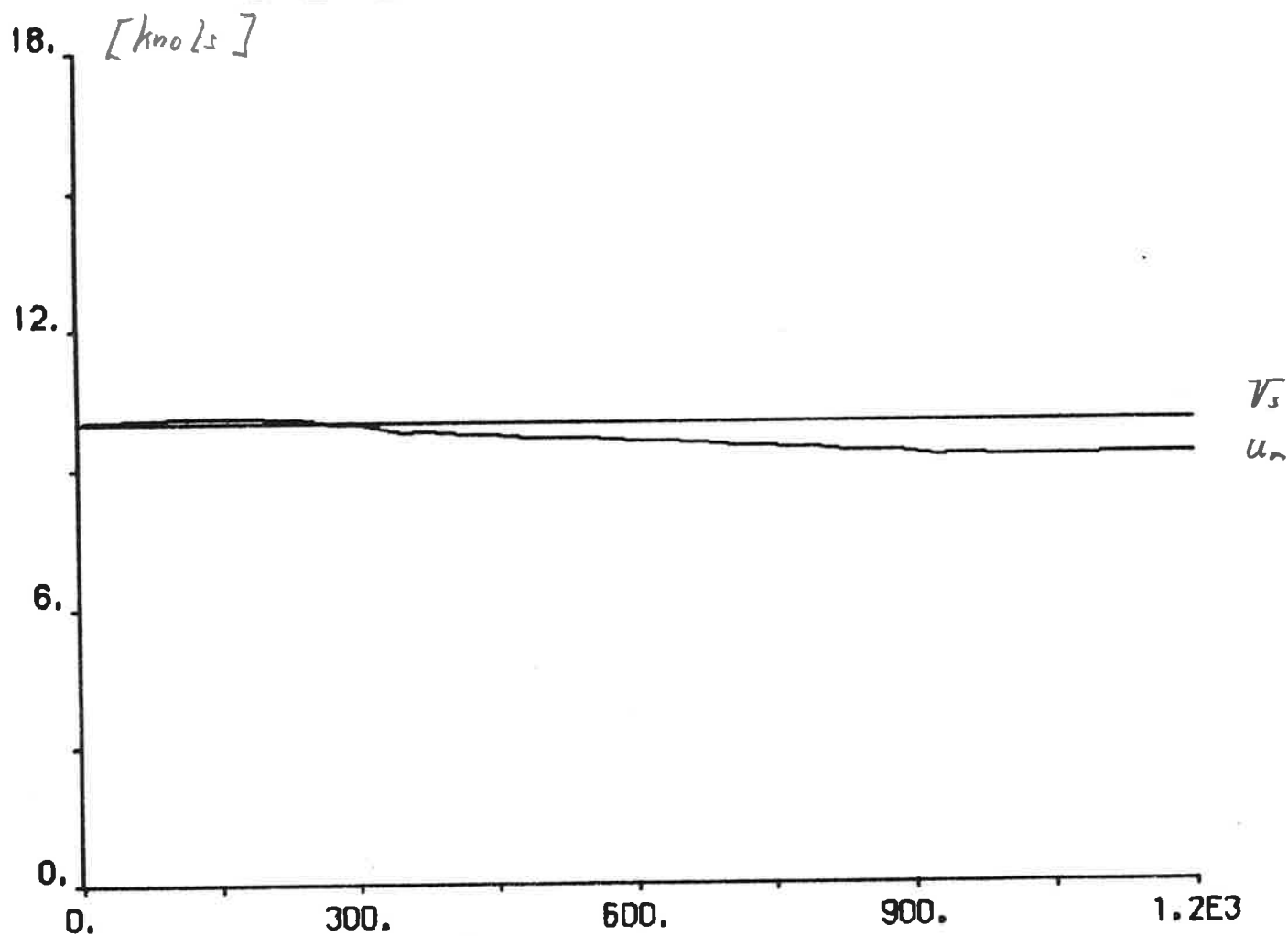
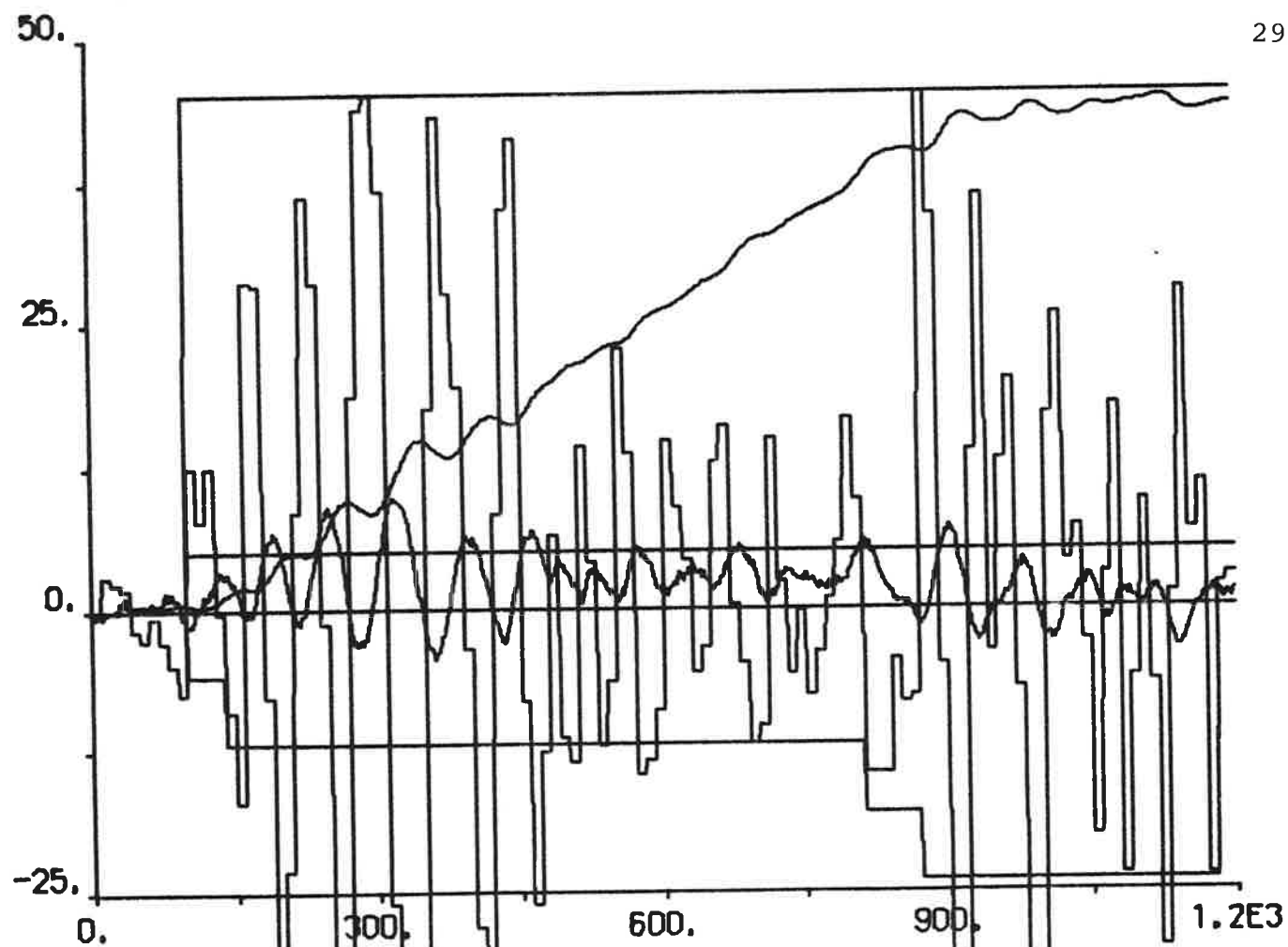


Fig. 4.84 - $T = 10.5$ m, $n_0 = 55.443$ rpm, $u_0 = 10$ knots, $\Delta\psi_{\text{ref}} = 45$ deg, $r_{\text{ref}} = 0.1$ deg/s, self-tuning regulator and yaw regulator using non-filtered measurements ($\bar{c}_2 = 63.25$ s).

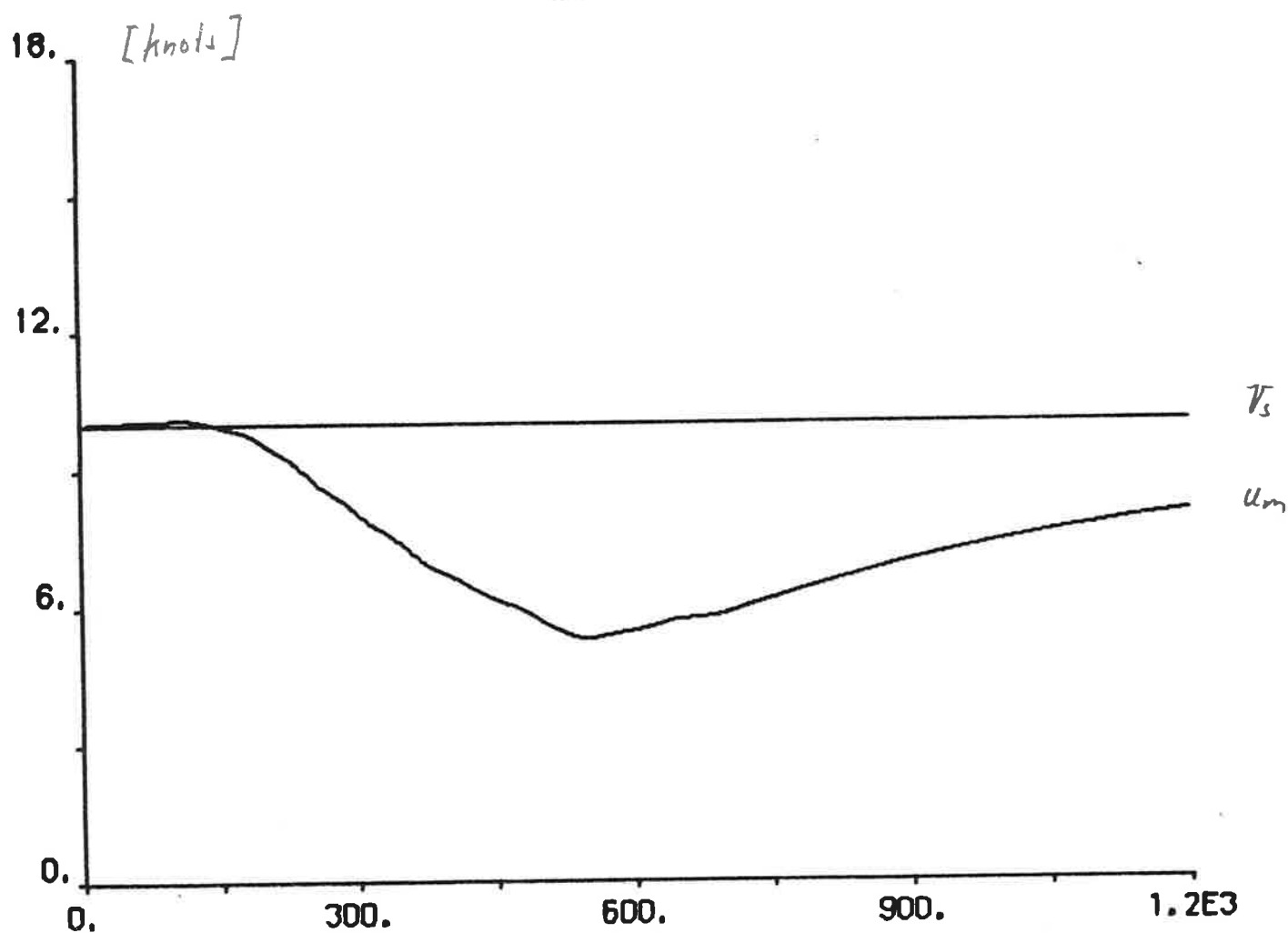
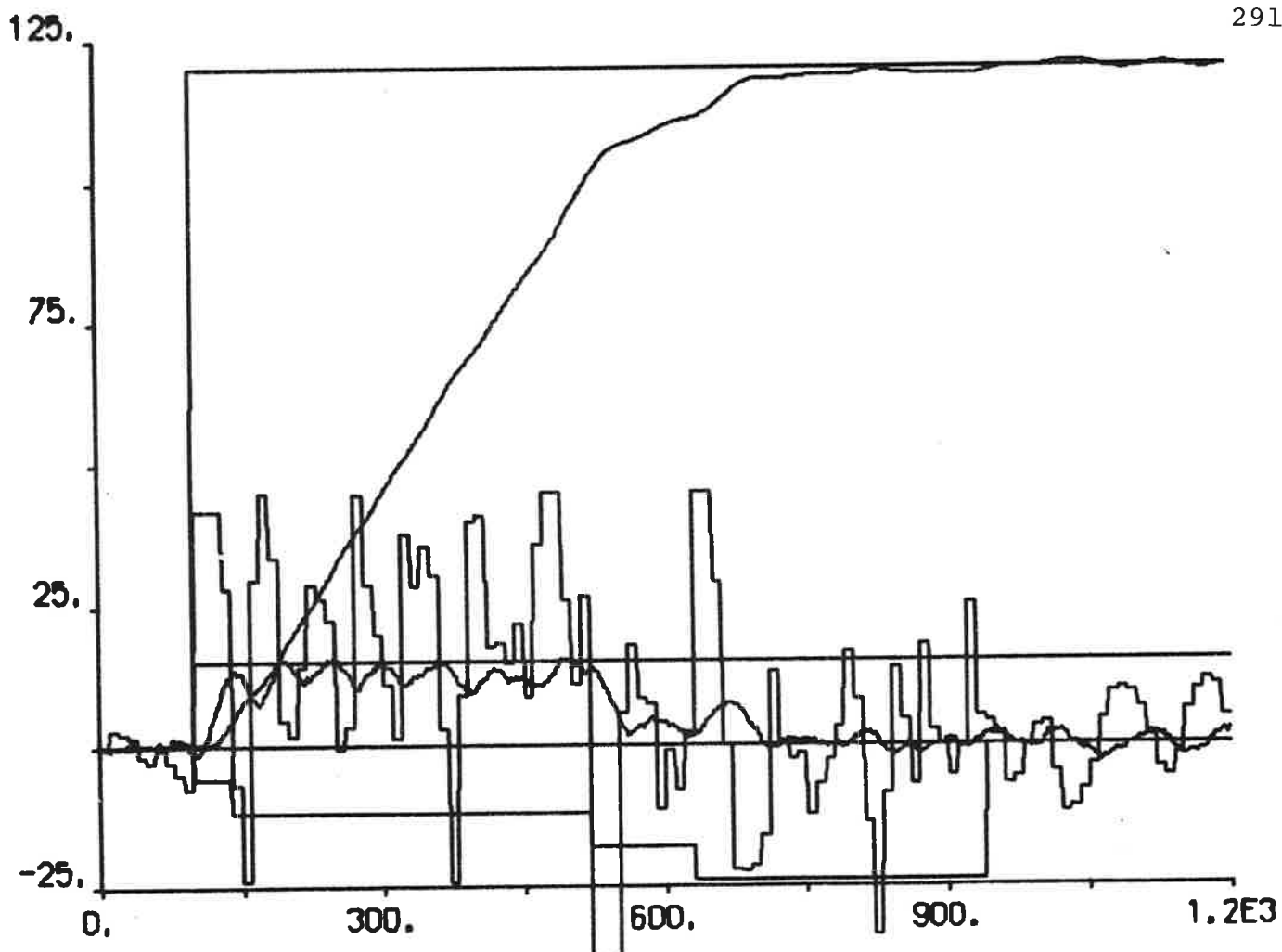


Fig. 4.85 - $T = 10.5$ m, $n_0 = 55.443$ rpm, $u_0 = 10$ knots, $\Delta\psi_{\text{ref}} = 120$ deg, $r_{\text{ref}} = 0.3$ deg/s, self-tuning regulator and yaw regulator using non-filtered measurements ($\bar{c}_2 = 63.25$ s).

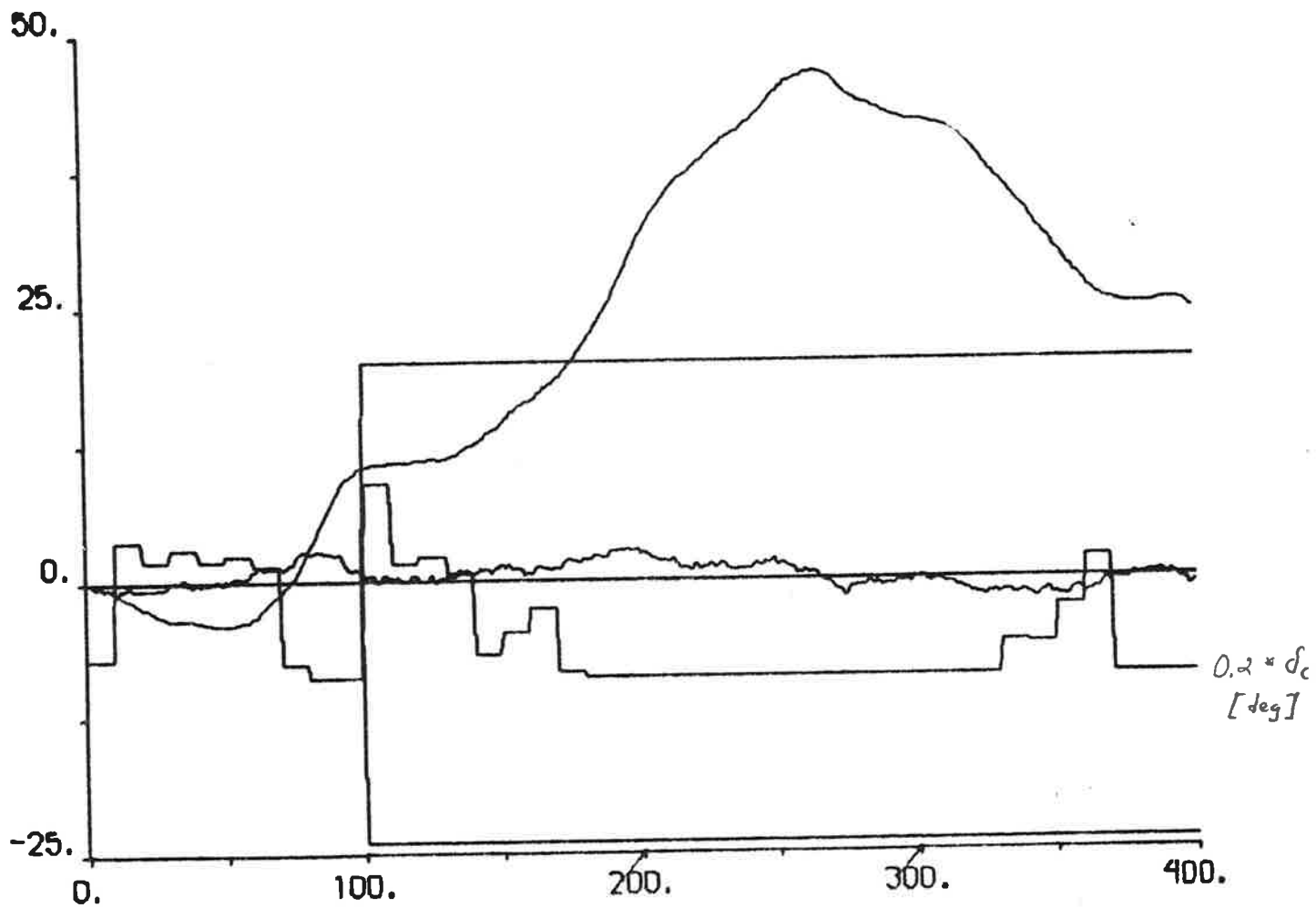


Fig. 4.86 - $T = 22.3$ m, $n_0 = 22.1772$ rpm, $u_0 = 4$ knots,
 $\Delta\psi_{\text{ref}} = 2$ deg, $r_{\text{ref}} = 0$ deg/s, self-tuning
 regulator and yaw regulator using estimates from
 the Kalman filter ($\bar{c}_2 = 100$ s).

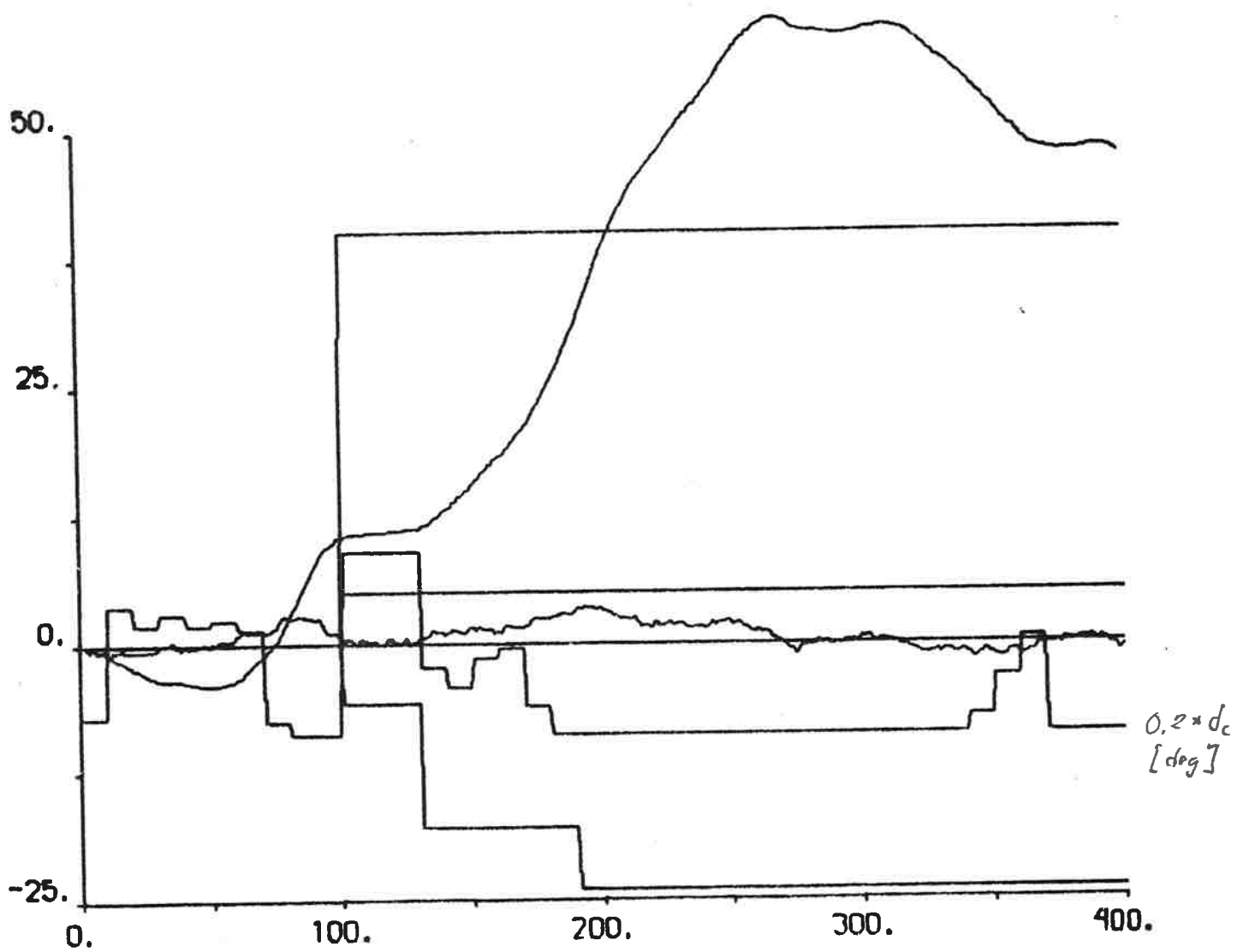


Fig. 4.87 - $T = 22.3$ m, $n_0 = 22.1772$ rpm, $u_0 = 4$ knots,
 $\Delta\psi_{\text{ref}} = 4$ deg, $r_{\text{ref}} = 0.1$ deg/s, self-tuning
 regulator and yaw regulator using estimates
 from the Kalman filter ($\bar{c}_2 = 100$ s).

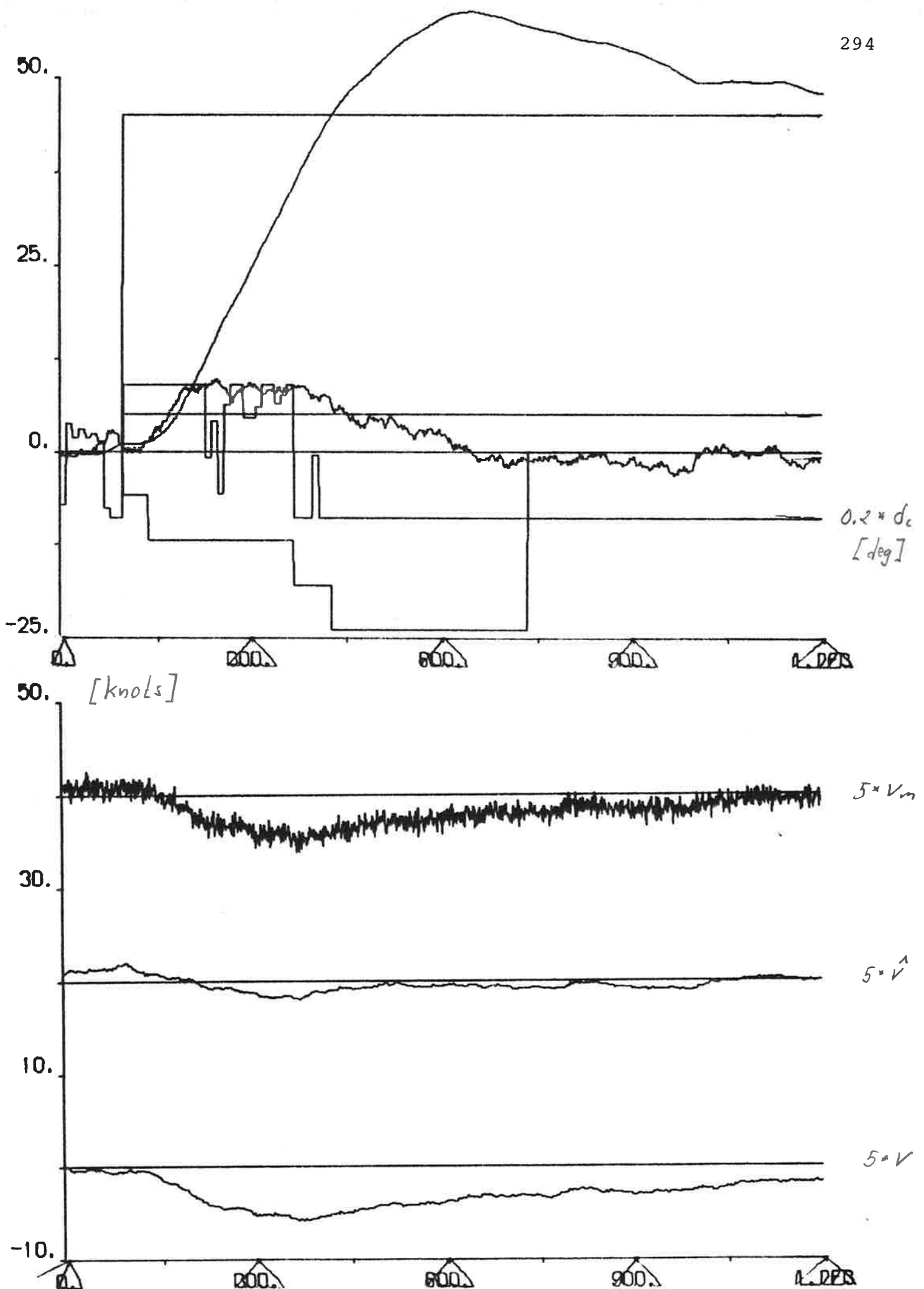


Fig. 4.88 a - $T = 22.3$ m, $n_0 = 22.1772$ rpm, $u_0 = 4$ knots, $\Delta\psi_{\text{ref}} = 45$ deg, $r_{\text{ref}} = 0.1$ deg/s, self-tuning regulator and yaw regulator using estimates from the Kalman filter ($c_2 = 100$ s).

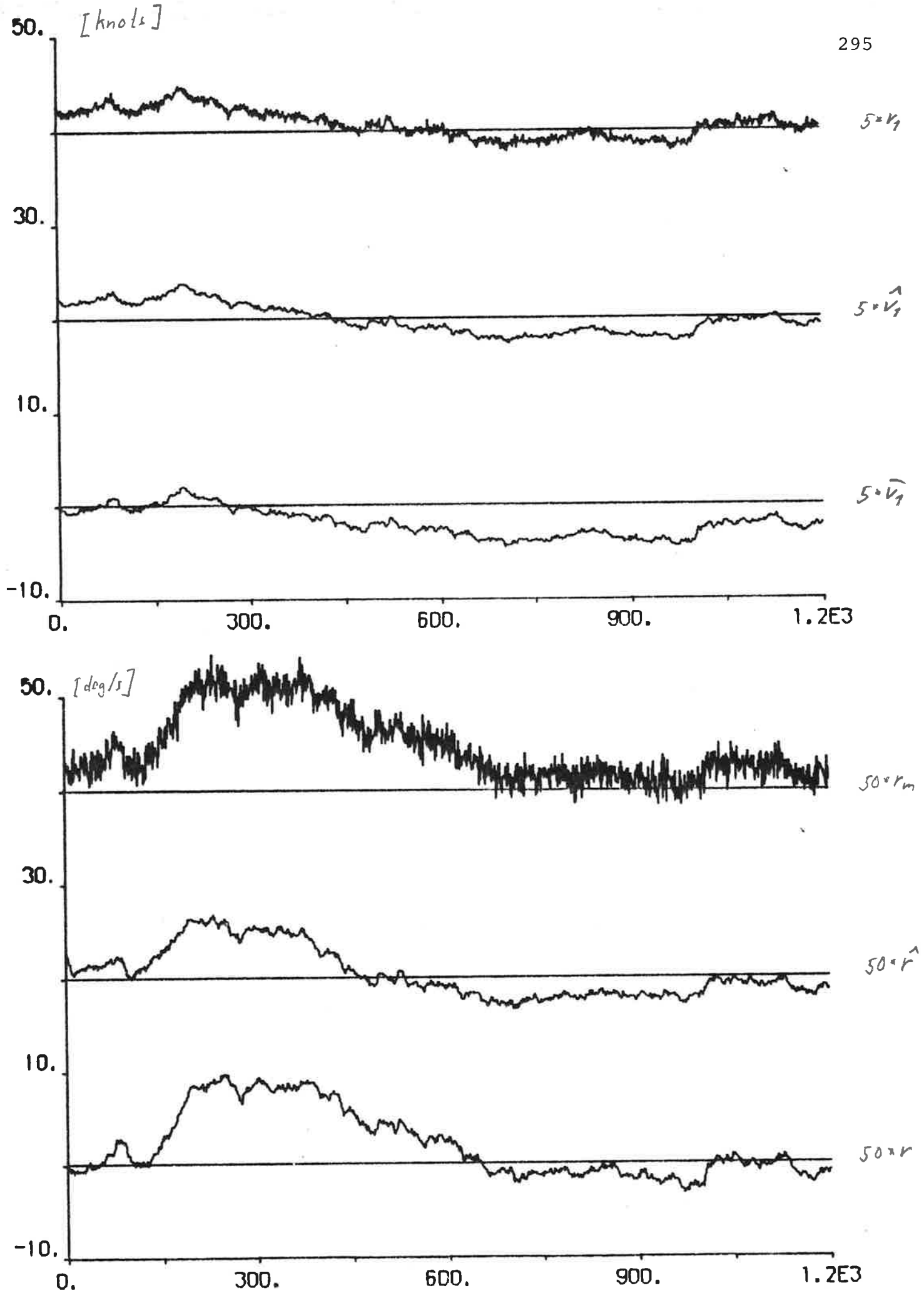


Fig. 4.88 b

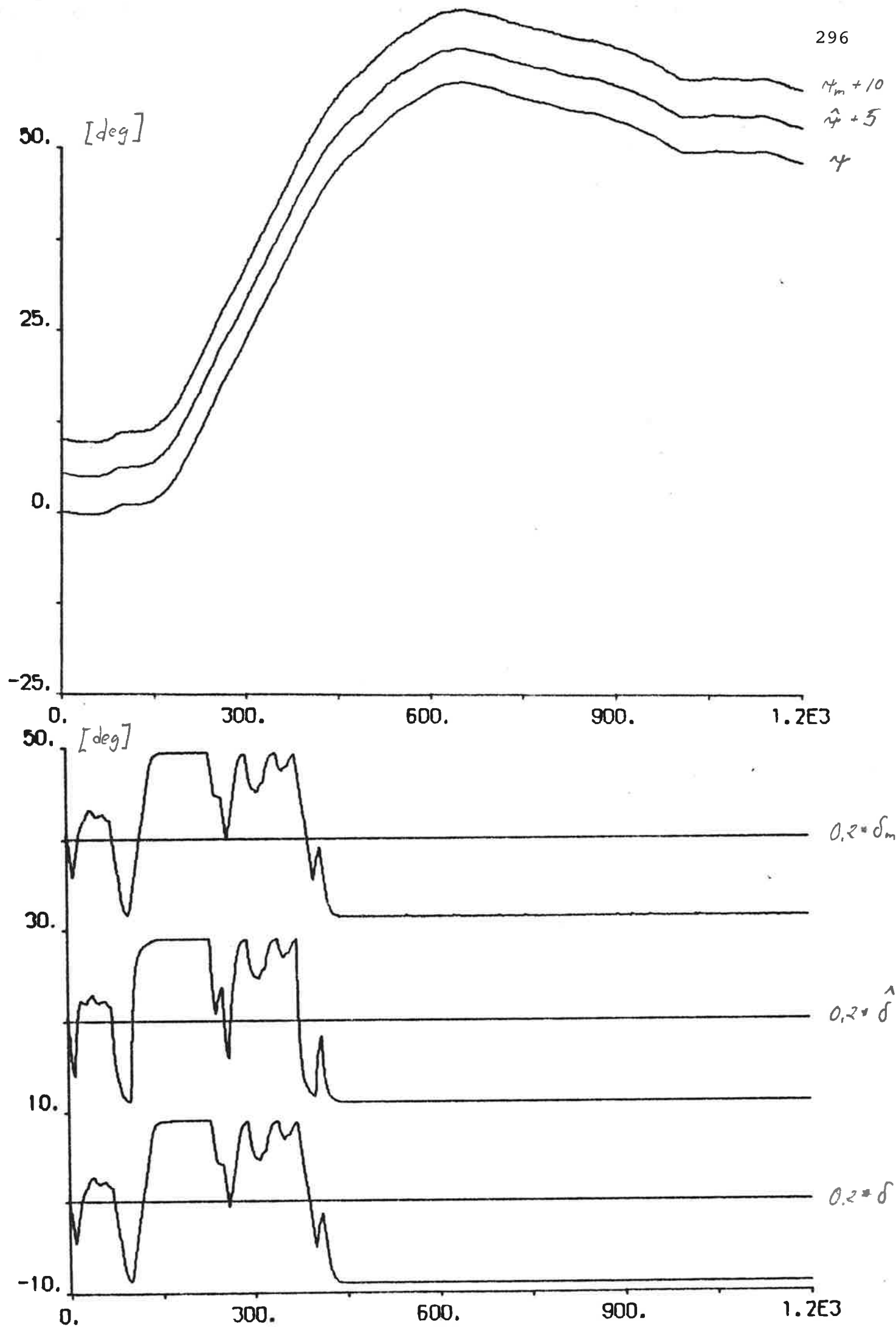


Fig. 4.88 c

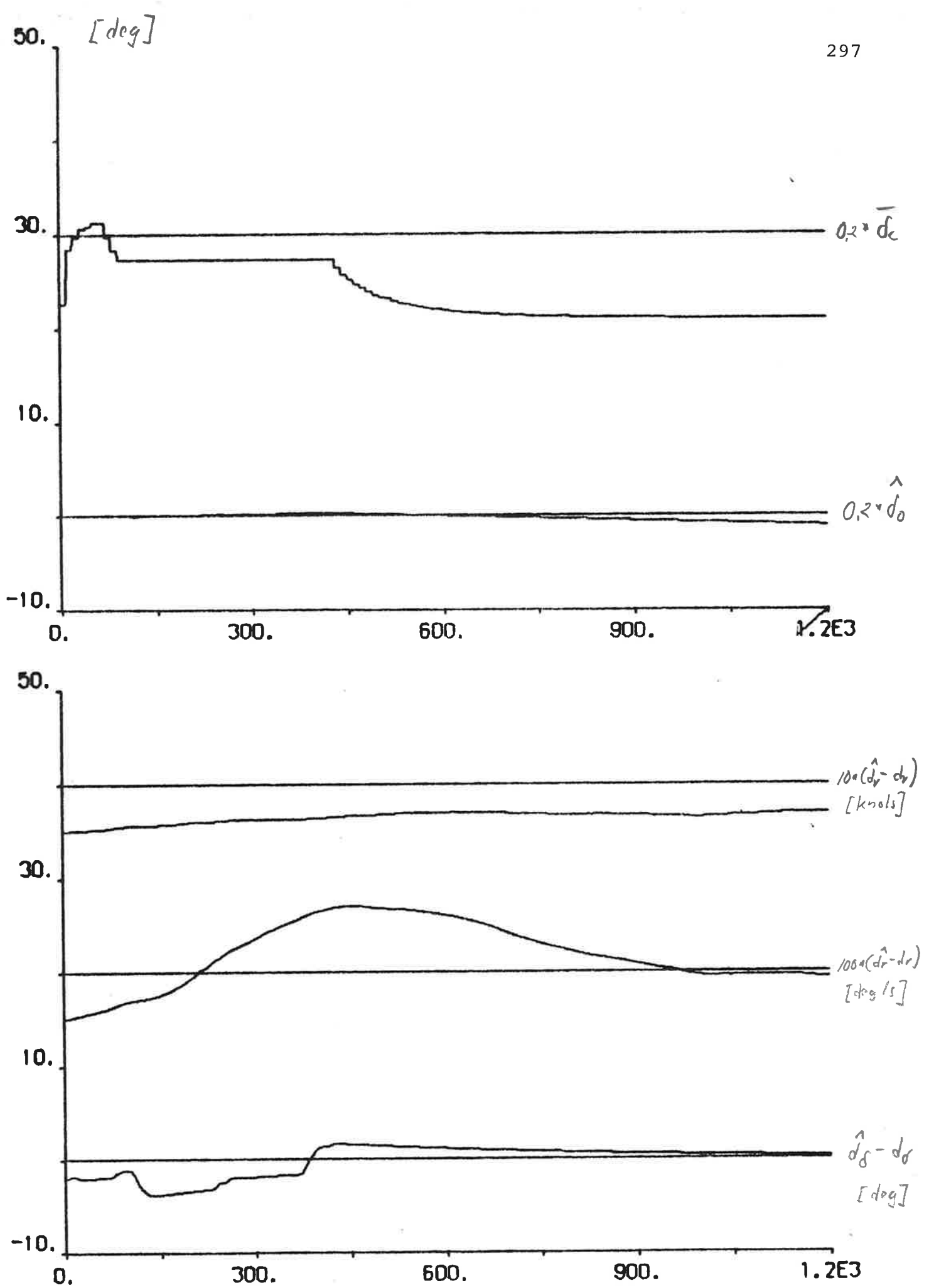


Fig. 4.88 d

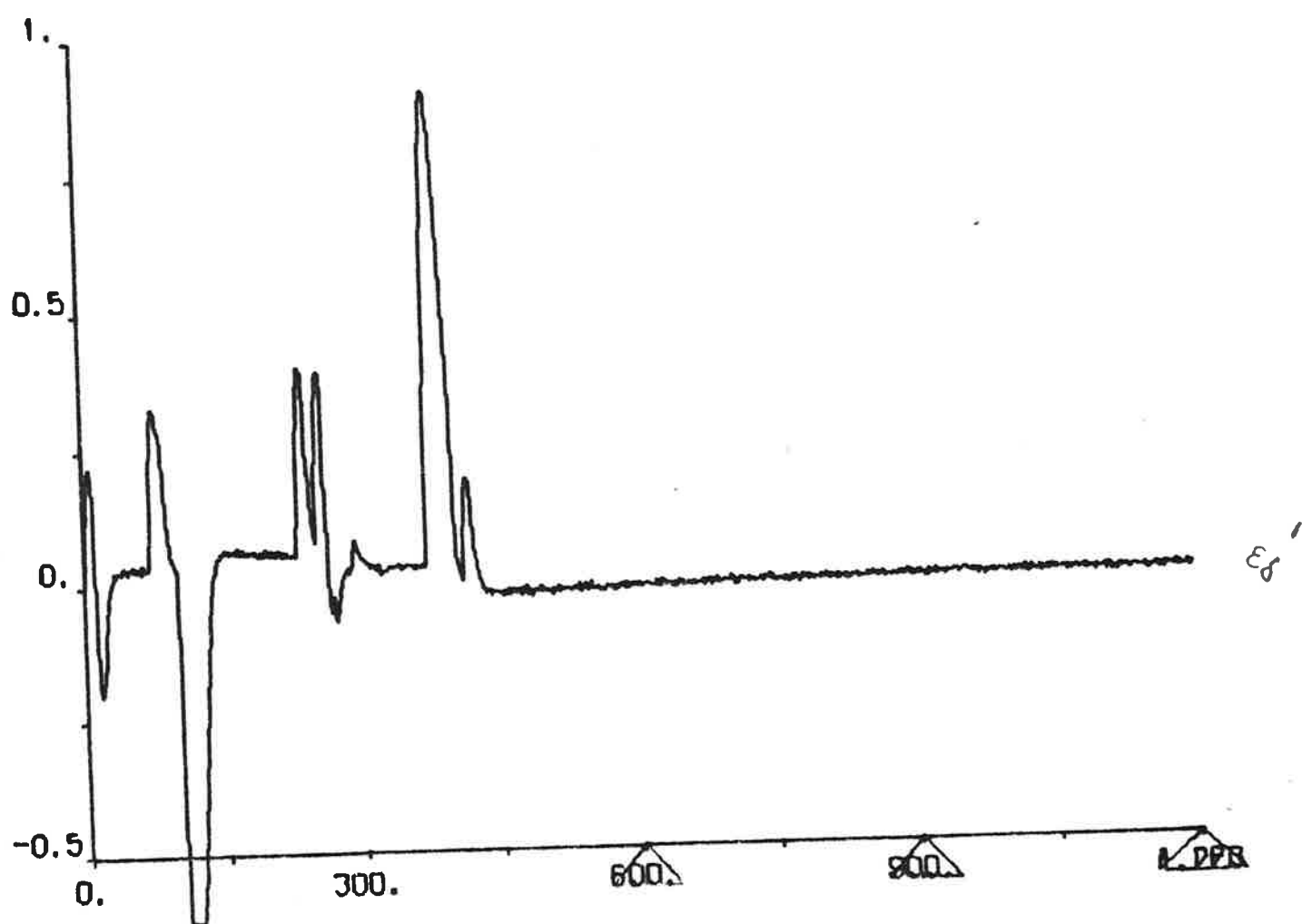
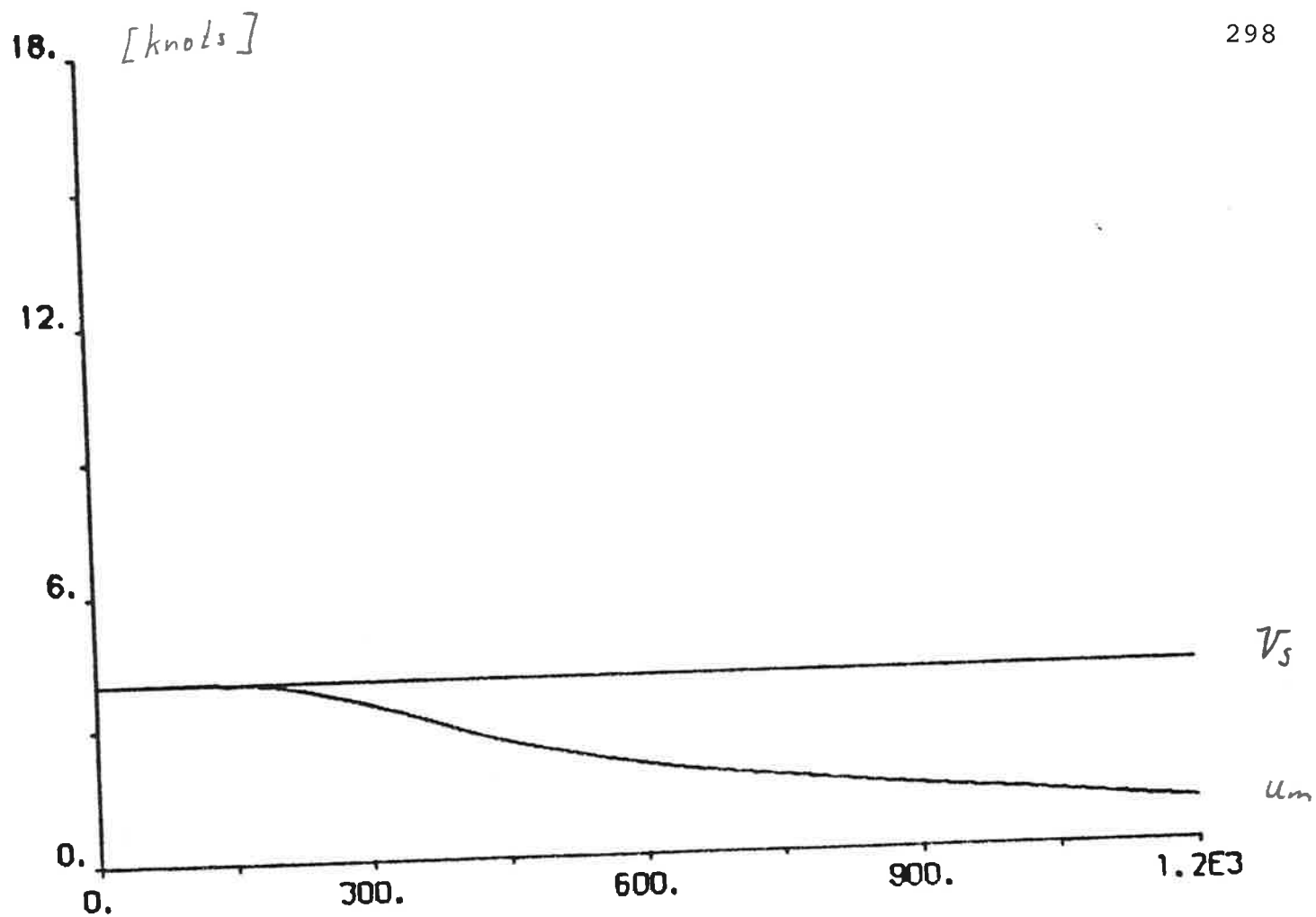


Fig. 4.88 e

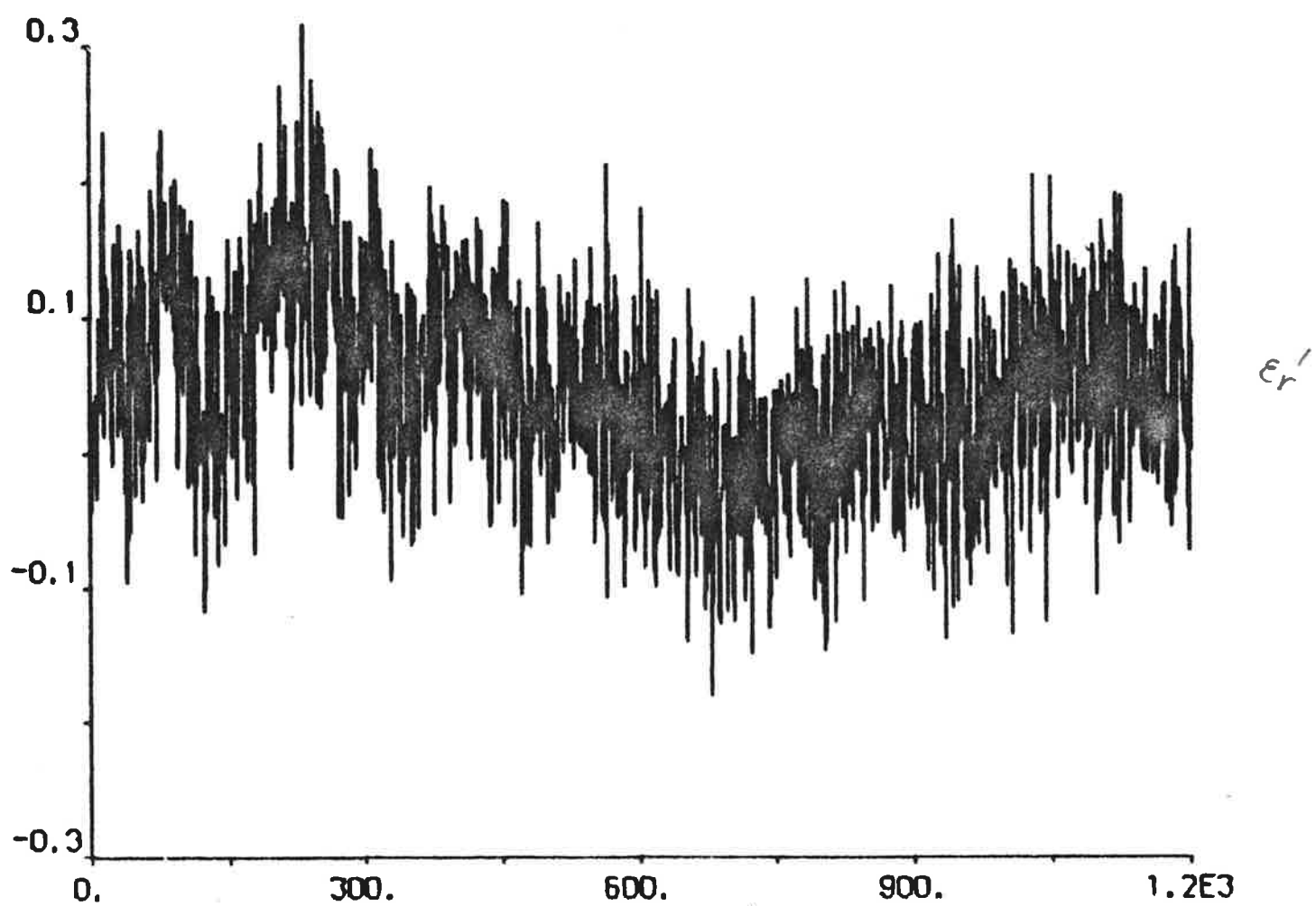
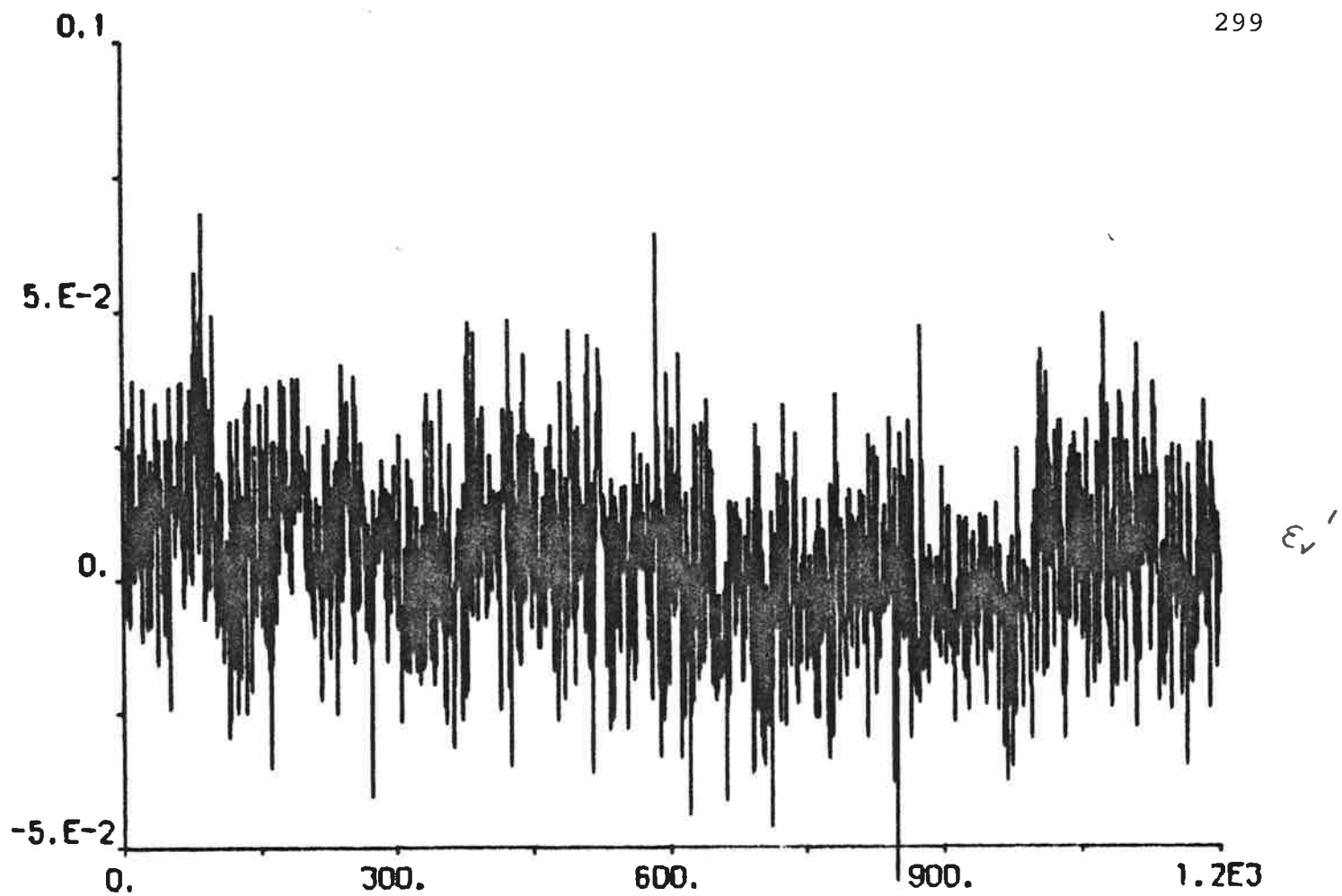


Fig. 4.88 f

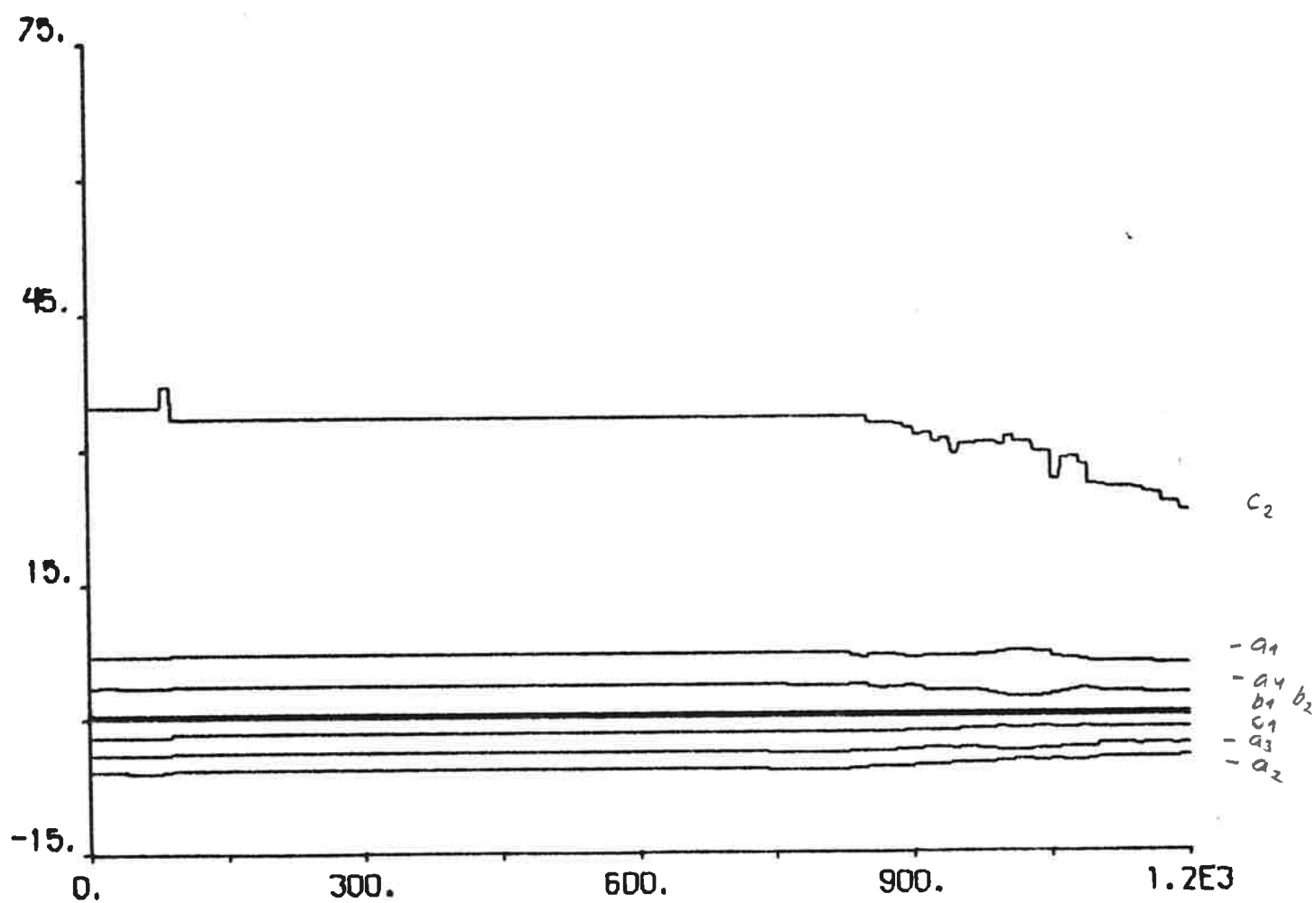
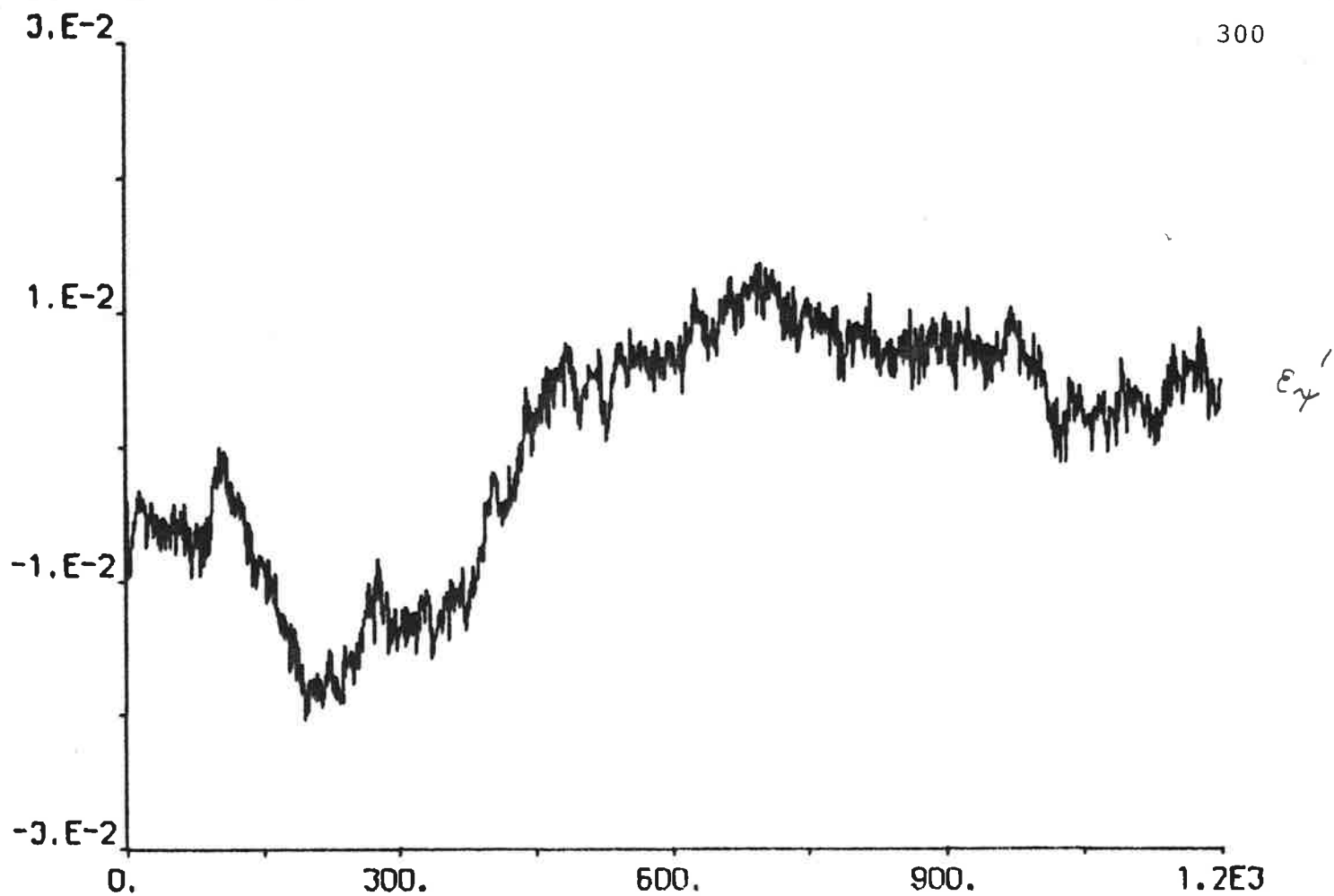


Fig. 4.88 a

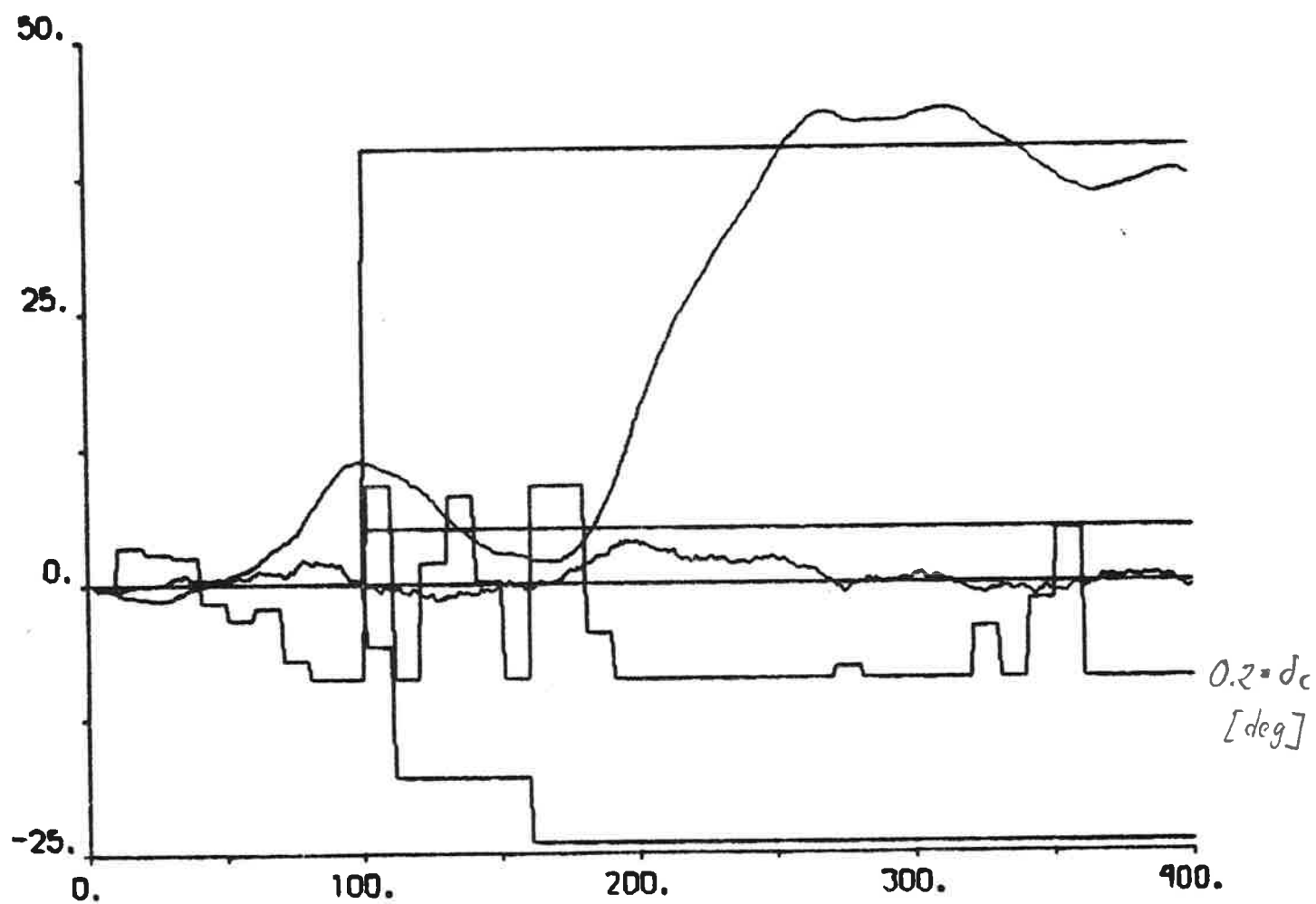


Fig. 4.89 - $T = 22.3$ m, $n_0 = 22.1772$ rpm, $u_0 = 4$ knots,
 $\Delta\psi_{\text{ref}} = 4$ deg, $r_{\text{ref}} = 0.1$ deg/s, self-tuning
 regulator and yaw regulator using non-filtered
 measurements ($\bar{c}_2 = 100$ s).

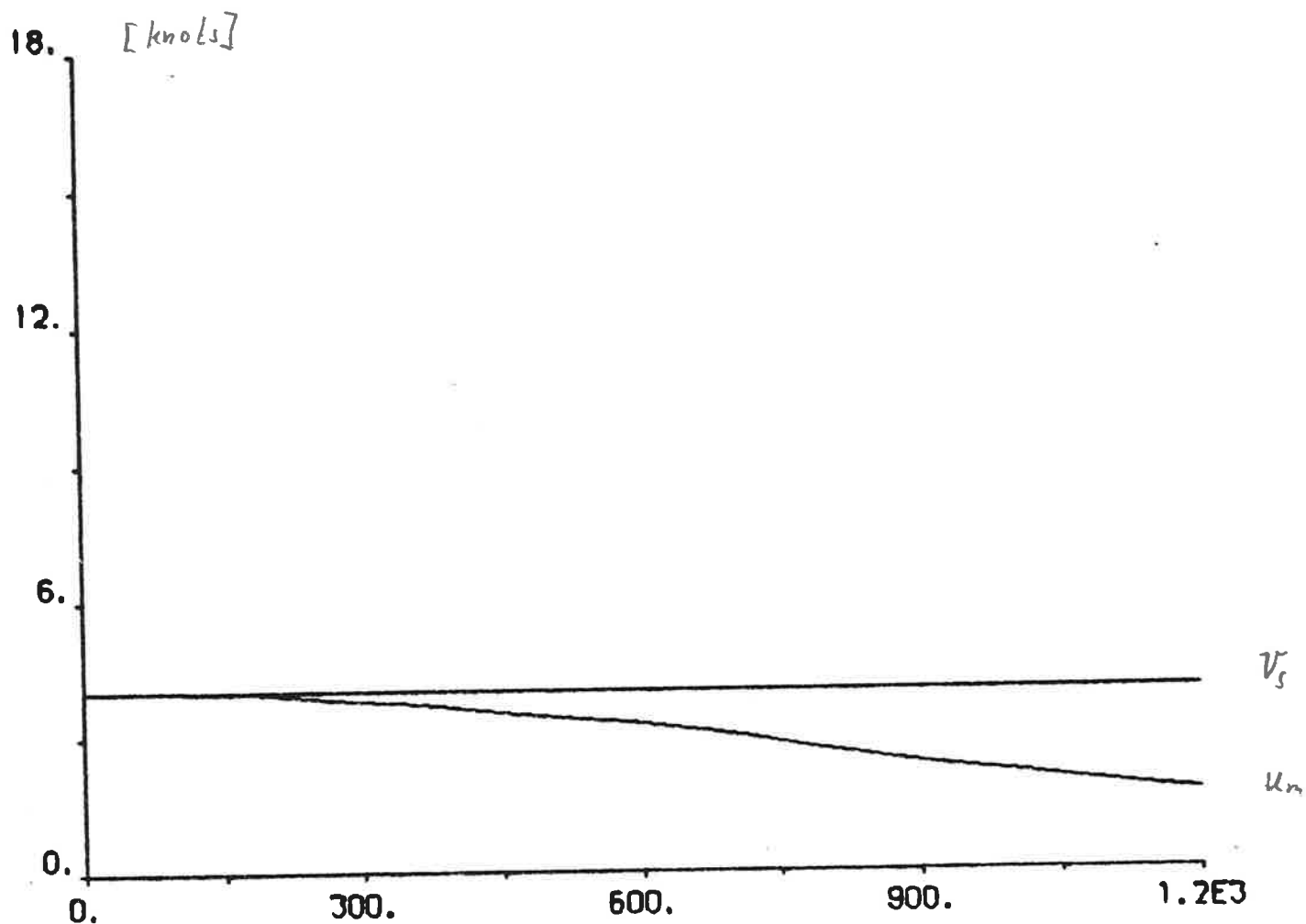
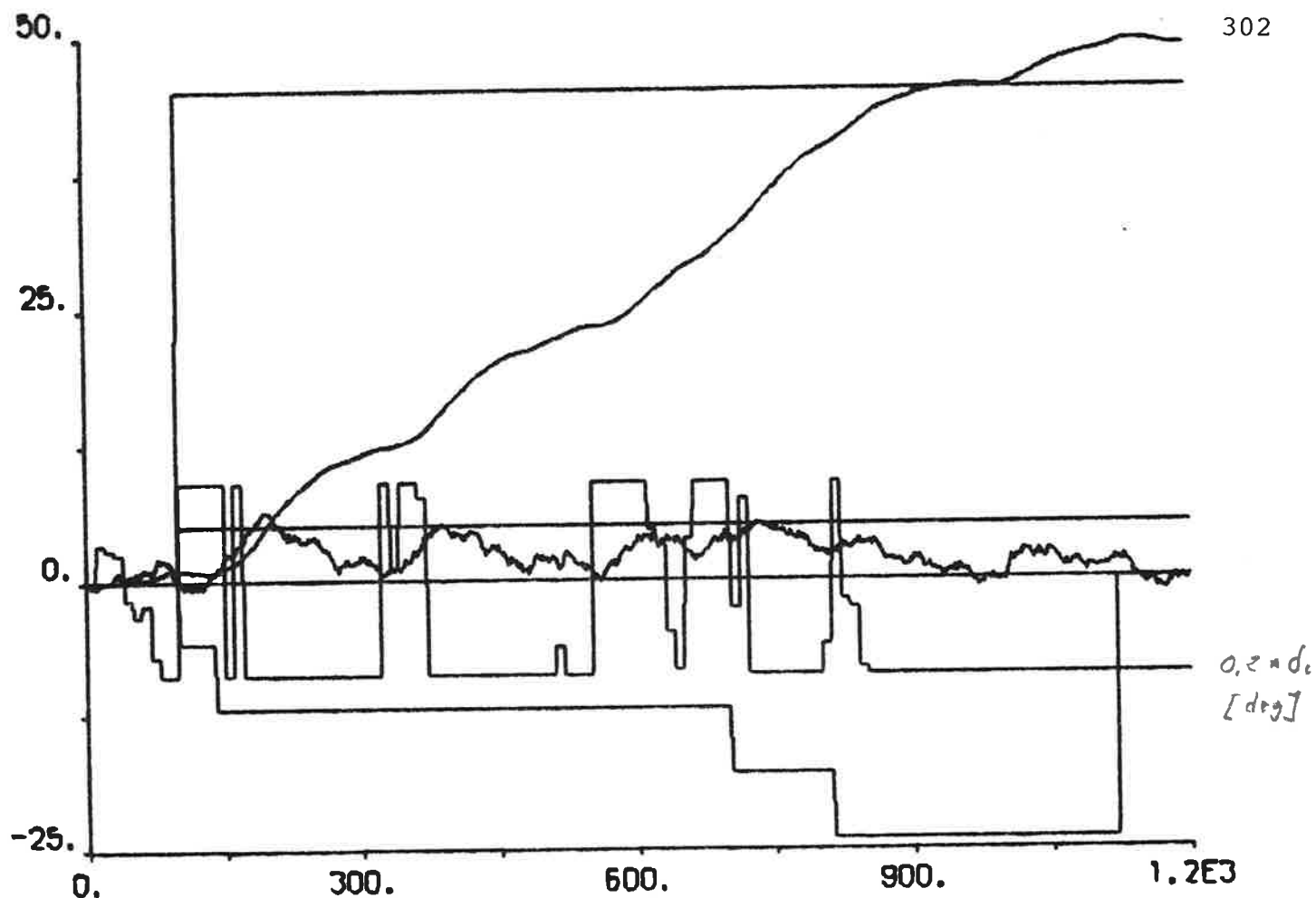


Fig. 4.90 - $T = 22.3$ m, $n_0 = 22.1772$ rpm, $u_0 = 4$ knots, $\Delta\psi_{\text{ref}} = 45$ deg, $r_{\text{ref}} = 0.1$ deg/s, self-tuning regulator and yaw regulator using non-filtered measurements ($\bar{c}_2 = 100$ s).

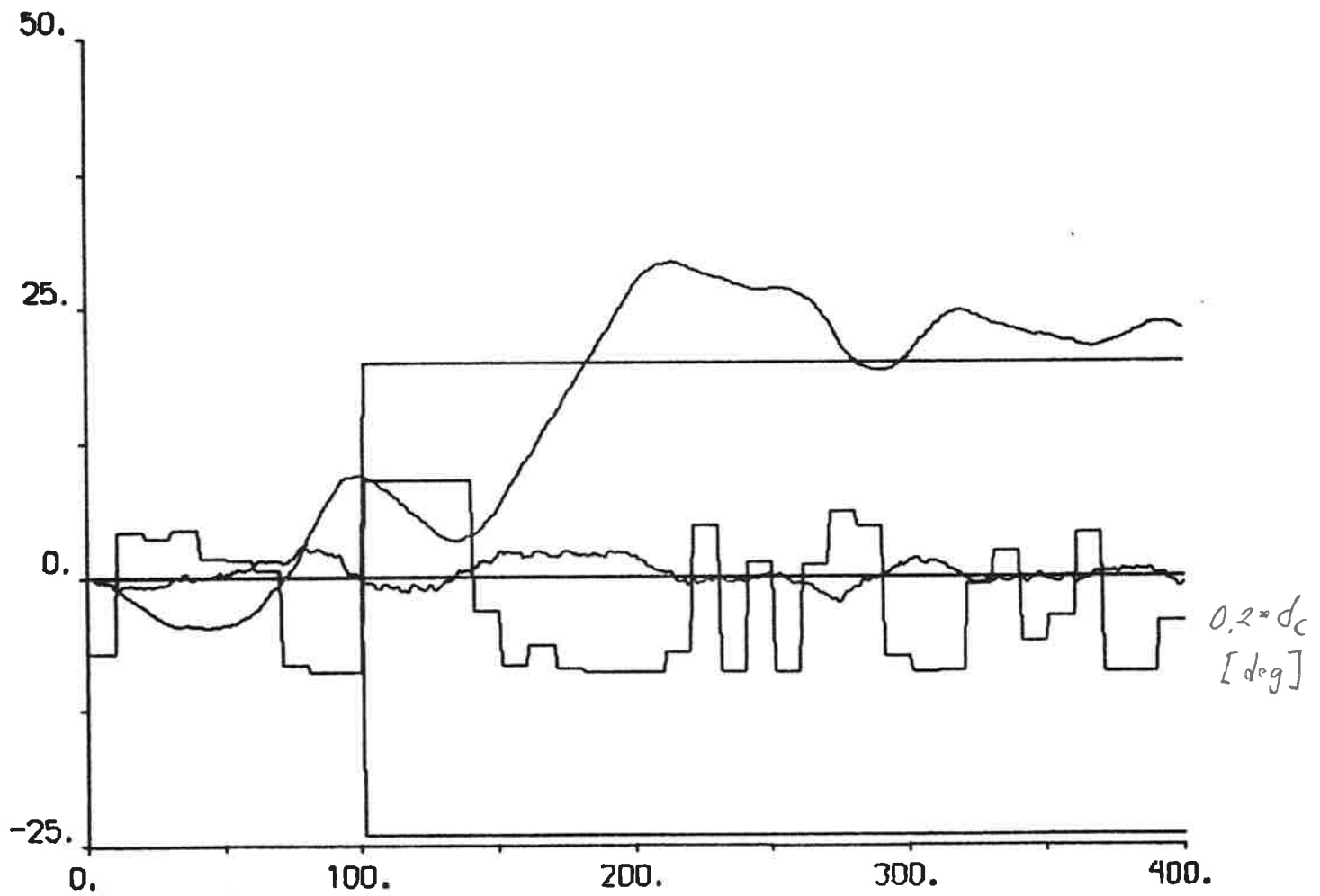


Fig. 4.91 - $T = 10.5$ m, $n_0 = 22.1772$ rpm, $u_0 = 4$ knots,
 $\Delta\psi_{\text{ref}} = 2$ deg, $r_{\text{ref}} = 0$ deg/s, self-tuning
 regulator and yaw regulator using estimates
 from the Kalman filter ($\bar{c}_2 = 100$ s).

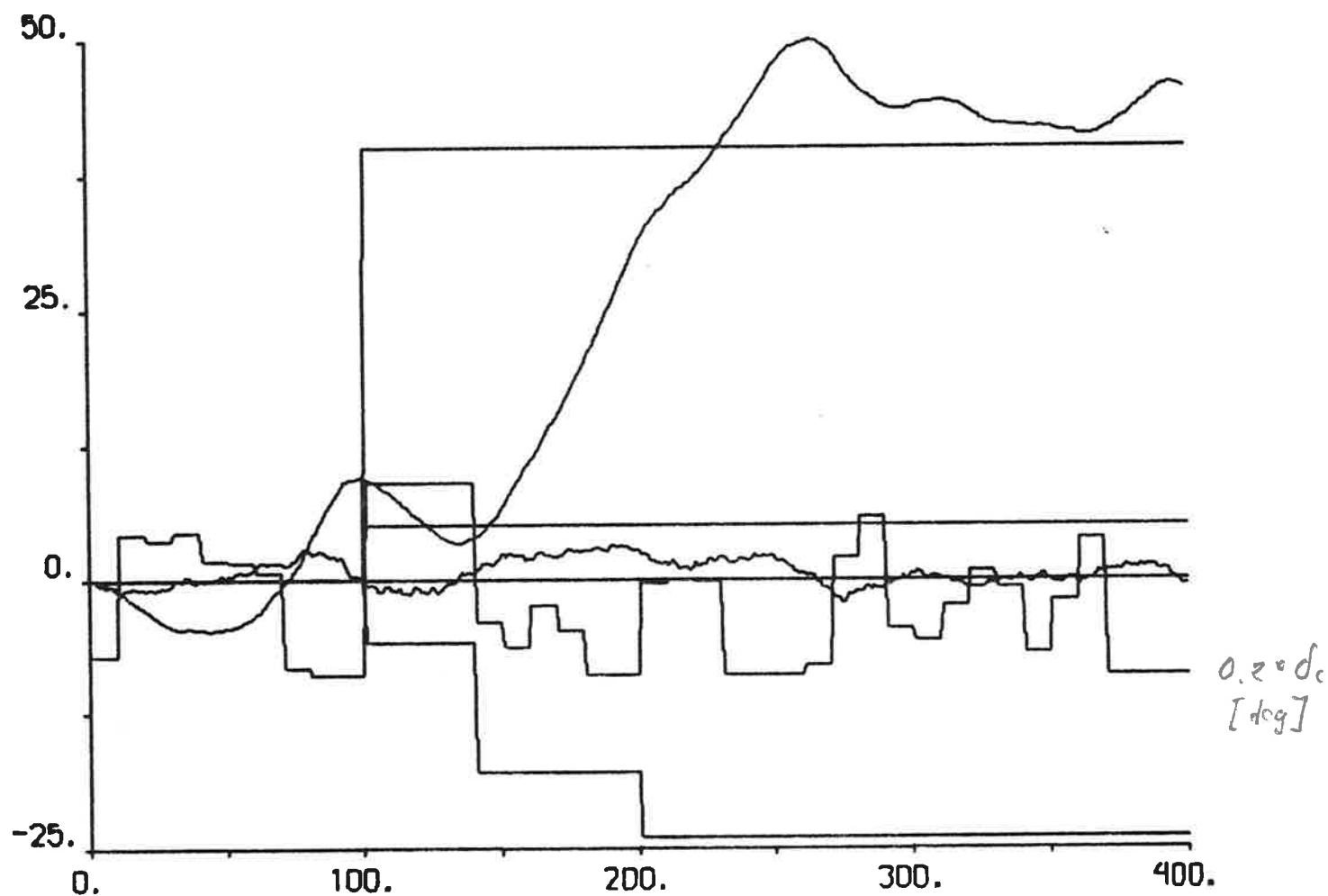


Fig. 4.92 - $T = 10.5$ m, $n_0 = 22.1772$ rpm, $u_0 = 4$ knots,
 $\Delta\psi_{\text{ref}} = 4$ deg, $r_{\text{ref}} = 0.1$ deg/s, self-tuning
 regulator and yaw regulator using estimates
 from the Kalman filter ($\bar{c}_2 = 100$ s).

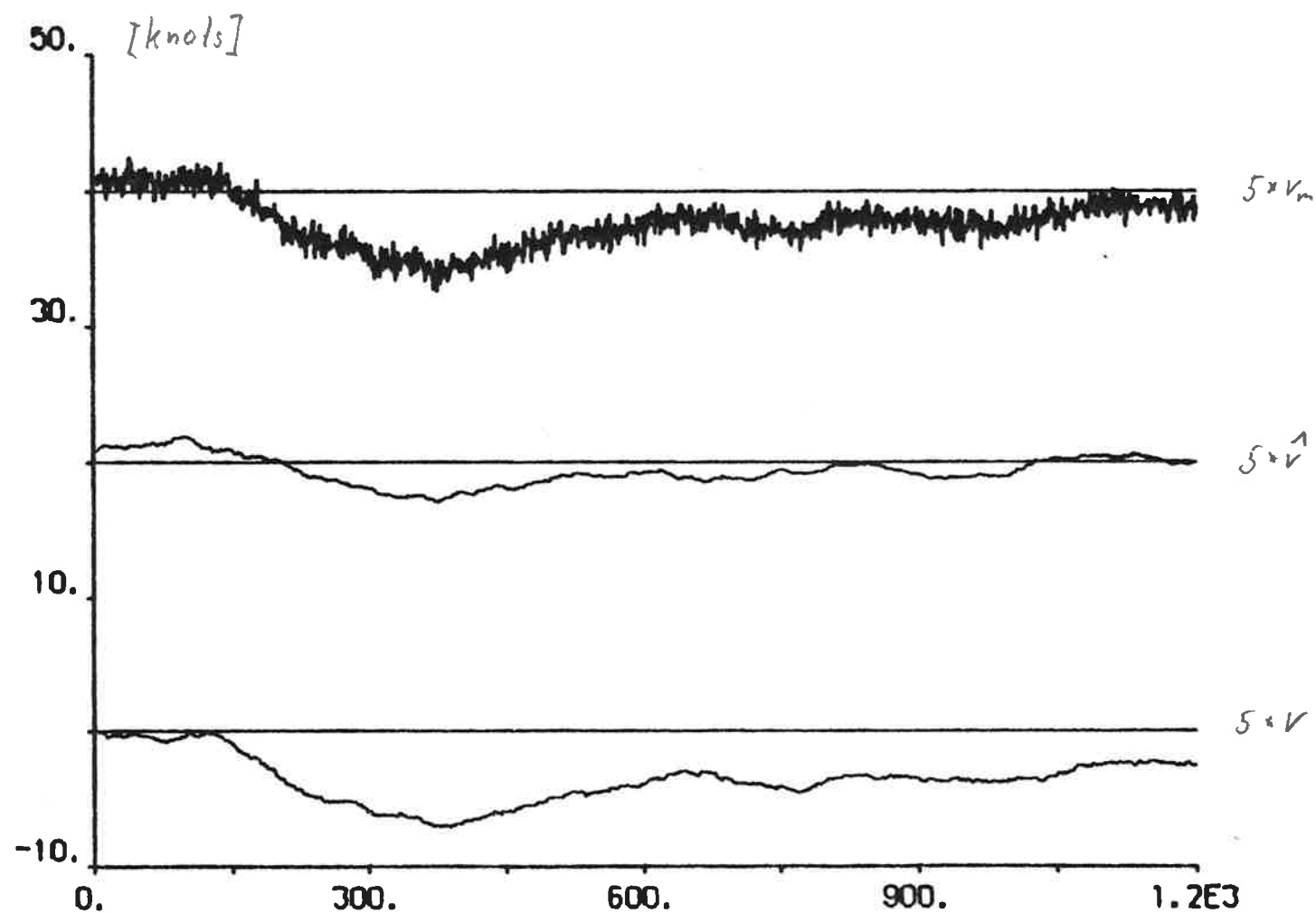
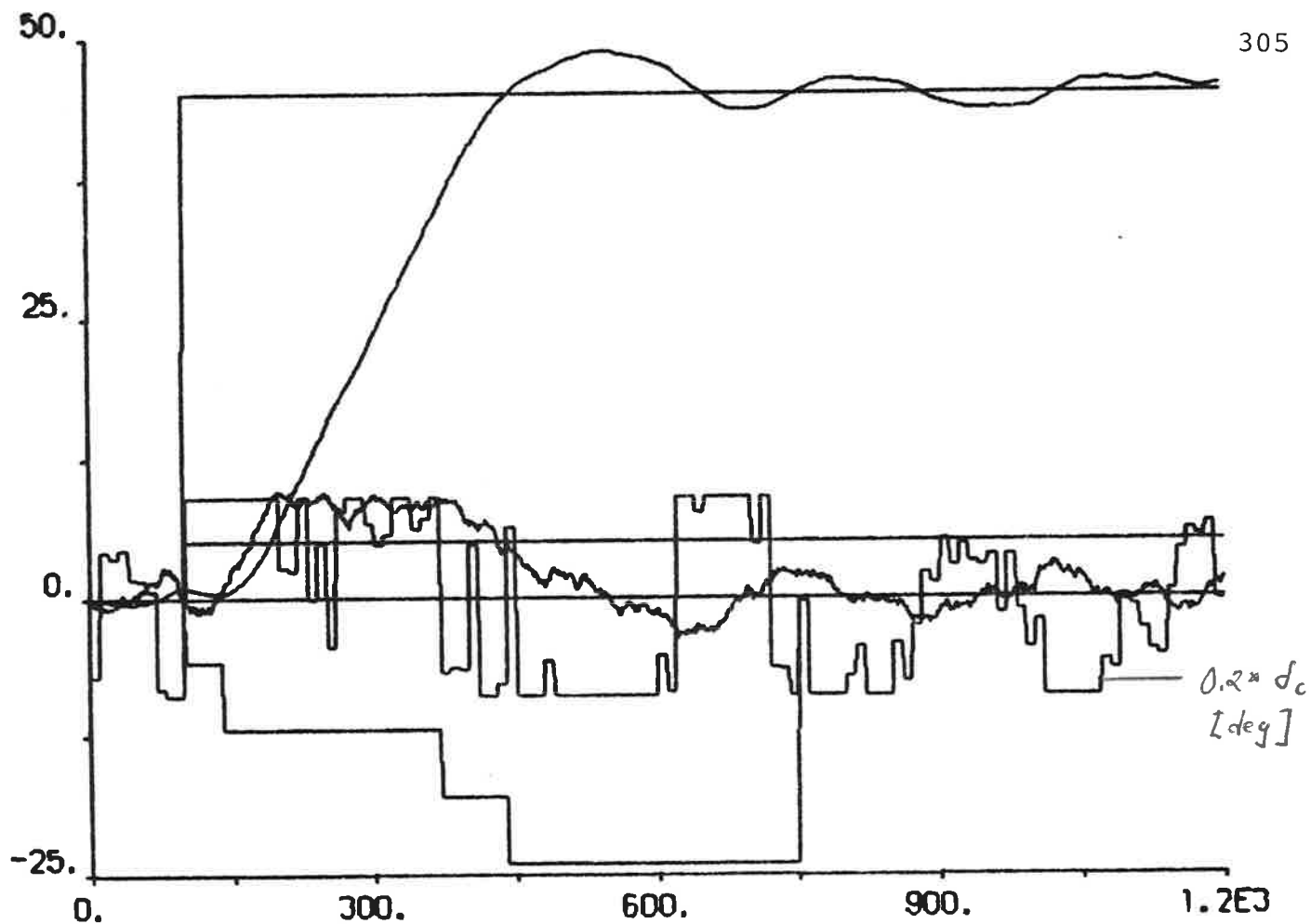


Fig. 4.93 a - $T = 10.5$ m, $n_0 = 22.1772$ rpm, $u_0 = 4$ knots, $\Delta\psi_{\text{ref}} = 45$ deg, $r_{\text{ref}} = 0.1$ deg/s, self-tuning regulator and yaw regulator using estimates from the Kalman filter ($\bar{c}_2 = 100$ s).

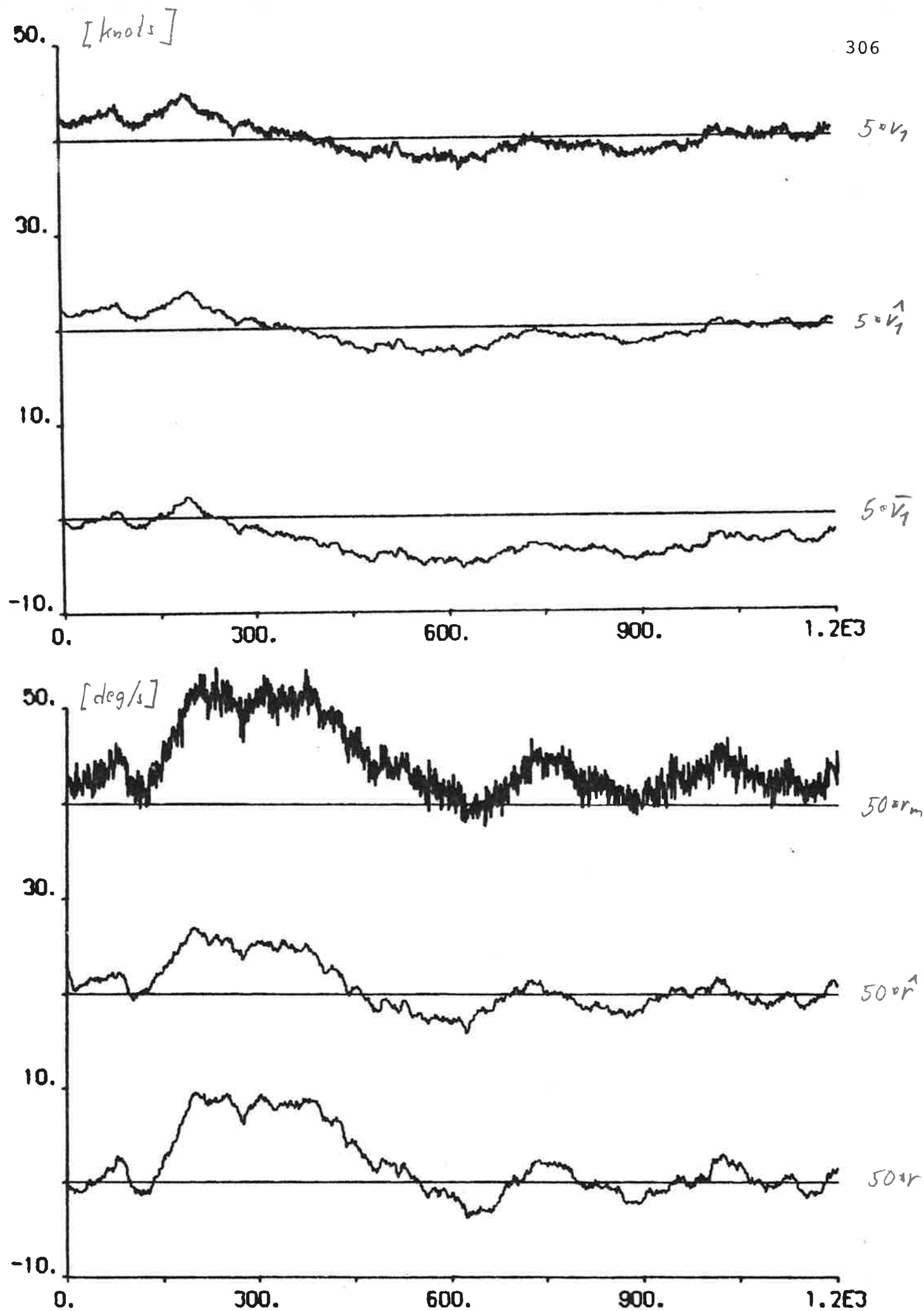


Fig. 4.93 b

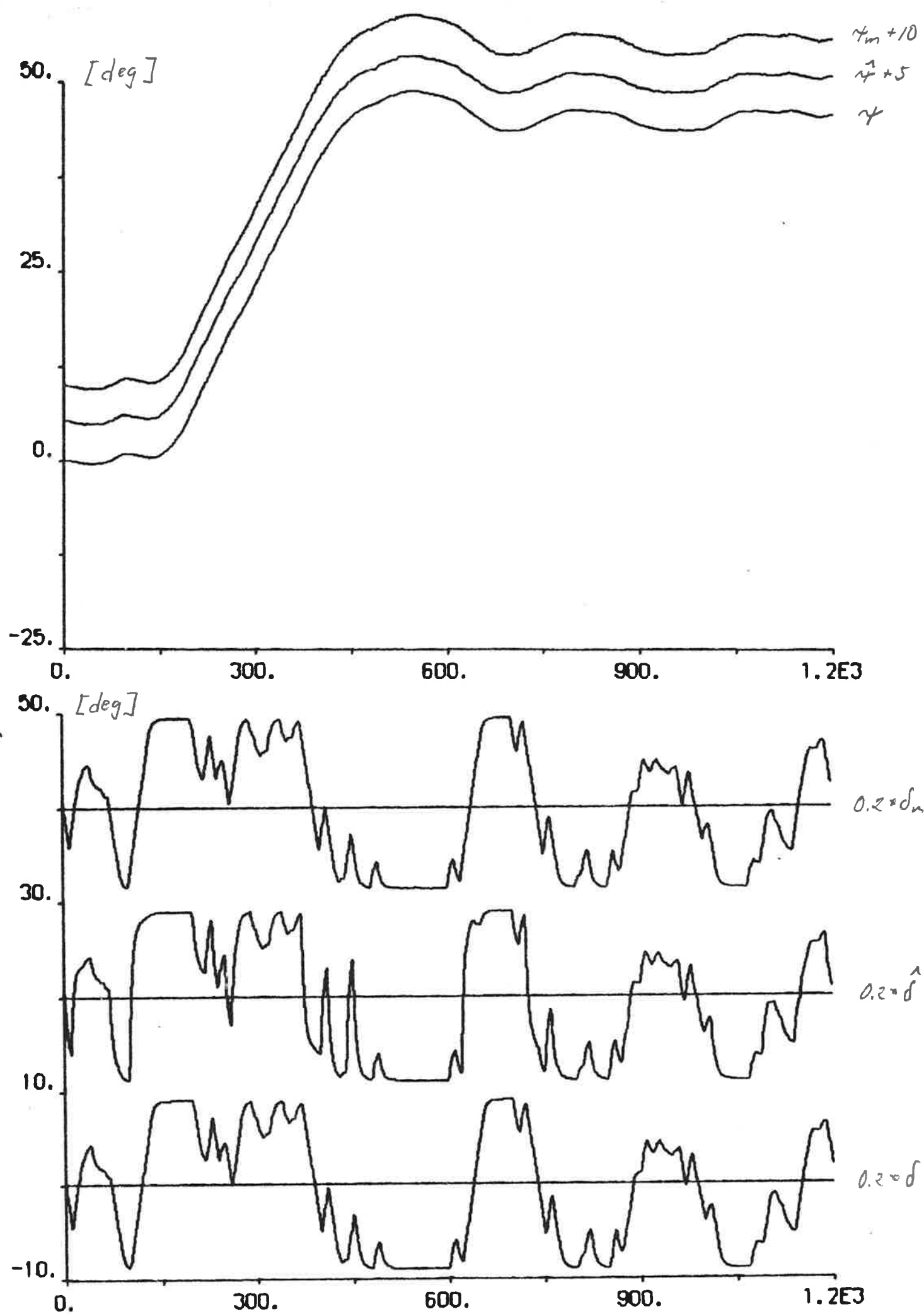


Fig. 4.93 c

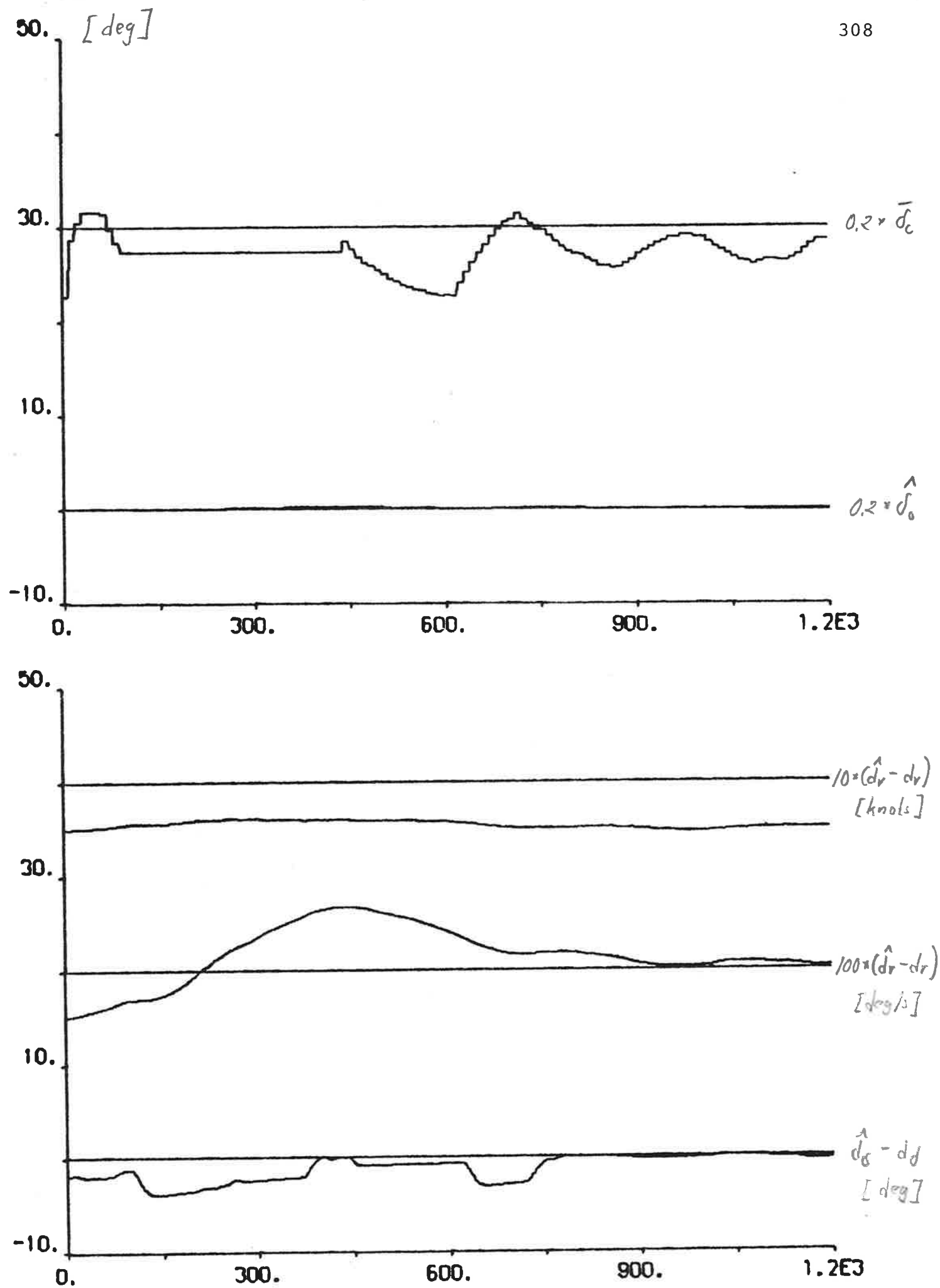


Fig. 4.93 d

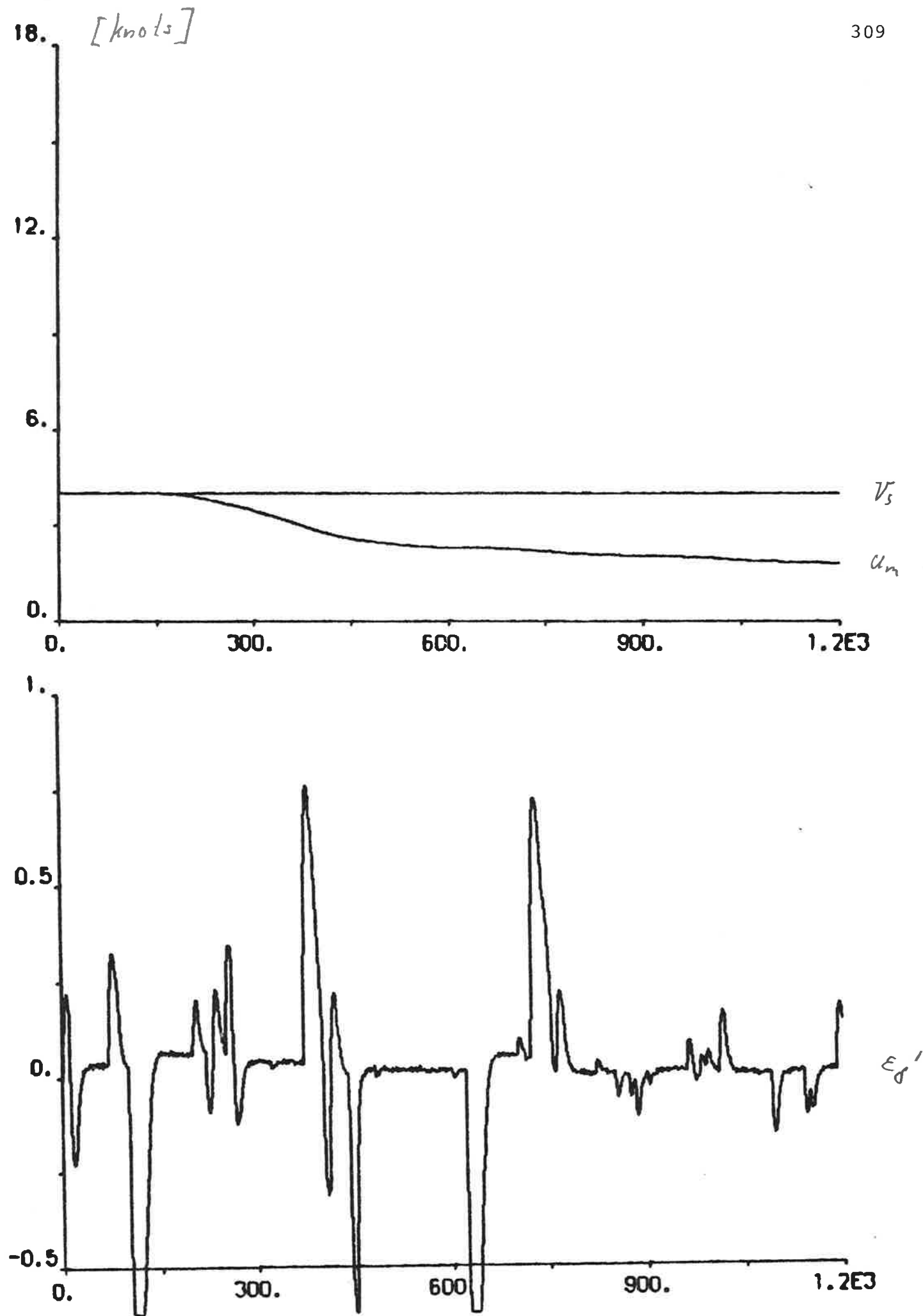


Fig. 4.93 e

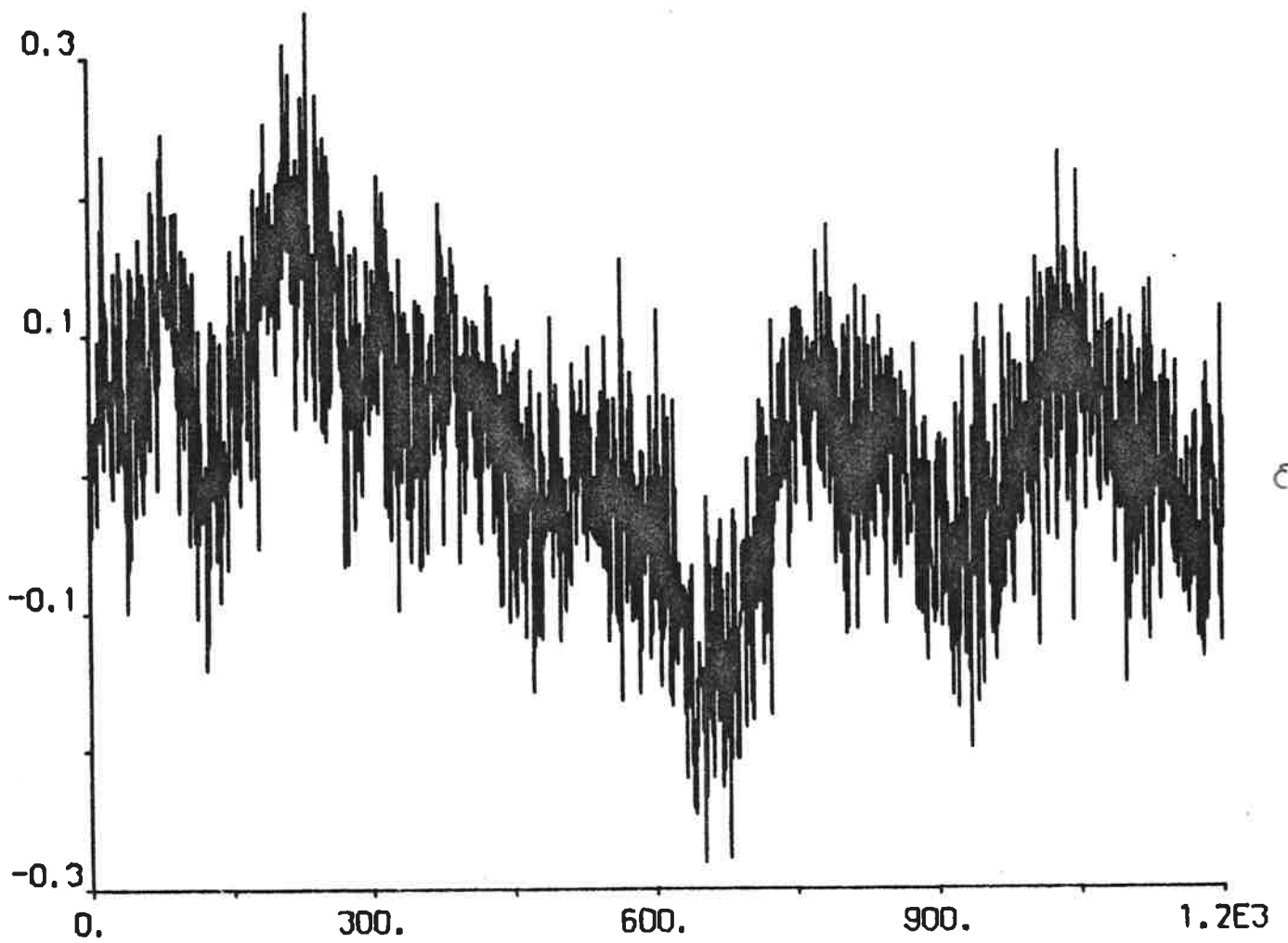
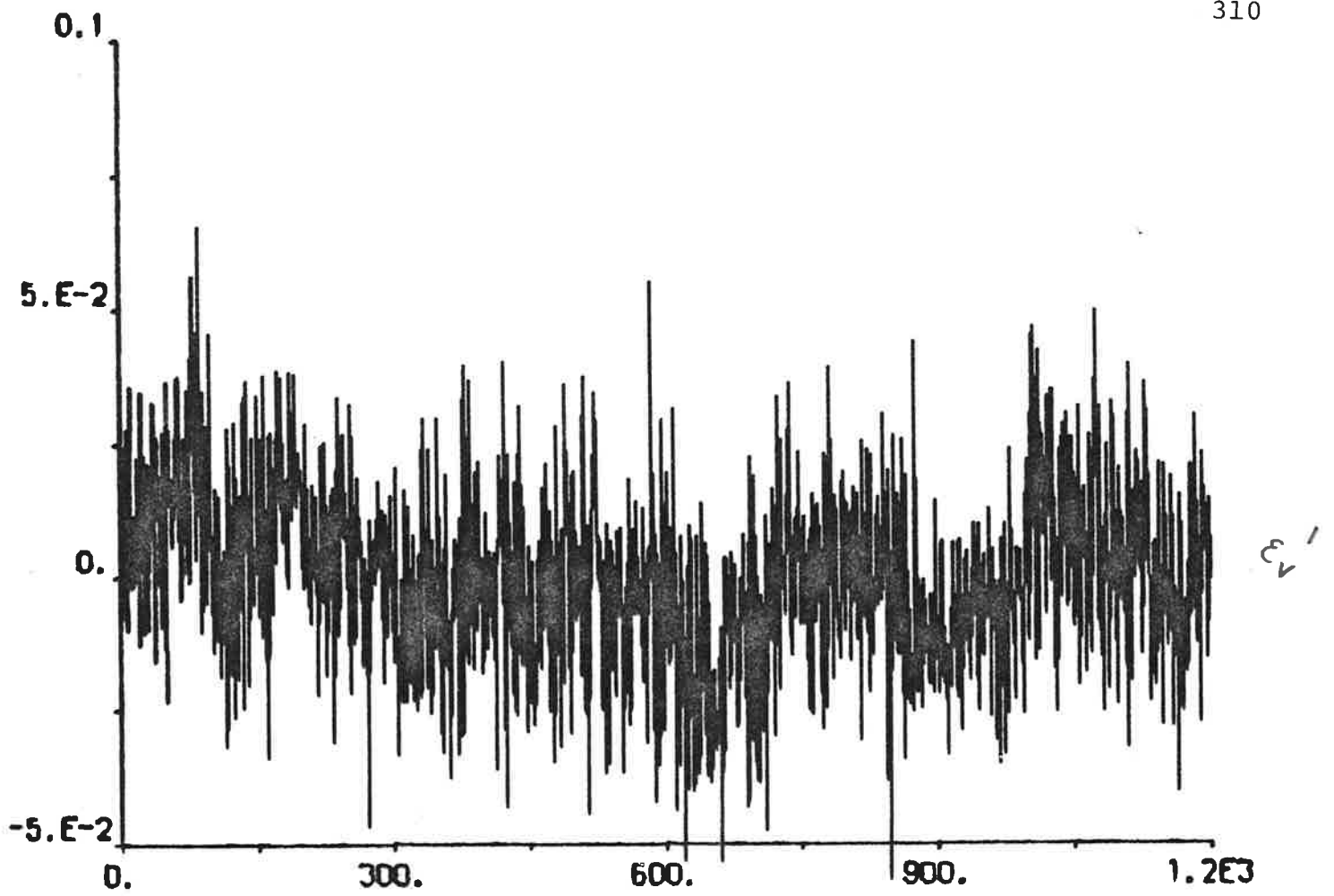


Fig. 4.93 f

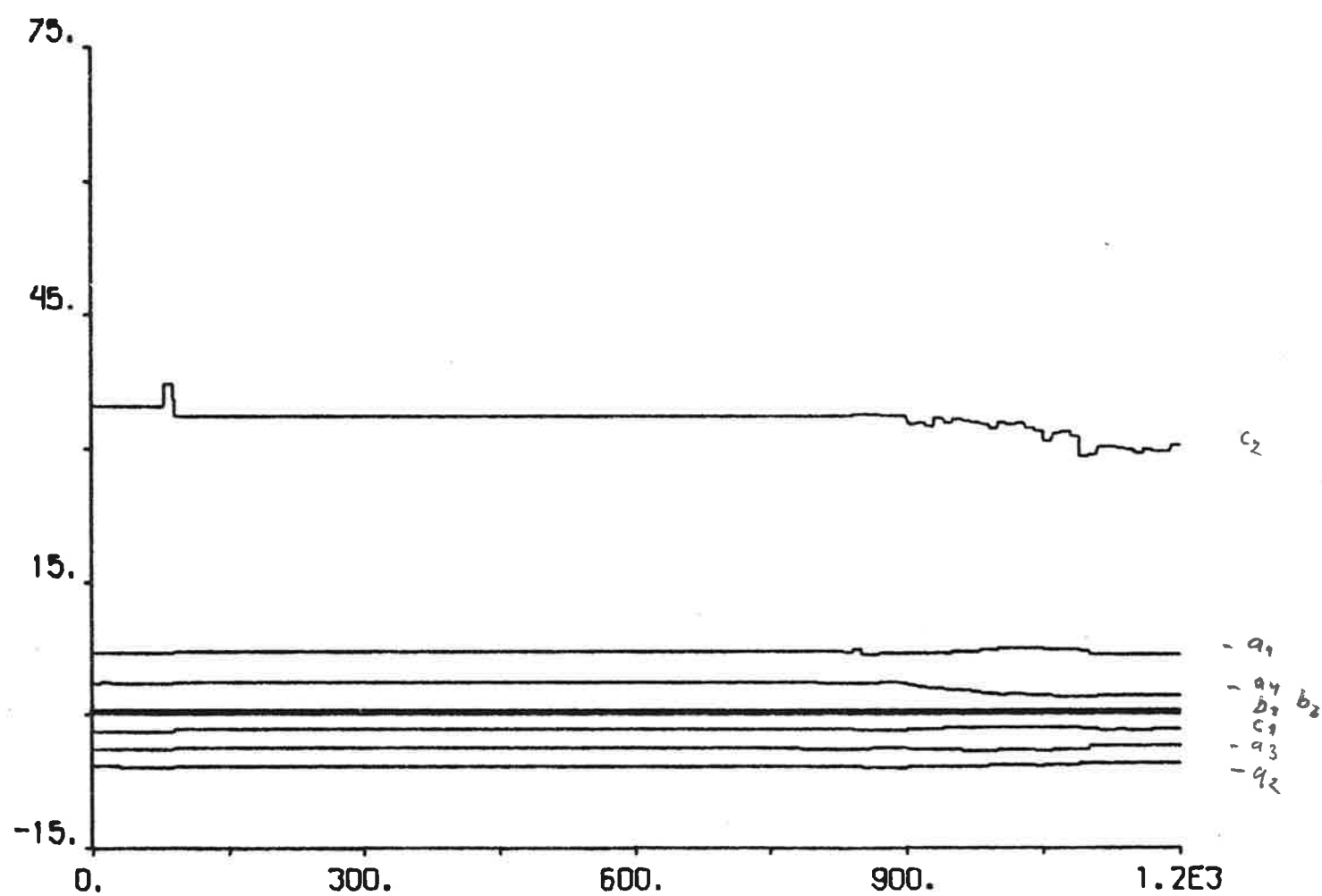
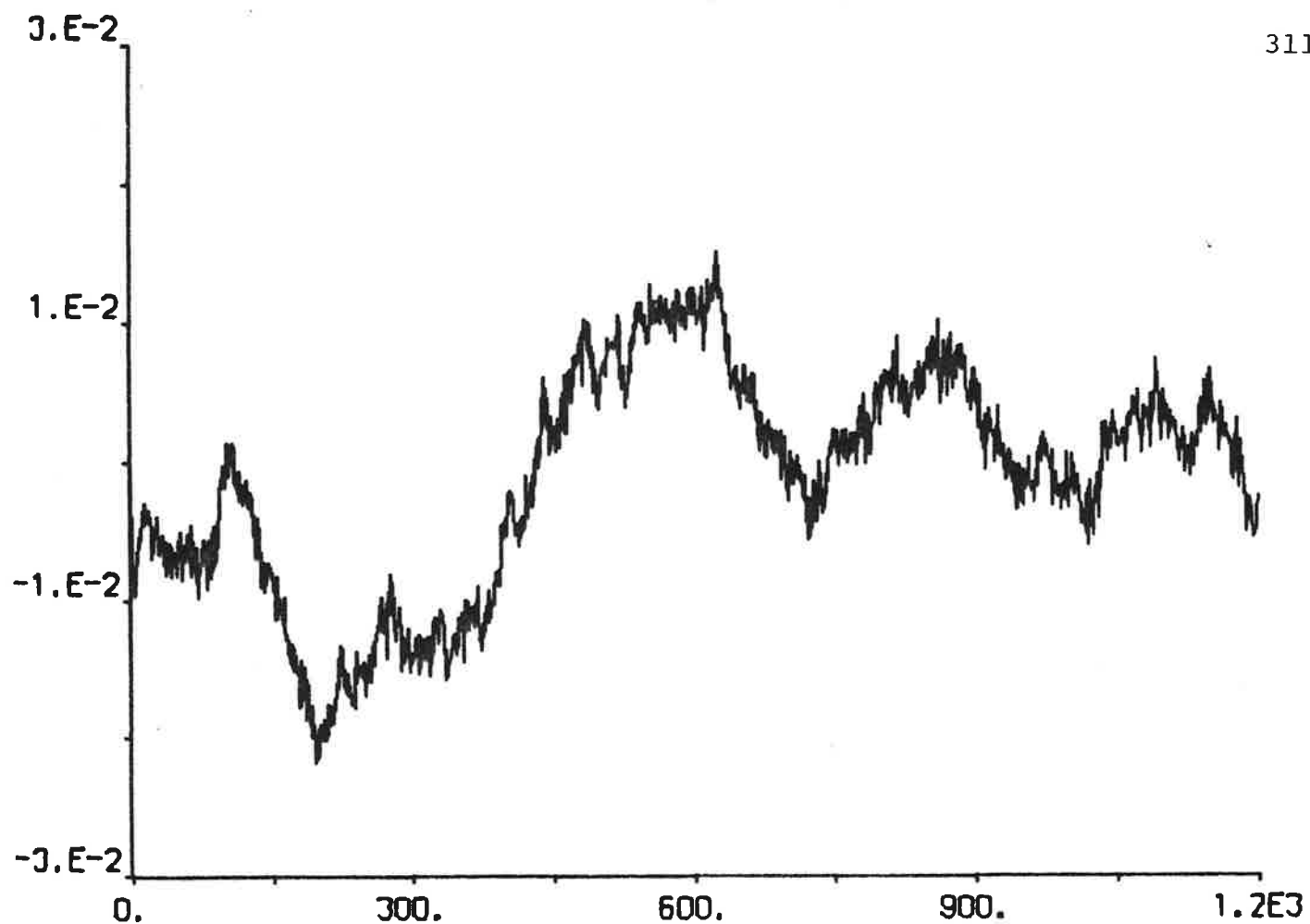


Fig. 4.93 g

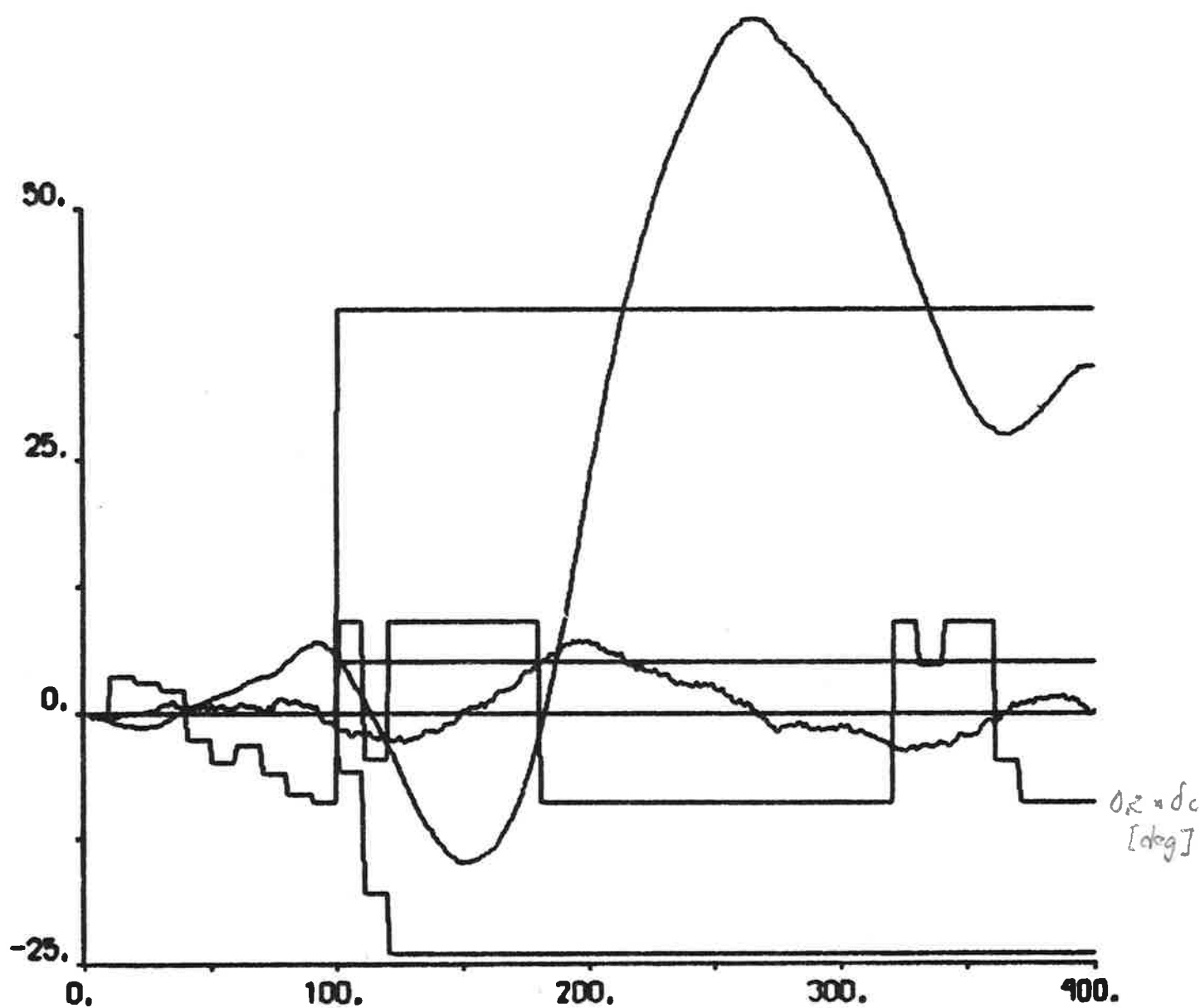


Fig. 4.94 - $T = 10.5$ m, $n_0 = 22.1772$ rpm, $u_0 = 4$ knots,
 $\Delta\psi_{\text{ref}} = 4$ deg, $r_{\text{ref}} = 0.1$ deg/s, self-tuning
 regulator and yaw regulator using non-filtered
 measurements ($\bar{c}_2 = 100$ s).

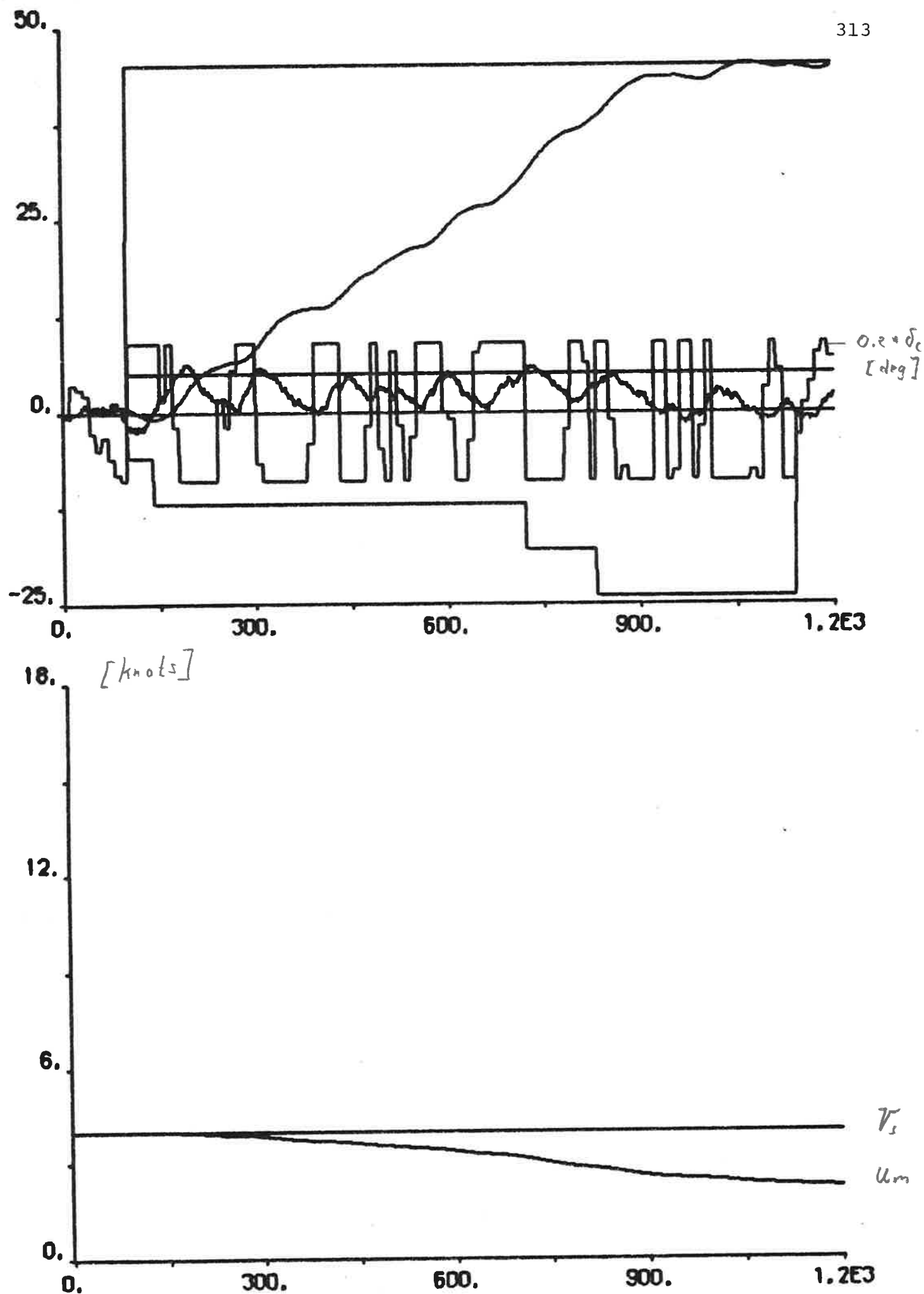


Fig. 4.95 - $T = 10.5$ m, $n_0 = 22.1772$ rpm, $u_0 = 4$ knots, $\Delta\psi_{\text{ref}} = 45$ deg, $r_{\text{ref}} = 0.1$ deg/s, self-tuning regulator and yaw regulator using non-filtered measurements ($\bar{c}_2 = 100$ s).

5. CONCLUSIONS

Simulations of straight course keeping and yawing of a 350 000 tdw tanker in full load condition as well as in ballast condition and with three different initial speeds (15.8, 10 and 4 knots) are presented in this report. The disturbances applied are equivalent to a rather rough weather condition to obtain decisive comparisons. The steering is performed by an autopilot, which consists of a Kalman filter, a self-tuning regulator and a yaw regulator. A PID-regulator is also implemented for comparison. It is possible to use either the Kalman filter estimates or the non-filtered measurements in the different regulators.

The performance of the Kalman filter is very good in full load condition as well as in ballast condition, when the initial speed is 15.8 or 10 knots. The quality of the filter estimates is decreased, although quite acceptable, when the initial speed is 4 knots. A difficulty is that the limited rudder turning rate has not been considered in the Kalman filter. This means rather poor estimates of the rudder angle when large rudder deviations are requested, which is the case quite often when the speed is low. The performance of the Kalman filter is approximately of the same quality during yawing as during straight course keeping. Notice, however, that it is suitable to skip the updating of the bias estimates of the filter during yaws.

The simulations show that the performance of both the self-tuning regulator and the PID-regulator is improved when Kalman filter estimates are used instead of non-filtered measurements. When Kalman filter estimates are used, the performance of the self-tuning regulator is significantly better than the performance of the PID-regulator, if the ship is full-loaded. However, the difference between the two regulators is hardly noticable in ballast condition. It

is not possible to make any conclusions when non-filtered measurements are used by the self-tuning regulator and the PID-regulator, since the simulation results are rather confusing. However, the self-tuning regulator using Kalman filter estimates is always significantly better than the PID-regulator using non-filtered measurements, and the PID-regulator using Kalman filter estimates is always significantly better than the self-tuning regulator using non-filtered measurements.

The performance of the yaw regulator, when Kalman filter estimates are used, is very good for different load conditions and speeds, with one exception: the performance is rather bad when the ship is full-loaded and the initial speed is 4 knots. One explanation is that the actual forward speed is increased to the extreme value 1 knot during one of the simulations. When non-filtered measurements are used instead of Kalman filter estimates, the performance quality of the yaw regulator is significantly decreased. The reason is, of course, that the yaw rate measurements are very noisy. In such a case, it is highly desired to filter the yaw rate signal in some way, to obtain a better performance. Another possibility is to perform a difference approximation of the heading angle measurement to obtain a yaw rate estimate. This requires, however, a rather good resolution of the heading measurement.

6. REFERENCES

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APPENDIX - PROGRAM LISTINGS

A 1

CONNECTING SYSTEM CON1

¢
¢AUTHOR, C.KALLSTROM 1976-03-24

¢
TIME T

¢
W1ATANK3R=E1ANOIS1R*NOI1
W2ATANK3R=E2ANOIS1R*NOI2
EE1ATANK3R=E3ANOIS1R
EE2ATANK3R=E4ANOIS1R
EE3ATANK3R=E5ANOIS1R
EE4ATANK3R=E6ANOIS1R
DELMAAUTR=DELMATANK3R
DELTAAUTR=DELMNATANK3R
V1AAUTR=V1MNATANK3R
RAAUTR=RMNATANK3R
PSIMAAUTR=PSIMATANK3R
PSIAAUTR=PSIMNATANK3R
UAAUTR=UMATANK3R
ANAAUTR=NMATANK3R
DELCATANK3R=DELCOAAUTR

¢
SCC1:0.
SCC2:0.
SCC3:0.
SCC4:0.
NOI1:1.E-5
NOI2:1.E-5

¢
END

CONTINUOUS SYSTEM TANK3

FAUTHOR, C.KALLSTROM 1976-03-17

F

FKOCKUMS CONVENTION OF THE RUDDER SIGN

F

INPUT DELC W1 W2 EE1 EE2 EE3 EE4

FDELC = RUDDER COMMAND ADEGA

FW1 = STATE NOISE AM/(S*S)A

FW2 = STATE NOISE A1/(S*S)A

FEE1 = MEASUREMENT NOISE ADEGA

FEE2 = MEASUREMENT NOISE AKNOTSÅ

FEE3 = MEASUREMENT NOISE ADEG/SA

FEE4 = MEASUREMENT NOISE ADEGA

OUTPUT DELM DELMN V1MN RMN PSIM PSIMN UM NM

F

FDELM = RUDDER ANGLE ADEGA

FDELMN = RUDDER ANGLE MEASUREMENT ADEGA

FV1MN = BOW SWAY VELOCITY MEASUREMENT AKNOTSÅ

FRMN = YAW RATE MEASUREMENT ADEG/SA

FPSIM = HEADING ADEGA

FPSIMN = HEADING MEASUREMENT ADEGA

FUM = FORWARD SPEED AKNOTSÅ

FNM = NUMBER OF PROPELLER REVOLUTIONS ARPMÅ

STATE DEL U V R PSI

FDEL = RUDDER ANGLE ARADA

FU = FORWARD VELOCITY AM/SA

FV = SWAY VELOCITY AM/SA

FR = YAW RATE A1/SA*100

FPSI = HEADING ARADA

DER DDEL DU DV DR DPSI

INITIAL

U=U0/CMK

SGL=SQRT(G*L)

F1=(22.3-TT)/11.8

F2=(TT-10.5)/11.8

N=N0/60.

UD1=N*(17.25*F1+15.8*F2)/(1.46*CMK)

TS1 = 1/TS

TS2 = TS1/CRG

DL1 = DL/CRG

XUD=XUD1*F1+XUD2*F2

XUDL=XUD*L

XUU=(XUU1*F1+XUU2*F2)/XUDL

XVR=(XVR1*F1+XVR2*F2)/XUD

XRR=(XRR1*F1+XRR2*F2)*L/XUD

XUV=XUVVV/(G*L*XUDL)

XUDD=(XUDD1*F1+XUDD2*F2)/XUDL

XT=X1T/XUD

YVD=YVD1*F1+YVD2*F2

YVDL=YVD*L

YRU1=YRU/YVD

$YRUU1 = YRUU / (SGL * YVD)$
 $YUV = (YUV1 * F1 + YUV2 * F2) / YVDL$
 $YUUV = (YUUV1 * F1 + YUUV2 * F2) / (SGL * YVDL)$
 $YVV = (YVV1 * F1 + YVV2 * F2) / YVDL$
 $YRAV = (YRAV1 * F1 + YRAV2 * F2) / YVD$
 $YARV = (YARV1 * F1 + YARV2 * F2) / YVD$
 $YUUD = (YUUD1 * F1 + YUUD2 * F2) / YVDL$
 $YTD1 = YTD / YVD$
 $KTY1 = KTY / YVD$

$NRDL = NRD * L$
 $NRDLL = NRDL * L$
 $NRU = (NRU1 * F1 + NRU2 * F2) / NRDL$
 $NRUU = (NRUU1 * F1 + NRUU2 * F2) / (SGL * NRDL)$
 $NUV = (NUV1 * F1 + NUV2 * F2) / NRDLL$
 $NUUV = (NUUV1 * F1 + NUUV2 * F2) / (SGL * NRDLL)$
 $NVV = (NVV1 * F1 + NVV2 * F2) / NRDLL$
 $NRR = (NRR1 * F1 + NRR2 * F2) / NRD$
 $NRAV = (NRAV1 * F1 + NRAV2 * F2) / NRDL$
 $NARV = (NARV1 * F1 + NARV2 * F2) / NRDL$
 $NUUD = (NUUD1 * F1 + NUUD2 * F2) / NRDLL$
 $NTD1 = NTD / NRDL$
 $KTN1 = KTN / NRDL$

$XF = FW / XUD$
 $YF = FW / YVD$
 $NF = FW * LV / NRDLL$

$JJ1 = (1 - W) / (N * D)$
 $DISPL = DISP1 * F1 + DISP2 * F2$
 $TT1 = N * N * D * D * D * D / DISPL$
 $JJ = UO1 * JJ1$
 $JJP = JJ / \sqrt{1 + JJ * JJ}$
 $KKT = -0.33 * JJP * JJP - 0.38 * JJP + 0.35$
 $TMO = KKT * (1 + JJ * JJ) * TT1$
 $LL1 = CMK * L1$
 $ALF1 = ALFA / CRG$

OUTPUT

$DELM = CRG * DEL$
 $DELS = SC35 * DELM + SC36$
 $DELMN = DELM + D3 + EE1$
 $DMNS = SC37 * DELMN + SC38$
 $D3S = SC39 * D3 + SC40$
 $VM = CMK * V$
 $VMS = SC41 * VM + SC42$
 $V1M = LL1 * R / 100. + VM$
 $V1MS = SC43 * V1M + SC44$
 $V1MN = V1M + D1 + EE2$
 $V1MNS = SC45 * V1MN + SC46$
 $D1S = SC47 * D1 + SC48$
 $RM = CRG * R / 100.$
 $RMS = SC49 * RM + SC50$
 $RMN = RM + D2 + EE3$
 $RMNS = SC51 * RMN + SC52$
 $D2S = SC53 * D2 + SC54$
 $PSIM = CRG * PSI$
 $PSIMS = SC55 * PSIM + SC56$
 $PSIMN = PSIM + EE4$
 $PSMNS = SC57 * PSIMN + SC58$
 $UM = CMK * U$
 $UMS = SC59 * UM + SC60$
 $NM = NO$

DYNAMICS

```

RR=R/100.
APSI=ALF1-PSI
SINW=SIN(APSI)
J=U*JJ1
JP=J/SQRT(1+J*J)
KT=-0.33*JP*JP-0.38*JP+0.35
TM=KT*(1+J*J)*TT1
TM1=IF TM<TMO THEN TM ELSE TMO
TMD =TM1*DEL
U2=U*U
AV=ABS(V)
AR=ABS(RR)
RU=RR*U
RU2=RU*U
UV=U*V
U2V=U*UV
VAV=V*AV
RAV=RR*AV
ARV=AR*V
U2D=U2*DEL
DDEL1=-TS1*DEL+TS2*DELC
DDEL=IF DDEL1<-DL1 THEN -DL1 ELSE IF DDEL1>DL1 THEN DL1 ELSE DDEL1

DU=XUU*U2+XVR*V+RR+XRR*RR*RR+XUV*UV*VAV+XUDD*U2D*DEL+XT*TM-XF*COS(APSI)

SL=YRU1*RU+YRUU1*RU2+YUV*UV+YUUV*U2V+YVV*VAV+YRAV*RAV
DV=YARV*ARV-YUUD*U2D-YTD1*TMD+KTY1*TM-YF*SINW+W1/YVD+SL
SL1=NRU*RU+NRUU*RU2+NUV*UV+NUUV*U2V+NVV*VAV+NRR*RR*AR
DR=(SL1+NRAV*RAV+NARV*ARV-NUUD*U2D-NTD1*TMD+KTN1*TM+NF*SINW+W2/NRD)*100.

DPSI=RR
G:9.80665
CMK:1.943844
CRG:57.2958
L:350.
U0:15.8
TT:22.3
TS:5.0
DL:2.32      F2 PUMPS
XUD1:
XUD2:
XUU1:
XUU2:
XVR1:
XVR2:
XRR1:
XRR2:
XUVV:
XUDD1:
XUDD2:
X1T:
YVD1:
YVD2:
YRU:
YRUU:
YUV1:
YUV2:
YUUV1:
YUUV2:
YVV1:
YVV2:

```

YRAV1:
YRAV2:
YARV1:
YARV2:
YUUD1:
YUUD2:
YTD:
KTY:
NRD:
NRU1:
NRU2:
NRUU1:
NRUU2:
NUV1:
NUV2:
NUUV1:
NUUV2:
NVV1:
NVV2:
NRR1:
NRR2:
NRAV1:
NRAV2:
NARV1:
NARV2:
NUUD1:
NUUD2:
NTD:
KTN:

FW:0.
LV:25.
W:
D:
DISP1:
DISP2:
L1:164.35
ALFA:0.
NO:87.6
D1:1.
D2:0.1
D3:2.
SC35:1.
SC36:0.
SC37:1.
SC38:0.
SC39:1.
SC40:0.
SC41:1.
SC42:0.
SC43:1.
SC44:0.
SC45:1.
SC46:0.
SC47:1.
SC48:0.
SC49:1.
SC50:0.
SC51:1.
SC52:0.
SC53:1.
SC54:0.
SC55:1.
SC56:0.

SC57:1.
SC58:0.
SC59:1.
SC60:0.

END

SIMNON -

AN INTERACTIVE SIMULATION PROGRAM
FOR NONLINEAR SYSTEMS

MAIN PROGRAM

AUTHOR HILDING ELMQVIST
REVISED, C.KALLSTROM 1976-03-24.

REFERENCE

H. ELMQVIST: SIMNON - AN INTERACTIVE SIMULATION
PROGRAM FOR NONLINEAR SYSTEMS -
USER'S MANUAL

DATA BASE

/PSCODE/ IPSEUD()
IPSEUD- PSEUDO CODE AREA

/VARTAB/ VARS(), IPNTS(), ITYPES()
VARS - IDENTIFIER TABLE
IPNTS - ADDRESS TABLE
ITYPES- TYPE TABLE
1: TIME
2: STATE
3: INPUT
4: OUTPUT
5: INIT
6: DER
7: NEW
8: TSAMP
9: PAR
10: VAR

/VALUES/ VALUE()
VALUE - VALUE TABLE AND LITTERAL TABLE

/SYSINF/ NASYST, ASYSTS(), IVARS(,2), INFSYS(), LENTRY(,3)
NASYST- NUMBER OF ACTIVE SYSTEMS
ASYSTS- SYSTEM IDENTIFIERS FOR ACTIVE SYSTEMS
IVARS - DEFINING THE POSITION OF THE VARIABLE
TABLE FOR EACH SYSTEM
INFSYS- SYSTEM TYPE
1: CONNECTING
2: CONTINUOUS
3: DISCRETE
4: CONTINUOUS (FORTRAN)
5: DISCRETE (FORTRAN)
LENTY- ENTRY POINTS FOR EACH ACTIVE SYSTEM
(,1): INITIAL-SECTION
OR THE NUMBER OF A FORTRAN-SYSTEM
(,2): OUTPUT- OR CONNECT-SECTION
(,3): DYNAMICS-SECTION

/EXTCON/ IEVAL, IERR, TYPE, SYSID, NEXTSY, NS
IEVAL - POINTER IN VARIABLE TABLE
IERR - ERROR INDICATOR
TYPE - SYSTEM TYPE FROM SUBROUTINE IDENT

```

C          'CONT' OR 'DISCR'
C          SYSD - SYSTEM IDENTIFIER FROM SUBROUTINE IDENT
C          NEXTSY- NUMBER OF EXTERNAL SYSTEMS
C          NS   - NUMBER OF ELEMENTS IN THE ALLOCATION AREA
C          /ENTRYS/ NTRINT,NTRDER,NTRSMP
C          NTRINT- ENTRY POINT FOR INITIAL COMPUTATIONS
C          NTRDER- ENTRY POINT FOR COMPUTATIONS OF DERIVATIVES
C          NTRSMP- ENTRY POINT FOR SAMPLING
C
C          /ENTRY/ LENTRY
C          LENTRY- ACTUAL ENTRY POINT FOR CALCUL
C
C          /PNTS/ NXC,NXD,KX( ),KDX( ),KXI( ),KTSAMP( )
C          NXC   - NUMBER OF STATES IN CONTINUOUS SYSTEMS
C          NXD   - NUMBER OF STATES IN DISCRETE SYSTEMS
C          KX    - POINTERS TO STATE VARIABLES
C          KDX   - POINTERS TO DER- AND NEW-VARIABLES
C          KXI   - POINTERS TO INIT-VARIABLES
C          KTSAMP- POINTERS TO TSAMP-VARIABLES
C
C          /CONINF/ (SEE INTRAC)
C
C          /MACINF/ (SEE INTRAC)
C
C          /MESS/ MESS
C          MESS   - MESSAGE INDICATOR
C
C          /SIHN/ NOSYST,OVFLO,IPLCOM,IEXIT,IWARN,ICOMPU,LDARK
C          ,NOCOUT,LLPCOM,INIDRA
C          NOSYST- TRUE IF NO SYSTEM DEFINED
C          OVFLO - TRUE IF OVERFLOW CHECK PERFORMED
C          IPLCOM- TRUE IF PLOT-COMMAND SHOULD BE WRITTEN
C          IEXIT - TRUE IF THE EDITOR IS TO MAKE
C                   AUTOMATIC EXIT (SYST)
C          IWARN - TRUE IF WARNINGS SHOULD BE WRITTEN
C          ICOMPU- TRUE IF MESSAGE ABOUT COMPUTATIONS
C                   IN OUTPUT-SECTION SHOULD BE GIVEN
C          LDARK - TRUE IF NOT VISABLE LINES AT SAMPLINGS
C          NOCOUT- TRUE IF CONTINUATION OF THE SIMULATION
C                   IS NOT POSSIBLE
C          LLPCOM- TRUE IF COMMANDS SHOULD BE ECHOED ON THE LP
C          INIDRA- TRUE IF INITIALIZATION OF DRAW
C
C          /PLT/ NPLT,IVADR( ),IHADR,PLTCOM( )
C          NPLT  - NUMBER OF PLOT-VARIABLES
C          IVADR - POINTERS TO VERTICAL VARIABLES
C          IHADR - POINTER TO HORIZONTAL VARIABLE
C          PLTCOM- BUFFER FOR PLOT-COMMAND
C
C          /STOVAR/ NSTV,IVARS( ),ISYSS( )
C          NSTV  - NUMBER OF VARIABLES TO BE STORED
C          IVARS - POINTERS TO VARIABLE NAMES
C          ISYSS - POINTERS TO SYSTEM IDENTIFIERS
C
C          /DATCON/ FILE,DTF
C          FILE  - STORE FILE NAME
C          DTF   - MINIMAL TIME INCREMENT
C
C          /SHOVAR/ NSHVAR
C          NSHVAR- NUMBER OF SHOWED VARIABLES SINCE AXES
C
C          /AX/ HMIN,DH,VMIN,DV
C          HMIN  - HORIZONTAL MINIMUM
C          DH    - HORIZONTAL VALUE PER CENTIMETER

```

```

C      VMIN - VERTICAL MINIMUM
C      DV   - VERTICAL VALUE PER CENTIMETER
C
C      /ERRWEI/ EPS,WEIGHT( )
C      EPS   - ERROR BOUND
C      WEIGHT- ERROR WEIGHTS
C
C      /ALG/ IALG
C      IALG  - SPECIFIES INTEGRATION ALGORITHM
C              1: HAMPC
C              2: RK
C              3: RKFIX
C
C      /MARKS/ IMARK,MRK,TRK,DTMRK
C      IMARK - TRUE IF MARKS WANTED
C      MRK   - SPECIFIES WHICH MARKS
C      TRK   - TIME FOR NEXT MARKS
C      DTMRK - TIME DISTANCE BETWEEN MARKS
C
C      /USER/ LSTOP,LDARK,LCALUS,NRESUM,LFIRST,NOPLT
C      LSTOP - TRUE IF SIMULATION SHOULD BE STOPPED
C      LDARK - TRUE IF DARK LINE
C      LCALUS- TRUE IF THE SUBROUTINE USRSUB SHOULD BE CALLED
C      NRESUM- NUMBER OF DISCRETE SYSTEMS THAT HASN'T
C              PRODUCED A DISCONTINUITY
C      LFIRST- TRUE IF SYSTS CALLED FIRST TIMES
C      NOPLT- IF TRUE NO PLOT
C
C      /DESTIN/ ISYST,IPART
C      ISYST - SYSTEM NUMBER
C      IPART - PART NUMBER
C
C      /NSYSTS/ NSYST
C      NSYST - NUMBER OF EXTERNAL SYSTEMS
C
C      /NALLOC/ NALL
C      NALL  - NUMBER OF ELEMENTS IN THE ALLOCATION AREA
C
C      /TIME/ T
C      T     - THE SIMULATION TIME
C
C      /STATES/ X( )
C      X     - STATES OF CONTINUOUS SYSTEMS
C
C      /DERS/ DX( )
C      DX    - DERIVATIVES OF THE STATES
C
C      /CMPVAR/ MODE,IASYST,ISYTP,IERR,IVAR1,IVAR2,IVAL1,IVAL2
C              L,LENT1,LENT2,LENT3
C      MODE  - COMPILER MODE
C              1: SYSTEM HEADING
C              2:
C              3: DECLARATIONS
C              4:
C              5: INITIAL-SECTION
C              6: OUTPUT-SECTION
C              7: DYNAMICS-SECTION
C              8: CONNECT-SECTION
C              9: END
C      IASYST- INDEX FOR ACTUAL SYSTEM
C      ISYTP- SYSTEM TYPE
C              1: CONNECTING
C              2: CONTINUOUS
C              3: DISCRETE

```

IERR - ERROR FLAG
 IVAR1 - INDEX FOR LOWER BOUND IN VARIABLE TABLE
 IVAR2 - INDEX FOR UPPER BOUND IN VARIABLE TABLE
 IVAL1 - POINTER IN THE VALUE TABLE
 IVAL2 - POINTER IN THE LITTERAL TABLE
 L - POINTER IN THE PSEUDO CODE AREA
 LENTR1 - POINTER TO INITIAL-SECTION
 LENTR2 - POINTER TO OUTPUT- OR CONNECT-SECTION
 LENTR3 - POINTER TO DYNAMICS-SECTION

/NXPNT/ NXP(,2)

NXP - SPECIFIES WHICH STATES THAT BELONGS
 TO EACH DISCRETE SYSTEM

/COND/ LSAMP,LSAMPS()

LSAMP - TRUE IF SAMPLING IS TO BE DONE
 LSAMPS - SPECIFIES WHICH SYSTEMS THAT IS TO BE SAMPLED

/LIMITS/ MPSC,MVAR,MVAL,MX

MPSC - NUMBER OF ELEMENTS IN PSEUDO CODE AREA
 MVAR - NUMBER OF ELEMENTS IN VARIABLE TABLE
 MVAL - NUMBER OF ELEMENTS IN VALUE TABLE
 MX - MAXIMUM NUMBER OF STATES

/SIMARG/ T1,T2,DT,LCONT,LMARK

T1 - START TIME
 T2 - STOP TIME
 DT - TIME INCREMENT
 LCONT - LOGICAL VARIABLE TO INDICATE IF CONTINUATION
 OF SIMULATION IS WANTED
 LMARK - LOGICAL VARIABLE INDICATING IF MARKS IS WANTED
 DURING THE PLOTTING

/ARGSAV/ H1,H2,V1,V2

H1 - LAST HORIZONTAL MINIMUM (AXES)
 H2 - LAST HORIZONTAL MAXIMUM
 V1 - LAST VERTICAL MINIMUM
 V2 - LAST VERTICAL MAXIMUM

/AXINF/ IX0,IY0,XAX,YAX

IX0,IY0 - ORIGO FOR AXES (TEKPOINTS)
 XAX,YAX - LENGTH OF AXES (CM)

SUBROUTINE REQUIRED

ISIMN
 ESIMN
 SIMNSY
 SIMU

COMMON/ALLCOM/ID0(3)

COMMON /PSCODE/ IDUM1(1500)

COMMON /VARTAB/ IDUM2(1000),DUM2(500)

COMMON /VALUES/ DUM3(300)

COMMON /SYSINF/ IDUM4(151),DUM4(25)

COMMON /EXTCOM/ IDUM5(4),DUM5(2)

COMMON /ENTRYS/ IDUM6(3)

COMMON /ENTRY/ IDUM7

COMMON /PNTS/ IDUM8(177)

COMMON /COMINF/ IDUM9(33),DUM9(41)

COMMON /MACINF/ IDUM10(191),DUM10(107)

COMMON /MESSS/ IDUM11

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COMMON /SIMN/      IDUM12(10)
COMMON /PLT/       IDUM13(12),DUM13(16)
COMMON /STOVAR/    IDUM15(101)
COMMON /DATCOM/    DUM136(2)
COMMON /SHOVAR/    IDU137
C ***** HCOFY *****
COMMON/HCPCOM/DUM138(10),IDU138(30)
C ***** HCOFY *****
COMMON /AX/        DUM14(4)
COMMON /ERRWE1/    DUM15(51)
COMMON /ALG/       IDUM16
COMMON /MARKS/     IDUM17(2),DUM17(2)
COMMON /USER/      IDUM18(6)
COMMON /DESTIN/    IDUM19(2)
COMMON /NSYSTS/    IDM191
COMMON /NALLOC/    IDM192
COMMON /TIME/      DUM20
COMMON /STATES/    DUM21(50)
COMMON /DERS/      DUM211(50)
COMMON /CMPVAR/    IDUM23(12)
COMMON /NXPNT/     IDUM24(50)
COMMON /COND/      IDUM25(26)
COMMON /LIMITS/    MPSC,IDM261,NVAL,MX
COMMON /SINHARG/    IDUM26(2),DUM26(3)
COMMON /ARGSAV/    DUM0(4)
COMMON /AXINF/     IDUM27(2),DUM27(2)

C
C
MPSC=1500
NVAL=300
MX=20
CALL LOGG(0)
CALL LPHDL(0)

C
CALL ISIMN

C
MODE=1
10 CALL ESIMN(MODE)
C
GOTO(1,2,3,4),MODE

C
1 CALL LPHDL(1)
CALL LOGG(1)
STOP

C
2 CALL SIMNSY
GO TO 10

C
3 CALL SIMU
GO TO 10

C
4 CALL LPHDL(2)
CALL LOGG(2)
STOP
END

```

SUBROUTINE SYSTS

AUTHOR/ C.KALLSTROM 1976-03-24.

DIMENSION S(88)

COMMON/DESTIN/ISYST,IDUM

COMMON/NSYSTS/NSYST

COMMON/NALLOC/NS

COMMON/SAVEAR/IS(9)

NSYST=2

NS=88

GO TO (1,2),ISYST

1 CALL SNOISE('NOIS1',IS(1),S)

RETURN

2 CALL AUT

RETURN

END

SUBROUTINE AUT

SYSTEM DEFINITION OF AN AUTOPILOT FOR SHIP.

AUTHOR, C.KALLSTROM 1976-03-18.

REVISED, C.KALLSTROM 1976-04-01.

SUBROUTINE REQUIRED

AUTP3

STUR

IDENT

INPUT

OUTPUT

TSAMP

PAR

PARV

VAR

VARV

DIMENSION AMSO(4),AMEAS(4),AMSUM(4),SC(34),PPD(10)

COMMON /DESTIN/ IDUM,IPART

COMMON /TIME/ T

COMMON /DATA/ ITIME,IDELC,MODYAW,IDEXP,ISTBD,IPTORT,

* IFLAG,IPRINT,INAUT,IKX,MEAS(4),

* DELCO,DELTA(2),V1(2),R(2),PS1(2),DELO,DELCOM,DELTAS,U,AN,

* P,VEST,PSIREF,RREF,PLIM,V(2),DELTAO,D1,D2,D3,TH(10),

* CGR,CKM,PI,PI2,AL,AL1,A11,A12,A14,A15,A21,A22,A24,A25,

* A31,A32,A34,A35,A44,A45,B11,B21,B31,B41,AKK(32),

* TEST(4),BB,PVD,RLV,AKVD,VCONST,VMIN,VMAX,VO,THO(10),

* PPD(10),RL,BD,B2,AK1,AK2,AK3,PSIMX1,PSIMX2,PSIMX3,

* EPS1Y,EPS2Y,EPS3Y,C1Y,C2Y,C3Y,AK1Y,AK2Y,AK3Y,AK4Y,

* AK5Y,AK6Y,AK7Y,AK8Y,BD,BV,ALAN,DELAMP,PSIO,AKID,

* IREGK,IKAL,IKMX,IMX,IVVC,IVV,IPEGV,IAKV,ILOS,IREGYT,IREG,IPID,

* IST,NAB,NC1,NC2,K,IREGY,IYAW,IT1Y,IT3Y,IT4Y,IPK,IPC,IPR,

* EPS(4),VV,PV,PP(55),EPSI1(2),EDEL(2),VLOS1(2),

* ENM1(2),CN,EDELTA,EPSI2,VLOS2,ENM2,

* MEASUM(4),IVV1,

* X(8),PS(4),DD(3),DDOLD,VVV(4),VV2,V0V,V0V2,AKV,

* PFO1,PFO2,DELOLD,VOLD,ROLD,DAT(46),SINT,RRF,AINT2,

* AINT4,STD,STV,SL1,SL2,SL3,SL4,SL5,DUM(10),

* IRK,IREV,INEAS(4),IRYT,IR,NC,NAB,NP,K1,NDAT,NDAT1,

* ND1,N1,IRY,ITIM1,ITIM3,ITIM4,IP,I,J,L

GO TO (100,200,300,400,500,600,700,800) , IPART

CALL IDENT('DISCR','AUT')

RETURN

CALL INPUT(DELM,'DELM')

CALL INPUT(DELTA(1),'DELTA')

CALL INPUT(V1(1),'V1')

CALL INPUT(R(1),'R')

CALL INPUT(PSIM,'PSIM')

CALL INPUT(PSII,'PSI')

CALL INPUT(U,'U')

CALL INPUT(AN,'AN')

CALL OUTPUT(DELCO,'DELCO')

CALL TSAMP(TS,'TS')

CALL PAR(DT,'DT')

CALL PAR(PREF1,'PREF1')

CALL PAR(PREF2,'PREF2')

CALL PAR(RD,'RD')

CALL PAR(TD,'TD')

CALL PAR(DLIM,'DLIM')

CALL PAR(BB,'BB')

CALL PAR(PVU,'PVU')

CALL PAR(RLV,'RLV')

CALL PAR(AKVO,'AKVO')

CALL PAR(VCONST,'VCONS')

CALL PAR(VMIN,'VMIN')

CALL PAR(VMAX,'VMAX')

CALL PAR(VO,'VO')

CALL PAR(RL,'RL')

CALL PAR(BD,'BD')

CALL PAR(Q2,'Q2')

CALL PAR(AK1,'AK1')

CALL PAR(AK2,'AK2')

CALL PAR(AK3,'AK3')

CALL PAR(PSIMX1,'PSIX1')

CALL PAR(PSIMX2,'PSIX2')

CALL PAR(PSIMX3,'PSIX3')

CALL PAR(EPS1Y,'EPS1Y')

CALL PAR(EPS2Y,'EPS2Y')

CALL PAR(EPS3Y,'EPS3Y')

CALL PAR(C1Y,'C1Y')

CALL PAR(C2Y,'C2Y')

CALL PAR(C3Y,'C3Y')

CALL PAR(AK1Y,'AK1Y')

CALL PAR(AK2Y,'AK2Y')

CALL PAR(AK3Y,'AK3Y')

CALL PAR(AK4Y,'AK4Y')

CALL PAR(AK5Y,'AK5Y')

CALL PAR(AK6Y,'AK6Y')

CALL PAR(AK7Y,'AK7Y')

CALL PAR(AK8Y,'AK8Y')

CALL PAR(BD,'BD')

CALL PAR(BV,'BV')

CALL PAR(ALAM,'ALAM')

CALL PAR(AIREGK,'IREGK')

CALL PAR(AIKAL,'IKAL')

CALL PAR(AIKMX,'IKMX')

CALL PAR(AIMX,'IMX')

CALL PAR(AIVVC,'IVVC')

CALL PAR(AIVV,'IVV')

CALL PAR(AIREGV,'IREGV')

CALL PAR(AIAKV,'IAKV')

CALL PAR(AIRYT,'IRYT')

CALL PAR(AIREG,'IREG')

CALL PAR(AIPID,'IPID')

CALL PAR(AIST,'IST')

CALL PAR(ANA,'NA')

CALL PAR(ANB,'NB')

CALL PAR(ANC1,'NC1')

CALL PAR(ANC2,'NC2')

CALL PAR(AK,'K')

CALL PAR(AIREGY,'IREGY')

CALL PAR(AIYAW,'IYAW')

CALL PAR(AIT1Y,'IT1Y')

```
CALL PAR(AIT3Y,'IT3Y')
CALL PAR(AIT4Y,'IT4Y')
CALL PAR(AILO,'ILO')
```

```
CALL PARV(AMSD,4,'MSD')
CALL PARV(AKK,32,'AKK')
CALL PARV(TEST,4,'TEST')
CALL PARV(THD,10,'THD')
CALL PARV(PPD,10,'PPD')
CALL PARV(SC,34,'SC')
```

```
CALL VAR(DELCS,'DELCS')
CALL VAR(DELES,'DELES')
CALL VAR(VCS,'VCS')
CALL VAR(VES,'VES')
CALL VAR(VIES,'VIES')
CALL VAR(RES,'RES')
CALL VAR(PSIES,'PSIES')
CALL VAR(PREFS,'PREFS')
CALL VAR(RREFS,'RREFS')
CALL VAR(VESTS,'VESTS')
CALL VAR(VVS,'VVS')
CALL VAR(DUES,'DOES')
CALL VAR(D1ES,'D1ES')
CALL VAR(D2ES,'D2ES')
CALL VAR(D3ES,'D3ES')
CALL VAR(AHODS,'HODY')
CALL VAR(EDELS,'EDELS')
CALL VAR(PV,'PV')
CALL VAR(CN,'CN')
CALL VAR(EPSI2,'EPSI2')
CALL VAR(VLOS2,'VLOS2')
CALL VAR(ENM2,'ENM2')
CALL VAR(AIVV1,'IVV1')
CALL VAR(EFSIM,'EPSIM')
CALL VAR(SPSIM,'SPSIM')
CALL VAR(EDELT,'EDELT')
CALL VAR(SDELT,'SDELT')
CALL VAR(ENM,'ENM')
CALL VAR(VL1,'VL1')
CALL VAR(VL2,'VL2')
CALL VAR(FLL,'FLL')
```

```
CALL VARV(AMEAS,4,'MEAS')
CALL VARV(TH,10,'TH')
CALL VARV(EPS,4,'EPS')
CALL VARV(PPD,10,'PPD')
CALL VARV(EPSI1,2,'EPI1')
CALL VARV(EDEL,2,'EDEL')
CALL VARV(VLOS1,2,'VLS1')
CALL VARV(ENM1,2,'ENM1')
CALL VARV(AMSUM,4,'MSUM')
```

```
RETURN
```

```
DT=1.
PREF1=0.
PREF2=45.
RC=0.1
TD=100000.
DLIM=35.
HB=0.806
PVD=1.
```

```

RLV=0.995
AKVQ=100.
VCONST=6.
VMIN=0.2
VMAX=12.
VQ=8.
RL=0.99
BU=1.
Q2=0.
AK1=5.
AK2=200.
AK3=0.005
PSIMX1=0.35
PSIMX2=2.5
PSIMX3=2.5
EPS1Y=0.
EPS2Y=0.02
EPS3Y=1.
C1Y=60.
C2Y=50.
C3Y=60.
AK1Y=5.
AK2Y=200.
AK3Y=0.005
AK4Y=200.
AK5Y=200.
AK6Y=8.
AK7Y=2.
AK8Y=200.
BD=0.05
BV=0.05
ALAN=0.0833333
AIKRGK=1.
AIKAL=1.
AIKMX=300.
AIXX=10.
AIVVC=1.
AIVV=2.
AIREGV=5.
AIAKV=0.
AIRYT=5.
AIREG=15.
AIPID=0.
AIST=2.
ANA=4.
ANB=2.
ANC1=1.
ANC2=1.
AK=5.
AIREGY=10.
AIYAA=2.
AIT1Y=30.
AIT3Y=100.
AIT4Y=300.
AII0=1201.
DO 302 I=1,4
302 AMS0(I)=0.
AKK(1)=-1.054E-4
AKK(2)=5.367E-4
AKK(3)=1.173E-5
AKK(4)=1.603E-2
AKK(5)=-2.172E-5
AKK(6)=-7.649E-5
AKK(7)=-2.793E-6

```

```

AKK(8)=2.542E-3
AKK(9)=0.2332
AKK(10)=0.3541
AKK(11)=4.458E-3
AKK(12)=9.306E-5
AKK(13)=-1.173E-5
AKK(14)=7.462E-3
AKK(15)=-9.884E-4
AKK(16)=1.360E-6
AKK(17)=-3.960E-2
AKK(18)=0.1157
AKK(19)=3.849E-3
AKK(20)=1.717E-3
AKK(21)=-1.692E-3
AKK(22)=9.062E-4
AKK(23)=2.932E-3
AKK(24)=2.507E-6
AKK(25)=-0.5298
AKK(26)=1.237
AKK(27)=0.3351
AKK(28)=1.843E-2
AKK(29)=-1.827E-2
AKK(30)=9.269E-3
AKK(31)=-5.871E-2
AKK(32)=2.599E-5
TEST(1)=0.0070
TEST(2)=0.0084
TEST(3)=0.033
TEST(4)=0.0022
THD(1)=10.0
THD(2)=-12.9
THD(3)=5.4
THD(4)=0.
THD(5)=0.45
THD(6)=0.30
THD(7)=-5.3
THD(8)=42.8
THD(9)=0.
THD(10)=0.
PPD(1)=1000.
PPD(2)=1000.
PPD(3)=1000.
PPD(4)=1000.
PPD(5)=1.
PPD(6)=1.
PPD(7)=1000.
PPD(8)=1000.
PPD(9)=0.
PPD(10)=0.
DO 310 I=1,33,2
SC(I)=1.
SC(I+1)=0.

```

310
C

```
RETURN
```

C
C

400

```

TS=T
IREGK=AIREGK+0.1
IKAL=AIKAL+0.1
IKMX=AIKMX+0.1
IMX=AIMX+0.1
IVVC=AIVVC+0.1
IVV=AIVV+0.1
IREGV=AIREGV+0.1

```

```

IAKV=AIAKV+0.1
IREGYT=AIREYT+0.1
IREG=AIREG+0.1
IFID=AIFID+0.1
IST=AIST+0.1
IA=ANA+0.1
IB=ANB+0.1
IC1=ANC1+0.1
IC2=ANC2+0.1
K=AK+0.1
IREGY=AIREGY+0.1
IYAK=A1YAK+0.1
IT1Y=A1T1Y+0.1
IT3Y=A1T3Y+0.1
IT4Y=A1T4Y+0.1
ILO=A1LO+0.1
DO 410 I=1,4
410 REAS(I)=AMSD(I)+0.1
C

```

```

ITIME=-1
IDEXP=0
INAUT=1
T01=TC-0.01
FLL=0.
SUM1=0.
SUM2=0.
SUM3=0.
SUM4=0.
SUM5=0.
CGR=0.0174533
CKM=0.514444
PI=3.141593
PI2=6.283185
AL=350.
AL1=164.35
A11=0.99163
A12=-0.0100810
A14=-0.00208207
A15=0.000212358
A21=-0.0759537
A22=0.96485
A24=0.0164706
A25=-0.00170991
A31=-0.000874514
A32=0.0224515
A34=0.000195418
A35=-0.0000132723
A44=0.81873
A45=-0.18127
B11=-0.000212358
B21=0.00170991
B31=0.0000132723
B41=0.18127
ILUS=1
IPK=0
IPC=1
IFR=0

```

```

RETURN

```

```

C
C
500 ITIME=ITIME+1
PSI(1)=PSII
IF(PSII .LT. 0.) PSI(1)=PSII+360.

```

```

P=0.34186C*AN
PREF=PREF1
IF(T .GE. T01) PREF=PREF2
PSIREF=PREF
IF(PSIREF .LT. 0.) PSIREF=PSIREF+360.
RREF=0.
IF(1 .GE. T01) RREF=R0

```

```

C
CALL AUTP3

```

```

C
PSIE=PSI(2)
IF(PSIE .GT. 180.) PSIE=PSIE-360.
DO 508 I=1,10
L=I*(I+1)/2
508 PPD(I)=PP(L)
IF(ILO) 516,510,510
510 IF(itime-ILO) 514,512,514
C

```

```

512 FLL=0.
SUM1=0.
SUM2=0.
SUM3=0.
SUM4=0.
SUM5=0.

```

```

C
514 FLL=FLL+1.
SL1=PSIE-PREF
IF(SL1 .LE. -180.) SL1=SL1+360.
IF(SL1 .GT. 180.) SL1=SL1-360.
SUM1=SUM1+SL1
SUM2=SUM2+SL1*SL1
SUM3=SUM3+DELM
SUM4=SUM4+DELM*DELM
SUM5=SUM5+P/(CKM*D*COS(CGR*SL1))

```

```

C
516 DELCS=SC(1)*DELCO+SC(2)
DELES=SC(3)*DELTA(2)+SC(4)
VCS=SC(5)*V(1)+SC(6)
VES=SC(7)*V(2)+SC(8)
V1ES=SC(9)*V1(2)+SC(10)
RES=SC(11)*R(2)+SC(12)
PSIES=SC(13)*PSIE+SC(14)
PREFS=SC(15)*PREF+SC(16)
RREFS=SC(17)*RREF+SC(18)
VESTS=SC(19)*VEST+SC(20)
VVS=SC(21)*VV/CKM+SC(22)
DOES=SC(23)*DELTA0+SC(24)
D1ES=SC(25)*D1+SC(26)
D2ES=SC(27)*D2+SC(28)
D3ES=SC(29)*D3+SC(30)
AMODS=SC(31)*FLOAT(MODYAW)+SC(32)
EDEL=SC(33)*EDELTA+SC(34)

```

```

C
DO 520 I=1,4
APEAS(I)=PEAS(I)
520 APSUM(I)=PEASUM(I)
AIVV1=IVV1

```

```

C
RETURN

```

```

C
C
C
600 TS=T+DT
RETURN
C

```

```

C
700  RETURN
C
C
800  EPSIM=SUM1/FLL
    VPSIM=SUM2/FLL-EPSIM*EPSIM
    SPSIM=-1.
    IF(VPSIM .GE. 0.) SPSIM=SQRT(VPSIM)
    EDDELT=SUM3/FLL
    VDDELT=SUM4/FLL-EDDELT*EDDELT
    SDDELT=-1.
    IF(VDDELT .GE. 0.) SDDELT=SQRT(VDDELT)
    ENM=SUM5/FLL
    VL2=SUM2/FLL+ALAM*VDDELT
    VL1=VL2+ALAM*EDDELT*EDDELT
C
    RETURN
C
C
    END

```

SUBROUTINE AUTP3

AUTOPILOT FOR SHIP, INCLUDING KALMAN FILTER,
SELF-TUNING REGULATOR AND PID-REGULATOR FOR
STRAIGHT COURSE KEEPING, AND YAW REGULATOR.

AUTHOR, C.KALLSTROM 1976-02-22.
REVISED, C.KALLSTROM 1976-04-01.

SUBROUTINE REQUIRED
STUR

COMMON /DATA/ ITIME, IDELC, MODYAW, IDEXP, ISTBD, IPORT,
* IFLAG, IPRINT, INAUT, IKX, MEAS(4),
* DELCO, DELTA(2), V1(2), R(2), PSI(2), DELU, DELCOM, DELTAS, U, AN,
* P, VEST, PSIREF, RREF, DLIM, V(2), DELTA0, D1, D2, D3, TH(10),
* CGR, CKM, PI, PI2, AL, AL1, A11, A12, A14, A15, A21, A22, A24, A25,
* A31, A32, A34, A35, A44, A45, B11, B21, B31, B41, AK(8,4),
* TEST(4), BB, PVC, RLV, AKVG, VCONST, VMIN, VMAX, VO, TH0(10),
* PPO(10), RL, B0, Q2, AK1, AK2, AK3, PSIMX1, PSIMX2, PSIMX3,
* EPS1Y, EPS2Y, EPS3Y, C1Y, C2Y, C3Y, AK1Y, AK2Y, AK3Y, AK4Y,
* AK5Y, AK6Y, AK7Y, AK8Y, BD, BV, ALAM, DELAMP, PSIU, AKID,
* IREGK, IKAL, IKX, IMX, IVVC, IVV, IREGV, IAKV, ILOS, IREGYT, IREG, IPID,
* IST, NA, NB, NC1, NC2, K, IREGY, IYAW, IT1Y, IT3Y, IT4Y, IPK, IPC, IPR,
* EPS(4), VV, PV, PP(55), EPSI1(2), EDEL(2), VLOS1(2),
* ENM1(2), CN, EDELTA, EPSI2, VLOS2, ENM2,
* MEASUM(4), IVV1,
* X(8), PS(4), DD(3), DDOLD, VVV(4), VV2, VOV, VOV2, AKV,
* PFO1, PFO2, DELOLD, VOLD, ROLD, DAT(46), SINT, RRF, AINT2,
* AINT4, STD, STV, SL1, SL2, SL3, SL4, SL5, DUM(10),
* IRK, IREV, INEAS(4), IRYT, IR, NC, NAB, NP, K1, NDAT, NDAT1,
* NU1, N1, IKY, ITIM1, ITIM3, ITIM4, IP, I, J, L

COMPUTE THE SWAY VELOCITY V(1).

$V(1) = V(1) - AL1 * R(1) * CGR / CKM$

IF(INAUT) 80, 80, 10

INITIALIZE IF INAUT=1.

IVV1=IVVC

VVV(1)=U*CKM

VV2=VVV(1)*VVV(1)

VEST=U

VVV(2)=VVV(1)

VVV(3)=AN*U.18*CKM

VVV(4)=VCONST

IF(VVV(IVV1)-VMIN) 10, 14, 14

IF(VVV(IVV1)-VMAX) 18, 18, 16

IVV1=IVV1+1

GO TO 12

VV=VVV(IVV1)

VOV=VO/VV

VOV2=VOV*VOV

IRK=IREGK

IKX=0

DELCOM=0.

DELCO=0.

IF(IKAL) 50, 50, 20

IF(MEAS(1)) 22, 22, 24

```

22      X(4)=DELTA(1)*CGR
      GO TO 28
24      X(4)=0.
26      IF(MEAS(3)) 28,28,30
28      X(2)=R(1)*AL*CGR/VV
      GO TO 32
30      X(2)=0.
      GO TO 36
32      IF(MEAS(2)) 34,34,36
34      X(1)=V(1)*CGR/VV
      GO TO 36
36      X(1)=0.
38      IF(MEAS(4)) 40,40,999
40      X(5)=PSI(1)*CGR
      X(5)=0.
      X(6)=0.
      X(7)=0.
      X(8)=0.
      DO 42 I=1,4
      IMEAS(I)=0
42      MEASUM(1)=0
C
50      DDOLD=0.
      PS(1)=PSI(1)
      PS(2)=PSI(1)
      PS(3)=PSI(1)
      PS(4)=PSI(1)
      DD(1)=0.
      DD(2)=0.
      DD(3)=0.
      PV=PVO
      IREV=IREGV
C
      IRYT=IREGYT
      PFOI=PSI(1)
      BODYAW=0
      IR=IREG
      NC=NC1+NC2
      NAB=NA+NB
      NP=NAB+NC
      K1=K+1
      NDAT=NAB+(K1+1)*(NC+2)-2
      NDATI=NDAT+1
      NU1=NA+K+2
      N1=NU1+K
      J=NU1-1
      SL1=PSI(1)-PSIREF
      IF(SL1 .LE. -180.) SL1=SL1+360.
      IF(SL1 .GT. 180.) SL1=SL1-360.
      DO 52 I=1,J
52      DAT(I)=SL1
      J=J+1
      DO 54 I=J,46
54      DAT(I)=0.
      DELOLD=0.
      VOLD=0.
      ROLD=0.
      SINT=0.
      DO 60 I=1,TO
      DO 60 J=1,I
      L=1*(I-1)/2+J
      IF(I-J) 58,58,58
56      PP(L)=PPO(1)
      GO TO 60

```

```

58 PP(L)=0.
59 CONTINUE
DO 62 I=1,10
62 TH(I)=TH0(I)
EDELTA=0.
STD=1.-80
EPSI2=0.
VLOS2=0.
ENM2=0.
STV=1.-8V
ISTBD=0
IPORT=0
IP=IPR
INAUT=0
80 IF(IREGK) 200,200,90
90 IF(IRK-IREGK) 180,100,200
C
C LOOP WITH SAMPLING INTERVAL IREGK.
C
100 IRK=1
IF(IRKAL) 140,140,102
C
C KALMAN FILTER.
C
102 SL5=CGR*DELCOF
SL1=A11*x(1)+A12*x(2)+A14*x(4)+A15*x(5)+B11*SL5
SL2=A21*x(1)+A22*x(2)+A24*x(4)+A25*x(5)+B21*SL5
SL3=A31*x(1)+A32*x(2)+x(3)+A34*x(4)+A35*x(5)+B31*SL5
SL4=A44*x(4)+A45*x(5)+B41*SL5
x(1)=SL1
x(2)=SL2
x(3)=SL3
x(4)=SL4
C
EPS(1)=DELTA(1)*CGR-x(4)-x(5)-x(8)
EPS(2)=V1(1)*CKM/VV-x(1)-x(2)*AL1/AL-x(6)
EPS(3)=R(1)*CGR*AL/VV-x(2)-x(7)
EPS(4)=PSI(1)*CGR-x(3)
IF(EPS(4) .LE. -PI) EPS(4)=EPS(4)+PI2
IF(EPS(4) .GT. PI) EPS(4)=EPS(4)-PI2
C
IF(IX-IXX) 104,108,108
104 IX=IXX+1
DO 106 I=1,4
106 IMEAS(I)=0
GO TO 122
C
108 DO 120 I=1,4
IF(MEAS(I)) 112,112,118
112 IF(ABS(EPS(I))-TEST(I)) 118,118,114
114 IMEAS(I)=IMEAS(I)+1
MEASUM(I)=MEASUM(I)+1
IF(IMEAS(I)-IMX) 120,116,116
116 MEAS(I)=1
118 IMEAS(I)=0
120 CONTINUE
C
122 DO 134 I=1,4
IF(MEAS(I)+IMEAS(I)) 130,130,134
130 DO 132 J=1,8
IF((J .EQ. 8) .AND. (MEAS(1) .GT. 0)) GO TO 132
IF((J .EQ. 8) .AND. (MEAS(2) .GT. 0)) GO TO 132
IF((J .EQ. 7) .AND. (MEAS(3) .GT. 0)) GO TO 132
X(J)=X(J)+AK(J,I)*EPS(I)

```

```

132 CONTINUE
134 CONTINUE
C
IF(X(3) .LT. 0.) X(3)=X(3)+PI2
IF(X(3) .GE. PI2) X(3)=X(3)-PI2
C
V(2)=VV*X(1)/CKM
R(2)=VV*X(2)/(AL*CGR)
PS1(2)=X(3)/CGR
DELTA(2)=(X(4)+X(5))/CGR
V1(2)=VV*(X(1)+X(2)*AL1/AL)/CKM
DELTA0=X(5)/CGR
D1=VV*X(6)/CKM
D2=VV*X(7)/(AL*CGR)
D3=X(8)/CGR
C
C COMPUTE THE FORWARD SPEED VV.
C
140 IF(IREGV) 160,160,141
141 IF(IREV-IREGV) 150,142,160
C
142 IREV=1
PS(1)=PSI(IVV)
DD(1)=DELTA(IVV)-DDOLD
DDOLD=DELTA(IVV)
SL1=FLOAT(IREGK*IREGV)
SL2=BB*SL1*SL1*DD(2)/(AL*AL)
SL3=1.+PV*SL2*SL2
AKV=PV*SL2/SL3
SL4=PS(1)-PS(4)
IF(SL4 .LE. -180.) SL4=SL4+360.
IF(SL4 .GT. 180.) SL4=SL4-360.
SL5=PS(3)-PS(2)
IF(SL5 .LE. -180.) SL5=SL5+360.
IF(SL5 .GT. 180.) SL5=SL5-360.
SL1=AKV
IF(IAKV .GT. 0) SL1=AKV0
VV2=VV2+SL1*(SL4+3.*SL5-SL2*VV2)
PV=(PV-AKV*AKV*SL3)/RLV
PS(4)=PS(3)
PS(3)=PS(2)
PS(2)=PS(1)
DD(2)=DD(1)
C
IF(VV2) 144,143,143
143 VVV(1)=SQRT(VV2)
GO TO 140
144 VVV(1)=-1.
140 VEST=VVV(1)/CKM
VVV(2)=C*CKM
VVV(3)=AN*0.18*CKM
VVV(4)=VCONST
C
148 IF(VVV(IVV1)-VMIN) 152,150,150
150 IF(VVV(IVV1)-VMAX) 154,154,152
152 IVV1=IVV1+1
GO TO 148
154 VV=VVV(IVV1)
VLV=V0/VV
V0V2=V0V*VVV
GO TO 160
C
156 IREV=IREV+1
C

```

```

C      COMPUTE THE LOSS FUNCTIONS.
C
160  IF(ILOS) 170,166,162
162  ILOS=0
      CN=0.
      DO 164 I=1,2
      EPSI1(I)=0.
      EDEL(I)=0.
      VLOS1(I)=L.
164  ENM1(I)=0.
166  CN=CN+1.
      SL1=(CN-1.)/CN
      DO 168 I=1,2
      SL2=PSI(I)-PSIREF
      IF(SL2 .LE. -180.) SL2=SL2+360.
      IF(SL2 .GT. 180.) SL2=SL2-360.
      EPSI1(I)=SL1*EPSI1(I)+SL2/CN
      EDEL(I)=SL1*EDEL(I)+DELTA(I)/CN
      VLOS1(I)=SL1*VLOS1(I)+(SL2*SL2+ALAM*(DELTA(I)-EDEL(I)))*
*      (DELTA(I)-EDEL(I)))/CN
168  ENM1(I)=SL1*ENM1(I)+P/(CK*U*COS(SL2*CGR)*CN)
C
170  IF(IPK .GT. 0) IPRINT=1
      GO TO 200
C
180  IRK=IRK+1
C
200  IF(IDEXP) 201,201,700
C
C      LOOP WITH SAMPLING INTERVAL IREGYT FOR YAW TEST.
C
201  IF(IRYT-IREGYT) 206,202,208
C
202  IRYT=1
      SL1=PSIREF-PF01
      PF01=PSIREF
      IF(MODYAW) 204,204,300
204  PF02=PSIREF-SL1
      IF(SL1 .LE. -180.) SL1=SL1+360.
      IF(SL1 .GT. 180.) SL1=SL1-360.
      IF(ABS(SL1)-PSINX1) 500,500,302
C
206  IRYT=IRYT+1
C
208  IF(MODYAW) 500,500,300
C
C      LOOP WITH SAMPLING INTERVAL IREGY FOR YAWING.
C
300  IF(IRY-IREGY) 309,302,392
C
302  IRY=1
      SL1=PSIREF-PF02
      PF02=PSIREF
      IF(SL1 .LE. -180.) SL1=SL1+360.
      IF(SL1 .GT. 180.) SL1=SL1-360.
      SL2=PSI(IYAW) - PSIREF
      IF(SL2 .LE. -180.) SL2=SL2+360.
      IF(SL2 .GT. 180.) SL2=SL2-360.
      IF(MODYAW) 309,309,304
304  IF(ABS(SL1)-PSINX3) 320,320,306
306  IF(MODYAW-2) 314,314,308
308  MODYAW=1
309  IF(SL2) 310,310,312
310  RRF=RREF

```

```

GO TO 320
312 RRF=-RRF
GO TO 320
314 IF(RRF) 316,316,318
316 IF(SL2) 308,320,320
318 IF(SL2) 320,320,308
C
320 RRF=RRF*ABS(RRF)/RRF
SL3=R(IYAW)-RRF
C
IF(MODYAW) 322,322,324
322 IF(ABS(SL1)-PSIMX2) 338,338,326
324 IF(MODYAW-2) 328,332,325
325 IF(MODYAW-4) 336,339,339
C
326 MODYAW=1
ITIM1=-IREGY
328 ITIM1=ITIM1+IREGY
IF(RRF .GE. 0. .AND. SL3 .GT. -EPS1Y) GO TO 330
IF(RRF .LT. 0. .AND. SL3 .LT. EPS1Y) GO TO 330
IF(ITIM1-IT1Y) 332,332,330
C
330 MODYAW=2
AINT2=0.
332 IF(RRF .GE. 0. .AND. -C2Y*R(IYAW) .LT. SL2) GO TO 334
IF(RRF .LT. 0. .AND. -C2Y*R(IYAW) .GT. SL2) GO TO 334
IF(MODYAW-1) 340,340,350
C
334 MODYAW=3
ITIM3=-IREGY
336 ITIM3=ITIM3+IREGY
IF(ABS(R(IYAW)) .LT. EPS2Y) GO TO 338
IF(RRF .GE. 0. .AND. SL2 .GT. -EPS3Y) GO TO 338
IF(RRF .LT. 0. .AND. SL2 .LT. EPS3Y) GO TO 338
IF(ITIM3-IT3Y) 360,360,338
C
338 MODYAW=4
ITIM4=-IREGY
AINT4=0.
339 ITIM4=ITIM4+IREGY
IF(ITIM4-IT4Y) 370,370,400
C
C YAW PHASE 1.
C
340 SL4=AK4Y*SL3
SL5=ABS(C1Y*RRF)
IF(SL4 .GT. SL5) SL4=SL5
IF(SL4 .LT. -SL5) SL4=-SL5
DELCON=-VOV2*SL4+EDELTA
GO TO 360
C
C YAW PHASE 2.
C
350 DELCON=-VOV2*(AK5Y*SL3+AK6Y*AINT2)+EDELTA
AINT2=AINT2+SL3*FLOAT(IREGY)
GO TO 330
C
C YAW PHASE 3.
C
360 SL4=AK7Y*SL2+AK8Y*R(IYAW)
SL5=ABS(C3Y*RRF)
IF(SL4 .GT. SL5) SL4=SL5
IF(SL4 .LT. -SL5) SL4=-SL5
DELCON=-VOV2*SL4

```

```

      GO TO 380
C
C      YAW PHASE 4.
C
370  DELCOM=-VGV2*(AK1Y*SL2+AK2Y*R(IYAW)+AK3Y*AI4T)
      AI4T=AI4T+SL2*FLOAT(IREGY)
C
C
380  IF(DELCOM .GT. DLIM) DELCOM=DLIM
      IF(DELCOM .LT. -DLIM) DELCOM=-DLIM
      GO TO 600
C
C
390  IRY=IRY+1
C
392  GO TO 900
C
C      INITIALIZING OF STRAIGHT COURSE KEEPING.
C
400  J=NU1-1
      SL1=PSI(IST)-PSIREF
      IF(SL1 .LE. -180.) SL1=SL1+360.
      IF(SL1 .GT. 180.) SL1=SL1-360.
      DO 402 I=1,J
402  DAT(I)=SL1
      J=J+1
      DO 404 I=J,46
404  DAT(I)=0.
      DELOLD=EDELTA
      VOLD=V(IST)
      ROLD=R(IST)
      BODYAW=0
      SINT=0.
      GO TO 502
C
C      LOOP WITH SAMPLING INTERVAL IREG FOR STRAIGHT COURSE KEEPING.
C
500  IF(IR-IREG) 540,502,542
C
502  IF=1
      SL1=PSI(IST)-PSIREF
      IF(SL1 .LE. -180.) SL1=SL1+360.
      IF(SL1 .GT. 180.) SL1=SL1-360.
      IF(IPID) 504,504,520
C
504  SL2=V(IST)-VOLD
      SL3=R(IST)-ROLD
      VOLD=V(IST)
      ROLD=R(IST)
      DAT(1)=SL1
      IF(NC-1) 514,506,508
      IF(NC1) 512,512,508
506  J=NAB+2*K+3
      DAT(J)=SL2*VV
      IF(NC-1) 514,514,510
510  J=NAB+3*K+5
      DAT(J)=SL3
      GO TO 514
512  J=NAB+2*K+5
      DAT(J)=SL3
C
514  CALL STOR(DAT,TH,PP,DUM,RL,NA,NAB,PP,K1,NDAT,NDAT1,NU1,N1)
C
      SL2=VGV2*EC/(EO*EO+Q2)
      DELCOM=SL2*DAT(NU1)+DELOLD

```

```

C
520  GO TO 530
C
520  DELCOM=-VQV2*(AK1*SL1+AK2*R(IST)+AK3*SINT)
      SINT=SINT+SL1*FLOAT(IREG)
C
530  IF(DELCOM .GT. DLIM) DELCOM=DLIM
      IF(DELCOM .LT. -DLIM) DELCOM=-DLIM
      IF(IDEXP) 531,531,800
531  IF(IPID) 532,532,534
532  DAT(NU1)=BC*(DELCOM-DELOLD)/VQV2
      DELOLD=DELCOM
534  GO TO 600
C
540  IR=IR+1
C
542  GO TO 900
C
      COMPUTE THE MEAN RUDDER COMMAND EDELTA AND THE LOSS FUNCTIONS.
C
600  IF(MODYAW) 604,604,602
602  IF(MODYAW-4) 800,604,604
604  EDELTA=EDELTA+(STD+BD)*(DELCOM-EDELTA)
      STD=(1.-BD)*STD/(1.-BD+STD)
      IF(MODYAW) 606,606,800
606  EPSI2=EPSI2+(STV+BV)*(SL1-EPSI2)
      SL2=DELCOM-EDELTA
      SL3=SL1*SL1+ALAM*SL2*SL2
      VLOS2=VLOS2+(STV+BV)*(SL3-VLOS2)
      SL2=P/(CKM*U*COS(SL1*CGR))
      ENM2=ENM2+(STV+BV)*(SL2-ENM2)
      STV=(1.-BV)*STV/(1.-BV+STV)
      GO TO 800
C
      IDENTIFICATION EXPERIMENT.
C
700  IF(IR-IREG) 720,702,900
C
702  IR=1
      IF(ISTBD+IPORT-1) 712,704,710
704  IF(ISTBD) 708,708,706
706  DEL0=DELAMP
      GO TO 712
708  DEL0=-DELAMP
      GO TO 712
710  DEL0=0.
712  ISTBD=0
      IPORT=0
      SL1=PSI(1)-PSIU
      IF(SL1 .LE. -180.) SL1=SL1+360.
      IF(SL1 .GT. 180.) SL1=SL1-360.
      DELCOM=DEL0-AK10*SL1
      GO TO 530
C
720  IR=IR+1
      GO TO 900
C
      INDICATE RUDDER CHANGE.
C
800  DELCO=DELCOM
      IDELC=1
C
      IF(IPC .GT. 0) IPRINT=1
C
900  IF(IPR) 999,999,902

```

```
C
902  IF(IP=IPR) 906,904,999
C
904  IP=1
      IPRINT=1
      GO TO 999
C
906  IP=IP+1
C
999  RETURN
      END
```

SUBROUTINE STOR(DAT,TH,P,DUM,RL,NA,NB,NC,K1,NDAT,NDAT1,NU1,N1)

SELF-TUNING REGULATOR BASED ON LEAST SQUARES IDENTIFICATION
AND MINIMUM VARIANCE CONTROL, ADMITS FEEDFORWARD AND
EXPLOITS SYMMETRY OF P.

AUTHOR, C. KALLSTROM 1976-02-18.

THE ALGORITHM IS BASED ON THE MODEL

$$Y(T) + A(1)*Y(T-K-1) + \dots + A(NA)*Y(T-K-NA) = \\ B(0)*(U(T-K-1) + B(1)*U(T-K-2) + \dots + B(NB)*U(T-K-NB-1)) + \\ C(1)*V1(T-K-1) + C(2)*V2(T-K-1) + \dots + C(NC)*VNC(T-K-1) + EPS(T)$$

AT EACH STEP THE LEAST SQUARES ESTIMATES OF THE PARAMETERS
OF THE MODEL ARE COMPUTED. THE CONTROL VARIABLE U(T) TO
BE APPLIED AT TIME T IS THEN COMPUTED FROM

$$US(T) = AE(1)*Y(T) + \dots + AE(NA)*Y(T-NA+1) \\ - BE(1)*US(T-1) - \dots - BE(NB)*US(T-NB) \\ - CE(1)*V1(T) - \dots - CE(NC)*VNC(T)$$

WHERE AE, BE AND CE ARE THE PARAMETER ESTIMATES
AND US THE SCALED CONTROL SIGNAL I.E. $US = B(0)*U$

WHEN USING THE ALGORITHM THE PROCESS OUTPUT Y(T) AND THE
FEEDFORWARD SIGNALS V(T) ARE READ AT TIME T AND THE CONTROL
SIGNAL U(T) TO BE APPLIED AT TIME T IS THEN COMPUTED

DAT- VECTOR OF DIMENSION $NA+NB+(K+2)*(NC+2)-2$ CONTAINING
PROCESS OUTPUTS Y, SCALED CONTROL VARIABLES U
AND FEED FORWARD SIGNALS V ORGANIZED AS FOLLOWS

DAT(1)=Y(T)	RETURNED AS Y(T)
DAT(2)=Y(T-1)	RETURNED AS Y(T)
DAT(3)=Y(T-2)	RETURNED AS Y(T-1)
•	
DAT(NA+K+1)=Y(T-K-NA)	RETURNED AS Y(T-K-NA+1)
DAT(NA+K+2)=US(T-1)	RETURNED AS US(T)
DAT(NA+K+3)=US(T-2)	RETURNED AS US(T-1)
•	
DAT(NA+NB+2*K+2)=US(T-K-NB-1)	RETURNED AS US(T-K-NB)
DAT(NA+NB+2*K+3)=V1(T)	RETURNED AS US(T-K-NB-1)
DAT(NA+NB+2*K+4)=V1(T-1)	RETURNED AS V1(T)
•	
DAT(NA+NB+3*K+4)=V1(T-K-1)	RETURNED AS V1(T-K)
•	
DAT(NA+NB+(K+2)*(NC+1)-1)=VNC(T)	RETURNED AS V(NC-1)(T-K-1)
•	
DAT(NA+NB+(K+2)*(NC+2)-2)=VNC(T-K-1)	RETURNED AS VNC(T-K)

TH- VECTOR OF DIMENSION $NP=NA+NB+NC$ CONTAINING THE PARAMETER
ESTIMATES ORGANIZED AS FOLLOWS

TH(1)=-AE(1)
TH(2)=-AE(2)
•
TH(NA)=-AE(NA)
TH(NA+1)=BE(1)
TH(NA+2)=BE(2)
•
TH(NA+NB)=BE(NB)
TH(NA+NB+1)=CE(1)
TH(NA+NB+2)=CE(2)

```

C      *
C      TH(NA+NB+NC)=CE(NC)
C
C
C      P= COVARIANCE MATRIX STORED AS FOLLOWS
C      P(1)=P(1,1)
C      P(2)=P(2,1)
C      P(3)=P(2,2)
C
C      *
C      P(I*(I-1)/2+J)=P(I,J)
C
C      *
C      P(NP*(NP+1)/2)=P(NP,NP)
C
C
C      DUM= DUMMY VECTOR OF DIMENSION NP
C      RL= BASE OF EXPONENTIAL WEIGHTING FACTOR
C      NA= NUMBER OF A-PARAMETERS (MAX 10, MIN 0)
C      NB= NUMBER OF B-PARAMETERS (MAX 10, MIN 0)
C      NC= NUMBER OF C-PARAMETERS (MAX 10, MIN 0)
C      K= NUMBER OF TIME DELAYS IN THE MODEL
C          (MAX ((46-NA-NB+2)/(NC+2))-2 , MIN 0)
C
C      NAB= NA+NB
C      NP= NA+NB+NC (MAX 10, MIN 1)
C      K1= K+1
C      NDAT= NAB+(K1+1)*(NC+2)-2
C      NDAT1= NDAT+1
C      NU1= NA+K+2
C      N1= NU1+K
C
C      SUBROUTINE REQUIRED
C          NONE
C
C      DIMENSION DAT(46),TH(10),P(55),DUM(10)
C
C      RES=DAT(1)-DAT(N1)
C      DENOM=1.
C      DO 12 I=1,NP
C      R=0.
C      DO 10 J=1,NP
C      L=I*(I-1)/2+J
C      IF (J.GT.I) L=J*(J-1)/2+I
C      M=K1+J
C      IF (J.GT.NA) M=M+K1
C      IF (J.GT.NAB) M=2*K1+(J-NAB)*(K1+1)+NAB
10      R=R+P(L)*DAT(M)
C      DUM(I)=R
C      M=K1+I
C      IF (I.GT.NA) M=M+K1
C      IF (I.GT.NAB) M=2*K1+(I-NAB)*(K1+1)+NAB
C      DENOM=DENOM+R*DAT(M)
12      RES=RES-DAT(M)*TH(I)
C
C      DO 20 I=1,NP
C      R=DUM(I)/DENOM
C      TH(I)=TH(I)+R*RES
C      DO 20 J=1,I
C      L=I*(I-1)/2+J
20      P(L)=(P(L)-R*DUM(J))/RL
C
C      R=0.
C      DO 30 I=1,NP
C      L=I
C      IF (I.GT.NA) L=L+K1
C      IF (I.GT.NAB) L=NAB+K1+(K1+1)*(I-NAB)

```

```
30      R=R-TH(I)*DAT(L)
C
      DO 32 I=2,NDAT
      L=NDAT1-I
32      DAT(L+1)=DAT(L)
      DAT(NDAT1)=R
C
      RETURN
      END
```