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Low Complexity Adaptive Channel Estimation and QR Decomposition for an LTE-A Downlink

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***Abstract*—This paper presents a link adaptive processor to perform low-complexity channel estimation and QR decomposition (QRD) in Long Term Evolution-Advanced (LTE-A) receivers. The processor utilizes frequency domain correlation of the propagation channel to adaptively avoid unnecessary computations in the received signal processing, achieving significant complexity reduction with negligible performance loss. More specifically, a windowed Discrete Fourier transform (DFT) algorithm is used to detect channel conditions and to compute a minimum number of sparse subcarrier channel estimates required for low complexity linear QRD interpolation. Furthermore, the sparsity of subcarrier channel estimates can be adaptively changed to handle different channel conditions. Simulation results demonstrate a reduction of 40%-80% in computational complexity for different channel models specified in the LTE-A standard.**

***Keywords*—Wireless communication, MIMO, Channel estimation, OFDM, Adaptive signal processing.**

I. Introduction

The requirement for high speed wireless communication, limited frequency bands and fluctuating channel conditions has made Multiple-Input Multiple-Output (MIMO) a widely adopted technique in many radio standards, including the 3GPP Long Term Evolution-Advanced (LTE-A). To fully utilize the capabilities of LTE-A systems, sophisticated signal processing operations are required. Among others, accurate channel estimation and the following channel matrix QR decomposition (QRD) are indispensable for advanced MIMO signal detectors, e.g., the K-Best detector [1], to recover transmitted information from noisy received signals. Different algorithms for channel estimation are presented in [2] [3] and the most frequently used QRD algorithms are detailed in [4]. These essential signal processing operations of a MIMO system come with the price of high computational complexity, preventing accurate implementation of the corresponding algorithms in power and area limited handheld devices.

MIMO is usually combined with Orthogonal Frequency Division Multiplexing (OFDM) to provide high spectral efficiency. As a result, channel estimation and QRD have to be performed on a tone-by-tone basis, making the complexity a more critical issue. To alleviate this complexity problem, authors in [5] present an interpolation technique, where channel estimation and QRD are performed only on pilot tones and the QRD for tones in between pilots is obtained by interpolation. However, the algorithm is implemented for a fixed interpolation distance and also requires a translation into another domain for performing QRD interpolation. Moreover, such a static interpolation strategy may degrade performance in highly frequency selective channels while resulting in unnecessary computations in channels with low frequency selectivity.

Mobile devices operate in fluctuating channel scenarios depending on their surroundings and the LTE-A standard classifies wireless channels into three main categories based on the frequency selectivity, namely the EPA, EVA and ETU. The EPA channel has a very low frequency selectivity and requires fewer subcarrier channel estimates than the highly frequency selective ETU channel to reach a target system bit error rate (BER). Hence, a fixed solution such as the tone-by-tone approach or the one presented in [5] is not efficient in terms of the number of computations performed. To alleviate the aforementioned problem, we propose an adaptive solution which takes into account the dynamic nature and frequency selectivity of the wireless channel. In detail, the proposed solution utilizes a windowed Discrete Fourier transform (DFT) based channel estimator to produce only a required number of subcarrier channel estimates enabling very low complexity linear QRD interpolation. The windowed DFT method also provides a simple way of detecting operating channel conditions, enabling the adaptive processor to optimize the interpolation distance, measured in subcarriers, to reach a desired BER with the lowest computational efforts. To verify the proposed scheme, we simulated a simplified LTE-A downlink system with a 4×4 MIMO setup. Simulations performed with the EVA and ETU channels show that the proposed method offers significant complexity saving over the traditional tone by tone method, with minor performance loss.

II. Background

A MIMO system with M transmitter (Tx) and receiver (Rx) antennas can be modelled as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \ , \tag{1}$$

where $\mathbf{y} = [y_1, y_2, ..., y_M]^T$ is the received data vector, $\mathbf{H} \in \mathbb{C}^{M \times M}$ is the channel gain matrix between the antennas, $\mathbf{x} = [x_1, x_2, ..., x_M]^T$ is transmit data vector and $\mathbf{n}$ is the additive white Gaussian noise (AWGN). To achieve a low BER, the MIMO symbol detector has to minimize the error $\| \mathbf{y} - \tilde{\mathbf{H}}\tilde{\mathbf{x}} \|_2$, where $\tilde{\mathbf{x}}$ is the estimate of the transmit vector and $\tilde{\mathbf{H}}$ is the estimated channel matrix obtained from a set of predefined pilot tones. Fig. 1 shows the structure of these predefined pilots in a 4×4 LTE-A system and the redundancy in pilots enables good performance even under high frequency selectivity.

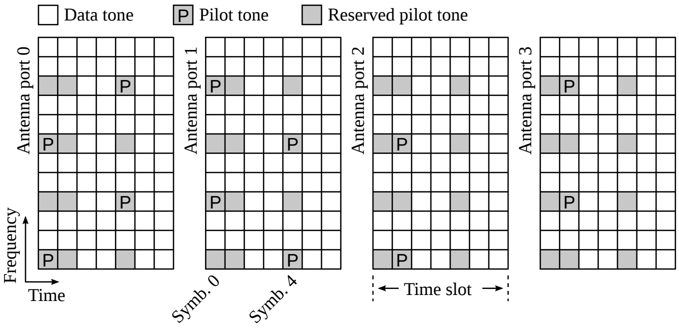

Fig. 1: Pilot structure of a 4×4 LTE-A system

Estimation of the channel gain matrix $\tilde{\mathbf{H}}$ is the first major step performed in order to recover data from (1) and accurate estimation enables data detection with a low BER. Furthermore, since there are M^2 paths between the Tx and Rx in $M \times M$ MIMO systems, estimators with lower complexity are preferred. Once the estimate $\tilde{\mathbf{H}}$ is obtained, advanced decoders such as the K-best decoder solve (1) by decomposing the channel gain matrix $\tilde{\mathbf{H}}$ into a product of a unitary $\mathbf{Q}$ and an upper triangular matrix $\mathbf{R}$. The unitary matrix is used to rotate the received vector and the resulting linear system is solved by using tree search techniques as

$$\begin{aligned} \mathbf{y} &= \mathbf{QRx} + \mathbf{n} \\ \mathbf{Q}^*\mathbf{y} &= \mathbf{Rx} + \mathbf{Q}^*\mathbf{n} \\ \hat{\mathbf{y}} &= \mathbf{Rx} + \hat{\mathbf{n}} \end{aligned} \tag{2}$$

where $\mathbf{Q}^*$ is the Hermitian transpose of the unitary $\mathbf{Q}$ matrix. The complexity of popular QRD algorithms, measured in number of multiplications, is $\mathcal{O}(M^3)$ [4]. Hence, in LTE-A receivers, algorithms producing accurate channel estimation and QRD with low complexity are needed to utilize the full potential of MIMO systems.

The traditional approach for solving the signal detection problem has been by considering channel estimation and QRD as separate entities as shown in Fig. 2(a). Channel estimation is performed as an independent operation followed by QRD for all the data tones. Several methods of channel estimation such as the Least Squares (LS), Robust Minimum Mean Square Estimator (RMMSE), DFT and Matching Pursuit (MP) [2] [3] haven been proposed. DFT based estimators are a class of low complexity estimators which utilize the DFT and inverse DFT operations to perform noise filtering, but suffer from a high noise floor due to spectral leakage [6]. Authors in [7] suggest a method to utilize windows to weigh the data to minimize the spectral leakage enabling better performance. The advantage of DFT based estimators is their efficient hardware implementation through Fast Fourier Transform (FFT) algorithms. The complexity of this implementation measured in terms of total multiplications is $\mathcal{O}\left(M^2\left(N_p log_2\left(N_p\right) + N_d log_2\left(N_d\right)\right)\right)$ where N_p is the number of pilot tones and N_d is the number of channel estimates produced for an $M \times M$ MIMO system.

QRD which is performed on the channel estimates can be implemented by several techniques such as the Given's rotation (GR) [8], the Gram-Schmidt (GS) [9] or the Householder transform , but all have a complexity $\mathcal{O}(M^3 N_d)$. Even though both

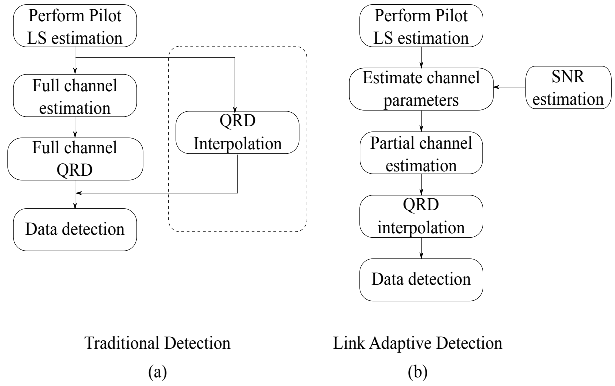

Fig. 2: Data detection flow: Traditional Vs Proposed

channel estimation and QRD are computationally intensive, it has to be noted that in a typical LTE-A system using DFT based channel estimators needs more computations for channel estimation than the QRD.

To reduce the high complexity of the above tone by tone approach, interpolation techniques have been used as shown in Fig. 2(a). The channel estimates at the pilot positions are estimated, followed by QRD interpolation for the data tones in between these pilot positions. A theoretical background when using the GS method to obtain lossless QRD interpolation is provided in [10] and a hardware implementation for an LTE-A frame structure utilizing only pilot positions to perform QRD interpolation is presented in [5]. These techniques rely on mapping the $\mathbf{Q}$ and $\mathbf{R}$ matrices into a sub space where polynomial interpolation can be applied and the de-mapping the interpolated matrices. Even though these methods are more efficient, they do not utilize channel properties such as frequency selectivity to further optimize the number of subcarrier channel estimates and QRDs performed. The fixed architecture of interpolating over pilot positions offers little flexibility and results in many redundant computations in channels with low frequency selectivity and a performance loss in highly frequency selective channels.

III. Link Adaptive QR decomposition

Mobile devices experience high frequency selectivity in rich multipath environments requiring channel estimation at data tones which are closely spaced whereas in low frequency selectivity environments subcarrier channel estimates can be produced at tones which are farther apart. Moreover, when operating in higher received Signal to Noise Ratios (SNRs), a more accurate channel estimation and QRD is required whereas in lower SNRs the gain obtained by performing accurate channel estimation and QRD is lost due to the high noise levels. Hence, by examining the current channel conditions, a reduction in the total number of computations can be obtained by tuning the channel estimator and QRD processor to execute only the minimum number of required computations to reach a desired level of system performance.

Fig. 2(b) depicts such an adaptive approach where the operating channel conditions such as the frequency selectivity are obtained by examining the LS estimates of the pilot tones with the Signal to Noise Ratio (SNR) estimates obtained from external processing units [11]. Based on these channel parameters, an adaptive channel estimator is utilized to produce estimates at tones much farther apart than the pilots when operating in an EPA channel or at tones much closer than the pilot positions when operating in the ETU channel. This estimated channel data is used by a low complexity QRD interpolation method to approximate the $\mathbf{Q}$ and $\mathbf{R}$ matrices of the intermediate data tones. The proposed link adaptive processor incorporates these ideas and the following sections introduce the different components of this processor along with the methodology used to select the optimal distances for the low complexity QRD interpolation. A DFT based channel estimator is used in the link adaptive processor due to its reconfigurability and efficient hardware implementation and a method of mitigating the spectral leakage is discussed next.

A. Windowed DFT channel estimator

DFT based estimators perform an inverse DFT operation on the pilot tones, weigh the taps according to a predefined strategy aimed at filtering noise and perform a DFT to produce the channel estimates at the data positions. Such an estimator producing N_d estimates with N_p pilots can be represented as

$$\mathbf{h_d} = \mathbf{F_d} \mathbf{W} \mathbf{F_p^H} \mathbf{h_p}, \tag{3}$$

where $\mathbf{h_d}$ is the vector of channel estimates at data tones, $\mathbf{F_d}$ is a $N_d \times N_d$ DFT matrix, $\mathbf{W}$ is the $N_d \times N_p$ filtering matrix, $\mathbf{F_p}$ is the $N_p \times N_p$ DFT matrix and $\mathbf{h_p}$ is the vector of channel estimates at pilot tones. These estimators perform well at lower SNRs and are hardware efficient when implemented using the FFT algorithm but suffer from a high noise floor due to spectral leakage at higher SNRs [6].

The basic requirement of the proposed link adaptive processor is the ability to recognize the operating channel conditions, which can be achieved by either analyzing the cyclic prefix or by an inverse DFT operation on the pilot tones. Fig. 3 shows the average power in the different taps of LTE-A channels obtained by using a 64 point inverse FFT operation on the pilot tones in a 5 MHz bandwidth downlink. We notice that by analyzing the distribution of power in the first few taps of the inverse FFT output, the current channel conditions can be detected. Furthermore, the inverse DFT operation is part of the DFT based estimator represented by (3) and enables the detection of channel conditions at no additional cost.

Fig. 3 also shows the spectral leakage due to the non sample spaced channels which causes degraded performance at higher SNRs. Utilizing windows to reduce spectral leakage is a well-known method and [7] describes a method to improve the performance of DFT based estimators at higher SNRs by using Hann windows. An inverse window operation is required to remove the effects of windowing and [7] uses a division by the Hann window to achieve this. The overlap-add method is another way of removing windowing effects and the link adaptive processor uses this method as depicted in Fig. 4. The data from the first and last pilots in the 5 MHz spectrum is extended to produce 128 points and the Hann windowing reduces spectral leakage. Finally, the windowing effects on the channel response is removed by using the overlap-add method to produce interpolated channel estimates at the data tones.

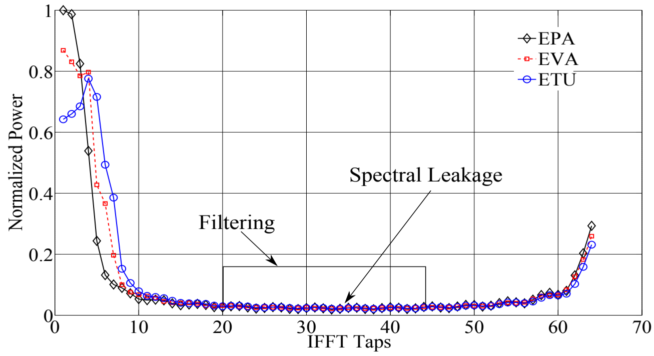

Fig. 3: Average power in taps of a 5 MHz LTE-A downlink

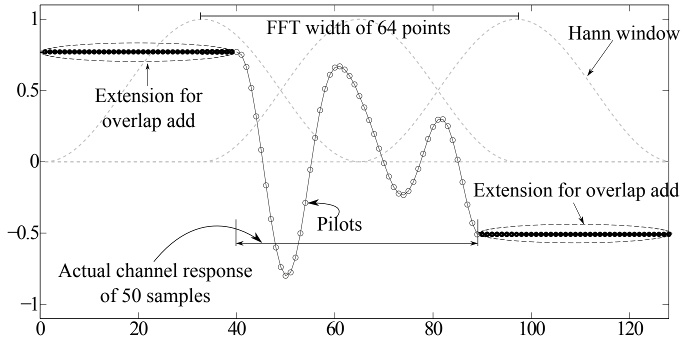

Fig. 4: Overlap add method to reduce spectral leakage

B. QR interpolation

The windowed DFT based channel estimator enables the detection of channel conditions such as frequency selectivity, which can be used to estimate the distance, measured in subcarriers, for QRD interpolation. The effects of interpolation errors on the system BER can be minimized by adaptively changing the interpolation distance depending on channel conditions, for example, by choosing subcarriers which are close when operating in highly frequency selective channels.

All unitary $\mathbf{Q}$ matrices of size $M \times M$ are part of the unitary group $\mathbf{U}(M)$ and any unitary matrix $\mathbf{Q}_1$ can be transformed into another unitary matrix $\mathbf{Q}_2$ by using a rotation matrix of the form $\mathbf{Q}_2\mathbf{Q}_1^*$. Authors in [12] present a method of obtaining the intermediate $\mathbf{Q}$ matrices between $\mathbf{Q}_1$ and $\mathbf{Q}_2$ using

$$\mathbf{Q}(s) = (\mathbf{Q_2}\mathbf{Q_1^*})^s \mathbf{Q_1} \qquad s \in \mathbb{R} \mid 0 \le s \le 1\,. \tag{4}$$

If $\parallel \mathbf{Q}_1 - \mathbf{Q}_2 \parallel_F = \epsilon$, where ϵ is a small constant, a linear interpolation of the form

$$\mathbf{Q}(s) = (1 - s) \times \mathbf{Q}_1 + s \times \mathbf{Q}_2, \tag{5}$$

can be used to approximate (4). The corresponding $\mathbf{R}(s)$ matrix can also be approximated by

$$\mathbf{R}(s) = (1 - s) \times \mathbf{R}_1 + s \times \mathbf{R}_2. \tag{6}$$

These approximations lead to errors in QRD interpolation and a strategy to choose the correct interpolation distances,

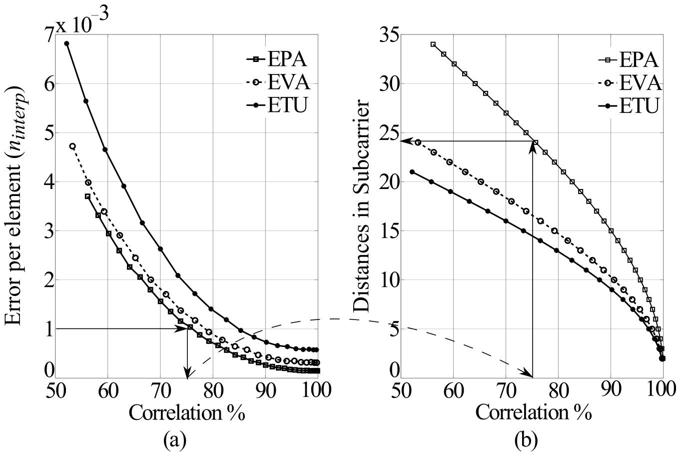

Fig. 5: Interpolation error, Subcarrier distances and Correlation

to minimize the effect of these errors on BER for different channel scenarios is presented in the next section.

C. Coherence Bandwidth and Interpolation error

The coherence band width (B_{coh}) of a wireless channel is defined as the bandwidth over which the correlation of channel gains is higher than a specified limit [13]. QRD of channel gain matrices $\mathbf{H}_1$ and $\mathbf{H}_2$ of two correlated subcarriers results in $\mathbf{Q}_1$ and $\mathbf{Q}_2$ such that $\parallel \mathbf{Q}_1 - \mathbf{Q}_2 \parallel_F = \epsilon$, where ϵ is a small constant. This enables B_{coh} to be used as a parameter to evaluate the interpolation error due to approximations in (6). The LTE-A standard uses three main channel models and each channel exhibits a different B_{coh} enabling the link adaptive processor to choose between different bandwidths over which QRD interpolation can be performed. Fig. 5(b) shows the distances in sub carriers for different levels of correlation for the three models. In the EPA model, gain matrices $\mathbf{H}_1$ and $\mathbf{H}_2$ which are 24 subcarriers apart show a correlation of 75% whereas the EVA and ETU models show 75% correlation for channels around 15 subcarriers apart.

The uncoded BER of a wireless system operating in a frequency selective channel affected by AWGN is inversely proportional to the SNR available at the receiver. The total noise n in a wireless receiver employing interpolation techniques can be expressed as

$$n = n_{sys} + n_{interp} \tag{7}$$

where n_{sys} is the system noise and n_{interp} is the noise introduced due to linear interpolation. A parameter

$$\gamma = \frac{n_{interp}}{n_{sys}}, \tag{8}$$

can be used to decide the amount of interpolation error that can be introduced depending on the receiver SNR. The effects of interpolation error on BER can be minimized by keeping γ small, which enables the link adaptive processor to increase interpolation distances at lower SNRs and to adaptively lower the distances at higher SNRs. Fig. 5(a) shows the dependence of the error n_{interp} obtained by interpolating $\mathbf{Q}$ matrices using (5) between sub carriers spaced at different correlation levels for the three channel models used in LTE-A. The average error due to interpolation in an EPA channel model is in the order of 10^{-3} at correlation levels of 75% whereas the error in the fast fading ETU model is almost double that of the EPA model.

Using Fig. 5(a) and Fig. 5(b) the following strategy illustrates how a link adaptive processor can be used to reduce total computational complexity. An SNR of 20 dB at the receiver in a 4×4 MIMO system receiving 16 QAM data would correspond to a n_{sys} of 10^{-2}. A link adaptive processor configured to operate with a value of $\gamma = 0.1$ can operate with $n_{interp} = 10^{-3}$. If the current operating channel is detected to be EPA, the processor will choose a correlation value of 75% in Fig. 5(a) corresponding to an interpolation distance of 24 subcarriers in Fig. 5(b). Using a similar strategy, if operating in an ETU channel, correlation of 85% is chosen leading to interpolation distances of around 10 subcarriers. A lower value of SNR will result in higher n_{sys} enabling the link adaptive processor to choose subcarriers which have a lower correlation value leading to increased interpolation distances.

IV. Results

A. Performance

The methodology described in the previous sections enables us to choose interpolation distances adaptively to reduce complexity while maintaining the required level of BER. Table I shows an example implementation with different interpolation distances chosen for the three LTE-A channel models depending on the SNR available at the receiver. The interpolation distances are chosen so that minimal loss to BER is introduced when compared to the performance with perfect channel state information (CSI).

TABLE I: Interpolation distances measured in subcarriers

Model \ SNR (dB)	≤ 10	11 - 15	16 - 20	21 - 25	26 - 30	> 30
EPA	48	32	24	16	8	8
EVA	32	24	16	8	8	4
ETU	32	24	16	8	4	2

Fig. 6 shows two sets of uncoded BER curves for a 4×4 MIMO system with a K-Best decoder with K=10 operating in the EVA channel. The first set of curves are obtained by using perfect CSI and full channel QRD along with the proposed adaptive QRD (AQRD) with distances chosen from Table I. These curves enable us to analyze the effects of n_{interp} on BER and it can be seen that the performance loss is negligible when choosing the proposed interpolation distances. The second set of curves are obtained by using different channel estimation techniques and QRD interpolation methods. The DFT based estimator [2] with the proposed adaptive QRD shows significant degradation, with an error floor visible at higher SNRs due to spectral leakage. Use of the proposed windowed DFT based estimator and adaptive QRD improves the performance at higher SNRs and is on par with performance of a receiver employing the RMMSE estimator [14] with the QRD interpolator from [5].

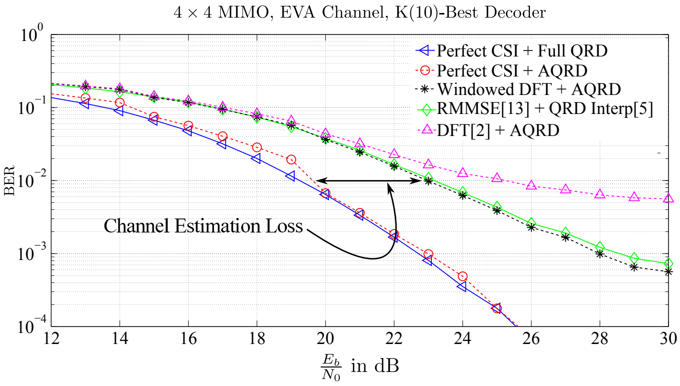

Fig. 6: BER of an uncoded 16QAM system

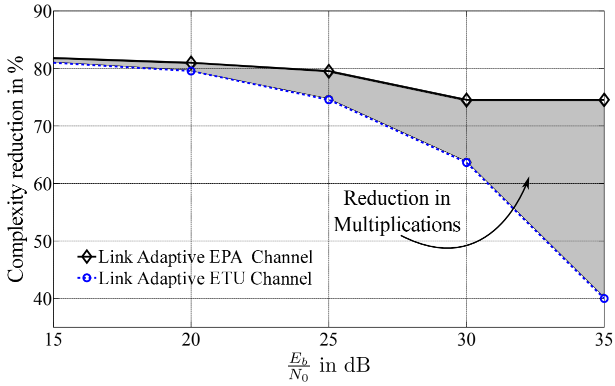

Fig. 7: Range of possible reduction in multiplications

B. Complexity analysis

A DFT based channel estimator used in the traditional flow of Fig.2(a) for an $M \times M$ MIMO system would require $(N_p log_2(N_p) + N_d log_2(N_d))M^2$ multiplications. Using the proposed windowed DFT instead and choosing the number of channel estimations X, changes the multiplication count of channel estimation to $(3N_p log_2(N_p) + X log_2(X))M^2$. Furthermore, the complexity of QRDs computed also reduces from $N_d M^3$ to XM^3.

Fig. 7 shows the savings obtained by using the link adaptive processor over the traditional tone by tone approach, with interpolation distances chosen from Table I. The link adaptive processor is designed to closely follow the BER obtained when operating with perfect CSI. Higher savings are obtained at lower SNRs as farther interpolation distances can be chosen, whereas this gain reduces at higher SNRs. Furthermore, EPA channels need significantly lower number of computations resulting in higher savings when compared to ETU channels. The shaded region in Fig. 7 indicates the range of possible reductions when operating in different channel conditions.

C. Hardware requirements

The main advantage of DFT based channel estimators is the efficient hardware implementation. A typical N_d point radix-2 FFT is implemented using a pipelined structure [15]. The order of the output samples from the pipelined decimation in frequency (DIF) FFT algorithm is bit reversed with the first $X = 2^t$ where $t \in \mathbb{Z}$ output bins spaced at distances of $\frac{N_d}{X}$ and the next X outputs resulting in all bins at a resolution of $\frac{N_d}{2X}$, enabling selective channel estimation at only the desired frequency bins. For example, when operating in the EPA channel with an SNR of 15 dB, from Table I, channel estimates which are spaced 32 subcarriers apart are needed. The first 16 outputs from a pipelined 512 point radix-2 DIF FFT are at the bins $[0, 32, 64 \ldots]$ corresponding to the desired channel estimates. This enables circuit level optimizations such as clock gating which can be activated once the desired channel estimates are calculated leading to power savings.

QRD interpolation can be implemented using the methods described in [8] [9] and the proposed linear QRD interpolation can be performed using only fixed multiplications and additions leading to a negligible increase in complexity.

V. Conclusion

The proposed link adaptive processor is capable of decreasing the complexity of both channel estimation and QRD, which are two important baseband signal processing operations. The processor utilizes a windowed DFT based channel estimator which not only suppresses the error floor due to spectral leakage but is also capable of producing channel estimates at specified interpolation distances. Channel properties are used to identify these interpolation distances which add a minimal loss to BER while enabling the use of a simple QRD interpolation technique. The BER performance of the proposed link adaptive processor is on par with existing systems while providing a reduction in complexity of upto 40% in higher SNRs and 80% in lower SNRs. Hence, the link adaptive processor is an attractive solution for adaptive processing in varying channel conditions for mobile LTE-A receivers.

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