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Frequency domain properties of Otto Smith regulators

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Frequency domain characteristics of Otto Smith regulators are investigated. It is shown that the regulator can be regarded as an ordinary regulator in cascade with a lead network with considerable lead.

1. Introduction

The idea of dead-time compensation introduced by Otto Smith (1957) had little use in analogue systems because of the difficulty of implementing such systems. Since the regulator can easily be implemented digitally it is now finding increasing use in practical control systems. This note presents a simple analysis of the frequency domain properties of the regulator. The note was inspired by a discussion of digital control systems given in a seminar to an industrial audience. Specifically it answers the question : 'This is all fine but where does the phase-lead come from ?' which was asked by one of the participants.

2. Otto Smith's regulator

Consider the system whose block diagram is shown in Fig. 1.

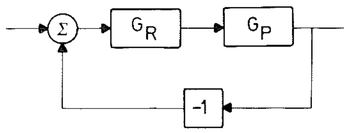

Figure 1.

Assume that the compensator G_R is chosen in such a way that a suitable performance of the closed-loop system is obtained. The closed-loop transfer function is then

$$G(s) = \frac{G_R G_P}{1 + G_R G_P}(s) \tag{1}$$

If the system has an extra time delay this transfer function is changed to

$$G(s) = \frac{G_R G_P \exp(-sT)}{1 + G_R G_P \exp(-sT)} \tag{2}$$

If T is sufficiently large the system will always be unstable. To avoid this difficulty Otto Smith (1958) proposed the regulator shown in Fig. 2.

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If the block diagram of Fig. 2 is redrawn as shown in Fig. 3 it is easily seen that the signals $-y$ and y_c will cancel each other and the closed-loop transfer function becomes

$$G(s) = \frac{G_R G_P \exp(-sT)}{1 + G_R G_P} \qquad (3)$$

The closed-loop system whose transfer function is given by eqn. (3) is clearly stable for any value of T. Apart from the factor $\exp(-sT)$ in the numerator of (3) the transfer function (3) is in fact identical to (1). This means that the regulator shown in the dashed block in Fig. 2 must give a significant phase lead. This will be explored further in the next section.

3. The regulator transfer function

The regulator in the dashed block in Fig. 2 has the transfer function

$$G_R' = \frac{G_R}{1 + G_R G_P[1 - \exp(-sT)]} = \frac{1}{1 + [1 - \exp(-sT)]G_0} G_R \qquad (4)$$

where

$$G_0(s) = G_R(s) G_P(s) \qquad (5)$$

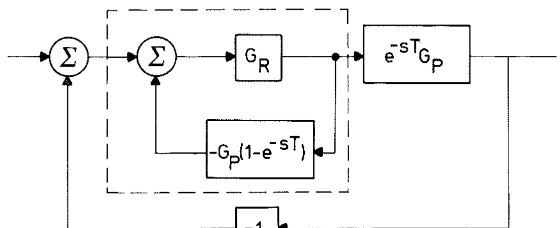

Figure 2

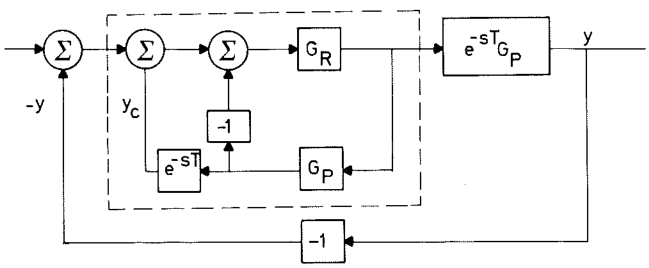

Figure 3.

The Otto Smith regulator can thus be considered as being a cascade connection of the ordinary regulator (G_R) with a compensator having the transfer function

$$G_c(s) = \frac{1}{1 + [1 - \exp(-sT)]G_0(s)} \qquad (6)$$

The properties of the transfer function G_c will now be explored.

For typical control loops the open-loop gain G_0 will be small for high frequencies and high for low frequencies. The compensator G_c will thus have a low gain at low frequencies. The low frequency gain will decrease with increasing time delay T. At high frequencies the gain of G_c will be equal to 1. The amplitude curve thus indicates that the general characteristics of G_c are those of a lead network. For frequencies such that $G_0(s) \approx -1$ it follows from (6) that

$$G_c(s) \approx \exp(sT) \quad (\text{for } G_0 \approx -1)$$

This indicates clearly that the network will give a considerable phase advance.

The approximative analysis thus indicates that the transfer function G_c corresponds to a lead network. The total phase advance will increase with increasing T.

4. An example

A specific example will now be investigated. Let the open-loop transfer function be

$$G_0 = \frac{K}{s(s+1)(s+2)}$$

A reasonable value of the gain is $K=1$. See, e.g. Åström (1967, p. 163). The transfer function G_c defined by (6) then has the properties

$$\lim_{s\to 0} G_c(s) = \frac{1}{1+kT/2}$$

$$\lim_{s\to\infty} G_c(s) = 1$$

The shapes of the Nyquist diagram of the transfer function G_c are indicated in Fig. 4.

Since both $G_c(0)$ and $G_c(i\infty)$ are real the total phase advance is a multiple of 2π. The frequency characteristics of G_c will now be explored further. The frequencies where the Nyquist curve intersects the negative real axis are given by

$$\arg\,[1-\exp(-i\omega T)]G_0(i\omega) = \pi$$

Hence

$$\frac{\pi}{2} - \frac{R[\omega T]}{2} - \frac{\pi}{2} - \text{arctg}\ \omega - \text{arctg}\ \omega/2 = \pi$$

or

$$-\frac{R[\omega T]}{2} = \pi + \gamma$$

where

$$\gamma = \text{arctg}\,\frac{3\omega}{2-\omega^2}$$

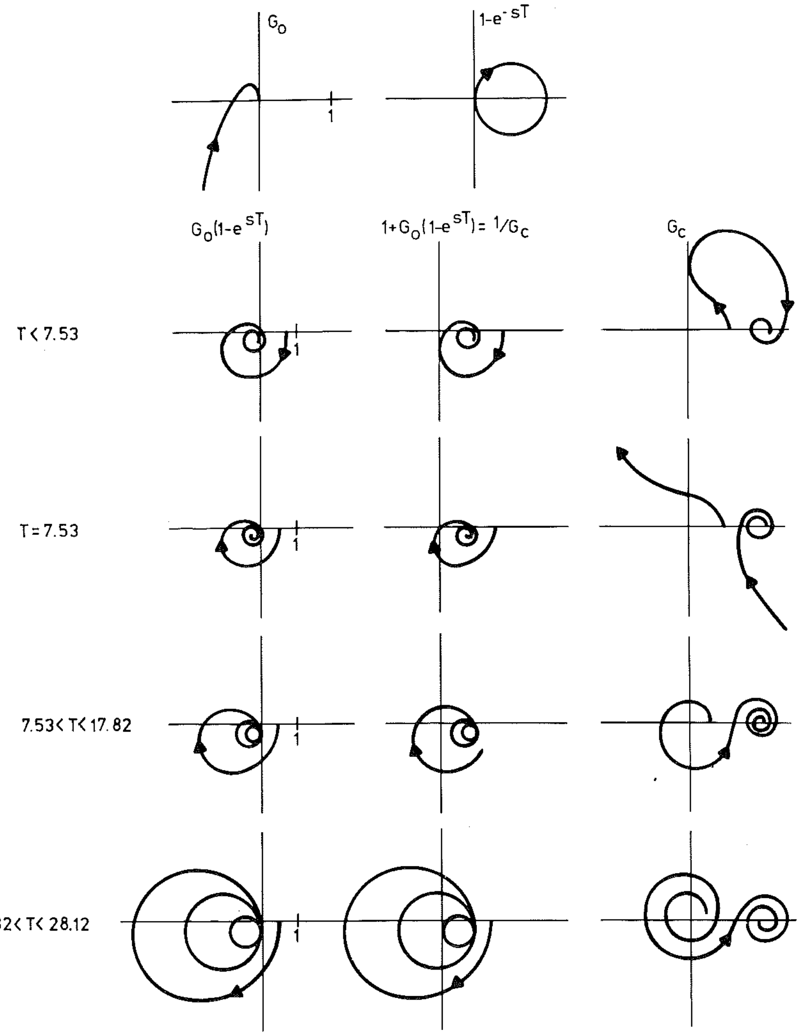

Figure 4.

and $R[\alpha]$ denotes the residue of α modulo 2π. The frequencies where the Nyquist curve intersects the negative real axis are thus given by

$$\text{tg}\ \omega T/2 = \frac{3\omega}{\omega^2 - 2} = -\text{tg}\ \gamma \tag{7}$$

The magnitudes of the transfer function G_c at these frequencies ω_0 are given by

$$|[1 - \exp(-i\omega_0 T)]G_0(i\omega_0)| = 2 \sin \gamma \frac{K}{\omega_0 \sqrt{[(1+\omega_0{}^2)(4+\omega_0{}^2)]}}$$

But

$$\sin \gamma = \frac{3\omega_0}{\sqrt{(\omega_0^4 + 5\omega_0^2 + 4)}} = \frac{3\omega_0}{\sqrt{[(\omega_0^2 + 1)(4 + \omega_0^2)]}}$$

Hence

$$|[1 - \exp(-i\omega_0 T)]G_0(i\omega_0)| = \frac{6K}{(1 + \omega_0^2)(4 + \omega_0^2)}$$

The intersection will be to the left of the point -1 if

$$6K > (1 + \omega_0^2)(4 + \omega_0^2)$$

For $K = 1$ this gives

$$\omega_0 < \sqrt{[(\sqrt{33} - 5)/2]} = 0{\cdot}6102$$

Furthermore, it follows from (7) that

$$\omega_0 T = 2\pi n - 2\gamma, \quad n = 1, 2, \ldots$$

Hence

$$\gamma = n\pi - \omega_0 T/2, \quad n = 1, 2, \ldots$$

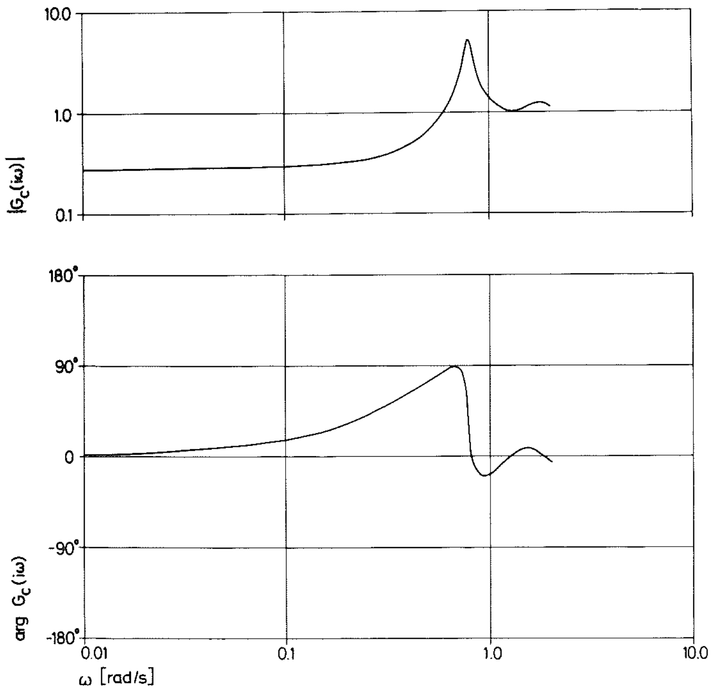

Figure 5. Bode diagram for the transfer function

$$G_c(s) = \frac{1}{1 + [1 - \exp(-sT)]G_0(s)}$$

for

$$G_0(s) = \frac{1}{s(s+1)(s+2)}$$

and $T = 5$.

Introducing $\omega_0 = 0{\cdot}6102$ and using (8) the following numerical values are obtained :

$$T = 10{\cdot}3n - 2{\cdot}77$$

The time delay corresponding to integer values of n are listed below.

n	T
1	7·53
2	17·82
3	28·12
4	38·42
5	48·72
6	59·01
7	69·31
8	79·61
9	89·90
10	100·2

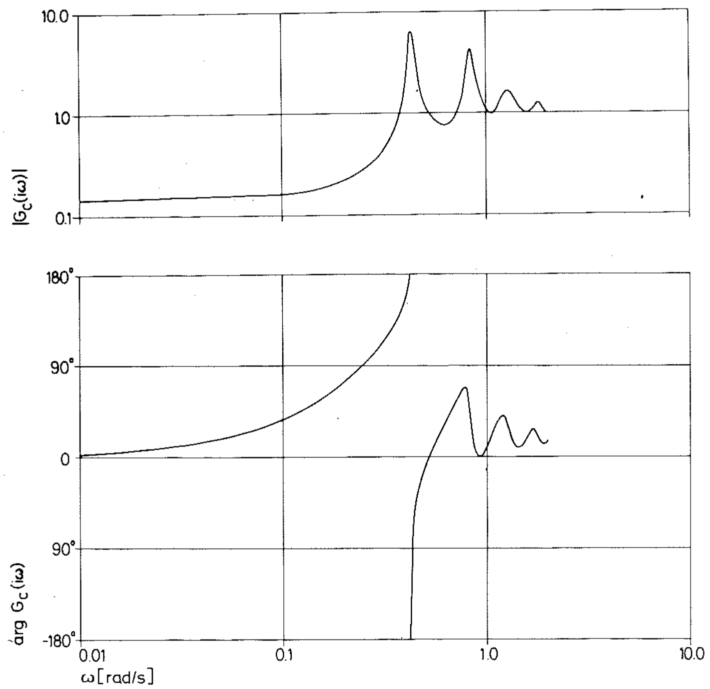

Figure 6. Bode diagram for the transfer function

$$G_c(s) = \frac{1}{1 + [1 - \exp(-sT)]G_0(s)}$$

for

$$G_0(s) = \frac{1}{s(s+1)(s+2)}$$

and $T = 12$.

This list gives the limits of the time delay for the Nyquist curve to make 1, 2, 3, ... revolutions around the origin. Notice that for the values of T given in the table above the function $1+[1-\exp(-sT)]G_0(s)$ will vanish for certain frequencies, which means that the transfer function G_c becomes infinite. In Fig. 5, Fig. 6 and Fig. 7 are shown the Bode diagrams for $T=5$, 12 and 22 corresponding to $n=0$, 1 and 2. Notice that the arguments are shown modulo 2π in Fig. 6 and Fig. 7.

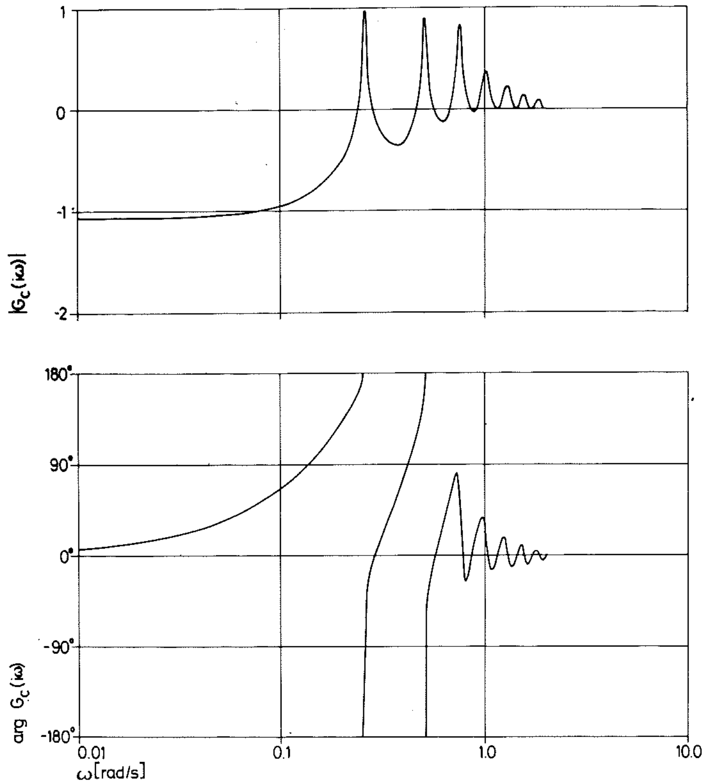

Figure 7. Bode diagram for the transfer function

$$G_c(s)=\frac{1}{1+[1-\exp(-sT)]G_0(s)}$$

for

$$G_0(s)=\frac{1}{s(s+1)(s+2)}$$

and $T=22$.

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