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On the Coherence of Higher Order Beliefs

Stefan Schubert and Erik J. Olsson

1. Introduction

For more than a decade epistemic coherence has been studied extensively within the framework of probability theory in a way that has significantly raised the level of the debate in terms of clarity and precision. Advancing beyond the usual vague characterizations of coherence, Bayesian coherence theorists have devised precise formulae by means of which the degree of coherence of a set can be measured (Shogenji 1999, Olsson 2002a), and some striking results have been proved concerning the relationship between coherence and truth or high probability. Being negative in nature, the most well-known of these – the so-called impossibility results – show that there cannot be a non-trivial measure of coherence that is truth conducive in the sense that more coherence implies a higher likelihood of truth. This observation has been taken by some researchers, e.g. Olsson (2005a), to disprove the coherence theory altogether while others, e.g. Bovens and Hartmann (2003), have taken them to be less damaging.[1]

The point of departure of the present paper is the observation that this literature has been exclusively concerned with "first order beliefs", i.e. with beliefs about the world, or concrete parts of it, such as the belief that it will rain tomorrow, or that the economy will recover. To the best of our knowledge, second order beliefs, which we take to be beliefs about the reliability of (first order) belief forming processes, have received no serious attention. And yet, as we shall see, second order beliefs play an important, or even

[1] More recently, a number of probabilistic findings have been reported that seem to support rather than undermine the coherence theory. For example, Angere (2008) shows that coherence increases the likelihood of truth in most scenarios, even if it falls short of doing it in all cases; Glass (2007) argues that coherence can be an important component in the proper understanding of the practice of inference to the best explanation; and the present authors have established that the so-called Shogenji measure (Shogenji 1999) is reliability conducive in the sense that, in a variety of contexts, such coherence raises the probability that the information sources are reliable (Olsson and Schubert, 2007, Schubert, 2010, Schubert, to appear).

crucial, role in respectable informal coherence theories of knowledge and justification. In what follows, we inquire into the possible extension of the probabilistic treatment of coherence to sets that include second order beliefs and reflect on the epistemological significance of this endeavor. Our main conclusion will be that while extending the framework to second order beliefs sheds doubt on the generality of the impossibility results and their relevance to informal coherence theories of knowledge and justification, another problem crops up that may be no less damaging to the coherentist project than those results were initially thought to be: facts of coherence turn out, in general, to be epistemically accessible only for agents having a good deal of insight into matters external to their own doxastic states.

2. Coherence as mutual support

The probabilistic study of coherence has made it increasingly clear that coherence can be understood in several different ways. In one sense, a set is coherent if its elements are in agreement. On this view, sets of equivalent propositions are maximally coherent. But coherence could also be understood as consisting in the agreement being particularly striking or salient. Agreement between vague or unspecific proposition will not be very striking while agreement between very specific items of information will. In yet another sense, coherence is determined by the elements' degree of mutual support.[2] These conceptions of coherence are presumably related, and some measures of coherence in the literature can be understood as explicating several in one swoop.

In this article we will focus exclusively on coherence as mutual support. Our justification for this choice is that some of the most influential explications of coherence have aimed at capturing coherence in this very sense. Cases in point include, for instance, A. C. Ewing's early definition of coherence as mutual derivability and C. I. Lewis's proposal that we should understand by coherence the property which a set of statements have "if they are so related that the antecedent probability of any one of them will be increased if the remainder of the set can be assumed as given premises" (Ewing 1934,

[2] For a discussion of the different senses of "coherence", see for instance Olsson (2002a, 2005a) and Meijs (2006).

229, Lewis 1946, 338). An intuitively appealing Bayesian treatment of mutual support can be found in Douven and Meijs (2007).[3] The basic idea is that the degree of coherence of an ordered set S of propositions equals the average mutual support of all non-empty and non-overlapping ordered subsets of S.[4] Consider, for instance, the ordered set $\langle A_1, A_2\rangle$. Its degree of mutual support, denoted $M(A_1, A_2)$, is given by

$$M(A_1, A_2) = \frac{S(A_1, A_2) + S(A_1, A_2)}{2}$$

where $S(A_1,A_2)$ is the degree of support that A_1 gives to A_2. The degree of coherence of the triple $\langle A_1, A_2, A_3\rangle$ is defined as the average over the following: $M(A_1, A_2)$, $M(A_1, A_3)$, $M(A_2, A_3)$, $M(A_1, A_2\&A_3)$, $M(A_1\& A_2, A_3)$, $M(A_1\& A_3, A_2)$. And so on.

In order to give more specific content to the Douven-Meijs approach something needs to be said abouit what kind of measure S is supposed to be. As the reader may already know, there is no shortage of possible measures to choose from in the extensive literature on scientific support or confirmation.[5] Fortunately, we need not rely on any particular support measure for the puporses of this paper: all our arguments will be "measure-insensitive" (Fitelson 2001) in the sense of relying only on the following principles:

(P1) B confirms A if and only if $P(A|B) > P(A)$

(P2) B disconfirms A if and only if $P(A|B) < P(A)$

(P3) B neither confirms nor disconfirms A if and only if $P(A|B) = P(A)$

(P4) If $P(B) = P(C)$, B confirms A more than C does if and only if $P(A|B) > P(A|C)$.

(P5) If $P(B) = P(C)$, A confirms B more than it confirms C if and only if $P(B|A) > P(C|A)$.

[3] In the following, we will present a version of their definition which is technically slightly different in ways that are of little epistemological significance. The relation to Douven and Meijs's original proposal is deferred to an appendix.

[4] For a justification of the use of ordered sets, see Olsson (2005, 17-18).

[5] A particularly popular measure is the difference measure (Gillies 1986 and Rosenkrantz 1994): $S_D(A,B) = P(A \mid B) - P(A)$. Another possible choice would be the ratio measure (Schlesinger 1995, Horwich 1998): $S_R(A, B) = P(A \mid B)/P(A)$.

These principles are satisfied by all prominent measures of support or confirmation.

3. A potential problem for the formal coherence theory

A promising starting point for getting an impression of the generality of a particular theory is to study the examples with reference to which the theory was originally motivated. Bayesian coherence theories are no exceptions to this general rule. Olsson (2002a) reasons around an example involving the two propositions "Robert has a gun" and "Robert is a professional criminal", while Bovens and Hartmann (2003) find it useful to reflect on a case featuring the two propositions "The culprit is French" and "The culprit drove away from the crime scene in a Renault". What is salient about these and other examples in the Bayesian literature is that they involve only first order propositions. This is troublesome since, arguably, most informal coherence theories have been more liberal in their understanding of what kind of propositions may figure in epistemologically relevant sets. That this is so follows already from (A) the rather obvious fact that we often do entertain second order beliefs and (B) the further observation that, with few exceptions, coherence theories are holistic so that the relevant type of coherence is supposed to be that which pertains to the whole belief system of a subject.

Bonjour's coherence theory, as developed in his influential 1985 book, is a case in point. On that theory, an observational or "cognitively spontaneous" belief, such as the belief that there is a red book on the table, is justified by the following justificatory argument (1985, 118):

(1) I have a cognitively spontaneous belief of kind K_1 that there is a red book on the table.

(2) Conditions C_1 obtain.

(3) Cognitively spontaneous visual beliefs of kind K_1 in context C_1 are very likely to be true.

Therefore, my belief that there is a red book on the desk is very likely to be true.

Therefore, (probably) there is a red book on the desk.

Bonjour seems to think that a subject normally believes all the premises of this justificatory argument (ibid.). But this means that since premise (3) is clearly a second order belief, being about the reliability of a first order belief forming process, Bonjour is committed to the view that people normally entertain second order beliefs.

Bonjour commits himself to holism in the following passage (ibid., 91):

> The epistemic issue on a particular occasion will usually be merely the justification of a single empirical belief, or small set of such beliefs, within the context of a cognitive system whose overall justification is (more or less) taken for granted; we may call this the *local* level of justification. But it is also possible, at least in principle, to raise the issue of the overall *global* level of justification. For the sort of coherence theory which will be developed here – and indeed, I would argue, for any comprehensive, nonskeptical epistemology – it is the issue of justification as it arises at the latter, global, level which is in the final analysis decisive for the determination of empirical justification in general.

The fact that formal coherence theory has focused exclusively on sets of first order beliefs raises the question whether it can accommodate more complex scenarios as well. If not, the worry is that at least some fruits of that theory may have been achieved at the cost of excluding the very kinds of belief sets that coherence theorists have been interested in. In the following we shall inquire into the seriousness of this *prima facie* legitimate concern.

4. A technical problem and a quick fix

While the main concern of coherence theorists is traditionally the coherence of beliefs, many researchers, following C. I. Lewis (1946), consider it useful to study coherence in the context of witness reports, if only to fix ideas. More precisely, the study of coherence has focused attention on the coherence of the *contents* of witness reports. Insights into the nature and role of coherence gained by studying witness scenarios are assumed to carry over to the doxastic case via the coherentist proposal that an epistemic subject should, initially, view her beliefs as mere “reports” from her belief system analogous to the

witness reports in court. It is the coherence of the contents of belief reports that is thought to be ultimately at stake.[6]

The paradigm witness case is one involving witnesses that deliver their reports independently. While the precise meaning of independence is open to some interpretation, a consensus has emerged on a number of intuitively appealing assumptions. Let us think of $A_1, \ldots, A_n$ as the content statements delivered by the witnesses in response to a query, and of $E_1, \ldots E_n$ as sentences expressing that witness 1 has said A_1, witness 2 that A_2, and so on. We will initially follow tradition in assuming that coherence is a property pertaining to content statements. Let R_i denote the proposition that the *i*:th witness is reliable, by which we will mean "fully reliable". We can now characterize the kind of witness scenario we have in mind using the language of probability: A *witness scenario is a set* **S** = $\{\langle E_1, A_1\rangle, \ldots, \langle E_n, A_n\rangle\}$, for n > 1, satisfying the following conditions:

(i) $P(E_i | A_i, R_i) = 1$ for $i = 1, \ldots, n.$

(ii) $P(E_i | \neg A_i, R_i) = 0$ for $i = 1, \ldots, n.$

(iii) $P(E_i | A_i, \neg R_i) = P(A_i) = P(E_i | \neg A_i, \neg R_i)$ for $i = 1, \ldots, n.$

(iv) $0 < P(A_i) < 1$ for $i = 1, \ldots, n.$

(v) $0 < P(R_i) < 1$ for $i = 1, \ldots, n.$

(vi) $\boldsymbol{E_i} \perp\!\!\!\perp \boldsymbol{E}_1, \boldsymbol{R}_1, \boldsymbol{A}_1, \ldots, \boldsymbol{E_{i-1}}, \boldsymbol{R_{i-1}}, \boldsymbol{A_{i-1}}, \boldsymbol{E_{i+1}}, \boldsymbol{R_{i+1}}, \boldsymbol{A_{i+1}}, \ldots, \boldsymbol{E_n}, \boldsymbol{R_n}, \boldsymbol{A_n} | \boldsymbol{R_i}, \boldsymbol{A_i}$ for $i = 1, \ldots, n.$[7]

(vii) $\boldsymbol{R_i} \perp\!\!\!\perp \boldsymbol{R}_1, \ldots, \boldsymbol{R_{i-1}}, \boldsymbol{R_{i+1}}, \ldots, \boldsymbol{R_n}, \boldsymbol{A}_1, \ldots, \boldsymbol{A_n}$ for $i = 1, \ldots, n.$

[6] See Bonjour (1985, 147-148) for the analogy between witness and belief reports.

[7] $\boldsymbol{E_i}$, $\boldsymbol{R_i}$ and $\boldsymbol{A_i}$ are random variables taking E_i and $\neg E_i$, R_i and $\neg R_i$ and A_i and $\neg A_i$, respectively, as their values.

Clause (i) states that if a certain proposition is true, then a reliable witness will report it. By (ii), if a certain proposition is false, then a reliable witness will not report it. Clause (iii) states that the probability that an unreliable witness will report a certain proposition equals the probability that the proposition is true. This clause obviously does not hold in general but it holds often enough to be a useful and popular idealization. Suppose, for instance, that we have a line-up of suspects, each equally likely to be the perpetrator. Assuming that an unreliable witness is equally likely to point out each suspect (i.e. that she is unbiased), the probability that she points out a particular suspect equals the probability that the suspect is in fact the perpetrator.[8]

According to clause (iv), the testimonial contents are neither certainly true nor certainly false, while clause (v) states that it is neither certainly true nor certainly false that the witnesses are reliable. The notation in (vi)-(vii) is borrowed from Pearl (2000), whereas the clauses themselves are natural generalizations of some principles adopted in Bovens and Hartmann (2003, 61). Clauses (vi) and (vii) abbreviate a great number of (conditional) independence statements. Intuitively, clause (vi) states that whether a witness will give a particular report is solely determined by the reliability of that witness and the truth value of the reported proposition. Hence, whether such a report will be forthcoming does not depend on what the other witnesses are reporting or on their reliability. By (vii), the reliability of a witness is independent of the reliability of other witnesses and the contents of the testimonies, so that learning that one witness is reliable (or unreliable) does not change your expectations regarding the reliability of some other witness. Likewise, learning what some witness is claiming shouldn't affect your confidence in that, or some other, witness's reliability.

Let us now move beyond this standard way of representing witness scenarios by considering a scenario involving not only first order but also second order testimonies, i.e. testimonies about the reliability of other testimonies. We assume that the first two witnesses both claim that Jones did it (A), the third witness that the first witness is reliable (R_1), and the fourth witness that the second witness is reliable (R_2). Let us call this set of testimonies T = $\{\langle E_1,A\rangle, \langle E_2,A\rangle, \langle E_3,R_1\rangle, \langle E_4,R_2\rangle\}$. In order to judge whether T

[8] Clause (iii) was introduced into the coherence debate by Olsson (2002b, 280) and later adopted by Bovens and Hartmann (2003, 115-116).

exhibits mutual support, we need to consider the relation between the content propositions. For instance, do the sets $\langle A,A\rangle$ and $\langle R_1,R_2\rangle$ mutually support each other? Intuitively, one would think that that they do. For if two witnesses give identical testimonies, the probability that they are reliable should increase (cf. Bovens and Hartmann, 2003, 62), and this is precisely what witness 3 and 4 are claiming. Conversely, if two witnesses are both reliable, as claimed by witness 3 and 4, the probability that they will both tell the truth and give identical testimonies increases.

Let us now see to what extent these expectations are sustained by formal coherence theory. A necessary preliminary is to check whether or not our seemingly unremarkable witness situation falls under the concept of a witness scenario as defined a moment ago. To repeat, T = $\{\langle E_1,A\rangle, \langle E_2,A\rangle, \langle E_3,R_1\rangle, \langle E_4,R_2\rangle\}$ where

A = "Jones did it",

R_1 = "witness 1 is reliable", and

R_2 = "witness 2 is reliable".

Unfortunately, this scenario does not come out as a witness scenario in the formal sense, the reason being that clause (vii) in the definition of a witness scenario is violated. For it follows from the definition of T and clause (vii) of the definition of a witness scenario that R_1 is independent of itself, which is impossible since, by clause (v) of the latter definition, $0 < P(R_1) < 1$.

Fortunately, the problem can be dealt with quite easily by weakening clause (vii) in the concept of a witness scenario so that it does not require the reliability of a witness to be independent of itself:

Definition 1: Let $Q_1, \ldots, Q_n$ denote first order propositions. A witness scenario is a set

$$\mathbf{S} = \{\langle E_1,A_1\rangle,\ldots,\langle E_n,A_n\rangle\}, \text{for } n > 1,$$

where $A_i \in \{Q_1,\ldots,Q_n, R_1, \neg R_1 \ldots, R_{i-1}, \neg R_{i-1}, R_{i+1}, \neg R_{i+1}, \ldots, R_n, \neg R_n\}$, satisfying (i)-(vi) and

(vii´) $\boldsymbol{R_i} \perp\!\!\!\perp \boldsymbol{R}_1,\ldots,\boldsymbol{R_{i-1}},\boldsymbol{R_{i+1}},\ldots,\boldsymbol{R_n},\boldsymbol{A}_1,\ldots,\boldsymbol{A_n}$ for $i = 1,\ldots,n$, where $\boldsymbol{R_i} \neq \boldsymbol{A_j}$ for $j = 1,\ldots,n$.

In other words: if the statements given by the witnesses can be second order, we cannot assume that the reliability of a witness is independent of what the witnesses say because what they say may be precisely that the witness in question is reliable. Clause (vii´) takes care of this problem by making the more modest claim that the reliability of a witness is independent of the reliability of the *other* witnesses (as well as of all first-order propositions).

While the move from (vii) to (vii´) does not strike us as particularly harmful, we have admittedly made a number of other assumptions that can be questioned from an intuitive standpoint. Thus, Definition 1 excludes at the outset the possibility of one reporter reporting on his or her own reliability, as well as reporters making complex statements having, say, both first order and second order parts. Indeed, the assumption that there are only two levels of beliefs is in itself an oversimplification. Why stop at second order beliefs? Why not consider also third order, forth order or, generally, *n*-order beliefs for an arbitrary *n*? Had our aim been complete faithfulness to the complexity of real life, none of these simplifying assumptions would have seemed justified. However, our goal is the more modest one of taking the first few steps toward addressing a problem that, as far as we know, has to date gone unnoticed. Hopefully, novelty, if that is what it is, will excuse lack of generality.

5. Coherence reconsidered

Unfortunately, this is not the end of the story. As we will now show, the amended definition of a witness scenario yields the wrong result given Douven and Meijs' definition of mutual support. Consider an alternative scenario T´ in which the first two witnesses, as before, claim that Jones did it but in which the third and fourth witnesses give testimonies that are entirely unrelated to the other testimonies, say, that it will be windy tomorrow (W) and that the US economy will fall into depression (D), respectively. Clearly, $\langle A,A\rangle$ and $\langle R_1,R_2\rangle$ support each other to a higher degree than $\langle A,A\rangle$ and $\langle W,D\rangle$. But by using the amended clause (vii´) and Douven and Meijs' definition of mutual support we can prove the opposite: that the sets have the same degree of mutual support. All the theorems that follow should be understood as being conditional on the

propositions mentioned being part of a suitable witness scenario in the sense of Definition 1.

Theorem 1: $P(A|R_1, R_2) = (A) = P(A|W, D)$

Theorem 2: $P(R_1, R_2|A) = (R_1, R_2) = P(R_1, R_2|W, D)$

Proof: By clause (vii´) $P(A, R_1, R_2) = P(A)P(R_1)P(R_2)$ and $P(R_1, R_2, W, D) = P(R_1, R_2)$. Hence $P(A|R_1, R_2) = (A) = P(A|W, D)$ and

$P(R_1, R_2|A) = (R_1, R_2) = P(R_1, R_2|W, D)$.

Intuitively, the way out of this predicament is to reconsider our understanding of coherence. In a purely first order scenario, whether or not the statements in question have been delivered by some witnesses does not affect the degree of coherence of the set of testimonies. For instance, suppose the first witness says (E_1) that Matilda is a feminist (F) and the second witness says (E_2) that she is a left-wing activist (L). Then the coherence of the set consisting of F and L does not in any way hinge on the fact that the witness reports were made. Rather, coherence is simply a matter of the relationship between F and L.

Let us introduce some informal notation for the purpose of expressing the triviality just mentioned. We let $C(A_1, \ldots, A_n \mid E_1, \ldots, E_n)$, the *conditional coherence* of $A_1, \ldots, A_n$ given $E_1, \ldots, E_n$, stand for "the degree of coherence of the set $A_1, \ldots, A_n$ given reports $E_1, \ldots, E_n$". What we just agreed on, we hope, was that $C(F, L \mid E_1, E_2) = C(F, L)$. And similarly for all sets of first order testimonies: their unconditional coherence equals their conditional coherence.

Things change once second order testimonies are taken into consideration. In such cases, it cannot be taken for granted anymore that the conditional coherence of a set equals its unconditional coherence, and in our example with T this is indeed not the case. We recall that T consists of $\langle E_1, A\rangle$, $\langle E_2, A\rangle$, $\langle E_3, R_1\rangle$, and $\langle E_4, R_2\rangle$ where A = "Jones did it", R_1 = "The first witness is reliable" and R_2 = "The second witness is reliable", and $E_1,\ldots,E_4$ are the corresponding reports. Consider the set $\langle A, A, R_1, R_2\rangle$. On closer examination, this set is no more coherent, or mutually supportive, than the alternative set

T´ which contained *A* twice plus two entirely irrelevant propositions. Two occurrences of "Jones did it" provide some coherence to the set, but the further addition that the first and second witnesses are reliable doesn't seem to add any coherence at all. It is only when it is assumed that the reports have been given that the matter changes, and changes radically. For then we also know that the witnesses that are univocally claiming *A* to be true are the very same witnesses as those claimed to be reliable by witness 3 and 4. Suddenly, the coherence of the set shots up to new heights.

This shows two things, it seems. First, it shows that for sets involving second-order testimonies, the conditional coherence of a set need not equal its unconditional coherence but, rather, that conditional coherence may be higher than the unconditional coherence. Second, our intuitions about coherence seem to be more oriented toward conditional coherence than toward unconditional coherence, suggesting the primacy of former over the latter. The fact that the Bayesian discussion so far has focused on unconditional coherence now emerges as a possible artifact of the decision to focus all attention on first order sets, for which unconditional and conditional coherence coincide. (As it turns out, this account of the situation is not exactly right, and we will soon have reason to question it, but it is still on the right track.)

How are we to make formal sense of this? The simplest idea would be to add the fact that $A_1,\ldots, A_n$ have been testified, i.e. $E_1, \ldots, E_n$, as *background knowledge.* This is very well in line with the way confirmation theorists think about confirmation, namely, as being relative to a body of background knowledge. Let us, therefore, consider what support $\langle R_1, R_2 \rangle$ and $\langle A, A \rangle$, two ordered subsets of T, give each other, given $E_1,\ldots,E_4$ as background knowledge:

Theorem 3: $P(A|R_1, R_2, E_1, E_2, E_3, E_4) > P(A_1, A_2|E_1, E_2, E_3, E_4)$

Proof: In appendix.

Theorem 4: $P(R_1, R_2|A, E_1, E_2, E_3, E_4) > P(R_1, R_2|E_1, E_2, E_3, E_4)$

Proof: In appendix.

As we see, the ordered subsets now come out as supporting each other, thus easing the observed tension between intuition and formalism.

But this still isn't quite what we want. Consider instead the set T´´ = $\{\langle E_1, A\rangle, \langle E_2, R_1\rangle\}$ where

A = "Jones did it" and

R_1 = "witness 1 is reliable"

Intuitively, there is no mutual support here. For while it is true that the second testimony strengthens the first one, the converse does not hold: our belief that witness 1 is reliable is not strengthened by hearing that she points out Jones.[9] But if we understand mutual support along the lines of the proposal just made, we get the untoward result that the testimonies do support each other:

Theorem 5: $P(A|R_1, E_1, E_2) > P(A|E_1, E_2)$. Also:

Theorem 6: $P(R_1|A, E_1, E_2) > P(R_1|E_1, E_2)$.

Proofs in appendix.

So, alas, simply adding $E_1, \ldots, E_n$, as background knowledge does not, after all, solve our problem in a way that we can rest content with. This takes us to our final proposal.

6. A final proposal

Let us look a bit closer at the set T´´. Why is it that we think that the second testimony strengthens the first one, whereas the first does not strengthen the second? Consider the support that the proposition that Jones did it receives from the fact that it has been stated by witness 1:

$P(A|E_1)$

[9] It is not generally true that a first order testimony should leave our credence in a second order testimony unaffected. Take for instance the case of one witness claming that the moon is made of French cheese and a second witness claiming the first witness to be reliable. In this case, the first testimony counts against the reliability of the witness delivering it, and hence also against the truth of the second testimony.

Now compare this with the support the proposition that Jones did it receives from the fact that it has been stated by witness 1 *and* the fact that witness 2 has said that witness 1 is reliable:

$$P(A|E_1, E_2)$$

Intuitively, the proposition that Jones did it should be more supported by the fact that it has been stated by witness 1 in conjunction with the fact that witness 2 has said that witness 1 is reliable than by the fact that it has been stated by witness 1 alone. And indeed:

Theorem 7: $P(A|E_1, E_2) > P(A_1|E_1)$

Proof in appendix.

Let us now look at the support the proposition that witness 1 is reliable receives. Clearly, given clause (iii) in our definition, the fact that witness 1 points out Jones does not increase the degree of support that this proposition receives beyond the support it has already received through the fact that it has been stated by witness 2. That is:

Theorem 8: $P(R_1|E_1, E_2) = P(R_1|E_2)$

Proof in appendix.

The bottom line is that we get the intuitively right results if we think of mutual support in the following way:

Definition 2: $\langle E_1, A_1 \rangle$ and $\langle E_2, A_2 \rangle$ mutually support each other iff $P(A_1|E_1, E_2) > P(A_1|E_1)$ and $P(A_2|E_1, E_2) > P(A_2|E_2)$.

We can now show:

Theorem 9: $P(A_1, A_2|E_1, E_2, E_3, E_4) > P(A_1, A_2|E_1, E_2)$

Theorem 10: $P(R_1, R_2|E_1, E_2, E_3, E_4) > P(R_1, R_2|E_3, E_4)$

Proofs in appendix.

According to Douven and Meijs' (2007) conception of mutual support, two testimonies A_1 and A_2 mutually support each other if and only if

$P(A_1|A_2) > P(A_1)$ and $P(A_2|A_1) > P(A_2)$.

It can now be proved that in first-order cases

Theorem 11: $P(A_1|E_1, E_2)$ is an increasing function of $P(A_1|A_2)$ given that $P(A_1)$ is kept fixed.

Proof: In appendix.

This means that, given that all witnesses have the same probability of reliability and $P(A_2) = P(A_3)$,

$P(A_1|E_1, E_2) > P(A_1|E_1, E_3)$ iff $P(A_1|A_2) > P(A_1|A_3)$

and

$P(A_2|E_1, E_2) > P(A_3|E_1, E_3)$ iff $P(A_2|A_1) > P(A_3|A_1)$

which in turn implies that, given (P4) and (P5),

$M(E_1, A_1, E_2, A_2) > M(E_1, A_1, E_3, A_3)$ iff $M(A_1, A_2) > M(A_1, A_3)$, provided that $P(A_2) = P(A_3)$.

Hence, in this case, our proposed definition orders sets of first-order testimonies in the same way as Douven and Meijs' original definition does. Thus it solves the puzzles we identified earlier for second order beliefs and testimonies in a way that can arguable be called conservative.

This proposal conceives of mutual support as a matter of all or nothing, whereas Douven and Meijs's definition yields a graded conception of the same concept. But we can easily take the further step of combining the present proposal with Douven and Meijs's general recipe, thus obtaining a revised measure of coherence as mutual support. What we need to adjust in that recipe is only the definition of the degree of mutual support between two proposition. More precisely, what we need to do is to redefine M as follows:

Definition 3: $M(E_1, A_1, E_2, A_2) = \dfrac{S(A_1, E_2 \mid E_1) + S(A_2, E_1 \mid E_2)}{2}$

The intended reading of $S(A_1, E_2 \mid E_1)$ is "the support confered by E_2 upon A_1 given E_1". As before, we choose to remain uncommitted as to the exact nature of the support measure.

7. Discussion and conclusion

It is time to reflect on the deeper epistemological significance of all this, if such there is. What our discussion so far suggests are two things: (1) that the impossibility results that have been taken to be damaging to the coherence theory are in fact more limited in scope than has been generally appreciated within the Bayesian community; and (2) that facts of coherence may be epistemically less accessible for an epistemic agent than what more traditional coherence theorists would have liked them to be. The first claim counts in favor of the coherence theory, while the latter has rather the opposite effect. Let us explain.

As we noted in the introduction, the most thought-provoking recent results about coherence and probability concern the possibility of finding a measure of coherence that is *truth conducive* in the following sense: if a set of beliefs K is more coherent than another set of beliefs K´, then the probability of K is higher than the probability of K´. Here it is assumed that the beliefs in question are partially reliable (to the same degree) and that they are independently held.[10] One would think that it wouldn't be that hard to a find a measure of the required sort given the fortunate circumstances of partial reliability and independence. Alas, it is not only hard to find such a measure; it is, as we saw, impossible.[11] These results give rise to a thought-provoking paradox. How can it be that we trust and rely on coherence reasoning, in everyday life and in science, when in fact coherence is not truth conducive? And what room is left for the coherence theory once it

[10] Finding such a measure was first stated as an open problem in Olsson (2002a).

[11] An impossibility result to that effect was first proved by Luc Bovens and Stephan Hartmann in their 2003 book. A different impossibility theorem was proved in Olsson (2005a). See Olsson (2005b) for a detailed discussion of that result, including a comparison with Bovens and Hartmann's theorem.

has been proved that coherence is not correlated with truth even in this apparently extremely weak sense?

Now an assumption underlying both these impossibility results is that coherence is merely a matter of the contents of beliefs so that coherence is independent of the sources of these beliefs or of the fact that they are believed. In other words, if we know the contents of a set of beliefs, we can in principle determine the coherence of that set if we also know the extent to which the different subsets of the set support each other. Perhaps it is not so unreasonable to think that a person can have access to the contents of her beliefs and be in a position to judge, with some degree of confidence, that those contents support each other in various degrees.

But what we have just suggested contradicts this harmonious picture altogether. For what we are now claiming is that in order to determine the coherence of a set of witness reports we need to consider not only the contents of those reports but also the fact that they have been reported. Translated into doxastic vocabulary, recalling the analogy between witness and belief reports, we need to consider the fact that they the propositions in question are believed.

This point can brought out most forcefully in connection with our revised definition of mutual support in the graded sense of that concept:

$$M(E_1, A_1, E_2, A_2) = \frac{S(A_1, E_2 \mid E_1) + S(A_2, E_1 \mid E_2)}{2}$$

From a doxastic perspective, this definition states that the degree of mutual support between two believed proposition A_1 and A_2 is a function not only of A_1 and A_2, and their probabilities or support relations, but also of the belief reports E_1 and E_2, which we may think of as stating, respectively, that A_1 and A_2 are believed by some person – Jenny, say.

Our second point, about the limited access to facts of doxastic coherence, is a direct consequence of what has been said already. To be concrete, let us think of A_1 as stating that Jim is rich and of A_2 as stating that the process by means of which Jenny formed her belief that A_1 is a reliable process, respectively. What our formula expresses is that the mutual support of these propositions is determined (i) by the extent to which the proposition that Jim is rich is supported by the proposition that Jenny believes that the process by means of which she formed her belief that Jim is rich is a reliable process,

given that she believes him to be rich; and (ii) by the extent to which the proposition that the process by means of which Jenny formed her belief that Jim is rich is a reliable process is supported by the proposition that she believes him to be rich, given that she believes that the process by means of which she formed her belief that Jim is rich is a reliable process. Hence, Jenny, or any subject in her position, cannot therefore in general estimate the mutual support of a set of her own beliefs unless she is also able to ascertain, with sufficient accuracy and detail, the impact of her having a particular belief (such as the belief that Jim is rich) has upon the probability of other propositions (such as the proposition that the process by means of which she formed her belief is reliable). What this boils down to is the fact that she will be unable to give such an estimation *unless she is able to ascertain her own reliability as a believer*.

But, traditionally, coherence theoriest have been reluctant, to say the least, to assume from the outset that epistemic subjects are endowed with accurate and detailed insight into the reliability of their beliefs or belief forming processes. Rather, the subject's insight into the reliability of her beliefs or belief forming processes is supposed to arise *ex post* as the subject's own best explanation of the fact that she has observed her beliefs to exhibit a high degree of coherence.

What this suggests, again, is that while extending the Bayesian treatment of coherence to include second order testimonies or beliefs apparently undermines the impossibility results of probabilistic coherence, coherence theorist should not take too much comfort in this fact. For the very same reasons that shed doubt on impossiblity theorems also show that grasping the coherence of one's own beliefs may be a task that requires epistemic access to facts traditionally thought to lie outside the scope of internalist first-person reflection.[12]

Appendix

[12] Having made the initial observation that formal coherence theory should take into account second order beliefs, Stefan Schubert wrote a draft on that problem which he presented at a research seminar at Lund University. The draft provided the starting point for the joint work that led to the present article. Stefan also proved all the theorems. We would like to thank our colleagues in Lund, in particular the participants of the Higher seminar in theoretical philosophy as well as of the Working seminar in philosophy of science, led by Bengt Hansson, for their patience and valuable input on several versions of this paper.

Doven and Meijs' definition in the case of *n* testimonies: A consequence of Douven and Meijs' original definition in terms of unordered sets of propositions is that the coherence of a set of equivalent testimonies reduces to coherence of a singleton set, for which coherence is undefined. Douven and Meijs argue that this is as it should be since otherwise a witness could increase the degree of coherence of his or her testimonies too easily by simply rephrasing the original testimony (Douven and Meijs 2007, 417). As pointed out in Schubert (to appear), this problem does not carry over to witness scenarios because in a witness scenario the testimonies are assumed to be independently given. Hence, there is nothing problematic with allowing coherence to be defined for sets of equivalent testimonies or, indeed, to allow for a significant degree of coherence in such cases. Formally, this is handled by defining coherence or mutual support for ordered sets. A modified definition was given in Schubert (to appear). It is that definition that is used here.

The general definition of a coherence measure given a measure of support *m* runs as follows. Let $\boldsymbol{S} = \langle A_1, ..., A_n \rangle$ be an ordered set of propositions. Let $[\boldsymbol{S}]$ be the set of ordered pairs of non-empty non-overlapping subsequences of $\boldsymbol{S}$. Let $\langle \hat{\boldsymbol{S}}_1, ..., \hat{\boldsymbol{S}}_{[\![\boldsymbol{S}]\!]} \rangle$ be an ordering of the members of $[\boldsymbol{S}]$. And, let $[\![\boldsymbol{S}]\!]$ indicate the cardinality of $[\boldsymbol{S}]$ Then:

$$C_m(\boldsymbol{S}) =_{\boldsymbol{df}} \frac{\sum_{\boldsymbol{i}=1}^{[\![\boldsymbol{S}]\!]} m(\hat{\boldsymbol{S}}_{\boldsymbol{i}})}{[\![\boldsymbol{S}]\!]}$$

This definition amounts to adding the degrees of support that the ordered pairs of subsets lend each other and dividing the result by the number of such pairs, resulting in the average degree of support over all non-empty non-overlapping subsets.

Theorem 3: $P(A|R_1, R_2, E_1, E_2, E_3, E_4) > P(A|E_1, E_2, E_3, E_4)$

Proof:

First, let us show the following lemma:

Lemma 1:

$$P(E_1,...,E_4)$$
$$=\sum_{R_1,R_2,R_3,R_4,A} P(R_1)P(R_2)P(R_3)P(R_4)P(A)P(E_1|A,R_1)P(E_2|A,R_2)P(E_3|R_1,R_3)P(E_4|R_2,R_4)$$

Proof:

$$P(E_1,E_2,E_3,E_4)=\sum_{R_1,R_2,R_3,R_4,A} P(E_1,E_2,E_3,E_4,R_1,R_2,R_3,R_4,A)$$

$$=\sum_{R_1,R_2,R_3,R_4,A} P(R_1,R_2,R_3,R_4,A)P(E_1|A,R_1)P(E_2|A,R_2)P(E_3|R_1,R_3)P(E_4|R_2,R_4) \quad \text{By (vi)}$$

$$=\sum_{R_1,R_2,R_3,R_4,A} P(R_1)P(R_2)P(R_3)P(R_4)P(A)P(E_1|A,R_1)P(E_2|A,R_2)P(E_3|R_1,R_3)P(E_4|R_2,R_4)$$

By (vii´)

Now:

$$P(A_1,A_2|R_1,R_2,E_1,E_2,E_3,E_4)=\frac{P(A,R_1,R_2,E_1,E_2,E_3,E_4)}{P(R_1,R_2,E_1,E_2,E_3,E_4)}$$

$$=\frac{P(A,R_1,R_2,E_1,E_2,E_3,E_4)}{P(A,R_1,R_2,E_1,E_2,E_3,E_4)}$$

For, since $P(E_i|\neg A_i,R_i)=0$ (clause ii), $P(\neg A_i,R_i,E_i)=0$ and

hence $P(\neg A,R_1,R_2,E_1,E_2,E_3,E_4)=0$.

$=\frac{P(A,R_1,R_2,E_1,E_2,E_3,E_4)}{P(A,R_1,R_2,E_1,E_2,E_3,E_4)}$=1, given that $P(A,R_1,R_2,E_1,E_2,E_3,E_4)\neq 0$.

Proof that $P(A, R_1, R_2, E_1, E_2, E_3, E_4) \neq 0$:

It suffices to show that

$$P(A, R_1, R_2, R_3, R_4, E_1, E_2, E_3, E_4) \neq 0$$

$$P(A, R_1, R_2, R_3, R_4, E_1, E_2, E_3, E_4) = P(R_1)P(R_2)P(R_3)P(R_4)P(A) \quad \text{By Lemma 1, (i)}$$

$$P(R_1)P(R_2)P(R_3)P(R_4)P(A) \neq 0 \quad \text{By (iv), (v)}$$

$$P(A|E_1, E_2, E_3, E_4) = \frac{P(A, E_1, E_2, E_3, E_4)}{P(E_1, E_2, E_3, E_4)} = \frac{P(A, E_1, E_2, E_3, E_4)}{\sum_A P(A, E_1, E_2, E_3, E_4)}$$

Since we want to prove that $\frac{P(A, E_1, E_2, E_3, E_4)}{\sum_A P(A, E_1, E_2, E_3, E_4)}$ <1, it suffices to show that

$$P(\neg A, E_1, E_2, E_3, E_4) > 0$$

It suffices to show that $P(\neg A, \neg R_1, \neg R_2, \neg R_3, \neg R_4, E_1, E_2, E_3, E_4) \neq 0$

$$P(\neg A, \neg R_1, \neg R_2, \neg R_3, \neg R_4, E_1, E_2, E_3, E_4)$$

$$= P(\neg R_1)P(\neg R_2)P(\neg R_3)P(\neg R_4)P(\neg A)P(A)P(A)P(R_1)P(R_2) \quad \text{By lemma 1, (iii)}$$

$P(\neg R_1)P(\neg R_2)P(\neg R_3)P(\neg R_4)P(\neg A)P(A)P(A)P(R_1)P(R_2) > 0$ By (iv), (v)

Hence $P(\neg A, E_1, E_2, E_3, E_4) > 0$

Hence $P(A|R_1, R_2, E_1, E_2, E_3, E_4) > P(A|E_1, E_2, E_3, E_4)$

Theorem 4: $P(R_1, R_2|A, E_1, E_2, E_3, E_4) > P(R_1, R_2|E_1, E_2, E_3, E_4)$

Proof:

$$P(R_1, R_2|A, E_1, E_2, E_3, E_4) = \frac{P(R_1, R_2, A, E_1, E_2, E_3, E_4)}{P(A, E_1, E_2, E_3, E_4)}$$

$$P(R_1, R_2|E_1, E_2, E_3, E_4) = \frac{P(R_1, R_2, E_1, E_2, E_3, E_4)}{P(E_1, E_2, E_3, E_4)}$$

$P(R_1, R_2, E_1, E_2, E_3, E_4) = P(A, R_1, R_2, E_1, E_2, E_3, E_4)$ See proof of theorem 3

Thus, in order to prove that $\frac{P(R_1, R_2, A, E_1, E_2, E_3, E_4)}{P(A, E_1, E_2, E_3, E_4)} > \frac{P(R_1, R_2, E_1, E_2, E_3, E_4)}{P(E_1, E_2, E_3, E_4)}$, it suffices to show that $P(E_1, E_2, E_3, E_4) > P(A, E_1, E_2, E_3, E_4)$ and that $P(A, E_1, E_2, E_3, E_4) \neq 0$. Since $P(A, R_1, R_2, E_1, E_2, E_3, E_4) \neq 0$ and $P(\neg A, E_1, E_2, E_3, E_4) > 0$, as we saw in the previous proof, that is indeed the case. Hence $P(R_1, R_2|A, E_1, E_2, E_3, E_4) > P(R_1, R_2|E_1, E_2, E_3, E_4)$.

Theorem 5: $P(A|R_1, E_1, E_2) > P(A|E_1, E_2)$

Proof:

$$P(A|R_1, E_1, E_2) = \frac{P(A, R_1, E_1, E_2)}{P(R_1, E_1, E_2)}$$

$$= \frac{P(A, R_1, E_1, E_2)}{P(A, R_1, E_1, E_2)}$$ See proof of theorem 3

$$= \frac{P(A, R_1, E_1, E_2)}{P(A, R_1, E_1, E_2)} = 1,$$ given that $P(A, R_1, E_1, E_2) \neq 0$.

Proof that $P(A, R_1, E_1, E_2) \neq 0$:

The following lemma is useful here:

Lemma 2: $P(E_1, E_2) = \sum_{A, R_1, R_2} P(R_1)P(R_2)P(A)P(E_1|A, R_1)P(E_2|R_1, R_2)$

Proof omitted (analogous to proof of lemma 1)

It suffices to show that $P(A, R_1, R_2, E_1, E_2) \neq 0$

$P(A, R_1, R_2, E_1, E_2) = P(R_1)P(R_2)P(A)$ By lemma 2, (i)

$P(A)P(R_2)P(R_1) > 0$ By (iv), (v)

$$P(A|E_1, E_2) = \frac{P(A, E_1, E_2)}{P(E_1, E_2)} = \frac{P(A, E_1, E_2)}{P(A, E_1, E_2) + P(\neg A, E_1, E_2)}$$

In order to show that $\frac{P(A, E_1, E_2)}{P(A, E_1, E_2) + P(\neg A, E_1, E_2)}$<1, it suffices to show $P(\neg A, E_1, E_2)$>0, for which it suffices to show that $P(\neg A, \neg R_1, \neg R_2, E_1, E_2)$>0

$P(\neg A, \neg R_1, \neg R_2, E_1, E_2)$

$= P(\neg R_1)P(\neg R_2)P(\neg A)P(A)P(R_1)$ By lemma 2, (iii)

$P(\neg R_1)P(\neg R_2)P(\neg A)P(A)P(R_1) \neq 0$ By (iv,v)

Hence $P(A|R_1, E_1, E_2) > P(A|E_1, E_2)$

Theorem 6: $P(R_1|A, E_1, E_2) > P(R_1|E_1, E_2)$

Proof:

$$P(R_1|A, E_1, E_2) = \frac{P(R_1, A, E_1, E_2)}{P(A, E_1, E_2)}$$

$$P(R_1|E_1, E_2) = \frac{P(R_1, E_1, E_2)}{P(E_1, E_2)}$$

As we saw in the proof of theorem 5, $P(R_1, A, E_1, E_2) = P(R_1, E_1, E_2)$.

Hence we only need to prove that $P(A, E_1, E_2) < P(E_1, E_2)$. Since we saw in the proof of theorem 5 that $P(\neg A, E_1, E_2) > 0$, $P(A, E_1, E_2) < P(E_1, E_2)$. Hence,
$P(R_1|A, E_1, E_2) > P(R_1|E_1, E_2)$

Theorem 7: $P(A|E_1, E_2) > P(A|E_1)$

Let:

$P(A) = a$, $P(A) = a$, $P(\neg A) = \neg a$, $P(\neg A_i) = \neg a_i$, $P(A_i A_j) = a_{ij}$ $P(A_i \neg A_j) = a_{i\neg j}$

$P(R_i) = r_i$, $P(\neg r_i) = \neg r_i$.

Proof:

$P(A, E_1, E_2) = \sum_{R_1, R_2} P(R_1)P(R_2)P(A)P(E_1|A, R_1)P(E_2|R_1, R_2)$ By lemma 2

$= r_1 r_2 a + r_1 \neg r_2 a r_1 + \neg r_1 \neg r_2 a^2 r_1$ By (i-vii´)

$P(\neg A, E_1, E_2) = \sum_{R_1, R_2} P(R_1)P(R_2)P(\neg A)P(E_1|\neg A, R_1)P(E_2|R_1, R_2)$ By lemma 2

$= \neg r_1 \neg r_2 a \neg a r_1$ By (i-vii´)

Hence $P(E_1, E_2) = r_1 r_2 a + r_1 \neg r_2 a r_1 + \neg r_1 \neg r_2 a r_1 = a r_1$

Hence $P(A|E_1, E_2) = \dfrac{r_1 r_2 a + r_1 \neg r_2 a r_1 + \neg r_1 \neg r_2 a^2 r_1}{a r_1} = r_2 \neg r_1 \neg a + r_1 + \neg r_1 a$

$P(A|E_1) = r_1 + \neg r_1 a$ Bovens and Hartmann, 2003, 62

Since $r_2 \neg r_1 \neg a > 0$, by (iv) and (v),

$$P(A|E_1, E_2) > P(A|E_1)$$

Theorem 8: $P(R_1|E_1, E_2) = P(R_1|E_1)$

$$P(R_1|E_1, E_2) = \frac{P(R_1, E_1, E_2)}{P(E_1, E_2)}$$

$$P(R_1, E_1, E_2) = \sum_{A,R_2} P(R_1)P(R_2)P(A)P(E_1|A, R_1)P(E_2|R_1, R_2)$$ By lemma 2

$$= r_1 a(r_2 + \neg r_2 r_1)$$ By (i)-(vii´)

As we saw above, $P(E_1, E_2) = r_1 a$

Hence $P(R_1|E_1, E_2) = r_2 + \neg r_2 r_1$

$$P(R_1|E_1) = r_2 + \neg r_2 r_1$$ Bovens and Hartmann, 2003, 62

Hence $P(R_1|E_1, E_2) = P(R_1|E_1)$

Theorem 9: $P(A|E_1, E_2, E_3, E_4) > P(A|E_1, E_2)$

$$P(A|E_1, E_2, E_3, E_4) = \frac{P(A, E_1, E_2, E_3, E_4)}{P(E_1, E_2, E_3, E_4)}$$

$$P(A, E_1, E_2, E_3, E_4) = ar_1r_2((r_3 + \neg r_3 r_1) + \neg r_1 \neg r_3 a(r_4 + \neg r_4 r_2))((r_4 + \neg r_4 r_2) + \neg r_2 \neg r_4 a((r_3 + \neg r_3 r_1)))$$

$$P(\neg A, E_1, E_2, E_3, E_4) = \neg r_1 \neg r_2 \neg r_3 \neg r_4 \neg a a^2 r_1 r_2$$ By lemma 1, (i-vii´)

$$P(A|E_1, E_2, E_3, E_4) = \frac{1}{1 + \frac{\neg a}{a} \frac{\neg r_1 \neg r_2 \neg r_3 \neg r_4 a^2}{(r_3 + \neg r_3 r_1 + \neg r_1 \neg r_3 a)(r_4 + \neg r_4 r_2 + \neg r_2 \neg r_4 a)}}$$

$$P(A, E_1, E_2) = a(r_1 r_2 + r_1 \neg r_2 a + \neg r_1 r_2 a + \neg r_1 \neg r_2 a^2)$$

$$P(\neg A, E_1, E_2) = \neg r_1 \neg r_2 a^2 \neg a$$ By lemma 2, (i-vii´)

$$P(A|E_1, E_2) == \frac{1}{1 + \frac{\neg a}{a} \frac{\neg r_1 \neg r_2 a^2}{(r_1 + \neg r_1 a)(r_2 + \neg r_2 a)}}$$

Thus it suffices to show that:

$$\frac{\neg r_1 \neg r_2 \neg r_3 \neg r_4 a^2}{(r_3 + \neg r_3 r_1 + \neg r_1 \neg r_3 a)(r_4 + \neg r_4 r_2 + \neg r_2 \neg r_4 a)} < \frac{\neg r_1 \neg r_2 a^2}{(r_1 + \neg r_1 a)(r_2 + \neg r_2 a)}$$

which is true iff

$$\neg r_3 \neg r_4 (r_1 + \neg r_1 a)(r_2 + \neg r_2 a) < (r_3 + \neg r_3 r_1 + \neg r_1 \neg r_3 a)(r_4 + \neg r_4 r_2 + \neg r_2 \neg r_4 a)$$

$0 < r_3 r_4 + \neg r_3 r_4 r_1 + r_3 \neg r_4 r_2 + \neg r_1 \neg r_3 r_4 a + \neg r_2 r_3 \neg r_4 a$, which is true, by (iv), (v)

Hence $P(A|E_1, E_2, E_3, E_4) > P(A|E_1, E_2)$

Theorem 10:

$$P(R_1, R_2|E_1, E_2, E_3, E_4) > P(R_1, R_2|E_3, E_4)$$

$$P(R_1, R_2|E_1, E_2, E_3, E_4) = \frac{P(R_1, R_2, E_1, E_2, E_3, E_4)}{P(E_1, E_2, E_3, E_4)}$$

$$P(R_1, R_2, E_1, E_2, E_3, E_4) = a r_1 r_2 (r_3 + \neg r_3 r_1)(r_4 + \neg r_4 r_2)$$ See proof of theorem 9

$$P(R_1, R_2|E_3, E_4) = \frac{P(R_1, R_2, E_3, E_4)}{P(E_3, E_4)}$$

$$P(R_1, R_2, E_3, E_4) = r_1 r_2 (r_3 + \neg r_3 r_1)(r_4 + \neg r_4 r_2)$$

Hence we need to show that

$$\frac{a r_1 r_2 (r_3 + \neg r_3 r_1)(r_4 + \neg r_4 r_2)}{P(E_1, E_2, E_3, E_4)} > \frac{r_1 r_2 (r_3 + \neg r_3 r_1)(r_4 + \neg r_4 r_2)}{P(E_3, E_4)}$$

which is true iff: $a > P(E_1, E_2 | E_3, E_4)$

$$P(E_1, E_2 | E_3, E_4) = \sum_{R_3, R_4} P(E_1, E_2, R_3, R_4 | E_3, E_4)$$

$$= aP(R_3, R_4 | E_3, E_4) + a^2 P((\neg R_3, R_4 | E_3, E_4) + P(R_3, \neg R_4 | E_3, E_4)) + a^2 P(\neg R_3, \neg R_4 | E_3, E_4)$$

By lemma 2, (i)-(vii´)

Hence it suffices to show that

$P(R_3, R_4 | E_3, E_4) < 1$, which is true (see proof of theorem 11)

Hence $P(R_1, R_2 | E_1, E_2, E_3, E_4) > P(R_1, R_2 | E_3, E_4)$

Theorem 11:

$P(A_1 | E_1, E_2)$ is an increasing function of $P(A_1 | A_2)$ given that $P(A_1)$ is kept fixed.

$$P(A_1 | E_1, E_2) = \frac{P(A_1, E_1, E_2)}{P(E_1, E_2)}$$

$$P(A_1, E_1, E_2) = r_1 r_2 a_{12} + \neg r_1 r_2 a_{12} a_1 + r_1 \neg r_2 a_2 a_1 + \neg r_1 \neg r_2 a_2 a_1^2$$

$$P(\neg A_1, E_1, E_2) = \neg r_1 r_2 a_{\neg 12} a_1 + \neg r_1 \neg r_2 a_{\neg 1} a_1 a_2$$

Hence:

$$P(E_1, E_2) = r_1 r_2 a_{12} + \neg r_1 r_2 a_1 a_2 + r_1 \neg r_2 a_1 a_2 + \neg r_1 \neg r_2 a_1 a_2$$

$$\frac{r_1 r_2 a_{12} + (1-r_1) r_2 a_{12} a_1 + r_1 (1-r_2) a_1 a_2 + (1-r_1)(1-r_2) a_1 a_1 a_2}{r_1 r_2 a_{12} + (1-r_1) r_2 a_1 a_2 + r_1 (1-r_2) a_1 a_2 + (1-r_1)(1-r_2) a_1 a_2}$$

$$= \frac{r_1 r_2 \frac{a_{12}}{a_2} + (1-r_1) r_2 \frac{a_{12}}{a_2} a_1 + r_1 (1-r_2) a_1 + (1-r_1)(1-r_2) a_1 a_1}{r_1 r_2 \frac{a_{12}}{a_2} + (1-r_1) r_2 a_1 + r_1 (1-r_2) a_1 + (1-r_1)(1-r_2) a_1}$$

Since $\neg r_1 r_2 a_{\neg 12} a_1 + \neg r_1 \neg r_2 a_{\neg 1} a_1 a_2 > 0$, by (iv) and (v), $\frac{P(A_1, E_1, E_2)}{P(E_1, E_2)} < 1$. Hence $\frac{P(A_1, A_2, E_1, E_2)}{P(E_1, E_2)} < 1$ and hence theorem 10 goes through.

Since $\frac{P(A_1, E_1, E_2)}{P(E_1, E_2)} < 1$ and $r_1 r_2 \frac{a_{12}}{a_2} + (1-r_1) r_2 \frac{a_{12}}{a_2} a_1 > r_1 r_2 \frac{a_{12}}{a_2}$, given clauses (iv) and (v), $\frac{P(A_1, E_1, E_2)}{P(E_1, E_2)}$ is a strictly increasing function of $P(A_1 | A_2)$.

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