Turn-off Reduction for the Cautious Regulator - a Simulation Study

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TURN-OFF REDUCTION FOR THE CAUTIOUS REGULATOR - A SIMULATION STUDY

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The cautious regulator based on one-step minimization of a quadratic loss function is easy to derive and seems to work well in many cases. A serious draw-back, however, is the possibility for turn-off. The present report is a small simulation study of a simple turn-off reducing scheme.

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INTRODUCTION

In controlling an unknown or time-varying system, parameter identification is needed to improve future control. This can be accomplished by using an actively adaptive control law. Then the conflict between identification and control is explicitly considered in determining the input.

A different approach is to neglect the need for identification in the input calculation. The cautious regulator falls in this category. The hope is then that parameter identification will be sufficiently good anyway. It is, however, well-known that this approach may give rise to the turn-off effect, see e.g. Åström and Wittenmark(1971), Hughes and Jacobs(1974) or Alster and Bélanger(1974). The input can then be close to zero for a long time. This makes identification poor, and the input will stay close to zero until there is a large disturbance in the system.

Apart from this built-in turn-off effect, the cautious regulator seems to work well. It could therefore be worthwhile to work out methods for avoiding turn-off. One approach in this direction is to add a perturbation signal to the input to improve identification. It is then necessary to determine a suitable amplitude for this signal, which may be difficult. Further discussion on this topic and some references are given in Sternby(1977).

Another simple method to avoid turn-off is suggested in this report. It is tested on a couple of simulated examples. An advantage is, that it uses no extra parameters apart from those in the identification algorithm. This simplifies the tuning.

THE CAUTIOUS REGULATOR AND THE TURN-OFF REDUCTION

Consider the system

$$y(t) + a_1(t)y(t-1) + \ldots\ldots + a_n(t)y(t-n) =$$

$$= b_1(t)u(t-1) + \ldots\ldots + b_m(t)u(t-m) + e(t) \qquad (1)$$

where $\{e(t)\}$ is a sequence of independent, Gaussian random variables with zero mean and standard deviation σ. With

$$\theta(t) = [\ a_1(t)\ldots\ldots a_n(t)\ \ b_1(t)\ \ b_2(t)\ldots\ldots b_m(t)\ ]^T$$

$$\tilde{\varphi}(t) = [-y(t-1)\ldots -y(t-n)\quad 0\quad u(t-2)\ldots\ldots u(t-m)]^T$$

$$\ell = [\quad 0\ldots\ldots\ldots 0\quad 1\quad 0\ldots\ldots\ldots 0\ ]^T$$

the system (1) can be written

$$y(t) = \theta(t)^T\tilde{\varphi}(t) + \theta(t)^T\ell u(t-1) + e(t) \qquad (2)$$

The parameter vector $\theta(t)$ is assumed to change according to

$$\theta(t+1) = \Phi\ \theta(t) + v(t+1) \qquad (3)$$

where $\{v(t)\}$ is a sequence of zero mean, independent and Gaussian random vectors with covariance matrix R. The sequences $\{e(t)\}$ and $\{v(t)\}$ and the initial value $\theta(0)$ are also assumed to be independent.

The cautious input at time t is defined to minimize $V_1(t)$ where

$$V_1(t) = E\left[\{y(t+1) - y_r\}^2\ \middle|\ F_t\right] \qquad (4)$$

In (4), y_r is the desired reference value for the output, and F_t is the σ-algebra generated by all previous inputs and outputs. When minimizing (4), the conditional means and covariances of the parameters are needed. They can be obtained from an ordinary Kalman filter, which corresponds to least-squares estimation.

Inserting (2) into (4) the minimizing input can be shown to be

$$u(t) = \frac{[\ell^T\hat{\theta}(t+1)][\ y_r - \hat{\theta}(t+1)^T\tilde{\varphi}(t+1)\ ] - \ell^T P(t+1)\tilde{\varphi}(t+1)}{[\ell^T\hat{\theta}(t+1)]^2 + \ell^T P(t+1)\ \ell} \qquad (5)$$

In this formula, $\hat{\theta}(t+1)$ is the vector of estimates at time t of the parameters $\theta(t+1)$. The corresponding covariance matrix is P(t+1).

Turn-off may occur if $u(t)$ for some reason is small for a while. The variance term in the denominator will then grow due to bad identification, and this will keep the input close to zero. This mechanism is likely to start if the true parameter $b_1(t)$ is close to zero.

The regulator (5) would be less cautious if the parameter values in the preceding step were exactly known. This would imply that

$$\hat{\theta}(t+1) = \Phi\,\theta(t)$$
$$P(t+1) = R$$

Since these old parameter values are not known in practise, such a regulator cannot be implemented. A middle way, however, is to keep $\hat{\theta}(t+1)$ in (5), but replace $P(t+1)$ by R. Then turn-off by the mechanism described above cannot take place.

This is the basic idea for a turn-off reducing device. Different variants could also be tried. It may e.g. be better to replace $P(t+1)$ in the denominator only, but keep it in the numerator. Also, it might be better to use some intermediate value, k, in between $\ell^T P(t+1)\ell$ and $\ell^T R \ell$.

SIMULATIONS

In the examples of this report just the basic idea will be tested, with $P(t+1)$ replaced by a constant k. Two examples will be considered, of first and second order respectively. The dynamics are assumed to be known, and only the b-parameter is unknown. Then the numerator of (5) will not contain the covariance matrix.

Four different regulators will be used for comparison with the turn-off reducing scheme. They are the basic cautious regulator, a cautious regulator with a perturbation signal added (amplitude $|u_e|$), the regulator given in Wittenmark(1975) with weighting λ of the variance term and the two-step regulator from Sternby(1977).

Example 1:

The first order system studied is

$$y(t) + ay(t-1) = b(t)u(t-1) + e(t)$$

$$b(t+1) = 0.9b(t) + v(t+1)$$

with

$$Ee(t)^2 = \sigma^2 = 0.25$$

$$Ev(t)^2 = R = 1$$

$$y_r = 1$$

Three values of a were used, a = 0, a = -0.9 and a = -1. For the case a=0, Åström and Wittenmark(1971) have derived the optimal control law numerically. It is therefore an interesting case for comparisons.

Figure 1 shows how turn-off may occur when the basic cautious regulator is used. There are frequent sign-changes in the b-parameter. $P(t)$ can then start to grow, and there are long intervals where $P(t)$ is large and $u(t)$ and $\hat{b}(t)$ are close to zero.

The left part of figure 2 shows the effects of the turn-off reducing scheme discussed here. $P(t+1)$ in the denominator of (5) has been replaced by k=R=1. There are still some tendencies in the variance to grow, but this lasts only for a few steps each time, and it does not show up in the input, output or b-parameter estimate. For comparison, the corresponding curves for the two-step regulator are shown in the right part of fig. 2. The two-step regulator is in this case almost optimal.

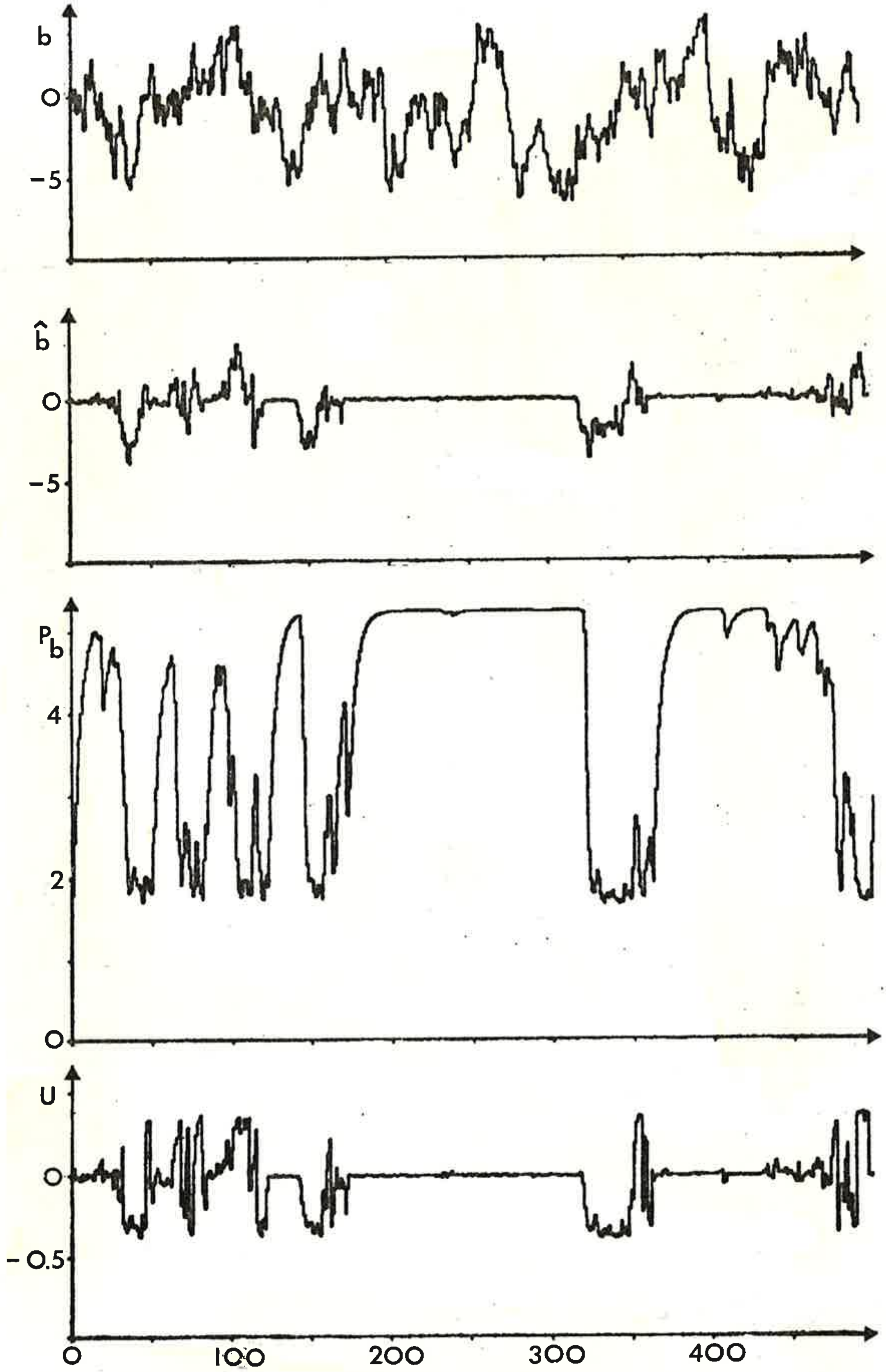

Figure 1 - The b-parameter, its estimate and variance and the input in one simulation of example 1 with a=0. The basic cautious controller is used, which leads to turn-off.

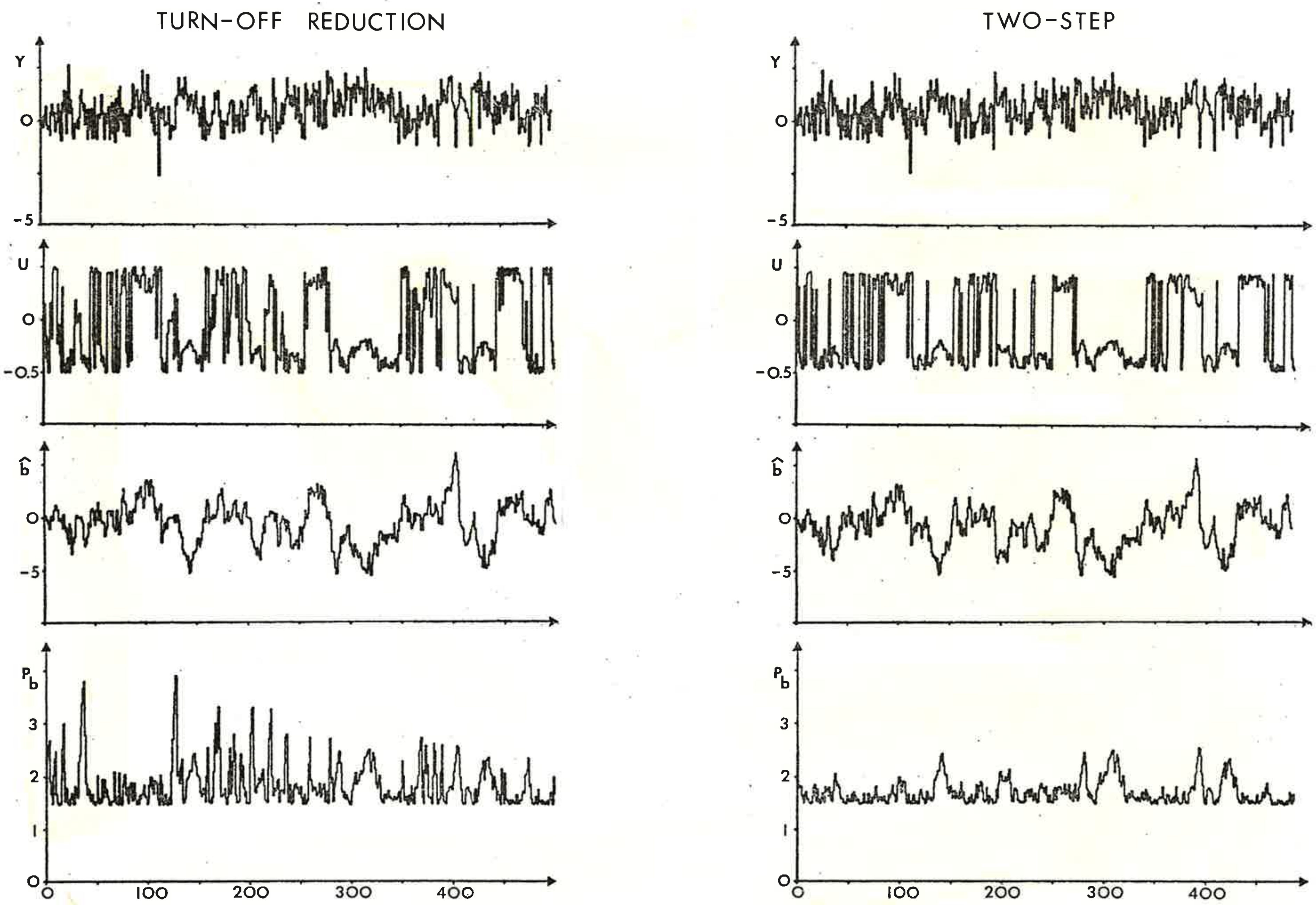

Figure 2 - Output, input, b-parameter estimate and variance in a simulation of example 1 with a=0 and the same noise sequence as in fig. 1. The turn-off reducing scheme (left) and the two-step regulator (right) were used.

To compare the performance of the different control laws, 50 simulations of 500 steps were done for each case. The average loss per step, m, and an estimated standard deviation, Σ, are shown in Table I.

Control law	a = 0 m	a = 0 Σ	a = -0.9 m	a = -0.9 Σ	a = -1 m	a = -1 Σ
Basic cautious	1.09§	0.11§	1.36§	-	1.68+	0.28+
Perturbed cautious $\lvert u_e \rvert = 0.14$	-	-	-	-	1.00+	0.15+
Wittenmark´s λ = 0.5, 0.5 and 0.3	0.83	0.07	0.76	0.11	0.91+	0.18+
Two-step	0.81	0.07	0.73	0.11	0.92+	0.25+
Turn-off reducer k = R = 1	0.85	0.06	0.76	0.14	1.01	0.21

Table I - Average loss per step and estimated standard deviation in example 1 from 50 runs of 500 steps each.

§This value is taken from Wittenmark(1975).
+This value is taken from Sternby(1977)

For the case with a = 0 the optimal loss is 0.83 per step. The last three regulators of Table I are thus all close to optimal. With a = -1 the open-loop system is on the stability limit. Turn-off is then no longer the only problem. Wittenmark´s and the two-step regulators seem to work slightly better than all the others in this case.

Other values for k than R(=1) were also tried. However, k = R proved to be best in all cases.

Example 2:

An extra integrator will now be added to increase the difficulties. The system is

$$y(t) - 2y(t-1) + y(t-2) = b(t)u(t-1) + e(t)$$

with

$$b(t+1) = 0.95 \cdot b(t) + v(t+1)$$

where

$$Ee(t)^2 = \sigma^2 = 0.0009$$

$$Ev(t)^2 = R = 0.09$$

$$y_r = 1$$

As discussed in Sternby(1977) long simulations are needed to get good estimates of the mean loss. Therefore, 25 runs of 5000 steps each were used. In this case $k = R$ (=0.09) is no longer the best choice. Table II shows the result for some different k-values.

k	average loss per step	estimated standard deviation
0.05	2.1	4.2
0.10	0.31	0.26
0.15	0.17	0.10
0.20	0.13	0.08
0.25	0.20	0.34
0.30	0.16	0.08

Table II - Average loss per step and estimated standard deviation from 25 runs of 5000 steps with different k-values.

The best result is with k=0.20, which is about twice the value of R. This may be explained by the fact that the risk for turn-off becomes smaller as the stability of the open-loop system is decreased. It is then advantageous to use a more cautious regulator by increasing k, since this can be done without risking turn-off.

This simple regulator with k=0.20 compares well with the more complicated regulators as Wittenmark´s or two-step. The values for Table III are taken from Sternby(1977), and show the average loss per step and an estimated standard deviation.

Regulator	Average loss per step	Estimated standard deviation
Basic cautious	0.34	0.27
Perturbed cautious $\lvert u_e \rvert = 0.027$	0.21	0.21
Wittenmark´s $\lambda = 0.16$	0.12	0.05
Two-step	0.13	0.08
Turn-off reducer $k = 0.20$	0.13	0.08

Table III - Results for double integrator example

In this example, no regulator is essentially better than the turn-off reduction method. But the k-value is now a parameter that has to be tuned. This tuning, however, should not be too difficult. It is known *a priori* that k should have the same magnitude as the variance of the leading b-parameter estimate.

CONCLUSIONS

In this report a simple scheme to avoid turn-off has been tested. It performs well on the simulated examples, even for the difficult double integrator example. For stable systems, no tuning of parameters seems to be needed apart from those used in the identification algorithm. This is an important advantage. For unstable systems, one additional parameter must be tuned, but with some *a priori* knowledge about its optimal value.

Sofar, no tests have been made of how it can handle systems with more than one unknown parameter. Also, other variants than the one used here could be tried. Since the covariance matrix of the parameter estimates is no longer needed in the control law, stochastic approximation could e.g. be used for the identification. This would give a very simple algorithm to handle these quite difficult systems.

It is also interesting from a theoretical point of view that such a simple scheme performs so well compared to the more elaborated regulators. This raises again the question: What is the true nature of the dual effects of the optimal regulator?

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