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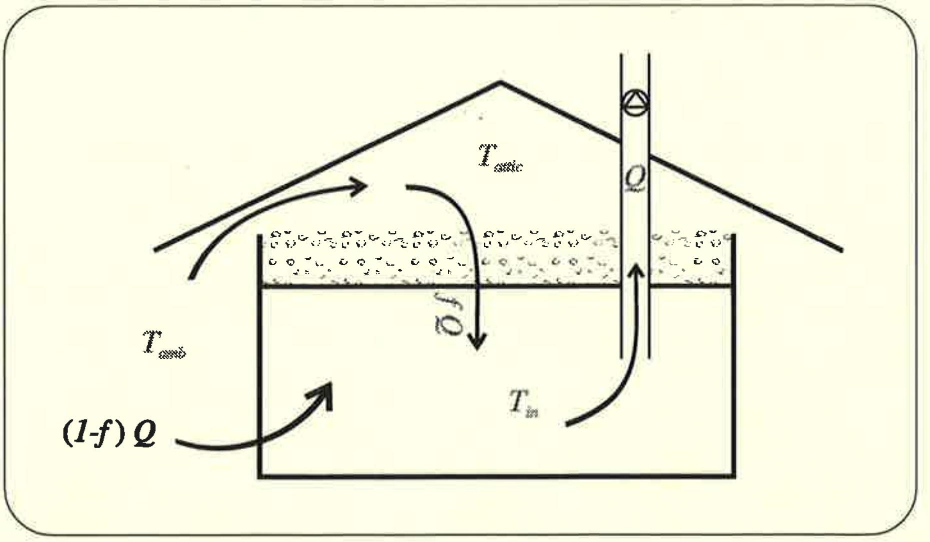
Report TABK--96/3034

Dynamic insulation



Analysis and field measurements

Petter Wallentén

Building Science

Lund University

Lund University, with eight faculties and a number of research centres and specialized institutes, is the largest establishment for research and higher education in Scandinavia. The main part of the University is situated in the small city of Lund which has about 94 000 inhabitants. A number of departments for research and education are, however, located in Malmö. Lind University was founded in 1666 and has today a total staff of 6 000 employees and 37 000 students attending 60 degree programmes and 850 subject courses offered by 170 departments.

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The Department of Buildning Science is part of the School of Architecture within the Faculty of Technology. The Department has two professorial chairs, Building Science and Building Services. Research at the Department is concentrated on energy management, climatic control and moisture problems. The main areas of research are:

- design and preformance of new low-energy buildings
- energy conservation in existing buildings
- utilization of solar heat
- climatic control
- climatic control in foreign climates
- moisture research

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Foreword

Skanska AB has developed a special concept -the ”optima” house - for the construction of energy efficient and healthy single family houses. In order that the performance of this house may be thoroughly evaluated, a special experimental house was designed and built by the company. The concept made use the principle of counterflow or *dynamic* insulation on the attic floor and of a heated crawl space foundation. The task of carrying out the evaluation was given to the Departments of Building Physics and Building Science at the University of Lund Institute of Technology. A comprehensive program was drawn up, and several of the researchers of these departments have actively participated in the project. The leaders of the project were Professor Bertil Fredlund from Dep. of Building Science and Professor Arne Elmroth from Dep. of Building Physics. The project is reported in Elmroth and Fredlund (1996). The task of studying the dynamic insulation was given to me. The measuring equipment I used was installed by Urban Lund and Rolf Lundberg who also kept the system running. I would like to express my thanks to my supervisor Professor Bertil Fredlund, Professor Lars Jensen, who has investigated dynamic insulation for a long time, Urban Lund, Rolf Lundberg and Hans Follin who helped me make this report. I would also like to thank those who have provided the finance, the Swedish Council of Building Research, the Swedish Building Industry Development Fund and Skanska AB.

Lund, March 1996

Petter Wallentén

Contents

Nomenclature

A_d	Area of the dynamic insulation	(m^2)
a	Diffusivity	(m^2/s)
c_a	Heat capacity of air	(J/kg°C)
c_i	Heat capacity of the insulation	(J/kg°C)
e_{dyn}	Dynamic energy efficiency for insulation	(-)
$e_{dyn,tot}$	Dynamic energy efficiency for whole house	(-)
$e_{dyn,\ amb}$	Measured dynamic energy efficiency with ambient temperature as T_{out}	(-)
$e_{dyn,\ attic}$	Measured dynamic energy efficiency with attic temperature as T_{out}	(-)
H	Height of insulation	(m)
L	Distance between T_0 and T_L	(m)
P_{dyn}	Specific heat loss for a house with dynamic insulation	(W/°C)
P_e	Specific heat loss for a house with heat recovery unit for the air that passes through the roof	(W/°C)
$P_{e,tot}$	Specific heat loss for a house with heat recovery unit for all air	(W/°C)
P_0	Specific heat loss for everything but the ventilation and conduction through the dynamic insulation	(W/°C)
P_U	Specific heat loss for a house without dynamic insulation	(W/°C)
P_{normal}	Specific heat loss for a house without dynamic insulation but with the same insulation thickness	(W/°C)
Q	Total exhaust air for the house	(m^3/s)
$R^2(\ )$	Least squares error function for the temperature distribution	(°C^2)

s	Standard deviation in temperature distribution	(°C)
t	Time variable	(s)
t_n	Time at time step n	(s)
t_0	Initial time for boundary condition	(s)
T	Temperature	(°C)
T^0	Temperature distribution at time t_0	(°C)
T_0	Temperature at x=0	(°C)
T_{amb}	Ambient temperature	(°C)
T_{attic}	Temperature in attic	(°C)
$T_i^{\Delta t}$	Measured temperature at $x=x_i$ integrated over Δt	(°C)
T_{in}	Temperature inside house	(°C)
T_L	Temperature at $x=L$	(°C)
T_{out}	Reference temperature in equations (3.6, 3.11)	(°C)
$\hat{T}_i^{\Delta t}$	Calculated temperature integrated over Δt	(°C)
T_i^n	Calculated temperature in finite difference calculation. Time step n and $x=x_i$	(°C)
$\overline{T}_i$	Temporary temperature used in finite difference calculation	(°C)
u	Air flow through the insulation	(m^3/m^2s)
u_e	Least squares estimate of the air flow	(m^3/m^2s)
U_{normal}	U value with zero air flow	(W/m^2°C)
U_{dyn}	Dynamic U value for a specific u	(W/m^2°C)
$U_{dyn,amb}$	Measured dynamic U value with ambient temperature as T_{out}.	(W/m^2°C)
$U_{dyn,attic}$	Measured dynamic U value with attic temperature as T_{out}.	(W/m^2°C)
x	Length coordinate	(m)
x_j	Coordinate for node j	(m)
Δt	Time used in integration of temperature	(s)
δt	Time step used in finite difference calculation	(s)
δx	Distance between nodes in finite difference calculation	(m)
λ	Heat transfer coefficient for the insulation	(W/m°C)
ρ_a	Density of air	(kg/m^3)
ρ_i	Density of the insulation	(kg/m^3)

Summary

This report describes measurements and calculations for a 'dynamic' insulation. The term dynamic insulation implies that part of the inlet or exhaust air passes through the insulation of a house. The main reasons for using dynamic insulation with inlet air are: the dynamic insulation is similar to a heat exchanger for the ventilation air, the insulation filters the air to a theoretically very high degree and the inlet air is preheated, thus providing a high degree of comfort in the house. Dynamic insulation can be used in any insulation in contact with ambient air. The energy efficiency of a dynamic insulation is measured by the 'dynamic U value' or an equivalent heat exchanger efficiency. The efficiency of a dynamic insulation depends on the amount of air that passes through the insulation. Typically, the air flow through the insulation is produced by keeping the inside of the house at a pressure lower than that of the ambient air.

A house with dynamic insulation in the ceiling was continuously measured for approximately a year and a half. The house was a single storey one family house built at Dalby in the south of Sweden. The house was constructed in conformity with the Swedish Building Code. The performance of the dynamic insulation was estimated by using hourly values of the temperature distribution inside the insulation. The air flow through the insulation was calculated as the air flow that best fitted the measured temperature distribution. In order that the temperature distribution for a given air flow may be calculated, the transient and steady state heat transfer equations were solved both analytically and numerically. The numerical solutions were used to make hourly calculations of the air flow. The analytical solutions were used to better understand the importance of the physical parameters and to verify the numerical solution. The method described above is here called the *gradient method*. The main

objective of the study was to investigate the performance of a dynamic insulation placed in a realistic environment. The only major exception to this was that the house was unoccupied during the measurement period.

The analytical calculations showed that for periods shorter than 6h it was necessary to use a true transient calculation. The most important parameter was the first term in the series solution of the transient heat transfer equation. The analytical solution clearly verified the numerical solution.

The air flow through the dynamic insulation as measured by the gradient method was 40% of the total inlet air. Results from a few tracer gas and direct measurements indicated a higher flow, but this could be expected since the gradient method was the only method that measured the flow in the insulation itself.

The dynamic U value for the insulation was about 0.05 W/m^2°C for the ceiling. The normal U value for the ceiling was 0.16 W/m^2°C. This corresponds to a dynamic energy efficiency for the insulation of 35%. This factor should be multiplied by the proportion of the total inlet air that passed through the insulation, i.e. 40%, to obtain the total energy efficiency for the ventilation system. The total energy efficiency calculated this way is 14%. A theoretical calculation shows that if 100% of the inlet air were to pass through the insulation, this would correspond to an energy recovery of the ventilation energy by 22% which for the OPTIMA house corresponds to a power saving of 15 W/°C or an energy saving of 1600 kWh/year.

The calculated 24h average air flow increased slightly with increasing difference between inside and outside temperature. Both the 24h and 2h average air flow decreased with increasing wind speed. The reduction was of the same order as reported in the literature. No obvious effect of wind direction was found.

The transient calculation showed a daily variation of the air flow that increased with increasing sunshine. This variation was typically ± 30% of the average flow with the maximum at midnight and the minimum at noon. This air flow variation could not be detected in the inlet terminals.

When the ventilation system was turned off and the inlet terminals completely closed, the air flow through the insulation changed from 0.2 mm/s downwards to 0.1 mm/s upwards. This could be a dangerous situation if the air moving upwards came from the living space. The air moving upwards could then create condensation in the insulation. If the air came from outside, this would only lead to a higher energy loss.

The measurements in the OPTIMA house showed that the use of dynamic insulation could not be justified by the energy savings alone. However, the inlet air was preheated and filtered and there were fewer ventilation ducts for transporting the inlet air than would be needed in a house with a heat exchanger system. Taken together, this makes the dynamic insulation an attractive choice, at least in theory. In practice the leakage in the rest of the house reduced the energy saving of the dynamic insulation to about 70% of the maximum value and only 40% of the inlet air was filtered by the dynamic insulation. The general conclusion from the measurements was that the dynamic insulation requires a house constructed to much more exacting standards in order to work properly.

1 Introduction

1.1 Dynamic insulation

In 'dynamic' insulation, part of the inlet or exhaust air passes through the insulation. Inlet air is normally used since there is a risk of condensation if exhaust air is used. This is also called counterflow insulation since the direction of air flow is opposite to that of heat flow. When exhaust air is used, this is called parallel flow insulation. There are three reasons for using inlet air with dynamic insulation:

1. Energy loss is less than in a house without heat recovery in the ventilation system.
2. The insulation filters the air to a theoretically very high degree.
3. The inlet air is preheated, thus providing a high degree of comfort in the house.

Dynamic insulation can be used in any insulation in contact with ambient air. Dynamic insulation is similar to a heat exchanger between the inlet and exhaust air, but a house with dynamic insulation with inlet air needs fewer ventilation ducts for transporting the inlet air than a house with a heat exchanger system. The energy efficiency of a dynamic insulation is measured by the 'dynamic U value' or an equivalent heat exchanger efficiency. The efficiency of dynamic insulation depends on the amount of air that passes through the insulation. Typically, the air flow through the insulation is produced by keeping the house at a pressure lower than the pressure of the ambient air.
A house with dynamic insulation is sometimes equipped with a heat pump working on the exhaust air.

Houses with dynamic insulation have been built in France, Finland, Norway, Sweden and Switzerland. The first systems were built at the end of the 70s. (Anderlind 1983, Arquis 1986, Humm 1994, Söderberg 1984). Thorén (1978) has a patent in Sweden for a dynamic insulation system. According to Anderlind, about 60 000 m^2 in Sweden and 500 000 m^2 in Finland were constructed in the year 1980. Most of these buildings were industrial buildings. Only few laboratory and full scale experiments have been made (Anderlind, Arquis, Roots 1994). These experiments mostly show good agreement with the theory. All experiments reported so far have been steady state ones, i.e. no variation with time has been studied. The most extensive studies have been performed in the laboratory. The main goal of the study presented here was to

- Analyse the steady state and transient behaviour of the dynamic insulation.

- Measure the performance of a dynamic insulation installed in a house exposed to realistic conditions.

- Measure the importance of wind speed, wind direction, ambient temperature and sunshine on the performance of the dynamic insulation.

1.2 Method

To achieve the first goal above, the transient and steady state heat transfer equations were solved analytically and expressions for the energy efficiency of the insulation was calculated. To achieve the second and third goal above , a house with dynamic insulation in the ceiling was continuously measured for approximately a year and a half. The house, which was called the OPTIMA house, was a single storey one family house of 116 m^2 built at Dalby in the south of Sweden.

1.3 Literature survey

Some articles and reports regarding dynamic insulation are listed below with comments.

Boman, C-A., & Matsson, M. (1981). *Testhouse with dynamic insulation, measurements.*

The report describes measurements, performed over a year and a half, of energy consumption and temperature in a house. The house had counterflow insulation in the external walls and parallel flow insulation in the floor. The authors stated that the calculated energy consumption for the house was close to the measured energy consumption. The energy saving due to the dynamic insulation was not given by the authors.

Jensen, L. (1982). *Dynamic insulation and heat recovery of ventilation air.*

This report presents a detailed theoretical calculation of the thermal performance of dynamic insulation. The dynamic insulation was compared with other heat recovery systems. The author showed that a counterflow insulation can never be better than a heat exchanger with the energy efficiency 50%. A combined parallel and counterflow insulation can reach the energy efficiency 100%.

Anderlind, G., & Johansson, B. (1983). *Dynamic insulation, A theoretical analysis of thermal insulation through which a gas or fluid flows.*

This report gives a full description of the steady state theory of dynamic insulation. Special attention was given to the temperature and moisture content of the insulation. The energy efficiency was described by the dynamic U value.

Söderberg, D., & Fagerstedt, A. (1984). *Dynamic insulation, State of the art of development, Calculations with respect to ventilation.*

The report is divided into two parts. The first part is a literature survey in the form of a state of the art report. The general conclusion from this was that the usefulness of dynamic insulation for dwellings was questionable due to the low ventilation rate but that the technique looked more promising for industrial buildings. The second part is a theoretical study of some important topics regarding dynamic insulation: the importance of leakage in the external walls, the effect of wind speed, the risk of non-uniform flow across the insulation and the moisture transport in the insulation.

Arquis, E., & Langlais, C. (1986). *What scope for 'dynamic insulation'?*

The article describes a French study of dynamic insulation including full scale laboratory tests. The energy efficiency was described by the steady state parameters: dynamic U value, energy efficiency for an equivalent heat exchanger and an overall heat recovery efficiency for the whole system. The conclusion from the full scale experiments was that a uniform air flow distribution is difficult to achieve in a vertical configuration. A non uniform air flow will always give a lower energy efficiency than a uniform air flow. The main conclusion was that conventional insulation techniques remained competitive for dwellings but dynamic insulation offers greater possibilities for industrial buildings with larger ventilation rates.

Humm, O. (1994). *Dynamic Insulation - Using transmission losses for building heating.*

The article describes two houses in Knonau in Switzerland built in 1990 and measured at least until 1994 when the article was written. The houses had dynamic insulation in the roof, which was a purlin roof. The measured energy for heating was 30% higher than the calculated energy. According to the author this was enough to justify the investment in dynamic insulation.

Roots, P. (1994). *Skanterra - optima, a study of the thermal performance of an exterior wall design.*

The report describes a steady state full scale laboratory measurement of a wall with dynamic insulation. The wall was an 'OPTIMA wall' which was constructed in a way similar to the OPTIMA roof described below. The dynamic U value was calculated from temperature measurements in the wall. When the living space side of the wall had a negative pressure compared with the ambient air side of the wall, 50% of the air passed through leakage paths in the wall and 50% through the insulation. When air was extracted directly from the inlet ducts, 30% passed through leakage paths in the wall and 70% through the insulation. The measured U value when the air flow was zero was higher than the calculated one. The author drew the conclusion that natural convection in the wall was responsible for this increase in the U value.

2 Physical Model for Heat and Air Flow

This chapter describes the equations that are the basis for this study. Some simple conclusions will be drawn from the analytical solutions. When no other value is stated, the values for the physical parameters apply for the OPTIMA house. These values are described in table 2.1.

Table 2.1 Physical parameters for the OPTIMA house.

Heat transfer coefficient for the insulation	λ = 0.042 W/m°C
Heat capacity of air	c_a = 1005 J/kg°C
Heat capacity of the insulation	c_i = 1000 J/kg°C
Density of air	ρ_a = 1.27 kg/m^3
Density of the insulation	ρ_i = 19 kg/m^3

2.1 Basic equations

The air passes through the loose fill insulation at low velocity (~ 0.2 mm/s) which permits the use of a simple laminar model. For the macroscopic behaviour it is convenient to describe the loose fill insulation as homogeneous and isotropic. With these assumptions the steady state heat transfer equation in one dimension is (Anderlind 1980):

$$\lambda \frac{d^2T(x)}{dx^2} - u\, \rho_a\, c_a \frac{dT(x)}{dx} = 0 \tag{2.1}$$

The boundary conditions are:

$$T(0) = T_0 \qquad T(L) = T_L \tag{2.2}$$

where T is the temperature in °C, λ is the heat transfer coefficient for the insulation, u~0.2 10^{-3} m/s (= 0.2 mm/s) is the air flow which is positive in the direction of increasing x (m), ρ_a is the density of air, c_a is the heat capacity of air and L is the height of the simulated part of the insulation. The upper part of the insulation is x=0 and the lower part is x=L. The air flow is downward towards the living space.

To be precise, u is not really the speed of the air but the total air flow that passes through one square metre of the insulation. The unit for u is m^3/m^2 s. The local speed in the material will thus be higher due to the porosity of the loose fill insulation. Only the macroscopic behaviour of the insulation will be studied here, and for the sake of convenience the units m/s or mm/s will be used throughout this report.

The transient heat transfer equation in one dimension is:

$$\lambda \frac{\partial^2 T(x,t)}{\partial x^2} - u\, \rho_a\, c_a \frac{\partial T(x,t)}{\partial x} = \rho_i\, c_i \frac{\partial T(x,t)}{\partial t}, \tag{2.3}$$

with the initial and boundary conditions:

$$T(x,t_0) = T^0(x) \qquad T(0,t) = T_0(t) \qquad T(L,t) = T_L(t) \tag{2.4}$$

where t is the time is seconds, ρ_i is the density of the insulation and c_i is the heat capacity of the insulation. Equation (2.3) can also be written:

$$a \frac{\partial^2 T(x,t)}{\partial x^2} - a\nu \frac{\partial T(x,t)}{\partial x} = \frac{\partial T(x,t)}{\partial t} \tag{2.5}$$

where a (m^2/s) is the diffusivity of the insulation. The ratio of convection to conduction is described by (m^{-1}). The product $a \cdot \nu$ is the *thermal velocity*. It describes at what rate the temperature would change if diffusion were neglected. For the OPTIMA house the thermal velocity was typically 0.01 mm/s, i.e. it would take roughly 8h for a thermal front to pass through an insulation of 0.3m if the conduction were neglected.

$$\nu = u\, \rho_a\, c_a / \lambda \approx 6\ \mathrm{m}^{-1}$$

$$a = \lambda / \rho_i\, c_i \approx 2.2 \cdot 10^{-6}\ \mathrm{m^2/s}$$

$$a\nu = u \frac{\rho_a c_a}{\rho_i c_i} \approx 13 \cdot 10^{-6}\ \mathrm{m/s} \tag{2.6}$$

2.2 Solution to the steady state equation

The analytical solution to the steady state temperature distribution (2.1) is straightforward:

$$T(x) = T_0 + (T_L - T_0)\frac{e^{xu\rho_a c_a/\lambda} - 1}{e^{Lu\rho_a c_a/\lambda} - 1} = T_0 + (T_L - T_0)\frac{e^{\nu x} - 1}{e^{\nu L} - 1} \tag{2.7}$$

The solution is thus an exponential function that will approach a straight line for decreasing u.

2.3 Solution to the transient calculation

The solution to the general transient temperature distribution is very complex. Instead of trying to solve the general problem the problem of zero initial conditions and a step change at x=0 is studied. The new initial and boundary conditions are thus:

$$T(x,0)=0, \quad T(0,t)=T_0, \quad T(L,t)=0 \tag{2.8}$$

The standard approach is to use the Laplace transform in the time domain. The calculations are shown in appendix A. Two solutions are possible; one short term and one long term solution. For a dynamic insulation the long term solution is of more importance. Only the long term solution of equation (2.5) is presented below.

$$T(x,t) = e^{x\nu/2} T_0 \left(\frac{\sinh \frac{\nu}{2}(L-x)}{\sinh \frac{\nu}{2} L} + \sum_{n=1}^{\infty} (-1)^n e^{-\left(\frac{a\nu^2}{4} + \frac{a\, n^2\pi^2}{L^2}\right)t} \cdot \frac{\sin \frac{n\pi}{L}(L-x)}{\frac{\nu^2 L^2}{8n\pi} + \frac{n\pi}{2}} \right) \tag{2.9}$$

The solution is thus the sum of an infinite series. The steady-state solution is approached for large t:

$$T(x,\infty) = e^{x\nu/2} T_0 \left(\frac{\sinh \frac{\nu}{2}(L-x)}{\sinh \frac{\nu}{2} L} \right) = T_0 \left(1 - \frac{e^{\nu x} - 1}{e^{\nu L} - 1} \right) \tag{2.10}$$

The terms in (2.9) decrease exponentially in time. The behaviour of the first term is:

$$e^{-\left(a\,\nu^2/4 \,+\, a\pi^2/L^2\right)t} = e^{-t/\tau}$$

$$\tau = \frac{1}{a\,\nu^2 / 4 \,+\, a\pi^2 / L^2} \approx \frac{1}{20 \cdot 10^{-6}\ \mathrm{s}^{-1} + 240 \cdot 10^{-6}\ \mathrm{s}^{-1}} \ (\mathrm{s}) \tag{2.11}$$

where τ (s) is the time constant that will be used in the following. The table below shows the value of for some different u, (L=0.3m):

Table 2.2 Relationship between τ and u for the OPTIMA house.

u (mm/s)	0.01	0.1	0.5	1.0	2.0
τ (min)	69	68	45	22	7

The ratio of the transient to the steady state temperature for u=0.05-0.5 mm/s is plotted in figure 2.1 as a function of t/τ. The relationship between the temperature and t/τ is fairly constant for different air flows u. It is therefore reasonable to use τ in describing how long the transient behaviour from a step change lasts in the insulation. The time constant depends heavily on the air flow: u. The ratio of the transient to the steady state temperature for x=0.05-0.25 m is plotted in figure 2.2 as a function of the dimensionless fraction t/τ.

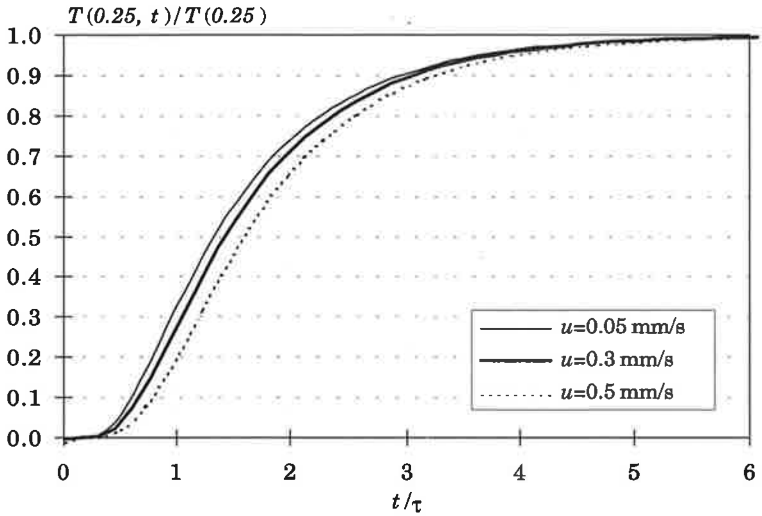

Figure 2.1 The ratio of the transient to the steady state temperature for u=0.05-0.5 mm/s, (x=0.25 m) as a function of t/τ.

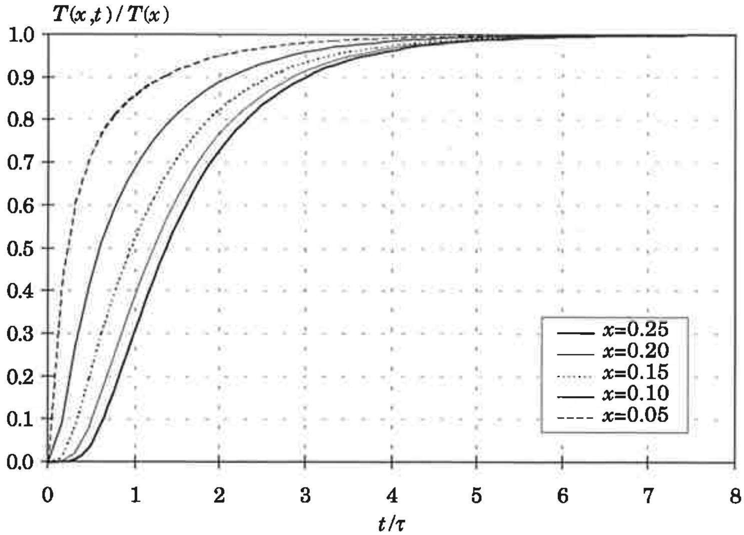

Figure 2.2 *The ratio of the transient to the steady state temperature for x=0.05-0.25 m, (u=0.2 mm / s) as a function of t / τ.*

If the air flow is 0.1 mm/s, about 30% of the effect of a step change will thus be reached in 1 hour at x=0.25m and about 90% will be reached in 3.5 hours. If conduction is neglected a thermal front will need 8h to pass through an insulation of 0.3m. Heat transfer by convection is thus somewhat smaller than the heat transfer by conduction in the dynamic insulation in the OPTIMA house.

The calculation above suggests that for time periods shorter than 5τ or approximately 6h it is necessary to use a transient calculation.

The ratio of the transient to the steady state temperature at x=0.25 for n=1,2 and 20 is plotted in figure 2.3. The solution (2.9) requires a smaller number of terms with increasing t. A solution with n=1 gives a relative error smaller than 10% for $t>\tau$ and a solution with n=2 gives a relative error smaller than 1%. The solution (2.9) with n=1 is:

$$T(x,t) = e^{x\nu/2}T_0\left(\frac{\sinh\frac{\nu}{2}(L-x)}{\sinh\frac{\nu}{2}L} - e^{-t/\tau}\cdot\frac{\sin\frac{\pi}{L}(L-x)}{\frac{\nu^2L^2}{8\pi}+\frac{\pi}{2}}\right) \qquad (2.12)$$

For times shorter than t=0.2τ, n=20 is not enough to give satisfactory relative accuracy. The short term solution presented in appendix A must then be used.

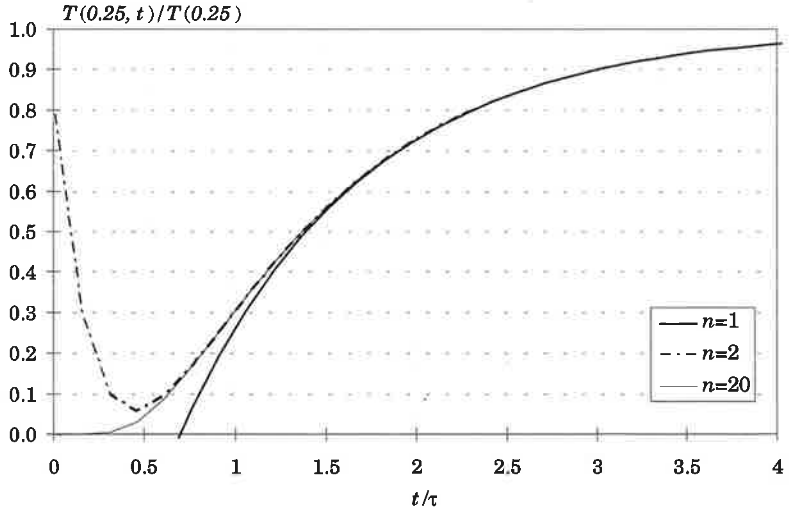

Figure 2.3 *The ratio of the transient to the steady state temperature at x=0.25m for* n=*1,2 and 20, (u=0.2 mm/s) as a function of* t/τ.

3 Energy Consumption

The energy consumption of a house with dynamic insulation can be described by the 'dynamic U value', dynamic energy efficiency or a total energy saving. The dynamic U value is used when the house is described as a normal house without heat recovery in the ventilation system. This leads to a low U value for the dynamic insulation. The dynamic energy efficiency is used when the house is described as a house with a heat exchanger between inlet and exhaust air. The total energy saving is used when the house is compared without dynamic insulation but with the same insulation thickness.

3.1 Dynamic U value: U_{dyn}

To calculate the dynamic U value two houses are compared: one house with dynamic insulation and one house with the same air flow but without any heat recovery system for the ventilation. The total energy consumption of these two houses is equal for a certain U value of the normal house. This U value is called the dynamic U value = U_{dyn}. The energy consumption for the house with dynamic insulation is:

$$A_d \cdot \left(\lambda \frac{dT(x)}{dx} + (T_{in} - T(x)) \frac{fQ}{A_d} \rho_a c_a\right)$$
$$+(1-f)\, Q \rho_a c_a \cdot (T_{in} - T_{out}) + P_0\,(T_{in} - T_{out}) = P_{dyn}\,(T_{in} - T_{out}) \tag{3.1}$$

where Q (m^3/s) is the total air flow , A_d (m^2) is the area of the dynamic insulation and f (-) is the proportion of the air flow that passes the dynamic insulation. The parameter P_0 describes all other specific energy losses in the house divided by the temperature difference between inside and outside. The parameter P_{dyn} describes the total specific energy loss for a house with dynamic insulation. The energy consumption for the house without dynamic insulation, with a different U value, is:

$$A_d \cdot U_{dyn} (T_{in} - T_{out}) + Q \rho_a c_a (T_{in} - T_{out}) + P_0 (T_{in} - T_{out}) \\ = P_U (T_{in} - T_{out}) \tag{3.2}$$

These two energy consumptions are equal for a certain U_{dyn}.

$$P_{dyn} = P_U \tag{3.3}$$

The air velocity through the insulation is:

$$u = \frac{fQ}{A_d} \tag{3.4}$$

The U value is now calculated as:

$$U_{dyn} = \left(\lambda \frac{dT(x)}{dx} + (T_{out} - T(x)) \frac{fQ}{A_d} \rho_a c_a \right) \frac{1}{(T_{in} - T_{out})} \tag{3.5}$$

The U value is independent of x. The choice of T_{out} and T_{in} defines what part of the construction U_{dyn} represents. If T_{in} is the inside temperature and T_{out} is the ambient temperature T_{amb}, the ceiling, insulation and roof are included in U_{dyn}. If T_{out} is chosen as the attic temperature T_{attic} only the ceiling and insulation are included. If steady state conditions are assumed (3.5) becomes, with the use of the air velocity u:

$$U_{dyn} = \frac{u \rho_a c_a}{T_{in} - T_{out}} \left(\frac{T_L - T_0}{e^{u \rho_a c_a L / \lambda} - 1} + T_{out} - T_0 \right) \tag{3.6}$$

where L is the distance between the points where the temperatures T_L and T_0 are measured. If it is assumed that the full height of the insulation is H and that all the thermal resistance occurs over the insulation, i.e. $T_{out}=T_0$ and $T_{in}=T_L$, equation (3.6) becomes:

$$U_{dyn} = \frac{u\rho_a c_a}{e^{u\rho_a c_a H/\lambda} - 1} = \frac{f Q\rho_a c_a}{A_d\left(e^{f Q\rho_a c_a H/(\lambda A_d)} - 1\right)} \tag{3.7}$$

Figure 3.1 shows U_{dyn} according to (3.7) and the flow u as a function of the proportion, f, of the inlet air that passes through the dynamic insulation.

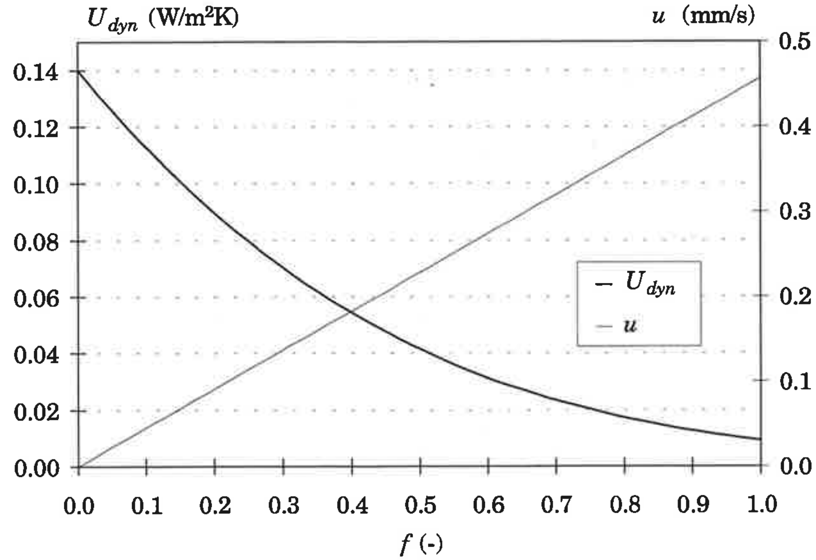

Figure 3.1 *The dynamic U value according to (3.7) and the flow u as a function of the proportion, f, of the inlet air that passes through the dynamic insulation (H=0.3m).*

3.2 Dynamic energy efficiency: e_{dyn}

Jensen (1982) and Arquis (1986) shows that the dynamic energy efficiency is perhaps a more adequate parameter for a dynamic insulation than the dynamic U value. To calculate the dynamic energy efficiency two houses are compared: one house with dynamic insulation and one house with the same air flow but with a heat exchanger between inlet and outlet air. The energy consumption of these two houses is equal for a certain value of the energy efficiency for the heat exchanger. This value is here called the dynamic energy efficiency = e_{dyn}. The energy consumption for the house with a heat exchanger is:

$$A_d \cdot U_{normal} (T_{in} - T_{out}) + (1 - e_{dyn}) f Q \rho_a c_a (T_{in} - T_{out}) + (1 - f) Q \rho_a c_a (T_{in} - T_{out}) + P_0 (T_{in} - T_{out}) = P_e (T_{in} - T_{out}) \tag{3.8}$$

Note that only the air that should have passed through the dynamic insulation ($f Q$) is used in the heat exchanger. In (3.8) U_{normal} is the U value of the investigated building component with the air flow set to zero. This energy consumption is equal to the energy consumption of the house with dynamic insulation:

$$P_{dyn} = P_e \tag{3.9}$$

The dynamic energy efficiency is now calculated as:

$$e_{dyn} = 1 - \frac{T_{in} - T(x)}{T_{in} - T_{out}} + \frac{1}{u \rho_a c_a} \left(U_{normal} - \frac{\lambda}{T_{in} - T_{out}} \frac{dT(x)}{dx} \right) \tag{3.10}$$

The energy efficiency should be independent of x. The choice of T_{out} and T_{in} defines what part of the construction e_{dyn} represents, as above for U_{dyn}. If steady state conditions are assumed (3.10) becomes:

$$e_{dyn} = \frac{U_{normal}}{u \rho_a c_a} + \frac{e^{u \rho_a c_a L/\lambda} (T_0 - T_{out}) + T_{out} - T_L}{\left(e^{u \rho_a c_a L/\lambda} - 1\right)(T_{in} - T_{out})} \tag{3.11}$$

where L is the distance between the points where the temperatures T_L and T_0 are measured. If we assume that the full height of the insulation is H and that all the thermal resistance occurs over the insulation, i.e. T_{out}=T_0 and T_{in}=T_L, equation (3.11) becomes:

$$e_{dyn} = \frac{\lambda}{u\rho_a c_a H} - \frac{1}{e^{u\rho_a c_a H/\lambda} - 1} \tag{3.12}$$

The energy efficiency can never be more than 0.5 for a dynamic insulation and this value is reached for air flows close to zero.

3.3 Total dynamic energy efficiency: $e_{dyn,tot}$

If the house with dynamic insulation is compared with a house with a heat exhchanger between *all* inlet and outlet air, the total dynamic energy efficiency: $e_{dyn,tot}$ is obtained. This is the total energy efficiency for the house seen as one system. The energy consumption for a house with a heat recovery unit on all air is:

$$\begin{aligned} &A_d \cdot U_{normal} (T_{in} - T_{out}) + (1 - e_{dyn,tot})\ Q\ \rho_a\ c_a (T_{in} - T_{out}) \\ &+P_0 (T_{in} - T_{out}) = P_{e,tot} (T_{in} - T_{out}) \end{aligned} \tag{3.13}$$

This energy consumption is equal to the energy consumption for the house with dynamic insulation for a certain $e_{dyn,tot}$:

$$P_{dyn} = P_{e,tot} \tag{3.14}$$

After some calculations we have:

$$e_{dyn,tot} = f \cdot e_{dyn} \tag{3.15}$$

The total energy efficiency takes into account the fact that only a fraction, f, of the inlet air has actually passed through the dynamic insulation. The total energy consumption of the house is the sum of the ventilation energy, the heat loss due to conduction through the dynamic insulation and the heat loss due to conduction in the normal external walls. The heat loss in the normal walls does not depend on f but the other two heat losses do so. In figure 3.2 the energy efficiency e_{dyn}, the total energy efficiency $e_{dyn,tot}$ are shown as functions of f.

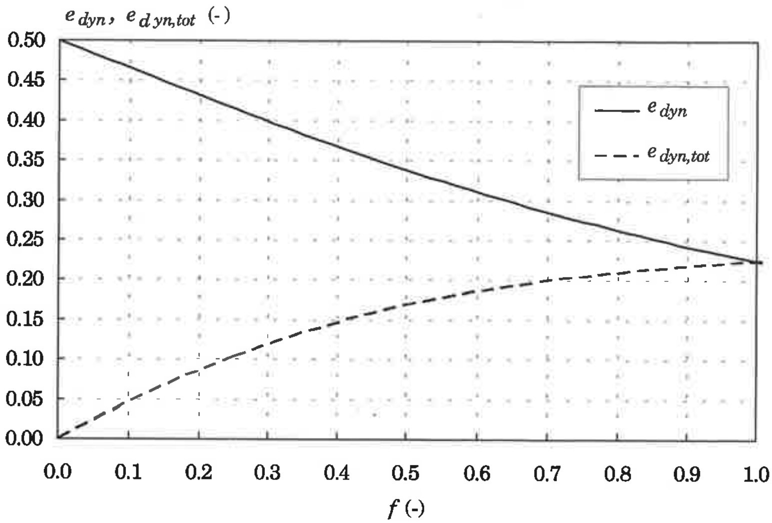

Figure 3.2 *The energy efficiency e_{dyn} and the total energy efficiency $e_{dyn,tot}$ for the OPTIMA house. The calculation is based on 53 l/s exhaust air for f=1.*

3.4 Relative energy saving: η

Two ways of describing a dynamic insulation have so far been presented: as a heat exchanger with equivalent energy efficiency e_{dyn} between inlet and outlet air or as a wall with an equivalent U-value U_{dyn}. A third way of describing a dynamic insulation is to calculate the energy saving compared with an identical house but with no treatment of the inlet and outlet air. This relative energy saving is denoted η:

$$\eta = \frac{P_{normal} - P_{dyn}}{P_{normal} - P_0} \tag{3.16}$$

Note that η does not depend on the heat losses in the rest of the house P_0.

It is possible to calculate h theoretically. With the same assumptions as in (3.7) and (3.12) h becomes:

$$\eta = \frac{1}{1+u\rho_a c_a H/\lambda}\left(1+\frac{u\rho_a c_a H/\lambda}{1-e^{u\rho_a c_a H/\lambda}}\right) \tag{3.17}$$

This function is shown in figure 3.3 as dependent on the dimensionless number $u\ \rho_a\ c_a\ H/\lambda$ which is the ratio of convection to conduction for a dynamic insulation of the height H.

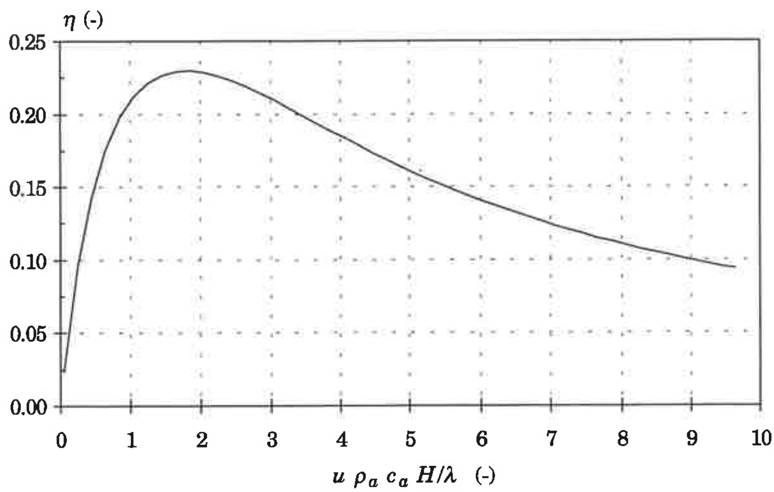

Figure 3.3. *The relative energy saving η as a function of the dimensionless number $u\ \rho_a\ c_a\ H/\lambda$.*

Obviously η has a maximum for $u\ \rho_a\ c_a\ H/\lambda = 1.79$ and this maximum is 0.23. This means that the relative energy saving with a dynamic insulation of counter flow type can never be more than 23%. Figure 3.4 shows the energy efficiency e_{dyn} the relative energy saving η and the energy loss through conduction and ventilation per square meter $(P_{dyn}-P_0)/A_d$ in the ceiling in the OPTIMA house as a function of the height of the insulation H.

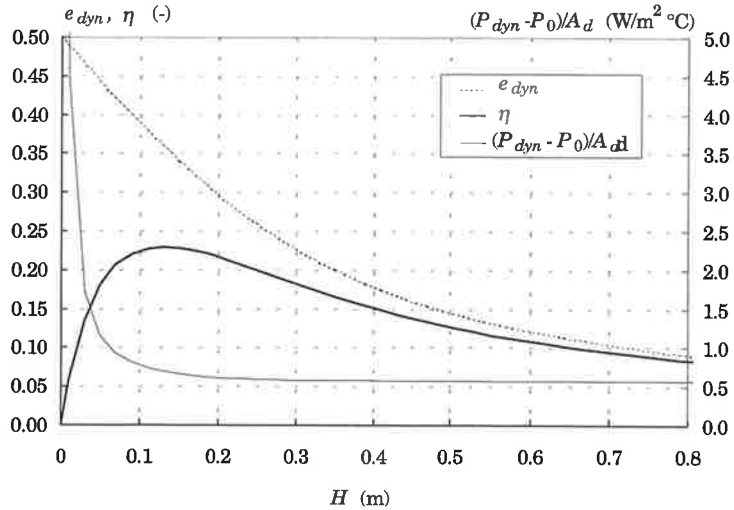

Figure 3.4. *The energy efficiency e_{dyn} the relative energy saving η and the energy loss through conduction and ventilation per square meter $(P_{dyn}-P_0)/A_d$ in the ceiling in the OPTIMA house as a function of the height of the insulation H. The calculations are based on f=100% and the total flow 53 l/s.*

From figure 3.4 it is clear that the energy loss rapidly decreases for the first 0.1 m of insulation and then only very slowly decreases when the insulation is increased even more. Clearly an, in some sense, optimal thickness of the insulation is about 0.2 m. The actual height in the OPTIMA house was 0.3 m.

If one assusmes that only the fraction f of the inlet air passes through the insulation equation 3.17 becomes:

$$\eta = \frac{1}{1+u\rho_a c_a H/(f\lambda)}\left(1+\frac{u\rho_a c_a H/\lambda}{1-e^{u\rho_a c_a H/\lambda}}\right) \qquad (3.18)$$

Figure 3.5 shows the total energy efficiency $e_{dyn,tot}$ the relative energy saving η and the energy loss through conduction and ventilation per square meter $(P_{dyn}-P_0)/A_d$ in the ceiling in the OPTIMA as a function of the fraction f.

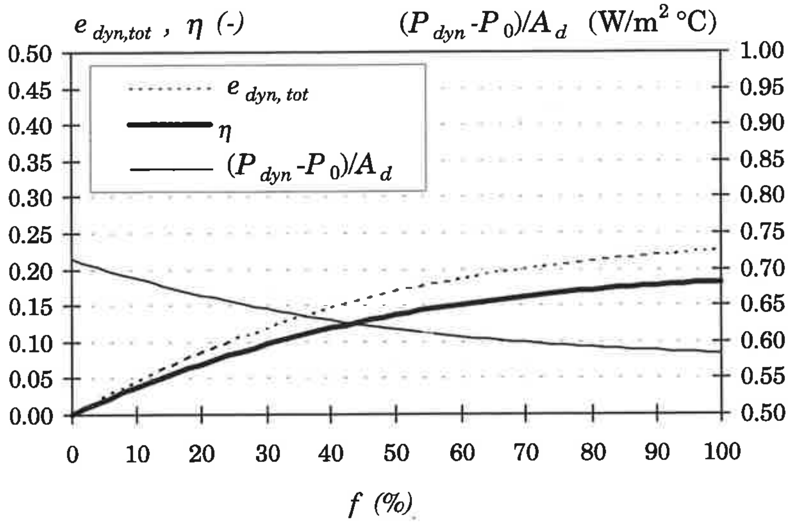

Figure 3.5. *The total energy efficiency $e_{dyn,tot}$ the relative energy saving η and the energy loss through conduction and ventilation per square meter $(P_{dyn}-P_0)/A_d$ in the ceiling in the OPTIMA house as a function of the fraction f of the inlet air that passes through the insulation. The calculations are based on H=0.3m, a total air flow Q= 53 l/s and an dynamic area A_d=116 m^2.*

From figure 3.5 it is clear that the maximal relative energy saving η for the OPTIMA house is about 18% or $e_{dyn,tot}$=23%. If 40% of the inlet air passes through the insulation the numbers are η=12% and $e_{dyn,tot}$=15%. The energy loss through conduction and convection per square meter $(P_{dyn}-P_0)/A_d$ decreases from 0.71 to 0.58 W/m^2°C when f incerases from 0 to 100%. According to Elmroth and Fredlund (1993), the heat losses in the normal walls amount to 82 W/°C in the OPTIMA house. If f is increased from 0 to 100 %, i.e. a "perfect" house, the total energy consumption for the house decreases from 82+116×0.714≈165 to 82+116×0.580≈150 W/°C, i.e. by 9 %.

4 The OPTIMA House

The OPTIMA house was a single storey one family house with a living area of 116 m^2. The house is described in detail in Elmroth and Fredlund (1993). Measurements have been carried out over approximately a year and a half. The house was unoccupied during this period.

In the OPTIMA house the ambient air passed from the outside into the attic through 0.3 m of loose fill insulation and into a small air space over a gypsum board. The air passed into the living space through five inlet terminals in the gypsum board. The exhaust air passed to a central fan through five air outlets. The ventilation rate was set to 60 l/s or 0.75 ach. The house had a negative pressure of approximately 12 Pa compared with outside. Figure 4.1 shows the paths that the air was supposed to take in the house.

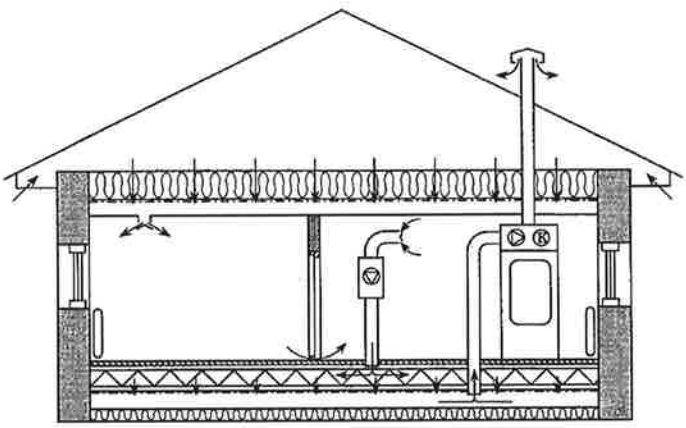

Figure 4.1 The design flow paths for the air in the OPTIMA house.

The U values for the different building components were:

Table 4.1 Design U values , areas of building components and fabric losses.

Building component	U value (W/m^2°C)	Area (m^2)	Fabric loss (W/°C)
Roof	(0.05)	116.0	(5.8)
Walls	0.30	99.0	29.7
Floor	0.14	116.0	16.2
Windows	1.80	17.2	31.0
Doors	1.00	5.0	5.0
		353.2	(87.7)

The design U value for the roof is the 'dynamic U value' which incorporates the energy savings from the ventilation air. The normal U values of different parts of the roof are set out in table 4.2:

Table 4.2 The U values of the different building components in the roof and ceiling.

Building component	U value (W/m^2°C)
Ceiling+insulation	0.163
Ceiling+insulation+joists	0.177
Ceiling+insulation+roof	0.158
Ceiling+insulation+roof+joists	0.17

5 The Gradient Method

The important question was whether the air passed through the insulation or through leakage paths in the rest of the house. Three different methods of measuring the air flow through the insulation were used in the OPTIMA house:

1. Measurement of the temperature gradient inside the insulation.
2. Tracer gas technique.
3. Direct measurements of air flow in the inlet terminals.

In the study presented here the first method was used. This is here called the *gradient method.* Comparisons with the other two methods will be presented.

The gradient method is an indirect method. The goal is to find the air flow for which the theoretically calculated temperature distribution fits the measured temperature distribution as closely as possible. The least squares method is used to determine this fit. The main difficulty in the gradient method is to calculate the temperature distribution for a given air flow and to minimize the least-squares error function. The temperature disribution can stem from either a steady state or transient calculation. Both techniques were used in the OPTIMA house.

5.1 Experimental set-up

The temperature distribution in the OPTIMA house was found by placing five thermocouples in a vertical column inside the insulation. The thermocouples were positioned 0.3 m from the nearest ceiling joist. The vertical distance between the thermocouples was 0.05 m. The temperatures were measured every third minute and integrated over one hour. Only hourly integrated values were thus available. Four such columns were placed in the insulation. They were evenly distributed over the area and placed so as not to be disturbed by installations in the attic. For column 1 to column 4, the distances to the nearest air inlet were 2.4 m, 3.4 m, 6.5 m and 2.7 m respectively.

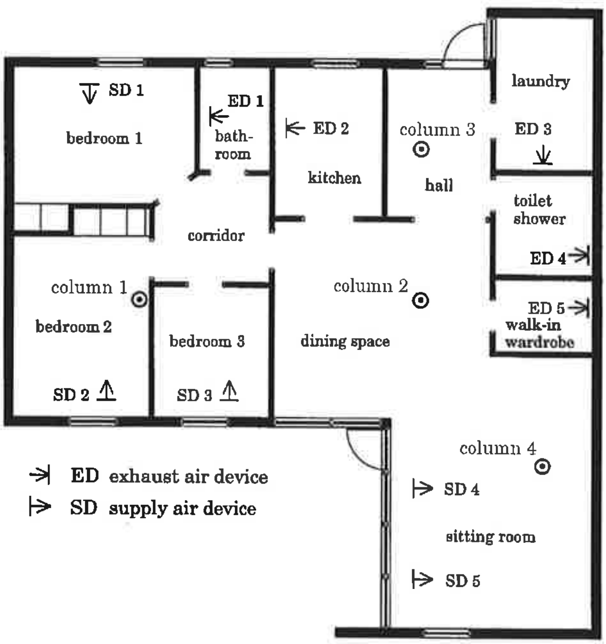

Figure 5.1 The positions of the inlet terminals and the columns with thermocouples.

5.2 Steady-state calculation

Steady state, i.e. time independent, conditions were assumed. The solution to the steady state heat transfer equation in one dimension (2.1) is (2.7). The temperatures in the insulation were measured at five points: $T_1^{\Delta t}$, $T_2^{\Delta t}$ ...$T_5^{\Delta t}$ with the x coordinates $x_1, x_2 \, . . \, x_5$. The temperatures were hourly values or an average over up to 24 hours. The average time is denoted by Δt. The temperatures $T_1^{\Delta t}$ and $T_5^{\Delta t}$ were used as boundary conditions for the analytic solution.

$$T_0 = T_1^{\Delta t} \qquad T_L = T_5^{\Delta t} \qquad x_1 = 0 \qquad x_5 = L \tag{5.1}$$

With $T(x,u)=T(x)$ for a given u, the nonlinear least squares error function $R^2(u)$ can now be constructed:

$$R^2(u) = \sum_{i=2}^{4} \left(T(x_i,u) - T_i\right)^2 \tag{5.2}$$

The problem is now to minimise $R^2(u)$ with respect to u. To find the minimum of $R^2(u)$ the Brent's method as written in Press et al (1986) is used. Brent's method is a mix between golden section search and parabolic interpolation. The value of u where this minimum occurs is called u_e and is thus a nonlinear least squares estimate of the true air flow. An approximate value of the standard deviation s of the temperatures $T(x_i,u_e)$-T_i is:

$$s = \sqrt{R^2(u_e)/2} \tag{5.3}$$

The unit for s is (°C). A low value of s indicates that the model (2.7) is physically realistic. In order to make the calculation as accurate as possible, the value of thermal conductivity according to Johansson and Löfsröm (1992) was used:

$$\lambda = 0.5/(13.02-0.082\, T) \tag{5.4}$$

where T is the temperature in °C.

5.3 Transient calculation

To solve equation (2.5) a finite difference model was used. The insulation was divided into a series of mesh nodes, typically 9 nodes. The temperature was calculated at these nodes at specific times. Each node was separated by the distance δx. The step in time was δt. The finite difference model used here was the MacCormack method, (Anderson 1984):

$$\text{Predictor: } \overline{T}_j^{n+1} = T_j^n - av\frac{\delta t}{\delta x}\left(T_{j+1}^n - T_j^n\right) + a\frac{\delta t}{\delta x^2}\left(T_{j+1}^n - 2T_j^n + T_{j-1}^n\right) \tag{5.5}$$

$$\text{Corrector: } T_j^{n+1} = \frac{1}{2}\left[T_j^n + \overline{T}_j^{n+1} - av\frac{\delta t}{\delta x}\left(\overline{T}_j^{n+1} - \overline{T}_{j-1}^{n+1}\right) + a\frac{\delta t}{\delta x^2}\left(\overline{T}_{j+1}^{n+1} - 2\overline{T}_j^{n+1} + \overline{T}_{j-1}^{n+1}\right)\right] \tag{5.6}$$

The temperature at time t, starting with given initial and boundary conditions, can thus be found by taking small steps in time. All the results presented here were calculated with 9 nodes. The measured temperatures were hourly integrated values. The initial condition $T^0(x)$ and boundary conditions $T_0(t)$ and $T_L(t)$ were therefore not known at specific times. The method used here was to guess an initial temperature distribution $T^0(x)$, use the integrated temperatures $T_1^{\Delta t}$ and $T_5^{\Delta t}$ as boundary conditions over one hour and then calculate temperatures with (5.5-6) up to the next averaging time step Δt. The calculated temperature at the nodes where the temperature was measured was integrated over Δt:

$$\hat{T}\left(x_j\right) = \frac{1}{\Delta t}\sum_{t_n=t_0}^{t_0+\Delta t} T_j^n \delta t \tag{5.7}$$

With $\hat{T}\left(x_j, u\right) = \hat{T}\left(x_j\right)$ for a given u, the least squares error function $R^2(u)$ is constructed:

$$R^2(u) = \sum_{i=2}^{4} \left(\hat{T}(x_i, u) - T_i \right)^2 \tag{5.8}$$

The function $R^2(u)$ is minimised as before with respect to u to find the least squares estimate u_e of the air flow. The temperature distribution for u_e is used as initial condition for the next calculation.

6 Verification of the Gradient Method

The calculation was verified with the analytical solution described in section 2.3 and an accurate numerical solution using 100 nodes instead of 9 which was used in the normal calculations. With the analytical and the accurate numerical solutions, files with data organised as the measured data were produced, i.e. hourly values at five locations in the loose fill insulation. The air flow was constant at 0.2 mm/s but the temperature at the upper surface varied sinusoidally with the amplitude 5°C. When the model with 9 nodes calculated the air flow from these files the agreement was very good. Figure 6.1 shows that the error after 12h was only 0.002 mm/s or 1%.

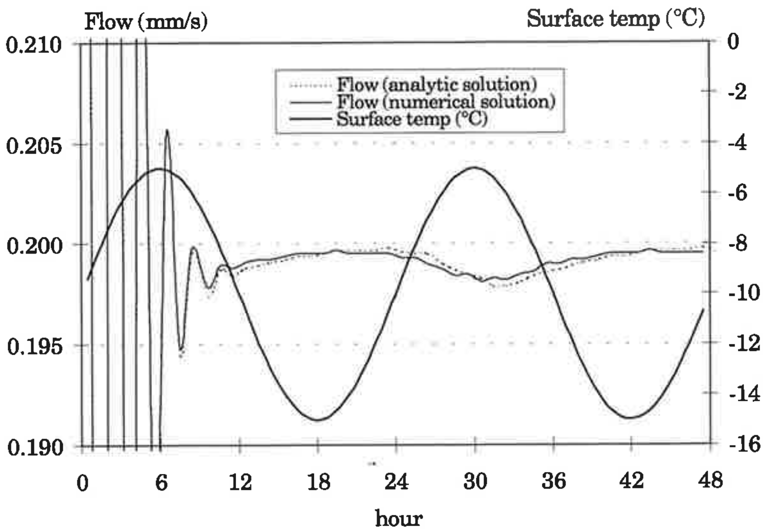

Figure 6.1 *The 2h average u_e based on the numerical and analytical solutions and the surface temperatures used for these cases. The air flow used in the simulations was 0.200 mm / s.*

The agreement between the analytical solution and the accurate numerical model was excellent. The accurate numerical model was then used to test more complicated situations with varying air flows and temperatures. The agreement was good, even for complicated patterns of temperatures and air flows. The absolute error in air flow was less than 0.025 mm/s with an average of 0.005 mm/s. The estimated standard deviation s was less than 0.05°C. The results when the difference between T_1 and T_5 was less than 4 degrees were not used due to an increase in the error as the temperature difference decreased.

The steady state method was used only for 24h averaged values. The transient method was used for 2h and 6h averaged values.

7 Results

The exhaust ventilation during the measurement period was approximately 60 l/s. If all air passed through the insulation the air flow would be $0.5 \cdot 10^{-3}$ m^3/m^2 s = 0.5 mm/s. The material parameters are described in table 7.1.

Table 7.1 Physical parameters for the OPTIMA house.

λ = 0.5/(13.02-0.082 T) W/m°C
c_a = 1005 J/kg°C
c_i = 1000 J/kg°C
ρ_a = 1.27 kg/m^3
ρ_i = 19 kg/m^3

These values are valid at 5°C except for λ which varied with temperature according to (5.4). The temperature gradient was measured with four columns of thermocouples.

7.1 Air flow 24h average

Figure 7.1 shows the average air flow u_e and the ambient temperature over the period 930224-930420. The air flow was calculated with the steady state method using the 24 hour average (Δt=24h). Figure 7.2 shows the average air flow u_e and the ambient temperature over the period 930828-931106. The average standard deviation s was in all cases less than 0.3°C.

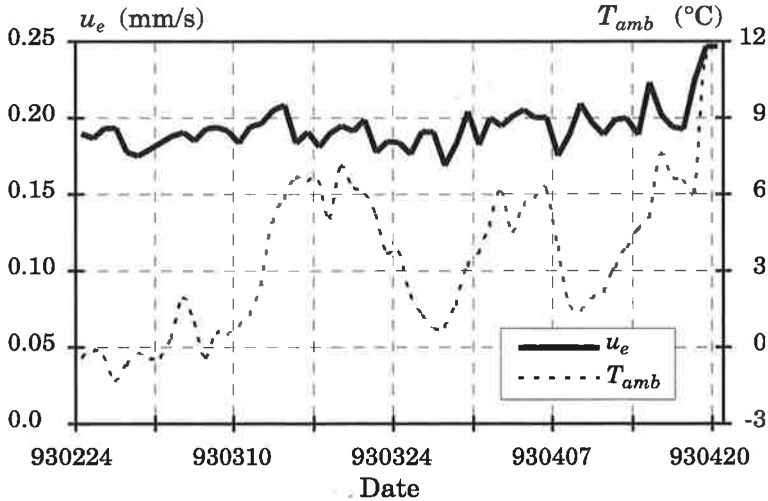

Figure 7.1 *The 24h average u_e and the ambient temperature for the period 930224-930420.*

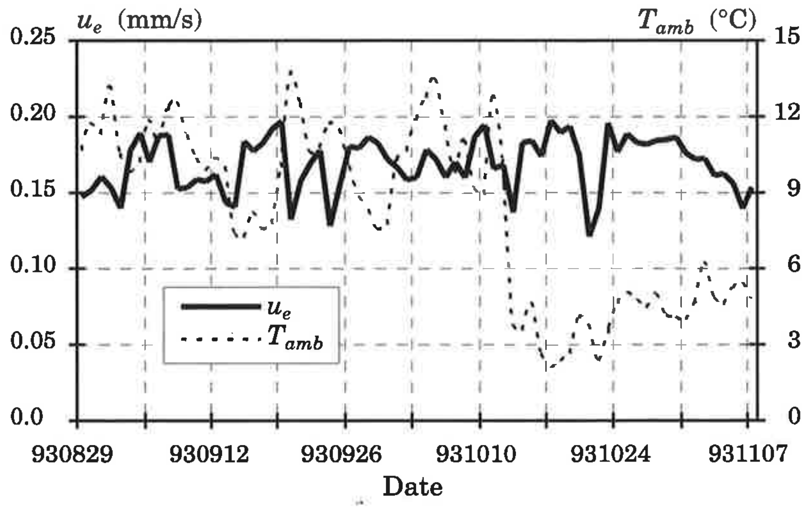

Figure 7.2 *The 24h average u_e and the ambient temperature for the period 930828-931106.*

The average u_e was 0.19 mm/s for the period 930224-930420 and 0.17 mm/s for the period 930828-931106. This represents a total airflow of 22 l/s and 19.7 l/s respectively. Measurements of the outlet air show a slight decrease over the year: from 62 l/s during the spring to 59 l/s during the autumn.

7.2 Tracer gas, direct and gradient methods

Direct measurements showed that the air flow to the air outlets was about 58 l/s. Measurement over an orifice plate in the duct showed that the exhaust air should be 63 l/s. Direct measurements showed that 27 l/s came in through the inlet terminals. The airtightness of the house was 1.7 ach at 50 Pa negative pressure.

During the period 930123-930206 tracer gas experiments were performed by Björn Hedin (1994). The inlet terminals were either modified to be as open as possible or were in the normal position. The results from the tracer gas, direct and gradient measurements are summarised in table 7.2 and figure 7.3. The flow in l/s is calculated by multiplying the air flow in mm/s by the area 116 m^2. This is probably an overestimation of the air flow since the joists decrease the insulated area. The total exhaust air given by the tracer gas experiment was 53 l/s. This takes into account the fact that part of the air that reached the air outlets was recirculated into the living space.

Table 7.2 Results from the different measurement techniques. The flow is expressed as l/s, mm/s and the percentage of the total exhaust air (53 l/s).

l/s (mm/s) (%)	Tracer gas	Direct	Gradient
Normal	33 (0.28) (62%)	27 (0.23) (51%)	21 (0.18) (40%)
Open	45 (0.39) (85%)	39 (0.34) (74%)	26 (0.22) (49%)

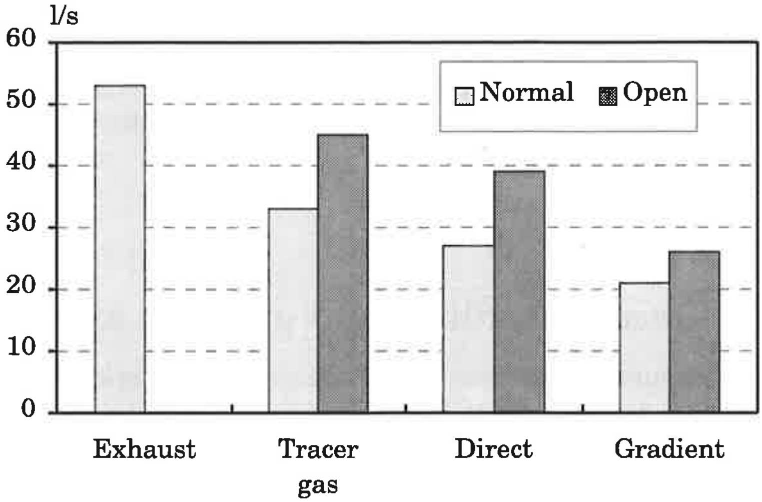

Figure 7.3 *The air flow through the insulation as measured by the three different methods for the case with normal and open inlet terminals..*

If the average of u_e over the four columns is a true average over the roof, 40% of the exhaust air actually passed through the dynamic insulation. Further, of the air that reached the inlet terminals about 80% had passed through the insulation. Roots (1994) measured the thermal behaviour of a dynamic insulation of the OPTIMA type but built as an external wall. His measurements were performed in a laboratory. The conclusion from Roots was that about 50% of the air passed through the dynamic insulation when the "inside" space had a negative pressure compared with the "ambient" space. When the air was extracted directly from the inlet terminals, 73% of the air passed through the dynamic insulation. The results from Roots are thus quite similar to the results as measured in the OPTIMA house. Figure 7.4 shows u_e calculated for the period when the tracer gas experiments were performed. One can clearly see when the terminals were changed from normal (N) to open (O) position.

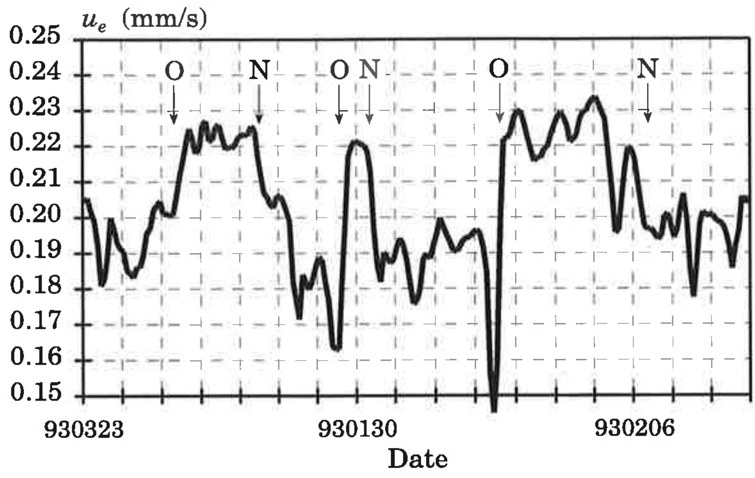

Figure 7.4 *The 6h average u_e during the tracer gas experiment. The times when the terminals were open (O) or normal (N) are marked in the figure.*

7.3 Dynamic U value and dynamic energy efficiency.

The dynamic U value incorporates the energy savings of the ventilation air. The normal U value for different parts of the roof is set out in table 7.3:

Table 7.3 U values of the different building components in the roof and ceiling. The correction is specified in the Swedish Building Code (1994)

Building component	U-value (W/m^2°C)	Correction (W/m^2°C)	Total U value (W/m^2°C)	Dynamic U-value
Ceiling + insulation	0.134	0.03	0.163	$U_{dyn,attic}$
Ceiling + insulation + joists	0.147	0.03	0.177	
Ceiling + insulation + roof	0.128	0.03	0.158	$U_{dyn,amb}$
Ceiling + insulation + roof + joists	0.140	0.03	0.17	

The U value is increased when the heat losses through the ceiling joists are included. The U value for ceiling+insulation corresponds to $U_{dyn,att}$ which is the dynamic U value when T_{out} in equation (3.5) is the attic temperature T_{attic}. The U value for ceiling+insulation+roof corresponds to $U_{dyn,amb}$ which is the dynamic U value when T_{out} is the ambient temperature T_{amb}.The average dynamic U values $U_{dyn,amb}$ and $U_{dyn,attic}$ for the four columns for the periods 930224-930420 and 930828-931106 are shown in figures 7.5-6. The dynamic U value $U_{dyn,amb}$ decreases with increasing insolation since the attic functions as a solar collector. The realistic average U value for the whole roof is higher than $U_{dyn,amb}$ due to the presence of the ceiling joists. The theoretical U_{dyn} from equation (3.7) is $U_{dyn}(u$=0.19mm/s)=0.052 and $U_{dyn}(u$=0.17mm/s)=0.059 W/m^{2}K for the air flow that was calculated for the spring and autumn respectively.

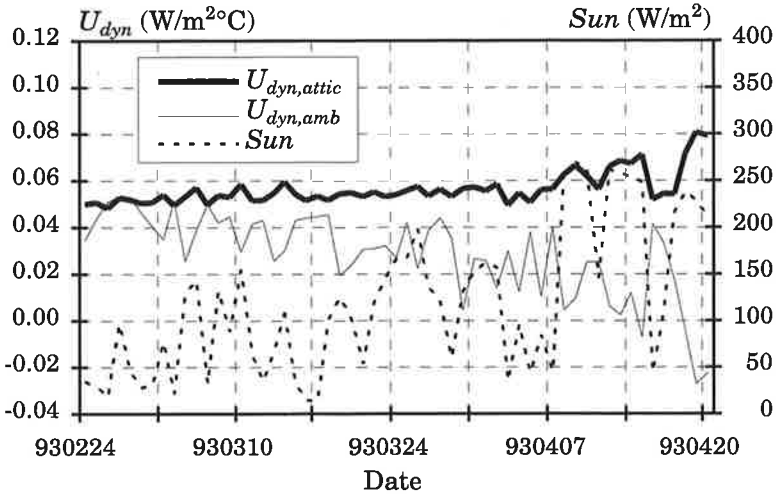

Figure 7.5 *The 24h average $U_{dyn,attic}$, $U_{dyn,amb}$ and the global sun for the period 930224-930420.*

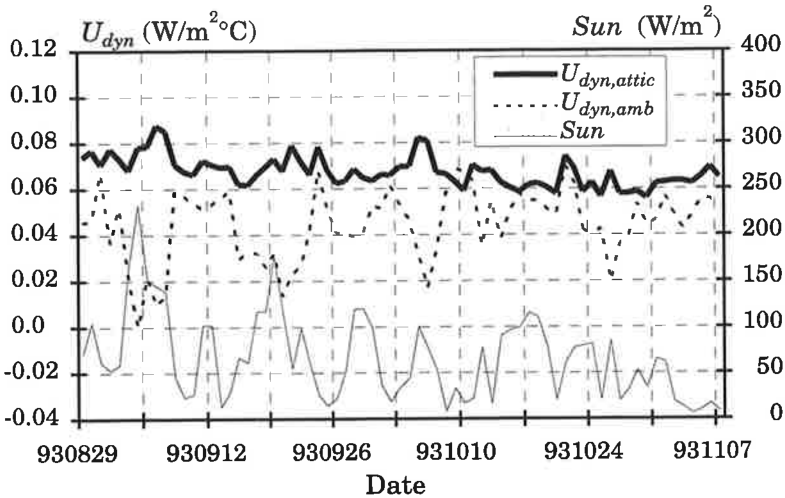

Figure 7.6 *The 24h average $U_{dyn,attic}$, $U_{dyn,amb}$ and the global sun for the period 930828-931106.*

The average dynamic energy efficiency e_{dyn} for the four columns, for the periods 930224-930420 and 930828-931106, is shown in figures 7.7-8. For a dynamic insulation without sunshine exposure the energy efficiency can never be above 0.5. The value 0.5 is only reached for air flows close to zero. The reason that e_{dyn} was as good as 0.35 when there was no sun is that the air flow in the OPTIMA house was extremely low. Obviously the roof acted as a solar collector when $e_{dyn,amb}$ was above 0.5. Note that this efficiency only affects the air that actually passes the insulation. If the previous calculation that about 40% of the total inlet air passes the insulation is true, the total energy efficiency should be $e_{dyn,tot} = e_{dyn,amb} \cdot 0.4$ or $e_{dyn,tot} =$ 16% for the spring period and 17% for the autumn period..

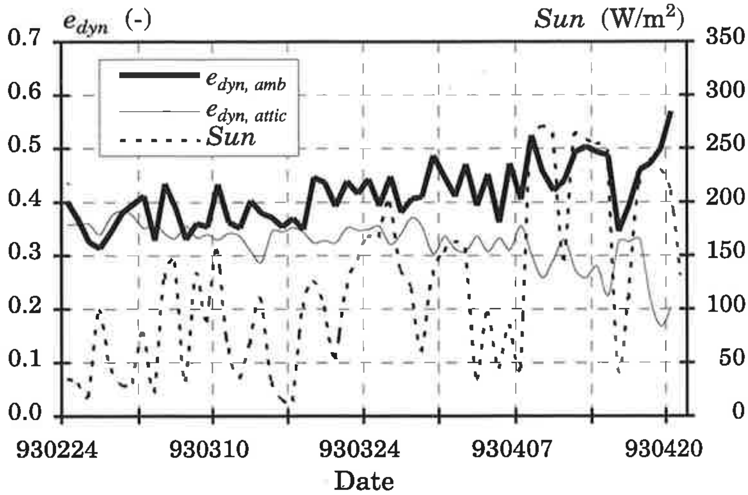

Figure 7.7 *The 24h average $e_{dyn,amb}$, $e_{dyn,attic}$ and the global sun for the period 930224-930420.*

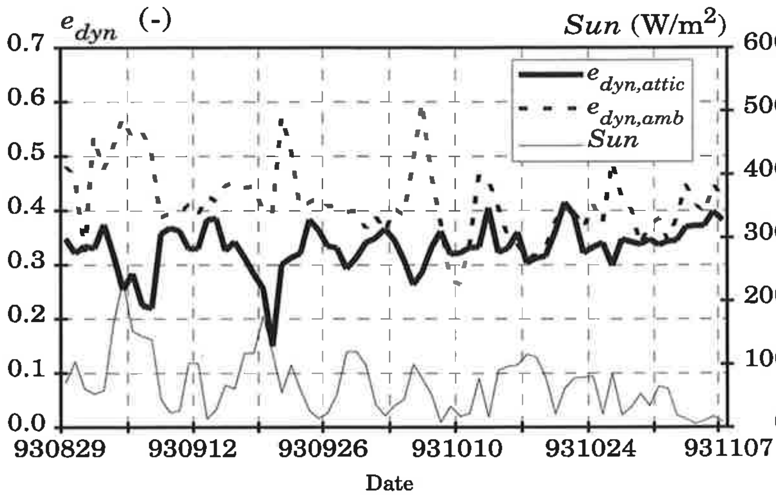

Figure 7.8 *The 24h average $e_{dyn,attic}$, $e_{dyn,amb}$ and the global sun for the period 930828-931106.*

7.4 Air flow 2h average

Figure 7.9 shows the 2h average air flow for the four columns for the period 930301-930311. The four columns gave similar results except column 1 which sometimes gave values up to 50 % higher. The measurements showed a daily variation of the air flow that increased with increasing sunshine. This variation was typically ± 30% of the average flow with maximum at midnight and minimum at noon, see figures 7.9-10. The outside temperature was not identified as responsible for this variation since the minimum of the temperature came before the maximum of flow. If the outside temperature had been responsible for the variation in the air flow this would have been a non causal behaviour. The only parameter that was in phase with this variation was the global sun. No other measurement has confirmed that this variation actually occurs. However, due to the positive verification of the model, obvious errors such as bugs in the program code can most certainly be excluded.

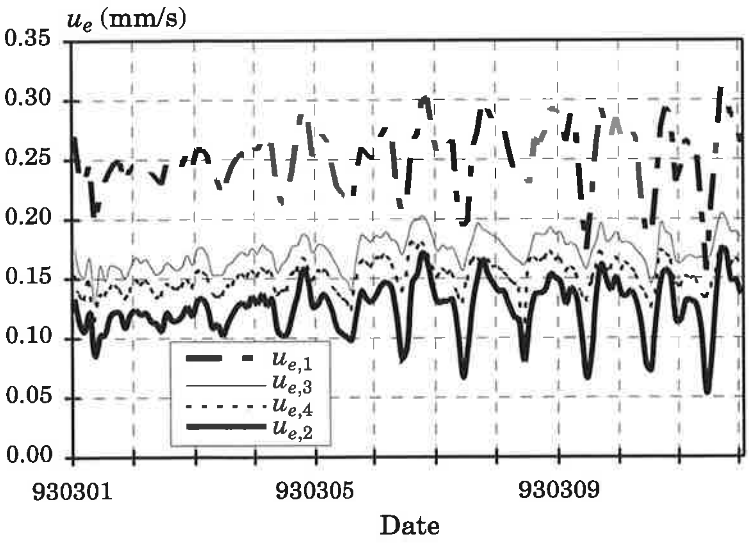

Figure 7.9 *The 2h average $u_{e,1}$ -u_{e4} for the period 930301-930311.*

For columns 1 to 4, the distances to the nearest air inlet were: 2.4m, 3.4m, 6.5m and 2.7m. It might perhaps be expected that columns 1 and 4 would have the highest flow and column 3 the lowest. This is not the case. Column 1 does give the highest flow, but the other three do not conform to this theory. It is not possible at this moment to explain why column 1 gave such a result. Figure 7.10 shows both the 2h and 6h average values for u_e.

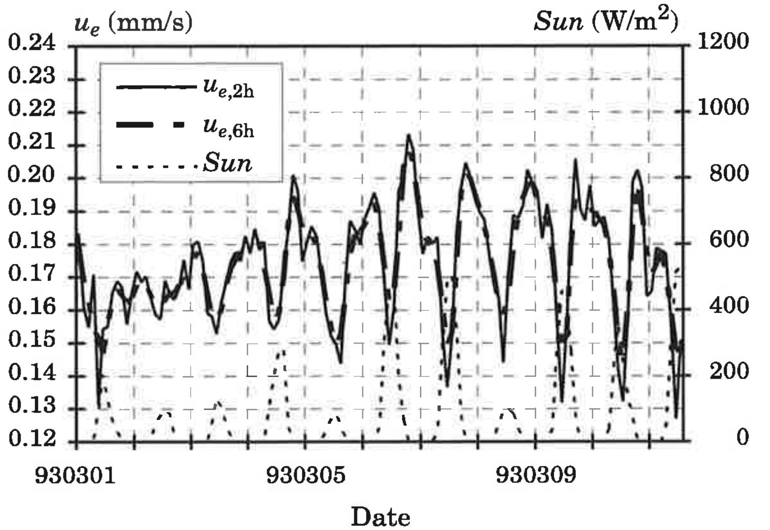

Figure 7.10 *The 2h and 6h average u_e, and the global sun for the period 930301-930311.*

7.5 The dependence of u_e on other parameters.

The calculated 24h average air flow increased slightly with increasing difference between inside and outside temperature. Both the 24h and 2h average air flow decreased with increasing wind speed. No obvious effect of wind direction was found.

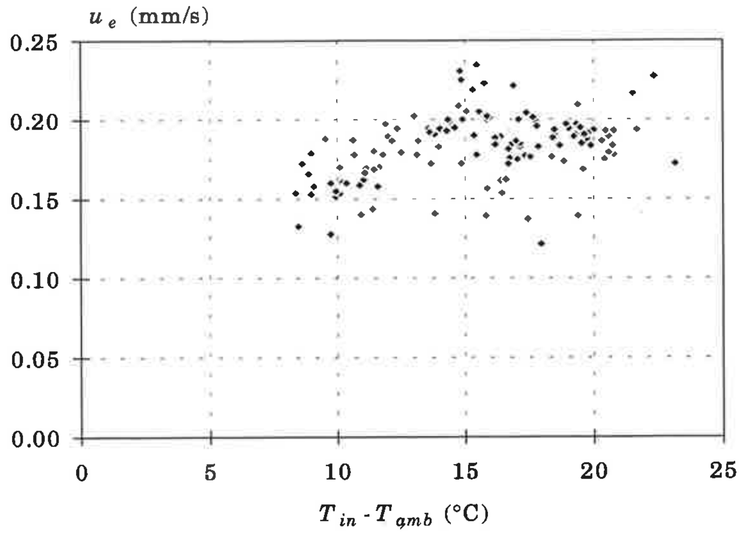

Figure 7.11 *The 24h average u_e versus (T_{in}-T_{amb}) for the period 930224-930420 and 930828-931106.*

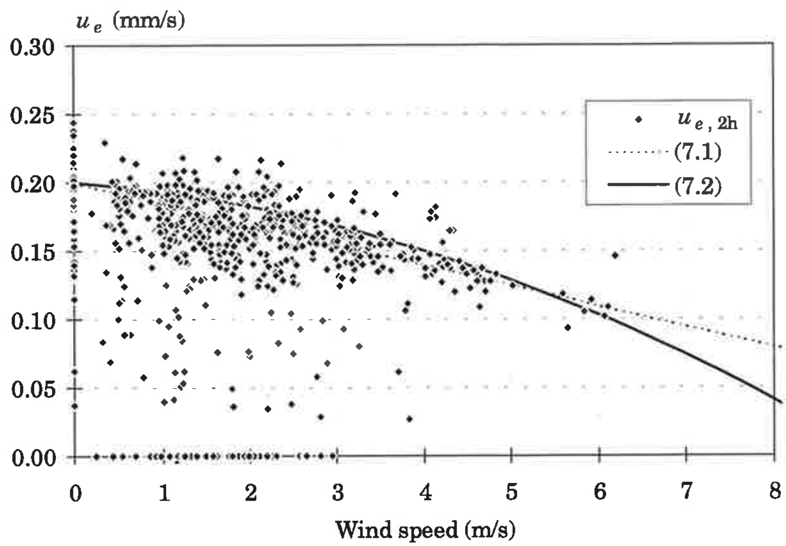

Figure 7.12 *The 2h average u_e versus wind speed for the period 930828-931106. The results from equation (7.1) and (7.2) are also included.*

From figure 7.12 it is clear that the 2h average air flow decreased slightly with increasing wind speed. A rough estimate is that the average air flow depends on the wind according to

$$u_e = 0.2(1\text{-}0.075\text{Wind}) \tag{7.1}$$

with the wind expressed as (m/s). Söderberg and Fagerstedt (1984) made detailed theoretical calculations of the sensitivity to wind speed. From Hedin (1994) most of the parameters used by Söderberg and Fagerstedt can be estimated. An approximation of the equations of Söderberg and Fagerstedt for a house similar to the OPTIMA house is thus:

$$u_e = 0.2(1\text{-}0.024\text{Wind-}\ 0.0097\text{Wind}^2) \tag{7.2}$$

It is not possible to verify equations (7.1) or (7.2) from figure 7.12. Both equations give reasonable results for the measured wind speeds.

7.6 Test with ventilation turned off.

During the period 930221-930223 the ventilation was turned off and the inlet terminals were closed so that no air could pass through. Figure 7.13 shows u_e calculated for this period. The air flow was reversed: from 0.2 mm/s downwards to 0.1 mm/s upwards. The total air flow thus decreased from 23 l/s downwards to 10 l/s upwards. This could be a dangerous situation if the air moving upwards came from the living space. The air moving upwards could then create condensation in the insulation. If the air came from outside, this would only lead to a higher energy loss. It is for the moment not known where the air came from.

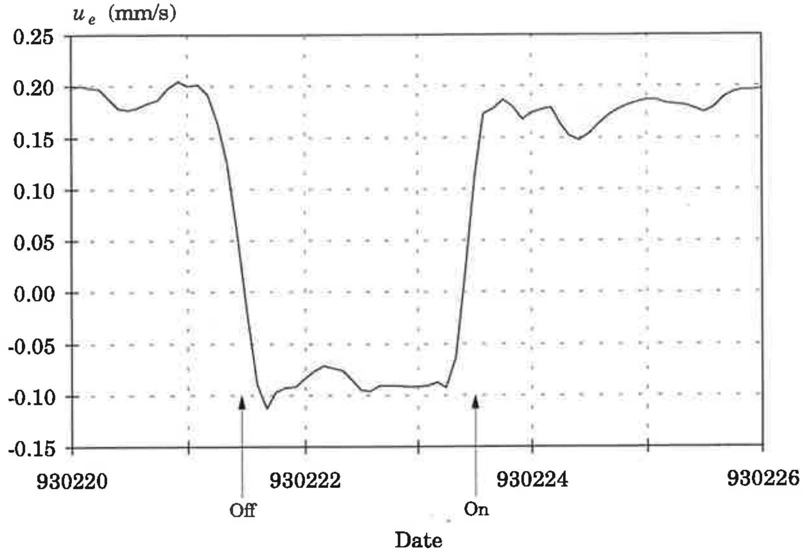

Figure 7.13 *The 6h average u_e during the period with the ventilation turned off.*

7.7 Test with the inlet terminals removed

During the period 931109-931123 the inlet terminals were removed. Figure 7.14 shows u_e calculated for this period. The air flow increased from 0.16 to 0.23 mm/s. This means an increase in the total air flow from 19 to 27 l/s, i.e. 8 l/s. This should be compared with the increase in the total air flow when the inlet terminals were modified to be as open as possible, which only resulted in an increase of 5 l/s. When the terminals were completely removed the pressure drop over the terminals decreased from 12 to 1 Pa. The negative pressure in the house gradually decreased from 13 to 4 Pa. When the inlet terminals were removed the air flow was much more sensitive to wind speed and other disturbances. On day 931115 and during the period 931119-931122, the wind was somewhat stronger and also changed direction many times. Research personnel also visited the house during this period.

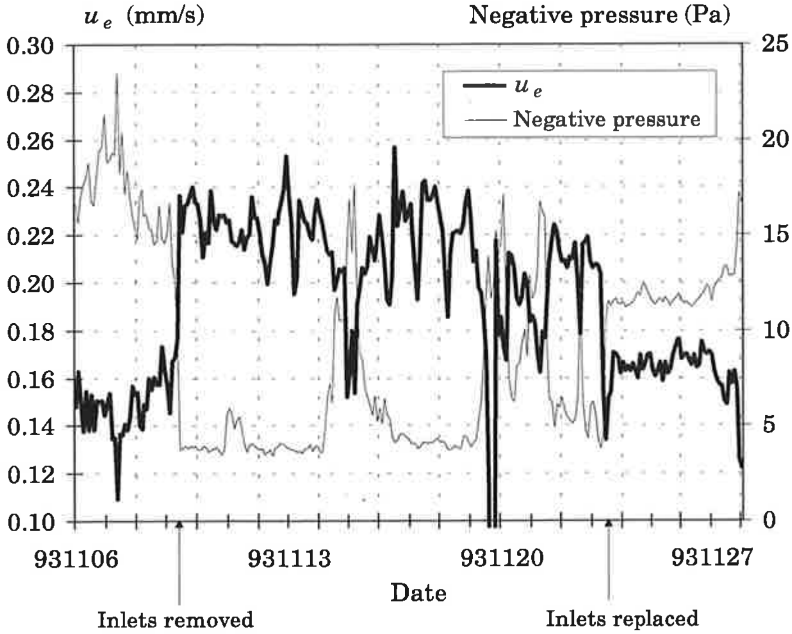

Figure 7.14 The 2h average u_e during the period with inlet terminals removed.

7.8 Inlet air temperature

Figures 7.15,16 show the 24h average inlet temperature for the five inlet terminals for the whole year 1993. During the period 930221-930223 the ventilation system was turned off, see figure 7.12. It is clear from these figures that the minimum inlet temperature was reached during the first days in January and the the maximum inlet temperature was reached in May. The inlet temperature was about two degrees lower than the inside temperature for most of the heating season. Due to the dynamic insulation, the inlet temperature is rather high during the summer. Figure 7.17 shows the hourly average temperature in the house (T_{in}), the ambient temperature (T_{amb}), the minimum and maximum temperature in the inlet terminals for the period 930101-930110. The minima and maxima are taken over the five inlet terminals. Figure 7.18 shows the same thing for the period 930515-930525. The minimum inlet temperature during the winter was about 16°C. This occurred when the ambient temperature was -7°C. The maximum inlet temperature during the summer was about 29°C. This occurred when the ambient temperature was 27°C.

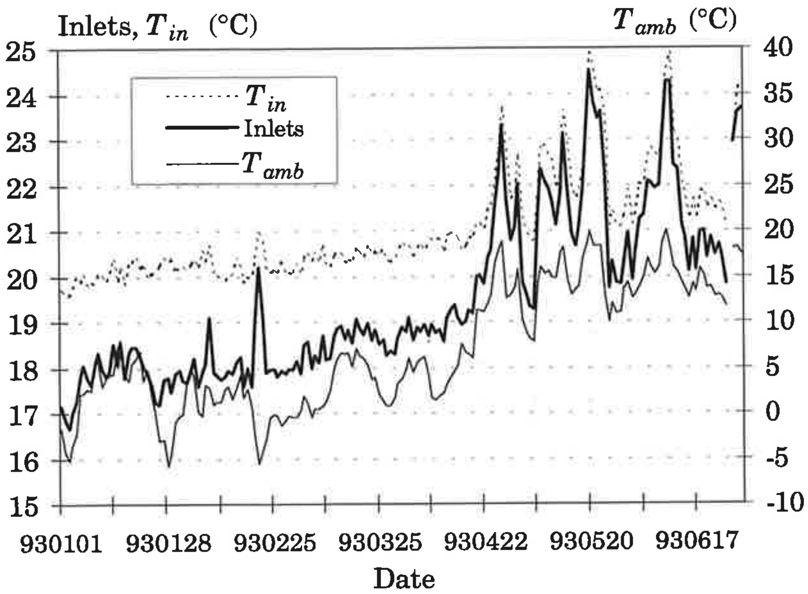

Figure 7.15 The 24h average inlet air temperature, The ambient temperature and the inside temperature at the four inlets for the period 930101-930701.

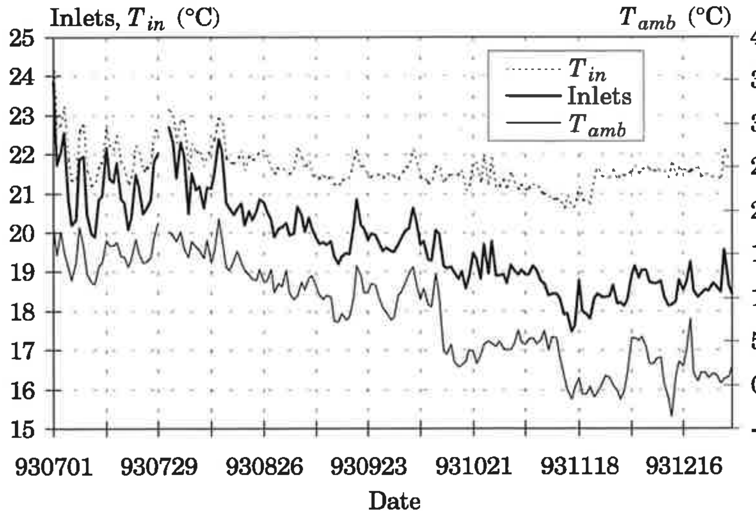

Figure 7.16 *The 24h average inlet air temperature, The ambient temperature and the inside temperature at the four inlets for the period 930701-931212.*

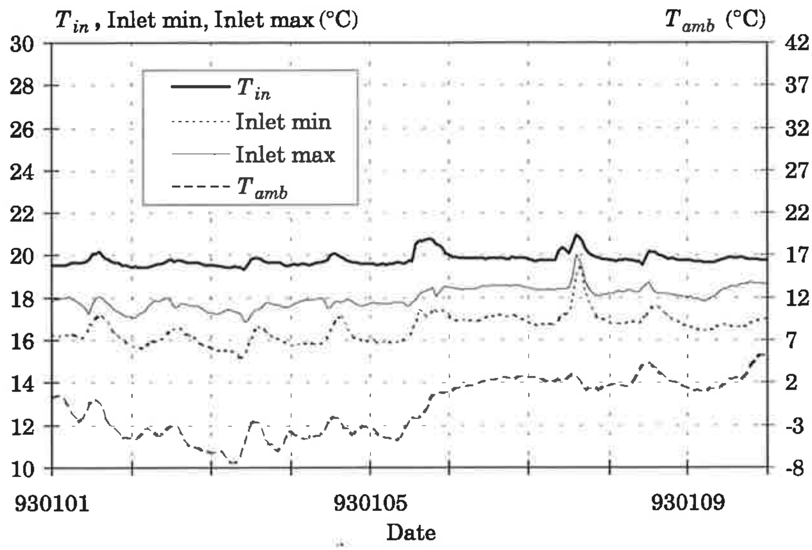

Figure 7.17 *The hourly average inlet air temperature, The ambient temperature and the inside temperature at the four inlets for the period 930101-930110.*

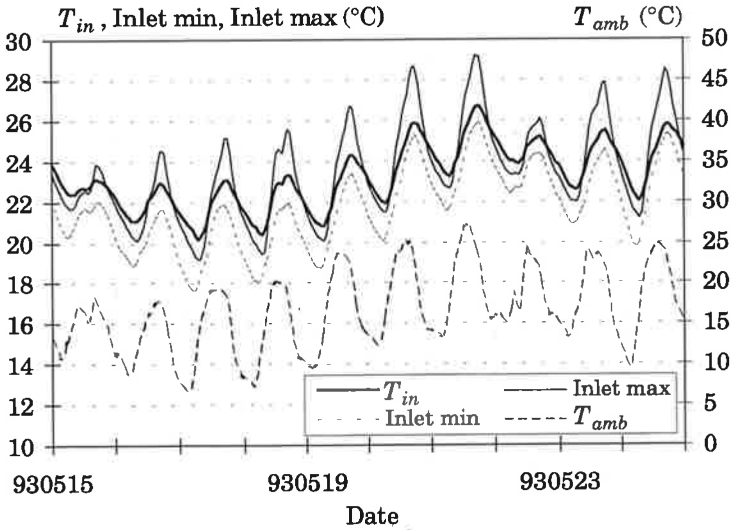

Figure 7.18 The hourly average inlet air temperature, The ambient temperature and the inside temperature at the four inlets for the period 930515-930525.

Figure 7.19 shows the temperature in the inlet terminals for the period 930301-930311. Note the peaks when there are more sunshine, see figure 7.10. It is noticeable that the temperature at inlet 1 does not follow the increase of the other four inlets when the sun shines. This could indicate that the air that reaches inlet 1 does not pass through the dynamic insulation but through leaks in the external walls.

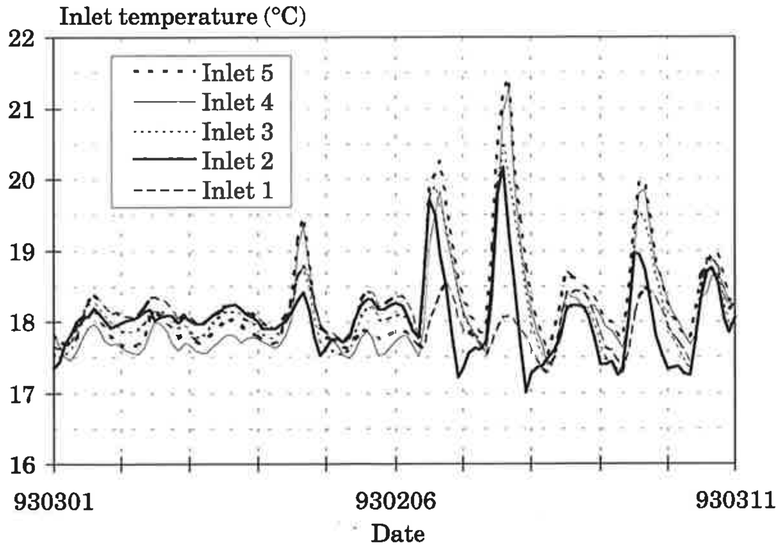

Figure 7.19 The 2h average inlet air temperature at the five inlets for the period 930301-930311.

8 Discussion

8.1 Sensitivity analysis

The mathematical model has been validated against an analytical and an accurate numerical simulation. Possible errors are to be found in the estimation of the physical parameters and in the model itself. The most sensitive parameters are λ, ρ_a and L. A 10% change in these parameters changes the air flow by about 10%. The direct dependence on ρ_i is zero for the steady state calculation and very small in the transient calculation. The heat capacity of air ρ_a varies only slowly with the temperature. When the temperature increases 10°C ρ_a decreases 4%. The error in the measurement of L is estimated as less than 4%. The temperature dependence of λ is accounted for by using an estimate from Johansson and Löfström (1992). They also concluded that λ decreases 5% when ρ_i increases from 18 kg/m^3 to 20 kg/m^3. The error in the temperature measurement was 0.1 K, with the thermocouples taken from the same series. This results in an error in the air flow of 3% for the transient calculation and 1% for the steady state calculation. If one assumes that the errors are uncorrelated a linear estimate of the accuracy of the steady state air flow calculation will be 8% or ±0.015 mm/s, i.e. an accuracy of the U value calculation of ±0.006 W/m^{2}K. The transient calculation will have a slightly higher error. The calculation of successive air flow variations will however be much more accurate, a relative error < 4%. There is some indication that the density of the insulation was less than 19 kg/m^3. This would introduce a systematic underestimation of the air flow. The dependence of λ on ρ_i indicates that this underestimation of the air flow would be of the order of 5%.

8.2 Comparison with other methods

The total inlet flow was approximately 53 l/s according to the tracer gas experiments. The air flow through the dynamic insulation as measured by the tracer gas, direct and gradient method was 33, 27 and 21 l/s respectively. This corresponds to 62%, 51% and 40% of the total inlet air. The methods do not measure exactly the same air flow. The tracer gas method measures all the air that passes from the attic to the living space regardless of which path the air took. If the air space over the gypsum board does not have a large leakage to the ambient air the tracer gas measurement should give the largest air flow of the three methods. The direct measurement over the inlet terminals does not show whether the air that passes through the air inlet has actually passed through the insulation. With the assumption of no large leakage this should give a lower value than the tracer gas and a higher value than the gradient method.

The gradient method is the only method that directly measures the air flow which passes through the insulation and should therefore give the smallest air flow. The problem with the gradient method is that the air flow is measured only at four specific locations in the whole attic. We do not know how representative this is for the average air flow. It is however reasonable to assume that this is the highest local air flow in the insulation since the thermocouple columns were placed in undisturbed loose fill insulation. The presence of ceiling joists and ventilation ducts introduces possible leakage in the insulation which will shortcircuit the air flow. Such shortcircuits would reduce the air flow that passes through the insulation. In the calculations of the total air flow the whole ceiling area of 116 m^2 has been used. The ceiling joists will in fact reduce the effective area used as dynamic insulation. In view of all this it is highly probable that the true air flow is lower than the calculated air flow.

8.3 How to increase the air flow

With the calibration curve for the air inlet it was possible to calculate the flow based on the estimated pressure drop over the air inlet. The measured negative pressure in the house was between 12 and 18 Pa. The larger part (90%) of the pressure drop occurred over the air inlet. This corresponded to an air flow through the inlet terminals ranging from 19 to 35 l/s. To make 50-60 l/s pass through the inlet terminals would require a negative pressure in the building of 30 Pa! There was therefore reason to believe that the inlet terminals were badly designed. This conclusion led to experiments with modified and removed inlet terminals to test how an air inlet with a smaller pressure drop would have performed.

When the inlet terminals were modified to be as open as possible the percentage of the total inlet air passing the insulation increased to 85% (12 l/s), 74%(12 l/s) and 49% (5 l/s) as measured by the tracer gas, direct and gradient method. The numbers in parentheses are the increase measured in l/s. When the inlet terminals were completely removed the air flow increased by 8 l/s according to the gradient method. This showed that 55% of the inlet air passed through the insulation, (with the assumption that the total inlet flow dropped to 49 l/s during the autumn). Unfortunately, direct measurements could not be made with removed inlet terminals due to the low air speed, and the tracer gas experiment did not test this either. In brief, it seems impossible to increase the part of the total inlet air that passes the dynamic insulation to more than 55% of the total inlet air with the present number of inlet terminals. The tracer gas and direct method reach numbers close to 85% and 75% with modified inlet terminals.

8.4 Daily variations

The measurements showed a daily variation in the air flow when the sun was more than 200 W/m^2. This variation was typically ±20% of the average flow with maximum at midnight and minimum at noon. No other measurement could confirm this, but due to the rigorous testing of the mathematical model obvious errors, such as bugs in the program code, can be excluded. A tentative hypothesis is that the sunshine creates convection inside the attic that is not noticeable in the inlet terminals. This could also be the reason that the energy efficiency for the insulation and ceiling $e_{dyn,attic}$ decreases with insolation and that the dynamic U value for the ceiling and insulation $U_{dyn,attic}$ increases with increasing insolation.

9 Summary and Conclusion

A nonlinear least squares method for continuous measurement of the air flow in 'dynamic insulation' has been presented. The method is called the *gradient method* to indicate that the temperature gradients in the insulation are used. The method includes one dimensional transient and steady state calculations of the heat transfer in the insulation. The model was verified by analytical and numerical simulations. With the use of this method the air flow through an insulation was continually measured in a single storey one family house with a living area of 116 m^2 during one year (1993). The house was built at Dalby, Sweden which has a yearly average temperature of 7.8°C. The exhaust ventilation was about 53 l/s. The air flow through the dynamic insulation as measured by the gradient method was 21 l/s or 40% of the total inlet air. The measurements showed a slightly higher air flow during the spring that during the autumn. This result corresponds well with laboratory measurements performed by Roots (1994) where 50% of the inlet air passed through the insulation. Roots measured an 'OPTIMA wall' where the wall instead of the ceiling was used as dynamic insulation.

Results from a few tracer gas and direct measurements indicated a higher flow, but this could be expected since the gradient method is the only method that measures the flow in the insulation itself. The air flows as measured by the tracer gas and direct method were 33 and 27 l/s or 62% and 51% of the total inlet air. This indicated that of the air that came through the inlet terminals 80% had passed through the insulation. A problem with the design of the house was that the pressure drop over the inlet terminals was too high and that the inlet terminals were too few. However, when the inlet terminals were modified to be as open as possible the flow through the insulation only increased to 50% of the total inlet air. Even when the inlet terminals were completely removed, i.e. there were just holes in the ceiling, the flow through the insulation only increased to about 55% of the total inlet air. The major reason for this low percentage was that other parts of the house leaked too much air. This is supported by calculations by Hedin (1994).

The dynamic U value for the insulation was about 0.05 W/m^2°C neglecting the fact that the sun heated the attic during the summer and also neglecting the ceiling joists. This corresponds to a dynamic energy efficiency for the insulation of 35%. This factor should be multiplied by the fraction of the total inlet air that passed through the insulation, i.e. 40% to get the total energy efficiency for the ventilation system. The total energy efficiency calculated this way is 14%. Even when heating of the attic by the sun during the spring and autumn is included this factor only increased to 17%. A theoretical calculation shows that if 100% of the inlet air passed through the insulation the total energy recovery would be 22% which for the OPTIMA house corresponds to a power saving of 15 W/°C or an energy saving of 1600 kWh/year.

The calculated 24h average air flow increased slightly with increasing difference between inside and outside temperature. Both the 24h and 2h average air flow decreased with increasing wind speed. The reduction was of the same order of magnitude as the reductions calculated by Södergren and Fagerstedt (1984). No obvious effect of wind direction was found.

The transient calculation showed a daily variation of the air flow that increased with increasing sunshine. This variation was typically ± 30% of the average flow with maximum at midnight and minimum at noon. This air flow variation could not be detected in the inlet terminals.

The inlet air was preheated to about two degrees lower than the inside temperature for most of the heating season. During the winter the average inlet temperature was about 18°C and during summer the average inlet temperature was about 22°C. The lowest measured inlet temperature was 15.5°C and the highest measured inlet temperature was 29°C.

When the ventilation system was turned off and the inlet terminals completely closed the air flow through the insulation changed from 0.2 mm/s downwards to 0.1 mm/s upwards. This could be a dangerous situation if the air moving upwards came from the living space. The air moving upwards could then create condensation in the insulation. If the air came from outside this would only lead to a higher energy loss.

The measurements in the OPTIMA house showed that the dynamic insulation could not be justified by the energy savings alone. However, the inlet air was preheated and filtered and there were fewer ventilation ducts for transporting the inlet air than would be needed in a house with a heat exchanger system. This taken together, makes the dynamic insulation an attractive choice, at least in theory. In practice the leakage in the rest of the house reduced the energy saving of the dynamic insulation to about 70% of the maximal value and only 40% of the inlet air was filtered by the dynamic insulation. The general conclusion from the measurements was that the dynamic insulation needs a house constructed to much more exacting standards to work properly.

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Appendix A

This chapter describes one short term and one long term solution to the transient heat flow equation.

A.1 Transient heat flow equation

The air passes through the loose fill insulation at low velocity (~2 m/s) which permits the use of a simple laminar model. For the macroscopic behaviour it is convenient to describe the loose fill insulation as homogeneous and isotropic. The transient heat transfer equation in one dimension is:

$$\lambda \frac{\partial^2 T(x,t)}{\partial x^2} - u \rho_a c_a \frac{\partial T(x,t)}{\partial x} = \rho_i c_i \frac{\partial T(x,t)}{\partial t}, \qquad \text{(A.1)}$$

with the initial and boundary conditions:

$$T(x,t_0) = T^0(x) \qquad T(0,t) = T_0(t) \qquad T(L,t) = T_L(t) \qquad \text{(A.2)}$$

where T is the temperature, λ the heat transfer coefficient for the insulation, u the air flow positive in the direction of increasing x, ρ_a the density of air, c_a the heat capacity of air, ρ_i the density of the insulation, c_i the heat capacity of the insulation, L the height of the simulated part of the insulation and t is the time. The upper part of the insulation is x=0 and the lower part is $x = L$. Equation (A.1) can also be written:

$$a\frac{\partial^2 T(x,t)}{\partial x^2} - a\nu\frac{\partial T(x,t)}{\partial x} = \frac{\partial T(x,t)}{\partial t} \tag{A.3}$$

where a (m^2/s) is the diffusivity for the insulation. The ratio of convection to conduction is described by (m^{-1}). In equation A.3 the following parameters are used.

$$\nu = u\,\rho_a\,c_a/\lambda \qquad a = \lambda/\rho_i\,c_i \qquad a\nu = u\frac{\rho_a c_a}{\rho_i c_i} \tag{A.4}$$

A.2 Laplace transform of the equation

The standard approach to solue the transient temperature equation is to use the Laplace transform in the time domain. If $f=f(x,t)$ is an arbitrary continuous and analytical function the one sided Laplace transform of f is defined:

$$\tilde{f}(x,s) = \mathrm{L}(f(x,t)) = \int_0^\infty f(x,t)\,e^{-st}\,dt \tag{A.5}$$

Here, s is a complex number whose real part is greater than zero, $Re(s)>0$. The Laplace transform of equation (5) is:

$$\frac{\partial^2\,\tilde{T}(x,s)}{\partial x^2} - \nu\frac{\partial\tilde{T}(x,s)}{\partial x} = \frac{1}{a}\left(s\,\tilde{T}(x,s) - T(x,0)\right) \tag{A.6}$$

or

$$\frac{s}{a}\,\tilde{T}(x,s) + \frac{\partial^2\tilde{T}(x,s)}{\partial x^2} - n\frac{\partial\tilde{T}(x,s)}{\partial x} = -\frac{1}{a}T(x,0) \tag{A.7}$$

where $T(x,0)$ is the temperature profile at t=0. If this profile is set to zero one gets:

$$\frac{s}{a}\ \tilde{T}(x,s) + \frac{\partial^2 \tilde{T}(x,s)}{\partial x^2} - \nu\ \frac{\partial \tilde{T}(x,s)}{\partial x} = 0 \tag{A.8}$$

This equation has the solution:

$$\tilde{T}(x,s) = A(s)\ e^{x\left(\nu/2+\sqrt{s/a+(\nu/2)^2}\right)} + B(s)e^{x\left(\nu/2-\sqrt{s/a+(\nu/2)^2}\right)} \tag{A.9}$$

The upper part of the dynamic insulation is exposed to almost the ambient temperature and will fluctuate over a 24h period. The lower part is strongly coupled to the inside temperature and will thus be more or less constant. It is therefore reasonable to use zero initial temperature and a temperature step at x=0 but with the temperature held constant at x=L:

$$T(0,t) = T_0 \qquad \Rightarrow \tilde{T}(0,s) = T_0 / s \tag{A.10}$$

$$T(L,t) = 0 \qquad \Rightarrow \tilde{T}(L,s) = 0$$

After some calculations the solution to (A.9) with boundary conditions (A.10) becomes:

$$\tilde{T}(x,s) = e^{x\,\nu/2}\ \frac{T_0}{s}\ \frac{\sinh\left((L-x)\sqrt{s/a+(\nu/2)^2}\right)}{\sinh\left(L\sqrt{s/a+(\nu/2)^2}\right)} \tag{A.11}$$

where sinh(x) is the hyperbolic function $(e^x\text{-}e^{-x})/2$. The solution in the time domain can now be calculated. The definition of the inverse Laplace transform is:

$$f(x,t) = \mathrm{L}^{-1}\left(\tilde{f}(x,s)\right) = \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty} e^{st}\ \tilde{f}(x,s)\ ds \tag{A.12}$$

where $\tilde{f}(x,s)$ must be defined for $-\sigma <$ Re $s < \sigma$. The solution in the time domain becomes:

$$T(x,t) = \frac{1}{2\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} e^{st}\; e^{x\,\nu/2}\; \frac{T_0}{s}\; \frac{\sinh\left((L-x)\sqrt{s/a + (\nu/2)^2}\right)}{\sinh\left(L\sqrt{s/a + (\nu/2)^2}\right)}\, ds \tag{A.13}$$

Based on Carslaw and Jaeger (1959) two possible solutions exist: one which is based on residue calculation and one which is based on series expansion of (A.11) into exponents. The residue calculation is better for longer times and the series expansion is better for shorter times.

A.3 Long term solution

The residue calculation is relatively straightforward. The integral (A.13) can be expanded into a contour integral from σ-$i\infty$ to σ+$i\infty$ plus a half circle in the left half-plane. The integral on the half circle will disappear if the circle is large enough. The residue calculus now states that the solution to this contour integral is the sum of the residues:

$$T(x,t) = \sum \mathrm{Res}\left(e^{st}\, \tilde{T}(x,s) \right) \tag{A.14}$$

The function (A.11) has one pole in s=0:

$$\underset{s=0}{\mathrm{Res}} \left(e^{st}\, \tilde{T}(x,s) \right) = \frac{\sinh\left(\frac{v}{2}(L-x) \right)}{\sinh\left(\frac{v}{2} L \right)} \tag{A.15}$$

This correspond to the steady state solution. There are also poles where

$$\sinh\left(L\sqrt{s/a+(\nu/2)^2}\right)=0 \tag{A.16}$$

that is:

$$s = s_n = -\frac{a\nu^2}{4} - \frac{a\,n^2\pi^2}{L^2} \qquad ;\ n = 0,1,2,\ldots \tag{A.17}$$

This results in the residue:

$$\operatorname*{Res}_{s=s_k}\left(e^{st}\,\tilde{T}(x,s)\right) =$$

$$e^{-\left(\frac{a\nu^2}{4}+\frac{a\,n^2\pi^2}{L^2}\right)t}\cdot\frac{\sin\frac{n\pi}{L}(L-x)}{\left(\frac{a\nu^2}{4}+\frac{a\,n^2\pi^2}{L^2}\right)\frac{L^2}{2an\pi}\cos n\pi} \tag{A.18}$$

The integral (A.13) thus becomes (the pole at n=0 is cancelled by the numerator.):

$$T(x,t) = e^{x\nu/2}\,T_0\cdot\left(\frac{\sinh\frac{\nu}{2}(L-x)}{\sinh\frac{\nu}{2}L}\right.$$

$$\left.+\sum_{n=1}^{\infty}(-1)^n e^{-\left(\frac{a\nu^2}{4}+\frac{a\,n^2\pi^2}{L^2}\right)t}\cdot\frac{\sin\frac{n\pi}{L}(L-x)}{\frac{\nu^2L^2}{8n\pi}+\frac{n\pi}{2}}\right) \tag{A.19}$$

The solution is thus the sum of an infinite series. The steady-state solution is approached for large t:

$$T(x,\infty) = e^{x\nu/2}\, T_0 \left(\frac{\sinh \frac{\nu}{2}(L-x)}{\sinh \frac{\nu}{2} L} \right) = T_0 \frac{e^{\nu L} - e^{\nu x}}{e^{\nu L} - 1} \tag{A.20}$$

A.4 Short term solution

The calculation above is not very accurate for short times: $t<0.2\tau$. The shorter the time the greater is the number of terms which must be included in the sum. The solution (A:11) can be expanded into a geometric series on which it is possible to perform a direct inverse transform using standard techniques (Carslaw and Jaeger):

$$\tilde{T}(x,s) = e^{x\nu/2}\, \frac{T_0}{s}\, \frac{\sinh\left((L-x)\sqrt{s/a+(\nu/2)^2}\right)}{\sinh\left(L\sqrt{s/a+(\nu/2)^2}\right)}$$

$$= T_0 \frac{e^{x\nu/2}}{s} \cdot \frac{e^{(L-x)\sqrt{s/a+(\nu/2)^2}} - e^{-(L-x)\sqrt{s/a+(\nu/2)^2}}}{e^{L\sqrt{s/a+(\nu/2)^2}}\left(1 - e^{-2L\sqrt{s/a+(\nu/2)^2}}\right)}$$

$$= T_0 \frac{e^{x\nu/2}}{s} \cdot \sum_{n=0}^{\infty} e^{-(2nL+x)\sqrt{s/a+(\nu/2)^2}} - T_0 \frac{e^{x\nu/2}}{s} \cdot \sum_{n=0}^{\infty} e^{-(2nL+2L-x)\sqrt{s/a+(\nu/2)^2}} \tag{A.21}$$

The transform of the complementary error function erfc() will be used:

$$\operatorname{erfc}(x) = \frac{1}{\sqrt{\pi}} \int_x^{\infty} e^{-\xi^2} d\xi \tag{A.22}$$

$$L(\operatorname{erfc}(x / 2\sqrt{at})) = \frac{e^{-x\sqrt{s/a}}}{s} \tag{A.23}$$

The complementary error function can be approximated with :

$$erfc(x) \approx \sqrt{\pi} \cdot e^{-x^2} \left(\frac{1}{x} - \frac{1}{2x^3} + \frac{3}{4x^5} \right) \tag{A.24}$$

With the use of the complementary error function the solution of (A.13) becomes:

$$\begin{aligned} T(x,t) = \frac{1}{2} \sum_{n=0}^{\infty} \Bigg[& e^{-2nL\nu/2} \cdot \operatorname{erfc}\left(\frac{2nL + x - \nu at}{\sqrt{4at}} \right) \\ & + e^{2nL\nu/2 + x\nu} \cdot \operatorname{erfc}\left(\frac{2nL + x + \nu at}{\sqrt{4at}} \right) \\ & - e^{-(2n+2)L\nu/2 + x\nu} \cdot \operatorname{erfc}\left(\frac{(2n+2)L - x - \nu at}{\sqrt{4at}} \right) \\ & - e^{-(2n+2)L\nu/2} \cdot \operatorname{erfc}\left(\frac{(2n+2)L - x + \nu at}{\sqrt{4at}} \right) \Bigg] \end{aligned} \tag{A.25}$$

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