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Abstract. It is attempted to solve the design challenge using a two-degree-of-freedom controller. A standard indirect adaptive controller based on modifed least squares estimation and pole placement design is used. Particular emphasis is given to selection of model structure, sampling period, filtering and selection of desired closed loop poles. These choices require some prior knowledge of the properties of the system. Methods to acquire the required apriori knowledge are discussed briefly. The controller satisfies the objective in all experiments.

1. Introduction

In this paper the problem is solved using an indirect self-tuner based on modified least squares estimation and pole placement design. Such an approach is quite natural for the given problem. Since we have a computer implementation in mind the design is carried out in discrete time. The behavior of the discrete time regulator will be arbitrarily close to a continuous time regulator by choosing the sampling period sufficiently small.

The design challenge has the following features: dynamics with dead time and right half plane zeros, actuator saturation, measurement noise and load disturbances. A particular difficulty is that the disturbance signals was specified so that a severe load disturbance occurs at the time when there is also a set point change.

The adaptive controller used is a direct self-tuner based on recursive least squares and a pole-placement design with a two-degree-of-freedom (2DOF) structure. The controller is described in Section 2. Some details of the controller design are given in Section 3. The simulation experiments were made using a newly developed Matlab toolbox for adaptive control. The results are presented in Section 4.

2. An Adaptive 2DOF Controller

There are a large variety of adaptive design methods. In this paper we have chosen to use direct adaptive pole placement. The basic controller structure used is a two-degree-of-freedom controller of the form suggested by Horowitz (1963).

Pole Placement Design

Pole placement is a design procedure which is well described in literature, see, e.g., Åström and Wittenmark (1984, 1989). It is a deceptively simple procedure, the desired closed loop poles are specified and a simple calculation gives the controller. The key issue is to choose the closed loop poles. This choice requires considerable insight, since controllers with widely different properties can be obtained by choosing different closed loop poles. To introduce some structure into the choice the closed loop poles can be separated into two categories, model poles and observer poles. The separation of model poles and observer poles is suggested by the two-degree-of-freedom structure. The model poles are controllable from the reference signal but the observer poles are not. A brief description of the design procedure is given here. The key idea is to find a feedback law such that the closed loop poles have the desired locations. Let the process to be controlled be described by

$$Ay = Bu + v \tag{1}$$

where u is the control variable, y the measured output and v a disturbance. The symbols A and B denote polynomials in the the forward shift operator q. It is assumed that the polynomial A is *monic*. Let the desired response from the reference signal y_r to the output be described by the dynamics

$$A_m y_m = B_m y_r \tag{2}$$

For simplicity it is assumed that no process zeros are canceled. Polynomial B must then divide B_m. Furthermore, let the closed loop poles be $A_o A_c$ where A_o is the observer polynomial. A general two-degree-of-freedom controller can be described by

$$Ru = Ty_r - Sy \tag{3}$$

Elimination of u between (1) and (3) gives

$$y = \frac{BT}{AR + BS} y_r + \frac{R}{AR + BS} v$$

To achieve the desired input-output response the following condition must hold

$$\frac{BT}{AR+BS}=\frac{B_m}{A_m}$$

The denominator $AR+BS$ is the closed loop characteristic polynomial. Since this was required to be A_oA_m, we get

$$AR+BS=A_oA_m \tag{4}$$

Since B divides B_m it follows from (4) that

$$T=A_oB_m/B \tag{5}$$

The pole placement design procedure can be summarized in the following algorithm.

ALGORITHM 1—Pole placement design
Data: Process model specified by A and B.

Specifications: Polynomials A_m, B_m A_c and A_o.

Compatibility conditions:

$$\begin{gathered}B \text{ divides } B_m\\ \deg A_m-\deg B_m\geq \deg A-\deg B\\ \deg A_o\geq 2\deg A-\deg A_m-1\end{gathered}$$

Step 1: Solve $AR+BS=A_oA_c$

Step 2: Form $T=A_oB_m/B$

The control law is

$$Ru=Ty_r-Sy \tag{6}$$

□

There are many variations of the pole placement procedure.

The Observer Polynomial

State feedback design combined with an observer gives a closed loop system where the observer dynamics is not controllable from the reference signal. This implies that in input output form there is a pole-zero cancellation. This is the reason why A_o is called the observer polynomial. This polynomial should be stable and fulfill the compatibility conditions. The observer polynomial influences the response of the closed loop system to load disturbances and measurement noise. It also influences the sensitivity of the closed loop system to unmodeled dynamics, see Åström and Wittenmark (1990). When the properties of the disturbances are known optimal filtering theory can be used to determine the optimal observer. For instance, if the disturbance is described by

$$v=Ce$$

where e is white noise then it follows from optimal control theory that A_o should be chosen as $A_o=C$, see Åström and Wittenmark (1990). If the disturbances are not known the observer polynomial can be regarded as a design parameter. Roughly speaking a slow observer makes the system less sensitive to measurement noise and unmodeled dynamics but also gives slower recovery to load disturbances. In this particular design challenge the sensitivity to unmodeled dynamics is important. Therefore we choose an observer that is slightly slower than A_m.

Sensitivity of the Design

The sensitivity of the closed loop system obtained with the method is discussed in Åström and Wittenmark (1990) where it is shown that the closed loop is stable if the modeling error satisfies the inequality

$$\left|\frac{\Delta H(z)}{H(z)}\right|<\left|\frac{A_m}{B_m}\frac{T}{S}\right| \tag{7}$$

Known Disturbances

The pole placement design can be modified to account for disturbances with known dynamics. Assume that the disturbance v is generated from the dynamical system

$$A_dv=e \tag{8}$$

where e is a pulse, a set of widely spread pulses or discrete time white noise. A step disturbance is, e.g., in discrete time systems generated by $A_d(q)=q-1$. The system model then becomes

$$AA_dy=BA_du+e \tag{9}$$

With the regulator (6) we find

$$\begin{aligned}y&=\frac{BT}{AR+BS}y_r+\frac{R}{A_d(AR+BS)}e\\ u&=\frac{AT}{AR+BS}y_r-\frac{S}{A_d(AR+BS)}e\end{aligned} \tag{10}$$

The closed loop characteristic polynomial thus contains the disturbance dynamics as a factor. This polynomial is typically unstable. It follows from (10) that in order to maintain a finite output in case of these disturbances it is necessary that A_d divides R. This would make y finite but the controlled input u may be infinite.

Response to Command Signals

An advantage of the given design is that the response to load disturbances and command signals are different. The response to load disturbances has dynamics given by $A_oA_mA_d$ and the response to command signals has response given by A_m.

Actuator Saturation and Anti-windup

The given design method is based on purely linear analysis. This is a good approximation but there is one nonlinear effect that must always be considered namely actuator saturation. If linear regulators are used without any precautions they may have a bad performance when the actuator saturates. The control law is therefore implemented as

$$A_o u = f\big(T y_r - S y + (A_o - R) u\big) \qquad (11)$$

where the nonlinear function f is a model of the actuator nonlinearity. For a simple actuator that saturates at u_{low} and u_{high} the function becomes

$$f(x) = \begin{cases} u_{low} & x \le u_{low} \\ x & u_{low} < x < u_{high} \\ u_{high} & u \ge u_{high} \end{cases}$$

This device will avoid integrator windup. For more detail see Åström and Wittenmark (1990).

Adaptive Pole Placement

A self-tuning controller is obtained in the usual way by estimating the unknown parameters of the model (1) and computing the controller parameters as if the estimates were correct (*certainty equivalence*). Many different estimation schemes can be used. In this paper recursive least squares is used. To carry out the parameter estimation the model (1) is rewritten as

$$y(t) = \varphi^T(t-1)\theta$$

where $d_0 = \deg A - \deg B$ and

$$\begin{aligned} \theta^T &= \big[b_0 \; b_1 \ldots b_m \; a_1 \ldots a_n\big] \\ \varphi^T(t-1) &= \big[u(t-1) \;\; \ldots u(t-m) \\ &\qquad - y(t-1) \;\; \ldots - y(t-n)\big] \end{aligned}$$

The least squares estimate of the parameters is then given by

$$\begin{aligned} \hat\theta(t) &= \hat\theta(t-1) + K(t)\varepsilon(t) \\ \varepsilon(t) &= y(t) - \varphi^T(t-1)\hat\theta(t-1) \\ K(t) &= \frac{P(t-1)\varphi(t-1)}{\lambda + \varphi^T(t-1)P(t-1)\varphi(t-1)} \\ P(t) &= \big(I - K(t)\varphi^T(t-1)\big)\, P(t-1) \,/\, \lambda \end{aligned} \qquad (12)$$

The adaptive algorithm can thus be described as follows:

Algorithm 2—Indirect self-tuning regulator
Data: Given specifications in the form of desired closed loop pulse transfer operator, B_m/A_m, a desired observer polynomial A_o and a disturbance rejection polynom A_d.

Step 1: Estimate the coefficients of the polynomials A and B in (1) recursively using the least squares method (12).

Step 2: Substitute A and B by the estimates obtained in Step 1 and apply Algorithm 1 for pole placement design. This means that polynomials R and S are solved from equation (4) and T is given by (5).

Step 3: Calculate the control signal from (11).

Repeat the steps 1–3 at each sampling period. □

For further details see Åström and Wittenmark (1989).

3. Controller Implementation

The adaptive control algorithm given in Section 2 will now be applied to the benchmark example.

Problem Features

The transfer function of the process is

$$G(s) = K e^{-s\tau} \frac{(1 - sT)\omega_0^2}{(1 + sT)(s^2 + 2\zeta\omega_0 s + \omega_0^2)} \qquad (13)$$

There is also an actuator that saturates when the control signal is greater than 25 in magnitude. The smallest value of the process gain is 0.5. It is required to keep the output at 10 and the maximum disturbance level at the process input is 5. This means that there will always be operating conditions where the control signal will be at the actuator limit. Since such a case is somewhat pathological the full range of variation of the gain parameter has not been used in all cases.

Process dynamics limits performance since the plant has a right half plane zero and a dead time. In the worst case the right half plane zero is at $s = 0.67$ and the dead-time is $\tau = 0.4$. It follows from a fundamental inequality in frequency response design that the achievable gain cross over frequency ω_{gc} is approximately given by

$$\arg G_{nmp}(i\omega_{gc}) < \pi - \phi_m - n_{gc}\frac{\pi}{2} \qquad (14)$$

where G_{nmp} is the non-minimum-phase part of the plant transfer function, ϕ_m the phase margin and n_{gc} the slope of the gain curve at the gain cross over. With $\phi_m = \pi/4$, $n_{gc} = 0.5$ and a process given by (13) we get

$$\omega_{gc}\tau + 2\arctan \omega_{gc}T < \frac{\pi}{2}$$

This gives $\omega_{gc} < 0.54$. Since the major limitation is due to the right half plane zero which is estimated it may be useful to let the desired bandwidth depend on the estimated zero. In the standard cases we choose a constant desired bandwidth.

Model structure

Selection of an appropriate discrete time model structure is a key issue in the design of an adaptive controller. It is necessary to choose a model that is flexible enough to capture the essential dynamics of the process. The model should, however, not have unnecessarily many parameters because this makes estimation difficult. With the given parameter values the dynamics given by Equation (13) is dominated by

$$G(s) = Ke^{-s\tau}\frac{1-sT}{1+sT} \quad (15)$$

This model will be taken as the starting point when finding an appropriate controller parameterization. Sampling the model gives a discrete time model with the structure

$$y(t) + ay(t-1) = b_d u(t-d) + \ldots + b_{d+2}u(t-d-2)$$

where the sampling period has been chosen as the time unit. Furthermore d is the integer part of $(\tau + h)/h$. Since the dead-time τ is unknown it is necessary to choose the sampling period and the number of parameters so that the model given by (15) includes the true process. To accommodate this a model with the structure

$$y(t) + ay(t-1) = b_2 u(t-2) + \ldots + b_n u(t-n) \quad (16)$$

is chosen. To include a model with maximum dead-time the parameters must be chosen so that $nh \geq \tau + 3h$. where h is the sampling period. With a sampling period $h = 0.2$ and the largest dead time $\tau = 0.4$ it follows that n must be 5. By having so many $b-$parameters it is also possible to capture part of the unmodeled dynamics. The advantage of having a model structure with a few a-parameters and many b-parameters was discussed in Åström (1980).

Filtering

The signals are band-pass filtered before they are introduced to the estimator because of the severe disturbances which are piecewise constant. The filter is chosen as second order Butterworth sections. After some experimentation the pass band was chosen as $0.01\pi < \omega h < 0.5\pi$. Experiments with different filters were carried out since this is an essential parameter in the design of an adaptive system. In a real design it is also necessary to consider anti-alias filters. This was not done in this investigation.

Integral action

To obtain a control law that can withstand the severe disturbances it is necessary to ensure that it has integral action. This is achieved by finding a solution to the Diophantine equation such that $q-1$ is a factor of $R(q)$. The polynomial A_d was thus chosen as $q-1$.

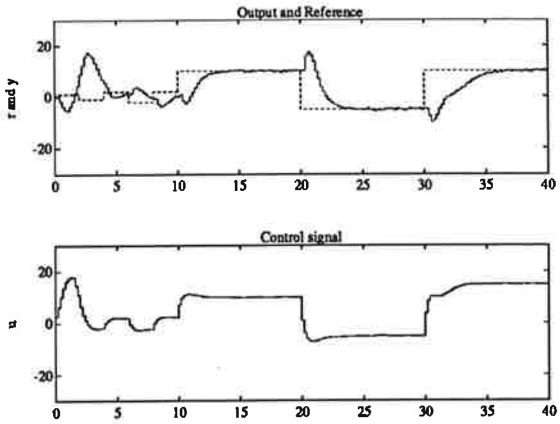

Figure 1. Process input and output in the nominal case

Design parameters

All control algorithms have design parameters that have to be chosen. In this case these parameters are the sampling period h, the filtering polynomial A_f and the polynomials A_o and A_m. To determine the degrees of the polynomials consider Equation (16) which implies that $degA = 5$.With a controller having integral action the minimal solution then has a polynomial S of degree five. The polynomials A_m and A_o can then also be of fifth order. Polynomial A_o is chosen to have two real zeros at $exp(-h/T_m)$ and the remaining zeros at the origin. The reason for choosing real zeros is that it was desired to have a set point response without overshoot. Polynomial A_o is chosen to have two zeros at $exp(-h/T_o)$ and the remaining zeros at the origin. Parameters T_m and T_o are chosen as 0.5 and 0.7 respectively. This choice was made after investigating the response to set point changes load disturbances, measurement noise and robustness as discussed previously.

4. Results

The results obtained when applying the controller to the benchmark problem will now be discussed. We focus on Experiments 3 and 6. The controller based on the reduced order model is uses in all cases. Load disturbance and measurement noise are present in all experiments shown.

Nominal Case

The initial sequence was chosen as a square wave with a frequency close to the limit cycle frequency under relay feedback. Figure 1 shows the time traces for the nominal plant. The figure shows that the adaptive controller operates quite well in the nominal case. Notice that the nonminimum phase nature of the plant is clearly noticeable in the responses. Also notice that the system copes well with the load disturbances and measurement noise. The parameter estimates are shown in Figure 2. The estimates

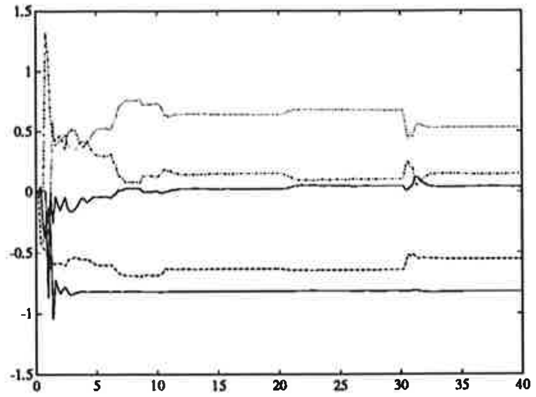

Figure 2. Parameter estimates in the nominal case

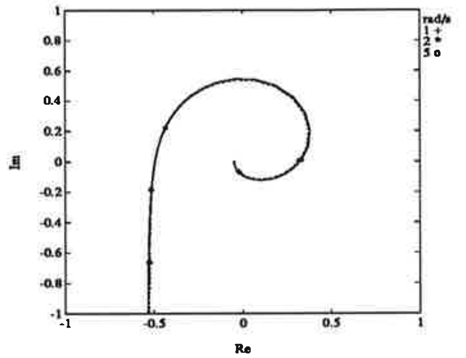

Figure 3. Nyquist curves of the loop transfer function computed from the controller parameters at time $t = 29.8$ and the true model (solid line) and from the estimated process model (dashed line).

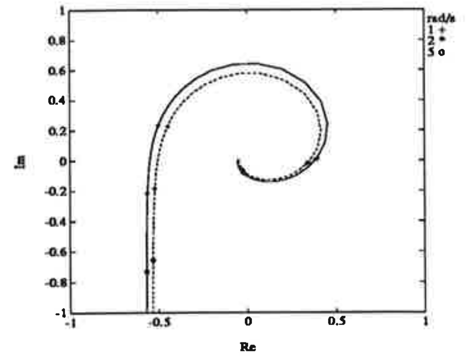

Figure 4. Nyquist curves of the loop transfer function computed from the controller parameters at time $t = 40$ and the true model (solid line) and from the estimated process model (dashed line).

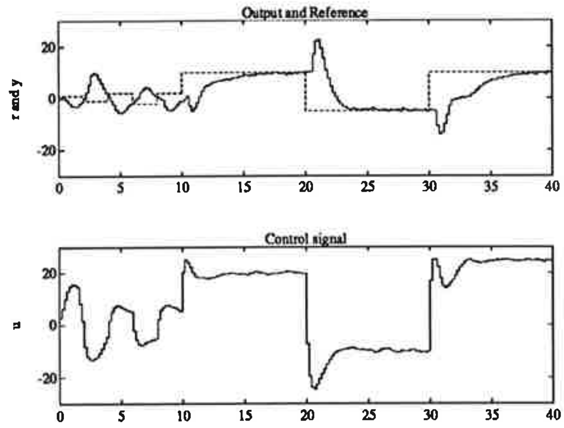

Figure 5. Process input and output in the worst case.

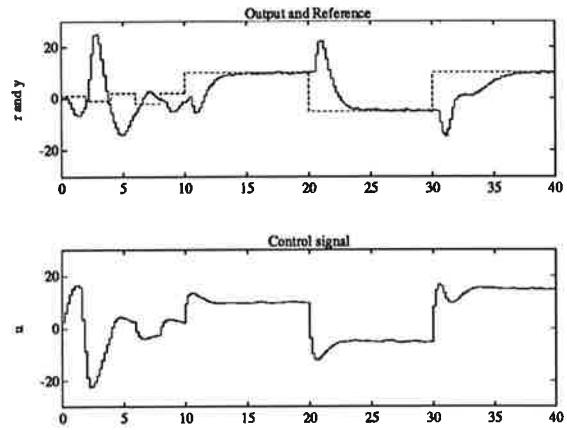

Figure 6. Process input and output when all parameters except K have their worst case values.

converge quite quickly. Naturally there are fluctuations in the estimates after the load disturbance at time 30. The parameter behavior depends critically on the regressor filter.

Another illustration of the parameter estimates is shown in Figure 3 which shows the Nyquist curve of the loop transfer function computed from the controller parameters obtained at time $t = 29.8$, i.e. just before the load disturbance. The figure shows clearly that the estimated parameters give a very good estimate of the loop gain at the frequencies of importance for the control design. The shape of the loop transfer function also indicates that the design is reasonable. Figure 4 shows the Nyquist curve of the loop transfer function at time $t = 40$ i.e. after the load disturbance. The figure shows that there are deviations due to the load disturbance.

Worst case

The worst case is when the parameter variations are such that τ and T have their largest values, and K and ω have their smallest values. Figure 5 shows a simulation of the adaptive controller in this case. Notice that the controller saturates at the high limit ($u = 25$) at time $t = 30$ when the load disturbance occurs but that the system in spite of this behaves quite well. This is an indication that the anti-windup is designed properly. Since it is more representative to show the curves for the case when the actuator does not saturate Figure 6 shows the response of the system when all parameters except K have their worst case values. The chosen adaptive control design can clearly cope with the worst case parameters. The primary control objective was to obtain an overhoot-free response with a 5% rise time in $4T$. This objective is satisfied in all cases as can be seen from Figures 1, 5 and 6.

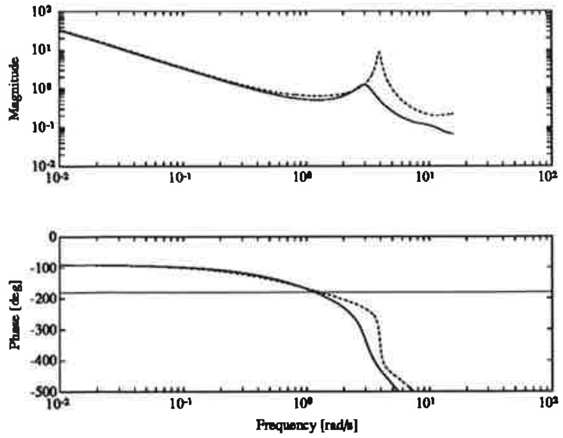

Figure 7. Bode diagrams of the loop transfer function in the nominal case (solid line) and in the worst case (dashed line).

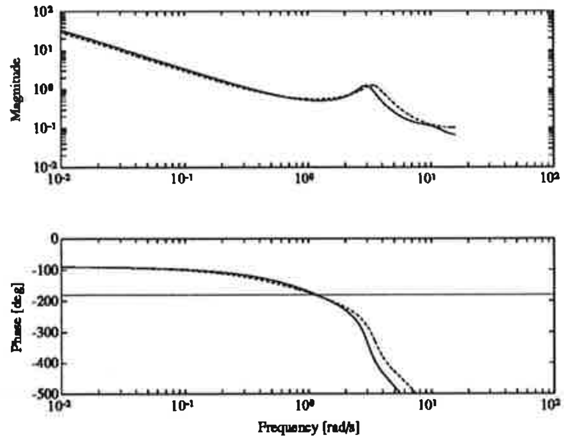

Figure 8. Same as Figure 7 but with slower observer, $T_o = 1$ instead of $T_o = 0.7$

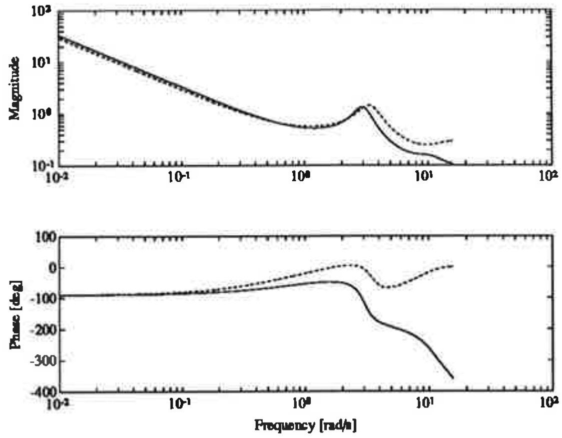

Figure 9. Bode diagram for the controller transfer function of the nominal (solid line) and worst case (dashed line) with $T_o = 1.0$

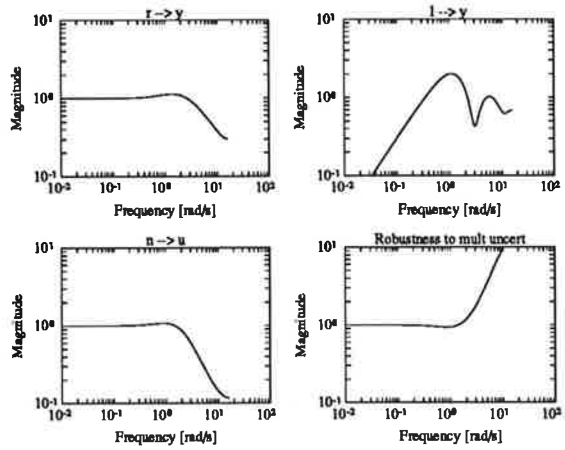

Figure 10. Bode diagrams of transfer functions from a) reference signal to process output, b) input load disturbance to process output, c) measurement noise to control signal and d) the robustness measure given by Equation (7).

Many tests have been performed with the chosen controller. They all indicate that the chosen design is well suited for the problem. An illustration of this is given in Figure 7 which shows the Bode diagrams of the loop transfer function for the nominal case and for the worst case.

The loop transfer functions are quite reasonable in the nominal case. It has, however, a high resonance peak in the worst case. This can be reduced significantly by chosing a slower observer. The loop transfer functions obtained in the nominal case and in the worst case are shown in Figure 8. A comparison with Figure 7 shows that the peak is reduced significantly. This indicates that it would be useful to let the observer polynomial depend on the estimated right half plane zero.

The Bode diagrams of the controllers in Figure 9 show that the controller for the worst case requires much more phase lead than the nominal controller.

Another illustration of the properties of the controller is given in Figure 10 which shows Bode diagrams of transfer functions that show the sensitivity of the design to load disturbances, measurement errors and modeling errors for the nominal design.

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